Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.1.2-g-cos-^p-a+b-sin-^m

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [653]. This is test number [70].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (653)	% 0.00 (0)
Mathematica	% 97.70 (638)	% 2.30 (15)
Maple	% 86.06 (562)	% 13.94 (91)
Maxima	% 44.10 (288)	% 55.90 (365)
Fricas	% 54.82 (358)	% 45.18 (295)
Sympy	% 14.70 (96)	% 85.30 (557)
Giac	% 42.57 (278)	% 57.43 (375)
Mupad	% 39.51 (258)	% 60.49 (395)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

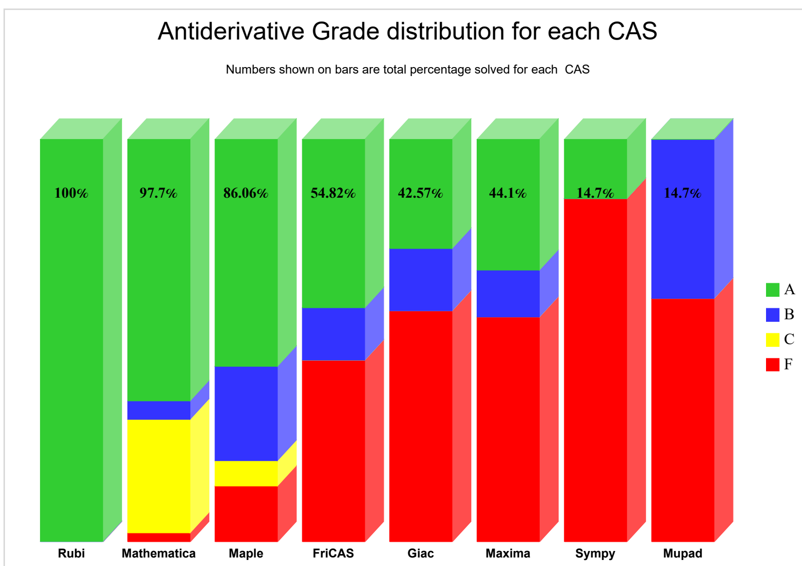
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

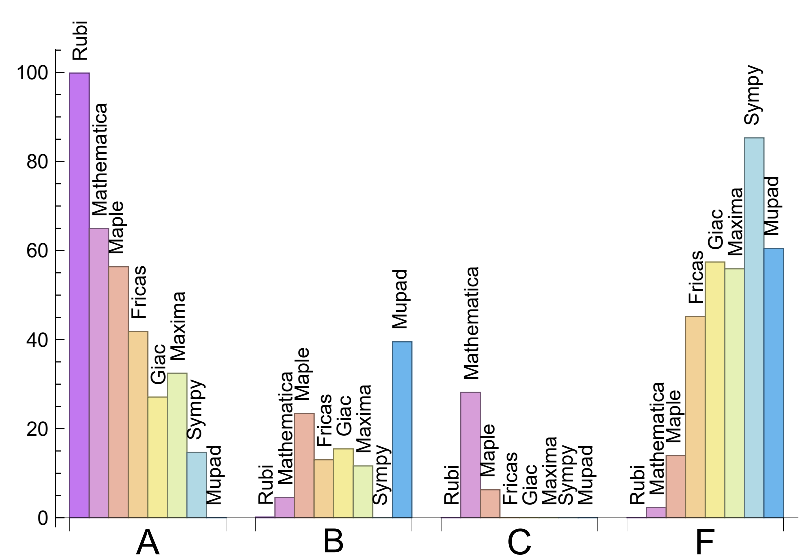
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.85	0.15	0.00	0.00
Mathematica	64.93	4.59	28.18	2.30
Maple	56.36	23.43	6.28	13.94
Maxima	32.47	11.64	0.00	55.90
Fricas	41.81	13.02	0.00	45.18
Sympy	14.70	0.00	0.00	85.30
Giac	27.11	15.47	0.00	57.43
Mupad	0.00	39.51	0.00	60.49

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	15	100.00 %	0.00 %	0.00 %
Maple	91	100.00 %	0.00 %	0.00 %
Maxima	365	81.64 %	7.40 %	10.96 %
Fricas	295	78.64 %	20.34 %	1.02 %
Sympy	557	31.42 %	68.58 %	0.00 %
Giac	375	69.07 %	28.27 %	2.67 %
Mupad	395	98.73 %	1.27 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

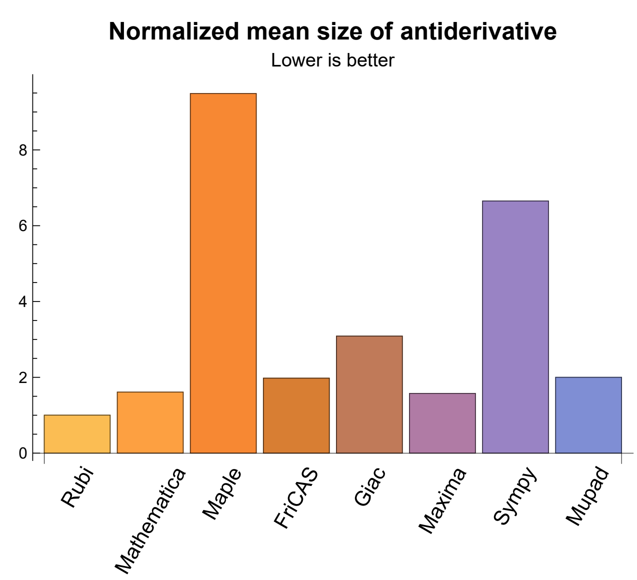
1.3 Performance

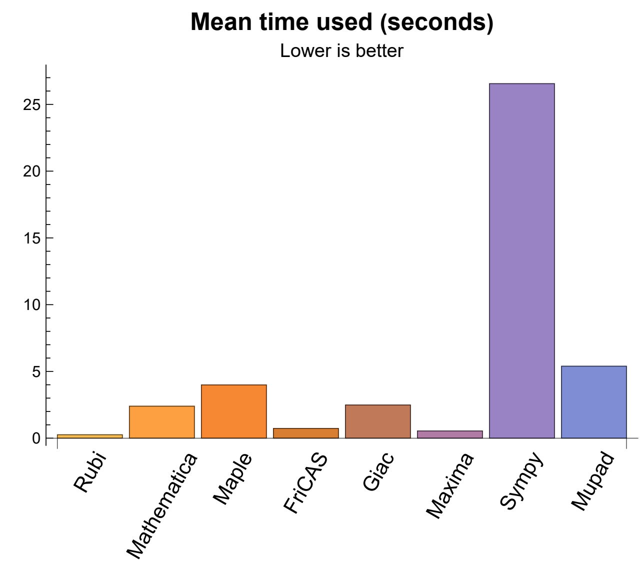
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	157.61	1.00	121.00	1.00
Mathematica	2.39	261.98	1.61	82.00	0.93
Maple	3.99	4781.19	9.49	173.50	1.42
Maxima	0.54	156.00	1.58	101.50	1.10
Fricas	0.73	264.55	1.98	106.50	1.30
Sympy	26.55	427.47	6.65	170.00	3.28
Giac	2.48	294.23	3.09	134.00	1.51
Mupad	5.39	283.19	2.00	105.00	1.26

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {615, 648}

Mathematica {112, 330, 352, 353, 467, 469, 470, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 628}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
           leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

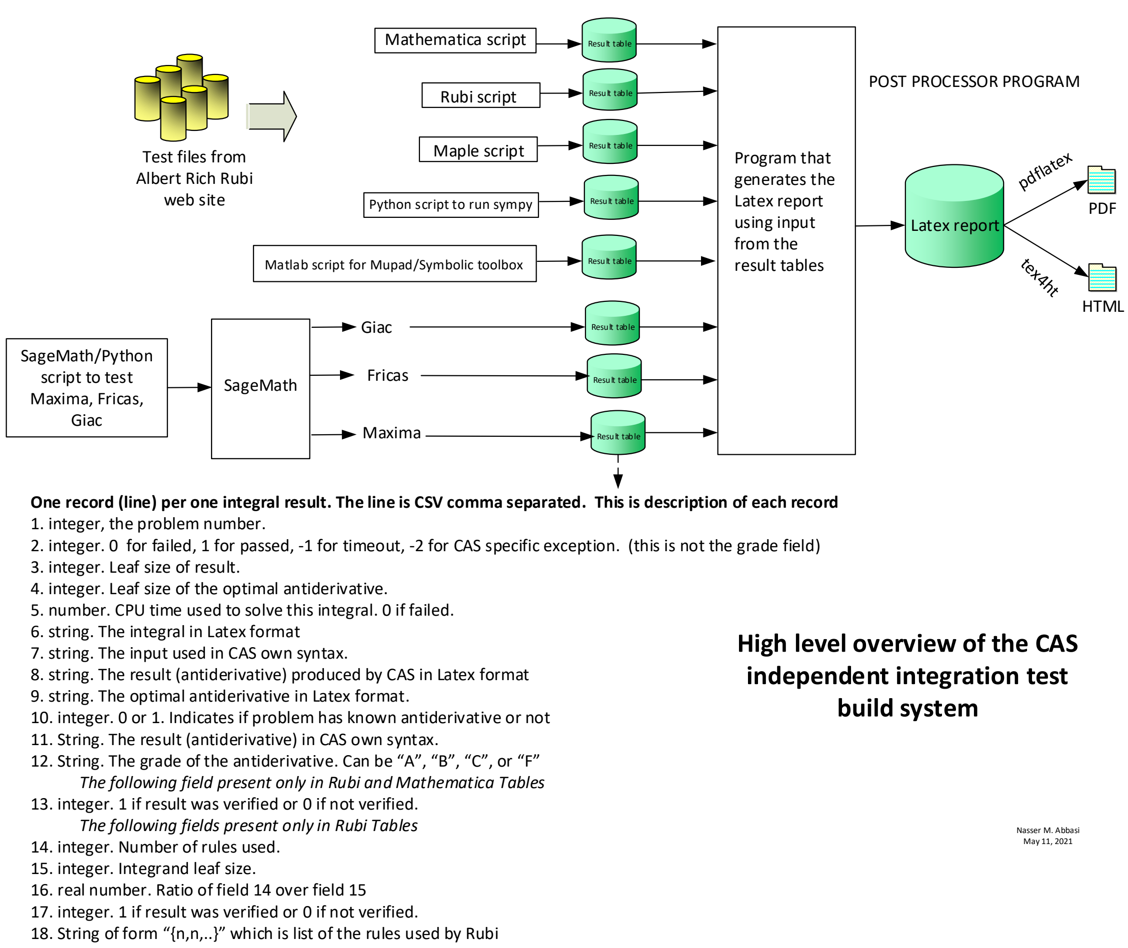
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

B grade: { 615 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 48, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 114, 115, 116, 117, 118, 119, 120, 121, 123, 127, 128, 129, 130, 131, 132, 133, 134, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 152, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 196, 198, 200, 202, 276, 277, 278, 279, 285, 286, 287, 288, 294, 295, 296, 297, 301, 302, 303, 304, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 347, 350, 351, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 403, 404, 406, 407, 408, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 447, 448, 449, 450, 451, 452, 453, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 522, 523, 524, 525, 526, 527, 528, 529, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 616, 623, 624, 625, 626, 627, 628, 630, 631, 632, 633, 634, 635, 636, 647, 648, 649, 650 }

B grade: { 18, 32, 45, 53, 55, 68, 88, 122, 135, 151, 348, 349, 389, 402, 405, 415, 431, 432, 433, 444, 445, 446, 456, 457, 467, 469, 470, 619, 620, 621 }

C grade: { 34, 47, 49, 79, 108, 109, 110, 111, 112, 113, 124, 125, 126, 137, 138, 148, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 181, 192, 193, 194, 195, 197, 199, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 298, 299, 300, 305, 306, 307, 313, 314, 315, 330, 338, 352, 353, 519, 520, 521, 530, 531, 532, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618 }

F grade: { 622, 629, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 651, 652, 653 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 24, 25, 26, 27, 29, 31, 32, 33, 34, 38, 44, 45, 46, 52, 54, 56, 57, 58, 59, 61, 63, 65, 67, 68, 69, 70, 71, 72, 73, 74, 76, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 214, 215, 216, 217, 218, 219, 220, 224, 225, 226, 227, 228, 233, 234, 235, 236, 237, 238, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 263, 264, 265, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 316, 317, 318, 319, 320, 346, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 406, 407, 408, 409, 410, 411, 412, 415, 416, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 434, 437, 438, 439, 440, 441, 442, 443, 446, 447, 449, 450, 451, 452, 453, 454, 455, 461, 462, 463, 464, 465, 466, 473, 474, 475, 476, 477, 483, 484, 485, 494, 495, 496, 506, 507, 508, 509, 510, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 539, 541, 542, 543, 544, 545, 551, 552, 559, 560, 568, 569, 633 }

B grade: { 21, 23, 28, 30, 35, 36, 37, 39, 40, 41, 42, 43, 47, 48, 49, 50, 51, 53, 55, 60, 62, 64, 66, 75, 77, 81, 90, 203, 211, 212, 213, 221, 222, 223, 229, 230, 231, 232, 239, 240, 241, 242, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 266, 267, 268, 269, 270, 271, 272, 292, 293, 314, 315, 391, 405, 413, 414, 417, 418, 424, 431, 432, 433, 435, 436, 444, 445, 448, 456, 457, 458, 459, 460, 467, 468, 469, 470, 471, 472, 478, 479, 480, 481, 482, 486, 487, 488, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 502, 503, 504, 505, 511, 512, 513, 514, 515, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 538, 540, 546, 547, 548, 549, 550, 553, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 570, 571, 572, 573, 615 }

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2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 42, 44, 45, 46, 47, 48, 49, 50, 52, 54, 56, 57, 59, 61, 63, 65, 67, 68, 69, 70, 72, 74, 78, 80, 82, 83, 85, 87, 91, 95, 96, 98, 100, 101, 103, 105, 107, 108, 110, 112, 114, 116, 118, 120, 121, 123, 125, 127, 129, 131, 132, 134, 136, 138, 139, 141, 143, 145, 146, 148, 150, 152, 154, 160, 162, 163, 165, 167, 169, 171, 173, 175, 176, 178, 180, 183, 185, 187, 189, 191, 192, 194, 346, 363, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 449, 450, 451, 452, 453, 461, 462, 464, 473, 474, 475, 483, 484, 485, 494, 495, 496, 506, 507, 508, 516, 517, 518, 527, 528, 529, 631, 632, 633 }

B grade: { 36, 41, 43, 51, 53, 55, 58, 60, 62, 64, 66, 71, 73, 75, 76, 77, 79, 81, 84, 86, 88, 89, 90, 92, 93, 94, 97, 99, 122, 133, 135, 147, 149, 151, 156, 158, 276, 277, 278, 279, 285, 286, 287, 288, 294, 295, 296, 297, 301, 302, 303, 304, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 343, 344, 345, 367, 368, 369, 413, 414, 454, 455, 463, 465, 466, 630 }

C grade: { }

F grade: { 102, 104, 106, 109, 111, 113, 115, 117, 119, 124, 126, 128, 130, 137, 140, 142, 144, 153, 155, 157, 159, 161, 164, 166, 168, 170, 172, 174, 177, 179, 181, 182, 184, 186, 188, 190, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 298, 299, 300, 305, 306, 307, 313, 314, 315, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 370, 371, 372, 373, 374, 375, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 456, 457, 458, 459, 460, 467, 468, 469, 470, 471, 472, 476, 477, 478, 479, 480, 481, 482, 486, 487, 488, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 502, 503, 504, 505, 509, 510, 511, 512, 513, 514, 515, 519, 520, 521, 522, 523, 524, 525, 526, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 40, 42, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 82, 84, 85, 86, 87, 91, 94, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 130, 132, 133, 134, 135, 136, 138, 139, 140, 142, 144, 146, 147, 148, 149, 150, 152, 154, 155, 156, 157, 158, 160, 162, 163, 165, 166, 167, 168, 169, 171, 173, 175, 176, 178, 179, 180, 181, 183, 185, 187, 189, 192, 193, 194, 195, 276, 277, 278, 279, 285, 286, 287, 288, 295, 296, 297, 301, 302, 303, 304, 309, 310, 311, 312, 318, 319, 320, 343, 344, 345, 346, 360, 361, 362, 363, 368, 369, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 444, 445, 446, 447, 448, 449, 450, 451, 456, 458, 461, 473, 474, 475, 483, 484, 494, 506, 507, 508, 516, 517, 518, 527, 528, 632, 633 }

B grade: { 18, 23, 32, 36, 39, 41, 43, 45, 76, 81, 83, 88, 89, 90, 92, 93, 95, 96, 98, 109, 124, 129, 131, 137, 141, 143, 145, 151, 153, 159, 161, 164, 170, 172, 174, 177, 182, 184, 186, 188, 190, 191, 294, 308, 316, 317, 367, 389, 402, 413, 414, 415, 442, 443, 452, 453, 454, 455, 457, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 476, 477, 485, 487, 495, 496, 497, 498, 499, 529, 530, 630, 631 }

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2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 41, 42, 43, 44, 45, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 89, 91, 93, 95, 107, 118, 120, 162, 175, 189, 191, 344, 345, 346, 376, 377, 378, 382, 383, 387, 388, 389, 393, 394, 395, 400, 401, 402, 406, 407, 413, 414, 415, 419, 427, 439, 440, 451, 452, 461, 462, 463, 464, 475, 484, 485, 508, 518, 528, 529, 633 }

B grade: { }

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F grade: { 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 33, 34, 35, 36, 37, 38, 39, 40, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 70, 71, 72, 73, 74, 75, 83, 84, 85, 86, 87, 88, 90, 92, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 384, 385, 386, 390, 391, 392, 396, 397, 398, 399, 403, 404, 405, 408, 409, 410, 411, 412, 416, 417, 418, 420, 421, 422, 423, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 453, 454, 455, 456, 457, 458, 459, 460, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 509, 510, 511, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

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B grade: { 8, 19, 26, 28, 33, 35, 37, 41, 43, 48, 50, 60, 76, 78, 90, 91, 101, 103, 105, 107, 110, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 127, 128, 129, 130, 131, 133, 139, 140, 141, 142, 143, 144, 145, 156, 157, 158, 159, 161, 163, 164, 165, 166, 169, 170, 171, 172, 174, 176, 177, 179, 180, 182, 183, 184, 185, 186, 187, 188, 190, 193, 194, 195, 345, 386, 399, 411, 412, 413, 414, 420, 424, 431, 435, 436, 444, 447, 448, 456, 457, 459, 460, 465, 466, 467, 468, 469, 470, 471, 472, 632 }

C grade: { }

F grade: { 108, 123, 124, 125, 126, 132, 134, 135, 136, 137, 138, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 167, 168, 178, 181, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 473, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 509, 510, 511, 512, 513, 514, 515, 516, 519, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.8 Mupad

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B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 107, 120, 122, 131, 133, 135, 145, 149, 151, 162, 175, 191, 200, 276, 277, 278, 279, 285, 286, 287, 288, 294, 295, 296, 297, 301, 302, 303, 304, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 343, 344, 345, 346, 360, 361, 362, 363, 367, 368, 369, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 475, 485, 496, 508, 518, 528, 529, 543, 630, 631, 632, 633 }

C grade: { }

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2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	74	56	92	62	105	118	90
normalized size	1	1.00	0.85	0.64	1.06	0.71	1.21	1.36	1.03
time (sec)	N/A	0.056	0.043	0.135	0.402	0.787	10.287	2.685	0.087

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	62	63	62	172	107	226
normalized size	1	1.00	0.66	0.71	0.72	0.71	1.98	1.23	2.60
time (sec)	N/A	0.061	0.158	0.145	0.626	0.619	6.702	1.292	8.231

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	60	46	70	51	83	88	68
normalized size	1	1.00	0.94	0.72	1.09	0.80	1.30	1.38	1.06
time (sec)	N/A	0.044	0.023	0.135	0.324	0.764	3.649	0.750	0.051

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	48	51	124	77	165
normalized size	1	1.00	0.95	0.80	0.74	0.78	1.91	1.18	2.54
time (sec)	N/A	0.045	0.084	0.143	0.327	0.612	2.176	0.694	7.991

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	36	48	39	60	48	46
normalized size	1	1.00	0.98	0.80	1.07	0.87	1.33	1.07	1.02
time (sec)	N/A	0.037	0.015	0.131	0.320	0.741	1.016	0.596	0.058

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	37	37	71	47	103
normalized size	1	1.00	1.07	0.95	0.86	0.86	1.65	1.09	2.40
time (sec)	N/A	0.033	0.048	0.083	0.324	0.565	0.534	0.805	6.761

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	39	25	20	25	34	25	20
normalized size	1	1.27	1.77	1.14	0.91	1.14	1.55	1.14	0.91
time (sec)	N/A	0.016	0.013	0.043	0.355	0.694	0.219	0.496	0.040

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	26	16	15	17	0	37	15
normalized size	1	1.00	1.53	0.94	0.88	1.00	0.00	2.18	0.88
time (sec)	N/A	0.020	0.014	0.083	0.353	0.673	0.000	0.935	0.047

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	40	0	19	19
normalized size	1	1.00	1.00	1.04	1.00	1.74	0.00	0.83	0.83
time (sec)	N/A	0.032	0.014	0.135	0.487	0.553	0.000	0.470	4.688

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	52	54	42	67	0	54	30
normalized size	1	1.00	1.33	1.38	1.08	1.72	0.00	1.38	0.77
time (sec)	N/A	0.042	0.018	0.168	0.328	1.046	0.000	0.544	0.061

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	35	52	0	66	63
normalized size	1	1.00	0.93	0.86	0.80	1.18	0.00	1.50	1.43
time (sec)	N/A	0.036	0.061	0.164	0.379	0.639	0.000	0.364	4.604

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	68	74	86	136	0	92	71
normalized size	1	1.00	0.81	0.88	1.02	1.62	0.00	1.10	0.85
time (sec)	N/A	0.064	0.079	0.175	0.528	0.796	0.000	0.515	4.501

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	171	129	115	85	398	123	461
normalized size	1	1.00	1.36	1.02	0.91	0.67	3.16	0.98	3.66
time (sec)	N/A	0.107	1.520	0.176	0.607	0.959	14.952	0.522	6.916

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	58	99	95	71	158	117	92
normalized size	1	1.00	0.87	1.48	1.42	1.06	2.36	1.75	1.37
time (sec)	N/A	0.061	0.074	0.172	0.491	0.711	8.340	2.271	4.536

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	151	109	89	72	287	106	349
normalized size	1	1.00	1.48	1.07	0.87	0.71	2.81	1.04	3.42
time (sec)	N/A	0.093	0.544	0.177	0.321	0.978	5.219	0.612	6.792

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	79	56	58	107	56	53
normalized size	1	1.00	1.02	1.76	1.24	1.29	2.38	1.24	1.18
time (sec)	N/A	0.046	0.084	0.165	0.408	0.725	3.051	0.886	0.064

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	131	87	65	59	180	72	237
normalized size	1	1.00	1.68	1.12	0.83	0.76	2.31	0.92	3.04
time (sec)	N/A	0.089	0.296	0.120	0.320	0.550	1.998	0.443	6.708

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	47	21	20	44	53	20	32
normalized size	1	1.00	2.14	0.95	0.91	2.00	2.41	0.91	1.45
time (sec)	N/A	0.024	0.021	0.071	0.342	0.690	0.818	0.743	4.545

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	29	53	30	32	0	91	26
normalized size	1	1.00	0.85	1.56	0.88	0.94	0.00	2.68	0.76
time (sec)	N/A	0.042	0.020	0.144	0.394	0.596	0.000	0.874	0.054

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	75	47	47	74	0	33	28
normalized size	1	1.00	1.97	1.24	1.24	1.95	0.00	0.87	0.74
time (sec)	N/A	0.082	0.055	0.183	0.666	0.596	0.000	0.555	4.561

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	32	75	18	19	0	30	18
normalized size	1	1.00	1.60	3.75	0.90	0.95	0.00	1.50	0.90
time (sec)	N/A	0.038	0.136	0.213	0.324	0.767	0.000	0.740	0.042

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	63	52	97	0	54	81
normalized size	1	1.00	0.92	1.00	0.83	1.54	0.00	0.86	1.29
time (sec)	N/A	0.069	0.009	0.216	0.605	0.593	0.000	1.687	4.558

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	144	71	125	0	77	58
normalized size	1	1.00	0.88	2.25	1.11	1.95	0.00	1.20	0.91
time (sec)	N/A	0.066	0.068	0.227	0.349	0.667	0.000	0.623	4.466

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	82	93	77	85	0	106	156
normalized size	1	1.00	1.28	1.45	1.20	1.33	0.00	1.66	2.44
time (sec)	N/A	0.056	0.011	0.223	0.372	0.609	0.000	0.649	4.634

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	85	190	108	203	0	119	94
normalized size	1	1.00	0.78	1.74	0.99	1.86	0.00	1.09	0.86
time (sec)	N/A	0.089	0.145	0.242	0.955	0.740	0.000	0.501	4.338

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	110	121	98	115	0	171	276
normalized size	1	1.00	1.34	1.48	1.20	1.40	0.00	2.09	3.37
time (sec)	N/A	0.059	0.024	0.237	0.332	0.596	0.000	0.650	5.066

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	181	163	141	98	439	157	501
normalized size	1	1.00	1.18	1.06	0.92	0.64	2.85	1.02	3.25
time (sec)	N/A	0.154	1.995	0.184	0.385	0.843	21.327	0.862	6.806

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	58	133	108	85	196	134	106
normalized size	1	1.00	0.87	1.99	1.61	1.27	2.93	2.00	1.58
time (sec)	N/A	0.066	0.093	0.171	0.369	0.645	13.396	0.985	4.539

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	161	143	115	85	335	123	389
normalized size	1	1.00	1.24	1.10	0.88	0.65	2.58	0.95	2.99
time (sec)	N/A	0.140	0.779	0.181	0.322	0.598	9.999	0.843	6.711

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	113	82	72	146	82	80
normalized size	1	1.00	0.96	2.51	1.82	1.60	3.24	1.82	1.78
time (sec)	N/A	0.047	0.151	0.171	0.438	0.694	5.778	0.713	4.472

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	141	121	91	72	226	89	277
normalized size	1	1.00	1.33	1.14	0.86	0.68	2.13	0.84	2.61
time (sec)	N/A	0.124	0.408	0.125	0.536	0.853	3.722	1.068	6.532

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	65	21	20	57	70	20	53
normalized size	1	1.00	2.95	0.95	0.91	2.59	3.18	0.91	2.41
time (sec)	N/A	0.025	0.027	0.073	0.320	0.761	1.114	0.828	0.056

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	69	43	45	0	128	36
normalized size	1	1.00	0.79	1.33	0.83	0.87	0.00	2.46	0.69
time (sec)	N/A	0.047	0.029	0.135	0.387	0.661	0.000	0.900	0.053

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	87	68	101	0	91	138
normalized size	1	1.00	1.10	1.74	1.36	2.02	0.00	1.82	2.76
time (sec)	N/A	0.136	0.034	0.220	0.415	0.685	0.000	0.545	4.776

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	59	128	33	51	0	92	35
normalized size	1	1.00	1.48	3.20	0.82	1.28	0.00	2.30	0.88
time (sec)	N/A	0.052	0.042	0.224	1.105	0.603	0.000	0.499	4.521

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	120	78	99	0	38	55
normalized size	1	1.00	0.90	3.87	2.52	3.19	0.00	1.23	1.77
time (sec)	N/A	0.085	0.025	0.263	0.477	0.668	0.000	1.518	4.592

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	35	146	28	30	0	63	18
normalized size	1	1.00	1.52	6.35	1.22	1.30	0.00	2.74	0.78
time (sec)	N/A	0.040	0.232	0.240	0.664	0.529	0.000	0.702	0.066

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	110	171	103	149	0	86	135
normalized size	1	1.00	1.20	1.86	1.12	1.62	0.00	0.93	1.47
time (sec)	N/A	0.093	0.018	0.261	0.427	0.523	0.000	0.821	4.694

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	67	238	96	185	0	90	81
normalized size	1	1.00	0.77	2.74	1.10	2.13	0.00	1.03	0.93
time (sec)	N/A	0.072	0.103	0.246	0.397	0.636	0.000	0.557	4.536

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	134	217	122	112	0	138	228
normalized size	1	1.00	1.35	2.19	1.23	1.13	0.00	1.39	2.30
time (sec)	N/A	0.083	0.012	0.271	0.477	0.535	0.000	0.519	4.865

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	58	513	173	149	558	219	134
normalized size	1	1.00	0.87	7.66	2.58	2.22	8.33	3.27	2.00
time (sec)	N/A	0.084	0.416	0.182	0.610	0.791	127.889	1.959	0.179

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	211	535	339	150	1280	208	684
normalized size	1	1.00	0.74	1.87	1.19	0.52	4.48	0.73	2.39
time (sec)	N/A	0.403	3.203	0.196	0.660	0.643	94.802	1.971	7.051

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	463	134	136	422	134	132
normalized size	1	1.00	0.96	10.29	2.98	3.02	9.38	2.98	2.93
time (sec)	N/A	0.047	1.089	0.189	0.325	0.746	50.556	1.569	0.117

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	191	480	319	137	1018	174	572
normalized size	1	1.00	0.73	1.83	1.22	0.52	3.89	0.66	2.18
time (sec)	N/A	0.374	1.494	0.141	0.377	0.744	37.274	1.503	7.258

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	147	21	20	122	148	20	118
normalized size	1	1.00	6.68	0.95	0.91	5.55	6.73	0.91	5.36
time (sec)	N/A	0.024	0.090	0.079	0.660	0.699	19.839	0.901	4.746

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	95	149	109	114	0	288	109
normalized size	1	1.00	0.59	0.92	0.67	0.70	0.00	1.78	0.67
time (sec)	N/A	0.077	0.170	0.167	0.626	0.710	0.000	0.954	4.648

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	55	389	331	231	0	231	513
normalized size	1	1.00	0.27	1.94	1.65	1.15	0.00	1.15	2.55
time (sec)	N/A	0.342	0.062	0.382	0.671	0.679	0.000	1.901	8.733

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	111	345	97	130	0	275	97
normalized size	1	1.00	0.92	2.85	0.80	1.07	0.00	2.27	0.80
time (sec)	N/A	0.094	0.270	0.279	0.310	0.681	0.000	0.938	4.623

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	59	478	311	247	0	200	437
normalized size	1	1.00	0.33	2.67	1.74	1.38	0.00	1.12	2.44
time (sec)	N/A	0.319	0.052	0.418	0.639	0.706	0.000	0.890	9.129

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	73	503	95	139	0	243	96
normalized size	1	1.00	0.66	4.57	0.86	1.26	0.00	2.21	0.87
time (sec)	N/A	0.091	0.444	0.279	0.348	0.642	0.000	0.815	4.602

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	141	245	258	50	1355	114	107
normalized size	1	1.00	1.93	3.36	3.53	0.68	18.56	1.56	1.47
time (sec)	N/A	0.068	0.799	0.146	0.665	1.111	34.038	0.608	8.155

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	45	47	37	530	47	54
normalized size	1	1.00	0.98	0.96	1.00	0.79	11.28	1.00	1.15
time (sec)	N/A	0.056	0.091	0.135	0.312	0.839	18.773	0.349	4.660

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	119	141	156	37	558	75	66
normalized size	1	1.00	2.43	2.88	3.18	0.76	11.39	1.53	1.35
time (sec)	N/A	0.055	0.320	0.138	0.597	0.649	11.380	0.856	6.874

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	28	25	25	158	25	22
normalized size	1	1.00	0.75	0.88	0.78	0.78	4.94	0.78	0.69
time (sec)	N/A	0.046	0.038	0.077	0.329	0.637	5.796	0.401	4.488

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	97	43	52	17	88	34	29
normalized size	1	1.00	5.11	2.26	2.74	0.89	4.63	1.79	1.53
time (sec)	N/A	0.043	0.122	0.125	0.596	0.603	3.018	0.594	4.532

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	18	16	24	19	16
normalized size	1	1.00	1.00	1.19	1.12	1.00	1.50	1.19	1.00
time (sec)	N/A	0.026	0.011	0.057	0.314	0.738	0.504	0.399	0.042

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	54	47	58	0	58	33
normalized size	1	1.00	0.81	1.46	1.27	1.57	0.00	1.57	0.89
time (sec)	N/A	0.051	0.040	0.145	0.394	0.648	0.000	0.449	0.066

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	45	70	129	49	0	67	71
normalized size	1	1.00	1.07	1.67	3.07	1.17	0.00	1.60	1.69
time (sec)	N/A	0.052	0.056	0.148	0.611	0.673	0.000	0.647	4.560

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	75	90	91	125	0	96	74
normalized size	1	1.00	0.97	1.17	1.18	1.62	0.00	1.25	0.96
time (sec)	N/A	0.076	0.099	0.161	0.320	0.568	0.000	0.853	4.656

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	66	130	294	75	0	119	125
normalized size	1	1.00	1.06	2.10	4.74	1.21	0.00	1.92	2.02
time (sec)	N/A	0.059	0.098	0.164	0.480	0.674	0.000	0.685	5.941

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	97	126	130	147	0	116	115
normalized size	1	1.00	0.81	1.05	1.08	1.22	0.00	0.97	0.96
time (sec)	N/A	0.107	0.154	0.168	0.301	0.640	0.000	0.753	0.142

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	151	415	393	60	2531	179	172
normalized size	1	1.00	1.45	3.99	3.78	0.58	24.34	1.72	1.65
time (sec)	N/A	0.112	1.116	0.225	0.629	0.808	148.509	0.802	8.223

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	45	47	47	1037	47	54
normalized size	1	1.00	0.98	0.96	1.00	1.00	22.06	1.00	1.15
time (sec)	N/A	0.052	0.162	0.187	0.347	0.729	92.863	1.011	4.662

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	131	279	267	50	1243	127	65
normalized size	1	1.00	1.64	3.49	3.34	0.62	15.54	1.59	0.81
time (sec)	N/A	0.100	0.509	0.179	0.725	0.596	58.826	0.647	4.746

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	34	19	35	37	394	35	32
normalized size	1	1.00	1.48	0.83	1.52	1.61	17.13	1.52	1.39
time (sec)	N/A	0.043	0.063	0.167	0.332	0.688	37.355	1.971	4.610

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	109	142	140	35	403	73	32
normalized size	1	1.00	1.95	2.54	2.50	0.62	7.20	1.30	0.57
time (sec)	N/A	0.085	0.176	0.189	0.489	0.730	22.409	0.362	4.653

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	33	30	27	150	54	27
normalized size	1	1.00	0.81	1.03	0.94	0.84	4.69	1.69	0.84
time (sec)	N/A	0.049	0.033	0.176	0.349	0.639	1.788	0.425	0.060

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	104	41	56	61	95	33	28
normalized size	1	1.00	3.06	1.21	1.65	1.79	2.79	0.97	0.82
time (sec)	N/A	0.043	0.184	0.200	0.762	0.863	6.925	0.488	4.641

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	31	21	20	21	32	20	18
normalized size	1	1.00	1.48	1.00	0.95	1.00	1.52	0.95	0.86
time (sec)	N/A	0.026	0.066	0.073	0.304	0.624	1.121	0.425	0.048

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	38	72	72	105	0	71	60
normalized size	1	1.00	0.63	1.20	1.20	1.75	0.00	1.18	1.00
time (sec)	N/A	0.058	0.097	0.185	0.808	0.742	0.000	0.748	4.519

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	53	98	204	79	0	93	156
normalized size	1	1.00	0.75	1.38	2.87	1.11	0.00	1.31	2.20
time (sec)	N/A	0.093	0.075	0.192	0.424	0.636	0.000	0.381	4.770

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	85	108	108	178	0	106	93
normalized size	1	1.00	0.82	1.04	1.04	1.71	0.00	1.02	0.89
time (sec)	N/A	0.082	0.115	0.232	0.368	0.731	0.000	0.806	0.103

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	78	158	396	103	0	145	276
normalized size	1	1.00	0.84	1.70	4.26	1.11	0.00	1.56	2.97
time (sec)	N/A	0.098	0.057	0.233	0.778	0.692	0.000	0.784	5.184

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	137	144	167	198	0	126	151
normalized size	1	1.00	0.94	0.99	1.14	1.36	0.00	0.86	1.03
time (sec)	N/A	0.110	0.333	0.233	0.373	0.851	0.000	0.536	0.186

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	141	313	310	60	0	140	81
normalized size	1	1.00	1.37	3.04	3.01	0.58	0.00	1.36	0.79
time (sec)	N/A	0.109	1.076	0.214	0.774	0.539	0.000	0.388	4.721

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	44	19	45	45	654	45	53
normalized size	1	1.00	1.91	0.83	1.96	1.96	28.43	1.96	2.30
time (sec)	N/A	0.043	0.160	0.178	0.449	0.550	161.609	0.745	4.548

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	121	177	184	45	690	88	57
normalized size	1	1.00	1.57	2.30	2.39	0.58	8.96	1.14	0.74
time (sec)	N/A	0.100	0.465	0.203	0.466	0.613	108.266	0.424	4.638

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	38	49	41	36	564	115	36
normalized size	1	1.00	0.76	0.98	0.82	0.72	11.28	2.30	0.72
time (sec)	N/A	0.050	0.055	0.185	0.309	0.732	64.192	2.669	4.560

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	59	64	139	78	478	80	69
normalized size	1	1.00	1.20	1.31	2.84	1.59	9.76	1.63	1.41
time (sec)	N/A	0.085	0.044	0.214	0.589	0.542	40.794	2.084	4.852

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	58	37	37	41	299	35	36
normalized size	1	1.00	1.49	0.95	0.95	1.05	7.67	0.90	0.92
time (sec)	N/A	0.051	0.059	0.199	0.368	0.796	1.947	1.252	4.543

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	28	55	99	95	153	36	53
normalized size	1	1.00	1.04	2.04	3.67	3.52	5.67	1.33	1.96
time (sec)	N/A	0.039	0.019	0.223	0.427	0.472	15.149	0.553	4.580

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	33	21	20	36	51	20	18
normalized size	1	1.00	1.50	0.95	0.91	1.64	2.32	0.91	0.82
time (sec)	N/A	0.025	0.059	0.070	0.384	0.704	1.893	2.911	4.460

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	61	90	98	154	0	81	83
normalized size	1	1.00	0.74	1.10	1.20	1.88	0.00	0.99	1.01
time (sec)	N/A	0.063	0.088	0.213	0.501	0.818	0.000	1.442	4.595

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	63	130	310	106	0	119	228
normalized size	1	1.00	0.64	1.31	3.13	1.07	0.00	1.20	2.30
time (sec)	N/A	0.135	0.109	0.219	0.455	0.816	0.000	0.476	5.079

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	126	146	228	0	116	129
normalized size	1	1.00	0.75	1.00	1.16	1.81	0.00	0.92	1.02
time (sec)	N/A	0.095	0.166	0.260	0.326	0.613	0.000	0.744	0.149

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	85	190	482	130	0	171	167
normalized size	1	1.00	0.69	1.54	3.92	1.06	0.00	1.39	1.36
time (sec)	N/A	0.145	0.114	0.268	0.444	0.617	0.000	0.749	5.563

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	145	162	188	248	0	136	173
normalized size	1	1.00	0.85	0.95	1.10	1.45	0.00	0.80	1.01
time (sec)	N/A	0.128	0.519	0.270	0.603	0.841	0.000	0.705	4.765

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	275	146	295	244	0	99	91
normalized size	1	1.00	2.17	1.15	2.32	1.92	0.00	0.78	0.72
time (sec)	N/A	0.182	6.081	0.283	0.641	0.757	0.000	1.179	7.770

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	55	74	82	2006	68	64
normalized size	1	1.00	0.78	1.53	2.06	2.28	55.72	1.89	1.78
time (sec)	N/A	0.046	0.080	0.250	0.324	0.625	42.650	1.398	0.072

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	36	145	375	239	0	125	118
normalized size	1	1.00	0.62	2.50	6.47	4.12	0.00	2.16	2.03
time (sec)	N/A	0.080	0.107	0.279	0.388	0.590	0.000	0.610	6.628

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	43	93	100	1120	137	54
normalized size	1	1.00	0.89	0.66	1.43	1.54	17.23	2.11	0.83
time (sec)	N/A	0.059	0.129	0.253	0.315	0.654	42.110	1.443	4.712

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	58	175	461	291	0	151	140
normalized size	1	1.00	0.49	1.48	3.91	2.47	0.00	1.28	1.19
time (sec)	N/A	0.168	0.090	0.300	0.425	0.690	0.000	0.596	7.148

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	33	96	105	493	28	28
normalized size	1	1.00	0.96	0.73	2.13	2.33	10.96	0.62	0.62
time (sec)	N/A	0.053	0.173	0.266	0.320	0.615	41.537	0.586	0.097

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	78	205	547	339	0	177	162
normalized size	1	1.00	0.43	1.12	2.99	1.85	0.00	0.97	0.89
time (sec)	N/A	0.272	0.130	0.302	0.521	0.731	0.000	0.782	8.117

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	33	21	20	108	128	20	18
normalized size	1	1.00	1.50	0.95	0.91	4.91	5.82	0.91	0.82
time (sec)	N/A	0.025	0.229	0.095	0.308	0.582	42.269	0.491	4.668

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	122	180	213	374	0	131	198
normalized size	1	1.00	0.63	0.93	1.10	1.93	0.00	0.68	1.02
time (sec)	N/A	0.112	0.833	0.294	0.323	0.520	0.000	0.470	0.303

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	113	280	740	225	0	249	233
normalized size	1	1.00	0.46	1.14	3.02	0.92	0.00	1.02	0.95
time (sec)	N/A	0.404	0.402	0.264	0.596	0.765	0.000	1.524	8.035

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	175	216	248	446	0	166	231
normalized size	1	1.00	0.74	0.91	1.04	1.87	0.00	0.70	0.97
time (sec)	N/A	0.171	1.830	0.341	0.537	0.837	0.000	0.574	0.489

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	125	340	866	249	0	301	277
normalized size	1	1.00	0.45	1.22	3.10	0.89	0.00	1.08	0.99
time (sec)	N/A	0.419	0.433	0.350	0.628	0.857	0.000	1.051	9.231

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	195	252	305	466	0	186	290
normalized size	1	1.00	0.69	0.89	1.07	1.64	0.00	0.65	1.02
time (sec)	N/A	0.216	2.610	0.361	0.978	0.949	0.000	0.624	0.798

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	74	57	72	88	0	249	-1
normalized size	1	1.00	0.76	0.59	0.74	0.91	0.00	2.57	-0.01
time (sec)	N/A	0.083	4.324	0.198	0.376	0.641	0.000	2.627	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	99	75	0	172	0	219	-1
normalized size	1	1.00	0.78	0.59	0.00	1.35	0.00	1.72	-0.01
time (sec)	N/A	0.258	3.943	0.200	0.000	0.670	0.000	1.119	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	64	41	55	68	0	189	-1
normalized size	1	1.00	0.88	0.56	0.75	0.93	0.00	2.59	-0.01
time (sec)	N/A	0.073	1.025	0.161	0.305	0.568	0.000	1.696	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	89	65	0	132	0	159	-1
normalized size	1	1.00	0.94	0.68	0.00	1.39	0.00	1.67	-0.01
time (sec)	N/A	0.179	0.768	0.197	0.000	0.630	0.000	1.094	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	54	31	38	46	0	129	-1
normalized size	1	1.00	1.10	0.63	0.78	0.94	0.00	2.63	-0.02
time (sec)	N/A	0.065	0.231	0.150	0.440	0.784	0.000	0.690	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	79	55	0	92	0	99	-1
normalized size	1	1.00	1.25	0.87	0.00	1.46	0.00	1.57	-0.02
time (sec)	N/A	0.111	0.170	0.181	0.000	0.646	0.000	0.398	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	44	21	20	25	58	68	20
normalized size	1	1.00	1.83	0.88	0.83	1.04	2.42	2.83	0.83
time (sec)	N/A	0.033	0.083	0.041	0.667	0.632	0.679	0.920	4.566

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	95	32	58	92	0	0	-1
normalized size	1	1.00	2.38	0.80	1.45	2.30	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.101	0.099	0.938	0.589	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	106	83	0	159	0	102	-1
normalized size	1	1.00	1.47	1.15	0.00	2.21	0.00	1.42	-0.01
time (sec)	N/A	0.079	0.231	0.232	0.000	0.548	0.000	0.523	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	271	90	117	99	0	282	-1
normalized size	1	1.00	2.85	0.95	1.23	1.04	0.00	2.97	-0.01
time (sec)	N/A	0.123	0.378	0.263	0.600	0.614	0.000	0.871	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	302	153	0	188	0	178	-1
normalized size	1	1.00	2.20	1.12	0.00	1.37	0.00	1.30	-0.01
time (sec)	N/A	0.159	0.418	0.243	0.000	0.695	0.000	0.773	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	179	118	168	121	0	442	-1
normalized size	1	1.00	1.20	0.79	1.13	0.81	0.00	2.97	-0.01
time (sec)	N/A	0.203	0.489	0.334	1.006	0.798	0.000	2.747	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	191	244	0	210	0	239	-1
normalized size	1	1.00	0.97	1.24	0.00	1.07	0.00	1.21	-0.01
time (sec)	N/A	0.294	0.645	0.349	0.000	0.617	0.000	2.531	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	61	57	72	110	0	505	-1
normalized size	1	1.00	0.63	0.59	0.74	1.13	0.00	5.21	-0.01
time (sec)	N/A	0.086	0.450	0.175	0.331	0.564	0.000	1.114	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	79	87	0	210	0	474	-1
normalized size	1	1.00	0.50	0.55	0.00	1.32	0.00	2.98	-0.01
time (sec)	N/A	0.302	0.656	0.206	0.000	0.571	0.000	1.340	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	41	55	88	0	381	-1
normalized size	1	1.00	0.70	0.56	0.75	1.21	0.00	5.22	-0.01
time (sec)	N/A	0.076	0.170	0.166	0.321	0.640	0.000	1.025	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	69	77	0	166	0	350	-1
normalized size	1	1.00	0.54	0.61	0.00	1.31	0.00	2.76	-0.01
time (sec)	N/A	0.235	0.161	0.185	0.000	0.476	0.000	1.185	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	31	38	66	252	257	-1
normalized size	1	1.00	0.84	0.63	0.78	1.35	5.14	5.24	-0.02
time (sec)	N/A	0.068	0.087	0.150	0.316	0.695	127.617	0.704	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	59	67	0	122	0	226	-1
normalized size	1	1.00	0.62	0.71	0.00	1.28	0.00	2.38	-0.01
time (sec)	N/A	0.167	0.150	0.266	0.000	0.646	0.000	0.636	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	40	90	133	20
normalized size	1	1.00	1.00	0.88	0.83	1.67	3.75	5.54	0.83
time (sec)	N/A	0.034	0.042	0.042	1.512	0.720	29.542	0.395	4.624

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	60	49	80	72	0	1021	-1
normalized size	1	1.00	0.97	0.79	1.29	1.16	0.00	16.47	-0.02
time (sec)	N/A	0.068	0.105	0.137	0.929	0.695	0.000	12.200	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	67	37	98	26	0	3663	37
normalized size	1	1.00	2.58	1.42	3.77	1.00	0.00	140.88	1.42
time (sec)	N/A	0.058	0.159	0.138	2.507	0.647	0.000	155.910	4.773

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	72	70	94	99	0	0	-1
normalized size	1	1.00	0.99	0.96	1.29	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.259	0.191	1.407	0.605	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	130	107	0	215	0	0	-1
normalized size	1	1.00	1.21	1.00	0.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.395	0.234	0.000	0.638	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	44	101	151	155	0	0	-1
normalized size	1	1.00	0.35	0.80	1.19	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.074	0.329	0.608	0.765	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	288	172	0	248	0	0	-1
normalized size	1	1.00	1.70	1.02	0.00	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.428	0.253	0.000	0.766	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	41	55	114	0	471	-1
normalized size	1	1.00	0.70	0.56	0.75	1.56	0.00	6.45	-0.01
time (sec)	N/A	0.074	0.224	0.165	0.520	0.756	0.000	2.765	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	79	87	0	219	0	438	-1
normalized size	1	1.00	0.50	0.55	0.00	1.38	0.00	2.75	-0.01
time (sec)	N/A	0.293	0.332	0.190	0.000	0.616	0.000	1.392	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	31	38	88	0	339	-1
normalized size	1	1.00	0.84	0.63	0.78	1.80	0.00	6.92	-0.02
time (sec)	N/A	0.067	0.141	0.161	0.474	0.776	0.000	4.542	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	69	77	0	167	0	306	-1
normalized size	1	1.00	0.54	0.61	0.00	1.31	0.00	2.41	-0.01
time (sec)	N/A	0.224	0.309	0.198	0.000	0.608	0.000	1.368	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	61	0	207	20
normalized size	1	1.00	1.00	0.88	0.83	2.54	0.00	8.62	0.83
time (sec)	N/A	0.034	0.066	0.044	0.565	0.727	0.000	0.555	4.782

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	73	66	97	89	0	0	-1
normalized size	1	1.00	0.85	0.77	1.13	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.204	0.167	0.519	0.572	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	45	191	41	0	6622	88
normalized size	1	1.00	0.65	0.82	3.47	0.75	0.00	120.40	1.60
time (sec)	N/A	0.117	4.617	0.168	1.075	0.734	0.000	54.374	5.460

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	75	66	94	102	0	0	-1
normalized size	1	1.00	1.09	0.96	1.36	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.459	0.197	0.532	0.683	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	69	47	184	43	0	0	225
normalized size	1	1.00	2.30	1.57	6.13	1.43	0.00	0.00	7.50
time (sec)	N/A	0.059	5.159	0.163	0.518	0.837	0.000	0.000	7.589

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	110	107	134	147	0	0	-1
normalized size	1	1.00	1.07	1.04	1.30	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.286	0.262	0.421	0.615	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	129	120	0	263	0	0	-1
normalized size	1	1.00	0.93	0.86	0.00	1.89	0.00	0.00	-0.01
time (sec)	N/A	0.197	5.307	0.300	0.000	0.664	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	44	113	185	208	0	0	-1
normalized size	1	1.00	0.28	0.71	1.16	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.109	0.500	0.730	0.680	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	64	57	72	154	0	669	-1
normalized size	1	1.00	0.66	0.59	0.74	1.59	0.00	6.90	-0.01
time (sec)	N/A	0.079	0.634	0.220	0.837	0.775	0.000	6.140	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	102	107	0	296	0	636	-1
normalized size	1	1.00	0.46	0.48	0.00	1.33	0.00	2.85	-0.00
time (sec)	N/A	0.430	0.547	0.232	0.000	0.658	0.000	8.968	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	41	55	128	0	537	-1
normalized size	1	1.00	0.74	0.56	0.75	1.75	0.00	7.36	-0.01
time (sec)	N/A	0.075	0.305	0.171	0.324	0.840	0.000	3.130	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	92	97	0	244	0	504	-1
normalized size	1	1.00	0.48	0.51	0.00	1.28	0.00	2.64	-0.01
time (sec)	N/A	0.365	0.259	0.203	0.000	0.698	0.000	2.281	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	31	38	102	0	405	-1
normalized size	1	1.00	0.90	0.63	0.78	2.08	0.00	8.27	-0.02
time (sec)	N/A	0.066	0.147	0.156	0.403	0.636	0.000	2.375	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	82	87	0	192	0	372	-1
normalized size	1	1.00	0.52	0.55	0.00	1.21	0.00	2.34	-0.01
time (sec)	N/A	0.292	0.229	0.202	0.000	0.450	0.000	1.850	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	74	0	273	20
normalized size	1	1.00	1.00	0.88	0.83	3.08	0.00	11.38	0.83
time (sec)	N/A	0.033	0.086	0.040	0.517	0.578	0.000	0.974	4.844

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	85	83	115	102	0	0	-1
normalized size	1	1.00	0.77	0.75	1.05	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.345	0.168	0.812	0.737	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	48	55	237	54	0	0	-1
normalized size	1	1.00	0.54	0.62	2.66	0.61	0.00	0.00	-0.01
time (sec)	N/A	0.174	5.485	0.175	0.547	0.726	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	42	83	112	116	0	0	-1
normalized size	1	1.00	0.46	0.91	1.23	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.097	0.266	0.482	0.725	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	82	57	320	57	0	0	118
normalized size	1	1.00	1.34	0.93	5.25	0.93	0.00	0.00	1.93
time (sec)	N/A	0.113	5.299	0.192	0.598	0.708	0.000	0.000	8.531

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	108	75	132	145	0	0	-1
normalized size	1	1.00	1.02	0.71	1.25	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.294	0.276	0.426	0.781	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	69	47	270	54	0	0	86
normalized size	1	1.00	2.30	1.57	9.00	1.80	0.00	0.00	2.87
time (sec)	N/A	0.057	5.272	0.178	0.533	0.678	0.000	0.000	8.427

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	120	144	168	193	0	0	-1
normalized size	1	1.00	0.89	1.07	1.24	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.555	0.352	0.464	0.642	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	139	139	0	312	0	0	-1
normalized size	1	1.00	0.81	0.81	0.00	1.82	0.00	0.00	-0.01
time (sec)	N/A	0.268	5.497	0.238	0.000	0.652	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	44	129	219	254	0	0	-1
normalized size	1	1.00	0.23	0.68	1.15	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.312	0.108	0.416	0.451	0.556	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	388	205	0	346	0	0	-1
normalized size	1	1.00	1.67	0.88	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.354	5.678	0.276	0.000	0.723	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	61	57	281	82	0	430	-1
normalized size	1	1.00	0.63	0.59	2.90	0.85	0.00	4.43	-0.01
time (sec)	N/A	0.076	0.298	0.158	0.409	0.693	0.000	2.699	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	59	64	0	155	0	402	-1
normalized size	1	1.00	0.62	0.67	0.00	1.63	0.00	4.23	-0.01
time (sec)	N/A	0.170	0.258	0.199	0.000	0.617	0.000	4.623	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	41	160	62	0	310	-1
normalized size	1	1.00	0.70	0.56	2.19	0.85	0.00	4.25	-0.01
time (sec)	N/A	0.067	0.142	0.154	0.717	0.811	0.000	1.588	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	54	0	115	0	278	-1
normalized size	1	1.00	0.78	0.86	0.00	1.83	0.00	4.41	-0.02
time (sec)	N/A	0.111	0.071	0.176	0.000	0.684	0.000	1.925	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	34	31	75	40	0	75	-1
normalized size	1	1.00	0.69	0.63	1.53	0.82	0.00	1.53	-0.02
time (sec)	N/A	0.063	0.062	0.142	0.531	0.679	0.000	1.164	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	44	0	71	0	143	-1
normalized size	1	1.00	1.00	1.47	0.00	2.37	0.00	4.77	-0.03
time (sec)	N/A	0.051	0.068	0.188	0.000	0.781	0.000	1.098	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	32	20	20
normalized size	1	1.00	1.00	0.95	0.91	0.91	1.45	0.91	0.91
time (sec)	N/A	0.030	0.026	0.026	0.551	0.667	1.212	0.845	4.824

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	39	54	78	90	0	211	-1
normalized size	1	1.00	0.65	0.90	1.30	1.50	0.00	3.52	-0.02
time (sec)	N/A	0.062	0.050	0.148	1.247	0.592	0.000	1.437	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	118	130	0	200	0	419	-1
normalized size	1	1.00	1.16	1.27	0.00	1.96	0.00	4.11	-0.01
time (sec)	N/A	0.093	0.263	0.204	0.000	0.684	0.000	2.319	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	42	107	132	145	0	587	-1
normalized size	1	1.00	0.36	0.92	1.14	1.25	0.00	5.06	-0.01
time (sec)	N/A	0.133	0.072	0.267	0.745	0.588	0.000	2.812	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	117	231	0	230	0	745	-1
normalized size	1	1.00	0.72	1.43	0.00	1.42	0.00	4.60	-0.01
time (sec)	N/A	0.218	0.635	0.245	0.000	0.843	0.000	4.085	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	44	135	183	167	0	0	-1
normalized size	1	1.00	0.25	0.77	1.05	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.263	0.082	0.346	0.814	0.597	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	140	308	0	250	0	0	-1
normalized size	1	1.00	0.63	1.39	0.00	1.13	0.00	0.00	-0.00
time (sec)	N/A	0.357	0.718	0.276	0.000	0.564	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	54	57	72	72	0	370	-1
normalized size	1	1.00	0.56	0.59	0.74	0.74	0.00	3.81	-0.01
time (sec)	N/A	0.082	0.239	0.156	0.310	0.565	0.000	2.564	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	57	0	142	0	340	-1
normalized size	1	1.00	0.78	0.90	0.00	2.25	0.00	5.40	-0.02
time (sec)	N/A	0.116	0.198	0.195	0.000	0.758	0.000	1.716	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	41	55	52	0	250	-1
normalized size	1	1.00	0.60	0.56	0.75	0.71	0.00	3.42	-0.01
time (sec)	N/A	0.075	0.098	0.156	0.411	0.727	0.000	2.209	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	42	47	0	98	0	199	-1
normalized size	1	1.00	1.40	1.57	0.00	3.27	0.00	6.63	-0.03
time (sec)	N/A	0.058	0.061	0.168	0.000	0.706	0.000	1.735	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	32	29	36	28	0	56	-1
normalized size	1	1.00	0.68	0.62	0.77	0.60	0.00	1.19	-0.02
time (sec)	N/A	0.067	0.053	0.132	0.376	0.552	0.000	1.678	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	84	94	0	196	0	190	-1
normalized size	1	1.00	1.11	1.24	0.00	2.58	0.00	2.50	-0.01
time (sec)	N/A	0.081	0.150	0.213	0.000	0.846	0.000	1.933	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	33	56	20	50
normalized size	1	1.00	1.00	0.95	0.91	1.50	2.55	0.91	2.27
time (sec)	N/A	0.034	0.032	0.023	0.459	0.567	3.461	1.882	4.876

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	41	71	91	132	0	379	-1
normalized size	1	1.00	0.46	0.80	1.02	1.48	0.00	4.26	-0.01
time (sec)	N/A	0.077	0.070	0.172	0.709	0.586	0.000	2.049	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	224	202	0	240	0	591	-1
normalized size	1	1.00	1.67	1.51	0.00	1.79	0.00	4.41	-0.01
time (sec)	N/A	0.162	0.315	0.247	0.000	0.696	0.000	2.655	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	42	124	146	187	0	0	-1
normalized size	1	1.00	0.28	0.83	0.97	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.077	0.270	0.751	0.807	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	334	289	0	270	0	914	-1
normalized size	1	1.00	1.71	1.48	0.00	1.38	0.00	4.69	-0.01
time (sec)	N/A	0.291	0.346	0.310	0.000	0.789	0.000	9.618	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	44	152	197	207	0	1076	-1
normalized size	1	1.00	0.21	0.72	0.93	0.98	0.00	5.10	-0.00
time (sec)	N/A	0.343	0.092	0.363	0.463	0.597	0.000	7.406	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	444	367	0	290	0	0	-1
normalized size	1	1.00	1.73	1.43	0.00	1.13	0.00	0.00	-0.00
time (sec)	N/A	0.429	1.509	0.292	0.000	0.637	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	59	67	0	201	0	526	-1
normalized size	1	1.00	0.62	0.71	0.00	2.12	0.00	5.54	-0.01
time (sec)	N/A	0.195	0.770	0.204	0.000	0.773	0.000	2.360	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	64	67	89	82	0	430	-1
normalized size	1	1.00	0.53	0.55	0.74	0.68	0.00	3.55	-0.01
time (sec)	N/A	0.090	0.290	0.166	0.666	0.531	0.000	2.949	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	57	0	161	0	402	-1
normalized size	1	1.00	0.78	0.90	0.00	2.56	0.00	6.38	-0.02
time (sec)	N/A	0.119	0.354	0.208	0.000	0.760	0.000	3.162	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	54	57	72	62	0	310	-1
normalized size	1	1.00	0.56	0.59	0.74	0.64	0.00	3.20	-0.01
time (sec)	N/A	0.083	0.188	0.170	1.311	0.608	0.000	10.721	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	42	47	0	117	0	255	-1
normalized size	1	1.00	1.40	1.57	0.00	3.90	0.00	8.50	-0.03
time (sec)	N/A	0.057	0.109	0.162	0.000	0.645	0.000	5.637	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	44	41	55	40	0	172	-1
normalized size	1	1.00	0.62	0.58	0.77	0.56	0.00	2.42	-0.01
time (sec)	N/A	0.075	0.072	0.368	0.325	0.670	0.000	5.243	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	96	112	0	215	0	255	-1
normalized size	1	1.00	0.89	1.04	0.00	1.99	0.00	2.36	-0.01
time (sec)	N/A	0.144	0.184	0.234	0.000	0.682	0.000	3.119	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	29	42	41	267	39	-1
normalized size	1	1.00	0.67	0.64	0.93	0.91	5.93	0.87	-0.02
time (sec)	N/A	0.068	0.061	0.134	1.156	0.829	26.903	1.446	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	100	123	0	252	0	293	-1
normalized size	1	1.00	1.33	1.64	0.00	3.36	0.00	3.91	-0.01
time (sec)	N/A	0.082	0.246	0.212	0.000	0.649	0.000	1.632	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	48	65	20	72
normalized size	1	1.00	1.00	0.88	0.83	2.00	2.71	0.83	3.00
time (sec)	N/A	0.035	0.047	0.025	0.614	0.774	25.471	1.429	7.521

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	41	88	114	169	0	0	-1
normalized size	1	1.00	0.36	0.78	1.01	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.090	0.169	1.433	0.739	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	284	266	0	280	0	751	-1
normalized size	1	1.00	1.70	1.59	0.00	1.68	0.00	4.50	-0.01
time (sec)	N/A	0.230	0.457	0.236	0.000	0.715	0.000	5.626	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	42	141	167	225	0	916	-1
normalized size	1	1.00	0.23	0.76	0.90	1.22	0.00	4.95	-0.01
time (sec)	N/A	0.275	0.093	0.296	1.226	0.570	0.000	11.917	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	394	355	0	308	0	1074	-1
normalized size	1	1.00	1.69	1.52	0.00	1.32	0.00	4.61	-0.00
time (sec)	N/A	0.364	0.552	0.289	0.000	0.750	0.000	9.341	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	98	249	0	0	0	0	-1
normalized size	1	1.00	0.79	2.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.630	0.662	0.000	0.606	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	264	214	0	0	0	0	-1
normalized size	1	1.00	2.78	2.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	2.838	0.643	0.000	0.815	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	179	0	0	0	0	-1
normalized size	1	1.00	0.79	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.377	0.615	0.000	0.708	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	260	120	0	0	0	0	-1
normalized size	1	1.00	4.13	1.90	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	1.061	0.601	0.000	0.654	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	103	0	0	0	0	45
normalized size	1	1.00	0.79	1.69	0.00	0.00	0.00	0.00	0.74
time (sec)	N/A	0.046	22.316	0.384	0.000	0.529	0.000	0.000	0.546

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	188	117	0	0	0	0	-1
normalized size	1	1.00	2.07	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.979	0.779	0.000	0.667	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	86	189	0	0	0	0	-1
normalized size	1	1.00	0.89	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.461	0.883	0.000	0.592	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	144	304	0	0	0	0	-1
normalized size	1	1.00	1.14	2.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	1.275	1.402	0.000	0.699	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	66	295	0	0	0	0	-1
normalized size	1	1.00	0.39	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.109	0.750	0.000	0.829	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	66	260	0	0	0	0	-1
normalized size	1	1.00	0.48	1.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.118	0.864	0.000	0.910	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	66	203	0	0	0	0	-1
normalized size	1	1.00	0.48	1.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.079	0.747	0.000	0.604	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	66	188	0	0	0	0	-1
normalized size	1	1.00	0.63	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.046	0.789	0.000	0.758	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	64	152	0	0	0	0	-1
normalized size	1	1.00	0.61	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.035	0.571	0.000	0.758	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	120	0	0	0	0	-1
normalized size	1	1.00	0.75	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.065	0.813	0.000	0.596	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	66	193	0	0	0	0	-1
normalized size	1	1.00	0.74	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.068	1.041	0.000	0.890	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	66	305	0	0	0	0	-1
normalized size	1	1.00	0.52	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.086	1.380	0.000	0.581	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	66	375	0	0	0	0	-1
normalized size	1	1.00	0.58	3.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.115	1.591	0.000	0.692	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	66	488	0	0	0	0	-1
normalized size	1	1.00	0.46	3.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.156	2.438	0.000	0.693	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	66	321	0	0	0	0	-1
normalized size	1	1.00	0.33	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.078	0.941	0.000	0.625	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	66	264	0	0	0	0	-1
normalized size	1	1.00	0.39	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.107	1.029	0.000	0.711	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	66	251	0	0	0	0	-1
normalized size	1	1.00	0.38	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.073	1.265	0.000	0.754	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	66	214	0	0	0	0	-1
normalized size	1	1.00	0.47	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.046	0.847	0.000	0.729	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	64	178	0	0	0	0	-1
normalized size	1	1.00	0.47	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.036	0.767	0.000	0.633	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	64	146	0	0	0	0	-1
normalized size	1	1.00	0.60	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.057	1.182	0.000	0.590	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	219	0	0	0	0	-1
normalized size	1	1.00	0.60	1.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.054	1.266	0.000	0.638	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	66	332	0	0	0	0	-1
normalized size	1	1.00	0.52	2.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.088	1.884	0.000	0.753	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	66	401	0	0	0	0	-1
normalized size	1	1.00	0.52	3.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.087	2.002	0.000	0.807	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	66	514	0	0	0	0	-1
normalized size	1	1.00	0.40	3.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.127	2.684	0.000	0.545	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	66	295	0	0	0	0	-1
normalized size	1	1.00	0.31	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	0.113	0.938	0.000	0.777	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	66	258	0	0	0	0	-1
normalized size	1	1.00	0.37	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.065	0.982	0.000	0.862	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	64	222	0	0	0	0	-1
normalized size	1	1.00	0.36	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.058	0.863	0.000	0.771	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	64	190	0	0	0	0	-1
normalized size	1	1.00	0.41	1.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.073	1.146	0.000	0.787	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	66	263	0	0	0	0	-1
normalized size	1	1.00	0.43	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.078	1.282	0.000	0.714	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	66	332	0	0	0	0	-1
normalized size	1	1.00	0.52	2.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.101	1.795	0.000	0.709	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	66	401	0	0	0	0	-1
normalized size	1	1.00	0.52	3.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.110	2.001	0.000	0.885	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	66	514	0	0	0	0	-1
normalized size	1	1.00	0.39	3.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.140	2.935	0.000	0.584	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	66	583	0	0	0	0	-1
normalized size	1	1.00	0.39	3.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.259	0.244	3.241	0.000	0.713	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	66	251	0	0	0	0	-1
normalized size	1	1.00	0.50	1.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.143	0.888	0.000	0.699	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	66	216	0	0	0	0	-1
normalized size	1	1.00	0.65	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.098	0.793	0.000	0.881	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	66	181	0	0	0	0	-1
normalized size	1	1.00	0.65	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.073	1.086	0.000	0.517	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	122	0	0	0	0	-1
normalized size	1	1.00	0.97	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.099	0.711	0.000	0.718	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	110	0	0	0	0	-1
normalized size	1	1.00	1.00	1.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.076	0.517	0.000	0.711	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	115	0	0	0	0	-1
normalized size	1	1.00	0.89	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.042	1.143	0.000	0.679	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	190	0	0	0	0	-1
normalized size	1	1.00	0.82	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.039	1.479	0.000	0.781	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	63	304	0	0	0	0	-1
normalized size	1	1.00	0.56	2.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.058	2.003	0.000	0.706	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	66	375	0	0	0	0	-1
normalized size	1	1.00	0.59	3.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.076	2.248	0.000	0.878	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	66	488	0	0	0	0	-1
normalized size	1	1.00	0.46	3.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.102	2.719	0.000	0.689	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	66	203	0	0	0	0	-1
normalized size	1	1.00	0.46	1.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.178	0.921	0.000	0.849	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	66	190	0	0	0	0	-1
normalized size	1	1.00	0.58	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.101	1.001	0.000	0.717	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	66	155	0	0	0	0	-1
normalized size	1	1.00	0.59	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.075	0.755	0.000	0.586	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	120	0	0	0	0	-1
normalized size	1	1.00	0.84	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.095	1.152	0.000	0.848	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	66	193	0	0	0	0	-1
normalized size	1	1.00	0.80	2.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.077	1.395	0.000	0.624	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	66	303	0	0	0	0	-1
normalized size	1	1.00	0.57	2.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.042	2.172	0.000	0.720	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	372	0	0	0	0	-1
normalized size	1	1.00	0.55	3.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.043	2.245	0.000	0.773	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	66	488	0	0	0	0	-1
normalized size	1	1.00	0.44	3.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.073	3.078	0.000	0.741	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	66	557	0	0	0	0	-1
normalized size	1	1.00	0.44	3.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.076	3.652	0.000	0.776	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	66	670	0	0	0	0	-1
normalized size	1	1.00	0.36	3.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.112	4.586	0.000	0.593	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	66	251	0	0	0	0	-1
normalized size	1	1.00	0.39	1.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.395	1.191	0.000	0.749	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	66	216	0	0	0	0	-1
normalized size	1	1.00	0.48	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.237	1.104	0.000	0.983	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	66	181	0	0	0	0	-1
normalized size	1	1.00	0.50	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.164	0.927	0.000	0.687	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	66	146	0	0	0	0	-1
normalized size	1	1.00	0.64	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.104	1.262	0.000	0.638	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	66	219	0	0	0	0	-1
normalized size	1	1.00	0.62	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.081	1.369	0.000	0.734	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	66	330	0	0	0	0	-1
normalized size	1	1.00	0.56	2.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.081	2.142	0.000	0.661	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	66	401	0	0	0	0	-1
normalized size	1	1.00	0.56	3.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.076	2.316	0.000	0.549	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	66	512	0	0	0	0	-1
normalized size	1	1.00	0.43	3.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.043	3.409	0.000	0.658	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	66	580	0	0	0	0	-1
normalized size	1	1.00	0.43	3.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.049	3.991	0.000	0.762	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	66	696	0	0	0	0	-1
normalized size	1	1.00	0.35	3.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.064	4.732	0.000	0.624	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	66	225	0	0	0	0	-1
normalized size	1	1.00	0.37	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.322	1.081	0.000	0.785	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	66	190	0	0	0	0	-1
normalized size	1	1.00	0.44	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.189	1.653	0.000	0.667	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	66	263	0	0	0	0	-1
normalized size	1	1.00	0.46	1.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.165	1.683	0.000	0.689	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	66	332	0	0	0	0	-1
normalized size	1	1.00	0.55	2.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.094	2.471	0.000	0.722	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	66	401	0	0	0	0	-1
normalized size	1	1.00	0.55	3.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.081	2.692	0.000	0.600	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	66	514	0	0	0	0	-1
normalized size	1	1.00	0.43	3.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.088	3.403	0.000	0.603	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	66	583	0	0	0	0	-1
normalized size	1	1.00	0.43	3.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.071	4.367	0.000	0.862	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	66	694	0	0	0	0	-1
normalized size	1	1.00	0.35	3.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	0.046	5.337	0.000	0.658	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	66	762	0	0	0	0	-1
normalized size	1	1.00	0.35	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.058	5.291	0.000	0.722	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	66	878	0	0	0	0	-1
normalized size	1	1.00	0.29	3.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.088	6.760	0.000	0.722	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	269	241	0	0	0	0	-1
normalized size	1	1.00	1.14	1.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.357	0.955	0.350	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	195	213	0	0	0	0	-1
normalized size	1	1.00	1.01	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.270	0.740	0.271	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	108	142	0	0	0	0	-1
normalized size	1	1.00	0.67	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.472	0.190	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	34	131	38	0	0	30
normalized size	1	1.00	1.00	1.00	3.85	1.12	0.00	0.00	0.88
time (sec)	N/A	0.068	0.113	0.206	0.965	0.936	0.000	0.000	5.311

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	46	44	206	48	0	0	61
normalized size	1	1.00	0.62	0.59	2.78	0.65	0.00	0.00	0.82
time (sec)	N/A	0.146	0.250	0.211	0.970	0.886	0.000	0.000	5.666

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	56	54	282	58	0	0	97
normalized size	1	1.00	0.49	0.47	2.45	0.50	0.00	0.00	0.84
time (sec)	N/A	0.224	0.344	0.211	0.983	1.076	0.000	0.000	6.060

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	74	70	357	70	0	0	129
normalized size	1	1.00	0.48	0.45	2.32	0.45	0.00	0.00	0.84
time (sec)	N/A	0.307	0.796	0.228	0.983	0.755	0.000	0.000	7.243

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	78	314	0	0	0	0	-1
normalized size	1	1.00	0.24	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.564	0.168	0.340	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	78	288	0	0	0	0	-1
normalized size	1	1.00	0.28	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.475	0.132	0.326	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	77	262	0	0	0	0	-1
normalized size	1	1.00	0.32	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	0.121	0.298	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	75	228	0	0	0	0	-1
normalized size	1	1.00	0.38	1.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.284	0.106	0.261	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	75	323	0	0	0	0	-1
normalized size	1	1.00	0.36	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.113	0.209	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	131	45	0	0	47
normalized size	1	1.00	1.00	0.94	3.64	1.25	0.00	0.00	1.31
time (sec)	N/A	0.073	0.096	0.178	1.745	0.843	0.000	0.000	5.622

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	44	207	69	0	0	71
normalized size	1	1.00	0.97	0.59	2.80	0.93	0.00	0.00	0.96
time (sec)	N/A	0.148	0.135	0.193	0.968	0.719	0.000	0.000	5.985

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	105	54	281	84	0	0	116
normalized size	1	1.00	0.93	0.48	2.49	0.74	0.00	0.00	1.03
time (sec)	N/A	0.230	0.202	0.201	1.129	0.659	0.000	0.000	6.810

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	74	70	357	98	0	0	261
normalized size	1	1.00	0.49	0.46	2.35	0.64	0.00	0.00	1.72
time (sec)	N/A	0.309	0.237	0.203	1.014	0.802	0.000	0.000	10.956

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	77	344	0	0	0	0	-1
normalized size	1	1.00	0.24	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.543	0.297	0.326	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	78	318	0	0	0	0	-1
normalized size	1	1.00	0.27	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	0.119	0.338	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	76	284	0	0	0	0	-1
normalized size	1	1.00	0.31	1.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.102	0.299	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	75	445	0	0	0	0	-1
normalized size	1	1.00	0.31	1.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	0.165	0.250	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	77	545	0	0	0	0	-1
normalized size	1	1.00	0.38	2.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	0.190	0.242	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	131	107	0	0	65
normalized size	1	1.00	1.00	0.94	3.64	2.97	0.00	0.00	1.81
time (sec)	N/A	0.075	0.144	0.177	1.151	1.215	0.000	0.000	6.058

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	54	44	207	75	0	0	96
normalized size	1	1.00	0.71	0.58	2.72	0.99	0.00	0.00	1.26
time (sec)	N/A	0.148	0.172	0.205	1.002	1.162	0.000	0.000	6.336

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	64	54	282	100	0	0	119
normalized size	1	1.00	0.57	0.48	2.50	0.88	0.00	0.00	1.05
time (sec)	N/A	0.223	0.232	0.199	1.009	0.992	0.000	0.000	6.694

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	74	70	357	120	0	0	232
normalized size	1	1.00	0.49	0.47	2.38	0.80	0.00	0.00	1.55
time (sec)	N/A	0.305	0.236	0.211	1.125	1.278	0.000	0.000	11.173

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	77	239	0	0	0	0	-1
normalized size	1	1.00	0.32	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	0.163	0.282	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	77	212	0	0	0	0	-1
normalized size	1	1.00	0.38	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	0.107	0.237	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	77	141	0	0	0	0	-1
normalized size	1	1.00	0.46	0.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.079	0.182	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	34	130	41	0	0	46
normalized size	1	1.00	1.00	1.00	3.82	1.21	0.00	0.00	1.35
time (sec)	N/A	0.060	0.066	0.174	0.808	0.820	0.000	0.000	5.662

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	46	44	210	67	0	0	77
normalized size	1	1.00	0.61	0.58	2.76	0.88	0.00	0.00	1.01
time (sec)	N/A	0.133	0.106	0.177	1.033	0.736	0.000	0.000	6.007

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	56	54	287	81	0	0	120
normalized size	1	1.00	0.49	0.47	2.50	0.70	0.00	0.00	1.04
time (sec)	N/A	0.209	0.090	0.196	1.021	0.576	0.000	0.000	6.680

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	66	70	363	93	0	0	261
normalized size	1	1.00	0.43	0.45	2.36	0.60	0.00	0.00	1.69
time (sec)	N/A	0.288	0.170	0.191	0.870	0.896	0.000	0.000	11.008

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	80	266	0	0	0	0	-1
normalized size	1	1.00	0.32	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	0.130	0.272	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	80	232	0	0	0	0	-1
normalized size	1	1.00	0.37	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.147	0.240	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	80	321	0	0	0	0	-1
normalized size	1	1.00	0.34	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.362	0.125	0.199	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	49	34	131	100	0	0	82
normalized size	1	1.00	1.36	0.94	3.64	2.78	0.00	0.00	2.28
time (sec)	N/A	0.064	0.069	0.187	0.833	0.989	0.000	0.000	5.992

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	59	44	211	71	0	0	95
normalized size	1	1.00	0.78	0.58	2.78	0.93	0.00	0.00	1.25
time (sec)	N/A	0.130	0.112	0.186	1.095	0.916	0.000	0.000	6.379

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	56	54	294	99	0	0	119
normalized size	1	1.00	0.49	0.47	2.56	0.86	0.00	0.00	1.03
time (sec)	N/A	0.210	0.102	0.173	0.894	1.041	0.000	0.000	6.823

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	66	70	373	115	0	0	230
normalized size	1	1.00	0.43	0.45	2.42	0.75	0.00	0.00	1.49
time (sec)	N/A	0.289	0.171	0.184	0.575	0.980	0.000	0.000	11.102

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	76	80	451	125	0	0	413
normalized size	1	1.00	0.39	0.41	2.34	0.65	0.00	0.00	2.14
time (sec)	N/A	0.372	0.300	0.215	1.036	0.517	0.000	0.000	11.651

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	80	282	0	0	0	0	-1
normalized size	1	1.00	0.31	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.469	0.176	0.285	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	80	443	0	0	0	0	-1
normalized size	1	1.00	0.33	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	0.122	0.234	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	80	545	0	0	0	0	-1
normalized size	1	1.00	0.37	2.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	0.138	0.224	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	49	34	131	70	0	0	102
normalized size	1	1.00	1.36	0.94	3.64	1.94	0.00	0.00	2.83
time (sec)	N/A	0.071	0.119	0.191	0.955	0.667	0.000	0.000	6.567

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	59	44	207	148	0	0	145
normalized size	1	1.00	0.78	0.58	2.72	1.95	0.00	0.00	1.91
time (sec)	N/A	0.131	0.097	0.200	0.698	0.635	0.000	0.000	7.047

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	69	54	287	98	0	0	137
normalized size	1	1.00	0.60	0.47	2.50	0.85	0.00	0.00	1.19
time (sec)	N/A	0.202	0.155	0.194	0.528	0.883	0.000	0.000	7.662

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	66	70	373	130	0	0	261
normalized size	1	1.00	0.43	0.45	2.42	0.84	0.00	0.00	1.69
time (sec)	N/A	0.297	0.152	0.189	0.540	0.538	0.000	0.000	11.167

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	76	80	451	144	0	0	379
normalized size	1	1.00	0.39	0.41	2.34	0.75	0.00	0.00	1.96
time (sec)	N/A	0.375	0.275	0.199	0.792	0.466	0.000	0.000	11.537

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.157	0.236	0.000	0.499	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.113	0.221	0.000	0.547	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.078	0.221	0.000	0.529	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.085	0.225	0.000	0.455	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.080	0.210	0.000	0.513	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.093	0.204	0.000	0.517	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.198	14.571	0.000	0.458	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.110	5.356	0.000	0.436	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.106	4.298	0.000	0.469	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	245	0	0	0	0	0	-1
normalized size	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	1.397	1.487	0.000	0.497	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.161	0.290	0.000	0.449	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.175	0.693	0.000	0.466	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.164	0.658	0.000	0.482	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.207	3.306	0.000	0.497	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	102	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.224	0.210	0.000	0.469	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	102	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.184	0.198	0.000	0.452	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	101	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.187	0.210	0.000	0.439	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	310	0	0	0	0	0	-1
normalized size	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	3.886	0.202	0.000	0.443	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.103	0.170	0.000	0.444	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.170	0.168	0.000	0.444	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	102	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.136	0.167	0.000	0.479	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	112	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.198	1.345	0.000	0.480	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	89	0	520	153	0	0	555
normalized size	1	1.00	0.82	0.00	4.77	1.40	0.00	0.00	5.09
time (sec)	N/A	0.086	0.702	6.771	0.710	0.493	0.000	0.000	10.480

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	68	0	266	102	5534	0	195
normalized size	1	1.00	0.84	0.00	3.28	1.26	68.32	0.00	2.41
time (sec)	N/A	0.069	0.320	3.224	0.816	0.467	172.269	0.000	1.971

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	0	111	61	1114	152	85
normalized size	1	1.00	0.95	0.00	2.02	1.11	20.25	2.76	1.55
time (sec)	N/A	0.058	0.118	1.738	0.934	0.577	21.470	1.471	0.726

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	28	80	26	29
normalized size	1	1.00	1.00	1.04	1.00	1.08	3.08	1.00	1.12
time (sec)	N/A	0.028	0.037	0.024	0.541	0.459	2.472	0.921	0.215

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	63	0	0	0	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.101	0.921	0.000	0.465	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	111	0	0	0	0	0	-1
normalized size	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.356	0.203	0.000	0.437	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	163	0	0	0	0	0	-1
normalized size	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.736	0.231	0.000	0.460	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.125	2.444	0.000	0.455	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.100	1.342	0.000	0.444	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	3917	0	0	0	0	0	-1
normalized size	1	1.00	53.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	16.116	0.156	0.000	0.474	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	9400	0	0	0	0	0	-1
normalized size	1	1.00	113.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	21.277	0.196	0.000	0.490	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.205	0.204	0.000	0.458	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.154	0.196	0.000	0.494	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.084	0.219	0.000	0.447	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	83	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.078	0.189	0.000	0.471	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.099	0.181	0.000	0.456	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.101	0.186	0.000	0.450	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	101	0	0	104	0	0	137
normalized size	1	1.00	0.50	0.00	0.00	0.52	0.00	0.00	0.68
time (sec)	N/A	0.321	0.193	0.344	0.000	0.476	0.000	0.000	6.828

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	76	0	0	75	0	0	103
normalized size	1	1.00	0.54	0.00	0.00	0.53	0.00	0.00	0.73
time (sec)	N/A	0.222	0.127	0.309	0.000	0.464	0.000	0.000	6.126

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	53	0	0	61	0	0	71
normalized size	1	1.00	0.60	0.00	0.00	0.69	0.00	0.00	0.80
time (sec)	N/A	0.124	0.116	0.320	0.000	0.469	0.000	0.000	5.605

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	65	39	0	0	34
normalized size	1	1.00	1.00	0.00	1.91	1.15	0.00	0.00	1.00
time (sec)	N/A	0.051	0.051	0.303	0.459	0.457	0.000	0.000	0.288

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	108	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.120	0.565	0.000	0.466	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.256	0.308	0.000	0.477	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	113	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.239	0.314	0.000	0.479	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	105	0	624	314	0	0	601
normalized size	1	1.00	0.70	0.00	4.16	2.09	0.00	0.00	4.01
time (sec)	N/A	0.240	0.387	1.678	1.386	0.481	0.000	0.000	13.477

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	72	0	351	170	0	0	241
normalized size	1	1.00	0.77	0.00	3.73	1.81	0.00	0.00	2.56
time (sec)	N/A	0.141	0.221	1.573	1.381	0.455	0.000	0.000	8.768

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	144	80	0	0	58
normalized size	1	1.00	0.98	0.00	3.27	1.82	0.00	0.00	1.32
time (sec)	N/A	0.055	0.150	2.197	0.954	0.486	0.000	0.000	5.582

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	0.062	1.332	0.000	0.472	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	76	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.132	1.368	0.000	0.494	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	96	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.204	1.843	0.000	0.472	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	96	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.195	1.711	0.000	0.473	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	90	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.083	0.753	0.000	0.468	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.218	1.287	0.000	0.474	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	46	70	51	83	88	68
normalized size	1	1.00	1.00	0.77	1.17	0.85	1.38	1.47	1.13
time (sec)	N/A	0.042	0.021	0.146	0.339	0.459	4.424	0.920	0.065

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	48	39	60	48	46
normalized size	1	1.00	1.00	0.82	1.09	0.89	1.36	1.09	1.05
time (sec)	N/A	0.032	0.013	0.143	0.316	0.471	1.225	0.444	0.057

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	39	25	20	25	34	25	23
normalized size	1	1.27	1.77	1.14	0.91	1.14	1.55	1.14	1.05
time (sec)	N/A	0.016	0.011	0.053	0.354	0.450	0.245	0.404	0.042

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	26	34	35	37	0	37	54
normalized size	1	1.00	0.60	0.79	0.81	0.86	0.00	0.86	1.26
time (sec)	N/A	0.040	0.011	0.099	0.356	0.456	0.000	0.746	0.072

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	52	54	53	67	0	55	44
normalized size	1	1.00	1.27	1.32	1.29	1.63	0.00	1.34	1.07
time (sec)	N/A	0.038	0.020	0.166	0.324	0.483	0.000	0.661	0.075

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	68	74	78	82	0	70	64
normalized size	1	1.00	1.11	1.21	1.28	1.34	0.00	1.15	1.05
time (sec)	N/A	0.044	0.149	0.168	0.328	0.459	0.000	0.579	5.145

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	48	51	124	77	111
normalized size	1	1.00	0.95	0.80	0.74	0.78	1.91	1.18	1.71
time (sec)	N/A	0.045	0.106	0.149	0.371	0.464	2.339	2.015	8.694

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	37	37	71	47	68
normalized size	1	1.00	1.07	0.95	0.86	0.86	1.65	1.09	1.58
time (sec)	N/A	0.033	0.061	0.087	0.333	0.431	0.643	0.347	7.381

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	22	0	33	22
normalized size	1	1.00	1.00	1.04	1.00	0.96	0.00	1.43	0.96
time (sec)	N/A	0.031	0.011	0.150	0.331	0.432	0.000	0.821	5.132

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	35	35	0	76	42
normalized size	1	1.00	0.93	0.86	0.80	0.80	0.00	1.73	0.95
time (sec)	N/A	0.036	0.073	0.159	0.329	0.433	0.000	0.413	5.265

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	48	48	50	0	120	75
normalized size	1	1.00	0.88	0.80	0.80	0.83	0.00	2.00	1.25
time (sec)	N/A	0.040	0.175	0.160	0.329	0.437	0.000	1.274	5.299

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	104	98	106	87	158	136	104
normalized size	1	1.00	1.05	0.99	1.07	0.88	1.60	1.37	1.05
time (sec)	N/A	0.089	0.205	0.202	0.321	0.482	7.448	0.748	0.074

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	78	73	69	107	80	74
normalized size	1	1.00	0.73	1.01	0.95	0.90	1.39	1.04	0.96
time (sec)	N/A	0.071	0.119	0.200	0.321	0.485	3.163	0.631	0.050

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	46	21	20	48	53	20	39
normalized size	1	1.00	2.09	0.95	0.91	2.18	2.41	0.91	1.77
time (sec)	N/A	0.027	0.013	0.085	0.330	0.490	0.770	0.663	0.058

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	60	62	0	62	50
normalized size	1	1.00	0.89	1.18	0.98	1.02	0.00	1.02	0.82
time (sec)	N/A	0.082	0.063	0.145	0.329	0.493	0.000	0.509	5.156

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	113	118	78	90	0	86	62
normalized size	1	1.00	1.92	2.00	1.32	1.53	0.00	1.46	1.05
time (sec)	N/A	0.061	0.923	0.254	0.325	0.473	0.000	1.152	0.107

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	166	165	115	118	0	118	93
normalized size	1	1.00	1.68	1.67	1.16	1.19	0.00	1.19	0.94
time (sec)	N/A	0.088	0.737	0.253	0.320	0.479	0.000	0.973	5.098

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	141	128	114	108	398	162	178
normalized size	1	1.00	0.97	0.88	0.78	0.74	2.73	1.11	1.22
time (sec)	N/A	0.134	0.345	0.227	0.328	0.485	13.093	0.620	5.466

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	133	108	88	89	287	123	134
normalized size	1	1.00	1.15	0.93	0.76	0.77	2.47	1.06	1.16
time (sec)	N/A	0.116	0.195	0.217	0.317	0.463	4.686	0.935	5.307

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	85	86	64	70	180	76	71
normalized size	1	1.00	0.99	1.00	0.74	0.81	2.09	0.88	0.83
time (sec)	N/A	0.095	0.231	0.153	0.334	0.448	1.410	1.007	5.380

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	55	46	46	45	0	63	53
normalized size	1	1.00	1.12	0.94	0.94	0.92	0.00	1.29	1.08
time (sec)	N/A	0.048	0.058	0.191	0.480	0.459	0.000	0.780	5.207

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	105	62	51	52	0	102	71
normalized size	1	1.00	1.40	0.83	0.68	0.69	0.00	1.36	0.95
time (sec)	N/A	0.096	0.327	0.283	0.330	0.430	0.000	0.510	5.258

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	84	92	76	77	0	181	103
normalized size	1	1.00	0.82	0.89	0.74	0.75	0.00	1.76	1.00
time (sec)	N/A	0.101	0.435	0.250	0.372	0.458	0.000	0.544	5.364

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	120	97	99	0	260	135
normalized size	1	1.00	0.85	0.93	0.75	0.77	0.00	2.02	1.05
time (sec)	N/A	0.123	0.811	0.296	0.322	0.448	0.000	0.405	5.547

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	120	135	144	117	202	185	141
normalized size	1	1.00	0.83	0.94	1.00	0.81	1.40	1.28	0.98
time (sec)	N/A	0.133	0.516	0.233	0.336	0.487	12.917	0.996	0.092

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	115	100	95	151	112	98
normalized size	1	1.00	0.73	1.49	1.30	1.23	1.96	1.45	1.27
time (sec)	N/A	0.080	0.141	0.230	0.322	0.445	4.868	0.499	5.128

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	57	21	20	71	73	20	55
normalized size	1	1.00	2.59	0.95	0.91	3.23	3.32	0.91	2.50
time (sec)	N/A	0.026	0.064	0.096	0.314	0.434	1.274	0.474	0.060

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	67	108	91	93	0	93	65
normalized size	1	1.00	0.84	1.35	1.14	1.16	0.00	1.16	0.81
time (sec)	N/A	0.105	0.118	0.184	0.330	0.477	0.000	0.527	5.129

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	176	154	98	112	0	114	99
normalized size	1	1.00	1.59	1.39	0.88	1.01	0.00	1.03	0.89
time (sec)	N/A	0.134	1.336	0.285	0.331	0.475	0.000	1.135	5.211

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	318	195	136	138	0	139	114
normalized size	1	1.00	3.38	2.07	1.45	1.47	0.00	1.48	1.21
time (sec)	N/A	0.080	4.037	0.268	0.324	0.482	0.000	0.789	5.166

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	182	145	117	117	348	173	474
normalized size	1	1.00	1.15	0.92	0.74	0.74	2.20	1.09	3.00
time (sec)	N/A	0.217	0.379	0.277	0.416	0.473	8.674	1.262	6.926

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	107	123	93	98	236	113	356
normalized size	1	1.00	0.82	0.94	0.71	0.75	1.80	0.86	2.72
time (sec)	N/A	0.194	0.514	0.205	1.017	0.498	3.157	0.622	6.627

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	68	89	70	70	0	123	103
normalized size	1	1.00	0.86	1.13	0.89	0.89	0.00	1.56	1.30
time (sec)	N/A	0.070	0.314	0.396	0.513	0.455	0.000	0.735	5.814

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	136	122	80	77	0	128	81
normalized size	1	1.00	1.62	1.45	0.95	0.92	0.00	1.52	0.96
time (sec)	N/A	0.088	0.422	0.404	0.333	0.466	0.000	1.921	5.249

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	190	173	105	101	0	243	119
normalized size	1	1.00	1.41	1.28	0.78	0.75	0.00	1.80	0.88
time (sec)	N/A	0.192	0.540	0.318	0.335	0.434	0.000	1.414	5.407

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	245	219	124	124	0	358	152
normalized size	1	1.00	1.48	1.33	0.75	0.75	0.00	2.17	0.92
time (sec)	N/A	0.207	0.897	0.366	0.335	0.453	0.000	1.884	5.604

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	299	265	145	146	0	473	275
normalized size	1	1.00	1.56	1.38	0.76	0.76	0.00	2.46	1.43
time (sec)	N/A	0.221	1.520	0.383	0.337	0.459	0.000	1.596	6.132

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	120	530	311	356	614	464	306
normalized size	1	1.00	0.83	3.68	2.16	2.47	4.26	3.22	2.12
time (sec)	N/A	0.221	1.991	0.266	0.328	0.563	119.111	3.939	5.454

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	480	233	310	468	272	231
normalized size	1	1.00	0.73	6.23	3.03	4.03	6.08	3.53	3.00
time (sec)	N/A	0.151	0.847	0.257	0.326	0.520	54.458	1.971	5.374

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	137	21	20	257	168	20	135
normalized size	1	1.00	6.23	0.95	0.91	11.68	7.64	0.91	6.14
time (sec)	N/A	0.026	0.361	0.109	0.341	0.508	20.961	2.932	5.266

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	227	465	317	327	0	378	212
normalized size	1	1.00	0.93	1.90	1.29	1.33	0.00	1.54	0.87
time (sec)	N/A	0.182	0.214	0.232	0.326	0.524	0.000	1.992	5.362

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	366	645	323	368	0	408	257
normalized size	1	1.00	1.29	2.27	1.14	1.30	0.00	1.44	0.90
time (sec)	N/A	0.242	2.347	0.334	0.332	0.542	0.000	0.596	5.389

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	514	760	348	366	0	429	305
normalized size	1	1.00	1.61	2.38	1.09	1.14	0.00	1.34	0.95
time (sec)	N/A	0.304	4.118	0.347	0.328	0.539	0.000	0.998	5.484

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	457	497	336	315	1115	364	467
normalized size	1	1.00	1.08	1.17	0.79	0.74	2.64	0.86	1.10
time (sec)	N/A	1.217	1.012	0.201	0.338	0.521	39.897	3.190	7.344

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	313	406	348	266	0	799	767
normalized size	1	1.00	0.90	1.16	1.00	0.76	0.00	2.29	2.20
time (sec)	N/A	0.563	1.089	0.422	0.432	0.516	0.000	0.783	7.734

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	414	495	328	268	0	684	726
normalized size	1	1.00	1.12	1.34	0.89	0.73	0.00	1.85	1.97
time (sec)	N/A	0.646	1.111	0.457	0.431	0.498	0.000	0.759	7.823

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	472	544	315	281	0	663	665
normalized size	1	1.00	1.24	1.43	0.83	0.74	0.00	1.74	1.75
time (sec)	N/A	0.724	1.334	0.523	0.443	0.502	0.000	1.188	7.598

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	479	567	310	306	0	726	546
normalized size	1	1.00	1.19	1.40	0.77	0.76	0.00	1.80	1.35
time (sec)	N/A	0.821	1.435	0.410	0.432	0.528	0.000	0.697	8.853

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	313	662	315	336	0	892	659
normalized size	1	1.00	1.33	2.81	1.33	1.42	0.00	3.78	2.79
time (sec)	N/A	0.385	4.460	0.395	0.334	0.513	0.000	1.922	6.680

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	103	163	108	107	0	120	109
normalized size	1	1.00	0.87	1.38	0.92	0.91	0.00	1.02	0.92
time (sec)	N/A	0.105	0.226	0.147	0.311	0.486	0.000	1.089	5.072

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	55	53	0	56	55
normalized size	1	1.00	0.89	1.18	0.90	0.87	0.00	0.92	0.90
time (sec)	N/A	0.066	0.065	0.144	0.310	0.474	0.000	0.796	0.077

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	18	41	19	18
normalized size	1	1.00	1.00	1.06	1.00	1.00	2.28	1.06	1.00
time (sec)	N/A	0.027	0.006	0.085	0.328	0.466	1.056	0.374	5.085

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	64	76	64	62	0	71	69
normalized size	1	1.00	0.85	1.01	0.85	0.83	0.00	0.95	0.92
time (sec)	N/A	0.083	0.056	0.149	0.329	0.467	0.000	0.409	5.133

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	170	164	139	153	0	177	148
normalized size	1	1.00	1.38	1.33	1.13	1.24	0.00	1.44	1.20
time (sec)	N/A	0.164	0.583	0.181	0.333	0.577	0.000	0.428	5.388

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	266	305	278	253	0	332	322
normalized size	1	1.00	1.36	1.56	1.43	1.30	0.00	1.70	1.65
time (sec)	N/A	0.255	0.948	0.179	0.337	0.641	0.000	0.472	0.586

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	2827	1055	0	483	0	496	3075
normalized size	1	1.00	15.04	5.61	0.00	2.57	0.00	2.64	16.36
time (sec)	N/A	0.462	6.306	0.164	0.000	0.504	0.000	0.490	7.655

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	428	450	0	332	0	226	364
normalized size	1	1.00	3.37	3.54	0.00	2.61	0.00	1.78	2.87
time (sec)	N/A	0.252	4.510	0.148	0.000	0.503	0.000	2.808	6.116

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	361	142	0	214	0	95	318
normalized size	1	1.00	5.16	2.03	0.00	3.06	0.00	1.36	4.54
time (sec)	N/A	0.114	1.411	0.145	0.000	0.496	0.000	0.966	5.373

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	152	117	0	305	0	107	149
normalized size	1	1.00	1.81	1.39	0.00	3.63	0.00	1.27	1.77
time (sec)	N/A	0.094	0.323	0.157	0.000	0.500	0.000	1.168	5.255

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	202	270	0	466	0	273	387
normalized size	1	1.00	1.47	1.97	0.00	3.40	0.00	1.99	2.82
time (sec)	N/A	0.251	1.340	0.188	0.000	0.506	0.000	0.527	7.946

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	370	525	0	666	0	584	774
normalized size	1	1.00	1.88	2.66	0.00	3.38	0.00	2.96	3.93
time (sec)	N/A	0.495	2.527	0.184	0.000	0.519	0.000	1.226	8.061

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	235	305	190	243	0	251	259
normalized size	1	1.00	1.28	1.66	1.03	1.32	0.00	1.36	1.41
time (sec)	N/A	0.173	0.476	0.246	0.315	0.542	0.000	0.977	0.121

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	127	174	116	156	0	150	118
normalized size	1	1.00	1.06	1.45	0.97	1.30	0.00	1.25	0.98
time (sec)	N/A	0.101	0.631	0.252	0.315	0.489	0.000	0.840	0.083

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	78	61	78	221	91	69
normalized size	1	1.00	0.83	1.24	0.97	1.24	3.51	1.44	1.10
time (sec)	N/A	0.065	0.035	0.239	0.311	0.464	1.882	1.693	0.085

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	20	51	20	20
normalized size	1	1.00	1.00	1.05	1.00	1.00	2.55	1.00	1.00
time (sec)	N/A	0.027	0.024	0.127	0.308	0.445	1.244	0.362	5.065

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	101	118	188	0	147	98
normalized size	1	1.00	0.98	0.97	1.13	1.81	0.00	1.41	0.94
time (sec)	N/A	0.115	0.211	0.265	0.317	0.536	0.000	1.285	0.210

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	222	192	275	381	0	244	227
normalized size	1	1.00	1.25	1.08	1.55	2.15	0.00	1.38	1.28
time (sec)	N/A	0.209	1.755	0.325	0.342	0.576	0.000	0.992	5.474

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	406	331	505	527	0	460	449
normalized size	1	1.00	1.51	1.23	1.88	1.96	0.00	1.71	1.67
time (sec)	N/A	0.321	6.109	0.309	0.338	0.766	0.000	0.437	5.944

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	3679	1021	0	599	0	469	2530
normalized size	1	1.00	19.67	5.46	0.00	3.20	0.00	2.51	13.53
time (sec)	N/A	0.371	6.523	0.254	0.000	0.539	0.000	2.499	7.583

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	448	385	0	411	0	235	601
normalized size	1	1.00	3.50	3.01	0.00	3.21	0.00	1.84	4.70
time (sec)	N/A	0.213	5.112	0.243	0.000	0.481	0.000	1.396	6.397

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	414	153	0	388	0	126	329
normalized size	1	1.00	4.93	1.82	0.00	4.62	0.00	1.50	3.92
time (sec)	N/A	0.110	2.692	0.231	0.000	0.502	0.000	1.376	5.559

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	162	222	0	538	0	271	303
normalized size	1	1.00	1.25	1.71	0.00	4.14	0.00	2.08	2.33
time (sec)	N/A	0.208	1.150	0.225	0.000	0.504	0.000	2.550	7.396

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	336	370	0	782	0	427	727
normalized size	1	1.00	1.74	1.92	0.00	4.05	0.00	2.21	3.77
time (sec)	N/A	0.367	1.863	0.318	0.000	0.526	0.000	0.667	8.511

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	282	320	200	304	0	245	234
normalized size	1	1.00	1.48	1.68	1.05	1.60	0.00	1.29	1.23
time (sec)	N/A	0.159	0.640	0.285	0.328	0.546	0.000	0.473	0.122

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	143	183	131	212	0	142	142
normalized size	1	1.00	1.13	1.44	1.03	1.67	0.00	1.12	1.12
time (sec)	N/A	0.105	0.984	0.289	0.317	0.481	0.000	0.514	5.136

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	85	76	110	398	62	80
normalized size	1	1.00	0.76	1.18	1.06	1.53	5.53	0.86	1.11
time (sec)	N/A	0.072	0.118	0.260	0.318	0.448	2.392	1.017	0.092

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	43	73	20	39
normalized size	1	1.00	1.00	0.95	0.91	1.95	3.32	0.91	1.77
time (sec)	N/A	0.026	0.027	0.110	0.311	0.420	1.971	0.408	0.058

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	135	166	223	462	0	242	169
normalized size	1	1.00	0.93	1.14	1.54	3.19	0.00	1.67	1.17
time (sec)	N/A	0.154	0.589	0.290	0.340	0.531	0.000	0.692	5.400

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	283	258	438	707	0	413	388
normalized size	1	1.00	1.25	1.14	1.94	3.13	0.00	1.83	1.72
time (sec)	N/A	0.277	3.952	0.324	0.356	0.743	0.000	1.376	5.776

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	388	398	725	895	0	575	688
normalized size	1	1.00	1.18	1.21	2.21	2.73	0.00	1.75	2.10
time (sec)	N/A	0.420	2.735	0.336	0.373	1.442	0.000	0.725	6.565

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	3889	1060	0	752	0	457	1226
normalized size	1	1.00	19.74	5.38	0.00	3.82	0.00	2.32	6.22
time (sec)	N/A	0.367	6.572	0.308	0.000	0.877	0.000	0.699	8.583

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	2641	560	0	716	0	272	1360
normalized size	1	1.00	19.00	4.03	0.00	5.15	0.00	1.96	9.78
time (sec)	N/A	0.208	6.275	0.306	0.000	0.776	0.000	0.421	7.550

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	93	443	0	501	0	207	282
normalized size	1	1.00	0.81	3.85	0.00	4.36	0.00	1.80	2.45
time (sec)	N/A	0.128	0.254	0.298	0.000	0.641	0.000	0.547	7.369

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	193	705	0	894	0	385	650
normalized size	1	1.00	1.01	3.67	0.00	4.66	0.00	2.01	3.39
time (sec)	N/A	0.391	3.060	0.276	0.000	0.738	0.000	4.780	8.820

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	380	854	0	1200	0	622	1167
normalized size	1	1.00	1.44	3.23	0.00	4.55	0.00	2.36	4.42
time (sec)	N/A	0.640	2.809	0.338	0.000	0.878	0.000	9.089	9.369

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	171	208	279	382	2530	215	276
normalized size	1	1.00	0.83	1.00	1.35	1.85	12.22	1.04	1.33
time (sec)	N/A	0.171	1.144	0.357	0.330	0.840	47.443	3.884	0.242

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	107	127	206	309	1425	117	206
normalized size	1	1.00	0.76	0.90	1.46	2.19	10.11	0.83	1.46
time (sec)	N/A	0.109	0.272	0.342	0.331	0.732	43.300	6.194	0.136

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	54	67	151	254	636	52	152
normalized size	1	1.00	0.70	0.87	1.96	3.30	8.26	0.68	1.97
time (sec)	N/A	0.072	0.190	0.335	1.518	0.711	41.333	4.464	5.217

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	218	167	20	119
normalized size	1	1.00	1.00	0.95	0.91	9.91	7.59	0.91	5.41
time (sec)	N/A	0.027	0.080	0.138	0.309	0.765	40.650	2.567	5.203

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	365	699	1160	3165	0	1010	937
normalized size	1	1.00	0.95	1.82	3.01	8.22	0.00	2.62	2.43
time (sec)	N/A	0.528	2.688	0.438	0.396	4.678	0.000	2.068	7.635

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	770	804	1670	3678	0	1327	1443
normalized size	1	1.00	1.46	1.53	3.17	6.98	0.00	2.52	2.74
time (sec)	N/A	0.736	6.748	0.481	0.438	8.070	0.000	4.381	9.887

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	6570	9454	0	3721	0	2326	9647
normalized size	1	1.00	13.38	19.25	0.00	7.58	0.00	4.74	19.65
time (sec)	N/A	1.274	8.505	0.424	0.000	1.872	0.000	9.879	32.217

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	386	6933	0	2250	0	1650	1868
normalized size	1	1.00	0.95	17.03	0.00	5.53	0.00	4.05	4.59
time (sec)	N/A	0.792	6.001	0.408	0.000	1.440	0.000	9.949	12.465

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	1167	9171	0	2657	0	1932	2184
normalized size	1	1.00	2.84	22.31	0.00	6.46	0.00	4.70	5.31
time (sec)	N/A	0.791	6.077	0.391	0.000	1.381	0.000	5.093	10.854

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	1896	11250	0	2972	0	2207	2440
normalized size	1	1.00	4.49	26.66	0.00	7.04	0.00	5.23	5.78
time (sec)	N/A	0.746	6.204	0.407	0.000	1.523	0.000	3.819	10.339

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	494	7675	0	3882	0	2610	3273
normalized size	1	1.00	0.93	14.51	0.00	7.34	0.00	4.93	6.19
time (sec)	N/A	1.765	5.248	0.385	0.000	1.710	0.000	7.973	53.316

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	597	7823	0	4500	0	3047	-1
normalized size	1	1.00	0.91	11.98	0.00	6.89	0.00	4.67	-0.00
time (sec)	N/A	2.137	5.995	0.587	0.000	2.232	0.000	20.509	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	117	126	116	142	0	0	-1
normalized size	1	1.00	0.76	0.82	0.75	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.344	0.605	0.321	0.777	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	55	61	78	0	78	-1
normalized size	1	1.00	0.70	0.66	0.73	0.94	0.00	0.94	-0.01
time (sec)	N/A	0.082	0.128	0.376	0.323	0.916	0.000	2.172	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	20	83	20	20
normalized size	1	1.00	1.00	0.88	0.83	0.83	3.46	0.83	0.83
time (sec)	N/A	0.036	0.016	0.043	0.332	1.022	0.553	1.447	5.204

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	63	0	1729	0	0	-1
normalized size	1	1.00	1.00	0.85	0.00	23.36	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.052	0.389	0.000	1.190	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	143	185	0	2101	0	0	-1
normalized size	1	1.00	1.15	1.49	0.00	16.94	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.706	0.609	0.000	1.320	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	224	509	0	0	0	0	-1
normalized size	1	1.00	1.08	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	1.525	0.880	0.000	1.613	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	233	1189	0	0	0	0	-1
normalized size	1	1.00	0.78	3.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.565	0.911	0.690	0.000	1.044	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	185	792	0	0	0	0	-1
normalized size	1	1.00	0.86	3.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	0.828	0.715	0.000	1.007	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	127	614	0	0	0	0	-1
normalized size	1	1.00	0.85	4.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	2.812	1.007	0.000	1.139	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	270	1259	0	0	0	0	-1
normalized size	1	1.00	1.09	5.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.371	3.362	0.846	0.000	0.764	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	131	126	116	184	0	0	-1
normalized size	1	1.00	0.85	0.82	0.75	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.718	0.696	0.345	0.785	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	55	61	111	314	0	-1
normalized size	1	1.00	0.70	0.66	0.73	1.34	3.78	0.00	-0.01
time (sec)	N/A	0.091	0.180	0.327	0.319	1.012	121.573	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	53	116	0	20
normalized size	1	1.00	1.00	0.88	0.83	2.21	4.83	0.00	0.83
time (sec)	N/A	0.039	0.022	0.045	0.323	0.936	26.321	0.000	5.410

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	89	218	0	0	0	0	-1
normalized size	1	1.00	0.95	2.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.095	0.477	0.000	1.898	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	121	279	0	1969	0	0	-1
normalized size	1	1.00	0.93	2.15	0.00	15.15	0.00	0.00	-0.01
time (sec)	N/A	0.274	0.700	0.683	0.000	0.977	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	297	409	0	0	0	0	-1
normalized size	1	1.00	1.58	2.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.318	2.604	1.019	0.000	1.107	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	278	1355	0	0	0	0	-1
normalized size	1	1.00	0.84	4.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.692	1.090	0.802	0.000	1.196	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	222	943	0	0	0	0	-1
normalized size	1	1.00	0.90	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.462	1.041	0.722	0.000	0.784	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	163	635	0	0	0	0	-1
normalized size	1	1.00	0.97	3.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.657	0.789	0.000	1.019	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	211	938	0	0	0	0	-1
normalized size	1	1.00	0.97	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.438	2.553	1.196	0.000	0.776	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	364	1519	0	0	0	0	-1
normalized size	1	1.00	1.10	4.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.698	6.278	1.035	0.000	1.275	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	113	126	116	224	0	0	-1
normalized size	1	1.00	0.73	0.82	0.75	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.576	0.642	0.328	1.001	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	55	61	143	0	0	-1
normalized size	1	1.00	0.70	0.66	0.73	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.098	0.485	0.361	1.076	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	77	0	0	20
normalized size	1	1.00	1.00	0.88	0.83	3.21	0.00	0.00	0.83
time (sec)	N/A	0.037	0.034	0.041	0.338	0.762	0.000	0.000	5.569

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	105	312	0	1937	0	0	-1
normalized size	1	1.00	0.90	2.67	0.00	16.56	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.161	0.522	0.000	3.096	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	147	356	0	2071	0	0	-1
normalized size	1	1.00	0.95	2.30	0.00	13.36	0.00	0.00	-0.01
time (sec)	N/A	0.270	0.890	0.672	0.000	1.743	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	307	538	0	2229	0	0	-1
normalized size	1	1.00	1.54	2.70	0.00	11.20	0.00	0.00	-0.01
time (sec)	N/A	0.282	3.384	1.787	0.000	1.534	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	321	1619	0	0	0	0	-1
normalized size	1	1.00	0.81	4.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.937	1.296	0.965	0.000	1.133	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	239	1190	0	0	0	0	-1
normalized size	1	1.00	0.80	3.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.669	1.037	0.888	0.000	1.019	0.000	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	203	1042	0	0	0	0	-1
normalized size	1	1.00	1.00	5.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.896	0.876	0.000	0.707	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	259	1249	0	0	0	0	-1
normalized size	1	1.00	1.09	5.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.394	3.520	0.910	0.000	0.628	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	351	1360	0	0	0	0	-1
normalized size	1	1.00	1.09	4.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.672	6.274	0.945	0.000	1.171	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	338	1888	0	0	0	0	-1
normalized size	1	1.00	0.77	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.944	4.441	8.633	0.000	0.878	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	118	126	160	111	0	0	-1
normalized size	1	1.00	0.78	0.83	1.05	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.315	0.334	0.340	0.777	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	58	55	75	54	0	75	-1
normalized size	1	1.00	0.72	0.68	0.93	0.67	0.00	0.93	-0.01
time (sec)	N/A	0.085	0.098	0.296	0.317	0.892	0.000	1.884	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	54	20	20
normalized size	1	1.00	1.00	0.95	0.91	0.91	2.45	0.91	0.91
time (sec)	N/A	0.035	0.013	0.021	0.318	1.316	1.144	1.790	6.224

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	62	0	0	0	0	-1
normalized size	1	1.00	1.00	0.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.054	0.473	0.000	0.683	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	176	218	0	0	0	0	-1
normalized size	1	1.00	1.22	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.305	0.512	0.816	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	244	618	0	0	0	0	-1
normalized size	1	1.00	1.06	2.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.389	1.903	1.112	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	219	942	0	0	0	0	-1
normalized size	1	1.00	0.89	3.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	1.073	0.853	0.000	1.368	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	145	462	0	0	0	0	-1
normalized size	1	1.00	0.83	2.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.816	0.743	0.000	0.809	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	177	640	0	0	0	0	-1
normalized size	1	1.00	0.97	3.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.634	0.798	0.000	0.974	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	306	1314	0	0	0	0	-1
normalized size	1	1.00	1.05	4.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	4.143	2.258	0.000	0.852	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	116	116	124	125	0	0	-1
normalized size	1	1.00	0.77	0.77	0.83	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.256	0.354	0.321	0.542	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	54	67	67	0	72	-1
normalized size	1	1.00	0.72	0.68	0.85	0.85	0.00	0.91	-0.01
time (sec)	N/A	0.094	0.064	0.326	0.325	0.799	0.000	0.563	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	32	56	20	51
normalized size	1	1.00	1.00	0.95	0.91	1.45	2.55	0.91	2.32
time (sec)	N/A	0.038	0.015	0.020	0.320	0.642	3.009	1.901	6.145

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	99	0	0	0	0	-1
normalized size	1	1.00	0.87	0.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.086	0.558	0.000	0.963	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	221	250	0	0	0	0	-1
normalized size	1	1.00	1.19	1.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.334	1.199	0.894	0.000	0.000	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	324	649	0	0	0	0	-1
normalized size	1	1.00	1.14	2.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.521	2.240	1.101	0.000	1.492	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	273	1195	0	0	0	0	-1
normalized size	1	1.00	0.87	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.539	1.512	0.825	0.000	0.916	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	187	797	0	0	0	0	-1
normalized size	1	1.00	0.82	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	1.121	0.697	0.000	0.930	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	125	434	0	0	0	0	-1
normalized size	1	1.00	0.78	2.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	2.770	0.749	0.000	0.776	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	205	1062	0	0	0	0	-1
normalized size	1	1.00	0.82	4.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	1.748	1.121	0.000	0.775	0.000	0.000	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	348	1646	0	0	0	0	-1
normalized size	1	1.00	0.97	4.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.613	3.110	4.731	0.000	0.733	0.000	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	117	116	122	147	0	0	-1
normalized size	1	1.00	0.78	0.77	0.81	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.303	0.432	0.754	0.807	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	56	55	64	91	304	61	1402
normalized size	1	1.00	0.71	0.70	0.81	1.15	3.85	0.77	17.75
time (sec)	N/A	0.094	0.056	0.392	0.733	0.900	23.930	0.856	11.891

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	55	87	20	157
normalized size	1	1.00	1.00	0.88	0.83	2.29	3.62	0.83	6.54
time (sec)	N/A	0.042	0.019	0.024	0.468	0.567	22.735	0.887	7.246

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	94	130	0	3225	0	0	-1
normalized size	1	1.00	0.68	0.94	0.00	23.20	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.081	0.747	0.000	1.654	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	245	283	0	0	0	0	-1
normalized size	1	1.00	1.06	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.416	0.881	0.934	0.000	1.592	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	296	682	0	0	0	0	-1
normalized size	1	1.00	0.87	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.645	3.432	1.054	0.000	2.356	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	356	2253	0	0	0	0	-1
normalized size	1	1.00	0.93	5.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.761	1.787	0.887	0.000	1.104	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	244	1642	0	0	0	0	-1
normalized size	1	1.00	0.83	5.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.513	1.221	0.783	0.000	0.867	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	174	1047	0	0	0	0	-1
normalized size	1	1.00	0.79	4.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	1.026	0.778	0.000	0.814	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	167	864	0	0	0	0	-1
normalized size	1	1.00	0.76	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.263	1.029	0.822	0.000	0.674	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	241	1653	0	0	0	0	-1
normalized size	1	1.00	0.74	5.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	1.893	3.851	0.000	0.743	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	341	2585	0	0	0	0	-1
normalized size	1	1.00	0.80	6.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.876	2.503	5.841	0.000	0.969	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	104	259	0	0	0	0	-1
normalized size	1	1.00	0.84	2.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.901	1.395	0.000	0.658	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	222	0	0	0	0	-1
normalized size	1	1.00	0.83	2.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.493	1.412	0.000	0.878	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	185	0	0	0	0	-1
normalized size	1	1.00	0.83	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.483	1.288	0.000	1.240	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	123	0	0	0	0	-1
normalized size	1	1.00	0.89	1.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.100	1.116	0.000	0.818	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	106	0	0	0	0	47
normalized size	1	1.00	0.82	1.74	0.00	0.00	0.00	0.00	0.77
time (sec)	N/A	0.052	0.199	0.844	0.000	0.988	0.000	0.000	6.494

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	54	119	0	0	0	0	-1
normalized size	1	1.00	0.59	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.109	1.623	0.000	0.907	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	55	193	0	0	0	0	-1
normalized size	1	1.00	0.57	1.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.145	2.084	0.000	0.723	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	70	310	0	0	0	0	-1
normalized size	1	1.00	0.56	2.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.345	3.237	0.000	0.616	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	160	473	0	0	0	0	-1
normalized size	1	1.00	0.85	2.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	1.978	1.751	0.000	0.788	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	113	408	0	0	0	0	-1
normalized size	1	1.00	0.76	2.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.918	1.514	0.000	0.982	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	115	343	0	0	0	0	-1
normalized size	1	1.00	0.77	2.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	1.179	1.725	0.000	1.005	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	80	251	0	0	0	0	-1
normalized size	1	1.00	0.73	2.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.335	1.727	0.000	0.844	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	75	210	0	0	0	0	-1
normalized size	1	1.00	0.69	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.428	1.069	0.000	1.086	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	71	197	0	0	0	0	-1
normalized size	1	1.00	0.63	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.241	1.799	0.000	0.732	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	72	333	0	0	0	0	-1
normalized size	1	1.00	0.61	2.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.256	2.181	0.000	0.739	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	105	564	0	0	0	0	-1
normalized size	1	1.00	0.66	3.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.560	3.995	0.000	0.714	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	205	618	0	0	0	0	-1
normalized size	1	1.00	0.86	2.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	2.087	3.471	0.000	0.829	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	150	534	0	0	0	0	-1
normalized size	1	1.00	0.76	2.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.286	1.413	3.090	0.000	1.010	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	153	450	0	0	0	0	-1
normalized size	1	1.00	0.78	2.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	1.448	2.625	0.000	0.920	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	101	339	0	0	0	0	-1
normalized size	1	1.00	0.65	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.241	0.630	2.122	0.000	1.277	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	94	279	0	0	0	0	-1
normalized size	1	1.00	0.62	1.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.790	1.760	0.000	0.885	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	98	248	0	0	0	0	-1
normalized size	1	1.00	0.61	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.241	0.412	2.693	0.000	1.086	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	103	384	0	0	0	0	-1
normalized size	1	1.00	0.63	2.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.709	2.600	0.000	0.760	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	126	618	0	0	0	0	-1
normalized size	1	1.00	0.67	3.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	0.735	5.149	0.000	0.830	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	140	750	0	0	0	0	-1
normalized size	1	1.00	0.74	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.653	5.731	0.000	1.423	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	251	863	0	0	0	0	-1
normalized size	1	1.00	0.82	2.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.551	4.805	3.413	0.000	0.777	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	209	776	0	0	0	0	-1
normalized size	1	1.00	0.81	3.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.508	2.151	2.999	0.000	1.022	0.000	0.000	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	189	639	0	0	0	0	-1
normalized size	1	1.00	0.73	2.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	2.810	2.756	0.000	0.869	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	137	525	0	0	0	0	-1
normalized size	1	1.00	0.65	2.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	1.098	2.593	0.000	1.269	0.000	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	130	412	0	0	0	0	-1
normalized size	1	1.00	0.62	1.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.445	1.129	2.022	0.000	0.741	0.000	0.000	0.000

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	135	378	0	0	0	0	-1
normalized size	1	1.00	0.62	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.438	0.599	3.015	0.000	0.606	0.000	0.000	0.000

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	137	575	0	0	0	0	-1
normalized size	1	1.00	0.63	2.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.445	1.189	2.661	0.000	0.623	0.000	0.000	0.000

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	152	874	0	0	0	0	-1
normalized size	1	1.00	0.64	3.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	0.628	5.958	0.000	0.775	0.000	0.000	0.000

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	177	1067	0	0	0	0	-1
normalized size	1	1.00	0.73	4.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	0.932	7.290	0.000	0.795	0.000	0.000	0.000

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	219	1416	0	0	0	0	-1
normalized size	1	1.00	0.83	5.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	1.623	9.879	0.000	0.850	0.000	0.000	0.000

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	2035	3711	0	0	0	0	-1
normalized size	1	1.00	3.83	6.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.908	28.305	5.135	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	446	446	834	2126	0	0	0	0	-1
normalized size	1	1.00	1.87	4.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.286	27.172	3.569	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	1955	2329	0	0	0	0	-1
normalized size	1	1.00	4.24	5.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.323	29.041	4.467	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	709	1131	0	0	0	0	-1
normalized size	1	1.00	1.85	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.870	21.806	3.668	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	219	1266	0	0	0	0	-1
normalized size	1	1.00	0.55	3.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.882	4.671	3.233	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	361	682	0	0	0	0	-1
normalized size	1	1.00	1.24	2.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.582	16.034	2.844	0.000	167.625	0.000	0.000	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	558	678	0	0	0	0	-1
normalized size	1	1.00	1.87	2.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.574	16.126	2.766	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	791	1103	0	0	0	0	-1
normalized size	1	1.00	1.92	2.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.928	22.854	4.299	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	1192	1083	0	0	0	0	-1
normalized size	1	1.00	2.75	2.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.999	24.597	4.960	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	881	2399	0	0	0	0	-1
normalized size	1	1.00	1.81	4.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.328	6.759	7.173	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	2030	19829	0	0	0	0	-1
normalized size	1	1.00	3.74	36.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.519	27.802	13.797	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	835	20346	0	0	0	0	-1
normalized size	1	1.00	1.82	44.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.120	26.817	10.696	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	1956	14392	0	0	0	0	-1
normalized size	1	1.00	4.14	30.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.118	27.039	10.765	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	371	13221	0	0	0	0	-1
normalized size	1	1.00	0.95	33.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.819	37.946	8.143	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	614	9301	0	0	0	0	-1
normalized size	1	1.00	1.52	23.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.892	12.530	8.237	0.000	123.481	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	787	7033	0	0	0	0	-1
normalized size	1	1.00	1.86	16.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.868	16.382	9.147	0.000	113.335	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	1181	4457	0	0	0	0	-1
normalized size	1	1.00	2.75	10.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.898	23.762	8.897	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	777	8216	0	0	0	0	-1
normalized size	1	1.00	1.58	16.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.219	6.411	13.270	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	1258	6022	0	0	0	0	-1
normalized size	1	1.00	2.45	11.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.309	24.344	17.258	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	949	10743	0	0	0	0	-1
normalized size	1	1.00	1.65	18.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.623	6.772	25.885	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	575	575	932	111631	0	0	0	0	-1
normalized size	1	1.00	1.62	194.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.415	27.052	36.489	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	589	589	2024	85607	0	0	0	0	-1
normalized size	1	1.00	3.44	145.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.487	27.621	39.143	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	483	483	777	85489	0	0	0	0	-1
normalized size	1	1.00	1.61	177.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.075	25.990	29.464	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	1954	65216	0	0	0	0	-1
normalized size	1	1.00	3.93	131.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.079	26.371	29.355	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	831	63272	0	0	0	0	-1
normalized size	1	1.00	1.65	125.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.105	24.058	31.065	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	1211	45147	0	0	0	0	-1
normalized size	1	1.00	2.33	86.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.138	23.748	30.135	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	837	36688	0	0	0	0	-1
normalized size	1	1.00	1.63	71.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.187	24.334	30.272	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	1226	25322	0	0	0	0	-1
normalized size	1	1.00	2.36	48.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.226	24.548	28.635	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	922	46134	0	0	0	0	-1
normalized size	1	1.00	1.55	77.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.593	6.727	57.541	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	1308	32645	0	0	0	0	-1
normalized size	1	1.00	2.13	53.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.721	23.490	78.670	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	1014	49016	0	0	0	0	-1
normalized size	1	1.00	1.48	71.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.028	6.904	111.396	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	671	671	2102	300244	0	0	0	0	-1
normalized size	1	1.00	3.13	447.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.832	27.814	139.890	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	937	180834	0	0	0	0	-1
normalized size	1	1.00	1.68	324.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.364	26.846	94.455	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	2020	144252	0	0	0	0	-1
normalized size	1	1.00	3.54	252.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.378	26.497	111.918	0.000	0.000	0.000	0.000	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	900	237416	0	0	0	0	-1
normalized size	1	1.00	1.52	401.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.430	26.826	100.678	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	1263	192036	0	0	0	0	-1
normalized size	1	1.00	2.12	321.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.520	24.063	114.161	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	892	179434	0	0	0	0	-1
normalized size	1	1.00	1.55	312.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.447	26.718	115.350	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	592	592	1263	138380	0	0	0	0	-1
normalized size	1	1.00	2.13	233.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.471	23.847	116.408	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	900	112960	0	0	0	0	-1
normalized size	1	1.00	1.55	195.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.532	6.630	110.043	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	593	593	1276	85165	0	0	0	0	-1
normalized size	1	1.00	2.15	143.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.570	24.457	104.026	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	996	150599	0	0	0	0	-1
normalized size	1	1.00	1.48	223.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.952	6.819	184.058	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	B	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	374	117	442	0	0	0	0	-1
normalized size	1	2.04	0.64	2.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.428	0.320	0.609	0.000	1.071	0.000	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	290	0	0	0	0	0	-1
normalized size	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.370	54.884	6.004	0.000	0.761	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	285	0	0	0	0	0	-1
normalized size	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	1.050	4.431	0.000	0.628	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	240	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.993	1.772	0.000	0.581	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	3815	0	0	0	0	0	-1
normalized size	1	1.00	24.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	20.267	1.054	0.000	0.734	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	4727	0	0	0	0	0	-1
normalized size	1	1.00	27.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	25.541	1.115	0.000	0.801	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	7781	0	0	0	0	0	-1
normalized size	1	1.00	45.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	28.256	1.398	0.000	0.744	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	67.681	8.120	0.000	1.039	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	7.464	0.276	0.000	0.954	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.791	0.219	0.000	0.668	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.962	0.215	0.000	0.714	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	185	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	1.076	0.174	0.000	0.634	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	185	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	2.748	0.197	0.000	0.756	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	3.038	0.181	0.000	0.936	0.000	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	2.412	1.331	0.000	0.837	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	459	0	558	814	0	0	1196
normalized size	1	1.00	1.81	0.00	2.20	3.20	0.00	0.00	4.71
time (sec)	N/A	0.164	6.114	0.735	0.997	0.921	0.000	0.000	19.094

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	169	0	286	381	0	0	641
normalized size	1	1.00	1.01	0.00	1.71	2.28	0.00	0.00	3.84
time (sec)	N/A	0.112	0.902	0.613	1.063	0.730	0.000	0.000	11.621

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	74	0	117	142	0	340	197
normalized size	1	1.00	0.80	0.00	1.27	1.54	0.00	3.70	2.14
time (sec)	N/A	0.072	0.177	0.489	0.784	0.766	0.000	0.355	7.512

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	33	99	26	26
normalized size	1	1.00	1.00	1.04	1.00	1.27	3.81	1.00	1.00
time (sec)	N/A	0.027	0.024	0.027	0.294	0.716	2.206	1.273	6.322

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	99	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.118	1.010	0.000	0.577	0.000	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	157	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.621	0.515	0.000	0.774	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	260	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.421	3.996	0.845	0.000	0.608	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	4.218	0.391	0.000	0.685	0.000	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	5.883	0.313	0.000	0.867	0.000	0.000	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	2.229	0.149	0.000	0.790	0.000	0.000	0.000

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	4.916	0.328	0.000	0.693	0.000	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	55.454	0.204	0.000	0.887	0.000	0.000	0.000

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	5.697	0.188	0.000	0.832	0.000	0.000	0.000

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	1.931	0.204	0.000	0.705	0.000	0.000	0.000

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	1.771	0.176	0.000	0.714	0.000	0.000	0.000

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	1.924	0.173	0.000	0.687	0.000	0.000	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	2.104	0.181	0.000	0.907	0.000	0.000	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	826	0	0	0	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.018	6.095	0.365	0.000	0.892	0.000	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	420	319	0	0	0	0	0	-1
normalized size	1	1.35	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.512	5.047	0.332	0.000	1.043	0.000	0.000	0.000

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	168	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.959	0.336	0.000	1.032	0.000	0.000	0.000

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.388	0.322	0.000	0.786	0.000	0.000	0.000

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	1.669	0.594	0.000	0.667	0.000	0.000	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	5.632	0.327	0.000	0.771	0.000	0.000	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	4.727	0.328	0.000	0.839	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [594] had the largest ratio of [.5200]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	19	0.105
2	A	5	3	1.00	19	0.158
3	A	3	2	1.00	19	0.105
4	A	4	3	1.00	19	0.158
5	A	3	2	1.00	19	0.105
6	A	3	3	1.00	19	0.158
7	A	2	1	1.27	17	0.059
8	A	2	2	1.00	17	0.118
9	A	3	3	1.00	19	0.158
10	A	4	3	1.00	19	0.158
11	A	3	2	1.00	19	0.105
12	A	4	3	1.00	19	0.158
13	A	6	4	1.00	21	0.190
14	A	3	2	1.00	21	0.095
15	A	5	4	1.00	21	0.190
16	A	3	2	1.00	21	0.095
17	A	4	4	1.00	21	0.190
18	A	2	2	1.00	19	0.105
19	A	3	2	1.00	19	0.105
20	A	3	3	1.00	21	0.143
21	A	2	2	1.00	21	0.095
22	A	3	3	1.00	21	0.143
23	A	4	3	1.00	21	0.143
24	A	3	2	1.00	21	0.095
25	A	4	3	1.00	21	0.143
26	A	3	2	1.00	21	0.095
27	A	7	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	3	2	1.00	21	0.095
29	A	6	4	1.00	21	0.190
30	A	3	2	1.00	21	0.095
31	A	5	4	1.00	21	0.190
32	A	2	2	1.00	19	0.105
33	A	3	2	1.00	19	0.105
34	A	4	4	1.00	21	0.190
35	A	3	2	1.00	21	0.095
36	A	2	2	1.00	21	0.095
37	A	2	2	1.00	21	0.095
38	A	4	3	1.00	21	0.143
39	A	4	3	1.00	21	0.143
40	A	4	3	1.00	21	0.143
41	A	3	2	1.00	21	0.095
42	A	11	4	1.00	21	0.190
43	A	3	2	1.00	21	0.095
44	A	10	4	1.00	21	0.190
45	A	2	2	1.00	19	0.105
46	A	3	2	1.00	19	0.105
47	A	9	6	1.00	21	0.286
48	A	3	2	1.00	21	0.095
49	A	8	6	1.00	21	0.286
50	A	3	2	1.00	21	0.095
51	A	4	3	1.00	21	0.143
52	A	3	2	1.00	21	0.095
53	A	3	3	1.00	21	0.143
54	A	2	1	1.00	21	0.048
55	A	2	2	1.00	21	0.095
56	A	2	2	1.00	19	0.105
57	A	4	3	1.00	19	0.158
58	A	3	3	1.00	21	0.143
59	A	4	3	1.00	21	0.143
60	A	3	2	1.00	21	0.095
61	A	4	3	1.00	21	0.143
62	A	5	4	1.00	21	0.190
63	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	4	4	1.00	21	0.190
65	A	2	2	1.00	21	0.095
66	A	3	3	1.00	21	0.143
67	A	3	2	1.00	21	0.095
68	A	2	2	1.00	21	0.095
69	A	2	2	1.00	19	0.105
70	A	4	3	1.00	19	0.158
71	A	4	3	1.00	21	0.143
72	A	4	3	1.00	21	0.143
73	A	4	2	1.00	21	0.095
74	A	4	3	1.00	21	0.143
75	A	5	4	1.00	21	0.190
76	A	2	2	1.00	21	0.095
77	A	4	4	1.00	21	0.190
78	A	3	2	1.00	21	0.095
79	A	3	3	1.00	21	0.143
80	A	3	2	1.00	21	0.095
81	A	1	1	1.00	21	0.048
82	A	2	2	1.00	19	0.105
83	A	4	3	1.00	19	0.158
84	A	5	3	1.00	21	0.143
85	A	4	3	1.00	21	0.143
86	A	5	2	1.00	21	0.095
87	A	4	3	1.00	21	0.143
88	A	5	2	1.00	21	0.095
89	A	2	2	1.00	21	0.095
90	A	2	2	1.00	21	0.095
91	A	3	2	1.00	21	0.095
92	A	4	2	1.00	21	0.095
93	A	3	2	1.00	21	0.095
94	A	6	2	1.00	21	0.095
95	A	2	2	1.00	19	0.105
96	A	4	3	1.00	19	0.158
97	A	10	3	1.00	21	0.143
98	A	4	3	1.00	21	0.143
99	A	10	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	4	3	1.00	21	0.143
101	A	3	2	1.00	23	0.087
102	A	4	2	1.00	23	0.087
103	A	3	2	1.00	23	0.087
104	A	3	2	1.00	23	0.087
105	A	3	2	1.00	23	0.087
106	A	2	2	1.00	23	0.087
107	A	2	2	1.00	21	0.095
108	A	3	3	1.00	21	0.143
109	A	3	3	1.00	23	0.130
110	A	5	5	1.00	23	0.217
111	A	5	5	1.00	23	0.217
112	A	7	6	1.00	23	0.261
113	A	7	6	1.00	23	0.261
114	A	3	2	1.00	23	0.087
115	A	5	2	1.00	23	0.087
116	A	3	2	1.00	23	0.087
117	A	4	2	1.00	23	0.087
118	A	3	2	1.00	23	0.087
119	A	3	2	1.00	23	0.087
120	A	2	2	1.00	21	0.095
121	A	4	4	1.00	21	0.190
122	A	1	1	1.00	23	0.043
123	A	4	4	1.00	23	0.174
124	A	4	3	1.00	23	0.130
125	A	6	5	1.00	23	0.217
126	A	6	5	1.00	23	0.217
127	A	3	2	1.00	23	0.087
128	A	5	2	1.00	23	0.087
129	A	3	2	1.00	23	0.087
130	A	4	2	1.00	23	0.087
131	A	2	2	1.00	21	0.095
132	A	5	4	1.00	21	0.190
133	A	2	2	1.00	23	0.087
134	A	4	4	1.00	23	0.174
135	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	5	4	1.00	23	0.174
137	A	5	3	1.00	23	0.130
138	A	7	5	1.00	23	0.217
139	A	3	2	1.00	23	0.087
140	A	7	2	1.00	23	0.087
141	A	3	2	1.00	23	0.087
142	A	6	2	1.00	23	0.087
143	A	3	2	1.00	23	0.087
144	A	5	2	1.00	23	0.087
145	A	2	2	1.00	21	0.095
146	A	6	4	1.00	21	0.190
147	A	3	2	1.00	23	0.087
148	A	5	5	1.00	23	0.217
149	A	2	2	1.00	23	0.087
150	A	5	5	1.00	23	0.217
151	A	1	1	1.00	23	0.043
152	A	6	4	1.00	23	0.174
153	A	6	3	1.00	23	0.130
154	A	8	5	1.00	23	0.217
155	A	8	5	1.00	23	0.217
156	A	3	2	1.00	23	0.087
157	A	3	2	1.00	23	0.087
158	A	3	2	1.00	23	0.087
159	A	2	2	1.00	23	0.087
160	A	3	2	1.00	23	0.087
161	A	1	1	1.00	23	0.043
162	A	2	2	1.00	21	0.095
163	A	4	4	1.00	21	0.190
164	A	4	4	1.00	23	0.174
165	A	6	5	1.00	23	0.217
166	A	6	5	1.00	23	0.217
167	A	8	6	1.00	23	0.261
168	A	8	5	1.00	23	0.217
169	A	3	2	1.00	23	0.087
170	A	2	2	1.00	23	0.087
171	A	3	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
172	A	1	1	1.00	23	0.043
173	A	3	2	1.00	23	0.087
174	A	3	3	1.00	23	0.130
175	A	2	2	1.00	21	0.095
176	A	5	4	1.00	21	0.190
177	A	5	5	1.00	23	0.217
178	A	7	6	1.00	23	0.261
179	A	7	5	1.00	23	0.217
180	A	9	6	1.00	23	0.261
181	A	9	5	1.00	23	0.217
182	A	3	2	1.00	23	0.087
183	A	3	2	1.00	23	0.087
184	A	2	2	1.00	23	0.087
185	A	3	2	1.00	23	0.087
186	A	1	1	1.00	23	0.043
187	A	3	2	1.00	23	0.087
188	A	4	3	1.00	23	0.130
189	A	3	2	1.00	23	0.087
190	A	3	3	1.00	23	0.130
191	A	2	2	1.00	21	0.095
192	A	6	4	1.00	21	0.190
193	A	6	5	1.00	23	0.217
194	A	8	6	1.00	23	0.261
195	A	8	5	1.00	23	0.217
196	A	5	4	1.00	23	0.174
197	A	4	4	1.00	23	0.174
198	A	4	4	1.00	23	0.174
199	A	3	3	1.00	23	0.130
200	A	3	3	1.00	23	0.130
201	A	4	4	1.00	23	0.174
202	A	4	4	1.00	23	0.174
203	A	5	4	1.00	23	0.174
204	A	6	5	1.00	25	0.200
205	A	5	5	1.00	25	0.200
206	A	5	5	1.00	25	0.200
207	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
208	A	4	4	1.00	25	0.160
209	A	4	4	1.00	25	0.160
210	A	4	4	1.00	25	0.160
211	A	5	5	1.00	25	0.200
212	A	4	4	1.00	25	0.160
213	A	5	4	1.00	25	0.160
214	A	7	5	1.00	25	0.200
215	A	6	5	1.00	25	0.200
216	A	6	5	1.00	25	0.200
217	A	5	4	1.00	25	0.160
218	A	5	4	1.00	25	0.160
219	A	5	5	1.00	25	0.200
220	A	5	5	1.00	25	0.200
221	A	5	5	1.00	25	0.200
222	A	5	5	1.00	25	0.200
223	A	6	5	1.00	25	0.200
224	A	7	5	1.00	25	0.200
225	A	6	4	1.00	25	0.160
226	A	6	4	1.00	25	0.160
227	A	6	5	1.00	25	0.200
228	A	6	5	1.00	25	0.200
229	A	5	4	1.00	25	0.160
230	A	5	4	1.00	25	0.160
231	A	6	6	1.00	25	0.240
232	A	6	6	1.00	25	0.240
233	A	5	4	1.00	25	0.160
234	A	4	4	1.00	25	0.160
235	A	4	4	1.00	25	0.160
236	A	3	3	1.00	25	0.120
237	A	3	3	1.00	25	0.120
238	A	3	3	1.00	25	0.120
239	A	3	3	1.00	25	0.120
240	A	4	4	1.00	25	0.160
241	A	4	4	1.00	25	0.160
242	A	5	4	1.00	25	0.160
243	A	5	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
244	A	4	4	1.00	25	0.160
245	A	4	4	1.00	25	0.160
246	A	3	3	1.00	25	0.120
247	A	3	3	1.00	25	0.120
248	A	4	4	1.00	25	0.160
249	A	4	4	1.00	25	0.160
250	A	5	5	1.00	25	0.200
251	A	5	5	1.00	25	0.200
252	A	6	5	1.00	25	0.200
253	A	6	5	1.00	25	0.200
254	A	5	5	1.00	25	0.200
255	A	5	5	1.00	25	0.200
256	A	4	4	1.00	25	0.160
257	A	4	4	1.00	25	0.160
258	A	4	4	1.00	25	0.160
259	A	4	4	1.00	25	0.160
260	A	5	4	1.00	25	0.160
261	A	5	4	1.00	25	0.160
262	A	6	5	1.00	25	0.200
263	A	6	4	1.00	25	0.160
264	A	5	4	1.00	25	0.160
265	A	5	4	1.00	25	0.160
266	A	4	3	1.00	25	0.120
267	A	4	3	1.00	25	0.120
268	A	5	5	1.00	25	0.200
269	A	5	5	1.00	25	0.200
270	A	6	4	1.00	25	0.160
271	A	6	4	1.00	25	0.160
272	A	7	5	1.00	25	0.200
273	A	8	8	1.00	27	0.296
274	A	7	7	1.00	27	0.259
275	A	6	6	1.00	27	0.222
276	A	1	1	1.00	27	0.037
277	A	2	2	1.00	27	0.074
278	A	3	2	1.00	27	0.074
279	A	4	2	1.00	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
280	A	10	9	1.00	27	0.333
281	A	9	8	1.00	27	0.296
282	A	8	7	1.00	27	0.259
283	A	7	7	1.00	27	0.259
284	A	7	7	1.00	27	0.259
285	A	1	1	1.00	27	0.037
286	A	2	2	1.00	27	0.074
287	A	3	2	1.00	27	0.074
288	A	4	2	1.00	27	0.074
289	A	10	8	1.00	27	0.296
290	A	9	7	1.00	27	0.259
291	A	8	7	1.00	27	0.259
292	A	8	8	1.00	27	0.296
293	A	7	7	1.00	27	0.259
294	A	1	1	1.00	27	0.037
295	A	2	2	1.00	27	0.074
296	A	3	2	1.00	27	0.074
297	A	4	2	1.00	27	0.074
298	A	8	8	1.00	27	0.296
299	A	7	7	1.00	27	0.259
300	A	6	6	1.00	27	0.222
301	A	1	1	1.00	27	0.037
302	A	2	2	1.00	27	0.074
303	A	3	2	1.00	27	0.074
304	A	4	2	1.00	27	0.074
305	A	8	8	1.00	27	0.296
306	A	7	7	1.00	27	0.259
307	A	8	8	1.00	27	0.296
308	A	1	1	1.00	27	0.037
309	A	2	2	1.00	27	0.074
310	A	3	2	1.00	27	0.074
311	A	4	2	1.00	27	0.074
312	A	5	2	1.00	27	0.074
313	A	9	9	1.00	27	0.333
314	A	8	8	1.00	27	0.296
315	A	7	7	1.00	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	1	1	1.00	27	0.037
317	A	2	2	1.00	27	0.074
318	A	3	2	1.00	27	0.074
319	A	4	2	1.00	27	0.074
320	A	5	2	1.00	27	0.074
321	A	3	3	1.00	27	0.111
322	A	3	3	1.00	27	0.111
323	A	3	3	1.00	27	0.111
324	A	3	3	1.00	27	0.111
325	A	3	3	1.00	27	0.111
326	A	3	3	1.00	27	0.111
327	A	2	2	1.00	23	0.087
328	A	2	2	1.00	23	0.087
329	A	2	2	1.00	23	0.087
330	A	2	2	1.00	21	0.095
331	A	2	2	1.00	23	0.087
332	A	2	2	1.00	23	0.087
333	A	2	2	1.00	23	0.087
334	A	2	2	1.00	23	0.087
335	A	3	3	1.00	25	0.120
336	A	3	3	1.00	25	0.120
337	A	3	3	1.00	25	0.120
338	A	3	3	1.00	25	0.120
339	A	3	3	1.00	25	0.120
340	A	3	3	1.00	25	0.120
341	A	3	3	1.00	25	0.120
342	A	3	3	1.00	23	0.130
343	A	3	2	1.00	21	0.095
344	A	3	2	1.00	21	0.095
345	A	3	2	1.00	21	0.095
346	A	2	2	1.00	19	0.105
347	A	2	2	1.00	19	0.105
348	A	2	2	1.00	21	0.095
349	A	2	2	1.00	21	0.095
350	A	3	3	1.00	21	0.143
351	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
352	A	3	3	1.00	21	0.143
353	A	3	3	1.00	21	0.143
354	A	3	3	1.00	25	0.120
355	A	3	3	1.00	25	0.120
356	A	3	3	1.00	25	0.120
357	A	3	3	1.00	25	0.120
358	A	3	3	1.00	25	0.120
359	A	3	3	1.00	25	0.120
360	A	4	2	1.00	27	0.074
361	A	3	2	1.00	27	0.074
362	A	2	2	1.00	27	0.074
363	A	1	1	1.00	27	0.037
364	A	3	3	1.00	25	0.120
365	A	3	3	1.00	27	0.111
366	A	3	3	1.00	27	0.111
367	A	3	2	1.00	27	0.074
368	A	2	2	1.00	27	0.074
369	A	1	1	1.00	27	0.037
370	A	3	3	1.00	27	0.111
371	A	3	3	1.00	27	0.111
372	A	4	4	1.00	27	0.148
373	A	4	4	1.00	27	0.148
374	A	4	4	1.00	25	0.160
375	A	4	4	1.00	27	0.148
376	A	4	3	1.00	19	0.158
377	A	3	2	1.00	19	0.105
378	A	2	1	1.27	17	0.059
379	A	4	3	1.00	17	0.176
380	A	3	3	1.00	19	0.158
381	A	4	4	1.00	19	0.210
382	A	4	3	1.00	19	0.158
383	A	3	3	1.00	19	0.158
384	A	3	3	1.00	19	0.158
385	A	3	2	1.00	19	0.105
386	A	3	2	1.00	19	0.105
387	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
388	A	3	2	1.00	21	0.095
389	A	2	2	1.00	19	0.105
390	A	6	4	1.00	19	0.210
391	A	3	3	1.00	21	0.143
392	A	4	4	1.00	21	0.190
393	A	6	4	1.00	21	0.190
394	A	5	4	1.00	21	0.190
395	A	4	4	1.00	21	0.190
396	A	3	2	1.00	21	0.095
397	A	4	4	1.00	21	0.190
398	A	4	3	1.00	21	0.143
399	A	4	3	1.00	21	0.143
400	A	3	2	1.00	21	0.095
401	A	3	2	1.00	21	0.095
402	A	2	2	1.00	19	0.105
403	A	6	4	1.00	19	0.210
404	A	6	5	1.00	21	0.238
405	A	4	4	1.00	21	0.190
406	A	6	5	1.00	21	0.238
407	A	5	5	1.00	21	0.238
408	A	2	2	1.00	21	0.095
409	A	5	5	1.00	21	0.238
410	A	5	5	1.00	21	0.238
411	A	5	4	1.00	21	0.190
412	A	5	4	1.00	21	0.190
413	A	3	2	1.00	21	0.095
414	A	3	2	1.00	21	0.095
415	A	2	2	1.00	19	0.105
416	A	6	4	1.00	19	0.210
417	A	7	5	1.00	21	0.238
418	A	8	6	1.00	21	0.286
419	A	10	5	1.00	21	0.238
420	A	7	3	1.00	21	0.143
421	A	7	4	1.00	21	0.190
422	A	7	4	1.00	21	0.190
423	A	7	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
424	A	10	6	1.00	21	0.286
425	A	3	2	1.00	21	0.095
426	A	3	2	1.00	21	0.095
427	A	2	2	1.00	19	0.105
428	A	6	4	1.00	19	0.210
429	A	4	3	1.00	21	0.143
430	A	5	4	1.00	21	0.190
431	A	7	6	1.00	21	0.286
432	A	6	6	1.00	21	0.286
433	A	5	5	1.00	21	0.238
434	A	5	5	1.00	21	0.238
435	A	6	6	1.00	21	0.286
436	A	7	6	1.00	21	0.286
437	A	3	2	1.00	21	0.095
438	A	3	2	1.00	21	0.095
439	A	3	2	1.00	21	0.095
440	A	2	2	1.00	19	0.105
441	A	4	3	1.00	19	0.158
442	A	4	3	1.00	21	0.143
443	A	5	4	1.00	21	0.190
444	A	7	6	1.00	21	0.286
445	A	6	6	1.00	21	0.286
446	A	5	5	1.00	21	0.238
447	A	6	6	1.00	21	0.286
448	A	7	6	1.00	21	0.286
449	A	3	2	1.00	21	0.095
450	A	3	2	1.00	21	0.095
451	A	3	2	1.00	21	0.095
452	A	2	2	1.00	19	0.105
453	A	4	3	1.00	19	0.158
454	A	4	3	1.00	21	0.143
455	A	5	4	1.00	21	0.190
456	A	7	7	1.00	21	0.333
457	A	6	6	1.00	21	0.286
458	A	6	6	1.00	21	0.286
459	A	7	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
460	A	8	7	1.00	21	0.333
461	A	3	2	1.00	21	0.095
462	A	3	2	1.00	21	0.095
463	A	3	2	1.00	21	0.095
464	A	2	2	1.00	19	0.105
465	A	4	3	1.00	19	0.158
466	A	4	3	1.00	21	0.143
467	A	11	7	1.00	21	0.333
468	A	11	7	1.00	21	0.333
469	A	11	7	1.00	21	0.333
470	A	11	6	1.00	21	0.286
471	A	12	7	1.00	21	0.333
472	A	13	7	1.00	21	0.333
473	A	3	2	1.00	23	0.087
474	A	3	2	1.00	23	0.087
475	A	2	2	1.00	21	0.095
476	A	5	4	1.00	21	0.190
477	A	6	5	1.00	23	0.217
478	A	7	6	1.00	23	0.261
479	A	8	8	1.00	23	0.348
480	A	7	7	1.00	23	0.304
481	A	7	7	1.00	23	0.304
482	A	7	7	1.00	23	0.304
483	A	3	2	1.00	23	0.087
484	A	3	2	1.00	23	0.087
485	A	2	2	1.00	21	0.095
486	A	6	5	1.00	21	0.238
487	A	6	5	1.00	23	0.217
488	A	7	6	1.00	23	0.261
489	A	8	7	1.00	23	0.304
490	A	7	7	1.00	23	0.304
491	A	6	6	1.00	23	0.261
492	A	7	7	1.00	23	0.304
493	A	8	7	1.00	23	0.304
494	A	3	2	1.00	23	0.087
495	A	3	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
496	A	2	2	1.00	21	0.095
497	A	7	6	1.00	21	0.286
498	A	7	6	1.00	23	0.261
499	A	7	6	1.00	23	0.261
500	A	9	8	1.00	23	0.348
501	A	8	8	1.00	23	0.348
502	A	7	7	1.00	23	0.304
503	A	7	7	1.00	23	0.304
504	A	8	8	1.00	23	0.348
505	A	9	8	1.00	23	0.348
506	A	3	2	1.00	23	0.087
507	A	3	2	1.00	23	0.087
508	A	2	2	1.00	21	0.095
509	A	5	4	1.00	21	0.190
510	A	6	5	1.00	23	0.217
511	A	7	6	1.00	23	0.261
512	A	7	7	1.00	23	0.304
513	A	6	6	1.00	23	0.261
514	A	6	6	1.00	23	0.261
515	A	7	7	1.00	23	0.304
516	A	3	2	1.00	23	0.087
517	A	3	2	1.00	23	0.087
518	A	2	2	1.00	21	0.095
519	A	6	5	1.00	21	0.238
520	A	7	6	1.00	23	0.261
521	A	8	7	1.00	23	0.304
522	A	8	7	1.00	23	0.304
523	A	7	7	1.00	23	0.304
524	A	6	6	1.00	23	0.261
525	A	7	7	1.00	23	0.304
526	A	8	7	1.00	23	0.304
527	A	3	2	1.00	23	0.087
528	A	3	2	1.00	23	0.087
529	A	2	2	1.00	21	0.095
530	A	7	6	1.00	21	0.286
531	A	8	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
532	A	9	7	1.00	23	0.304
533	A	9	8	1.00	23	0.348
534	A	8	8	1.00	23	0.348
535	A	7	7	1.00	23	0.304
536	A	7	7	1.00	23	0.304
537	A	8	8	1.00	23	0.348
538	A	9	8	1.00	23	0.348
539	A	5	4	1.00	23	0.174
540	A	4	4	1.00	23	0.174
541	A	4	4	1.00	23	0.174
542	A	3	3	1.00	23	0.130
543	A	3	3	1.00	23	0.130
544	A	4	4	1.00	23	0.174
545	A	4	4	1.00	23	0.174
546	A	5	4	1.00	23	0.174
547	A	6	5	1.00	25	0.200
548	A	5	5	1.00	25	0.200
549	A	5	5	1.00	25	0.200
550	A	4	4	1.00	25	0.160
551	A	4	4	1.00	25	0.160
552	A	4	4	1.00	25	0.160
553	A	4	4	1.00	25	0.160
554	A	5	5	1.00	25	0.200
555	A	7	6	1.00	25	0.240
556	A	6	6	1.00	25	0.240
557	A	6	6	1.00	25	0.240
558	A	5	5	1.00	25	0.200
559	A	5	5	1.00	25	0.200
560	A	5	5	1.00	25	0.200
561	A	5	5	1.00	25	0.200
562	A	5	5	1.00	25	0.200
563	A	5	5	1.00	25	0.200
564	A	8	6	1.00	25	0.240
565	A	7	6	1.00	25	0.240
566	A	7	6	1.00	25	0.240
567	A	6	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
568	A	6	5	1.00	25	0.200
569	A	6	5	1.00	25	0.200
570	A	6	5	1.00	25	0.200
571	A	6	6	1.00	25	0.240
572	A	6	6	1.00	25	0.240
573	A	6	5	1.00	25	0.200
574	A	15	12	1.00	25	0.480
575	A	14	12	1.00	25	0.480
576	A	14	12	1.00	25	0.480
577	A	13	11	1.00	25	0.440
578	A	13	11	1.00	25	0.440
579	A	9	7	1.00	25	0.280
580	A	9	7	1.00	25	0.280
581	A	13	11	1.00	25	0.440
582	A	13	11	1.00	25	0.440
583	A	14	12	1.00	25	0.480
584	A	15	12	1.00	25	0.480
585	A	14	12	1.00	25	0.480
586	A	14	12	1.00	25	0.480
587	A	13	11	1.00	25	0.440
588	A	13	11	1.00	25	0.440
589	A	13	11	1.00	25	0.440
590	A	13	11	1.00	25	0.440
591	A	14	12	1.00	25	0.480
592	A	14	12	1.00	25	0.480
593	A	15	12	1.00	25	0.480
594	A	15	13	1.00	25	0.520
595	A	15	13	1.00	25	0.520
596	A	14	12	1.00	25	0.480
597	A	14	12	1.00	25	0.480
598	A	14	12	1.00	25	0.480
599	A	14	12	1.00	25	0.480
600	A	14	12	1.00	25	0.480
601	A	14	12	1.00	25	0.480
602	A	15	13	1.00	25	0.520
603	A	15	13	1.00	25	0.520

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
604	A	16	13	1.00	25	0.520
605	A	16	13	1.00	25	0.520
606	A	15	12	1.00	25	0.480
607	A	15	12	1.00	25	0.480
608	A	15	13	1.00	25	0.520
609	A	15	13	1.00	25	0.520
610	A	15	12	1.00	25	0.480
611	A	15	12	1.00	25	0.480
612	A	15	12	1.00	25	0.480
613	A	15	12	1.00	25	0.480
614	A	16	13	1.00	25	0.520
615	B	2	2	2.04	27	0.074
616	A	4	4	1.00	23	0.174
617	A	3	3	1.00	23	0.130
618	A	2	2	1.00	21	0.095
619	A	1	1	1.00	23	0.043
620	A	1	1	1.00	23	0.043
621	A	1	1	1.00	23	0.043
622	A	1	1	1.00	23	0.043
623	A	2	2	1.00	25	0.080
624	A	2	2	1.00	25	0.080
625	A	2	2	1.00	25	0.080
626	A	2	2	1.00	25	0.080
627	A	2	2	1.00	25	0.080
628	A	2	2	1.00	25	0.080
629	A	2	2	1.00	23	0.087
630	A	3	2	1.00	21	0.095
631	A	3	2	1.00	21	0.095
632	A	3	2	1.00	21	0.095
633	A	2	2	1.00	19	0.105
634	A	5	3	1.00	19	0.158
635	A	6	4	1.00	21	0.190
636	A	7	5	1.00	21	0.238
637	A	2	2	1.00	21	0.095
638	A	2	2	1.00	21	0.095
639	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
640	A	2	2	1.00	21	0.095
641	A	2	2	1.00	25	0.080
642	A	2	2	1.00	25	0.080
643	A	2	2	1.00	25	0.080
644	A	2	2	1.00	25	0.080
645	A	2	2	1.00	25	0.080
646	A	2	2	1.00	25	0.080
647	A	9	7	1.00	27	0.259
648	A	5	5	1.35	27	0.185
649	A	3	3	1.00	27	0.111
650	A	1	1	1.00	27	0.037
651	A	2	2	1.00	25	0.080
652	A	2	2	1.00	27	0.074
653	A	2	2	1.00	27	0.074

Chapter 3

Listing of integrals

3.1 $\int \cos^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{(a\sin(c+dx)+a)^8}{8a^7d}+\frac{6(a\sin(c+dx)+a)^7}{7a^6d}-\frac{2(a\sin(c+dx)+a)^6}{a^5d}+\frac{8(a\sin(c+dx)+a)^5}{5a^4d}$$

```
[Out] 8/5*(a+a*sin(d*x+c))^5/a^4/d-2*(a+a*sin(d*x+c))^6/a^5/d+6/7*(a+a*sin(d*x+c)
)^7/a^6/d-1/8*(a+a*sin(d*x+c))^8/a^7/d
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2667, 43\}$

$$-\frac{(a\sin(c+dx)+a)^8}{8a^7d}+\frac{6(a\sin(c+dx)+a)^7}{7a^6d}-\frac{2(a\sin(c+dx)+a)^6}{a^5d}+\frac{8(a\sin(c+dx)+a)^5}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^5)/(5*a^4*d) - (2*(a + a*Sin[c + d*x])^6)/(a^5*d) +
 (6*(a + a*Sin[c + d*x])^7)/(7*a^6*d) - (a + a*Sin[c + d*x])^8/(8*a^7*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^4-12a^2(a+x)^5+6a(a+x)^6-(a+x)^7\right)\,dx, x, a\sin\right.}{a^7 d}$$
$$= \frac{8(a+a\sin(c+dx))^5}{5a^4 d} - \frac{2(a+a\sin(c+dx))^6}{a^5 d} + \frac{6(a+a\sin(c+dx))^7}{7a^6 d} - \frac{(a}{}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.85

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cos[c + d*x]^8)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (3*
a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

fricas [A] time = 0.79, size = 62, normalized size = 0.71

$$-\frac{35\,a\cos\left(dx+c\right)^8-8\left(5\,a\cos\left(dx+c\right)^6+6\,a\cos\left(dx+c\right)^4+8\,a\cos\left(dx+c\right)^2+16\,a\right)\sin\left(dx+c\right)}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/280*(35*a*cos(d*x + c)^8 - 8*(5*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 +
8*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

giac [A] time = 2.69, size = 118, normalized size = 1.36

$$-\frac{a\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a\cos\left(6\,dx+6\,c\right)}{128\,d}-\frac{7\,a\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{7\,a\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{a\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{7\,a\sin\left(5\,dx+\right.}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1024*a*cos(8*d*x + 8*c)/d - 1/128*a*cos(6*d*x + 6*c)/d - 7/256*a*cos(4*d
*x + 4*c)/d - 7/128*a*cos(2*d*x + 2*c)/d + 1/448*a*sin(7*d*x + 7*c)/d + 7/3
20*a*sin(5*d*x + 5*c)/d + 7/64*a*sin(3*d*x + 3*c)/d + 35/64*a*sin(d*x + c)/
d
```

maple [A] time = 0.14, size = 56, normalized size = 0.64

$$\frac{-\frac{a\left(\cos^8(dx+c)\right)}{8}+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/8*a*cos(d*x+c)^8+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(
d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.40, size = 92, normalized size = 1.06

$$-\frac{35\,a\sin\left(dx+c\right)^{8}+40\,a\sin\left(dx+c\right)^{7}-140\,a\sin\left(dx+c\right)^{6}-168\,a\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}+280\,a}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/280*(35*a*sin(d*x + c)^8 + 40*a*sin(d*x + c)^7 - 140*a*sin(d*x + c)^6 -
168*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 + 280*a*sin(d*x + c)^3 - 140*a*
sin(d*x + c)^2 - 280*a*sin(d*x + c))/d
```

mupad [B] time = 0.09, size = 90, normalized size = 1.03

$$\frac{-\frac{a\sin(c+d\,x)^8}{8}-\frac{a\sin(c+d\,x)^7}{7}+\frac{a\sin(c+d\,x)^6}{2}+\frac{3\,a\sin(c+d\,x)^5}{5}-\frac{3\,a\sin(c+d\,x)^4}{4}-a\sin\left(c+d\,x\right)^3+\frac{a\sin(c+d\,x)^2}{2}+a\,\sin\left(c+d\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) + (a*sin(c + d*x)^2)/2 - a*sin(c + d*x)^3 - (3*a*sin(c + d*
x)^4)/4 + (3*a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/2 - (a*sin(c + d*x)^7
)/7 - (a*sin(c + d*x)^8)/8)/d
```

sympy [A] time = 10.29, size = 105, normalized size = 1.21

$$\begin{cases} \frac{16a\sin^{7}(c+dx)}{35d}+\frac{8a\sin^{5}(c+dx)\cos^{2}(c+dx)}{5d}+\frac{2a\sin^{3}(c+dx)\cos^{4}(c+dx)}{d}+\frac{a\sin(c+dx)\cos^{6}(c+dx)}{d}-\frac{a\cos^{8}(c+dx)}{8d} & \text{for } d\neq 0 \\ x\left(a\sin(c)+a\right)\cos^{7}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**7/(35*d) + 8*a*sin(c + d*x)**5*cos(c + d*x)**
2/(5*d) + 2*a*sin(c + d*x)**3*cos(c + d*x)**4/d + a*sin(c + d*x)*cos(c + d*
x)**6/d - a*cos(c + d*x)**8/(8*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**7,
True))
```

3.2 $\int \cos^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}$$

```
[Out] 5/16*a*x-1/7*a*cos(d*x+c)^7/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x
+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 2635, 8}

$$-\frac{a\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/16 - (a*Cos[c + d*x]^7)/(7*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16
*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c +
d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+a\sin(c+dx))\,dx &= -\frac{a\cos^7(c+dx)}{7d}+a\int \cos^6(c+dx)\,dx\\ &= -\frac{a\cos^7(c+dx)}{7d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}(5a)\int \cos^4(c+dx)\,dx\\ &= -\frac{a\cos^7(c+dx)}{7d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+dx)\sin(c+d}{6d}\\ &= -\frac{a\cos^7(c+dx)}{7d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+d}{24d}\\ &= \frac{5ax}{16}-\frac{a\cos^7(c+dx)}{7d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(}{24d}\end{aligned}$$

Mathematica [A] time = 0.16, size = 57, normalized size = 0.66

$$\frac{a\left(7(45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))+60c+60dx)-192\cos^7(c+dx)\right)}{1344d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-192*Cos[c + d*x]^7 + 7*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*
(c + d*x)] + Sin[6*(c + d*x)])))/(1344*d)
```

fricas [A] time = 0.62, size = 62, normalized size = 0.71

$$-\frac{48\,a\cos\left(dx+c\right)^{7}-105\,adx-7\left(8\,a\cos\left(dx+c\right)^{5}+10\,a\cos\left(dx+c\right)^{3}+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/336*(48*a*cos(d*x + c)^7 - 105*a*d*x - 7*(8*a*cos(d*x + c)^5 + 10*a*cos(
d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 1.29, size = 107, normalized size = 1.23

$$\frac{5}{16}\,ax-\frac{a\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{a\cos\left(5\,dx+5\,c\right)}{64\,d}-\frac{3\,a\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{5\,a\cos\left(dx+c\right)}{64\,d}+\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{3\,a\sin\left(4\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*a*x - 1/448*a*cos(7*d*x + 7*c)/d - 1/64*a*cos(5*d*x + 5*c)/d - 3/64*a*
cos(3*d*x + 3*c)/d - 5/64*a*cos(d*x + c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 3
/64*a*sin(4*d*x + 4*c)/d + 15/64*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.14, size = 62, normalized size = 0.71

$$\frac{-\frac{\left(\cos^{7}\left(dx+c\right)\right)a}{7}+a\left(\frac{\left(\cos^{5}\left(dx+c\right)+\frac{5\left(\cos^{3}\left(dx+c\right)\right)}{4}+\frac{15\cos\left(dx+c\right)}{8}\right)\sin\left(dx+c\right)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/7*cos(d*x+c)^7*a+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x
+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

maxima [A] time = 0.63, size = 63, normalized size = 0.72

$$-\frac{192\,a\cos\left(dx+c\right)^{7}+7\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a}{1344\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/1344*(192*a*cos(d*x + c)^7 + 7*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 8.23, size = 226, normalized size = 2.60

$$\frac{5\,a\,x}{16}+\frac{-\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+\left(\frac{a\,\left(735\,c+735\,d\,x-672\right)}{336}-\frac{35\,a\,\left(c+d\,x\right)}{16}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}-\frac{85\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\left(\frac{a\,(\ldots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x)),x)
```

```
[Out] (5*a*x)/16 + ((a*(105*c + 105*d*x - 96))/336 + (11*a*tan(c/2 + (d*x)/2))/8
- (5*a*(c + d*x))/16 + tan(c/2 + (d*x)/2)^12*((a*(735*c + 735*d*x - 672))/3
36 - (35*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^4*((a*(2205*c + 2205*d*x - 2
016))/336 - (105*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^8*((a*(3675*c + 3675
*d*x - 3360))/336 - (175*a*(c + d*x))/16) + (7*a*tan(c/2 + (d*x)/2)^3)/6 +
(85*a*tan(c/2 + (d*x)/2)^5)/24 - (85*a*tan(c/2 + (d*x)/2)^9)/24 - (7*a*tan(
c/2 + (d*x)/2)^11)/6 - (11*a*tan(c/2 + (d*x)/2)^13)/8)/(d*(tan(c/2 + (d*x)/
2)^2 + 1)^7)
```

sympy [A] time = 6.70, size = 172, normalized size = 1.98

$$\begin{cases} \frac{5ax\sin^{6}(c+dx)}{16} + \frac{15ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16} + \frac{15ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16} + \frac{5ax\cos^{6}(c+dx)}{16} + \frac{5a\sin^{5}(c+dx)\cos(c+dx)}{16d} + \frac{5a\sin^{3}(c+dx}{6\cdots} \\ x\left(a\sin(c)+a\right)\cos^{6}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16
 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c + d*
x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - a*cos(c + d*x)**7/
(7*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**6, True))
```

3.3 $\int \cos^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=64

$$\frac{(a\sin(c+dx)+a)^6}{6a^5d}-\frac{4(a\sin(c+dx)+a)^5}{5a^4d}+\frac{(a\sin(c+dx)+a)^4}{a^3d}$$

```
[Out] (a+a*sin(d*x+c))^4/a^3/d-4/5*(a+a*sin(d*x+c))^5/a^4/d+1/6*(a+a*sin(d*x+c))^
6/a^5/d
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^6}{6a^5d}-\frac{4(a\sin(c+dx)+a)^5}{5a^4d}+\frac{(a\sin(c+dx)+a)^4}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a + a*Sin[c + d*x])^4/(a^3*d) - (4*(a + a*Sin[c + d*x])^5)/(5*a^4*d) + (a
+ a*Sin[c + d*x])^6/(6*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^3-4a(a+x)^4+(a+x)^5\right)dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{(a+a\sin(c+dx))^4}{a^3d}-\frac{4(a+a\sin(c+dx))^5}{5a^4d}+\frac{(a+a\sin(c+dx))^6}{6a^5d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.94

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cos[c + d*x]^6)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.76, size = 51, normalized size = 0.80

$$-\frac{5\,a\cos\left(dx+c\right)^{6}-2\,\left(3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(5*a*cos(d*x + c)^6 - 2*(3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*
a)*sin(d*x + c))/d
```

giac [A] time = 0.75, size = 88, normalized size = 1.38

$$-\frac{a\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{a\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*a*cos(6*d*x + 6*c)/d - 1/32*a*cos(4*d*x + 4*c)/d - 5/64*a*cos(2*d*x
+ 2*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 5/8*a*si
n(d*x + c)/d
```

maple [A] time = 0.14, size = 46, normalized size = 0.72

$$\frac{-\frac{\left(\cos^{6}(dx+c)\right)a}{6}+\frac{a\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/6*cos(d*x+c)^6*a+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+
c))
```

maxima [A] time = 0.32, size = 70, normalized size = 1.09

$$\frac{5\,a\sin\left(dx+c\right)^{6}+6\,a\sin\left(dx+c\right)^{5}-15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}+15\,a\sin\left(dx+c\right)^{2}+30\,a\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 - 15*a*sin(d*x + c)^4 - 20*a*
sin(d*x + c)^3 + 15*a*sin(d*x + c)^2 + 30*a*sin(d*x + c))/d
```

mupad [B] time = 0.05, size = 68, normalized size = 1.06

$$\frac{\frac{a\sin(c+d\,x)^6}{6}+\frac{a\sin(c+d\,x)^5}{5}-\frac{a\sin(c+d\,x)^4}{2}-\frac{2\,a\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}+a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) + (a*sin(c + d*x)^2)/2 - (2*a*sin(c + d*x)^3)/3 - (a*sin(c
+ d*x)^4)/2 + (a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/6)/d
```

sympy [A] time = 3.65, size = 83, normalized size = 1.30

$$\begin{cases} \frac{8a\sin^5(c+dx)}{15d} + \frac{4a\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^4(c+dx)}{d} - \frac{a\cos^6(c+dx)}{6d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d - a*cos(c + d*x)**6/(6*d), Ne(d,
0)), (x*(a*sin(c) + a)*cos(c)**5, True))
```

3.4 $\int \cos^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[Out] 3/8*a*x-1/5*a*cos(d*x+c)^5/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2669, 2635, 8}

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (a*Cos[c + d*x]^5)/(5*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c+dx)(a+a\sin(c+dx))\,dx &= -\frac{a\cos^5(c+dx)}{5d}+a\int \cos^4(c+dx)\,dx\\
&= -\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3a)\int \cos^2(c+dx)\,dx\\
&= -\frac{a\cos^5(c+dx)}{5d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}\\
&= \frac{3ax}{8}-\frac{a\cos^5(c+dx)}{5d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}-\frac{a\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (a*Cos[c + d*x]^5)/(5*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.61, size = 51, normalized size = 0.78

$$-\frac{8\,a\cos\left(dx+c\right)^{5}-15\,adx-5\left(2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40*(8*a*cos(d*x + c)^5 - 15*a*d*x - 5*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x
+ c))*sin(d*x + c))/d
```

giac [A] time = 0.69, size = 77, normalized size = 1.18

$$\frac{3}{8}\,ax-\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x - 1/80*a*cos(5*d*x + 5*c)/d - 1/16*a*cos(3*d*x + 3*c)/d - 1/8*a*cos
(d*x + c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.14, size = 52, normalized size = 0.80

$$\frac{-\frac{\left(\cos^{5}\left(dx+c\right)\right)a}{5}+a\left(\frac{\left(\cos^{3}\left(dx+c\right)+\frac{3\cos\left(dx+c\right)}{2}\right)\sin\left(dx+c\right)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/5*cos(d*x+c)^5*a+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
8*d*x+3/8*c))
```

maxima [A] time = 0.33, size = 48, normalized size = 0.74

$$-\frac{32\,a\cos\left(dx+c\right)^{5}-5\,\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/160*(32*a*cos(d*x + c)^5 - 5*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2
*d*x + 2*c))*a)/d
```

mupad [B] time = 7.99, size = 165, normalized size = 2.54

$$\frac{3\,a\,x}{8}+\frac{-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\left(\frac{a\,\left(75\,c+75\,d\,x-80\right)}{40}-\frac{15\,a\,\left(c+d\,x\right)}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\left(\frac{a\,\left(150\,c+150\,d\,x-160\right)}{40}-\frac{15\,a\,(c+}{4}\right.}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + ((a*(15*c + 15*d*x - 16))/40 + (5*a*tan(c/2 + (d*x)/2))/4 - (3*
a*(c + d*x))/8 + tan(c/2 + (d*x)/2)^8*((a*(75*c + 75*d*x - 80))/40 - (15*a*
(c + d*x))/8) + tan(c/2 + (d*x)/2)^4*((a*(150*c + 150*d*x - 160))/40 - (15*
a*(c + d*x))/4) + (a*tan(c/2 + (d*x)/2)^3)/2 - (a*tan(c/2 + (d*x)/2)^7)/2 -
 (5*a*tan(c/2 + (d*x)/2)^9)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 2.18, size = 124, normalized size = 1.91

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{a\cos^5(c+dx)}{5d} & \text{fo} \\ x(a\sin(c)+a)\cos^4(c) & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) - a*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x
*(a*sin(c) + a)*cos(c)**4, True))
```

3.5 $\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=45

$$\frac{2(a\sin(c+dx)+a)^3}{3a^2d}-\frac{(a\sin(c+dx)+a)^4}{4a^3d}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^3/a^2/d-1/4*(a+a*sin(d*x+c))^4/a^3/d
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^3}{3a^2d}-\frac{(a\sin(c+dx)+a)^4}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - (a + a*Sin[c + d*x])^4/(4*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(2a(a+x)^2-(a+x)^3\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{2(a+a\sin(c+dx))^3}{3a^2d}-\frac{(a+a\sin(c+dx))^4}{4a^3d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.98

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*(a*Cos[c + d*x]^4)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.74, size = 39, normalized size = 0.87

$$-\frac{3\,a\cos\left(dx+c\right)^{4}-4\,\left(a\cos\left(dx+c\right)^{2}+2\,a\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*cos(d*x + c)^4 - 4*(a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

giac [A] time = 0.60, size = 48, normalized size = 1.07

$$-\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-6\,a\sin\left(dx+c\right)^{2}-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

maple [A] time = 0.13, size = 36, normalized size = 0.80

$$\frac{-\frac{\left(\cos^{4}\left(dx+c\right)\right)a}{4}+\frac{a\left(2+\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/4*cos(d*x+c)^4*a+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 48, normalized size = 1.07

$$-\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-6\,a\sin\left(dx+c\right)^{2}-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

mupad [B] time = 0.06, size = 46, normalized size = 1.02

$$\frac{-\frac{a\sin(c+d\,x)^4}{4}-\frac{a\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}+a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) + (a*sin(c + d*x)^2)/2 - (a*sin(c + d*x)^3)/3 - (a*sin(c +
d*x)^4)/4)/d
```

sympy [A] time = 1.02, size = 60, normalized size = 1.33

$$\begin{cases} \frac{2a\sin^{3}\left(c+dx\right)}{3d}+\frac{a\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}-\frac{a\cos^{4}\left(c+dx\right)}{4d} & \text{for}\; d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)\cos^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d - a
*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**3, True))
```

3.6 $\int \cos^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] 1/2*a*x-1/3*a*cos(d*x+c)^3/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 2635, 8}

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/2 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(a+a\sin(c+dx))\,dx &= -\frac{a\cos^3(c+dx)}{3d}+a\int \cos^2(c+dx)\,dx\\
&= -\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\
&= \frac{ax}{2}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 1.07

$$\frac{a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}-\frac{a\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) - (a*Cos[c + d*x]^3)/(3*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.56, size = 37, normalized size = 0.86

$$-\frac{2\,a\cos\left(dx+c\right)^{3}-3\,adx-3\,a\cos\left(dx+c\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*cos(d*x + c)^3 - 3*a*d*x - 3*a*cos(d*x + c)*sin(d*x + c))/d
```

giac [A] time = 0.80, size = 47, normalized size = 1.09

$$\frac{1}{2}\,ax-\frac{a\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{a\cos\left(dx+c\right)}{4\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x - 1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d + 1/4*a*sin(2*d*
x + 2*c)/d
```

maple [A] time = 0.08, size = 41, normalized size = 0.95

$$\frac{-\frac{\left(\cos^{3}(dx+c)\right)a}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/3*cos(d*x+c)^3*a+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.32, size = 37, normalized size = 0.86

$$-\frac{4\,a\cos\left(dx+c\right)^{3}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*a*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 6.76, size = 103, normalized size = 2.40

$$\frac{a\,x}{2}+\frac{-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+\left(\frac{a\,(9\,c+9\,d\,x-12)}{6}-\frac{3\,a\,(c+d\,x)}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\,(3\,c+3\,d\,x-4)}{6}-\frac{a\,(c+d\,x)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/2 + ((a*(3*c + 3*d*x - 4))/6 + a*tan(c/2 + (d*x)/2) - (a*(c + d*x))/2
 + tan(c/2 + (d*x)/2)^4*((a*(9*c + 9*d*x - 12))/6 - (3*a*(c + d*x))/2) - a*
tan(c/2 + (d*x)/2)^5)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

sympy [A] time = 0.53, size = 71, normalized size = 1.65

$$\begin{cases} \frac{ax\sin^2{(c+dx)}}{2} + \frac{ax\cos^2{(c+dx)}}{2} + \frac{a\sin{(c+dx)}\cos{(c+dx)}}{2d} - \frac{a\cos^3{(c+dx)}}{3d} & \text{for } d \neq 0 \\ x\left(a\sin{(c)} + a\right)\cos^2{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) - a*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)*c
os(c)**2, True))
```

3.7 $\int \cos(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=22

$$\frac{(a\sin(c+dx)+a)^2}{2ad}$$

```
[Out] 1/2*(a+a*sin(d*x+c))^2/a/d
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2667}

$$\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{a\sin(c+dx)}{d}+\frac{a\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.77

$$-\frac{a\cos^2(c+dx)}{2d}+\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*(a*Cos[c + d*x]^2)/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

fricas [A] time = 0.69, size = 25, normalized size = 1.14

$$-\frac{a\cos\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)^2 - 2*a*sin(d*x + c))/d
```

giac [A] time = 0.50, size = 25, normalized size = 1.14

$$\frac{a\sin(dx+c)^2 + 2\,a\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*(a*sin(d*x + c)^2 + 2*a*sin(d*x + c))/d
```

maple [A] time = 0.04, size = 25, normalized size = 1.14

$$\frac{\frac{a\left(\sin^2(dx+c)\right)}{2} + a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c)),x)
[Out] 1/d*(1/2*a*sin(d*x+c)^2+a*sin(d*x+c))
```

maxima [A] time = 0.35, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^2}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(a*sin(d*x + c) + a)^2/(a*d)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.91

$$\frac{a\,\sin(c+d\,x)\,\left(\sin(c+d\,x)+2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x)),x)
[Out] (a*sin(c + d*x)*(sin(c + d*x) + 2))/(2*d)
```

sympy [A] time = 0.22, size = 34, normalized size = 1.55

$$\begin{cases} \frac{a\sin^2(c+dx)}{2d} + \frac{a\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x\,(a\sin(c)+a)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x)
[Out] Piecewise((a*sin(c + d*x)**2/(2*d) + a*sin(c + d*x)/d, Ne(d, 0)), (x*(a*sin
(c) + a)*cos(c), True))
```

3.8 $\int \sec(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=17

$$-\frac{a\log(1-\sin(c+dx))}{d}$$

```
[Out] -a*ln(1-sin(d*x+c))/d
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2667, 31}

$$-\frac{a\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[1 - Sin[c + d*x]])/d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a\,\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{a\log(1-\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.53

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d
```

fricas [A] time = 0.67, size = 17, normalized size = 1.00

$$-\frac{a\log\left(-\sin\left(dx+c\right)+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -a*log(-sin(d*x + c) + 1)/d
```

giac [B] time = 0.94, size = 37, normalized size = 2.18

$$\frac{a \log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1\right) - 2\,a \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))
)/d
```

maple [A] time = 0.08, size = 16, normalized size = 0.94

$$-\frac{a \ln\left(\sin\left(dx + c\right) - 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a*ln(sin(d*x+c)-1)
```

maxima [A] time = 0.35, size = 15, normalized size = 0.88

$$-\frac{a \log\left(\sin\left(dx + c\right) - 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -a*log(sin(d*x + c) - 1)/d
```

mupad [B] time = 0.05, size = 15, normalized size = 0.88

$$-\frac{a\,\ln\left(\sin\left(c + d\,x\right) - 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/cos(c + d*x),x)
```

```
[Out] -(a*log(sin(c + d*x) - 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \sin\left(c + dx\right) \sec\left(c + dx\right)\, dx + \int \sec\left(c + dx\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

3.9 $\int \sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=23

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

```
[Out] a*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a\sec(c+dx)}{d}+a\int \sec^2(c+dx)\,dx\\ &= \frac{a\sec(c+dx)}{d}-\frac{a\,\mathrm{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

fricas [A] time = 0.55, size = 40, normalized size = 1.74

$$\frac{a\cos\left(dx+c\right)+a\sin\left(dx+c\right)+a}{d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*cos(d*x + c) + a*sin(d*x + c) + a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.47, size = 19, normalized size = 0.83

$$-\frac{2\,a}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*a/(d*(tan(1/2*d*x + 1/2*c) - 1))
```

maple [A] time = 0.14, size = 24, normalized size = 1.04

$$\frac{\frac{a}{\cos(dx+c)}+a\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a/cos(d*x+c)+a*tan(d*x+c))
```

maxima [A] time = 0.49, size = 23, normalized size = 1.00

$$\frac{a\tan\left(dx+c\right)+\frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*tan(d*x + c) + a/cos(d*x + c))/d
```

mupad [B] time = 4.69, size = 19, normalized size = 0.83

$$-\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/cos(c + d*x)^2,x)
```

```
[Out] -(2*a)/(d*(tan(c/2 + (d*x)/2) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\int\sec^{2}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x)
)
```

3.10 $\int \sec^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=39

$$\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+1/2*a^2/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + a^2/(2*d*(a - a*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \sec^3(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a^3\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^2}+\frac{1}{2a\left(a^2-x^2\right)}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx,x,a\sin(c+dx)\right)}{2d}\\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2}{2d(a-a\sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.33

$$\frac{a\sec^2(c+dx)}{2d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]^2)/(2*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

fricas [A] time = 1.05, size = 67, normalized size = 1.72

$$\frac{(a\sin(dx+c)-a)\log(\sin(dx+c)+1)-(a\sin(dx+c)-a)\log(-\sin(dx+c)+1)-2a}{4(d\sin(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((a*sin(d*x + c) - a)*log(sin(d*x + c) + 1) - (a*sin(d*x + c) - a)*log(
-sin(d*x + c) + 1) - 2*a)/(d*sin(d*x + c) - d)
```

giac [A] time = 0.54, size = 54, normalized size = 1.38

$$\frac{a\log(|\sin(dx+c)+1|)-a\log(|\sin(dx+c)-1|)+\frac{a\sin(dx+c)-3a}{\sin(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(a*log(abs(sin(d*x + c) + 1)) - a*log(abs(sin(d*x + c) - 1)) + (a*sin(d
*x + c) - 3*a)/(sin(d*x + c) - 1))/d
```

maple [A] time = 0.17, size = 54, normalized size = 1.38

$$\frac{a}{2d\cos(dx+c)^2}+\frac{a\sec(dx+c)\tan(dx+c)}{2d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a/cos(d*x+c)^2+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+ta
n(d*x+c))
```

maxima [A] time = 0.33, size = 42, normalized size = 1.08

$$\frac{a\log(\sin(dx+c)+1)-a\log(\sin(dx+c)-1)-\frac{2a}{\sin(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*a/(sin(d*x + c)
- 1))/d
```

mupad [B] time = 0.06, size = 30, normalized size = 0.77

$$\frac{a\,\mathrm{atanh}(\sin(c+d\,x))}{2\,d}-\frac{a}{2\,d\,(\sin(c+d\,x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(sin(c + d*x)))/(2*d) - a/(2*d*(sin(c + d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\int \sec^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x)
)
```

3.11 $\int \sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}$$

```
[Out] 1/3*a*sec(d*x+c)^3/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2669, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a\sec^3(c+dx)}{3d}+a\int \sec^4(c+dx)\,dx\\ &= \frac{a\sec^3(c+dx)}{3d}-\frac{a\,\mathrm{Subst}\left(\int \left(1+x^2\right)dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{a\sec^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 41, normalized size = 0.93

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{a\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.64, size = 52, normalized size = 1.18

$$-\frac{2\,a\cos\left(dx+c\right)^2+2\,a\sin\left(dx+c\right)-a}{3\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*cos(d*x + c)^2 + 2*a*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x +
c) - d*cos(d*x + c))
```

giac [A] time = 0.36, size = 66, normalized size = 1.50

$$-\frac{\frac{3a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}+\frac{9a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7a}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*a/(tan(1/2*d*x + 1/2*c) + 1) + (9*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*t
an(1/2*d*x + 1/2*c) + 7*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

maple [A] time = 0.16, size = 38, normalized size = 0.86

$$\frac{\frac{a}{3\cos(dx+c)^3}-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a/cos(d*x+c)^3-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

maxima [A] time = 0.38, size = 35, normalized size = 0.80

$$\frac{\left(\tan(dx+c)^3+3\tan(dx+c)\right)a+\frac{a}{\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a + a/cos(d*x + c)^3)/d
```

mupad [B] time = 4.60, size = 63, normalized size = 1.43

$$\frac{2a\left(\cos(c+dx)+2\sin(c+dx)+\cos(2c+2dx)-\frac{\sin(2c+2dx)}{2}\right)}{3d\left(2\cos(c+dx)-\sin(2c+2dx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (2*a*(cos(c + d*x) + 2*sin(c + d*x) + cos(2*c + 2*d*x) - sin(2*c + 2*d*x)/2
))/(3*d*(2*cos(c + d*x) - sin(2*c + 2*d*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin(c+dx)\sec^4(c+dx)\,dx+\int\sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x)
)
```

3.12 $\int \sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=84

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2+1/4*a^2/d/(a-a*sin
(d*x+c))-1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2667, 44, 206}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/
(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^3}+\frac{1}{4a^3(a-x)^2}+\frac{1}{8a^3(a+x)^2}+\frac{3}{8a^3\left(a^2-x^2\right)}\right)dx,x,a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a+a\sin(c+dx))}+$$

$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{4d(a-a\sin(c+dx))}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 0.81

$$\frac{a\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

fricas [A] time = 0.80, size = 136, normalized size = 1.62

$$-\frac{6\,a\cos\left(dx+c\right)^2-3\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(a\cos\left(dx+c\right)^2\right.}{16\left(d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-d\cos\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) + 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 6*a*sin(d*x + c) - 2*a)/(d*cos(d*x + c)^2*
sin(d*x + c) - d*cos(d*x + c)^2)
```

giac [A] time = 0.51, size = 92, normalized size = 1.10

$$\frac{6\,a\log\left(|\sin\left(dx+c\right)+1|\right)-6\,a\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\,(3\,a\sin(dx+c)+5\,a)}{\sin(dx+c)+1}+\frac{9\,a\sin(dx+c)^2-26\,a\sin(dx+c)+21\,a}{(\sin(dx+c)-1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*a*log(abs(sin(d*x + c) + 1)) - 6*a*log(abs(sin(d*x + c) - 1)) - 2*(
3*a*sin(d*x + c) + 5*a)/(sin(d*x + c) + 1) + (9*a*sin(d*x + c)^2 - 26*a*sin
(d*x + c) + 21*a)/(sin(d*x + c) - 1)^2)/d
```

maple [A] time = 0.18, size = 74, normalized size = 0.88

$$\frac{a}{4d\cos\left(dx+c\right)^4}+\frac{a\tan\left(dx+c\right)\left(\sec^3\left(dx+c\right)\right)}{4d}+\frac{3a\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right.\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/cos(d*x+c)^4+1/4/d*a*tan(d*x+c)*sec(d*x+c)^3+3/8*a*sec(d*x+c)*tan(d
*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.53, size = 86, normalized size = 1.02

$$\frac{3\,a\log\left(\sin\left(dx+c\right)+1\right)-3\,a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,a\sin(dx+c)^2-3\,a\sin(dx+c)-2\,a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*a*log(sin(d*x + c) + 1) - 3*a*log(sin(d*x + c) - 1) - 2*(3*a*sin(d*
x + c)^2 - 3*a*sin(d*x + c) - 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d
*x + c) + 1))/d
```

mupad [B] time = 4.50, size = 71, normalized size = 0.85

$$\frac{3\,a\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{8\,d}-\frac{-\frac{3\,a\,\sin\left(c+d\,x\right)^2}{8}+\frac{3\,a\,\sin\left(c+d\,x\right)}{8}+\frac{a}{4}}{d\,\left(-\sin\left(c+d\,x\right)^3+\sin\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(sin(c + d*x)))/(8*d) - (a/4 + (3*a*sin(c + d*x))/8 - (3*a*sin(c
+ d*x)^2)/8)/(d*(sin(c + d*x) + sin(c + d*x)^2 - sin(c + d*x)^3 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin\left(c+dx\right)\sec^5\left(c+dx\right)dx+\int \sec^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**5, x)
)
```

3.13 $\int \cos^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$-\frac{9a^2\cos^7(c+dx)}{56d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{8d}+\frac{3a^2\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{15a^2\sin(c+dx)\cos^3(c}{64d}$$

```
[Out] 45/128*a^2*x-9/56*a^2*cos(d*x+c)^7/d+45/128*a^2*cos(d*x+c)*sin(d*x+c)/d+15/
64*a^2*cos(d*x+c)^3*sin(d*x+c)/d+3/16*a^2*cos(d*x+c)^5*sin(d*x+c)/d-1/8*cos
(d*x+c)^7*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A] time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2678, 2669, 2635, 8}

$$-\frac{9a^2\cos^7(c+dx)}{56d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{8d}+\frac{3a^2\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{15a^2\sin(c+dx)\cos^3(c}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (45*a^2*x)/128 - (9*a^2*Cos[c + d*x]^7)/(56*d) + (45*a^2*Cos[c + d*x]*Sin[c
 + d*x])/(128*d) + (15*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (3*a^2*Cos
[c + d*x]^5*Sin[c + d*x])/(16*d) - (Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x])
)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{8d}+\frac{1}{8}(9a)\int \cos^6(c+dx)(a+a\sin(c$$
$$= -\frac{9a^2\cos^7(c+dx)}{56d}-\frac{\cos^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{8d}+\frac{1}{8}\left(9a^2\right)\int \cos$$
$$= -\frac{9a^2\cos^7(c+dx)}{56d}+\frac{3a^2\cos^5(c+dx)\sin(c+dx)}{16d}-\frac{\cos^7(c+dx)\left(a^2+a\right.}{8d}$$
$$= -\frac{9a^2\cos^7(c+dx)}{56d}+\frac{15a^2\cos^3(c+dx)\sin(c+dx)}{64d}+\frac{3a^2\cos^5(c+dx)\,\mathrm{si}}{16d}$$
$$= -\frac{9a^2\cos^7(c+dx)}{56d}+\frac{45a^2\cos(c+dx)\sin(c+dx)}{128d}+\frac{15a^2\cos^3(c+dx)\,\mathrm{s}}{64d}$$
$$= \frac{45a^2x}{128}-\frac{9a^2\cos^7(c+dx)}{56d}+\frac{45a^2\cos(c+dx)\sin(c+dx)}{128d}+\frac{15a^2\cos^3(c}{}$$

Mathematica [A] time = 1.52, size = 171, normalized size = 1.36

$$-\frac{a^2\left(630\sqrt{1-\sin(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\,\left(112\sin^8(c+dx)+144\sin^7(c+dx)-424\sin\right.\right.}{896d(\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/896*(a^2*Cos[c + d*x]^7*(630*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt
[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(256 - 837*Sin[c + d*x] - 187*S
in[c + d*x]^2 + 978*Sin[c + d*x]^3 + 558*Sin[c + d*x]^4 - 600*Sin[c + d*x]^
5 - 424*Sin[c + d*x]^6 + 144*Sin[c + d*x]^7 + 112*Sin[c + d*x]^8)))/(d*(-1
+ Sin[c + d*x])^4*(1 + Sin[c + d*x])^(7/2))
```

fricas [A] time = 0.96, size = 85, normalized size = 0.67

$$-\frac{256\,a^2\cos\left(dx+c\right)^7-315\,a^2dx+7\left(16\,a^2\cos\left(dx+c\right)^7-24\,a^2\cos\left(dx+c\right)^5-30\,a^2\cos\left(dx+c\right)^3-45\,a^2\cos\left(d\right.\right.}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/896*(256*a^2*cos(d*x + c)^7 - 315*a^2*d*x + 7*(16*a^2*cos(d*x + c)^7 - 2
4*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x + c)^3 - 45*a^2*cos(d*x + c))*sin(d*x
 + c))/d
```

giac [A] time = 0.52, size = 123, normalized size = 0.98

$$\frac{45}{128}\,a^2x-\frac{a^2\cos\left(7\,dx+7\,c\right)}{224\,d}-\frac{a^2\cos\left(5\,dx+5\,c\right)}{32\,d}-\frac{3\,a^2\cos\left(3\,dx+3\,c\right)}{32\,d}-\frac{5\,a^2\cos\left(dx+c\right)}{32\,d}-\frac{a^2\sin\left(8\,dx+8\,c\right)}{1024\,d}+\frac{5\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 45/128*a^2*x - 1/224*a^2*cos(7*d*x + 7*c)/d - 1/32*a^2*cos(5*d*x + 5*c)/d -
 3/32*a^2*cos(3*d*x + 3*c)/d - 5/32*a^2*cos(d*x + c)/d - 1/1024*a^2*sin(8*d
*x + 8*c)/d + 5/128*a^2*sin(4*d*x + 4*c)/d + 1/4*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.18, size = 129, normalized size = 1.02

$$\frac{a^2\left(-\frac{\sin(dx+c)(\cos^7(dx+c))}{8}+\frac{\left(\cos^5(dx+c)+\frac{5(\cos^3(dx+c))}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{2a^2(\cos^7(dx+c))}{7}+a^2\left(\left(\cos^5(dx+c)+\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-2/7*a^2*cos(d*x+c)^7+a^2*(1/
6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*
c))
```

maxima [A] time = 0.61, size = 115, normalized size = 0.91

$$-\frac{6144\,a^2\cos(dx+c)^7-7\left(64\,\sin(2\,dx+2\,c)^3+120\,dx+120\,c-3\,\sin(8\,dx+8\,c)-24\,\sin(4\,dx+4\,c)\right)a^2+}{21504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/21504*(6144*a^2*cos(d*x + c)^7 - 7*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 12
0*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a^2 + 112*(4*sin(2*d*x + 2*
c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2)/d
```

mupad [B] time = 6.92, size = 461, normalized size = 3.66

$$\frac{45\,a^2\,x}{128}-\frac{\frac{815\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{64}-\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{64}-\frac{815\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{64}-\frac{295\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{64}+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}+\frac{295\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (45*a^2*x)/128 - ((815*a^2*tan(c/2 + (d*x)/2)^9)/64 - (3*a^2*tan(c/2 + (d*x
)/2)^5)/64 - (815*a^2*tan(c/2 + (d*x)/2)^7)/64 - (295*a^2*tan(c/2 + (d*x)/2
)^3)/64 + (3*a^2*tan(c/2 + (d*x)/2)^11)/64 + (295*a^2*tan(c/2 + (d*x)/2)^13
)/64 + (83*a^2*tan(c/2 + (d*x)/2)^15)/64 + (a^2*(315*c + 315*d*x))/896 - (a
^2*(315*c + 315*d*x - 512))/896 + tan(c/2 + (d*x)/2)^2*((a^2*(315*c + 315*d
*x))/112 - (a^2*(2520*c + 2520*d*x - 512))/896) + tan(c/2 + (d*x)/2)^14*((a
^2*(315*c + 315*d*x))/112 - (a^2*(2520*c + 2520*d*x - 3584))/896) + tan(c/2
 + (d*x)/2)^12*((a^2*(315*c + 315*d*x))/32 - (a^2*(8820*c + 8820*d*x - 3584
))/896) + tan(c/2 + (d*x)/2)^4*((a^2*(315*c + 315*d*x))/32 - (a^2*(8820*c +
 8820*d*x - 10752))/896) + tan(c/2 + (d*x)/2)^6*((a^2*(315*c + 315*d*x))/16
 - (a^2*(17640*c + 17640*d*x - 10752))/896) + tan(c/2 + (d*x)/2)^10*((a^2*(
315*c + 315*d*x))/16 - (a^2*(17640*c + 17640*d*x - 17920))/896) + tan(c/2 +
 (d*x)/2)^8*((5*a^2*(315*c + 315*d*x))/64 - (a^2*(22050*c + 22050*d*x - 179
20))/896) - (83*a^2*tan(c/2 + (d*x)/2))/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^8
)
```

sympy [A] time = 14.95, size = 398, normalized size = 3.16

$$\begin{cases}\frac{5a^2x\sin^8(c+dx)}{128}+\frac{5a^2x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{5a^2x\sin^6(c+dx)}{16}+\frac{15a^2x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16}+ \\ x\left(a\sin(c)+a\right)^2\cos^6(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**8/128 + 5*a**2*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + 5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c
+ d*x)**4/64 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 5*a**2*x*sin(
c + d*x)**2*cos(c + d*x)**6/32 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/
16 + 5*a**2*x*cos(c + d*x)**8/128 + 5*a**2*x*cos(c + d*x)**6/16 + 5*a**2*si
n(c + d*x)**7*cos(c + d*x)/(128*d) + 55*a**2*sin(c + d*x)**5*cos(c + d*x)**
3/(384*d) + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 73*a**2*sin(c + d*
x)**3*cos(c + d*x)**5/(384*d) + 5*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - 5*a**2*sin(c + d*x)*cos(c + d*x)**7/(128*d) + 11*a**2*sin(c + d*x)*cos(
c + d*x)**5/(16*d) - 2*a**2*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin(c)
+ a)**2*cos(c)**6, True))
```

3.14 $\int \cos^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=67

$$\frac{(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(a\sin(c+dx)+a)^6}{3a^4d}+\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

```
[Out] 4/5*(a+a*sin(d*x+c))^5/a^3/d-2/3*(a+a*sin(d*x+c))^6/a^4/d+1/7*(a+a*sin(d*x+
c))^7/a^5/d
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(a\sin(c+dx)+a)^6}{3a^4d}+\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(a + a*Sin[c + d*x])^6)/(3*a^4*d)
 + (a + a*Sin[c + d*x])^7/(7*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^2\,dx&=\frac{\text{Subst}\left(\int (a-x)^2(a+x)^4\,dx,x,a\sin(c+dx)\right)}{a^5d}\\&=\frac{\text{Subst}\left(\int \left(4a^2(a+x)^4-4a(a+x)^5+(a+x)^6\right)\,dx,x,a\sin(c+dx)\right)}{a^5d}\\&=\frac{4(a+a\sin(c+dx))^5}{5a^3d}-\frac{2(a+a\sin(c+dx))^6}{3a^4d}+\frac{(a+a\sin(c+dx))^7}{7a^5d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 58, normalized size = 0.87

$$-\frac{a^2(\sin(c+dx)+1)^2\left(15\sin^2(c+dx)-40\sin(c+dx)+29\right)\cos^6(c+dx)}{105d(\sin(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/105*(a^2*Cos[c + d*x]^6*(1 + Sin[c + d*x])^2*(29 - 40*Sin[c + d*x] + 15*
Sin[c + d*x]^2))/(d*(-1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.71, size = 71, normalized size = 1.06

$$-\frac{35\,a^2\cos\left(dx+c\right)^6+\left(15\,a^2\cos\left(dx+c\right)^6-24\,a^2\cos\left(dx+c\right)^4-32\,a^2\cos\left(dx+c\right)^2-64\,a^2\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(35*a^2*cos(d*x + c)^6 + (15*a^2*cos(d*x + c)^6 - 24*a^2*cos(d*x + c
)^4 - 32*a^2*cos(d*x + c)^2 - 64*a^2)*sin(d*x + c))/d
```

giac [A] time = 2.27, size = 117, normalized size = 1.75

$$-\frac{a^2\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{a^2\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{5\,a^2\cos\left(2\,dx+2\,c\right)}{32\,d}-\frac{a^2\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{a^2\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{19\,a^2\sin\left(3\right.}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*a^2*cos(6*d*x + 6*c)/d - 1/16*a^2*cos(4*d*x + 4*c)/d - 5/32*a^2*cos(2
*d*x + 2*c)/d - 1/448*a^2*sin(7*d*x + 7*c)/d + 1/320*a^2*sin(5*d*x + 5*c)/d
 + 19/192*a^2*sin(3*d*x + 3*c)/d + 45/64*a^2*sin(d*x + c)/d
```

maple [A] time = 0.17, size = 99, normalized size = 1.48

$$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{\left(\cos^6(dx+c)\right)a^2}{3}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/3*cos(d*x+c)^6*a^2+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.49, size = 95, normalized size = 1.42

$$\frac{15\,a^2\sin\left(dx+c\right)^7+35\,a^2\sin\left(dx+c\right)^6-21\,a^2\sin\left(dx+c\right)^5-105\,a^2\sin\left(dx+c\right)^4-35\,a^2\sin\left(dx+c\right)^3+105\,a^2\,\mathrm{s}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*a^2*sin(d*x + c)^7 + 35*a^2*sin(d*x + c)^6 - 21*a^2*sin(d*x + c)^
5 - 105*a^2*sin(d*x + c)^4 - 35*a^2*sin(d*x + c)^3 + 105*a^2*sin(d*x + c)^2
 + 105*a^2*sin(d*x + c))/d
```

mupad [B] time = 4.54, size = 92, normalized size = 1.37

$$\frac{\frac{a^2\sin(c+d\,x)^7}{7}+\frac{a^2\sin(c+d\,x)^6}{3}-\frac{a^2\sin(c+d\,x)^5}{5}-a^2\sin\left(c+d\,x\right)^4-\frac{a^2\sin(c+d\,x)^3}{3}+a^2\sin\left(c+d\,x\right)^2+a^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x) + a^2*sin(c + d*x)^2 - (a^2*sin(c + d*x)^3)/3 - a^2*sin(c
 + d*x)^4 - (a^2*sin(c + d*x)^5)/5 + (a^2*sin(c + d*x)^6)/3 + (a^2*sin(c +
d*x)^7)/7)/d
```

sympy [A] time = 8.34, size = 158, normalized size = 2.36

$$\begin{cases} \frac{8a^2\sin^7(c+dx)}{105d} + \frac{4a^2\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{8a^2\sin^5(c+dx)}{15d} + \frac{a^2\sin^3(c+dx)\cos^4(c+dx)}{3d} + \frac{4a^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^2\sin(c+dx \\ x\left(a\sin(c)+a\right)^2\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**7/(105*d) + 4*a**2*sin(c + d*x)**5*cos(c +
d*x)**2/(15*d) + 8*a**2*sin(c + d*x)**5/(15*d) + a**2*sin(c + d*x)**3*cos(c
 + d*x)**4/(3*d) + 4*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + a**2*sin(
c + d*x)*cos(c + d*x)**4/d - a**2*cos(c + d*x)**6/(3*d), Ne(d, 0)), (x*(a*s
in(c) + a)**2*cos(c)**5, True))
```

3.15 $\int \cos^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=102

$$-\frac{7a^2\cos^5(c+dx)}{30d}-\frac{\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{6d}+\frac{7a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{16d}$$

```
[Out] 7/16*a^2*x-7/30*a^2*cos(d*x+c)^5/d+7/16*a^2*cos(d*x+c)*sin(d*x+c)/d+7/24*a^
2*cos(d*x+c)^3*sin(d*x+c)/d-1/6*cos(d*x+c)^5*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2678, 2669, 2635, 8}

$$-\frac{7a^2\cos^5(c+dx)}{30d}-\frac{\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{6d}+\frac{7a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*a^2*x)/16 - (7*a^2*Cos[c + d*x]^5)/(30*d) + (7*a^2*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + (7*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (Cos[c + d*x]^5
*(a^2 + a^2*Sin[c + d*x]))/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{6d}+\frac{1}{6}(7a)\int \cos^4(c+dx)(a+a\sin$$
$$=-\frac{7a^2\cos^5(c+dx)}{30d}-\frac{\cos^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{6d}+\frac{1}{6}\left(7a^2\right)\int$$
$$=-\frac{7a^2\cos^5(c+dx)}{30d}+\frac{7a^2\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{\cos^5(c+dx)\left(a^2\right.}{6}$$
$$=-\frac{7a^2\cos^5(c+dx)}{30d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{7a^2\cos^3(c+dx)\sin}{24d}$$
$$=\frac{7a^2x}{16}-\frac{7a^2\cos^5(c+dx)}{30d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{7a^2\cos^3(c}{}$$

Mathematica [A] time = 0.54, size = 151, normalized size = 1.48

$$-\frac{a^2\left(\sqrt{\sin(c+dx)+1}\left(40\sin^6(c+dx)+56\sin^5(c+dx)-106\sin^4(c+dx)-182\sin^3(c+dx)+57\sin^2(c+\right.\right.}{240d(\sin(c+dx)-1)^3(\sin(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/240*(a^2*Cos[c + d*x]^5*(-210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqr
t[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-96 + 231*Sin[c + d*x] + 57*S
in[c + d*x]^2 - 182*Sin[c + d*x]^3 - 106*Sin[c + d*x]^4 + 56*Sin[c + d*x]^5
 + 40*Sin[c + d*x]^6)))/(d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

fricas [A] time = 0.98, size = 72, normalized size = 0.71

$$-\frac{96\,a^2\cos\left(dx+c\right)^5-105\,a^2dx+5\left(8\,a^2\cos\left(dx+c\right)^5-14\,a^2\cos\left(dx+c\right)^3-21\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(96*a^2*cos(d*x + c)^5 - 105*a^2*d*x + 5*(8*a^2*cos(d*x + c)^5 - 14*
a^2*cos(d*x + c)^3 - 21*a^2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.61, size = 106, normalized size = 1.04

$$\frac{7}{16}\,a^2x-\frac{a^2\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{a^2\cos\left(3\,dx+3\,c\right)}{8\,d}-\frac{a^2\cos\left(dx+c\right)}{4\,d}-\frac{a^2\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{a^2\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{17\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 7/16*a^2*x - 1/40*a^2*cos(5*d*x + 5*c)/d - 1/8*a^2*cos(3*d*x + 3*c)/d - 1/4
*a^2*cos(d*x + c)/d - 1/192*a^2*sin(6*d*x + 6*c)/d + 1/64*a^2*sin(4*d*x + 4
*c)/d + 17/64*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.18, size = 109, normalized size = 1.07

$$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{2\left(\cos^5(dx+c)\right)a^2}{5}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(d}{4}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-2/5*cos(d*x+c)^5*a^2+a^2*(1/4*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.32, size = 89, normalized size = 0.87

$$-\frac{384\,a^2\cos\left(dx+c\right)^5-5\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^2-30\left(12\,dx+12\,c+\sin\left(4\,dx\right.\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/960*(384*a^2*cos(d*x + c)^5 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c -
3*sin(4*d*x + 4*c))*a^2 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*
x + 2*c))*a^2)/d
```

mupad [B] time = 6.79, size = 349, normalized size = 3.42

$$\frac{7\,a^2\,x}{16}-\frac{\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{89\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}-\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{89\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}+\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{a^2\left(105\,c+105\,d\,x\right)}{240}-\frac{a^2\left(1\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (7*a^2*x)/16 - ((11*a^2*tan(c/2 + (d*x)/2)^5)/4 - (89*a^2*tan(c/2 + (d*x)/2
)^3)/24 - (11*a^2*tan(c/2 + (d*x)/2)^7)/4 + (89*a^2*tan(c/2 + (d*x)/2)^9)/2
4 + (9*a^2*tan(c/2 + (d*x)/2)^11)/8 + (a^2*(105*c + 105*d*x))/240 - (a^2*(1
05*c + 105*d*x - 192))/240 + tan(c/2 + (d*x)/2)^2*((a^2*(105*c + 105*d*x))/
40 - (a^2*(630*c + 630*d*x - 192))/240) + tan(c/2 + (d*x)/2)^10*((a^2*(105*
c + 105*d*x))/40 - (a^2*(630*c + 630*d*x - 960))/240) + tan(c/2 + (d*x)/2)^
8*((a^2*(105*c + 105*d*x))/16 - (a^2*(1575*c + 1575*d*x - 960))/240) + tan(
c/2 + (d*x)/2)^4*((a^2*(105*c + 105*d*x))/16 - (a^2*(1575*c + 1575*d*x - 19
20))/240) + tan(c/2 + (d*x)/2)^6*((a^2*(105*c + 105*d*x))/12 - (a^2*(2100*c
 + 2100*d*x - 1920))/240) - (9*a^2*tan(c/2 + (d*x)/2))/8)/(d*(tan(c/2 + (d*
x)/2)^2 + 1)^6)
```

sympy [A] time = 5.22, size = 287, normalized size = 2.81

$$\begin{cases} \frac{a^2x\sin^6\left(c+dx\right)}{16}+\frac{3a^2x\sin^4\left(c+dx\right)\cos^2\left(c+dx\right)}{16}+\frac{3a^2x\sin^4\left(c+dx\right)}{8}+\frac{3a^2x\sin^2\left(c+dx\right)\cos^4\left(c+dx\right)}{16}+\frac{3a^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{a^2x\cos^6\left(\right.}{16} \\ x\left(a\sin\left(c\right)+a\right)^2\cos^4\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/16 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**2*x*cos(c + d*x)*
*6/16 + 3*a**2*x*cos(c + d*x)**4/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*
d) + a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 3*a**2*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*a**2*sin(c
+ d*x)*cos(c + d*x)**3/(8*d) - 2*a**2*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*
(a*sin(c) + a)**2*cos(c)**4, True))
```

3.16 $\int \cos^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=45

$$\frac{(a\sin(c+dx)+a)^4}{2a^2d}-\frac{(a\sin(c+dx)+a)^5}{5a^3d}$$

```
[Out] 1/2*(a+a*sin(d*x+c))^4/a^2/d-1/5*(a+a*sin(d*x+c))^5/a^3/d
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^4}{2a^2d}-\frac{(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a + a*Sin[c + d*x])^4/(2*a^2*d) - (a + a*Sin[c + d*x])^5/(5*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^2\,dx&=\frac{\text{Subst}\left(\int (a-x)(a+x)^3\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{\text{Subst}\left(\int \left(2a(a+x)^3-(a+x)^4\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{(a+a\sin(c+dx))^4}{2a^2d}-\frac{(a+a\sin(c+dx))^5}{5a^3d}\end{aligned}$$

Mathematica [A] time = 0.08, size = 46, normalized size = 1.02

$$-\frac{a^2\sin(c+dx)\left(2\sin^4(c+dx)+5\sin^3(c+dx)-10\sin(c+dx)-10\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/10*(a^2*Sin[c + d*x]*(-10 - 10*Sin[c + d*x] + 5*Sin[c + d*x]^3 + 2*Sin[c
 + d*x]^4))/d
```

fricas [A] time = 0.73, size = 58, normalized size = 1.29

$$-\frac{5\,a^2\cos\left(dx+c\right)^4+2\,\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2-4\,a^2\right)\sin\left(dx+c\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/10*(5*a^2*cos(d*x + c)^4 + 2*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2
- 4*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.89, size = 56, normalized size = 1.24

$$-\frac{2\,a^2\sin\left(dx+c\right)^5+5\,a^2\sin\left(dx+c\right)^4-10\,a^2\sin\left(dx+c\right)^2-10\,a^2\sin\left(dx+c\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*a^2*sin(d*x + c)^5 + 5*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^2
- 10*a^2*sin(d*x + c))/d
```

maple [A] time = 0.16, size = 79, normalized size = 1.76

$$\frac{a^2\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{\left(\cos^4(dx+c)\right)a^2}{2}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
2*cos(d*x+c)^4*a^2+1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.41, size = 56, normalized size = 1.24

$$-\frac{2\,a^2\sin\left(dx+c\right)^5+5\,a^2\sin\left(dx+c\right)^4-10\,a^2\sin\left(dx+c\right)^2-10\,a^2\sin\left(dx+c\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*a^2*sin(d*x + c)^5 + 5*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^2
- 10*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.06, size = 53, normalized size = 1.18

$$\frac{-\frac{a^2\sin(c+d\,x)^5}{5}-\frac{a^2\sin(c+d\,x)^4}{2}+a^2\sin\left(c+d\,x\right)^2+a^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x) + a^2*sin(c + d*x)^2 - (a^2*sin(c + d*x)^4)/2 - (a^2*sin(
c + d*x)^5)/5)/d
```

sympy [A] time = 3.05, size = 107, normalized size = 2.38

$$\begin{cases} \frac{2a^2\sin^5(c+dx)}{15d} + \frac{a^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{2a^2\sin^3(c+dx)}{3d} + \frac{a^2\sin(c+dx)\cos^2(c+dx)}{d} - \frac{a^2\cos^4(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^2\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**5/(15*d) + a**2*sin(c + d*x)**3*cos(c + d*x
)**2/(3*d) + 2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)*
*2/d - a**2*cos(c + d*x)**4/(2*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*cos(c)**
3, True))
```

3.17 $\int \cos^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=78

$$-\frac{5a^2\cos^3(c+dx)}{12d}-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{5a^2x}{8}$$

```
[Out] 5/8*a^2*x-5/12*a^2*cos(d*x+c)^3/d+5/8*a^2*cos(d*x+c)*sin(d*x+c)/d-1/4*cos(d
*x+c)^3*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2678, 2669, 2635, 8}

$$-\frac{5a^2\cos^3(c+dx)}{12d}-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{5a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/8 - (5*a^2*Cos[c + d*x]^3)/(12*d) + (5*a^2*Cos[c + d*x]*Sin[c + d
*x])/(8*d) - (Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x]))/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^3(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{4d} + \frac{1}{4}(5a)\int \cos^2(c+dx)(a+a\sin$$

$$= -\frac{5a^2\cos^3(c+dx)}{12d} - \frac{\cos^3(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{4d} + \frac{1}{4}\left(5a^2\right)\int$$

$$= -\frac{5a^2\cos^3(c+dx)}{12d} + \frac{5a^2\cos(c+dx)\sin(c+dx)}{8d} - \frac{\cos^3(c+dx)\left(a^2+\right.}{4d}$$

$$= \frac{5a^2x}{8} - \frac{5a^2\cos^3(c+dx)}{12d} + \frac{5a^2\cos(c+dx)\sin(c+dx)}{8d} - \frac{\cos^3(c+dx}{}$$

Mathematica [A] time = 0.30, size = 131, normalized size = 1.68

$$-\frac{a^2\left(30\sqrt{1-\sin(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\,\left(6\sin^4(c+dx)+10\sin^3(c+dx)-7\sin^2(c\right.\right.}{24d(\sin(c+dx)-1)^2(\sin(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/24*(a^2*Cos[c + d*x]^3*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1
 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(16 - 25*Sin[c + d*x] - 7*Sin[c +
 d*x]^2 + 10*Sin[c + d*x]^3 + 6*Sin[c + d*x]^4)))/(d*(-1 + Sin[c + d*x])^2*
(1 + Sin[c + d*x])^(3/2))
```

fricas [A] time = 0.55, size = 59, normalized size = 0.76

$$-\frac{16\,a^2\cos\left(dx+c\right)^3-15\,a^2dx+3\left(2\,a^2\cos\left(dx+c\right)^3-5\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(16*a^2*cos(d*x + c)^3 - 15*a^2*d*x + 3*(2*a^2*cos(d*x + c)^3 - 5*a^2
*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.44, size = 72, normalized size = 0.92

$$\frac{5}{8}\,a^2x-\frac{a^2\cos\left(3\,dx+3\,c\right)}{6\,d}-\frac{a^2\cos\left(dx+c\right)}{2\,d}-\frac{a^2\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/8*a^2*x - 1/6*a^2*cos(3*d*x + 3*c)/d - 1/2*a^2*cos(d*x + c)/d - 1/32*a^2*
sin(4*d*x + 4*c)/d + 1/4*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.12, size = 87, normalized size = 1.12

$$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{2\left(\cos^3(dx+c)\right)a^2}{3}+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

[Out] 1/d*(a^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-2/3*cos(d*x+c)^3*a^2+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

maxima [A] time = 0.32, size = 65, normalized size = 0.83

$$-\frac{64\,a^2\cos\left(dx+c\right)^3-3\,(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right))a^2-24\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^2}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/96*(64*a^2*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2)/d

mupad [B] time = 6.71, size = 237, normalized size = 3.04

$$\frac{5\,a^2\,x}{8}-\frac{\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{a^2\,(15\,c+15\,d\,x)}{24}-\frac{a^2\,(15\,c+15\,d\,x-32)}{24}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^2\,(15\,c+}{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^2,x)

[Out] (5*a^2*x)/8 - ((11*a^2*tan(c/2 + (d*x)/2)^5)/4 - (11*a^2*tan(c/2 + (d*x)/2)^3)/4 + (3*a^2*tan(c/2 + (d*x)/2)^7)/4 + (a^2*(15*c + 15*d*x))/24 - (a^2*(15*c + 15*d*x - 32))/24 + tan(c/2 + (d*x)/2)^2*((a^2*(15*c + 15*d*x))/6 - (a^2*(60*c + 60*d*x - 32))/24) + tan(c/2 + (d*x)/2)^6*((a^2*(15*c + 15*d*x))/6 - (a^2*(60*c + 60*d*x - 96))/24) + tan(c/2 + (d*x)/2)^4*((a^2*(15*c + 15*d*x))/4 - (a^2*(90*c + 90*d*x - 96))/24) - (3*a^2*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)

sympy [A] time = 2.00, size = 180, normalized size = 2.31

$$\begin{cases}\frac{a^2x\sin^4\left(c+dx\right)}{8}+\frac{a^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{a^2x\sin^2\left(c+dx\right)}{2}+\frac{a^2x\cos^4\left(c+dx\right)}{8}+\frac{a^2x\cos^2\left(c+dx\right)}{2}+\frac{a^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{a^2\sin(c+}{} \\ x\left(a\sin\left(c\right)+a\right)^2\cos^2\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**2,x)

[Out] Piecewise((a**2*x*sin(c + d*x)**4/8 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**4/8 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + a**2*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**2*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*cos(c)**2, True))

3.18 $\int \cos(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=22

$$\frac{(a\sin(c+dx)+a)^3}{3ad}$$

```
[Out] 1/3*(a+a*sin(d*x+c))^3/a/d
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2667, 32}

$$\frac{(a\sin(c+dx)+a)^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a + a*Sin[c + d*x])^3/(3*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^3}{3ad}\end{aligned}$$

Mathematica [B] time = 0.02, size = 47, normalized size = 2.14

$$\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/d + (a^2*Sin[c + d*x]^3)/(3*d)
```

fricas [B] time = 0.69, size = 44, normalized size = 2.00

$$-\frac{3\,a^2\cos\left(dx+c\right)^2+\left(a^2\cos\left(dx+c\right)^2-4\,a^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(d*x + c)^2 + (a^2*cos(d*x + c)^2 - 4*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.74, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^3}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(a*sin(d*x + c) + a)^3/(a*d)
```

maple [A] time = 0.07, size = 21, normalized size = 0.95

$$\frac{(a+a\sin(dx+c))^3}{3da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/3*(a+a*sin(d*x+c))^3/d/a
```

maxima [A] time = 0.34, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^3}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*sin(d*x + c) + a)^3/(a*d)
```

mupad [B] time = 4.55, size = 32, normalized size = 1.45

$$\frac{a^2\,\sin(c+d\,x)\,\left(\sin(c+d\,x)^2+3\,\sin(c+d\,x)+3\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x)*(3*sin(c + d*x) + sin(c + d*x)^2 + 3))/(3*d)
```

sympy [A] time = 0.82, size = 53, normalized size = 2.41

$$\begin{cases} \frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d} & \text{for } d\neq 0 \\ x\,(a\sin(c)+a)^2\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)**2/d + a**2*sin(c
 + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**2*cos(c), True))
```

3.19 $\int \sec(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=34

$$-\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\log(1-\sin(c+dx))}{d}$$

```
[Out] -2*a^2*ln(1-sin(d*x+c))/d-a^2*sin(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$-\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Log[1 - Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{a\,\text{Subst}\left(\int \frac{a+x}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a\,\text{Subst}\left(\int \left(-1+\frac{2a}{a-x}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= -\frac{2a^2\log(1-\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.85

$$\frac{a^2(-\sin(c+dx)-2\log(1-\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-2*Log[1 - Sin[c + d*x]] - Sin[c + d*x]))/d
```

fricas [A] time = 0.60, size = 32, normalized size = 0.94

$$-\frac{2\,a^2\log\left(-\sin\left(dx+c\right)+1\right)+a^2\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] -(2*a^2*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/d
```

giac [B] time = 0.87, size = 91, normalized size = 2.68

$$\frac{2\left(a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] 2*(a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*a^2*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - (a^2*tan(1/2*d*x + 1/2*c)^2 + a^2*tan(1/2*d*x + 1/2*c) + a^2)/(tan
(1/2*d*x + 1/2*c)^2 + 1))/d
```

maple [A] time = 0.14, size = 53, normalized size = 1.56

$$-\frac{a^2\sin\left(dx+c\right)}{d}+\frac{2a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}-\frac{2a^2\ln\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^2,x)
[Out] -a^2*sin(d*x+c)/d+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))-2/d*a^2*ln(cos(d*x+c))
```

maxima [A] time = 0.39, size = 30, normalized size = 0.88

$$-\frac{2\,a^2\log\left(\sin\left(dx+c\right)-1\right)+a^2\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] -(2*a^2*log(sin(d*x + c) - 1) + a^2*sin(d*x + c))/d
```

mupad [B] time = 0.05, size = 26, normalized size = 0.76

$$-\frac{a^2\,\left(2\,\ln\left(\sin\left(c+d\,x\right)-1\right)+\sin\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x),x)
[Out] -(a^2*(2*log(sin(c + d*x) - 1) + sin(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**2,x)
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x)**2*s
ec(c + d*x), x) + Integral(sec(c + d*x), x))
```

3.20 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=38

$$\frac{2a^4\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}-a^2x$$

```
[Out] -a^2*x+2*a^4*cos(d*x+c)/d/(a^2-a^2*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2670, 2680, 8}

$$\frac{2a^4\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (2*a^4*Cos[c + d*x])/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)(a+a\sin(c+dx))^2\,dx &= a^4\int \frac{\cos^2(c+dx)}{(a-a\sin(c+dx))^2}\,dx\\
&= \frac{2a^4\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}-a^2\int 1\,dx\\
&= -a^2x+\frac{2a^4\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 1.97

$$\frac{2a^2\sqrt{\sin(c+dx)+1}\left(\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\right)\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sec[c + d*x]*Sqrt[1 + Sin[c + d*x]]*(ArcSin[Sqrt[1 - Sin[c + d*x]]/S
qrt[2]]*Sqrt[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]))/d
```

fricas [A] time = 0.60, size = 74, normalized size = 1.95

$$-\frac{a^2dx - 2\,a^2 + \left(a^2dx - 2\,a^2\right)\cos\left(dx + c\right) - \left(a^2dx + 2\,a^2\right)\sin\left(dx + c\right)}{d\cos\left(dx + c\right) - d\sin\left(dx + c\right) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x - 2*a^2 + (a^2*d*x - 2*a^2)*cos(d*x + c) - (a^2*d*x + 2*a^2)*sin(
d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [A] time = 0.55, size = 33, normalized size = 0.87

$$-\frac{(dx + c)a^2 + \frac{4\,a^2}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^2 + 4*a^2/(tan(1/2*d*x + 1/2*c) - 1))/d
```

maple [A] time = 0.18, size = 47, normalized size = 1.24

$$\frac{a^2\left(\tan\left(dx + c\right) - dx - c\right) + \frac{2a^2}{\cos(dx+c)} + a^2\tan\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a^2/cos(d*x+c)+a^2*tan(d*x+c))
```

maxima [A] time = 0.67, size = 47, normalized size = 1.24

$$-\frac{(dx + c - \tan\left(dx + c\right))a^2 - a^2\tan\left(dx + c\right) - \frac{2\,a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a^2 - a^2*tan(d*x + c) - 2*a^2/cos(d*x + c))/d
```

mupad [B] time = 4.56, size = 28, normalized size = 0.74

$$-a^2\,x - \frac{4\,a^2}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] - a^2*x - (4*a^2)/(d*(tan(c/2 + (d*x)/2) - 1))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\sec^2(c+dx)\,dx+\int \sin^2(c+dx)\sec^2(c+dx)\,dx+\int \sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

3.21 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=20

$$\frac{a^3}{d(a-a\sin(c+dx))}$$

```
[Out] a^3/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2667, 32\}$

$$\frac{a^3}{d(a-a\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^3/(d*(a - a*Sin[c + d*x]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{a^3\,\text{Subst}\left(\int \frac{1}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^3}{d(a-a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.14, size = 32, normalized size = 1.60

$$\frac{a^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2)
```

fricas [A] time = 0.77, size = 19, normalized size = 0.95

$$-\frac{a^2}{d\sin(dx+c)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] -a^2/(d*sin(d*x + c) - d)
```

giac [A] time = 0.74, size = 30, normalized size = 1.50

$$\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] 2*a^2*tan(1/2*d*x + 1/2*c)/(d*(tan(1/2*d*x + 1/2*c) - 1)^2)
```

maple [B] time = 0.21, size = 75, normalized size = 3.75

$$\frac{a^2\left(\sin^3\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^2}+\frac{a^2\sin\left(dx+c\right)}{2d}+\frac{a^2}{d\cos\left(dx+c\right)^2}+\frac{a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
[Out] 1/2/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*a^2*sin(d*x+c)/d+1/d*a^2/cos(d*x+c)
^2+1/2/d*a^2*sec(d*x+c)*tan(d*x+c)
```

maxima [A] time = 0.32, size = 18, normalized size = 0.90

$$-\frac{a^2}{d(\sin\left(dx+c\right)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] -a^2/(d*(sin(d*x + c) - 1))
```

mupad [B] time = 0.04, size = 18, normalized size = 0.90

$$-\frac{a^2}{d\,\left(\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^3,x)
[Out] -a^2/(d*(sin(c + d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x)**3, x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x))
```

3.22 $\int \sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=63

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{a^4\cos(c+dx)}{3d\left(a^2-a^2\sin(c+dx)\right)}$$

```
[Out] 1/3*a^4*cos(d*x+c)/d/(a-a*sin(d*x+c))^2+1/3*a^4*cos(d*x+c)/d/(a^2-a^2*sin(d
*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = $\{2670, 2650, 2648\}$

$$\frac{a^4\cos(c+dx)}{3d\left(a^2-a^2\sin(c+dx)\right)}+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^4*Cos[c + d*x])/(3*d*(a - a*Sin[c + d*x])^2) + (a^4*Cos[c + d*x])/(3*d*(
a^2 - a^2*Sin[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sin(c+dx))^2\,dx &= a^4\int \frac{1}{(a-a\sin(c+dx))^2}\,dx\\ &= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{1}{3}a^3\int \frac{1}{a-a\sin(c+dx)}\,dx\\ &= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{a^3\cos(c+dx)}{3d(a-a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.01, size = 58, normalized size = 0.92

$$-\frac{a^2\tan^3(c+dx)}{3d}+\frac{2a^2\sec^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/d - (a^2*T
an[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.59, size = 97, normalized size = 1.54

$$-\frac{a^2 \cos(dx+c)^2 + 2a^2\cos(dx+c) + a^2 - \left(a^2\cos(dx+c) - a^2\right)\sin(dx+c)}{3\left(d\cos(dx+c)^2 - d\cos(dx+c) + (d\cos(dx+c)+2d)\sin(dx+c) - 2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2 - (a^2*cos(d*x + c) - a
^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*
d)*sin(d*x + c) - 2*d)
```

giac [A] time = 1.69, size = 54, normalized size = 0.86

$$-\frac{2\left(3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 2a^2\right)}{3d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c) + 2*a^2)/(d
*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

maple [A] time = 0.22, size = 63, normalized size = 1.00

$$\frac{\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3} + \frac{2a^2}{3\cos(dx+c)^3} - a^2\left(-\frac{2}{3} - \frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*a^2*sin(d*x+c)^3/cos(d*x+c)^3+2/3*a^2/cos(d*x+c)^3-a^2*(-2/3-1/3*s
ec(d*x+c)^2)*tan(d*x+c))
```

maxima [A] time = 0.60, size = 52, normalized size = 0.83

$$\frac{a^2\tan(dx+c)^3 + \left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^2 + \frac{2a^2}{\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 2*a^2/cos
(d*x + c)^3)/d
```

mupad [B] time = 4.56, size = 81, normalized size = 1.29

$$-\frac{2a^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right) + \frac{2a^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2-3\right)}{3}}{d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] -(2*a^2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + (2*a^2*cos(c/2 + (d*x)/2)
*(cos(c/2 + (d*x)/2)^2 - 3))/3)/(d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)
)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\sec^4(c+dx)\,dx + \int \sin^2(c+dx)\sec^4(c+dx)\,dx + \int \sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x)**4, x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

3.23 $\int \sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=64

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{a^3}{4d(a-a\sin(c+dx))}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

```
[Out] 1/4*a^2*arctanh(sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d*x+c))^2+1/4*a^3/d/(a-a*s
in(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{a^3}{4d(a-a\sin(c+dx))}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + a^3/
(4*d*(a - a*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \sec^5(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{a^5\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^5\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^3}+\frac{1}{4a^2(a-x)^2}+\frac{1}{4a^2\left(a^2-x^2\right)}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{a^3}{4d(a-a\sin(c+dx))}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx
\\
&= \frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{a^3}{4d(a-a\sin(c+d
\end{aligned}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 0.88

$$\frac{a^2(\sin(c+dx)+1)^2\sec^4(c+dx)\left(-\sin(c+dx)+(\sin(c+dx)-1)^2\tanh^{-1}(\sin(c+dx))+2\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c + d*x]^4*(2 + ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])^2 - Sin[
c + d*x])*(1 + Sin[c + d*x])^2)/(4*d)
```

fricas [B] time = 0.67, size = 125, normalized size = 1.95

$$\frac{2\,a^2\sin\left(dx+c\right)-4\,a^2+\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2\cos\left(dx+c\right)^2+2\right.}{8\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a^2*sin(d*x + c) - 4*a^2 + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
- 2*a^2)*log(sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) -
 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [A] time = 0.62, size = 77, normalized size = 1.20

$$\frac{2\,a^2\log\left(|\sin\left(dx+c\right)+1|\right)-2\,a^2\log\left(|\sin\left(dx+c\right)-1|\right)+\frac{3\,a^2\sin(dx+c)^2-10\,a^2\sin(dx+c)+11\,a^2}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 2*a^2*log(abs(sin(d*x + c) - 1)) +
 (3*a^2*sin(d*x + c)^2 - 10*a^2*sin(d*x + c) + 11*a^2)/(sin(d*x + c) - 1)^2
)/d
```

maple [B] time = 0.23, size = 144, normalized size = 2.25

$$\frac{a^2\left(\sin^3\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}+\frac{a^2\left(\sin^3\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^2}+\frac{a^2\sin\left(dx+c\right)}{8d}+\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{4d}+\frac{a^2}{2d\cos\left(dx+c\right)^4}+\frac{a^2\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8
*a^2*sin(d*x+c)/d+1/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2/cos(d*x+c)^
4+1/4/d*a^2*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^2*sec(d*x+c)*tan(d*x+c)
```

maxima [A] time = 0.35, size = 71, normalized size = 1.11

$$\frac{a^2\log\left(\sin\left(dx+c\right)+1\right)-a^2\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(a^2\sin(dx+c)-2\,a^2\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - a^2*log(sin(d*x + c) - 1) - 2*(a^2*sin(d*x
 + c) - 2*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 4.47, size = 58, normalized size = 0.91

$$\frac{a^2 \operatorname{atanh}(\sin(c+d\,x))}{4\,d}-\frac{\frac{a^2\,\sin(c+d\,x)}{4}-\frac{a^2}{2}}{d\,\left(\sin(c+d\,x)^2-2\,\sin(c+d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^5,x)
```

```
[Out] (a^2*atanh(sin(c + d*x)))/(4*d) - ((a^2*sin(c + d*x))/4 - a^2/2)/(d*(sin(c
+ d*x)^2 - 2*sin(c + d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.24 $\int \sec^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=64

$$\frac{a^2\tan^3(c+dx)}{5d}+\frac{3a^2\tan(c+dx)}{5d}+\frac{2\sec^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{5d}$$

```
[Out] 2/5*sec(d*x+c)^5*(a^2+a^2*sin(d*x+c))/d+3/5*a^2*tan(d*x+c)/d+1/5*a^2*tan(d*
x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2676, 3767}

$$\frac{a^2\tan^3(c+dx)}{5d}+\frac{3a^2\tan(c+dx)}{5d}+\frac{2\sec^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(5*d) + (3*a^2*Tan[c + d*x])/(5
*d) + (a^2*Tan[c + d*x]^3)/(5*d)
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{2\sec^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{5d}+\frac{1}{5}\left(3a^2\right)\int \sec^4(c+dx)\,dx\\ &= \frac{2\sec^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{5d}-\frac{\left(3a^2\right)\text{Subst}\left(\int\left(1+x^2\right)dx,x,-\text{ta}\right.}{5d}\\ &= \frac{2\sec^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{5d}+\frac{3a^2\tan(c+dx)}{5d}+\frac{a^2\tan^3(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.28

$$\frac{2a^2\tan^5(c+dx)}{5d}+\frac{2a^2\sec^5(c+dx)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{d}-\frac{a^2\tan^3(c+dx)\sec^2(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^4*Tan[c + d*x])/d - (a^2*S
ec[c + d*x]^2*Tan[c + d*x]^3)/d + (2*a^2*Tan[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.61, size = 85, normalized size = 1.33

$$-\frac{4\,a^2\cos\left(dx+c\right)^2-2\,a^2-\left(2\,a^2\cos\left(dx+c\right)^2-3\,a^2\right)\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/5*(4*a^2*cos(d*x + c)^2 - 2*a^2 - (2*a^2*cos(d*x + c)^2 - 3*a^2)*sin(d*x
 + c))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c)
)
```

giac [A] time = 0.65, size = 106, normalized size = 1.66

$$-\frac{\frac{5\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-90\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21\,a^2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(5*a^2/(tan(1/2*d*x + 1/2*c) + 1) + (35*a^2*tan(1/2*d*x + 1/2*c)^4 -
90*a^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*tan(1/2*d*x + 1/2*c)^2 - 70*a^2*tan
(1/2*d*x + 1/2*c) + 21*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^5)/d
```

maple [A] time = 0.22, size = 93, normalized size = 1.45

$$\frac{a^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{2a^2}{5\cos(dx+c)^5}-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+2/5
*a^2/cos(d*x+c)^5-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)
)
```

maxima [A] time = 0.37, size = 77, normalized size = 1.20

$$\frac{\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)a^2+\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)a^2+\frac{6\,a^2}{\cos(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + (3*tan
(d*x + c)^5 + 5*tan(d*x + c)^3)*a^2 + 6*a^2/cos(d*x + c)^5)/d
```

mupad [B] time = 4.63, size = 156, normalized size = 2.44

$$\frac{2\,a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+10\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-10\,\cos}{5\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^6,x)
```

```
[Out] (2*a^2*cos(c/2 + (d*x)/2)*(2*cos(c/2 + (d*x)/2)^5 + 5*sin(c/2 + (d*x)/2)^5
- 10*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^4 - 3*cos(c/2 + (d*x)/2)^4*sin(c
/2 + (d*x)/2) + 10*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^3))/(5*d*(cos(c/
2 + (d*x)/2) - sin(c/2 + (d*x)/2))^5*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/
2)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.25 $\int \sec^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=109

$$\frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+dx))^2}+\frac{3a^3}{16d(a-a\sin(c+dx))}-\frac{a^3}{16d(a\sin(c+dx)+a)}+\frac{a^2\tanh^{-1}(\mathrm{si}}{4d}$$

```
[Out] 1/4*a^2*arctanh(sin(d*x+c))/d+1/12*a^5/d/(a-a*sin(d*x+c))^3+1/8*a^4/d/(a-a*
sin(d*x+c))^2+3/16*a^3/d/(a-a*sin(d*x+c))-1/16*a^3/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2667, 44, 206}

$$\frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+dx))^2}+\frac{3a^3}{16d(a-a\sin(c+dx))}-\frac{a^3}{16d(a\sin(c+dx)+a)}+\frac{a^2\tanh^{-1}(\mathrm{si}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^5/(12*d*(a - a*Sin[c + d*x])^3) + a^4
/(8*d*(a - a*Sin[c + d*x])^2) + (3*a^3)/(16*d*(a - a*Sin[c + d*x])) - a^3/(
16*d*(a + a*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \sec^7(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{a^7\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^7\,\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^4}+\frac{1}{4a^3(a-x)^3}+\frac{3}{16a^4(a-x)^2}+\frac{1}{16a^4(a+x)^2}+\frac{1}{4a^4\left(a^2-x^2\right)}\right)\right.}{d}\\
&= \frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+dx))^2}+\frac{3a^3}{16d(a-a\sin(c+}\\
&= \frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 85, normalized size = 0.78

$$-\frac{a^2(\sin(c+dx)+1)^2\sec^6(c+dx)\left(-3\sin^3(c+dx)+6\sin^2(c+dx)-\sin(c+dx)+3(\sin(c+dx)+1)(\sin(c+d\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/12*(a^2*Sec[c + d*x]^6*(1 + Sin[c + d*x])^2*(-4 - Sin[c + d*x] + 6*Sin[c
 + d*x]^2 - 3*Sin[c + d*x]^3 + 3*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])^
3*(1 + Sin[c + d*x])))/d
```

fricas [A] time = 0.74, size = 203, normalized size = 1.86

$$-\frac{12\,a^2\cos(dx+c)^2-4\,a^2-3\left(a^2\cos(dx+c)^4+2\,a^2\cos(dx+c)^2\sin(dx+c)-2\,a^2\cos(dx+c)^2\right)\log(\sin(dx}{24\left(d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^2*cos(d*x + c)^2 - 4*a^2 - 3*(a^2*cos(d*x + c)^4 + 2*a^2*cos(d*
x + c)^2*sin(d*x + c) - 2*a^2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*(a^
2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*cos(d*x + c)^2
)*log(-sin(d*x + c) + 1) - 2*(3*a^2*cos(d*x + c)^2 - 4*a^2)*sin(d*x + c))/(
d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2*sin(d*x + c) - 2*d*cos(d*x + c)^2)
```

giac [A] time = 0.50, size = 119, normalized size = 1.09

$$\frac{6\,a^2\log(|\sin(dx+c)+1|)-6\,a^2\log(|\sin(dx+c)-1|)-\frac{3\left(2\,a^2\sin(dx+c)+3\,a^2\right)}{\sin(dx+c)+1}+\frac{11\,a^2\sin(dx+c)^3-42\,a^2\sin(dx+c)^2+57\,a^2\sin(}{(\sin(dx+c)-1)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(6*a^2*log(abs(sin(d*x + c) + 1)) - 6*a^2*log(abs(sin(d*x + c) - 1)) -
 3*(2*a^2*sin(d*x + c) + 3*a^2)/(sin(d*x + c) + 1) + (11*a^2*sin(d*x + c)^3
 - 42*a^2*sin(d*x + c)^2 + 57*a^2*sin(d*x + c) - 30*a^2)/(sin(d*x + c) - 1)
^3)/d
```

maple [A] time = 0.24, size = 190, normalized size = 1.74

$$\frac{a^2\left(\sin^3(dx+c)\right)}{6d\cos(dx+c)^6}+\frac{a^2\left(\sin^3(dx+c)\right)}{8d\cos(dx+c)^4}+\frac{a^2\left(\sin^3(dx+c)\right)}{16d\cos(dx+c)^2}+\frac{a^2\sin(dx+c)}{16d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{4d}+\frac{}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/6/d*a^2*sin(d*x+c)^3/cos(d*x+c)^6+1/8/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1/1
6/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/16*a^2*sin(d*x+c)/d+1/4/d*a^2*ln(sec(d*
x+c)+tan(d*x+c))+1/3/d*a^2/cos(d*x+c)^6+1/6/d*a^2*tan(d*x+c)*sec(d*x+c)^5+5
/24/d*a^2*tan(d*x+c)*sec(d*x+c)^3+5/16/d*a^2*sec(d*x+c)*tan(d*x+c)
```

maxima [A] time = 0.95, size = 108, normalized size = 0.99

$$\frac{3\,a^2\log(\sin(dx+c)+1)-3\,a^2\log(\sin(dx+c)-1)-\frac{2\left(3\,a^2\sin(dx+c)^3-6\,a^2\sin(dx+c)^2+a^2\sin(dx+c)+4\,a^2\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^3+2\,\sin(dx+c)-1}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^2*log(sin(d*x + c) + 1) - 3*a^2*log(sin(d*x + c) - 1) - 2*(3*a^2*
sin(d*x + c)^3 - 6*a^2*sin(d*x + c)^2 + a^2*sin(d*x + c) + 4*a^2)/(sin(d*x
+ c)^4 - 2*sin(d*x + c)^3 + 2*sin(d*x + c) - 1))/d
```

mupad [B] time = 4.34, size = 94, normalized size = 0.86

$$\frac{a^2\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{4\,d}-\frac{\frac{a^2\,{\sin\left(c+d\,x\right)}^3}{4}-\frac{a^2\,{\sin\left(c+d\,x\right)}^2}{2}+\frac{a^2\,\sin\left(c+d\,x\right)}{12}+\frac{a^2}{3}}{d\,\left({\sin\left(c+d\,x\right)}^4-2\,{\sin\left(c+d\,x\right)}^3+2\,\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^7,x)
```

```
[Out] (a^2*atanh(sin(c + d*x)))/(4*d) - ((a^2*sin(c + d*x))/12 + a^2/3 - (a^2*sin
(c + d*x)^2)/2 + (a^2*sin(c + d*x)^3)/4)/(d*(2*sin(c + d*x) - 2*sin(c + d*x
)^3 + sin(c + d*x)^4 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.26 $\int \sec^8(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=82

$$\frac{a^2\tan^5(c+dx)}{7d}+\frac{10a^2\tan^3(c+dx)}{21d}+\frac{5a^2\tan(c+dx)}{7d}+\frac{2\sec^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{7d}$$

```
[Out] 2/7*sec(d*x+c)^7*(a^2+a^2*sin(d*x+c))/d+5/7*a^2*tan(d*x+c)/d+10/21*a^2*tan(
d*x+c)^3/d+1/7*a^2*tan(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2676, 3767}

$$\frac{a^2\tan^5(c+dx)}{7d}+\frac{10a^2\tan^3(c+dx)}{21d}+\frac{5a^2\tan(c+dx)}{7d}+\frac{2\sec^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(7*d) + (5*a^2*Tan[c + d*x])/(7
*d) + (10*a^2*Tan[c + d*x]^3)/(21*d) + (a^2*Tan[c + d*x]^5)/(7*d)
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{2\sec^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{7d}+\frac{1}{7}\left(5a^2\right)\int \sec^6(c+dx)\,dx\\ &= \frac{2\sec^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{7d}-\frac{\left(5a^2\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx,\right.}{7d}\\ &= \frac{2\sec^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{7d}+\frac{5a^2\tan(c+dx)}{7d}+\frac{10a^2\tan^3(c+}{21d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 110, normalized size = 1.34

$$-\frac{8a^2\tan^7(c+dx)}{21d}+\frac{2a^2\sec^7(c+dx)}{7d}+\frac{a^2\tan(c+dx)\sec^6(c+dx)}{d}-\frac{5a^2\tan^3(c+dx)\sec^4(c+dx)}{3d}+\frac{4a^2\tan^5(c+d}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sec[c + d*x]^7)/(7*d) + (a^2*Sec[c + d*x]^6*Tan[c + d*x])/d - (5*a^2
*Sec[c + d*x]^4*Tan[c + d*x]^3)/(3*d) + (4*a^2*Sec[c + d*x]^2*Tan[c + d*x]^
5)/(3*d) - (8*a^2*Tan[c + d*x]^7)/(21*d)
```

fricas [A] time = 0.60, size = 115, normalized size = 1.40

$$-\frac{16\,a^2\cos\left(dx+c\right)^4-8\,a^2\cos\left(dx+c\right)^2-2\,a^2-\left(8\,a^2\cos\left(dx+c\right)^4-12\,a^2\cos\left(dx+c\right)^2-5\,a^2\right)\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)^5+2\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/21*(16*a^2*cos(d*x + c)^4 - 8*a^2*cos(d*x + c)^2 - 2*a^2 - (8*a^2*cos(d*
x + c)^4 - 12*a^2*cos(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5 +
 2*d*cos(d*x + c)^3*sin(d*x + c) - 2*d*cos(d*x + c)^3)
```

giac [B] time = 0.65, size = 171, normalized size = 2.09

$$-\frac{\frac{7\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}+\frac{273\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2450\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2870\,a^2\tan\left(\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^7}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/168*(7*(9*a^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*tan(1/2*d*x + 1/2*c) + 8*a
^2)/(tan(1/2*d*x + 1/2*c) + 1)^3 + (273*a^2*tan(1/2*d*x + 1/2*c)^6 - 1155*a
^2*tan(1/2*d*x + 1/2*c)^5 + 2450*a^2*tan(1/2*d*x + 1/2*c)^4 - 2870*a^2*tan(
1/2*d*x + 1/2*c)^3 + 2037*a^2*tan(1/2*d*x + 1/2*c)^2 - 791*a^2*tan(1/2*d*x
+ 1/2*c) + 152*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

maple [A] time = 0.24, size = 121, normalized size = 1.48

$$\frac{a^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{2a^2}{7\cos(dx+c)^7}-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\mathrm{ta}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/10
5*sin(d*x+c)^3/cos(d*x+c)^3)+2/7*a^2/cos(d*x+c)^7-a^2*(-16/35-1/7*sec(d*x+c
)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

maxima [A] time = 0.33, size = 98, normalized size = 1.20

$$\frac{\left(15\tan\left(dx+c\right)^7+42\tan\left(dx+c\right)^5+35\tan\left(dx+c\right)^3\right)a^2+3\left(5\tan\left(dx+c\right)^7+21\tan\left(dx+c\right)^5+35\tan\left(d\right.\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^2 + 3*
(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c)
)*a^2 + 30*a^2/cos(d*x + c)^7)/d
```

mupad [B] time = 5.07, size = 276, normalized size = 3.37

$$\frac{2\,a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+76\cos\left(\frac{c}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^8,x)
```

```
[Out] (2*a^2*cos(c/2 + (d*x)/2)*(6*cos(c/2 + (d*x)/2)^9 + 21*sin(c/2 + (d*x)/2)^9
 - 42*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^8 - 3*cos(c/2 + (d*x)/2)^8*sin(
c/2 + (d*x)/2) + 28*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^7 + 56*cos(c/2
+ (d*x)/2)^3*sin(c/2 + (d*x)/2)^6 - 42*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)
/2)^5 - 28*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^4 + 76*cos(c/2 + (d*x)/2
)^6*sin(c/2 + (d*x)/2)^3 - 24*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^2))/(
21*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^7*(cos(c/2 + (d*x)/2) + sin(
c/2 + (d*x)/2))^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.27 $\int \cos^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=154

$$-\frac{11a^3\cos^7(c+dx)}{56d}-\frac{11\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{72d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{55a^3\sin(c+dx)\,\mathrm{co}}{192d}$$

```
[Out] 55/128*a^3*x-11/56*a^3*cos(d*x+c)^7/d+55/128*a^3*cos(d*x+c)*sin(d*x+c)/d+55
/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d+11/48*a^3*cos(d*x+c)^5*sin(d*x+c)/d-1/9*
a*cos(d*x+c)^7*(a+a*sin(d*x+c))^2/d-11/72*cos(d*x+c)^7*(a^3+a^3*sin(d*x+c))
/d
```

Rubi [A] time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2678, 2669, 2635, 8}

$$-\frac{11a^3\cos^7(c+dx)}{56d}-\frac{11\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{72d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{55a^3\sin(c+dx)\,\mathrm{co}}{192d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (55*a^3*x)/128 - (11*a^3*Cos[c + d*x]^7)/(56*d) + (55*a^3*Cos[c + d*x]*Sin[
c + d*x])/(128*d) + (55*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (11*a^3*
Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a*Cos[c + d*x]^7*(a + a*Sin[c + d*x]
)^2)/(9*d) - (11*Cos[c + d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(72*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d} + \frac{1}{9}(11a)\int \cos^6(c+dx)(a+a\sin(c$$
$$= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d} - \frac{11\cos^7(c+dx)\left(a^3+a^3\sin(c+d\right.}{72d}$$
$$= -\frac{11a^3\cos^7(c+dx)}{56d} - \frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d} - \frac{11\cos^7(c+d}{}$$
$$= -\frac{11a^3\cos^7(c+dx)}{56d} + \frac{11a^3\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{a\cos^7(c+dx)(a+}{9d}$$
$$= -\frac{11a^3\cos^7(c+dx)}{56d} + \frac{55a^3\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{11a^3\cos^5(c+dx)}{48d}$$
$$= -\frac{11a^3\cos^7(c+dx)}{56d} + \frac{55a^3\cos(c+dx)\sin(c+dx)}{128d} + \frac{55a^3\cos^3(c+dx)}{192d}$$
$$= \frac{55a^3x}{128} - \frac{11a^3\cos^7(c+dx)}{56d} + \frac{55a^3\cos(c+dx)\sin(c+dx)}{128d} + \frac{55a^3\cos^3(}{}$$

Mathematica [A] time = 2.00, size = 181, normalized size = 1.18

$$-\frac{a^3\left(6930\sqrt{1-\sin(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\,\left(896\sin^9(c+dx)+2128\sin^8(c+dx)-2000\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/8064*(a^3*Cos[c + d*x]^7*(6930*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sq
rt[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(3712 - 8311*Sin[c + d*x] - 5
641*Sin[c + d*x]^2 + 7174*Sin[c + d*x]^3 + 11514*Sin[c + d*x]^4 - 1224*Sin[
c + d*x]^5 - 8248*Sin[c + d*x]^6 - 2000*Sin[c + d*x]^7 + 2128*Sin[c + d*x]^
8 + 896*Sin[c + d*x]^9)))/(d*(-1 + Sin[c + d*x])^4*(1 + Sin[c + d*x])^(7/2)
)
```

fricas [A] time = 0.84, size = 98, normalized size = 0.64

$$\frac{896\,a^3\cos(dx+c)^9 - 4608\,a^3\cos(dx+c)^7 + 3465\,a^3dx - 21\left(144\,a^3\cos(dx+c)^7 - 88\,a^3\cos(dx+c)^5 - 110\,a^3\right.}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8064*(896*a^3*cos(d*x + c)^9 - 4608*a^3*cos(d*x + c)^7 + 3465*a^3*d*x - 2
1*(144*a^3*cos(d*x + c)^7 - 88*a^3*cos(d*x + c)^5 - 110*a^3*cos(d*x + c)^3
- 165*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.86, size = 157, normalized size = 1.02

$$\frac{55}{128}\,a^3x + \frac{a^3\cos(9\,dx+9\,c)}{2304\,d} - \frac{9\,a^3\cos(7\,dx+7\,c)}{1792\,d} - \frac{3\,a^3\cos(5\,dx+5\,c)}{64\,d} - \frac{29\,a^3\cos(3\,dx+3\,c)}{192\,d} - \frac{33\,a^3\cos(dx+c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 55/128*a^3*x + 1/2304*a^3*cos(9*d*x + 9*c)/d - 9/1792*a^3*cos(7*d*x + 7*c)/
d - 3/64*a^3*cos(5*d*x + 5*c)/d - 29/192*a^3*cos(3*d*x + 3*c)/d - 33/128*a^
```

```
3*cos(d*x + c)/d - 3/1024*a^3*sin(8*d*x + 8*c)/d - 1/96*a^3*sin(6*d*x + 6*c
)/d + 3/128*a^3*sin(4*d*x + 4*c)/d + 9/32*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.18, size = 163, normalized size = 1.06

$$\frac{a^3\left(-\frac{(\sin^2(dx+c))(\cos^7(dx+c))}{9}-\frac{2(\cos^7(dx+c))}{63}\right)+3a^3\left(-\frac{\sin(dx+c)(\cos^7(dx+c))}{8}+\frac{\left(\cos^5(dx+c)+\frac{5(\cos^3(dx+c))}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48} - \cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+3*a^3*(-1/8*sin
(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
in(d*x+c)+5/128*d*x+5/128*c)-3/7*a^3*cos(d*x+c)^7+a^3*(1/6*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

maxima [A] time = 0.38, size = 141, normalized size = 0.92

$$-\frac{27648\,a^3\cos(dx+c)^7-1024\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^3-63\left(64\sin(2\,dx+2\,c)^3+120\,dx+120\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/64512*(27648*a^3*cos(d*x + c)^7 - 1024*(7*cos(d*x + c)^9 - 9*cos(d*x + c
)^7)*a^3 - 63*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
 - 24*sin(4*d*x + 4*c))*a^3 + 336*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3)/d
```

mupad [B] time = 6.81, size = 501, normalized size = 3.25

$$\frac{55\,a^3\,x}{128}-\frac{\frac{17\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{32}-\frac{949\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96}-\frac{699\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{32}+\frac{699\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32}-\frac{17\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}+\frac{949\,a^3\tan\left(\frac{c}{2}+\cdots\right.}{96}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (55*a^3*x)/128 - ((17*a^3*tan(c/2 + (d*x)/2)^5)/32 - (949*a^3*tan(c/2 + (d*
x)/2)^3)/96 - (699*a^3*tan(c/2 + (d*x)/2)^7)/32 + (699*a^3*tan(c/2 + (d*x)/
2)^11)/32 - (17*a^3*tan(c/2 + (d*x)/2)^13)/32 + (949*a^3*tan(c/2 + (d*x)/2)
^15)/96 + (73*a^3*tan(c/2 + (d*x)/2)^17)/64 + (a^3*(3465*c + 3465*d*x))/806
4 - (a^3*(3465*c + 3465*d*x - 7424))/8064 + tan(c/2 + (d*x)/2)^2*((a^3*(346
5*c + 3465*d*x))/896 - (a^3*(31185*c + 31185*d*x - 18432))/8064) + tan(c/2
+ (d*x)/2)^16*((a^3*(3465*c + 3465*d*x))/896 - (a^3*(31185*c + 31185*d*x -
48384))/8064) + tan(c/2 + (d*x)/2)^14*((a^3*(3465*c + 3465*d*x))/224 - (a^3
*(124740*c + 124740*d*x - 129024))/8064) + tan(c/2 + (d*x)/2)^4*((a^3*(3465
*c + 3465*d*x))/224 - (a^3*(124740*c + 124740*d*x - 138240))/8064) + tan(c/
2 + (d*x)/2)^12*((a^3*(3465*c + 3465*d*x))/96 - (a^3*(291060*c + 291060*d*x
 - 236544))/8064) + tan(c/2 + (d*x)/2)^6*((a^3*(3465*c + 3465*d*x))/96 - (a
^3*(291060*c + 291060*d*x - 387072))/8064) + tan(c/2 + (d*x)/2)^8*((a^3*(34
65*c + 3465*d*x))/64 - (a^3*(436590*c + 436590*d*x - 290304))/8064) + tan(c
/2 + (d*x)/2)^10*((a^3*(3465*c + 3465*d*x))/64 - (a^3*(436590*c + 436590*d*
x - 645120))/8064) - (73*a^3*tan(c/2 + (d*x)/2))/64)/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^9)
```

sympy [A] time = 21.33, size = 439, normalized size = 2.85

$$\begin{cases} \frac{15a^3x\sin^8(c+dx)}{128} + \frac{15a^3x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{5a^3x\sin^6(c+dx)}{16} + \frac{45a^3x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{15a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a}{} \\ x(a\sin(c)+a)^3\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((15*a**3*x*sin(c + d*x)**8/128 + 15*a**3*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 5*a**3*x*sin(c + d*x)**6/16 + 45*a**3*x*sin(c + d*x)**4*cos(
c + d*x)**4/64 + 15*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*a**3*x*s
in(c + d*x)**2*cos(c + d*x)**6/32 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)*
*4/16 + 15*a**3*x*cos(c + d*x)**8/128 + 5*a**3*x*cos(c + d*x)**6/16 + 15*a*
*3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*a**3*sin(c + d*x)**5*cos(c + d
*x)**3/(128*d) + 5*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 73*a**3*sin(c
 + d*x)**3*cos(c + d*x)**5/(128*d) + 5*a**3*sin(c + d*x)**3*cos(c + d*x)**3
/(6*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 15*a**3*sin(c + d*x)*
cos(c + d*x)**7/(128*d) + 11*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 2*a
**3*cos(c + d*x)**9/(63*d) - 3*a**3*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a
*sin(c) + a)**3*cos(c)**6, True))
```

3.28 $\int \cos^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{(a\sin(c+dx)+a)^8}{8a^5d}-\frac{4(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^6/a^3/d-4/7*(a+a*sin(d*x+c))^7/a^4/d+1/8*(a+a*sin(d*x+
c))^8/a^5/d
```

Rubi [A] time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^8}{8a^5d}-\frac{4(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (4*(a + a*Sin[c + d*x])^7)/(7*a^4*d)
 + (a + a*Sin[c + d*x])^8/(8*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^5\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^5-4a(a+x)^6+(a+x)^7\right)dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{2(a+a\sin(c+dx))^6}{3a^3d}-\frac{4(a+a\sin(c+dx))^7}{7a^4d}+\frac{(a+a\sin(c+dx))^8}{8a^5d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 58, normalized size = 0.87

$$-\frac{a^3(\sin(c+dx)+1)^3\left(21\sin^2(c+dx)-54\sin(c+dx)+37\right)\cos^6(c+dx)}{168d(\sin(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/168*(a^3*Cos[c + d*x]^6*(1 + Sin[c + d*x])^3*(37 - 54*Sin[c + d*x] + 21*
Sin[c + d*x]^2))/(d*(-1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.64, size = 85, normalized size = 1.27

$$\frac{21\,a^3\cos\left(dx+c\right)^8-112\,a^3\cos\left(dx+c\right)^6-8\left(9\,a^3\cos\left(dx+c\right)^6-6\,a^3\cos\left(dx+c\right)^4-8\,a^3\cos\left(dx+c\right)^2-16\,a^3\right)\mathrm{s}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/168*(21*a^3*cos(d*x + c)^8 - 112*a^3*cos(d*x + c)^6 - 8*(9*a^3*cos(d*x +
c)^6 - 6*a^3*cos(d*x + c)^4 - 8*a^3*cos(d*x + c)^2 - 16*a^3)*sin(d*x + c))/
d
```

giac [B] time = 0.98, size = 134, normalized size = 2.00

$$\frac{a^3\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{5\,a^3\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{25\,a^3\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{33\,a^3\cos\left(2\,dx+2\,c\right)}{128\,d}-\frac{3\,a^3\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{a^3\,\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1024*a^3*cos(8*d*x + 8*c)/d - 5/384*a^3*cos(6*d*x + 6*c)/d - 25/256*a^3*c
os(4*d*x + 4*c)/d - 33/128*a^3*cos(2*d*x + 2*c)/d - 3/448*a^3*sin(7*d*x + 7
*c)/d - 1/64*a^3*sin(5*d*x + 5*c)/d + 17/192*a^3*sin(3*d*x + 3*c)/d + 55/64
*a^3*sin(d*x + c)/d
```

maple [B] time = 0.17, size = 133, normalized size = 1.99

$$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+3a^3\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{\left(\cos^6(dx+c\right.}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+3*a^3*(-1/7*sin
(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1
/2*cos(d*x+c)^6*a^3+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.37, size = 108, normalized size = 1.61

$$\frac{21\,a^3\sin\left(dx+c\right)^8+72\,a^3\sin\left(dx+c\right)^7+28\,a^3\sin\left(dx+c\right)^6-168\,a^3\sin\left(dx+c\right)^5-210\,a^3\sin\left(dx+c\right)^4+56\,a^3\,\mathrm{s}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/168*(21*a^3*sin(d*x + c)^8 + 72*a^3*sin(d*x + c)^7 + 28*a^3*sin(d*x + c)^
6 - 168*a^3*sin(d*x + c)^5 - 210*a^3*sin(d*x + c)^4 + 56*a^3*sin(d*x + c)^3
 + 252*a^3*sin(d*x + c)^2 + 168*a^3*sin(d*x + c))/d
```

mupad [B] time = 4.54, size = 106, normalized size = 1.58

$$\frac{\frac{a^3\sin(c+d\,x)^8}{8}+\frac{3\,a^3\sin(c+d\,x)^7}{7}+\frac{a^3\sin(c+d\,x)^6}{6}-a^3\sin\left(c+d\,x\right)^5-\frac{5\,a^3\sin(c+d\,x)^4}{4}+\frac{a^3\sin(c+d\,x)^3}{3}+\frac{3\,a^3\sin(c+d\,x)^2}{2}+a^3\,\sin\left(c\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x) + (3*a^3*sin(c + d*x)^2)/2 + (a^3*sin(c + d*x)^3)/3 - (5*
a^3*sin(c + d*x)^4)/4 - a^3*sin(c + d*x)^5 + (a^3*sin(c + d*x)^6)/6 + (3*a^
3*sin(c + d*x)^7)/7 + (a^3*sin(c + d*x)^8)/8)/d
```

sympy [A] time = 13.40, size = 196, normalized size = 2.93

$$\begin{cases} \frac{8a^3\sin^7(c+dx)}{35d} + \frac{4a^3\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{8a^3\sin^5(c+dx)}{15d} + \frac{a^3\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{4a^3\sin^3(c+dx)\cos^2(c+dx)}{3d} - \frac{a^3\sin^2(c+d}{} \\ x\left(a\sin(c)+a\right)^3\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**7/(35*d) + 4*a**3*sin(c + d*x)**5*cos(c + d
*x)**2/(5*d) + 8*a**3*sin(c + d*x)**5/(15*d) + a**3*sin(c + d*x)**3*cos(c +
 d*x)**4/d + 4*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - a**3*sin(c + d*
x)**2*cos(c + d*x)**6/(6*d) + a**3*sin(c + d*x)*cos(c + d*x)**4/d - a**3*co
s(c + d*x)**8/(24*d) - a**3*cos(c + d*x)**6/(2*d), Ne(d, 0)), (x*(a*sin(c)
+ a)**3*cos(c)**5, True))
```

3.29 $\int \cos^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=130

$$-\frac{3a^3\cos^5(c+dx)}{10d}-\frac{3\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{14d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{9a^3\sin(c+dx)\cos(c+d}{16d}$$

```
[Out] 9/16*a^3*x-3/10*a^3*cos(d*x+c)^5/d+9/16*a^3*cos(d*x+c)*sin(d*x+c)/d+3/8*a^3
*cos(d*x+c)^3*sin(d*x+c)/d-1/7*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^2/d-3/14*cos
(d*x+c)^5*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2678, 2669, 2635, 8}

$$-\frac{3a^3\cos^5(c+dx)}{10d}-\frac{3\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{14d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{9a^3\sin(c+dx)\cos(c+d}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (9*a^3*x)/16 - (3*a^3*Cos[c + d*x]^5)/(10*d) + (9*a^3*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^
5*(a + a*Sin[c + d*x])^2)/(7*d) - (3*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]
))/(14*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d} + \frac{1}{7}(9a)\int \cos^4(c+dx)(a+a\sin$$
$$= -\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d} - \frac{3\cos^5(c+dx)\left(a^3+a^3\sin(c+\right.}{14d}$$
$$= -\frac{3a^3\cos^5(c+dx)}{10d} - \frac{a\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d} - \frac{3\cos^5(c+}{}$$
$$= -\frac{3a^3\cos^5(c+dx)}{10d} + \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^5(c+dx)(a}{7}$$
$$= -\frac{3a^3\cos^5(c+dx)}{10d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{3a^3\cos^3(c+dx)\,s}{8d}$$
$$= \frac{9a^3x}{16} - \frac{3a^3\cos^5(c+dx)}{10d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{3a^3\cos^3(c}{}$$

Mathematica [A] time = 0.78, size = 161, normalized size = 1.24

$$-\frac{a^3\left(\sqrt{\sin(c+dx)+1}\left(80\sin^7(c+dx)+200\sin^6(c+dx)-72\sin^5(c+dx)-558\sin^4(c+dx)-306\sin^3(c+\right.\right.}{560d(\sin(c+dx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/560*(a^3*Cos[c + d*x]^5*(-630*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqr
t[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-368 + 613*Sin[c + d*x] + 411
*Sin[c + d*x]^2 - 306*Sin[c + d*x]^3 - 558*Sin[c + d*x]^4 - 72*Sin[c + d*x]
^5 + 200*Sin[c + d*x]^6 + 80*Sin[c + d*x]^7)))/(d*(-1 + Sin[c + d*x])^3*(1
+ Sin[c + d*x])^(5/2))
```

fricas [A] time = 0.60, size = 85, normalized size = 0.65

$$\frac{80\,a^3\cos\left(dx+c\right)^7 - 448\,a^3\cos\left(dx+c\right)^5 + 315\,a^3dx - 35\left(8\,a^3\cos\left(dx+c\right)^5 - 6\,a^3\cos\left(dx+c\right)^3 - 9\,a^3\cos\left(dx\right.\right.}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(80*a^3*cos(d*x + c)^7 - 448*a^3*cos(d*x + c)^5 + 315*a^3*d*x - 35*(8
*a^3*cos(d*x + c)^5 - 6*a^3*cos(d*x + c)^3 - 9*a^3*cos(d*x + c))*sin(d*x +
c))/d
```

giac [A] time = 0.84, size = 123, normalized size = 0.95

$$\frac{9}{16}a^3x + \frac{a^3\cos\left(7\,dx+7\,c\right)}{448\,d} - \frac{11\,a^3\cos\left(5\,dx+5\,c\right)}{320\,d} - \frac{13\,a^3\cos\left(3\,dx+3\,c\right)}{64\,d} - \frac{27\,a^3\cos\left(dx+c\right)}{64\,d} - \frac{a^3\sin\left(6\,dx+6\,c\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 9/16*a^3*x + 1/448*a^3*cos(7*d*x + 7*c)/d - 11/320*a^3*cos(5*d*x + 5*c)/d -
 13/64*a^3*cos(3*d*x + 3*c)/d - 27/64*a^3*cos(d*x + c)/d - 1/64*a^3*sin(6*d
*x + 6*c)/d - 1/64*a^3*sin(4*d*x + 4*c)/d + 19/64*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.18, size = 143, normalized size = 1.10

$$\frac{a^3\left(-\frac{(\sin^2(dx+c))(\cos^5(dx+c))}{7}-\frac{2(\cos^5(dx+c))}{35}\right)+3a^3\left(-\frac{\sin(dx+c)(\cos^5(dx+c))}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{3(}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a^3*(-1/6*sin
(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x
+1/16*c)-3/5*cos(d*x+c)^5*a^3+a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*
x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.32, size = 115, normalized size = 0.88

$$-\frac{1344\,a^3\cos(dx+c)^5-64\left(5\,\cos(dx+c)^7-7\,\cos(dx+c)^5\right)a^3-35\left(4\,\sin(2\,dx+2\,c)^3+12\,dx+12\,c-3\,\sin(\right.}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2240*(1344*a^3*cos(d*x + c)^5 - 64*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)
*a^3 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3 -
 70*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

mupad [B] time = 6.71, size = 389, normalized size = 2.99

$$\frac{9\,a^3\,x}{16}-\frac{\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8}-\frac{17\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{8}+\frac{17\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{2}+\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+\frac{a^3\,(315\,c+315\,d\,x)}{560}-\frac{a^3\,(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (9*a^3*x)/16 - ((13*a^3*tan(c/2 + (d*x)/2)^5)/8 - (17*a^3*tan(c/2 + (d*x)/2
)^3)/2 - (13*a^3*tan(c/2 + (d*x)/2)^9)/8 + (17*a^3*tan(c/2 + (d*x)/2)^11)/2
 + (7*a^3*tan(c/2 + (d*x)/2)^13)/8 + (a^3*(315*c + 315*d*x))/560 - (a^3*(31
5*c + 315*d*x - 736))/560 + tan(c/2 + (d*x)/2)^2*((a^3*(315*c + 315*d*x))/8
0 - (a^3*(2205*c + 2205*d*x - 1792))/560) + tan(c/2 + (d*x)/2)^12*((a^3*(31
5*c + 315*d*x))/80 - (a^3*(2205*c + 2205*d*x - 3360))/560) + tan(c/2 + (d*x
)/2)^4*((3*a^3*(315*c + 315*d*x))/80 - (a^3*(6615*c + 6615*d*x - 6496))/560
) + tan(c/2 + (d*x)/2)^10*((3*a^3*(315*c + 315*d*x))/80 - (a^3*(6615*c + 66
15*d*x - 8960))/560) + tan(c/2 + (d*x)/2)^8*((a^3*(315*c + 315*d*x))/16 - (
a^3*(11025*c + 11025*d*x - 7840))/560) + tan(c/2 + (d*x)/2)^6*((a^3*(315*c
+ 315*d*x))/16 - (a^3*(11025*c + 11025*d*x - 17920))/560) - (7*a^3*tan(c/2
+ (d*x)/2))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 10.00, size = 335, normalized size = 2.58

$$\begin{cases}\frac{3a^3x\sin^6(c+dx)}{16}+\frac{9a^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3a^3x\sin^4(c+dx)}{8}+\frac{9a^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^3x\cos}{1}\\ x\,(a\sin(c)+a)^3\cos^4(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**6/16 + 9*a**3*x*sin(c + d*x)**4*cos(c + d
*x)**2/16 + 3*a**3*x*sin(c + d*x)**4/8 + 9*a**3*x*sin(c + d*x)**2*cos(c + d
*x)**4/16 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**3*x*cos(c + d
*x)**6/16 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**5*cos(c + d*x
)/(16*d) + a**3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) + 3*a**3*sin(c + d*x)
**3*cos(c + d*x)/(8*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**
3*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*a**3*sin(c + d*x)*cos(c + d*x)**3
/(8*d) - 2*a**3*cos(c + d*x)**7/(35*d) - 3*a**3*cos(c + d*x)**5/(5*d), Ne(d
, 0)), (x*(a*sin(c) + a)**3*cos(c)**4, True))
```

3.30 $\int \cos^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=45

$$\frac{2(a\sin(c+dx)+a)^5}{5a^2d}-\frac{(a\sin(c+dx)+a)^6}{6a^3d}$$

```
[Out] 2/5*(a+a*sin(d*x+c))^5/a^2/d-1/6*(a+a*sin(d*x+c))^6/a^3/d
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = $\{2667, 43\}$

$$\frac{2(a\sin(c+dx)+a)^5}{5a^2d}-\frac{(a\sin(c+dx)+a)^6}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^5)/(5*a^2*d) - (a + a*Sin[c + d*x])^6/(6*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^3\,dx&=\frac{\text{Subst}\left(\int (a-x)(a+x)^4\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{\text{Subst}\left(\int \left(2a(a+x)^4-(a+x)^5\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{2(a+a\sin(c+dx))^5}{5a^2d}-\frac{(a+a\sin(c+dx))^6}{6a^3d}\end{aligned}$$

Mathematica [A] time = 0.15, size = 43, normalized size = 0.96

$$-\frac{a^3(5\sin(c+dx)-7)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^{10}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/30*(a^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^10*(-7 + 5*Sin[c + d*x]))/
d
```

fricas [A] time = 0.69, size = 72, normalized size = 1.60

$$\frac{5\,a^{3}\cos\left(dx+c\right)^{6}-30\,a^{3}\cos\left(dx+c\right)^{4}-2\left(9\,a^{3}\cos\left(dx+c\right)^{4}-8\,a^{3}\cos\left(dx+c\right)^{2}-16\,a^{3}\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(5*a^3*cos(d*x + c)^6 - 30*a^3*cos(d*x + c)^4 - 2*(9*a^3*cos(d*x + c)^
4 - 8*a^3*cos(d*x + c)^2 - 16*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.71, size = 82, normalized size = 1.82

$$-\frac{5\,a^{3}\sin\left(dx+c\right)^{6}+18\,a^{3}\sin\left(dx+c\right)^{5}+15\,a^{3}\sin\left(dx+c\right)^{4}-20\,a^{3}\sin\left(dx+c\right)^{3}-45\,a^{3}\sin\left(dx+c\right)^{2}-30\,a^{3}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/30*(5*a^3*sin(d*x + c)^6 + 18*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 20*a^3*sin(d*x + c)^3 - 45*a^3*sin(d*x + c)^2 - 30*a^3*sin(d*x + c))/d
```

maple [B] time = 0.17, size = 113, normalized size = 2.51

$$\frac{a^{3}\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{4}(dx+c)\right)}{6}-\frac{\left(\cos^{4}(dx+c)\right)}{12}\right)+3a^{3}\left(-\frac{\left(\cos^{4}(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3\left(\cos^{4}(dx+c)\right)a^{3}}{4}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a^3*(-1/5*cos
(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/4*cos(d*x+c)^4*a^3
+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.44, size = 82, normalized size = 1.82

$$-\frac{5\,a^{3}\sin\left(dx+c\right)^{6}+18\,a^{3}\sin\left(dx+c\right)^{5}+15\,a^{3}\sin\left(dx+c\right)^{4}-20\,a^{3}\sin\left(dx+c\right)^{3}-45\,a^{3}\sin\left(dx+c\right)^{2}-30\,a^{3}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/30*(5*a^3*sin(d*x + c)^6 + 18*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 20*a^3*sin(d*x + c)^3 - 45*a^3*sin(d*x + c)^2 - 30*a^3*sin(d*x + c))/d
```

mupad [B] time = 4.47, size = 80, normalized size = 1.78

$$\frac{-\frac{a^{3}\sin(c+d\,x)^{6}}{6}-\frac{3\,a^{3}\sin(c+d\,x)^{5}}{5}-\frac{a^{3}\sin(c+d\,x)^{4}}{2}+\frac{2\,a^{3}\sin(c+d\,x)^{3}}{3}+\frac{3\,a^{3}\sin(c+d\,x)^{2}}{2}+a^{3}\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x) + (3*a^3*sin(c + d*x)^2)/2 + (2*a^3*sin(c + d*x)^3)/3 - (
a^3*sin(c + d*x)^4)/2 - (3*a^3*sin(c + d*x)^5)/5 - (a^3*sin(c + d*x)^6)/6)/
d
```

sympy [A] time = 5.78, size = 146, normalized size = 3.24

$$\begin{cases} \frac{2a^3\sin^5(c+dx)}{5d} + \frac{a^3\sin^3(c+dx)\cos^2(c+dx)}{d} + \frac{2a^3\sin^3(c+dx)}{3d} - \frac{a^3\sin^2(c+dx)\cos^4(c+dx)}{4d} + \frac{a^3\sin(c+dx)\cos^2(c+dx)}{d} - \frac{a^3\cos^6(c+dx)}{12d} - \frac{3 \\ x\left(a\sin(c)+a\right)^3\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**5/(5*d) + a**3*sin(c + d*x)**3*cos(c + d*x)
**2/d + 2*a**3*sin(c + d*x)**3/(3*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**4
/(4*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d - a**3*cos(c + d*x)**6/(12*d)
- 3*a**3*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*cos(c)**3,
True))
```

3.31 $\int \cos^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=106

$$-\frac{7a^3\cos^3(c+dx)}{12d}-\frac{7\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{20d}+\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^3x}{8}-\frac{a\cos^3(c+dx)(a}{5}$$

```
[Out] 7/8*a^3*x-7/12*a^3*cos(d*x+c)^3/d+7/8*a^3*cos(d*x+c)*sin(d*x+c)/d-1/5*a*cos
(d*x+c)^3*(a+a*sin(d*x+c))^2/d-7/20*cos(d*x+c)^3*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A] time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = $\{2678, 2669, 2635, 8\}$

$$-\frac{7a^3\cos^3(c+dx)}{12d}-\frac{7\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{20d}+\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^3x}{8}-\frac{a\cos^3(c+dx)(a}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*x)/8 - (7*a^3*Cos[c + d*x]^3)/(12*d) + (7*a^3*Cos[c + d*x]*Sin[c + d
*x])/(8*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(5*d) - (7*Cos[c + d
*x]^3*(a^3 + a^3*Sin[c + d*x]))/(20*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d} + \frac{1}{5}(7a)\int \cos^2(c+dx)(a+a\sin(c$$
$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d} - \frac{7\cos^3(c+dx)\left(a^3+a^3\sin(c+dx\right.}{20d}$$
$$= -\frac{7a^3\cos^3(c+dx)}{12d} - \frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d} - \frac{7\cos^3(c+dx)}{}$$
$$= -\frac{7a^3\cos^3(c+dx)}{12d} + \frac{7a^3\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)(a+a}{5d}$$
$$= \frac{7a^3x}{8} - \frac{7a^3\cos^3(c+dx)}{12d} + \frac{7a^3\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)}{}$$

Mathematica [A] time = 0.41, size = 141, normalized size = 1.33

$$-\frac{a^3\left(210\sqrt{1-\sin(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\,\left(24\sin^5(c+dx)+66\sin^4(c+dx)+22\sin^3(c\right.\right.}{120d(\sin(c+dx)-1)^2(\sin(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/120*(a^3*Cos[c + d*x]^3*(210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt
[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(136 - 151*Sin[c + d*x] - 97*Si
n[c + d*x]^2 + 22*Sin[c + d*x]^3 + 66*Sin[c + d*x]^4 + 24*Sin[c + d*x]^5)))
/(d*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(3/2))
```

fricas [A] time = 0.85, size = 72, normalized size = 0.68

$$\frac{24\,a^3\cos\left(dx+c\right)^5-160\,a^3\cos\left(dx+c\right)^3+105\,a^3dx-15\left(6\,a^3\cos\left(dx+c\right)^3-7\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(24*a^3*cos(d*x + c)^5 - 160*a^3*cos(d*x + c)^3 + 105*a^3*d*x - 15*(6
*a^3*cos(d*x + c)^3 - 7*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 1.07, size = 89, normalized size = 0.84

$$\frac{7}{8}\,a^3x+\frac{a^3\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{13\,a^3\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{7\,a^3\cos\left(dx+c\right)}{8\,d}-\frac{3\,a^3\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a^3\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 7/8*a^3*x + 1/80*a^3*cos(5*d*x + 5*c)/d - 13/48*a^3*cos(3*d*x + 3*c)/d - 7/
8*a^3*cos(d*x + c)/d - 3/32*a^3*sin(4*d*x + 4*c)/d + 1/4*a^3*sin(2*d*x + 2*
c)/d
```

maple [A] time = 0.12, size = 121, normalized size = 1.14

$$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3a^3\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\left(\cos^3\left(dx+c\right)\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a^3*(-1/4*sin
(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-cos(d*x+c)^3*
a^3+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.54, size = 91, normalized size = 0.86

$$-\frac{480\,a^{3}\cos\left(dx+c\right)^{3}-32\left(3\,\cos\left(dx+c\right)^{5}-5\,\cos\left(dx+c\right)^{3}\right)a^{3}-45\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^{3}-120\left(2\,\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/480*(480*a^3*cos(d*x + c)^3 - 32*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a
^3 - 45*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^3 - 120*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^3)/d
```

mupad [B] time = 6.53, size = 277, normalized size = 2.61

$$\frac{7\,a^3\,x}{8}-\frac{\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{a^3\,\left(105\,c+105\,d\,x\right)}{120}-\frac{a^3\,\left(105\,c+105\,d\,x-272\right)}{120}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (7*a^3*x)/8 - ((13*a^3*tan(c/2 + (d*x)/2)^7)/2 - (13*a^3*tan(c/2 + (d*x)/2)
^3)/2 + (a^3*tan(c/2 + (d*x)/2)^9)/4 + (a^3*(105*c + 105*d*x))/120 - (a^3*(
105*c + 105*d*x - 272))/120 + tan(c/2 + (d*x)/2)^2*((a^3*(105*c + 105*d*x))
/24 - (a^3*(525*c + 525*d*x - 640))/120) + tan(c/2 + (d*x)/2)^8*((a^3*(105*
c + 105*d*x))/24 - (a^3*(525*c + 525*d*x - 720))/120) + tan(c/2 + (d*x)/2)^
4*((a^3*(105*c + 105*d*x))/12 - (a^3*(1050*c + 1050*d*x - 800))/120) + tan(
c/2 + (d*x)/2)^6*((a^3*(105*c + 105*d*x))/12 - (a^3*(1050*c + 1050*d*x - 19
20))/120) - (a^3*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 3.72, size = 226, normalized size = 2.13

$$\begin{cases} \frac{3a^3x\sin^4(c+dx)}{8}+\frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{a^3x\sin^2(c+dx)}{2}+\frac{3a^3x\cos^4(c+dx)}{8}+\frac{a^3x\cos^2(c+dx)}{2}+\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d}- \\ x\left(a\sin(c)+a\right)^3\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**4/8 + a**3*x*co
s(c + d*x)**2/2 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a**3*sin(c +
d*x)**2*cos(c + d*x)**3/(3*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) +
 a**3*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**3*cos(c + d*x)**5/(15*d) - a**
3*cos(c + d*x)**3/d, Ne(d, 0)), (x*(a*sin(c) + a)**3*cos(c)**2, True))
```

3.32 $\int \cos(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=22

$$\frac{(a\sin(c+dx)+a)^4}{4ad}$$

```
[Out] 1/4*(a+a*sin(d*x+c))^4/a/d
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2667, 32\}$

$$\frac{(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a + a*Sin[c + d*x])^4/(4*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^4}{4ad}\end{aligned}$$

Mathematica [B] time = 0.03, size = 65, normalized size = 2.95

$$\frac{a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/
d + (a^3*Sin[c + d*x]^4)/(4*d)
```

fricas [B] time = 0.76, size = 57, normalized size = 2.59

$$\frac{a^3\cos(dx+c)^4-8\,a^3\cos(dx+c)^2-4\left(a^3\cos(dx+c)^2-2\,a^3\right)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a^3*cos(d*x + c)^4 - 8*a^3*cos(d*x + c)^2 - 4*(a^3*cos(d*x + c)^2 - 2*
a^3)*sin(d*x + c))/d
```

giac [A] time = 0.83, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^4}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*sin(d*x + c) + a)^4/(a*d)
```

maple [A] time = 0.07, size = 21, normalized size = 0.95

$$\frac{(a+a\sin(dx+c))^4}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4*(a+a*sin(d*x+c))^4/d/a
```

maxima [A] time = 0.32, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^4}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*sin(d*x + c) + a)^4/(a*d)
```

mupad [B] time = 0.06, size = 53, normalized size = 2.41

$$\frac{\frac{a^3\sin(c+d\,x)^4}{4}+a^3\sin(c+d\,x)^3+\frac{3\,a^3\sin(c+d\,x)^2}{2}+a^3\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x) + (3*a^3*sin(c + d*x)^2)/2 + a^3*sin(c + d*x)^3 + (a^3*si
n(c + d*x)^4)/4)/d
```

sympy [A] time = 1.11, size = 70, normalized size = 3.18

$$\begin{cases} \frac{a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d} & \text{for}\ d\neq 0\\ x\,(a\sin(c)+a)^3\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**4/(4*d) + a**3*sin(c + d*x)**3/d + 3*a**3*sin
(c + d*x)**2/(2*d) + a**3*sin(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**3*c
os(c), True))
```

3.33 $\int \sec(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=52

$$-\frac{a^3\sin^2(c+dx)}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{4a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] -4*a^3*ln(1-sin(d*x+c))/d-3*a^3*sin(d*x+c)/d-1/2*a^3*sin(d*x+c)^2/d
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$-\frac{a^3\sin^2(c+dx)}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{4a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^3*Log[1 - Sin[c + d*x]])/d - (3*a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*
x]^2)/(2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^3\,dx&=\frac{a\,\text{Subst}\left(\int \frac{(a+x)^2}{a-x}\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{a\,\text{Subst}\left(\int \left(-3a+\frac{4a^2}{a-x}-x\right)\,dx,x,a\sin(c+dx)\right)}{d}\\&=-\frac{4a^3\log(1-\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.79

$$\frac{a^3\left(-\frac{1}{2}\sin^2(c+dx)-3\sin(c+dx)-4\log(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-4*Log[1 - Sin[c + d*x]] - 3*Sin[c + d*x] - Sin[c + d*x]^2/2))/d
```

fricas [A] time = 0.66, size = 45, normalized size = 0.87

$$\frac{a^3 \cos(dx+c)^2 - 8\,a^3 \log(-\sin(dx+c)+1) - 6\,a^3 \sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*cos(d*x + c)^2 - 8*a^3*log(-sin(d*x + c) + 1) - 6*a^3*sin(d*x + c)
)/d
```

giac [B] time = 0.90, size = 128, normalized size = 2.46

$$\frac{2\left(2\,a^3 \log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1\right) - 4\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{3\,a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 3\,a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 7\,a^3 \tan(}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 +\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 4*a^3*log(abs(tan(1/2*d*x + 1/2*
c) - 1)) - (3*a^3*tan(1/2*d*x + 1/2*c)^4 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 7
*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 3*a^3)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.14, size = 69, normalized size = 1.33

$$-\frac{a^3\left(\sin^2(dx+c)\right)}{2d} - \frac{4a^3 \ln(\cos(dx+c))}{d} - \frac{3a^3 \sin(dx+c)}{d} + \frac{4a^3 \ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2*a^3*sin(d*x+c)^2/d-4/d*a^3*ln(cos(d*x+c))-3*a^3*sin(d*x+c)/d+4/d*a^3*l
n(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.39, size = 43, normalized size = 0.83

$$-\frac{a^3 \sin(dx+c)^2 + 8\,a^3 \log(\sin(dx+c)-1) + 6\,a^3 \sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^3*sin(d*x + c)^2 + 8*a^3*log(sin(d*x + c) - 1) + 6*a^3*sin(d*x + c)
)/d
```

mupad [B] time = 0.05, size = 36, normalized size = 0.69

$$-\frac{a^3\,\left(8\,\ln(\sin(c+d\,x)-1) + 6\,\sin(c+d\,x) + \sin(c+d\,x)^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] -(a^3*(8*log(sin(c + d*x) - 1) + 6*sin(c + d*x) + sin(c + d*x)^2))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int 3 \sin(c + dx) \sec(c + dx) \, dx + \int 3 \sin^2(c + dx) \sec(c + dx) \, dx + \int \sin^3(c + dx) \sec(c + dx) \, dx + \int \sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x), x) + Integral(3*sin(c + d*x)**2
*sec(c + d*x), x) + Integral(sin(c + d*x)**3*sec(c + d*x), x) + Integral(se
c(c + d*x), x))
```

3.34 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=50

$$\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}+\frac{3a^3\cos(c+dx)}{d}-3a^3x$$

```
[Out] -3*a^3*x+3*a^3*cos(d*x+c)/d+2*a^5*cos(d*x+c)^3/d/(a-a*sin(d*x+c))^2
```

Rubi [A] time = 0.14, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2670, 2680, 2682, 8}

$$\frac{3a^3\cos(c+dx)}{d}+\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -3*a^3*x + (3*a^3*Cos[c + d*x])/d + (2*a^5*Cos[c + d*x]^3)/(d*(a - a*Sin[c
+ d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))^3\,dx &= a^6\int \frac{\cos^4(c+dx)}{(a-a\sin(c+dx))^3}\,dx\\ &= \frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-\left(3a^4\right)\int \frac{\cos^2(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= \frac{3a^3\cos(c+dx)}{d}+\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-\left(3a^3\right)\int 1\,dx\\ &= -3a^3x+\frac{3a^3\cos(c+dx)}{d}+\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}\end{aligned}$$

Mathematica [C] time = 0.03, size = 55, normalized size = 1.10

$$\frac{4\sqrt{2}\,a^3\sqrt{\sin(c+dx)+1}\,\sec(c+dx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Sqrt[2]*a^3*Hypergeometric2F1[-3/2, -1/2, 1/2, (1 - Sin[c + d*x])/2]*Sec
[c + d*x]*Sqrt[1 + Sin[c + d*x]])/d
```

fricas [A] time = 0.68, size = 101, normalized size = 2.02

$$-\frac{3\,a^3dx-a^3\cos\left(dx+c\right)^2-4\,a^3+\left(3\,a^3dx-5\,a^3\right)\cos\left(dx+c\right)-\left(3\,a^3dx-a^3\cos\left(dx+c\right)+4\,a^3\right)\sin\left(dx+c\right)}{d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*a^3*d*x - a^3*cos(d*x + c)^2 - 4*a^3 + (3*a^3*d*x - 5*a^3)*cos(d*x + c)
 - (3*a^3*d*x - a^3*cos(d*x + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c) - d
*sin(d*x + c) + d)
```

giac [A] time = 0.54, size = 91, normalized size = 1.82

$$-\frac{3\,(dx+c)a^3+\frac{2\left(4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)*a^3 + 2*(4*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2
*c) + 5*a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x
 + 1/2*c) - 1))/d
```

maple [A] time = 0.22, size = 87, normalized size = 1.74

$$\frac{a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2\left(dx+c\right)\right)\cos\left(dx+c\right)\right)+3a^3\left(\tan\left(dx+c\right)-dx-c\right)+\frac{3a^3}{\cos(dx+c)}+a^3\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(tan(d
*x+c)-d*x-c)+3*a^3/cos(d*x+c)+a^3*tan(d*x+c))
```

maxima [A] time = 0.41, size = 68, normalized size = 1.36

$$-\frac{3\left(dx+c-\tan\left(dx+c\right)\right)a^{3}-a^{3}\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)-a^{3}\tan\left(dx+c\right)-\frac{3a^{3}}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*(d*x + c - tan(d*x + c))*a^3 - a^3*(1/cos(d*x + c) + cos(d*x + c)) - a^
3*tan(d*x + c) - 3*a^3/cos(d*x + c))/d
```

mupad [B] time = 4.78, size = 138, normalized size = 2.76

$$-3\,a^{3}\,x-\frac{3\,a^{3}\,\left(c+d\,x\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^{3}\,\left(c+d\,x\right)-a^{3}\,\left(3\,c+3\,d\,x-2\right)\right)-a^{3}\,\left(3\,c+3\,d\,x-10\right)+\tan\left(\frac{c}{2}+\frac{d}{}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] - 3*a^3*x - (3*a^3*(c + d*x) - tan(c/2 + (d*x)/2)*(3*a^3*(c + d*x) - a^3*(3
*c + 3*d*x - 2)) - a^3*(3*c + 3*d*x - 10) + tan(c/2 + (d*x)/2)^2*(3*a^3*(c
+ d*x) - a^3*(3*c + 3*d*x - 8)))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*
x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\sin\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\int 3\sin^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\int\sin^{3}\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x)**2, x) + Integral(3*sin(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**2, x) + In
tegral(sec(c + d*x)**2, x))
```

3.35 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=40

$$\frac{2a^4}{d(a-a\sin(c+dx))}+\frac{a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] a^3*ln(1-sin(d*x+c))/d+2*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{2a^4}{d(a-a\sin(c+dx))}+\frac{a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[1 - Sin[c + d*x]])/d + (2*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \sec^3(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{a^3\operatorname{Subst}\left(\int \frac{a+x}{(a-x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\operatorname{Subst}\left(\int \left(\frac{2a}{(a-x)^2}+\frac{1}{-a+x}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\log(1-\sin(c+dx))}{d}+\frac{2a^4}{d(a-a\sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 1.48

$$\frac{a^3(1-\sin(c+dx))(\sin(c+dx)+1)\sec^2(c+dx)\left(\frac{2}{1-\sin(c+dx)}+\log(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(Log[1 - Sin[c + d*x]] + 2/(1 - Sin[c + d*x]))*(1 - Sin
[c + d*x])*(1 + Sin[c + d*x]))/d
```

fricas [A] time = 0.60, size = 51, normalized size = 1.28

$$-\frac{2\,a^3 - \left(a^3 \sin\left(dx+c\right) - a^3\right)\log\left(-\sin\left(dx+c\right)+1\right)}{d\sin\left(dx+c\right) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*a^3 - (a^3*sin(d*x + c) - a^3)*log(-sin(d*x + c) + 1))/(d*sin(d*x + c)
- d)
```

giac [B] time = 0.50, size = 92, normalized size = 2.30

$$-\frac{a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right) - 2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*a^3*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (3*a^3*tan(1/2*d*x + 1/2*c)^2 - 10*a^3*tan(1/2*d*x + 1/2*c) + 3*a^3
)/(tan(1/2*d*x + 1/2*c) - 1)^2)/d
```

maple [B] time = 0.22, size = 128, normalized size = 3.20

$$\frac{a^3\left(\tan^2\left(dx+c\right)\right)}{2d} + \frac{a^3\ln\left(\cos\left(dx+c\right)\right)}{d} + \frac{3a^3\left(\sin^3\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^2} + \frac{3a^3\sin\left(dx+c\right)}{2d} - \frac{a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d*a^3*tan(d*x+c)^2+1/d*a^3*ln(cos(d*x+c))+3/2/d*a^3*sin(d*x+c)^3/cos(d*
x+c)^2+3/2*a^3*sin(d*x+c)/d-1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a^3/cos
(d*x+c)^2+1/2/d*a^3*sec(d*x+c)*tan(d*x+c)
```

maxima [A] time = 1.11, size = 33, normalized size = 0.82

$$\frac{a^3\log\left(\sin\left(dx+c\right)-1\right) - \frac{2\,a^3}{\sin(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (a^3*log(sin(d*x + c) - 1) - 2*a^3/(sin(d*x + c) - 1))/d
```

mupad [B] time = 4.52, size = 35, normalized size = 0.88

$$\frac{a^3\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{d} - \frac{2\,a^3}{d\,\left(\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (a^3*log(sin(c + d*x) - 1))/d - (2*a^3)/(d*(sin(c + d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin(c+dx)\sec^3(c+dx)\,dx+\int 3\sin^2(c+dx)\sec^3(c+dx)\,dx+\int \sin^3(c+dx)\sec^3(c+dx)\,dx+\int \mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x)**3, x) + Integral(3*sin(c + d*x)
**2*sec(c + d*x)**3, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(sec(c + d*x)**3, x))
```

3.36 $\int \sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=31

$$\frac{a^6\cos^3(c+dx)}{3d(a-a\sin(c+dx))^3}$$

```
[Out] 1/3*a^6*cos(d*x+c)^3/d/(a-a*sin(d*x+c))^3
```

Rubi [A] time = 0.08, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2670, 2671}

$$\frac{a^6\cos^3(c+dx)}{3d(a-a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^6*Cos[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^3)
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sin(c+dx))^3\,dx &= a^6\int \frac{\cos^2(c+dx)}{(a-a\sin(c+dx))^3}\,dx\\ &= \frac{a^6\cos^3(c+dx)}{3d(a-a\sin(c+dx))^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.90

$$\frac{a^3(\sin(c+dx)+1)^3\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^3*(1 + Sin[c + d*x])^3)/(3*d)
```

fricas [B] time = 0.67, size = 99, normalized size = 3.19

$$\frac{a^3\cos(dx+c)^2-a^3\cos(dx+c)-2a^3-\left(a^3\cos(dx+c)+2a^3\right)\sin(dx+c)}{3\left(d\cos(dx+c)^2-d\cos(dx+c)+(d\cos(dx+c)+2d)\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3 - (a^3*cos(d*x + c) + 2*
a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2
*d)*sin(d*x + c) - 2*d)
```

giac [A] time = 1.52, size = 38, normalized size = 1.23

$$-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\right)}{3\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^3*tan(1/2*d*x + 1/2*c)^2 + a^3)/(d*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

maple [B] time = 0.26, size = 120, normalized size = 3.87

$$\frac{a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+\frac{a^3\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{a^3}{\cos(dx+c)^3}-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+
sin(d*x+c)^2)*cos(d*x+c))+a^3/cos(d*x+c)^3*sin(d*x+c)^3+a^3/cos(d*x+c)^3-a^
3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

maxima [B] time = 0.48, size = 78, normalized size = 2.52

$$\frac{3\,a^3\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3-\frac{\left(3\,\cos(dx+c)^2-1\right)a^3}{\cos(dx+c)^3}+\frac{3\,a^3}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - (3*cos(
d*x + c)^2 - 1)*a^3/cos(d*x + c)^3 + 3*a^3/cos(d*x + c)^3)/d
```

mupad [B] time = 4.59, size = 55, normalized size = 1.77

$$-\frac{2\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\right)}{3\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^4,x)
```

```
[Out] -(2*a^3*cos(c/2 + (d*x)/2)*(2*cos(c/2 + (d*x)/2)^2 - 3))/(3*d*(cos(c/2 + (d
*x)/2) - sin(c/2 + (d*x)/2))^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.37 $\int \sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=23

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}$$

```
[Out] 1/2*a^5/d/(a-a*sin(d*x+c))^2
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2667, 32\}$

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^5/(2*d*(a - a*Sin[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{a^5\,\text{Subst}\left(\int \frac{1}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^5}{2d(a-a\sin(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.23, size = 35, normalized size = 1.52

$$\frac{a^3}{2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3/(2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4)
```

fricas [A] time = 0.53, size = 30, normalized size = 1.30

$$-\frac{a^3}{2\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*a^3/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [B] time = 0.70, size = 63, normalized size = 2.74

$$\frac{2\left(a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*
x + 1/2*c))/(d*(tan(1/2*d*x + 1/2*c) - 1)^4)
```

maple [B] time = 0.24, size = 146, normalized size = 6.35

$$\frac{a^{3}\left(\sin^{4}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}+\frac{3a^{3}\left(\sin^{3}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}+\frac{3a^{3}\left(\sin^{3}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}+\frac{3a^{3}\sin\left(dx+c\right)}{8d}+\frac{3a^{3}}{4d\cos\left(dx+c\right)^{4}}+\frac{a^{3}\tan\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4+3/8
/d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+3/8*a^3*sin(d*x+c)/d+3/4/d*a^3/cos(d*x+c)^
4+1/4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*sec(d*x+c)*tan(d*x+c)
```

maxima [A] time = 0.66, size = 28, normalized size = 1.22

$$\frac{a^{3}}{2\left(\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3/((sin(d*x + c)^2 - 2*sin(d*x + c) + 1)*d)
```

mupad [B] time = 0.07, size = 18, normalized size = 0.78

$$\frac{a^{3}}{2\,d\,\left(\sin\left(c+d\,x\right)-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^5,x)
```

```
[Out] a^3/(2*d*(sin(c + d*x) - 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.38 $\int \sec^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=92

$$\frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{2a^6\cos(c+dx)}{15d\left(a^3-a^3\sin(c+dx)\right)}$$

```
[Out] 1/5*a^6*cos(d*x+c)/d/(a-a*sin(d*x+c))^3+2/15*a^5*cos(d*x+c)/d/(a-a*sin(d*x+
c))^2+2/15*a^6*cos(d*x+c)/d/(a^3-a^3*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2670, 2650, 2648}

$$\frac{2a^6\cos(c+dx)}{15d\left(a^3-a^3\sin(c+dx)\right)}+\frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^6*Cos[c + d*x])/(5*d*(a - a*Sin[c + d*x])^3) + (2*a^5*Cos[c + d*x])/(15*
d*(a - a*Sin[c + d*x])^2) + (2*a^6*Cos[c + d*x])/(15*d*(a^3 - a^3*Sin[c + d
*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+a\sin(c+dx))^3\,dx &= a^6\int \frac{1}{(a-a\sin(c+dx))^3}\,dx\\ &= \frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{1}{5}\left(2a^5\right)\int \frac{1}{(a-a\sin(c+dx))^2}\,dx\\ &= \frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{1}{15}\left(2a^4\right)\int \frac{1}{a-a\sin}\\ &= \frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{2a^4\cos(c+dx)}{15d(a-a\sin(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 110, normalized size = 1.20

$$\frac{2a^3\tan^5(c+dx)}{15d}+\frac{7a^3\sec^5(c+dx)}{15d}+\frac{a^3\tan(c+dx)\sec^4(c+dx)}{d}+\frac{a^3\tan^2(c+dx)\sec^3(c+dx)}{3d}-\frac{a^3\tan^3(c+dx)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*Sec[c + d*x]^5)/(15*d) + (a^3*Sec[c + d*x]^4*Tan[c + d*x])/d + (a^3*
Sec[c + d*x]^3*Tan[c + d*x]^2)/(3*d) - (a^3*Sec[c + d*x]^2*Tan[c + d*x]^3)/
(3*d) + (2*a^3*Tan[c + d*x]^5)/(15*d)
```

fricas [A] time = 0.52, size = 149, normalized size = 1.62

$$\frac{2\,a^3\cos(dx+c)^3-4\,a^3\cos(dx+c)^2-9\,a^3\cos(dx+c)-3\,a^3+\left(2\,a^3\cos(dx+c)^2+6\,a^3\cos(dx+c)-3\,a^3\right)}{15\left(d\cos(dx+c)^3+3\,d\cos(dx+c)^2-2\,d\cos(dx+c)-\left(d\cos(dx+c)^2-2\,d\cos(dx+c)-4\,d\right)\sin(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(2*a^3*cos(d*x + c)^3 - 4*a^3*cos(d*x + c)^2 - 9*a^3*cos(d*x + c) - 3*
a^3 + (2*a^3*cos(d*x + c)^2 + 6*a^3*cos(d*x + c) - 3*a^3)*sin(d*x + c))/(d*
cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2
- 2*d*cos(d*x + c) - 4*d)*sin(d*x + c) - 4*d)
```

giac [A] time = 0.82, size = 86, normalized size = 0.93

$$-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^3\right)}{15\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^3*tan(1/2*d*x + 1/2*c)^4 - 30*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a
^3*tan(1/2*d*x + 1/2*c)^2 - 20*a^3*tan(1/2*d*x + 1/2*c) + 7*a^3)/(d*(tan(1/
2*d*x + 1/2*c) - 1)^5)
```

maple [A] time = 0.26, size = 171, normalized size = 1.86

$$\frac{a^3\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)+3a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{3a^3}{5\cos(dx+c)^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15
*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/5*sin(d
*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+3/5*a^3/cos(d*x+c)^5-a
^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

maxima [A] time = 0.43, size = 103, normalized size = 1.12

$$\frac{\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)a^3+3\left(3\,\tan(dx+c)^5+5\,\tan(dx+c)^3\right)a^3-\frac{\left(5\,\cos(dx+c)\right.}{\cos(dx+}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 3*(3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*a^3 - (5*cos(d*x + c)^2 - 3)*a^3/cos(d*x
+ c)^5 + 9*a^3/cos(d*x + c)^5)/d
```

mupad [B] time = 4.69, size = 135, normalized size = 1.47

$$\frac{2\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(7\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-20\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-30\,\cos\left(\right.\right.}{15\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^6,x)
```

```
[Out] (2*a^3*cos(c/2 + (d*x)/2)*(7*cos(c/2 + (d*x)/2)^4 + 15*sin(c/2 + (d*x)/2)^4
 - 30*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3 - 20*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2))/(15*d*(cos
(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.39 $\int \sec^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=87

$$\frac{a^6}{6d(a-a\sin(c+dx))^3}+\frac{a^5}{8d(a-a\sin(c+dx))^2}+\frac{a^4}{8d(a-a\sin(c+dx))}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] 1/8*a^3*arctanh(sin(d*x+c))/d+1/6*a^6/d/(a-a*sin(d*x+c))^3+1/8*a^5/d/(a-a*s
in(d*x+c))^2+1/8*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2667, 44, 206\}$

$$\frac{a^6}{6d(a-a\sin(c+dx))^3}+\frac{a^5}{8d(a-a\sin(c+dx))^2}+\frac{a^4}{8d(a-a\sin(c+dx))}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/(8*d) + a^6/(6*d*(a - a*Sin[c + d*x])^3) + a^5/
(8*d*(a - a*Sin[c + d*x])^2) + a^4/(8*d*(a - a*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec^7(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{a^7\operatorname{Subst}\left(\int \frac{1}{(a-x)^4(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^7\operatorname{Subst}\left(\int \left(\frac{1}{2a(a-x)^4}+\frac{1}{4a^2(a-x)^3}+\frac{1}{8a^3(a-x)^2}+\frac{1}{8a^3\left(a^2-x^2\right)}\right)dx,x,a\sin(c\right.}{d}\\ &= \frac{a^6}{6d(a-a\sin(c+dx))^3}+\frac{a^5}{8d(a-a\sin(c+dx))^2}+\frac{a^4}{8d(a-a\sin(c+dx}\\ &= \frac{a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^6}{6d(a-a\sin(c+dx))^3}+\frac{a^5}{8d(a-a\sin(c+d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 0.77

$$-\frac{a^3(\sin(c+dx)+1)^3\sec^6(c+dx)\left(-3\sin^2(c+dx)+9\sin(c+dx)+3(\sin(c+dx)-1)^3\tanh^{-1}(\sin(c+dx))-1\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/24*(a^3*Sec[c + d*x]^6*(1 + Sin[c + d*x])^3*(-10 + 3*ArcTanh[Sin[c + d*x
]]*(-1 + Sin[c + d*x])^3 + 9*Sin[c + d*x] - 3*Sin[c + d*x]^2))/d
```

fricas [B] time = 0.64, size = 185, normalized size = 2.13

$$\frac{6\,a^3\cos(dx+c)^2+18\,a^3\sin(dx+c)-26\,a^3+3\left(3\,a^3\cos(dx+c)^2-4\,a^3-\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)\right.}{48\left(3\,d\cos(dx+c)^2-\left(d\cos(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(6*a^3*cos(d*x + c)^2 + 18*a^3*sin(d*x + c) - 26*a^3 + 3*(3*a^3*cos(d*
x + c)^2 - 4*a^3 - (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c))*log(sin(d*x +
 c) + 1) - 3*(3*a^3*cos(d*x + c)^2 - 4*a^3 - (a^3*cos(d*x + c)^2 - 4*a^3)*s
in(d*x + c))*log(-sin(d*x + c) + 1))/(3*d*cos(d*x + c)^2 - (d*cos(d*x + c)^
2 - 4*d)*sin(d*x + c) - 4*d)
```

giac [A] time = 0.56, size = 90, normalized size = 1.03

$$\frac{6\,a^3\log\left(|\sin(dx+c)+1|\right)-6\,a^3\log\left(|\sin(dx+c)-1|\right)+\frac{11\,a^3\sin(dx+c)^3-45\,a^3\sin(dx+c)^2+69\,a^3\sin(dx+c)-51\,a^3}{(\sin(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*a^3*log(abs(sin(d*x + c) + 1)) - 6*a^3*log(abs(sin(d*x + c) - 1)) +
 (11*a^3*sin(d*x + c)^3 - 45*a^3*sin(d*x + c)^2 + 69*a^3*sin(d*x + c) - 51*
a^3)/(sin(d*x + c) - 1)^3)/d
```

maple [B] time = 0.25, size = 238, normalized size = 2.74

$$\frac{a^3\left(\sin^4(dx+c)\right)}{6d\cos(dx+c)^6}+\frac{a^3\left(\sin^4(dx+c)\right)}{12d\cos(dx+c)^4}+\frac{a^3\left(\sin^3(dx+c)\right)}{2d\cos(dx+c)^6}+\frac{3a^3\left(\sin^3(dx+c)\right)}{8d\cos(dx+c)^4}+\frac{3a^3\left(\sin^3(dx+c)\right)}{16d\cos(dx+c)^2}+\frac{3a^3\sin(dx+}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/6/d*a^3*sin(d*x+c)^4/cos(d*x+c)^6+1/12/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4+1/
2/d*a^3*sin(d*x+c)^3/cos(d*x+c)^6+3/8/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4+3/16/
d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+3/16*a^3*sin(d*x+c)/d+1/8/d*a^3*ln(sec(d*x+
c)+tan(d*x+c))+1/2/d*a^3/cos(d*x+c)^6+1/6/d*a^3*tan(d*x+c)*sec(d*x+c)^5+5/2
4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+5/16/d*a^3*sec(d*x+c)*tan(d*x+c)
```

maxima [A] time = 0.40, size = 96, normalized size = 1.10

$$\frac{3\,a^3\log(\sin(dx+c)+1)-3\,a^3\log(\sin(dx+c)-1)-\frac{2\left(3\,a^3\sin(dx+c)^2-9\,a^3\sin(dx+c)+10\,a^3\right)}{\sin(dx+c)^3-3\,\sin(dx+c)^2+3\,\sin(dx+c)-1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^3*log(sin(d*x + c) + 1) - 3*a^3*log(sin(d*x + c) - 1) - 2*(3*a^3*
sin(d*x + c)^2 - 9*a^3*sin(d*x + c) + 10*a^3)/(sin(d*x + c)^3 - 3*sin(d*x +
 c)^2 + 3*sin(d*x + c) - 1))/d
```

mupad [B] time = 4.54, size = 81, normalized size = 0.93

$$\frac{a^3 \operatorname{atanh}(\sin(c+d\,x))}{8\,d}-\frac{\frac{a^3\sin(c+d\,x)^2}{8}-\frac{3\,a^3\,\sin(c+d\,x)}{8}+\frac{5\,a^3}{12}}{d\,\left(\sin(c+d\,x)^3-3\sin(c+d\,x)^2+3\,\sin(c+d\,x)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^7,x)
```

```
[Out] (a^3*atanh(sin(c + d*x)))/(8*d) - ((5*a^3)/12 - (3*a^3*sin(c + d*x))/8 + (a
^3*sin(c + d*x)^2)/8)/(d*(3*sin(c + d*x) - 3*sin(c + d*x)^2 + sin(c + d*x)^
3 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.40 $\int \sec^8(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=99

$$\frac{3a^3\tan^5(c+dx)}{35d}+\frac{2a^3\tan^3(c+dx)}{7d}+\frac{3a^3\tan(c+dx)}{7d}+\frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a\sin(c+dx)+a)^2}{7d}$$

```
[Out] 3/35*a^3*sec(d*x+c)^5/d+2/7*a*sec(d*x+c)^7*(a+a*sin(d*x+c))^2/d+3/7*a^3*tan
(d*x+c)/d+2/7*a^3*tan(d*x+c)^3/d+3/35*a^3*tan(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2676, 2669, 3767\}$

$$\frac{3a^3\tan^5(c+dx)}{35d}+\frac{2a^3\tan^3(c+dx)}{7d}+\frac{3a^3\tan(c+dx)}{7d}+\frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a\sin(c+dx)+a)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*a^3*Sec[c + d*x]^5)/(35*d) + (2*a*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2)
/(7*d) + (3*a^3*Tan[c + d*x])/(7*d) + (2*a^3*Tan[c + d*x]^3)/(7*d) + (3*a^3
*Tan[c + d*x]^5)/(35*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{1}{7}\left(3a^2\right)\int \sec^6(c+dx)(a+a\sin(c\\ &= \frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{1}{7}\left(3a^3\right)\int \sec^6\\ &= \frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{\left(3a^3\right)\text{Subst}\left(\int\right.}{}\\ &= \frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{3a^3\tan(c+dx}{7d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 134, normalized size = 1.35

$$-\frac{8a^3\tan^7(c+dx)}{35d}+\frac{13a^3\sec^7(c+dx)}{35d}+\frac{a^3\tan(c+dx)\sec^6(c+dx)}{d}+\frac{a^3\tan^2(c+dx)\sec^5(c+dx)}{5d}-\frac{a^3\tan^3(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*a^3*Sec[c + d*x]^7)/(35*d) + (a^3*Sec[c + d*x]^6*Tan[c + d*x])/d + (a^3
*Sec[c + d*x]^5*Tan[c + d*x]^2)/(5*d) - (a^3*Sec[c + d*x]^4*Tan[c + d*x]^3)
/d + (4*a^3*Sec[c + d*x]^2*Tan[c + d*x]^5)/(5*d) - (8*a^3*Tan[c + d*x]^7)/(
35*d)
```

fricas [A] time = 0.54, size = 112, normalized size = 1.13

$$\frac{8\,a^3\cos\left(dx+c\right)^4-36\,a^3\cos\left(dx+c\right)^2+15\,a^3+4\left(6\,a^3\cos\left(dx+c\right)^2-5\,a^3\right)\sin\left(dx+c\right)}{35\left(3\,d\cos\left(dx+c\right)^3-4\,d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^3-4\,d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/35*(8*a^3*cos(d*x + c)^4 - 36*a^3*cos(d*x + c)^2 + 15*a^3 + 4*(6*a^3*cos(
d*x + c)^2 - 5*a^3)*sin(d*x + c))/(3*d*cos(d*x + c)^3 - 4*d*cos(d*x + c) -
(d*cos(d*x + c)^3 - 4*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.52, size = 138, normalized size = 1.39

$$-\frac{\frac{35\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{525\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1960\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4025\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-4480\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3143\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35*a^3/(tan(1/2*d*x + 1/2*c) + 1) + (525*a^3*tan(1/2*d*x + 1/2*c)^6
 - 1960*a^3*tan(1/2*d*x + 1/2*c)^5 + 4025*a^3*tan(1/2*d*x + 1/2*c)^4 - 4480
*a^3*tan(1/2*d*x + 1/2*c)^3 + 3143*a^3*tan(1/2*d*x + 1/2*c)^2 - 1176*a^3*ta
n(1/2*d*x + 1/2*c) + 243*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

maple [B] time = 0.27, size = 217, normalized size = 2.19

$$\frac{a^3\left(\frac{\sin^4(dx+c)}{7\cos(dx+c)^7}+\frac{3\left(\sin^4(dx+c)\right)}{35\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{35\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{35}\right)+3a^3\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35
*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^
2)*cos(d*x+c))+3*a^3*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d
*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+3/7*a^3/cos(d*x+c)^7-a^3*(-16/35-1
/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

maxima [A] time = 0.48, size = 122, normalized size = 1.23

$$\frac{\left(15\,\tan\left(dx+c\right)^7+42\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3\right)a^3+\left(5\,\tan\left(dx+c\right)^7+21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/35*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^3 + (5*
tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*a
^3 - (7*cos(d*x + c)^2 - 5)*a^3/cos(d*x + c)^7 + 15*a^3/cos(d*x + c)^7)/d
```

mupad [B] time = 4.86, size = 228, normalized size = 2.30

$$\frac{2\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(13\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-43\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+77\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-7\cos\left(\frac{c}{2}\right.\right.}{35\,d\,\left(\cos\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^8,x)
```

```
[Out] (2*a^3*cos(c/2 + (d*x)/2)*(13*cos(c/2 + (d*x)/2)^7 + 35*sin(c/2 + (d*x)/2)^
7 - 105*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^6 - 43*cos(c/2 + (d*x)/2)^6*s
in(c/2 + (d*x)/2) + 175*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^5 - 105*cos
(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^4 - 7*cos(c/2 + (d*x)/2)^4*sin(c/2 + (
d*x)/2)^3 + 77*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^2))/(35*d*(cos(c/2 +
 (d*x)/2) - sin(c/2 + (d*x)/2))^7*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.41 $\int \cos^5(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=67

$$\frac{(a\sin(c+dx)+a)^{13}}{13a^5d}-\frac{(a\sin(c+dx)+a)^{12}}{3a^4d}+\frac{4(a\sin(c+dx)+a)^{11}}{11a^3d}$$

```
[Out] 4/11*(a+a*sin(d*x+c))^11/a^3/d-1/3*(a+a*sin(d*x+c))^12/a^4/d+1/13*(a+a*sin(
d*x+c))^13/a^5/d
```

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^{13}}{13a^5d}-\frac{(a\sin(c+dx)+a)^{12}}{3a^4d}+\frac{4(a\sin(c+dx)+a)^{11}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^11)/(11*a^3*d) - (a + a*Sin[c + d*x])^12/(3*a^4*d)
+ (a + a*Sin[c + d*x])^13/(13*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^8\,dx &= \frac{\operatorname{Subst}\left(\int (a-x)^2(a+x)^{10}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\operatorname{Subst}\left(\int \left(4a^2(a+x)^{10}-4a(a+x)^{11}+(a+x)^{12}\right)dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{4(a+a\sin(c+dx))^{11}}{11a^3d}-\frac{(a+a\sin(c+dx))^{12}}{3a^4d}+\frac{(a+a\sin(c+dx))^{13}}{13a^5d}\end{aligned}$$

Mathematica [A] time = 0.42, size = 58, normalized size = 0.87

$$-\frac{a^8(\sin(c+dx)+1)^8\left(33\sin^2(c+dx)-77\sin(c+dx)+46\right)\cos^6(c+dx)}{429d(\sin(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]
```

[Out] -1/429*(a^8*Cos[c + d*x]^6*(1 + Sin[c + d*x])^8*(46 - 77*Sin[c + d*x] + 33*
Sin[c + d*x]^2))/(d*(-1 + Sin[c + d*x])^3)

fricas [B] time = 0.79, size = 149, normalized size = 2.22

$$\frac{286 a^8 \cos(dx+c)^{12} - 3432 a^8 \cos(dx+c)^{10} + 10296 a^8 \cos(dx+c)^8 - 9152 a^8 \cos(dx+c)^6 + \left(33 a^8 \cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="fricas")

[Out] 1/429*(286*a^8*cos(d*x + c)^12 - 3432*a^8*cos(d*x + c)^10 + 10296*a^8*cos(d
*x + c)^8 - 9152*a^8*cos(d*x + c)^6 + (33*a^8*cos(d*x + c)^12 - 1212*a^8*co
s(d*x + c)^10 + 6280*a^8*cos(d*x + c)^8 - 8512*a^8*cos(d*x + c)^6 + 768*a^8
*cos(d*x + c)^4 + 1024*a^8*cos(d*x + c)^2 + 2048*a^8)*sin(d*x + c))/d

giac [B] time = 1.96, size = 219, normalized size = 3.27

$$\frac{a^8 \cos(12\,dx+12\,c)}{3072\,d} - \frac{3\,a^8 \cos(10\,dx+10\,c)}{256\,d} + \frac{27\,a^8 \cos(8\,dx+8\,c)}{512\,d} + \frac{155\,a^8 \cos(6\,dx+6\,c)}{768\,d} - \frac{475\,a^8 \cos(4\,dx+4}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="giac")

[Out] 1/3072*a^8*cos(12*d*x + 12*c)/d - 3/256*a^8*cos(10*d*x + 10*c)/d + 27/512*a
^8*cos(8*d*x + 8*c)/d + 155/768*a^8*cos(6*d*x + 6*c)/d - 475/1024*a^8*cos(4
*d*x + 4*c)/d - 323/128*a^8*cos(2*d*x + 2*c)/d + 1/53248*a^8*sin(13*d*x + 1
3*c)/d - 115/45056*a^8*sin(11*d*x + 11*c)/d + 205/6144*a^8*sin(9*d*x + 9*c)
/d - 7/2048*a^8*sin(7*d*x + 7*c)/d - 2033/4096*a^8*sin(5*d*x + 5*c)/d - 613
7/12288*a^8*sin(3*d*x + 3*c)/d + 4845/1024*a^8*sin(d*x + c)/d

maple [B] time = 0.18, size = 513, normalized size = 7.66

$$\frac{a^8 \left(-\frac{(\sin^7(dx+c))(\cos^6(dx+c))}{13} - \frac{7(\sin^5(dx+c))(\cos^6(dx+c))}{143} - \frac{35(\sin^3(dx+c))(\cos^6(dx+c))}{1287} - \frac{5\sin(dx+c)(\cos^6(dx+c))}{429} + \frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(c}{}\right.}{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x)

[Out] 1/d*(a^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-3
5/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*sin(d*x+c)*cos(d*x+c)^6+1/429*(8/3+c
os(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+8*a^8*(-1/12*sin(d*x+c)^6*cos(d*x
+c)^6-1/20*sin(d*x+c)^4*cos(d*x+c)^6-1/40*sin(d*x+c)^2*cos(d*x+c)^6-1/120*c
os(d*x+c)^6)+28*a^8*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(
d*x+c)^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+
c)^2)*sin(d*x+c))+56*a^8*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2
*cos(d*x+c)^6-1/60*cos(d*x+c)^6)+70*a^8*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/2
1*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))+56*a^8*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+28*a^8*(-1/7
*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c
))-4/3*a^8*cos(d*x+c)^6+1/5*a^8*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))

maxima [B] time = 0.61, size = 173, normalized size = 2.58

$$\frac{33 a^8 \sin(dx+c)^{13} + 286 a^8 \sin(dx+c)^{12} + 1014 a^8 \sin(dx+c)^{11} + 1716 a^8 \sin(dx+c)^{10} + 715 a^8 \sin(dx+c)^9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/429*(33*a^8*sin(d*x + c)^13 + 286*a^8*sin(d*x + c)^12 + 1014*a^8*sin(d*x
+ c)^11 + 1716*a^8*sin(d*x + c)^10 + 715*a^8*sin(d*x + c)^9 - 2574*a^8*sin(
d*x + c)^8 - 5148*a^8*sin(d*x + c)^7 - 3432*a^8*sin(d*x + c)^6 + 1287*a^8*s
in(d*x + c)^5 + 4290*a^8*sin(d*x + c)^4 + 3718*a^8*sin(d*x + c)^3 + 1716*a^
8*sin(d*x + c)^2 + 429*a^8*sin(d*x + c))/d
```

mupad [B] time = 0.18, size = 134, normalized size = 2.00

$$\frac{a^8\,\sin\left(c+d\,x\right)\,\left(33\,\sin\left(c+d\,x\right)^{12}+286\,\sin\left(c+d\,x\right)^{11}+1014\,\sin\left(c+d\,x\right)^{10}+1716\,\sin\left(c+d\,x\right)^{9}+715\,\sin\left(c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^8,x)
```

```
[Out] (a^8*sin(c + d*x)*(1716*sin(c + d*x) + 3718*sin(c + d*x)^2 + 4290*sin(c + d
*x)^3 + 1287*sin(c + d*x)^4 - 3432*sin(c + d*x)^5 - 5148*sin(c + d*x)^6 - 2
574*sin(c + d*x)^7 + 715*sin(c + d*x)^8 + 1716*sin(c + d*x)^9 + 1014*sin(c
+ d*x)^10 + 286*sin(c + d*x)^11 + 33*sin(c + d*x)^12 + 429))/(429*d)
```

sympy [A] time = 127.89, size = 558, normalized size = 8.33

$$\begin{cases} \frac{8a^8\sin^{13}(c+dx)}{1287d} + \frac{4a^8\sin^{11}(c+dx)\cos^2(c+dx)}{99d} + \frac{32a^8\sin^{11}(c+dx)}{99d} + \frac{a^8\sin^9(c+dx)\cos^4(c+dx)}{9d} + \frac{16a^8\sin^9(c+dx)\cos^2(c+dx)}{9d} + \frac{16a^8 s}{} \\ x\left(a\sin(c)+a\right)^8\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((8*a**8*sin(c + d*x)**13/(1287*d) + 4*a**8*sin(c + d*x)**11*cos(c
 + d*x)**2/(99*d) + 32*a**8*sin(c + d*x)**11/(99*d) + a**8*sin(c + d*x)**9*
cos(c + d*x)**4/(9*d) + 16*a**8*sin(c + d*x)**9*cos(c + d*x)**2/(9*d) + 16*
a**8*sin(c + d*x)**9/(9*d) + 4*a**8*sin(c + d*x)**7*cos(c + d*x)**4/d + 8*a
**8*sin(c + d*x)**7*cos(c + d*x)**2/d + 32*a**8*sin(c + d*x)**7/(15*d) - 4*
a**8*sin(c + d*x)**6*cos(c + d*x)**6/(3*d) + 14*a**8*sin(c + d*x)**5*cos(c
+ d*x)**4/d + 112*a**8*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + 8*a**8*sin(
c + d*x)**5/(15*d) - a**8*sin(c + d*x)**4*cos(c + d*x)**8/d - 28*a**8*sin(c
 + d*x)**4*cos(c + d*x)**6/(3*d) + 28*a**8*sin(c + d*x)**3*cos(c + d*x)**4/
(3*d) + 4*a**8*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - 2*a**8*sin(c + d*x)*
*2*cos(c + d*x)**10/(5*d) - 14*a**8*sin(c + d*x)**2*cos(c + d*x)**8/(3*d) -
 28*a**8*sin(c + d*x)**2*cos(c + d*x)**6/(3*d) + a**8*sin(c + d*x)*cos(c +
d*x)**4/d - a**8*cos(c + d*x)**12/(15*d) - 14*a**8*cos(c + d*x)**10/(15*d)
- 7*a**8*cos(c + d*x)**8/(3*d) - 4*a**8*cos(c + d*x)**6/(3*d), Ne(d, 0)), (
x*(a*sin(c) + a)**8*cos(c)**5, True))
```

3.42 $\int \cos^4(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=286

$$-\frac{4199a^8\cos^5(c+dx)}{1920d}-\frac{4199\cos^5(c+dx)\left(a^8\sin(c+dx)+a^8\right)}{2688d}+\frac{4199a^8\sin(c+dx)\cos^3(c+dx)}{1536d}+\frac{4199a^8\sin(c+ \cdots}{10\cdots}$$

```
[Out] 4199/1024*a^8*x-4199/1920*a^8*cos(d*x+c)^5/d+4199/1024*a^8*cos(d*x+c)*sin(d
*x+c)/d+4199/1536*a^8*cos(d*x+c)^3*sin(d*x+c)/d-323/1320*a^3*cos(d*x+c)^5*(
a+a*sin(d*x+c))^5/d-19/132*a^2*cos(d*x+c)^5*(a+a*sin(d*x+c))^6/d-1/12*a*cos
(d*x+c)^5*(a+a*sin(d*x+c))^7/d-4199/6336*a^2*cos(d*x+c)^5*(a^2+a^2*sin(d*x+
c))^3/d-323/792*cos(d*x+c)^5*(a^2+a^2*sin(d*x+c))^4/d-4199/4032*cos(d*x+c)^
5*(a^4+a^4*sin(d*x+c))^2/d-4199/2688*cos(d*x+c)^5*(a^8+a^8*sin(d*x+c))/d
```

Rubi [A] time = 0.40, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2678, 2669, 2635, 8}

$$-\frac{4199a^8\cos^5(c+dx)}{1920d}+\frac{4199a^8\sin(c+dx)\cos^3(c+dx)}{1536d}-\frac{323a^3\cos^5(c+dx)(a\sin(c+dx)+a)^5}{1320d}-\frac{19a^2\cos^5(c+ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (4199*a^8*x)/1024 - (4199*a^8*Cos[c + d*x]^5)/(1920*d) + (4199*a^8*Cos[c +
d*x]*Sin[c + d*x])/(1024*d) + (4199*a^8*Cos[c + d*x]^3*Sin[c + d*x])/(1536*
d) - (323*a^3*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^5)/(1320*d) - (19*a^2*Cos
[c + d*x]^5*(a + a*Sin[c + d*x])^6)/(132*d) - (a*Cos[c + d*x]^5*(a + a*Sin[
c + d*x])^7)/(12*d) - (4199*a^2*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x])^3)/
(6336*d) - (323*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x])^4)/(792*d) - (4199*
Cos[c + d*x]^5*(a^4 + a^4*Sin[c + d*x])^2)/(4032*d) - (4199*Cos[c + d*x]^5*
(a^8 + a^8*Sin[c + d*x]))/(2688*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
```

```
*m, 2*p]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^8\,dx = -\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^7}{12d}+\frac{1}{12}(19a)\int \cos^4(c+dx)(a+a\,\mathrm{s}$$
$$= -\frac{19a^2\cos^5(c+dx)(a+a\sin(c+dx))^6}{132d}-\frac{a\cos^5(c+dx)(a+a\sin(c+}{12d}$$
$$= -\frac{323a^3\cos^5(c+dx)(a+a\sin(c+dx))^5}{1320d}-\frac{19a^2\cos^5(c+dx)(a+a\,\mathrm{si}}{132d}$$
$$= -\frac{323a^3\cos^5(c+dx)(a+a\sin(c+dx))^5}{1320d}-\frac{19a^2\cos^5(c+dx)(a+a\,\mathrm{si}}{132d}$$
$$= -\frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d}-\frac{323a^3\cos^5(c+dx)(a+a}{1320d}$$
$$= -\frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d}-\frac{323a^3\cos^5(c+dx)(a+a}{1320d}$$
$$= -\frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d}-\frac{323a^3\cos^5(c+dx)(a+a}{1320d}$$
$$= -\frac{4199a^8\cos^5(c+dx)}{1920d}-\frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d}-3$$
$$= -\frac{4199a^8\cos^5(c+dx)}{1920d}+\frac{4199a^8\cos^3(c+dx)\sin(c+dx)}{1536d}-\frac{4199a^5\,\mathrm{co}}{}$$
$$= -\frac{4199a^8\cos^5(c+dx)}{1920d}+\frac{4199a^8\cos(c+dx)\sin(c+dx)}{1024d}+\frac{4199a^8\,\mathrm{co}}{}$$
$$= \frac{4199a^8x}{1024}-\frac{4199a^8\cos^5(c+dx)}{1920d}+\frac{4199a^8\cos(c+dx)\sin(c+dx)}{1024d}+$$

Mathematica [A] time = 3.20, size = 211, normalized size = 0.74

$$-\frac{a^8\left(\sqrt{\sin(c+dx)+1}\,\left(295680\sin^{12}(c+dx)+2284800\sin^{11}(c+dx)+6969984\sin^{10}(c+dx)+9086336\,\mathrm{sin}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/3548160*(a^8*Cos[c + d*x]^5*(-29099070*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqr
t[2]]*Sqrt[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-22470656 + 11469281
*Sin[c + d*x] + 13958687*Sin[c + d*x]^2 + 20459158*Sin[c + d*x]^3 + 1428311
4*Sin[c + d*x]^4 - 8321928*Sin[c + d*x]^5 - 26346616*Sin[c + d*x]^6 - 20428
112*Sin[c + d*x]^7 - 1239728*Sin[c + d*x]^8 + 9086336*Sin[c + d*x]^9 + 6969
984*Sin[c + d*x]^10 + 2284800*Sin[c + d*x]^11 + 295680*Sin[c + d*x]^12)))/(
d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

fricas [A] time = 0.64, size = 150, normalized size = 0.52

$$\frac{2580480\,a^8\cos(dx+c)^{11}-31539200\,a^8\cos(dx+c)^9+97320960\,a^8\cos(dx+c)^7-90832896\,a^8\cos(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/3548160*(2580480*a^8*cos(d*x + c)^11 - 31539200*a^8*cos(d*x + c)^9 + 9732
0960*a^8*cos(d*x + c)^7 - 90832896*a^8*cos(d*x + c)^5 + 14549535*a^8*d*x +
231*(1280*a^8*cos(d*x + c)^11 - 47744*a^8*cos(d*x + c)^9 + 253488*a^8*cos(d
*x + c)^7 - 359624*a^8*cos(d*x + c)^5 + 41990*a^8*cos(d*x + c)^3 + 62985*a^
8*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 1.97, size = 208, normalized size = 0.73

$$\frac{4199}{1024}a^8x+\frac{a^8\cos(11\,dx+11\,c)}{1408\,d}-\frac{31\,a^8\cos(9\,dx+9\,c)}{1152\,d}+\frac{139\,a^8\cos(7\,dx+7\,c)}{896\,d}+\frac{171\,a^8\cos(5\,dx+5\,c)}{640\,d}-\frac{323\,a^8\cos}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 4199/1024*a^8*x + 1/1408*a^8*cos(11*d*x + 11*c)/d - 31/1152*a^8*cos(9*d*x +
 9*c)/d + 139/896*a^8*cos(7*d*x + 7*c)/d + 171/640*a^8*cos(5*d*x + 5*c)/d -
 323/192*a^8*cos(3*d*x + 3*c)/d - 323/64*a^8*cos(d*x + c)/d + 1/24576*a^8*s
in(12*d*x + 12*c)/d - 29/5120*a^8*sin(10*d*x + 10*c)/d + 673/8192*a^8*sin(8
*d*x + 8*c)/d - 361/3072*a^8*sin(6*d*x + 6*c)/d - 8721/8192*a^8*sin(4*d*x +
 4*c)/d + 323/512*a^8*sin(2*d*x + 2*c)/d
```

maple [B] time = 0.20, size = 535, normalized size = 1.87

$$a^8\left(-\frac{(\sin^7(dx+c))(\cos^5(dx+c))}{12}-\frac{7(\sin^5(dx+c))(\cos^5(dx+c))}{120}-\frac{7(\sin^3(dx+c))(\cos^5(dx+c))}{192}-\frac{7\sin(dx+c)(\cos^5(dx+c))}{384}+\frac{7\left(\cos^3(dx+c)+\frac{3\cos(d}{2}\right.}{1536}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/12*sin(d*x+c)^7*cos(d*x+c)^5-7/120*sin(d*x+c)^5*cos(d*x+c)^5-7
/192*sin(d*x+c)^3*cos(d*x+c)^5-7/384*sin(d*x+c)*cos(d*x+c)^5+7/1536*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)+8*a^8*(-1/11*sin(d*x
+c)^6*cos(d*x+c)^5-2/33*sin(d*x+c)^4*cos(d*x+c)^5-8/231*sin(d*x+c)^2*cos(d*
x+c)^5-16/1155*cos(d*x+c)^5)+28*a^8*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*s
in(d*x+c)^3*cos(d*x+c)^5-1/32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3
/2*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+56*a^8*(-1/9*sin(d*x+c)^4*cos(
d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5)+70*a^8*(-1/8*si
n(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2
*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+56*a^8*(-1/7*sin(d*x+c)^2*cos(d*
x+c)^5-2/35*cos(d*x+c)^5)+28*a^8*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-8/5*cos(d*x+c)^5*a^8+a^8
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.66, size = 339, normalized size = 1.19

$$-\frac{45416448\,a^8\cos(dx+c)^5-196608\left(105\,\cos(dx+c)^{11}-385\,\cos(dx+c)^9+495\,\cos(dx+c)^7-231\,\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/28385280*(45416448*a^8*cos(d*x + c)^5 - 196608*(105*cos(d*x + c)^11 - 38
5*cos(d*x + c)^9 + 495*cos(d*x + c)^7 - 231*cos(d*x + c)^5)*a^8 + 5046272*(
35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*a^8 - 45416448*(
5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^8 + 231*(384*sin(2*d*x + 2*c)^5 + 20
*sin(4*d*x + 4*c)^3 - 840*d*x - 840*c - 15*sin(8*d*x + 8*c) + 240*sin(4*d*x
```

```
 + 4*c))*a^8 + 77616*(32*sin(2*d*x + 2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x
 + 8*c) + 40*sin(4*d*x + 4*c))*a^8 - 4139520*(4*sin(2*d*x + 2*c)^3 + 12*d*x
 + 12*c - 3*sin(4*d*x + 4*c))*a^8 - 1940400*(24*d*x + 24*c + sin(8*d*x + 8*
c) - 8*sin(4*d*x + 4*c))*a^8 - 887040*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8
*sin(2*d*x + 2*c))*a^8)/d
```

mupad [B] time = 7.05, size = 684, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^8,x)
```

```
[Out] (4199*a^8*x)/1024 - ((1543*a^8*tan(c/2 + (d*x)/2)^3)/512 - (1068767*a^8*tan
(c/2 + (d*x)/2)^5)/2560 - (3297279*a^8*tan(c/2 + (d*x)/2)^7)/2560 - (168283
*a^8*tan(c/2 + (d*x)/2)^9)/3840 + (256139*a^8*tan(c/2 + (d*x)/2)^11)/256 -
(256139*a^8*tan(c/2 + (d*x)/2)^13)/256 + (168283*a^8*tan(c/2 + (d*x)/2)^15)
/3840 + (3297279*a^8*tan(c/2 + (d*x)/2)^17)/2560 + (1068767*a^8*tan(c/2 + (
d*x)/2)^19)/2560 - (1543*a^8*tan(c/2 + (d*x)/2)^21)/512 - (3175*a^8*tan(c/2
 + (d*x)/2)^23)/512 + a^8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((4199*c)
/1024 + (4199*d*x)/1024 - 43888/3465) + tan(c/2 + (d*x)/2)^22*(12*a^8*((419
9*c)/1024 + (4199*d*x)/1024) - a^8*((12597*c)/256 + (12597*d*x)/256 - 16))
+ tan(c/2 + (d*x)/2)^2*(12*a^8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((12
597*c)/256 + (12597*d*x)/256 - 157072/1155)) + tan(c/2 + (d*x)/2)^20*(66*a^
8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((138567*c)/512 + (138567*d*x)/51
2 - 336)) + tan(c/2 + (d*x)/2)^4*(66*a^8*((4199*c)/1024 + (4199*d*x)/1024)
- a^8*((138567*c)/512 + (138567*d*x)/512 - 52496/105)) + tan(c/2 + (d*x)/2)
^18*(220*a^8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((230945*c)/256 + (230
945*d*x)/256 - 5584/3)) + tan(c/2 + (d*x)/2)^6*(220*a^8*((4199*c)/1024 + (4
199*d*x)/1024) - a^8*((230945*c)/256 + (230945*d*x)/256 - 58288/63)) + tan(
c/2 + (d*x)/2)^14*(792*a^8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((415701
*c)/128 + (415701*d*x)/128 - 17696/5)) + tan(c/2 + (d*x)/2)^10*(792*a^8*((4
199*c)/1024 + (4199*d*x)/1024) - a^8*((415701*c)/128 + (415701*d*x)/128 - 2
27232/35)) + tan(c/2 + (d*x)/2)^12*(924*a^8*((4199*c)/1024 + (4199*d*x)/102
4) - a^8*((969969*c)/256 + (969969*d*x)/256 - 87776/15)) + tan(c/2 + (d*x)/
2)^16*(495*a^8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((2078505*c)/1024 +
(2078505*d*x)/1024 - 3504)) + tan(c/2 + (d*x)/2)^8*(495*a^8*((4199*c)/1024
+ (4199*d*x)/1024) - a^8*((2078505*c)/1024 + (2078505*d*x)/1024 - 19360/7))
 + (3175*a^8*tan(c/2 + (d*x)/2))/512)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^12)
```

sympy [A] time = 94.80, size = 1280, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((7*a**8*x*sin(c + d*x)**12/1024 + 21*a**8*x*sin(c + d*x)**10*cos(
c + d*x)**2/512 + 21*a**8*x*sin(c + d*x)**10/64 + 105*a**8*x*sin(c + d*x)**
8*cos(c + d*x)**4/1024 + 105*a**8*x*sin(c + d*x)**8*cos(c + d*x)**2/64 + 10
5*a**8*x*sin(c + d*x)**8/64 + 35*a**8*x*sin(c + d*x)**6*cos(c + d*x)**6/256
 + 105*a**8*x*sin(c + d*x)**6*cos(c + d*x)**4/32 + 105*a**8*x*sin(c + d*x)*
*6*cos(c + d*x)**2/16 + 7*a**8*x*sin(c + d*x)**6/4 + 105*a**8*x*sin(c + d*x
)**4*cos(c + d*x)**8/1024 + 105*a**8*x*sin(c + d*x)**4*cos(c + d*x)**6/32 +
 315*a**8*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 21*a**8*x*sin(c + d*x)**4*
cos(c + d*x)**2/4 + 3*a**8*x*sin(c + d*x)**4/8 + 21*a**8*x*sin(c + d*x)**2*
cos(c + d*x)**10/512 + 105*a**8*x*sin(c + d*x)**2*cos(c + d*x)**8/64 + 105*
a**8*x*sin(c + d*x)**2*cos(c + d*x)**6/16 + 21*a**8*x*sin(c + d*x)**2*cos(c
 + d*x)**4/4 + 3*a**8*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 7*a**8*x*cos(c
```

```
+ d*x)**12/1024 + 21*a**8*x*cos(c + d*x)**10/64 + 105*a**8*x*cos(c + d*x)**
8/64 + 7*a**8*x*cos(c + d*x)**6/4 + 3*a**8*x*cos(c + d*x)**4/8 + 7*a**8*sin
(c + d*x)**11*cos(c + d*x)/(1024*d) + 119*a**8*sin(c + d*x)**9*cos(c + d*x)
**3/(3072*d) + 21*a**8*sin(c + d*x)**9*cos(c + d*x)/(64*d) - 281*a**8*sin(c
 + d*x)**7*cos(c + d*x)**5/(2560*d) + 49*a**8*sin(c + d*x)**7*cos(c + d*x)*
*3/(32*d) + 105*a**8*sin(c + d*x)**7*cos(c + d*x)/(64*d) - 8*a**8*sin(c + d
*x)**6*cos(c + d*x)**5/(5*d) - 231*a**8*sin(c + d*x)**5*cos(c + d*x)**7/(25
60*d) - 14*a**8*sin(c + d*x)**5*cos(c + d*x)**5/(5*d) + 385*a**8*sin(c + d*
x)**5*cos(c + d*x)**3/(64*d) + 7*a**8*sin(c + d*x)**5*cos(c + d*x)/(4*d) -
48*a**8*sin(c + d*x)**4*cos(c + d*x)**7/(35*d) - 56*a**8*sin(c + d*x)**4*co
s(c + d*x)**5/(5*d) - 119*a**8*sin(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 4
9*a**8*sin(c + d*x)**3*cos(c + d*x)**7/(32*d) - 385*a**8*sin(c + d*x)**3*co
s(c + d*x)**5/(64*d) + 14*a**8*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 3*a*
*8*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 64*a**8*sin(c + d*x)**2*cos(c + d*x
)**9/(105*d) - 32*a**8*sin(c + d*x)**2*cos(c + d*x)**7/(5*d) - 56*a**8*sin(
c + d*x)**2*cos(c + d*x)**5/(5*d) - 7*a**8*sin(c + d*x)*cos(c + d*x)**11/(1
024*d) - 21*a**8*sin(c + d*x)*cos(c + d*x)**9/(64*d) - 105*a**8*sin(c + d*x
)*cos(c + d*x)**7/(64*d) - 7*a**8*sin(c + d*x)*cos(c + d*x)**5/(4*d) + 5*a*
*8*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 128*a**8*cos(c + d*x)**11/(1155*d)
- 64*a**8*cos(c + d*x)**9/(45*d) - 16*a**8*cos(c + d*x)**7/(5*d) - 8*a**8*c
os(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c)**4, True))
```

3.43 $\int \cos^3(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=45

$$\frac{(a\sin(c+dx)+a)^{10}}{5a^2d}-\frac{(a\sin(c+dx)+a)^{11}}{11a^3d}$$

```
[Out] 1/5*(a+a*sin(d*x+c))^10/a^2/d-1/11*(a+a*sin(d*x+c))^11/a^3/d
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^{10}}{5a^2d}-\frac{(a\sin(c+dx)+a)^{11}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a + a*Sin[c + d*x])^10/(5*a^2*d) - (a + a*Sin[c + d*x])^11/(11*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^9\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a(a+x)^9-(a+x)^{10}\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{(a+a\sin(c+dx))^{10}}{5a^2d}-\frac{(a+a\sin(c+dx))^{11}}{11a^3d}\end{aligned}$$

Mathematica [A] time = 1.09, size = 43, normalized size = 0.96

$$-\frac{a^8(5\sin(c+dx)-6)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^{20}}{55d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/55*(a^8*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^20*(-6 + 5*Sin[c + d*x]))/
d
```

fricas [B] time = 0.75, size = 136, normalized size = 3.02

$$\frac{44\,a^8\cos(dx+c)^{10} - 550\,a^8\cos(dx+c)^8 + 1760\,a^8\cos(dx+c)^6 - 1760\,a^8\cos(dx+c)^4 + \left(5\,a^8\cos(dx+c)^{10} - \right.}{55}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/55*(44*a^8*cos(d*x + c)^10 - 550*a^8*cos(d*x + c)^8 + 1760*a^8*cos(d*x +
c)^6 - 1760*a^8*cos(d*x + c)^4 + (5*a^8*cos(d*x + c)^10 - 190*a^8*cos(d*x +
 c)^8 + 1040*a^8*cos(d*x + c)^6 - 1568*a^8*cos(d*x + c)^4 + 256*a^8*cos(d*x
 + c)^2 + 512*a^8)*sin(d*x + c))/d
```

giac [B] time = 1.57, size = 134, normalized size = 2.98

$$-\frac{5\,a^8\sin(dx+c)^{11} + 44\,a^8\sin(dx+c)^{10} + 165\,a^8\sin(dx+c)^9 + 330\,a^8\sin(dx+c)^8 + 330\,a^8\sin(dx+c)^7 - 46}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/55*(5*a^8*sin(d*x + c)^11 + 44*a^8*sin(d*x + c)^10 + 165*a^8*sin(d*x + c
)^9 + 330*a^8*sin(d*x + c)^8 + 330*a^8*sin(d*x + c)^7 - 462*a^8*sin(d*x + c
)^5 - 660*a^8*sin(d*x + c)^4 - 495*a^8*sin(d*x + c)^3 - 220*a^8*sin(d*x + c
)^2 - 55*a^8*sin(d*x + c))/d
```

maple [B] time = 0.19, size = 463, normalized size = 10.29

$$a^8\left(-\frac{(\sin^7(dx+c))(\cos^4(dx+c))}{11} - \frac{7(\sin^5(dx+c))(\cos^4(dx+c))}{99} - \frac{5(\sin^3(dx+c))(\cos^4(dx+c))}{99} - \frac{(\cos^4(dx+c))\sin(dx+c)}{33} + \frac{(2+\cos^2(dx+c))\sin(dx}{99}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*sin(d*x+c)^5*cos(d*x+c)^4-5/
99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*cos(d*x+c)^4*sin(d*x+c)+1/99*(2+cos(d*x+c
)^2)*sin(d*x+c))+8*a^8*(-1/10*sin(d*x+c)^6*cos(d*x+c)^4-3/40*sin(d*x+c)^4*c
os(d*x+c)^4-1/20*sin(d*x+c)^2*cos(d*x+c)^4-1/40*cos(d*x+c)^4)+28*a^8*(-1/9*
sin(d*x+c)^5*cos(d*x+c)^4-5/63*sin(d*x+c)^3*cos(d*x+c)^4-1/21*cos(d*x+c)^4*
sin(d*x+c)+1/63*(2+cos(d*x+c)^2)*sin(d*x+c))+56*a^8*(-1/8*sin(d*x+c)^4*cos(
d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1/24*cos(d*x+c)^4)+70*a^8*(-1/7*sin
(d*x+c)^3*cos(d*x+c)^4-3/35*cos(d*x+c)^4*sin(d*x+c)+1/35*(2+cos(d*x+c)^2)*s
in(d*x+c))+56*a^8*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+28*a^8
*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2*cos(d*x+
c)^4*a^8+1/3*a^8*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [B] time = 0.33, size = 134, normalized size = 2.98

$$-\frac{5\,a^8\sin(dx+c)^{11} + 44\,a^8\sin(dx+c)^{10} + 165\,a^8\sin(dx+c)^9 + 330\,a^8\sin(dx+c)^8 + 330\,a^8\sin(dx+c)^7 - 46}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/55*(5*a^8*sin(d*x + c)^11 + 44*a^8*sin(d*x + c)^10 + 165*a^8*sin(d*x + c
)^9 + 330*a^8*sin(d*x + c)^8 + 330*a^8*sin(d*x + c)^7 - 462*a^8*sin(d*x + c
)^5 - 660*a^8*sin(d*x + c)^4 - 495*a^8*sin(d*x + c)^3 - 220*a^8*sin(d*x + c
)^2 - 55*a^8*sin(d*x + c))/d
```

mupad [B] time = 0.12, size = 132, normalized size = 2.93

$$\frac{-\frac{a^8 \sin(c+d x)^{11}}{11} - \frac{4 a^8 \sin(c+d x)^{10}}{5} - 3 a^8 \sin(c+d x)^9 - 6 a^8 \sin(c+d x)^8 - 6 a^8 \sin(c+d x)^7 + \frac{42 a^8 \sin(c+d x)^5}{5} + 12}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^8,x)
```

```
[Out] (a^8*sin(c + d*x) + 4*a^8*sin(c + d*x)^2 + 9*a^8*sin(c + d*x)^3 + 12*a^8*si
n(c + d*x)^4 + (42*a^8*sin(c + d*x)^5)/5 - 6*a^8*sin(c + d*x)^7 - 6*a^8*sin
(c + d*x)^8 - 3*a^8*sin(c + d*x)^9 - (4*a^8*sin(c + d*x)^10)/5 - (a^8*sin(c
 + d*x)^11)/11)/d
```

sympy [A] time = 50.56, size = 422, normalized size = 9.38

$$\begin{cases} \frac{2a^8 \sin^{11}(c+dx)}{99d} + \frac{a^8 \sin^9(c+dx)\cos^2(c+dx)}{9d} + \frac{8a^8 \sin^9(c+dx)}{9d} + \frac{4a^8 \sin^7(c+dx)\cos^2(c+dx)}{d} + \frac{4a^8 \sin^7(c+dx)}{d} - \frac{2a^8 \sin^6(c+dx)\cos^4(c+}{d} \\ x(a\sin(c)+a)^8 \cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((2*a**8*sin(c + d*x)**11/(99*d) + a**8*sin(c + d*x)**9*cos(c + d*
x)**2/(9*d) + 8*a**8*sin(c + d*x)**9/(9*d) + 4*a**8*sin(c + d*x)**7*cos(c +
 d*x)**2/d + 4*a**8*sin(c + d*x)**7/d - 2*a**8*sin(c + d*x)**6*cos(c + d*x)
**4/d + 14*a**8*sin(c + d*x)**5*cos(c + d*x)**2/d + 56*a**8*sin(c + d*x)**5
/(15*d) - 2*a**8*sin(c + d*x)**4*cos(c + d*x)**6/d - 14*a**8*sin(c + d*x)**
4*cos(c + d*x)**4/d + 28*a**8*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 2*a**
8*sin(c + d*x)**3/(3*d) - a**8*sin(c + d*x)**2*cos(c + d*x)**8/d - 28*a**8*
sin(c + d*x)**2*cos(c + d*x)**6/(3*d) - 14*a**8*sin(c + d*x)**2*cos(c + d*x
)**4/d + a**8*sin(c + d*x)*cos(c + d*x)**2/d - a**8*cos(c + d*x)**10/(5*d)
- 7*a**8*cos(c + d*x)**8/(3*d) - 14*a**8*cos(c + d*x)**6/(3*d) - 2*a**8*cos
(c + d*x)**4/d, Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c)**3, True))
```

3.44 $\int \cos^2(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=262

$$-\frac{2431a^8\cos^3(c+dx)}{384d}-\frac{2431\cos^3(c+dx)\left(a^8\sin(c+dx)+a^8\right)}{640d}+\frac{2431a^8\sin(c+dx)\cos(c+dx)}{256d}+\frac{2431a^8x}{256}-\frac{2431}{}$$

```
[Out] 2431/256*a^8*x-2431/384*a^8*cos(d*x+c)^3/d+2431/256*a^8*cos(d*x+c)*sin(d*x+
c)/d-17/48*a^3*cos(d*x+c)^3*(a+a*sin(d*x+c))^5/d-17/90*a^2*cos(d*x+c)^3*(a+
a*sin(d*x+c))^6/d-1/10*a*cos(d*x+c)^3*(a+a*sin(d*x+c))^7/d-2431/2016*a^2*co
s(d*x+c)^3*(a^2+a^2*sin(d*x+c))^3/d-221/336*cos(d*x+c)^3*(a^2+a^2*sin(d*x+c
))^4/d-2431/1120*cos(d*x+c)^3*(a^4+a^4*sin(d*x+c))^2/d-2431/640*cos(d*x+c)^
3*(a^8+a^8*sin(d*x+c))/d
```

Rubi [A] time = 0.37, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2678, 2669, 2635, 8}

$$-\frac{2431a^8\cos^3(c+dx)}{384d}-\frac{17a^3\cos^3(c+dx)(a\sin(c+dx)+a)^5}{48d}-\frac{17a^2\cos^3(c+dx)(a\sin(c+dx)+a)^6}{90d}-\frac{2431a^2\cos^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (2431*a^8*x)/256 - (2431*a^8*Cos[c + d*x]^3)/(384*d) + (2431*a^8*Cos[c + d*
x]*Sin[c + d*x])/(256*d) - (17*a^3*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^5)/(
48*d) - (17*a^2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^6)/(90*d) - (a*Cos[c +
d*x]^3*(a + a*Sin[c + d*x])^7)/(10*d) - (2431*a^2*Cos[c + d*x]^3*(a^2 + a^2
*Sin[c + d*x])^3)/(2016*d) - (221*Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x])^4
)/(336*d) - (2431*Cos[c + d*x]^3*(a^4 + a^4*Sin[c + d*x])^2)/(1120*d) - (24
31*Cos[c + d*x]^3*(a^8 + a^8*Sin[c + d*x]))/(640*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^8\,dx = -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^7}{10d} + \frac{1}{10}(17a)\int \cos^2(c+dx)(a+a\,\mathrm{s}$$

$$= -\frac{17a^2\cos^3(c+dx)(a+a\sin(c+dx))^6}{90d} - \frac{a\cos^3(c+dx)(a+a\sin(c+}{10d}$$

$$= -\frac{17a^3\cos^3(c+dx)(a+a\sin(c+dx))^5}{48d} - \frac{17a^2\cos^3(c+dx)(a+a\sin}{90d}$$

$$= -\frac{17a^3\cos^3(c+dx)(a+a\sin(c+dx))^5}{48d} - \frac{17a^2\cos^3(c+dx)(a+a\sin}{90d}$$

$$= -\frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d} - \frac{17a^3\cos^3(c+dx)(a+a\,\mathrm{s}}{48d}$$

$$= -\frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d} - \frac{17a^3\cos^3(c+dx)(a+a\,\mathrm{s}}{48d}$$

$$= -\frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d} - \frac{17a^3\cos^3(c+dx)(a+a\,\mathrm{s}}{48d}$$

$$= -\frac{2431a^8\cos^3(c+dx)}{384d} - \frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d} - \frac{17}{}$$

$$= -\frac{2431a^8\cos^3(c+dx)}{384d} + \frac{2431a^8\cos(c+dx)\sin(c+dx)}{256d} - \frac{2431a^5\cos}{}$$

$$= \frac{2431a^8x}{256} - \frac{2431a^8\cos^3(c+dx)}{384d} + \frac{2431a^8\cos(c+dx)\sin(c+dx)}{256d} - \frac{2}{}$$

Mathematica [A] time = 1.49, size = 191, normalized size = 0.73

$$-\frac{a^8\left(1531530\sqrt{1-\sin(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\,\left(8064\sin^{10}(c+dx)+63616\sin^9(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/80640*(a^8*Cos[c + d*x]^3*(1531530*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]
]*Sqrt[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(1193984 - 508859*Sin[c +
 d*x] - 410693*Sin[c + d*x]^2 - 543442*Sin[c + d*x]^3 - 492846*Sin[c + d*x]
^4 - 130728*Sin[c + d*x]^5 + 257704*Sin[c + d*x]^6 + 353648*Sin[c + d*x]^7
+ 209552*Sin[c + d*x]^8 + 63616*Sin[c + d*x]^9 + 8064*Sin[c + d*x]^10)))/(d
*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(3/2))
```

fricas [A] time = 0.74, size = 137, normalized size = 0.52

$$\frac{71680\,a^8\cos\left(dx+c\right)^9 - 921600\,a^8\cos\left(dx+c\right)^7 + 3096576\,a^8\cos\left(dx+c\right)^5 - 3440640\,a^8\cos\left(dx+c\right)^3 + 765}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/80640*(71680*a^8*cos(d*x + c)^9 - 921600*a^8*cos(d*x + c)^7 + 3096576*a^8
*cos(d*x + c)^5 - 3440640*a^8*cos(d*x + c)^3 + 765765*a^8*d*x + 63*(128*a^8
*cos(d*x + c)^9 - 4976*a^8*cos(d*x + c)^7 + 28328*a^8*cos(d*x + c)^5 - 4651
0*a^8*cos(d*x + c)^3 + 12155*a^8*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 1.50, size = 174, normalized size = 0.66

$$\frac{2431}{256}\,a^8x+\frac{a^8\cos\left(9\,dx+9\,c\right)}{288\,d}-\frac{33\,a^8\cos\left(7\,dx+7\,c\right)}{224\,d}+\frac{51\,a^8\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{17\,a^8\cos\left(3\,dx+3\,c\right)}{8\,d}-\frac{221\,a^8\cos\left(dx\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2431/256*a^8*x + 1/288*a^8*cos(9*d*x + 9*c)/d - 33/224*a^8*cos(7*d*x + 7*c)
/d + 51/40*a^8*cos(5*d*x + 5*c)/d - 17/8*a^8*cos(3*d*x + 3*c)/d - 221/16*a^
8*cos(d*x + c)/d + 1/5120*a^8*sin(10*d*x + 10*c)/d - 59/2048*a^8*sin(8*d*x
+ 8*c)/d + 527/1024*a^8*sin(6*d*x + 6*c)/d - 561/256*a^8*sin(4*d*x + 4*c)/d
 - 663/512*a^8*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.14, size = 480, normalized size = 1.83

$$a^8\left(-\frac{\left(\sin^7(dx+c)\right)\left(\cos^3(dx+c)\right)}{10}-\frac{7\left(\sin^5(dx+c)\right)\left(\cos^3(dx+c)\right)}{80}-\frac{7\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{96}-\frac{7\sin(dx+c)\left(\cos^3(dx+c)\right)}{128}+\frac{7\cos(dx+c)\sin(dx+c}{256}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*sin(d*x+c)^5*cos(d*x+c)^3-7/
96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*sin(d*x+c)*cos(d*x+c)^3+7/256*cos(d*x+c)
*sin(d*x+c)+7/256*d*x+7/256*c)+8*a^8*(-1/9*sin(d*x+c)^6*cos(d*x+c)^3-2/21*s
in(d*x+c)^4*cos(d*x+c)^3-8/105*sin(d*x+c)^2*cos(d*x+c)^3-16/315*cos(d*x+c)^
3)+28*a^8*(-1/8*sin(d*x+c)^5*cos(d*x+c)^3-5/48*sin(d*x+c)^3*cos(d*x+c)^3-5/
64*sin(d*x+c)*cos(d*x+c)^3+5/128*cos(d*x+c)*sin(d*x+c)+5/128*d*x+5/128*c)+5
6*a^8*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/105*
cos(d*x+c)^3)+70*a^8*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x
+c)^3+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+56*a^8*(-1/5*sin(d*x+c)^2
*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+28*a^8*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*c
os(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-8/3*cos(d*x+c)^3*a^8+a^8*(1/2*cos(d*x+c
)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.38, size = 319, normalized size = 1.22

$$-\frac{1720320\,a^8\cos\left(dx+c\right)^3-16384\left(35\,\cos\left(dx+c\right)^9-135\,\cos\left(dx+c\right)^7+189\,\cos\left(dx+c\right)^5-105\,\cos\left(dx+c\right)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/645120*(1720320*a^8*cos(d*x + c)^3 - 16384*(35*cos(d*x + c)^9 - 135*cos(
d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(d*x + c)^3)*a^8 + 344064*(15*cos(
d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^8 - 2408448*(3*cos(d*
x + c)^5 - 5*cos(d*x + c)^3)*a^8 - 21*(96*sin(2*d*x + 2*c)^5 - 640*sin(2*d*
x + 2*c)^3 + 840*d*x + 840*c - 45*sin(8*d*x + 8*c) - 120*sin(4*d*x + 4*c))*
a^8 + 5880*(64*sin(2*d*x + 2*c)^3 - 120*d*x - 120*c + 3*sin(8*d*x + 8*c) +
24*sin(4*d*x + 4*c))*a^8 + 235200*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3
*sin(4*d*x + 4*c))*a^8 - 564480*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^8 - 1612
80*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^8)/d
```

mupad [B] time = 7.26, size = 572, normalized size = 2.18

$$\frac{2431\,a^8\,x}{256}-\frac{\frac{11809\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128}-\frac{23647\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160}-\frac{40749\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{32}-\frac{70499\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{64}+\frac{70499\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}+\dots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^8,x)
```

```
[Out] (2431*a^8*x)/256 - ((11809*a^8*tan(c/2 + (d*x)/2)^3)/128 - (23647*a^8*tan(c
/2 + (d*x)/2)^5)/160 - (40749*a^8*tan(c/2 + (d*x)/2)^7)/32 - (70499*a^8*tan
(c/2 + (d*x)/2)^9)/64 + (70499*a^8*tan(c/2 + (d*x)/2)^11)/64 + (40749*a^8*t
an(c/2 + (d*x)/2)^13)/32 + (23647*a^8*tan(c/2 + (d*x)/2)^15)/160 - (11809*a
^8*tan(c/2 + (d*x)/2)^17)/128 - (2175*a^8*tan(c/2 + (d*x)/2)^19)/128 + a^8*
((2431*c)/256 + (2431*d*x)/256) - a^8*((2431*c)/256 + (2431*d*x)/256 - 9328
/315) + tan(c/2 + (d*x)/2)^18*(10*a^8*((2431*c)/256 + (2431*d*x)/256) - a^8
*((12155*c)/128 + (12155*d*x)/128 - 16)) + tan(c/2 + (d*x)/2)^2*(10*a^8*((2
431*c)/256 + (2431*d*x)/256) - a^8*((12155*c)/128 + (12155*d*x)/128 - 17648
/63)) + tan(c/2 + (d*x)/2)^14*(120*a^8*((2431*c)/256 + (2431*d*x)/256) - a^
8*((36465*c)/32 + (36465*d*x)/32 - 1984)) + tan(c/2 + (d*x)/2)^6*(120*a^8*(
(2431*c)/256 + (2431*d*x)/256) - a^8*((36465*c)/32 + (36465*d*x)/32 - 32960
/21)) + tan(c/2 + (d*x)/2)^16*(45*a^8*((2431*c)/256 + (2431*d*x)/256) - a^8
*((109395*c)/256 + (109395*d*x)/256 - 336)) + tan(c/2 + (d*x)/2)^4*(45*a^8*
((2431*c)/256 + (2431*d*x)/256) - a^8*((109395*c)/256 + (109395*d*x)/256 -
6976/7)) + tan(c/2 + (d*x)/2)^10*(252*a^8*((2431*c)/256 + (2431*d*x)/256) -
 a^8*((153153*c)/64 + (153153*d*x)/64 - 18656/5)) + tan(c/2 + (d*x)/2)^12*(
210*a^8*((2431*c)/256 + (2431*d*x)/256) - a^8*((255255*c)/128 + (255255*d*x
)/128 - 4288)) + tan(c/2 + (d*x)/2)^8*(210*a^8*((2431*c)/256 + (2431*d*x)/2
56) - a^8*((255255*c)/128 + (255255*d*x)/128 - 5792/3)) + (2175*a^8*tan(c/2
 + (d*x)/2))/128)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

sympy [A] time = 37.27, size = 1018, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((7*a**8*x*sin(c + d*x)**10/256 + 35*a**8*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 35*a**8*x*sin(c + d*x)**8/32 + 35*a**8*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 35*a**8*x*sin(c + d*x)**6*cos(c + d*x)**2/8 + 35*a**8*x
*sin(c + d*x)**6/8 + 35*a**8*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 105*a*
*8*x*sin(c + d*x)**4*cos(c + d*x)**4/16 + 105*a**8*x*sin(c + d*x)**4*cos(c
+ d*x)**2/8 + 7*a**8*x*sin(c + d*x)**4/2 + 35*a**8*x*sin(c + d*x)**2*cos(c
+ d*x)**8/256 + 35*a**8*x*sin(c + d*x)**2*cos(c + d*x)**6/8 + 105*a**8*x*si
n(c + d*x)**2*cos(c + d*x)**4/8 + 7*a**8*x*sin(c + d*x)**2*cos(c + d*x)**2
+ a**8*x*sin(c + d*x)**2/2 + 7*a**8*x*cos(c + d*x)**10/256 + 35*a**8*x*cos(
c + d*x)**8/32 + 35*a**8*x*cos(c + d*x)**6/8 + 7*a**8*x*cos(c + d*x)**4/2 +
 a**8*x*cos(c + d*x)**2/2 + 7*a**8*sin(c + d*x)**9*cos(c + d*x)/(256*d) - 7
9*a**8*sin(c + d*x)**7*cos(c + d*x)**3/(384*d) + 35*a**8*sin(c + d*x)**7*co
s(c + d*x)/(32*d) - 8*a**8*sin(c + d*x)**6*cos(c + d*x)**3/(3*d) - 7*a**8*s
in(c + d*x)**5*cos(c + d*x)**5/(30*d) - 511*a**8*sin(c + d*x)**5*cos(c + d*
x)**3/(96*d) + 35*a**8*sin(c + d*x)**5*cos(c + d*x)/(8*d) - 16*a**8*sin(c +
 d*x)**4*cos(c + d*x)**5/(5*d) - 56*a**8*sin(c + d*x)**4*cos(c + d*x)**3/(3
*d) - 49*a**8*sin(c + d*x)**3*cos(c + d*x)**7/(384*d) - 385*a**8*sin(c + d*
x)**3*cos(c + d*x)**5/(96*d) - 35*a**8*sin(c + d*x)**3*cos(c + d*x)**3/(3*d
) + 7*a**8*sin(c + d*x)**3*cos(c + d*x)/(2*d) - 64*a**8*sin(c + d*x)**2*cos
(c + d*x)**7/(35*d) - 224*a**8*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 56*
a**8*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 7*a**8*sin(c + d*x)*cos(c + d*
x)**9/(256*d) - 35*a**8*sin(c + d*x)*cos(c + d*x)**7/(32*d) - 35*a**8*sin(c
 + d*x)*cos(c + d*x)**5/(8*d) - 7*a**8*sin(c + d*x)*cos(c + d*x)**3/(2*d) +
 a**8*sin(c + d*x)*cos(c + d*x)/(2*d) - 128*a**8*cos(c + d*x)**9/(315*d) -
64*a**8*cos(c + d*x)**7/(15*d) - 112*a**8*cos(c + d*x)**5/(15*d) - 8*a**8*c
os(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c)**2, True))
```

3.45 $\int \cos(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=22

$$\frac{(a\sin(c+dx)+a)^9}{9ad}$$

```
[Out] 1/9*(a+a*sin(d*x+c))^9/a/d
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2667, 32\}$

$$\frac{(a\sin(c+dx)+a)^9}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a + a*Sin[c + d*x])^9/(9*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a+x)^8\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^9}{9ad}\end{aligned}$$

Mathematica [B] time = 0.09, size = 147, normalized size = 6.68

$$\frac{a^8\sin^9(c+dx)}{9d}+\frac{a^8\sin^8(c+dx)}{d}+\frac{4a^8\sin^7(c+dx)}{d}+\frac{28a^8\sin^6(c+dx)}{3d}+\frac{14a^8\sin^5(c+dx)}{d}+\frac{14a^8\sin^4(c+dx)}{d}+\frac{28}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*Sin[c + d*x])/d + (4*a^8*Sin[c + d*x]^2)/d + (28*a^8*Sin[c + d*x]^3)/(
3*d) + (14*a^8*Sin[c + d*x]^4)/d + (14*a^8*Sin[c + d*x]^5)/d + (28*a^8*Sin[
c + d*x]^6)/(3*d) + (4*a^8*Sin[c + d*x]^7)/d + (a^8*Sin[c + d*x]^8)/d + (a^
8*Sin[c + d*x]^9)/(9*d)
```

fricas [B] time = 0.70, size = 122, normalized size = 5.55

$$\frac{9\,a^8\cos(dx+c)^8-120\,a^8\cos(dx+c)^6+432\,a^8\cos(dx+c)^4-576\,a^8\cos(dx+c)^2+\left(a^8\cos(dx+c)^8-40\,a^8\,\mathrm{c}\right.}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a^8*cos(d*x + c)^8 - 120*a^8*cos(d*x + c)^6 + 432*a^8*cos(d*x + c)^4
 - 576*a^8*cos(d*x + c)^2 + (a^8*cos(d*x + c)^8 - 40*a^8*cos(d*x + c)^6 + 2
40*a^8*cos(d*x + c)^4 - 448*a^8*cos(d*x + c)^2 + 256*a^8)*sin(d*x + c))/d
```

giac [A] time = 0.90, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^9}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/9*(a*sin(d*x + c) + a)^9/(a*d)
```

maple [A] time = 0.08, size = 21, normalized size = 0.95

$$\frac{(a+a\sin(dx+c))^9}{9da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/9*(a+a*sin(d*x+c))^9/d/a
```

maxima [A] time = 0.66, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^9}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/9*(a*sin(d*x + c) + a)^9/(a*d)
```

mupad [B] time = 4.75, size = 118, normalized size = 5.36

$$\frac{\frac{a^8\sin(c+d\,x)^9}{9}+a^8\sin(c+d\,x)^8+4\,a^8\sin(c+d\,x)^7+\frac{28\,a^8\sin(c+d\,x)^6}{3}+14\,a^8\sin(c+d\,x)^5+14\,a^8\sin(c+d\,x)^4+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^8,x)
```

```
[Out] (a^8*sin(c + d*x) + 4*a^8*sin(c + d*x)^2 + (28*a^8*sin(c + d*x)^3)/3 + 14*a
^8*sin(c + d*x)^4 + 14*a^8*sin(c + d*x)^5 + (28*a^8*sin(c + d*x)^6)/3 + 4*a
^8*sin(c + d*x)^7 + a^8*sin(c + d*x)^8 + (a^8*sin(c + d*x)^9)/9)/d
```

sympy [A] time = 19.84, size = 148, normalized size = 6.73

$$\begin{cases}\frac{a^8\sin^9(c+dx)}{9d}+\frac{a^8\sin^8(c+dx)}{d}+\frac{4a^8\sin^7(c+dx)}{d}+\frac{28a^8\sin^6(c+dx)}{3d}+\frac{14a^8\sin^5(c+dx)}{d}+\frac{14a^8\sin^4(c+dx)}{d}+\frac{28a^8\sin^3(c+dx)}{3d}+\frac{4a^8 s}{} \\ x(a\sin(c)+a)^8\cos(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((a**8*sin(c + d*x)**9/(9*d) + a**8*sin(c + d*x)**8/d + 4*a**8*sin
(c + d*x)**7/d + 28*a**8*sin(c + d*x)**6/(3*d) + 14*a**8*sin(c + d*x)**5/d
+ 14*a**8*sin(c + d*x)**4/d + 28*a**8*sin(c + d*x)**3/(3*d) + 4*a**8*sin(c
+ d*x)**2/d + a**8*sin(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c),
True))
```

3.46 $\int \sec(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=162

$$-\frac{64a^8\sin(c+dx)}{d}-\frac{128a^8\log(1-\sin(c+dx))}{d}-\frac{16a^5(a\sin(c+dx)+a)^3}{3d}-\frac{16\left(a^4\sin(c+dx)+a^4\right)^2}{d}-\frac{4a^3(a\sin$$

```
[Out] -128*a^8*ln(1-sin(d*x+c))/d-64*a^8*sin(d*x+c)/d-16/3*a^5*(a+a*sin(d*x+c))^3
/d-4/5*a^3*(a+a*sin(d*x+c))^5/d-1/3*a^2*(a+a*sin(d*x+c))^6/d-1/7*a*(a+a*sin
(d*x+c))^7/d-2*(a^2+a^2*sin(d*x+c))^4/d-16*(a^4+a^4*sin(d*x+c))^2/d
```

Rubi [A] time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2667, 43}

$$-\frac{64a^8\sin(c+dx)}{d}-\frac{16a^5(a\sin(c+dx)+a)^3}{3d}-\frac{4a^3(a\sin(c+dx)+a)^5}{5d}-\frac{a^2(a\sin(c+dx)+a)^6}{3d}-\frac{2\left(a^2\sin(c+dx\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-128*a^8*Log[1 - Sin[c + d*x]])/d - (64*a^8*Sin[c + d*x])/d - (16*a^5*(a +
 a*Sin[c + d*x])^3)/(3*d) - (4*a^3*(a + a*Sin[c + d*x])^5)/(5*d) - (a^2*(a
+ a*Sin[c + d*x])^6)/(3*d) - (a*(a + a*Sin[c + d*x])^7)/(7*d) - (2*(a^2 + a
^2*Sin[c + d*x])^4)/d - (16*(a^4 + a^4*Sin[c + d*x])^2)/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^8\,dx &= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^7}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a\,\text{Subst}\left(\int \left(-64a^6+\frac{128a^7}{a-x}-32a^5(a+x)-16a^4(a+x)^2-8a^3(a+x)^3\right.\right.}{d}\\ &= -\frac{128a^8\log(1-\sin(c+dx))}{d}-\frac{64a^8\sin(c+dx)}{d}-\frac{16a^5(a+a\sin(c+d\right.}{3d}\end{aligned}$$

Mathematica [A] time = 0.17, size = 95, normalized size = 0.59

$$\frac{a^8\left(-\frac{1}{7}\sin^7(c+dx)-\frac{4}{3}\sin^6(c+dx)-\frac{29}{5}\sin^5(c+dx)-16\sin^4(c+dx)-33\sin^3(c+dx)-60\sin^2(c+dx)-1\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*(-128*Log[1 - Sin[c + d*x]] - 127*Sin[c + d*x] - 60*Sin[c + d*x]^2 - 3
3*Sin[c + d*x]^3 - 16*Sin[c + d*x]^4 - (29*Sin[c + d*x]^5)/5 - (4*Sin[c + d
*x]^6)/3 - Sin[c + d*x]^7/7))/d
```

fricas [A] time = 0.71, size = 114, normalized size = 0.70

$$\frac{140\,a^8\cos(dx+c)^6 - 2100\,a^8\cos(dx+c)^4 + 10080\,a^8\cos(dx+c)^2 - 13440\,a^8\log(-\sin(dx+c)+1) + 3\,(5\,a^8\,c}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(140*a^8*cos(d*x + c)^6 - 2100*a^8*cos(d*x + c)^4 + 10080*a^8*cos(d*x
 + c)^2 - 13440*a^8*log(-sin(d*x + c) + 1) + 3*(5*a^8*cos(d*x + c)^6 - 218*
a^8*cos(d*x + c)^4 + 1576*a^8*cos(d*x + c)^2 - 5808*a^8)*sin(d*x + c))/d
```

giac [A] time = 0.95, size = 288, normalized size = 1.78

$$2\left(6720\,a^8\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right) - 13440\,a^8\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{17424\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14} + 13335\,a^8\tan\left(\frac{1}{2}\,d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/105*(6720*a^8*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 13440*a^8*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - (17424*a^8*tan(1/2*d*x + 1/2*c)^14 + 13335*a^8*tan(1/
2*d*x + 1/2*c)^13 + 134568*a^8*tan(1/2*d*x + 1/2*c)^12 + 93870*a^8*tan(1/2*
d*x + 1/2*c)^11 + 442344*a^8*tan(1/2*d*x + 1/2*c)^10 + 265209*a^8*tan(1/2*d
*x + 1/2*c)^9 + 780640*a^8*tan(1/2*d*x + 1/2*c)^8 + 370308*a^8*tan(1/2*d*x
+ 1/2*c)^7 + 780640*a^8*tan(1/2*d*x + 1/2*c)^6 + 265209*a^8*tan(1/2*d*x + 1
/2*c)^5 + 442344*a^8*tan(1/2*d*x + 1/2*c)^4 + 93870*a^8*tan(1/2*d*x + 1/2*c
)^3 + 134568*a^8*tan(1/2*d*x + 1/2*c)^2 + 13335*a^8*tan(1/2*d*x + 1/2*c) +
17424*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

maple [A] time = 0.17, size = 149, normalized size = 0.92

$$-\frac{a^8\left(\sin^7(dx+c)\right)}{7d} - \frac{4a^8\left(\sin^6(dx+c)\right)}{3d} - \frac{29a^8\left(\sin^5(dx+c)\right)}{5d} - \frac{16a^8\left(\sin^4(dx+c)\right)}{d} - \frac{33a^8\left(\sin^3(dx+c)\right)}{d} - \frac{60a^8\left(s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^8,x)
```

```
[Out] -1/7/d*a^8*sin(d*x+c)^7-4/3/d*a^8*sin(d*x+c)^6-29/5*a^8*sin(d*x+c)^5/d-16*a
^8*sin(d*x+c)^4/d-33*a^8*sin(d*x+c)^3/d-60*a^8*sin(d*x+c)^2/d-127*a^8*sin(d
*x+c)/d-128/d*a^8*ln(cos(d*x+c))+128/d*a^8*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.63, size = 109, normalized size = 0.67

$$-\frac{15\,a^8\sin(dx+c)^7 + 140\,a^8\sin(dx+c)^6 + 609\,a^8\sin(dx+c)^5 + 1680\,a^8\sin(dx+c)^4 + 3465\,a^8\sin(dx+c)^3 +}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/105*(15*a^8*sin(d*x + c)^7 + 140*a^8*sin(d*x + c)^6 + 609*a^8*sin(d*x +
c)^5 + 1680*a^8*sin(d*x + c)^4 + 3465*a^8*sin(d*x + c)^3 + 6300*a^8*sin(d*x
 + c)^2 + 13440*a^8*log(sin(d*x + c) - 1) + 13335*a^8*sin(d*x + c))/d
```

mupad [B] time = 4.65, size = 109, normalized size = 0.67

$$-\frac{128\,a^8\,\ln\left(\sin\left(c+d\,x\right)-1\right)+127\,a^8\,\sin\left(c+d\,x\right)+60\,a^8\,\sin\left(c+d\,x\right)^2+33\,a^8\,\sin\left(c+d\,x\right)^3+16\,a^8\,\sin\left(c+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^8/cos(c + d*x),x)
```

```
[Out] -(128*a^8*log(sin(c + d*x) - 1) + 127*a^8*sin(c + d*x) + 60*a^8*sin(c + d*x
)^2 + 33*a^8*sin(c + d*x)^3 + 16*a^8*sin(c + d*x)^4 + (29*a^8*sin(c + d*x)^
5)/5 + (4*a^8*sin(c + d*x)^6)/3 + (a^8*sin(c + d*x)^7)/7)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.47 $\int \sec^2(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=201

$$\frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7}+\frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5}+\frac{1001a^8\cos^5(c+dx)}{10d}-\frac{1001a^8\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{3003a^8\sin(\cdots}{}$$

```
[Out] -3003/16*a^8*x+1001/10*a^8*cos(d*x+c)^5/d-3003/16*a^8*cos(d*x+c)*sin(d*x+c)
/d-1001/8*a^8*cos(d*x+c)^3*sin(d*x+c)/d+2*a^15*cos(d*x+c)^13/d/(a-a*sin(d*x
+c))^7+26*a^13*cos(d*x+c)^11/d/(a-a*sin(d*x+c))^5+286/3*a^14*cos(d*x+c)^9/d
/(a^2-a^2*sin(d*x+c))^3+143/2*a^16*cos(d*x+c)^7/d/(a^8-a^8*sin(d*x+c))
```

Rubi [A] time = 0.34, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2670, 2680, 2679, 2682, 2635, 8}

$$\frac{1001a^8\cos^5(c+dx)}{10d}+\frac{143a^{16}\cos^7(c+dx)}{2d\left(a^8-a^8\sin(c+dx)\right)}+\frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7}+\frac{286a^{14}\cos^9(c+dx)}{3d\left(a^2-a^2\sin(c+dx)\right)^3}+\frac{26a^{13}\cos^{11}(c+\cdots}{d(a-a\sin(c+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-3003*a^8*x)/16 + (1001*a^8*Cos[c + d*x]^5)/(10*d) - (3003*a^8*Cos[c + d*x
]*Sin[c + d*x])/(16*d) - (1001*a^8*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) + (2*
a^15*Cos[c + d*x]^13)/(d*(a - a*Sin[c + d*x])^7) + (26*a^13*Cos[c + d*x]^11
)/(d*(a - a*Sin[c + d*x])^5) + (286*a^14*Cos[c + d*x]^9)/(3*d*(a^2 - a^2*Si
n[c + d*x])^3) + (143*a^16*Cos[c + d*x]^7)/(2*d*(a^8 - a^8*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^8\,dx = a^{16}\int \frac{\cos^{14}(c+dx)}{(a-a\sin(c+dx))^8}\,dx$$
$$= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} - \left(13a^{14}\right)\int \frac{\cos^{12}(c+dx)}{(a-a\sin(c+dx))^6}\,dx$$
$$= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5} - \left(143a^{12}\right)\int \frac{\cos^{10}(}{(a-a\sin}$$
$$= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5} + \frac{286a^{11}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^3}$$
$$= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5} + \frac{286a^{11}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^3}$$
$$= \frac{1001a^8\cos^5(c+dx)}{10d} + \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5}$$
$$= \frac{1001a^8\cos^5(c+dx)}{10d} - \frac{1001a^8\cos^3(c+dx)\sin(c+dx)}{8d} + \frac{2a^{15}\cos^{13}}{d(a-a\sin(}$$
$$= \frac{1001a^8\cos^5(c+dx)}{10d} - \frac{3003a^8\cos(c+dx)\sin(c+dx)}{16d} - \frac{1001a^8\cos^3(}{}$$
$$= -\frac{3003a^8x}{16} + \frac{1001a^8\cos^5(c+dx)}{10d} - \frac{3003a^8\cos(c+dx)\sin(c+dx)}{16d} -$$

Mathematica [C] time = 0.06, size = 55, normalized size = 0.27

$$\frac{128\sqrt{2}\,a^8\sqrt{\sin(c+dx)+1}\,\sec(c+dx)\,{}_2F_1\left(-\frac{13}{2},-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (128*Sqrt[2]*a^8*Hypergeometric2F1[-13/2, -1/2, 1/2, (1 - Sin[c + d*x])/2]*
Sec[c + d*x]*Sqrt[1 + Sin[c + d*x]])/d
```

fricas [A] time = 0.68, size = 231, normalized size = 1.15

$$40\,a^8\cos(dx+c)^7 + 384\,a^8\cos(dx+c)^6 - 1526\,a^8\cos(dx+c)^5 - 6400\,a^8\cos(dx+c)^4 + 11865\,a^8\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(40*a^8*cos(d*x + c)^7 + 384*a^8*cos(d*x + c)^6 - 1526*a^8*cos(d*x +
c)^5 - 6400*a^8*cos(d*x + c)^4 + 11865*a^8*cos(d*x + c)^3 - 45045*a^8*d*x +
 46080*a^8*cos(d*x + c)^2 + 30720*a^8 - 15*(3003*a^8*d*x - 4027*a^8)*cos(d*
x + c) + (40*a^8*cos(d*x + c)^6 - 344*a^8*cos(d*x + c)^5 - 1870*a^8*cos(d*x
 + c)^4 + 4530*a^8*cos(d*x + c)^3 + 45045*a^8*d*x + 16395*a^8*cos(d*x + c)^
2 - 29685*a^8*cos(d*x + c) + 30720*a^8)*sin(d*x + c))/(d*cos(d*x + c) - d*s
in(d*x + c) + d)
```

giac [A] time = 1.90, size = 231, normalized size = 1.15

$$-\frac{45045\,(dx+c)a^8 + \frac{61440\,a^8}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1} + \frac{2\left(14565\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 28800\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} + 50855\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} - 174720\,a^8\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/240*(45045*(d*x + c)*a^8 + 61440*a^8/(tan(1/2*d*x + 1/2*c) - 1) + 2*(145
65*a^8*tan(1/2*d*x + 1/2*c)^11 - 28800*a^8*tan(1/2*d*x + 1/2*c)^10 + 50855*
a^8*tan(1/2*d*x + 1/2*c)^9 - 174720*a^8*tan(1/2*d*x + 1/2*c)^8 + 36930*a^8*
tan(1/2*d*x + 1/2*c)^7 - 400640*a^8*tan(1/2*d*x + 1/2*c)^6 - 36930*a^8*tan(
1/2*d*x + 1/2*c)^5 - 426240*a^8*tan(1/2*d*x + 1/2*c)^4 - 50855*a^8*tan(1/2*
d*x + 1/2*c)^3 - 211584*a^8*tan(1/2*d*x + 1/2*c)^2 - 14565*a^8*tan(1/2*d*x
+ 1/2*c) - 40064*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [B] time = 0.38, size = 389, normalized size = 1.94

$$\frac{a^8\left(\frac{\sin^9(dx+c)}{\cos(dx+c)} + \left(\sin^7(dx+c) + \frac{7\left(\sin^5(dx+c)\right)}{6} + \frac{35\left(\sin^3(dx+c)\right)}{24} + \frac{35\sin(dx+c)}{16}\right)\cos(dx+c) - \frac{35dx}{16} - \frac{35c}{16}\right) + 8a^8\left(\frac{\sin^8(dx+c)}{\cos(dx+c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(sin(d*x+c)^9/cos(d*x+c)+(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(
d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-35/16*d*x-35/16*c)+8*a^8*(sin(d*x+c)^
8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+
c))+28*a^8*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin
(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+56*a^8*(sin(d*x+c)^6/cos(d*x+c)+(8/3+s
in(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+70*a^8*(sin(d*x+c)^5/cos(d*x+c)+(
sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+56*a^8*(sin(d*x+c)^4
/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+28*a^8*(tan(d*x+c)-d*x-c)+8*a^8/co
s(d*x+c)+a^8*tan(d*x+c))
```

maxima [A] time = 0.67, size = 331, normalized size = 1.65

$$\frac{384\left(\cos(dx+c)^5 - 5\cos(dx+c)^3 + \frac{5}{\cos(dx+c)} + 15\cos(dx+c)\right)a^8 - 4480\left(\cos(dx+c)^3 - \frac{3}{\cos(dx+c)} - 6\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/240*(384*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/cos(d*x + c) + 15*cos(d*x
 + c))*a^8 - 4480*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^8 -
5*(105*d*x + 105*c - (87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x +
```

```
 c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*
x + c))*a^8 - 840*(15*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan
(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 8*tan(d*x + c))*a^8 - 8400*(3*d*x + 3
*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^8 - 6720*(d*x +
c - tan(d*x + c))*a^8 + 13440*a^8*(1/cos(d*x + c) + cos(d*x + c)) + 240*a^8
*tan(d*x + c) + 1920*a^8/cos(d*x + c))/d
```

mupad [B] time = 8.73, size = 513, normalized size = 2.55

$$-\frac{3003\,a^8\,x}{16}-\frac{\frac{3003\,a^8\,(c+d\,x)}{16}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3003\,a^8\,(c+d\,x)}{16}-\frac{a^8\,(45045\,c+45045\,d\,x-50998)}{240}\right)-\frac{a^8\,(45045\,c+45045\,d\,x-141568)}{240}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^8/cos(c + d*x)^2,x)
```

```
[Out] - (3003*a^8*x)/16 - ((3003*a^8*(c + d*x))/16 - tan(c/2 + (d*x)/2)*((3003*a^
8*(c + d*x))/16 - (a^8*(45045*c + 45045*d*x - 50998))/240) - (a^8*(45045*c
+ 45045*d*x - 141568))/240 + tan(c/2 + (d*x)/2)^12*((3003*a^8*(c + d*x))/16
 - (a^8*(45045*c + 45045*d*x - 90570))/240) - tan(c/2 + (d*x)/2)^11*((9009*
a^8*(c + d*x))/8 - (a^8*(270270*c + 270270*d*x - 86730))/240) - tan(c/2 + (
d*x)/2)^3*((9009*a^8*(c + d*x))/8 - (a^8*(270270*c + 270270*d*x - 321458))/
240) + tan(c/2 + (d*x)/2)^10*((9009*a^8*(c + d*x))/8 - (a^8*(270270*c + 270
270*d*x - 527950))/240) + tan(c/2 + (d*x)/2)^2*((9009*a^8*(c + d*x))/8 - (a
^8*(270270*c + 270270*d*x - 762678))/240) - tan(c/2 + (d*x)/2)^9*((45045*a^
8*(c + d*x))/16 - (a^8*(675675*c + 675675*d*x - 451150))/240) - tan(c/2 + (
d*x)/2)^5*((45045*a^8*(c + d*x))/16 - (a^8*(675675*c + 675675*d*x - 778620)
)/240) - tan(c/2 + (d*x)/2)^7*((15015*a^8*(c + d*x))/4 - (a^8*(900900*c + 9
00900*d*x - 875140))/240) + tan(c/2 + (d*x)/2)^8*((45045*a^8*(c + d*x))/16
- (a^8*(675675*c + 675675*d*x - 1344900))/240) + tan(c/2 + (d*x)/2)^4*((450
45*a^8*(c + d*x))/16 - (a^8*(675675*c + 675675*d*x - 1672370))/240) + tan(c
/2 + (d*x)/2)^6*((15015*a^8*(c + d*x))/4 - (a^8*(900900*c + 900900*d*x - 19
56220))/240))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.48 $\int \sec^3(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=121

$$\frac{64a^9}{d(a-a\sin(c+dx))}+\frac{a^8\sin^5(c+dx)}{5d}+\frac{2a^8\sin^4(c+dx)}{d}+\frac{10a^8\sin^3(c+dx)}{d}+\frac{36a^8\sin^2(c+dx)}{d}+\frac{129a^8\sin(c+dx)}{d}$$

```
[Out] 192*a^8*ln(1-sin(d*x+c))/d+129*a^8*sin(d*x+c)/d+36*a^8*sin(d*x+c)^2/d+10*a^
8*sin(d*x+c)^3/d+2*a^8*sin(d*x+c)^4/d+1/5*a^8*sin(d*x+c)^5/d+64*a^9/d/(a-a*
sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2667, 43\}$

$$\frac{a^8\sin^5(c+dx)}{5d}+\frac{2a^8\sin^4(c+dx)}{d}+\frac{10a^8\sin^3(c+dx)}{d}+\frac{36a^8\sin^2(c+dx)}{d}+\frac{64a^9}{d(a-a\sin(c+dx))}+\frac{129a^8\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (192*a^8*Log[1 - Sin[c + d*x]])/d + (129*a^8*Sin[c + d*x])/d + (36*a^8*Sin[
c + d*x]^2)/d + (10*a^8*Sin[c + d*x]^3)/d + (2*a^8*Sin[c + d*x]^4)/d + (a^8
*Sin[c + d*x]^5)/(5*d) + (64*a^9)/(d*(a - a*Sin[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sin(c+dx))^8\,dx &= \frac{a^3\,\mathrm{Subst}\left(\int \frac{(a+x)^6}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^3\,\mathrm{Subst}\left(\int \left(129a^4+\frac{64a^6}{(a-x)^2}-\frac{192a^5}{a-x}+72a^3x+30a^2x^2+8ax^3+x^4\right)dx, x\right.}{d}\\ &= \frac{192a^8\log(1-\sin(c+dx))}{d}+\frac{129a^8\sin(c+dx)}{d}+\frac{36a^8\sin^2(c+dx)}{d}+\frac{1}{}\end{aligned}$$

Mathematica [A] time = 0.27, size = 111, normalized size = 0.92

$$\frac{a^8(1-\sin(c+dx))(\sin(c+dx)+1)\sec^2(c+dx)\left(\frac{1}{5}\sin^5(c+dx)+2\sin^4(c+dx)+10\sin^3(c+dx)+36\sin^2(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c + d*x]^2*(1 - Sin[c + d*x])*(1 + Sin[c + d*x])*(192*Log[1 - Sin[
c + d*x]] + 64/(1 - Sin[c + d*x]) + 129*Sin[c + d*x] + 36*Sin[c + d*x]^2 +
10*Sin[c + d*x]^3 + 2*Sin[c + d*x]^4 + Sin[c + d*x]^5/5))/d
```

fricas [A] time = 0.68, size = 130, normalized size = 1.07

$$-\frac{4\,a^8\cos\left(dx+c\right)^6-172\,a^8\cos\left(dx+c\right)^4+2192\,a^8\cos\left(dx+c\right)^2-1119\,a^8-3840\left(a^8\sin\left(dx+c\right)-a^8\right)\log\left(-\right.}{20\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/20*(4*a^8*cos(d*x + c)^6 - 172*a^8*cos(d*x + c)^4 + 2192*a^8*cos(d*x + c
)^2 - 1119*a^8 - 3840*(a^8*sin(d*x + c) - a^8)*log(-sin(d*x + c) + 1) - (36
*a^8*cos(d*x + c)^4 - 592*a^8*cos(d*x + c)^2 - 2399*a^8)*sin(d*x + c))/(d*s
in(d*x + c) - d)
```

giac [B] time = 0.94, size = 275, normalized size = 2.27

$$-\frac{2\left(480\,a^8\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-960\,a^8\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{160\left(9\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-20\,a^8\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/5*(480*a^8*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 960*a^8*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) + 160*(9*a^8*tan(1/2*d*x + 1/2*c)^2 - 20*a^8*tan(1/2*d*x +
1/2*c) + 9*a^8)/(tan(1/2*d*x + 1/2*c) - 1)^2 - (1096*a^8*tan(1/2*d*x + 1/2*
c)^10 + 645*a^8*tan(1/2*d*x + 1/2*c)^9 + 5840*a^8*tan(1/2*d*x + 1/2*c)^8 +
2780*a^8*tan(1/2*d*x + 1/2*c)^7 + 12120*a^8*tan(1/2*d*x + 1/2*c)^6 + 4286*a
^8*tan(1/2*d*x + 1/2*c)^5 + 12120*a^8*tan(1/2*d*x + 1/2*c)^4 + 2780*a^8*tan
(1/2*d*x + 1/2*c)^3 + 5840*a^8*tan(1/2*d*x + 1/2*c)^2 + 645*a^8*tan(1/2*d*x
 + 1/2*c) + 1096*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [B] time = 0.28, size = 345, normalized size = 2.85

$$\frac{a^8\left(\sin^7\left(dx+c\right)\right)}{2d}+\frac{4a^8\left(\sin^6\left(dx+c\right)\right)}{d}+\frac{147a^8\left(\sin^5\left(dx+c\right)\right)}{10d}+\frac{34a^8\left(\sin^4\left(dx+c\right)\right)}{d}+\frac{119a^8\left(\sin^3\left(dx+c\right)\right)}{2d}+\frac{68}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/2/d*a^8*sin(d*x+c)^7+4/d*a^8*sin(d*x+c)^6+147/10*a^8*sin(d*x+c)^5/d+34*a^
8*sin(d*x+c)^4/d+119/2*a^8*sin(d*x+c)^3/d+68*a^8*sin(d*x+c)^2/d+385/2*a^8*s
in(d*x+c)/d+1/2/d*a^8*sin(d*x+c)^9/cos(d*x+c)^2+4/d*a^8*sin(d*x+c)^8/cos(d*
x+c)^2+14/d*a^8*sin(d*x+c)^7/cos(d*x+c)^2+192/d*a^8*ln(cos(d*x+c))-192/d*a^
8*ln(sec(d*x+c)+tan(d*x+c))+28/d*a^8*sin(d*x+c)^6/cos(d*x+c)^2+35/d*a^8*sin
(d*x+c)^5/cos(d*x+c)^2+14/d*a^8*sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*a^8*sec(d*x
+c)*tan(d*x+c)+28/d*a^8*tan(d*x+c)^2+4/d*a^8/cos(d*x+c)^2
```

maxima [A] time = 0.31, size = 97, normalized size = 0.80

$$\frac{a^8\sin\left(dx+c\right)^5+10\,a^8\sin\left(dx+c\right)^4+50\,a^8\sin\left(dx+c\right)^3+180\,a^8\sin\left(dx+c\right)^2+960\,a^8\log\left(\sin\left(dx+c\right)-1\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(a^8*sin(d*x + c)^5 + 10*a^8*sin(d*x + c)^4 + 50*a^8*sin(d*x + c)^3 + 1
80*a^8*sin(d*x + c)^2 + 960*a^8*log(sin(d*x + c) - 1) + 645*a^8*sin(d*x + c
) - 320*a^8/(sin(d*x + c) - 1))/d
```

mupad [B] time = 4.62, size = 97, normalized size = 0.80

$$\frac{192\,a^8\,\ln\left(\sin\left(c+d\,x\right)-1\right)-\frac{64\,a^8}{\sin(c+d\,x)-1}+129\,a^8\,\sin\left(c+d\,x\right)+36\,a^8\,\sin\left(c+d\,x\right)^2+10\,a^8\,\sin\left(c+d\,x\right)^3+2\,a^8\,\mathrm{si}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^8/cos(c + d*x)^3,x)
```

```
[Out] (192*a^8*log(sin(c + d*x) - 1) - (64*a^8)/(sin(c + d*x) - 1) + 129*a^8*sin(
c + d*x) + 36*a^8*sin(c + d*x)^2 + 10*a^8*sin(c + d*x)^3 + 2*a^8*sin(c + d*
x)^4 + (a^8*sin(c + d*x)^5)/5)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.49 $\int \sec^4(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=179

$$\frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5}-\frac{385a^8\cos^3(c+dx)}{4d}+\frac{1155a^8\sin(c+dx)\cos(c+dx)}{8d}+\frac{1155a^8x}{8}$$

```
[Out] 1155/8*a^8*x-385/4*a^8*cos(d*x+c)^3/d+1155/8*a^8*cos(d*x+c)*sin(d*x+c)/d+2/
3*a^15*cos(d*x+c)^11/d/(a-a*sin(d*x+c))^7-22/3*a^13*cos(d*x+c)^9/d/(a-a*sin
(d*x+c))^5-66*a^14*cos(d*x+c)^7/d/(a^2-a^2*sin(d*x+c))^3-231/4*a^16*cos(d*x
+c)^5/d/(a^8-a^8*sin(d*x+c))
```

Rubi [A] time = 0.32, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2670, 2680, 2679, 2682, 2635, 8}

$$-\frac{385a^8\cos^3(c+dx)}{4d}-\frac{231a^{16}\cos^5(c+dx)}{4d\left(a^8-a^8\sin(c+dx)\right)}+\frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{66a^{14}\cos^7(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)^3}-\frac{22a^{13}\cos^9(}{3d(a-a\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (1155*a^8*x)/8 - (385*a^8*Cos[c + d*x]^3)/(4*d) + (1155*a^8*Cos[c + d*x]*Si
n[c + d*x])/(8*d) + (2*a^15*Cos[c + d*x]^11)/(3*d*(a - a*Sin[c + d*x])^7) -
 (22*a^13*Cos[c + d*x]^9)/(3*d*(a - a*Sin[c + d*x])^5) - (66*a^14*Cos[c + d
*x]^7)/(d*(a^2 - a^2*Sin[c + d*x])^3) - (231*a^16*Cos[c + d*x]^5)/(4*d*(a^8
 - a^8*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)(a+a\sin(c+dx))^8\,dx &= a^{16}\int \frac{\cos^{12}(c+dx)}{(a-a\sin(c+dx))^8}\,dx \\
&= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{1}{3}\left(11a^{14}\right)\int \frac{\cos^{10}(c+dx)}{(a-a\sin(c+dx))^6}\,dx \\
&= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5}+\left(33a^{12}\right)\int \frac{\cos^8(c+}{(a-a\sin(c} \\
&= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5}-\frac{66a^{11}\cos^7(c+dx)}{d(a-a\sin(c+dx))^3}+ \\
&= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5}-\frac{66a^{11}\cos^7(c+dx)}{d(a-a\sin(c+dx))^3}- \\
&= -\frac{385a^8\cos^3(c+dx)}{4d}+\frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5}- \\
&= -\frac{385a^8\cos^3(c+dx)}{4d}+\frac{1155a^8\cos(c+dx)\sin(c+dx)}{8d}+\frac{2a^{15}\cos^{11}(c}{3d(a-a\sin(c} \\
&= \frac{1155a^8x}{8}-\frac{385a^8\cos^3(c+dx)}{4d}+\frac{1155a^8\cos(c+dx)\sin(c+dx)}{8d}+\frac{2a^1}{3d(a}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 59, normalized size = 0.33

$$\frac{64\sqrt{2}\,a^8(\sin(c+dx)+1)^{3/2}\sec^3(c+dx)\,{}_2F_1\left(-\frac{11}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (64*Sqrt[2]*a^8*Hypergeometric2F1[-11/2, -3/2, -1/2, (1 - Sin[c + d*x])/2]*
Sec[c + d*x]^3*(1 + Sin[c + d*x])^(3/2))/(3*d)
```

fricas [A] time = 0.71, size = 247, normalized size = 1.38

$$-\frac{6\,a^8\cos(dx+c)^6-52\,a^8\cos(dx+c)^5-317\,a^8\cos(dx+c)^4+1286\,a^8\cos(dx+c)^3+6930\,a^8dx+512\,a^8-(3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^8*cos(d*x + c)^6 - 52*a^8*cos(d*x + c)^5 - 317*a^8*cos(d*x + c)^
4 + 1286*a^8*cos(d*x + c)^3 + 6930*a^8*d*x + 512*a^8 - (3465*a^8*d*x + 5641
*a^8)*cos(d*x + c)^2 + (3465*a^8*d*x - 6674*a^8)*cos(d*x + c) - (6*a^8*cos(
d*x + c)^5 + 58*a^8*cos(d*x + c)^4 - 259*a^8*cos(d*x + c)^3 + 6930*a^8*d*x
- 1545*a^8*cos(d*x + c)^2 - 512*a^8 + (3465*a^8*d*x - 7186*a^8)*cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d
)*sin(d*x + c) - 2*d)
```

giac [A] time = 0.89, size = 200, normalized size = 1.12

$$\frac{3465\,(dx+c)a^8 + \frac{1024\left(6\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3} + \frac{2\left(369\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1728\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+393\,a^8\tan\cdots\right.}{\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/24*(3465*(d*x + c)*a^8 + 1024*(6*a^8*tan(1/2*d*x + 1/2*c)^2 - 15*a^8*tan(
1/2*d*x + 1/2*c) + 7*a^8)/(tan(1/2*d*x + 1/2*c) - 1)^3 + 2*(369*a^8*tan(1/2
*d*x + 1/2*c)^7 - 1728*a^8*tan(1/2*d*x + 1/2*c)^6 + 393*a^8*tan(1/2*d*x + 1
/2*c)^5 - 5568*a^8*tan(1/2*d*x + 1/2*c)^4 - 393*a^8*tan(1/2*d*x + 1/2*c)^3
- 5696*a^8*tan(1/2*d*x + 1/2*c)^2 - 369*a^8*tan(1/2*d*x + 1/2*c) - 1856*a^8
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [B] time = 0.42, size = 478, normalized size = 2.67

$$\frac{a^8\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3} - \frac{2\left(\sin^9(dx+c)\right)}{\cos(dx+c)} - 2\left(\sin^7(dx+c) + \frac{7\left(\sin^5(dx+c)\right)}{6} + \frac{35\left(\sin^3(dx+c)\right)}{24} + \frac{35\sin(dx+c)}{16}\right)\cos(dx+c) + \frac{35dx}{8} + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(1/3*sin(d*x+c)^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*
x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/
8*d*x+35/8*c)+8*a^8*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x
+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+2
8*a^8*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d
*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+56*a^8*
(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/
3*sin(d*x+c)^2)*cos(d*x+c))+70*a^8*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+56*a
^8*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*
x+c)^2)*cos(d*x+c))+28/3*a^8/cos(d*x+c)^3*sin(d*x+c)^3+8/3*a^8/cos(d*x+c)^3
-a^8*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

maxima [A] time = 0.64, size = 311, normalized size = 1.74

$$\frac{224\,a^8\tan(dx+c)^3 + 64\left(\cos(dx+c)^3 - \frac{9\,\cos(dx+c)^2-1}{\cos(dx+c)^3} - 9\,\cos(dx+c)\right)a^8 + \left(8\,\tan(dx+c)^3 + 105\,dx + 105\,\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/24*(224*a^8*tan(d*x + c)^3 + 64*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/
cos(d*x + c)^3 - 9*cos(d*x + c))*a^8 + (8*tan(d*x + c)^3 + 105*d*x + 105*c
```

```
- 3*(13*tan(d*x + c)^3 + 11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^
2 + 1) - 72*tan(d*x + c))*a^8 + 112*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*t
an(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c))*a^8 + 560*(tan(d*x + c)
^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^8 + 8*(tan(d*x + c)^3 + 3*tan(d*x + c)
)*a^8 - 448*a^8*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) -
448*(3*cos(d*x + c)^2 - 1)*a^8/cos(d*x + c)^3 + 64*a^8/cos(d*x + c)^3)/d
```

mupad [B] time = 9.13, size = 437, normalized size = 2.44

$$\frac{1155\,a^8\,x}{8}+\frac{\frac{1155\,a^8\,(c+d\,x)}{8}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3465\,a^8\,(c+d\,x)}{8}-\frac{a^8\,(10395\,c+10395\,d\,x-25758)}{24}\right)-\frac{a^8\,(3465\,c+3465\,d\,x-10880)}{24}+\tan\left(\frac{c}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^8/cos(c + d*x)^4,x)
```

```
[Out] (1155*a^8*x)/8 + ((1155*a^8*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((3465*a^8*(c
 + d*x))/8 - (a^8*(10395*c + 10395*d*x - 25758))/24) - (a^8*(3465*c + 3465*
d*x - 10880))/24 + tan(c/2 + (d*x)/2)^10*((3465*a^8*(c + d*x))/8 - (a^8*(10
395*c + 10395*d*x - 6882))/24) - tan(c/2 + (d*x)/2)^9*((8085*a^8*(c + d*x))
/8 - (a^8*(24255*c + 24255*d*x - 21030))/24) + tan(c/2 + (d*x)/2)^2*((8085*
a^8*(c + d*x))/8 - (a^8*(24255*c + 24255*d*x - 55130))/24) + tan(c/2 + (d*x
)/2)^8*((15015*a^8*(c + d*x))/8 - (a^8*(45045*c + 45045*d*x - 45112))/24) -
 tan(c/2 + (d*x)/2)^3*((15015*a^8*(c + d*x))/8 - (a^8*(45045*c + 45045*d*x
- 96328))/24) - tan(c/2 + (d*x)/2)^7*((10395*a^8*(c + d*x))/4 - (a^8*(62370
*c + 62370*d*x - 86040))/24) + tan(c/2 + (d*x)/2)^4*((10395*a^8*(c + d*x))/
4 - (a^8*(62370*c + 62370*d*x - 109800))/24) + tan(c/2 + (d*x)/2)^6*((12705
*a^8*(c + d*x))/4 - (a^8*(76230*c + 76230*d*x - 103972))/24) - tan(c/2 + (d
*x)/2)^5*((12705*a^8*(c + d*x))/4 - (a^8*(76230*c + 76230*d*x - 135388))/24
))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.50 $\int \sec^5(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=110

$$\frac{16a^{10}}{d(a-a\sin(c+dx))^2}-\frac{80a^9}{d(a-a\sin(c+dx))}-\frac{a^8\sin^3(c+dx)}{3d}-\frac{4a^8\sin^2(c+dx)}{d}-\frac{31a^8\sin(c+dx)}{d}-\frac{80a^8\log(1-$$

```
[Out] -80*a^8*ln(1-sin(d*x+c))/d-31*a^8*sin(d*x+c)/d-4*a^8*sin(d*x+c)^2/d-1/3*a^8
*sin(d*x+c)^3/d+16*a^10/d/(a-a*sin(d*x+c))^2-80*a^9/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$-\frac{a^8\sin^3(c+dx)}{3d}-\frac{4a^8\sin^2(c+dx)}{d}+\frac{16a^{10}}{d(a-a\sin(c+dx))^2}-\frac{80a^9}{d(a-a\sin(c+dx))}-\frac{31a^8\sin(c+dx)}{d}-\frac{80a^8\log(1$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-80*a^8*Log[1 - Sin[c + d*x]])/d - (31*a^8*Sin[c + d*x])/d - (4*a^8*Sin[c
+ d*x]^2)/d - (a^8*Sin[c + d*x]^3)/(3*d) + (16*a^10)/(d*(a - a*Sin[c + d*x]
)^2) - (80*a^9)/(d*(a - a*Sin[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))^8\,dx &= \frac{a^5\operatorname{Subst}\left(\int \frac{(a+x)^5}{(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^5\operatorname{Subst}\left(\int \left(-31a^2+\frac{32a^5}{(a-x)^3}-\frac{80a^4}{(a-x)^2}+\frac{80a^3}{a-x}-8ax-x^2\right)dx,x,a\sin(c\right.}{d}\\ &= -\frac{80a^8\log(1-\sin(c+dx))}{d}-\frac{31a^8\sin(c+dx)}{d}-\frac{4a^8\sin^2(c+dx)}{d}-\end{aligned}$$

Mathematica [A] time = 0.44, size = 73, normalized size = 0.66

$$\frac{a^8\left(-\frac{1}{3}\sin^3(c+dx)-4\sin^2(c+dx)-31\sin(c+dx)+\frac{16(5\sin(c+dx)-4)}{(\sin(c+dx)-1)^2}-80\log(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]

[Out] (a^8*(-80*Log[1 - Sin[c + d*x]] - 31*Sin[c + d*x] - 4*Sin[c + d*x]^2 - Sin[c + d*x]^3/3 + (16*(-4 + 5*Sin[c + d*x]))/(-1 + Sin[c + d*x])^2))/d

fricas [A] time = 0.64, size = 139, normalized size = 1.26

$$\frac{10\,a^{8}\cos\left(dx+c\right)^{4}+160\,a^{8}\cos\left(dx+c\right)^{2}+16\,a^{8}-240\,\left(a^{8}\cos\left(dx+c\right)^{2}+2\,a^{8}\sin\left(dx+c\right)-2\,a^{8}\right)\log\left(-\sin\left(dx\right.\right.}{3\,\left(d\cos\left(dx+c\right)^{2}+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="fricas")

[Out] 1/3*(10*a^8*cos(d*x + c)^4 + 160*a^8*cos(d*x + c)^2 + 16*a^8 - 240*(a^8*cos(d*x + c)^2 + 2*a^8*sin(d*x + c) - 2*a^8)*log(-sin(d*x + c) + 1) + (a^8*cos(d*x + c)^4 - 72*a^8*cos(d*x + c)^2 - 64*a^8)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)

giac [B] time = 0.81, size = 243, normalized size = 2.21

$$2\left(120\,a^{8}\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)-240\,a^{8}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{220\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+93\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+68\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="giac")

[Out] 2/3*(120*a^8*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 240*a^8*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - (220*a^8*tan(1/2*d*x + 1/2*c)^6 + 93*a^8*tan(1/2*d*x + 1/2*c)^5 + 684*a^8*tan(1/2*d*x + 1/2*c)^4 + 190*a^8*tan(1/2*d*x + 1/2*c)^3 + 684*a^8*tan(1/2*d*x + 1/2*c)^2 + 93*a^8*tan(1/2*d*x + 1/2*c) + 220*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3 + 4*(125*a^8*tan(1/2*d*x + 1/2*c)^4 - 536*a^8*tan(1/2*d*x + 1/2*c)^3 + 846*a^8*tan(1/2*d*x + 1/2*c)^2 - 536*a^8*tan(1/2*d*x + 1/2*c) + 125*a^8)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d

maple [B] time = 0.28, size = 503, normalized size = 4.57

$$-\frac{4a^{8}\left(\sin^{6}\left(dx+c\right)\right)}{d}+\frac{a^{8}\left(\sin^{9}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}+\frac{2a^{8}\left(\sin^{8}\left(dx+c\right)\right)}{d\cos\left(dx+c\right)^{4}}+\frac{7a^{8}\left(\sin^{7}\left(dx+c\right)\right)}{d\cos\left(dx+c\right)^{4}}+\frac{35a^{8}\left(\sin^{5}\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^{4}}+\frac{7a^{8}\left(\sin\cdots\right.}{d\cos\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x)

[Out] -637/8*a^8*sin(d*x+c)/d-12*a^8*sin(d*x+c)^2/d-665/24*a^8*sin(d*x+c)^3/d-6*a^8*sin(d*x+c)^4/d-91/8*a^8*sin(d*x+c)^5/d+2/d*a^8/cos(d*x+c)^4+14/d*a^8*tan(d*x+c)^4-5/8/d*a^8*sin(d*x+c)^7-4/d*a^8*sin(d*x+c)^6-80/d*a^8*ln(cos(d*x+c))+80/d*a^8*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^8*sin(d*x+c)^9/cos(d*x+c)^4+2/d*a^8*sin(d*x+c)^8/cos(d*x+c)^4+7/d*a^8*sin(d*x+c)^7/cos(d*x+c)^4+35/2/d*a^8*sin(d*x+c)^5/cos(d*x+c)^4+7/d*a^8*sin(d*x+c)^3/cos(d*x+c)^4+1/4/d*a^8*tan(d*x+c)*sec(d*x+c)^3+14/d*a^8/cos(d*x+c)^4*sin(d*x+c)^4-28/d*a^8*tan(d*x+c)^2-5/8/d*a^8*sin(d*x+c)^9/cos(d*x+c)^2-4/d*a^8*sin(d*x+c)^8/cos(d*x+c)^2-21/2/d*a^8*sin(d*x+c)^7/cos(d*x+c)^2-35/4/d*a^8*sin(d*x+c)^5/cos(d*x+c)^2+7/2/d*a^8*sin(d*x+c)^3/cos(d*x+c)^2+3/8/d*a^8*sec(d*x+c)*tan(d*x+c)

maxima [A] time = 0.35, size = 95, normalized size = 0.86

$$-\frac{a^{8}\sin\left(dx+c\right)^{3}+12\,a^{8}\sin\left(dx+c\right)^{2}+240\,a^{8}\log\left(\sin\left(dx+c\right)-1\right)+93\,a^{8}\sin\left(dx+c\right)-\frac{48\,\left(5\,a^{8}\sin\left(dx+c\right)-4\,a^{8}\right)}{\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3*(a^8*sin(d*x + c)^3 + 12*a^8*sin(d*x + c)^2 + 240*a^8*log(sin(d*x + c)
 - 1) + 93*a^8*sin(d*x + c) - 48*(5*a^8*sin(d*x + c) - 4*a^8)/(sin(d*x + c)
^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 4.60, size = 96, normalized size = 0.87

$$-\frac{80\,a^8\,\ln\left(\sin\left(c+d\,x\right)-1\right)+31\,a^8\,\sin\left(c+d\,x\right)-\frac{80\,a^8\,\sin(c+d\,x)-64\,a^8}{\sin(c+d\,x)^2-2\,\sin(c+d\,x)+1}+4\,a^8\,\sin\left(c+d\,x\right)^2+\frac{a^8\,\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^8/cos(c + d*x)^5,x)
```

```
[Out] -(80*a^8*log(sin(c + d*x) - 1) + 31*a^8*sin(c + d*x) - (80*a^8*sin(c + d*x)
 - 64*a^8)/(sin(c + d*x)^2 - 2*sin(c + d*x) + 1) + 4*a^8*sin(c + d*x)^2 + (
a^8*sin(c + d*x)^3)/3)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.51 $\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

```
[Out] 3/8*x/a+1/5*cos(d*x+c)^5/a/d+3/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c)^3
*sin(d*x+c)/a/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2682, 2635, 8}

$$\frac{\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) + Cos[c + d*x]^5/(5*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d
) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cos^5(c+dx)}{5ad}+\frac{\int \cos^4(c+dx)\,dx}{a}\\
&= \frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{3\int \cos^2(c+dx)\,dx}{4a}\\
&= \frac{\cos^5(c+dx)}{5ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{3\int 1\,dx}{8a}\\
&= \frac{3x}{8a}+\frac{\cos^5(c+dx)}{5ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}
\end{aligned}$$

Mathematica [A] time = 0.80, size = 141, normalized size = 1.93

$$-\frac{\left(30\sqrt{1-\sin(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\,\left(8\sin^5(c+dx)-18\sin^4(c+dx)-6\sin^3(c+dx)\right.\right.}{40ad(\sin(c+dx)-1)^4(\sin(c+dx)+1)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/40*(Cos[c + d*x]^7*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - S
in[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-8 - 17*Sin[c + d*x] + 41*Sin[c + d*
x]^2 - 6*Sin[c + d*x]^3 - 18*Sin[c + d*x]^4 + 8*Sin[c + d*x]^5)))/(a*d*(-1
+ Sin[c + d*x])^4*(1 + Sin[c + d*x])^(7/2))
```

fricas [A] time = 1.11, size = 50, normalized size = 0.68

$$\frac{8 \cos\left(dx+c\right)^{5}+15\,dx+5\left(2 \cos\left(dx+c\right)^{3}+3 \cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*cos(d*x + c)^5 + 15*d*x + 5*(2*cos(d*x + c)^3 + 3*cos(d*x + c))*sin
(d*x + c))/(a*d)
```

giac [A] time = 0.61, size = 114, normalized size = 1.56

$$\frac{\frac{15\,(dx+c)}{a}-\frac{2\left(25 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-40 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+10 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-80 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-25 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}a}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*(d*x + c)/a - 2*(25*tan(1/2*d*x + 1/2*c)^9 - 40*tan(1/2*d*x + 1/2*
c)^8 + 10*tan(1/2*d*x + 1/2*c)^7 - 80*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d
*x + 1/2*c)^3 - 25*tan(1/2*d*x + 1/2*c) - 8)/((tan(1/2*d*x + 1/2*c)^2 + 1)^
5*a))/d
```

maple [B] time = 0.15, size = 245, normalized size = 3.36

$$-\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{2\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{4\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] -5/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9+2/a/d/(1+tan(1/2*d
*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8-1/2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(
1/2*d*x+1/2*c)^7+4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4+1/2/
a/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+5/4/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^5*tan(1/2*d*x+1/2*c)+2/5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^5+3/4/a/d*a
rctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.67, size = 258, normalized size = 3.53

$$\frac{\frac{\frac{25\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{80\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{10\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{40\,\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{25\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+8}{a+\frac{5\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{5\,a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{15\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/20*((25*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 80*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 10*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 40*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 25*sin(d*x + c)^9
/(cos(d*x + c) + 1)^9 + 8)/(a + 5*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
0*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 8.15, size = 107, normalized size = 1.47

$$\frac{3\,x}{8\,a}+\frac{-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{2}{5}}{a\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x)),x)
```

```
[Out] (3*x)/(8*a) + ((5*tan(c/2 + (d*x)/2))/4 + tan(c/2 + (d*x)/2)^3/2 + 4*tan(c/
2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^7/2 + 2*tan(c/2 + (d*x)/2)^8 - (5*tan(c
/2 + (d*x)/2)^9)/4 + 2/5)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 34.04, size = 1355, normalized size = 18.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**10/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a
*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*
x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) + 75*d*x*tan(c/2 + d*x/2)**
8/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(
c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2
 + 40*a*d) + 150*d*x*tan(c/2 + d*x/2)**6/(40*a*d*tan(c/2 + d*x/2)**10 + 200
*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 +
d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) + 150*d*x*tan(c/2 + d*x/2
)**4/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*t
an(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)
**2 + 40*a*d) + 75*d*x*tan(c/2 + d*x/2)**2/(40*a*d*tan(c/2 + d*x/2)**10 + 2
00*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2
+ d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) + 15*d*x/(40*a*d*tan(c/
2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6
+ 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) - 50*
tan(c/2 + d*x/2)**9/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)
**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*t
an(c/2 + d*x/2)**2 + 40*a*d) + 80*tan(c/2 + d*x/2)**8/(40*a*d*tan(c/2 + d*x
/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a
*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) - 20*tan(c/2
 + d*x/2)**7/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 4
00*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2
+ d*x/2)**2 + 40*a*d) + 160*tan(c/2 + d*x/2)**4/(40*a*d*tan(c/2 + d*x/2)**1
0 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan
(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) + 20*tan(c/2 + d*x
/2)**3/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d
*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/
2)**2 + 40*a*d) + 50*tan(c/2 + d*x/2)/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*
```

```
d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x
/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) + 16/(40*a*d*tan(c/2 + d*x/2
)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d
*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d), Ne(d, 0)), (x
*cos(c)**6/(a*sin(c) + a), True))
```

3.52 $\int \frac{\cos^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=47

$$\frac{(a-a\sin(c+dx))^4}{4a^5d}-\frac{2(a-a\sin(c+dx))^3}{3a^4d}$$

```
[Out] -2/3*(a-a*sin(d*x+c))^3/a^4/d+1/4*(a-a*sin(d*x+c))^4/a^5/d
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a-a\sin(c+dx))^4}{4a^5d}-\frac{2(a-a\sin(c+dx))^3}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*(a - a*Sin[c + d*x])^3)/(3*a^4*d) + (a - a*Sin[c + d*x])^4/(4*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(2a(a-x)^2-(a-x)^3\right)\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= -\frac{2(a-a\sin(c+dx))^3}{3a^4d}+\frac{(a-a\sin(c+dx))^4}{4a^5d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 0.98

$$\frac{\sin(c+dx)\left(3\sin^3(c+dx)-4\sin^2(c+dx)-6\sin(c+dx)+12\right)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(12 - 6*Sin[c + d*x] - 4*Sin[c + d*x]^2 + 3*Sin[c + d*x]^3))/
(12*a*d)
```

fricas [A] time = 0.84, size = 37, normalized size = 0.79

$$\frac{3 \cos\left(dx+c\right)^{4} + 4 \left(\cos\left(dx+c\right)^{2} + 2\right) \sin\left(dx+c\right)}{12 \, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(d*x + c)^4 + 4*(cos(d*x + c)^2 + 2)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.35, size = 47, normalized size = 1.00

$$\frac{3 \, \sin\left(dx+c\right)^{4} - 4 \, \sin\left(dx+c\right)^{3} - 6 \, \sin\left(dx+c\right)^{2} + 12 \, \sin\left(dx+c\right)}{12 \, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 - 6*sin(d*x + c)^2 + 12*sin(d*x +
 c))/(a*d)
```

maple [A] time = 0.14, size = 45, normalized size = 0.96

$$\frac{\frac{\left(\sin^{4}\left(dx+c\right)\right)}{4} - \frac{\left(\sin^{3}\left(dx+c\right)\right)}{3} - \frac{\left(\sin^{2}\left(dx+c\right)\right)}{2} + \sin\left(dx+c\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/4*sin(d*x+c)^4-1/3*sin(d*x+c)^3-1/2*sin(d*x+c)^2+sin(d*x+c))
```

maxima [A] time = 0.31, size = 47, normalized size = 1.00

$$\frac{3 \, \sin\left(dx+c\right)^{4} - 4 \, \sin\left(dx+c\right)^{3} - 6 \, \sin\left(dx+c\right)^{2} + 12 \, \sin\left(dx+c\right)}{12 \, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 - 6*sin(d*x + c)^2 + 12*sin(d*x +
 c))/(a*d)
```

mupad [B] time = 4.66, size = 54, normalized size = 1.15

$$\frac{\frac{\sin(c+d\,x)}{a} - \frac{\sin(c+d\,x)^2}{2\,a} - \frac{\sin(c+d\,x)^3}{3\,a} + \frac{\sin(c+d\,x)^4}{4\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)/a - sin(c + d*x)^2/(2*a) - sin(c + d*x)^3/(3*a) + sin(c + d*x
)^4/(4*a))/d
```

sympy [A] time = 18.77, size = 530, normalized size = 11.28

$$\begin{cases} \frac{6 \tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad \tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad \tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad \tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad \tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} - \frac{6 \tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad \tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad \tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad \tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+\cdots} \\ \frac{x \cos^{5}\left(c\right)}{a \sin\left(c\right)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((6*tan(c/2 + d*x/2)**7/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/
2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3
*a*d) - 6*tan(c/2 + d*x/2)**6/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 +
 d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*
d) + 10*tan(c/2 + d*x/2)**5/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d
*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d)
 + 10*tan(c/2 + d*x/2)**3/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d*x
/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) -
 6*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d*x/2)
**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) + 6*
tan(c/2 + d*x/2)/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d*x/2)**6 +
18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d), Ne(d, 0))
, (x*cos(c)**5/(a*sin(c) + a), True))
```

3.53 $\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

```
[Out] 1/2*x/a+1/3*cos(d*x+c)^3/a/d+1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2682, 2635, 8}

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(2*a) + Cos[c + d*x]^3/(3*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx&=\frac{\cos^3(c+dx)}{3ad}+\frac{\int \cos^2(c+dx)\,dx}{a}\\&=\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}+\frac{\int 1\,dx}{2a}\\&=\frac{x}{2a}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}\end{aligned}$$

Mathematica [B] time = 0.32, size = 119, normalized size = 2.43

$$-\frac{\left(\sqrt{\sin(c+dx)+1}\left(2\sin^3(c+dx)-5\sin^2(c+dx)+\sin(c+dx)+2\right)-6\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+d}\right.}{6ad(\sin(c+dx)-1)^3(\sin(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(Cos[c + d*x]^5*(-6*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Si
n[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(2 + Sin[c + d*x] - 5*Sin[c + d*x]^2 +
 2*Sin[c + d*x]^3)))/(a*d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

fricas [A] time = 0.65, size = 37, normalized size = 0.76

$$\frac{2 \cos(dx+c)^3 + 3\,dx + 3 \cos(dx+c)\sin(dx+c)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c)^3 + 3*d*x + 3*cos(d*x + c)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.86, size = 75, normalized size = 1.53

$$\frac{\frac{3\,(dx+c)}{a} - \frac{2\left(3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^5 - 6*tan(1/2*d*x + 1/2*c)^4
 - 3*tan(1/2*d*x + 1/2*c) - 2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

maple [B] time = 0.14, size = 141, normalized size = 2.88

$$-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} + \frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} + \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} + \frac{2}{3ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} + \frac{\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+2/a/d/(1+tan(1/2*d*x
+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4+1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*
d*x+1/2*c)+2/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3+1/a/d*arctan(tan(1/2*d*x+1/2*
c))
```

maxima [B] time = 0.60, size = 156, normalized size = 3.18

$$\frac{\frac{\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{6 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + 2}{a + \frac{3\,a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3\,a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{3 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(a + 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6) + 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/
d
```

mupad [B] time = 6.87, size = 66, normalized size = 1.35

$$\frac{x}{2\,a}+\frac{-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{2}{3}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x)),x)
```

```
[Out] x/(2*a) + (tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)
^5 + 2/3)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

sympy [A] time = 11.38, size = 558, normalized size = 11.39

$$\begin{cases}\frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}+\frac{9dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}+\frac{}{6ad\tan^6\left(\frac{c}{2}\right.} \\ \frac{x\cos^4(c)}{a\sin(c)+a}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*ta
n(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 9*d*x*tan(c/2 + d
*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*t
an(c/2 + d*x/2)**2 + 6*a*d) + 9*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*
x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d)
+ 3*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*ta
n(c/2 + d*x/2)**2 + 6*a*d) - 6*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)*
*6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 12*
tan(c/2 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4
 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 6*tan(c/2 + d*x/2)/(6*a*d*tan(c/2
+ d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a
*d) + 4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*ta
n(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*cos(c)**4/(a*sin(c) + a), True))
```

3.54 $\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d
```

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2667}

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.04, size = 24, normalized size = 0.75

$$-\frac{(\sin(c+dx)-2)\sin(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
[Out] -1/2*((-2 + Sin[c + d*x])*Sin[c + d*x])/(a*d)
```

fricas [A] time = 0.64, size = 25, normalized size = 0.78

$$\frac{\cos(dx+c)^2+2\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(cos(d*x + c)^2 + 2*sin(d*x + c))/(a*d)
```

giac [A] time = 0.40, size = 25, normalized size = 0.78

$$-\frac{\sin(dx+c)^2 - 2\,\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

maple [A] time = 0.08, size = 28, normalized size = 0.88

$$-\frac{\frac{\left(\sin^2(dx+c)\right)}{2} - \sin(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
[Out] -1/a/d*(1/2*sin(d*x+c)^2-sin(d*x+c))
```

maxima [A] time = 0.33, size = 25, normalized size = 0.78

$$-\frac{\sin(dx+c)^2 - 2\,\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

mupad [B] time = 4.49, size = 22, normalized size = 0.69

$$-\frac{\sin(c+d\,x)\,\left(\sin(c+d\,x)-2\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x)),x)
[Out] -(sin(c + d*x)*(sin(c + d*x) - 2))/(2*a*d)
```

sympy [A] time = 5.80, size = 158, normalized size = 4.94

$$\begin{cases} \frac{2\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^3(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
[Out] Piecewise((2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2 +
 d*x/2)**2 + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*
tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**4 +
2*a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a), T
rue))
```

3.55 $\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=19

$$\frac{\cos(c+dx)}{ad}+\frac{x}{a}$$

```
[Out] x/a+cos(d*x+c)/a/d
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2682, 8}

$$\frac{\cos(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Cos[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cos(c+dx)}{ad}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}+\frac{\cos(c+dx)}{ad}\end{aligned}$$

Mathematica [B] time = 0.12, size = 97, normalized size = 5.11

$$-\frac{\left(2\sqrt{1-\sin(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+(\sin(c+dx)-1)\sqrt{\sin(c+dx)+1}\right)\cos^3(c+dx)}{ad(\sin(c+dx)-1)^2(\sin(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^3*(2*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
+ d*x]] + (-1 + Sin[c + d*x])*Sqrt[1 + Sin[c + d*x]]))/(a*d*(-1 + Sin[c + d
*x])^2*(1 + Sin[c + d*x])^(3/2)))
```

fricas [A] time = 0.60, size = 17, normalized size = 0.89

$$\frac{dx+\cos(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x + cos(d*x + c))/(a*d)
```

giac [A] time = 0.59, size = 34, normalized size = 1.79

$$\frac{\frac{dx+c}{a} + \frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a + 2/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

maple [B] time = 0.12, size = 43, normalized size = 2.26

$$\frac{2}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} + \frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/a/d/(1+tan(1/2*d*x+1/2*c)^2)+2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.60, size = 52, normalized size = 2.74

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2))/d
```

mupad [B] time = 4.53, size = 29, normalized size = 1.53

$$\frac{x}{a} + \frac{2}{a\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

```
[Out] x/a + 2/(a*d*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [A] time = 3.02, size = 88, normalized size = 4.63

$$\begin{cases} \frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + d*x/(a
*d*tan(c/2 + d*x/2)**2 + a*d) + 2/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)
), (x*cos(c)**2/(a*sin(c) + a), True))
```

3.56 $\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=16

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] ln(1+sin(d*x+c))/a/d
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 31}

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{\log(1+\sin(c+dx))}{ad}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

fricas [A] time = 0.74, size = 16, normalized size = 1.00

$$\frac{\log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] log(sin(d*x + c) + 1)/(a*d)
```

giac [A] time = 0.40, size = 19, normalized size = 1.19

$$\frac{\log\left(|a\sin\left(dx+c\right)+a|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] log(abs(a*sin(d*x + c) + a))/(a*d)
```

maple [A] time = 0.06, size = 19, normalized size = 1.19

$$\frac{\ln\left(a+a\sin\left(dx+c\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] 1/d*ln(a+a*sin(d*x+c))/a
```

maxima [A] time = 0.31, size = 18, normalized size = 1.12

$$\frac{\log\left(a\sin\left(dx+c\right)+a\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] log(a*sin(d*x + c) + a)/(a*d)
```

mupad [B] time = 0.04, size = 16, normalized size = 1.00

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x)),x)
[Out] log(sin(c + d*x) + 1)/(a*d)
```

sympy [A] time = 0.50, size = 24, normalized size = 1.50

$$\begin{cases} \frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad} & \text{for}\ d\neq 0 \\ \frac{x\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d), Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)
, True))
```

3.57 $\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a\sin(c+dx)+a)}$$

```
[Out] 1/2*arctanh(sin(d*x+c))/a/d-1/2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - 1/(2*d*(a + a*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a\left(a^2-x^2\right)}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= -\frac{1}{2d(a+a\sin(c+dx))}+\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx,x,a\sin(c+dx)\right)}{2d}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a+a\sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 0.81

$$\frac{\tanh^{-1}(\sin(c+dx))-\frac{1}{\sin(c+dx)+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]] - (1 + Sin[c + d*x])^(-1))/(2*a*d)
```

fricas [A] time = 0.65, size = 58, normalized size = 1.57

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)-(\sin(dx+c)+1)\log(-\sin(dx+c)+1)-2}{4\,(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (sin(d*x + c) + 1)*log(-sin
(d*x + c) + 1) - 2)/(a*d*sin(d*x + c) + a*d)
```

giac [A] time = 0.45, size = 58, normalized size = 1.57

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{a}-\frac{\log(|\sin(dx+c)-1|)}{a}-\frac{\sin(dx+c)+3}{a(\sin(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c) - 1))/a - (sin(d*x
 + c) + 3)/(a*(sin(d*x + c) + 1)))/d
```

maple [A] time = 0.14, size = 54, normalized size = 1.46

$$-\frac{\ln(\sin(dx+c)-1)}{4ad}-\frac{1}{2ad\,(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/a/d*ln(sin(d*x+c)-1)-1/2/a/d/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.39, size = 47, normalized size = 1.27

$$\frac{\frac{\log(\sin(dx+c)+1)}{a}-\frac{\log(\sin(dx+c)-1)}{a}-\frac{2}{a\sin(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a - 2/(a*sin(d*x + c)
+ a))/d
```

mupad [B] time = 0.07, size = 33, normalized size = 0.89

$$\frac{\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{2\,a\,d}-\frac{1}{2\,d\,\left(a+a\,\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] atanh(sin(c + d*x))/(2*a*d) - 1/(2*d*(a + a*sin(c + d*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x) + 1), x)/a
```

3.58 $\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=42

$$\frac{2\tan(c+dx)}{3ad}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

```
[Out] -1/3*sec(d*x+c)/d/(a+a*sin(d*x+c))+2/3*tan(d*x+c)/a/d
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2672, 3767, 8}

$$\frac{2\tan(c+dx)}{3ad}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Sec[c + d*x]/(3*d*(a + a*Sin[c + d*x])) + (2*Tan[c + d*x])/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}+\frac{2\int \sec^2(c+dx)\,dx}{3a}\\ &= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}-\frac{2\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{3ad}\\ &= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}+\frac{2\tan(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.06, size = 45, normalized size = 1.07

$$\frac{2\tan(c+dx)-\cos(2(c+dx))\sec(c+dx)}{3ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-(Cos[2*(c + d*x)]*Sec[c + d*x]) + 2*Tan[c + d*x])/(3*a*d*(1 + Sin[c + d*x
]))
```

fricas [A] time = 0.67, size = 49, normalized size = 1.17

$$-\frac{2\cos(dx+c)^2 - 2\sin(dx+c) - 1}{3(ad\cos(dx+c)\sin(dx+c) + ad\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(d*x + c)^2 - 2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)*sin(d*x + c)
 + a*d*cos(d*x + c))
```

giac [A] time = 0.65, size = 67, normalized size = 1.60

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)} + \frac{9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) + (9*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 7)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

maple [A] time = 0.15, size = 70, normalized size = 1.67

$$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} + \frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/d/a*(-1/4/(tan(1/2*d*x+1/2*c)-1)-1/3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(tan(1/
2*d*x+1/2*c)+1)^2-3/4/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.61, size = 129, normalized size = 3.07

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + \frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - 1\right)}{3\left(a + \frac{2a\sin(dx+c)}{\cos(dx+c)+1} - \frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/((a + 2*a*sin(d*x + c)/(cos(
d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4)*d)
```

mupad [B] time = 4.56, size = 71, normalized size = 1.69

$$-\frac{2\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*(tan(c/2 + (d*x)/2) + 3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^3 -
 1))/(3*a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^3)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.59 $\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=77

$$-\frac{a}{8d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{1}{4d(a\sin(c+dx)+a)}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}$$

```
[Out] 3/8*arctanh(sin(d*x+c))/a/d+1/8/d/(a-a*sin(d*x+c))-1/8*a/d/(a+a*sin(d*x+c))
^2-1/4/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2667, 44, 206\}$

$$-\frac{a}{8d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{1}{4d(a\sin(c+dx)+a)}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + 1/(8*d*(a - a*Sin[c + d*x])) - a/(8*d*(
a + a*Sin[c + d*x])^2) - 1/(4*d*(a + a*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{a^3\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3\left(a^2-x^2\right)}\right) dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{1}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+a\sin(c+dx))^2}-\frac{1}{4d(a+a\sin(c+dx))}+\frac{3\,\text{Subst}\left(\int\right.}{}\\
&= \frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+a\sin(c+dx))^2}-\frac{}{4d(a+a\,\text{s}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 75, normalized size = 0.97

$$-\frac{\sec^2(c+dx)\left(-3\sin^2(c+dx)-3\sin(c+dx)+3(\sin(c+dx)-1)(\sin(c+dx)+1)^2\tanh^{-1}(\sin(c+dx))+2\right)}{8ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(Sec[c + d*x]^2*(2 - 3*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 3*ArcTanh[Sin
[c + d*x]]*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^2))/(a*d*(1 + Sin[c + d*x
]))
```

fricas [A] time = 0.57, size = 125, normalized size = 1.62

$$-\frac{6\cos(dx+c)^2-3\left(\cos(dx+c)^2\sin(dx+c)+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)+3\left(\cos(dx+c)^2\sin(dx+\right.}{16\left(ad\cos(dx+c)^2\sin(dx+c)+ad\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*cos(d*x + c)^2 - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*
log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*lo
g(-sin(d*x + c) + 1) - 6*sin(d*x + c) - 2)/(a*d*cos(d*x + c)^2*sin(d*x + c)
 + a*d*cos(d*x + c)^2)
```

giac [A] time = 0.85, size = 96, normalized size = 1.25

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a}-\frac{6\log(|\sin(dx+c)-1|)}{a}+\frac{2(3\sin(dx+c)-5)}{a(\sin(dx+c)-1)}-\frac{9\sin(dx+c)^2+26\sin(dx+c)+21}{a(\sin(dx+c)+1)^2}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c) - 1))/a + 2*(
3*sin(d*x + c) - 5)/(a*(sin(d*x + c) - 1)) - (9*sin(d*x + c)^2 + 26*sin(d*x
 + c) + 21)/(a*(sin(d*x + c) + 1)^2))/d
```

maple [A] time = 0.16, size = 90, normalized size = 1.17

$$-\frac{1}{8ad(\sin(dx+c)-1)}-\frac{3\ln(\sin(dx+c)-1)}{16ad}-\frac{1}{8ad(1+\sin(dx+c))^2}-\frac{1}{4ad(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx}{16ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/8/a/d/(sin(d*x+c)-1)-3/16/a/d*ln(sin(d*x+c)-1)-1/8/a/d/(1+sin(d*x+c))^2-
1/4/a/d/(1+sin(d*x+c))+3/16*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.32, size = 91, normalized size = 1.18

$$-\frac{\frac{2\left(3\sin(dx+c)^2+3\sin(dx+c)-2\right)}{a\sin(dx+c)^3+a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{3\log(\sin(dx+c)+1)}{a}+\frac{3\log(\sin(dx+c)-1)}{a}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(
d*x + c)^2 - a*sin(d*x + c) - a) - 3*log(sin(d*x + c) + 1)/a + 3*log(sin(d*
x + c) - 1)/a)/d
```

mupad [B] time = 4.66, size = 74, normalized size = 0.96

$$\frac{3\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{8\,a\,d}+\frac{\frac{3\,{\sin\left(c+d\,x\right)}^2}{8}+\frac{3\,\sin\left(c+d\,x\right)}{8}-\frac{1}{4}}{d\,\left(-a\,{\sin\left(c+d\,x\right)}^3-a\,{\sin\left(c+d\,x\right)}^2+a\,\sin\left(c+d\,x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atanh(sin(c + d*x)))/(8*a*d) + ((3*sin(c + d*x))/8 + (3*sin(c + d*x)^2)/
8 - 1/4)/(d*(a + a*sin(c + d*x) - a*sin(c + d*x)^2 - a*sin(c + d*x)^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

3.60 $\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{4\tan^3(c+dx)}{15ad}+\frac{4\tan(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{5d(a\sin(c+dx)+a)}$$

```
[Out] -1/5*sec(d*x+c)^3/d/(a+a*sin(d*x+c))+4/5*tan(d*x+c)/a/d+4/15*tan(d*x+c)^3/a
/d
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 3767}

$$\frac{4\tan^3(c+dx)}{15ad}+\frac{4\tan(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{5d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Sec[c + d*x]^3/(5*d*(a + a*Sin[c + d*x])) + (4*Tan[c + d*x])/(5*a*d) + (4*
Tan[c + d*x]^3)/(15*a*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\sec^3(c+dx)}{5d(a+a\sin(c+dx))}+\frac{4\int \sec^4(c+dx)\,dx}{5a}\\ &= -\frac{\sec^3(c+dx)}{5d(a+a\sin(c+dx))}-\frac{4\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{5ad}\\ &= -\frac{\sec^3(c+dx)}{5d(a+a\sin(c+dx))}+\frac{4\tan(c+dx)}{5ad}+\frac{4\tan^3(c+dx)}{15ad}\end{aligned}$$

Mathematica [A] time = 0.10, size = 66, normalized size = 1.06

$$-\frac{\sec^3(c+dx)(-2(3\sin(c+dx)+\sin(3(c+dx)))+2\cos(2(c+dx))+\cos(4(c+dx)))}{15ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

[Out] -1/15*(Sec[c + d*x]^3*(2*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] - 2*(3*Sin[c + d*x] + Sin[3*(c + d*x)])))/(a*d*(1 + Sin[c + d*x]))

fricas [A] time = 0.67, size = 75, normalized size = 1.21

$$-\frac{8\cos(dx+c)^4-4\cos(dx+c)^2-4\left(2\cos(dx+c)^2+1\right)\sin(dx+c)-1}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/15*(8*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + 1)*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)

giac [B] time = 0.69, size = 119, normalized size = 1.92

$$-\frac{\frac{5\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}+\frac{165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+400\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+113}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 13)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) + (165*tan(1/2*d*x + 1/2*c)^4 + 480*tan(1/2*d*x + 1/2*c)^3 + 650*tan(1/2*d*x + 1/2*c)^2 + 400*tan(1/2*d*x + 1/2*c) + 113)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d

maple [B] time = 0.16, size = 130, normalized size = 2.10

$$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{2\left(\tan\left(\frac{dx}{2}\right.\right.}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c)),x)

[Out] 2/d/a*(-1/12/(tan(1/2*d*x+1/2*c)-1)^3-1/8/(tan(1/2*d*x+1/2*c)-1)^2-5/16/(tan(1/2*d*x+1/2*c)-1)-1/5/(tan(1/2*d*x+1/2*c)+1)^5+1/2/(tan(1/2*d*x+1/2*c)+1)^4-5/6/(tan(1/2*d*x+1/2*c)+1)^3+3/4/(tan(1/2*d*x+1/2*c)+1)^2-11/16/(tan(1/2*d*x+1/2*c)+1))

maxima [B] time = 0.48, size = 294, normalized size = 4.74

$$\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{13\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{25\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-3\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 2/15*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 13*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 25*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 5*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -

```
 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

mupad [B] time = 5.94, size = 125, normalized size = 2.02

$$-\frac{2\left(15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+21\tan\left(\frac{c}{2}+\right.\right.}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*(9*tan(c/2 + (d*x)/2) + 21*tan(c/2 + (d*x)/2)^2 + 13*tan(c/2 + (d*x)/2)
^3 - 25*tan(c/2 + (d*x)/2)^4 - 5*tan(c/2 + (d*x)/2)^5 + 15*tan(c/2 + (d*x)/
2)^6 + 15*tan(c/2 + (d*x)/2)^7 - 3))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(ta
n(c/2 + (d*x)/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

3.61 $\int \frac{\sec^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=120

$$-\frac{a^2}{24d(a\sin(c+dx)+a)^3}+\frac{a}{32d(a-a\sin(c+dx))^2}-\frac{3a}{32d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{}{16d(a\sin(}$$

```
[Out] 5/16*arctanh(sin(d*x+c))/a/d+1/32*a/d/(a-a*sin(d*x+c))^2+1/8/d/(a-a*sin(d*x
+c))-1/24*a^2/d/(a+a*sin(d*x+c))^3-3/32*a/d/(a+a*sin(d*x+c))^2-3/16/d/(a+a*
sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^2}{24d(a\sin(c+dx)+a)^3}+\frac{a}{32d(a-a\sin(c+dx))^2}-\frac{3a}{32d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{}{16d(a\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(16*a*d) + a/(32*d*(a - a*Sin[c + d*x])^2) + 1/(8
*d*(a - a*Sin[c + d*x])) - a^2/(24*d*(a + a*Sin[c + d*x])^3) - (3*a)/(32*d*
(a + a*Sin[c + d*x])^2) - 3/(16*d*(a + a*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^3} + \frac{1}{8a^5(a-x)^2} + \frac{1}{8a^3(a+x)^4} + \frac{3}{16a^4(a+x)^3} + \frac{3}{16a^5(a+x)^2} + \frac{5}{16a^5\left(a^2-x^2\right)}\right) dx, x\right.}{d}$$

$$= \frac{a}{32d(a-a\sin(c+dx))^2} + \frac{1}{8d(a-a\sin(c+dx))} - \frac{a^2}{24d(a+a\sin(c+dx))^3} - \frac{}{32d(a+a}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{16ad} + \frac{a}{32d(a-a\sin(c+dx))^2} + \frac{1}{8d(a-a\sin(c+dx))} - \frac{a}{24d(a+a\sin}$$

Mathematica [A] time = 0.15, size = 97, normalized size = 0.81

$$\frac{\sec^4(c+dx)\left(-15\sin^4(c+dx) - 15\sin^3(c+dx) + 25\sin^2(c+dx) + 25\sin(c+dx) + 15(\sin(c+dx)-1)^2(\sin(c+\right.}{48ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^4*(-8 + 25*Sin[c + d*x] + 25*Sin[c + d*x]^2 - 15*Sin[c + d*x]
^3 - 15*Sin[c + d*x]^4 + 15*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])^2*(1
+ Sin[c + d*x])^3))/(48*a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.64, size = 147, normalized size = 1.22

$$-\frac{30\cos(dx+c)^4 - 10\cos(dx+c)^2 - 15\left(\cos(dx+c)^4\sin(dx+c) + \cos(dx+c)^4\right)\log(\sin(dx+c)+1) + 15\left(\right.}{96\left(ad\cos(dx+c)^4\sin(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/96*(30*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 15*(cos(d*x + c)^4*sin(d*x +
 c) + cos(d*x + c)^4)*log(sin(d*x + c) + 1) + 15*(cos(d*x + c)^4*sin(d*x +
c) + cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 10*(3*cos(d*x + c)^2 + 2)*sin
(d*x + c) - 4)/(a*d*cos(d*x + c)^4*sin(d*x + c) + a*d*cos(d*x + c)^4)
```

giac [A] time = 0.75, size = 116, normalized size = 0.97

$$\frac{\frac{30\log(|\sin(dx+c)+1|)}{a} - \frac{30\log(|\sin(dx+c)-1|)}{a} + \frac{3\left(15\sin(dx+c)^2 - 38\sin(dx+c) + 25\right)}{a(\sin(dx+c)-1)^2} - \frac{55\sin(dx+c)^3 + 201\sin(dx+c)^2 + 255\sin(dx+c) + 117}{a(\sin(dx+c)+1)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(30*log(abs(sin(d*x + c) + 1))/a - 30*log(abs(sin(d*x + c) - 1))/a +
3*(15*sin(d*x + c)^2 - 38*sin(d*x + c) + 25)/(a*(sin(d*x + c) - 1)^2) - (55
*sin(d*x + c)^3 + 201*sin(d*x + c)^2 + 255*sin(d*x + c) + 117)/(a*(sin(d*x
+ c) + 1)^3))/d
```

maple [A] time = 0.17, size = 126, normalized size = 1.05

$$\frac{1}{32ad(\sin(dx+c)-1)^2} - \frac{1}{8ad(\sin(dx+c)-1)} - \frac{5\ln(\sin(dx+c)-1)}{32ad} - \frac{1}{24ad(1+\sin(dx+c))^3} - \frac{3}{32ad(1+\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/32/a/d/(sin(d*x+c)-1)^2-1/8/a/d/(sin(d*x+c)-1)-5/32/a/d*ln(sin(d*x+c)-1)-
1/24/a/d/(1+sin(d*x+c))^3-3/32/a/d/(1+sin(d*x+c))^2-3/16/a/d/(1+sin(d*x+c))
+5/32*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.30, size = 130, normalized size = 1.08

$$-\frac{\frac{2\left(15\sin(dx+c)^4+15\sin(dx+c)^3-25\sin(dx+c)^2-25\sin(dx+c)+8\right)}{a\sin(dx+c)^5+a\sin(dx+c)^4-2a\sin(dx+c)^3-2a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-1)}{a}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*sin(d*x + c)^4 + 15*sin(d*x + c)^3 - 25*sin(d*x + c)^2 - 25*si
n(d*x + c) + 8)/(a*sin(d*x + c)^5 + a*sin(d*x + c)^4 - 2*a*sin(d*x + c)^3 -
 2*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 15*log(sin(d*x + c) + 1)/a + 15
*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 0.14, size = 115, normalized size = 0.96

$$\frac{5\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{16\,a\,d}-\frac{\frac{5\sin(c+d\,x)^4}{16}+\frac{5\sin(c+d\,x)^3}{16}-\frac{25\sin(c+d\,x)^2}{48}-\frac{25\sin(c+d\,x)}{48}+\frac{1}{6}}{d\,\left(a\sin\left(c+d\,x\right)^5+a\sin\left(c+d\,x\right)^4-2\,a\sin\left(c+d\,x\right)^3-2\,a\sin\left(c+d\,x\right)^2+a\,\sin(c+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*atanh(sin(c + d*x)))/(16*a*d) - ((5*sin(c + d*x)^3)/16 - (25*sin(c + d*x
)^2)/48 - (25*sin(c + d*x))/48 + (5*sin(c + d*x)^4)/16 + 1/6)/(d*(a + a*sin
(c + d*x) - 2*a*sin(c + d*x)^2 - 2*a*sin(c + d*x)^3 + a*sin(c + d*x)^4 + a*
sin(c + d*x)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^5(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

3.62 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=104

$$\frac{7\cos^5(c+dx)}{30a^2d}+\frac{\cos^7(c+dx)}{6d\left(a^2\sin(c+dx)+a^2\right)}+\frac{7\sin(c+dx)\cos^3(c+dx)}{24a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{7x}{16a^2}$$

```
[Out] 7/16*x/a^2+7/30*cos(d*x+c)^5/a^2/d+7/16*cos(d*x+c)*sin(d*x+c)/a^2/d+7/24*co
s(d*x+c)^3*sin(d*x+c)/a^2/d+1/6*cos(d*x+c)^7/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2679, 2682, 2635, 8}

$$\frac{7\cos^5(c+dx)}{30a^2d}+\frac{\cos^7(c+dx)}{6d\left(a^2\sin(c+dx)+a^2\right)}+\frac{7\sin(c+dx)\cos^3(c+dx)}{24a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{7x}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*x)/(16*a^2) + (7*Cos[c + d*x]^5)/(30*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*
x])/(16*a^2*d) + (7*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^2*d) + Cos[c + d*x]^
7/(6*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\cos^7(c+dx)}{6d\left(a^2+a^2\sin(c+dx)\right)} + \frac{7\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}dx}{6a}$$
$$= \frac{7\cos^5(c+dx)}{30a^2d} + \frac{\cos^7(c+dx)}{6d\left(a^2+a^2\sin(c+dx)\right)} + \frac{7\int \cos^4(c+dx)\,dx}{6a^2}$$
$$= \frac{7\cos^5(c+dx)}{30a^2d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^2d} + \frac{\cos^7(c+dx)}{6d\left(a^2+a^2\sin(c+dx)\right)} + \frac{7\int \cos$$
$$= \frac{7\cos^5(c+dx)}{30a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^2d} + \frac{c}{6d\left(a^2\right.}$$
$$= \frac{7x}{16a^2} + \frac{7\cos^5(c+dx)}{30a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^2d} +$$

Mathematica [A] time = 1.12, size = 151, normalized size = 1.45

$$-\frac{\left(\sqrt{\sin(c+dx)+1}\,\left(40\sin^6(c+dx)-136\sin^5(c+dx)+86\sin^4(c+dx)+202\sin^3(c+dx)-327\sin^2(c+d\right.\right.}{240a^2d(\sin(c+dx)-1)^5(\sin(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/240*(Cos[c + d*x]^9*(-210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1
- Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(96 + 39*Sin[c + d*x] - 327*Sin[c
+ d*x]^2 + 202*Sin[c + d*x]^3 + 86*Sin[c + d*x]^4 - 136*Sin[c + d*x]^5 + 40
*Sin[c + d*x]^6)))/(a^2*d*(-1 + Sin[c + d*x])^5*(1 + Sin[c + d*x])^(9/2))
```

fricas [A] time = 0.81, size = 60, normalized size = 0.58

$$\frac{96\cos(dx+c)^5 + 105\,dx - 5\left(8\cos(dx+c)^5 - 14\cos(dx+c)^3 - 21\cos(dx+c)\right)\sin(dx+c)}{240\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(96*cos(d*x + c)^5 + 105*d*x - 5*(8*cos(d*x + c)^5 - 14*cos(d*x + c)^
3 - 21*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.80, size = 179, normalized size = 1.72

$$\frac{\frac{105\,(dx+c)}{a^2} - \frac{2\left(135\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11} - 480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10} + 445\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9} - 480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8} - 330\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7} - 960\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(105*(d*x + c)/a^2 - 2*(135*tan(1/2*d*x + 1/2*c)^11 - 480*tan(1/2*d*x
 + 1/2*c)^10 + 445*tan(1/2*d*x + 1/2*c)^9 - 480*tan(1/2*d*x + 1/2*c)^8 - 33
0*tan(1/2*d*x + 1/2*c)^7 - 960*tan(1/2*d*x + 1/2*c)^6 + 330*tan(1/2*d*x + 1
/2*c)^5 - 960*tan(1/2*d*x + 1/2*c)^4 - 445*tan(1/2*d*x + 1/2*c)^3 - 96*tan(
```

```
1/2*d*x + 1/2*c)^2 - 135*tan(1/2*d*x + 1/2*c) - 96)/((tan(1/2*d*x + 1/2*c)^
2 + 1)^6*a^2))/d
```

maple [B] time = 0.22, size = 415, normalized size = 3.99

$$-\frac{9\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{89\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{4\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -9/8/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+4/a^2/d/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10-89/24/a^2/d/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)^9+4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1
/2*c)^8+11/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+8/a^2/d/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6-11/4/a^2/d/(1+tan(1/2*d*x+1
/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+8/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*
d*x+1/2*c)^4+89/24/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+4/
5/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2+9/8/a^2/d/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)+4/5/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^6+7
/8/a^2/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.63, size = 393, normalized size = 3.78

$$\frac{\frac{\frac{135\sin(dx+c)}{\cos(dx+c)+1}+\frac{96\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{445\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{960\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{330\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{330\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{480\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{445\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{480\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\cdots}{a^2+\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}\cdots}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*((135*sin(d*x + c)/(cos(d*x + c) + 1) + 96*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 445*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 960*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 330*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 960*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 330*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 480
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 445*sin(d*x + c)^9/(cos(d*x + c) + 1
)^9 + 480*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 135*sin(d*x + c)^11/(cos(
d*x + c) + 1)^11 + 96)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/(cos(d*x
+ c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*sin(d*x +
c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) +
105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 8.22, size = 172, normalized size = 1.65

$$\frac{7\,x}{16\,a^2}+\frac{-\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{89\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-\cdots}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (7*x)/(16*a^2) + ((9*tan(c/2 + (d*x)/2))/8 + (4*tan(c/2 + (d*x)/2)^2)/5 + (
89*tan(c/2 + (d*x)/2)^3)/24 + 8*tan(c/2 + (d*x)/2)^4 - (11*tan(c/2 + (d*x)/
```

```
2)^5)/4 + 8*tan(c/2 + (d*x)/2)^6 + (11*tan(c/2 + (d*x)/2)^7)/4 + 4*tan(c/2
+ (d*x)/2)^8 - (89*tan(c/2 + (d*x)/2)^9)/24 + 4*tan(c/2 + (d*x)/2)^10 - (9*
tan(c/2 + (d*x)/2)^11)/8 + 4/5)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 148.51, size = 2531, normalized size = 24.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((105*d*x*tan(c/2 + d*x/2)**12/(240*a**2*d*tan(c/2 + d*x/2)**12 +
1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a
**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*t
an(c/2 + d*x/2)**2 + 240*a**2*d) + 630*d*x*tan(c/2 + d*x/2)**10/(240*a**2*d
*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(
c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d
*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 1575*d*x*tan(c/2
 + d*x/2)**8/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2
)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 +
 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a*
*2*d) + 2100*d*x*tan(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 + d*x/2)**12 + 144
0*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2
*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(
c/2 + d*x/2)**2 + 240*a**2*d) + 1575*d*x*tan(c/2 + d*x/2)**4/(240*a**2*d*ta
n(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2
 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/
2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 630*d*x*tan(c/2 + d
*x/2)**2/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**1
0 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 360
0*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d
) + 105*d*x/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)
**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 +
3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**
2*d) - 270*tan(c/2 + d*x/2)**11/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**
2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*ta
n(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 +
 d*x/2)**2 + 240*a**2*d) + 960*tan(c/2 + d*x/2)**10/(240*a**2*d*tan(c/2 + d
*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)
**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1
440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) - 890*tan(c/2 + d*x/2)**9/(240
*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2
*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(
c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 960*tan(c
/2 + d*x/2)**8/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x
/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6
 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*
a**2*d) + 660*tan(c/2 + d*x/2)**7/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a
**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*
tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2
 + d*x/2)**2 + 240*a**2*d) + 1920*tan(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 +
 d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/
2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 +
 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) - 660*tan(c/2 + d*x/2)**5/(2
40*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a*
*2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*ta
n(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 1920*ta
n(c/2 + d*x/2)**4/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 +
d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)
**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 2
```

```
40*a**2*d) + 890*tan(c/2 + d*x/2)**3/(240*a**2*d*tan(c/2 + d*x/2)**12 + 144
0*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2
*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(
c/2 + d*x/2)**2 + 240*a**2*d) + 192*tan(c/2 + d*x/2)**2/(240*a**2*d*tan(c/2
 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*
x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4
 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 270*tan(c/2 + d*x/2)/(24
0*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**
2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan
(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 192/(240
*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2
*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(
c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d), Ne(d, 0)),
 (x*cos(c)**8/(a*sin(c) + a)**2, True))
```

3.63 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{(a-a\sin(c+dx))^5}{5a^7d}-\frac{(a-a\sin(c+dx))^4}{2a^6d}$$

```
[Out] -1/2*(a-a*sin(d*x+c))^4/a^6/d+1/5*(a-a*sin(d*x+c))^5/a^7/d
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a-a\sin(c+dx))^5}{5a^7d}-\frac{(a-a\sin(c+dx))^4}{2a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a - a*Sin[c + d*x])^4/(2*a^6*d) + (a - a*Sin[c + d*x])^5/(5*a^7*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)\,dx,x,a\sin(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int \left(2a(a-x)^3-(a-x)^4\right)\,dx,x,a\sin(c+dx)\right)}{a^7d}\\
&= -\frac{(a-a\sin(c+dx))^4}{2a^6d}+\frac{(a-a\sin(c+dx))^5}{5a^7d}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 46, normalized size = 0.98

$$-\frac{\sin(c+dx)\left(2\sin^4(c+dx)-5\sin^3(c+dx)+10\sin(c+dx)-10\right)}{10a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/10*(Sin[c + d*x]*(-10 + 10*Sin[c + d*x] - 5*Sin[c + d*x]^3 + 2*Sin[c + d
*x]^4))/(a^2*d)
```

fricas [A] time = 0.73, size = 47, normalized size = 1.00

$$\frac{5\,\cos\left(dx+c\right)^{4}-2\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}-4\right)\sin\left(dx+c\right)}{10\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/10*(5*cos(d*x + c)^4 - 2*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - 4)*sin(d*x
+ c))/(a^2*d)
```

giac [A] time = 1.01, size = 47, normalized size = 1.00

$$-\frac{2\,\sin\left(dx+c\right)^{5}-5\,\sin\left(dx+c\right)^{4}+10\,\sin\left(dx+c\right)^{2}-10\,\sin\left(dx+c\right)}{10\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*sin(d*x + c)^5 - 5*sin(d*x + c)^4 + 10*sin(d*x + c)^2 - 10*sin(d*x
 + c))/(a^2*d)
```

maple [A] time = 0.19, size = 45, normalized size = 0.96

$$\frac{-\frac{\left(\sin^{5}(dx+c)\right)}{5}+\frac{\left(\sin^{4}(dx+c)\right)}{2}-\left(\sin^{2}\left(dx+c\right)\right)+\sin\left(dx+c\right)}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/5*sin(d*x+c)^5+1/2*sin(d*x+c)^4-sin(d*x+c)^2+sin(d*x+c))
```

maxima [A] time = 0.35, size = 47, normalized size = 1.00

$$-\frac{2\,\sin\left(dx+c\right)^{5}-5\,\sin\left(dx+c\right)^{4}+10\,\sin\left(dx+c\right)^{2}-10\,\sin\left(dx+c\right)}{10\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*sin(d*x + c)^5 - 5*sin(d*x + c)^4 + 10*sin(d*x + c)^2 - 10*sin(d*x
 + c))/(a^2*d)
```

mupad [B] time = 4.66, size = 54, normalized size = 1.15

$$\frac{\frac{\sin(c+d\,x)}{a^2}-\frac{\sin(c+d\,x)^2}{a^2}+\frac{\sin(c+d\,x)^4}{2\,a^2}-\frac{\sin(c+d\,x)^5}{5\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)/a^2 - sin(c + d*x)^2/a^2 + sin(c + d*x)^4/(2*a^2) - sin(c + d
*x)^5/(5*a^2))/d
```

sympy [A] time = 92.86, size = 1037, normalized size = 22.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((10*tan(c/2 + d*x/2)**9/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*a**2*
d*tan(c/2 + d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(c/2 +
 d*x/2)**4 + 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d) - 20*tan(c/2 + d*x/2
)**8/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*a**2*d*tan(c/2 + d*x/2)**8 + 50*a*
*2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(c/2 + d*x/2)**4 + 25*a**2*d*tan(c/
2 + d*x/2)**2 + 5*a**2*d) + 40*tan(c/2 + d*x/2)**7/(5*a**2*d*tan(c/2 + d*x/
2)**10 + 25*a**2*d*tan(c/2 + d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6 + 50
*a**2*d*tan(c/2 + d*x/2)**4 + 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d) - 2
0*tan(c/2 + d*x/2)**6/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*a**2*d*tan(c/2 +
d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(c/2 + d*x/2)**4 +
 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d) + 28*tan(c/2 + d*x/2)**5/(5*a**2
*d*tan(c/2 + d*x/2)**10 + 25*a**2*d*tan(c/2 + d*x/2)**8 + 50*a**2*d*tan(c/2
 + d*x/2)**6 + 50*a**2*d*tan(c/2 + d*x/2)**4 + 25*a**2*d*tan(c/2 + d*x/2)**
2 + 5*a**2*d) - 20*tan(c/2 + d*x/2)**4/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*
a**2*d*tan(c/2 + d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(
c/2 + d*x/2)**4 + 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d) + 40*tan(c/2 +
d*x/2)**3/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*a**2*d*tan(c/2 + d*x/2)**8 +
50*a**2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(c/2 + d*x/2)**4 + 25*a**2*d*t
an(c/2 + d*x/2)**2 + 5*a**2*d) - 20*tan(c/2 + d*x/2)**2/(5*a**2*d*tan(c/2 +
 d*x/2)**10 + 25*a**2*d*tan(c/2 + d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6
 + 50*a**2*d*tan(c/2 + d*x/2)**4 + 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d
) + 10*tan(c/2 + d*x/2)/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*a**2*d*tan(c/2
+ d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(c/2 + d*x/2)**4
 + 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d), Ne(d, 0)), (x*cos(c)**7/(a*si
n(c) + a)**2, True))
```

3.64 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=80

$$\frac{5\cos^3(c+dx)}{12a^2d}+\frac{\cos^5(c+dx)}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{5x}{8a^2}$$

```
[Out] 5/8*x/a^2+5/12*cos(d*x+c)^3/a^2/d+5/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d
*x+c)^5/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2679, 2682, 2635, 8}

$$\frac{5\cos^3(c+dx)}{12a^2d}+\frac{\cos^5(c+dx)}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{5x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*x)/(8*a^2) + (5*Cos[c + d*x]^3)/(12*a^2*d) + (5*Cos[c + d*x]*Sin[c + d*x
])/(8*a^2*d) + Cos[c + d*x]^5/(4*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\cos^5(c+dx)}{4d\left(a^2+a^2\sin(c+dx)\right)} + \frac{5\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{4a}$$
$$= \frac{5\cos^3(c+dx)}{12a^2d} + \frac{\cos^5(c+dx)}{4d\left(a^2+a^2\sin(c+dx)\right)} + \frac{5\int \cos^2(c+dx)\,dx}{4a^2}$$
$$= \frac{5\cos^3(c+dx)}{12a^2d} + \frac{5\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^5(c+dx)}{4d\left(a^2+a^2\sin(c+dx)\right)} + \frac{5\int 1\,dx}{8a^2}$$
$$= \frac{5x}{8a^2} + \frac{5\cos^3(c+dx)}{12a^2d} + \frac{5\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^5(c+dx)}{4d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.51, size = 131, normalized size = 1.64

$$-\frac{\left(30\sqrt{1-\sin(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\,\left(6\sin^4(c+dx)-22\sin^3(c+dx)+25\sin^2(c+\right.\right.}{24a^2d(\sin(c+dx)-1)^4(\sin(c+dx)+1)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/24*(Cos[c + d*x]^7*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - S
in[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-16 + 7*Sin[c + d*x] + 25*Sin[c + d*
x]^2 - 22*Sin[c + d*x]^3 + 6*Sin[c + d*x]^4)))/(a^2*d*(-1 + Sin[c + d*x])^4
*(1 + Sin[c + d*x])^(7/2))
```

fricas [A] time = 0.60, size = 50, normalized size = 0.62

$$\frac{16\cos(dx+c)^3+15\,dx-3\left(2\cos(dx+c)^3-5\cos(dx+c)\right)\sin(dx+c)}{24\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*cos(d*x + c)^3 + 15*d*x - 3*(2*cos(d*x + c)^3 - 5*cos(d*x + c))*si
n(d*x + c))/(a^2*d)
```

giac [A] time = 0.65, size = 127, normalized size = 1.59

$$\frac{\frac{15\,(dx+c)}{a^2}-\frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-48\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+33\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-48\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-33\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*(d*x + c)/a^2 - 2*(9*tan(1/2*d*x + 1/2*c)^7 - 48*tan(1/2*d*x + 1/2
*c)^6 + 33*tan(1/2*d*x + 1/2*c)^5 - 48*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*
d*x + 1/2*c)^3 - 16*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 16)/(
(tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^2))/d
```

maple [B] time = 0.18, size = 279, normalized size = 3.49

$$-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{11\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+4/a^2/d/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6-11/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)
^4*tan(1/2*d*x+1/2*c)^5+4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*
c)^4+11/4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+4/3/a^2/d/(
1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+3/4/a^2/d/(1+tan(1/2*d*x+1/2
*c)^2)^4*tan(1/2*d*x+1/2*c)+4/3/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^4+5/4/a^2/d*
arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.73, size = 267, normalized size = 3.34

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{48\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{48\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+16}{a^2+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 48*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 16)/(a^2 + 4
*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 4.75, size = 65, normalized size = 0.81

$$\frac{5\,x}{8\,a^2}+\frac{2\cos\left(c+d\,x\right)^3}{3\,a^2\,d}-\frac{\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,a^2\,d}+\frac{5\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (5*x)/(8*a^2) + (2*cos(c + d*x)^3)/(3*a^2*d) - (cos(c + d*x)^3*sin(c + d*x)
)/(4*a^2*d) + (5*cos(c + d*x)*sin(c + d*x))/(8*a^2*d)
```

sympy [A] time = 58.83, size = 1243, normalized size = 15.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**8/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a
**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(
c/2 + d*x/2)**2 + 24*a**2*d) + 60*d*x*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/
2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)
**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 90*d*x*tan(c/2 + d*x/2)*
*4/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**
2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 60*d
*x*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 +
 d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2
 + 24*a**2*d) + 15*d*x/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 +
 d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2
```

```
 + 24*a**2*d) - 18*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*
a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan
(c/2 + d*x/2)**2 + 24*a**2*d) + 96*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/2 +
 d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4
 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 66*tan(c/2 + d*x/2)**5/(24*
a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan
(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 96*tan(c/2
+ d*x/2)**4/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6
+ 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*
d) + 66*tan(c/2 + d*x/2)**3/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(
c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/
2)**2 + 24*a**2*d) + 32*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8
+ 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*
d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 18*tan(c/2 + d*x/2)/(24*a**2*d*tan(c/2
 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)*
*4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 32/(24*a**2*d*tan(c/2 + d
*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 +
 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d), Ne(d, 0)), (x*cos(c)**6/(a*sin
(c) + a)**2, True))
```

3.65 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=23

$$-\frac{(a-a\sin(c+dx))^3}{3a^5d}$$

```
[Out] -1/3*(a-a*sin(d*x+c))^3/a^5/d
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2667, 32\}$

$$-\frac{(a-a\sin(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a - a*Sin[c + d*x])^3/(3*a^5*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2\,dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= -\frac{(a-a\sin(c+dx))^3}{3a^5d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 1.48

$$\frac{\sin(c+dx)\left(\sin^2(c+dx)-3\sin(c+dx)+3\right)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]*(3 - 3*Sin[c + d*x] + Sin[c + d*x]^2))/(3*a^2*d)
```

fricas [A] time = 0.69, size = 37, normalized size = 1.61

$$\frac{3\,\cos\left(dx+c\right)^2-\left(\cos\left(dx+c\right)^2-4\right)\sin\left(dx+c\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/3*(3*cos(d*x + c)^2 - (cos(d*x + c)^2 - 4)*sin(d*x + c))/(a^2*d)
```

giac [A] time = 1.97, size = 35, normalized size = 1.52

$$\frac{\sin(dx+c)^3 - 3\sin(dx+c)^2 + 3\sin(dx+c)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 3*sin(d*x + c))/(a^2*d)
```

maple [A] time = 0.17, size = 19, normalized size = 0.83

$$\frac{(\sin(dx+c)-1)^3}{3d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
[Out] 1/3/d/a^2*(sin(d*x+c)-1)^3
```

maxima [A] time = 0.33, size = 35, normalized size = 1.52

$$\frac{\sin(dx+c)^3 - 3\sin(dx+c)^2 + 3\sin(dx+c)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 3*sin(d*x + c))/(a^2*d)
```

mupad [B] time = 4.61, size = 32, normalized size = 1.39

$$\frac{\sin(c+d\,x)\,\left(\sin(c+d\,x)^2 - 3\sin(c+d\,x) + 3\right)}{3\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^2,x)
[Out] (sin(c + d*x)*(sin(c + d*x)^2 - 3*sin(c + d*x) + 3))/(3*a^2*d)
```

sympy [A] time = 37.35, size = 394, normalized size = 17.13

$$\begin{cases} \frac{6\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d} - \frac{12\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d} + \frac{}{3a^2d\tan} \\ \frac{x\cos^5(c)}{(a\sin(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
[Out] Piecewise((6*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*t
an(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 12*tan(c/2
+ d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 +
```

```
9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 20*tan(c/2 + d*x/2)**3/(3*a**2*d
*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*
x/2)**2 + 3*a**2*d) - 12*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**6
+ 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) +
 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d), Ne(d, 0)), (x*cos(c)**5/(
a*sin(c) + a)**2, True))
```

3.66 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$\frac{3\cos(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{2d\left(a^2\sin(c+dx)+a^2\right)}+\frac{3x}{2a^2}$$

```
[Out] 3/2*x/a^2+3/2*cos(d*x+c)/a^2/d+1/2*cos(d*x+c)^3/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2679, 2682, 8}

$$\frac{3\cos(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{2d\left(a^2\sin(c+dx)+a^2\right)}+\frac{3x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/(2*a^2) + (3*Cos[c + d*x])/(2*a^2*d) + Cos[c + d*x]^3/(2*d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\cos^3(c+dx)}{2d\left(a^2+a^2\sin(c+dx)\right)}+\frac{3\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{2a}\\ &= \frac{3\cos(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{2d\left(a^2+a^2\sin(c+dx)\right)}+\frac{3\int 1\,dx}{2a^2}\\ &= \frac{3x}{2a^2}+\frac{3\cos(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{2d\left(a^2+a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.18, size = 109, normalized size = 1.95

$$-\frac{\left(\sqrt{\sin(c+dx)+1}\left(\sin^2(c+dx)-5\sin(c+dx)+4\right)-6\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}\right)\cos^5(c+dx)}{2a^2d(\sin(c+dx)-1)^3(\sin(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(Cos[c + d*x]^5*(-6*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Si
n[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(4 - 5*Sin[c + d*x] + Sin[c + d*x]^2))
)/(a^2*d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

fricas [A] time = 0.73, size = 35, normalized size = 0.62

$$\frac{3\,dx-\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,\cos\left(dx+c\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*d*x - cos(d*x + c)*sin(d*x + c) + 4*cos(d*x + c))/(a^2*d)
```

giac [A] time = 0.36, size = 73, normalized size = 1.30

$$\frac{\frac{3\,(dx+c)}{a^2}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)/a^2 + 2*(tan(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^2
 - tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

maple [B] time = 0.19, size = 142, normalized size = 2.54

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{3\,\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+4/a^2/d/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan
(1/2*d*x+1/2*c)+4/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^2+3/a^2/d*arctan(tan(1/2*d
*x+1/2*c))
```

maxima [B] time = 0.49, size = 140, normalized size = 2.50

$$-\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-4}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) - 4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 4)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [**B**] time = 4.65, size = 32, normalized size = 0.57

$$\frac{4\cos(c+d\,x)-\frac{\sin(2\,c+2\,d\,x)}{2}+3\,d\,x}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (4*cos(c + d*x) - sin(2*c + 2*d*x)/2 + 3*d*x)/(2*a^2*d)
```

sympy [**A**] time = 22.41, size = 403, normalized size = 7.20

$$\begin{cases} \frac{3dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d}+\frac{6dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d}+\frac{3dx}{2a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d}+ \\ \frac{x\cos^4(c)}{(a\sin(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**4/(2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2
*d*tan(c/2 + d*x/2)**2 + 2*a**2*d) + 6*d*x*tan(c/2 + d*x/2)**2/(2*a**2*d*ta
n(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d) + 3*d*x/(2*a**
2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d) + 2*tan(
c/2 + d*x/2)**3/(2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**
2 + 2*a**2*d) + 8*tan(c/2 + d*x/2)**2/(2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**
2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d) - 2*tan(c/2 + d*x/2)/(2*a**2*d*tan(c/2
+ d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d) + 8/(2*a**2*d*tan(c/
2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d), Ne(d, 0)), (x*cos
(c)**4/(a*sin(c) + a)**2, True))
```

3.67 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=32

$$\frac{2\log(\sin(c+dx)+1)}{a^2d}-\frac{\sin(c+dx)}{a^2d}$$

```
[Out] 2*ln(1+sin(d*x+c))/a^2/d-sin(d*x+c)/a^2/d
```

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{2\log(\sin(c+dx)+1)}{a^2d}-\frac{\sin(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Log[1 + Sin[c + d*x]])/(a^2*d) - Sin[c + d*x]/(a^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx&=\frac{\text{Subst}\left(\int \frac{a-x}{a+x}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{\text{Subst}\left(\int \left(-1+\frac{2a}{a+x}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{2\log(1+\sin(c+dx))}{a^2d}-\frac{\sin(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.81

$$-\frac{\sin(c+dx)-2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((-2*Log[1 + Sin[c + d*x]] + Sin[c + d*x])/(a^2*d))
```

fricas [A] time = 0.64, size = 27, normalized size = 0.84

$$\frac{2 \log\left(\sin\left(dx+c\right)+1\right)-\sin\left(dx+c\right)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] (2*log(sin(d*x + c) + 1) - sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.43, size = 54, normalized size = 1.69

$$-\frac{\frac{2 \log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}+\frac{a\sin(dx+c)+a}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] -(2*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a^2 + (a*s
in(d*x + c) + a)/a^3)/d
```

maple [A] time = 0.18, size = 33, normalized size = 1.03

$$\frac{2 \ln\left(1+\sin\left(dx+c\right)\right)}{a^2 d}-\frac{\sin\left(dx+c\right)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
[Out] 2*ln(1+sin(d*x+c))/a^2/d-sin(d*x+c)/a^2/d
```

maxima [A] time = 0.35, size = 30, normalized size = 0.94

$$\frac{\frac{2 \log(\sin(dx+c)+1)}{a^2}-\frac{\sin(dx+c)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] (2*log(sin(d*x + c) + 1)/a^2 - sin(d*x + c)/a^2)/d
```

mupad [B] time = 0.06, size = 27, normalized size = 0.84

$$\frac{2 \ln\left(\sin\left(c+d\,x\right)+1\right)-\sin\left(c+d\,x\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^2,x)
[Out] (2*log(sin(c + d*x) + 1) - sin(c + d*x))/(a^2*d)
```

sympy [A] time = 1.79, size = 150, normalized size = 4.69

$$\begin{cases} \frac{2 \log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{a^2 d\sin\left(c+dx\right)+a^2 d}+\frac{2 \log\left(\sin\left(c+dx\right)+1\right)}{a^2 d\sin\left(c+dx\right)+a^2 d}-\frac{2\sin^2\left(c+dx\right)}{a^2 d\sin\left(c+dx\right)+a^2 d}-\frac{\cos^2\left(c+dx\right)}{a^2 d\sin\left(c+dx\right)+a^2 d}+\frac{2}{a^2 d\sin\left(c+dx\right)+a^2 d} & \text{for}\: d\neq 0 \\ \frac{x\cos^3\left(c\right)}{\left(a\sin\left(c\right)+a\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*log(sin(c + d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2
*d) + 2*log(sin(c + d*x) + 1)/(a**2*d*sin(c + d*x) + a**2*d) - 2*sin(c + d*
x)**2/(a**2*d*sin(c + d*x) + a**2*d) - cos(c + d*x)**2/(a**2*d*sin(c + d*x)
 + a**2*d) + 2/(a**2*d*sin(c + d*x) + a**2*d), Ne(d, 0)), (x*cos(c)**3/(a*s
in(c) + a)**2, True))
```

3.68 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=34

$$-\frac{2\cos(c+dx)}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{x}{a^2}$$

```
[Out] -x/a^2-2*cos(d*x+c)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2680, 8}

$$-\frac{2\cos(c+dx)}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Cos[c + d*x])/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= -\frac{2\cos(c+dx)}{d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\int 1\,dx}{a^2}\\ &= -\frac{x}{a^2}-\frac{2\cos(c+dx)}{d\left(a^2+a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [B] time = 0.18, size = 104, normalized size = 3.06

$$\frac{2\left(\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}\,(\sin(c+dx)+1)+\sqrt{\sin(c+dx)+1}\,(\sin(c+dx)-1)\right)\cos^3(c+dx)}{a^2d(\sin(c+dx)-1)^2(\sin(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x]^3*((-1 + Sin[c + d*x])*Sqrt[1 + Sin[c + d*x]] + ArcSin[Sqrt
[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])))/(a^
2*d*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(5/2))
```

fricas [A] time = 0.86, size = 61, normalized size = 1.79

$$-\frac{dx+(dx+2)\cos(dx+c)+(dx-2)\sin(dx+c)+2}{a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(d*x + (d*x + 2)*cos(d*x + c) + (d*x - 2)*sin(d*x + c) + 2)/(a^2*d*cos(d*x
 + c) + a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.49, size = 33, normalized size = 0.97

$$-\frac{\frac{dx+c}{a^2}+\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)/a^2 + 4/(a^2*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.20, size = 41, normalized size = 1.21

$$-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}-\frac{4}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/a^2/d*arctan(tan(1/2*d*x+1/2*c))-4/a^2/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.76, size = 56, normalized size = 1.65

$$-\frac{2\left(\frac{2}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*(2/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1)) + arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 4.64, size = 28, normalized size = 0.82

$$-\frac{x}{a^2}-\frac{4}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - x/a^2 - 4/(a^2*d*(tan(c/2 + (d*x)/2) + 1))
```

sympy [A] time = 6.93, size = 95, normalized size = 2.79

$$\begin{cases} -\frac{dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{dx}{a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{4}{a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)/(a**2*d*tan(c/2 + d*x/2) + a**2*d) - d*x/(
a**2*d*tan(c/2 + d*x/2) + a**2*d) - 4/(a**2*d*tan(c/2 + d*x/2) + a**2*d), N
e(d, 0)), (x*cos(c)**2/(a*sin(c) + a)**2, True))
```

3.69 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] -1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 32}

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(1/(d*(a^2 + a^2*Sin[c + d*x])))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= -\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 31, normalized size = 1.48

$$-\frac{1}{a^2 d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(1/(a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))
```

fricas [A] time = 0.62, size = 21, normalized size = 1.00

$$-\frac{1}{a^2 d\sin\left(dx+c\right)+a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] -1/(a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.42, size = 20, normalized size = 0.95

$$-\frac{1}{(a\sin(dx+c)+a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] -1/((a*sin(d*x + c) + a)*a*d)
```

maple [A] time = 0.07, size = 21, normalized size = 1.00

$$-\frac{1}{d\left(a+a\sin(dx+c)\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^2,x)
[Out] -1/d/(a+a*sin(d*x+c))/a
```

maxima [A] time = 0.30, size = 20, normalized size = 0.95

$$-\frac{1}{(a\sin(dx+c)+a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] -1/((a*sin(d*x + c) + a)*a*d)
```

mupad [B] time = 0.05, size = 18, normalized size = 0.86

$$-\frac{1}{a^2\,d\,\left(\sin(c+d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^2,x)
[Out] -1/(a^2*d*(sin(c + d*x) + 1))
```

sympy [A] time = 1.12, size = 32, normalized size = 1.52

$$\begin{cases} -\frac{1}{a^2 d\sin(c+dx)+a^2 d} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**2,x)
[Out] Piecewise((-1/(a**2*d*sin(c + d*x) + a**2*d), Ne(d, 0)), (x*cos(c)/(a*sin(c
) + a)**2, True))
```

3.70 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{1}{4d(a\sin(c+dx)+a)^2}$$

```
[Out] 1/4*arctanh(sin(d*x+c))/a^2/d-1/4/d/(a+a*sin(d*x+c))^2-1/4/d/(a^2+a^2*sin(d
*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$-\frac{1}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{1}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) - 1/(4*d*(a + a*Sin[c + d*x])^2) - 1/(4*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{a\,\mathrm{Subst}\left(\int \frac{1}{(a-x)(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a\,\mathrm{Subst}\left(\int \left(\frac{1}{2a(a+x)^3}+\frac{1}{4a^2(a+x)^2}+\frac{1}{4a^2(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\sin(c+dx)\right)}+\frac{\mathrm{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c\right.}{4ad}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.10, size = 38, normalized size = 0.63

$$\frac{\tanh^{-1}(\sin(c+dx))-\frac{\sin(c+dx)+2}{(\sin(c+dx)+1)^2}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (ArcTanh[Sin[c + d*x]] - (2 + Sin[c + d*x])/(1 + Sin[c + d*x])^2)/(4*a^2*d)
```

fricas [A] time = 0.74, size = 105, normalized size = 1.75

$$\frac{\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(\sin(dx+c)+1)-\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(-\sin(dx+}{8\left(a^2d\cos(dx+c)^2-2a^2d\sin(dx+c)-2a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - (cos(d*x
 + c)^2 - 2*sin(d*x + c) - 2)*log(-sin(d*x + c) + 1) + 2*sin(d*x + c) + 4)/
(a^2*d*cos(d*x + c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

giac [A] time = 0.75, size = 71, normalized size = 1.18

$$\frac{\frac{2\log(|\sin(dx+c)+1|)}{a^2}-\frac{2\log(|\sin(dx+c)-1|)}{a^2}-\frac{3\sin(dx+c)^2+10\sin(dx+c)+11}{a^2(\sin(dx+c)+1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(sin(d*x + c) + 1))/a^2 - 2*log(abs(sin(d*x + c) - 1))/a^2 -
 (3*sin(d*x + c)^2 + 10*sin(d*x + c) + 11)/(a^2*(sin(d*x + c) + 1)^2))/d
```

maple [A] time = 0.18, size = 72, normalized size = 1.20

$$-\frac{\ln(\sin(dx+c)-1)}{8a^2d}-\frac{1}{4a^2d(1+\sin(dx+c))^2}-\frac{1}{4a^2d(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/8/a^2/d*ln(sin(d*x+c)-1)-1/4/a^2/d/(1+sin(d*x+c))^2-1/4/a^2/d/(1+sin(d*x
+c))+1/8*ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.81, size = 72, normalized size = 1.20

$$-\frac{\frac{2\,(\sin(dx+c)+2)}{a^2\sin(dx+c)^2+2\,a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c)-1)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(sin(d*x + c) + 2)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c) + a^2)
- log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c) - 1)/a^2)/d
```

mupad [B] time = 4.52, size = 60, normalized size = 1.00

$$\frac{\operatorname{atanh}(\sin(c+d\,x))}{4\,a^2\,d}-\frac{\frac{\sin(c+d\,x)}{4}+\frac{1}{2}}{d\,\left(a^2\sin(c+d\,x)^2+2\,a^2\,\sin(c+d\,x)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] atanh(sin(c + d*x))/(4*a^2*d) - (sin(c + d*x)/4 + 1/2)/(d*(2*a^2*sin(c + d*
x) + a^2 + a^2*sin(c + d*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.71 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan(c+dx)}{5a^2d}-\frac{\sec(c+dx)}{5d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\sec(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

```
[Out] -1/5*sec(d*x+c)/d/(a+a*sin(d*x+c))^2-1/5*sec(d*x+c)/d/(a^2+a^2*sin(d*x+c))+
2/5*tan(d*x+c)/a^2/d
```

Rubi [A] time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2672, 3767, 8}

$$\frac{2\tan(c+dx)}{5a^2d}-\frac{\sec(c+dx)}{5d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\sec(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Sec[c + d*x]/(5*d*(a + a*Sin[c + d*x])^2) - Sec[c + d*x]/(5*d*(a^2 + a^2*S
in[c + d*x])) + (2*Tan[c + d*x])/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}+\frac{3\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{5a}\\
&= -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{\sec(c+dx)}{5d\left(a^2+a^2\sin(c+dx)\right)}+\frac{2\int \sec^2(c+dx)\,dx}{5a^2}\\
&= -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{\sec(c+dx)}{5d\left(a^2+a^2\sin(c+dx)\right)}-\frac{2\,\text{Subst}(\int 1\,dx,x,-\tan(c+
}{5a^2d}\\
&= -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{\sec(c+dx)}{5d\left(a^2+a^2\sin(c+dx)\right)}+\frac{2\tan(c+dx)}{5a^2d}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 53, normalized size = 0.75

$$-\frac{\sec(c+dx)(-5\sin(c+dx)+\sin(3(c+dx))+4\cos(2(c+dx)))}{10a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/10*(Sec[c + d*x]*(4*Cos[2*(c + d*x)] - 5*Sin[c + d*x] + Sin[3*(c + d*x)]
))/(a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.64, size = 79, normalized size = 1.11

$$\frac{4\cos(dx+c)^2+\left(2\cos(dx+c)^2-3\right)\sin(dx+c)-2}{5\left(a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)\sin(dx+c)-2a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5*(4*cos(d*x + c)^2 + (2*cos(d*x + c)^2 - 3)*sin(d*x + c) - 2)/(a^2*d*cos
(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

giac [A] time = 0.38, size = 93, normalized size = 1.31

$$-\frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}+\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+90\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+70\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+21}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) + (35*tan(1/2*d*x + 1/2*c)^4 + 90
*tan(1/2*d*x + 1/2*c)^3 + 120*tan(1/2*d*x + 1/2*c)^2 + 70*tan(1/2*d*x + 1/2
*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.19, size = 98, normalized size = 1.38

$$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{7}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(-1/8/(tan(1/2*d*x+1/2*c)-1)-2/5/(tan(1/2*d*x+1/2*c)+1)^5+1/(tan(1/
2*d*x+1/2*c)+1)^4-3/2/(tan(1/2*d*x+1/2*c)+1)^3+5/4/(tan(1/2*d*x+1/2*c)+1)^2
-7/8/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.42, size = 204, normalized size = 2.87

$$-\frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{10\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{5\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2/5*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 - 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 5*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 2)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - 4*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x
+ c) + 1)^6)*d)
```

mupad [B] time = 4.77, size = 156, normalized size = 2.20

$$\frac{2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(-2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+10\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+10\cos\left(\right.\right.}{5\,a^2\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)*(5*sin(c/2 + (d*x)/2)^5 - 2*cos(c/2 + (d*x)/2)^5 + 10
*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^4 - 3*cos(c/2 + (d*x)/2)^4*sin(c/2 +
 (d*x)/2) + 10*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^3))/(5*a^2*d*(cos(c/
2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)
)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.72 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{3}{16d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{a}{12d(a\sin(c+dx)+a)^3}-\frac{1}{8d(a\sin(c+$$

```
[Out] 1/4*arctanh(sin(d*x+c))/a^2/d-1/12*a/d/(a+a*sin(d*x+c))^3-1/8/d/(a+a*sin(d*
x+c))^2+1/16/d/(a^2-a^2*sin(d*x+c))-3/16/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{3}{16d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{a}{12d(a\sin(c+dx)+a)^3}-\frac{1}{8d(a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) - a/(12*d*(a + a*Sin[c + d*x])^3) - 1/(8*d*
(a + a*Sin[c + d*x])^2) + 1/(16*d*(a^2 - a^2*Sin[c + d*x])) - 3/(16*d*(a^2
+ a^2*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{a^3\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^4}\,dx,x,a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^2}+\frac{1}{4a^2(a+x)^4}+\frac{1}{4a^3(a+x)^3}+\frac{3}{16a^4(a+x)^2}+\frac{1}{4a^4(a^2-x^2)}\right)dx,x,a\sin(\right.}{d}$$

$$= -\frac{a}{12d(a+a\sin(c+dx))^3}-\frac{1}{8d(a+a\sin(c+dx))^2}+\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}-$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{a}{12d(a+a\sin(c+dx))^3}-\frac{1}{8d(a+a\sin(c+dx))^2}+\frac{}{16d\left(a\right.}$$

Mathematica [A] time = 0.12, size = 85, normalized size = 0.82

$$-\frac{\sec^2(c+dx)\left(-3\sin^3(c+dx)-6\sin^2(c+dx)-\sin(c+dx)+3(\sin(c+dx)-1)(\sin(c+dx)+1)^3\tanh^{-1}(\sin\right.}{12a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/12*(Sec[c + d*x]^2*(4 - Sin[c + d*x] - 6*Sin[c + d*x]^2 - 3*Sin[c + d*x]
^3 + 3*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^3))/(a^
2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.73, size = 178, normalized size = 1.71

$$\frac{12\cos(dx+c)^2+3\left(\cos(dx+c)^4-2\cos(dx+c)^2\sin(dx+c)-2\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left(\right.}{24\left(a^2d\cos(dx+c)^4-2a^2d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*cos(d*x + c)^2 + 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c)
 - 2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^4 - 2*cos(d*x
+ c)^2*sin(d*x + c) - 2*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(3*cos(d
*x + c)^2 - 4)*sin(d*x + c) - 4)/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x +
c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

giac [A] time = 0.81, size = 106, normalized size = 1.02

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^2}-\frac{6\log(|\sin(dx+c)-1|)}{a^2}+\frac{3(2\sin(dx+c)-3)}{a^2(\sin(dx+c)-1)}-\frac{11\sin(dx+c)^3+42\sin(dx+c)^2+57\sin(dx+c)+30}{a^2(\sin(dx+c)+1)^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(6*log(abs(sin(d*x + c) + 1))/a^2 - 6*log(abs(sin(d*x + c) - 1))/a^2 +
 3*(2*sin(d*x + c) - 3)/(a^2*(sin(d*x + c) - 1)) - (11*sin(d*x + c)^3 + 42*
sin(d*x + c)^2 + 57*sin(d*x + c) + 30)/(a^2*(sin(d*x + c) + 1)^3))/d
```

maple [A] time = 0.23, size = 108, normalized size = 1.04

$$-\frac{1}{16a^2d(\sin(dx+c)-1)}-\frac{\ln(\sin(dx+c)-1)}{8a^2d}-\frac{1}{12a^2d(1+\sin(dx+c))^3}-\frac{1}{8a^2d(1+\sin(dx+c))^2}-\frac{}{16a^2d(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/16/a^2/d/(sin(d*x+c)-1)-1/8/a^2/d*ln(sin(d*x+c)-1)-1/12/a^2/d/(1+sin(d*x
+c))^3-1/8/a^2/d/(1+sin(d*x+c))^2-3/16/a^2/d/(1+sin(d*x+c))+1/8*ln(1+sin(d*
x+c))/a^2/d
```

maxima [A] time = 0.37, size = 108, normalized size = 1.04

$$-\frac{\frac{2\left(3\sin(dx+c)^3+6\sin(dx+c)^2+\sin(dx+c)-4\right)}{a^2\sin(dx+c)^4+2a^2\sin(dx+c)^3-2a^2\sin(dx+c)-a^2}-\frac{3\log(\sin(dx+c)+1)}{a^2}+\frac{3\log(\sin(dx+c)-1)}{a^2}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/24*(2*(3*sin(d*x + c)^3 + 6*sin(d*x + c)^2 + sin(d*x + c) - 4)/(a^2*sin(
d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2*a^2*sin(d*x + c) - a^2) - 3*log(sin(d
*x + c) + 1)/a^2 + 3*log(sin(d*x + c) - 1)/a^2)/d
```

mupad [B] time = 0.10, size = 93, normalized size = 0.89

$$\frac{\frac{\sin(c+dx)^3}{4}+\frac{\sin(c+dx)^2}{2}+\frac{\sin(c+dx)}{12}-\frac{1}{3}}{d\left(-a^2\sin(c+dx)^4-2a^2\sin(c+dx)^3+2a^2\sin(c+dx)+a^2\right)}+\frac{\operatorname{atanh}(\sin(c+dx))}{4a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (sin(c + d*x)/12 + sin(c + d*x)^2/2 + sin(c + d*x)^3/4 - 1/3)/(d*(2*a^2*sin
(c + d*x) + a^2 - 2*a^2*sin(c + d*x)^3 - a^2*sin(c + d*x)^4)) + atanh(sin(c
 + d*x))/(4*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.73 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=93

$$\frac{4\tan^3(c+dx)}{21a^2d}+\frac{4\tan(c+dx)}{7a^2d}-\frac{\sec^3(c+dx)}{7d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\sec^3(c+dx)}{7d(a\sin(c+dx)+a)^2}$$

```
[Out] -1/7*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^2-1/7*sec(d*x+c)^3/d/(a^2+a^2*sin(d*x+
c))+4/7*tan(d*x+c)/a^2/d+4/21*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 3767}

$$\frac{4\tan^3(c+dx)}{21a^2d}+\frac{4\tan(c+dx)}{7a^2d}-\frac{\sec^3(c+dx)}{7d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\sec^3(c+dx)}{7d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Sec[c + d*x]^3/(7*d*(a + a*Sin[c + d*x])^2) - Sec[c + d*x]^3/(7*d*(a^2 + a
^2*Sin[c + d*x])) + (4*Tan[c + d*x])/(7*a^2*d) + (4*Tan[c + d*x]^3)/(21*a^2
*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}+\frac{5\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{7a}\\ &= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{7d\left(a^2+a^2\sin(c+dx)\right)}+\frac{4\int \sec^4(c+dx)\,dx}{7a^2}\\ &= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{7d\left(a^2+a^2\sin(c+dx)\right)}-\frac{4\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\right.}{7a^2d}\\ &= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{7d\left(a^2+a^2\sin(c+dx)\right)}+\frac{4\tan(c+dx)}{7a^2d}+\frac{4\tan^3(c+}{21a^2d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.84

$$-\frac{\left(8\sin^5(c+dx)+16\sin^4(c+dx)-4\sin^3(c+dx)-24\sin^2(c+dx)-9\sin(c+dx)+6\right)\sec^3(c+dx)}{21a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/21*(Sec[c + d*x]^3*(6 - 9*Sin[c + d*x] - 24*Sin[c + d*x]^2 - 4*Sin[c + d
*x]^3 + 16*Sin[c + d*x]^4 + 8*Sin[c + d*x]^5))/(a^2*d*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.69, size = 103, normalized size = 1.11

$$\frac{16\cos(dx+c)^4 - 8\cos(dx+c)^2 + \left(8\cos(dx+c)^4 - 12\cos(dx+c)^2 - 5\right)\sin(dx+c) - 2}{21\left(a^2d\cos(dx+c)^5 - 2a^2d\cos(dx+c)^3\sin(dx+c) - 2a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/21*(16*cos(d*x + c)^4 - 8*cos(d*x + c)^2 + (8*cos(d*x + c)^4 - 12*cos(d*x
 + c)^2 - 5)*sin(d*x + c) - 2)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)
^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.78, size = 145, normalized size = 1.56

$$-\frac{\frac{7\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3} + \frac{273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 + 1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 2870\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 2037\tan}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{168d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 8)/(a^2*(ta
n(1/2*d*x + 1/2*c) - 1)^3) + (273*tan(1/2*d*x + 1/2*c)^6 + 1155*tan(1/2*d*x
 + 1/2*c)^5 + 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*tan(1/2*d*x + 1/2*c)^3 + 2
037*tan(1/2*d*x + 1/2*c)^2 + 791*tan(1/2*d*x + 1/2*c) + 152)/(a^2*(tan(1/2*
d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.23, size = 158, normalized size = 1.70

$$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3} - \frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} - \frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7} + \frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6} - \frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5} + \frac{5}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(-1/24/(tan(1/2*d*x+1/2*c)-1)^3-1/16/(tan(1/2*d*x+1/2*c)-1)^2-3/16/
(tan(1/2*d*x+1/2*c)-1)-2/7/(tan(1/2*d*x+1/2*c)+1)^7+1/(tan(1/2*d*x+1/2*c)+1
)^6-2/(tan(1/2*d*x+1/2*c)+1)^5+5/2/(tan(1/2*d*x+1/2*c)+1)^4-55/24/(tan(1/2*
d*x+1/2*c)+1)^3+23/16/(tan(1/2*d*x+1/2*c)+1)^2-13/16/(tan(1/2*d*x+1/2*c)+1)
)
```

maxima [B] time = 0.78, size = 396, normalized size = 4.26

$$-\frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1} - \frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{76\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{28\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{42\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{56\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{28\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{42}{(\cos}\right.}{21\left(a^2 + \frac{4a^2\sin(dx+c)}{\cos(dx+c)+1} + \frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8} -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2/21*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 76*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 28*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 42*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 56*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 28*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 42*sin(d*x
+ c)^8/(cos(d*x + c) + 1)^8 - 21*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 6)/(
(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x +
 c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x + c)^1
0/(cos(d*x + c) + 1)^10)*d)
```

mupad [B] time = 5.18, size = 276, normalized size = 2.97

$$2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(-6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+76\cos\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)*(21*sin(c/2 + (d*x)/2)^9 - 6*cos(c/2 + (d*x)/2)^9 + 4
2*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^8 - 3*cos(c/2 + (d*x)/2)^8*sin(c/2
+ (d*x)/2) + 28*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^7 - 56*cos(c/2 + (d
*x)/2)^3*sin(c/2 + (d*x)/2)^6 - 42*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^
5 + 28*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^4 + 76*cos(c/2 + (d*x)/2)^6*
sin(c/2 + (d*x)/2)^3 + 24*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^2))/(21*a
^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(
c/2 + (d*x)/2))^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{4}{\left(c+dx \right)}}{\sin^{2}{\left(c+dx \right)}+2\sin{\left(c+dx \right)}+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.74 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2}{32d(a\sin(c+dx)+a)^4}+\frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{15\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{}{16d(a\,\mathrm{s}}$$

```
[Out] 15/64*arctanh(sin(d*x+c))/a^2/d+1/64/d/(a-a*sin(d*x+c))^2-1/32*a^2/d/(a+a*s
in(d*x+c))^4-1/16*a/d/(a+a*sin(d*x+c))^3-3/32/d/(a+a*sin(d*x+c))^2+5/64/d/(
a^2-a^2*sin(d*x+c))-5/32/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^2}{32d(a\sin(c+dx)+a)^4}+\frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{15\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{}{16d(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + 1/(64*d*(a - a*Sin[c + d*x])^2) - a
^2/(32*d*(a + a*Sin[c + d*x])^4) - a/(16*d*(a + a*Sin[c + d*x])^3) - 3/(32*
d*(a + a*Sin[c + d*x])^2) + 5/(64*d*(a^2 - a^2*Sin[c + d*x])) - 5/(32*d*(a^
2 + a^2*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{a^5\operatorname{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\operatorname{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^3}+\frac{5}{64a^6(a-x)^2}+\frac{1}{8a^3(a+x)^5}+\frac{3}{16a^4(a+x)^4}+\frac{3}{16a^5(a+x)^3}+\frac{5}{32a^6(a+x)^2}\right.\right.}{d}$$

$$= \frac{1}{64d(a-a\sin(c+dx))^2}-\frac{a^2}{32d(a+a\sin(c+dx))^4}-\frac{a}{16d(a+a\sin(c+dx))^3}-32$$

$$= \frac{15\tanh^{-1}(\sin(c+dx))}{64a^2d}+\frac{1}{64d(a-a\sin(c+dx))^2}-\frac{a^2}{32d(a+a\sin(c+dx))^4}-16$$

Mathematica [A] time = 0.33, size = 137, normalized size = 0.94

$$\frac{(1-\sin(c+dx))^2(\sin(c+dx)+1)^2\sec^4(c+dx)\left(\frac{5}{64(1-\sin(c+dx))}-\frac{5}{32(\sin(c+dx)+1)}+\frac{1}{64(1-\sin(c+dx))^2}-\frac{3}{32(\sin(c+dx)+1)}\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^4*(1 - Sin[c + d*x])^2*(1 + Sin[c + d*x])^2*((15*ArcTanh[Sin[
c + d*x]])/64 + 1/(64*(1 - Sin[c + d*x])^2) + 5/(64*(1 - Sin[c + d*x])) - 1
/(32*(1 + Sin[c + d*x])^4) - 1/(16*(1 + Sin[c + d*x])^3) - 3/(32*(1 + Sin[c
 + d*x])^2) - 5/(32*(1 + Sin[c + d*x]))))/(a^2*d)
```

fricas [A] time = 0.85, size = 198, normalized size = 1.36

$$\frac{60\cos(dx+c)^4-20\cos(dx+c)^2+15\left(\cos(dx+c)^6-2\cos(dx+c)^4\sin(dx+c)-2\cos(dx+c)^4\right)\log(\mathrm{si}}{128\left(a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/128*(60*cos(d*x + c)^4 - 20*cos(d*x + c)^2 + 15*(cos(d*x + c)^6 - 2*cos(d
*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 15*(cos(
d*x + c)^6 - 2*cos(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(-sin(d*x
 + c) + 1) + 2*(15*cos(d*x + c)^4 - 20*cos(d*x + c)^2 - 12)*sin(d*x + c) -
8)/(a^2*d*cos(d*x + c)^6 - 2*a^2*d*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*d*co
s(d*x + c)^4)
```

giac [A] time = 0.54, size = 126, normalized size = 0.86

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^2}-\frac{60\log(|\sin(dx+c)-1|)}{a^2}+\frac{2\left(45\sin(dx+c)^2-110\sin(dx+c)+69\right)}{a^2(\sin(dx+c)-1)^2}-\frac{125\sin(dx+c)^4+580\sin(dx+c)^3+1038\sin(dx+c)^2+}{a^2(\sin(dx+c)+1)^4}}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*(60*log(abs(sin(d*x + c) + 1))/a^2 - 60*log(abs(sin(d*x + c) - 1))/a^
2 + 2*(45*sin(d*x + c)^2 - 110*sin(d*x + c) + 69)/(a^2*(sin(d*x + c) - 1)^2
) - (125*sin(d*x + c)^4 + 580*sin(d*x + c)^3 + 1038*sin(d*x + c)^2 + 868*si
n(d*x + c) + 301)/(a^2*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.23, size = 144, normalized size = 0.99

$$\frac{1}{64a^2d(\sin(dx+c)-1)^2}-\frac{5}{64a^2d(\sin(dx+c)-1)}-\frac{15\ln(\sin(dx+c)-1)}{128a^2d}-\frac{1}{32a^2d(1+\sin(dx+c))^4}-\frac{}{16a^2d(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/64/a^2/d/(sin(d*x+c)-1)^2-5/64/a^2/d/(sin(d*x+c)-1)-15/128/a^2/d*ln(sin(d
*x+c)-1)-1/32/a^2/d/(1+sin(d*x+c))^4-1/16/a^2/d/(1+sin(d*x+c))^3-3/32/a^2/d
/(1+sin(d*x+c))^2-5/32/a^2/d/(1+sin(d*x+c))+15/128*ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.37, size = 167, normalized size = 1.14

$$-\frac{\frac{2\left(15\sin(dx+c)^5+30\sin(dx+c)^4-10\sin(dx+c)^3-50\sin(dx+c)^2-17\sin(dx+c)+16\right)}{a^2\sin(dx+c)^6+2a^2\sin(dx+c)^5-a^2\sin(dx+c)^4-4a^2\sin(dx+c)^3-a^2\sin(dx+c)^2+2a^2\sin(dx+c)+a^2}-\frac{15\log(\sin(dx+c)+1)}{a^2}+\frac{15\log(\sin(dx+c)-1)}{a^2}}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/128*(2*(15*sin(d*x + c)^5 + 30*sin(d*x + c)^4 - 10*sin(d*x + c)^3 - 50*s
in(d*x + c)^2 - 17*sin(d*x + c) + 16)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x +
 c)^5 - a^2*sin(d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 + 2*
a^2*sin(d*x + c) + a^2) - 15*log(sin(d*x + c) + 1)/a^2 + 15*log(sin(d*x + c
) - 1)/a^2)/d
```

mupad [B] time = 0.19, size = 151, normalized size = 1.03

$$\frac{15\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{64\,a^2\,d}+\frac{-\frac{15\sin(c+d\,x)^5}{64}-\frac{15\sin(c+d\,x)^4}{32}+\frac{5\sin(c+d\,x)^3}{32}+\frac{25\sin(c+d\,x)^2}{32}+\frac{17\sin(c}{64}}{d\,\left(a^2\sin\left(c+d\,x\right)^6+2\,a^2\sin\left(c+d\,x\right)^5-a^2\sin\left(c+d\,x\right)^4-4\,a^2\sin\left(c+d\,x\right)^3-a^2\sin(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (15*atanh(sin(c + d*x)))/(64*a^2*d) + ((17*sin(c + d*x))/64 + (25*sin(c + d
*x)^2)/32 + (5*sin(c + d*x)^3)/32 - (15*sin(c + d*x)^4)/32 - (15*sin(c + d*
x)^5)/64 - 1/4)/(d*(2*a^2*sin(c + d*x) + a^2 - a^2*sin(c + d*x)^2 - 4*a^2*s
in(c + d*x)^3 - a^2*sin(c + d*x)^4 + 2*a^2*sin(c + d*x)^5 + a^2*sin(c + d*x
)^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.75 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\sin(c+dx)\cos^3(c+dx)}{12a^3d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{7x}{8a^3}+\frac{2\cos^7(c+dx)}{3ad(a\sin(c+dx)+a)^2}$$

```
[Out] 7/8*x/a^3+7/15*cos(d*x+c)^5/a^3/d+7/8*cos(d*x+c)*sin(d*x+c)/a^3/d+7/12*cos(
d*x+c)^3*sin(d*x+c)/a^3/d+2/3*cos(d*x+c)^7/a/d/(a+a*sin(d*x+c))^2
```

Rubi [A] time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2680, 2682, 2635, 8}

$$\frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\sin(c+dx)\cos^3(c+dx)}{12a^3d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{7x}{8a^3}+\frac{2\cos^7(c+dx)}{3ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*x)/(8*a^3) + (7*Cos[c + d*x]^5)/(15*a^3*d) + (7*Cos[c + d*x]*Sin[c + d*x
])/(8*a^3*d) + (7*Cos[c + d*x]^3*Sin[c + d*x])/(12*a^3*d) + (2*Cos[c + d*x]
^7)/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{2\cos^7(c+dx)}{3ad(a+a\sin(c+dx))^2} + \frac{7\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx}{3a^2}$$
$$= \frac{7\cos^5(c+dx)}{15a^3d} + \frac{2\cos^7(c+dx)}{3ad(a+a\sin(c+dx))^2} + \frac{7\int \cos^4(c+dx)\,dx}{3a^3}$$
$$= \frac{7\cos^5(c+dx)}{15a^3d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{12a^3d} + \frac{2\cos^7(c+dx)}{3ad(a+a\sin(c+dx))^2} + \frac{7\int \cos^2(c}{4a^3}$$
$$= \frac{7\cos^5(c+dx)}{15a^3d} + \frac{7\cos(c+dx)\sin(c+dx)}{8a^3d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{12a^3d} + \frac{2\cos}{3ad(a+a}$$
$$= \frac{7x}{8a^3} + \frac{7\cos^5(c+dx)}{15a^3d} + \frac{7\cos(c+dx)\sin(c+dx)}{8a^3d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{12a^3d} + \frac{}{3a}$$

Mathematica [A] time = 1.08, size = 141, normalized size = 1.37

$$-\frac{\left(\sqrt{\sin(c+dx)+1}\,\left(24\sin^5(c+dx)-114\sin^4(c+dx)+202\sin^3(c+dx)-127\sin^2(c+dx)-121\sin(c+dx)+\right.\right.}{120a^3d(\sin(c+dx)-1)^5(\sin(c+dx)+1)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/120*(Cos[c + d*x]^9*(-210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1
- Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(136 - 121*Sin[c + d*x] - 127*Sin[
c + d*x]^2 + 202*Sin[c + d*x]^3 - 114*Sin[c + d*x]^4 + 24*Sin[c + d*x]^5)))
/(a^3*d*(-1 + Sin[c + d*x])^5*(1 + Sin[c + d*x])^(9/2))
```

fricas [A] time = 0.54, size = 60, normalized size = 0.58

$$-\frac{24\cos(dx+c)^5-160\cos(dx+c)^3-105\,dx+15\left(6\cos(dx+c)^3-7\cos(dx+c)\right)\sin(dx+c)}{120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/120*(24*cos(d*x + c)^5 - 160*cos(d*x + c)^3 - 105*d*x + 15*(6*cos(d*x +
c)^3 - 7*cos(d*x + c))*sin(d*x + c))/(a^3*d)
```

giac [A] time = 0.39, size = 140, normalized size = 1.36

$$\frac{\frac{105\,(dx+c)}{a^3} - \frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-360\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+390\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-960\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-400\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-390\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-320\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^5a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*(d*x + c)/a^3 - 2*(15*tan(1/2*d*x + 1/2*c)^9 - 360*tan(1/2*d*x +
 1/2*c)^8 + 390*tan(1/2*d*x + 1/2*c)^7 - 960*tan(1/2*d*x + 1/2*c)^6 - 400*t
an(1/2*d*x + 1/2*c)^4 - 390*tan(1/2*d*x + 1/2*c)^3 - 320*tan(1/2*d*x + 1/2*
c)^2 - 15*tan(1/2*d*x + 1/2*c) - 136)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^3))
/d
```

maple [B] time = 0.21, size = 313, normalized size = 3.04

$$-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{6\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{13\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{16\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{3}d\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9+6/a^3/d/(1+tan(1
/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8-13/2/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)^7+16/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2
*c)^6+20/3/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4+13/2/a^3/d
/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+16/3/a^3/d/(1+tan(1/2*d*x+
1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2+1/4/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1
/2*d*x+1/2*c)+34/15/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^5+7/4/a^3/d*arctan(tan(1
/2*d*x+1/2*c))
```

maxima [B] time = 0.77, size = 310, normalized size = 3.01

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{390\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{400\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{960\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{390\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{360\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{15\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+136}{a^{3}+\frac{5a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{5a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{105\arctan\left(\frac{\sin(d}{\cos(dx}\right.}{a^{3}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 390*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 400*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 360*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 + 136)/(a^3 + 5*a^3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
+ a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*arctan(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 4.72, size = 81, normalized size = 0.79

$$\frac{7\,x}{8\,a^3}+\frac{4\cos\left(c+d\,x\right)^3}{3\,a^3\,d}-\frac{\cos\left(c+d\,x\right)^5}{5\,a^3\,d}-\frac{3\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,a^3\,d}+\frac{7\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (7*x)/(8*a^3) + (4*cos(c + d*x)^3)/(3*a^3*d) - cos(c + d*x)^5/(5*a^3*d) - (
3*cos(c + d*x)^3*sin(c + d*x))/(4*a^3*d) + (7*cos(c + d*x)*sin(c + d*x))/(8
*a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.76 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=23

$$-\frac{(a-a\sin(c+dx))^4}{4a^7d}$$

```
[Out] -1/4*(a-a*sin(d*x+c))^4/a^7/d
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$-\frac{(a-a\sin(c+dx))^4}{4a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a - a*Sin[c + d*x])^4/(4*a^7*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3\,dx, x, a\sin(c+dx)\right)}{a^7d}\\ &= -\frac{(a-a\sin(c+dx))^4}{4a^7d}\end{aligned}$$

Mathematica [A] time = 0.16, size = 44, normalized size = 1.91

$$-\frac{\sin(c+dx)\left(\sin^3(c+dx)-4\sin^2(c+dx)+6\sin(c+dx)-4\right)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*(Sin[c + d*x]*(-4 + 6*Sin[c + d*x] - 4*Sin[c + d*x]^2 + Sin[c + d*x]^3
))/(a^3*d)
```

fricas [B] time = 0.55, size = 45, normalized size = 1.96

$$-\frac{\cos(dx+c)^4-8\cos(dx+c)^2+4\left(\cos(dx+c)^2-2\right)\sin(dx+c)}{4a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x +
c))/(a^3*d)
```

giac [B] time = 0.74, size = 45, normalized size = 1.96

$$-\frac{\sin\left(dx+c\right)^{4}-4\,\sin\left(dx+c\right)^{3}+6\,\sin\left(dx+c\right)^{2}-4\,\sin\left(dx+c\right)}{4\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 4*sin(d*x + c)
)/(a^3*d)
```

maple [A] time = 0.18, size = 19, normalized size = 0.83

$$-\frac{\left(\sin\left(dx+c\right)-1\right)^{4}}{4d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d/a^3*(sin(d*x+c)-1)^4
```

maxima [B] time = 0.45, size = 45, normalized size = 1.96

$$-\frac{\sin\left(dx+c\right)^{4}-4\,\sin\left(dx+c\right)^{3}+6\,\sin\left(dx+c\right)^{2}-4\,\sin\left(dx+c\right)}{4\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 4*sin(d*x + c)
)/(a^3*d)
```

mupad [B] time = 4.55, size = 53, normalized size = 2.30

$$\frac{\frac{\sin(c+d\,x)}{a^3}-\frac{3\sin(c+d\,x)^2}{2\,a^3}+\frac{\sin(c+d\,x)^3}{a^3}-\frac{\sin(c+d\,x)^4}{4\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (sin(c + d*x)/a^3 - (3*sin(c + d*x)^2)/(2*a^3) + sin(c + d*x)^3/a^3 - sin(c
 + d*x)^4/(4*a^3))/d
```

sympy [A] time = 161.61, size = 654, normalized size = 28.43

$$\begin{cases} \frac{2\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^{3}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}d}-\frac{6\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^{3}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+4 \\ \frac{x\cos^{7}(c)}{(a\sin(c)+a)^{3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*tan(c/2 + d*x/2)**7/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan
(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)
**2 + a**3*d) - 6*tan(c/2 + d*x/2)**6/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*
d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d
*x/2)**2 + a**3*d) + 14*tan(c/2 + d*x/2)**5/(a**3*d*tan(c/2 + d*x/2)**8 + 4
*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c
/2 + d*x/2)**2 + a**3*d) - 16*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)*
*8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d
*tan(c/2 + d*x/2)**2 + a**3*d) + 14*tan(c/2 + d*x/2)**3/(a**3*d*tan(c/2 + d
*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*
a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 6*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/
2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4
 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 2*tan(c/2 + d*x/2)/(a**3*d*tan(
c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)*
*4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d), Ne(d, 0)), (x*cos(c)**7/(a*sin
(c) + a)**3, True))
```

3.77 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=77

$$\frac{5\cos^3(c+dx)}{3a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{5x}{2a^3}+\frac{2\cos^5(c+dx)}{ad(a\sin(c+dx)+a)^2}$$

```
[Out] 5/2*x/a^3+5/3*cos(d*x+c)^3/a^3/d+5/2*cos(d*x+c)*sin(d*x+c)/a^3/d+2*cos(d*x+
c)^5/a/d/(a+a*sin(d*x+c))^2
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2680, 2682, 2635, 8}

$$\frac{5\cos^3(c+dx)}{3a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{5x}{2a^3}+\frac{2\cos^5(c+dx)}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(2*a^3) + (5*Cos[c + d*x]^3)/(3*a^3*d) + (5*Cos[c + d*x]*Sin[c + d*x]
)/(2*a^3*d) + (2*Cos[c + d*x]^5)/(a*d*(a + a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2} + \frac{5\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{5\cos^3(c+dx)}{3a^3d} + \frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2} + \frac{5\int \cos^2(c+dx)\,dx}{a^3}$$
$$= \frac{5\cos^3(c+dx)}{3a^3d} + \frac{5\cos(c+dx)\sin(c+dx)}{2a^3d} + \frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2} + \frac{5\int 1\,dx}{2a^3}$$
$$= \frac{5x}{2a^3} + \frac{5\cos^3(c+dx)}{3a^3d} + \frac{5\cos(c+dx)\sin(c+dx)}{2a^3d} + \frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2}$$

Mathematica [A] time = 0.47, size = 121, normalized size = 1.57

$$-\frac{\left(30\sqrt{1-\sin(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\,\left(2\sin^3(c+dx)-11\sin^2(c+dx)+31\sin(c+dx)\right.\right.}{6a^3d(\sin(c+dx)-1)^4(\sin(c+dx)+1)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*(Cos[c + d*x]^7*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Si
n[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-22 + 31*Sin[c + d*x] - 11*Sin[c + d*
x]^2 + 2*Sin[c + d*x]^3)))/(a^3*d*(-1 + Sin[c + d*x])^4*(1 + Sin[c + d*x])^
(7/2))
```

fricas [A] time = 0.61, size = 45, normalized size = 0.58

$$-\frac{2\cos\left(dx+c\right)^3-15\,dx+9\cos\left(dx+c\right)\sin\left(dx+c\right)-24\cos\left(dx+c\right)}{6\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*cos(d*x + c)^3 - 15*d*x + 9*cos(d*x + c)*sin(d*x + c) - 24*cos(d*x
+ c))/(a^3*d)
```

giac [A] time = 0.42, size = 88, normalized size = 1.14

$$\frac{\frac{15\,(dx+c)}{a^3}+\frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+48\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+22\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)/a^3 + 2*(9*tan(1/2*d*x + 1/2*c)^5 + 18*tan(1/2*d*x + 1/2*
c)^4 + 48*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 22)/((tan(1/2*d
*x + 1/2*c)^2 + 1)^3*a^3))/d
```

maple [B] time = 0.20, size = 177, normalized size = 2.30

$$\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{6\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^3d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{}{3a^3d\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 3/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+6/a^3/d/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4+16/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^3*ta
n(1/2*d*x+1/2*c)^2-3/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)+22
/3/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)^3+5/a^3/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.47, size = 184, normalized size = 2.39

$$-\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{9\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-22}{a^3+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 9*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - 22)/(a^3 + 3*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6) - 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 4.64, size = 57, normalized size = 0.74

$$\frac{5\,x}{2\,a^3}+\frac{4\,\cos\left(c+d\,x\right)}{a^3\,d}-\frac{{\cos\left(c+d\,x\right)}^3}{3\,a^3\,d}-\frac{3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*x)/(2*a^3) + (4*cos(c + d*x))/(a^3*d) - cos(c + d*x)^3/(3*a^3*d) - (3*co
s(c + d*x)*sin(c + d*x))/(2*a^3*d)
```

sympy [A] time = 108.27, size = 690, normalized size = 8.96

$$\begin{cases} \frac{15dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^3d}+\frac{45dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^3d}+\cdots \\ \frac{x\cos^6(c)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**6/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a*
*3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d) + 45*d
*x*tan(c/2 + d*x/2)**4/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 +
d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d) + 45*d*x*tan(c/2 + d*
x/2)**2/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*
a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d) + 15*d*x/(6*a**3*d*tan(c/2 + d*x/2)*
*6 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3
*d) + 18*tan(c/2 + d*x/2)**5/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(
c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d) + 36*tan(c/2 +
d*x/2)**4/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 1
8*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d) + 96*tan(c/2 + d*x/2)**2/(6*a**3*d
```

```
*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 +
d*x/2)**2 + 6*a**3*d) - 18*tan(c/2 + d*x/2)/(6*a**3*d*tan(c/2 + d*x/2)**6 +
 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d)
+ 44/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**
3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d), Ne(d, 0)), (x*cos(c)**6/(a*sin(c) + a)
**3, True))
```

3.78 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=50

$$\frac{\sin^2(c+dx)}{2a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-3*sin(d*x+c)/a^3/d+1/2*sin(d*x+c)^2/a^3/d
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{\sin^2(c+dx)}{2a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Log[1 + Sin[c + d*x]])/(a^3*d) - (3*Sin[c + d*x])/(a^3*d) + Sin[c + d*x]
^2/(2*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{a+x}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(-3a+x+\frac{4a^2}{a+x}\right)dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{4\log(1+\sin(c+dx))}{a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{\sin^2(c+dx)}{2a^3d}\end{aligned}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 0.76

$$\frac{\sin^2(c+dx)-6\sin(c+dx)+8\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (8*Log[1 + Sin[c + d*x]] - 6*Sin[c + d*x] + Sin[c + d*x]^2)/(2*a^3*d)
```

fricas [A] time = 0.73, size = 36, normalized size = 0.72

$$-\frac{\cos\left(dx+c\right)^{2}-8\,\log\left(\sin\left(dx+c\right)+1\right)+6\,\sin\left(dx+c\right)}{2\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(cos(d*x + c)^2 - 8*log(sin(d*x + c) + 1) + 6*sin(d*x + c))/(a^3*d)
```

giac [B] time = 2.67, size = 115, normalized size = 2.30

$$-\frac{2\left(\frac{2\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{a^{3}}-\frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(2*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 4*log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/a^3 - (3*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/
2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 3)/((tan(1/2*d*x + 1/2*c)^2 + 1
)^2*a^3))/d
```

maple [A] time = 0.18, size = 49, normalized size = 0.98

$$\frac{4\ln\left(1+\sin\left(dx+c\right)\right)}{a^{3}d}-\frac{3\sin\left(dx+c\right)}{a^{3}d}+\frac{\sin^{2}\left(dx+c\right)}{2a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-3*sin(d*x+c)/a^3/d+1/2*sin(d*x+c)^2/a^3/d
```

maxima [A] time = 0.31, size = 41, normalized size = 0.82

$$\frac{\frac{\sin\left(dx+c\right)^{2}-6\,\sin\left(dx+c\right)}{a^{3}}+\frac{8\,\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((sin(d*x + c)^2 - 6*sin(d*x + c))/a^3 + 8*log(sin(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 4.56, size = 36, normalized size = 0.72

$$\frac{8\,\ln\left(\sin\left(c+d\,x\right)+1\right)-6\,\sin\left(c+d\,x\right)+{\sin\left(c+d\,x\right)}^2}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (8*log(sin(c + d*x) + 1) - 6*sin(c + d*x) + sin(c + d*x)^2)/(2*a^3*d)
```

sympy [A] time = 64.19, size = 564, normalized size = 11.28

$$\begin{cases} \frac{8\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}+\frac{16\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}+\frac{8\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}-\frac{4\log}{a^3d\tan} \\ \frac{x\cos^5(c)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2
+ d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 16*log(tan(c/2 + d*x
/2) + 1)*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2
 + d*x/2)**2 + a**3*d) + 8*log(tan(c/2 + d*x/2) + 1)/(a**3*d*tan(c/2 + d*x/
2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 4*log(tan(c/2 + d*x/2)**2
+ 1)*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d
*x/2)**2 + a**3*d) - 8*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(a*
*3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 4*log(t
an(c/2 + d*x/2)**2 + 1)/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*
x/2)**2 + a**3*d) - 6*tan(c/2 + d*x/2)**3/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a
**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 2*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2
 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 6*tan(c/2 + d*x/2)/
(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d), Ne(d,
 0)), (x*cos(c)**5/(a*sin(c) + a)**3, True))
```

3.79 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=49

$$-\frac{3\cos(c+dx)}{a^3d}-\frac{3x}{a^3}-\frac{2\cos^3(c+dx)}{ad(a\sin(c+dx)+a)^2}$$

```
[Out] -3*x/a^3-3*cos(d*x+c)/a^3/d-2*cos(d*x+c)^3/a/d/(a+a*sin(d*x+c))^2
```

Rubi [A] time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2680, 2682, 8}

$$-\frac{3\cos(c+dx)}{a^3d}-\frac{3x}{a^3}-\frac{2\cos^3(c+dx)}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 - (3*Cos[c + d*x])/(a^3*d) - (2*Cos[c + d*x]^3)/(a*d*(a + a*Sin[
c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= -\frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}-\frac{3\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{a^2}\\ &= -\frac{3\cos(c+dx)}{a^3d}-\frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}-\frac{3\int 1\,dx}{a^3}\\ &= -\frac{3x}{a^3}-\frac{3\cos(c+dx)}{a^3d}-\frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}\end{aligned}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 1.20

$$-\frac{\cos^5(c+dx)\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\sqrt{2}\,a^3d(\sin(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/5*(Cos[c + d*x]^5*Hypergeometric2F1[3/2, 5/2, 7/2, (1 - Sin[c + d*x])/2]
)/(Sqrt[2]*a^3*d*(1 + Sin[c + d*x])^(5/2))
```

fricas [A] time = 0.54, size = 78, normalized size = 1.59

$$-\frac{3\,dx + (3\,dx + 5)\cos(dx + c) + \cos(dx + c)^2 + (3\,dx + \cos(dx + c) - 4)\sin(dx + c) + 4}{a^3 d\cos(dx + c) + a^3 d\sin(dx + c) + a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*d*x + (3*d*x + 5)*cos(d*x + c) + cos(d*x + c)^2 + (3*d*x + cos(d*x + c)
 - 4)*sin(d*x + c) + 4)/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

giac [A] time = 2.08, size = 80, normalized size = 1.63

$$-\frac{\frac{3\,(dx+c)}{a^3} + \frac{2\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)/a^3 + 2*(4*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 5)
/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) +
 1)*a^3))/d
```

maple [A] time = 0.21, size = 64, normalized size = 1.31

$$-\frac{2}{a^3 d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3 d}-\frac{8}{a^3 d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/a^3/d/(1+tan(1/2*d*x+1/2*c)^2)-6/a^3/d*arctan(tan(1/2*d*x+1/2*c))-8/a^3/
d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.59, size = 139, normalized size = 2.84

$$-\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+5}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{3\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2*((sin(d*x + c)/(cos(d*x + c) + 1) + 4*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 5)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1) + a^3*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 4.85, size = 69, normalized size = 1.41

$$-\frac{3\,x}{a^3}-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+10}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (3*x)/a^3 - (2*tan(c/2 + (d*x)/2) + 8*tan(c/2 + (d*x)/2)^2 + 10)/(a^3*d*(
tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [A] time = 40.79, size = 478, normalized size = 9.76

$$\begin{cases} -\frac{3dx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}-\frac{3dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}-\frac{3\cdots}{a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\cdots} \\ \frac{x\cos^4(c)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**3/(a**3*d*tan(c/2 + d*x/2)**3 + a**3*d*
tan(c/2 + d*x/2)**2 + a**3*d*tan(c/2 + d*x/2) + a**3*d) - 3*d*x*tan(c/2 + d
*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**3 + a**3*d*tan(c/2 + d*x/2)**2 + a**3*d*
tan(c/2 + d*x/2) + a**3*d) - 3*d*x*tan(c/2 + d*x/2)/(a**3*d*tan(c/2 + d*x/2
)**3 + a**3*d*tan(c/2 + d*x/2)**2 + a**3*d*tan(c/2 + d*x/2) + a**3*d) - 3*d
*x/(a**3*d*tan(c/2 + d*x/2)**3 + a**3*d*tan(c/2 + d*x/2)**2 + a**3*d*tan(c/
2 + d*x/2) + a**3*d) - 8*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**3 +
a**3*d*tan(c/2 + d*x/2)**2 + a**3*d*tan(c/2 + d*x/2) + a**3*d) - 2*tan(c/2
+ d*x/2)/(a**3*d*tan(c/2 + d*x/2)**3 + a**3*d*tan(c/2 + d*x/2)**2 + a**3*d*
tan(c/2 + d*x/2) + a**3*d) - 10/(a**3*d*tan(c/2 + d*x/2)**3 + a**3*d*tan(c/
2 + d*x/2)**2 + a**3*d*tan(c/2 + d*x/2) + a**3*d), Ne(d, 0)), (x*cos(c)**4/
(a*sin(c) + a)**3, True))
```

3.80 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=39

$$-\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] -ln(1+sin(d*x+c))/a^3/d-2/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$-\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Log[1 + Sin[c + d*x]]/(a^3*d)) - 2/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{a-x}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{2a}{(a+x)^2}\right)dx, x, a\sin(c+dx)\right)}{a^3d}\\ &= -\frac{\log(1+\sin(c+dx))}{a^3d}-\frac{2}{d\left(a^3+a^3\sin(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 1.49

$$-\frac{\sin(c+dx)\log(\sin(c+dx)+1)+\log(\sin(c+dx)+1)+2}{a^3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2 + Log[1 + Sin[c + d*x]] + Log[1 + Sin[c + d*x]]*Sin[c + d*x])/(a^3*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))
```

fricas [A] time = 0.80, size = 41, normalized size = 1.05

$$-\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)+2}{a^3 d\sin(dx+c)+a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) + 2)/(a^3*d*sin(d*x + c) + a^3*d
)
```

giac [A] time = 1.25, size = 35, normalized size = 0.90

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a^3}+\frac{2}{a^3(\sin(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(log(abs(sin(d*x + c) + 1))/a^3 + 2/(a^3*(sin(d*x + c) + 1)))/d
```

maple [A] time = 0.20, size = 37, normalized size = 0.95

$$-\frac{\ln(1+\sin(dx+c))}{a^3 d}-\frac{2}{a^3 d(1+\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -ln(1+sin(d*x+c))/a^3/d-2/a^3/d/(1+sin(d*x+c))
```

maxima [A] time = 0.37, size = 37, normalized size = 0.95

$$-\frac{\frac{2}{a^3\sin(dx+c)+a^3}+\frac{\log(\sin(dx+c)+1)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(2/(a^3*sin(d*x + c) + a^3) + log(sin(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 4.54, size = 36, normalized size = 0.92

$$-\frac{2}{a^3\,d\,\left(\sin\left(c+d\,x\right)+1\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - 2/(a^3*d*(sin(c + d*x) + 1)) - log(sin(c + d*x) + 1)/(a^3*d)
```

sympy [A] time = 1.95, size = 299, normalized size = 7.67

$$\begin{cases} -\frac{2\log(\sin(c+dx)+1)\sin^2(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{4\log(\sin(c+dx)+1)\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{2\log(\sin(c+dx)+1)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{}{2a^3d\sin^2(} \\ \frac{x\cos^3(c)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-2*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 4*log(sin(c + d*x) + 1)*sin(c + d*
x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 2*log(si
n(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*
d) - 2*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a
**3*d) - cos(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x)
+ 2*a**3*d) - 2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*
d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a)**3, True))
```

3.81 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=27

$$-\frac{\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

```
[Out] -1/3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^3
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2671}

$$-\frac{\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Cos[c + d*x]^3/(3*d*(a + a*Sin[c + d*x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.04

$$-\frac{\cos^3(c+dx)}{3a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/3*Cos[c + d*x]^3/(a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [B] time = 0.47, size = 95, normalized size = 3.52

$$-\frac{\cos(dx+c)^2 + (\cos(dx+c)+2)\sin(dx+c) - \cos(dx+c) - 2}{3\left(a^3d\cos(dx+c)^2 - a^3d\cos(dx+c) - 2\,a^3d - \left(a^3d\cos(dx+c) + 2\,a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^2 + (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)/
(a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c)
+ 2*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.55, size = 36, normalized size = 1.33

$$-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{3\,a^{3}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^3)
```

maple [B] time = 0.22, size = 55, normalized size = 2.04

$$\frac{-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(-4/3/(tan(1/2*d*x+1/2*c)+1)^3+2/(tan(1/2*d*x+1/2*c)+1)^2-1/(tan(1/
2*d*x+1/2*c)+1))
```

maxima [B] time = 0.43, size = 99, normalized size = 3.67

$$-\frac{2\left(\frac{3\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{3\left(a^{3}+\frac{3\,a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/3*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/((a^3 + 3*a^3*sin(d*x + c)
/(cos(d*x + c) + 1) + 3*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3)*d)
```

mupad [B] time = 4.58, size = 53, normalized size = 1.96

$$\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-3\right)}{3\,a^{3}\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)*(2*cos(c/2 + (d*x)/2)^2 - 3))/(3*a^3*d*(cos(c/2 + (d*
x)/2) + sin(c/2 + (d*x)/2))^3)
```

sympy [A] time = 15.15, size = 153, normalized size = 5.67

$$\begin{cases} -\frac{6\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{3}d\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{3}d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{3}d} - \frac{2}{3a^{3}d\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{3}d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{3}d} & \text{for } d \neq \\ \frac{x\cos^{2}(c)}{(a\sin(c)+a)^{3}} & \text{otherv} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-6*tan(c/2 + d*x/2)**2/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*
tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) - 2/(3*a**3*d*t
an(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/
2) + 3*a**3*d), Ne(d, 0)), (x*cos(c)**2/(a*sin(c) + a)**3, True))
```

3.82 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] -1/2/a/d/(a+a*sin(d*x+c))^2
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2667, 32\}$

$$-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/(2*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= -\frac{1}{2ad(a+a\sin(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 33, normalized size = 1.50

$$-\frac{1}{2a^3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*1/(a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [A] time = 0.70, size = 36, normalized size = 1.64

$$\frac{1}{2\left(a^3d\cos(dx+c)^2-2\,a^3d\sin(dx+c)-2\,a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] 1/2/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

giac [A] time = 2.91, size = 20, normalized size = 0.91

$$-\frac{1}{2\left(a\sin\left(dx+c\right)+a\right)^{2}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] -1/2/((a*sin(d*x + c) + a)^2*a*d)
```

maple [A] time = 0.07, size = 21, normalized size = 0.95

$$-\frac{1}{2ad\left(a+a\sin\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^3,x)
[Out] -1/2/a/d/(a+a*sin(d*x+c))^2
```

maxima [A] time = 0.38, size = 20, normalized size = 0.91

$$-\frac{1}{2\left(a\sin\left(dx+c\right)+a\right)^{2}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2/((a*sin(d*x + c) + a)^2*a*d)
```

mupad [B] time = 4.46, size = 18, normalized size = 0.82

$$-\frac{1}{2\,a^3\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^3,x)
[Out] -1/(2*a^3*d*(sin(c + d*x) + 1)^2)
```

sympy [A] time = 1.89, size = 51, normalized size = 2.32

$$\begin{cases} -\frac{1}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d} & \text{for}\ d\neq 0\\ \frac{x\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**3,x)
[Out] Piecewise((-1/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d)
, Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)**3, True))
```

3.83 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{8ad(a\sin(c+dx)+a)^2}-\frac{1}{6d(a\sin(c+dx)+a)^3}$$

```
[Out] 1/8*arctanh(sin(d*x+c))/a^3/d-1/6/d/(a+a*sin(d*x+c))^3-1/8/a/d/(a+a*sin(d*x
+c))^2-1/8/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{8ad(a\sin(c+dx)+a)^2}-\frac{1}{6d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(8*a^3*d) - 1/(6*d*(a + a*Sin[c + d*x])^3) - 1/(8*a*d
*(a + a*Sin[c + d*x])^2) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^4}+\frac{1}{4a^2(a+x)^3}+\frac{1}{8a^3(a+x)^2}+\frac{1}{8a^3(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\sin(c+dx)\right)}+ \text{Sub}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{}{8d\left(a^3+\right.}$$

Mathematica [A] time = 0.09, size = 61, normalized size = 0.74

$$\frac{-\frac{1}{8(\sin(c+dx)+1)}-\frac{1}{8(\sin(c+dx)+1)^2}-\frac{1}{6(\sin(c+dx)+1)^3}+\frac{1}{8}\tanh^{-1}(\sin(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]/8 - 1/(6*(1 + Sin[c + d*x])^3) - 1/(8*(1 + Sin[c + d
*x])^2) - 1/(8*(1 + Sin[c + d*x])))/(a^3*d)
```

fricas [B] time = 0.82, size = 154, normalized size = 1.88

$$-\frac{6\cos(dx+c)^2-3\left(3\cos(dx+c)^2+\left(\cos(dx+c)^2-4\right)\sin(dx+c)-4\right)\log(\sin(dx+c)+1)+3\left(3\cos(dx\right.}{48\left(3a^3d\cos(dx+c)^2-4a^3d+\left(a^3d\cos(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(6*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*
x + c) - 4)*log(sin(d*x + c) + 1) + 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 -
 4)*sin(d*x + c) - 4)*log(-sin(d*x + c) + 1) - 18*sin(d*x + c) - 26)/(3*a^3
*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 4*a^3*d)*sin(d*x + c)
)
```

giac [A] time = 1.44, size = 81, normalized size = 0.99

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^3}-\frac{6\log(|\sin(dx+c)-1|)}{a^3}-\frac{11\sin(dx+c)^3+45\sin(dx+c)^2+69\sin(dx+c)+51}{a^3(\sin(dx+c)+1)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c) - 1))/a^3 -
 (11*sin(d*x + c)^3 + 45*sin(d*x + c)^2 + 69*sin(d*x + c) + 51)/(a^3*(sin(d
*x + c) + 1)^3))/d
```

maple [A] time = 0.21, size = 90, normalized size = 1.10

$$-\frac{\ln(\sin(dx+c)-1)}{16a^3d}-\frac{1}{6a^3d(1+\sin(dx+c))^3}-\frac{1}{8a^3d(1+\sin(dx+c))^2}-\frac{1}{8a^3d(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx}{16a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/16/a^3/d*ln(sin(d*x+c)-1)-1/6/a^3/d/(1+sin(d*x+c))^3-1/8/a^3/d/(1+sin(d*
x+c))^2-1/8/a^3/d/(1+sin(d*x+c))+1/16*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.50, size = 98, normalized size = 1.20

$$-\frac{\frac{2\left(3\sin(dx+c)^2+9\sin(dx+c)+10\right)}{a^3\sin(dx+c)^3+3a^3\sin(dx+c)^2+3a^3\sin(dx+c)+a^3}-\frac{3\log(\sin(dx+c)+1)}{a^3}+\frac{3\log(\sin(dx+c)-1)}{a^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(3*sin(d*x + c)^2 + 9*sin(d*x + c) + 10)/(a^3*sin(d*x + c)^3 + 3*a
^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) - 3*log(sin(d*x + c) + 1)/a^3
 + 3*log(sin(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 4.59, size = 83, normalized size = 1.01

$$\frac{\operatorname{atanh}(\sin(c+dx))}{8a^3d}-\frac{\frac{\sin(c+dx)^2}{8}+\frac{3\sin(c+dx)}{8}+\frac{5}{12}}{d\left(a^3\sin(c+dx)^3+3a^3\sin(c+dx)^2+3a^3\sin(c+dx)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] atanh(sin(c + d*x))/(8*a^3*d) - ((3*sin(c + d*x))/8 + sin(c + d*x)^2/8 + 5/
12)/(d*(3*a^3*sin(c + d*x) + a^3 + 3*a^3*sin(c + d*x)^2 + a^3*sin(c + d*x)^
3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x)
 + 1), x)/a**3
```

3.84 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^3} \, dx$

Optimal. Leaf size=99

$$\frac{8\tan(c+dx)}{35a^3d}-\frac{4\sec(c+dx)}{35d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4\sec(c+dx)}{35ad(a\sin(c+dx)+a)^2}-\frac{\sec(c+dx)}{7d(a\sin(c+dx)+a)^3}$$

```
[Out] -1/7*sec(d*x+c)/d/(a+a*sin(d*x+c))^3-4/35*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^2
-4/35*sec(d*x+c)/d/(a^3+a^3*sin(d*x+c))+8/35*tan(d*x+c)/a^3/d
```

Rubi [A] time = 0.13, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2672, 3767, 8}

$$\frac{8\tan(c+dx)}{35a^3d}-\frac{4\sec(c+dx)}{35d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4\sec(c+dx)}{35ad(a\sin(c+dx)+a)^2}-\frac{\sec(c+dx)}{7d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Sec[c + d*x]/(7*d*(a + a*Sin[c + d*x])^3) - (4*Sec[c + d*x])/(35*a*d*(a +
a*Sin[c + d*x])^2) - (4*Sec[c + d*x])/(35*d*(a^3 + a^3*Sin[c + d*x])) + (8*
Tan[c + d*x])/(35*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} + \frac{4\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{7a}$$

$$= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} + \frac{12\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{35a^2}$$

$$= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{4\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)} +$$

$$= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{4\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)} -$$

$$= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{4\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)} +$$

Mathematica [A] time = 0.11, size = 63, normalized size = 0.64

$$\frac{\sec(c+dx)(14\sin(c+dx)-6\sin(3(c+dx))-14\cos(2(c+dx))+\cos(4(c+dx)))}{35a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(-14*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + 14*Sin[c + d*x] -
6*Sin[3*(c + d*x)]))/(35*a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.82, size = 106, normalized size = 1.07

$$-\frac{8\cos(dx+c)^4-36\cos(dx+c)^2-4\left(6\cos(dx+c)^2-5\right)\sin(dx+c)+15}{35\left(3a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)+\left(a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(8*cos(d*x + c)^4 - 36*cos(d*x + c)^2 - 4*(6*cos(d*x + c)^2 - 5)*sin(
d*x + c) + 15)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(
d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.48, size = 119, normalized size = 1.20

$$-\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)} + \frac{525\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+1960\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+4025\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+4480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3143\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+117}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) + (525*tan(1/2*d*x + 1/2*c)^6 +
 1960*tan(1/2*d*x + 1/2*c)^5 + 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*tan(1/2*d
*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 + 1176*tan(1/2*d*x + 1/2*c) + 2
43)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.22, size = 130, normalized size = 1.31

$$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7} + \frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6} - \frac{38}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5} + \frac{9}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4} - \frac{15}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} + \frac{17}{4\left(\tan\left(\frac{dx}{2}\right.\right.}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(-1/16/(tan(1/2*d*x+1/2*c)-1)-4/7/(tan(1/2*d*x+1/2*c)+1)^7+2/(tan(1
/2*d*x+1/2*c)+1)^6-19/5/(tan(1/2*d*x+1/2*c)+1)^5+9/2/(tan(1/2*d*x+1/2*c)+1)
^4-15/4/(tan(1/2*d*x+1/2*c)+1)^3+17/8/(tan(1/2*d*x+1/2*c)+1)^2-15/16/(tan(1
/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.46, size = 310, normalized size = 3.13

$$-\frac{2\left(\frac{43\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{105\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{175\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{105\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{35\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+13\right)}{35\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/35*(43*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 105*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 175*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 105*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 35*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 13)/((a^
3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6
*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(cos(d*x + c)
 + 1)^8)*d)
```

mupad [B] time = 5.08, size = 228, normalized size = 2.30

$$-\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(13\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7+43\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+77\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+7\cos\left(\frac{c}{2}\right.\right.}{35\,a^3\,d\left(\cos\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(2*cos(c/2 + (d*x)/2)*(13*cos(c/2 + (d*x)/2)^7 - 35*sin(c/2 + (d*x)/2)^7 -
 105*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^6 + 43*cos(c/2 + (d*x)/2)^6*sin(
c/2 + (d*x)/2) - 175*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^5 - 105*cos(c/
2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^4 + 7*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x
)/2)^3 + 77*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^2))/(35*a^3*d*(cos(c/2
+ (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^
7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

3.85 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$\frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)}-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{5\tanh^{-1}(\sin(c+dx))}{32a^3d}-\frac{a}{16d(a\sin(c+dx)+a)^4}-\frac{}{12d(a\sin}$$

```
[Out] 5/32*arctanh(sin(d*x+c))/a^3/d-1/16*a/d/(a+a*sin(d*x+c))^4-1/12/d/(a+a*sin(
d*x+c))^3-3/32/a/d/(a+a*sin(d*x+c))^2+1/32/d/(a^3-a^3*sin(d*x+c))-1/8/d/(a^
3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$\frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)}-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{5\tanh^{-1}(\sin(c+dx))}{32a^3d}-\frac{a}{16d(a\sin(c+dx)+a)^4}-\frac{}{12d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(32*a^3*d) - a/(16*d*(a + a*Sin[c + d*x])^4) - 1/
(12*d*(a + a*Sin[c + d*x])^3) - 3/(32*a*d*(a + a*Sin[c + d*x])^2) + 1/(32*d
*(a^3 - a^3*Sin[c + d*x])) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\,\mathrm{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^2} + \frac{1}{4a^2(a+x)^5} + \frac{1}{4a^3(a+x)^4} + \frac{3}{16a^4(a+x)^3} + \frac{1}{8a^5(a+x)^2} + \frac{5}{32a^5\left(a^2-x^2\right)}\right) dx\right.}{d}$$

$$= -\frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{12d(a+a\sin(c+dx))^3} - \frac{3}{32ad(a+a\sin(c+dx))^2} + \frac{}{32}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{32a^3d} - \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{12d(a+a\sin(c+dx))^3} - \frac{}{32ad(a}$$

Mathematica [A] time = 0.17, size = 95, normalized size = 0.75

$$-\frac{\sec^2(c+dx)\left(-15\sin^4(c+dx) - 45\sin^3(c+dx) - 35\sin^2(c+dx) + 15\sin(c+dx) + 15(\sin(c+dx)-1)(\sin(c\right.}{96a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/96*(Sec[c + d*x]^2*(32 + 15*Sin[c + d*x] - 35*Sin[c + d*x]^2 - 45*Sin[c
+ d*x]^3 - 15*Sin[c + d*x]^4 + 15*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])
*(1 + Sin[c + d*x])^4))/(a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.61, size = 228, normalized size = 1.81

$$-\frac{30\cos(dx+c)^4 - 130\cos(dx+c)^2 - 15\left(3\cos(dx+c)^4 - 4\cos(dx+c)^2 + \left(\cos(dx+c)^4 - 4\cos(dx+c)^2\right)\right.}{192\left(3a^3d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(30*cos(d*x + c)^4 - 130*cos(d*x + c)^2 - 15*(3*cos(d*x + c)^4 - 4*c
os(d*x + c)^2 + (cos(d*x + c)^4 - 4*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d
*x + c) + 1) + 15*(3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4 -
4*cos(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 30*(3*cos(d*x + c)
^2 - 2)*sin(d*x + c) + 36)/(3*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2
 + (a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 0.74, size = 116, normalized size = 0.92

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^3} - \frac{60\log(|\sin(dx+c)-1|)}{a^3} + \frac{12(5\sin(dx+c)-7)}{a^3(\sin(dx+c)-1)} - \frac{125\sin(dx+c)^4+596\sin(dx+c)^3+1110\sin(dx+c)^2+996\sin(dx+c)+405}{a^3(\sin(dx+c)+1)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(60*log(abs(sin(d*x + c) + 1))/a^3 - 60*log(abs(sin(d*x + c) - 1))/a^
3 + 12*(5*sin(d*x + c) - 7)/(a^3*(sin(d*x + c) - 1)) - (125*sin(d*x + c)^4
+ 596*sin(d*x + c)^3 + 1110*sin(d*x + c)^2 + 996*sin(d*x + c) + 405)/(a^3*(
sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.26, size = 126, normalized size = 1.00

$$-\frac{1}{32a^3d(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{64a^3d}-\frac{1}{16a^3d(1+\sin(dx+c))^4}-\frac{1}{12a^3d(1+\sin(dx+c))^3}-\frac{}{32a^3d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/32/a^3/d/(sin(d*x+c)-1)-5/64/a^3/d*ln(sin(d*x+c)-1)-1/16/a^3/d/(1+sin(d*
x+c))^4-1/12/a^3/d/(1+sin(d*x+c))^3-3/32/a^3/d/(1+sin(d*x+c))^2-1/8/a^3/d/(
1+sin(d*x+c))+5/64*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.33, size = 146, normalized size = 1.16

$$-\frac{\frac{2\left(15\sin(dx+c)^4+45\sin(dx+c)^3+35\sin(dx+c)^2-15\sin(dx+c)-32\right)}{a^3\sin(dx+c)^5+3a^3\sin(dx+c)^4+2a^3\sin(dx+c)^3-2a^3\sin(dx+c)^2-3a^3\sin(dx+c)-a^3}-\frac{15\log(\sin(dx+c)+1)}{a^3}+\frac{15\log(\sin(dx+c)-1)}{a^3}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(15*sin(d*x + c)^4 + 45*sin(d*x + c)^3 + 35*sin(d*x + c)^2 - 15*s
in(d*x + c) - 32)/(a^3*sin(d*x + c)^5 + 3*a^3*sin(d*x + c)^4 + 2*a^3*sin(d*
x + c)^3 - 2*a^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - a^3) - 15*log(sin(d*
x + c) + 1)/a^3 + 15*log(sin(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 0.15, size = 129, normalized size = 1.02

$$\frac{5\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{32\,a^3\,d}+\frac{\frac{5\sin(c+d\,x)^4}{32}+\frac{15\sin(c+d\,x)^3}{32}+\frac{35\sin(c+d\,x)^2}{96}-\frac{5\sin(c+d\,x)}{32}-\frac{1}{3}}{d\left(-a^3\sin\left(c+d\,x\right)^5-3\,a^3\sin\left(c+d\,x\right)^4-2\,a^3\sin\left(c+d\,x\right)^3+2\,a^3\sin\left(c+d\,x\right)^2+3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (5*atanh(sin(c + d*x)))/(32*a^3*d) + ((35*sin(c + d*x)^2)/96 - (5*sin(c + d
*x))/32 + (15*sin(c + d*x)^3)/32 + (5*sin(c + d*x)^4)/32 - 1/3)/(d*(3*a^3*s
in(c + d*x) + a^3 + 2*a^3*sin(c + d*x)^2 - 2*a^3*sin(c + d*x)^3 - 3*a^3*sin
(c + d*x)^4 - a^3*sin(c + d*x)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

3.86 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=123

$$\frac{8\tan^3(c+dx)}{63a^3d}+\frac{8\tan(c+dx)}{21a^3d}-\frac{2\sec^3(c+dx)}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{2\sec^3(c+dx)}{21ad(a\sin(c+dx)+a)^2}-\frac{\sec^3(c+dx)}{9d(a\sin(c+dx)+a)^3}$$

```
[Out] -1/9*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^3-2/21*sec(d*x+c)^3/a/d/(a+a*sin(d*x+c
))^2-2/21*sec(d*x+c)^3/d/(a^3+a^3*sin(d*x+c))+8/21*tan(d*x+c)/a^3/d+8/63*ta
n(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 3767}

$$\frac{8\tan^3(c+dx)}{63a^3d}+\frac{8\tan(c+dx)}{21a^3d}-\frac{2\sec^3(c+dx)}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{2\sec^3(c+dx)}{21ad(a\sin(c+dx)+a)^2}-\frac{\sec^3(c+dx)}{9d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Sec[c + d*x]^3/(9*d*(a + a*Sin[c + d*x])^3) - (2*Sec[c + d*x]^3)/(21*a*d*(
a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x]^3)/(21*d*(a^3 + a^3*Sin[c + d*x]))
 + (8*Tan[c + d*x])/(21*a^3*d) + (8*Tan[c + d*x]^3)/(63*a^3*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}+\frac{2\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{3a}\\
&= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}+\frac{10\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{21a^2}\\
&= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)}+\frac{8}{\ldots}\\
&= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)}-\frac{8\ldots}{\ldots}\\
&= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)}+\frac{8\ldots}{\ldots}\end{aligned}$$

Mathematica [A] time = 0.11, size = 85, normalized size = 0.69

$$\frac{\sec^3(c+dx)(36\sin(c+dx)+2\sin(3(c+dx))-6\sin(5(c+dx))-27\cos(2(c+dx))-12\cos(4(c+dx))+\cos}{126a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-27*Cos[2*(c + d*x)] - 12*Cos[4*(c + d*x)] + Cos[6*(c + d*
x)] + 36*Sin[c + d*x] + 2*Sin[3*(c + d*x)] - 6*Sin[5*(c + d*x)]))/(126*a^3*
d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.62, size = 130, normalized size = 1.06

$$-\frac{16\cos(dx+c)^6-72\cos(dx+c)^4+30\cos(dx+c)^2-2\left(24\cos(dx+c)^4-20\cos(dx+c)^2-7\right)\sin(dx+}{63\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/63*(16*cos(d*x + c)^6 - 72*cos(d*x + c)^4 + 30*cos(d*x + c)^2 - 2*(24*co
s(d*x + c)^4 - 20*cos(d*x + c)^2 - 7)*sin(d*x + c) + 7)/(3*a^3*d*cos(d*x +
c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c
)^3)*sin(d*x + c))
```

giac [A] time = 0.75, size = 171, normalized size = 1.39

$$-\frac{\frac{21\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+19\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}+\frac{3591\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+19656\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+56196\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+95760\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{a^3(\mathrm{t}}}{2016d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2016*(21*(21*tan(1/2*d*x + 1/2*c)^2 - 36*tan(1/2*d*x + 1/2*c) + 19)/(a^3
*(tan(1/2*d*x + 1/2*c) - 1)^3) + (3591*tan(1/2*d*x + 1/2*c)^8 + 19656*tan(1
/2*d*x + 1/2*c)^7 + 56196*tan(1/2*d*x + 1/2*c)^6 + 95760*tan(1/2*d*x + 1/2*
c)^5 + 107730*tan(1/2*d*x + 1/2*c)^4 + 79464*tan(1/2*d*x + 1/2*c)^3 + 38484
*tan(1/2*d*x + 1/2*c)^2 + 10944*tan(1/2*d*x + 1/2*c) + 1615)/(a^3*(tan(1/2*
d*x + 1/2*c) + 1)^9))/d
```

maple [A] time = 0.27, size = 190, normalized size = 1.54

$$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{7}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{68}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{}{3(\tan}}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(-1/48/(tan(1/2*d*x+1/2*c)-1)^3-1/32/(tan(1/2*d*x+1/2*c)-1)^2-7/64/
(tan(1/2*d*x+1/2*c)-1)-4/9/(tan(1/2*d*x+1/2*c)+1)^9+2/(tan(1/2*d*x+1/2*c)+1
)^8-34/7/(tan(1/2*d*x+1/2*c)+1)^7+23/3/(tan(1/2*d*x+1/2*c)+1)^6-35/4/(tan(1
/2*d*x+1/2*c)+1)^5+59/8/(tan(1/2*d*x+1/2*c)+1)^4-19/4/(tan(1/2*d*x+1/2*c)+1
)^3+9/4/(tan(1/2*d*x+1/2*c)+1)^2-57/64/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.44, size = 482, normalized size = 3.92

$$-\frac{2\left(\frac{51\sin(dx+c)}{\cos(dx+c)+1}+\frac{39\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{235\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{450\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{306\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{294\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{378\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{63}{(\cos}\right.}{63\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c}{(\cos(dx+c)+1)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/63*(51*sin(d*x + c)/(cos(d*x + c) + 1) + 39*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 235*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 450*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 306*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 294*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 378*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 63*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 - 273*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
 - 189*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 63*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 + 19)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(
cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

mupad [B] time = 5.56, size = 167, normalized size = 1.36

$$\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{63\cos\left(\frac{5c}{2}+\frac{5d\,x}{2}\right)}{8}-\frac{171\cos\left(\frac{3c}{2}+\frac{3d\,x}{2}\right)}{8}-\frac{145\cos\left(\frac{7c}{2}+\frac{7d\,x}{2}\right)}{16}+\frac{49\cos\left(\frac{9c}{2}+\frac{9d\,x}{2}\right)}{16}+\frac{\cos\left(\frac{11c}{2}+\frac{11d\,x}{2}\right)}{2}+\frac{617\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-\right.}{2016\,a^3\,d\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)^9\cos\left(\frac{c}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((63*cos((5*c)/2 + (5*d*x)/2))/8 - (171*cos((3*c)/2 + (
3*d*x)/2))/8 - (145*cos((7*c)/2 + (7*d*x)/2))/16 + (49*cos((9*c)/2 + (9*d*x
)/2))/16 + cos((11*c)/2 + (11*d*x)/2)/2 + (617*sin(c/2 + (d*x)/2))/16 - (32
9*sin((3*c)/2 + (3*d*x)/2))/16 + (145*sin((5*c)/2 + (5*d*x)/2))/32 - (113*s
in((7*c)/2 + (7*d*x)/2))/32 - (115*sin((9*c)/2 + (9*d*x)/2))/32 + (19*sin((
11*c)/2 + (11*d*x)/2))/32))/(2016*a^3*d*cos(c/2 - pi/4 + (d*x)/2)^9*cos(c/2
 + pi/4 + (d*x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

3.87 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=171

$$\frac{3}{64d\left(a^3-a^3\sin(c+dx)\right)}-\frac{15}{128d\left(a^3\sin(c+dx)+a^3\right)}+\frac{21\tanh^{-1}(\sin(c+dx))}{128a^3d}-\frac{a^2}{40d(a\sin(c+dx)+a)^5}-\frac{\cdots}{64d(\cdots}$$

```
[Out] 21/128*arctanh(sin(d*x+c))/a^3/d+1/128/a/d/(a-a*sin(d*x+c))^2-1/40*a^2/d/(a
+a*sin(d*x+c))^5-3/64*a/d/(a+a*sin(d*x+c))^4-1/16/d/(a+a*sin(d*x+c))^3-5/64
/a/d/(a+a*sin(d*x+c))^2+3/64/d/(a^3-a^3*sin(d*x+c))-15/128/d/(a^3+a^3*sin(d
*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^2}{40d(a\sin(c+dx)+a)^5}+\frac{3}{64d\left(a^3-a^3\sin(c+dx)\right)}-\frac{15}{128d\left(a^3\sin(c+dx)+a^3\right)}+\frac{21\tanh^{-1}(\sin(c+dx))}{128a^3d}-\frac{\cdots}{64\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (21*ArcTanh[Sin[c + d*x]])/(128*a^3*d) + 1/(128*a*d*(a - a*Sin[c + d*x])^2)
 - a^2/(40*d*(a + a*Sin[c + d*x])^5) - (3*a)/(64*d*(a + a*Sin[c + d*x])^4)
- 1/(16*d*(a + a*Sin[c + d*x])^3) - 5/(64*a*d*(a + a*Sin[c + d*x])^2) + 3/(
64*d*(a^3 - a^3*Sin[c + d*x])) - 15/(128*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{a^5\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\mathrm{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^3} + \frac{3}{64a^7(a-x)^2} + \frac{1}{8a^3(a+x)^6} + \frac{3}{16a^4(a+x)^5} + \frac{3}{16a^5(a+x)^4} + \frac{5}{32a^6(a+x)^3} + \right.\right.}{d}$$

$$= \frac{1}{128ad(a-a\sin(c+dx))^2} - \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{3a}{64d(a+a\sin(c+dx))^4} - $$

$$= \frac{21\tanh^{-1}(\sin(c+dx))}{128a^3d} + \frac{1}{128ad(a-a\sin(c+dx))^2} - \frac{a^2}{40d(a+a\sin(c+dx))^5} - $$

Mathematica [A] time = 0.52, size = 145, normalized size = 0.85

$$\frac{\sec^4(c+dx)\left(-105\sin^6(c+dx) - 315\sin^5(c+dx) - 140\sin^4(c+dx) + 420\sin^3(c+dx) + 469\sin^2(c+dx) + 7\,\mathrm{s}\right.}{640a^3d(\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^4*(-176 + 105*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^10 + 7*Sin[c + d*x] +
469*Sin[c + d*x]^2 + 420*Sin[c + d*x]^3 - 140*Sin[c + d*x]^4 - 315*Sin[c +
d*x]^5 - 105*Sin[c + d*x]^6))/(640*a^3*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.84, size = 248, normalized size = 1.45

$$-\frac{210\cos(dx+c)^6 - 910\cos(dx+c)^4 + 252\cos(dx+c)^2 - 105\left(3\cos(dx+c)^6 - 4\cos(dx+c)^4 + \left(\cos(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1280*(210*cos(d*x + c)^6 - 910*cos(d*x + c)^4 + 252*cos(d*x + c)^2 - 105
*(3*cos(d*x + c)^6 - 4*cos(d*x + c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)
*sin(d*x + c))*log(sin(d*x + c) + 1) + 105*(3*cos(d*x + c)^6 - 4*cos(d*x +
c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)*sin(d*x + c))*log(-sin(d*x + c)
+ 1) - 14*(45*cos(d*x + c)^4 - 30*cos(d*x + c)^2 - 16)*sin(d*x + c) + 96)/(
3*a^3*d*cos(d*x + c)^6 - 4*a^3*d*cos(d*x + c)^4 + (a^3*d*cos(d*x + c)^6 - 4
*a^3*d*cos(d*x + c)^4)*sin(d*x + c))
```

giac [A] time = 0.71, size = 136, normalized size = 0.80

$$\frac{\frac{420\log(|\sin(dx+c)+1|)}{a^3} - \frac{420\log(|\sin(dx+c)-1|)}{a^3} + \frac{10\left(63\sin(dx+c)^2 - 150\sin(dx+c) + 91\right)}{a^3(\sin(dx+c)-1)^2} - \frac{959\sin(dx+c)^5 + 5395\sin(dx+c)^4 + 12390\sin(dx+c)^3}{a^3(\sin(dx+c)}}{5120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/5120*(420*log(abs(sin(d*x + c) + 1))/a^3 - 420*log(abs(sin(d*x + c) - 1))
/a^3 + 10*(63*sin(d*x + c)^2 - 150*sin(d*x + c) + 91)/(a^3*(sin(d*x + c) -
```

```
1)^2) - (959*sin(d*x + c)^5 + 5395*sin(d*x + c)^4 + 12390*sin(d*x + c)^3 +
14710*sin(d*x + c)^2 + 9275*sin(d*x + c) + 2647)/(a^3*(sin(d*x + c) + 1)^5)
)/d
```

maple [A] time = 0.27, size = 162, normalized size = 0.95

$$\frac{1}{128a^3d(\sin(dx+c)-1)^2}-\frac{3}{64a^3d(\sin(dx+c)-1)}-\frac{21\ln(\sin(dx+c)-1)}{256a^3d}-\frac{1}{40a^3d(1+\sin(dx+c))^5}-\frac{}{64a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/128/a^3/d/(sin(d*x+c)-1)^2-3/64/a^3/d/(sin(d*x+c)-1)-21/256/a^3/d*ln(sin(
d*x+c)-1)-1/40/a^3/d/(1+sin(d*x+c))^5-3/64/a^3/d/(1+sin(d*x+c))^4-1/16/a^3/
d/(1+sin(d*x+c))^3-5/64/a^3/d/(1+sin(d*x+c))^2-15/128/a^3/d/(1+sin(d*x+c))+
21/256*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.60, size = 188, normalized size = 1.10

$$-\frac{\frac{2\left(105\sin(dx+c)^6+315\sin(dx+c)^5+140\sin(dx+c)^4-420\sin(dx+c)^3-469\sin(dx+c)^2-7\sin(dx+c)+176\right)}{a^3\sin(dx+c)^7+3a^3\sin(dx+c)^6+a^3\sin(dx+c)^5-5a^3\sin(dx+c)^4-5a^3\sin(dx+c)^3+a^3\sin(dx+c)^2+3a^3\sin(dx+c)+a^3}-\frac{105\log(\sin(dx+c)+1)}{a^3}+}{1280d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(2*(105*sin(d*x + c)^6 + 315*sin(d*x + c)^5 + 140*sin(d*x + c)^4 -
420*sin(d*x + c)^3 - 469*sin(d*x + c)^2 - 7*sin(d*x + c) + 176)/(a^3*sin(d*
x + c)^7 + 3*a^3*sin(d*x + c)^6 + a^3*sin(d*x + c)^5 - 5*a^3*sin(d*x + c)^4
 - 5*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) -
105*log(sin(d*x + c) + 1)/a^3 + 105*log(sin(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 4.77, size = 173, normalized size = 1.01

$$\frac{21\,\mathrm{atanh}(\sin(c+d\,x))}{128\,a^3\,d}-\frac{\frac{21\sin(c+d\,x)^6}{128}+\frac{63\sin(c+d\,x)^5}{128}+\frac{7\sin(c+d\,x)^4}{32}-\frac{21\sin(c+d\,x)^3}{32}-\frac{46}{}}{d\left(a^3\sin(c+d\,x)^7+3\,a^3\sin(c+d\,x)^6+a^3\sin(c+d\,x)^5-5\,a^3\sin(c+d\,x)^4-5\,a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (21*atanh(sin(c + d*x)))/(128*a^3*d) - ((7*sin(c + d*x)^4)/32 - (469*sin(c
+ d*x)^2)/640 - (21*sin(c + d*x)^3)/32 - (7*sin(c + d*x))/640 + (63*sin(c +
 d*x)^5)/128 + (21*sin(c + d*x)^6)/128 + 11/40)/(d*(3*a^3*sin(c + d*x) + a^
3 + a^3*sin(c + d*x)^2 - 5*a^3*sin(c + d*x)^3 - 5*a^3*sin(c + d*x)^4 + a^3*
sin(c + d*x)^5 + 3*a^3*sin(c + d*x)^6 + a^3*sin(c + d*x)^7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

3.88 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=127

$$\frac{2\cos(c+dx)}{d\left(a^8\sin(c+dx)+a^8\right)}+\frac{x}{a^8}+\frac{2\cos^5(c+dx)}{5a^3d(a\sin(c+dx)+a)^5}-\frac{2\cos^3(c+dx)}{3a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{2\cos^7(c+dx)}{7ad(a\sin(c+dx)+a)^7}$$

```
[Out] x/a^8-2/7*cos(d*x+c)^7/a/d/(a+a*sin(d*x+c))^7+2/5*cos(d*x+c)^5/a^3/d/(a+a*s
in(d*x+c))^5-2/3*cos(d*x+c)^3/a^2/d/(a^2+a^2*sin(d*x+c))^3+2*cos(d*x+c)/d/(
a^8+a^8*sin(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2680, 8}

$$\frac{2\cos^5(c+dx)}{5a^3d(a\sin(c+dx)+a)^5}-\frac{2\cos^3(c+dx)}{3a^2d\left(a^2\sin(c+dx)+a^2\right)^3}+\frac{2\cos(c+dx)}{d\left(a^8\sin(c+dx)+a^8\right)}+\frac{x}{a^8}-\frac{2\cos^7(c+dx)}{7ad(a\sin(c+dx)+a)^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] x/a^8 - (2*Cos[c + d*x]^7)/(7*a*d*(a + a*Sin[c + d*x])^7) + (2*Cos[c + d*x]
^5)/(5*a^3*d*(a + a*Sin[c + d*x])^5) - (2*Cos[c + d*x]^3)/(3*a^2*d*(a^2 + a
^2*Sin[c + d*x])^3) + (2*Cos[c + d*x])/(d*(a^8 + a^8*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^8}\,dx &= -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7}-\frac{\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{a^2}\\
&= -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7}+\frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5}+\frac{\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx}{a^4}\\
&= -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7}+\frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5}-\frac{2\cos^3(c+dx)}{3a^5d(a+a\sin(c+dx))^3}-\int\\
&= -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7}+\frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5}-\frac{2\cos^3(c+dx)}{3a^5d(a+a\sin(c+dx))^3}+\frac{}{d}\\
&= \frac{x}{a^8}-\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7}+\frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5}-\frac{2\cos^3(c+dx)}{3a^5d(a+a\sin(c+dx))^3}
\end{aligned}$$

Mathematica [B] time = 6.08, size = 275, normalized size = 2.17

$$-\frac{2\sqrt{2}\left(\frac{1}{2}(1-\sin(c+dx))-1\right)^4\left(\frac{\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}}{\sqrt{2}\sqrt{\frac{1}{2}(\sin(c+dx)-1)+1}}+\frac{(1-\sin(c+dx))^4}{112\left(\frac{1}{2}(1-\sin(c+dx))-1\right)^4}+\frac{(1-\sin(c+dx))^3}{40\left(\frac{1}{2}(1-\sin(c+dx))-1\right)^3}+\frac{}{12(}\right.}{a^8d\left(\frac{1}{2}(\sin(c+dx)-1)+1\right)^{7/2}(1-\sin(c+dx))^5(\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-2*Sqrt[2]*Cos[c + d*x]^9*(-1 + (1 - Sin[c + d*x])/2)^4*((ArcSin[Sqrt[1 -
Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[2]*Sqrt[1 + (-1 + Sin[
c + d*x])/2]) + (1 - Sin[c + d*x])/(2*(-1 + (1 - Sin[c + d*x])/2)) + (1 - S
in[c + d*x])^2/(12*(-1 + (1 - Sin[c + d*x])/2)^2) + (1 - Sin[c + d*x])^3/(4
0*(-1 + (1 - Sin[c + d*x])/2)^3) + (1 - Sin[c + d*x])^4/(112*(-1 + (1 - Sin
[c + d*x])/2)^4)))/(a^8*d*(1 + (-1 + Sin[c + d*x])/2)^(7/2)*(1 - Sin[c + d*
x])^5*(1 + Sin[c + d*x])^(9/2))
```

fricas [B] time = 0.76, size = 244, normalized size = 1.92

$$\frac{(105\,dx-352)\cos(dx+c)^4-(315\,dx+568)\cos(dx+c)^3-24\,(35\,dx-31)\cos(dx+c)^2+840\,dx+60\,(7\,dx}{105\left(a^8d\cos(dx+c)^4-3\,a^8d\cos(dx+c)^3-8\,a^8d\cos(dx+c)^2+4\,a^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*((105*d*x - 352)*cos(d*x + c)^4 - (315*d*x + 568)*cos(d*x + c)^3 - 24
*(35*d*x - 31)*cos(d*x + c)^2 + 840*d*x + 60*(7*d*x + 12)*cos(d*x + c) - ((
105*d*x + 352)*cos(d*x + c)^3 + 12*(35*d*x - 18)*cos(d*x + c)^2 - 840*d*x -
 60*(7*d*x + 16)*cos(d*x + c) - 240)*sin(d*x + c) - 240)/(a^8*d*cos(d*x + c
)^4 - 3*a^8*d*cos(d*x + c)^3 - 8*a^8*d*cos(d*x + c)^2 + 4*a^8*d*cos(d*x + c
) + 8*a^8*d - (a^8*d*cos(d*x + c)^3 + 4*a^8*d*cos(d*x + c)^2 - 4*a^8*d*cos(
d*x + c) - 8*a^8*d)*sin(d*x + c))
```

giac [A] time = 1.18, size = 99, normalized size = 0.78

$$\frac{\frac{105\,(dx+c)}{a^8}+\frac{16\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+175\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+490\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+294\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+133\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+19\right)}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(105*(d*x + c)/a^8 + 16*(105*tan(1/2*d*x + 1/2*c)^5 + 175*tan(1/2*d*x
 + 1/2*c)^4 + 490*tan(1/2*d*x + 1/2*c)^3 + 294*tan(1/2*d*x + 1/2*c)^2 + 133
*tan(1/2*d*x + 1/2*c) + 19)/(a^8*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

maple [A] time = 0.28, size = 146, normalized size = 1.15

$$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^8d}-\frac{256}{7a^8d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{128}{a^8d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{896}{5a^8d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{}{a^8d\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/a^8/d*arctan(tan(1/2*d*x+1/2*c))-256/7/a^8/d/(tan(1/2*d*x+1/2*c)+1)^7+128
/a^8/d/(tan(1/2*d*x+1/2*c)+1)^6-896/5/a^8/d/(tan(1/2*d*x+1/2*c)+1)^5+128/a^
8/d/(tan(1/2*d*x+1/2*c)+1)^4-160/3/a^8/d/(tan(1/2*d*x+1/2*c)+1)^3+16/a^8/d/
(tan(1/2*d*x+1/2*c)+1)^2
```

maxima [B] time = 0.64, size = 295, normalized size = 2.32

$$\frac{2\left(\frac{8\left(\frac{133\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{294\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{490\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{175\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{105\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+19\right)}{a^8+\frac{7\,a^8\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\,a^8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{35\,a^8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{35\,a^8\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21\,a^8\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{7\,a^8\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^8\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{105\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^8}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 2/105*(8*(133*sin(d*x + c)/(cos(d*x + c) + 1) + 294*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 490*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 175*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 105*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 19)/(a^8
+ 7*a^8*sin(d*x + c)/(cos(d*x + c) + 1) + 21*a^8*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 35*a^8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 35*a^8*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 21*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*a
^8*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^8*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7) + 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^8)/d
```

mupad [B] time = 7.77, size = 91, normalized size = 0.72

$$\frac{x}{a^8}+\frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{80\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{224\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{224\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{304\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{304}{105}}{a^8\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + a*sin(c + d*x))^8,x)
```

```
[Out] x/a^8 + ((304*tan(c/2 + (d*x)/2))/15 + (224*tan(c/2 + (d*x)/2)^2)/5 + (224*
tan(c/2 + (d*x)/2)^3)/3 + (80*tan(c/2 + (d*x)/2)^4)/3 + 16*tan(c/2 + (d*x)/
2)^5 + 304/105)/(a^8*d*(tan(c/2 + (d*x)/2) + 1)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.89 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=36

$$-\frac{(a-a\sin(c+dx))^4}{8d\left(a^3\sin(c+dx)+a^3\right)^4}$$

```
[Out] -1/8*(a-a*sin(d*x+c))^4/d/(a^3+a^3*sin(d*x+c))^4
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 37}

$$-\frac{(a-a\sin(c+dx))^4}{8d\left(a^3\sin(c+dx)+a^3\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(a - a*Sin[c + d*x])^4/(8*d*(a^3 + a^3*Sin[c + d*x])^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^3}{(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^7 d}\\ &= -\frac{(a-a\sin(c+dx))^4}{8d\left(a^3+a^3\sin(c+dx)\right)^4}\end{aligned}$$

Mathematica [A] time = 0.08, size = 28, normalized size = 0.78

$$-\frac{\cos^8(c+dx)}{8a^8 d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/8*Cos[c + d*x]^8/(a^8*d*(1 + Sin[c + d*x])^8)
```

fricas [B] time = 0.62, size = 82, normalized size = 2.28

$$-\frac{\left(\cos\left(dx+c\right)^{2}-2\right)\sin\left(dx+c\right)}{a^{8}d\cos\left(dx+c\right)^{4}-8\,a^{8}d\cos\left(dx+c\right)^{2}+8\,a^{8}d-4\left(a^{8}d\cos\left(dx+c\right)^{2}-2\,a^{8}d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c)^2 - 2)*sin(d*x + c)/(a^8*d*cos(d*x + c)^4 - 8*a^8*d*cos(d*x
+ c)^2 + 8*a^8*d - 4*(a^8*d*cos(d*x + c)^2 - 2*a^8*d)*sin(d*x + c))
```

giac [A] time = 1.40, size = 68, normalized size = 1.89

$$\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{8}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2*(tan(1/2*d*x + 1/2*c)^7 + 7*tan(1/2*d*x + 1/2*c)^5 + 7*tan(1/2*d*x + 1/2*
c)^3 + tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^8)
```

maple [A] time = 0.25, size = 55, normalized size = 1.53

$$\frac{-\frac{3}{\left(1+\sin\left(dx+c\right)\right)^{2}}+\frac{1}{1+\sin\left(dx+c\right)}-\frac{2}{\left(1+\sin\left(dx+c\right)\right)^{4}}+\frac{4}{\left(1+\sin\left(dx+c\right)\right)^{3}}}{d\,a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d/a^8*(-3/(1+sin(d*x+c))^2+1/(1+sin(d*x+c))-2/(1+sin(d*x+c))^4+4/(1+sin(d
*x+c))^3)
```

maxima [B] time = 0.32, size = 74, normalized size = 2.06

$$\frac{\sin\left(dx+c\right)^{3}+\sin\left(dx+c\right)}{\left(a^{8}\sin\left(dx+c\right)^{4}+4\,a^{8}\sin\left(dx+c\right)^{3}+6\,a^{8}\sin\left(dx+c\right)^{2}+4\,a^{8}\sin\left(dx+c\right)+a^{8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] (sin(d*x + c)^3 + sin(d*x + c))/((a^8*sin(d*x + c)^4 + 4*a^8*sin(d*x + c)^3
 + 6*a^8*sin(d*x + c)^2 + 4*a^8*sin(d*x + c) + a^8)*d)
```

mupad [B] time = 0.07, size = 64, normalized size = 1.78

$$\frac{\frac{1}{a^{8}\left(\sin(c+d\,x)+1\right)}-\frac{3}{a^{8}\left(\sin(c+d\,x)+1\right)^{2}}+\frac{4}{a^{8}\left(\sin(c+d\,x)+1\right)^{3}}-\frac{2}{a^{8}\left(\sin(c+d\,x)+1\right)^{4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^8,x)
```

```
[Out] (1/(a^8*(sin(c + d*x) + 1)) - 3/(a^8*(sin(c + d*x) + 1)^2) + 4/(a^8*(sin(c
+ d*x) + 1)^3) - 2/(a^8*(sin(c + d*x) + 1)^4))/d
```

sympy [A] time = 42.65, size = 2006, normalized size = 55.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((16*sin(c + d*x)**6/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c
 + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 122
5*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c +
d*x) + 35*a**8*d) + 77*sin(c + d*x)**5/(35*a**8*d*sin(c + d*x)**7 + 245*a**
8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)
**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d
*sin(c + d*x) + 35*a**8*d) - 8*sin(c + d*x)**4*cos(c + d*x)**2/(35*a**8*d*s
in(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 +
1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(
c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + 155*sin(c + d*x)**4/(3
5*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c +
d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a
**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) - 21*sin(c + d
*x)**3*cos(c + d*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)
**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*
d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) +
35*a**8*d) + 168*sin(c + d*x)**3/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*si
n(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 +
1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c
 + d*x) + 35*a**8*d) + 6*sin(c + d*x)**2*cos(c + d*x)**4/(35*a**8*d*sin(c +
 d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a
**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*
x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) - 19*sin(c + d*x)**2*cos(c + d
*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d
*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**
3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + 104
*sin(c + d*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 +
735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(
c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**
8*d) + 7*sin(c + d*x)*cos(c + d*x)**4/(35*a**8*d*sin(c + d*x)**7 + 245*a**8
*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)*
*4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*
sin(c + d*x) + 35*a**8*d) - 7*sin(c + d*x)*cos(c + d*x)**2/(35*a**8*d*sin(c
 + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225
*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c +
d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + 35*sin(c + d*x)/(35*a**8*d
*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5
+ 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*si
n(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) - 5*cos(c + d*x)**6/(3
5*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c +
d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a
**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + cos(c + d*x)
**4/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*si
n(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 +
 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) - cos(c
+ d*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**
8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x
)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) +
5/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(
c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 7
35*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d), Ne(d, 0))
, (x*cos(c)**7/(a*sin(c) + a)**8, True))
```

3.90 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=58

$$-\frac{\cos^7(c+dx)}{63ad(a\sin(c+dx)+a)^7}-\frac{\cos^7(c+dx)}{9d(a\sin(c+dx)+a)^8}$$

```
[Out] -1/9*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^8-1/63*cos(d*x+c)^7/a/d/(a+a*sin(d*x+c
))^7
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 2671}

$$-\frac{\cos^7(c+dx)}{63ad(a\sin(c+dx)+a)^7}-\frac{\cos^7(c+dx)}{9d(a\sin(c+dx)+a)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -Cos[c + d*x]^7/(9*d*(a + a*Sin[c + d*x])^8) - Cos[c + d*x]^7/(63*a*d*(a +
a*Sin[c + d*x])^7)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^8}\,dx &= -\frac{\cos^7(c+dx)}{9d(a+a\sin(c+dx))^8}+\frac{\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{9a}\\ &= -\frac{\cos^7(c+dx)}{9d(a+a\sin(c+dx))^8}-\frac{\cos^7(c+dx)}{63ad(a+a\sin(c+dx))^7}\end{aligned}$$

Mathematica [A] time = 0.11, size = 36, normalized size = 0.62

$$-\frac{(\sin(c+dx)+8)\cos^7(c+dx)}{63a^8d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/63*(Cos[c + d*x]^7*(8 + Sin[c + d*x]))/(a^8*d*(1 + Sin[c + d*x])^8)
```

fricas [B] time = 0.59, size = 239, normalized size = 4.12

$$\frac{\cos(dx+c)^5 - 4\cos(dx+c)^4 + 19\cos(dx+c)^3 + 52\cos(dx+c)^2 - \left(\cos(dx+c)^4 + 5\cdots\right.}{63\left(a^8 d\cos(dx+c)^5 + 5a^8 d\cos(dx+c)^4 - 8a^8 d\cos(dx+c)^3 - 20a^8 d\cos(dx+c)^2 + 8a^8 d\cos(dx+c) + 1\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/63*(cos(d*x + c)^5 - 4*cos(d*x + c)^4 + 19*cos(d*x + c)^3 + 52*cos(d*x +
c)^2 - (cos(d*x + c)^4 + 5*cos(d*x + c)^3 + 24*cos(d*x + c)^2 - 28*cos(d*x
+ c) - 56)*sin(d*x + c) - 28*cos(d*x + c) - 56)/(a^8*d*cos(d*x + c)^5 + 5*a
^8*d*cos(d*x + c)^4 - 8*a^8*d*cos(d*x + c)^3 - 20*a^8*d*cos(d*x + c)^2 + 8*
a^8*d*cos(d*x + c) + 16*a^8*d + (a^8*d*cos(d*x + c)^4 - 4*a^8*d*cos(d*x + c
)^3 - 12*a^8*d*cos(d*x + c)^2 + 8*a^8*d*cos(d*x + c) + 16*a^8*d)*sin(d*x +
c))
```

giac [B] time = 0.61, size = 125, normalized size = 2.16

$$-\frac{2\left(63\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^8 + 63\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 483\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^6 + 315\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 693\tan\left(\frac{1}{2}\cdots\right.\right.}{63a^8 d\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/63*(63*tan(1/2*d*x + 1/2*c)^8 + 63*tan(1/2*d*x + 1/2*c)^7 + 483*tan(1/2*
d*x + 1/2*c)^6 + 315*tan(1/2*d*x + 1/2*c)^5 + 693*tan(1/2*d*x + 1/2*c)^4 +
189*tan(1/2*d*x + 1/2*c)^3 + 225*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1
/2*c) + 8)/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^9)
```

maple [B] time = 0.28, size = 145, normalized size = 2.50

$$\frac{-\frac{172}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} - \frac{256}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9} - \frac{1856}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7} + \frac{14}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1} + \frac{152}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4} - \frac{272}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}\cdots}{d\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(-86/3/(tan(1/2*d*x+1/2*c)+1)^3-128/9/(tan(1/2*d*x+1/2*c)+1)^9-928/
7/(tan(1/2*d*x+1/2*c)+1)^7+7/(tan(1/2*d*x+1/2*c)+1)^2-1/(tan(1/2*d*x+1/2*c)
+1)+76/(tan(1/2*d*x+1/2*c)+1)^4-136/(tan(1/2*d*x+1/2*c)+1)^5+64/(tan(1/2*d*
x+1/2*c)+1)^8+496/3/(tan(1/2*d*x+1/2*c)+1)^6)
```

maxima [B] time = 0.39, size = 375, normalized size = 6.47

$$-\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1} + \frac{225\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{189\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{693\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{315\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{483\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{63\sin\cdots}{(\cos\cdots}\right.}{63\left(a^8 + \frac{9a^8\sin(dx+c)}{\cos(dx+c)+1} + \frac{36a^8\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{84a^8\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{126a^8\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{126a^8\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{84a^8\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{36a^8\sin\cdots}{(\cos\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/63*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 225*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 189*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 693*sin(d*x + c)^4/(cos
```

```
(d*x + c) + 1)^4 + 315*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 483*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 63*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 63*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 8)/((a^8 + 9*a^8*sin(d*x + c)/(cos(d*x +
 c) + 1) + 36*a^8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 84*a^8*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 126*a^8*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 126
*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 84*a^8*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 36*a^8*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*a^8*sin(d*x + c
)^8/(cos(d*x + c) + 1)^8 + a^8*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*d)
```

mupad [B] time = 6.63, size = 118, normalized size = 2.03

$$-\frac{\sqrt{2}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{63\,\sin(c+d\,x)}{2}-\frac{257\,\cos(c+d\,x)}{8}-\frac{113\,\cos(2\,c+2\,d\,x)}{4}+\frac{37\,\cos(3\,c+3\,d\,x)}{8}+\frac{7\,\cos(4\,c+4\,d\,x)}{16}-\frac{63\,\sin(2\,c+2\,d\,x)}{8}-\frac{9}{}\right.}{1008\,a^8\,d\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^8,x)
```

```
[Out] -(2^(1/2)*cos(c/2 + (d*x)/2)*((63*sin(c + d*x))/2 - (257*cos(c + d*x))/8 -
(113*cos(2*c + 2*d*x))/4 + (37*cos(3*c + 3*d*x))/8 + (7*cos(4*c + 4*d*x))/1
6 - (63*sin(2*c + 2*d*x))/8 - (9*sin(3*c + 3*d*x))/2 + (9*sin(4*c + 4*d*x))
/16 + 1013/16))/(1008*a^8*d*cos(c/2 - pi/4 + (d*x)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.91 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{3a^5d(a\sin(c+dx)+a)^3}-\frac{4}{5a^3d(a\sin(c+dx)+a)^5}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^4}$$

```
[Out] -4/5/a^3/d/(a+a*sin(d*x+c))^5-1/3/a^5/d/(a+a*sin(d*x+c))^3+1/d/(a^2+a^2*sin
(d*x+c))^4
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$-\frac{1}{3a^5d(a\sin(c+dx)+a)^3}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{4}{5a^3d(a\sin(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -4/(5*a^3*d*(a + a*Sin[c + d*x])^5) - 1/(3*a^5*d*(a + a*Sin[c + d*x])^3) +
1/(d*(a^2 + a^2*Sin[c + d*x])^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{4a^2}{(a+x)^6}-\frac{4a}{(a+x)^5}+\frac{1}{(a+x)^4}\right)dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= -\frac{4}{5a^3d(a+a\sin(c+dx))^5}-\frac{1}{3a^5d(a+a\sin(c+dx))^3}+\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)^4}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 58, normalized size = 0.89

$$\frac{\left(5\sin^2(c+dx)-5\sin(c+dx)+2\right)\cos^6(c+dx)}{15a^8d(\sin(c+dx)-1)^3(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Cos[c + d*x]^6*(2 - 5*Sin[c + d*x] + 5*Sin[c + d*x]^2))/(15*a^8*d*(-1 + Si
n[c + d*x])^3*(1 + Sin[c + d*x])^8)
```

fricas [A] time = 0.65, size = 100, normalized size = 1.54

$$\frac{5\cos(dx+c)^2 + 5\sin(dx+c) - 7}{15\left(5a^8d\cos(dx+c)^4 - 20a^8d\cos(dx+c)^2 + 16a^8d + \left(a^8d\cos(dx+c)^4 - 12a^8d\cos(dx+c)^2 + 16a^8d\right)\sin(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/15*(5*cos(d*x + c)^2 + 5*sin(d*x + c) - 7)/(5*a^8*d*cos(d*x + c)^4 - 20*a
^8*d*cos(d*x + c)^2 + 16*a^8*d + (a^8*d*cos(d*x + c)^4 - 12*a^8*d*cos(d*x +
 c)^2 + 16*a^8*d)*sin(d*x + c))
```

giac [B] time = 1.44, size = 137, normalized size = 2.11

$$\frac{2\left(15\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 30\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^8 + 140\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 170\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^6 + 282\tan\left(\frac{1}{2}dx + \right.\right.}{15a^8d\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^9 + 30*tan(1/2*d*x + 1/2*c)^8 + 140*tan(1/2*d
*x + 1/2*c)^7 + 170*tan(1/2*d*x + 1/2*c)^6 + 282*tan(1/2*d*x + 1/2*c)^5 + 1
70*tan(1/2*d*x + 1/2*c)^4 + 140*tan(1/2*d*x + 1/2*c)^3 + 30*tan(1/2*d*x + 1
/2*c)^2 + 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^10)
```

maple [A] time = 0.25, size = 43, normalized size = 0.66

$$\frac{-\frac{1}{3(1+\sin(dx+c))^3} + \frac{1}{(1+\sin(dx+c))^4} - \frac{4}{5(1+\sin(dx+c))^5}}{d\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d/a^8*(-1/3/(1+sin(d*x+c))^3+1/(1+sin(d*x+c))^4-4/5/(1+sin(d*x+c))^5)
```

maxima [A] time = 0.32, size = 93, normalized size = 1.43

$$-\frac{5\sin(dx+c)^2 - 5\sin(dx+c) + 2}{15\left(a^8\sin(dx+c)^5 + 5a^8\sin(dx+c)^4 + 10a^8\sin(dx+c)^3 + 10a^8\sin(dx+c)^2 + 5a^8\sin(dx+c) + a^8\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(5*sin(d*x + c)^2 - 5*sin(d*x + c) + 2)/((a^8*sin(d*x + c)^5 + 5*a^8*
sin(d*x + c)^4 + 10*a^8*sin(d*x + c)^3 + 10*a^8*sin(d*x + c)^2 + 5*a^8*sin(
d*x + c) + a^8)*d)
```

mupad [B] time = 4.71, size = 54, normalized size = 0.83

$$\frac{1}{a^8\,d\,(\sin(c+d\,x)+1)^4} - \frac{1}{3\,a^8\,d\,(\sin(c+d\,x)+1)^3} - \frac{4}{5\,a^8\,d\,(\sin(c+d\,x)+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^8,x)
```

```
[Out] 1/(a^8*d*(sin(c + d*x) + 1)^4) - 1/(3*a^8*d*(sin(c + d*x) + 1)^3) - 4/(5*a^
8*d*(sin(c + d*x) + 1)^5)
```

sympy [A] time = 42.11, size = 1120, normalized size = 17.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((-8*sin(c + d*x)**4/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(
c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3
675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c
 + d*x) + 105*a**8*d) - 21*sin(c + d*x)**3/(105*a**8*d*sin(c + d*x)**7 + 73
5*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c
+ d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735
*a**8*d*sin(c + d*x) + 105*a**8*d) + 12*sin(c + d*x)**2*cos(c + d*x)**2/(10
5*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c +
 d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205
*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) - 19*sin(c
+ d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a
**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d
*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d
) + 14*sin(c + d*x)*cos(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*
d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)*
*4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d
*sin(c + d*x) + 105*a**8*d) - 7*sin(c + d*x)/(105*a**8*d*sin(c + d*x)**7 +
735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(
c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 7
35*a**8*d*sin(c + d*x) + 105*a**8*d) - 15*cos(c + d*x)**4/(105*a**8*d*sin(c
 + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 367
5*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c
+ d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) + 2*cos(c + d*x)**2/(105*
a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d
*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a
**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) - 1/(105*a**8
*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)*
*5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*
d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d), Ne(d, 0)), (x*co
s(c)**5/(a*sin(c) + a)**8, True))
```

3.92 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=118

$$-\frac{2\cos^5(c+dx)}{1155a^3d(a\sin(c+dx)+a)^5}-\frac{2\cos^5(c+dx)}{231a^2d(a\sin(c+dx)+a)^6}-\frac{\cos^5(c+dx)}{33ad(a\sin(c+dx)+a)^7}-\frac{\cos^5(c+dx)}{11d(a\sin(c+dx)+a)^8}$$

```
[Out] -1/11*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^8-1/33*cos(d*x+c)^5/a/d/(a+a*sin(d*x+
c))^7-2/231*cos(d*x+c)^5/a^2/d/(a+a*sin(d*x+c))^6-2/1155*cos(d*x+c)^5/a^3/d
/(a+a*sin(d*x+c))^5
```

Rubi [A] time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2672, 2671\}$

$$-\frac{2\cos^5(c+dx)}{1155a^3d(a\sin(c+dx)+a)^5}-\frac{2\cos^5(c+dx)}{231a^2d(a\sin(c+dx)+a)^6}-\frac{\cos^5(c+dx)}{33ad(a\sin(c+dx)+a)^7}-\frac{\cos^5(c+dx)}{11d(a\sin(c+dx)+a)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -Cos[c + d*x]^5/(11*d*(a + a*Sin[c + d*x])^8) - Cos[c + d*x]^5/(33*a*d*(a +
 a*Sin[c + d*x])^7) - (2*Cos[c + d*x]^5)/(231*a^2*d*(a + a*Sin[c + d*x])^6)
 - (2*Cos[c + d*x]^5)/(1155*a^3*d*(a + a*Sin[c + d*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx &= -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8}+\frac{3\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{11a}\\
&= -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8}-\frac{\cos^5(c+dx)}{33ad(a+a\sin(c+dx))^7}+\frac{2\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{33a^2}\\
&= -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8}-\frac{\cos^5(c+dx)}{33ad(a+a\sin(c+dx))^7}-\frac{2\cos^5(c+dx)}{231a^2d(a+a\sin(c+dx))^6}+\\
&= -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8}-\frac{\cos^5(c+dx)}{33ad(a+a\sin(c+dx))^7}-\frac{2\cos^5(c+dx)}{231a^2d(a+a\sin(c+dx))^6}-
\end{aligned}$$

Mathematica [A] time = 0.09, size = 58, normalized size = 0.49

$$-\frac{\left(2\sin^3(c+dx)+16\sin^2(c+dx)+61\sin(c+dx)+152\right)\cos^5(c+dx)}{1155a^8d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/1155*(Cos[c + d*x]^5*(152 + 61*Sin[c + d*x] + 16*Sin[c + d*x]^2 + 2*Sin[
c + d*x]^3))/(a^8*d*(1 + Sin[c + d*x])^8)
```

fricas [B] time = 0.69, size = 291, normalized size = 2.47

$$\frac{2\cos(dx+c)^6+12\cos(dx+c)^5-25\cos(dx+c)^4-70\cos(dx+c)^3-245\cos(dx+c}{1155\left(a^8d\cos(dx+c)^6-5a^8d\cos(dx+c)^5-18a^8d\cos(dx+c)^4+20a^8d\cos(dx+c)^3+48a^8d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/1155*(2*cos(d*x + c)^6 + 12*cos(d*x + c)^5 - 25*cos(d*x + c)^4 - 70*cos(d
*x + c)^3 - 245*cos(d*x + c)^2 + (2*cos(d*x + c)^5 - 10*cos(d*x + c)^4 - 35
*cos(d*x + c)^3 + 35*cos(d*x + c)^2 - 210*cos(d*x + c) - 420)*sin(d*x + c)
+ 210*cos(d*x + c) + 420)/(a^8*d*cos(d*x + c)^6 - 5*a^8*d*cos(d*x + c)^5 -
18*a^8*d*cos(d*x + c)^4 + 20*a^8*d*cos(d*x + c)^3 + 48*a^8*d*cos(d*x + c)^2
 - 16*a^8*d*cos(d*x + c) - 32*a^8*d - (a^8*d*cos(d*x + c)^5 + 6*a^8*d*cos(d
*x + c)^4 - 12*a^8*d*cos(d*x + c)^3 - 32*a^8*d*cos(d*x + c)^2 + 16*a^8*d*co
s(d*x + c) + 32*a^8*d)*sin(d*x + c))
```

giac [A] time = 0.60, size = 151, normalized size = 1.28

$$-\frac{2\left(1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+3465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+13860\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+23100\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/1155*(1155*tan(1/2*d*x + 1/2*c)^10 + 3465*tan(1/2*d*x + 1/2*c)^9 + 13860
*tan(1/2*d*x + 1/2*c)^8 + 23100*tan(1/2*d*x + 1/2*c)^7 + 37422*tan(1/2*d*x
+ 1/2*c)^6 + 32802*tan(1/2*d*x + 1/2*c)^5 + 27060*tan(1/2*d*x + 1/2*c)^4 +
11220*tan(1/2*d*x + 1/2*c)^3 + 4895*tan(1/2*d*x + 1/2*c)^2 + 517*tan(1/2*d*
x + 1/2*c) + 152)/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^11)
```

maple [A] time = 0.30, size = 175, normalized size = 1.48

$$\frac{\frac{584}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{576}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{60}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{256}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}+\frac{14}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1024}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}-\frac{4752}{7\left(\tan\left(\frac{dx}{2}+\right.\right.}}{d\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(292/(tan(1/2*d*x+1/2*c)+1)^6+288/(tan(1/2*d*x+1/2*c)+1)^8-30/(tan(
1/2*d*x+1/2*c)+1)^3-128/11/(tan(1/2*d*x+1/2*c)+1)^11+7/(tan(1/2*d*x+1/2*c)+
1)^2-512/3/(tan(1/2*d*x+1/2*c)+1)^9-2376/7/(tan(1/2*d*x+1/2*c)+1)^7+64/(tan
```

```
(1/2*d*x+1/2*c)+1)^10-1/(tan(1/2*d*x+1/2*c)+1)+88/(tan(1/2*d*x+1/2*c)+1)^4-
932/5/(tan(1/2*d*x+1/2*c)+1)^5)
```

maxima [B] time = 0.42, size = 461, normalized size = 3.91

$$-\frac{2\left(\frac{517\sin(dx+c)}{\cos(dx+c)+1}+\frac{4895\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{11220\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{27060\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{32802\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{37422\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{231}{(\text{c}}\right.}{1155\left(a^8+\frac{11a^8\sin(dx+c)}{\cos(dx+c)+1}+\frac{55a^8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{165a^8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{330a^8\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{462a^8\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{462a^8\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{330a}{(\text{co}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/1155*(517*sin(d*x + c)/(cos(d*x + c) + 1) + 4895*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 11220*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 27060*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 32802*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 374
22*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 23100*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 13860*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3465*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9 + 1155*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 152)/((a
^8 + 11*a^8*sin(d*x + c)/(cos(d*x + c) + 1) + 55*a^8*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 165*a^8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 330*a^8*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 462*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 + 462*a^8*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 330*a^8*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 + 165*a^8*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 55*a^8*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 + 11*a^8*sin(d*x + c)^10/(cos(d*x + c) +
 1)^10 + a^8*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*d)
```

mupad [B] time = 7.15, size = 140, normalized size = 1.19

$$-\frac{\sqrt{2}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{7623\sin(c+d\,x)}{4}-697\cos\left(c+d\,x\right)-\frac{3977\cos(2\,c+2\,d\,x)}{4}+\frac{3203\cos(3\,c+3\,d\,x)}{16}+\frac{461\cos(4\,c+4\,d\,x)}{8}-\frac{75\cos}{}\right.}{36960\,a^8\,d\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^8,x)
```

```
[Out] -(2^(1/2)*cos(c/2 + (d*x)/2)*((7623*sin(c + d*x))/4 - 697*cos(c + d*x) - (3
977*cos(2*c + 2*d*x))/4 + (3203*cos(3*c + 3*d*x))/16 + (461*cos(4*c + 4*d*x
))/8 - (75*cos(5*c + 5*d*x))/16 - 462*sin(2*c + 2*d*x) - (4983*sin(3*c + 3*
d*x))/16 + (187*sin(4*c + 4*d*x))/4 + (77*sin(5*c + 5*d*x))/16 + 12721/8))/
(36960*a^8*d*cos(c/2 - pi/4 + (d*x)/2)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.93 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=45

$$\frac{1}{5a^3d(a\sin(c+dx)+a)^5}-\frac{1}{3a^2d(a\sin(c+dx)+a)^6}$$

```
[Out] -1/3/a^2/d/(a+a*sin(d*x+c))^6+1/5/a^3/d/(a+a*sin(d*x+c))^5
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = $\{2667, 43\}$

$$\frac{1}{5a^3d(a\sin(c+dx)+a)^5}-\frac{1}{3a^2d(a\sin(c+dx)+a)^6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/(3*a^2*d*(a + a*Sin[c + d*x])^6) + 1/(5*a^3*d*(a + a*Sin[c + d*x])^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{a-x}{(a+x)^7}\,dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{2a}{(a+x)^7}-\frac{1}{(a+x)^6}\right)dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= -\frac{1}{3a^2d(a+a\sin(c+dx))^6}+\frac{1}{5a^3d(a+a\sin(c+dx))^5}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 43, normalized size = 0.96

$$\frac{3\sin(c+dx)-2}{15a^8d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-2 + 3*Sin[c + d*x])/(15*a^8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^12)
```

fricas [B] time = 0.61, size = 105, normalized size = 2.33

$$-\frac{3\sin(dx+c)-2}{15\left(a^8d\cos(dx+c)^6-18a^8d\cos(dx+c)^4+48a^8d\cos(dx+c)^2-32a^8d-2\left(3a^8d\cos(dx+c)^4-16a^8d\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/15*(3*sin(d*x + c) - 2)/(a^8*d*cos(d*x + c)^6 - 18*a^8*d*cos(d*x + c)^4
+ 48*a^8*d*cos(d*x + c)^2 - 32*a^8*d - 2*(3*a^8*d*cos(d*x + c)^4 - 16*a^8*d
*cos(d*x + c)^2 + 16*a^8*d)*sin(d*x + c))
```

giac [A] time = 0.59, size = 28, normalized size = 0.62

$$\frac{3\sin(dx+c)-2}{15a^8d(\sin(dx+c)+1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(d*x + c) - 2)/(a^8*d*(sin(d*x + c) + 1)^6)
```

maple [A] time = 0.27, size = 33, normalized size = 0.73

$$\frac{\frac{1}{5(1+\sin(dx+c))^5}-\frac{1}{3(1+\sin(dx+c))^6}}{d\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d/a^8*(1/5/(1+sin(d*x+c))^5-1/3/(1+sin(d*x+c))^6)
```

maxima [B] time = 0.32, size = 96, normalized size = 2.13

$$\frac{3\sin(dx+c)-2}{15\left(a^8\sin(dx+c)^6+6a^8\sin(dx+c)^5+15a^8\sin(dx+c)^4+20a^8\sin(dx+c)^3+15a^8\sin(dx+c)^2+6a^8\sin(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(d*x + c) - 2)/((a^8*sin(d*x + c)^6 + 6*a^8*sin(d*x + c)^5 + 15*
a^8*sin(d*x + c)^4 + 20*a^8*sin(d*x + c)^3 + 15*a^8*sin(d*x + c)^2 + 6*a^8*
sin(d*x + c) + a^8)*d)
```

mupad [B] time = 0.10, size = 28, normalized size = 0.62

$$\frac{3\sin(c+d\,x)-2}{15\,a^8\,d\,(\sin(c+d\,x)+1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^8,x)
```

```
[Out] (3*sin(c + d*x) - 2)/(15*a^8*d*(sin(c + d*x) + 1)^6)
```

sympy [A] time = 41.54, size = 493, normalized size = 10.96

$$\begin{cases} \frac{6\sin^2(c+dx)}{105a^8d\sin^7(c+dx)+735a^8d\sin^6(c+dx)+2205a^8d\sin^5(c+dx)+3675a^8d\sin^4(c+dx)+3675a^8d\sin^3(c+dx)+2205a^8d\sin^2(c+dx)+735a^8d\sin(c+dx} \\ \frac{x\cos^3(c)}{(a\sin(c)+a)^8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((6*sin(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c
 + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 36
75*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c
+ d*x) + 105*a**8*d) + 7*sin(c + d*x)/(105*a**8*d*sin(c + d*x)**7 + 735*a**
8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x
)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8
*d*sin(c + d*x) + 105*a**8*d) - 15*cos(c + d*x)**2/(105*a**8*d*sin(c + d*x)
**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*
d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)*
*2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) + 1/(105*a**8*d*sin(c + d*x)**7
+ 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*si
n(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 +
 735*a**8*d*sin(c + d*x) + 105*a**8*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) +
 a)**8, True))
```

3.94 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=183

$$-\frac{20\cos^3(c+dx)}{3003a^3d(a\sin(c+dx)+a)^5}-\frac{8\cos^3(c+dx)}{9009a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{8\cos^3(c+dx)}{3003d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{20\cos^3(c+}{1287a^2d(a\sin(c+}$$

```
[Out] -1/13*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^8-5/143*cos(d*x+c)^3/a/d/(a+a*sin(d*x
+c))^7-20/1287*cos(d*x+c)^3/a^2/d/(a+a*sin(d*x+c))^6-20/3003*cos(d*x+c)^3/a
^3/d/(a+a*sin(d*x+c))^5-8/3003*cos(d*x+c)^3/d/(a^2+a^2*sin(d*x+c))^4-8/9009
*cos(d*x+c)^3/a^2/d/(a^2+a^2*sin(d*x+c))^3
```

Rubi [A] time = 0.27, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 2671}

$$-\frac{8\cos^3(c+dx)}{9009a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{8\cos^3(c+dx)}{3003d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{20\cos^3(c+dx)}{3003a^3d(a\sin(c+dx)+a)^5}-\frac{20\cos^3(c+}{1287a^2d(a\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -Cos[c + d*x]^3/(13*d*(a + a*Sin[c + d*x])^8) - (5*Cos[c + d*x]^3)/(143*a*d
*(a + a*Sin[c + d*x])^7) - (20*Cos[c + d*x]^3)/(1287*a^2*d*(a + a*Sin[c + d
*x])^6) - (20*Cos[c + d*x]^3)/(3003*a^3*d*(a + a*Sin[c + d*x])^5) - (8*Cos[
c + d*x]^3)/(3003*d*(a^2 + a^2*Sin[c + d*x])^4) - (8*Cos[c + d*x]^3)/(9009*
a^2*d*(a^2 + a^2*Sin[c + d*x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} + \frac{5\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{13a}$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} + \frac{20\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{143a^2}$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+dx$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+dx$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+dx$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+dx$$

Mathematica [A] time = 0.13, size = 78, normalized size = 0.43

$$-\frac{\left(8\sin^5(c+dx)+64\sin^4(c+dx)+236\sin^3(c+dx)+544\sin^2(c+dx)+911\sin(c+dx)+1240\right)\cos^3(c+d}{9009a^8d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/9009*(Cos[c + d*x]^3*(1240 + 911*Sin[c + d*x] + 544*Sin[c + d*x]^2 + 236
*Sin[c + d*x]^3 + 64*Sin[c + d*x]^4 + 8*Sin[c + d*x]^5))/(a^8*d*(1 + Sin[c
+ d*x])^8)
```

fricas [A] time = 0.73, size = 339, normalized size = 1.85

$$\frac{8\cos(dx+c)^7-48\cos(dx+c)^6-196\cos(dx+c)^5+280\cos(dx+c)^4+735\cos(dx}{9009\left(a^8d\cos(dx+c)^7+7a^8d\cos(dx+c)^6-18a^8d\cos(dx+c)^5-56a^8d\cos(dx+c)^4+48a^8d\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9009*(8*cos(d*x + c)^7 - 48*cos(d*x + c)^6 - 196*cos(d*x + c)^5 + 280*cos
(d*x + c)^4 + 735*cos(d*x + c)^3 - 378*cos(d*x + c)^2 - (8*cos(d*x + c)^6 +
 56*cos(d*x + c)^5 - 140*cos(d*x + c)^4 - 420*cos(d*x + c)^3 + 315*cos(d*x
+ c)^2 + 693*cos(d*x + c) + 1386)*sin(d*x + c) + 693*cos(d*x + c) + 1386)/(
a^8*d*cos(d*x + c)^7 + 7*a^8*d*cos(d*x + c)^6 - 18*a^8*d*cos(d*x + c)^5 - 5
6*a^8*d*cos(d*x + c)^4 + 48*a^8*d*cos(d*x + c)^3 + 112*a^8*d*cos(d*x + c)^2
 - 32*a^8*d*cos(d*x + c) - 64*a^8*d + (a^8*d*cos(d*x + c)^6 - 6*a^8*d*cos(d
*x + c)^5 - 24*a^8*d*cos(d*x + c)^4 + 32*a^8*d*cos(d*x + c)^3 + 80*a^8*d*co
s(d*x + c)^2 - 32*a^8*d*cos(d*x + c) - 64*a^8*d)*sin(d*x + c))
```

giac [A] time = 0.78, size = 177, normalized size = 0.97

$$-\frac{2\left(9009\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}+45045\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+183183\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+435435\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/9009*(9009*tan(1/2*d*x + 1/2*c)^12 + 45045*tan(1/2*d*x + 1/2*c)^11 + 183
183*tan(1/2*d*x + 1/2*c)^10 + 435435*tan(1/2*d*x + 1/2*c)^9 + 810810*tan(1/
2*d*x + 1/2*c)^8 + 1051050*tan(1/2*d*x + 1/2*c)^7 + 1076790*tan(1/2*d*x + 1
/2*c)^6 + 785070*tan(1/2*d*x + 1/2*c)^5 + 451165*tan(1/2*d*x + 1/2*c)^4 + 1
71457*tan(1/2*d*x + 1/2*c)^3 + 51675*tan(1/2*d*x + 1/2*c)^2 + 7111*tan(1/2*
d*x + 1/2*c) + 1240)/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^13)
```

maple [A] time = 0.30, size = 205, normalized size = 1.12

$$-\frac{\frac{480}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{864}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}+\frac{1472}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{12}}-\frac{4544}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}-\frac{11680}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}-\frac{905}{7\left(\tan\left(\frac{dx}{2}+\right.\right.}}{d\,a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(-240/(tan(1/2*d*x+1/2*c)+1)^5+432/(tan(1/2*d*x+1/2*c)+1)^10+736/(t
an(1/2*d*x+1/2*c)+1)^8+64/(tan(1/2*d*x+1/2*c)+1)^12-2272/11/(tan(1/2*d*x+1/
2*c)+1)^11-5840/9/(tan(1/2*d*x+1/2*c)+1)^9-4528/7/(tan(1/2*d*x+1/2*c)+1)^7+
1336/3/(tan(1/2*d*x+1/2*c)+1)^6+7/(tan(1/2*d*x+1/2*c)+1)^2-1/(tan(1/2*d*x+1
/2*c)+1)+100/(tan(1/2*d*x+1/2*c)+1)^4-94/3/(tan(1/2*d*x+1/2*c)+1)^3-128/13/
(tan(1/2*d*x+1/2*c)+1)^13)
```

maxima [B] time = 0.52, size = 547, normalized size = 2.99

$$-\frac{2\left(\frac{7111\sin(dx+c)}{\cos(dx+c)+1}+\frac{51675\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{171457\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{451165\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{785070\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{1076790\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\cdots\right.}{9009\left(a^{8}+\frac{13a^{8}\sin(dx+c)}{\cos(dx+c)+1}+\frac{78a^{8}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{286a^{8}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{715a^{8}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{1287a^{8}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{1716a^{8}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/9009*(7111*sin(d*x + c)/(cos(d*x + c) + 1) + 51675*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 171457*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 451165*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 785070*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
+ 1076790*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1051050*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 + 810810*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 435435*sin(
d*x + c)^9/(cos(d*x + c) + 1)^9 + 183183*sin(d*x + c)^10/(cos(d*x + c) + 1)
^10 + 45045*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 9009*sin(d*x + c)^12/(c
os(d*x + c) + 1)^12 + 1240)/((a^8 + 13*a^8*sin(d*x + c)/(cos(d*x + c) + 1)
+ 78*a^8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 286*a^8*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 715*a^8*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1287*a^8*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1716*a^8*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + 1716*a^8*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1287*a^8*sin(d*x + c
)^8/(cos(d*x + c) + 1)^8 + 715*a^8*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 28
6*a^8*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 78*a^8*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11 + 13*a^8*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + a^8*sin(d*
x + c)^13/(cos(d*x + c) + 1)^13)*d)
```

mupad [B] time = 8.12, size = 162, normalized size = 0.89

$$\frac{\sqrt{2}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{14983\,\cos(c+d\,x)}{2}-\frac{63921\,\sin(c+d\,x)}{2}+17605\,\cos(2\,c+2\,d\,x)-\frac{15365\,\cos(3\,c+3\,d\,x)}{4}-\frac{6943\,\cos(4\,c+4\,d\,x)}{4}+\cdots\right.}{576576\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^8,x)
```

```
[Out] (2^(1/2)*cos(c/2 + (d*x)/2)*((14983*cos(c + d*x))/2 - (63921*sin(c + d*x))/
2 + 17605*cos(2*c + 2*d*x) - (15365*cos(3*c + 3*d*x))/4 - (6943*cos(4*c + 4
*d*x))/4 + (937*cos(5*c + 5*d*x))/4 + (77*cos(6*c + 6*d*x))/4 + (28743*sin(
2*c + 2*d*x))/4 + (27027*sin(3*c + 3*d*x))/4 - (5005*sin(4*c + 4*d*x))/4 -
(1079*sin(5*c + 5*d*x))/4 + (39*sin(6*c + 6*d*x))/2 - 21013))/(576576*a^8*d
*cos(c/2 - pi/4 + (d*x)/2)^13)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.95 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^8} dx$

Optimal. Leaf size=22

$$-\frac{1}{7ad(a\sin(c+dx)+a)^7}$$

[Out] -1/7/a/d/(a+a*sin(d*x+c))^7

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2667, 32\}$

$$-\frac{1}{7ad(a\sin(c+dx)+a)^7}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^8,x]

[Out] -1/(7*a*d*(a + a*Sin[c + d*x])^7)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^8}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= -\frac{1}{7ad(a+a\sin(c+dx))^7}\end{aligned}$$

Mathematica [A] time = 0.23, size = 33, normalized size = 1.50

$$-\frac{1}{7a^8d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^{14}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^8,x]

[Out] -1/7*1/(a^8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^14)

fricas [B] time = 0.58, size = 108, normalized size = 4.91

$$\frac{1}{7\left(7\,a^8d\cos\left(dx+c\right)^6-56\,a^8d\cos\left(dx+c\right)^4+112\,a^8d\cos\left(dx+c\right)^2-64\,a^8d+\left(a^8d\cos\left(dx+c\right)^6-24\,a^8d\cos\left(dx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/7/(7*a^8*d*cos(d*x + c)^6 - 56*a^8*d*cos(d*x + c)^4 + 112*a^8*d*cos(d*x +
 c)^2 - 64*a^8*d + (a^8*d*cos(d*x + c)^6 - 24*a^8*d*cos(d*x + c)^4 + 80*a^8
*d*cos(d*x + c)^2 - 64*a^8*d)*sin(d*x + c))
```

giac [A] time = 0.49, size = 20, normalized size = 0.91

$$-\frac{1}{7 \, {\left(a \sin \left(d x + c\right) + a\right)}^{7} a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/7/((a*sin(d*x + c) + a)^7*a*d)
```

maple [A] time = 0.10, size = 21, normalized size = 0.95

$$-\frac{1}{7 a d \left(a +a \sin \left(d x +c \right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^8,x)
```

```
[Out] -1/7/a/d/(a+a*sin(d*x+c))^7
```

maxima [A] time = 0.31, size = 20, normalized size = 0.91

$$-\frac{1}{7 \, {\left(a \sin \left(d x + c\right) + a\right)}^{7} a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/7/((a*sin(d*x + c) + a)^7*a*d)
```

mupad [B] time = 4.67, size = 18, normalized size = 0.82

$$-\frac{1}{7\,a^8\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^8,x)
```

```
[Out] -1/(7*a^8*d*(sin(c + d*x) + 1)^7)
```

sympy [A] time = 42.27, size = 128, normalized size = 5.82

$$\begin{cases} - \frac{1}{7 a^{8} d \sin^{7}{\left(c + d x \right)} + 49 a^{8} d \sin^{6}{\left(c + d x \right)} + 147 a^{8} d \sin^{5}{\left(c + d x \right)} + 245 a^{8} d \sin^{4}{\left(c + d x \right)} + 245 a^{8} d \sin^{3}{\left(c + d x \right)} + 147 a^{8} d \sin^{2}{\left(c + d x \right)} + 49 a^{8} d \sin{\left(c + d x \right)} + 7 a^{8} d} \\ \frac{x \cos{\left(c \right)}}{\left(a \sin{\left(c \right)} + a\right)^{8}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((-1/(7*a**8*d*sin(c + d*x)**7 + 49*a**8*d*sin(c + d*x)**6 + 147*a
**8*d*sin(c + d*x)**5 + 245*a**8*d*sin(c + d*x)**4 + 245*a**8*d*sin(c + d*x
)**3 + 147*a**8*d*sin(c + d*x)**2 + 49*a**8*d*sin(c + d*x) + 7*a**8*d), Ne(
d, 0)), (x*cos(c)/(a*sin(c) + a)**8, True))
```

3.96 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=194

$$-\frac{1}{256d\left(a^8\sin(c+dx)+a^8\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{256a^8d}-\frac{1}{192a^5d(a\sin(c+dx)+a)^3}-\frac{1}{256d\left(a^4\sin(c+dx)+a^4\right)^2}-80a$$

```
[Out] 1/256*arctanh(sin(d*x+c))/a^8/d-1/16/d/(a+a*sin(d*x+c))^8-1/28/a/d/(a+a*sin
(d*x+c))^7-1/48/a^2/d/(a+a*sin(d*x+c))^6-1/80/a^3/d/(a+a*sin(d*x+c))^5-1/19
2/a^5/d/(a+a*sin(d*x+c))^3-1/128/d/(a^2+a^2*sin(d*x+c))^4-1/256/d/(a^4+a^4*
sin(d*x+c))^2-1/256/d/(a^8+a^8*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2667, 44, 206}

$$-\frac{1}{256d\left(a^8\sin(c+dx)+a^8\right)}-\frac{1}{256d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{1}{192a^5d(a\sin(c+dx)+a)^3}-\frac{1}{128d\left(a^2\sin(c+dx)+a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(256*a^8*d) - 1/(16*d*(a + a*Sin[c + d*x])^8) - 1/(28
*a*d*(a + a*Sin[c + d*x])^7) - 1/(48*a^2*d*(a + a*Sin[c + d*x])^6) - 1/(80*
a^3*d*(a + a*Sin[c + d*x])^5) - 1/(192*a^5*d*(a + a*Sin[c + d*x])^3) - 1/(1
28*d*(a^2 + a^2*Sin[c + d*x])^4) - 1/(256*d*(a^4 + a^4*Sin[c + d*x])^2) - 1
/(256*d*(a^8 + a^8*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^9}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^9} + \frac{1}{4a^2(a+x)^8} + \frac{1}{8a^3(a+x)^7} + \frac{1}{16a^4(a+x)^6} + \frac{1}{32a^5(a+x)^5} + \frac{1}{64a^6(a+x)^4} + \right.\right.}{d}$$

$$= -\frac{1}{16d(a+a\sin(c+dx))^8} - \frac{1}{28ad(a+a\sin(c+dx))^7} - \frac{1}{48a^2d(a+a\sin(c+dx))^6}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{256a^8d} - \frac{1}{16d(a+a\sin(c+dx))^8} - \frac{1}{28ad(a+a\sin(c+dx))^7} - \frac{}{48a^2}$$

Mathematica [A] time = 0.83, size = 122, normalized size = 0.63

$$-\frac{105\sin^7(c+dx) + 840\sin^6(c+dx) + 2975\sin^5(c+dx) + 6160\sin^4(c+dx) + 8351\sin^3(c+dx) + 8008\sin^2}{26880a^8d(\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/26880*(4096 - 105*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^16 + 5993*Sin[c + d*x] + 8008*Sin[c + d*x]^2 + 8351*Sin[c + d*x]^3 +
6160*Sin[c + d*x]^4 + 2975*Sin[c + d*x]^5 + 840*Sin[c + d*x]^6 + 105*Sin[c
+ d*x]^7)/(a^8*d*(1 + Sin[c + d*x])^8)
```

fricas [B] time = 0.52, size = 374, normalized size = 1.93

$$\frac{1680\cos(dx+c)^6 - 17360\cos(dx+c)^4 + 45696\cos(dx+c)^2 + 105\left(\cos(dx+c)^8 - 32\cos(dx+c)^6 + 160\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/53760*(1680*cos(d*x + c)^6 - 17360*cos(d*x + c)^4 + 45696*cos(d*x + c)^2
+ 105*(cos(d*x + c)^8 - 32*cos(d*x + c)^6 + 160*cos(d*x + c)^4 - 256*cos(d*
x + c)^2 - 8*(cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 24*cos(d*x + c)^2 - 16)*
sin(d*x + c) + 128)*log(sin(d*x + c) + 1) - 105*(cos(d*x + c)^8 - 32*cos(d*
x + c)^6 + 160*cos(d*x + c)^4 - 256*cos(d*x + c)^2 - 8*(cos(d*x + c)^6 - 10
*cos(d*x + c)^4 + 24*cos(d*x + c)^2 - 16)*sin(d*x + c) + 128)*log(-sin(d*x
+ c) + 1) + 2*(105*cos(d*x + c)^6 - 3290*cos(d*x + c)^4 + 14616*cos(d*x + c
)^2 - 17424)*sin(d*x + c) - 38208)/(a^8*d*cos(d*x + c)^8 - 32*a^8*d*cos(d*x
 + c)^6 + 160*a^8*d*cos(d*x + c)^4 - 256*a^8*d*cos(d*x + c)^2 + 128*a^8*d -
 8*(a^8*d*cos(d*x + c)^6 - 10*a^8*d*cos(d*x + c)^4 + 24*a^8*d*cos(d*x + c)^
2 - 16*a^8*d)*sin(d*x + c))
```

giac [A] time = 0.47, size = 131, normalized size = 0.68

$$\frac{\frac{840\log(|\sin(dx+c)+1|)}{a^8} - \frac{840\log(|\sin(dx+c)-1|)}{a^8} - \frac{2283\sin(dx+c)^8 + 19944\sin(dx+c)^7 + 77364\sin(dx+c)^6 + 175448\sin(dx+c)^5 + 258370\sin(d}{a^8(\sin(dx+c)+1)^8}}{430080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/430080*(840*log(abs(sin(d*x + c) + 1))/a^8 - 840*log(abs(sin(d*x + c) - 1
))/a^8 - (2283*sin(d*x + c)^8 + 19944*sin(d*x + c)^7 + 77364*sin(d*x + c)^6
 + 175448*sin(d*x + c)^5 + 258370*sin(d*x + c)^4 + 261464*sin(d*x + c)^3 +
192052*sin(d*x + c)^2 + 114152*sin(d*x + c) + 67819)/(a^8*(sin(d*x + c) + 1
)^8))/d
```

maple [A] time = 0.29, size = 180, normalized size = 0.93

$$-\frac{\ln(\sin(dx+c)-1)}{512a^8d}-\frac{1}{16a^8d(1+\sin(dx+c))^8}-\frac{1}{28a^8d(1+\sin(dx+c))^7}-\frac{1}{48a^8d(1+\sin(dx+c))^6}-\frac{}{80a^8d(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^8,x)
```

```
[Out] -1/512/a^8/d*ln(sin(d*x+c)-1)-1/16/a^8/d/(1+sin(d*x+c))^8-1/28/a^8/d/(1+sin
(d*x+c))^7-1/48/a^8/d/(1+sin(d*x+c))^6-1/80/a^8/d/(1+sin(d*x+c))^5-1/128/a^
8/d/(1+sin(d*x+c))^4-1/192/a^8/d/(1+sin(d*x+c))^3-1/256/a^8/d/(1+sin(d*x+c)
)^2-1/256/a^8/d/(1+sin(d*x+c))+1/512/a^8/d*ln(1+sin(d*x+c))
```

maxima [A] time = 0.32, size = 213, normalized size = 1.10

$$-\frac{\frac{2\left(105\sin(dx+c)^7+840\sin(dx+c)^6+2975\sin(dx+c)^5+6160\sin(dx+c)^4+8351\sin(dx+c)^3+8008\sin(dx+c)^2+5993\sin(dx+c)+4096\right)}{a^8\sin(dx+c)^8+8a^8\sin(dx+c)^7+28a^8\sin(dx+c)^6+56a^8\sin(dx+c)^5+70a^8\sin(dx+c)^4+56a^8\sin(dx+c)^3+28a^8\sin(dx+c)^2+8a^8\sin(dx+c)+a^8}-\frac{105}{}}{53760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/53760*(2*(105*sin(d*x + c)^7 + 840*sin(d*x + c)^6 + 2975*sin(d*x + c)^5
+ 6160*sin(d*x + c)^4 + 8351*sin(d*x + c)^3 + 8008*sin(d*x + c)^2 + 5993*si
n(d*x + c) + 4096)/(a^8*sin(d*x + c)^8 + 8*a^8*sin(d*x + c)^7 + 28*a^8*sin(
d*x + c)^6 + 56*a^8*sin(d*x + c)^5 + 70*a^8*sin(d*x + c)^4 + 56*a^8*sin(d*x
 + c)^3 + 28*a^8*sin(d*x + c)^2 + 8*a^8*sin(d*x + c) + a^8) - 105*log(sin(d
*x + c) + 1)/a^8 + 105*log(sin(d*x + c) - 1)/a^8)/d
```

mupad [B] time = 0.30, size = 198, normalized size = 1.02

$$\frac{\operatorname{atanh}(\sin(c+d\,x))}{256\,a^8\,d}-\frac{\frac{\sin(c+d\,x)^7}{256}+\frac{\sin(c+d\,x)^6}{32}+\frac{85\sin(c+d\,x)^5}{768}+\frac{11\sin(c+d\,x)^4}{48}+\frac{1193}{}}{d\left(a^8\sin(c+d\,x)^8+8\,a^8\sin(c+d\,x)^7+28\,a^8\sin(c+d\,x)^6+56\,a^8\sin(c+d\,x)^5+70\,a^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^8),x)
```

```
[Out] atanh(sin(c + d*x))/(256*a^8*d) - ((5993*sin(c + d*x))/26880 + (143*sin(c +
 d*x)^2)/480 + (1193*sin(c + d*x)^3)/3840 + (11*sin(c + d*x)^4)/48 + (85*si
n(c + d*x)^5)/768 + sin(c + d*x)^6/32 + sin(c + d*x)^7/256 + 16/105)/(d*(8*
a^8*sin(c + d*x) + a^8 + 28*a^8*sin(c + d*x)^2 + 56*a^8*sin(c + d*x)^3 + 70
*a^8*sin(c + d*x)^4 + 56*a^8*sin(c + d*x)^5 + 28*a^8*sin(c + d*x)^6 + 8*a^8
*sin(c + d*x)^7 + a^8*sin(c + d*x)^8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.97 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=245

$$\frac{128\tan(c+dx)}{12155a^8d}-\frac{64\sec(c+dx)}{12155d\left(a^8\sin(c+dx)+a^8\right)}-\frac{64\sec(c+dx)}{12155d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{168\sec(c+dx)}{12155a^3d(a\sin(c+dx)+a)^5}-$$

```
[Out] -1/17*sec(d*x+c)/d/(a+a*sin(d*x+c))^8-3/85*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^
7-24/1105*sec(d*x+c)/a^2/d/(a+a*sin(d*x+c))^6-168/12155*sec(d*x+c)/a^3/d/(a
+a*sin(d*x+c))^5-112/12155*sec(d*x+c)/d/(a^2+a^2*sin(d*x+c))^4-16/2431*sec(
d*x+c)/a^2/d/(a^2+a^2*sin(d*x+c))^3-64/12155*sec(d*x+c)/d/(a^4+a^4*sin(d*x+
c))^2-64/12155*sec(d*x+c)/d/(a^8+a^8*sin(d*x+c))+128/12155*tan(d*x+c)/a^8/d
```

Rubi [A] time = 0.40, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2672, 3767, 8}

$$\frac{128\tan(c+dx)}{12155a^8d}-\frac{64\sec(c+dx)}{12155d\left(a^8\sin(c+dx)+a^8\right)}-\frac{64\sec(c+dx)}{12155d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{16\sec(c+dx)}{2431a^2d\left(a^2\sin(c+dx)+a^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -Sec[c + d*x]/(17*d*(a + a*Sin[c + d*x])^8) - (3*Sec[c + d*x])/(85*a*d*(a +
 a*Sin[c + d*x])^7) - (24*Sec[c + d*x])/(1105*a^2*d*(a + a*Sin[c + d*x])^6)
 - (168*Sec[c + d*x])/(12155*a^3*d*(a + a*Sin[c + d*x])^5) - (112*Sec[c + d
*x])/(12155*d*(a^2 + a^2*Sin[c + d*x])^4) - (16*Sec[c + d*x])/(2431*a^2*d*(
a^2 + a^2*Sin[c + d*x])^3) - (64*Sec[c + d*x])/(12155*d*(a^4 + a^4*Sin[c +
d*x])^2) - (64*Sec[c + d*x])/(12155*d*(a^8 + a^8*Sin[c + d*x])) + (128*Tan[
c + d*x])/(12155*a^8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} + \frac{9\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{17a}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} + \frac{24\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{85a^2}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6}$$

Mathematica [**A**] time = 0.40, size = 113, normalized size = 0.46

$$\frac{\sec(c+dx)(4862\sin(c+dx) - 6188\sin(3(c+dx)) + 1700\sin(5(c+dx)) - 119\sin(7(c+dx)) + \sin(9(c+dx)) -}{24310a^8d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]*(-7072*Cos[2*(c + d*x)] + 3808*Cos[4*(c + d*x)] - 544*Cos[6*(
c + d*x)] + 16*Cos[8*(c + d*x)] + 4862*Sin[c + d*x] - 6188*Sin[3*(c + d*x)]
 + 1700*Sin[5*(c + d*x)] - 119*Sin[7*(c + d*x)] + Sin[9*(c + d*x)]))/(24310
*a^8*d*(1 + Sin[c + d*x])^8)
```

fricas [**A**] time = 0.77, size = 225, normalized size = 0.92

$$\frac{1024\cos(dx+c)^8 - 10752\cos(dx+c)^6 + 29568\cos(dx+c)^4 - 27456\cos(dx+c)^2 + \left(128\cos(dx+\right.}{12155\left(a^8d\cos(dx+c)^9 - 32a^8d\cos(dx+c)^7 + 160a^8d\cos(dx+c)^5 - 256a^8d\cos(dx+c)^3 + 128a^8d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/12155*(1024*cos(d*x + c)^8 - 10752*cos(d*x + c)^6 + 29568*cos(d*x + c)^4
- 27456*cos(d*x + c)^2 + (128*cos(d*x + c)^8 - 4032*cos(d*x + c)^6 + 18480*
```

```
cos(d*x + c)^4 - 24024*cos(d*x + c)^2 + 6435)*sin(d*x + c) + 5720)/(a^8*d*c
os(d*x + c)^9 - 32*a^8*d*cos(d*x + c)^7 + 160*a^8*d*cos(d*x + c)^5 - 256*a^
8*d*cos(d*x + c)^3 + 128*a^8*d*cos(d*x + c) - 8*(a^8*d*cos(d*x + c)^7 - 10*
a^8*d*cos(d*x + c)^5 + 24*a^8*d*cos(d*x + c)^3 - 16*a^8*d*cos(d*x + c))*sin
(d*x + c))
```

giac [A] time = 1.52, size = 249, normalized size = 1.02

$$-\frac{\frac{12155}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}+\frac{6211205\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{16}+55791450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}+303072770\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{14}+1091397450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/3111680*(12155/(a^8*(tan(1/2*d*x + 1/2*c) - 1)) + (6211205*tan(1/2*d*x +
 1/2*c)^16 + 55791450*tan(1/2*d*x + 1/2*c)^15 + 303072770*tan(1/2*d*x + 1/2
*c)^14 + 1091397450*tan(1/2*d*x + 1/2*c)^13 + 2909561798*tan(1/2*d*x + 1/2*
c)^12 + 5901218466*tan(1/2*d*x + 1/2*c)^11 + 9405145178*tan(1/2*d*x + 1/2*c
)^10 + 11877161010*tan(1/2*d*x + 1/2*c)^9 + 12017308160*tan(1/2*d*x + 1/2*c
)^8 + 9710430158*tan(1/2*d*x + 1/2*c)^7 + 6263238566*tan(1/2*d*x + 1/2*c)^6
 + 3172666718*tan(1/2*d*x + 1/2*c)^5 + 1247921210*tan(1/2*d*x + 1/2*c)^4 +
365303990*tan(1/2*d*x + 1/2*c)^3 + 77883902*tan(1/2*d*x + 1/2*c)^2 + 104982
14*tan(1/2*d*x + 1/2*c) + 982907)/(a^8*(tan(1/2*d*x + 1/2*c) + 1)^17))/d
```

maple [A] time = 0.26, size = 280, normalized size = 1.14

$$\frac{-\frac{1}{256\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{256}{17\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{17}}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{16}}-\frac{2752}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{15}}+\frac{1568}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{14}}-\frac{42800}{13\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{13}}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(-1/512/(tan(1/2*d*x+1/2*c)-1)-128/17/(tan(1/2*d*x+1/2*c)+1)^17+64/
(tan(1/2*d*x+1/2*c)+1)^16-1376/5/(tan(1/2*d*x+1/2*c)+1)^15+784/(tan(1/2*d*x
+1/2*c)+1)^14-21400/13/(tan(1/2*d*x+1/2*c)+1)^13+2692/(tan(1/2*d*x+1/2*c)+1
)^12-38954/11/(tan(1/2*d*x+1/2*c)+1)^11+19109/5/(tan(1/2*d*x+1/2*c)+1)^10-6
847/2/(tan(1/2*d*x+1/2*c)+1)^9+10241/4/(tan(1/2*d*x+1/2*c)+1)^8-12799/8/(ta
n(1/2*d*x+1/2*c)+1)^7+13313/16/(tan(1/2*d*x+1/2*c)+1)^6-57083/160/(tan(1/2*
d*x+1/2*c)+1)^5+7937/64/(tan(1/2*d*x+1/2*c)+1)^4-4351/128/(tan(1/2*d*x+1/2*
c)+1)^3+1793/256/(tan(1/2*d*x+1/2*c)+1)^2-511/512/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.60, size = 740, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/12155*(18181*sin(d*x + c)/(cos(d*x + c) + 1) + 128384*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 545224*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1667360*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3612364*sin(d*x + c)^5/(cos(d*x + c) +
1)^5 + 5742464*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 6271096*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 3928496*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 85085
0*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 5289856*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10 - 7137416*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 5989984*sin(d*
x + c)^12/(cos(d*x + c) + 1)^12 - 3607604*sin(d*x + c)^13/(cos(d*x + c) + 1
```

```
)^13 - 1555840*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 486200*sin(d*x + c)^
15/(cos(d*x + c) + 1)^15 - 97240*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 12
155*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 1896)/((a^8 + 16*a^8*sin(d*x +
c)/(cos(d*x + c) + 1) + 119*a^8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 544*a
^8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1700*a^8*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 + 3808*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 6188*a^8*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 7072*a^8*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 + 4862*a^8*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4862*a^8*sin(d*x + c)^10
/(cos(d*x + c) + 1)^10 - 7072*a^8*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 6
188*a^8*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 3808*a^8*sin(d*x + c)^13/(c
os(d*x + c) + 1)^13 - 1700*a^8*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 544*
a^8*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 119*a^8*sin(d*x + c)^16/(cos(d*
x + c) + 1)^16 - 16*a^8*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - a^8*sin(d*x
 + c)^18/(cos(d*x + c) + 1)^18)*d)
```

mupad [B] time = 8.04, size = 233, normalized size = 0.95

$$\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{519571\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{16}-\frac{576147\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{16}+\frac{213707\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}-\frac{183243\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}-\frac{18207\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{16}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^8),x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((519571*cos((5*c)/2 + (5*d*x)/2))/16 - (576147*cos((3*
c)/2 + (3*d*x)/2))/16 + (213707*cos((7*c)/2 + (7*d*x)/2))/16 - (183243*cos(
(9*c)/2 + (9*d*x)/2))/16 - (18207*cos((11*c)/2 + (11*d*x)/2))/16 + (13855*c
os((13*c)/2 + (13*d*x)/2))/16 + (493*cos((15*c)/2 + (15*d*x)/2))/32 - (237*
cos((17*c)/2 + (17*d*x)/2))/32 + (56425*sin(c/2 + (d*x)/2))/2 - (51563*sin(
(3*c)/2 + (3*d*x)/2))/2 - (53191*sin((5*c)/2 + (5*d*x)/2))/2 + (47003*sin((
7*c)/2 + (7*d*x)/2))/2 + (9403*sin((9*c)/2 + (9*d*x)/2))/2 - (7703*sin((11*
c)/2 + (11*d*x)/2))/2 - (355*sin((13*c)/2 + (13*d*x)/2))/2 + 118*sin((15*c)
/2 + (15*d*x)/2) + sin((17*c)/2 + (17*d*x)/2)/2))/(3111680*a^8*d*cos(c/2 -
pi/4 + (d*x)/2)^17*cos(c/2 + pi/4 + (d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.98 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=238

$$\frac{1}{1024d\left(a^8-a^8\sin(c+dx)\right)}-\frac{9}{1024d\left(a^8\sin(c+dx)+a^8\right)}+\frac{5\tanh^{-1}(\sin(c+dx))}{512a^8d}-\frac{7}{768a^5d(a\sin(c+dx)+a)^3}$$

```
[Out] 5/512*arctanh(sin(d*x+c))/a^8/d-1/36*a/d/(a+a*sin(d*x+c))^9-1/32/d/(a+a*sin
(d*x+c))^8-3/112/a/d/(a+a*sin(d*x+c))^7-1/48/a^2/d/(a+a*sin(d*x+c))^6-1/64/
a^3/d/(a+a*sin(d*x+c))^5-7/768/a^5/d/(a+a*sin(d*x+c))^3-3/256/d/(a^2+a^2*si
n(d*x+c))^4-1/128/d/(a^4+a^4*sin(d*x+c))^2+1/1024/d/(a^8-a^8*sin(d*x+c))-9/
1024/d/(a^8+a^8*sin(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$\frac{1}{1024d\left(a^8-a^8\sin(c+dx)\right)}-\frac{9}{1024d\left(a^8\sin(c+dx)+a^8\right)}-\frac{1}{128d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{3}{256d\left(a^2\sin(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(512*a^8*d) - a/(36*d*(a + a*Sin[c + d*x])^9) - 1
/(32*d*(a + a*Sin[c + d*x])^8) - 3/(112*a*d*(a + a*Sin[c + d*x])^7) - 1/(48
*a^2*d*(a + a*Sin[c + d*x])^6) - 1/(64*a^3*d*(a + a*Sin[c + d*x])^5) - 7/(7
68*a^5*d*(a + a*Sin[c + d*x])^3) - 3/(256*d*(a^2 + a^2*Sin[c + d*x])^4) - 1
/(128*d*(a^4 + a^4*Sin[c + d*x])^2) + 1/(1024*d*(a^8 - a^8*Sin[c + d*x])) -
 9/(1024*d*(a^8 + a^8*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{a^3\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{10}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{1024a^{10}(a-x)^2} + \frac{1}{4a^2(a+x)^{10}} + \frac{1}{4a^3(a+x)^9} + \frac{3}{16a^4(a+x)^8} + \frac{1}{8a^5(a+x)^7} + \frac{5}{64a^6(a+x)^6} + \right.\right.}{}$$

$$= -\frac{a}{36d(a+a\sin(c+dx))^9} - \frac{1}{32d(a+a\sin(c+dx))^8} - \frac{3}{112ad(a+a\sin(c+dx))^7} - \frac{}{4}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{512a^8d} - \frac{a}{36d(a+a\sin(c+dx))^9} - \frac{1}{32d(a+a\sin(c+dx))^8} - \frac{}{112ad(}$$

Mathematica [A] time = 1.83, size = 175, normalized size = 0.74

$$-\frac{\sec^2(c+dx)\left(-315\sin^9(c+dx) - 2520\sin^8(c+dx) - 8610\sin^7(c+dx) - 15960\sin^6(c+dx) - 16128\sin^5(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/32256*(Sec[c + d*x]^2*(5120 - 315*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^18 + 9019*Sin
[c + d*x] + 11736*Sin[c + d*x]^2 + 7074*Sin[c + d*x]^3 - 5544*Sin[c + d*x]^
4 - 16128*Sin[c + d*x]^5 - 15960*Sin[c + d*x]^6 - 8610*Sin[c + d*x]^7 - 252
0*Sin[c + d*x]^8 - 315*Sin[c + d*x]^9))/(a^8*d*(1 + Sin[c + d*x])^8)
```

fricas [B] time = 0.84, size = 446, normalized size = 1.87

$$\frac{5040\cos(dx+c)^8 - 52080\cos(dx+c)^6 + 137088\cos(dx+c)^4 - 114624\cos(dx+c)^2 + 315\left(\cos(dx+c)^{10} - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/64512*(5040*cos(d*x + c)^8 - 52080*cos(d*x + c)^6 + 137088*cos(d*x + c)^4
 - 114624*cos(d*x + c)^2 + 315*(cos(d*x + c)^10 - 32*cos(d*x + c)^8 + 160*c
os(d*x + c)^6 - 256*cos(d*x + c)^4 + 128*cos(d*x + c)^2 - 8*(cos(d*x + c)^8
 - 10*cos(d*x + c)^6 + 24*cos(d*x + c)^4 - 16*cos(d*x + c)^2)*sin(d*x + c))
*log(sin(d*x + c) + 1) - 315*(cos(d*x + c)^10 - 32*cos(d*x + c)^8 + 160*cos
(d*x + c)^6 - 256*cos(d*x + c)^4 + 128*cos(d*x + c)^2 - 8*(cos(d*x + c)^8 -
 10*cos(d*x + c)^6 + 24*cos(d*x + c)^4 - 16*cos(d*x + c)^2)*sin(d*x + c))*l
og(-sin(d*x + c) + 1) + 2*(315*cos(d*x + c)^8 - 9870*cos(d*x + c)^6 + 43848
*cos(d*x + c)^4 - 52272*cos(d*x + c)^2 + 8960)*sin(d*x + c) + 14336)/(a^8*d
*cos(d*x + c)^10 - 32*a^8*d*cos(d*x + c)^8 + 160*a^8*d*cos(d*x + c)^6 - 256
*a^8*d*cos(d*x + c)^4 + 128*a^8*d*cos(d*x + c)^2 - 8*(a^8*d*cos(d*x + c)^8
- 10*a^8*d*cos(d*x + c)^6 + 24*a^8*d*cos(d*x + c)^4 - 16*a^8*d*cos(d*x + c)
^2)*sin(d*x + c))
```

giac [A] time = 0.57, size = 166, normalized size = 0.70

$$\frac{\frac{2520\log(|\sin(dx+c)+1|)}{a^8} - \frac{2520\log(|\sin(dx+c)-1|)}{a^8} + \frac{504(5\sin(dx+c)-6)}{a^8(\sin(dx+c)-1)} - \frac{7129\sin(dx+c)^9 + 68697\sin(dx+c)^8 + 296964\sin(dx+c)^7 + 758772\sin}{}}{516096\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/516096*(2520*log(abs(sin(d*x + c) + 1))/a^8 - 2520*log(abs(sin(d*x + c) -
 1))/a^8 + 504*(5*sin(d*x + c) - 6)/(a^8*(sin(d*x + c) - 1)) - (7129*sin(d*
x + c)^9 + 68697*sin(d*x + c)^8 + 296964*sin(d*x + c)^7 + 758772*sin(d*x +
c)^6 + 1271214*sin(d*x + c)^5 + 1465758*sin(d*x + c)^4 + 1191540*sin(d*x +
c)^3 + 693828*sin(d*x + c)^2 + 295425*sin(d*x + c) + 89553)/(a^8*(sin(d*x +
 c) + 1)^9))/d
```

maple [A] time = 0.34, size = 216, normalized size = 0.91

$$-\frac{1}{1024a^8d(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{1024a^8d}-\frac{1}{36a^8d(1+\sin(dx+c))^9}-\frac{1}{32a^8d(1+\sin(dx+c))^8}-\frac{}{112a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x)
```

```
[Out] -1/1024/a^8/d/(sin(d*x+c)-1)-5/1024/a^8/d*ln(sin(d*x+c)-1)-1/36/a^8/d/(1+si
n(d*x+c))^9-1/32/a^8/d/(1+sin(d*x+c))^8-3/112/a^8/d/(1+sin(d*x+c))^7-1/48/a
^8/d/(1+sin(d*x+c))^6-1/64/a^8/d/(1+sin(d*x+c))^5-3/256/a^8/d/(1+sin(d*x+c)
)^4-7/768/a^8/d/(1+sin(d*x+c))^3-1/128/a^8/d/(1+sin(d*x+c))^2-9/1024/a^8/d/
(1+sin(d*x+c))+5/1024/a^8/d*ln(1+sin(d*x+c))
```

maxima [A] time = 0.54, size = 248, normalized size = 1.04

$$-\frac{\frac{2\left(315\sin(dx+c)^9+2520\sin(dx+c)^8+8610\sin(dx+c)^7+15960\sin(dx+c)^6+16128\sin(dx+c)^5+5544\sin(dx+c)^4-7074\sin(dx+c)^3-11736\sin(dx\right.}{a^8\sin(dx+c)^{10}+8a^8\sin(dx+c)^9+27a^8\sin(dx+c)^8+48a^8\sin(dx+c)^7+42a^8\sin(dx+c)^6-42a^8\sin(dx+c)^4-48a^8\sin(dx+c)^3-27a^8\sin(dx+c}}{64512d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/64512*(2*(315*sin(d*x + c)^9 + 2520*sin(d*x + c)^8 + 8610*sin(d*x + c)^7
 + 15960*sin(d*x + c)^6 + 16128*sin(d*x + c)^5 + 5544*sin(d*x + c)^4 - 7074
*sin(d*x + c)^3 - 11736*sin(d*x + c)^2 - 9019*sin(d*x + c) - 5120)/(a^8*sin
(d*x + c)^10 + 8*a^8*sin(d*x + c)^9 + 27*a^8*sin(d*x + c)^8 + 48*a^8*sin(d*
x + c)^7 + 42*a^8*sin(d*x + c)^6 - 42*a^8*sin(d*x + c)^4 - 48*a^8*sin(d*x +
 c)^3 - 27*a^8*sin(d*x + c)^2 - 8*a^8*sin(d*x + c) - a^8) - 315*log(sin(d*x
 + c) + 1)/a^8 + 315*log(sin(d*x + c) - 1)/a^8)/d
```

mupad [B] time = 0.49, size = 231, normalized size = 0.97

$$\frac{\frac{5\sin(c+dx)^9}{512}+\frac{5\sin(c+dx)^8}{64}+\frac{205\sin(c+dx)^7}{768}+\frac{95\sin(c+dx)^6}{192}+\frac{\sin(c+dx)^5}{2}+\frac{11\sin(c+dx}{64}}{d\left(-a^8\sin(c+dx)^{10}-8a^8\sin(c+dx)^9-27a^8\sin(c+dx)^8-48a^8\sin(c+dx)^7-42a^8\sin(c+dx)^6+42\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^8),x)
```

```
[Out] ((11*sin(c + d*x)^4)/64 - (163*sin(c + d*x)^2)/448 - (393*sin(c + d*x)^3)/1
792 - (9019*sin(c + d*x))/32256 + sin(c + d*x)^5/2 + (95*sin(c + d*x)^6)/19
2 + (205*sin(c + d*x)^7)/768 + (5*sin(c + d*x)^8)/64 + (5*sin(c + d*x)^9)/5
12 - 10/63)/(d*(8*a^8*sin(c + d*x) + a^8 + 27*a^8*sin(c + d*x)^2 + 48*a^8*s
in(c + d*x)^3 + 42*a^8*sin(c + d*x)^4 - 42*a^8*sin(c + d*x)^6 - 48*a^8*sin(
c + d*x)^7 - 27*a^8*sin(c + d*x)^8 - 8*a^8*sin(c + d*x)^9 - a^8*sin(c + d*x
)^10)) + (5*atanh(sin(c + d*x)))/(512*a^8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.99 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=279

$$\frac{128\tan^3(c+dx)}{12597a^8d}+\frac{128\tan(c+dx)}{4199a^8d}-\frac{32\sec^3(c+dx)}{4199d\left(a^8\sin(c+dx)+a^8\right)}-\frac{32\sec^3(c+dx)}{4199d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{66\sec^3(}{4199a^3d(a\sin}$$

```
[Out] -1/19*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^8-11/323*sec(d*x+c)^3/a/d/(a+a*sin(d*
x+c))^7-22/969*sec(d*x+c)^3/a^2/d/(a+a*sin(d*x+c))^6-66/4199*sec(d*x+c)^3/a
^3/d/(a+a*sin(d*x+c))^5-48/4199*sec(d*x+c)^3/d/(a^2+a^2*sin(d*x+c))^4-112/1
2597*sec(d*x+c)^3/a^2/d/(a^2+a^2*sin(d*x+c))^3-32/4199*sec(d*x+c)^3/d/(a^4+
a^4*sin(d*x+c))^2-32/4199*sec(d*x+c)^3/d/(a^8+a^8*sin(d*x+c))+128/4199*tan(
d*x+c)/a^8/d+128/12597*tan(d*x+c)^3/a^8/d
```

Rubi [A] time = 0.42, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 3767}

$$\frac{128\tan^3(c+dx)}{12597a^8d}+\frac{128\tan(c+dx)}{4199a^8d}-\frac{32\sec^3(c+dx)}{4199d\left(a^8\sin(c+dx)+a^8\right)}-\frac{32\sec^3(c+dx)}{4199d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{112\,\mathrm{se}}{12597a^2d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -Sec[c + d*x]^3/(19*d*(a + a*Sin[c + d*x])^8) - (11*Sec[c + d*x]^3)/(323*a*
d*(a + a*Sin[c + d*x])^7) - (22*Sec[c + d*x]^3)/(969*a^2*d*(a + a*Sin[c + d
*x])^6) - (66*Sec[c + d*x]^3)/(4199*a^3*d*(a + a*Sin[c + d*x])^5) - (48*Sec
[c + d*x]^3)/(4199*d*(a^2 + a^2*Sin[c + d*x])^4) - (112*Sec[c + d*x]^3)/(12
597*a^2*d*(a^2 + a^2*Sin[c + d*x])^3) - (32*Sec[c + d*x]^3)/(4199*d*(a^4 +
a^4*Sin[c + d*x])^2) - (32*Sec[c + d*x]^3)/(4199*d*(a^8 + a^8*Sin[c + d*x])
) + (128*Tan[c + d*x])/(4199*a^8*d) + (128*Tan[c + d*x]^3)/(12597*a^8*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} + \frac{11\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{19a}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} + \frac{110\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{323a^2}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6}$$

Mathematica [A] time = 0.43, size = 125, normalized size = 0.45

$$\frac{\sec^3(c+dx)(8398\sin(c+dx) - 5814\sin(3(c+dx)) - 2907\sin(5(c+dx)) + 1463\sin(7(c+dx)) - 117\sin(9(c+}{50388a^8d(\sin(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^3*(-10336*Cos[2*(c + d*x)] + 2736*Cos[6*(c + d*x)] - 512*Cos[
8*(c + d*x)] + 16*Cos[10*(c + d*x)] + 8398*Sin[c + d*x] - 5814*Sin[3*(c + d
*x)] - 2907*Sin[5*(c + d*x)] + 1463*Sin[7*(c + d*x)] - 117*Sin[9*(c + d*x)]
 + Sin[11*(c + d*x)]))/(50388*a^8*d*(1 + Sin[c + d*x])^8)
```

fricas [A] time = 0.86, size = 249, normalized size = 0.89

$$\frac{2048\cos(dx+c)^{10} - 21504\cos(dx+c)^8 + 59136\cos(dx+c)^6 - 54912\cos(dx+c)^4 + 11440\cos(dx+c)^2 +}{12597\left(a^8d\cos(dx+c)^{11} - 32a^8d\cos(dx+c)^9 + 160a^8d\cos(dx+c)^7 - 256a^8d\cos(dx+c)^5 + 1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/12597*(2048*cos(d*x + c)^10 - 21504*cos(d*x + c)^8 + 59136*cos(d*x + c)^6
 - 54912*cos(d*x + c)^4 + 11440*cos(d*x + c)^2 + (256*cos(d*x + c)^10 - 806
```

```
4*cos(d*x + c)^8 + 36960*cos(d*x + c)^6 - 48048*cos(d*x + c)^4 + 12870*cos(
d*x + c)^2 + 2431)*sin(d*x + c) + 1768)/(a^8*d*cos(d*x + c)^11 - 32*a^8*d*c
os(d*x + c)^9 + 160*a^8*d*cos(d*x + c)^7 - 256*a^8*d*cos(d*x + c)^5 + 128*a
^8*d*cos(d*x + c)^3 - 8*(a^8*d*cos(d*x + c)^9 - 10*a^8*d*cos(d*x + c)^7 + 2
4*a^8*d*cos(d*x + c)^5 - 16*a^8*d*cos(d*x + c)^3)*sin(d*x + c))
```

giac [A] time = 1.05, size = 301, normalized size = 1.08

$$-\frac{\frac{4199\left(18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-33\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+17\right)}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}+\frac{12823746\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{18}+140368371\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17}+879644311\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{16}+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/6449664*(4199*(18*tan(1/2*d*x + 1/2*c)^2 - 33*tan(1/2*d*x + 1/2*c) + 17)
/(a^8*(tan(1/2*d*x + 1/2*c) - 1)^3) + (12823746*tan(1/2*d*x + 1/2*c)^18 + 1
40368371*tan(1/2*d*x + 1/2*c)^17 + 879644311*tan(1/2*d*x + 1/2*c)^16 + 3693
272440*tan(1/2*d*x + 1/2*c)^15 + 11467502592*tan(1/2*d*x + 1/2*c)^14 + 2740
3194676*tan(1/2*d*x + 1/2*c)^13 + 51919375300*tan(1/2*d*x + 1/2*c)^12 + 791
83835016*tan(1/2*d*x + 1/2*c)^11 + 98304418212*tan(1/2*d*x + 1/2*c)^10 + 99
750226290*tan(1/2*d*x + 1/2*c)^9 + 82860874122*tan(1/2*d*x + 1/2*c)^8 + 561
10430792*tan(1/2*d*x + 1/2*c)^7 + 30766700912*tan(1/2*d*x + 1/2*c)^6 + 1346
2452660*tan(1/2*d*x + 1/2*c)^5 + 4616712644*tan(1/2*d*x + 1/2*c)^4 + 119785
1960*tan(1/2*d*x + 1/2*c)^3 + 226248618*tan(1/2*d*x + 1/2*c)^2 + 27911475*t
an(1/2*d*x + 1/2*c) + 2143959)/(a^8*(tan(1/2*d*x + 1/2*c) + 1)^19))/d
```

maple [A] time = 0.35, size = 340, normalized size = 1.22

$$\frac{-\frac{1}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3}{256\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{256}{19\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{19}}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{18}}-\frac{10496}{17\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{17}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(-1/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/1024/(tan(1/2*d*x+1/2*c)-1)^2-3
/512/(tan(1/2*d*x+1/2*c)-1)-128/19/(tan(1/2*d*x+1/2*c)+1)^19+64/(tan(1/2*d*
x+1/2*c)+1)^18-5248/17/(tan(1/2*d*x+1/2*c)+1)^17+992/(tan(1/2*d*x+1/2*c)+1)
^16-7096/3/(tan(1/2*d*x+1/2*c)+1)^15+4428/(tan(1/2*d*x+1/2*c)+1)^14-87508/1
3/(tan(1/2*d*x+1/2*c)+1)^13+25468/3/(tan(1/2*d*x+1/2*c)+1)^12-18011/2/(tan(
1/2*d*x+1/2*c)+1)^11+32417/4/(tan(1/2*d*x+1/2*c)+1)^10-6215/(tan(1/2*d*x+1/
2*c)+1)^9+32525/8/(tan(1/2*d*x+1/2*c)+1)^8-72425/32/(tan(1/2*d*x+1/2*c)+1)^
7+204605/192/(tan(1/2*d*x+1/2*c)+1)^6-26871/64/(tan(1/2*d*x+1/2*c)+1)^5+217
7/16/(tan(1/2*d*x+1/2*c)+1)^4-54229/1536/(tan(1/2*d*x+1/2*c)+1)^3+7181/1024
/(tan(1/2*d*x+1/2*c)+1)^2-509/512/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.63, size = 866, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/12597*(19787*sin(d*x + c)/(cos(d*x + c) + 1) + 136032*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 540806*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1483064*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2552175*sin(d*x + c)^5/(cos(d*x + c) +
1)^5 + 2356608*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1108536*sin(d*x + c)^7
```

```
/(cos(d*x + c) + 1)^7 - 6930288*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10934
842*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 7793344*sin(d*x + c)^10/(cos(d*x
+ c) + 1)^10 + 1058148*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 9204208*sin(
d*x + c)^12/(cos(d*x + c) + 1)^12 + 9985222*sin(d*x + c)^13/(cos(d*x + c) +
 1)^13 + 4837248*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 1108536*sin(d*x +
c)^15/(cos(d*x + c) + 1)^15 - 3527160*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
 - 2985489*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 1478048*sin(d*x + c)^18/
(cos(d*x + c) + 1)^18 - 495482*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 1007
76*sin(d*x + c)^20/(cos(d*x + c) + 1)^20 - 12597*sin(d*x + c)^21/(cos(d*x +
 c) + 1)^21 + 2024)/((a^8 + 16*a^8*sin(d*x + c)/(cos(d*x + c) + 1) + 117*a^
8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 512*a^8*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 1463*a^8*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2736*a^8*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 + 2907*a^8*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
- 5814*a^8*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10336*a^8*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9 - 8398*a^8*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 8398
*a^8*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 10336*a^8*sin(d*x + c)^13/(cos
(d*x + c) + 1)^13 + 5814*a^8*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 2907*a
^8*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 2736*a^8*sin(d*x + c)^17/(cos(d*
x + c) + 1)^17 - 1463*a^8*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 512*a^8*s
in(d*x + c)^19/(cos(d*x + c) + 1)^19 - 117*a^8*sin(d*x + c)^20/(cos(d*x + c
) + 1)^20 - 16*a^8*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 - a^8*sin(d*x + c)
^22/(cos(d*x + c) + 1)^22)*d)
```

mupad [B] time = 9.23, size = 277, normalized size = 0.99

$$\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{896971\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{64}-\frac{1062347\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{64}-\frac{40375\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{40375\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}+\frac{412471\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{128}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^8),x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((896971*cos((5*c)/2 + (5*d*x)/2))/64 - (1062347*cos((3
*c)/2 + (3*d*x)/2))/64 - (40375*cos((7*c)/2 + (7*d*x)/2))/16 + (40375*cos((
9*c)/2 + (9*d*x)/2))/16 + (412471*cos((11*c)/2 + (11*d*x)/2))/128 - (324919
*cos((13*c)/2 + (13*d*x)/2))/128 - (11305*cos((15*c)/2 + (15*d*x)/2))/32 +
(7209*cos((17*c)/2 + (17*d*x)/2))/32 + (765*cos((19*c)/2 + (19*d*x)/2))/128
 - (253*cos((21*c)/2 + (21*d*x)/2))/128 + (65033*sin(c/2 + (d*x)/2))/4 - (5
6635*sin((3*c)/2 + (3*d*x)/2))/4 - 6271*sin((5*c)/2 + (5*d*x)/2) + (9635*si
n((7*c)/2 + (7*d*x)/2))/2 - (9635*sin((9*c)/2 + (9*d*x)/2))/2 + (16363*sin(
(11*c)/2 + (11*d*x)/2))/4 + (10537*sin((13*c)/2 + (13*d*x)/2))/8 - (7611*si
n((15*c)/2 + (15*d*x)/2))/8 - (485*sin((17*c)/2 + (17*d*x)/2))/8 + (251*sin
((19*c)/2 + (19*d*x)/2))/8 + sin((21*c)/2 + (21*d*x)/2)/4))/(12899328*a^8*d
*cos(c/2 - pi/4 + (d*x)/2)^19*cos(c/2 + pi/4 + (d*x)/2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.100 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=284

$$\frac{11}{4096d\left(a^8-a^8\sin(c+dx)\right)}-\frac{55}{4096d\left(a^8\sin(c+dx)+a^8\right)}+\frac{33\tanh^{-1}(\sin(c+dx))}{2048a^8d}-\frac{3}{256a^5d(a\sin(c+dx)+a)^{}}$$

```
[Out] 33/2048*arctanh(sin(d*x+c))/a^8/d-1/80*a^2/d/(a+a*sin(d*x+c))^10-1/48*a/d/(
a+a*sin(d*x+c))^9-3/128/d/(a+a*sin(d*x+c))^8-5/224/a/d/(a+a*sin(d*x+c))^7-5
/256/a^2/d/(a+a*sin(d*x+c))^6-21/1280/a^3/d/(a+a*sin(d*x+c))^5-3/256/a^5/d/
(a+a*sin(d*x+c))^3-7/512/d/(a^2+a^2*sin(d*x+c))^4+1/4096/d/(a^4-a^4*sin(d*x
+c))^2-45/4096/d/(a^4+a^4*sin(d*x+c))^2+11/4096/d/(a^8-a^8*sin(d*x+c))-55/4
096/d/(a^8+a^8*sin(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2667, 44, 206}

$$-\frac{a^2}{80d(a\sin(c+dx)+a)^{10}}+\frac{11}{4096d\left(a^8-a^8\sin(c+dx)\right)}-\frac{55}{4096d\left(a^8\sin(c+dx)+a^8\right)}+\frac{1}{4096d\left(a^4-a^4\sin(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (33*ArcTanh[Sin[c + d*x]])/(2048*a^8*d) - a^2/(80*d*(a + a*Sin[c + d*x])^10
) - a/(48*d*(a + a*Sin[c + d*x])^9) - 3/(128*d*(a + a*Sin[c + d*x])^8) - 5/
(224*a*d*(a + a*Sin[c + d*x])^7) - 5/(256*a^2*d*(a + a*Sin[c + d*x])^6) - 2
1/(1280*a^3*d*(a + a*Sin[c + d*x])^5) - 3/(256*a^5*d*(a + a*Sin[c + d*x])^3
) - 7/(512*d*(a^2 + a^2*Sin[c + d*x])^4) + 1/(4096*d*(a^4 - a^4*Sin[c + d*x
])^2) - 45/(4096*d*(a^4 + a^4*Sin[c + d*x])^2) + 11/(4096*d*(a^8 - a^8*Sin[
c + d*x])) - 55/(4096*d*(a^8 + a^8*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{1}{2048a^{11}(a-x)^3} + \frac{11}{4096a^{12}(a-x)^2} + \frac{1}{8a^3(a+x)^{11}} + \frac{3}{16a^4(a+x)^{10}} + \frac{3}{16a^5(a+x)^9} + \frac{5}{32a^6(a-}\right.\right.}{}$$

$$= -\frac{a^2}{80d(a+a\sin(c+dx))^{10}} - \frac{a}{48d(a+a\sin(c+dx))^9} - \frac{3}{128d(a+a\sin(c+dx))^8} - \frac{}{2}$$

$$= \frac{33\tanh^{-1}(\sin(c+dx))}{2048a^8d} - \frac{a^2}{80d(a+a\sin(c+dx))^{10}} - \frac{a}{48d(a+a\sin(c+dx))^9} - \frac{}{128a}$$

Mathematica [A] time = 2.61, size = 195, normalized size = 0.69

$$\frac{\sec^4(c+dx)\left(-3465\sin^{11}(c+dx) - 27720\sin^{10}(c+dx) - 91245\sin^9(c+dx) - 147840\sin^8(c+dx) - 82698\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^4*(-34816 + 3465*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^20 - 66953*Sin[c +
d*x] - 72776*Sin[c + d*x]^2 + 21395*Sin[c + d*x]^3 + 190080*Sin[c + d*x]^4
+ 255222*Sin[c + d*x]^5 + 114576*Sin[c + d*x]^6 - 82698*Sin[c + d*x]^7 - 14
7840*Sin[c + d*x]^8 - 91245*Sin[c + d*x]^9 - 27720*Sin[c + d*x]^10 - 3465*S
in[c + d*x]^11))/(215040*a^8*d*(1 + Sin[c + d*x])^8)
```

fricas [A] time = 0.95, size = 466, normalized size = 1.64

$$\frac{55440\cos(dx+c)^{10} - 572880\cos(dx+c)^8 + 1507968\cos(dx+c)^6 - 1260864\cos(dx+c)^4 + 157696\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/430080*(55440*cos(d*x + c)^10 - 572880*cos(d*x + c)^8 + 1507968*cos(d*x +
 c)^6 - 1260864*cos(d*x + c)^4 + 157696*cos(d*x + c)^2 + 3465*(cos(d*x + c)
^12 - 32*cos(d*x + c)^10 + 160*cos(d*x + c)^8 - 256*cos(d*x + c)^6 + 128*co
s(d*x + c)^4 - 8*(cos(d*x + c)^10 - 10*cos(d*x + c)^8 + 24*cos(d*x + c)^6 -
 16*cos(d*x + c)^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - 3465*(cos(d*x + c
)^12 - 32*cos(d*x + c)^10 + 160*cos(d*x + c)^8 - 256*cos(d*x + c)^6 + 128*c
os(d*x + c)^4 - 8*(cos(d*x + c)^10 - 10*cos(d*x + c)^8 + 24*cos(d*x + c)^6
- 16*cos(d*x + c)^4)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(3465*cos(d*x
 + c)^10 - 108570*cos(d*x + c)^8 + 482328*cos(d*x + c)^6 - 574992*cos(d*x +
 c)^4 + 98560*cos(d*x + c)^2 + 32256)*sin(d*x + c) + 43008)/(a^8*d*cos(d*x
+ c)^12 - 32*a^8*d*cos(d*x + c)^10 + 160*a^8*d*cos(d*x + c)^8 - 256*a^8*d*c
os(d*x + c)^6 + 128*a^8*d*cos(d*x + c)^4 - 8*(a^8*d*cos(d*x + c)^10 - 10*a^
8*d*cos(d*x + c)^8 + 24*a^8*d*cos(d*x + c)^6 - 16*a^8*d*cos(d*x + c)^4)*sin
(d*x + c))
```

giac [A] time = 0.62, size = 186, normalized size = 0.65

$$\frac{\frac{27720\log(|\sin(dx+c)+1|)}{a^8} - \frac{27720\log(|\sin(dx+c)-1|)}{a^8} + \frac{420\left(99\sin(dx+c)^2 - 220\sin(dx+c)+123\right)}{a^8(\sin(dx+c)-1)^2} - \frac{81191\sin(dx+c)^{10} + 858110\sin(dx+c)^9 + 410}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3440640*(27720*log(abs(sin(d*x + c) + 1))/a^8 - 27720*log(abs(sin(d*x + c
) - 1))/a^8 + 420*(99*sin(d*x + c)^2 - 220*sin(d*x + c) + 123)/(a^8*(sin(d*
x + c) - 1)^2) - (81191*sin(d*x + c)^10 + 858110*sin(d*x + c)^9 + 4107195*s
in(d*x + c)^8 + 11748840*sin(d*x + c)^7 + 22318590*sin(d*x + c)^6 + 2958354
0*sin(d*x + c)^5 + 27983550*sin(d*x + c)^4 + 19002600*sin(d*x + c)^3 + 9206
235*sin(d*x + c)^2 + 3108990*sin(d*x + c) + 648327)/(a^8*(sin(d*x + c) + 1)
^10))/d
```

maple [A] time = 0.36, size = 252, normalized size = 0.89

$$\frac{1}{4096a^8d\left(\sin\left(dx+c\right)-1\right)^2}-\frac{11}{4096a^8d\left(\sin\left(dx+c\right)-1\right)}-\frac{33\ln\left(\sin\left(dx+c\right)-1\right)}{4096a^8d}-\frac{1}{80a^8d\left(1+\sin\left(dx+c\right)\right)^{10}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/4096/a^8/d/(sin(d*x+c)-1)^2-11/4096/a^8/d/(sin(d*x+c)-1)-33/4096/a^8/d*ln
(sin(d*x+c)-1)-1/80/a^8/d/(1+sin(d*x+c))^10-1/48/a^8/d/(1+sin(d*x+c))^9-3/1
28/a^8/d/(1+sin(d*x+c))^8-5/224/a^8/d/(1+sin(d*x+c))^7-5/256/a^8/d/(1+sin(d
*x+c))^6-21/1280/a^8/d/(1+sin(d*x+c))^5-7/512/a^8/d/(1+sin(d*x+c))^4-3/256/
a^8/d/(1+sin(d*x+c))^3-45/4096/a^8/d/(1+sin(d*x+c))^2-55/4096/a^8/d/(1+sin(
d*x+c))+33/4096/a^8/d*ln(1+sin(d*x+c))
```

maxima [A] time = 0.98, size = 305, normalized size = 1.07

$$-\frac{\frac{2\left(3465\sin(dx+c)^{11}+27720\sin(dx+c)^{10}+91245\sin(dx+c)^{9}+147840\sin(dx+c)^{8}+82698\sin(dx+c)^{7}-114576\sin(dx+c)^{6}-255222\sin(dx+c)^{5}-\cdots\right.}{a^8\sin(dx+c)^{12}+8a^8\sin(dx+c)^{11}+26a^8\sin(dx+c)^{10}+40a^8\sin(dx+c)^{9}+15a^8\sin(dx+c)^{8}-48a^8\sin(dx+c)^{7}-84a^8\sin(dx+c)^{6}-48a^8\sin(dx+\cdots}}{43008\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/430080*(2*(3465*sin(d*x + c)^11 + 27720*sin(d*x + c)^10 + 91245*sin(d*x
+ c)^9 + 147840*sin(d*x + c)^8 + 82698*sin(d*x + c)^7 - 114576*sin(d*x + c)
^6 - 255222*sin(d*x + c)^5 - 190080*sin(d*x + c)^4 - 21395*sin(d*x + c)^3 +
 72776*sin(d*x + c)^2 + 66953*sin(d*x + c) + 34816)/(a^8*sin(d*x + c)^12 +
8*a^8*sin(d*x + c)^11 + 26*a^8*sin(d*x + c)^10 + 40*a^8*sin(d*x + c)^9 + 15
*a^8*sin(d*x + c)^8 - 48*a^8*sin(d*x + c)^7 - 84*a^8*sin(d*x + c)^6 - 48*a^
8*sin(d*x + c)^5 + 15*a^8*sin(d*x + c)^4 + 40*a^8*sin(d*x + c)^3 + 26*a^8*s
in(d*x + c)^2 + 8*a^8*sin(d*x + c) + a^8) - 3465*log(sin(d*x + c) + 1)/a^8
+ 3465*log(sin(d*x + c) - 1)/a^8)/d
```

mupad [B] time = 0.80, size = 290, normalized size = 1.02

$$\frac{33\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{2048\,a^8\,d}-\frac{\frac{33\sin(c+d\,x)^{11}}{2048}+\frac{33\sin(c+d\,x)^{10}}{256}+\frac{869\sin(c+d\,x\cdots}{2048}\cdots}{d\,\left(a^8\sin\left(c+d\,x\right)^{12}+8\,a^8\sin\left(c+d\,x\right)^{11}+26\,a^8\sin\left(c+d\,x\right)^{10}+40\,a^8\sin\left(c+d\,x\right)^{9}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^8),x)
```

```
[Out] (33*atanh(sin(c + d*x)))/(2048*a^8*d) - ((66953*sin(c + d*x))/215040 + (909
7*sin(c + d*x)^2)/26880 - (4279*sin(c + d*x)^3)/43008 - (99*sin(c + d*x)^4)
/112 - (42537*sin(c + d*x)^5)/35840 - (341*sin(c + d*x)^6)/640 + (1969*sin(
c + d*x)^7)/5120 + (11*sin(c + d*x)^8)/16 + (869*sin(c + d*x)^9)/2048 + (33
*sin(c + d*x)^10)/256 + (33*sin(c + d*x)^11)/2048 + 17/105)/(d*(8*a^8*sin(c
```

```
 + d*x) + a^8 + 26*a^8*sin(c + d*x)^2 + 40*a^8*sin(c + d*x)^3 + 15*a^8*sin(
c + d*x)^4 - 48*a^8*sin(c + d*x)^5 - 84*a^8*sin(c + d*x)^6 - 48*a^8*sin(c +
 d*x)^7 + 15*a^8*sin(c + d*x)^8 + 40*a^8*sin(c + d*x)^9 + 26*a^8*sin(c + d*
x)^10 + 8*a^8*sin(c + d*x)^11 + a^8*sin(c + d*x)^12))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**8,x)
[Out] Timed out
```

3.101 $\int \cos^7(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{15/2}}{15a^7d}+\frac{12(a\sin(c+dx)+a)^{13/2}}{13a^6d}-\frac{24(a\sin(c+dx)+a)^{11/2}}{11a^5d}+\frac{16(a\sin(c+dx)+a)^{9/2}}{9a^4d}$$

```
[Out] 16/9*(a+a*sin(d*x+c))^(9/2)/a^4/d-24/11*(a+a*sin(d*x+c))^(11/2)/a^5/d+12/13
*(a+a*sin(d*x+c))^(13/2)/a^6/d-2/15*(a+a*sin(d*x+c))^(15/2)/a^7/d
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{15/2}}{15a^7d}+\frac{12(a\sin(c+dx)+a)^{13/2}}{13a^6d}-\frac{24(a\sin(c+dx)+a)^{11/2}}{11a^5d}+\frac{16(a\sin(c+dx)+a)^{9/2}}{9a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(9/2))/(9*a^4*d) - (24*(a + a*Sin[c + d*x])^(11/2)
)/(11*a^5*d) + (12*(a + a*Sin[c + d*x])^(13/2))/(13*a^6*d) - (2*(a + a*Sin[
c + d*x])^(15/2))/(15*a^7*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{7/2}\,dx, x, a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{7/2}-12a^2(a+x)^{9/2}+6a(a+x)^{11/2}-(a+x)^{13/2}\right)\right.}{a^7d}\\ &= \frac{16(a+a\sin(c+dx))^{9/2}}{9a^4d}-\frac{24(a+a\sin(c+dx))^{11/2}}{11a^5d}+\frac{12(a+a\sin(c+\right.}{13a^6d}\end{aligned}$$

Mathematica [A] time = 4.32, size = 74, normalized size = 0.76

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^8(-10755\sin(c+dx)+429\sin(3(c+dx))-3366\cos}{12870d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Sqrt[a*(1 + Sin[c + d*x])]*(8330 -
 3366*Cos[2*(c + d*x)] - 10755*Sin[c + d*x] + 429*Sin[3*(c + d*x)]))/(12870
*d)
```

fricas [A] time = 0.64, size = 88, normalized size = 0.91

$$\frac{2\left(33\,\cos\left(dx+c\right)^{6}+56\,\cos\left(dx+c\right)^{4}+128\,\cos\left(dx+c\right)^{2}+\left(429\,\cos\left(dx+c\right)^{6}+504\,\cos\left(dx+c\right)^{4}+640\,\cos\left(d\right.\right.\right.}{6435\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/6435*(33*cos(d*x + c)^6 + 56*cos(d*x + c)^4 + 128*cos(d*x + c)^2 + (429*c
os(d*x + c)^6 + 504*cos(d*x + c)^4 + 640*cos(d*x + c)^2 + 1024)*sin(d*x + c
) + 1024)*sqrt(a*sin(d*x + c) + a)/d
```

giac [B] time = 2.63, size = 249, normalized size = 2.57

$$\frac{1}{411840}\sqrt{2}\sqrt{a}\left(\frac{495\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{4}\,\pi+\frac{13}{2}\,dx+\frac{13}{2}\,c\right)}{d}+\frac{5005\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/411840*sqrt(2)*sqrt(a)*(495*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*p
i + 13/2*d*x + 13/2*c)/d + 5005*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4
*pi + 9/2*d*x + 9/2*c)/d + 27027*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/
4*pi + 5/2*d*x + 5/2*c)/d + 225225*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(
1/4*pi + 1/2*d*x + 1/2*c)/d + 429*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 15/2*d*x + 15/2*c)/d + 4095*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(-1/4*pi + 11/2*d*x + 11/2*c)/d + 19305*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin(-1/4*pi + 7/2*d*x + 7/2*c)/d + 75075*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*sin(-1/4*pi + 3/2*d*x + 3/2*c)/d)
```

maple [A] time = 0.20, size = 57, normalized size = 0.59

$$\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{9}{2}}\left(429\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-1683\left(\cos^{2}\left(dx+c\right)\right)-2796\sin\left(dx+c\right)+2924\right)}{6435a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/6435/a^4*(a+a*sin(d*x+c))^(9/2)*(429*cos(d*x+c)^2*sin(d*x+c)-1683*cos(d*x
+c)^2-2796*sin(d*x+c)+2924)/d
```

maxima [A] time = 0.38, size = 72, normalized size = 0.74

$$-\frac{2\left(429\left(a\sin\left(dx+c\right)+a\right)^{\frac{15}{2}}-2970\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}a+7020\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}a^{2}-5720\left(a\sin\left(dx+c\right)\right.\right.}{6435\,a^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/6435*(429*(a*sin(d*x + c) + a)^(15/2) - 2970*(a*sin(d*x + c) + a)^(13/2)
*a + 7020*(a*sin(d*x + c) + a)^(11/2)*a^2 - 5720*(a*sin(d*x + c) + a)^(9/2)
*a^3)/(a^7*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^7 \sqrt{a+a\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.102 $\int \cos^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=127

$$-\frac{256a^4\cos^7(c+dx)}{3003d(a\sin(c+dx)+a)^{7/2}}-\frac{64a^3\cos^7(c+dx)}{429d(a\sin(c+dx)+a)^{5/2}}-\frac{24a^2\cos^7(c+dx)}{143d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^7(c+dx)}{13d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -256/3003*a^4*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)-64/429*a^3*cos(d*x+c)^7
/d/(a+a*sin(d*x+c))^(5/2)-24/143*a^2*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(3/2)-
2/13*a*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{24a^2\cos^7(c+dx)}{143d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^7(c+dx)}{429d(a\sin(c+dx)+a)^{5/2}}-\frac{256a^4\cos^7(c+dx)}{3003d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{13d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^7)/(3003*d*(a + a*Sin[c + d*x])^(7/2)) - (64*a^3*Cos
[c + d*x]^7)/(429*d*(a + a*Sin[c + d*x])^(5/2)) - (24*a^2*Cos[c + d*x]^7)/(
143*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^7)/(13*d*Sqrt[a + a*S
in[c + d*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= -\frac{2a\cos^7(c+dx)}{13d\sqrt{a+a\sin(c+dx)}}+\frac{1}{13}(12a)\int \frac{\cos^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx\\ &= -\frac{24a^2\cos^7(c+dx)}{143d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^7(c+dx)}{13d\sqrt{a+a\sin(c+dx)}}+\frac{1}{143}\left(96a^2\right)\int\\ &= -\frac{64a^3\cos^7(c+dx)}{429d(a+a\sin(c+dx))^{5/2}}-\frac{24a^2\cos^7(c+dx)}{143d(a+a\sin(c+dx))^{3/2}}-\\ &= -\frac{256a^4\cos^7(c+dx)}{3003d(a+a\sin(c+dx))^{7/2}}-\frac{64a^3\cos^7(c+dx)}{429d(a+a\sin(c+dx))^{5/2}}-\end{aligned}$$

Mathematica [A] time = 3.94, size = 99, normalized size = 0.78

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7(-6377\sin(c+dx)+231\sin(3(c+dx))+1890\cos(}{6006d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*Sqrt[a*(1 + Sin[c + d*x])]*(-5230
+ 1890*Cos[2*(c + d*x)] - 6377*Sin[c + d*x] + 231*Sin[3*(c + d*x)]))/(6006*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.67, size = 172, normalized size = 1.35

$$-\frac{2\left(231\cos(dx+c)^7-21\cos(dx+c)^6+28\cos(dx+c)^5-40\cos(dx+c)^4+64\cos(dx+c)^3-128\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3003*(231*cos(d*x + c)^7 - 21*cos(d*x + c)^6 + 28*cos(d*x + c)^5 - 40*co
s(d*x + c)^4 + 64*cos(d*x + c)^3 - 128*cos(d*x + c)^2 - (231*cos(d*x + c)^6
 + 252*cos(d*x + c)^5 + 280*cos(d*x + c)^4 + 320*cos(d*x + c)^3 + 384*cos(d
*x + c)^2 + 512*cos(d*x + c) + 1024)*sin(d*x + c) + 512*cos(d*x + c) + 1024
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [A] time = 1.12, size = 219, normalized size = 1.72

$$\frac{1}{96096}\sqrt{2}\sqrt{a}\left(\frac{273\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{4}\pi+\frac{11}{2}dx+\frac{11}{2}c\right)}{d}+\frac{2574\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96096*sqrt(2)*sqrt(a)*(273*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi
 + 11/2*d*x + 11/2*c)/d + 2574*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*
pi + 7/2*d*x + 7/2*c)/d + 15015*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4
*pi + 3/2*d*x + 3/2*c)/d + 231*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 13/2*d*x + 13/2*c)/d + 2002*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 9/2*d*x + 9/2*c)/d + 9009*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(
-1/4*pi + 5/2*d*x + 5/2*c)/d + 60060*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(-1/4*pi + 1/2*d*x + 1/2*c)/d)
```

maple [A] time = 0.20, size = 75, normalized size = 0.59

$$-\frac{2(1+\sin(dx+c))\,a\,(\sin(dx+c)-1)^4\left(231\left(\sin^3(dx+c)\right)+945\left(\sin^2(dx+c)\right)+1421\sin(dx+c)+835\right)}{3003\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/3003*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^4*(231*sin(d*x+c)^3+945*sin(d*x+c)^
2+1421*sin(d*x+c)+835)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^6\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.103 $\int \cos^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^5d}-\frac{8(a\sin(c+dx)+a)^{9/2}}{9a^4d}+\frac{8(a\sin(c+dx)+a)^{7/2}}{7a^3d}$$

```
[Out] 8/7*(a+a*sin(d*x+c))^(7/2)/a^3/d-8/9*(a+a*sin(d*x+c))^(9/2)/a^4/d+2/11*(a+a
*sin(d*x+c))^(11/2)/a^5/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^5d}-\frac{8(a\sin(c+dx)+a)^{9/2}}{9a^4d}+\frac{8(a\sin(c+dx)+a)^{7/2}}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(7/2))/(7*a^3*d) - (8*(a + a*Sin[c + d*x])^(9/2))/(
9*a^4*d) + (2*(a + a*Sin[c + d*x])^(11/2))/(11*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{5/2}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{5/2}-4a(a+x)^{7/2}+(a+x)^{9/2}\right)dx,x,a\sin(c+dx\right.}{a^5d}\\ &= \frac{8(a+a\sin(c+dx))^{7/2}}{7a^3d}-\frac{8(a+a\sin(c+dx))^{9/2}}{9a^4d}+\frac{2(a+a\sin(c+dx}{11a^5d}\end{aligned}$$

Mathematica [A] time = 1.02, size = 64, normalized size = 0.88

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6(364\sin(c+dx)+63\cos(2(c+dx))-365)}{693d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/693*((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*Sqrt[a*(1 + Sin[c + d*x])]*
(-365 + 63*Cos[2*(c + d*x)] + 364*Sin[c + d*x]))/d
```

fricas [A] time = 0.57, size = 68, normalized size = 0.93

$$\frac{2\left(7\,\cos\left(dx+c\right)^{4}+16\,\cos\left(dx+c\right)^{2}+\left(63\,\cos\left(dx+c\right)^{4}+80\,\cos\left(dx+c\right)^{2}+128\right)\sin\left(dx+c\right)+128\right)\sqrt{a\sin\left(dx\right.}}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(7*cos(d*x + c)^4 + 16*cos(d*x + c)^2 + (63*cos(d*x + c)^4 + 80*cos(d
*x + c)^2 + 128)*sin(d*x + c) + 128)*sqrt(a*sin(d*x + c) + a)/d
```

giac [B] time = 1.70, size = 189, normalized size = 2.59

$$\frac{1}{11088}\sqrt{2}\sqrt{a}\left(\frac{77\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{4}\,\pi+\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}{d}+\frac{693\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/11088*sqrt(2)*sqrt(a)*(77*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi
+ 9/2*d*x + 9/2*c)/d + 693*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
 5/2*d*x + 5/2*c)/d + 6930*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
 1/2*d*x + 1/2*c)/d + 63*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
11/2*d*x + 11/2*c)/d + 495*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
+ 7/2*d*x + 7/2*c)/d + 2310*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
 + 3/2*d*x + 3/2*c)/d)
```

maple [A] time = 0.16, size = 41, normalized size = 0.56

$$-\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{7}{2}}\left(63\left(\cos^{2}\left(dx+c\right)\right)+182\sin\left(dx+c\right)-214\right)}{693a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/693/a^3*(a+a*sin(d*x+c))^(7/2)*(63*cos(d*x+c)^2+182*sin(d*x+c)-214)/d
```

maxima [A] time = 0.30, size = 55, normalized size = 0.75

$$\frac{2\left(63\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-308\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a+396\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a^{2}\right)}{693\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(a*sin(d*x + c) + a)^(11/2) - 308*(a*sin(d*x + c) + a)^(9/2)*a +
396*(a*sin(d*x + c) + a)^(7/2)*a^2)/(a^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.104 $\int \cos^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{64a^3\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}-\frac{16a^2\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^5(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -64/315*a^3*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-16/63*a^2*cos(d*x+c)^5/d/
(a+a*sin(d*x+c))^(3/2)-2/9*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2674, 2673}

$$-\frac{16a^2\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) - (16*a^2*Cos[c
 + d*x]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^5)/(9*d*Sq
rt[a + a*Sin[c + d*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= -\frac{2a\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{1}{9}(8a)\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx\\ &= -\frac{16a^2\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{1}{63}\left(32a^2\right)\int \frac{c}{(a+}\\ &= -\frac{64a^3\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}}-\frac{16a^2\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^5(c+}{9d\sqrt{a+a\sin(}}\end{aligned}$$

Mathematica [A] time = 0.77, size = 89, normalized size = 0.94

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(220\sin(c+dx)-35\cos(2(c+dx))+249)}{315d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/315*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*
(249 - 35*Cos[2*(c + d*x)] + 220*Sin[c + d*x]))/(d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]))
```

fricas [A] time = 0.63, size = 132, normalized size = 1.39

$$-\frac{2\left(35\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{4}+8\cos\left(dx+c\right)^{3}-16\cos\left(dx+c\right)^{2}-\left(35\cos\left(dx+c\right)^{4}+40\cos\left(dx+\right.\right.\right.}{315\left(d\cos\left(dx+c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 - 5*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 16*cos(d*
x + c)^2 - (35*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 48*cos(d*x + c)^2 + 64*
cos(d*x + c) + 128)*sin(d*x + c) + 64*cos(d*x + c) + 128)*sqrt(a*sin(d*x +
c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [A] time = 1.09, size = 159, normalized size = 1.67

$$\frac{1}{2520}\sqrt{2}\sqrt{a}\left(\frac{45\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{4}\pi+\frac{7}{2}dx+\frac{7}{2}c\right)}{d}+\frac{420\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\mathrm{si}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*sqrt(a)*(45*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
 7/2*d*x + 7/2*c)/d + 420*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
3/2*d*x + 3/2*c)/d + 35*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 9
/2*d*x + 9/2*c)/d + 252*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5
/2*d*x + 5/2*c)/d + 1890*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)/d)
```

maple [A] time = 0.20, size = 65, normalized size = 0.68

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a\left(\sin\left(dx+c\right)-1\right)^{3}\left(35\left(\sin^{2}\left(dx+c\right)\right)+110\sin\left(dx+c\right)+107\right)}{315\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^3*(35*sin(d*x+c)^2+110*sin(d*x+c)+107
)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\cos^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**4, x)
```

3.105 $\int \cos^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{4(a\sin(c+dx)+a)^{5/2}}{5a^2d}-\frac{2(a\sin(c+dx)+a)^{7/2}}{7a^3d}$$

```
[Out] 4/5*(a+a*sin(d*x+c))^(5/2)/a^2/d-2/7*(a+a*sin(d*x+c))^(7/2)/a^3/d
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{5/2}}{5a^2d}-\frac{2(a\sin(c+dx)+a)^{7/2}}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(5/2))/(5*a^2*d) - (2*(a + a*Sin[c + d*x])^(7/2))/(
7*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^{3/2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a(a+x)^{3/2}-(a+x)^{5/2}\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{4(a+a\sin(c+dx))^{5/2}}{5a^2d}-\frac{2(a+a\sin(c+dx))^{7/2}}{7a^3d}\end{aligned}$$

Mathematica [A] time = 0.23, size = 54, normalized size = 1.10

$$-\frac{2(5\sin(c+dx)-9)\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Sqrt[a*(1 + Sin[c + d*x])]*(-9
+ 5*Sin[c + d*x]))/(35*d)
```

fricas [A] time = 0.78, size = 46, normalized size = 0.94

$$\frac{2\left(\cos\left(dx+c\right)^{2}+\left(5\,\cos\left(dx+c\right)^{2}+8\right)\sin\left(dx+c\right)+8\right)\sqrt{a\sin\left(dx+c\right)+a}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(cos(d*x + c)^2 + (5*cos(d*x + c)^2 + 8)*sin(d*x + c) + 8)*sqrt(a*sin(
d*x + c) + a)/d
```

giac [B] time = 0.69, size = 129, normalized size = 2.63

$$\frac{1}{140}\sqrt{2}\sqrt{a}\left(\frac{7\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{105\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{4}\,\pi\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/140*sqrt(2)*sqrt(a)*(7*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 5
/2*d*x + 5/2*c)/d + 105*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 1/
2*d*x + 1/2*c)/d + 5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 7/2*
d*x + 7/2*c)/d + 35*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 3/2*d
*x + 3/2*c)/d)
```

maple [A] time = 0.15, size = 31, normalized size = 0.63

$$-\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{5}{2}}\left(5\sin\left(dx+c\right)-9\right)}{35a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/35/a^2*(a+a*sin(d*x+c))^(5/2)*(5*sin(d*x+c)-9)/d
```

maxima [A] time = 0.44, size = 38, normalized size = 0.78

$$-\frac{2\left(5\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}-14\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a\right)}{35\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/35*(5*(a*sin(d*x + c) + a)^(7/2) - 14*(a*sin(d*x + c) + a)^(5/2)*a)/(a^3
*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^3\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.106 $\int \cos^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^3(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -8/15*a^2*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/5*a*cos(d*x+c)^3/d/(a+a*s
in(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{8a^2\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^3(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(15*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*
x]^3)/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx&=-\frac{2a\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}+\frac{1}{5}(4a)\int\frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx\\&=-\frac{8a^2\cos^3(c+dx)}{15d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 79, normalized size = 1.25

$$-\frac{2(3\sin(c+dx)+7)\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}{15d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

[Out] (-2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(7 + 3*Sin[c + d*x]))/(15*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))

fricas [A] time = 0.65, size = 92, normalized size = 1.46

$$-\frac{2\left(3\,\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}-\left(3\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)+4\,\cos\left(dx+c\right)+8\right)\cdot}{15\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -2/15*(3*cos(d*x + c)^3 - cos(d*x + c)^2 - (3*cos(d*x + c)^2 + 4*cos(d*x + c) + 8)*sin(d*x + c) + 4*cos(d*x + c) + 8)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)

giac [A] time = 0.40, size = 99, normalized size = 1.57

$$\frac{1}{30}\sqrt{2}\sqrt{a}\left(\frac{5\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}{d}+\frac{3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/30*sqrt(2)*sqrt(a)*(5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d + 3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d + 30*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)

maple [A] time = 0.18, size = 55, normalized size = 0.87

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)a\left(\sin\left(dx+c\right)-1\right)^{2}\left(3\sin\left(dx+c\right)+7\right)}{15\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)

[Out] -2/15*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(3*sin(d*x+c)+7)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\cos\left(c+d\,x\right)^{2}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2),x)

```
[Out] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\cos^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**2, x)
```

3.107 $\int \cos(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=24

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{3ad}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^(3/2)/a/d
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2667, 32}

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(3*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{3/2}}{3ad}\end{aligned}$$

Mathematica [A] time = 0.08, size = 44, normalized size = 1.83

$$\frac{2\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Sqrt[a*(1 + Sin[c + d*x])])/(3*d
)
```

fricas [A] time = 0.63, size = 25, normalized size = 1.04

$$\frac{2\sqrt{a\sin(dx+c)+a}\,(\sin(dx+c)+1)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*sin(d*x + c) + a)*(sin(d*x + c) + 1)/d
```

giac [B] time = 0.92, size = 68, normalized size = 2.83

$$\frac{1}{3}\sqrt{2}\sqrt{a}\left(\frac{3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{d}+\frac{\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{3}{2}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*sqrt(a)*(3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 1/2
*d*x + 1/2*c)/d + sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 3/2*d*x
 + 3/2*c)/d)
```

maple [A] time = 0.04, size = 21, normalized size = 0.88

$$\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{3}{2}}}{3da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(a+a*sin(d*x+c))^(3/2)/d/a
```

maxima [A] time = 0.67, size = 20, normalized size = 0.83

$$\frac{2\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a*sin(d*x + c) + a)^(3/2)/(a*d)
```

mupad [B] time = 4.57, size = 20, normalized size = 0.83

$$\frac{2\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)^{3/2}}{3\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(3/2))/(3*a*d)
```

sympy [A] time = 0.68, size = 58, normalized size = 2.42

$$\begin{cases} \frac{2\sqrt{a\sin\left(c+dx\right)+a}\,\sin\left(c+dx\right)}{3d}+\frac{2\sqrt{a\sin\left(c+dx\right)+a}}{3d} & \text{for } d\neq 0\\ x\sqrt{a\sin\left(c\right)+a}\,\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)/(3*d) + 2*sqrt(a*sin(c +
 d*x) + a)/(3*d), Ne(d, 0)), (x*sqrt(a*sin(c) + a)*cos(c), True))
```

3.108 $\int \sec(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[Out] arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 63, 206}

$$\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\sin(c+dx)}\right)}{d}\\
&= \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 95, normalized size = 2.38

$$-\frac{(2-2i)\sqrt[4]{-1}\sqrt{a(\sin(c+dx)+1)}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\sin\left(\frac{1}{4}(2c+dx)\right)+\cos\left(\frac{1}{4}(2c+dx)\right)\right)\right)}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-2 + 2*I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*
c + d*x)/4] + Sin[(2*c + d*x)/4])]*Sqrt[a*(1 + Sin[c + d*x])])/(d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.59, size = 92, normalized size = 2.30

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)}{2d}, -\frac{\sqrt{2}\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}}{\sqrt{a\sin(dx+c)+a}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
 a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1))/d, -sqrt(2)*sqrt(-a)*arctan(sqrt(2)*
sqrt(-a)/sqrt(a*sin(d*x + c) + a))/d]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)-sqrt(2*a)*sign(cos(1/2*(d*x+c)-1/4*
pi))*ln(abs(tan(1/2*(1/2*d*x+1/4*(2*c-pi)))))/d
```

maple [A] time = 0.10, size = 32, normalized size = 0.80

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\sqrt{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

maxima [A] time = 0.94, size = 58, normalized size = 1.45

$$-\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqr
t(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x),x)
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x), x)
```

3.109 $\int \sec^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{2}d}$$

```
[Out] -1/2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)
/d*2^(1/2)+sec(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2675, 2649, 206}

$$\frac{\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(Sqrt[2]*d)) + (Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}+\frac{1}{2}a\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx\\
&= \frac{\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}-\frac{a\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{2}d}+\frac{\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}
\end{aligned}$$

Mathematica [C] time = 0.23, size = 106, normalized size = 1.47

$$\frac{\sec(c+dx)\sqrt{a(\sin(c+dx)+1)}\left(1-(1+i)(-1)^{3/4}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]*(1 - (1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d
*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])]*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]))*Sqrt[a*(1 + Sin[c + d*x])])/d
```

fricas [B] time = 0.55, size = 159, normalized size = 2.21

$$\frac{\sqrt{2}\sqrt{a}\cos(dx+c)\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right.}{4d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(a)*cos(d*x + c)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a
*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x
+ c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*
x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*sqrt(a*sin(d*x + c) + a))
/(d*cos(d*x + c))
```

giac [A] time = 0.52, size = 102, normalized size = 1.42

$$\frac{\sqrt{2}\left(\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)\mathrm{sg}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(log(abs(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c)) - log(abs(sin(-1/4*pi + 1/2*d*x + 1/2*c) - 1))*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c)) - 2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/sin(-1/4
*pi + 1/2*d*x + 1/2*c))*sqrt(a)/d
```

maple [A] time = 0.23, size = 83, normalized size = 1.15

$$-\frac{(1+\sin(dx+c))\left(\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)a\sqrt{a-a\sin(dx+c)}\ -2a^{\frac{3}{2}}\right)}{2\sqrt{a}\ \cos(dx+c)\sqrt{a+a\sin(dx+c)}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*(1+sin(d*x+c))*(2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(
1/2)/a^(1/2))*a*(a-a*sin(d*x+c))^(1/2)-2*a^(3/2))/cos(d*x+c)/(a+a*sin(d*x+c
))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin(dx+c)+a}\ \sec(dx+c)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^2,x)
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x)**2, x)
```

3.110 $\int \sec^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{3a}{4d\sqrt{a\sin(c+dx)+a}}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,d}+\frac{\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

```
[Out] 3/8*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-3
/4*a/d/(a+a*sin(d*x+c))^(1/2)+1/2*sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2675, 2667, 51, 63, 206}

$$-\frac{3a}{4d\sqrt{a\sin(c+dx)+a}}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,d}+\frac{\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(4*Sqrt[2]*
d) - (3*a)/(4*d*Sqrt[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^2*Sqrt[a + a*Sin[
c + d*x]])/(2*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{1}{4}(3a)\int \frac{\sec(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= \frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{(3a^2)\operatorname{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, a\sin\right.}{4d}$$
$$= -\frac{3a}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{(3a)\operatorname{Subst}}{}$$
$$= -\frac{3a}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{(3a)\operatorname{Subst}}{}$$
$$= \frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} - \frac{3a}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^2(c+dx)\sqrt{a-}}{2a}$$

Mathematica [C] time = 0.38, size = 271, normalized size = 2.85

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\frac{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{dx}{2}\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + (-3+3i)\sqrt[4]{}\right.}{4d\left(\sin\left(\frac{1}{2}(c+dx)\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-2 - (3 - 3*I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Co
s[(2*c + d*x)/4] + Sin[(2*c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + (2*Sin[(d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/((Cos[c/2] - Si
n[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + ((Cos[c/2] + Sin[c/2])*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])))*Sqrt[a*(1 + Sin[c + d*x])])/(4*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2)
```

fricas [A] time = 0.61, size = 99, normalized size = 1.04

$$\frac{3\sqrt{2}\sqrt{a}\cos(dx+c)^2\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)+4\sqrt{a\sin(dx+c)+a}\,(3\sin(dx+c)-1)}{16\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*sqrt(a)*cos(d*x + c)^2*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqr
t(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*sqrt(a*sin(d*x
 + c) + a)*(3*sin(d*x + c) - 1))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.87, size = 282, normalized size = 2.97

$$-\frac{\sqrt{2}\left(6\log\left(\frac{\left|-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|}{\left|\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1}\right)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/32*sqrt(2)*(6*log(abs(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/abs(cos(-1/4*
pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - (cos(-1/4
*pi + 1/2*d*x + 1/2*c) - 1)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/(cos(-1/4*p
i + 1/2*d*x + 1/2*c) + 1) + (14*(cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))/(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) - 3*(cos
(-1/4*pi + 1/2*d*x + 1/2*c) - 1)^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/(cos
(-1/4*pi + 1/2*d*x + 1/2*c) + 1)^2 + sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))/(
(cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)/(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) +
 (cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)^2/(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1
)^2))*sqrt(a)/d
```

maple [A] time = 0.26, size = 90, normalized size = 0.95

$$\frac{2a^3\left(-\frac{1}{4a^2\sqrt{a+a\sin(dx+c)}}-\frac{\frac{\sqrt{a+a\sin(dx+c)}}{2a\sin(dx+c)-2a}-\frac{3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{4\sqrt{a}}}{4a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*a^3*(-1/4/a^2/(a+a*sin(d*x+c))^(1/2)-1/4/a^2*(1/2*(a+a*sin(d*x+c))^(1/2)/
(a*sin(d*x+c)-a)-3/4*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(
1/2)/a^(1/2))))/d
```

maxima [A] time = 0.60, size = 117, normalized size = 1.23

$$-\frac{3\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(3(a\sin(dx+c)+a)a^2-4a^3\right)}{(a\sin(dx+c)+a)^{\frac{3}{2}}-2\sqrt{a\sin(dx+c)+a}\,a}}{16ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(3*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/
(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(3*(a*sin(d*x + c) + a)*a
^2 - 4*a^3)/((a*sin(d*x + c) + a)^(3/2) - 2*sqrt(a*sin(d*x + c) + a)*a))/(a
*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{\cos(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x)**3, x)
```

3.111 $\int \sec^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{5a^2\cos(c+dx)}{8d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}+\frac{5a\sec(c+dx)}{6d\sqrt{a\sin(c+dx)+a}}-\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{2}d}$$

```
[Out] -5/8*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-5/16*arctanh(1/2*cos(d*x+c)*a^
(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)+5/6*a*sec(d*x+c)/d/
(a+a*sin(d*x+c))^(1/2)+1/3*sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2687, 2650, 2649, 206}

$$-\frac{5a^2\cos(c+dx)}{8d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}+\frac{5a\sec(c+dx)}{6d\sqrt{a\sin(c+dx)+a}}-\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(8*Sqrt[2]*d) - (5*a^2*Cos[c + d*x])/(8*d*(a + a*Sin[c + d*x])^(3/2))
+ (5*a*Sec[c + d*x])/(6*d*Sqrt[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^3*Sqrt[
a + a*Sin[c + d*x]])/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
```

```
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{1}{6}(5a)\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= \frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{1}{4}\left(5a^2\right)\int \frac{}{(a}$$

$$= -\frac{5a^2\cos(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^3(c+dx)\sqrt{a+a}}{3d}$$

$$= -\frac{5a^2\cos(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^3(c+dx)\sqrt{a+a}}{3d}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{2}d} - \frac{5a^2\cos(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{5a\sec(}{6d\sqrt{a+a\,s}}$$

Mathematica [C] time = 0.42, size = 302, normalized size = 2.20

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\frac{12\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{3\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}{\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)}\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (((6*Sin[(d*x)/2])/(Cos[c/2] + Sin[c/2]) - (3*(Cos[c/2] - Sin[c/2])*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[c/2] + Sin[c/2]) - (15 + 15*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(
2*c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (12*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
))*Sqrt[a*(1 + Sin[c + d*x])])/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
3)
```

fricas [A] time = 0.69, size = 188, normalized size = 1.37

$$\frac{15\sqrt{2}\sqrt{a}\cos(dx+c)^3\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{2}\right)\sqrt{a}+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right.}{96d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*sqrt(a)*cos(d*x + c)^3*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*s
in(d*x + c) + a)*(sqrt(2)*cos(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sq
rt(a) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(
d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(15*c
os(d*x + c)^2 + 10*sin(d*x + c) - 2)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x +
 c)^3)
```

giac [A] time = 0.77, size = 178, normalized size = 1.30

$$\frac{\sqrt{2}\left(15\,\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-15\,\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(2)*(15*log(abs(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c)) - 15*log(abs(sin(-1/4*pi + 1/2*d*x + 1/2*c) - 1))*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 6*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*s
in(-1/4*pi + 1/2*d*x + 1/2*c)/(sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1) - 4*(6
*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 + sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c)))/sin(-1/4*pi + 1/2*d*x + 1/2*c)^3)*sqrt(a)
/d
```

maple [A] time = 0.24, size = 153, normalized size = 1.12

$$\frac{\sin\left(dx+c\right)\left(15\left(a-a\sin\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a-20a^{\frac{5}{2}}\right)-30a^{\frac{5}{2}}\left(\cos^{2}\left(dx+c\right)\right)+15\left(a-\right.}{48a^{\frac{3}{2}}\left(\sin\left(dx+c\right)-1\right)\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/48/a^(3/2)*(sin(d*x+c)*(15*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-20*a^(5/2))-30*a^(5/2)*cos(d*x+c)^2+
15*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2
)/a^(1/2))*a+4*a^(5/2))/(sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sec^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x)**4, x)
```

3.112 $\int \sec^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=149

$$-\frac{35a^2}{96d(a\sin(c+dx)+a)^{3/2}}-\frac{35a}{64d\sqrt{a\sin(c+dx)+a}}+\frac{35\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}+\frac{\sec^4(c+dx)\sqrt{a\sin(c+dx)+}}{4d}$$

```
[Out] -35/96*a^2/d/(a+a*sin(d*x+c))^(3/2)+35/128*arctanh(1/2*(a+a*sin(d*x+c))^(1/
2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-35/64*a/d/(a+a*sin(d*x+c))^(1/2)+7/16
*a*sec(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)+1/4*sec(d*x+c)^4*(a+a*sin(d*x+c))^
(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2675, 2687, 2667, 51, 63, 206}

$$-\frac{35a^2}{96d(a\sin(c+dx)+a)^{3/2}}-\frac{35a}{64d\sqrt{a\sin(c+dx)+a}}+\frac{35\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}+\frac{\sec^4(c+dx)\sqrt{a\sin(c+dx)+}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (35*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2
]*d) - (35*a^2)/(96*d*(a + a*Sin[c + d*x])^(3/2)) - (35*a)/(64*d*Sqrt[a + a
*Sin[c + d*x]]) + (7*a*Sec[c + d*x]^2)/(16*d*Sqrt[a + a*Sin[c + d*x]]) + (S
ec[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]])/(4*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\sec^4(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}+\frac{1}{8}(7a)\int\frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= \frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^4(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}+\frac{1}{32}\left(35a^2\right.$$

$$= \frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^4(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}+\frac{\left(35a^3\right)S}{}$$

$$= -\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}}+\frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^4(c+dx)\sqrt{}}{}$$

$$= -\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}}-\frac{35a}{64d\sqrt{a+a\sin(c+dx)}}+\frac{7a\sec^2(c}{16d\sqrt{a+a\sin}}$$

$$= -\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}}-\frac{35a}{64d\sqrt{a+a\sin(c+dx)}}+\frac{7a\sec^2(c}{16d\sqrt{a+a\sin}}$$

$$= \frac{35\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}-\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}}-\frac{}{64d\sqrt{a+}}$$

Mathematica [C] time = 0.49, size = 179, normalized size = 1.20

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\frac{329\sin(c+dx)+105\sin(3(c+dx))-70\cos(2(c+dx))-102}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}-(420-420i)\sqrt[4]{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(\right.\right.\right.}{768d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((-420 + 420*I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*
(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] + Sin[(2*c + d*x)/4])]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^3 + (-102 - 70*Cos[2*(c + d*x)] + 329*Sin[c +
 d*x] + 105*Sin[3*(c + d*x)])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4))/(76
8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [A] time = 0.80, size = 121, normalized size = 0.81

$$\frac{105\sqrt{2}\sqrt{a}\cos(dx+c)^4\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(35\cos(dx+c)^2-7\left(15\cos(dx+c)^2+8\right)\right.}{768\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(105*sqrt(2)*sqrt(a)*cos(d*x + c)^4*log(-(a*sin(d*x + c) + 2*sqrt(2)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(35*cos(d*x
 + c)^2 - 7*(15*cos(d*x + c)^2 + 8)*sin(d*x + c) + 8)*sqrt(a*sin(d*x + c) +
 a))/(d*cos(d*x + c)^4)
```

giac [B] time = 2.75, size = 442, normalized size = 2.97

$$-\frac{\sqrt{2}\left(420\log\left(\frac{\left|-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|}{\left|\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{3\left(\frac{24\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1}-21\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3072*sqrt(2)*(420*log(abs(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/abs(cos(-
1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) + 3*(24
*(cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/(
cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) - 210*(cos(-1/4*pi + 1/2*d*x + 1/2*c) -
 1)^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/(cos(-1/4*pi + 1/2*d*x + 1/2*c) +
 1)^2 - sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))*(cos(-1/4*pi + 1/2*d*x + 1/2*c
) + 1)^2/(cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)^2 - 72*(cos(-1/4*pi + 1/2*d*x
 + 1/2*c) - 1)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/(cos(-1/4*pi + 1/2*d*x +
 1/2*c) + 1) + 3*(cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)^2*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))/(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)^2 + 256*(9*(cos(-1/4*
pi + 1/2*d*x + 1/2*c) - 1)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/(cos(-1/4*pi
 + 1/2*d*x + 1/2*c) + 1) + 6*(cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)^2*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))/(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)^2 + 5*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c)))/((cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)/(co
s(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + 1)^3)*sqrt(a)/d
```

maple [A] time = 0.33, size = 118, normalized size = 0.79

$$-\frac{2a^5\left(\frac{3}{16a^4\sqrt{a+a\sin(dx+c)}}+\frac{1}{24a^3(a+a\sin(dx+c))^{\frac{3}{2}}}+\frac{\frac{\sqrt{a+a\sin(dx+c)}\,a(11\sin(dx+c)-15)}{8(a\sin(dx+c)-a)^2}-\frac{35\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16\sqrt{a}}}{16a^4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*a^5*(3/16/a^4/(a+a*sin(d*x+c))^(1/2)+1/24/a^3/(a+a*sin(d*x+c))^(3/2)+1/1
6/a^4*(1/8*(a+a*sin(d*x+c))^(1/2)*a*(11*sin(d*x+c)-15)/(a*sin(d*x+c)-a)^2-3
5/16*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))))/
d
```

maxima [A] time = 1.01, size = 168, normalized size = 1.13

$$-\frac{105\,\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(105\,(a\sin(dx+c)+a)^{3}a^{2}-350\,(a\sin(dx+c)+a)^{2}a^{3}+224\,(a\sin(dx+c)+a)a^{4}+64\,a^{5}\right)}{(a\sin(dx+c)+a)^{\frac{7}{2}}-4\,(a\sin(dx+c)+a)^{\frac{5}{2}}a+4\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^{2}}}{768\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/768*(105*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(105*(a*sin(d*x + c) +
 a)^3*a^2 - 350*(a*sin(d*x + c) + a)^2*a^3 + 224*(a*sin(d*x + c) + a)*a^4 +
 64*a^5)/((a*sin(d*x + c) + a)^(7/2) - 4*(a*sin(d*x + c) + a)^(5/2)*a + 4*(
a*sin(d*x + c) + a)^(3/2)*a^2))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.113 $\int \sec^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$-\frac{63a^2\cos(c+dx)}{128d(a\sin(c+dx)+a)^{3/2}}-\frac{21a^2\sec(c+dx)}{80d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^5(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}+\frac{3a\sec^3(c+dx)}{10d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -63/128*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-21/80*a^2*sec(d*x+c)/d/(a+a
*sin(d*x+c))^(3/2)-63/256*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d
*x+c))^(1/2))*a^(1/2)/d*2^(1/2)+21/32*a*sec(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
+3/10*a*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+1/5*sec(d*x+c)^5*(a+a*sin(d*x
+c))^(1/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2675, 2687, 2681, 2650, 2649, 206}

$$-\frac{63a^2\cos(c+dx)}{128d(a\sin(c+dx)+a)^{3/2}}-\frac{21a^2\sec(c+dx)}{80d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^5(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}+\frac{3a\sec^3(c+dx)}{10d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-63*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x
]])])/(128*Sqrt[2]*d) - (63*a^2*Cos[c + d*x])/(128*d*(a + a*Sin[c + d*x])^(
3/2)) - (21*a^2*Sec[c + d*x])/(80*d*(a + a*Sin[c + d*x])^(3/2)) + (21*a*Sec
[c + d*x])/(32*d*Sqrt[a + a*Sin[c + d*x]]) + (3*a*Sec[c + d*x]^3)/(10*d*Sqr
t[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\sec^5(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{1}{10}(9a)\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= \frac{3a\sec^3(c+dx)}{10d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^5(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{1}{20}\left(21a^2\right.$$

$$= -\frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{3a\sec^3(c+dx)}{10d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^5(c+dx)\sqrt{}}{}$$

$$= -\frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{21a\sec(c+dx)}{32d\sqrt{a+a\sin(c+dx)}} + \frac{3a\sec^3(c}{10d\sqrt{a+a\sin}}$$

$$= -\frac{63a^2\cos(c+dx)}{128d(a+a\sin(c+dx))^{3/2}} - \frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{21a\sec}{32d\sqrt{a+a}}$$

$$= -\frac{63a^2\cos(c+dx)}{128d(a+a\sin(c+dx))^{3/2}} - \frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{21a\sec}{32d\sqrt{a+a}}$$

$$= -\frac{63\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{2}d} - \frac{63a^2\cos(c+dx)}{128d(a+a\sin(c+dx))^{3/2}} - \frac{}{80d(}$$

Mathematica [C] time = 0.65, size = 191, normalized size = 0.97

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\frac{1572\sin(c+dx)+420\sin(3(c+dx))+1092\cos(2(c+dx))+315\cos(4(c+dx))+649}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5} - (2520+2520i)(-1)^{3/4}\left(\sin\right.\right.}{5120d\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((-2520 - 2520*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2
)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])]*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^4 + (649 + 1092*Cos[2*(c + d*x)] + 315*Cos[
4*(c + d*x)] + 1572*Sin[c + d*x] + 420*Sin[3*(c + d*x)])/(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^5))/(5120*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [A] time = 0.62, size = 210, normalized size = 1.07

$$\frac{315\sqrt{2}\sqrt{a}\cos(dx+c)^5\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{2}\right)\sqrt{a}+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right.}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2560*(315*sqrt(2)*sqrt(a)*cos(d*x + c)^5*log(-(a*cos(d*x + c)^2 - 2*sqrt(
a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))
*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(c
os(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(3
15*cos(d*x + c)^4 - 42*cos(d*x + c)^2 + 6*(35*cos(d*x + c)^2 + 24)*sin(d*x
+ c) - 16)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^5)
```

giac [A] time = 2.53, size = 239, normalized size = 1.21

$$\sqrt{2}\left(315\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-315\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2560*sqrt(2)*(315*log(abs(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c)) - 315*log(abs(sin(-1/4*pi + 1/2*d*x + 1/2*c) - 1)
)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 10*(15*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 17*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 -
1)^2 - 16*(30*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1
/2*c)^4 + 5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2
*c)^2 + sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))/sin(-1/4*pi + 1/2*d*x + 1/2*c)
^5)*sqrt(a)/d
```

maple [A] time = 0.35, size = 244, normalized size = 1.24

$$-\frac{-420a^{\frac{9}{2}}\sin(dx+c)\left(\cos^2(dx+c)\right)+\left(630(a-a\sin(dx+c))^{\frac{5}{2}}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)a^2-288a^{\frac{9}{2}}\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/1280/a^(7/2)*(-420*a^(9/2)*sin(d*x+c)*cos(d*x+c)^2+(630*(a-a*sin(d*x+c))
^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-288*
a^(9/2))*sin(d*x+c)-630*a^(9/2)*cos(d*x+c)^4+(-315*(a-a*sin(d*x+c))^(5/2)*2
^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2+84*a^(9/2))*
cos(d*x+c)^2+630*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c)
)^(1/2)*2^(1/2)/a^(1/2))*a^2+32*a^(9/2))/(sin(d*x+c)-1)^2/(1+sin(d*x+c))/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin(dx+c)+a}\,\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sec(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^6,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.114 $\int \cos^7(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{17/2}}{17a^7d}+\frac{4(a\sin(c+dx)+a)^{15/2}}{5a^6d}-\frac{24(a\sin(c+dx)+a)^{13/2}}{13a^5d}+\frac{16(a\sin(c+dx)+a)^{11/2}}{11a^4d}$$

```
[Out] 16/11*(a+a*sin(d*x+c))^(11/2)/a^4/d-24/13*(a+a*sin(d*x+c))^(13/2)/a^5/d+4/5
*(a+a*sin(d*x+c))^(15/2)/a^6/d-2/17*(a+a*sin(d*x+c))^(17/2)/a^7/d
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{17/2}}{17a^7d}+\frac{4(a\sin(c+dx)+a)^{15/2}}{5a^6d}-\frac{24(a\sin(c+dx)+a)^{13/2}}{13a^5d}+\frac{16(a\sin(c+dx)+a)^{11/2}}{11a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(11/2))/(11*a^4*d) - (24*(a + a*Sin[c + d*x])^(13/
2))/(13*a^5*d) + (4*(a + a*Sin[c + d*x])^(15/2))/(5*a^6*d) - (2*(a + a*Sin[
c + d*x])^(17/2))/(17*a^7*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{9/2}\,dx,x,a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{9/2}-12a^2(a+x)^{11/2}+6a(a+x)^{13/2}-(a+x)^{15/2}\right)\right.}{a^7d}\\ &= \frac{16(a+a\sin(c+dx))^{11/2}}{11a^4d}-\frac{24(a+a\sin(c+dx))^{13/2}}{13a^5d}+\frac{4(a+a\sin(c+\right.}{5a^6d}\end{aligned}$$

Mathematica [A] time = 0.45, size = 61, normalized size = 0.63

$$-\frac{2(\sin(c+dx)+1)^4\left(715\sin^3(c+dx)-2717\sin^2(c+dx)+3641\sin(c+dx)-1767\right)(a(\sin(c+dx)+1))^{3/2}}{12155d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(1 + Sin[c + d*x])^4*(a*(1 + Sin[c + d*x]))^(3/2)*(-1767 + 3641*Sin[c +
 d*x] - 2717*Sin[c + d*x]^2 + 715*Sin[c + d*x]^3))/(12155*d)
```

fricas [A] time = 0.56, size = 110, normalized size = 1.13

$$-\frac{2\left(715\,a\cos\left(dx+c\right)^{8}-66\,a\cos\left(dx+c\right)^{6}-112\,a\cos\left(dx+c\right)^{4}-256\,a\cos\left(dx+c\right)^{2}-2\left(429\,a\cos\left(dx+c\right)^{6}\right.\right.}{12155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/12155*(715*a*cos(d*x + c)^8 - 66*a*cos(d*x + c)^6 - 112*a*cos(d*x + c)^4
 - 256*a*cos(d*x + c)^2 - 2*(429*a*cos(d*x + c)^6 + 504*a*cos(d*x + c)^4 +
640*a*cos(d*x + c)^2 + 1024*a)*sin(d*x + c) - 2048*a)*sqrt(a*sin(d*x + c) +
 a)/d
```

giac [B] time = 1.11, size = 505, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/14002560*sqrt(2)*(7293*a*cos(1/4*pi + 15/2*d*x + 15/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d + 59670*a*cos(1/4*pi + 11/2*d*x + 11/2*c)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))/d + 218790*a*cos(1/4*pi + 7/2*d*x + 7/2*c)*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))/d + 510510*a*cos(1/4*pi + 3/2*d*x + 3/2*c)*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 6435*a*cos(-1/4*pi + 17/2*d*x + 17/2*
c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 50490*a*cos(-1/4*pi + 13/2*d*x +
 13/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 170170*a*cos(-1/4*pi + 9/2
*d*x + 9/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 306306*a*cos(-1/4*pi
+ 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 16830*a*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 13/2*d*x + 13/2*c)/d - 170170*a*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 9/2*d*x + 9/2*c)/d - 918918*
a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 5/2*d*x + 5/2*c)/d - 765
7650*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 1/2*d*x + 1/2*c)/d
 - 14586*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 15/2*d*x + 15/2
*c)/d - 139230*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 11/2*d*x
 + 11/2*c)/d - 656370*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 7
/2*d*x + 7/2*c)/d - 2552550*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*
pi + 3/2*d*x + 3/2*c)/d)*sqrt(a)
```

maple [A] time = 0.18, size = 57, normalized size = 0.59

$$\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{11}{2}}\left(715\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-2717\left(\cos^{2}\left(dx+c\right)\right)-4356\sin\left(dx+c\right)+4484\right)}{12155a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/12155/a^4*(a+a*sin(d*x+c))^(11/2)*(715*cos(d*x+c)^2*sin(d*x+c)-2717*cos(d
*x+c)^2-4356*sin(d*x+c)+4484)/d
```

maxima [A] time = 0.33, size = 72, normalized size = 0.74

$$-\frac{2\left(715\left(a\sin\left(dx+c\right)+a\right)^{\frac{17}{2}}-4862\left(a\sin\left(dx+c\right)+a\right)^{\frac{15}{2}}a+11220\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}a^{2}-8840\left(a\sin\left(dx\right.\right.\right.}{12155\,a^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/12155*(715*(a*sin(d*x + c) + a)^(17/2) - 4862*(a*sin(d*x + c) + a)^(15/2
)*a + 11220*(a*sin(d*x + c) + a)^(13/2)*a^2 - 8840*(a*sin(d*x + c) + a)^(11
/2)*a^3)/(a^7*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^7 (a+a\sin(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.115 $\int \cos^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=159

$$-\frac{4096a^5\cos^7(c+dx)}{45045d(a\sin(c+dx)+a)^{7/2}}-\frac{1024a^4\cos^7(c+dx)}{6435d(a\sin(c+dx)+a)^{5/2}}-\frac{128a^3\cos^7(c+dx)}{715d(a\sin(c+dx)+a)^{3/2}}-\frac{32a^2\cos^7(c+dx)}{195d\sqrt{a\sin(c+dx)+}}$$

```
[Out] -4096/45045*a^5*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)-1024/6435*a^4*cos(d*x
+c)^7/d/(a+a*sin(d*x+c))^(5/2)-128/715*a^3*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^
(3/2)-32/195*a^2*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(1/2)-2/15*a*cos(d*x+c)^7*
(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.30, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2674, 2673}

$$-\frac{32a^2\cos^7(c+dx)}{195d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^7(c+dx)}{715d(a\sin(c+dx)+a)^{3/2}}-\frac{1024a^4\cos^7(c+dx)}{6435d(a\sin(c+dx)+a)^{5/2}}-\frac{4096a^5\cos^7(c+dx)}{45045d(a\sin(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4096*a^5*Cos[c + d*x]^7)/(45045*d*(a + a*Sin[c + d*x])^(7/2)) - (1024*a^4
*Cos[c + d*x]^7)/(6435*d*(a + a*Sin[c + d*x])^(5/2)) - (128*a^3*Cos[c + d*x
]^7)/(715*d*(a + a*Sin[c + d*x])^(3/2)) - (32*a^2*Cos[c + d*x]^7)/(195*d*Sq
rt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^7*Sqrt[a + a*Sin[c + d*x]])/(15
*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2a\cos^7(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}+\frac{1}{15}(16a)\int \cos^6(c+dx)\sqrt{a+}\\
&= -\frac{32a^2\cos^7(c+dx)}{195d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^7(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}+\\
&= -\frac{128a^3\cos^7(c+dx)}{715d(a+a\sin(c+dx))^{3/2}}-\frac{32a^2\cos^7(c+dx)}{195d\sqrt{a+a\sin(c+dx)}}-2a\cos^7(c\\
&= -\frac{1024a^4\cos^7(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}}-\frac{128a^3\cos^7(c+dx)}{715d(a+a\sin(c+dx))^{3/2}}-\frac{32a}{195d\sqrt{}}\\
&= -\frac{4096a^5\cos^7(c+dx)}{45045d(a+a\sin(c+dx))^{7/2}}-\frac{1024a^4\cos^7(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}}-\frac{1}{715}
\end{aligned}$$

Mathematica [A] time = 0.66, size = 79, normalized size = 0.50

$$-\frac{2\left(3003\sin^4(c+dx)+15708\sin^3(c+dx)+33138\sin^2(c+dx)+34748\sin(c+dx)+16363\right)\cos^7(c+dx)(a(\mathrm{si}}{45045d(\sin(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7*(a*(1 + Sin[c + d*x]))^(3/2)*(16363 + 34748*Sin[c + d*x]
 + 33138*Sin[c + d*x]^2 + 15708*Sin[c + d*x]^3 + 3003*Sin[c + d*x]^4))/(450
45*d*(1 + Sin[c + d*x])^5)
```

fricas [A] time = 0.57, size = 210, normalized size = 1.32

$$-\frac{2\left(3003\,a\cos\left(dx+c\right)^{8}+6699\,a\cos\left(dx+c\right)^{7}-336\,a\cos\left(dx+c\right)^{6}+448\,a\cos\left(dx+c\right)^{5}-640\,a\cos\left(dx+c\right)^{4}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45045*(3003*a*cos(d*x + c)^8 + 6699*a*cos(d*x + c)^7 - 336*a*cos(d*x + c
)^6 + 448*a*cos(d*x + c)^5 - 640*a*cos(d*x + c)^4 + 1024*a*cos(d*x + c)^3 -
 2048*a*cos(d*x + c)^2 + 8192*a*cos(d*x + c) + (3003*a*cos(d*x + c)^7 - 369
6*a*cos(d*x + c)^6 - 4032*a*cos(d*x + c)^5 - 4480*a*cos(d*x + c)^4 - 5120*a
*cos(d*x + c)^3 - 6144*a*cos(d*x + c)^2 - 8192*a*cos(d*x + c) - 16384*a)*si
n(d*x + c) + 16384*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x
+ c) + d)
```

giac [B] time = 1.34, size = 474, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2882880*sqrt(2)*(3465*a*cos(1/4*pi + 13/2*d*x + 13/2*c)*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))/d + 25025*a*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))/d + 81081*a*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))/d + 225225*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))/d + 3003*a*cos(-1/4*pi + 15/2*d*x + 15/2*c)*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 20475*a*cos(-1/4*pi + 11/2*d*x + 11/
2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 57915*a*cos(-1/4*pi + 7/2*d*x
+ 7/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 75075*a*cos(-1/4*pi + 3/2*
d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 8190*a*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))*sin(1/4*pi + 11/2*d*x + 11/2*c)/d - 77220*a*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 7/2*d*x + 7/2*c)/d - 450450*a*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d - 6930*a*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 13/2*d*x + 13/2*c)/d - 60060*
a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 9/2*d*x + 9/2*c)/d - 27
0270*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d
 - 1801800*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/
2*c)/d)*sqrt(a)
```

maple [A] time = 0.21, size = 87, normalized size = 0.55

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)^{4}\left(3003\left(\sin^{4}\left(dx+c\right)\right)+15708\left(\sin^{3}\left(dx+c\right)\right)+33138\left(\sin^{2}\left(dx+c\right)\right)+\right.}{45045\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/45045*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^4*(3003*sin(d*x+c)^4+15708*sin(d
*x+c)^3+33138*sin(d*x+c)^2+34748*sin(d*x+c)+16363)/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^6\,(a+a\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.116 $\int \cos^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^5d}-\frac{8(a\sin(c+dx)+a)^{11/2}}{11a^4d}+\frac{8(a\sin(c+dx)+a)^{9/2}}{9a^3d}$$

```
[Out] 8/9*(a+a*sin(d*x+c))^(9/2)/a^3/d-8/11*(a+a*sin(d*x+c))^(11/2)/a^4/d+2/13*(a
+a*sin(d*x+c))^(13/2)/a^5/d
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{2667, 43\}$

$$\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^5d}-\frac{8(a\sin(c+dx)+a)^{11/2}}{11a^4d}+\frac{8(a\sin(c+dx)+a)^{9/2}}{9a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(9/2))/(9*a^3*d) - (8*(a + a*Sin[c + d*x])^(11/2))/
(11*a^4*d) + (2*(a + a*Sin[c + d*x])^(13/2))/(13*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{7/2}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{7/2}-4a(a+x)^{9/2}+(a+x)^{11/2}\right) dx, x, a\sin(c+dx\right.}{a^5d}\\ &= \frac{8(a+a\sin(c+dx))^{9/2}}{9a^3d}-\frac{8(a+a\sin(c+dx))^{11/2}}{11a^4d}+\frac{2(a+a\sin(c+dx)}{13a^5d}\end{aligned}$$

Mathematica [A] time = 0.17, size = 51, normalized size = 0.70

$$\frac{2(\sin(c+dx)+1)^3\left(99\sin^2(c+dx)-270\sin(c+dx)+203\right)(a(\sin(c+dx)+1))^{3/2}}{1287d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(1 + Sin[c + d*x])^3*(a*(1 + Sin[c + d*x]))^(3/2)*(203 - 270*Sin[c + d*x
] + 99*Sin[c + d*x]^2))/(1287*d)
```

fricas [A] time = 0.64, size = 88, normalized size = 1.21

$$-\frac{2\left(99\,a\cos\left(dx+c\right)^{6}-14\,a\cos\left(dx+c\right)^{4}-32\,a\cos\left(dx+c\right)^{2}-2\left(63\,a\cos\left(dx+c\right)^{4}+80\,a\cos\left(dx+c\right)^{2}+12\right.}{1287\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/1287*(99*a*cos(d*x + c)^6 - 14*a*cos(d*x + c)^4 - 32*a*cos(d*x + c)^2 -
2*(63*a*cos(d*x + c)^4 + 80*a*cos(d*x + c)^2 + 128*a)*sin(d*x + c) - 256*a)
*sqrt(a*sin(d*x + c) + a)/d
```

giac [B] time = 1.03, size = 381, normalized size = 5.22

$$-\frac{1}{288288}\sqrt{2}\left(\frac{819\,a\cos\left(\frac{1}{4}\,\pi+\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{5148\,a\cos\left(\frac{1}{4}\,\pi+\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\mathrm{sg}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/288288*sqrt(2)*(819*a*cos(1/4*pi + 11/2*d*x + 11/2*c)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))/d + 5148*a*cos(1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))/d + 15015*a*cos(1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))/d + 693*a*cos(-1/4*pi + 13/2*d*x + 13/2*c)*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))/d + 4004*a*cos(-1/4*pi + 9/2*d*x + 9/2*c)*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))/d + 9009*a*cos(-1/4*pi + 5/2*d*x + 5/2*c)*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 2002*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*sin(1/4*pi + 9/2*d*x + 9/2*c)/d - 18018*a*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*sin(1/4*pi + 5/2*d*x + 5/2*c)/d - 180180*a*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))*sin(1/4*pi + 1/2*d*x + 1/2*c)/d - 1638*a*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*sin(-1/4*pi + 11/2*d*x + 11/2*c)/d - 12870*a*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*sin(-1/4*pi + 7/2*d*x + 7/2*c)/d - 60060*a*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 3/2*d*x + 3/2*c)/d)*sqrt(a)
```

maple [A] time = 0.17, size = 41, normalized size = 0.56

$$-\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{9}{2}}\left(99\left(\cos^{2}\left(dx+c\right)\right)+270\sin\left(dx+c\right)-302\right)}{1287a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/1287/a^3*(a+a*sin(d*x+c))^(9/2)*(99*cos(d*x+c)^2+270*sin(d*x+c)-302)/d
```

maxima [A] time = 0.32, size = 55, normalized size = 0.75

$$\frac{2\left(99\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}-468\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}a+572\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a^{2}\right)}{1287\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1287*(99*(a*sin(d*x + c) + a)^(13/2) - 468*(a*sin(d*x + c) + a)^(11/2)*a
+ 572*(a*sin(d*x + c) + a)^(9/2)*a^2)/(a^5*d)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.117 $\int \cos^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{256a^4\cos^5(c+dx)}{1155d(a\sin(c+dx)+a)^{5/2}}-\frac{64a^3\cos^5(c+dx)}{231d(a\sin(c+dx)+a)^{3/2}}-\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos^5(c+dx)\sqrt{a\sin(c+}}{11d}$$

```
[Out] -256/1155*a^4*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-64/231*a^3*cos(d*x+c)^5
/d/(a+a*sin(d*x+c))^(3/2)-8/33*a^2*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-2/
11*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^5(c+dx)}{231d(a\sin(c+dx)+a)^{3/2}}-\frac{256a^4\cos^5(c+dx)}{1155d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)\sqrt{a\sin(c+}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(1155*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(231*d*(a + a*Sin[c + d*x])^(3/2)) - (8*a^2*Cos[c + d*x]^5)/(3
3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]
])/(11*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d}+\frac{1}{11}(12a)\int \cos^4(c+dx)\sqrt{a+}\\ &= -\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d}+\frac{1}{33}\\ &= -\frac{64a^3\cos^5(c+dx)}{231d(a+a\sin(c+dx))^{3/2}}-\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^5(c+}{}\\ &= -\frac{256a^4\cos^5(c+dx)}{1155d(a+a\sin(c+dx))^{5/2}}-\frac{64a^3\cos^5(c+dx)}{231d(a+a\sin(c+dx))^{3/2}}-\frac{8a^2}{33d\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 69, normalized size = 0.54

$$-\frac{2\left(105\sin^3(c+dx)+455\sin^2(c+dx)+755\sin(c+dx)+533\right)\cos^5(c+dx)(a(\sin(c+dx)+1))^{3/2}}{1155d(\sin(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(3/2)*(533 + 755*Sin[c + d*x] + 4
55*Sin[c + d*x]^2 + 105*Sin[c + d*x]^3))/(1155*d*(1 + Sin[c + d*x])^4)
```

fricas [A] time = 0.48, size = 166, normalized size = 1.31

$$-\frac{2\left(105\,a\cos\left(dx+c\right)^{6}+245\,a\cos\left(dx+c\right)^{5}-20\,a\cos\left(dx+c\right)^{4}+32\,a\cos\left(dx+c\right)^{3}-64\,a\cos\left(dx+c\right)^{2}+256\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(105*a*cos(d*x + c)^6 + 245*a*cos(d*x + c)^5 - 20*a*cos(d*x + c)^4
+ 32*a*cos(d*x + c)^3 - 64*a*cos(d*x + c)^2 + 256*a*cos(d*x + c) + (105*a*c
os(d*x + c)^5 - 140*a*cos(d*x + c)^4 - 160*a*cos(d*x + c)^3 - 192*a*cos(d*x
 + c)^2 - 256*a*cos(d*x + c) - 512*a)*sin(d*x + c) + 512*a)*sqrt(a*sin(d*x
+ c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 1.19, size = 350, normalized size = 2.76

$$-\frac{1}{55440}\sqrt{2}\left(\frac{385\,a\cos\left(\frac{1}{4}\pi+\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{2079\,a\cos\left(\frac{1}{4}\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\mathrm{sgn}\left(\mathrm{co}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/55440*sqrt(2)*(385*a*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d + 2079*a*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))/d + 6930*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))/d + 315*a*cos(-1/4*pi + 11/2*d*x + 11/2*c)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))/d + 1485*a*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))/d + 2310*a*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))/d - 990*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
sin(1/4*pi + 7/2*d*x + 7/2*c)/d - 9240*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin(1/4*pi + 3/2*d*x + 3/2*c)/d - 770*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*sin(-1/4*pi + 9/2*d*x + 9/2*c)/d - 5544*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d - 41580*a*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.18, size = 77, normalized size = 0.61

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)^{3}\left(105\left(\sin^{3}\left(dx+c\right)\right)+455\left(\sin^{2}\left(dx+c\right)\right)+755\sin\left(dx+c\right)+533\right)}{1155\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/1155*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^3*(105*sin(d*x+c)^3+455*sin(d*x+c)
^2+755*sin(d*x+c)+533)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.118 $\int \cos^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=49

$$\frac{4(a\sin(c+dx)+a)^{7/2}}{7a^2d}-\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^3d}$$

```
[Out] 4/7*(a+a*sin(d*x+c))^(7/2)/a^2/d-2/9*(a+a*sin(d*x+c))^(9/2)/a^3/d
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{7/2}}{7a^2d}-\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(7/2))/(7*a^2*d) - (2*(a + a*Sin[c + d*x])^(9/2))/(
9*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^{5/2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(2a(a+x)^{5/2}-(a+x)^{7/2}\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{4(a+a\sin(c+dx))^{7/2}}{7a^2d}-\frac{2(a+a\sin(c+dx))^{9/2}}{9a^3d}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 41, normalized size = 0.84

$$-\frac{2(\sin(c+dx)+1)^2(7\sin(c+dx)-11)(a(\sin(c+dx)+1))^{3/2}}{63d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(1 + Sin[c + d*x])^2*(a*(1 + Sin[c + d*x]))^(3/2)*(-11 + 7*Sin[c + d*x]
))/(63*d)
```

fricas [A] time = 0.70, size = 66, normalized size = 1.35

$$-\frac{2\left(7\,a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}-2\left(5\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)-16\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - 2*(5*a*cos(d*x + c)^2 + 8*
a)*sin(d*x + c) - 16*a)*sqrt(a*sin(d*x + c) + a)/d
```

giac [B] time = 0.70, size = 257, normalized size = 5.24

$$-\frac{1}{2520}\sqrt{2}\left(\frac{45\,a\cos\left(\frac{1}{4}\,\pi+\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{210\,a\cos\left(\frac{1}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\mathrm{sgn}\left(\cos\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2520*sqrt(2)*(45*a*cos(1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))/d + 210*a*cos(1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))/d + 35*a*cos(-1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d + 126*a*cos(-1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))/d - 126*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
 5/2*d*x + 5/2*c)/d - 1890*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi
 + 1/2*d*x + 1/2*c)/d - 90*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*p
i + 7/2*d*x + 7/2*c)/d - 630*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 3/2*d*x + 3/2*c)/d)*sqrt(a)
```

maple [A] time = 0.15, size = 31, normalized size = 0.63

$$-\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{7}{2}}\left(7\sin\left(dx+c\right)-11\right)}{63a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/63/a^2*(a+a*sin(d*x+c))^(7/2)*(7*sin(d*x+c)-11)/d
```

maxima [A] time = 0.32, size = 38, normalized size = 0.78

$$-\frac{2\left(7\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-18\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a\right)}{63\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/63*(7*(a*sin(d*x + c) + a)^(9/2) - 18*(a*sin(d*x + c) + a)^(7/2)*a)/(a^3
*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\cos\left(c+d\,x\right)^{3}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [A] time = 127.62, size = 252, normalized size = 5.14

$$\begin{cases} \frac{8a\sqrt{a\sin(c+dx)+a}\,\sin^4(c+dx)}{45d} + \frac{152a\sqrt{a\sin(c+dx)+a}\,\sin^3(c+dx)}{315d} + \frac{2a\sqrt{a\sin(c+dx)+a}\,\sin^2(c+dx)\cos^2(c+dx)}{5d} + \frac{8a\sqrt{a\sin(c+dx)+a}\,\sin^2(c+ \\ x\left(a\sin(c)+a\right)^{\frac{3}{2}}\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((8*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**4/(45*d) + 152*a*sqrt
(a*sin(c + d*x) + a)*sin(c + d*x)**3/(315*d) + 2*a*sqrt(a*sin(c + d*x) + a)
*sin(c + d*x)**2*cos(c + d*x)**2/(5*d) + 8*a*sqrt(a*sin(c + d*x) + a)*sin(c
 + d*x)**2/(21*d) + 4*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)*cos(c + d*x)*
*2/(5*d) + 8*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)/(315*d) + 2*a*sqrt(a*s
in(c + d*x) + a)*cos(c + d*x)**2/(5*d) - 16*a*sqrt(a*sin(c + d*x) + a)/(315
*d), Ne(d, 0)), (x*(a*sin(c) + a)**(3/2)*cos(c)**3, True))
```

3.119 $\int \cos^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{64a^3\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}-\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}$$

```
[Out] -64/105*a^3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-16/35*a^2*cos(d*x+c)^3/d/
(a+a*sin(d*x+c))^(1/2)-2/7*a*cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2674, 2673}

$$-\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) - (16*a^2*Cos[c
 + d*x]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^3*Sqrt[a + a
*Sin[c + d*x]])/(7*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d}+\frac{1}{7}(8a)\int \cos^2(c+dx)\sqrt{a+a}$$
$$= -\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d}+\frac{1}{35}$$
$$= -\frac{64a^3\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}}-\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^3(c+}{}$$

Mathematica [A] time = 0.15, size = 59, normalized size = 0.62

$$-\frac{2\left(15\sin^2(c+dx)+54\sin(c+dx)+71\right)\cos^3(c+dx)(a(\sin(c+dx)+1))^{3/2}}{105d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3*(a*(1 + Sin[c + d*x]))^(3/2)*(71 + 54*Sin[c + d*x] + 15*
Sin[c + d*x]^2))/(105*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.65, size = 122, normalized size = 1.28

$$-\frac{2\left(15\,a\cos\left(dx+c\right)^{4}+39\,a\cos\left(dx+c\right)^{3}-8\,a\cos\left(dx+c\right)^{2}+32\,a\cos\left(dx+c\right)+\left(15\,a\cos\left(dx+c\right)^{3}-24\,a\cos\left(\right.\right.}{105\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 8*a*cos(d*x + c)^2 + 32
*a*cos(d*x + c) + (15*a*cos(d*x + c)^3 - 24*a*cos(d*x + c)^2 - 32*a*cos(d*x
 + c) - 64*a)*sin(d*x + c) + 64*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c)
 + d*sin(d*x + c) + d)
```

giac [B] time = 0.64, size = 226, normalized size = 2.38

$$-\frac{1}{420}\sqrt{2}\left(\frac{21\,a\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{105\,a\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/420*sqrt(2)*(21*a*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))/d + 105*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))/d + 15*a*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))/d + 35*a*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d - 70*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/
2*d*x + 3/2*c)/d - 42*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5
/2*d*x + 5/2*c)/d - 420*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.27, size = 67, normalized size = 0.71

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)^{2}\left(15\left(\sin^{2}\left(dx+c\right)\right)+54\sin\left(dx+c\right)+71\right)}{105\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/105*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(15*sin(d*x+c)^2+54*sin(d*x+c)+7
1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(c+d\,x\right)^{2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{3}{2}}\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*cos(c + d*x)**2, x)
```

3.120 $\int \cos(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5ad}$$

```
[Out] 2/5*(a+a*sin(d*x+c))^(5/2)/a/d
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2667, 32\}$

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{5/2}}{5ad}\end{aligned}$$

Mathematica [A] time = 0.04, size = 24, normalized size = 1.00

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*a*d)
```

fricas [A] time = 0.72, size = 40, normalized size = 1.67

$$-\frac{2\left(a\cos\left(dx+c\right)^{2}-2\,a\sin\left(dx+c\right)-2\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a*cos(d*x + c)^2 - 2*a*sin(d*x + c) - 2*a)*sqrt(a*sin(d*x + c) + a)/d
```

giac [B] time = 0.39, size = 133, normalized size = 5.54

$$-\frac{1}{30}\sqrt{2}\left(\frac{5\,a\cos\left(\frac{1}{4}\,\pi + \frac{3}{2}\,dx + \frac{3}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{d} + \frac{3\,a\cos\left(-\frac{1}{4}\,\pi + \frac{5}{2}\,dx + \frac{5}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(2)*(5*a*cos(1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))/d + 3*a*cos(-1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))/d - 30*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 1/2*d*x
 + 1/2*c)/d - 10*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 3/2*d*
x + 3/2*c)/d)*sqrt(a)
```

maple [A] time = 0.04, size = 21, normalized size = 0.88

$$\frac{2\left(a + a\sin\left(dx + c\right)\right)^{\frac{5}{2}}}{5da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5*(a+a*sin(d*x+c))^(5/2)/d/a
```

maxima [A] time = 1.51, size = 20, normalized size = 0.83

$$\frac{2\left(a\sin\left(dx + c\right) + a\right)^{\frac{5}{2}}}{5\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(a*sin(d*x + c) + a)^(5/2)/(a*d)
```

mupad [B] time = 4.62, size = 20, normalized size = 0.83

$$\frac{2\left(a\ \left(\sin\left(c + d\,x\right) + 1\right)\right)^{5/2}}{5\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(5/2))/(5*a*d)
```

sympy [A] time = 29.54, size = 90, normalized size = 3.75

$$\begin{cases} \frac{2a\sqrt{a\sin\left(c+dx\right)+a}\ \sin^{2}\left(c+dx\right)}{5d} + \frac{4a\sqrt{a\sin\left(c+dx\right)+a}\ \sin\left(c+dx\right)}{5d} + \frac{2a\sqrt{a\sin\left(c+dx\right)+a}}{5d} & \text{for}\ d \neq 0 \\ x\left(a\sin\left(c\right) + a\right)^{\frac{3}{2}}\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((2*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**2/(5*d) + 4*a*sqrt(a*
sin(c + d*x) + a)*sin(c + d*x)/(5*d) + 2*a*sqrt(a*sin(c + d*x) + a)/(5*d),
Ne(d, 0)), (x*(a*sin(c) + a)**(3/2)*cos(c), True))
```

3.121 $\int \sec(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2a\sqrt{a\sin(c+dx)+a}}{d}$$

```
[Out] 2*a^(3/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-2*a
*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2667, 50, 63, 206}

$$\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2a\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (2*a*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{2a\sqrt{a+a\sin(c+dx)}}{d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin(c+dx\right.}{d}$$
$$= -\frac{2a\sqrt{a+a\sin(c+dx)}}{d} + \frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\sin(c+\right.}{d}$$
$$= \frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{2a\sqrt{a+a\sin(c+dx)}}{d}$$

Mathematica [A] time = 0.10, size = 60, normalized size = 0.97

$$\frac{2a\left(\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right) - \sqrt{a\sin(c+dx)+a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a*(Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])] -
 Sqrt[a + a*Sin[c + d*x]]))/d
```

fricas [A] time = 0.70, size = 72, normalized size = 1.16

$$\frac{\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right) - 2\sqrt{a\sin(dx+c)+a}\,a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*a^(3/2)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*
sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 2*sqrt(a*sin(d*x + c) + a)*a)/d
```

giac [B] time = 12.20, size = 1021, normalized size = 16.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*sqrt(a)*(sqrt(2)*(6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
tan(1/2*c)^3*tan(1/4*c)^5 - 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/2*c)^2*tan(1/4*c)^6 - 20*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*tan(1/2*c)^3*tan(1/4*c)^3 + 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*tan(1/2*c)^2*tan(1/4*c)^4 - 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*tan(1/2*c)*tan(1/4*c)^5 + sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*tan(1/4*c)^6 + 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*
c)^3*tan(1/4*c) - 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*
c)^2*tan(1/4*c)^2 + 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
2*c)*tan(1/4*c)^3 - 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
4*c)^4 + 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2 - 18*
sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c) + 15*sq
rt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 - sqrt(2)*a*sgn(co
```

```
s(-1/4*pi + 1/2*d*x + 1/2*c)))*log(abs(2*tan(1/4*d*x + c)*tan(1/2*c)^3 + 6*
tan(1/4*d*x + c)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 - 2*sqrt(2)*(tan(1/2*c)^2 +
1)^(3/2) - 6*tan(1/4*d*x + c)*tan(1/2*c) + 6*tan(1/2*c)^2 - 2*tan(1/4*d*x +
 c) + 6*tan(1/2*c) - 2)/abs(2*tan(1/4*d*x + c)*tan(1/2*c)^3 + 6*tan(1/4*d*x
 + c)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 + 2*sqrt(2)*(tan(1/2*c)^2 + 1)^(3/2) -
6*tan(1/4*d*x + c)*tan(1/2*c) + 6*tan(1/2*c)^2 - 2*tan(1/4*d*x + c) + 6*tan
(1/2*c) - 2))/((tan(1/4*c)^6 + 3*tan(1/4*c)^4 + 3*tan(1/4*c)^2 + 1)*(tan(1/
2*c)^2 + 1)^(3/2)) - 4*(a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x +
 c)*tan(1/4*c)^6 - 6*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)
*tan(1/4*c)^5 + a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^6 - 15*a*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^4 + 6*a*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^5 + 20*a*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^3 - 15*a*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))*tan(1/4*c)^4 + 15*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*
d*x + c)*tan(1/4*c)^2 - 20*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)
^3 - 6*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c) +
15*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 - a*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))*tan(1/4*d*x + c) + 6*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*tan(1/4*c) - a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))/((sqrt(2)*tan(1/4*c)
^6 + 3*sqrt(2)*tan(1/4*c)^4 + 3*sqrt(2)*tan(1/4*c)^2 + sqrt(2))*(tan(1/4*d*
x + c)^2 + 1)))/d
```

maple [A] time = 0.14, size = 49, normalized size = 0.79

$$-\frac{2a\left(\sqrt{a+a\sin\left(dx+c\right)}-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2*a*((a+a*sin(d*x+c))^(1/2)-a^(1/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(
1/2)*2^(1/2)/a^(1/2)))/d
```

maxima [A] time = 0.93, size = 80, normalized size = 1.29

$$-\frac{\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin\left(dx+c\right)+a}}\right)+2\sqrt{a\sin\left(dx+c\right)+a}\,a^{2}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt(2
)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 2*sqrt(a*sin(d*x + c) + a)*a^2)/(a
*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.122 $\int \sec^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=26

$$\frac{2a\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}$$

```
[Out] 2*a*sec(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$\frac{2a\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{2a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}$$

Mathematica [B] time = 0.16, size = 67, normalized size = 2.58

$$\frac{2(a(\sin(c+dx)+1))^{3/2}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(3/2))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [A] time = 0.65, size = 26, normalized size = 1.00

$$\frac{2\sqrt{a\sin(dx+c)+a}\,a}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*sin(d*x + c) + a)*a/(d*cos(d*x + c))
```

giac [B] time = 155.91, size = 3663, normalized size = 140.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(a)*(sqrt(2)*(sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/2*c)^3*tan(1/4*c)^6 - 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*tan(1/2*c)^3*tan(1/4*c)^4 + 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*tan(1/2*c)^2*tan(1/4*c)^5 - 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/2*c)*tan(1/4*c)^6 + 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/2*c)^3*tan(1/4*c)^2 - 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 + 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*tan(1/4*c)^5 - sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1
/2*c)^3 + 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan
(1/4*c) - 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1
/4*c)^2 + 20*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^3 + 3
*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c) - 6*sqrt(2)*a*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c))*log(abs(2*tan(1/4*d*x + c)*tan
(1/2*c)^3 + 6*tan(1/4*d*x + c)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 - 2*sqrt(2)*(t
an(1/2*c)^2 + 1)^(3/2) - 6*tan(1/4*d*x + c)*tan(1/2*c) + 6*tan(1/2*c)^2 - 2
*tan(1/4*d*x + c) + 6*tan(1/2*c) - 2)/abs(2*tan(1/4*d*x + c)*tan(1/2*c)^3 +
 6*tan(1/4*d*x + c)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 + 2*sqrt(2)*(tan(1/2*c)^2
 + 1)^(3/2) - 6*tan(1/4*d*x + c)*tan(1/2*c) + 6*tan(1/2*c)^2 - 2*tan(1/4*d*
x + c) + 6*tan(1/2*c) - 2))/((tan(1/4*c)^6 + 3*tan(1/4*c)^4 + 3*tan(1/4*c)^
2 + 1)*(tan(1/2*c)^2 + 1)^(3/2)) - 4*(6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^6*tan(1/4*c)^5 - 3*sqrt(2)*a*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^5*tan(1/4*c)^6 - 1
8*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)
^5*tan(1/4*c)^5 + 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c
)^6*tan(1/4*c)^5 + 9*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*
d*x + c)*tan(1/2*c)^4*tan(1/4*c)^6 - 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*tan(1/2*c)^5*tan(1/4*c)^6 - 20*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^6*tan(1/4*c)^3 + 45*sqrt(2)*a*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^5*tan(1/4*c)^4 -
 36*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*
c)^4*tan(1/4*c)^5 + 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
2*c)^5*tan(1/4*c)^5 + 10*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(
1/4*d*x + c)*tan(1/2*c)^3*tan(1/4*c)^6 - 9*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c)^6 + 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^5*tan(1/4*c)^3 - 20*sqrt(2)*a*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^6*tan(1/4*c)^3 - 135*sqrt(2)*
a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^4*tan(1/4
*c)^4 + 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^5*tan(1
/4*c)^4 + 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)
*tan(1/2*c)^3*tan(1/4*c)^5 - 36*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*tan(1/2*c)^4*tan(1/4*c)^5 - 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^6 + 10*sqrt(2)*a*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^6 + 6*sqrt(2)*a*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^6*tan(1/4*c) - 45*sqrt(
2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^5*tan(
1/4*c)^2 + 120*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x +
c)*tan(1/2*c)^4*tan(1/4*c)^3 - 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*tan(1/2*c)^5*tan(1/4*c)^3 - 150*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^3*tan(1/4*c)^4 + 135*sqrt(2)*a*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c)^4 + 54*sqrt(2)*a*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^5 -
60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^5
- 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*
c)*tan(1/4*c)^6 + 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c
)^2*tan(1/4*c)^6 - 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4
```

*d*x + c)*tan(1/2*c)^5*tan(1/4*c) + 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^6*tan(1/4*c) + 135*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^4*tan(1/4*c)^2 - 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^5*tan(1/4*c)^2 - 200*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^3*tan(1/4*c)^3 + 120*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c)^3 + 90*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^4 - 150*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^4 - 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)*tan(1/4*c)^5 + 54*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^5 + sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^6 - 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^6 + 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^5 - 36*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^4*tan(1/4*c) + 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^5*tan(1/4*c) + 150*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^3*tan(1/4*c)^2 - 135*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c)^2 - 180*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^3 + 200*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^3 + 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)*tan(1/4*c)^4 - 90*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^4 + 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^5 - sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^6 - 9*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^4 + 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^5 + 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^3*tan(1/4*c) - 36*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c) - 90*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^2 + 150*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^2 + 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)*tan(1/4*c)^3 - 180*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 - 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^4 + 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 10*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^3 + 9*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^4 + 54*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c) - 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c) - 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)*tan(1/4*c)^2 + 90*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^2 - 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^3 + 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^4 + 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2 - 10*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3 - 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)*tan(1/4*c) + 54*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c) + 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^2 - 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^2 + 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c) - 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2 + 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c) - 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 - sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c) + 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c) + sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))/((tan(1/2*c)^3*tan(1/4*c)^6 + 3*tan(1/2*c)^2*tan(1/4*c)^6 + 3*tan(1/2*c)^3*tan(1/4*c)^4 - 3*tan(1/2*c)*tan(1/4*c)^6 + 9*tan(1/2*c)^2*tan(1/4*c)^4 - tan(1/4*c)^6 + 3*tan(1/2*c)^3*tan(1/4*c)^2 - 9*tan(1/2*c)*tan(1/4*c)^4 + 9*tan(1/2*c)^2*

```
tan(1/4*c)^2 - 3*tan(1/4*c)^4 + tan(1/2*c)^3 - 9*tan(1/2*c)*tan(1/4*c)^2 +
3*tan(1/2*c)^2 - 3*tan(1/4*c)^2 - 3*tan(1/2*c) - 1)*(tan(1/4*d*x + c)^2*tan
(1/2*c)^3 + 3*tan(1/4*d*x + c)^2*tan(1/2*c)^2 - 2*tan(1/4*d*x + c)*tan(1/2*
c)^3 - 3*tan(1/4*d*x + c)^2*tan(1/2*c) + 6*tan(1/4*d*x + c)*tan(1/2*c)^2 -
tan(1/2*c)^3 - tan(1/4*d*x + c)^2 + 6*tan(1/4*d*x + c)*tan(1/2*c) - 3*tan(1
/2*c)^2 - 2*tan(1/4*d*x + c) + 3*tan(1/2*c) + 1)))/d
```

maple [A] time = 0.14, size = 37, normalized size = 1.42

$$\frac{2a^2\left(1+\sin\left(dx+c\right)\right)}{\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2*a^2*(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [B] time = 2.51, size = 98, normalized size = 3.77

$$-\frac{2\left(a^{\frac{3}{2}}+\frac{2a^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^{\frac{3}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(a^(3/2) + 2*a^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^(3/2)*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(3/2))
```

mupad [B] time = 4.77, size = 37, normalized size = 1.42

$$\frac{4\,a\,\cos\left(c+d\,x\right)\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

```
[Out] (4*a*cos(c + d*x)*(a*(sin(c + d*x) + 1))^(1/2))/(d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.123 $\int \sec^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{2d}$$

```
[Out] 1/2*sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d+1/4*a^(3/2)*arctanh(1/2*(a+a*sin(
d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2675, 2667, 63, 206}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*d)
 + (Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(2*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d} + \frac{1}{4}a\int \sec(c+dx)\sqrt{a+a\sin(c$$
$$= \frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\,\mathrm{si}\right.}{4d}$$
$$= \frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a}\right.}{2d}$$
$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} + \frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d}$$

Mathematica [A] time = 0.26, size = 72, normalized size = 0.99

$$\frac{a\left(\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right)-\frac{2\sqrt{a(\sin(c+dx)+1)}}{\sin(c+dx)-1}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*(Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])] -
 (2*Sqrt[a*(1 + Sin[c + d*x])])/(-1 + Sin[c + d*x])))/(4*d)
```

fricas [A] time = 0.61, size = 99, normalized size = 1.36

$$\frac{\left(\sqrt{2}\,a\sin(dx+c)-\sqrt{2}\,a\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\sqrt{a\sin(dx+c)+a}\,a}{8\,(d\sin(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*((sqrt(2)*a*sin(d*x + c) - sqrt(2)*a)*sqrt(a)*log(-(a*sin(d*x + c) + 2*
sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*sqr
t(a*sin(d*x + c) + a)*a)/(d*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.19, size = 70, normalized size = 0.96

$$\frac{2a^3\left(-\frac{\sqrt{a+a\sin(dx+c)}}{4a(a\sin(dx+c)-a)}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2*a^3*(-1/4*(a+a*sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)+1/8/a^(3/2)*2^(1/2)*a
rctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

maxima [A] time = 1.41, size = 94, normalized size = 1.29

$$-\frac{\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\,\sqrt{a\sin(dx+c)+a}\,a^3}{a\sin(dx+c)-a}}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sq
rt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*sqrt(a*sin(d*x + c) + a)*a^3
/(a*sin(d*x + c) - a))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{3/2}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.124 $\int \sec^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=107

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}+\frac{a\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

```
[Out] 1/3*sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d-1/4*a^(3/2)*arctanh(1/2*cos(d*x+c
)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)+1/2*a*sec(d*x+c)*(a+a*s
in(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2675, 2649, 206}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}+\frac{a\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(2*Sqrt[2]*d) + (a*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d) + (Sec[c
 + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} + \frac{1}{2}a\int \sec^2(c+dx)\sqrt{a+a\sin(c+
$$
$$= \frac{a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} +$$
$$= \frac{a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} -$$
$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}d} + \frac{a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{s}{}$$

Mathematica [C] time = 0.40, size = 130, normalized size = 1.21

$$\frac{\left(\frac{1}{12}+\frac{i}{12}\right)a\sec^3(c+dx)\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(6(-1)^{3/4}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((1/12 + I/12)*a*Sec[c + d*x]^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Sqr
t[a*(1 + Sin[c + d*x])]*(6*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 +
Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 - (1 - I)*(-5 +
3*Sin[c + d*x])))/d
```

fricas [B] time = 0.64, size = 215, normalized size = 2.01

$$\frac{3\left(\sqrt{2}\,a\cos(dx+c)\sin(dx+c)-\sqrt{2}\,a\cos(dx+c)\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{}\right.}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(}\right.}{24\,(d\cos(dx+c)\sin(dx+c)-d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(sqrt(2)*a*cos(d*x + c)*sin(d*x + c) - sqrt(2)*a*cos(d*x + c))*sqrt
(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x +
c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) + 4*(3*a*sin(d*x + c) - 5*a)*sqrt(a*sin(d*x +
 c) + a))/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.23, size = 107, normalized size = 1.00

$$\frac{(1+\sin(dx+c))\left(3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)a^2\,(a-a\sin(dx+c))^{\frac{3}{2}}-10a^{\frac{7}{2}}+6a^{\frac{7}{2}}\sin(dx+c)\right)}{12a^{\frac{3}{2}}\,(\sin(dx+c)-1)\cos(dx+c)\,\sqrt{a+a\sin(dx+c)}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/12/a^(3/2)*(1+sin(d*x+c))/(sin(d*x+c)-1)*(3*2^(1/2)*arctanh(1/2*(a-a*sin(
d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin(d*x+c))^(3/2)-10*a^(7/2)+6*a^(7
/2)*sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{3/2}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.125 $\int \sec^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=127

$$\frac{15a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}-\frac{15a^2}{32d\sqrt{a\sin(c+dx)+a}}+\frac{\sec^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{4d}+\frac{5a\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{16d}$$

```
[Out] 1/4*sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2)/d+15/64*a^(3/2)*arctanh(1/2*(a+a*si
n(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-15/32*a^2/d/(a+a*sin(d*x+c))^(1/
2)+5/16*a*sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2667, 51, 63, 206}

$$-\frac{15a^2}{32d\sqrt{a\sin(c+dx)+a}}+\frac{15a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{4d}+\frac{5a\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (15*a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2
]*d) - (15*a^2)/(32*d*Sqrt[a + a*Sin[c + d*x]]) + (5*a*Sec[c + d*x]^2*Sqrt[
a + a*Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2))/(
4*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{4d}+\frac{1}{8}(5a)\int \sec^3(c+dx)\sqrt{a+a\sin\\ &= \frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c+dx)}{4d}\\ &= \frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c+dx)}{4d}\\ &= -\frac{15a^2}{32d\sqrt{a+a\sin(c+dx)}}+\frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d}+\frac{\mathrm{se}}{}\\ &= -\frac{15a^2}{32d\sqrt{a+a\sin(c+dx)}}+\frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d}+\frac{\mathrm{se}}{}\\ &= \frac{15a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}-\frac{15a^2}{32d\sqrt{a+a\sin(c+dx)}}+\frac{5a\sec^2(c}{}\end{aligned}$$

Mathematica [C] time = 0.07, size = 44, normalized size = 0.35

$$-\frac{a^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{4d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/4*(a^2*Hypergeometric2F1[-1/2, 3, 1/2, (1 + Sin[c + d*x])/2])/(d*Sqrt[a
+ a*Sin[c + d*x]])
```

fricas [A] time = 0.77, size = 155, normalized size = 1.22

$$\frac{15\left(\sqrt{2}\,a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-\sqrt{2}\,a\cos\left(dx+c\right)^2\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(15a\right.}{128\left(d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(15*(sqrt(2)*a*cos(d*x + c)^2*sin(d*x + c) - sqrt(2)*a*cos(d*x + c)^2
)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 + 3*a)/(sin(d*x + c) - 1)) - 4*(15*a*cos(d*x + c)^2 + 20*a*sin(d*x + c) -
12*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x
+ c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.33, size = 101, normalized size = 0.80

$$-\frac{2a^{5}\left(\frac{1}{8a^{3}\sqrt{a+a\sin(dx+c)}}+\frac{\frac{\sqrt{a+a\sin(dx+c)}\,a(7\sin(dx+c)-11)}{8(a\sin(dx+c)-a)^{2}}-\frac{15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16\sqrt{a}}}{8a^{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x)
[Out] -2*a^5*(1/8/a^3/(a+a*sin(d*x+c))^(1/2)+1/8/a^3*(1/8*(a+a*sin(d*x+c))^(1/2)*
a*(7*sin(d*x+c)-11)/(a*sin(d*x+c)-a)^2-15/16*2^(1/2)/a^(1/2)*arctanh(1/2*(a
+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))))/d
```

maxima [A] time = 0.61, size = 151, normalized size = 1.19

$$-\frac{15\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(15\,(a\sin(dx+c)+a)^{2}a^{3}-50\,(a\sin(dx+c)+a)a^{4}+32\,a^{5}\right)}{(a\sin(dx+c)+a)^{\frac{5}{2}}-4\,(a\sin(dx+c)+a)^{\frac{3}{2}}a+4\,\sqrt{a\sin(dx+c)+a}\,a^{2}}}{128\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -1/128*(15*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(15*(a*sin(d*x + c) + a
)^2*a^3 - 50*(a*sin(d*x + c) + a)*a^4 + 32*a^5)/((a*sin(d*x + c) + a)^(5/2)
 - 4*(a*sin(d*x + c) + a)^(3/2)*a + 4*sqrt(a*sin(d*x + c) + a)*a^2))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{3/2}}{\cos(c+d\,x)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^5,x)
[Out] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.126 $\int \sec^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=169

$$-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,d}-\frac{7a^3\cos(c+dx)}{16d(a\sin(c+dx)+a)^{3/2}}+\frac{7a^2\sec(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}+\frac{\sec^5(c+dx)(a\sin(c+d}{5d}$$

```
[Out] -7/16*a^3*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)+1/5*sec(d*x+c)^5*(a+a*sin(d*x
+c))^(3/2)/d-7/32*a^(3/2)*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d
*x+c))^(1/2))/d*2^(1/2)+7/12*a^2*sec(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+7/30*a
*sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2687, 2650, 2649, 206}

$$-\frac{7a^3\cos(c+dx)}{16d(a\sin(c+dx)+a)^{3/2}}+\frac{7a^2\sec(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,d}+\frac{\sec^5(c+dx)(a\sin(c+d}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-7*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(16*Sqrt[2]*d) - (7*a^3*Cos[c + d*x])/(16*d*(a + a*Sin[c + d*x])^(3/2)
) + (7*a^2*Sec[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) + (7*a*Sec[c + d*x
]^3*Sqrt[a + a*Sin[c + d*x]])/(30*d) + (Sec[c + d*x]^5*(a + a*Sin[c + d*x])
^(3/2))/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\sec^5(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} + \frac{1}{10}(7a)\int \sec^4(c+dx)\sqrt{a+a\sin(}$$

$$= \frac{7a\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}$$

$$= \frac{7a^2\sec(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{30d} + \frac{\sec^5(c}{}$$

$$= -\frac{7a^3\cos(c+dx)}{16d(a+a\sin(c+dx))^{3/2}} + \frac{7a^2\sec(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^3(c+dx)}{}$$

$$= -\frac{7a^3\cos(c+dx)}{16d(a+a\sin(c+dx))^{3/2}} + \frac{7a^2\sec(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^3(c+dx)}{}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,d} - \frac{7a^3\cos(c+dx)}{16d(a+a\sin(c+dx))^{3/2}} + \frac{7a^2}{12d\sqrt{a}}$$

Mathematica [C] time = 0.43, size = 288, normalized size = 1.70

$$\frac{(a(\sin(c+dx)+1))^{3/2}\left(30\sin\left(\frac{1}{2}(c+dx)\right) + \frac{90\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{40\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{24\left(\sin\left(\frac{1}{2}(c+\right.\right.}{\left(\cos\left(\frac{1}{2}(c+\right.\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((30*Sin[(c + d*x)/2] - 15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (105 + 1
05*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (24*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5 + (40*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (90*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))*(a
*(1 + Sin[c + d*x]))^(3/2))/(240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [A] time = 0.77, size = 248, normalized size = 1.47

$$\frac{105\left(\sqrt{2}\,a\cos(dx+c)^3\sin(dx+c) - \sqrt{2}\,a\cos(dx+c)^3\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c) - \sqrt{2}\sin(dx+\right.}{\cos(dx+c)^2 - (\cos(dx+c)+}\right.}{960\left(d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/960*(105*(sqrt(2)*a*cos(d*x + c)^3*sin(d*x + c) - sqrt(2)*a*cos(d*x + c)^
3)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos
```

```
(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a
*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) +
2)*sin(d*x + c) - cos(d*x + c) - 2)) - 4*(175*a*cos(d*x + c)^2 - 21*(5*a*co
s(d*x + c)^2 - 4*a)*sin(d*x + c) - 36*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d
*x + c)^3*sin(d*x + c) - d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.25, size = 172, normalized size = 1.02

$$-\frac{210a^{\frac{7}{2}}\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)+\left(105\left(a-a\sin\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)a-168a^{\frac{7}{2}}\right)\sin}{480a^{\frac{3}{2}}\left(\sin\left(dx+c\right)-1\right)^{2}\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x)
[Out] -1/480/a^(3/2)*(210*a^(7/2)*sin(d*x+c)*cos(d*x+c)^2+(105*(a-a*sin(d*x+c))^(
5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-168*a^(7
/2))*sin(d*x+c)-350*a^(7/2)*cos(d*x+c)^2+105*(a-a*sin(d*x+c))^(5/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+72*a^(7/2))/(sin(d*x
+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^6,x)
[Out] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.127 $\int \cos^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{15/2}}{15a^5d}-\frac{8(a\sin(c+dx)+a)^{13/2}}{13a^4d}+\frac{8(a\sin(c+dx)+a)^{11/2}}{11a^3d}$$

```
[Out] 8/11*(a+a*sin(d*x+c))^(11/2)/a^3/d-8/13*(a+a*sin(d*x+c))^(13/2)/a^4/d+2/15*
(a+a*sin(d*x+c))^(15/2)/a^5/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{15/2}}{15a^5d}-\frac{8(a\sin(c+dx)+a)^{13/2}}{13a^4d}+\frac{8(a\sin(c+dx)+a)^{11/2}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(11/2))/(11*a^3*d) - (8*(a + a*Sin[c + d*x])^(13/2)
)/(13*a^4*d) + (2*(a + a*Sin[c + d*x])^(15/2))/(15*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{9/2}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{9/2}-4a(a+x)^{11/2}+(a+x)^{13/2}\right)dx,x,a\sin(c+d\right.}{a^5d}\\ &= \frac{8(a+a\sin(c+dx))^{11/2}}{11a^3d}-\frac{8(a+a\sin(c+dx))^{13/2}}{13a^4d}+\frac{2(a+a\sin(c+dx}{15a^5d}\end{aligned}$$

Mathematica [A] time = 0.22, size = 51, normalized size = 0.70

$$\frac{2(\sin(c+dx)+1)^3\left(143\sin^2(c+dx)-374\sin(c+dx)+263\right)(a(\sin(c+dx)+1))^{5/2}}{2145d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

[Out] (2*(1 + Sin[c + d*x])^3*(a*(1 + Sin[c + d*x]))^(5/2)*(263 - 374*Sin[c + d*x
] + 143*Sin[c + d*x]^2))/(2145*d)

fricas [A] time = 0.76, size = 114, normalized size = 1.56

$$-\frac{2\left(341\,a^2\cos\left(dx+c\right)^6-28\,a^2\cos\left(dx+c\right)^4-64\,a^2\cos\left(dx+c\right)^2-512\,a^2+\left(143\,a^2\cos\left(dx+c\right)^6-252\,a^2\cos\right.\right.}{2145\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -2/2145*(341*a^2*cos(d*x + c)^6 - 28*a^2*cos(d*x + c)^4 - 64*a^2*cos(d*x +
c)^2 - 512*a^2 + (143*a^2*cos(d*x + c)^6 - 252*a^2*cos(d*x + c)^4 - 320*a^2
*cos(d*x + c)^2 - 512*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/d

giac [B] time = 2.77, size = 471, normalized size = 6.45

$$-\frac{1}{2882880}\sqrt{2}\left(\frac{16380\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{102960\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{7}{2}\,dx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] -1/2882880*sqrt(2)*(16380*a^2*cos(1/4*pi + 11/2*d*x + 11/2*c)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))/d + 102960*a^2*cos(1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))/d + 300300*a^2*cos(1/4*pi + 3/2*d*x + 3/2*c)*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 13860*a^2*cos(-1/4*pi + 13/2*d*x + 1
3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 80080*a^2*cos(-1/4*pi + 9/2*
d*x + 9/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 180180*a^2*cos(-1/4*pi
 + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 3465*a^2*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 13/2*d*x + 13/2*c)/d - 5005*a^2*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 9/2*d*x + 9/2*c)/d - 17117
1*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 5/2*d*x + 5/2*c)/d -
 2027025*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 1/2*d*x + 1/2
*c)/d + 3003*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 15/2*d*x
 + 15/2*c)/d - 4095*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
1/2*d*x + 11/2*c)/d - 122265*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1
/4*pi + 7/2*d*x + 7/2*c)/d - 675675*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*sin(-1/4*pi + 3/2*d*x + 3/2*c)/d)*sqrt(a)

maple [A] time = 0.16, size = 41, normalized size = 0.56

$$-\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{11}{2}}\left(143\left(\cos^2\left(dx+c\right)\right)+374\sin\left(dx+c\right)-406\right)}{2145a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x)

[Out] -2/2145/a^3*(a+a*sin(d*x+c))^(11/2)*(143*cos(d*x+c)^2+374*sin(d*x+c)-406)/d

maxima [A] time = 0.52, size = 55, normalized size = 0.75

$$\frac{2\left(143\left(a\sin\left(dx+c\right)+a\right)^{\frac{15}{2}}-660\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}a+780\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}a^2\right)}{2145\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/2145*(143*(a*sin(d*x + c) + a)^(15/2) - 660*(a*sin(d*x + c) + a)^(13/2)*a
 + 780*(a*sin(d*x + c) + a)^(11/2)*a^2)/(a^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^5 (a+a\sin(c+dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.128 $\int \cos^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=159

$$-\frac{4096a^5\cos^5(c+dx)}{15015d(a\sin(c+dx)+a)^{5/2}}-\frac{1024a^4\cos^5(c+dx)}{3003d(a\sin(c+dx)+a)^{3/2}}-\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a\sin(c+dx)+a}}-\frac{32a^2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{143d}$$

```
[Out] -4096/15015*a^5*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-1024/3003*a^4*cos(d*x
+c)^5/d/(a+a*sin(d*x+c))^(3/2)-2/13*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2)/d
-128/429*a^3*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-32/143*a^2*cos(d*x+c)^5*
(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{32a^2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{143d}-\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a\sin(c+dx)+a}}-\frac{1024a^4\cos^5(c+dx)}{3003d(a\sin(c+dx)+a)^{3/2}}-\frac{4096a^5\cos^5(c+dx)}{15015d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4096*a^5*Cos[c + d*x]^5)/(15015*d*(a + a*Sin[c + d*x])^(5/2)) - (1024*a^4
*Cos[c + d*x]^5)/(3003*d*(a + a*Sin[c + d*x])^(3/2)) - (128*a^3*Cos[c + d*x
]^5)/(429*d*Sqrt[a + a*Sin[c + d*x]]) - (32*a^2*Cos[c + d*x]^5*Sqrt[a + a*S
in[c + d*x]])/(143*d) - (2*a*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13
*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= -\frac{2a\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{13d}+\frac{1}{13}(16a)\int \cos^4(c+dx)(a \\
&= -\frac{32a^2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d}-\frac{2a\cos^5(c+dx)(a+a\sin(\\
&= -\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a+a\sin(c+dx)}}-\frac{32a^2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d}- \\
&= -\frac{1024a^4\cos^5(c+dx)}{3003d(a+a\sin(c+dx))^{3/2}}-\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a+a\sin(c+dx)}}-\frac{32a^2\cos \\
&= -\frac{4096a^5\cos^5(c+dx)}{15015d(a+a\sin(c+dx))^{5/2}}-\frac{1024a^4\cos^5(c+dx)}{3003d(a+a\sin(c+dx))^{3/2}}-\frac{12}{429d}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 79, normalized size = 0.50

$$-\frac{2\left(1155\sin^4(c+dx)+6300\sin^3(c+dx)+14210\sin^2(c+dx)+16700\sin(c+dx)+9683\right)\cos^5(c+dx)(a(\sin(c}{15015d(\sin(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(5/2)*(9683 + 16700*Sin[c + d*x]
+ 14210*Sin[c + d*x]^2 + 6300*Sin[c + d*x]^3 + 1155*Sin[c + d*x]^4))/(15015
*d*(1 + Sin[c + d*x])^5)
```

fricas [A] time = 0.62, size = 219, normalized size = 1.38

$$\frac{2\left(1155\,a^2\cos\left(dx+c\right)^7-2835\,a^2\cos\left(dx+c\right)^6-6230\,a^2\cos\left(dx+c\right)^5+320\,a^2\cos\left(dx+c\right)^4-512\,a^2\cos\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*a^2*cos(d*x + c)^7 - 2835*a^2*cos(d*x + c)^6 - 6230*a^2*cos(d
*x + c)^5 + 320*a^2*cos(d*x + c)^4 - 512*a^2*cos(d*x + c)^3 + 1024*a^2*cos(
d*x + c)^2 - 4096*a^2*cos(d*x + c) - 8192*a^2 - (1155*a^2*cos(d*x + c)^6 +
3990*a^2*cos(d*x + c)^5 - 2240*a^2*cos(d*x + c)^4 - 2560*a^2*cos(d*x + c)^3
 - 3072*a^2*cos(d*x + c)^2 - 4096*a^2*cos(d*x + c) - 8192*a^2)*sin(d*x + c)
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 1.39, size = 438, normalized size = 2.75

$$-\frac{1}{1441440}\sqrt{2}\left(\frac{20020\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{108108\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/1441440*sqrt(2)*(20020*a^2*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d + 108108*a^2*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))/d + 360360*a^2*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 16380*a^2*cos(-1/4*pi + 11/2*d*x + 11/
2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 77220*a^2*cos(-1/4*pi + 7/2*d*
x + 7/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 120120*a^2*cos(-1/4*pi +
 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 4095*a^2*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 11/2*d*x + 11/2*c)/d - 12870*a^2*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 7/2*d*x + 7/2*c)/d - 255255
*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d +
3465*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 13/2*d*x + 13/2*
c)/d - 10010*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 9/2*d*x
+ 9/2*c)/d - 153153*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5
/2*d*x + 5/2*c)/d - 1261260*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/
4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.19, size = 87, normalized size = 0.55

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^3\left(\sin\left(dx+c\right)-1\right)^3\left(1155\left(\sin^4\left(dx+c\right)\right)+6300\left(\sin^3\left(dx+c\right)\right)+14210\left(\sin^2\left(dx+c\right)\right)+16\right.}{15015\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15015*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)^3*(1155*sin(d*x+c)^4+6300*sin(d*x
+c)^3+14210*sin(d*x+c)^2+16700*sin(d*x+c)+9683)/cos(d*x+c)/(a+a*sin(d*x+c))
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.129 $\int \cos^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=49

$$\frac{4(a\sin(c+dx)+a)^{9/2}}{9a^2d}-\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^3d}$$

```
[Out] 4/9*(a+a*sin(d*x+c))^(9/2)/a^2/d-2/11*(a+a*sin(d*x+c))^(11/2)/a^3/d
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{9/2}}{9a^2d}-\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(9/2))/(9*a^2*d) - (2*(a + a*Sin[c + d*x])^(11/2))/
(11*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^{7/2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a(a+x)^{7/2}-(a+x)^{9/2}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{4(a+a\sin(c+dx))^{9/2}}{9a^2d}-\frac{2(a+a\sin(c+dx))^{11/2}}{11a^3d}\end{aligned}$$

Mathematica [A] time = 0.14, size = 41, normalized size = 0.84

$$-\frac{2(\sin(c+dx)+1)^2(9\sin(c+dx)-13)(a(\sin(c+dx)+1))^{5/2}}{99d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(1 + Sin[c + d*x])^2*(a*(1 + Sin[c + d*x]))^(5/2)*(-13 + 9*Sin[c + d*x]
))/(99*d)
```

fricas [B] time = 0.78, size = 88, normalized size = 1.80

$$-\frac{2\left(23\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2-32\,a^2+\left(9\,a^2\cos\left(dx+c\right)^4-20\,a^2\cos\left(dx+c\right)^2-32\,a^2\right)\sin\left(dx\right.}{99\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/99*(23*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 - 32*a^2 + (9*a^2*cos(d
*x + c)^4 - 20*a^2*cos(d*x + c)^2 - 32*a^2)*sin(d*x + c))*sqrt(a*sin(d*x +
c) + a)/d
```

giac [B] time = 4.54, size = 339, normalized size = 6.92

$$-\frac{1}{55440}\sqrt{2}\left(\frac{1980\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{9240\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\mathrm{sg}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/55440*sqrt(2)*(1980*a^2*cos(1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))/d + 9240*a^2*cos(1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))/d + 1540*a^2*cos(-1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))/d + 5544*a^2*cos(-1/4*pi + 5/2*d*x + 5/2*c)*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 385*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*sin(1/4*pi + 9/2*d*x + 9/2*c)/d - 2079*a^2*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*sin(1/4*pi + 5/2*d*x + 5/2*c)/d - 48510*a^2*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*sin(1/4*pi + 1/2*d*x + 1/2*c)/d + 315*a^2*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))*sin(-1/4*pi + 11/2*d*x + 11/2*c)/d - 1485*a^2*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 7/2*d*x + 7/2*c)/d - 16170*a^2*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 3/2*d*x + 3/2*c)/d)*sqrt(a)
```

maple [A] time = 0.16, size = 31, normalized size = 0.63

$$-\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{9}{2}}\left(9\sin\left(dx+c\right)-13\right)}{99a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/99/a^2*(a+a*sin(d*x+c))^(9/2)*(9*sin(d*x+c)-13)/d
```

maxima [A] time = 0.47, size = 38, normalized size = 0.78

$$-\frac{2\left(9\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-22\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a\right)}{99\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/99*(9*(a*sin(d*x + c) + a)^(11/2) - 22*(a*sin(d*x + c) + a)^(9/2)*a)/(a^
3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.130 $\int \cos^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=127

$$-\frac{256a^4\cos^3(c+dx)}{315d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{8a^2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos^3(c+dx)(a$$

```
[Out] -256/315*a^4*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/9*a*cos(d*x+c)^3*(a+a*
sin(d*x+c))^(3/2)/d-64/105*a^3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-8/21*a
^2*cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2674, 2673}

$$-\frac{256a^4\cos^3(c+dx)}{315d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{8a^2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos^3(c+dx)(a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^3)/(315*d*(a + a*Sin[c + d*x])^(3/2)) - (64*a^3*Cos[
c + d*x]^3)/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (8*a^2*Cos[c + d*x]^3*Sqrt[a
 + a*Sin[c + d*x]])/(21*d) - (2*a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2)
)/(9*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= -\frac{2a\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9d}+\frac{1}{3}(4a)\int \cos^2(c+dx)(a+\\ &= -\frac{8a^2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2a\cos^3(c+dx)(a+a\sin(c\\ &= -\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{8a^2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\\ &= -\frac{256a^4\cos^3(c+dx)}{315d(a+a\sin(c+dx))^{3/2}}-\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{8a^2\cos^3(c\end{aligned}$$

Mathematica [A] time = 0.31, size = 69, normalized size = 0.54

$$-\frac{2\left(35\sin^3(c+dx)+165\sin^2(c+dx)+321\sin(c+dx)+319\right)\cos^3(c+dx)(a(\sin(c+dx)+1))^{5/2}}{315d(\sin(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3*(a*(1 + Sin[c + d*x]))^(5/2)*(319 + 321*Sin[c + d*x] + 1
65*Sin[c + d*x]^2 + 35*Sin[c + d*x]^3))/(315*d*(1 + Sin[c + d*x])^4)
```

fricas [A] time = 0.61, size = 167, normalized size = 1.31

$$\frac{2\left(35\,a^{2}\cos\left(dx+c\right)^{5}-95\,a^{2}\cos\left(dx+c\right)^{4}-226\,a^{2}\cos\left(dx+c\right)^{3}+32\,a^{2}\cos\left(dx+c\right)^{2}-128\,a^{2}\cos\left(dx+c\right)-256\cdots\right.}{315\,(d\cos\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^5 - 95*a^2*cos(d*x + c)^4 - 226*a^2*cos(d*x + c)
^3 + 32*a^2*cos(d*x + c)^2 - 128*a^2*cos(d*x + c) - 256*a^2 - (35*a^2*cos(d
*x + c)^4 + 130*a^2*cos(d*x + c)^3 - 96*a^2*cos(d*x + c)^2 - 128*a^2*cos(d*
x + c) - 256*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) +
d*sin(d*x + c) + d)
```

giac [B] time = 1.37, size = 306, normalized size = 2.41

$$-\frac{1}{2520}\,\sqrt{2}\left(\frac{252\,a^{2}\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{1260\,a^{2}\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\mathrm{sgn}\left(\cdots\right.}{d}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/2520*sqrt(2)*(252*a^2*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))/d + 1260*a^2*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))/d + 180*a^2*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))/d + 420*a^2*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))/d + 45*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
sin(1/4*pi + 7/2*d*x + 7/2*c)/d - 420*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d + 35*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*sin(-1/4*pi + 9/2*d*x + 9/2*c)/d - 252*a^2*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d - 3150*a^2*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.20, size = 77, normalized size = 0.61

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)a^{3}\left(\sin\left(dx+c\right)-1\right)^{2}\left(35\left(\sin^{3}\left(dx+c\right)\right)+165\left(\sin^{2}\left(dx+c\right)\right)+321\sin\left(dx+c\right)+319\right)}{315\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/315*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)^2*(35*sin(d*x+c)^3+165*sin(d*x+c)^
2+321*sin(d*x+c)+319)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.131 $\int \cos(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7ad}$$

```
[Out] 2/7*(a+a*sin(d*x+c))^(7/2)/a/d
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(7/2))/(7*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= \frac{\operatorname{Subst}\left(\int (a+x)^{5/2}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{7/2}}{7ad}\end{aligned}$$

Mathematica [A] time = 0.07, size = 24, normalized size = 1.00

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(7/2))/(7*a*d)
```

fricas [B] time = 0.73, size = 61, normalized size = 2.54

$$-\frac{2\left(3\,a^2\cos\left(dx+c\right)^2-4\,a^2+\left(a^2\cos\left(dx+c\right)^2-4\,a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*(3*a^2*cos(d*x + c)^2 - 4*a^2 + (a^2*cos(d*x + c)^2 - 4*a^2)*sin(d*x +
 c))*sqrt(a*sin(d*x + c) + a)/d
```

giac [B] time = 0.56, size = 207, normalized size = 8.62

$$-\frac{1}{420}\sqrt{2}\left(\frac{140\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{84\,a^2\cos\left(-\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\operatorname{sgn}\left(c\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/420*sqrt(2)*(140*a^2*cos(1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d + 84*a^2*cos(-1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))/d + 21*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi
 + 5/2*d*x + 5/2*c)/d - 525*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4
*pi + 1/2*d*x + 1/2*c)/d + 15*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 7/2*d*x + 7/2*c)/d - 175*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*s
in(-1/4*pi + 3/2*d*x + 3/2*c)/d)*sqrt(a)
```

maple [A] time = 0.04, size = 21, normalized size = 0.88

$$\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{7}{2}}}{7da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/7*(a+a*sin(d*x+c))^(7/2)/d/a
```

maxima [A] time = 0.56, size = 20, normalized size = 0.83

$$\frac{2\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}}{7\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(a*sin(d*x + c) + a)^(7/2)/(a*d)
```

mupad [B] time = 4.78, size = 20, normalized size = 0.83

$$\frac{2\,{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)}^{7/2}}{7\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(7/2))/(7*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.132 $\int \sec(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=86

$$\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{4a^2\sqrt{a\sin(c+dx)+a}}{d}-\frac{2a(a\sin(c+dx)+a)^{3/2}}{3d}$$

```
[Out] -2/3*a*(a+a*sin(d*x+c))^(3/2)/d+4*a^(5/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2
)*2^(1/2)/a^(1/2))*2^(1/2)/d-4*a^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = $\{2667, 50, 63, 206\}$

$$-\frac{4a^2\sqrt{a\sin(c+dx)+a}}{d}+\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2a(a\sin(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (4*a^2*Sqrt[a + a*Sin[c + d*x]])/d - (2*a*(a + a*Sin[c + d*x])^(3/2))/(3*d
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{a\,\text{Subst}\left(\int \frac{(a+x)^{3/2}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{2a(a+a\sin(c+dx))^{3/2}}{3d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{4a^2\sqrt{a+a\sin(c+dx)}}{d} - \frac{2a(a+a\sin(c+dx))^{3/2}}{3d} + \frac{(4a^3)\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{4a^2\sqrt{a+a\sin(c+dx)}}{d} - \frac{2a(a+a\sin(c+dx))^{3/2}}{3d} + \frac{(8a^3)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{4a^2\sqrt{a+a\sin(c+dx)}}{d} - \frac{2a(a+a}{}$$

Mathematica [A] time = 0.20, size = 73, normalized size = 0.85

$$\frac{12\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right) - 2a^2(\sin(c+dx)+7)\sqrt{a(\sin(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (12*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])] -
 2*a^2*Sqrt[a*(1 + Sin[c + d*x])]*(7 + Sin[c + d*x]))/(3*d)
```

fricas [A] time = 0.57, size = 89, normalized size = 1.03

$$\frac{2\left(3\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right) - \left(a^2\sin(dx+c)+7a^2\right)\sqrt{a\sin(dx+c)+a}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*a^(5/2)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - (a^2*sin(d*x + c) + 7*a^2)*sqrt(
a*sin(d*x + c) + a))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.17, size = 66, normalized size = 0.77

$$-\frac{2a\left(\frac{(a+a\sin(dx+c))^{\frac{3}{2}}}{3} + 2a\sqrt{a+a\sin(dx+c)} - 2a^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2*a*(1/3*(a+a*sin(d*x+c))^(3/2)+2*a*(a+a*sin(d*x+c))^(1/2)-2*a^(3/2)*2^(1/
2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

maxima [A] time = 0.52, size = 97, normalized size = 1.13

$$-\frac{2\left(3\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+(a\sin(dx+c)+a)^{\frac{3}{2}}a^2+6\sqrt{a\sin(dx+c)+a}\,a^3\right)}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(
sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + (a*sin(d*x + c) + a)^(3/2)*a
^2 + 6*sqrt(a*sin(d*x + c) + a)*a^3)/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.133 $\int \sec^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=55

$$\frac{8a^2\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{2a\sec(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}$$

```
[Out] -2*a*sec(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d+8*a^2*sec(d*x+c)*(a+a*sin(d*x+c))^
(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$\frac{8a^2\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{2a\sec(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*a^2*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d - (2*a*Sec[c + d*x]*(a + a*
Sin[c + d*x])^(3/2))/d
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx=-\frac{2a\sec(c+dx)(a+a\sin(c+dx))^{3/2}}{d}+(4a)\int \sec^2(c+dx)(a+a\,\mathrm{si}$$
$$=\frac{8a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}-\frac{2a\sec(c+dx)(a+a\sin(c+d}{d}$$

Mathematica [A] time = 4.62, size = 36, normalized size = 0.65

$$-\frac{2a^2(\sin(c+dx)-3)\sec(c+dx)\sqrt{a(\sin(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*Sec[c + d*x]*(-3 + Sin[c + d*x])*Sqrt[a*(1 + Sin[c + d*x])])/d
```

fricas [A] time = 0.73, size = 41, normalized size = 0.75

$$-\frac{2\left(a^{2}\sin\left(dx+c\right)-3\,a^{2}\right)\sqrt{a\sin\left(dx+c\right)+a}}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2*(a^2*sin(d*x + c) - 3*a^2)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c))
```

giac [B] time = 54.37, size = 6622, normalized size = 120.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*sqrt(a)*(sqrt(2)*(sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
tan(1/2*c)^3*tan(1/4*c)^6 - 15*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*tan(1/2*c)^3*tan(1/4*c)^4 + 18*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*tan(1/2*c)^2*tan(1/4*c)^5 - 3*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))*tan(1/2*c)*tan(1/4*c)^6 + 15*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^2 - 60*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 + 45*sqrt(2)*a^2*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 6*sqrt(2)*a^2*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))*tan(1/4*c)^5 - sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*tan(1/2*c)^3 + 18*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/2*c)^2*tan(1/4*c) - 45*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*tan(1/2*c)*tan(1/4*c)^2 + 20*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*tan(1/4*c)^3 + 3*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1
/2*c) - 6*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c))*log(a
bs(-2*tan(1/4*d*x + c)*tan(1/2*c)^3 - 6*tan(1/4*d*x + c)*tan(1/2*c)^2 + 2*t
an(1/2*c)^3 - 2*sqrt(2)*(tan(1/2*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(1
/2*c) - 6*tan(1/2*c)^2 + 2*tan(1/4*d*x + c) - 6*tan(1/2*c) + 2)/abs(-2*tan(
1/4*d*x + c)*tan(1/2*c)^3 - 6*tan(1/4*d*x + c)*tan(1/2*c)^2 + 2*tan(1/2*c)^
3 + 2*sqrt(2)*(tan(1/2*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(1/2*c) - 6*
tan(1/2*c)^2 + 2*tan(1/4*d*x + c) - 6*tan(1/2*c) + 2))/((tan(1/4*c)^6 + 3*t
an(1/4*c)^4 + 3*tan(1/4*c)^2 + 1)*(tan(1/2*c)^2 + 1)^(3/2)) - 2*(sqrt(2)*a^
2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^6*tan(1
/4*c)^6 - 6*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c
)^3*tan(1/2*c)^6*tan(1/4*c)^5 + 12*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^5*tan(1/4*c)^6 - 3*sqrt(2)*a^2*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^6*tan(1/4*c)^6
- 15*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan
(1/2*c)^6*tan(1/4*c)^4 + 72*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/4*d*x + c)^3*tan(1/2*c)^5*tan(1/4*c)^5 - 18*sqrt(2)*a^2*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^6*tan(1/4*c)^5 - 15*s
qrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c
)^4*tan(1/4*c)^6 + sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*
d*x + c)*tan(1/2*c)^6*tan(1/4*c)^6 + 20*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^6*tan(1/4*c)^3 - 180*sqrt(2)*a^2
*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^5*tan(1/
4*c)^4 + 45*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c
)^2*tan(1/2*c)^6*tan(1/4*c)^4 + 90*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^4*tan(1/4*c)^5 - 30*sqrt(2)*a^2*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^6*tan(1/4*c)^5 -
 40*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(
1/2*c)^3*tan(1/4*c)^6 + 45*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
tan(1/4*d*x + c)^2*tan(1/2*c)^4*tan(1/4*c)^6 + sqrt(2)*a^2*sgn(cos(-1/4*pi
```

+ 1/2*d*x + 1/2*c))*tan(1/2*c)^6*tan(1/4*c)^6 + 15*sqrt(2)*a^2*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^6*tan(1/4*c)^2 - 240*
sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*
c)^5*tan(1/4*c)^3 + 60*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(
1/4*d*x + c)^2*tan(1/2*c)^6*tan(1/4*c)^3 + 225*sqrt(2)*a^2*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^4*tan(1/4*c)^4 - 15*sqrt(
2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^6*ta
n(1/4*c)^4 - 240*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*
x + c)^3*tan(1/2*c)^3*tan(1/4*c)^5 + 270*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^4*tan(1/4*c)^5 - 18*sqrt(2)*a^2
*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^6*tan(1/4*c)^5 + 15*sqrt(2)
*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^2*ta
n(1/4*c)^6 - 51*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x
 + c)*tan(1/2*c)^4*tan(1/4*c)^6 + 12*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*tan(1/2*c)^5*tan(1/4*c)^6 - 6*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^6*tan(1/4*c) + 180*sqrt(2)*a^2*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^5*tan(1/4*
c)^2 - 45*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^
2*tan(1/2*c)^6*tan(1/4*c)^2 - 300*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^4*tan(1/4*c)^3 + 100*sqrt(2)*a^2*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^6*tan(1/4*c)^3 +
 600*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan
(1/2*c)^3*tan(1/4*c)^4 - 675*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*tan(1/4*d*x + c)^2*tan(1/2*c)^4*tan(1/4*c)^4 - 15*sqrt(2)*a^2*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^6*tan(1/4*c)^4 - 90*sqrt(2)*a^2*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^2*tan(1/4*c)^5 +
 234*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1
/2*c)^4*tan(1/4*c)^5 - 72*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*t
an(1/2*c)^5*tan(1/4*c)^5 + 12*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*tan(1/4*d*x + c)^3*tan(1/2*c)*tan(1/4*c)^6 - 45*sqrt(2)*a^2*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^2*tan(1/4*c)^6 + 21*s
qrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c)^6 -
sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*
c)^6 + 72*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^
3*tan(1/2*c)^5*tan(1/4*c) - 18*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*tan(1/4*d*x + c)^2*tan(1/2*c)^6*tan(1/4*c) - 225*sqrt(2)*a^2*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^4*tan(1/4*c)^2 + 15
*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c
)^6*tan(1/4*c)^2 + 800*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(
1/4*d*x + c)^3*tan(1/2*c)^3*tan(1/4*c)^3 - 900*sqrt(2)*a^2*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^4*tan(1/4*c)^3 + 60*sqrt(
2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^6*tan(1/4*c)^3 - 225*
sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*
c)^2*tan(1/4*c)^4 + 765*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan
(1/4*d*x + c)*tan(1/2*c)^4*tan(1/4*c)^4 - 180*sqrt(2)*a^2*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))*tan(1/2*c)^5*tan(1/4*c)^4 + 72*sqrt(2)*a^2*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)*tan(1/4*c)^5 - 270*sqr
t(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^
2*tan(1/4*c)^5 + 54*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2
*c)^4*tan(1/4*c)^5 - sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
4*d*x + c)^3*tan(1/4*c)^6 + 39*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^6 - 40*sqrt(2)*a^2*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^6 - 12*sqrt(2)*a^2*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^5 + 3*sqrt(2)*a^
2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^6 + 90*
sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*
c)^4*tan(1/4*c) - 30*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
4*d*x + c)*tan(1/2*c)^6*tan(1/4*c) - 600*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^3*tan(1/4*c)^2 + 675*sqrt(2)*a^

2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^4*tan(1
/4*c)^2 + 15*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^6*t
an(1/4*c)^2 + 300*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d
*x + c)^3*tan(1/2*c)^2*tan(1/4*c)^3 - 780*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^4*tan(1/4*c)^3 + 240*sqrt(2)*a^2
*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^5*tan(1/4*c)^3 - 180*sqrt(2
)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)*tan
(1/4*c)^4 + 675*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x
 + c)^2*tan(1/2*c)^2*tan(1/4*c)^4 - 315*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c)^4 + 6*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/4*c)^5 - 306*sqrt(2)*a^2*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^5 + 240
*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^5
 + 3*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(
1/4*c)^6 - 9*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*t
an(1/4*c)^6 + 15*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*
x + c)^3*tan(1/2*c)^4 - sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan
(1/4*d*x + c)*tan(1/2*c)^6 - 240*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^3*tan(1/4*c) + 270*sqrt(2)*a^2*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^4*tan(1/4*c) - 18
*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^6*tan(1/4*c) +
225*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(
1/2*c)^2*tan(1/4*c)^2 - 765*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/4*d*x + c)*tan(1/2*c)^4*tan(1/4*c)^2 + 180*sqrt(2)*a^2*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^5*tan(1/4*c)^2 - 240*sqrt(2)*a^2*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)*tan(1/4*c)^3 + 90
0*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/
2*c)^2*tan(1/4*c)^3 - 180*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*t
an(1/2*c)^4*tan(1/4*c)^3 + 15*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*tan(1/4*d*x + c)^3*tan(1/4*c)^4 - 585*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^4 + 600*sqrt(2)*a^2*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^4 + 18*sqrt(2)*
a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/4*c)^5 - 1
26*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^
5 - 5*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(
1/4*c)^6 + 12*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*ta
n(1/4*c)^6 + 40*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x
 + c)^3*tan(1/2*c)^3 - 45*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*t
an(1/4*d*x + c)^2*tan(1/2*c)^4 - sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/2*c)^6 - 90*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan
(1/4*d*x + c)^3*tan(1/2*c)^2*tan(1/4*c) + 234*sqrt(2)*a^2*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^4*tan(1/4*c) - 72*sqrt(2)*a^
2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^5*tan(1/4*c) + 180*sqrt(2)
*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)*tan(
1/4*c)^2 - 675*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x
+ c)^2*tan(1/2*c)^2*tan(1/4*c)^2 + 315*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c)^2 - 20*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/4*c)^3 + 1020*sqrt(2)*a^2*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^3 - 80
0*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^3
 - 45*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*ta
n(1/4*c)^4 + 135*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)
^2*tan(1/4*c)^4 + 6*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4
*d*x + c)*tan(1/4*c)^5 - 72*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/2*c)*tan(1/4*c)^5 + 3*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*tan(1/4*c)^6 - 15*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4
*d*x + c)^3*tan(1/2*c)^2 + 51*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*tan(1/4*d*x + c)*tan(1/2*c)^4 - 12*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))*tan(1/2*c)^5 + 72*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)

```
)*tan(1/4*d*x + c)^3*tan(1/2*c)*tan(1/4*c) - 270*sqrt(2)*a^2*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^2*tan(1/4*c) + 54*sqrt(
2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c) - 15*sqr
t(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/4*c)^
2 + 585*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*ta
n(1/2*c)^2*tan(1/4*c)^2 - 600*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*tan(1/2*c)^3*tan(1/4*c)^2 - 60*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*tan(1/4*d*x + c)^2*tan(1/4*c)^3 + 420*sqrt(2)*a^2*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 + 75*sqrt(2)*a^2*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^4 - 180*sqrt(2)*a^2*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 6*sqrt(2)*a^2*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^5 - 12*sqrt(2)*a^2*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c) + 45*sqrt(2)*a^2*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^2 - 21*sqrt(2)
*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^4 + 6*sqrt(2)*a^2*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/4*c) - 306*sqrt(2)*
a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1
/4*c) + 240*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*ta
n(1/4*c) + 45*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x +
 c)^2*tan(1/4*c)^2 - 135*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*ta
n(1/2*c)^2*tan(1/4*c)^2 - 20*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*tan(1/4*d*x + c)*tan(1/4*c)^3 + 240*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))*tan(1/2*c)*tan(1/4*c)^3 - 45*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))*tan(1/4*c)^4 + sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/4*d*x + c)^3 - 39*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*ta
n(1/4*d*x + c)*tan(1/2*c)^2 + 40*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/2*c)^3 + 18*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan
(1/4*d*x + c)^2*tan(1/4*c) - 126*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/2*c)^2*tan(1/4*c) - 75*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^2 + 180*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^2 + 20*sqrt(2)*a^2*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))*tan(1/4*c)^3 - 3*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*tan(1/4*d*x + c)^2 + 9*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*tan(1/2*c)^2 + 6*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1
/4*d*x + c)*tan(1/4*c) - 72*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/2*c)*tan(1/4*c) + 45*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/4*c)^2 + 5*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d
*x + c) - 12*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c) - 6
*sqrt(2)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - 3*sqrt(2)*a^2
*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))/((tan(1/2*c)^3*tan(1/4*c)^6 + 3*tan(1
/2*c)^2*tan(1/4*c)^6 + 3*tan(1/2*c)^3*tan(1/4*c)^4 - 3*tan(1/2*c)*tan(1/4*c
)^6 + 9*tan(1/2*c)^2*tan(1/4*c)^4 - tan(1/4*c)^6 + 3*tan(1/2*c)^3*tan(1/4*c
)^2 - 9*tan(1/2*c)*tan(1/4*c)^4 + 9*tan(1/2*c)^2*tan(1/4*c)^2 - 3*tan(1/4*c
)^4 + tan(1/2*c)^3 - 9*tan(1/2*c)*tan(1/4*c)^2 + 3*tan(1/2*c)^2 - 3*tan(1/4
*c)^2 - 3*tan(1/2*c) - 1)*(tan(1/4*d*x + c)^4*tan(1/2*c)^3 + 3*tan(1/4*d*x
+ c)^4*tan(1/2*c)^2 - 2*tan(1/4*d*x + c)^3*tan(1/2*c)^3 - 3*tan(1/4*d*x + c
)^4*tan(1/2*c) + 6*tan(1/4*d*x + c)^3*tan(1/2*c)^2 - tan(1/4*d*x + c)^4 + 6
*tan(1/4*d*x + c)^3*tan(1/2*c) - 2*tan(1/4*d*x + c)*tan(1/2*c)^3 - 2*tan(1/
4*d*x + c)^3 + 6*tan(1/4*d*x + c)*tan(1/2*c)^2 - tan(1/2*c)^3 + 6*tan(1/4*d
*x + c)*tan(1/2*c) - 3*tan(1/2*c)^2 - 2*tan(1/4*d*x + c) + 3*tan(1/2*c) + 1
)))/d
```

maple [A] time = 0.17, size = 45, normalized size = 0.82

$$-\frac{2a^3 (1+\sin(dx+c))(\sin(dx+c)-3)}{\cos(dx+c)\sqrt{a+a\sin(dx+c)}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2*a^3*(1+sin(d*x+c))*(sin(d*x+c)-3)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [B] time = 1.07, size = 191, normalized size = 3.47

$$-\frac{2\left(3\,a^{\frac{5}{2}}-\frac{2\,a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\,a^{\frac{5}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{9\,a^{\frac{5}{2}}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{2\,a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{3\,a^{\frac{5}{2}}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2*(3*a^(5/2) - 2*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 9*a^(5/2)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 4*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 9*a^(5/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*a^(5/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 + 3*a^(5/2)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)/(
d*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 1)^(5/2))
```

mupad [B] time = 5.46, size = 88, normalized size = 1.60

$$\frac{2\,a^2\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(-22\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2+4\,\sin\left(2\,c+2\,d\,x\right)+12\right)}{d\,\left(-4\sin\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

```
[Out] (2*a^2*(a*(sin(c + d*x) + 1))^(1/2)*(4*sin(2*c + 2*d*x) - 22*sin(c/2 + (d*x
)/2)^2 - 2*sin((3*c)/2 + (3*d*x)/2)^2 + 12))/(d*(sin(c + d*x) + sin(3*c + 3
*d*x) - 4*sin(c + d*x)^2 + 4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.134 $\int \sec^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=69

$$\frac{a\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}$$

```
[Out] a*sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d-1/2*a^(5/2)*arctanh(1/2*(a+a*sin(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2676, 2667, 63, 206}

$$\frac{a\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*d)
) + (a*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d} - \frac{1}{2}a^2\int \sec(c+dx)\sqrt{a+a\sin(c+\ldots}$$

$$= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin\ldots\right.}{2d}$$

$$= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\,\text{si}\ldots}\right.}{d}$$

$$= -\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d} + \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d}$$

Mathematica [A] time = 0.46, size = 75, normalized size = 1.09

$$\frac{a^2\left(-\frac{2\sqrt{a(\sin(c+dx)+1)}}{\sin(c+dx)-1} - \sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(-(Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a]
)]) - (2*Sqrt[a*(1 + Sin[c + d*x])])/(-1 + Sin[c + d*x])))/(2*d)
```

fricas [A] time = 0.68, size = 102, normalized size = 1.48

$$\frac{\sqrt{2}\left(a^2\sin(dx+c) - a^2\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c) - 2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a} + 3a}{\sin(dx+c)-1}\right) - 4\sqrt{a\sin(dx+c)+a}\,a^2}{4\left(d\sin(dx+c) - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a^2*sin(d*x + c) - a^2)*sqrt(a)*log(-(a*sin(d*x + c) - 2*sqrt
(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*sqrt(a*
sin(d*x + c) + a)*a^2)/(d*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 66, normalized size = 0.96

$$-\frac{a^3\left(\frac{\sqrt{a+a\sin(dx+c)}}{a\sin(dx+c)-a} + \frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -a^3*((a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)+1/2*2^(1/2)/a^(1/2)*arctanh(1
/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

maxima [A] time = 0.53, size = 94, normalized size = 1.36

$$\frac{\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)-\frac{4\,\sqrt{a\sin(dx+c)+a}\,a^{4}}{a\sin(dx+c)-a}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqr
t(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) - 4*sqrt(a*sin(d*x + c) + a)*a^4/
(a*sin(d*x + c) - a))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{5/2}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.135 $\int \sec^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=30

$$\frac{2a\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}$$

```
[Out] 2/3*a*sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$\frac{2a\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx=\frac{2a\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}$$

Mathematica [B] time = 5.16, size = 69, normalized size = 2.30

$$\frac{2(a(\sin(c+dx)+1))^{5/2}}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(5/2))/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [A] time = 0.84, size = 43, normalized size = 1.43

$$-\frac{2\sqrt{a\sin(dx+c)+a}\,a^2}{3\left(d\cos(dx+c)\sin(dx+c)-d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(a*sin(d*x + c) + a)*a^2/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x
+ c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.16, size = 47, normalized size = 1.57

$$-\frac{2a^{3}\left(1+\sin\left(dx+c\right)\right)}{3\left(\sin\left(dx+c\right)-1\right)\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x)
[Out] -2/3*a^3*(1+sin(d*x+c))/(sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [B] time = 0.52, size = 184, normalized size = 6.13

$$-\frac{2\left(a^{\frac{5}{2}}+\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,a^{\frac{5}{2}}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{\frac{5}{2}}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)}{3\,d\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-1\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] -2/3*(a^(5/2) + 4*a^(5/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^(5/2)*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^(5/2)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + a^(5/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/(d*(3*sin(d*x + c)/(
cos(d*x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 - 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(5/2))
```

mupad [B] time = 7.59, size = 225, normalized size = 7.50

$$\frac{4\,a^{2}\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(\sin\left(c+d\,x\right)^{2}\,4\mathrm{i}+\sin\left(c+d\,x\right)\,1\mathrm{i}-2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^{2}-2\,\sin\left(2\right.\right.}{3\,d\,\left(8\sin\left(c+d\,x\right)^{2}+4\,\sin\left(c+d\,x\right)-2\sin\left(2\,c+2\,d\,x\right)^{2}+4\,\sin\left(3\,c+3\,d\,x\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^4,x)
[Out] (4*a^2*(a*(sin(c + d*x) + 1))^(1/2)*(sin(c + d*x)*1i - 2*sin(2*c + 2*d*x) +
 sin(3*c + 3*d*x)*1i - 2*sin(c/2 + (d*x)/2)^2 + 2*sin((3*c)/2 + (3*d*x)/2)^
2 + sin(c + d*x)^2*4i - 4i))/(3*d*(4*sin(c + d*x) + 4*sin(3*c + 3*d*x) - 2*
sin(2*c + 2*d*x)^2 + 8*sin(c + d*x)^2 - 8)) + (4*a^2*(a*(sin(c + d*x) + 1))
^(1/2)*(sin(2*c + 2*d*x) + 4*sin(c/2 + (d*x)/2)^2 - sin(c + d*x)^2*2i - (2
- 2i)))/(3*d*(sin(c + d*x) + sin(3*c + 3*d*x) + 4*sin(c + d*x)^2 - 4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.136 $\int \sec^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=103

$$\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{4d}+\frac{3a\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{16d}$$

```
[Out] 3/16*a*sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d+1/4*sec(d*x+c)^4*(a+a*sin(d*x+
c))^(5/2)/d+3/32*a^(5/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)*2^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2675, 2667, 63, 206}

$$\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{4d}+\frac{3a\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]
*d) + (3*a*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(16*d) + (Sec[c + d*x
]^4*(a + a*Sin[c + d*x])^(5/2))/(4*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{4d} + \frac{1}{8}(3a)\int \sec^3(c+dx)(a+a\sin$$
$$= \frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d} + \frac{\sec^4(c+dx)(a+a\sin(c+d$$
$$= \frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d} + \frac{\sec^4(c+dx)(a+a\sin(c+d$$
$$= \frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d} + \frac{\sec^4(c+dx)(a+a\sin(c+d$$
$$= \frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d} + \frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d}$$

Mathematica [A] time = 0.29, size = 110, normalized size = 1.07

$$\frac{3\sqrt{2}a^{5/2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right)+2a^2(7-3\sin(c+dx))\sqrt{a(\sin(c+dx)}}{32d(\sin(c+dx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (3*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^4 + 2*a^2*(7 - 3*Sin[c + d*x])*Sqrt[a*(
1 + Sin[c + d*x])])/(32*d*(-1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.61, size = 147, normalized size = 1.43

$$\frac{3\left(\sqrt{2}a^2\cos(dx+c)^2+2\sqrt{2}a^2\sin(dx+c)-2\sqrt{2}a^2\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)+4\left(3a^2\mathrm{s}\right.}{64\left(d\cos(dx+c)^2+2d\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(sqrt(2)*a^2*cos(d*x + c)^2 + 2*sqrt(2)*a^2*sin(d*x + c) - 2*sqrt(2
)*a^2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sq
rt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(3*a^2*sin(d*x + c) - 7*a^2)*sqrt(a*si
n(d*x + c) + a))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.26, size = 107, normalized size = 1.04

$$-\frac{2a^5\left(-\frac{\sqrt{a+a\sin(dx+c)}}{8a(a\sin(dx+c)-a)^2}-\frac{3\left(-\frac{\sqrt{a+a\sin(dx+c)}}{4a(a\sin(dx+c)-a)}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{8a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2*a^5*(-1/8*(a+a*sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)^2-3/8/a*(-1/4*(a+a*s
in(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)+1/8/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*si
n(d*x+c))^(1/2)*2^(1/2)/a^(1/2))))/d
```

maxima [A] time = 0.42, size = 134, normalized size = 1.30

$$-\frac{3\,\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin\left(dx+c\right)+a}}\right)+\frac{4\left(3\,\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{4}-10\,\sqrt{a\sin\left(dx+c\right)+a}\,a^{5}\right)}{\left(a\sin\left(dx+c\right)+a\right)^{2}-4\,\left(a\sin\left(dx+c\right)+a\right)a+4\,a^{2}}}{64\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/64*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/
(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(3*(a*sin(d*x + c) + a)^(
3/2)*a^4 - 10*sqrt(a*sin(d*x + c) + a)*a^5)/((a*sin(d*x + c) + a)^2 - 4*(a*
sin(d*x + c) + a)*a + 4*a^2))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.137 $\int \sec^6(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=139

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{2}d}+\frac{a^2\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{5d}+\frac{a\sec^3}{}$$

```
[Out] 1/6*a*sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d+1/5*sec(d*x+c)^5*(a+a*sin(d*x+c
))^(5/2)/d-1/8*a^(5/2)*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+
c))^(1/2))/d*2^(1/2)+1/4*a^2*sec(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2675, 2649, 206}

$$\frac{a^2\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{2}d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{5d}+\frac{a\sec^3(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(4*Sqrt[2]*d) + (a^2*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d) + (a*S
ec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(6*d) + (Sec[c + d*x]^5*(a + a*Si
n[c + d*x])^(5/2))/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{5d} + \frac{1}{2}a\int \sec^4(c+dx)(a+a\sin(c+d$$
$$= \frac{a\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{6d} + \frac{\sec^5(c+dx)(a+a\sin(c+dx))^{5/}}{5d}$$
$$= \frac{a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} + \frac{a\sec^3(c+dx)(a+a\sin(c+dx))^{3/}}{6d}$$
$$= \frac{a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} + \frac{a\sec^3(c+dx)(a+a\sin(c+dx))^{3/}}{6d}$$
$$= -\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{2}d} + \frac{a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} +$$

Mathematica [C] time = 5.31, size = 129, normalized size = 0.93

$$\frac{(a(\sin(c+dx)+1))^{5/2}\left(\frac{-80\sin(c+dx)-15\cos(2(c+dx))+89}{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5} + (15+15i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(c+dx)\right)-\right.\right.\right.}{60d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((15 + 15*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + (89 - 15*Cos[2*(c + d*x)] - 80*Sin[c + d*x])/(2*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^5))*(a*(1 + Sin[c + d*x]))^(5/2))/(60*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^5)
```

fricas [B] time = 0.66, size = 263, normalized size = 1.89

$$\frac{15\left(\sqrt{2}a^2\cos(dx+c)^3 + 2\sqrt{2}a^2\cos(dx+c)\sin(dx+c) - 2\sqrt{2}a^2\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c}}{}\right.}{240\left(d\cos(dx+c)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/240*(15*(sqrt(2)*a^2*cos(d*x + c)^3 + 2*sqrt(2)*a^2*cos(d*x + c)*sin(d*x
+ c) - 2*sqrt(2)*a^2*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(
a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))
*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(c
os(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(1
5*a^2*cos(d*x + c)^2 + 40*a^2*sin(d*x + c) - 52*a^2)*sqrt(a*sin(d*x + c) +
a))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.30, size = 120, normalized size = 0.86

$$\frac{(1+\sin(dx+c))\left(-30a^{\frac{11}{2}}\left(\cos^2(dx+c)\right)-80a^{\frac{11}{2}}\sin(dx+c)+104a^{\frac{11}{2}}-15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3}{120a^{\frac{5}{2}}(\sin(dx+c)-1)^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/120*(1+sin(d*x+c))*(-30*a^(11/2)*cos(d*x+c)^2-80*a^(11/2)*sin(d*x+c)+104*
a^(11/2)-15*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3
*(a-a*sin(d*x+c))^(5/2))/a^(5/2)/(sin(d*x+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c
))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{5/2}}{\cos(c+d\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^6,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.138 $\int \sec^7(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=159

$$\frac{35a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d}-\frac{35a^3}{128d\sqrt{a\sin(c+dx)+a}}+\frac{35a^2\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{192d}+\frac{\sec^6(c+dx)(a\,\mathrm{si}}{6}$$

```
[Out] 7/48*a*sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2)/d+1/6*sec(d*x+c)^6*(a+a*sin(d*x+
c))^(5/2)/d+35/256*a^(5/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))*2^(1/2)/d-35/128*a^3/d/(a+a*sin(d*x+c))^(1/2)+35/192*a^2*sec(d*x+c)^2*(
a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.24, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2667, 51, 63, 206}

$$-\frac{35a^3}{128d\sqrt{a\sin(c+dx)+a}}+\frac{35a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d}+\frac{35a^2\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{192d}+\frac{\sec^6(c+dx)(a\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (35*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[
2]*d) - (35*a^3)/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (35*a^2*Sec[c + d*x]^2*
Sqrt[a + a*Sin[c + d*x]])/(192*d) + (7*a*Sec[c + d*x]^4*(a + a*Sin[c + d*x]
)^(3/2))/(48*d) + (Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2))/(6*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{\sec^6(c+dx)(a+a\sin(c+dx))^{5/2}}{6d} + \frac{1}{12}(7a)\int \sec^5(c+dx)(a+a\sin$$

$$= \frac{7a\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{48d} + \frac{\sec^6(c+dx)(a+a\sin(c+d$$

$$= \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d} + \frac{7a\sec^4(c+dx)(a+a\sin(c}{48d}$$

$$= \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d} + \frac{7a\sec^4(c+dx)(a+a\sin(c}{48d}$$

$$= -\frac{35a^3}{128d\sqrt{a+a\sin(c+dx)}} + \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d} +$$

$$= -\frac{35a^3}{128d\sqrt{a+a\sin(c+dx)}} + \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d} +$$

$$= \frac{35a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\,d} - \frac{35a^3}{128d\sqrt{a+a\sin(c+dx)}} + \frac{35a^2\sec}{}$$

Mathematica [C] time = 0.11, size = 44, normalized size = 0.28

$$-\frac{a^3\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{8d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/8*(a^3*Hypergeometric2F1[-1/2, 4, 1/2, (1 + Sin[c + d*x])/2])/(d*Sqrt[a
+ a*Sin[c + d*x]])
```

fricas [A] time = 0.68, size = 208, normalized size = 1.31

$$\frac{105\left(\sqrt{2}\,a^2\cos(dx+c)^4+2\sqrt{2}\,a^2\cos(dx+c)^2\sin(dx+c)-2\sqrt{2}\,a^2\cos(dx+c)^2\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{}}{\sin(}\right.}{1536\left(d\cos(dx+c)^4+2d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(105*(sqrt(2)*a^2*cos(d*x + c)^4 + 2*sqrt(2)*a^2*cos(d*x + c)^2*sin(
d*x + c) - 2*sqrt(2)*a^2*cos(d*x + c)^2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*s
qrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(245
*a^2*cos(d*x + c)^2 - 160*a^2 - 7*(15*a^2*cos(d*x + c)^2 - 32*a^2)*sin(d*x
```

```
+ c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2*sin(
d*x + c) - 2*d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.50, size = 113, normalized size = 0.71

$$\frac{2a^{7}\left(-\frac{1}{16a^{4}\sqrt{a+a\sin(dx+c)}}-\frac{-\frac{a^{2}\sqrt{a+a\sin(dx+c)}\left(57\left(\cos^{2}(dx+c)\right)+158\sin(dx+c)-190\right)}{48(a\sin(dx+c)-a)^{3}}-\frac{35\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{32\sqrt{a}}}{16a^{4}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2*a^7*(-1/16/a^4/(a+a*sin(d*x+c))^(1/2)-1/16/a^4*(-1/48*a^2*(a+a*sin(d*x+c)
)^(1/2)*(57*cos(d*x+c)^2+158*sin(d*x+c)-190)/(a*sin(d*x+c)-a)^3-35/32*2^(1/
2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))))/d
```

maxima [A] time = 0.73, size = 185, normalized size = 1.16

$$-\frac{105\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(105\,(a\sin(dx+c)+a)^{3}a^{4}-560\,(a\sin(dx+c)+a)^{2}a^{5}+924\,(a\sin(dx+c)+a)a^{6}-384\,a^{7}\right)}{(a\sin(dx+c)+a)^{\frac{7}{2}}-6\,(a\sin(dx+c)+a)^{\frac{5}{2}}a+12\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^{2}-8\,\sqrt{a\sin(dx+c)+a}\,a^{3}}}{1536\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/1536*(105*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(105*(a*sin(d*x + c)
+ a)^3*a^4 - 560*(a*sin(d*x + c) + a)^2*a^5 + 924*(a*sin(d*x + c) + a)*a^6
- 384*a^7)/((a*sin(d*x + c) + a)^(7/2) - 6*(a*sin(d*x + c) + a)^(5/2)*a + 1
2*(a*sin(d*x + c) + a)^(3/2)*a^2 - 8*sqrt(a*sin(d*x + c) + a)*a^3))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{5/2}}{\cos(c+d\,x)^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^7,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.139 $\int \cos^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{21/2}}{21a^7d}+\frac{12(a\sin(c+dx)+a)^{19/2}}{19a^6d}-\frac{24(a\sin(c+dx)+a)^{17/2}}{17a^5d}+\frac{16(a\sin(c+dx)+a)^{15/2}}{15a^4d}$$

```
[Out] 16/15*(a+a*sin(d*x+c))^(15/2)/a^4/d-24/17*(a+a*sin(d*x+c))^(17/2)/a^5/d+12/
19*(a+a*sin(d*x+c))^(19/2)/a^6/d-2/21*(a+a*sin(d*x+c))^(21/2)/a^7/d
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{21/2}}{21a^7d}+\frac{12(a\sin(c+dx)+a)^{19/2}}{19a^6d}-\frac{24(a\sin(c+dx)+a)^{17/2}}{17a^5d}+\frac{16(a\sin(c+dx)+a)^{15/2}}{15a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(15/2))/(15*a^4*d) - (24*(a + a*Sin[c + d*x])^(17/
2))/(17*a^5*d) + (12*(a + a*Sin[c + d*x])^(19/2))/(19*a^6*d) - (2*(a + a*Si
n[c + d*x])^(21/2))/(21*a^7*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{13/2}\,dx, x, a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{13/2}-12a^2(a+x)^{15/2}+6a(a+x)^{17/2}-(a+x)^{1}\right.\right.}{a^7d}\\ &= \frac{16(a+a\sin(c+dx))^{15/2}}{15a^4d}-\frac{24(a+a\sin(c+dx))^{17/2}}{17a^5d}+\frac{12(a+a\sin(}{19a}\end{aligned}$$

Mathematica [A] time = 0.63, size = 64, normalized size = 0.66

$$-\frac{2a^3(\sin(c+dx)+1)^7\left(1615\sin^3(c+dx)-5865\sin^2(c+dx)+7365\sin(c+dx)-3243\right)\sqrt{a(\sin(c+dx)+1)}}{33915d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[c + d*x])^7*Sqrt[a*(1 + Sin[c + d*x])]*(-3243 + 7365*Sin[c
 + d*x] - 5865*Sin[c + d*x]^2 + 1615*Sin[c + d*x]^3))/(33915*d)
```

fricas [A] time = 0.78, size = 154, normalized size = 1.59

$$\frac{2\left(1615\,a^3\cos\left(dx+c\right)^{10}-8300\,a^3\cos\left(dx+c\right)^{8}+264\,a^3\cos\left(dx+c\right)^{6}+448\,a^3\cos\left(dx+c\right)^{4}+1024\,a^3\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/33915*(1615*a^3*cos(d*x + c)^10 - 8300*a^3*cos(d*x + c)^8 + 264*a^3*cos(d
*x + c)^6 + 448*a^3*cos(d*x + c)^4 + 1024*a^3*cos(d*x + c)^2 + 8192*a^3 - 8
*(680*a^3*cos(d*x + c)^8 - 429*a^3*cos(d*x + c)^6 - 504*a^3*cos(d*x + c)^4
- 640*a^3*cos(d*x + c)^2 - 1024*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)
/d
```

giac [B] time = 6.14, size = 669, normalized size = 6.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/7449361920*sqrt(2)*(765765*a^3*cos(1/4*pi + 19/2*d*x + 19/2*c)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))/d - 7759752*a^3*cos(1/4*pi + 15/2*d*x + 15/2*c)*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 91265265*a^3*cos(1/4*pi + 11/2*d*x +
 11/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 365816880*a^3*cos(1/4*pi +
 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 882671790*a^3*cos
(1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 692835*a
^3*cos(-1/4*pi + 21/2*d*x + 21/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d -
 6846840*a^3*cos(-1/4*pi + 17/2*d*x + 17/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))/d - 77224455*a^3*cos(-1/4*pi + 13/2*d*x + 13/2*c)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))/d - 284524240*a^3*cos(-1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))/d - 529603074*a^3*cos(-1/4*pi + 5/2*d*x + 5/2*c
)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 5135130*a^3*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))*sin(1/4*pi + 17/2*d*x + 17/2*c)/d - 24622290*a^3*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 13/2*d*x + 13/2*c)/d + 12932920*a^3*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 9/2*d*x + 9/2*c)/d + 488864
376*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 5/2*d*x + 5/2*c)/d
 + 5296030740*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 1/2*d*x
+ 1/2*c)/d - 4594590*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
19/2*d*x + 19/2*c)/d - 21339318*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
(-1/4*pi + 15/2*d*x + 15/2*c)/d + 10581480*a^3*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*sin(-1/4*pi + 11/2*d*x + 11/2*c)/d + 349188840*a^3*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*sin(-1/4*pi + 7/2*d*x + 7/2*c)/d + 1765343580*a^3*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 3/2*d*x + 3/2*c)/d)*sqrt(a)
```

maple [A] time = 0.22, size = 57, normalized size = 0.59

$$\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{15}{2}}\left(1615\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-5865\left(\cos^{2}\left(dx+c\right)\right)-8980\sin\left(dx+c\right)+9108\right)}{33915a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2/33915/a^4*(a+a*sin(d*x+c))^(15/2)*(1615*cos(d*x+c)^2*sin(d*x+c)-5865*cos(
d*x+c)^2-8980*sin(d*x+c)+9108)/d
```

maxima [A] time = 0.84, size = 72, normalized size = 0.74

$$-\frac{2\left(1615\,(a\sin(dx+c)+a)^{\frac{21}{2}}-10710\,(a\sin(dx+c)+a)^{\frac{19}{2}}a+23940\,(a\sin(dx+c)+a)^{\frac{17}{2}}a^2-18088\,(a\sin(\right.}{33915\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/33915*(1615*(a*sin(d*x + c) + a)^(21/2) - 10710*(a*sin(d*x + c) + a)^(19
/2)*a + 23940*(a*sin(d*x + c) + a)^(17/2)*a^2 - 18088*(a*sin(d*x + c) + a)^
(15/2)*a^3)/(a^7*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^7\,(a+a\,\sin(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.140 $\int \cos^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=223

$$-\frac{131072a^7\cos^7(c+dx)}{969969d(a\sin(c+dx)+a)^{7/2}}-\frac{32768a^6\cos^7(c+dx)}{138567d(a\sin(c+dx)+a)^{5/2}}-\frac{12288a^5\cos^7(c+dx)}{46189d(a\sin(c+dx)+a)^{3/2}}-\frac{1024a^4\cos^7(c+\cdots}{4199d\sqrt{a\sin(c+\cdots}}$$

```
[Out] -131072/969969*a^7*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)-32768/138567*a^6*c
os(d*x+c)^7/d/(a+a*sin(d*x+c))^(5/2)-12288/46189*a^5*cos(d*x+c)^7/d/(a+a*si
n(d*x+c))^(3/2)-48/323*a^2*cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2)/d-2/19*a*cos
(d*x+c)^7*(a+a*sin(d*x+c))^(5/2)/d-1024/4199*a^4*cos(d*x+c)^7/d/(a+a*sin(d*
x+c))^(1/2)-64/323*a^3*cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.43, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2674, 2673}

$$-\frac{48a^2\cos^7(c+dx)(a\sin(c+dx)+a)^{3/2}}{323d}-\frac{64a^3\cos^7(c+dx)\sqrt{a\sin(c+dx)+a}}{323d}-\frac{1024a^4\cos^7(c+dx)}{4199d\sqrt{a\sin(c+dx)+a}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-131072*a^7*Cos[c + d*x]^7)/(969969*d*(a + a*Sin[c + d*x])^(7/2)) - (32768
*a^6*Cos[c + d*x]^7)/(138567*d*(a + a*Sin[c + d*x])^(5/2)) - (12288*a^5*Cos
[c + d*x]^7)/(46189*d*(a + a*Sin[c + d*x])^(3/2)) - (1024*a^4*Cos[c + d*x]^
7)/(4199*d*Sqrt[a + a*Sin[c + d*x]]) - (64*a^3*Cos[c + d*x]^7*Sqrt[a + a*Si
n[c + d*x]])/(323*d) - (48*a^2*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2))/(
323*d) - (2*a*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(5/2))/(19*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = -\frac{2a\cos^7(c+dx)(a+a\sin(c+dx))^{5/2}}{19d} + \frac{1}{19}(24a)\int \cos^6(c+dx)(a$$
$$= -\frac{48a^2\cos^7(c+dx)(a+a\sin(c+dx))^{3/2}}{323d} - \frac{2a\cos^7(c+dx)(a+a\sin}{19d}$$
$$= -\frac{64a^3\cos^7(c+dx)\sqrt{a+a\sin(c+dx)}}{323d} - \frac{48a^2\cos^7(c+dx)(a+a\sin}{323d}$$
$$= -\frac{1024a^4\cos^7(c+dx)}{4199d\sqrt{a+a\sin(c+dx)}} - \frac{64a^3\cos^7(c+dx)\sqrt{a+a\sin(c+dx)}}{323d}$$
$$= -\frac{12288a^5\cos^7(c+dx)}{46189d(a+a\sin(c+dx))^{3/2}} - \frac{1024a^4\cos^7(c+dx)}{4199d\sqrt{a+a\sin(c+dx)}} - 64a^3\,c$$
$$= -\frac{32768a^6\cos^7(c+dx)}{138567d(a+a\sin(c+dx))^{5/2}} - \frac{12288a^5\cos^7(c+dx)}{46189d(a+a\sin(c+dx))^{3/2}} - \frac{}{41}$$
$$= -\frac{131072a^7\cos^7(c+dx)}{969969d(a+a\sin(c+dx))^{7/2}} - \frac{32768a^6\cos^7(c+dx)}{138567d(a+a\sin(c+dx))^{5/2}} - \frac{}{4}$$

Mathematica [A] time = 0.55, size = 102, normalized size = 0.46

$$-\frac{2a^3\left(51051\sin^6(c+dx)+378378\sin^5(c+dx)+1222221\sin^4(c+dx)+2244396\sin^3(c+dx)+2546901\sin\right.}{969969d(\sin(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^7*Sqrt[a*(1 + Sin[c + d*x])]*(646739 + 1778602*Sin[c +
 d*x] + 2546901*Sin[c + d*x]^2 + 2244396*Sin[c + d*x]^3 + 1222221*Sin[c + d
*x]^4 + 378378*Sin[c + d*x]^5 + 51051*Sin[c + d*x]^6))/(969969*d*(1 + Sin[c
 + d*x])^4)
```

fricas [A] time = 0.66, size = 296, normalized size = 1.33

$$\frac{2\left(51051\,a^3\cos(dx+c)^{10}+225225\,a^3\cos(dx+c)^9-270270\,a^3\cos(dx+c)^8-562716\,a^3\cos(dx+c)^7+107\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/969969*(51051*a^3*cos(d*x + c)^10 + 225225*a^3*cos(d*x + c)^9 - 270270*a^
3*cos(d*x + c)^8 - 562716*a^3*cos(d*x + c)^7 + 10752*a^3*cos(d*x + c)^6 - 1
4336*a^3*cos(d*x + c)^5 + 20480*a^3*cos(d*x + c)^4 - 32768*a^3*cos(d*x + c)
^3 + 65536*a^3*cos(d*x + c)^2 - 262144*a^3*cos(d*x + c) - 524288*a^3 + (510
51*a^3*cos(d*x + c)^9 - 174174*a^3*cos(d*x + c)^8 - 444444*a^3*cos(d*x + c)
^7 + 118272*a^3*cos(d*x + c)^6 + 129024*a^3*cos(d*x + c)^5 + 143360*a^3*cos
(d*x + c)^4 + 163840*a^3*cos(d*x + c)^3 + 196608*a^3*cos(d*x + c)^2 + 26214
4*a^3*cos(d*x + c) + 524288*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*
cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 8.97, size = 636, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/1241560320*sqrt(2)*(285285*a^3*cos(1/4*pi + 17/2*d*x + 17/2*c)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))/d - 3357585*a^3*cos(1/4*pi + 13/2*d*x + 13/2*c)*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 32332300*a^3*cos(1/4*pi + 9/2*d*x +
9/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 112516404*a^3*cos(1/4*pi + 5
/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 320089770*a^3*cos(1
/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 255255*a^3
*cos(-1/4*pi + 19/2*d*x + 19/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 2
909907*a^3*cos(-1/4*pi + 15/2*d*x + 15/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))/d - 26453700*a^3*cos(-1/4*pi + 11/2*d*x + 11/2*c)*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))/d - 80368860*a^3*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))/d - 106696590*a^3*cos(-1/4*pi + 3/2*d*x + 3/2*c)*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 1939938*a^3*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))*sin(1/4*pi + 15/2*d*x + 15/2*c)/d - 7054320*a^3*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))*sin(1/4*pi + 11/2*d*x + 11/2*c)/d + 16628040*a^3*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 7/2*d*x + 7/2*c)/d + 232792560*
a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d - 1
711710*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 17/2*d*x + 17/
2*c)/d - 5969040*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 13/2
*d*x + 13/2*c)/d + 12932920*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/
4*pi + 9/2*d*x + 9/2*c)/d + 139675536*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d + 1066965900*a^3*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.23, size = 107, normalized size = 0.48

$$-\frac{2\left(1+\sin \left(dx+c\right)\right) a^{4} \left(\sin \left(dx+c\right)-1\right)^{4} \left(51051 \left(\sin^{6}\left(dx+c\right)\right)+378378 \left(\sin^{5}\left(dx+c\right)\right)+1222221 \left(\sin^{4}\left(dx+\right.\right.\right.}{969969 \cos \left(dx+c\right) \sqrt{a+a \sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/969969*(1+sin(d*x+c))*a^4*(sin(d*x+c)-1)^4*(51051*sin(d*x+c)^6+378378*si
n(d*x+c)^5+1222221*sin(d*x+c)^4+2244396*sin(d*x+c)^3+2546901*sin(d*x+c)^2+1
778602*sin(d*x+c)+646739)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(dx+c\right)+a\right)^{\frac{7}{2}} \cos \left(dx+c\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos \left(c+d\,x\right)}^6\,{\left(a+a\,\sin \left(c+d\,x\right)\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.141 $\int \cos^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{17/2}}{17a^5d}-\frac{8(a\sin(c+dx)+a)^{15/2}}{15a^4d}+\frac{8(a\sin(c+dx)+a)^{13/2}}{13a^3d}$$

```
[Out] 8/13*(a+a*sin(d*x+c))^(13/2)/a^3/d-8/15*(a+a*sin(d*x+c))^(15/2)/a^4/d+2/17*
(a+a*sin(d*x+c))^(17/2)/a^5/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{17/2}}{17a^5d}-\frac{8(a\sin(c+dx)+a)^{15/2}}{15a^4d}+\frac{8(a\sin(c+dx)+a)^{13/2}}{13a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(13/2))/(13*a^3*d) - (8*(a + a*Sin[c + d*x])^(15/2)
)/(15*a^4*d) + (2*(a + a*Sin[c + d*x])^(17/2))/(17*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{11/2}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{11/2}-4a(a+x)^{13/2}+(a+x)^{15/2}\right)dx,x,a\sin(c+d\right.}{a^5d}\\ &= \frac{8(a+a\sin(c+dx))^{13/2}}{13a^3d}-\frac{8(a+a\sin(c+dx))^{15/2}}{15a^4d}+\frac{2(a+a\sin(c+dx}{17a^5d}\end{aligned}$$

Mathematica [A] time = 0.30, size = 54, normalized size = 0.74

$$\frac{2a^3(\sin(c+dx)+1)^6\left(195\sin^2(c+dx)-494\sin(c+dx)+331\right)\sqrt{a(\sin(c+dx)+1)}}{3315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*(1 + Sin[c + d*x])^6*Sqrt[a*(1 + Sin[c + d*x])]*(331 - 494*Sin[c + d
*x] + 195*Sin[c + d*x]^2))/(3315*d)
```

fricas [B] time = 0.84, size = 128, normalized size = 1.75

$$\frac{2\left(195\,a^{3}\cos\left(dx+c\right)^{8}-1072\,a^{3}\cos\left(dx+c\right)^{6}+56\,a^{3}\cos\left(dx+c\right)^{4}+128\,a^{3}\cos\left(dx+c\right)^{2}+1024\,a^{3}-4\left(169\,a\right.\right.}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*a^3*cos(d*x + c)^8 - 1072*a^3*cos(d*x + c)^6 + 56*a^3*cos(d*x +
 c)^4 + 128*a^3*cos(d*x + c)^2 + 1024*a^3 - 4*(169*a^3*cos(d*x + c)^6 - 126
*a^3*cos(d*x + c)^4 - 160*a^3*cos(d*x + c)^2 - 256*a^3)*sin(d*x + c))*sqrt(
a*sin(d*x + c) + a)/d
```

giac [B] time = 3.13, size = 537, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/98017920*sqrt(2)*(51051*a^3*cos(1/4*pi + 15/2*d*x + 15/2*c)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))/d - 696150*a^3*cos(1/4*pi + 11/2*d*x + 11/2*c)*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))/d - 5469750*a^3*cos(1/4*pi + 7/2*d*x + 7/2*c
)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 16846830*a^3*cos(1/4*pi + 3/2*d*x
 + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 45045*a^3*cos(-1/4*pi + 1
7/2*d*x + 17/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 589050*a^3*cos(-1
/4*pi + 13/2*d*x + 13/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 4254250*
a^3*cos(-1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d -
10108098*a^3*cos(-1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))/d - 353430*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 13/2*d
*x + 13/2*c)/d - 850850*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi
+ 9/2*d*x + 9/2*c)/d + 5207202*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(
1/4*pi + 5/2*d*x + 5/2*c)/d + 84234150*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*sin(1/4*pi + 1/2*d*x + 1/2*c)/d - 306306*a^3*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*sin(-1/4*pi + 15/2*d*x + 15/2*c)/d - 696150*a^3*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))*sin(-1/4*pi + 11/2*d*x + 11/2*c)/d + 3719430*a^3*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 7/2*d*x + 7/2*c)/d + 28078050*a^
3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 3/2*d*x + 3/2*c)/d)*sqr
t(a)
```

maple [A] time = 0.17, size = 41, normalized size = 0.56

$$-\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{13}{2}}\left(195\left(\cos^{2}\left(dx+c\right)\right)+494\sin\left(dx+c\right)-526\right)}{3315a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/3315/a^3*(a+a*sin(d*x+c))^(13/2)*(195*cos(d*x+c)^2+494*sin(d*x+c)-526)/d
```

maxima [A] time = 0.32, size = 55, normalized size = 0.75

$$\frac{2\left(195\left(a\sin\left(dx+c\right)+a\right)^{\frac{17}{2}}-884\left(a\sin\left(dx+c\right)+a\right)^{\frac{15}{2}}a+1020\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}a^{2}\right)}{3315\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3315*(195*(a*sin(d*x + c) + a)^(17/2) - 884*(a*sin(d*x + c) + a)^(15/2)*a
 + 1020*(a*sin(d*x + c) + a)^(13/2)*a^2)/(a^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^5\,(a+a\,\sin(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.142 $\int \cos^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=191

$$-\frac{16384a^6\cos^5(c+dx)}{45045d(a\sin(c+dx)+a)^{5/2}}-\frac{4096a^5\cos^5(c+dx)}{9009d(a\sin(c+dx)+a)^{3/2}}-\frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^5(c+dx)\sqrt{a}}{429d}$$

```
[Out] -16384/45045*a^6*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-4096/9009*a^5*cos(d*
x+c)^5/d/(a+a*sin(d*x+c))^(3/2)-8/39*a^2*cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2
)/d-2/15*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2)/d-512/1287*a^4*cos(d*x+c)^5/
d/(a+a*sin(d*x+c))^(1/2)-128/429*a^3*cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.37, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{16384a^6\cos^5(c+dx)}{45045d(a\sin(c+dx)+a)^{5/2}}-\frac{4096a^5\cos^5(c+dx)}{9009d(a\sin(c+dx)+a)^{3/2}}-\frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^5(c+dx)\sqrt{a}}{429d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-16384*a^6*Cos[c + d*x]^5)/(45045*d*(a + a*Sin[c + d*x])^(5/2)) - (4096*a^
5*Cos[c + d*x]^5)/(9009*d*(a + a*Sin[c + d*x])^(3/2)) - (512*a^4*Cos[c + d*
x]^5)/(1287*d*Sqrt[a + a*Sin[c + d*x]]) - (128*a^3*Cos[c + d*x]^5*Sqrt[a +
a*Sin[c + d*x]])/(429*d) - (8*a^2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2)
)/(39*d) - (2*a*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(15*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = -\frac{2a\cos^5(c+dx)(a+a\sin(c+dx))^{5/2}}{15d} + \frac{1}{3}(4a)\int \cos^4(c+dx)(a+a\,\mathrm{si}$$
$$= -\frac{8a^2\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{39d} - \frac{2a\cos^5(c+dx)(a+a\sin(c+}{15d}$$
$$= -\frac{128a^3\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{429d} - \frac{8a^2\cos^5(c+dx)(a+a\sin(c}{39d}$$
$$= -\frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} - \frac{128a^3\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{429d} -$$
$$= -\frac{4096a^5\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}} - \frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} - \frac{128a^3\cos^5}{}$$
$$= -\frac{16384a^6\cos^5(c+dx)}{45045d(a+a\sin(c+dx))^{5/2}} - \frac{4096a^5\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}} - \frac{512a}{1287d\sqrt{}}$$

Mathematica [A] time = 0.26, size = 92, normalized size = 0.48

$$-\frac{2a^3\left(3003\sin^5(c+dx)+19635\sin^4(c+dx)+55230\sin^3(c+dx)+86870\sin^2(c+dx)+81815\sin(c+dx)+4\right.}{45045d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^5*Sqrt[a*(1 + Sin[c + d*x])]*(41735 + 81815*Sin[c + d*
x] + 86870*Sin[c + d*x]^2 + 55230*Sin[c + d*x]^3 + 19635*Sin[c + d*x]^4 + 3
003*Sin[c + d*x]^5))/(45045*d*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.70, size = 244, normalized size = 1.28

$$\frac{2\left(3003\,a^3\cos(dx+c)^8+13629\,a^3\cos(dx+c)^7-17346\,a^3\cos(dx+c)^6-36932\,a^3\cos(dx+c)^5+1280\,a^3\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*a^3*cos(d*x + c)^8 + 13629*a^3*cos(d*x + c)^7 - 17346*a^3*cos
(d*x + c)^6 - 36932*a^3*cos(d*x + c)^5 + 1280*a^3*cos(d*x + c)^4 - 2048*a^3
*cos(d*x + c)^3 + 4096*a^3*cos(d*x + c)^2 - 16384*a^3*cos(d*x + c) - 32768*
a^3 + (3003*a^3*cos(d*x + c)^7 - 10626*a^3*cos(d*x + c)^6 - 27972*a^3*cos(d
*x + c)^5 + 8960*a^3*cos(d*x + c)^4 + 10240*a^3*cos(d*x + c)^3 + 12288*a^3*
cos(d*x + c)^2 + 16384*a^3*cos(d*x + c) + 32768*a^3)*sin(d*x + c))*sqrt(a*s
in(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 2.28, size = 504, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/2882880*sqrt(2)*(3465*a^3*cos(1/4*pi + 13/2*d*x + 13/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d - 55055*a^3*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))/d - 351351*a^3*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 1216215*a^3*cos(1/4*pi + 1/2*d*x + 1/2*
```

```
c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 3003*a^3*cos(-1/4*pi + 15/2*d*x
+ 15/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 45045*a^3*cos(-1/4*pi + 1
1/2*d*x + 11/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 250965*a^3*cos(-1
/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 405405*a^3
*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 245
70*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 11/2*d*x + 11/2*c)/
d - 25740*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 7/2*d*x + 7/
2*c)/d + 570570*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*
x + 3/2*c)/d - 20790*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
13/2*d*x + 13/2*c)/d - 20020*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1
/4*pi + 9/2*d*x + 9/2*c)/d + 342342*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d + 3243240*a^3*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.20, size = 97, normalized size = 0.51

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^{4}\left(\sin\left(dx+c\right)-1\right)^{3}\left(3003\left(\sin^{5}\left(dx+c\right)\right)+19635\left(\sin^{4}\left(dx+c\right)\right)+55230\left(\sin^{3}\left(dx+c\right)\right)\right.}{45045\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2/45045*(1+sin(d*x+c))*a^4*(sin(d*x+c)-1)^3*(3003*sin(d*x+c)^5+19635*sin(d*
x+c)^4+55230*sin(d*x+c)^3+86870*sin(d*x+c)^2+81815*sin(d*x+c)+41735)/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(c+d\,x\right)^{4}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.143 $\int \cos^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=49

$$\frac{4(a\sin(c+dx)+a)^{11/2}}{11a^2d}-\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^3d}$$

```
[Out] 4/11*(a+a*sin(d*x+c))^(11/2)/a^2/d-2/13*(a+a*sin(d*x+c))^(13/2)/a^3/d
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{11/2}}{11a^2d}-\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(11/2))/(11*a^2*d) - (2*(a + a*Sin[c + d*x])^(13/2)
)/(13*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^{9/2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a(a+x)^{9/2}-(a+x)^{11/2}\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{4(a+a\sin(c+dx))^{11/2}}{11a^2d}-\frac{2(a+a\sin(c+dx))^{13/2}}{13a^3d}\end{aligned}$$

Mathematica [A] time = 0.15, size = 44, normalized size = 0.90

$$\frac{2\left(26a(a\sin(c+dx)+a)^{11/2}-11(a\sin(c+dx)+a)^{13/2}\right)}{143a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(26*a*(a + a*Sin[c + d*x])^(11/2) - 11*(a + a*Sin[c + d*x])^(13/2)))/(14
3*a^3*d)
```

fricas [B] time = 0.64, size = 102, normalized size = 2.08

$$\frac{2\left(11\,a^3\cos\left(dx+c\right)^6-68\,a^3\cos\left(dx+c\right)^4+8\,a^3\cos\left(dx+c\right)^2+64\,a^3-8\left(5\,a^3\cos\left(dx+c\right)^4-5\,a^3\cos\left(dx+c\right)\right.\right.}{143\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/143*(11*a^3*cos(d*x + c)^6 - 68*a^3*cos(d*x + c)^4 + 8*a^3*cos(d*x + c)^2
 + 64*a^3 - 8*(5*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^2 - 8*a^3)*sin(d*x
 + c))*sqrt(a*sin(d*x + c) + a)/d
```

giac [B] time = 2.38, size = 405, normalized size = 8.27

$$\frac{1}{480480}\sqrt{2}\left(\frac{1365\,a^3\cos\left(\frac{1}{4}\,\pi+\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{25740\,a^3\cos\left(\frac{1}{4}\,\pi+\frac{7}{2}\,dx+\frac{7}{2}\,c\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/480480*sqrt(2)*(1365*a^3*cos(1/4*pi + 11/2*d*x + 11/2*c)*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))/d - 25740*a^3*cos(1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))/d - 135135*a^3*cos(1/4*pi + 3/2*d*x + 3/2*c)*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))/d + 1155*a^3*cos(-1/4*pi + 13/2*d*x + 13/2*c
)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 20020*a^3*cos(-1/4*pi + 9/2*d*x +
 9/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 81081*a^3*cos(-1/4*pi + 5/2
*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 10010*a^3*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 9/2*d*x + 9/2*c)/d + 6006*a^3*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 5/2*d*x + 5/2*c)/d + 540540*a^3*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 1/2*d*x + 1/2*c)/d - 8190*a
^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 11/2*d*x + 11/2*c)/d +
 4290*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 7/2*d*x + 7/2*c
)/d + 180180*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 3/2*d*x
+ 3/2*c)/d)*sqrt(a)
```

maple [A] time = 0.16, size = 31, normalized size = 0.63

$$-\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{11}{2}}\left(11\sin\left(dx+c\right)-15\right)}{143a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/143/a^2*(a+a*sin(d*x+c))^(11/2)*(11*sin(d*x+c)-15)/d
```

maxima [A] time = 0.40, size = 38, normalized size = 0.78

$$-\frac{2\left(11\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}-26\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}a\right)}{143\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/143*(11*(a*sin(d*x + c) + a)^(13/2) - 26*(a*sin(d*x + c) + a)^(11/2)*a)/
(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(c+d\,x)^3\,(a+a\,\sin(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(7/2),x)
[Out] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

3.144 $\int \cos^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=159

$$-\frac{4096a^5\cos^3(c+dx)}{3465d(a\sin(c+dx)+a)^{3/2}}-\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{231d}-\frac{32a^2\cos^3(c$$

```
[Out] -4096/3465*a^5*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-32/99*a^2*cos(d*x+c)^3
*(a+a*sin(d*x+c))^(3/2)/d-2/11*a*cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2)/d-1024
/1155*a^4*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-128/231*a^3*cos(d*x+c)^3*(a
+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{4096a^5\cos^3(c+dx)}{3465d(a\sin(c+dx)+a)^{3/2}}-\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{231d}-\frac{32a^2\cos^3(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-4096*a^5*Cos[c + d*x]^3)/(3465*d*(a + a*Sin[c + d*x])^(3/2)) - (1024*a^4*
Cos[c + d*x]^3)/(1155*d*Sqrt[a + a*Sin[c + d*x]]) - (128*a^3*Cos[c + d*x]^3
*Sqrt[a + a*Sin[c + d*x]])/(231*d) - (32*a^2*Cos[c + d*x]^3*(a + a*Sin[c +
d*x])^(3/2))/(99*d) - (2*a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2))/(11*d
)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= -\frac{2a\cos^3(c+dx)(a+a\sin(c+dx))^{5/2}}{11d}+\frac{1}{11}(16a)\int \cos^2(c+dx)(a\\
&= -\frac{32a^2\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{99d}-\frac{2a\cos^3(c+dx)(a+a\sin\\
&= -\frac{128a^3\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{231d}-\frac{32a^2\cos^3(c+dx)(a+a\,s\\
&= -\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a+a\sin(c+dx)}}-\frac{128a^3\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{231d}\\
&= -\frac{4096a^5\cos^3(c+dx)}{3465d(a+a\sin(c+dx))^{3/2}}-\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a+a\sin(c+dx)}}-\frac{128a^3\,c
\end{aligned}$$

Mathematica [A] time = 0.23, size = 82, normalized size = 0.52

$$-\frac{2a^3\left(315\sin^4(c+dx)+1820\sin^3(c+dx)+4530\sin^2(c+dx)+6396\sin(c+dx)+5419\right)\cos^3(c+dx)\sqrt{a(\sin(c}}{3465d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^3*Sqrt[a*(1 + Sin[c + d*x])]*(5419 + 6396*Sin[c + d*x]
 + 4530*Sin[c + d*x]^2 + 1820*Sin[c + d*x]^3 + 315*Sin[c + d*x]^4))/(3465*d
*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.45, size = 192, normalized size = 1.21

$$\frac{2\left(315\,a^3\cos\left(dx+c\right)^6+1505\,a^3\cos\left(dx+c\right)^5-2150\,a^3\cos\left(dx+c\right)^4-4876\,a^3\cos\left(dx+c\right)^3+512\,a^3\cos\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*a^3*cos(d*x + c)^6 + 1505*a^3*cos(d*x + c)^5 - 2150*a^3*cos(d*x
 + c)^4 - 4876*a^3*cos(d*x + c)^3 + 512*a^3*cos(d*x + c)^2 - 2048*a^3*cos(d
*x + c) - 4096*a^3 + (315*a^3*cos(d*x + c)^5 - 1190*a^3*cos(d*x + c)^4 - 33
40*a^3*cos(d*x + c)^3 + 1536*a^3*cos(d*x + c)^2 + 2048*a^3*cos(d*x + c) + 4
096*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x
 + c) + d)
```

giac [B] time = 1.85, size = 372, normalized size = 2.34

$$\frac{1}{55440}\sqrt{2}\left(\frac{385\,a^3\cos\left(\frac{1}{4}\,\pi+\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{9009\,a^3\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\mathrm{sgn}\left(\mathrm{co}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/55440*sqrt(2)*(385*a^3*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))/d - 9009*a^3*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))/d - 48510*a^3*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))/d + 315*a^3*cos(-1/4*pi + 11/2*d*x + 11/2*c)*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))/d - 6435*a^3*cos(-1/4*pi + 7/2*d*x + 7/2*c)*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 16170*a^3*cos(-1/4*pi + 3/2*d*x + 3
/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 2970*a^3*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))*sin(1/4*pi + 7/2*d*x + 7/2*c)/d + 9240*a^3*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d - 2310*a^3*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 9/2*d*x + 9/2*c)/d + 5544*a^3*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d + 97020*a^3*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a
)
```

maple [A] time = 0.20, size = 87, normalized size = 0.55

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)a^4\left(\sin\left(dx+c\right)-1\right)^2\left(315\left(\sin^4\left(dx+c\right)\right)+1820\left(\sin^3\left(dx+c\right)\right)+4530\left(\sin^2\left(dx+c\right)\right)+639}{3465\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/3465*(1+sin(d*x+c))*a^4*(sin(d*x+c)-1)^2*(315*sin(d*x+c)^4+1820*sin(d*x+
c)^3+4530*sin(d*x+c)^2+6396*sin(d*x+c)+5419)/cos(d*x+c)/(a+a*sin(d*x+c))^(1
/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.145 $\int \cos(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9ad}$$

```
[Out] 2/9*(a+a*sin(d*x+c))^(9/2)/a/d
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(9/2))/(9*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{7/2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{9/2}}{9ad}\end{aligned}$$

Mathematica [A] time = 0.09, size = 24, normalized size = 1.00

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(9/2))/(9*a*d)
```

fricas [B] time = 0.58, size = 74, normalized size = 3.08

$$\frac{2\left(a^3\cos\left(dx+c\right)^4-8\,a^3\cos\left(dx+c\right)^2+8\,a^3-4\left(a^3\cos\left(dx+c\right)^2-2\,a^3\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/9*(a^3*cos(d*x + c)^4 - 8*a^3*cos(d*x + c)^2 + 8*a^3 - 4*(a^3*cos(d*x + c
)^2 - 2*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/d
```

giac [B] time = 0.97, size = 273, normalized size = 11.38

$$\frac{1}{2520}\sqrt{2}\left(\frac{45\,a^3\cos\left(\frac{1}{4}\,\pi+\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{1470\,a^3\cos\left(\frac{1}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\operatorname{sgn}\left(\mathrm{co}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(45*a^3*cos(1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))/d - 1470*a^3*cos(1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))/d + 35*a^3*cos(-1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d - 882*a^3*cos(-1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))/d - 378*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(1/4*pi + 5/2*d*x + 5/2*c)/d + 4410*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin(1/4*pi + 1/2*d*x + 1/2*c)/d - 270*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*sin(-1/4*pi + 7/2*d*x + 7/2*c)/d + 1470*a^3*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*sin(-1/4*pi + 3/2*d*x + 3/2*c)/d)*sqrt(a)
```

maple [A] time = 0.04, size = 21, normalized size = 0.88

$$\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{9}{2}}}{9da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2/9*(a+a*sin(d*x+c))^(9/2)/d/a
```

maxima [A] time = 0.52, size = 20, normalized size = 0.83

$$\frac{2\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/9*(a*sin(d*x + c) + a)^(9/2)/(a*d)
```

mupad [B] time = 4.84, size = 20, normalized size = 0.83

$$\frac{2\left(a\ \left(\sin\left(c+d\,x\right)+1\right)\right)^{9/2}}{9\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(9/2))/(9*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.146 $\int \sec(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=110

$$\frac{8\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{8a^3\sqrt{a\sin(c+dx)+a}}{d}-\frac{4a^2(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{2a(a\sin(c+dx)+a)^{5/2}}{5d}$$

```
[Out] -4/3*a^2*(a+a*sin(d*x+c))^(3/2)/d-2/5*a*(a+a*sin(d*x+c))^(5/2)/d+8*a^(7/2)*
arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-8*a^3*(a+a*si
n(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2667, 50, 63, 206}

$$-\frac{8a^3\sqrt{a\sin(c+dx)+a}}{d}-\frac{4a^2(a\sin(c+dx)+a)^{3/2}}{3d}+\frac{8\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a(a\sin(c+dx)+a)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (8*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (8*a^3*Sqrt[a + a*Sin[c + d*x]])/d - (4*a^2*(a + a*Sin[c + d*x])^(3/2))/(3
*d) - (2*a*(a + a*Sin[c + d*x])^(5/2))/(5*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{a\,\text{Subst}\left(\int \frac{(a+x)^{5/2}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{2a(a+a\sin(c+dx))^{5/2}}{5d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{(a+x)^{3/2}}{a-x}\,dx, x, a\sin(c+dx\right.}{d}$$

$$= -\frac{4a^2(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a(a+a\sin(c+dx))^{5/2}}{5d} + \frac{(4a^3)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{8a^3\sqrt{a+a\sin(c+dx)}}{d} - \frac{4a^2(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a(a+a\sin(c}{5d}$$

$$= -\frac{8a^3\sqrt{a+a\sin(c+dx)}}{d} - \frac{4a^2(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a(a+a\sin(c}{5d}$$

$$= \frac{8\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{8a^3\sqrt{a+a\sin(c+dx)}}{d} - \frac{4a^2(a+}{}$$

Mathematica [A] time = 0.35, size = 85, normalized size = 0.77

$$\frac{120\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right) - 2a^3\left(3\sin^2(c+dx)+16\sin(c+dx)+73\right)\sqrt{a(\sin(c+dx)+1)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (120*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]
- 2*a^3*Sqrt[a*(1 + Sin[c + d*x])]*(73 + 16*Sin[c + d*x] + 3*Sin[c + d*x]^2
))/(15*d)
```

fricas [A] time = 0.74, size = 102, normalized size = 0.93

$$\frac{2\left(30\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right) + \left(3a^3\cos(dx+c)^2 - 16a^3\sin(dx+c) - 76a^3\right)\sqrt{a\sin(}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(30*sqrt(2)*a^(7/2)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x +
c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + (3*a^3*cos(d*x + c)^2 - 16*a^3
*sin(d*x + c) - 76*a^3)*sqrt(a*sin(d*x + c) + a))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.17, size = 83, normalized size = 0.75

$$-\frac{2a\left(\frac{(a+a\sin(dx+c))^{\frac{5}{2}}}{5} + \frac{2(a+a\sin(dx+c))^{\frac{3}{2}}a}{3} + 4a^2\sqrt{a+a\sin(dx+c)} - 4a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2*a*(1/5*(a+a*sin(d*x+c))^(5/2)+2/3*(a+a*sin(d*x+c))^(3/2)*a+4*a^2*(a+a*si
n(d*x+c))^(1/2)-4*a^(5/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2
)/a^(1/2)))/d
```

maxima [A] time = 0.81, size = 115, normalized size = 1.05

$$-\frac{2\left(30\sqrt{2}a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+3\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a^{2}+10\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{3}+60\sqrt{a\sin\left(dx\right.}\right.}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/15*(30*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 3*(a*sin(d*x + c) + a)^(5/
2)*a^2 + 10*(a*sin(d*x + c) + a)^(3/2)*a^3 + 60*sqrt(a*sin(d*x + c) + a)*a^
4)/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.147 $\int \sec^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=89

$$\frac{64a^3\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{16a^2\sec(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{2a\sec(c+dx)(a\sin(c+dx)+a)^{5/2}}{3d}$$

```
[Out] -16/3*a^2*sec(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-2/3*a*sec(d*x+c)*(a+a*sin(d*x
+c))^(5/2)/d+64/3*a^3*sec(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2674, 2673}

$$\frac{64a^3\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{16a^2\sec(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{2a\sec(c+dx)(a\sin(c+dx)+a)^{5/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (64*a^3*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*d) - (16*a^2*Sec[c + d*x]
*(a + a*Sin[c + d*x])^(3/2))/(3*d) - (2*a*Sec[c + d*x]*(a + a*Sin[c + d*x])
^(5/2))/(3*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= -\frac{2a\sec(c+dx)(a+a\sin(c+dx))^{5/2}}{3d}+\frac{1}{3}(8a)\int \sec^2(c+dx)(a+a\\ &= -\frac{16a^2\sec(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}-\frac{2a\sec(c+dx)(a+a\sin(c\\ &= \frac{64a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}-\frac{16a^2\sec(c+dx)(a+a\sin(c\end{aligned}$$

Mathematica [A] time = 5.48, size = 48, normalized size = 0.54

$$\frac{a^3\sec(c+dx)\sqrt{a(\sin(c+dx)+1)}\,(-20\sin(c+dx)+\cos(2(c+dx))+45)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (a^3*Sec[c + d*x]*(45 + Cos[2*(c + d*x)] - 20*Sin[c + d*x])*Sqrt[a*(1 + Sin
[c + d*x])])/(3*d)
```

fricas [A] time = 0.73, size = 54, normalized size = 0.61

$$\frac{2\left(a^{3}\cos\left(dx+c\right)^{2}-10\,a^{3}\sin\left(dx+c\right)+22\,a^{3}\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^3*cos(d*x + c)^2 - 10*a^3*sin(d*x + c) + 22*a^3)*sqrt(a*sin(d*x + c)
 + a)/(d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.18, size = 55, normalized size = 0.62

$$-\frac{2a^{4}\left(1+\sin\left(dx+c\right)\right)\left(\sin^{2}\left(dx+c\right)+10\sin\left(dx+c\right)-23\right)}{3\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/3*a^4*(1+sin(d*x+c))*(sin(d*x+c)^2+10*sin(d*x+c)-23)/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

maxima [B] time = 0.55, size = 237, normalized size = 2.66

$$-\frac{2\left(23\,a^{\frac{7}{2}}-\frac{20\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{88\,a^{\frac{7}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{60\,a^{\frac{7}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{130\,a^{\frac{7}{2}}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{60\,a^{\frac{7}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{88\,a^{\frac{7}{2}}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{20\,a^{\frac{7}{2}}}{(\cos}\right.}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3*(23*a^(7/2) - 20*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 88*a^(7/2)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 60*a^(7/2)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 130*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 60*a^(7/2)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 88*a^(7/2)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 - 20*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23*a^(7/2)*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(7/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2}}{{\cos\left(c+d\,x\right)}^{2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^2,x)
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

3.148 $\int \sec^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=91

$$-\frac{3\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{3a^3\sqrt{a\sin(c+dx)+a}}{d}+\frac{a\sec^2(c+dx)(a\sin(c+dx)+a)^{5/2}}{d}$$

```
[Out] a*sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2)/d-3*a^(7/2)*arctanh(1/2*(a+a*sin(d*x+
c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+3*a^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2676, 2667, 50, 63, 206}

$$\frac{3a^3\sqrt{a\sin(c+dx)+a}}{d}-\frac{3\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{a\sec^2(c+dx)(a\sin(c+dx)+a)^{5/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-3*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
+ (3*a^3*Sqrt[a + a*Sin[c + d*x]])/d + (a*Sec[c + d*x]^2*(a + a*Sin[c + d*x
])^(5/2))/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
```

```
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \sec^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - \frac{1}{2}\left(3a^2\right)\int \sec(c+dx)(a+a\,\mathrm{s}\\
&= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - \frac{\left(3a^3\right)\mathrm{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx,x,a\,\mathrm{s}\right.}{2d}\\
&= \frac{3a^3\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - \frac{\left(3a^4\right.}{}\\
&= \frac{3a^3\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - \frac{\left(6a^4\right.}{}\\
&= -\frac{3\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{3a^3\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\sec^2}{}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 42, normalized size = 0.46

$$\frac{a(a\sin(c+dx)+a)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (a*Hypergeometric2F1[2, 5/2, 7/2, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*x]
)^(5/2))/(10*d)
```

fricas [A] time = 0.73, size = 116, normalized size = 1.27

$$\frac{3\sqrt{2}\left(a^3\sin(dx+c)-a^3\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)+4\left(a^3\sin(dx+c)-2a^3\right)\sqrt{a\sin(dx+}}{2\left(d\sin(dx+c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(2)*(a^3*sin(d*x + c) - a^3)*sqrt(a)*log(-(a*sin(d*x + c) - 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(a^3*
sin(d*x + c) - 2*a^3)*sqrt(a*sin(d*x + c) + a))/(d*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.27, size = 83, normalized size = 0.91

$$\frac{2a^{3}\left(\sqrt{a+a\sin\left(dx+c\right)}+4a\left(-\frac{\sqrt{a+a\sin(dx+c)}}{4(a\sin(dx+c)-a)}-\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2*a^3*((a+a*sin(d*x+c))^(1/2)+4*a*(-1/4*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c
)-a)-3/8*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)))/d
```

maxima [A] time = 0.48, size = 112, normalized size = 1.23

$$\frac{3\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+4\sqrt{a\sin\left(dx+c\right)+a}\,a^{4}-\frac{4\sqrt{a\sin(dx+c)+a}\,a^{5}}{a\sin(dx+c)-a}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(s
qrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*sqrt(a*sin(d*x + c) + a)*a^
4 - 4*sqrt(a*sin(d*x + c) + a)*a^5/(a*sin(d*x + c) - a))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(c+d\,x\right)\right)^{7/2}}{\cos\left(c+d\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.149 $\int \sec^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=61

$$\frac{2a\sec^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{d}-\frac{8a^2\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}$$

```
[Out] -8/3*a^2*sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d+2*a*sec(d*x+c)^3*(a+a*sin(d*
x+c))^(5/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$\frac{2a\sec^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{d}-\frac{8a^2\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-8*a^2*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d) + (2*a*Sec[c + d*x
]^3*(a + a*Sin[c + d*x])^(5/2))/d
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx=\frac{2a\sec^3(c+dx)(a+a\sin(c+dx))^{5/2}}{d}-(4a)\int \sec^4(c+dx)(a+a\sin$$

$$=-\frac{8a^2\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}+\frac{2a\sec^3(c+dx)(a+a\sin(}{d}$$

Mathematica [A] time = 5.30, size = 82, normalized size = 1.34

$$\frac{2a^3(3\sin(c+dx)-1)\sqrt{a(\sin(c+dx)+1)}}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Sqrt[a*(1 + Sin[c + d*x])]*(-1 + 3*Sin[c + d*x]))/(3*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.71, size = 57, normalized size = 0.93

$$-\frac{2\left(3\,a^{3}\sin\left(dx+c\right)-a^{3}\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*a^3*sin(d*x + c) - a^3)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c)*si
n(d*x + c) - d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.19, size = 57, normalized size = 0.93

$$-\frac{2a^{4}\left(1+\sin\left(dx+c\right)\right)\left(3\sin\left(dx+c\right)-1\right)}{3\left(\sin\left(dx+c\right)-1\right)\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/3*a^4*(1+sin(d*x+c))/(sin(d*x+c)-1)*(3*sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

maxima [B] time = 0.60, size = 320, normalized size = 5.25

$$\frac{2\left(a^{\frac{7}{2}}-\frac{6\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^{\frac{7}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{24\,a^{\frac{7}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{10\,a^{\frac{7}{2}}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36\,a^{\frac{7}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{10\,a^{\frac{7}{2}}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{24\,a^{\frac{7}{2}}\sin(dx+c}{(\cos(dx+c)+1)}\right.}{3\,d\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-1\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^(7/2) - 6*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^(7/2)*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - 24*a^(7/2)*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 10*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^(7/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 + 10*a^(7/2)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 - 24*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5*a^(7/2)*sin(d*x + c)^
8/(cos(d*x + c) + 1)^8 - 6*a^(7/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + a^
(7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)/(d*(3*sin(d*x + c)/(cos(d*x +
c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 - 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(7/2))
```

mupad [B] time = 8.53, size = 118, normalized size = 1.93

$$-\frac{a^{3}\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\sqrt{a+a\,\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(3-3\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,2\mathrm{i}\right)\,4\mathrm{i}}{3\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\mathrm{i}\right)\,{\left(1+\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^4,x)
```

```
[Out] -(a^3*exp(c*1i + d*x*1i)*(a + a*((exp(- c*1i - d*x*1i)*1i)/2 - (exp(c*1i +
d*x*1i)*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*2i - 3*exp(c*2i + d*x*2i) + 3)*4i
)/(3*d*(exp(c*1i + d*x*1i) + 1i)*(exp(c*1i + d*x*1i)*1i + 1)^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.150 $\int \sec^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=106

$$-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d}-\frac{a^2\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{8d}+\frac{a\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{2d}$$

```
[Out] -1/8*a^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d+1/2*a*sec(d*x+c)^4*(a+a*sin(
d*x+c))^(5/2)/d-1/16*a^(7/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(
1/2))*2^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2676, 2675, 2667, 63, 206}

$$-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d}-\frac{a^2\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{8d}+\frac{a\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -(a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*d
) - (a^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(8*d) + (a*Sec[c + d*x]
^4*(a + a*Sin[c + d*x])^(5/2))/(2*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{2d} - \frac{1}{4}a^2\int \sec^3(c+dx)(a+a\sin($$

$$= -\frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d} + \frac{a\sec^4(c+dx)(a+a\sin(c+$$

$$= -\frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d} + \frac{a\sec^4(c+dx)(a+a\sin(c+$$

$$= -\frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d} + \frac{a\sec^4(c+dx)(a+a\sin(c+$$

$$= -\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d} - \frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d}$$

Mathematica [A] time = 0.29, size = 108, normalized size = 1.02

$$\frac{2a^3(\sin(c+dx)+3)\sqrt{a(\sin(c+dx)+1)} - \sqrt{2}a^{7/2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right.}{16d(\sin(c+dx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-(Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + 2*a^3*Sqrt[a*(1 + Sin[c + d*x])]*(
3 + Sin[c + d*x]))/(16*d*(-1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.78, size = 145, normalized size = 1.37

$$\frac{\left(\sqrt{2}\,a^3\cos(dx+c)^2 + 2\sqrt{2}\,a^3\sin(dx+c) - 2\sqrt{2}\,a^3\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right) - 4\left(a^3\sin(\right.}{32\left(d\cos(dx+c)^2 + 2d\sin(dx+c) - 2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/32*((sqrt(2)*a^3*cos(d*x + c)^2 + 2*sqrt(2)*a^3*sin(d*x + c) - 2*sqrt(2)*
a^3)*sqrt(a)*log(-(a*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt
(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(a^3*sin(d*x + c) + 3*a^3)*sqrt(a*sin(d*
x + c) + a))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.28, size = 75, normalized size = 0.71

$$-\frac{2a^5\left(-\frac{\sqrt{a+a\sin(dx+c)}\,(3+\sin(dx+c))}{16(a\sin(dx+c)-a)^2}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{32a^{\frac{3}{2}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2*a^5*(-1/16*(a+a*sin(d*x+c))^(1/2)*(3+sin(d*x+c))/(a*sin(d*x+c)-a)^2+1/32
/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

maxima [A] time = 0.43, size = 132, normalized size = 1.25

$$\frac{\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left((a\sin(dx+c)+a)^{\frac{3}{2}}a^5+2\,\sqrt{a\sin(dx+c)+a}\,a^6\right)}{(a\sin(dx+c)+a)^2-4\,(a\sin(dx+c)+a)a+4\,a^2}}{32\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sq
rt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*((a*sin(d*x + c) + a)^(3/2)*
a^5 + 2*sqrt(a*sin(d*x + c) + a)*a^6)/((a*sin(d*x + c) + a)^2 - 4*(a*sin(d*
x + c) + a)*a + 4*a^2))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{7/2}}{\cos(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.151 $\int \sec^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=30

$$\frac{2a\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{5d}$$

```
[Out] 2/5*a*sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2673}

$$\frac{2a\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(5*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{2a\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{5d}$$

Mathematica [B] time = 5.27, size = 69, normalized size = 2.30

$$\frac{2(a(\sin(c+dx)+1))^{7/2}}{5d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(7/2))/(5*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

fricas [B] time = 0.68, size = 54, normalized size = 1.80

$$-\frac{2\sqrt{a\sin(dx+c)+a}\,a^3}{5\left(d\cos(dx+c)^3+2\,d\cos(dx+c)\sin(dx+c)-2\,d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(a*sin(d*x + c) + a)*a^3/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)*sin(
d*x + c) - 2*d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.18, size = 47, normalized size = 1.57

$$\frac{2a^4 (1 + \sin (dx + c))}{5 (\sin (dx + c) - 1)^2 \cos (dx + c) \sqrt{a + a \sin (dx + c)}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2/5*a^4*(1+sin(d*x+c))/(sin(d*x+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [B] time = 0.53, size = 270, normalized size = 9.00

$$-\frac{2 \left(a^{\frac{7}{2}} + \frac{6 a^{\frac{7}{2}} \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{15 a^{\frac{7}{2}} \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{20 a^{\frac{7}{2}} \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{15 a^{\frac{7}{2}} \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{6 a^{\frac{7}{2}} \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{a^{\frac{7}{2}} \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)}{5 d \left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} - \frac{10 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{5 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - 1\right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a^(7/2) + 6*a^(7/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^(7/2)*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^(7/2)*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15*a^(7/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^(7/2)*sin(
d*x + c)^10/(cos(d*x + c) + 1)^10 + a^(7/2)*sin(d*x + c)^12/(cos(d*x + c) +
 1)^12)/(d*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)*(sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 1)^(7/2))
```

mupad [B] time = 8.43, size = 86, normalized size = 2.87

$$-\frac{16\, a^3\, e^{c\, 3i + d\, x\, 3i} \sqrt{a + a \left(\frac{e^{-c\, 1i - d\, x\, 1i}\, 1i}{2} - \frac{e^{c\, 1i + d\, x\, 1i}\, 1i}{2}\right)}}{5\, d \left(e^{c\, 1i + d\, x\, 1i} - i\right)^5 \left(e^{c\, 1i + d\, x\, 1i} + 1i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^6,x)
```

```
[Out] -(16*a^3*exp(c*3i + d*x*3i)*(a + a*((exp(- c*1i - d*x*1i)*1i)/2 - (exp(c*1i
 + d*x*1i)*1i)/2))^(1/2))/(5*d*(exp(c*1i + d*x*1i) - 1i)^5*(exp(c*1i + d*x*
1i) + 1i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.152 $\int \sec^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=135

$$\frac{5a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}+\frac{5a^2\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{64d}+\frac{\sec^6(c+dx)(a\sin(c+dx)+a)^{7/2}}{6d}+\frac{5a\,\text{se}}{}$$

```
[Out] 5/64*a^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d+5/48*a*sec(d*x+c)^4*(a+a*sin
(d*x+c))^(5/2)/d+1/6*sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2)/d+5/128*a^(7/2)*ar
ctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2675, 2667, 63, 206}

$$\frac{5a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}+\frac{5a^2\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{64d}+\frac{\sec^6(c+dx)(a\sin(c+dx)+a)^{7/2}}{6d}+\frac{5a\,\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (5*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2]
*d) + (5*a^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(64*d) + (5*a*Sec[c
 + d*x]^4*(a + a*Sin[c + d*x])^(5/2))/(48*d) + (Sec[c + d*x]^6*(a + a*Sin[c
 + d*x])^(7/2))/(6*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\sec^6(c+dx)(a+a\sin(c+dx))^{7/2}}{6d} + \frac{1}{12}(5a)\int \sec^5(c+dx)(a+a\sin(c$$
$$= \frac{5a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{48d} + \frac{\sec^6(c+dx)(a+a\sin(c+dx))^{7}}{6d}$$
$$= \frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d} + \frac{5a\sec^4(c+dx)(a+a\sin(c+a}{48d}$$
$$= \frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d} + \frac{5a\sec^4(c+dx)(a+a\sin(c+a}{48d}$$
$$= \frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d} + \frac{5a\sec^4(c+dx)(a+a\sin(c+a}{48d}$$
$$= \frac{5a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} + \frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d} +$$

Mathematica [A] time = 0.56, size = 120, normalized size = 0.89

$$-\frac{15\sqrt{2}a^{7/2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right)+2a^3\left(15\sin^2(c+dx)-50\sin(c+dx)+\right.}{384d(\sin(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -1/384*(15*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt
[a])]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6 + 2*a^3*Sqrt[a*(1 + Sin[c + d
*x])]*(67 - 50*Sin[c + d*x] + 15*Sin[c + d*x]^2))/(d*(-1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.64, size = 193, normalized size = 1.43

$$\frac{15\left(3\sqrt{2}a^3\cos(dx+c)^2-4\sqrt{2}a^3-\left(\sqrt{2}a^3\cos(dx+c)^2-4\sqrt{2}a^3\right)\sin(dx+c)\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(d}}{\sin(dx+c)-1}\right.}{768\left(3d\cos(dx+c)^2-\left(d\cos(dx+c)^2-4d\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/768*(15*(3*sqrt(2)*a^3*cos(d*x + c)^2 - 4*sqrt(2)*a^3 - (sqrt(2)*a^3*cos(
d*x + c)^2 - 4*sqrt(2)*a^3)*sin(d*x + c))*sqrt(a)*log(-(a*sin(d*x + c) + 2*
sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(15
*a^3*cos(d*x + c)^2 + 50*a^3*sin(d*x + c) - 82*a^3)*sqrt(a*sin(d*x + c) + a
))/(3*d*cos(d*x + c)^2 - (d*cos(d*x + c)^2 - 4*d)*sin(d*x + c) - 4*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.35, size = 144, normalized size = 1.07

$$\frac{2a^7\left(-\frac{\sqrt{a+a\sin(dx+c)}}{12a(a\sin(dx+c)-a)^3}-\frac{5\left(-\frac{\sqrt{a+a\sin(dx+c)}}{8a(a\sin(dx+c)-a)^2}-\frac{3\left(-\frac{\sqrt{a+a\sin(dx+c)}}{4a(a\sin(dx+c)-a)}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{8a}\right)}{12a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2*a^7*(-1/12*(a+a*sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)^3-5/12/a*(-1/8*(a+a*
sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)^2-3/8/a*(-1/4*(a+a*sin(d*x+c))^(1/2)/a
/(a*sin(d*x+c)-a)+1/8/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^
(1/2)/a^(1/2)))))/d
```

maxima [A] time = 0.46, size = 168, normalized size = 1.24

$$-\frac{15\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(15\,(a\sin(dx+c)+a)^{\frac{5}{2}}a^5-80\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^6+132\,\sqrt{a\sin(dx+c)+a}\,a^7\right)}{(a\sin(dx+c)+a)^3-6\,(a\sin(dx+c)+a)^2a+12\,(a\sin(dx+c)+a)a^2-8\,a^3}}{768\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/768*(15*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(15*(a*sin(d*x + c) + a
)^(5/2)*a^5 - 80*(a*sin(d*x + c) + a)^(3/2)*a^6 + 132*sqrt(a*sin(d*x + c) +
 a)*a^7)/((a*sin(d*x + c) + a)^3 - 6*(a*sin(d*x + c) + a)^2*a + 12*(a*sin(d
*x + c) + a)*a^2 - 8*a^3))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{7/2}}{\cos(c+d\,x)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^7,x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.153 $\int \sec^8(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=171

$$-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{2}d}+\frac{a^3\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{8d}+\frac{a^2\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{12d}+\frac{\sec^7(c$$

```
[Out] 1/12*a^2*sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d+1/10*a*sec(d*x+c)^5*(a+a*sin
(d*x+c))^(5/2)/d+1/7*sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2)/d-1/16*a^(7/2)*arc
tanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)+1/8*a
^3*sec(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2675, 2649, 206}

$$\frac{a^2\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{12d}+\frac{a^3\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{8d}-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{2}d}+\frac{\sec^7(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -(a^(7/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(8*Sqrt[2]*d) + (a^3*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(8*d) + (a^2
*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(12*d) + (a*Sec[c + d*x]^5*(a +
 a*Sin[c + d*x])^(5/2))/(10*d) + (Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2)
)/(7*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
 + 1/2, 2*p]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\sec^7(c+dx)(a+a\sin(c+dx))^{7/2}}{7d} + \frac{1}{2}a\int \sec^6(c+dx)(a+a\sin(c$$
$$= \frac{a\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{10d} + \frac{\sec^7(c+dx)(a+a\sin(c+dx)}{7d}$$
$$= \frac{a^2\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{12d} + \frac{a\sec^5(c+dx)(a+a\sin(c+}{10d}$$
$$= \frac{a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{8d} + \frac{a^2\sec^3(c+dx)(a+a\sin(c+dx}{12d}$$
$$= \frac{a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{8d} + \frac{a^2\sec^3(c+dx)(a+a\sin(c+dx}{12d}$$
$$= -\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{2}\,d} + \frac{a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{8d}$$

Mathematica [C] time = 5.50, size = 139, normalized size = 0.81

$$\frac{(a(\sin(c+dx)+1))^{7/2}\left(\frac{-2471\sin(c+dx)+105\sin(3(c+dx))-770\cos(2(c+dx))+2286}{4\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7} + (105+105i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-\right.\right.}{840d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(7/2)*((105 + 105*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)
*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + (2286 - 770*Cos[2*(c + d*x)] - 2471*
Sin[c + d*x] + 105*Sin[3*(c + d*x)])/(4*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^7)))/(840*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

fricas [B] time = 0.65, size = 312, normalized size = 1.82

$$105\left(3\sqrt{2}a^3\cos(dx+c)^3 - 4\sqrt{2}a^3\cos(dx+c) - \left(\sqrt{2}a^3\cos(dx+c)^3 - 4\sqrt{2}a^3\cos(dx+c)\right)\sin(dx+c)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3360*(105*(3*sqrt(2)*a^3*cos(d*x + c)^3 - 4*sqrt(2)*a^3*cos(d*x + c) - (s
qrt(2)*a^3*cos(d*x + c)^3 - 4*sqrt(2)*a^3*cos(d*x + c))*sin(d*x + c))*sqrt(
a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x + c
) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*x
 + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d
*x + c) - cos(d*x + c) - 2)) + 4*(385*a^3*cos(d*x + c)^2 - 764*a^3 - 7*(15*
a^3*cos(d*x + c)^2 - 92*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(3*d*c
os(d*x + c)^3 - 4*d*cos(d*x + c) - (d*cos(d*x + c)^3 - 4*d*cos(d*x + c))*si
n(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.24, size = 139, normalized size = 0.81

$$\frac{(1+\sin(dx+c))\left(-210a^{\frac{15}{2}}\sin(dx+c)\left(\cos^2(dx+c)\right)+770a^{\frac{15}{2}}\left(\cos^2(dx+c)\right)+1288a^{\frac{15}{2}}\sin(dx+c)-1528a^{\frac{15}{2}}\right.}{1680a^{\frac{7}{2}}(\sin(dx+c)-1)^3\cos(dx+c)\sqrt{a+a\sin(dx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 1/1680/a^(7/2)*(1+sin(d*x+c))/(sin(d*x+c)-1)^3*(-210*a^(15/2)*sin(d*x+c)*co
s(d*x+c)^2+770*a^(15/2)*cos(d*x+c)^2+1288*a^(15/2)*sin(d*x+c)-1528*a^(15/2)
+105*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4*(a-a*s
in(d*x+c))^(7/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(c+dx))^{7/2}}{\cos(c+dx)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^8,x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.154 $\int \sec^9(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=191

$$\frac{315a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}\,d}-\frac{315a^4}{2048d\sqrt{a\sin(c+dx)+a}}+\frac{105a^3\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{1024d}+\frac{21a^2\sec^4(}{}$$

```
[Out] 21/256*a^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2)/d+3/32*a*sec(d*x+c)^6*(a+a*s
in(d*x+c))^(5/2)/d+1/8*sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2)/d+315/4096*a^(7/
2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-315/2048*a
^4/d/(a+a*sin(d*x+c))^(1/2)+105/1024*a^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2
)/d
```

Rubi [A] time = 0.31, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2675, 2667, 51, 63, 206}

$$-\frac{315a^4}{2048d\sqrt{a\sin(c+dx)+a}}+\frac{315a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}\,d}+\frac{21a^2\sec^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{256d}+\frac{105a^3\,\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (315*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2048*Sqr
t[2]*d) - (315*a^4)/(2048*d*Sqrt[a + a*Sin[c + d*x]]) + (105*a^3*Sec[c + d*
x]^2*Sqrt[a + a*Sin[c + d*x]])/(1024*d) + (21*a^2*Sec[c + d*x]^4*(a + a*Sin
[c + d*x])^(3/2))/(256*d) + (3*a*Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2))
/(32*d) + (Sec[c + d*x]^8*(a + a*Sin[c + d*x])^(7/2))/(8*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rubi steps

$$\int \sec^9(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\sec^8(c+dx)(a+a\sin(c+dx))^{7/2}}{8d} + \frac{1}{16}(9a)\int \sec^7(c+dx)(a+a\sin(c$$
$$= \frac{3a\sec^6(c+dx)(a+a\sin(c+dx))^{5/2}}{32d} + \frac{\sec^8(c+dx)(a+a\sin(c+dx))^{7}}{8d}$$
$$= \frac{21a^2\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{256d} + \frac{3a\sec^6(c+dx)(a+a\sin(c+}{32d}$$
$$= \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d} + \frac{21a^2\sec^4(c+dx)(a+a\sin(c}{256d}$$
$$= \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d} + \frac{21a^2\sec^4(c+dx)(a+a\sin(c}{256d}$$
$$= -\frac{315a^4}{2048d\sqrt{a+a\sin(c+dx)}} + \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d} +$$
$$= -\frac{315a^4}{2048d\sqrt{a+a\sin(c+dx)}} + \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d} +$$
$$= \frac{315a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}\,d} - \frac{315a^4}{2048d\sqrt{a+a\sin(c+dx)}} + \frac{105a^3\,\mathrm{se}}{}$$

Mathematica [**C**] time = 0.11, size = 44, normalized size = 0.23

$$-\frac{a^4\,{}_2F_1\left(-\frac{1}{2},5;\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{16d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -1/16*(a^4*Hypergeometric2F1[-1/2, 5, 1/2, (1 + Sin[c + d*x])/2])/(d*Sqrt[a
 + a*Sin[c + d*x]])
```

fricas [**A**] time = 0.56, size = 254, normalized size = 1.33

$$\frac{315\left(3\sqrt{2}\,a^3\cos(dx+c)^4 - 4\sqrt{2}\,a^3\cos(dx+c)^2 - \left(\sqrt{2}\,a^3\cos(dx+c)^4 - 4\sqrt{2}\,a^3\cos(dx+c)^2\right)\sin(dx+c)\right)\sqrt{a}}{8192\left(3\,d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/8192*(315*(3*sqrt(2)*a^3*cos(d*x + c)^4 - 4*sqrt(2)*a^3*cos(d*x + c)^2 -
(sqrt(2)*a^3*cos(d*x + c)^4 - 4*sqrt(2)*a^3*cos(d*x + c)^2)*sin(d*x + c))*s
qrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) +
3*a)/(sin(d*x + c) - 1)) + 4*(315*a^3*cos(d*x + c)^4 - 1722*a^3*cos(d*x + c
)^2 + 896*a^3 + 6*(175*a^3*cos(d*x + c)^2 - 192*a^3)*sin(d*x + c))*sqrt(a*s
in(d*x + c) + a))/(3*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 - (d*cos(d*x + c
)^4 - 4*d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.42, size = 129, normalized size = 0.68

$$-\frac{2a^9\left(\frac{1}{32a^5\sqrt{a+a\sin(dx+c)}}+\frac{-\frac{\sqrt{a+a\sin(dx+c)}\,a^3\left(187\left(\cos^2(dx+c)\right)\sin(dx+c)-725\left(\cos^2(dx+c)\right)-1236\sin(dx+c)+1364\right)}{128(a\sin(dx+c)-a)^4}-\frac{315\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{256\sqrt{a}}}{32a^5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2*a^9*(1/32/a^5/(a+a*sin(d*x+c))^(1/2)+1/32/a^5*(-1/128*(a+a*sin(d*x+c))^(
1/2)*a^3*(187*cos(d*x+c)^2*sin(d*x+c)-725*cos(d*x+c)^2-1236*sin(d*x+c)+1364
)/(a*sin(d*x+c)-a)^4-315/256*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(
1/2)*2^(1/2)/a^(1/2))))/d
```

maxima [A] time = 0.45, size = 219, normalized size = 1.15

$$-\frac{315\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(315\,(a\sin(dx+c)+a)^4a^5-2310\,(a\sin(dx+c)+a)^3a^6+6132\,(a\sin(dx+c)+a)^2a^7-6696\,(a\sin\right.}{(a\sin(dx+c)+a)^{\frac{9}{2}}-8\,(a\sin(dx+c)+a)^{\frac{7}{2}}a+24\,(a\sin(dx+c)+a)^{\frac{5}{2}}a^2-32\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^3+}}{8192\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/8192*(315*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(315*(a*sin(d*x + c)
+ a)^4*a^5 - 2310*(a*sin(d*x + c) + a)^3*a^6 + 6132*(a*sin(d*x + c) + a)^2*
a^7 - 6696*(a*sin(d*x + c) + a)*a^8 + 2048*a^9)/((a*sin(d*x + c) + a)^(9/2)
 - 8*(a*sin(d*x + c) + a)^(7/2)*a + 24*(a*sin(d*x + c) + a)^(5/2)*a^2 - 32*
(a*sin(d*x + c) + a)^(3/2)*a^3 + 16*sqrt(a*sin(d*x + c) + a)*a^4))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{7/2}}{\cos(c+d\,x)^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^9,x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^9, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.155 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=233

$$-\frac{11a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{64\sqrt{2}d}-\frac{11a^5\cos(c+dx)}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{11a^4\sec(c+dx)}{48d\sqrt{a\sin(c+dx)+a}}+\frac{11a^3\sec^3(c+dx)\sqrt{a\sin}}{120d}$$

```
[Out] -11/64*a^5*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)+11/140*a^2*sec(d*x+c)^5*(a+a
*sin(d*x+c))^(3/2)/d+11/126*a*sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2)/d+1/9*sec
(d*x+c)^9*(a+a*sin(d*x+c))^(7/2)/d-11/128*a^(7/2)*arctanh(1/2*cos(d*x+c)*a^
(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)+11/48*a^4*sec(d*x+c)/d/(a+a
*sin(d*x+c))^(1/2)+11/120*a^3*sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.35, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2687, 2650, 2649, 206}

$$-\frac{11a^5\cos(c+dx)}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{11a^2\sec^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{140d}+\frac{11a^3\sec^3(c+dx)\sqrt{a\sin(c+dx)+a}}{120d}+\frac{}{48}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-11*a^(7/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x
]])])/(64*Sqrt[2]*d) - (11*a^5*Cos[c + d*x])/(64*d*(a + a*Sin[c + d*x])^(3/
2)) + (11*a^4*Sec[c + d*x])/(48*d*Sqrt[a + a*Sin[c + d*x]]) + (11*a^3*Sec[c
 + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(120*d) + (11*a^2*Sec[c + d*x]^5*(a + a
*Sin[c + d*x])^(3/2))/(140*d) + (11*a*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(
5/2))/(126*d) + (Sec[c + d*x]^9*(a + a*Sin[c + d*x])^(7/2))/(9*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\sec^9(c+dx)(a+a\sin(c+dx))^{7/2}}{9d} + \frac{1}{18}(11a)\int \sec^8(c+dx)(a+a\sin$$
$$= \frac{11a\sec^7(c+dx)(a+a\sin(c+dx))^{5/2}}{126d} + \frac{\sec^9(c+dx)(a+a\sin(c+dx)}{9d}$$
$$= \frac{11a^2\sec^5(c+dx)(a+a\sin(c+dx))^{3/2}}{140d} + \frac{11a\sec^7(c+dx)(a+a\sin(c}{126d}$$
$$= \frac{11a^3\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{120d} + \frac{11a^2\sec^5(c+dx)(a+a\sin(c+}{140d}$$
$$= \frac{11a^4\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{11a^3\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{120d} + \frac{11a^2}{}$$
$$= -\frac{11a^5\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} + \frac{11a^4\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{11a^3\sec^3(c+}{}$$
$$= -\frac{11a^5\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} + \frac{11a^4\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{11a^3\sec^3(c+}{}$$
$$= -\frac{11a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{2}d} - \frac{11a^5\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} + \frac{11}{48d\sqrt{}}$$

Mathematica [C] time = 5.68, size = 388, normalized size = 1.67

$$(a(\sin(c+dx)+1))^{7/2}\left(630\sin\left(\frac{1}{2}(c+dx)\right) + \frac{3150\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{1680\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{1512(}{(\cos}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] ((630*Sin[(c + d*x)/2] - 315*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (3465
+ 3465*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])
]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (1120*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^9 + (1440*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7 + (15
12*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^5 + (1680*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^3 + (3150*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]))*(a*(1 + Sin[c + d*x]))^(7/2))/(20160*d*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^9)
```

fricas [A] time = 0.72, size = 346, normalized size = 1.48

$$3465\left(3\sqrt{2}a^3\cos(dx+c)^5 - 4\sqrt{2}a^3\cos(dx+c)^3 - \left(\sqrt{2}a^3\cos(dx+c)^5 - 4\sqrt{2}a^3\cos(dx+c)^3\right)\sin(dx+c)\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/80640*(3465*(3*sqrt(2)*a^3*cos(d*x + c)^5 - 4*sqrt(2)*a^3*cos(d*x + c)^3
- (sqrt(2)*a^3*cos(d*x + c)^5 - 4*sqrt(2)*a^3*cos(d*x + c)^3)*sin(d*x + c))
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos(d
*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a*c
os(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)
*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(12705*a^3*cos(d*x + c)^4 - 25212*a^
3*cos(d*x + c)^2 + 3920*a^3 - 77*(45*a^3*cos(d*x + c)^4 - 276*a^3*cos(d*x +
 c)^2 + 80*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(3*d*cos(d*x + c)^5
 - 4*d*cos(d*x + c)^3 - (d*cos(d*x + c)^5 - 4*d*cos(d*x + c)^3)*sin(d*x + c
))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.28, size = 205, normalized size = 0.88

$$-\frac{-6930a^{\frac{11}{2}}\sin(dx+c)\left(\cos^4(dx+c)\right)+42504a^{\frac{11}{2}}\sin(dx+c)\left(\cos^2(dx+c)\right)+385\left(9(a-a\sin(dx+c))^{\frac{9}{2}}\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -1/40320/a^(3/2)*(-6930*a^(11/2)*sin(d*x+c)*cos(d*x+c)^4+42504*a^(11/2)*sin
(d*x+c)*cos(d*x+c)^2+385*(9*(a-a*sin(d*x+c))^(9/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-32*a^(11/2))*sin(d*x+c)+25410*a^(11/2
)*cos(d*x+c)^4-50424*a^(11/2)*cos(d*x+c)^2+3465*(a-a*sin(d*x+c))^(9/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+7840*a^(11/2))/(s
in(d*x+c)-1)^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\,\sin(c+d\,x))^{7/2}}{\cos(c+d\,x)^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^10,x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^10, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.156 $\int \frac{\cos^7(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^7d}+\frac{12(a\sin(c+dx)+a)^{11/2}}{11a^6d}-\frac{8(a\sin(c+dx)+a)^{9/2}}{3a^5d}+\frac{16(a\sin(c+dx)+a)^{7/2}}{7a^4d}$$

```
[Out] 16/7*(a+a*sin(d*x+c))^(7/2)/a^4/d-8/3*(a+a*sin(d*x+c))^(9/2)/a^5/d+12/11*(a
+a*sin(d*x+c))^(11/2)/a^6/d-2/13*(a+a*sin(d*x+c))^(13/2)/a^7/d
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^7d}+\frac{12(a\sin(c+dx)+a)^{11/2}}{11a^6d}-\frac{8(a\sin(c+dx)+a)^{9/2}}{3a^5d}+\frac{16(a\sin(c+dx)+a)^{7/2}}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(7/2))/(7*a^4*d) - (8*(a + a*Sin[c + d*x])^(9/2))/
(3*a^5*d) + (12*(a + a*Sin[c + d*x])^(11/2))/(11*a^6*d) - (2*(a + a*Sin[c +
 d*x])^(13/2))/(13*a^7*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{5/2}\,dx, x, a\sin(c+dx)\right)}{a^7d} \\
&= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{5/2}-12a^2(a+x)^{7/2}+6a(a+x)^{9/2}-(a+x)^{11/2}\right)dx, x, a\sin(c\right.}{a^7d} \\
&= \frac{16(a+a\sin(c+dx))^{7/2}}{7a^4d}-\frac{8(a+a\sin(c+dx))^{9/2}}{3a^5d}+\frac{12(a+a\sin(c+dx))^{11/2}}{11a^6d}-2(
\end{aligned}$$

Mathematica [A] time = 0.30, size = 61, normalized size = 0.63

$$-\frac{2(\sin(c+dx)+1)^4\left(231\sin^3(c+dx)-945\sin^2(c+dx)+1421\sin(c+dx)-835\right)}{3003d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(1 + Sin[c + d*x])^4*(-835 + 1421*Sin[c + d*x] - 945*Sin[c + d*x]^2 + 2
31*Sin[c + d*x]^3))/(3003*d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.69, size = 82, normalized size = 0.85

$$\frac{2\left(231\cos(dx+c)^6 + 28\cos(dx+c)^4 + 64\cos(dx+c)^2 + 4\left(63\cos(dx+c)^4 + 80\cos(dx+c)^2 + 128\right)\sin(d\ldots}{3003\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*cos(d*x + c)^6 + 28*cos(d*x + c)^4 + 64*cos(d*x + c)^2 + 4*(63*
cos(d*x + c)^4 + 80*cos(d*x + c)^2 + 128)*sin(d*x + c) + 512)*sqrt(a*sin(d*
x + c) + a)/(a*d)
```

giac [B] time = 2.70, size = 430, normalized size = 4.43

$$2\left(\frac{835\,a^6}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)} + \left(\frac{3003\,a^6}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)} + \left(\frac{3926\,a^6}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)} + \left(\frac{6006\,a^6}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)} + \left(\frac{15301\,a^6}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)} + \left(\ldots\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3003*(835*a^6/sgn(tan(1/2*d*x + 1/2*c) + 1) + (3003*a^6/sgn(tan(1/2*d*x +
 1/2*c) + 1) + (3926*a^6/sgn(tan(1/2*d*x + 1/2*c) + 1) + (6006*a^6/sgn(tan(
1/2*d*x + 1/2*c) + 1) + (15301*a^6/sgn(tan(1/2*d*x + 1/2*c) + 1) + (21021*a
^6/sgn(tan(1/2*d*x + 1/2*c) + 1) + (15444*a^6/sgn(tan(1/2*d*x + 1/2*c) + 1)
 + (15444*a^6/sgn(tan(1/2*d*x + 1/2*c) + 1) + (21021*a^6/sgn(tan(1/2*d*x +
1/2*c) + 1) + (15301*a^6/sgn(tan(1/2*d*x + 1/2*c) + 1) + (6006*a^6/sgn(tan(
1/2*d*x + 1/2*c) + 1) + (3926*a^6/sgn(tan(1/2*d*x + 1/2*c) + 1) + (835*a^6*
tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) + 3003*a^6/sgn(tan(1/2*d
*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x +
 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*
tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d
*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x +
1/2*c)^2 + a)^(13/2)*d)
```

maple [A] time = 0.16, size = 57, normalized size = 0.59

$$\frac{2\,(a + a\sin(dx+c))^{\frac{7}{2}}\left(231\left(\cos^2(dx+c)\right)\sin(dx+c) - 945\left(\cos^2(dx+c)\right) - 1652\sin(dx+c) + 1780\right)}{3003a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3003/a^4*(a+a*sin(d*x+c))^(7/2)*(231*cos(d*x+c)^2*sin(d*x+c)-945*cos(d*x+
c)^2-1652*sin(d*x+c)+1780)/d
```

maxima [B] time = 0.41, size = 281, normalized size = 2.90

$$2\left(15015\sqrt{a\sin(dx+c)+a} - \frac{3003\left(3\,(a\sin(dx+c)+a)^{\frac{5}{2}} - 10\,(a\sin(dx+c)+a)^{\frac{3}{2}}a + 15\sqrt{a\sin(dx+c)+a}\,a^2\right)}{a^2} + \frac{143\left(35\,(a\sin(dx+c)+a)^{\frac{9}{2}} - 180\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(a*sin(d*x + c) + a) - 3003*(3*(a*sin(d*x + c) + a)^(5/2
) - 10*(a*sin(d*x + c) + a)^(3/2)*a + 15*sqrt(a*sin(d*x + c) + a)*a^2)/a^2
+ 143*(35*(a*sin(d*x + c) + a)^(9/2) - 180*(a*sin(d*x + c) + a)^(7/2)*a + 3
78*(a*sin(d*x + c) + a)^(5/2)*a^2 - 420*(a*sin(d*x + c) + a)^(3/2)*a^3 + 31
5*sqrt(a*sin(d*x + c) + a)*a^4)/a^4 - 5*(231*(a*sin(d*x + c) + a)^(13/2) -
1638*(a*sin(d*x + c) + a)^(11/2)*a + 5005*(a*sin(d*x + c) + a)^(9/2)*a^2 -
8580*(a*sin(d*x + c) + a)^(7/2)*a^3 + 9009*(a*sin(d*x + c) + a)^(5/2)*a^4 -
 6006*(a*sin(d*x + c) + a)^(3/2)*a^5 + 3003*sqrt(a*sin(d*x + c) + a)*a^6)/a
^6)/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^7}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.157 $\int \frac{\cos^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{64a^3\cos^7(c+dx)}{693d(a\sin(c+dx)+a)^{7/2}}-\frac{16a^2\cos^7(c+dx)}{99d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^7(c+dx)}{11d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -64/693*a^3*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)-16/99*a^2*cos(d*x+c)^7/d/
(a+a*sin(d*x+c))^(5/2)-2/11*a*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{16a^2\cos^7(c+dx)}{99d(a\sin(c+dx)+a)^{5/2}}-\frac{64a^3\cos^7(c+dx)}{693d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{11d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^7)/(693*d*(a + a*Sin[c + d*x])^(7/2)) - (16*a^2*Cos[c
 + d*x]^7)/(99*d*(a + a*Sin[c + d*x])^(5/2)) - (2*a*Cos[c + d*x]^7)/(11*d*(
a + a*Sin[c + d*x])^(3/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{2a\cos^7(c+dx)}{11d(a+a\sin(c+dx))^{3/2}}+\frac{1}{11}(8a)\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx\\ &= -\frac{16a^2\cos^7(c+dx)}{99d(a+a\sin(c+dx))^{5/2}}-\frac{2a\cos^7(c+dx)}{11d(a+a\sin(c+dx))^{3/2}}+\frac{1}{99}\left(32a^2\right)\int \frac{\cos^6(c+}{(a+a\sin(c+}\\ &= -\frac{64a^3\cos^7(c+dx)}{693d(a+a\sin(c+dx))^{7/2}}-\frac{16a^2\cos^7(c+dx)}{99d(a+a\sin(c+dx))^{5/2}}-\frac{2a\cos^7(c+dx)}{11d(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.26, size = 59, normalized size = 0.62

$$-\frac{2\left(63\sin^2(c+dx)+182\sin(c+dx)+151\right)\cos^7(c+dx)}{693d(\sin(c+dx)+1)^3\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^7*(151 + 182*Sin[c + d*x] + 63*Sin[c + d*x]^2))/(693*d*(1
+ Sin[c + d*x])^3*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.62, size = 155, normalized size = 1.63

$$\frac{2\left(63\cos\left(dx+c\right)^{6}-7\cos\left(dx+c\right)^{5}+10\cos\left(dx+c\right)^{4}-16\cos\left(dx+c\right)^{3}+32\cos\left(dx+c\right)^{2}+\left(63\cos\left(dx+\right.\right.\right.}{693}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*cos(d*x + c)^6 - 7*cos(d*x + c)^5 + 10*cos(d*x + c)^4 - 16*cos(d*
x + c)^3 + 32*cos(d*x + c)^2 + (63*cos(d*x + c)^5 + 70*cos(d*x + c)^4 + 80*
cos(d*x + c)^3 + 96*cos(d*x + c)^2 + 128*cos(d*x + c) + 256)*sin(d*x + c) -
 128*cos(d*x + c) - 256)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*s
in(d*x + c) + a*d)
```

giac [B] time = 4.62, size = 402, normalized size = 4.23

$$2\left(\frac{256\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}}-\frac{\frac{151\,a^{5}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{693\,a^{5}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{1177\,a^{5}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{1155\,a^{5}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{1782\,a^{5}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}\right.\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/693*(256*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(a) - (151*a^5/sgn(tan
(1/2*d*x + 1/2*c) + 1) - (693*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (1177*a^5
/sgn(tan(1/2*d*x + 1/2*c) + 1) - (1155*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) -
(1782*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (3234*a^5/sgn(tan(1/2*d*x + 1/2*c
) + 1) - (3234*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (1782*a^5/sgn(tan(1/2*d*
x + 1/2*c) + 1) - (1155*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (1177*a^5/sgn(t
an(1/2*d*x + 1/2*c) + 1) + (151*a^5*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x +
1/2*c) + 1) - 693*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))*
tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d
*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*
c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))/(a*tan(1/2*d*x + 1/2*c)^2 +
 a)^(11/2))/d
```

maple [A] time = 0.20, size = 64, normalized size = 0.67

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{4}\left(63\left(\sin^{2}\left(dx+c\right)\right)+182\sin\left(dx+c\right)+151\right)}{693\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/693*(1+sin(d*x+c))*(sin(d*x+c)-1)^4*(63*sin(d*x+c)^2+182*sin(d*x+c)+151)
/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{6}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^6}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.158 $\int \frac{\cos^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^5d}-\frac{8(a\sin(c+dx)+a)^{7/2}}{7a^4d}+\frac{8(a\sin(c+dx)+a)^{5/2}}{5a^3d}$$

```
[Out] 8/5*(a+a*sin(d*x+c))^(5/2)/a^3/d-8/7*(a+a*sin(d*x+c))^(7/2)/a^4/d+2/9*(a+a*
sin(d*x+c))^(9/2)/a^5/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{2667, 43\}$

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^5d}-\frac{8(a\sin(c+dx)+a)^{7/2}}{7a^4d}+\frac{8(a\sin(c+dx)+a)^{5/2}}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(5/2))/(5*a^3*d) - (8*(a + a*Sin[c + d*x])^(7/2))/(
7*a^4*d) + (2*(a + a*Sin[c + d*x])^(9/2))/(9*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{3/2}-4a(a+x)^{5/2}+(a+x)^{7/2}\right)dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{8(a+a\sin(c+dx))^{5/2}}{5a^3d}-\frac{8(a+a\sin(c+dx))^{7/2}}{7a^4d}+\frac{2(a+a\sin(c+dx))^{9/2}}{9a^5d}\end{aligned}$$

Mathematica [A] time = 0.14, size = 51, normalized size = 0.70

$$\frac{2(\sin(c+dx)+1)^3\left(35\sin^2(c+dx)-110\sin(c+dx)+107\right)}{315d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(1 + Sin[c + d*x])^3*(107 - 110*Sin[c + d*x] + 35*Sin[c + d*x]^2))/(315*
d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.81, size = 62, normalized size = 0.85

$$\frac{2\left(35\cos\left(dx+c\right)^{4}+8\cos\left(dx+c\right)^{2}+8\left(5\cos\left(dx+c\right)^{2}+8\right)\sin\left(dx+c\right)+64\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*cos(d*x + c)^4 + 8*cos(d*x + c)^2 + 8*(5*cos(d*x + c)^2 + 8)*sin(
d*x + c) + 64)*sqrt(a*sin(d*x + c) + a)/(a*d)
```

giac [B] time = 1.59, size = 310, normalized size = 4.25

$$2\left(\frac{107\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{315\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{324\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{420\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{882\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(107*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (315*a^4/sgn(tan(1/2*d*x + 1
/2*c) + 1) + (324*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (420*a^4/sgn(tan(1/2*
d*x + 1/2*c) + 1) + (882*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (882*a^4/sgn(t
an(1/2*d*x + 1/2*c) + 1) + (420*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (324*a^
4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (107*a^4*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2
*d*x + 1/2*c) + 1) + 315*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1
/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*ta
n(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x
 + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2)*d)
```

maple [A] time = 0.15, size = 41, normalized size = 0.56

$$-\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{5}{2}}\left(35\left(\cos^{2}\left(dx+c\right)\right)+110\sin\left(dx+c\right)-142\right)}{315a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/315/a^3*(a+a*sin(d*x+c))^(5/2)*(35*cos(d*x+c)^2+110*sin(d*x+c)-142)/d
```

maxima [B] time = 0.72, size = 160, normalized size = 2.19

$$\frac{2\left(315\sqrt{a\sin\left(dx+c\right)+a}-\frac{42\left(3\left(a\sin(dx+c)+a\right)^{\frac{5}{2}}-10\left(a\sin(dx+c)+a\right)^{\frac{3}{2}}a+15\sqrt{a\sin(dx+c)+a}\,a^{2}\right)}{a^{2}}+\frac{35\left(a\sin(dx+c)+a\right)^{\frac{9}{2}}-180\left(a\sin(dx+\right.}{}\right.}{315\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(a*sin(d*x + c) + a) - 42*(3*(a*sin(d*x + c) + a)^(5/2) - 10
*(a*sin(d*x + c) + a)^(3/2)*a + 15*sqrt(a*sin(d*x + c) + a)*a^2)/a^2 + (35*
```

```
(a*sin(d*x + c) + a)^(9/2) - 180*(a*sin(d*x + c) + a)^(7/2)*a + 378*(a*sin(
d*x + c) + a)^(5/2)*a^2 - 420*(a*sin(d*x + c) + a)^(3/2)*a^3 + 315*sqrt(a*s
in(d*x + c) + a)*a^4)/a^4)/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^5}{\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.159 $\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^5(c+dx)}{35d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -8/35*a^2*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-2/7*a*cos(d*x+c)^5/d/(a+a*s
in(d*x+c))^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{8a^2\cos^5(c+dx)}{35d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^5)/(35*d*(a + a*Sin[c + d*x])^(5/2)) - (2*a*Cos[c + d*
x]^5)/(7*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{2a\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}+\frac{1}{7}(4a)\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx\\ &= -\frac{8a^2\cos^5(c+dx)}{35d(a+a\sin(c+dx))^{5/2}}-\frac{2a\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 49, normalized size = 0.78

$$-\frac{2(5\sin(c+dx)+9)\cos^5(c+dx)}{35d(\sin(c+dx)+1)^2\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^5*(9 + 5*Sin[c + d*x]))/(35*d*(1 + Sin[c + d*x])^2*Sqrt[a*
(1 + Sin[c + d*x])])
```

fricas [B] time = 0.68, size = 115, normalized size = 1.83

$$\frac{2\left(5\cos\left(dx+c\right)^{4}-\cos\left(dx+c\right)^{3}+2\cos\left(dx+c\right)^{2}+\left(5\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)+16\right.}{35\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 - cos(d*x + c)^3 + 2*cos(d*x + c)^2 + (5*cos(d*x + c
)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sin(d*x + c) - 8*cos(d*x + c)
 - 16)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

giac [B] time = 1.93, size = 278, normalized size = 4.41

$$2\left(\frac{16\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}}-\frac{\frac{9\,a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{35\,a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{49\,a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{35\,a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\left(\frac{35\,a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(16*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(a) - (9*a^3/sgn(tan(1/2
*d*x + 1/2*c) + 1) - (35*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (49*a^3/sgn(ta
n(1/2*d*x + 1/2*c) + 1) - (35*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (35*a^3/s
gn(tan(1/2*d*x + 1/2*c) + 1) - (49*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) + (9*a
^3*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 35*a^3/sgn(tan(1/2*
d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x
+ 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))
/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

maple [A] time = 0.18, size = 54, normalized size = 0.86

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{3}\left(5\sin\left(dx+c\right)+9\right)}{35\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/35*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(5*sin(d*x+c)+9)/cos(d*x+c)/(a+a*sin(d
*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{4}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\cos\left(c+d\,x\right)^{4}}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**4/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.160 $\int \frac{\cos^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=49

$$\frac{4(a\sin(c+dx)+a)^{3/2}}{3a^2d}-\frac{2(a\sin(c+dx)+a)^{5/2}}{5a^3d}$$

```
[Out] 4/3*(a+a*sin(d*x+c))^(3/2)/a^2/d-2/5*(a+a*sin(d*x+c))^(5/2)/a^3/d
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{3/2}}{3a^2d}-\frac{2(a\sin(c+dx)+a)^{5/2}}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(3/2))/(3*a^2*d) - (2*(a + a*Sin[c + d*x])^(5/2))/(
5*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int (a-x)\sqrt{a+x}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a\sqrt{a+x}-(a+x)^{3/2}\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{4(a+a\sin(c+dx))^{3/2}}{3a^2d}-\frac{2(a+a\sin(c+dx))^{5/2}}{5a^3d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.69

$$-\frac{2(3\sin(c+dx)-7)(a(\sin(c+dx)+1))^{3/2}}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(a*(1 + Sin[c + d*x]))^(3/2)*(-7 + 3*Sin[c + d*x]))/(15*a^2*d)
```

fricas [A] time = 0.68, size = 40, normalized size = 0.82

$$\frac{2\left(3\cos\left(dx+c\right)^{2}+4\sin\left(dx+c\right)+4\right)\sqrt{a\sin\left(dx+c\right)+a}}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(d*x + c)^2 + 4*sin(d*x + c) + 4)*sqrt(a*sin(d*x + c) + a)/(a*d)
```

giac [A] time = 1.16, size = 75, normalized size = 1.53

$$\frac{2\left(15\sqrt{a\sin\left(dx+c\right)+a}-\frac{3\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}-10\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a+15\sqrt{a\sin\left(dx+c\right)+a}\,a^{2}}{a^{2}}\right)}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(a*sin(d*x + c) + a) - (3*(a*sin(d*x + c) + a)^(5/2) - 10*(a*s
in(d*x + c) + a)^(3/2)*a + 15*sqrt(a*sin(d*x + c) + a)*a^2)/a^2)/(a*d)
```

maple [A] time = 0.14, size = 31, normalized size = 0.63

$$-\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{3}{2}}\left(3\sin\left(dx+c\right)-7\right)}{15a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/15/a^2*(a+a*sin(d*x+c))^(3/2)*(3*sin(d*x+c)-7)/d
```

maxima [A] time = 0.53, size = 75, normalized size = 1.53

$$\frac{2\left(15\sqrt{a\sin\left(dx+c\right)+a}-\frac{3\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}-10\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a+15\sqrt{a\sin\left(dx+c\right)+a}\,a^{2}}{a^{2}}\right)}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(a*sin(d*x + c) + a) - (3*(a*sin(d*x + c) + a)^(5/2) - 10*(a*s
in(d*x + c) + a)^(3/2)*a + 15*sqrt(a*sin(d*x + c) + a)*a^2)/a^2)/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^{3}}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.161 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{2a\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -2/3*a*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$-\frac{2a\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x]^3)/(3*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2a\cos^3(c+dx)}{3d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.07, size = 30, normalized size = 1.00

$$-\frac{2a\cos^3(c+dx)}{3d(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x]^3)/(3*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.78, size = 71, normalized size = 2.37

$$\frac{2\left(\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(d*x + c)^2 + (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)*s
qrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

giac [B] time = 1.10, size = 143, normalized size = 4.77

$$\frac{2\left(\frac{2\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}}+\frac{\left(\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{3\,a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\,a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(a) + (((a*tan(1/2*d*x + 1
/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 3*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*ta
n(1/2*d*x + 1/2*c) + 3*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c
) - a/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2))/
d
```

maple [A] time = 0.19, size = 44, normalized size = 1.47

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{2}}{3\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/3*(1+sin(d*x+c))*(sin(d*x+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)^{2}}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.162 $\int \frac{\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=22

$$\frac{2\sqrt{a\sin(c+dx)+a}}{ad}$$

```
[Out] 2*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2667, 32\}$

$$\frac{2\sqrt{a\sin(c+dx)+a}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2\sqrt{a+a\sin(c+dx)}}{ad}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{2\sqrt{a\sin(c+dx)+a}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(a*d)
```

fricas [A] time = 0.67, size = 20, normalized size = 0.91

$$\frac{2\sqrt{a\sin(dx+c)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*sin(d*x + c) + a)/(a*d)
```

giac [A]   time = 0.84, size = 20, normalized size = 0.91

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a*sin(d*x + c) + a)/(a*d)
```

maple [A]   time = 0.03, size = 21, normalized size = 0.95

$$\frac{2\sqrt{a+a\sin\left(dx+c\right)}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*(a+a*sin(d*x+c))^(1/2)/d/a
```

maxima [A]   time = 0.55, size = 20, normalized size = 0.91

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a*sin(d*x + c) + a)/(a*d)
```

mupad [B]   time = 4.82, size = 20, normalized size = 0.91

$$\frac{2\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(1/2))/(a*d)
```

sympy [A]   time = 1.21, size = 32, normalized size = 1.45

$$\begin{cases} \frac{2\sqrt{a\sin\left(c+dx\right)+a}}{ad} & \text{for}\: d \neq 0 \\ \frac{x\cos\left(c\right)}{\sqrt{a\sin\left(c\right)+a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(a*sin(c + d*x) + a)/(a*d), Ne(d, 0)), (x*cos(c)/sqrt(a*si
n(c) + a), True))
```

3.163 $\int \frac{\sec(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{1}{d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] 1/2*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-1
/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2667, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{1}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) - 1
/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{1}{d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{2d}$$
$$= -\frac{1}{d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\sin(c+dx)}\right)}{d}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} - \frac{1}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.05, size = 39, normalized size = 0.65

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sin[c + d*x])/2]/(d*Sqrt[a + a*Sin[c
 + d*x]]))
```

fricas [A] time = 0.59, size = 90, normalized size = 1.50

$$\frac{\frac{\sqrt{2}\,(a\sin(dx+c)+a)\log\left(-\frac{\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}}{\sqrt{a}}+\sin(dx+c)+3}{\sin(dx+c)-1}\right)}{\sqrt{a}} - 4\sqrt{a\sin(dx+c)+a}}{4\,(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 1/4*(sqrt(2)*(a*sin(d*x + c) + a)*log(-(2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/
sqrt(a) + sin(d*x + c) + 3)/(sin(d*x + c) - 1))/sqrt(a) - 4*sqrt(a*sin(d*x
+ c) + a))/(a*d*sin(d*x + c) + a*d)
```

giac [B] time = 1.44, size = 211, normalized size = 3.52

$$-\frac{\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}-\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\text{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)} + \frac{2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan}\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] -(sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a) - sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*
```

```
c) + 1)) + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a) - sqrt(a))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))*sqrt(a) - a)*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.15, size = 54, normalized size = 0.90

$$-\frac{2a\left(\frac{1}{2a\sqrt{a+a\sin(dx+c)}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{3}{2}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*a*(1/2/a/(a+a*sin(d*x+c))^(1/2)-1/4/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*sin
(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

maxima [A] time = 1.25, size = 78, normalized size = 1.30

$$-\frac{\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\,a}{\sqrt{a\sin(dx+c)+a}}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sq
rt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*a/sqrt(a*sin(d*x + c) + a))/
(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos(c+d\,x)\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.164 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=102

$$-\frac{3a\cos(c+dx)}{4d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}\,d}$$

```
[Out] -3/4*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-3/8*arctanh(1/2*cos(d*x+c)*a^(1/
2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+sec(d*x+c)/d/(a+a*sin(
d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2687, 2650, 2649, 206}

$$-\frac{3a\cos(c+dx)}{4d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(4*
Sqrt[2]*Sqrt[a]*d) - (3*a*Cos[c + d*x])/(4*d*(a + a*Sin[c + d*x])^(3/2)) +
Sec[c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{1}{2}(3a)\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= -\frac{3a\cos(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{3}{8}\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{3a\cos(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{3\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+}}\right)}{4d}$$
$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}\,d}-\frac{3a\cos(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.26, size = 118, normalized size = 1.16

$$-\frac{\sec(c+dx)\left(-3\sin(c+dx)+(-3-3i)(-1)^{3/4}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}{4d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/4*(Sec[c + d*x]*(-1 - (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4
)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^2 - 3*Sin[c + d*x]))/(d*Sqrt[a*(1 + Sin[c + d*x
])])
```

fricas [B] time = 0.68, size = 200, normalized size = 1.96

$$\frac{3\sqrt{2}\left(\cos(dx+c)\sin(dx+c)+\cos(dx+c)\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(}\right.}{16\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*sqrt(a)*log(-(a*
cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
 sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c)
 + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) -
2)) + 4*sqrt(a*sin(d*x + c) + a)*(3*sin(d*x + c) + 1))/(a*d*cos(d*x + c)*si
n(d*x + c) + a*d*cos(d*x + c))
```

giac [B] time = 2.32, size = 419, normalized size = 4.11

$$\frac{\frac{3\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\text{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{4\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}\right.}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x +
 1/2*c) + 1)) + 4*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a) + sqrt(a))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))*sqrt(a) - a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(3*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 + (sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - (sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + a^(3/2))/(
((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a)
 - a)^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.20, size = 130, normalized size = 1.27

$$-\frac{\sin\left(dx+c\right)\left(3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)a\sqrt{a-a\sin\left(dx+c\right)}-6a^{\frac{3}{2}}\right)+3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right.}{8a^{\frac{3}{2}}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/8*(sin(d*x+c)*(3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1
/2))*a*(a-a*sin(d*x+c))^(1/2)-6*a^(3/2))+3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x
+c))^(1/2)*2^(1/2)/a^(1/2))*a*(a-a*sin(d*x+c))^(1/2)-2*a^(3/2))/a^(3/2)/cos
(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{2}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.165 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=116

$$-\frac{5}{8d\sqrt{a\sin(c+dx)+a}}-\frac{5a}{12d(a\sin(c+dx)+a)^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d}+\frac{\sec^2(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -5/12*a/d/(a+a*sin(d*x+c))^(3/2)+5/16*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^
(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-5/8/d/(a+a*sin(d*x+c))^(1/2)+1/2*sec(d*x+c
)^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2687, 2667, 51, 63, 206}

$$-\frac{5}{8d\sqrt{a\sin(c+dx)+a}}-\frac{5a}{12d(a\sin(c+dx)+a)^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d}+\frac{\sec^2(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (5*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*Sqrt[a]*
d) - (5*a)/(12*d*(a + a*Sin[c + d*x])^(3/2)) - 5/(8*d*Sqrt[a + a*Sin[c + d*
x]]) + Sec[c + d*x]^2/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{1}{4}(5a)\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{(5a^2)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{4d}$$
$$= -\frac{5a}{12d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{(5a)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,d\right.}{8d}$$
$$= -\frac{5a}{12d(a+a\sin(c+dx))^{3/2}} - \frac{5}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{5\,S}{}$$
$$= -\frac{5a}{12d(a+a\sin(c+dx))^{3/2}} - \frac{5}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{5\,S}{}$$
$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d} - \frac{5a}{12d(a+a\sin(c+dx))^{3/2}} - \frac{5}{8d\sqrt{a+a\sin(c+dx)}} + \frac{}{2}$$

Mathematica [C] time = 0.07, size = 42, normalized size = 0.36

$$-\frac{a\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{6d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/6*(a*Hypergeometric2F1[-3/2, 2, -1/2, (1 + Sin[c + d*x])/2])/(d*(a + a*S
in[c + d*x])^(3/2))
```

fricas [A] time = 0.59, size = 145, normalized size = 1.25

$$\frac{15\sqrt{2}\left(\cos(dx+c)^2\sin(dx+c)+\cos(dx+c)^2\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(15\cos(dx\right.}{96\left(ad\cos(dx+c)^2\sin(dx+c)+ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*sqrt(a)*log
(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(
d*x + c) - 1)) - 4*(15*cos(d*x + c)^2 - 10*sin(d*x + c) - 2)*sqrt(a*sin(d*x
 + c) + a))/(a*d*cos(d*x + c)^2*sin(d*x + c) + a*d*cos(d*x + c)^2)
```

giac [B] time = 2.81, size = 587, normalized size = 5.06

$$-\frac{\frac{15\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}-\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{6\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^3-\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\,\mathrm{ta}\cdots}\right.\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\cdots\right.\right.\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(15*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a) - sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*
x + 1/2*c) + 1)) + 6*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^3 - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))*a - a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 1
6*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5
+ 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*s
qrt(a) - 4*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^3*a - 12*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^2*a^(3/2) + 9*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))*a^2 - 2*a^(5/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^3*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.27, size = 107, normalized size = 0.92

$$\frac{2a^3\left(-\frac{1}{4a^3\sqrt{a+a\sin(dx+c)}}-\frac{1}{12a^2(a+a\sin(dx+c))^{\frac{3}{2}}}-\frac{\frac{\sqrt{a+a\sin(dx+c)}}{4a\sin(dx+c)-4a}-\frac{5\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8\sqrt{a}}}{4a^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*a^3*(-1/4/a^3/(a+a*sin(d*x+c))^(1/2)-1/12/a^2/(a+a*sin(d*x+c))^(3/2)-1/4/
a^3*(1/4*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)-5/8*2^(1/2)/a^(1/2)*arctan
h(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))))/d
```

maxima [A] time = 0.74, size = 132, normalized size = 1.14

$$-\frac{15\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(15(a\sin(dx+c)+a)^2a-20(a\sin(dx+c)+a)a^2-8a^3\right)}{(a\sin(dx+c)+a)^{\frac{5}{2}}-2(a\sin(dx+c)+a)^{\frac{3}{2}}a}}{96ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(15*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(15*(a*sin(d*x + c) + a)
^2*a - 20*(a*sin(d*x + c) + a)*a^2 - 8*a^3)/((a*sin(d*x + c) + a)^(5/2) - 2
*(a*sin(d*x + c) + a)^(3/2)*a))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.166 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=162

$$-\frac{35a\cos(c+dx)}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{35\sec(c+dx)}{48d\sqrt{a\sin(c+dx)+a}}-\frac{7a\sec(c+dx)}{24d(a\sin(c+dx)+a)^{3/2}}-\frac{35\tanh^{-}}{6}$$

```
[Out] -35/64*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-7/24*a*sec(d*x+c)/d/(a+a*sin(d
*x+c))^(3/2)-35/128*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))
^(1/2))*2^(1/2)/d/a^(1/2)+35/48*sec(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+1/3*sec
(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2687, 2681, 2650, 2649, 206}

$$-\frac{35a\cos(c+dx)}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{35\sec(c+dx)}{48d\sqrt{a\sin(c+dx)+a}}-\frac{7a\sec(c+dx)}{24d(a\sin(c+dx)+a)^{3/2}}-\frac{35\tanh^{-}}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-35*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(6
4*Sqrt[2]*Sqrt[a]*d) - (35*a*Cos[c + d*x])/(64*d*(a + a*Sin[c + d*x])^(3/2)
) - (7*a*Sec[c + d*x])/(24*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x]
)/(48*d*Sqrt[a + a*Sin[c + d*x]]) + Sec[c + d*x]^3/(3*d*Sqrt[a + a*Sin[c +
d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^3(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{1}{6}(7a)\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}}+\frac{\sec^3(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{35}{48}\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}}+\frac{35\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^3(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+$$

$$= -\frac{35a\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}}-\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}}+\frac{35\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{35a\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}}-\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}}+\frac{35\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}\,d}-\frac{35a\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}}-\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx}$$

Mathematica [C] time = 0.63, size = 117, normalized size = 0.72

$$\frac{\sec^3(c+dx)(329\sin(c+dx)+105\sin(3(c+dx))+70\cos(2(c+dx))+102)+(420+420i)(-1)^{3/4}\left(\sin\left(\frac{1}{2}(c+\right.\right.}{768d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((420 + 420*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x
)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + Sec[c + d*x]^3*(102 + 70*Cos
[2*(c + d*x)] + 329*Sin[c + d*x] + 105*Sin[3*(c + d*x)]))/(768*d*Sqrt[a*(1
+ Sin[c + d*x])])
```

fricas [A] time = 0.84, size = 230, normalized size = 1.42

$$\frac{105\sqrt{2}\left(\cos(dx+c)^3\sin(dx+c)+\cos(dx+c)^3\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx}\right.}{768\left(ad\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(105*sqrt(2)*(cos(d*x + c)^3*sin(d*x + c) + cos(d*x + c)^3)*sqrt(a)*l
og(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x
 + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d
*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)) + 4*(35*cos(d*x + c)^2 + 7*(15*cos(d*x + c)^2 + 8)*sin(d*x + c)
+ 8)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d
*x + c)^3)
```

giac [B] time = 4.09, size = 745, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(105*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d
*x + 1/2*c) + 1)) + 16*(15*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^5 - 33*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^4*sqrt(a) - 22*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^3*a + 66*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) + 51*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 + 11*a^(5/2))/(((sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^3*sgn(tan(1/
2*d*x + 1/2*c) + 1)) + 6*(53*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^7 + 179*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*sqrt(a) + 127*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a - 195*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 7*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 + 121*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 67*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 + 15*a^(7/2))/((
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a)
- a)^4*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.24, size = 231, normalized size = 1.43

$$\frac{-210a^{\frac{7}{2}}\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)+\left(210\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}\left(a-a\sin\left(dx+c\right)\right)^{\frac{3}{2}}-112a^{\frac{7}{2}}\right)\sin\left(d\right.}{384a^{\frac{7}{2}}\left(\sin\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/384*(-210*a^(7/2)*sin(d*x+c)*cos(d*x+c)^2+(210*2^(1/2)*arctanh(1/2*(a-a*s
in(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin(d*x+c))^(3/2)-112*a^(7/2))*s
in(d*x+c)+(-105*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))
*a^2*(a-a*sin(d*x+c))^(3/2)-70*a^(7/2))*cos(d*x+c)^2+210*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin(d*x+c))^(3/2)-16*a^(
7/2))/a^(7/2)/(sin(d*x+c)-1)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{4}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{4}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.167 $\int \frac{\sec^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=175

$$-\frac{63}{128d\sqrt{a\sin(c+dx)+a}}-\frac{21a}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{63\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d}+\frac{\sec^4(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{63}{160d\sqrt{}}$$

```
[Out] -21/64*a/d/(a+a*sin(d*x+c))^(3/2)-9/40*a*sec(d*x+c)^2/d/(a+a*sin(d*x+c))^(3
/2)+63/256*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^
(1/2)-63/128/d/(a+a*sin(d*x+c))^(1/2)+63/160*sec(d*x+c)^2/d/(a+a*sin(d*x+c)
)^(1/2)+1/4*sec(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2687, 2681, 2667, 51, 63, 206}

$$-\frac{63}{128d\sqrt{a\sin(c+dx)+a}}-\frac{21a}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{63\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d}+\frac{\sec^4(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{63}{160d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (63*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[2]*Sqrt[
a]*d) - (21*a)/(64*d*(a + a*Sin[c + d*x])^(3/2)) - (9*a*Sec[c + d*x]^2)/(40
*d*(a + a*Sin[c + d*x])^(3/2)) - 63/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (63*
Sec[c + d*x]^2)/(160*d*Sqrt[a + a*Sin[c + d*x]]) + Sec[c + d*x]^4/(4*d*Sqrt
[a + a*Sin[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}+\frac{1}{8}(9a)\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx\\
&= -\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}}+\frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}+\frac{63}{80}\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,d\\
&= -\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}}+\frac{63\sec^2(c+dx)}{160d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}+\\
&= -\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}}+\frac{63\sec^2(c+dx)}{160d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}+\\
&= -\frac{21a}{64d(a+a\sin(c+dx))^{3/2}}-\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}}+\frac{63\sec^2(c+dx)}{160d\sqrt{a+a\sin(c+dx)}}\\
&= -\frac{21a}{64d(a+a\sin(c+dx))^{3/2}}-\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}}-\frac{63}{128d\sqrt{a+a\sin(c+dx)}}\\
&= -\frac{21a}{64d(a+a\sin(c+dx))^{3/2}}-\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}}-\frac{63}{128d\sqrt{a+a\sin(c+dx)}}\\
&= \frac{63\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d}-\frac{21a}{64d(a+a\sin(c+dx))^{3/2}}-\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 44, normalized size = 0.25

$$-\frac{a^2\,{}_2F_1\left(-\frac{5}{2},3;-\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{20d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/20*(a^2*Hypergeometric2F1[-5/2, 3, -3/2, (1 + Sin[c + d*x])/2])/(d*(a +
a*Sin[c + d*x])^(5/2))
```

fricas [A] time = 0.60, size = 167, normalized size = 0.95

$$\frac{315 \sqrt{2} \left(\cos(dx+c)^4 \sin(dx+c) + \cos(dx+c)^4\right) \sqrt{a} \log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right) - 4\left(315\cos(dx+\right.}{2560\left(ad\cos(dx+c)^4\sin(dx+c)+ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2560*(315*sqrt(2)*(cos(d*x + c)^4*sin(d*x + c) + cos(d*x + c)^4)*sqrt(a)*
log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(s
in(d*x + c) - 1)) - 4*(315*cos(d*x + c)^4 - 42*cos(d*x + c)^2 - 6*(35*cos(d
*x + c)^2 + 24)*sin(d*x + c) - 16)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x +
 c)^4*sin(d*x + c) + a*d*cos(d*x + c)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos((d*t_nostep+c)/2-pi/4))]Discontinuities at zeroe
s of cos((d*t_nostep+c)/2-pi/4) were not checkedWarning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep+1)]Evaluation time: 0.46Not invertible Error: Bad Ar
gument Value
```

maple [A] time = 0.35, size = 135, normalized size = 0.77

$$-\frac{2a^5\left(\frac{3}{16a^5\sqrt{a+a\sin(dx+c)}}+\frac{1}{16a^4(a+a\sin(dx+c))^{\frac{3}{2}}}+\frac{1}{40a^3(a+a\sin(dx+c))^{\frac{5}{2}}}+\frac{\frac{\sqrt{a+a\sin(dx+c)}\,a(15\sin(dx+c)-19)}{16(a\sin(dx+c)-a)^2}-\frac{63\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right.}{32\sqrt{a}}}{16a^5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*a^5*(3/16/a^5/(a+a*sin(d*x+c))^(1/2)+1/16/a^4/(a+a*sin(d*x+c))^(3/2)+1/4
0/a^3/(a+a*sin(d*x+c))^(5/2)+1/16/a^5*(1/16*(a+a*sin(d*x+c))^(1/2)*a*(15*si
n(d*x+c)-19)/(a*sin(d*x+c)-a)^2-63/32*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(
d*x+c))^(1/2)*2^(1/2)/a^(1/2))))/d
```

maxima [A] time = 0.81, size = 183, normalized size = 1.05

$$-\frac{315\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(315(a\sin(dx+c)+a)^4a-1050(a\sin(dx+c)+a)^3a^2+672(a\sin(dx+c)+a)^2a^3+192(a\sin(dx+c)\right.}{(a\sin(dx+c)+a)^{\frac{9}{2}}-4(a\sin(dx+c)+a)^{\frac{7}{2}}a+4(a\sin(dx+c)+a)^{\frac{5}{2}}a^2}}{2560\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2560*(315*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(315*(a*sin(d*x + c)
+ a)^4*a - 1050*(a*sin(d*x + c) + a)^3*a^2 + 672*(a*sin(d*x + c) + a)^2*a^3
 + 192*(a*sin(d*x + c) + a)*a^4 + 128*a^5)/((a*sin(d*x + c) + a)^(9/2) - 4*
(a*sin(d*x + c) + a)^(7/2)*a + 4*(a*sin(d*x + c) + a)^(5/2)*a^2))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}{\left(c+dx\right)}}{\sqrt{a\left(\sin{\left(c+dx\right)}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.168 $\int \frac{\sec^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=221

$$-\frac{231a\cos(c+dx)}{512d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^5(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{11\sec^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}-\frac{11a\sec^3(c+dx)}{60d(a\sin(c+dx)+a)^{3/2}}+\frac{77}{128d\sqrt{a}}$$

```
[Out] -231/512*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-77/320*a*sec(d*x+c)/d/(a+a*s
in(d*x+c))^(3/2)-11/60*a*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-231/1024*arc
tanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/d/a^(1/
2)+77/128*sec(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+11/40*sec(d*x+c)^3/d/(a+a*sin
(d*x+c))^(1/2)+1/5*sec(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2687, 2681, 2650, 2649, 206}

$$-\frac{231a\cos(c+dx)}{512d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^5(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{11\sec^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}-\frac{11a\sec^3(c+dx)}{60d(a\sin(c+dx)+a)^{3/2}}+\frac{77}{128d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-231*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
512*Sqrt[2]*Sqrt[a]*d) - (231*a*Cos[c + d*x])/(512*d*(a + a*Sin[c + d*x])^(
3/2)) - (77*a*Sec[c + d*x])/(320*d*(a + a*Sin[c + d*x])^(3/2)) - (11*a*Sec[
c + d*x]^3)/(60*d*(a + a*Sin[c + d*x])^(3/2)) + (77*Sec[c + d*x])/(128*d*Sq
rt[a + a*Sin[c + d*x]]) + (11*Sec[c + d*x]^3)/(40*d*Sqrt[a + a*Sin[c + d*x]
]) + Sec[c + d*x]^5/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
```

```
IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^5(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{1}{10}(11a)\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= -\frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^5(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{33}{40}\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,d$$
$$= -\frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^5(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{1}{8}$$
$$= -\frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}}$$
$$= -\frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{77\sec(c+dx)}{128d\sqrt{a+a\sin(c+dx}}$$
$$= -\frac{231a\cos(c+dx)}{512d(a+a\sin(c+dx))^{3/2}} - \frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))}$$
$$= -\frac{231a\cos(c+dx)}{512d(a+a\sin(c+dx))^{3/2}} - \frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))}$$
$$= -\frac{231\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{512\sqrt{2}\sqrt{a}\,d} - \frac{231a\cos(c+dx)}{512d(a+a\sin(c+dx))^{3/2}} - \frac{77a\sec(c+d}{320d(a+a\sin(c+}$$

Mathematica [C] time = 0.72, size = 140, normalized size = 0.63

$$\frac{\frac{1}{16}\sec^5(c+dx)(36850\sin(c+dx)+17787\sin(3(c+dx))+3465\sin(5(c+dx))+11352\cos(2(c+dx))+231}{76}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((3465 + 3465*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d
*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (Sec[c + d*x]^5*(11090 + 1
1352*Cos[2*(c + d*x)] + 2310*Cos[4*(c + d*x)] + 36850*Sin[c + d*x] + 17787*
Sin[3*(c + d*x)] + 3465*Sin[5*(c + d*x)]))/16)/(7680*d*Sqrt[a*(1 + Sin[c +
d*x])])
```

fricas [A] time = 0.56, size = 250, normalized size = 1.13

$$\frac{3465\sqrt{2}\left(\cos(dx+c)^5\sin(dx+c)+\cos(dx+c)^5\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30720*(3465*sqrt(2)*(cos(d*x + c)^5*sin(d*x + c) + cos(d*x + c)^5)*sqrt(a
)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(
d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*si
n(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d
*x + c) - 2)) + 4*(1155*cos(d*x + c)^4 + 264*cos(d*x + c)^2 + 11*(315*cos(d
*x + c)^4 + 168*cos(d*x + c)^2 + 128)*sin(d*x + c) + 128)*sqrt(a*sin(d*x +
c) + a))/(a*d*cos(d*x + c)^5*sin(d*x + c) + a*d*cos(d*x + c)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos((d*t_nostep+c)/2-pi/4))]Discontinuities at zeroe
s of cos((d*t_nostep+c)/2-pi/4) were not checkedWarning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep+1)]Evaluation time: 0.47Not invertible Error: Bad Ar
gument Value
```

maple [A] time = 0.28, size = 308, normalized size = 1.39

$$-\frac{-6930a^{\frac{11}{2}}\sin\left(dx+c\right)\left(\cos^{4}\left(dx+c\right)\right)+\left(-3696a^{\frac{11}{2}}-3465\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{3}\left(a-a\sin\left(dx+c\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/15360*(-6930*a^(11/2)*sin(d*x+c)*cos(d*x+c)^4+(-3696*a^(11/2)-3465*2^(1/
2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*(a-a*sin(d*x+c))
^(5/2))*cos(d*x+c)^2*sin(d*x+c)+(-2816*a^(11/2)+13860*2^(1/2)*arctanh(1/2*(
a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*(a-a*sin(d*x+c))^(5/2))*sin(d*x+
c)-2310*a^(11/2)*cos(d*x+c)^4+(-528*a^(11/2)-10395*2^(1/2)*arctanh(1/2*(a-a
*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*(a-a*sin(d*x+c))^(5/2))*cos(d*x+c)^
2-256*a^(11/2)+13860*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(
1/2))*a^3*(a-a*sin(d*x+c))^(5/2))/a^(11/2)/(sin(d*x+c)-1)^2/(1+sin(d*x+c))^
2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{6}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^6/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{6}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^6*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.169 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^7d}+\frac{4(a\sin(c+dx)+a)^{9/2}}{3a^6d}-\frac{24(a\sin(c+dx)+a)^{7/2}}{7a^5d}+\frac{16(a\sin(c+dx)+a)^{5/2}}{5a^4d}$$

```
[Out] 16/5*(a+a*sin(d*x+c))^(5/2)/a^4/d-24/7*(a+a*sin(d*x+c))^(7/2)/a^5/d+4/3*(a+
a*sin(d*x+c))^(9/2)/a^6/d-2/11*(a+a*sin(d*x+c))^(11/2)/a^7/d
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^7d}+\frac{4(a\sin(c+dx)+a)^{9/2}}{3a^6d}-\frac{24(a\sin(c+dx)+a)^{7/2}}{7a^5d}+\frac{16(a\sin(c+dx)+a)^{5/2}}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(5/2))/(5*a^4*d) - (24*(a + a*Sin[c + d*x])^(7/2))
/(7*a^5*d) + (4*(a + a*Sin[c + d*x])^(9/2))/(3*a^6*d) - (2*(a + a*Sin[c + d
*x])^(11/2))/(11*a^7*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{3/2}\,dx,x,a\sin(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{3/2}-12a^2(a+x)^{5/2}+6a(a+x)^{7/2}-(a+x)^{9/2}\right)dx,x,a\sin(c+\right.}{a^7d}\\
&= \frac{16(a+a\sin(c+dx))^{5/2}}{5a^4d}-\frac{24(a+a\sin(c+dx))^{7/2}}{7a^5d}+\frac{4(a+a\sin(c+dx))^{9/2}}{3a^6d}-\frac{2(a}{}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 54, normalized size = 0.56

$$-\frac{2\left(105\sin^3(c+dx)-455\sin^2(c+dx)+755\sin(c+dx)-533\right)(a(\sin(c+dx)+1))^{5/2}}{1155a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a*(1 + Sin[c + d*x]))^(5/2)*(-533 + 755*Sin[c + d*x] - 455*Sin[c + d*x
]^2 + 105*Sin[c + d*x]^3))/(1155*a^4*d)
```

fricas [A] time = 0.57, size = 72, normalized size = 0.74

$$\frac{2\left(245\,\cos\left(dx+c\right)^{4}+32\,\cos\left(dx+c\right)^{2}-\left(105\,\cos\left(dx+c\right)^{4}-160\,\cos\left(dx+c\right)^{2}-256\right)\sin\left(dx+c\right)+256\right)\sqrt{a}}{1155\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(245*cos(d*x + c)^4 + 32*cos(d*x + c)^2 - (105*cos(d*x + c)^4 - 160*
cos(d*x + c)^2 - 256)*sin(d*x + c) + 256)*sqrt(a*sin(d*x + c) + a)/(a^2*d)
```

giac [B] time = 2.56, size = 370, normalized size = 3.81

$$2\left(\frac{533\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{1155\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{1199\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{3465\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{5874\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2/1155*(533*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (1155*a^4/sgn(tan(1/2*d*x +
 1/2*c) + 1) + (1199*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (3465*a^4/sgn(tan(
1/2*d*x + 1/2*c) + 1) + (5874*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (4158*a^4
/sgn(tan(1/2*d*x + 1/2*c) + 1) + (4158*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) +
(5874*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (3465*a^4/sgn(tan(1/2*d*x + 1/2*c
) + 1) + (1199*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (533*a^4*tan(1/2*d*x + 1
/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) + 1155*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1
))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/
2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1
/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))/((
a*tan(1/2*d*x + 1/2*c)^2 + a)^(11/2)*d)
```

maple [A] time = 0.16, size = 57, normalized size = 0.59

$$\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{5}{2}}\left(105\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-455\left(\cos^{2}\left(dx+c\right)\right)-860\sin\left(dx+c\right)+988\right)}{1155a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/1155/a^4*(a+a*sin(d*x+c))^(5/2)*(105*cos(d*x+c)^2*sin(d*x+c)-455*cos(d*x+
c)^2-860*sin(d*x+c)+988)/d
```

maxima [A] time = 0.31, size = 72, normalized size = 0.74

$$-\frac{2\left(105\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-770\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a+1980\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a^{2}-1848\left(a\sin\left(dx+c\right)\right.\right.}{1155\,a^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(105*(a*sin(d*x + c) + a)^(11/2) - 770*(a*sin(d*x + c) + a)^(9/2)*a
 + 1980*(a*sin(d*x + c) + a)^(7/2)*a^2 - 1848*(a*sin(d*x + c) + a)^(5/2)*a^
3)/(a^7*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^7}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.170 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^7(c+dx)}{63d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{9d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] -8/63*a^2*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)-2/9*a*cos(d*x+c)^7/d/(a+a*s
in(d*x+c))^(5/2)
```

Rubi [A] time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{8a^2\cos^7(c+dx)}{63d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{9d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^7)/(63*d*(a + a*Sin[c + d*x])^(7/2)) - (2*a*Cos[c + d*
x]^7)/(9*d*(a + a*Sin[c + d*x])^(5/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{2a\cos^7(c+dx)}{9d(a+a\sin(c+dx))^{5/2}}+\frac{1}{9}(4a)\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx\\ &= -\frac{8a^2\cos^7(c+dx)}{63d(a+a\sin(c+dx))^{7/2}}-\frac{2a\cos^7(c+dx)}{9d(a+a\sin(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.20, size = 49, normalized size = 0.78

$$-\frac{2(7\sin(c+dx)+11)\cos^7(c+dx)}{63d(\sin(c+dx)+1)^2(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7*(11 + 7*Sin[c + d*x]))/(63*d*(1 + Sin[c + d*x])^2*(a*(1
+ Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.76, size = 142, normalized size = 2.25

$$\frac{2\left(7\cos(dx+c)^5+17\cos(dx+c)^4-2\cos(dx+c)^3+4\cos(dx+c)^2-\left(7\cos(dx+c)^4-10\cos(dx+c)^3-1\right.\right.}{63\left(a^2d\cos(dx+c)+a^2d\sin(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*cos(d*x + c)^5 + 17*cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 4*cos(d*x +
 c)^2 - (7*cos(d*x + c)^4 - 10*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 16*cos(
d*x + c) - 32)*sin(d*x + c) - 16*cos(d*x + c) - 32)*sqrt(a*sin(d*x + c) + a
)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

giac [B] time = 1.72, size = 340, normalized size = 5.40

$$2\left(\frac{32\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}}-\frac{\frac{11a^3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{63a^3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{144a^3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{168a^3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{126a^3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right.\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2/63*(32*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(3/2) - (11*a^3/sgn(tan(1/
2*d*x + 1/2*c) + 1) - (63*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (144*a^3/sgn(
tan(1/2*d*x + 1/2*c) + 1) - (168*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (126*a
^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (126*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) -
 (168*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (144*a^3/sgn(tan(1/2*d*x + 1/2*c)
 + 1) + (11*a^3*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 63*a^3
/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))
*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*
d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))/(a*tan(1/2*d*x +
1/2*c)^2 + a)^(9/2))/d
```

maple [A] time = 0.20, size = 57, normalized size = 0.90

$$-\frac{2\left(1+\sin(dx+c)\right)\left(\sin(dx+c)-1\right)^4\left(7\sin(dx+c)+11\right)}{63a\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/63/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^4*(7*sin(d*x+c)+11)/cos(d*x+c)/(a+a*s
in(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(dx+c)^6}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+dx)^6}{(a+a\sin(c+dx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.171 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7a^5d}-\frac{8(a\sin(c+dx)+a)^{5/2}}{5a^4d}+\frac{8(a\sin(c+dx)+a)^{3/2}}{3a^3d}$$

```
[Out] 8/3*(a+a*sin(d*x+c))^(3/2)/a^3/d-8/5*(a+a*sin(d*x+c))^(5/2)/a^4/d+2/7*(a+a*
sin(d*x+c))^(7/2)/a^5/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7a^5d}-\frac{8(a\sin(c+dx)+a)^{5/2}}{5a^4d}+\frac{8(a\sin(c+dx)+a)^{3/2}}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(3/2))/(3*a^3*d) - (8*(a + a*Sin[c + d*x])^(5/2))/(
5*a^4*d) + (2*(a + a*Sin[c + d*x])^(7/2))/(7*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2\sqrt{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2\sqrt{a+x}-4a(a+x)^{3/2}+(a+x)^{5/2}\right)dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{8(a+a\sin(c+dx))^{3/2}}{3a^3d}-\frac{8(a+a\sin(c+dx))^{5/2}}{5a^4d}+\frac{2(a+a\sin(c+dx))^{7/2}}{7a^5d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 44, normalized size = 0.60

$$\frac{2\left(15\sin^2(c+dx)-54\sin(c+dx)+71\right)(a(\sin(c+dx)+1))^{3/2}}{105a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(3/2)*(71 - 54*Sin[c + d*x] + 15*Sin[c + d*x]^2))
/(105*a^3*d)
```

fricas [A] time = 0.73, size = 52, normalized size = 0.71

$$\frac{2\left(39\cos(dx+c)^2-\left(15\cos(dx+c)^2-32\right)\sin(dx+c)+32\right)\sqrt{a\sin(dx+c)+a}}{105\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(39*cos(d*x + c)^2 - (15*cos(d*x + c)^2 - 32)*sin(d*x + c) + 32)*sqrt
(a*sin(d*x + c) + a)/(a^2*d)
```

giac [B] time = 2.21, size = 250, normalized size = 3.42

$$2\left(\left(\left(\left(\left(\left(\frac{71\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{105\,a^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{91\,a^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{}{\mathrm{sgn}}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(((((((71*a^2*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) + 10
5*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 91*a^2/sgn(tan(
1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 245*a^2/sgn(tan(1/2*d*x + 1/2
*c) + 1))*tan(1/2*d*x + 1/2*c) + 245*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan
(1/2*d*x + 1/2*c) + 91*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2
*c) + 105*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 71*a^2/
sgn(tan(1/2*d*x + 1/2*c) + 1))/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2)*d)
```

maple [A] time = 0.16, size = 41, normalized size = 0.56

$$-\frac{2\left(a+a\sin(dx+c)\right)^{\frac{3}{2}}\left(15\left(\cos^2(dx+c)\right)+54\sin(dx+c)-86\right)}{105a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/105/a^3*(a+a*sin(d*x+c))^(3/2)*(15*cos(d*x+c)^2+54*sin(d*x+c)-86)/d
```

maxima [A] time = 0.41, size = 55, normalized size = 0.75

$$\frac{2\left(15\left(a\sin(dx+c)+a\right)^{\frac{7}{2}}-84\left(a\sin(dx+c)+a\right)^{\frac{5}{2}}a+140\left(a\sin(dx+c)+a\right)^{\frac{3}{2}}a^2\right)}{105\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(a*sin(d*x + c) + a)^(7/2) - 84*(a*sin(d*x + c) + a)^(5/2)*a + 14
0*(a*sin(d*x + c) + a)^(3/2)*a^2)/(a^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^5}{\left(a+a\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.172 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{2a\cos^5(c+dx)}{5d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] -2/5*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)
```

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$-\frac{2a\cos^5(c+dx)}{5d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[c + d*x]^5)/(5*d*(a + a*Sin[c + d*x])^(5/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2a\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}}$$

Mathematica [A] time = 0.06, size = 42, normalized size = 1.40

$$-\frac{2\cos^5(c+dx)\sqrt{a(\sin(c+dx)+1)}}{5a^2d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5*Sqrt[a*(1 + Sin[c + d*x])])/(5*a^2*d*(1 + Sin[c + d*x])^
3)
```

fricas [B] time = 0.71, size = 98, normalized size = 3.27

$$\frac{2\left(\cos(dx+c)^3+3\cos(dx+c)^2-\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a\,\mathrm{s}}}{5\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 - (cos(d*x + c)^2 - 2*cos(d*x + c) -
 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a*sin(d*x + c) + a)/(a^2*d*cos(
d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

giac [B] time = 1.73, size = 199, normalized size = 6.63

$$\frac{2\left(\frac{4\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{5\,a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{10\,a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{10}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,d\right.\right.}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{5}{2}}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(4*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(3/2) + (((((a*tan(1/2*d*x +
 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 5*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*
tan(1/2*d*x + 1/2*c) + 10*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/
2*c) - 10*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 5*a/sgn(t
an(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - a/sgn(tan(1/2*d*x + 1/2*c)
 + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

maple [A] time = 0.17, size = 47, normalized size = 1.57

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{3}}{5a\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{4}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\cos\left(c+d\,x\right)^{4}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^{4}\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**4/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.173 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{4\sqrt{a\sin(c+dx)+a}}{a^2d}-\frac{2(a\sin(c+dx)+a)^{3/2}}{3a^3d}$$

```
[Out] -2/3*(a+a*sin(d*x+c))^(3/2)/a^3/d+4*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4\sqrt{a\sin(c+dx)+a}}{a^2d}-\frac{2(a\sin(c+dx)+a)^{3/2}}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (4*Sqrt[a + a*Sin[c + d*x]])/(a^2*d) - (2*(a + a*Sin[c + d*x])^(3/2))/(3*a^
3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{a-x}{\sqrt{a+x}}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{2a}{\sqrt{a+x}}-\sqrt{a+x}\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{4\sqrt{a+a\sin(c+dx)}}{a^2d}-\frac{2(a+a\sin(c+dx))^{3/2}}{3a^3d}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 32, normalized size = 0.68

$$-\frac{2(\sin(c+dx)-5)\sqrt{a(\sin(c+dx)+1)}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(-5 + Sin[c + d*x])*Sqrt[a*(1 + Sin[c + d*x])])/(3*a^2*d)
```

fricas [A] time = 0.55, size = 28, normalized size = 0.60

$$-\frac{2\sqrt{a\sin(dx+c)+a}\,(\sin(dx+c)-5)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -2/3*sqrt(a*sin(d*x + c) + a)*(sin(d*x + c) - 5)/(a^2*d)
```

giac [A] time = 1.68, size = 56, normalized size = 1.19

$$\frac{2\left(3\sqrt{a\sin(dx+c)+a}-\frac{(a\sin(dx+c)+a)^{\frac{3}{2}}-3\sqrt{a\sin(dx+c)+a}\,a}{a}\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] 2/3*(3*sqrt(a*sin(d*x + c) + a) - ((a*sin(d*x + c) + a)^(3/2) - 3*sqrt(a*si
n(d*x + c) + a)*a)/a)/(a^2*d)
```

maple [A] time = 0.13, size = 29, normalized size = 0.62

$$-\frac{2\sqrt{a+a\sin(dx+c)}\;(\sin(dx+c)-5)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
[Out] -2/3/a^2*(a+a*sin(d*x+c))^(1/2)*(sin(d*x+c)-5)/d
```

maxima [A] time = 0.38, size = 36, normalized size = 0.77

$$-\frac{2\left((a\sin(dx+c)+a)^{\frac{3}{2}}-6\sqrt{a\sin(dx+c)+a}\,a\right)}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/3*((a*sin(d*x + c) + a)^(3/2) - 6*sqrt(a*sin(d*x + c) + a)*a)/(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)^3}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.174 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{2\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[Out] -2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d
*2^(1/2)+2*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2679, 2649, 206}

$$\frac{2\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(3/2)*d) + (2*Cos[c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}+\frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}\\
&= \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}-\frac{4\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad}\\
&= -\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 84, normalized size = 1.11

$$\frac{2\cos^3(c+dx)\left(\sqrt{1-\sin(c+dx)}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\right)}{d(1-\sin(c+dx))^{3/2}(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c + d*x]^3*(-(Sqrt[2]*ArcTanh[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]) + Sqr
t[1 - Sin[c + d*x]]))/(d*(1 - Sin[c + d*x])^(3/2)*(a*(1 + Sin[c + d*x]))^(3
/2))
```

fricas [B] time = 0.85, size = 196, normalized size = 2.58

$$\frac{\frac{\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+2\sqrt{a\sin(dx+}}{a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 - (cos(
d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x +
c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos
(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 2*sqrt(a*sin(d*x
 + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a^2*d*cos(d*x + c) + a^2*d*s
in(d*x + c) + a^2*d)
```

giac [B] time = 1.93, size = 190, normalized size = 2.50

$$-\frac{2\left(\frac{\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{1}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+\frac{\sqrt{2}\left(2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}\,a^{\frac{3}{2}}}-\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.}{2}\right.}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx\right.\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2*((tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 1/(a*sgn(tan(
1/2*d*x + 1/2*c) + 1)))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(2)*(2*sqr
t(a)*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sq
rt(-a)*a^(3/2)) - 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sg
n(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.21, size = 94, normalized size = 1.24

$$\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(\sqrt{a-a\sin\left(dx+c\right)}-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{a^2\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/a^2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*((a-a*sin(d*x+c))^(1/2)-a^(1
/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/cos(d*x+c)
/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.175 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2}{ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2667, 32\}$

$$-\frac{2}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -2/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= -\frac{2}{ad\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$-\frac{2}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -2/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

fricas [A] time = 0.57, size = 33, normalized size = 1.50

$$-\frac{2\sqrt{a\sin(dx+c)+a}}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(a*sin(d*x + c) + a)/(a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 1.88, size = 20, normalized size = 0.91

$$-\frac{2}{\sqrt{a \sin(dx+c)+a}\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(a*sin(d*x + c) + a)*a*d)
```

maple [A] time = 0.02, size = 21, normalized size = 0.95

$$-\frac{2}{ad\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
[Out] -2/a/d/(a+a*sin(d*x+c))^(1/2)
```

maxima [A] time = 0.46, size = 20, normalized size = 0.91

$$-\frac{2}{\sqrt{a \sin(dx+c)+a}\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(a*sin(d*x + c) + a)*a*d)
```

mupad [B] time = 4.88, size = 50, normalized size = 2.27

$$-\frac{4\sqrt{a\,(\sin(c+d\,x)+1)}\,\,(\sin(c+d\,x)+1)}{a^2\,d\,\left(2\sin(c+d\,x)^2+4\,\sin(c+d\,x)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^(3/2),x)
[Out] -(4*(a*(sin(c + d*x) + 1))^(1/2)*(sin(c + d*x) + 1))/(a^2*d*(4*sin(c + d*x)
 + 2*sin(c + d*x)^2 + 2))
```

sympy [A] time = 3.46, size = 56, normalized size = 2.55

$$\begin{cases} \text{NaN} & \text{for}\ \left(c=\frac{3\pi}{2} \vee c=-dx+\frac{3\pi}{2}\right) \wedge \left(c=-dx+\frac{3\pi}{2} \vee d=0\right) \\ \frac{x\cos(c)}{(a\sin(c)+a)^{\frac{3}{2}}} & \text{for}\ d=0 \\ -\frac{2}{ad\sqrt{a\sin(c+dx)+a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((nan, (Eq(d, 0) | Eq(c, -d*x + 3*pi/2)) & (Eq(c, 3*pi/2) | Eq(c,
-d*x + 3*pi/2))), (x*cos(c)/(a*sin(c) + a)**(3/2), Eq(d, 0)), (-2/(a*d*sqrt
(a*sin(c + d*x) + a)), True))
```

3.176 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{1}{2ad\sqrt{a\sin(c+dx)+a}}-\frac{1}{3d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -1/3/d/(a+a*sin(d*x+c))^(3/2)+1/4*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2
)/a^(1/2))/a^(3/2)/d*2^(1/2)-1/2/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2667, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{1}{2ad\sqrt{a\sin(c+dx)+a}}-\frac{1}{3d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) -
 1/(3*d*(a + a*Sin[c + d*x])^(3/2)) - 1/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{1}{3d(a+a\sin(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{2d}$$
$$= -\frac{1}{3d(a+a\sin(c+dx))^{3/2}} - \frac{1}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\,\text{si}\right.}{4ad}$$
$$= -\frac{1}{3d(a+a\sin(c+dx))^{3/2}} - \frac{1}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a}\right.}{2ad}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{1}{3d(a+a\sin(c+dx))^{3/2}} - \frac{1}{2ad\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.07, size = 41, normalized size = 0.46

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{3d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sin[c + d*x])/2]/(d*(a + a*Sin[c
 + d*x])^(3/2))
```

fricas [A] time = 0.59, size = 132, normalized size = 1.48

$$\frac{3\sqrt{2}\left(\cos(dx+c)^2 - 2\sin(dx+c) - 2\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right) + 4\sqrt{a\sin(dx+c)+a}\,(3\,\text{si}}{24\left(a^2d\cos(dx+c)^2 - 2a^2d\sin(dx+c) - 2a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*sqrt(a)*log(-(a*sin(d
*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) -
 1)) + 4*sqrt(a*sin(d*x + c) + a)*(3*sin(d*x + c) + 5))/(a^2*d*cos(d*x + c)
^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

giac [B] time = 2.05, size = 379, normalized size = 4.26

$$-\frac{\frac{3\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}-\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\text{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)} + \frac{2\left(9\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^5+15\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\,\text{t}}\right.\right.}{\left(\left(\sqrt{a}\tan\left(\right.\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a) - sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*
x + 1/2*c) + 1)) + 2*(9*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^5 + 15*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^4*sqrt(a) - 10*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^3*a - 30*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) + 21*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 - 5*a^(5/2))/(((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^3*a*sgn(tan(1/2*
d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.17, size = 71, normalized size = 0.80

$$-\frac{a\left(\frac{1}{2a^{2}\sqrt{a+a\sin(dx+c)}}+\frac{1}{3a(a+a\sin(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{5}{2}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -a*(1/2/a^2/(a+a*sin(d*x+c))^(1/2)+1/3/a/(a+a*sin(d*x+c))^(3/2)-1/4/a^(5/2)
*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

maxima [A] time = 0.71, size = 91, normalized size = 1.02

$$-\frac{\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\,(3\,a\sin(dx+c)+5\,a)}{(a\sin(dx+c)+a)^{\frac{3}{2}}}}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt(2)
*sqrt(a) + sqrt(a*sin(d*x + c) + a)))/sqrt(a) + 4*(3*a*sin(d*x + c) + 5*a)/
(a*sin(d*x + c) + a)^(3/2))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.177 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{32\sqrt{2}a^{3/2}d}-\frac{15\cos(c+dx)}{32d(a\sin(c+dx)+a)^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec(c+dx)}{4d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -15/32*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-1/4*sec(d*x+c)/d/(a+a*sin(d*x+c)
)^(3/2)-15/64*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2)
)/a^(3/2)/d*2^(1/2)+5/8*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2681, 2687, 2650, 2649, 206}

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{32\sqrt{2}a^{3/2}d}-\frac{15\cos(c+dx)}{32d(a\sin(c+dx)+a)^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec(c+dx)}{4d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-15*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(3
2*Sqrt[2]*a^(3/2)*d) - (15*Cos[c + d*x])/(32*d*(a + a*Sin[c + d*x])^(3/2))
- Sec[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(3/2)) + (5*Sec[c + d*x])/(8*a*d*S
qrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
```

```
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}} + \frac{5\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{8a}$$

$$= -\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}} + \frac{5\sec(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{15}{16}\int \frac{1}{(a+a\sin(c+dx))^{3}\cdots}$$

$$= -\frac{15\cos(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}} + \frac{5\sec(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{15\cos(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}} + \frac{5\sec(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{32\sqrt{2}\,a^{3/2}d} - \frac{15\cos(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec(c+dx)}{4d(a+a\sin(c+dx\cdots}$$

Mathematica [C] time = 0.32, size = 224, normalized size = 1.67

$$\frac{\frac{8\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} - 7\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2 + 14\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4 + (8*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 14*Sin[(
c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 7*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^2 + (15 + 15*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*
(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (8*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/
(32*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.70, size = 240, normalized size = 1.79

$$\frac{15\sqrt{2}\left(\cos(dx+c)^3 - 2\cos(dx+c)\sin(dx+c) - 2\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos\cdots}{\cos(dx+c)^2-\cdots}\right.}{128\left(a^2d\cos(dx+c)^3 - 2a^2d\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(15*sqrt(2)*(cos(d*x + c)^3 - 2*cos(d*x + c)*sin(d*x + c) - 2*cos(d*x
 + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*
sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x +
 c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x
 + c) - cos(d*x + c) - 2)) + 4*(15*cos(d*x + c)^2 - 20*sin(d*x + c) - 12)*s
qrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d
*x + c) - 2*a^2*d*cos(d*x + c))
```

giac [B] time = 2.65, size = 591, normalized size = 4.41

$$\frac{15\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{16\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}\right.}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(15*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d
*x + 1/2*c) + 1)) + 16*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a) + sqrt(a))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))*sqrt(a) - a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(41*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7 + 127*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(
a) + 91*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^5*a - 143*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^4*a^(3/2) + 3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^3*a^2 + 93*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^2*a^(5/2) - 47*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))*a^3 + 11*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^4*a*sgn(tan(1/2*d*x + 1/2*c
) + 1)))/d
```

maple [A] time = 0.25, size = 202, normalized size = 1.51

$$-\frac{\sin(dx+c)\left(30\sqrt{a-a\sin(dx+c)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}-40a^{\frac{5}{2}}\right)+\left(-15\sqrt{a-a\sin(dx+c)}\,\sqrt{2}\,\mathrm{a}\right.}{64a^{\frac{7}{2}}\left(1+\sin(dx+c)\right)\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/64/a^(7/2)*(sin(d*x+c)*(30*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a
-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-40*a^(5/2))+(-15*(a-a*sin(d*x+c))
^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2+30*a
^(5/2))*cos(d*x+c)^2+30*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-24*a^(5/2))/(1+sin(d*x+c))/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(dx+c)^{2}}{\left(a\sin(dx+c)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}{\left(c + dx \right)}}{\left(a \left(\sin{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.178 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d}-\frac{7}{16ad\sqrt{a\sin(c+dx)+a}}-\frac{7}{24d(a\sin(c+dx)+a)^{3/2}}+\frac{7\sec^2(c+dx)}{20ad\sqrt{a\sin(c+dx)+a}}-\frac{\text{se}}{5d(a\,\text{sir}}$$

```
[Out] -7/24/d/(a+a*sin(d*x+c))^(3/2)-1/5*sec(d*x+c)^2/d/(a+a*sin(d*x+c))^(3/2)+7/
32*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)-7/
16/a/d/(a+a*sin(d*x+c))^(1/2)+7/20*sec(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2681, 2687, 2667, 51, 63, 206}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d}-\frac{7}{16ad\sqrt{a\sin(c+dx)+a}}-\frac{7}{24d(a\sin(c+dx)+a)^{3/2}}+\frac{7\sec^2(c+dx)}{20ad\sqrt{a\sin(c+dx)+a}}-\frac{\text{se}}{5d(a\,\text{sir}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (7*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*a^(3/2)
*d) - 7/(24*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^2/(5*d*(a + a*Sin[
c + d*x])^(3/2)) - 7/(16*a*d*Sqrt[a + a*Sin[c + d*x]]) + (7*Sec[c + d*x]^2)
/(20*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{10a}$$

$$= -\frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\sec^2(c+dx)}{20ad\sqrt{a+a\sin(c+dx)}} + \frac{7}{8}\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3}}$$

$$= -\frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\sec^2(c+dx)}{20ad\sqrt{a+a\sin(c+dx)}} + \frac{(7a)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/}}\right.}{8d}$$

$$= -\frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\sec^2(c+dx)}{20ad\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} - \frac{7}{16ad\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} - \frac{7}{16ad\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d} - \frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.08, size = 42, normalized size = 0.28

$$-\frac{a\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{10d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/10*(a*Hypergeometric2F1[-5/2, 2, -3/2, (1 + Sin[c + d*x])/2])/(d*(a + a*
Sin[c + d*x])^(5/2))
```

fricas [A] time = 0.81, size = 187, normalized size = 1.25

$$\frac{105\sqrt{2}\left(\cos(dx+c)^4 - 2\cos(dx+c)^2\sin(dx+c) - 2\cos(dx+c)^2\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}}{\sin(dx+c)-1}\right.}{960\left(a^2d\cos(dx+c)^4 - 2a^2d\cos(dx+c)^2\sin(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/960*(105*sqrt(2)*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c) - 2*cos(
d*x + c)^2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(175*cos(d*x + c)^2 + 21*(5*cos(d
*x + c)^2 - 4)*sin(d*x + c) - 36)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x
+ c)^4 - 2*a^2*d*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos((d*t_nostep+c)/2-pi/4))]Discontinuities at zeroe
s of cos((d*t_nostep+c)/2-pi/4) were not checkedWarning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep+1)]Evaluation time: 0.56Not invertible Error: Bad Ar
gument Value
```

maple [A] time = 0.27, size = 124, normalized size = 0.83

$$\frac{2a^{3}\left(-\frac{3}{16a^{4}\sqrt{a+a\sin(dx+c)}}-\frac{1}{12a^{3}(a+a\sin(dx+c))^{\frac{3}{2}}}-\frac{1}{20a^{2}(a+a\sin(dx+c))^{\frac{5}{2}}}-\frac{\frac{\sqrt{a+a\sin(dx+c)}}{2a\sin(dx+c)-2a}-\frac{7\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4\sqrt{a}}}{16a^{4}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2*a^3*(-3/16/a^4/(a+a*sin(d*x+c))^(1/2)-1/12/a^3/(a+a*sin(d*x+c))^(3/2)-1/2
0/a^2/(a+a*sin(d*x+c))^(5/2)-1/16/a^4*(1/2*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*
x+c)-a)-7/4*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1
/2))))/d
```

maxima [A] time = 0.75, size = 146, normalized size = 0.97

$$-\frac{\frac{105\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\left(105\,(a\sin(dx+c)+a)^{3}-140\,(a\sin(dx+c)+a)^{2}a-56\,(a\sin(dx+c)+a)a^{2}-48\,a^{3}\right)}{(a\sin(dx+c)+a)^{\frac{7}{2}}-2\,(a\sin(dx+c)+a)^{\frac{5}{2}}a}}{960\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/960*(105*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt
(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a)))/sqrt(a) + 4*(105*(a*sin(d*x + c) +
 a)^3 - 140*(a*sin(d*x + c) + a)^2*a - 56*(a*sin(d*x + c) + a)*a^2 - 48*a^3
)/((a*sin(d*x + c) + a)^(7/2) - 2*(a*sin(d*x + c) + a)^(5/2)*a))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}{\left(c + dx \right)}}{\left(a \left(\sin{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**3/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.179 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{105\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{256\sqrt{2}a^{3/2}d}-\frac{105\cos(c+dx)}{256d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^3(c+dx)}{6d(a\sin(c+dx)+a)^{3/2}}+$$

```
[Out] -105/256*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-7/32*sec(d*x+c)/d/(a+a*sin(d*x
+c))^(3/2)-1/6*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-105/512*arctanh(1/2*co
s(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+35/64*se
c(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)+1/4*sec(d*x+c)^3/a/d/(a+a*sin(d*x+c))^(
1/2)
```

Rubi [A] time = 0.29, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2681, 2687, 2650, 2649, 206}

$$-\frac{105\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{256\sqrt{2}a^{3/2}d}-\frac{105\cos(c+dx)}{256d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^3(c+dx)}{6d(a\sin(c+dx)+a)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-105*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
256*Sqrt[2]*a^(3/2)*d) - (105*Cos[c + d*x])/(256*d*(a + a*Sin[c + d*x])^(3/
2)) - (7*Sec[c + d*x])/(32*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^3/(
6*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x])/(64*a*d*Sqrt[a + a*Sin[
c + d*x]]) + Sec[c + d*x]^3/(4*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{3\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^3(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} + \frac{7}{8}\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}$$

$$= -\frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^3(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{35\sec(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{105\cos(c+dx)}{256d(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}}$$

$$= -\frac{105\cos(c+dx)}{256d(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}}$$

$$= -\frac{105\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} - \frac{105\cos(c+dx)}{256d(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+d}{32d(a+a\sin(c-}$$

Mathematica [C] time = 0.35, size = 334, normalized size = 1.71

$$\frac{\frac{192\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{32\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - 123\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2 + 246\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-68 + (64*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 32/(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (136*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 246*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]) - 123*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (315 + 315*I)*
(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^3 + (32*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (192*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(768*d*(a*(1 + Sin[
c + d*x]))^(3/2))
```

fricas [A] time = 0.79, size = 270, normalized size = 1.38

$$\frac{315\sqrt{2}\left(\cos(dx+c)^5 - 2\cos(dx+c)^3\sin(dx+c) - 2\cos(dx+c)^3\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}}{\cos(dx+}\right.}{3072\left(a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(315*sqrt(2)*(cos(d*x + c)^5 - 2*cos(d*x + c)^3*sin(d*x + c) - 2*cos
(d*x + c)^3)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos
(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*s
in(d*x + c) - cos(d*x + c) - 2)) + 4*(315*cos(d*x + c)^4 - 252*cos(d*x + c)
^2 - 12*(35*cos(d*x + c)^2 + 16)*sin(d*x + c) - 64)*sqrt(a*sin(d*x + c) + a
))/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)^3*sin(d*x + c) - 2*a^2*d*co
s(d*x + c)^3)
```

giac [B] time = 9.62, size = 914, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/768*(315*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2
*d*x + 1/2*c) + 1)) + 64*(9*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^5 - 21*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^4*sqrt(a) - 14*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^3*a + 42*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) + 33*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 + 7*a^(5/2))/(((sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^3*a*sgn(tan(
1/2*d*x + 1/2*c) + 1)) + 2*(933*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^11 + 5847*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(a) + 13605*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^9*a + 5595*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*a^(3/2) - 17214*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7*a^2 - 8474*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(5/2) + 20
250*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a
^3 - 2250*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^4*a^(7/2) - 8695*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^3*a^4 + 6195*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2*a^(9/2) - 1743*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))*a^5 + 223*a^(11/2))/(((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^6*a*sgn(tan(1/2*d*x
 + 1/2*c) + 1)))/d
```

maple [A] time = 0.31, size = 289, normalized size = 1.48

$$\frac{\left(-840a^{\frac{9}{2}}-315\left(a-a\sin\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{3}\right)\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)+\left(-384a^{\frac{9}{2}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/1536/a^(11/2)*((-840*a^(9/2)-315*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)*cos(d*x+c)^2+(-3
84*a^(9/2)+1260*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))
```

```
^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)+630*a^(9/2)*cos(d*x+c)^4+(-504*a^(9
/2)-945*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2
^(1/2)/a^(1/2))*a^3)*cos(d*x+c)^2-128*a^(9/2)+1260*(a-a*sin(d*x+c))^(3/2)*2
^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(sin(d*x+c)
-1)/(1+sin(d*x+c))^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}{\left(c + dx \right)}}{\left(a \left(\sin{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**4/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.180 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=211

$$\frac{99\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d}-\frac{99}{256ad\sqrt{a\sin(c+dx)+a}}-\frac{33}{128d(a\sin(c+dx)+a)^{3/2}}+\frac{11\sec^4(c+dx)}{56ad\sqrt{a\sin(c+dx)+a}}-\frac{}{7d(\ldots}$$

```
[Out] -33/128/d/(a+a*sin(d*x+c))^(3/2)-99/560*sec(d*x+c)^2/d/(a+a*sin(d*x+c))^(3/
2)-1/7*sec(d*x+c)^4/d/(a+a*sin(d*x+c))^(3/2)+99/512*arctanh(1/2*(a+a*sin(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)-99/256/a/d/(a+a*sin(d*x+c))^
(1/2)+99/320*sec(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)+11/56*sec(d*x+c)^4/a/d
/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2681, 2687, 2667, 51, 63, 206}

$$\frac{99\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d}-\frac{99}{256ad\sqrt{a\sin(c+dx)+a}}-\frac{33}{128d(a\sin(c+dx)+a)^{3/2}}+\frac{11\sec^4(c+dx)}{56ad\sqrt{a\sin(c+dx)+a}}-\frac{}{7d(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (99*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(256*Sqrt[2]*a^(3/
2)*d) - 33/(128*d*(a + a*Sin[c + d*x])^(3/2)) - (99*Sec[c + d*x]^2)/(560*d*
(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^4/(7*d*(a + a*Sin[c + d*x])^(3/2
)) - 99/(256*a*d*Sqrt[a + a*Sin[c + d*x]]) + (99*Sec[c + d*x]^2)/(320*a*d*S
qrt[a + a*Sin[c + d*x]]) + (11*Sec[c + d*x]^4)/(56*a*d*Sqrt[a + a*Sin[c + d
*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{11\int \frac{\sec^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{14a}$$

$$= -\frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^4(c+dx)}{56ad\sqrt{a+a\sin(c+dx)}} + \frac{99}{112}\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx}$$

$$= -\frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^4(c+dx)}{56ad\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{99\sec^2(c+dx)}{320ad\sqrt{a+a\sin(c+}}$$

$$= -\frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{99\sec^2(c+dx)}{320ad\sqrt{a+a\sin(c+}}$$

$$= -\frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx)}$$

$$= -\frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx)}$$

$$= -\frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx)}$$

$$= \frac{99\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d} - \frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx}$$

Mathematica [C] time = 0.09, size = 44, normalized size = 0.21

$$-\frac{a^2\,{}_2F_1\left(-\frac{7}{2},3;-\frac{5}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{28d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/28*(a^2*Hypergeometric2F1[-7/2, 3, -5/2, (1 + Sin[c + d*x])/2])/(d*(a +
a*Sin[c + d*x])^(7/2))
```

fricas [A] time = 0.60, size = 207, normalized size = 0.98

$$\frac{3465\,\sqrt{2}\left(\cos\left(dx+c\right)^{6}-2\,\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)^{4}\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\,\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,\sqrt{a}+3}{\sin(dx+c)-1}\right.}{35840\left(a^{2}d\cos\left(dx+c\right)^{6}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35840*(3465*sqrt(2)*(cos(d*x + c)^6 - 2*cos(d*x + c)^4*sin(d*x + c) - 2*c
os(d*x + c)^4)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(5775*cos(d*x + c)^4 - 1188*co
s(d*x + c)^2 + 11*(315*cos(d*x + c)^4 - 252*cos(d*x + c)^2 - 160)*sin(d*x +
 c) - 480)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^6 - 2*a^2*d*cos(d*
x + c)^4*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^4)
```

giac [B] time = 7.41, size = 1076, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8960*(3465*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a) - sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(
1/2*d*x + 1/2*c) + 1)) + 70*(77*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^7 - 283*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) + 199*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a + 299*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 15*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 - 177*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 107*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 - 23*a^(7/2
))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2
- 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqr
t(a) - a)^4*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 32*(735*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^13 + 5985*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*sqrt(a) + 18830*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^11*a + 16730
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*a^(
3/2) - 32403*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^9*a^2 - 61397*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^8*a^(5/2) + 28244*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^7*a^3 + 69692*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(7/2) - 40663*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a^4 - 32697*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(9/2) + 41342*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^5 - 17654*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(11/
2) + 3563*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))*a^6 - 307*a^(13/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))*sqrt(a) - a)^7*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.36, size = 152, normalized size = 0.72

$$-\frac{2a^5\left(\frac{5}{32a^6\sqrt{a+a\sin(dx+c)}}+\frac{1}{16a^5(a+a\sin(dx+c))^{\frac{3}{2}}}+\frac{3}{80a^4(a+a\sin(dx+c))^{\frac{5}{2}}}+\frac{1}{56a^3(a+a\sin(dx+c))^{\frac{7}{2}}}+\frac{\frac{\sqrt{a+a\sin(dx+c)}\,a(19\sin(dx+c)-23)}{16(a\sin(dx+c)-a)^2}-\cdots}{32\cdots}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2*a^5*(5/32/a^6/(a+a*sin(d*x+c))^(1/2)+1/16/a^5/(a+a*sin(d*x+c))^(3/2)+3/8
0/a^4/(a+a*sin(d*x+c))^(5/2)+1/56/a^3/(a+a*sin(d*x+c))^(7/2)+1/32/a^6*(1/16
*(a+a*sin(d*x+c))^(1/2)*a*(19*sin(d*x+c)-23)/(a*sin(d*x+c)-a)^2-99/32*2^(1/
2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))))/d
```

maxima [A] time = 0.46, size = 197, normalized size = 0.93

$$-\frac{\frac{3465\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\left(3465\,(a\sin(dx+c)+a)^5-11550\,(a\sin(dx+c)+a)^4a+7392\,(a\sin(dx+c)+a)^3a^2+2112\,(a\sin(dx+c)+a)^2a\cdots\right.}{(a\sin(dx+c)+a)^{\frac{11}{2}}-4\,(a\sin(dx+c)+a)^{\frac{9}{2}}a+4\,(a\sin(dx+c)+a)^{\frac{7}{2}}a^2}}{35840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/35840*(3465*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(s
qrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a)))/sqrt(a) + 4*(3465*(a*sin(d*x +
c) + a)^5 - 11550*(a*sin(d*x + c) + a)^4*a + 7392*(a*sin(d*x + c) + a)^3*a^
2 + 2112*(a*sin(d*x + c) + a)^2*a^3 + 1408*(a*sin(d*x + c) + a)*a^4 + 1280*
a^5)/((a*sin(d*x + c) + a)^(11/2) - 4*(a*sin(d*x + c) + a)^(9/2)*a + 4*(a*s
in(d*x + c) + a)^(7/2)*a^2))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.181 $\int \frac{\sec^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=256

$$-\frac{3003\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8192\sqrt{2}a^{3/2}d}-\frac{3003\cos(c+dx)}{8192d(a\sin(c+dx)+a)^{3/2}}+\frac{13\sec^5(c+dx)}{80ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^5(c+dx)}{8d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -3003/8192*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-1001/5120*sec(d*x+c)/d/(a+a*
sin(d*x+c))^(3/2)-143/960*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-1/8*sec(d*x
+c)^5/d/(a+a*sin(d*x+c))^(3/2)-3003/16384*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^
(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+1001/2048*sec(d*x+c)/a/d/(a
+a*sin(d*x+c))^(1/2)+143/640*sec(d*x+c)^3/a/d/(a+a*sin(d*x+c))^(1/2)+13/80*
sec(d*x+c)^5/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2681, 2687, 2650, 2649, 206}

$$-\frac{3003\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8192\sqrt{2}a^{3/2}d}-\frac{3003\cos(c+dx)}{8192d(a\sin(c+dx)+a)^{3/2}}+\frac{13\sec^5(c+dx)}{80ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^5(c+dx)}{8d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3003*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/
(8192*Sqrt[2]*a^(3/2)*d) - (3003*Cos[c + d*x])/(8192*d*(a + a*Sin[c + d*x])
^(3/2)) - (1001*Sec[c + d*x])/(5120*d*(a + a*Sin[c + d*x])^(3/2)) - (143*Se
c[c + d*x]^3)/(960*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^5/(8*d*(a +
 a*Sin[c + d*x])^(3/2)) + (1001*Sec[c + d*x])/(2048*a*d*Sqrt[a + a*Sin[c +
d*x]]) + (143*Sec[c + d*x]^3)/(640*a*d*Sqrt[a + a*Sin[c + d*x]]) + (13*Sec[
c + d*x]^5)/(80*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
```

```
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{13\int \frac{\sec^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{16a}$$

$$= -\frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{13\sec^5(c+dx)}{80ad\sqrt{a+a\sin(c+dx)}} + \frac{143}{160}\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx \ldots}$$

$$= -\frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{13\sec^5(c+dx)}{80ad\sqrt{a+a\sin(c+dx \ldots}}$$

$$= -\frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{143\sec^3(c+dx)}{640ad\sqrt{a+a\sin(c+ \ldots}}$$

$$= -\frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx \ldots}$$

$$= -\frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx \ldots}$$

$$= -\frac{3003\cos(c+dx)}{8192d(a+a\sin(c+dx))^{3/2}} - \frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(c+ \ldots}{960d(a+a\sin(c \ldots}$$

$$= -\frac{3003\cos(c+dx)}{8192d(a+a\sin(c+dx))^{3/2}} - \frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(c+ \ldots}{960d(a+a\sin(c \ldots}$$

$$= -\frac{3003\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{8192\sqrt{2}a^{3/2}d} - \frac{3003\cos(c+dx)}{8192d(a+a\sin(c+dx))^{3/2}} - \frac{1001\sec \ldots}{5120d(a+a\sin \ldots}$$

Mathematica [C] time = 1.51, size = 444, normalized size = 1.73

$$\frac{\frac{28800\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{6400\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{1536\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5} - 16245\left(\sin\left(\frac{1}{2}(c \ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8860 + (3840*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 -
1920/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (9920*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 4960/(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2 + (17720*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) +
32490*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 16245*(Cos[(
```

```
c + d*x)/2] + Sin[(c + d*x)/2])^2 + (45045 + 45045*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3 + (1536*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^5 + (6400*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (28800*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(122880*d*(a*(1 + Sin[
c + d*x]))^(3/2))
```

fricas [A] time = 0.64, size = 290, normalized size = 1.13

$$\frac{45045 \sqrt{2} \left(\cos(dx+c)^7 - 2\cos(dx+c)^5 \sin(dx+c) - 2\cos(dx+c)^5\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}}{\cos(dx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/491520*(45045*sqrt(2)*(cos(d*x + c)^7 - 2*cos(d*x + c)^5*sin(d*x + c) - 2
*cos(d*x + c)^5)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x
+ c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a
*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) +
2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(45045*cos(d*x + c)^6 - 36036*cos(
d*x + c)^4 - 9152*cos(d*x + c)^2 - 156*(385*cos(d*x + c)^4 + 176*cos(d*x +
c)^2 + 128)*sin(d*x + c) - 4608)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x +
 c)^7 - 2*a^2*d*cos(d*x + c)^5*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos((d*t_nostep+c)/2-pi/4))]Discontinuities at zeroe
s of cos((d*t_nostep+c)/2-pi/4) were not checkedWarning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep+1)]Evaluation time: 0.71Not invertible Error: Bad Ar
gument Value
```

maple [A] time = 0.29, size = 367, normalized size = 1.43

$$-\frac{-120120 a^{\frac{13}{2}} \sin(dx+c)\left(\cos^4(dx+c)\right) + \left(-54912 a^{\frac{13}{2}} - 180180\left(a - a\sin(dx+c)\right)^{\frac{5}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{2\sqrt{a}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/245760/a^(15/2)*(-120120*a^(13/2)*sin(d*x+c)*cos(d*x+c)^4+(-54912*a^(13/
2)-180180*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))*a^4)*cos(d*x+c)^2*sin(d*x+c)+(-39936*a^(13/2)+360360*(a-a
*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))*a^4)*sin(d*x+c)+90090*a^(13/2)*cos(d*x+c)^6+9009*(-8*a^(13/2)+5*(a-a*si
n(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))
```

```
*a^4)*cos(d*x+c)^4+(-18304*a^(13/2)-360360*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(d*x+c)^2-9216*a
^(13/2)+360360*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^
(1/2)*2^(1/2)/a^(1/2))*a^4)/(sin(d*x+c)-1)^2/(1+sin(d*x+c))^3/cos(d*x+c)/(a
+a*sin(d*x+c))^(1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^6\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^6*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**6/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.182 $\int \frac{\cos^{10}(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{64a^3\cos^{11}(c+dx)}{2145d(a\sin(c+dx)+a)^{11/2}}-\frac{16a^2\cos^{11}(c+dx)}{195d(a\sin(c+dx)+a)^{9/2}}-\frac{2a\cos^{11}(c+dx)}{15d(a\sin(c+dx)+a)^{7/2}}$$

```
[Out] -64/2145*a^3*cos(d*x+c)^11/d/(a+a*sin(d*x+c))^(11/2)-16/195*a^2*cos(d*x+c)^
11/d/(a+a*sin(d*x+c))^(9/2)-2/15*a*cos(d*x+c)^11/d/(a+a*sin(d*x+c))^(7/2)
```

Rubi [A] time = 0.19, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2674, 2673}

$$-\frac{16a^2\cos^{11}(c+dx)}{195d(a\sin(c+dx)+a)^{9/2}}-\frac{64a^3\cos^{11}(c+dx)}{2145d(a\sin(c+dx)+a)^{11/2}}-\frac{2a\cos^{11}(c+dx)}{15d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^10/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^11)/(2145*d*(a + a*Sin[c + d*x])^(11/2)) - (16*a^2*Co
s[c + d*x]^11)/(195*d*(a + a*Sin[c + d*x])^(9/2)) - (2*a*Cos[c + d*x]^11)/(
15*d*(a + a*Sin[c + d*x])^(7/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{10}(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= -\frac{2a\cos^{11}(c+dx)}{15d(a+a\sin(c+dx))^{7/2}}+\frac{1}{15}(8a)\int \frac{\cos^{10}(c+dx)}{(a+a\sin(c+dx))^{7/2}}\,dx\\ &= -\frac{16a^2\cos^{11}(c+dx)}{195d(a+a\sin(c+dx))^{9/2}}-\frac{2a\cos^{11}(c+dx)}{15d(a+a\sin(c+dx))^{7/2}}+\frac{1}{195}\left(32a^2\right)\int \frac{\cos^{10}(}{(a+a\sin}\\ &= -\frac{64a^3\cos^{11}(c+dx)}{2145d(a+a\sin(c+dx))^{11/2}}-\frac{16a^2\cos^{11}(c+dx)}{195d(a+a\sin(c+dx))^{9/2}}-\frac{2a\cos^{11}(c+dx)}{15d(a+a\sin(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.77, size = 59, normalized size = 0.62

$$-\frac{2\left(143\sin^2(c+dx)+374\sin(c+dx)+263\right)\cos^{11}(c+dx)}{2145d(\sin(c+dx)+1)^3(a(\sin(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^10/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^11*(263 + 374*Sin[c + d*x] + 143*Sin[c + d*x]^2))/(2145*d*
(1 + Sin[c + d*x])^3*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [B] time = 0.77, size = 201, normalized size = 2.12

$$-\frac{2\left(143\cos\left(dx+c\right)^{8}-341\cos\left(dx+c\right)^{7}-736\cos\left(dx+c\right)^{6}+28\cos\left(dx+c\right)^{5}-40\cos\left(dx+c\right)^{4}+64\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/2145*(143*cos(d*x + c)^8 - 341*cos(d*x + c)^7 - 736*cos(d*x + c)^6 + 28*
cos(d*x + c)^5 - 40*cos(d*x + c)^4 + 64*cos(d*x + c)^3 - 128*cos(d*x + c)^2
 + (143*cos(d*x + c)^7 + 484*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 280*cos(
d*x + c)^4 - 320*cos(d*x + c)^3 - 384*cos(d*x + c)^2 - 512*cos(d*x + c) - 1
024)*sin(d*x + c) + 512*cos(d*x + c) + 1024)*sqrt(a*sin(d*x + c) + a)/(a^3*
d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

giac [B] time = 2.36, size = 526, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/2145*(1024*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(5/2) - (263*a^5/sgn(t
an(1/2*d*x + 1/2*c) + 1) - (2145*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (7335*
a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (13585*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1
) - (15795*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (17589*a^5/sgn(tan(1/2*d*x +
 1/2*c) + 1) - (29315*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (45045*a^5/sgn(ta
n(1/2*d*x + 1/2*c) + 1) - (45045*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (29315
*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (17589*a^5/sgn(tan(1/2*d*x + 1/2*c) +
1) - (15795*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) - (13585*a^5/sgn(tan(1/2*d*x
+ 1/2*c) + 1) - (7335*a^5/sgn(tan(1/2*d*x + 1/2*c) + 1) + (263*a^5*tan(1/2*
d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 2145*a^5/sgn(tan(1/2*d*x + 1/2
*c) + 1))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))
*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*
d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2
*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(
1/2*d*x + 1/2*c))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(15/2))/d
```

maple [A] time = 0.20, size = 67, normalized size = 0.71

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\left(\sin\left(dx+c\right)-1\right)^{6}\left(143\left(\sin^{2}\left(dx+c\right)\right)+374\sin\left(dx+c\right)+263\right)}{2145a^{2}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/2145/a^2*(1+sin(d*x+c))*(sin(d*x+c)-1)^6*(143*sin(d*x+c)^2+374*sin(d*x+c
)+263)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{10}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^10/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{10}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^10/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^10/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**10/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.183 $\int \frac{\cos^9(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^9d}-\frac{16(a\sin(c+dx)+a)^{11/2}}{11a^8d}+\frac{16(a\sin(c+dx)+a)^{9/2}}{3a^7d}-\frac{64(a\sin(c+dx)+a)^{7/2}}{7a^6d}+\frac{32(a\sin}{}$$

```
[Out] 32/5*(a+a*sin(d*x+c))^(5/2)/a^5/d-64/7*(a+a*sin(d*x+c))^(7/2)/a^6/d+16/3*(a
+a*sin(d*x+c))^(9/2)/a^7/d-16/11*(a+a*sin(d*x+c))^(11/2)/a^8/d+2/13*(a+a*si
n(d*x+c))^(13/2)/a^9/d
```

Rubi [A] time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^9d}-\frac{16(a\sin(c+dx)+a)^{11/2}}{11a^8d}+\frac{16(a\sin(c+dx)+a)^{9/2}}{3a^7d}-\frac{64(a\sin(c+dx)+a)^{7/2}}{7a^6d}+\frac{32(a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (32*(a + a*Sin[c + d*x])^(5/2))/(5*a^5*d) - (64*(a + a*Sin[c + d*x])^(7/2))
/(7*a^6*d) + (16*(a + a*Sin[c + d*x])^(9/2))/(3*a^7*d) - (16*(a + a*Sin[c +
 d*x])^(11/2))/(11*a^8*d) + (2*(a + a*Sin[c + d*x])^(13/2))/(13*a^9*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^9(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int (a-x)^4(a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{a^9d}\\ &= \frac{\text{Subst}\left(\int \left(16a^4(a+x)^{3/2}-32a^3(a+x)^{5/2}+24a^2(a+x)^{7/2}-8a(a+x)^{9/2}+(a+\right.\right.}{a^9d}\\ &= \frac{32(a+a\sin(c+dx))^{5/2}}{5a^5d}-\frac{64(a+a\sin(c+dx))^{7/2}}{7a^6d}+\frac{16(a+a\sin(c+dx))^{9/2}}{3a^7d}-\end{aligned}$$

Mathematica [A] time = 0.29, size = 64, normalized size = 0.53

$$\frac{2\left(1155\sin^4(c+dx)-6300\sin^3(c+dx)+14210\sin^2(c+dx)-16700\sin(c+dx)+9683\right)(a(\sin(c+dx)+1)}{15015a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(5/2)*(9683 - 16700*Sin[c + d*x] + 14210*Sin[c + d*x]^2 - 6300*Sin[c + d*x]^3 + 1155*Sin[c + d*x]^4))/(15015*a^5*d)
```

fricas [A] time = 0.53, size = 82, normalized size = 0.68

$$-\frac{2\left(1155\cos\left(dx+c\right)^{6}-6230\cos\left(dx+c\right)^{4}-512\cos\left(dx+c\right)^{2}+2\left(1995\cos\left(dx+c\right)^{4}-1280\cos\left(dx+c\right)^{2}-2\right.}{15015\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15015*(1155*cos(d*x + c)^6 - 6230*cos(d*x + c)^4 - 512*cos(d*x + c)^2 + 2*(1995*cos(d*x + c)^4 - 1280*cos(d*x + c)^2 - 2048)*sin(d*x + c) - 4096)*sqrt(a*sin(d*x + c) + a)/(a^3*d)
```

giac [B] time = 2.95, size = 430, normalized size = 3.55

$$2\left(\frac{9683\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{15015\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{25402\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{90090\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\frac{107393\,a^{4}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\left(\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/15015*(9683*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (15015*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (25402*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (90090*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (107393*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (93093*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (183612*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (183612*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (93093*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (107393*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (90090*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (25402*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1) + (9683*a^4*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) + 15015*a^4/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(13/2)*d)
```

maple [A] time = 0.17, size = 67, normalized size = 0.55

$$\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{5}{2}}\left(1155\left(\cos^{4}\left(dx+c\right)\right)+6300\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-16520\left(\cos^{2}\left(dx+c\right)\right)-23000\,\mathrm{s}\right.}{15015a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15015/a^5*(a+a*sin(d*x+c))^(5/2)*(1155*cos(d*x+c)^4+6300*cos(d*x+c)^2*sin(d*x+c)-16520*cos(d*x+c)^2-23000*sin(d*x+c)+25048)/d
```

maxima [A] time = 0.67, size = 89, normalized size = 0.74

$$\frac{2\left(1155\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}-10920\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}a+40040\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a^{2}-68640\left(a\sin\left(dx+\right.\right.}{15015\,a^{9}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*(a*sin(d*x + c) + a)^(13/2) - 10920*(a*sin(d*x + c) + a)^(11/
2)*a + 40040*(a*sin(d*x + c) + a)^(9/2)*a^2 - 68640*(a*sin(d*x + c) + a)^(7
/2)*a^3 + 48048*(a*sin(d*x + c) + a)^(5/2)*a^4)/(a^9*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^9}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^9/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^9/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.184 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^{5/2}} \, dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^9(c+dx)}{99d(a\sin(c+dx)+a)^{9/2}}-\frac{2a\cos^9(c+dx)}{11d(a\sin(c+dx)+a)^{7/2}}$$

```
[Out] -8/99*a^2*cos(d*x+c)^9/d/(a+a*sin(d*x+c))^(9/2)-2/11*a*cos(d*x+c)^9/d/(a+a*
sin(d*x+c))^(7/2)
```

Rubi [A] time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2674, 2673}

$$-\frac{8a^2\cos^9(c+dx)}{99d(a\sin(c+dx)+a)^{9/2}}-\frac{2a\cos^9(c+dx)}{11d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^9)/(99*d*(a + a*Sin[c + d*x])^(9/2)) - (2*a*Cos[c + d*
x]^9)/(11*d*(a + a*Sin[c + d*x])^(7/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= -\frac{2a\cos^9(c+dx)}{11d(a+a\sin(c+dx))^{7/2}}+\frac{1}{11}(4a)\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^{7/2}}\,dx\\ &= -\frac{8a^2\cos^9(c+dx)}{99d(a+a\sin(c+dx))^{9/2}}-\frac{2a\cos^9(c+dx)}{11d(a+a\sin(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.35, size = 49, normalized size = 0.78

$$-\frac{2(9\sin(c+dx)+13)\cos^9(c+dx)}{99d(\sin(c+dx)+1)^2(a(\sin(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^(5/2),x]
```

[Out] (-2*Cos[c + d*x]^9*(13 + 9*Sin[c + d*x]))/(99*d*(1 + Sin[c + d*x])^2*(a*(1 + Sin[c + d*x]))^(5/2))

fricas [B] time = 0.76, size = 161, normalized size = 2.56

$$-\frac{2\left(9\cos(dx+c)^{6}-23\cos(dx+c)^{5}-52\cos(dx+c)^{4}+4\cos(dx+c)^{3}-8\cos(dx+c)^{2}+\left(9\cos(dx+c\right.\right.}{99\left(a^{3}d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -2/99*(9*cos(d*x + c)^6 - 23*cos(d*x + c)^5 - 52*cos(d*x + c)^4 + 4*cos(d*x + c)^3 - 8*cos(d*x + c)^2 + (9*cos(d*x + c)^5 + 32*cos(d*x + c)^4 - 20*cos(d*x + c)^3 - 24*cos(d*x + c)^2 - 32*cos(d*x + c) - 64)*sin(d*x + c) + 32*cos(d*x + c) + 64)*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)

giac [B] time = 3.16, size = 402, normalized size = 6.38

$$2\left(\frac{64\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}}-\frac{\frac{13a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{99a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{319a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{561a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\left(\frac{594a^{3}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}\right.\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] 2/99*(64*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(5/2) - (13*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (99*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (319*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (561*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (594*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (462*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (462*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (594*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (561*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) - (319*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1) + (13*a^3*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 99*a^3/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(11/2))/d

maple [A] time = 0.21, size = 57, normalized size = 0.90

$$\frac{2\left(1+\sin(dx+c)\right)\left(\sin(dx+c)-1\right)^{5}\left(9\sin(dx+c)+13\right)}{99a^{2}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x)

[Out] 2/99/a^2*(1+sin(d*x+c))*(sin(d*x+c)-1)^5*(9*sin(d*x+c)+13)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(dx+c)^{8}}{\left(a\sin(dx+c)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^8/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+dx)^8}{(a+a\sin(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + a*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^8/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.185 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^7d}+\frac{12(a\sin(c+dx)+a)^{7/2}}{7a^6d}-\frac{24(a\sin(c+dx)+a)^{5/2}}{5a^5d}+\frac{16(a\sin(c+dx)+a)^{3/2}}{3a^4d}$$

```
[Out] 16/3*(a+a*sin(d*x+c))^(3/2)/a^4/d-24/5*(a+a*sin(d*x+c))^(5/2)/a^5/d+12/7*(a
+a*sin(d*x+c))^(7/2)/a^6/d-2/9*(a+a*sin(d*x+c))^(9/2)/a^7/d
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^7d}+\frac{12(a\sin(c+dx)+a)^{7/2}}{7a^6d}-\frac{24(a\sin(c+dx)+a)^{5/2}}{5a^5d}+\frac{16(a\sin(c+dx)+a)^{3/2}}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(3/2))/(3*a^4*d) - (24*(a + a*Sin[c + d*x])^(5/2))
/(5*a^5*d) + (12*(a + a*Sin[c + d*x])^(7/2))/(7*a^6*d) - (2*(a + a*Sin[c +
d*x])^(9/2))/(9*a^7*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx=\frac{\text{Subst}\left(\int (a-x)^3\sqrt{a+x}\,dx,x,a\sin(c+dx)\right)}{a^7d}$$
$$=\frac{\text{Subst}\left(\int \left(8a^3\sqrt{a+x}-12a^2(a+x)^{3/2}+6a(a+x)^{5/2}-(a+x)^{7/2}\right)dx,x,a\sin(c\right.}{a^7d}$$
$$=\frac{16(a+a\sin(c+dx))^{3/2}}{3a^4d}-\frac{24(a+a\sin(c+dx))^{5/2}}{5a^5d}+\frac{12(a+a\sin(c+dx))^{7/2}}{7a^6d}-$$

Mathematica [A] time = 0.19, size = 54, normalized size = 0.56

$$-\frac{2\left(35\sin^3(c+dx)-165\sin^2(c+dx)+321\sin(c+dx)-319\right)(a(\sin(c+dx)+1))^{3/2}}{315a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a*(1 + Sin[c + d*x]))^(3/2)*(-319 + 321*Sin[c + d*x] - 165*Sin[c + d*x
]^2 + 35*Sin[c + d*x]^3))/(315*a^4*d)
```

fricas [A] time = 0.61, size = 62, normalized size = 0.64

$$-\frac{2\left(35\,\cos\left(dx+c\right)^{4}-226\,\cos\left(dx+c\right)^{2}+2\left(65\,\cos\left(dx+c\right)^{2}-64\right)\sin\left(dx+c\right)-128\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*cos(d*x + c)^4 - 226*cos(d*x + c)^2 + 2*(65*cos(d*x + c)^2 - 64)
*sin(d*x + c) - 128)*sqrt(a*sin(d*x + c) + a)/(a^3*d)
```

giac [B] time = 10.72, size = 310, normalized size = 3.20

$$2\left(\left(\left(\left(\left(\left(\left(\left(\frac{319\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{315\,a^{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{648\,a^{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{}{\mathrm{sgn}(}\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*((((((((319*a^2*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) +
 315*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 648*a^2/sgn(
tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 1680*a^2/sgn(tan(1/2*d*x
+ 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 1134*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1
))*tan(1/2*d*x + 1/2*c) + 1134*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d
*x + 1/2*c) + 1680*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c)
+ 648*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 315*a^2/sgn
(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 319*a^2/sgn(tan(1/2*d*x
+ 1/2*c) + 1))/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2)*d)
```

maple [A] time = 0.17, size = 57, normalized size = 0.59

$$\frac{2\left(a+a\sin\left(dx+c\right)\right)^{\frac{3}{2}}\left(35\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-165\left(\cos^{2}\left(dx+c\right)\right)-356\sin\left(dx+c\right)+484\right)}{315a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/315/a^4*(a+a*sin(d*x+c))^(3/2)*(35*cos(d*x+c)^2*sin(d*x+c)-165*cos(d*x+c)
^2-356*sin(d*x+c)+484)/d
```

maxima [A] time = 1.31, size = 72, normalized size = 0.74

$$-\frac{2\left(35\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-270\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a+756\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a^{2}-840\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a\right.}{315\,a^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*(a*sin(d*x + c) + a)^(9/2) - 270*(a*sin(d*x + c) + a)^(7/2)*a +
756*(a*sin(d*x + c) + a)^(5/2)*a^2 - 840*(a*sin(d*x + c) + a)^(3/2)*a^3)/(a
^7*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^7}{(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.186 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{2a\cos^7(c+dx)}{7d(a\sin(c+dx)+a)^{7/2}}$$

```
[Out] -2/7*a*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)
```

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$-\frac{2a\cos^7(c+dx)}{7d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*Cos[c + d*x]^7)/(7*d*(a + a*Sin[c + d*x])^(7/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2a\cos^7(c+dx)}{7d(a+a\sin(c+dx))^{7/2}}$$

Mathematica [A] time = 0.11, size = 42, normalized size = 1.40

$$-\frac{2\cos^7(c+dx)\sqrt{a(\sin(c+dx)+1)}}{7a^3d(\sin(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7*Sqrt[a*(1 + Sin[c + d*x])])/(7*a^3*d*(1 + Sin[c + d*x])^
4)
```

fricas [B] time = 0.64, size = 117, normalized size = 3.90

$$-\frac{2\left(\cos(dx+c)^4 - 3\cos(dx+c)^3 - 8\cos(dx+c)^2 + \left(\cos(dx+c)^3 + 4\cos(dx+c)^2 - 4\cos(dx+c) - 8\right)\sin(\ldots}{7\left(a^3d\cos(dx+c) + a^3d\sin(dx+c) + a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*(cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 8*cos(d*x + c)^2 + (cos(d*x + c)^
3 + 4*cos(d*x + c)^2 - 4*cos(d*x + c) - 8)*sin(d*x + c) + 4*cos(d*x + c) +
8)*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*
d)
```

giac [B] time = 5.64, size = 255, normalized size = 8.50

$$2\left(\frac{8\sqrt{2}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{5}{2}}}+\frac{\left(\left(\left(\left(\left(\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{7\,a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{21\,a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/7*(8*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(5/2) + ((((((a*tan(1/2*d*x
 + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 7*a/sgn(tan(1/2*d*x + 1/2*c) + 1)
)*tan(1/2*d*x + 1/2*c) + 21*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x +
1/2*c) - 35*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 35*a/sg
n(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 21*a/sgn(tan(1/2*d*x +
1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 7*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(
1/2*d*x + 1/2*c) - a/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)
^2 + a)^(7/2))/d
```

maple [A] time = 0.16, size = 47, normalized size = 1.57

$$-\frac{2\,(1+\sin(dx+c))\,(\sin(dx+c)-1)^{4}}{7a^{2}\cos(dx+c)\,\sqrt{a+a\sin(dx+c)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/7/a^2*(1+sin(d*x+c))*(sin(d*x+c)-1)^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(dx+c)^{6}}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\cos(c+d\,x)^{6}}{(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.187 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5a^5d}-\frac{8(a\sin(c+dx)+a)^{3/2}}{3a^4d}+\frac{8\sqrt{a\sin(c+dx)+a}}{a^3d}$$

```
[Out] -8/3*(a+a*sin(d*x+c))^(3/2)/a^4/d+2/5*(a+a*sin(d*x+c))^(5/2)/a^5/d+8*(a+a*s
in(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5a^5d}-\frac{8(a\sin(c+dx)+a)^{3/2}}{3a^4d}+\frac{8\sqrt{a\sin(c+dx)+a}}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*Sqrt[a + a*Sin[c + d*x]])/(a^3*d) - (8*(a + a*Sin[c + d*x])^(3/2))/(3*a^
4*d) + (2*(a + a*Sin[c + d*x])^(5/2))/(5*a^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{4a^2}{\sqrt{a+x}}-4a\sqrt{a+x}+(a+x)^{3/2}\right)dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{8\sqrt{a+a\sin(c+dx)}}{a^3d}-\frac{8(a+a\sin(c+dx))^{3/2}}{3a^4d}+\frac{2(a+a\sin(c+dx))^{5/2}}{5a^5d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 44, normalized size = 0.62

$$\frac{2\left(3\sin^2(c+dx)-14\sin(c+dx)+43\right)\sqrt{a(\sin(c+dx)+1)}}{15a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])]*(43 - 14*Sin[c + d*x] + 3*Sin[c + d*x]^2))/(1
5*a^3*d)
```

fricas [A] time = 0.67, size = 40, normalized size = 0.56

$$-\frac{2\left(3\cos\left(dx+c\right)^{2}+14\sin\left(dx+c\right)-46\right)\sqrt{a\sin\left(dx+c\right)+a}}{15\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*cos(d*x + c)^2 + 14*sin(d*x + c) - 46)*sqrt(a*sin(d*x + c) + a)/(a
^3*d)
```

giac [B] time = 5.24, size = 172, normalized size = 2.42

$$\frac{2\left(\left(\left(\left(\left(\frac{43\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{15}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{70}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{\quad}{\mathrm{sgn}(\mathrm{ta}}\right.\right.\right.}{15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(((((43*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) + 15/sgn(ta
n(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 70/sgn(tan(1/2*d*x + 1/2*c)
 + 1))*tan(1/2*d*x + 1/2*c) + 70/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x
 + 1/2*c) + 15/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 43/sgn
(tan(1/2*d*x + 1/2*c) + 1))/((a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2)*d)
```

maple [A] time = 0.37, size = 41, normalized size = 0.58

$$-\frac{2\sqrt{a+a\sin\left(dx+c\right)}\,\left(3\left(\cos^{2}\left(dx+c\right)\right)+14\sin\left(dx+c\right)-46\right)}{15a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/15/a^3*(a+a*sin(d*x+c))^(1/2)*(3*cos(d*x+c)^2+14*sin(d*x+c)-46)/d
```

maxima [A] time = 0.33, size = 55, normalized size = 0.77

$$\frac{2\left(3\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}-20\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a+60\sqrt{a\sin\left(dx+c\right)+a}\,a^{2}\right)}{15\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(a*sin(d*x + c) + a)^(5/2) - 20*(a*sin(d*x + c) + a)^(3/2)*a + 60*s
qrt(a*sin(d*x + c) + a)*a^2)/(a^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.188 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] 2/3*cos(d*x+c)^3/a/d/(a+a*sin(d*x+c))^(3/2)-4*arctanh(1/2*cos(d*x+c)*a^(1/2
)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+4*cos(d*x+c)/a^2/d/(a+a
*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2679, 2649, 206}

$$\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (2*Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) +
(4*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} + \frac{2\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{a}$$
$$= \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \frac{4\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$
$$= \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{8\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(}{\sqrt{a+a\sin}}\right.}{a^2d}$$
$$= -\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+}}$$

Mathematica [A] time = 0.18, size = 96, normalized size = 0.89

$$-\frac{2\cos(c+dx)\left(\sqrt{1-\sin(c+dx)}\,(\sin(c+dx)-7)+6\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\right)}{3a^2d\sqrt{1-\sin(c+dx)}\,\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]*(6*Sqrt[2]*ArcTanh[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]] + Sqrt[
1 - Sin[c + d*x]]*(-7 + Sin[c + d*x])))/(3*a^2*d*Sqrt[1 - Sin[c + d*x]]*Sqr
t[a*(1 + Sin[c + d*x])])
```

fricas [B] time = 0.68, size = 215, normalized size = 1.99

$$\frac{2\left(\frac{3\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}} - (\cos(dx+\right.}{3\left(a^3d\cos(dx+c)+a^3d\sin(dx+c)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - (cos(d*x +
 c)^2 + (cos(d*x + c) + 8)*sin(d*x + c) - 7*cos(d*x + c) - 8)*sqrt(a*sin(d*
x + c) + a))/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

giac [B] time = 3.12, size = 255, normalized size = 2.36

$$-\frac{2\left(\frac{\left(\left(\frac{7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{9}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{9}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{7}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}+\frac{4\sqrt{2}\left(3a\arctan\right.}{}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*((((7*tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 9/(a*sg
n(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) + 9/(a*sgn(tan(1/2*d*x +
 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) - 7/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/
(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) + 4*sqrt(2)*(3*a*arctan(sqrt(a)/sqrt(-
a)) + 2*sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^3) - 12
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1
/2*c) + 1)))/d
```

maple [A] time = 0.23, size = 112, normalized size = 1.04

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(6a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-\left(a-a\sin\left(dx+c\right)\right)^{\frac{3}{2}}-6a\sqrt{\quad}\right.}{3a^{4}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(6*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(3/2)-6*a*(a-a*s
in(d*x+c))^(1/2))/a^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.189 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2\sqrt{a\sin(c+dx)+a}}{a^3d}-\frac{4}{a^2d\sqrt{a\sin(c+dx)+a}}$$

[Out] -4/a^2/d/(a+a*sin(d*x+c))^(1/2)-2*(a+a*sin(d*x+c))^(1/2)/a^3/d

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2\sqrt{a\sin(c+dx)+a}}{a^3d}-\frac{4}{a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]

[Out] -4/(a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Sqrt[a + a*Sin[c + d*x]])/(a^3*d)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{a-x}{(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{2a}{(a+x)^{3/2}}-\frac{1}{\sqrt{a+x}}\right)dx, x, a\sin(c+dx)\right)}{a^3d}\\ &= -\frac{4}{a^2d\sqrt{a+a\sin(c+dx)}}-\frac{2\sqrt{a+a\sin(c+dx)}}{a^3d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 30, normalized size = 0.67

$$-\frac{2(\sin(c+dx)+3)}{a^2d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]

```
[Out] (-2*(3 + Sin[c + d*x]))/(a^2*d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.83, size = 41, normalized size = 0.91

$$-\frac{2\sqrt{a\sin\left(dx+c\right)+a}\left(\sin\left(dx+c\right)+3\right)}{a^{3}d\sin\left(dx+c\right)+a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -2*sqrt(a*sin(d*x + c) + a)*(sin(d*x + c) + 3)/(a^3*d*sin(d*x + c) + a^3*d)
```

giac [A] time = 1.45, size = 39, normalized size = 0.87

$$-\frac{2\left(\frac{\sqrt{a\sin(dx+c)+a}}{a^{3}}+\frac{2}{\sqrt{a\sin(dx+c)+a}\,a^{2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] -2*(sqrt(a*sin(d*x + c) + a)/a^3 + 2/(sqrt(a*sin(d*x + c) + a)*a^2))/d
```

maple [A] time = 0.13, size = 29, normalized size = 0.64

$$-\frac{2\left(3+\sin\left(dx+c\right)\right)}{a^{2}\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
[Out] -2/a^2/(a+a*sin(d*x+c))^(1/2)*(3+sin(d*x+c))/d
```

maxima [A] time = 1.16, size = 42, normalized size = 0.93

$$-\frac{2\left(\frac{\sqrt{a\sin(dx+c)+a}}{a^{2}}+\frac{2}{\sqrt{a\sin(dx+c)+a}\,a}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] -2*(sqrt(a*sin(d*x + c) + a)/a^2 + 2/(sqrt(a*sin(d*x + c) + a)*a))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cos\left(c+d\,x\right)}^3}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [A] time = 26.90, size = 267, normalized size = 5.93

$$\begin{cases} \text{NaN} \\ \frac{x\cos^{3}(c)}{(a\sin(c)+a)^{\frac{5}{2}}} \\ -\frac{8\sqrt{a\sin(c+dx)+a}\,\sin^{2}(c+dx)}{3a^{3}d\sin^{2}(c+dx)+6a^{3}d\sin(c+dx)+3a^{3}d}-\frac{24\sqrt{a\sin(c+dx)+a}\,\sin(c+dx)}{3a^{3}d\sin^{2}(c+dx)+6a^{3}d\sin(c+dx)+3a^{3}d}-\frac{2\sqrt{a\sin(c+dx)+a}\,\cos^{2}(c+dx)}{3a^{3}d\sin^{2}(c+dx)+6a^{3}d\sin(c+dx)+3a^{3}d}-\frac{1}{3a^{3}d\sin^{2}(} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((nan, (Eq(d, 0) | Eq(c, -d*x + 3*pi/2)) & (Eq(c, 3*pi/2) | Eq(c,
-d*x + 3*pi/2))), (x*cos(c)**3/(a*sin(c) + a)**(5/2), Eq(d, 0)), (-8*sqrt(a
*sin(c + d*x) + a)*sin(c + d*x)**2/(3*a**3*d*sin(c + d*x)**2 + 6*a**3*d*sin
(c + d*x) + 3*a**3*d) - 24*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)/(3*a**3*d*
sin(c + d*x)**2 + 6*a**3*d*sin(c + d*x) + 3*a**3*d) - 2*sqrt(a*sin(c + d*x)
 + a)*cos(c + d*x)**2/(3*a**3*d*sin(c + d*x)**2 + 6*a**3*d*sin(c + d*x) + 3
*a**3*d) - 16*sqrt(a*sin(c + d*x) + a)/(3*a**3*d*sin(c + d*x)**2 + 6*a**3*d
*sin(c + d*x) + 3*a**3*d), True))
```

3.190 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{2}\,a^{5/2}d}-\frac{\cos(c+dx)}{ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(3/2)+1/2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2
^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2680, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{2}\,a^{5/2}d}-\frac{\cos(c+dx)}{ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]/(Sqrt[2]
*a^(5/2)*d) - Cos[c + d*x]/(a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= -\frac{\cos(c+dx)}{ad(a+a\sin(c+dx))^{3/2}}-\frac{\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}\\ &= -\frac{\cos(c+dx)}{ad(a+a\sin(c+dx))^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^2d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}-\frac{\cos(c+dx)}{ad(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.25, size = 100, normalized size = 1.33

$$\frac{\sec(c+dx)\left(2(\sin(c+dx)-1)+\sqrt{2-2\sin(c+dx)}\tanh^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\right)}{2a^2d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]*(ArcTanh[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2*Sqrt[2 - 2*Sin[c + d*x]] + 2*(-1 + Sin[c + d*x])))/(2*a
^2*d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [B] time = 0.65, size = 252, normalized size = 3.36

$$\frac{\frac{\sqrt{2}\left(a\cos(dx+c)^2-a\cos(dx+c)-(a\cos(dx+c)+2a)\sin(dx+c)-2a\right)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos\cdots}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}}{4\left(a^3d\cos(dx+c)^2-a^3d\cos(dx+c)-2a^3d-\left(a^3d\cos(dx+c)+2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a*cos(d*x + c)^2 - a*cos(d*x + c) - (a*cos(d*x + c) + 2*a)*si
n(d*x + c) - 2*a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) +
2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a
) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) -
 cos(d*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - si
n(d*x + c) + 1))/(a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^
3*d*cos(d*x + c) + 2*a^3*d)*sin(d*x + c))
```

giac [B] time = 1.63, size = 293, normalized size = 3.91

$$-\frac{\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{2\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^3+\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\cdots}\right.\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\cdots\right.\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x +
1/2*c) + 1)) - 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.21, size = 123, normalized size = 1.64

$$-\frac{\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a\sin(dx+c)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a+2\sqrt{a-a\sin(dx+c)}\,\sqrt{a}\right)}{2a^{\frac{7}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/2/a^(7/2)*(-2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*
a*sin(d*x+c)-2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+
2*(a-a*sin(d*x+c))^(1/2)*a^(1/2))*(-a*(sin(d*x+c)-1))^(1/2)/cos(d*x+c)/(a+a
*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.191 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=24

$$-\frac{2}{3ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -2/3/a/d/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2667, 32}

$$-\frac{2}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= -\frac{2}{3ad(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 1.00

$$-\frac{2}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*a*d*(a + a*Sin[c + d*x])^(3/2))
```

fricas [B] time = 0.77, size = 48, normalized size = 2.00

$$\frac{2\sqrt{a\sin(dx+c)+a}}{3\left(a^3d\cos(dx+c)^2-2\,a^3d\sin(dx+c)-2\,a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) -
 2*a^3*d)
```

giac [A] time = 1.43, size = 20, normalized size = 0.83

$$-\frac{2}{3\,(a\sin\left(dx+c\right)+a)^{\frac{3}{2}}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((a*sin(d*x + c) + a)^(3/2)*a*d)
```

maple [A] time = 0.02, size = 21, normalized size = 0.88

$$-\frac{2}{3ad\,(a+a\sin\left(dx+c\right))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3/a/d/(a+a*sin(d*x+c))^(3/2)
```

maxima [A] time = 0.61, size = 20, normalized size = 0.83

$$-\frac{2}{3\,(a\sin\left(dx+c\right)+a)^{\frac{3}{2}}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((a*sin(d*x + c) + a)^(3/2)*a*d)
```

mupad [B] time = 7.52, size = 72, normalized size = 3.00

$$\frac{8\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,\sqrt{a+a\,\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}{3\,a^3\,d\,{\left(-1+\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] (8*exp(c*2i + d*x*2i)*(a + a*((exp(- c*1i - d*x*1i)*1i)/2 - (exp(c*1i + d*x
*1i)*1i)/2))^(1/2))/(3*a^3*d*(exp(c*1i + d*x*1i)*1i - 1)^4)
```

sympy [A] time = 25.47, size = 65, normalized size = 2.71

$$\begin{cases} -\frac{2}{3a^2d\sqrt{a\sin\left(c+dx\right)+a}\sin\left(c+dx\right)+3a^2d\sqrt{a\sin\left(c+dx\right)+a}} & \text{for } d\neq 0 \\ \frac{x\cos\left(c\right)}{(a\sin\left(c\right)+a)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((-2/(3*a**2*d*sqrt(a*sin(c + d*x) + a)*sin(c + d*x) + 3*a**2*d*sq
rt(a*sin(c + d*x) + a)), Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)**(5/2), True))
```

3.192 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}-\frac{1}{4a^2d\sqrt{a\sin(c+dx)+a}}-\frac{1}{6ad(a\sin(c+dx)+a)^{3/2}}-\frac{1}{5d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] -1/5/d/(a+a*sin(d*x+c))^(5/2)-1/6/a/d/(a+a*sin(d*x+c))^(3/2)+1/8*arctanh(1/
2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-1/4/a^2/d/(a+a*
sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2667, 51, 63, 206}

$$-\frac{1}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}-\frac{1}{6ad(a\sin(c+dx)+a)^{3/2}}-\frac{1}{5d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d) -
 1/(5*d*(a + a*Sin[c + d*x])^(5/2)) - 1/(6*a*d*(a + a*Sin[c + d*x])^(3/2))
- 1/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{2d}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x,\right.}{4ad}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} - \frac{1}{4a^2d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} - \frac{1}{4a^2d\sqrt{a+a\sin(c+dx)}} +$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} - \frac{}{4a}$$

Mathematica [C] time = 0.09, size = 41, normalized size = 0.36

$$-\frac{{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{5d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sin[c + d*x])/2]/(d*(a + a*Sin[c
 + d*x])^(5/2))
```

fricas [A] time = 0.74, size = 169, normalized size = 1.50

$$\frac{15\sqrt{2}\left(3\cos(dx+c)^2+\left(\cos(dx+c)^2-4\right)\sin(dx+c)-4\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(15\right.}{240\left(3a^3d\cos(dx+c)^2-4a^3d+\left(a^3d\cos(dx+c)^2-4a^3d\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/240*(15*sqrt(2)*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x + c) - 4
)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 + 3*a)/(sin(d*x + c) - 1)) - 4*(15*cos(d*x + c)^2 - 40*sin(d*x + c) - 52)*
sqrt(a*sin(d*x + c) + a))/(3*a^3*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*
x + c)^2 - 4*a^3*d)*sin(d*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
```

```
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos((d*t_nostep+c)/2-pi/4))]Discontinuities at zeroe
s of cos((d*t_nostep+c)/2-pi/4) were not checkedWarning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep+1)]Evaluation time: 0.46Not invertible Error: Bad Ar
gument Value
```

maple [A] time = 0.17, size = 88, normalized size = 0.78

$$-\frac{2a\left(\frac{1}{8a^3\sqrt{a+a\sin(dx+c)}}+\frac{1}{12a^2(a+a\sin(dx+c))^{\frac{3}{2}}}+\frac{1}{10a(a+a\sin(dx+c))^{\frac{5}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{7}{2}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2*a*(1/8/a^3/(a+a*sin(d*x+c))^(1/2)+1/12/a^2/(a+a*sin(d*x+c))^(3/2)+1/10/a
/(a+a*sin(d*x+c))^(5/2)-1/16/a^(7/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(
1/2)*2^(1/2)/a^(1/2)))/d
```

maxima [A] time = 1.43, size = 114, normalized size = 1.01

$$-\frac{\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)}{a^{\frac{3}{2}}}+\frac{4\left(15\,(a\sin(dx+c)+a)^2+10\,(a\sin(dx+c)+a)a+12\,a^2\right)}{(a\sin(dx+c)+a)^{\frac{5}{2}}\,a}}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/240*(15*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt(
2)*sqrt(a) + sqrt(a*sin(d*x + c) + a)))/a^(3/2) + 4*(15*(a*sin(d*x + c) + a
)^2 + 10*(a*sin(d*x + c) + a)*a + 12*a^2)/((a*sin(d*x + c) + a)^(5/2)*a))/(
a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.193 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{2}a^{5/2}d}+\frac{35\sec(c+dx)}{96a^2d\sqrt{a\sin(c+dx)+a}}-\frac{35\cos(c+dx)}{128ad(a\sin(c+dx)+a)^{3/2}}-\frac{7\sec(c+dx)}{48ad(a\sin(c+dx)+a)^{3/}}$$

```
[Out] -1/6*sec(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)-35/128*cos(d*x+c)/a/d/(a+a*sin(d*x
+c))^(3/2)-7/48*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^(3/2)-35/256*arctanh(1/2*co
s(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+35/96*se
c(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2681, 2687, 2650, 2649, 206}

$$\frac{35\sec(c+dx)}{96a^2d\sqrt{a\sin(c+dx)+a}}-\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{2}a^{5/2}d}-\frac{35\cos(c+dx)}{128ad(a\sin(c+dx)+a)^{3/2}}-\frac{7\sec(c+dx)}{48ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-35*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
28*Sqrt[2]*a^(5/2)*d) - Sec[c + d*x]/(6*d*(a + a*Sin[c + d*x])^(5/2)) - (35
*Cos[c + d*x])/(128*a*d*(a + a*Sin[c + d*x])^(3/2)) - (7*Sec[c + d*x])/(48*
a*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x])/(96*a^2*d*Sqrt[a + a*Si
n[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}}+\frac{7\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{12a}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}}-\frac{7\sec(c+dx)}{48ad(a+a\sin(c+dx))^{3/2}}+\frac{35\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{96a^2}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}}-\frac{7\sec(c+dx)}{48ad(a+a\sin(c+dx))^{3/2}}+\frac{35\sec(c+dx)}{96a^2d\sqrt{a+a\sin(c+d}}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}}-\frac{35\cos(c+dx)}{128ad(a+a\sin(c+dx))^{3/2}}-\frac{7\sec(c+dx)}{48ad(a+a\sin(c+dx}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}}-\frac{35\cos(c+dx)}{128ad(a+a\sin(c+dx))^{3/2}}-\frac{7\sec(c+dx)}{48ad(a+a\sin(c+dx}$$

$$= -\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d}-\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}}-\frac{35\cos(c+dx}{128ad(a+a\sin(c+}$$

Mathematica [C] time = 0.46, size = 284, normalized size = 1.70

$$\frac{\frac{48\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-57\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4+114\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32 + (64*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 88*Sin
[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 44*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^2 + 114*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3 - 57*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (105 + 105*I)*(-1)
^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^5 + (48*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(384*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [A] time = 0.71, size = 280, normalized size = 1.68

$$\frac{105\sqrt{2}\left(3\cos(dx+c)^3+\left(\cos(dx+c)^3-4\cos(dx+c)\right)\sin(dx+c)-4\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2}{}\right.}{1536\left(3a^3d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(105*sqrt(2)*(3*cos(d*x + c)^3 + (cos(d*x + c)^3 - 4*cos(d*x + c))*s
in(d*x + c) - 4*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sq
```

```
rt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(
d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (co
s(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(245*cos(d*x + c)^2 +
 7*(15*cos(d*x + c)^2 - 32)*sin(d*x + c) - 160)*sqrt(a*sin(d*x + c) + a))/(
3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a
^3*d*cos(d*x + c))*sin(d*x + c))
```

giac [B] time = 5.63, size = 751, normalized size = 4.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(105*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1
/2*d*x + 1/2*c) + 1)) + 96*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a) + sqrt(a))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2
*(615*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^1
1 + 3501*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^10*sqrt(a) + 7911*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^9*a + 2841*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^8*a^(3/2) - 10122*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^7*a^2 - 5054*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(5/2) + 12222*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a^3 - 846*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(7/2) - 5389*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^4 + 3681*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(9/2)
 - 981*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*
a^5 + 133*a^(11/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))*sqrt(a) - a)^6*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.24, size = 266, normalized size = 1.59

$$-\frac{\left(210a^{\frac{7}{2}}-105\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{3}\right)\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)+\left(-448a^{\frac{7}{2}}+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/768/a^(11/2)*((210*a^(7/2)-105*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)*cos(d*x+c)^2+(-44
8*a^(7/2)+420*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(
1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)+(490*a^(7/2)-315*(a-a*sin(d*x+c))^(1/
2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*cos(d*x
+c)^2-320*a^(7/2)+420*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d
*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(1+sin(d*x+c))^2/cos(d*x+c)/(a+a*sin(d*x
+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{2}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.194 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d}-\frac{9}{32a^2d\sqrt{a\sin(c+dx)+a}}+\frac{9\sec^2(c+dx)}{40a^2d\sqrt{a\sin(c+dx)+a}}-\frac{3}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{70ad}{}$$

```
[Out] -1/7*sec(d*x+c)^2/d/(a+a*sin(d*x+c))^(5/2)-3/16/a/d/(a+a*sin(d*x+c))^(3/2)-
9/70*sec(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(3/2)+9/64*arctanh(1/2*(a+a*sin(d*x+
c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-9/32/a^2/d/(a+a*sin(d*x+c))^(1
/2)+9/40*sec(d*x+c)^2/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2681, 2687, 2667, 51, 63, 206}

$$-\frac{9}{32a^2d\sqrt{a\sin(c+dx)+a}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d}+\frac{9\sec^2(c+dx)}{40a^2d\sqrt{a\sin(c+dx)+a}}-\frac{3}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{70}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (9*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*a^(5/2)
*d) - Sec[c + d*x]^2/(7*d*(a + a*Sin[c + d*x])^(5/2)) - 3/(16*a*d*(a + a*Si
n[c + d*x])^(3/2)) - (9*Sec[c + d*x]^2)/(70*a*d*(a + a*Sin[c + d*x])^(3/2))
 - 9/(32*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (9*Sec[c + d*x]^2)/(40*a^2*d*Sqr
t[a + a*Sin[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{9\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{14a}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} + \frac{9\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{20a^2}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} + \frac{9\sec^2(c+dx)}{40a^2d\sqrt{a+a\sin(c+d}}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} + \frac{9\sec^2(c+dx)}{40a^2d\sqrt{a+a\sin(c+d}}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx)}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx)}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx)}$$

$$= \frac{9\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} - \frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.09, size = 42, normalized size = 0.23

$$-\frac{a\,{}_2F_1\left(-\frac{7}{2},2;-\frac{5}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{14d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

[Out] -1/14*(a*Hypergeometric2F1[-7/2, 2, -5/2, (1 + Sin[c + d*x])/2])/(d*(a + a*
Sin[c + d*x])^(7/2))

fricas [A] time = 0.57, size = 225, normalized size = 1.22

$$\frac{315\sqrt{2}\left(3\cos\left(dx+c\right)^{4}-4\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-4\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{\cdots}}{\sin\cdots}\right.}{4480\left(3a^{3}d\cos\left(dx+c\right)^{4}-4a^{3}d\cos\left(dx\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(315*sqrt(2)*(3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4
- 4*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(315*cos(d*
x + c)^4 - 1092*cos(d*x + c)^2 - 120*(7*cos(d*x + c)^2 - 3)*sin(d*x + c) +
200)*sqrt(a*sin(d*x + c) + a))/(3*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x +
c)^2 + (a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [B] time = 11.92, size = 916, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/1120*(315*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a) - sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan
(1/2*d*x + 1/2*c) + 1)) + 70*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^3 - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))*a - a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a^2*sgn(tan(1/2*d*x + 1/2*
c) + 1)) + 8*(455*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^13 + 3395*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^12*sqrt(a) + 10290*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^11*a + 8750*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^10*a^(3/2) - 16807*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^9*a^2 - 31423*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*a^(5/2) + 14076*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7*a^3 + 33908*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(7/
2) - 19607*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^5*a^4 - 15883*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^4*a^(9/2) + 19698*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^3*a^5 - 8386*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^2*a^(11/2) + 1687*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^6 - 153*a^(13/2))/(((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^7*a^2*sgn
(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.30, size = 141, normalized size = 0.76

$$\frac{2a^{3}\left(-\frac{1}{8a^{5}\sqrt{a+a\sin(dx+c)}}-\frac{1}{16a^{4}(a+a\sin(dx+c))^{\frac{3}{2}}}-\frac{1}{20a^{3}(a+a\sin(dx+c))^{\frac{5}{2}}}-\frac{1}{28a^{2}(a+a\sin(dx+c))^{\frac{7}{2}}}-\frac{\frac{\sqrt{a+a\sin(dx+c)}}{4a\sin(dx+c)-4a}-\frac{9\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(\cdots}}{2\sqrt{\cdots}}\right.}{8\sqrt{a}}}{16a^{5}}\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2*a^3*(-1/8/a^5/(a+a*sin(d*x+c))^(1/2)-1/16/a^4/(a+a*sin(d*x+c))^(3/2)-1/20
/a^3/(a+a*sin(d*x+c))^(5/2)-1/28/a^2/(a+a*sin(d*x+c))^(7/2)-1/16/a^5*(1/4*(
a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)-9/8*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a
*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))))/d
```

maxima [A] time = 1.23, size = 167, normalized size = 0.90

$$-\frac{\frac{4\left(315\,(a\sin(dx+c)+a)^4-420\,(a\sin(dx+c)+a)^3a-168\,(a\sin(dx+c)+a)^2a^2-144\,(a\sin(dx+c)+a)a^3-160\,a^4\right)}{(a\sin(dx+c)+a)^{\frac{9}{2}}a-2\,(a\sin(dx+c)+a)^{\frac{7}{2}}a^2}+\frac{315\,\sqrt{2}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)}{a^{\frac{3}{2}}}}{4480\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4480*(4*(315*(a*sin(d*x + c) + a)^4 - 420*(a*sin(d*x + c) + a)^3*a - 168
*(a*sin(d*x + c) + a)^2*a^2 - 144*(a*sin(d*x + c) + a)*a^3 - 160*a^4)/((a*s
in(d*x + c) + a)^(9/2)*a - 2*(a*sin(d*x + c) + a)^(7/2)*a^2) + 315*sqrt(2)*
log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(a
*sin(d*x + c) + a)))/a^(3/2))/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.195 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{1155\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4096\sqrt{2}a^{5/2}d}+\frac{11\sec^3(c+dx)}{64a^2d\sqrt{a\sin(c+dx)+a}}+\frac{385\sec(c+dx)}{1024a^2d\sqrt{a\sin(c+dx)+a}}-\frac{1155\cos(c+dx)}{4096ad(a\sin(c+dx)+\cdots}$$

```
[Out] -1/8*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^(5/2)-1155/4096*cos(d*x+c)/a/d/(a+a*si
n(d*x+c))^(3/2)-77/512*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^(3/2)-11/96*sec(d*x+
c)^3/a/d/(a+a*sin(d*x+c))^(3/2)-1155/8192*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^
(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+385/1024*sec(d*x+c)/a^2/d/(
a+a*sin(d*x+c))^(1/2)+11/64*sec(d*x+c)^3/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2681, 2687, 2650, 2649, 206}

$$\frac{11\sec^3(c+dx)}{64a^2d\sqrt{a\sin(c+dx)+a}}+\frac{385\sec(c+dx)}{1024a^2d\sqrt{a\sin(c+dx)+a}}-\frac{1155\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4096\sqrt{2}a^{5/2}d}-\frac{1155\cos(c+dx)}{4096ad(a\sin(c+dx)+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-1155*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/
(4096*Sqrt[2]*a^(5/2)*d) - Sec[c + d*x]^3/(8*d*(a + a*Sin[c + d*x])^(5/2))
- (1155*Cos[c + d*x])/(4096*a*d*(a + a*Sin[c + d*x])^(3/2)) - (77*Sec[c + d
*x])/(512*a*d*(a + a*Sin[c + d*x])^(3/2)) - (11*Sec[c + d*x]^3)/(96*a*d*(a
+ a*Sin[c + d*x])^(3/2)) + (385*Sec[c + d*x])/(1024*a^2*d*Sqrt[a + a*Sin[c
+ d*x]]) + (11*Sec[c + d*x]^3)/(64*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
```

```
IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} + \frac{11\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{16a}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{11\sec^3(c+dx)}{96ad(a+a\sin(c+dx))^{3/2}} + \frac{33\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{64a^2}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{11\sec^3(c+dx)}{96ad(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^3(c+dx)}{64a^2d\sqrt{a+a\sin(c+d}}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{77\sec(c+dx)}{512ad(a+a\sin(c+dx))^{3/2}} - \frac{11\sec^3(c+dx)}{96ad(a+a\sin(c+dx}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{77\sec(c+dx)}{512ad(a+a\sin(c+dx))^{3/2}} - \frac{11\sec^3(c+dx)}{96ad(a+a\sin(c+dx}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{1155\cos(c+dx)}{4096ad(a+a\sin(c+dx))^{3/2}} - \frac{77\sec(c+dx}{512ad(a+a\sin(c+}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{1155\cos(c+dx)}{4096ad(a+a\sin(c+dx))^{3/2}} - \frac{77\sec(c+dx}{512ad(a+a\sin(c+}$$

$$= -\frac{1155\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d} - \frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{1155\cos(c+}{4096ad(a+a\sin(}$$

Mathematica [C] time = 0.55, size = 394, normalized size = 1.69

$$\frac{\frac{1920\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{256\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - 1545\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4 + 309}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-736 + (768*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 38
4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (1472*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]) + 2072*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]) - 1036*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 3090*Sin
[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 1545*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^4 + (3465 + 3465*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*
(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5
 + (256*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(Cos[(c + d*x)/2] - Sin[(c
```

```
 + d*x)/2])^3 + (1920*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]))/(12288*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [A] time = 0.75, size = 308, normalized size = 1.32

$$\frac{3465\sqrt{2}\left(3\cos\left(dx+c\right)^{5}-4\cos\left(dx+c\right)^{3}+\left(\cos\left(dx+c\right)^{5}-4\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2}{}\right.}{49}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/49152*(3465*sqrt(2)*(3*cos(d*x + c)^5 - 4*cos(d*x + c)^3 + (cos(d*x + c)^
5 - 4*cos(d*x + c)^3)*sin(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt
(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*
a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2
 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(8085*cos(d*x +
 c)^4 - 5280*cos(d*x + c)^2 + 11*(315*cos(d*x + c)^4 - 672*cos(d*x + c)^2 -
 256)*sin(d*x + c) - 1280)*sqrt(a*sin(d*x + c) + a))/(3*a^3*d*cos(d*x + c)^
5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3
)*sin(d*x + c))
```

giac [B] time = 9.34, size = 1074, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/12288*(3465*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(ta
n(1/2*d*x + 1/2*c) + 1)) + 256*(21*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^5 - 51*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(a) - 34*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a + 102*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) + 81*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 + 17*a^(5/2))/(((sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 2*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^3*a
^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(18423*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^15 + 165753*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^14*sqrt(a) + 644313*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^13*a + 1072899*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^12*a^(3/2)
+ 94635*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^11*a^2 - 1907635*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^10*a^(5/2) - 875803*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^9*a^3 + 2261311*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*a^(7/2) + 723029*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7*a^4 - 2030229*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(9/2) + 509147*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a^5 + 6
88777*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4
*a^(11/2) - 599223*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^3*a^6 + 219151*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*a^(13/2) - 40793*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^7 + 3701*a^(15/2))/(((sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^8*a^2*sgn(tan(1
/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.29, size = 355, normalized size = 1.52

$$6930a^{\frac{11}{2}}\sin(dx+c)\left(\cos^4(dx+c)\right)-924\left(16a^{\frac{11}{2}}+15(a-a\sin(dx+c))^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)a^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/24576/a^(15/2)*(6930*a^(11/2)*sin(d*x+c)*cos(d*x+c)^4-924*(16*a^(11/2)+15
*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/
a^(1/2))*a^4)*cos(d*x+c)^2*sin(d*x+c)+(-5632*a^(11/2)+27720*(a-a*sin(d*x+c)
)^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4)*si
n(d*x+c)+(16170*a^(11/2)+3465*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a
-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(d*x+c)^4-1320*(8*a^(11/2)+21
*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/
a^(1/2))*a^4)*cos(d*x+c)^2-2560*a^(11/2)+27720*(a-a*sin(d*x+c))^(3/2)*2^(1/
2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4)/(sin(d*x+c)-1)/
(1+sin(d*x+c))^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^4\,\left(a+a\,\sin(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\left(a\left(\sin(c+dx)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.196 $\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=124

$$\frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{10ae^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{2a(e\cos(c+dx))^{9/2}}{9de}+\frac{2ae\sin(c+dx)(e\,\mathrm{co}}{7d}$$

```
[Out] -2/9*a*(e*cos(d*x+c))^(9/2)/d/e+2/7*a*e*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/d+1
0/21*a*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+10/21*a*e^3*s
in(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2635, 2642, 2641}

$$\frac{10ae^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}+\frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{9/2}}{9de}+\frac{2ae\sin(c+dx)(e\,\mathrm{co}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(9/2))/(9*d*e) + (10*a*e^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (10*a*e^3*Sqrt[e*Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))\,dx = -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + a\int (e\cos(c+dx))^{7/2}\,dx$$
$$= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{2ae(e\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}\Big(5ae$$
$$= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d} + 2ae(e$$
$$= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d} + 2ae(e$$
$$= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}} + 10a$$

Mathematica [A] time = 0.63, size = 98, normalized size = 0.79

$$-\frac{ae^3\sqrt{e\cos(c+dx)}\left(\sqrt{\cos(c+dx)}(-138\sin(c+dx)-18\sin(3(c+dx))+28\cos(2(c+dx))+7\cos(4(c+dx\right.}{252d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/252*(a*e^3*Sqrt[e*Cos[c + d*x]]*(-120*EllipticF[(c + d*x)/2, 2] + Sqrt[C
os[c + d*x]]*(21 + 28*Cos[2*(c + d*x)] + 7*Cos[4*(c + d*x)] - 138*Sin[c + d
*x] - 18*Sin[3*(c + d*x)])))/(d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ae^3\cos(dx+c)^3\sin(dx+c)+ae^3\cos(dx+c)^3\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*e^3*cos(d*x + c)^3*sin(d*x + c) + a*e^3*cos(d*x + c)^3)*sqrt(e*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a), x)
```

maple [A] time = 0.66, size = 249, normalized size = 2.01

$$-\frac{2a\,e^4\left(-224\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+144\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+560\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-216\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] -2/63/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^4*(-224*si
n(1/2*d*x+1/2*c)^11+144*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+560*sin(1/2
*d*x+1/2*c)^9-216*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-560*sin(1/2*d*x+1
/2*c)^7+168*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+280*sin(1/2*d*x+1/2*c)^
5+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)-48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-70
*sin(1/2*d*x+1/2*c)^3+7*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{7/2}\,(a+a\,\sin(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.197 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{6ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{7/2}}{7de}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{3/2}}{5d}$$

```
[Out] -2/7*a*(e*cos(d*x+c))^(7/2)/d/e+2/5*a*e*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/d+6
/5*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2669, 2635, 2640, 2639}

$$\frac{6ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{7/2}}{7de}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(7/2))/(7*d*e) + (6*a*e^2*Sqrt[e*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*e*(e*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))\,dx = -\frac{2a(e\cos(c+dx))^{7/2}}{7de} + a\int (e\cos(c+dx))^{5/2}\,dx$$
$$= -\frac{2a(e\cos(c+dx))^{7/2}}{7de} + \frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(3ae^2\right)$$
$$= -\frac{2a(e\cos(c+dx))^{7/2}}{7de} + \frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left(3ae^2\sqrt{e\,c\right.}$$
$$= -\frac{2a(e\cos(c+dx))^{7/2}}{7de} + \frac{6ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2ae(e}{}$$

Mathematica [C] time = 2.84, size = 264, normalized size = 2.78

$$\frac{ae^3\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(168(\cos(dx)-i\sin(dx))\sqrt{i\sin(2(c+dx))+\cos(2(c+dx))+1}\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2idx}(\cos(c)+i\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*e^3*Csc[c/2]*Sec[c/2]*(-154*Cos[d*x] - 182*Cos[2*c + d*x] + 14*Cos[2*c +
 3*d*x] - 14*Cos[4*c + 3*d*x] - 30*Sin[c] + 168*Hypergeometric2F1[-1/4, 1/2
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*Sqrt
[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + 56*Hypergeometric2F1[1/2, 3/4
, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*Sqrt
[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + 20*Sin[c + 2*d*x] - 20*Sin[3*
c + 2*d*x] + 5*Sin[3*c + 4*d*x] - 5*Sin[5*c + 4*d*x]))/(560*d*Sqrt[e*Cos[c
+ d*x]])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ae^2\cos(dx+c)^2\sin(dx+c)+ae^2\cos(dx+c)^2\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*cos(d*x + c)^2*sin(d*x + c) + a*e^2*cos(d*x + c)^2)*sqrt(e*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a), x)
```

maple [A] time = 0.64, size = 214, normalized size = 2.25

$$\frac{2a\,e^3\left(-80\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+56\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+160\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-56\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] 2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^3*(-80*sin(
1/2*d*x+1/2*c)^9+56*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+160*sin(1/2*d*x
+1/2*c)^7-56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-120*sin(1/2*d*x+1/2*c)
^5+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+14*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+4
0*sin(1/2*d*x+1/2*c)^3-5*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.198 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae\sin(c+dx)\sqrt{e\cos(c+dx)}}{3d}$$

```
[Out] -2/5*a*(e*cos(d*x+c))^(5/2)/d/e+2/3*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*
cos(d*x+c))^(1/2)+2/3*a*e*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2635, 2642, 2641}

$$\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae\sin(c+dx)\sqrt{e\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(5/2))/(5*d*e) + (2*a*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) + (2*a*e*Sqrt[e*Cos[c + d*x]
]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))\,dx = -\frac{2a(e\cos(c+dx))^{5/2}}{5de} + a\int (e\cos(c+dx))^{3/2}\,dx$$
$$= -\frac{2a(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\left(ae^2\right)$$
$$= -\frac{2a(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{\left(ae^2\sqrt{\cos}\right.}{}$$
$$= -\frac{2a(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}} + \frac{2ae}{}$$

Mathematica [A] time = 0.38, size = 75, normalized size = 0.79

$$-\frac{a(e\cos(c+dx))^{3/2}\left(\sqrt{\cos(c+dx)}\,(-10\sin(c+dx)+3\cos(2(c+dx))+3)-10F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x]),x]
[Out] -1/15*(a*(e*Cos[c + d*x])^(3/2)*(-10*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c
 + d*x]]*(3 + 3*Cos[2*(c + d*x)] - 10*Sin[c + d*x])))/(d*Cos[c + d*x]^(3/2)
)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left((ae\cos(dx+c)\sin(dx+c)+ae\cos(dx+c))\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((a*e*cos(d*x + c)*sin(d*x + c) + a*e*cos(d*x + c))*sqrt(e*cos(d*x
+ c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(a\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a), x)
```

maple [A] time = 0.62, size = 179, normalized size = 1.88

$$-\frac{2a\,e^2\left(-24\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+36\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{Ellipti}\right.}{15\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] -2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^2*(-24*sin
(1/2*d*x+1/2*c)^7+20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+36*sin(1/2*d*x
+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)-18*sin(1/2*d*x+1/2*c)^3+3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}} (a\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{3/2}\,(a+a\,\sin(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.199 $\int \sqrt{e \cos(c+dx)}\,(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=63

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{3/2}}{3de}$$

```
[Out] -2/3*a*(e*cos(d*x+c))^(3/2)/d/e+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2669, 2640, 2639}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{3/2}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(3/2))/(3*d*e) + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))\,dx &= -\frac{2a(e\cos(c+dx))^{3/2}}{3de}+a\int \sqrt{e\cos(c+dx)}\,dx\\ &= -\frac{2a(e\cos(c+dx))^{3/2}}{3de}+\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{2a(e\cos(c+dx))^{3/2}}{3de}+\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 1.06, size = 260, normalized size = 4.13

$$a\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)(\cos(dx)+i\sin(dx))\sqrt{e\cos(c+dx)}\left(6(\cos(dx)-i\sin(dx))\sqrt{i\sin(2(c+dx))+\cos(2(c+dx))}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sqrt[e*Cos[c + d*x]]*Csc[c/2]*Sec[c/2]*(Cos[d*x] + I*Sin[d*x])*(-6*Cos[d
*x] - 6*Cos[2*c + d*x] - 2*Sin[c] + 6*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E
^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*Sqrt[1 + Cos[2
*(c + d*x)] + I*Sin[2*(c + d*x)]] + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^
((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*Sqrt[1 + Cos[2*
(c + d*x)] + I*Sin[2*(c + d*x)]] + Sin[c + 2*d*x] - Sin[3*c + 2*d*x]))/(6*d
*((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c]))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a), x)
```

maple [A] time = 0.60, size = 120, normalized size = 1.90

$$\frac{2ae\left(-4\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}+4\left(\sin^{3}\left(\frac{dx}{2}+\right.\right.\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e*(-4*sin(1/2*
d*x+1/2*c)^5+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+4*sin(1/2*d*x+1/2*c)^3-sin(1/2*d*x+
1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+a\,\sin\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sqrt{e\cos\left(c+dx\right)}\,dx+\int \sqrt{e\cos\left(c+dx\right)}\,\sin\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*cos(c + d*x)), x) + Integral(sqrt(e*cos(c + d*x))*sin(c
+ d*x), x))
```

3.200 $\int \frac{a+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}}{de}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-2*a*(e*cos(d*x+c))^(
1/2)/d/e
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2669, 2642, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*a*Sqrt[e*Cos[c + d*x]])/(d*e) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx&=-\frac{2a\sqrt{e\cos(c+dx)}}{de}+a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx\\&=-\frac{2a\sqrt{e\cos(c+dx)}}{de}+\frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}}\\&=-\frac{2a\sqrt{e\cos(c+dx)}}{de}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 22.32, size = 48, normalized size = 0.79

$$-\frac{2a\left(\cos(c+dx)-\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
[Out] (-2*a*(Cos[c + d*x] - Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]))/(d*Sqr
t[e*Cos[c + d*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)}{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)/(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a\sin(dx+c)+a}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

maple [A] time = 0.38, size = 103, normalized size = 1.69

$$-\frac{2a\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}-2\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x)
[Out] -2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)-2*sin(1/2*d*x+1/2*c)^3+sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a\sin(dx+c)+a}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

mupad [B] time = 0.55, size = 45, normalized size = 0.74

$$-\frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\sqrt{\cos\left(c+d\,x\right)}-\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d\,\sqrt{e\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*a*cos(c + d*x)^(1/2)*(cos(c + d*x)^(1/2) - ellipticF(c/2 + (d*x)/2, 2))
)/(d*(e*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\sqrt{e\cos\left(c+dx\right)}}\,dx+\int \frac{\sin\left(c+dx\right)}{\sqrt{e\cos\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*cos(c + d*x)), x) + Integral(sin(c + d*x)/sqrt(e*cos(c
 + d*x)), x))
```

3.201 $\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*a/d/e/(e*cos(d*x+c))^(1/2)+2*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(1/2)-2*a*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2640, 2639}

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a)/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(d*e*Sqrt[e*Co
s[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{2a}{de\sqrt{e\cos(c+dx)}}+a\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{a\int \sqrt{e\cos(c+dx)}\,dx}{e^2} \\
&= \frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{e^2\sqrt{\cos(c+dx)}} \\
&= \frac{2a}{de\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 0.98, size = 188, normalized size = 2.07

$$
-\frac{a\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(3(\cos(dx)-i\sin(dx))\sqrt{i\sin(2(c+dx))+\cos(2(c+dx))+1}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2idx}(\cos(c)+i\sin\right.\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] -1/6*(a*Csc[c/2]*Sec[c/2]*(-6*(Cos[d*x] + Sin[c]) + 3*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x]
)*Sqrt[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + Hypergeometric2F1[1/2,
3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*S
qrt[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]]))/(d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)}{e^2\cos\left(dx+c\right)^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)/(e^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{a\sin\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

maple [A] time = 0.78, size = 117, normalized size = 1.29

$$
-\frac{2\left(\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sin\left(\right.\right.}{e\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\;\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))*a/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sin(dx+c) + a}{(e \cos(dx+c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a\,\sin(c + d\,x)}{(e\,\cos(c + d\,x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{1}{(e \cos(c+dx))^{\frac{3}{2}}}\, dx + \int \frac{\sin(c+dx)}{(e \cos(c+dx))^{\frac{3}{2}}}\, dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*cos(c + d*x))**(-3/2), x) + Integral(sin(c + d*x)/(e*cos(c +
 d*x))**(3/2), x))
```

3.202 $\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*a/d/e/(e*cos(d*x+c))^(3/2)+2/3*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(3/2)+2/
3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2642, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a)/(3*d*e*(e*Cos[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*d*e*(
e*Cos[c + d*x])^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2a}{3de(e\cos(c+dx))^{3/2}} + a\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2a}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$
$$= \frac{2a}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}} + \frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2\sqrt{e\cos(c+dx)}}$$
$$= \frac{2a}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.46, size = 86, normalized size = 0.89

$$\frac{2a\left(\cos(c+dx)-(\sin(c+dx)-1)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3de^2\sqrt{e\cos(c+dx)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
[Out] (2*a*(Cos[c + d*x] - Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(-1 + Sin
[c + d*x])))/(3*d*e^2*Sqrt[e*Cos[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)}{e^3\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)/(e^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

maple [A] time = 0.88, size = 189, normalized size = 1.95

$$-\frac{2\left(2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\mathrm{I}}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c)
)*a/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sin (dx + c) + a}{(e \cos (dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a\,\sin (c + d\,x)}{(e\,\cos (c + d\,x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.203 $\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}$$

```
[Out] 2/5*a/d/e/(e*cos(d*x+c))^(5/2)+2/5*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(5/2)+6/
5*a*sin(d*x+c)/d/e^3/(e*cos(d*x+c))^(1/2)-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(
1/2)/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2640, 2639}

$$\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*a)/(5*d*e*(e*Cos[c + d*x])^(5/2)) - (6*a*Sqrt[e*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(5*d*e*(
e*Cos[c + d*x])^(5/2)) + (6*a*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2a}{5de(e\cos(c+dx))^{5/2}} + a\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2a}{5de(e\cos(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} + \frac{(3a)\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2}$$
$$= \frac{2a}{5de(e\cos(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} + \frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} - \frac{(3a)\int \sqrt{e\cos(c}{5e^4}$$
$$= \frac{2a}{5de(e\cos(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} + \frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} - \frac{\left(3a\sqrt{e\cos(c+d}{5e^4\sqrt{}}$$
$$= \frac{2a}{5de(e\cos(c+dx))^{5/2}} - \frac{6a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} + \frac{}{5d}$$

Mathematica [C] time = 1.27, size = 144, normalized size = 1.14

$$\frac{2ae^{i(c+dx)}\left(i\sqrt{1+e^{2i(c+dx)}}\left(e^{i(c+dx)}-i\right)^2 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-6e^{i(c+dx)}-3ie^{2i(c+dx)}+i\right)}{5de^3\left(e^{i(c+dx)}-i\right)^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*a*E^(I*(c + d*x))*(I - 6*E^(I*(c + d*x)) - (3*I)*E^((2*I)*(c + d*x)) + I
*(-I + E^(I*(c + d*x)))^2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1
/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(5*d*e^3*(-I + E^(I*(c + d*x)))^2*Sqr
t[e*Cos[c + d*x]])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)}{e^4\cos(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)/(e^4*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

maple [B] time = 1.40, size = 304, normalized size = 2.41

$$-\frac{2\left(12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2
*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-sin(1/2
*d*x+1/2*c))*a/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sin(dx+c) + a}{(e \cos(dx+c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a\,\sin(c + d\,x)}{(e\,\cos(c + d\,x))^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.204 $\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$\frac{130a^2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{130a^2e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}-\frac{2\left(a^2\sin(c+\right.}{}$$

```
[Out] -26/99*a^2*(e*cos(d*x+c))^(9/2)/d/e+26/77*a^2*e*(e*cos(d*x+c))^(5/2)*sin(d*
x+c)/d-2/11*(e*cos(d*x+c))^(9/2)*(a^2+a^2*sin(d*x+c))/d/e+130/231*a^2*e^4*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+130/231*a^2*e^3*sin(d*x+c
)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 2669, 2635, 2642, 2641}

$$\frac{130a^2e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{130a^2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}-\frac{2\left(a^2\sin(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-26*a^2*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (130*a^2*e^4*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (130*a^2*e^3*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (26*a^2*e*(e*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(77*d) - (2*(e*Cos[c + d*x])^(9/2)*(a^2 + a^2*Sin[c + d*x]))/
(11*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2\,dx = -\frac{2(e\cos(c+dx))^{9/2}\left(a^2+a^2\sin(c+dx)\right)}{11de}+\frac{1}{11}(13a)\int (e\cos(c+$$

$$= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}-\frac{2(e\cos(c+dx))^{9/2}\left(a^2+a^2\sin(c+dx)\right)}{11de}$$

$$= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{26a^2e(e\cos(c+dx))^{5/2}\sin(c+dx)}{77d}-$$

$$= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{130a^2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{231d}+$$

$$= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{130a^2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{231d}+$$

$$= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{130a^2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}$$

Mathematica [C] time = 0.11, size = 66, normalized size = 0.39

$$-\frac{32\sqrt[4]{2}\,a^2(e\cos(c+dx))^{9/2}\,{}_2F_1\left(-\frac{13}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-32*2^(1/4)*a^2*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[-13/4, 9/4, 13/4,
 (1 - Sin[c + d*x])/2])/(9*d*e*(1 + Sin[c + d*x])^(9/4))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(a^2e^3\cos(dx+c)^5-2\,a^2e^3\cos(dx+c)^3\sin(dx+c)-2\,a^2e^3\cos(dx+c)^3\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*e^3*cos(d*x + c)^5 - 2*a^2*e^3*cos(d*x + c)^3*sin(d*x + c) -
 2*a^2*e^3*cos(d*x + c)^3)*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}\,(a\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^2, x)
```

maple [A] time = 0.75, size = 295, normalized size = 1.76

$$-\frac{2a^2e^4\left(-4032\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10080\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4928\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-82\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/693/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e^4*(-403
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+10080*sin(1/2*d*x+1/2*c)^10*cos(
1/2*d*x+1/2*c)-4928*sin(1/2*d*x+1/2*c)^11-8208*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8+12320*sin(1/2*d*x+1/2*c)^9+2232*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)-12320*sin(1/2*d*x+1/2*c)^7+924*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+6160*sin(1/2*d*x+1/2*c)^5+195*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-498*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-1540*sin(1/2*d*x+1/2*c)^3+154*sin(1/2*d*x+1/2*
c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}\,(a\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{7/2}\,(a+a\,\sin(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.205 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=137

$$\frac{22a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}-\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{7/2}}{9de}+\frac{22a}{}$$

```
[Out] -22/63*a^2*(e*cos(d*x+c))^(7/2)/d/e+22/45*a^2*e*(e*cos(d*x+c))^(3/2)*sin(d*
x+c)/d-2/9*(e*cos(d*x+c))^(7/2)*(a^2+a^2*sin(d*x+c))/d/e+22/15*a^2*e^2*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 2669, 2635, 2640, 2639}

$$\frac{22a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}-\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{7/2}}{9de}+\frac{22a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-22*a^2*(e*Cos[c + d*x])^(7/2))/(63*d*e) + (22*a^2*e^2*Sqrt[e*Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (22*a^2*e*(e*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(45*d) - (2*(e*Cos[c + d*x])^(7/2)*(a^2 + a^2*
Sin[c + d*x]))/(9*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
```

```
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2\,dx = -\frac{2(e\cos(c+dx))^{7/2}\left(a^2+a^2\sin(c+dx)\right)}{9de}+\frac{1}{9}(11a)\int (e\cos(c+dx)$$

$$= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}-\frac{2(e\cos(c+dx))^{7/2}\left(a^2+a^2\sin(c+dx)\right)}{9de}+$$

$$= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}+\frac{22a^2e(e\cos(c+dx))^{3/2}\sin(c+dx)}{45d}-\frac{2(e}{}$$

$$= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}+\frac{22a^2e(e\cos(c+dx))^{3/2}\sin(c+dx)}{45d}-\frac{2(e}{}$$

$$= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}+\frac{22a^2e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+$$

Mathematica [C] time = 0.12, size = 66, normalized size = 0.48

$$-\frac{16\,2^{3/4}a^2(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{11}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-16*2^(3/4)*a^2*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-11/4, 7/4, 11/4,
 (1 - Sin[c + d*x])/2])/(7*d*e*(1 + Sin[c + d*x])^(7/4))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2e^2\cos(dx+c)^4-2\,a^2e^2\cos(dx+c)^2\sin(dx+c)-2\,a^2e^2\cos(dx+c)^2\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*e^2*cos(d*x + c)^4 - 2*a^2*e^2*cos(d*x + c)^2*sin(d*x + c) -
 2*a^2*e^2*cos(d*x + c)^2)*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^2, x)
```

maple [A] time = 0.86, size = 260, normalized size = 1.90

$$2a^2e^3\left(-1120\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1440\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1064\left(\text{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e^3*(-1120
*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+2240*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8-1440*sin(1/2*d*x+1/2*c)^9-1064*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+2880*sin(1/2*d*x+1/2*c)^7-56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)-2160*sin(1/2*d*x+1/2*c)^5+231*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+84*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+720*sin(1/2*d*x+1/2*c)^3-90*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}} (a\sin(dx+c)+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{5/2}\,(a+a\,\sin(c+d\,x))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.206 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=137

$$\frac{6a^2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{5/2}}{7de}+\frac{6a^2e\sin(c$$

```
[Out] -18/35*a^2*(e*cos(d*x+c))^(5/2)/d/e-2/7*(e*cos(d*x+c))^(5/2)*(a^2+a^2*sin(d
*x+c))/d/e+6/7*a^2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+6
/7*a^2*e*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 2669, 2635, 2642, 2641}

$$\frac{6a^2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{5/2}}{7de}+\frac{6a^2e\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-18*a^2*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (6*a^2*e^2*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2])/(7*d*Sqrt[e*Cos[c + d*x]]) + (6*a^2*e*Sqrt[e*Cos[
c + d*x]]*Sin[c + d*x])/(7*d) - (2*(e*Cos[c + d*x])^(5/2)*(a^2 + a^2*Sin[c
+ d*x]))/(7*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
```

```
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2\,dx = -\frac{2(e\cos(c+dx))^{5/2}\left(a^2+a^2\sin(c+dx)\right)}{7de}+\frac{1}{7}(9a)\int (e\cos(c+d$$

$$= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}-\frac{2(e\cos(c+dx))^{5/2}\left(a^2+a^2\sin(c+dx)\right)}{7de}$$

$$= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}+\frac{6a^2e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7d}-\frac{2(e\,c}{}$$

$$= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}+\frac{6a^2e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7d}-\frac{2(e\,c}{}$$

$$= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}+\frac{6a^2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}+$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.48

$$-\frac{16\sqrt[4]{2}\,a^2(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{9}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-16*2^(1/4)*a^2*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-9/4, 5/4, 9/4, (
1 - Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(5/4))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2e\cos(dx+c)^3-2\,a^2e\cos(dx+c)\sin(dx+c)-2\,a^2e\cos(dx+c)\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*e*cos(d*x + c)^3 - 2*a^2*e*cos(d*x + c)*sin(d*x + c) - 2*a^2
*e*cos(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^2, x)
```

maple [A] time = 0.75, size = 203, normalized size = 1.48

$$-\frac{2a^2e^2\left(-80\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-112\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+168\left(\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e^2*(-80*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)-112*sin(1/2*d*x+1/2*c)^7+168*sin(1/2*d*x+1/2*c)^5+15*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)-20*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-84*sin(1/2*d*x+1/2*c)
^3+14*sin(1/2*d*x+1/2*c))/d
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}} (a\sin(dx+c)+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{3/2}\,(a+a\,\sin(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.207 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=105

$$-\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{3/2}}{5de}+\frac{14a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -14/15*a^2*(e*cos(d*x+c))^(3/2)/d/e-2/5*(e*cos(d*x+c))^(3/2)*(a^2+a^2*sin(d
*x+c))/d/e+14/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2678, 2669, 2640, 2639}

$$-\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{3/2}}{5de}+\frac{14a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-14*a^2*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (14*a^2*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*(e*Cos[c + d*x])^(3/
2)*(a^2 + a^2*Sin[c + d*x]))/(5*d*e)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2\,dx = -\frac{2(e\cos(c+dx))^{3/2}\left(a^2+a^2\sin(c+dx)\right)}{5de}+\frac{1}{5}(7a)\int \sqrt{e\cos(c+dx)}\,($$

$$= -\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}-\frac{2(e\cos(c+dx))^{3/2}\left(a^2+a^2\sin(c+dx)\right)}{5de}+$$

$$= -\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}-\frac{2(e\cos(c+dx))^{3/2}\left(a^2+a^2\sin(c+dx)\right)}{5de}+$$

$$= -\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}+\frac{14a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}-\frac{2(e}{}$$

Mathematica [C] time = 0.05, size = 66, normalized size = 0.63

$$-\frac{8\,2^{3/4}a^2(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-8*2^(3/4)*a^2*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-7/4, 3/4, 7/4, (1
 - Sin[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(3/4))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos\left(dx+c\right)^2-2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^2, x)
```

maple [A] time = 0.79, size = 188, normalized size = 1.79

$$\frac{2a^2e\left(-24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+24\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-40\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\,\text{EllipticE}\left(\text{co}\right.\right.}{15\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e*(-24*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
```

```
*c)-40*sin(1/2*d*x+1/2*c)^5+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+40*sin(1/2*d*x+1/2*c)^3-10*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos(c+d\,x)}\,\left(a+a\,\sin(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*(e*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.208 $\int \frac{(a+a\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)\sqrt{e\cos(c+dx)}}{3de}+\frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}$$

```
[Out] 10/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-10/3*a^2*(e*cos
(d*x+c))^(1/2)/d/e-2/3*(a^2+a^2*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A] time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2678, 2669, 2642, 2641}

$$-\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)\sqrt{e\cos(c+dx)}}{3de}+\frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-10*a^2*Sqrt[e*Cos[c + d*x]])/(3*d*e) + (10*a^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) - (2*Sqrt[e*Cos[c + d*x]]*(a
^2 + a^2*Sin[c + d*x]))/(3*d*e)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c+dx)\right)}{3de} + \frac{1}{3}(5a)\int \frac{a+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{10a^2\sqrt{e\cos(c+dx)}}{3de} - \frac{2\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c+dx)\right)}{3de} + \frac{1}{3}\left(5a^2\right)\int \frac{}{\sqrt{e\,c}}$$

$$= -\frac{10a^2\sqrt{e\cos(c+dx)}}{3de} - \frac{2\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c+dx)\right)}{3de} + \frac{\left(5a^2\sqrt{\cos(c+d}\right.}{3\sqrt{e\,c}}$$

$$= -\frac{10a^2\sqrt{e\cos(c+dx)}}{3de} + \frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}} - \frac{2\sqrt{e\cos(c+dx)}\,(a}{3c}$$

Mathematica [C] time = 0.03, size = 64, normalized size = 0.61

$$-\frac{8\sqrt[4]{2}\,a^2\sqrt{e\cos(c+dx)}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-8*2^(1/4)*a^2*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-5/4, 1/4, 5/4, (1 -
 Sin[c + d*x])/2])/(d*e*(1 + Sin[c + d*x])^(1/4))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2\right)\sqrt{e\cos(dx+c)}}{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^2}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/sqrt(e*cos(d*x + c)), x)
```

maple [A] time = 0.57, size = 152, normalized size = 1.45

$$-\frac{2a^2\left(-4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*(-4*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+2*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^3+6*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \sin(dx+c)+a)^2}{\sqrt{e \cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^2}{\sqrt{e\,\cos(c+d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.209 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}$$

```
[Out] 4*a^4*(e*cos(d*x+c))^(3/2)/d/e^3/(a^2-a^2*sin(d*x+c))-6*a^2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*c
os(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2670, 2680, 2640, 2639}

$$\frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-6*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c +
 d*x]]) + (4*a^4*(e*Cos[c + d*x])^(3/2))/(d*e^3*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx = \frac{a^4\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^2}\,dx}{e^4}$$
$$= \frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)} - \frac{(3a^2)\int\sqrt{e\cos(c+dx)}\,dx}{e^2}$$
$$= \frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)} - \frac{\left(3a^2\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{e^2\sqrt{\cos(c+dx)}}$$
$$= -\frac{6a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}} + \frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}$$

Mathematica [C] time = 0.06, size = 64, normalized size = 0.75

$$\frac{4\,2^{3/4}a^2\sqrt[4]{\sin(c+dx)+1}\;{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*2^(3/4)*a^2*Hypergeometric2F1[-3/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(1/4))/(d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2\right)\sqrt{e\cos(dx+c)}}{e^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(3/2), x)
```

maple [A] time = 0.81, size = 120, normalized size = 1.41

$$-\frac{2\left(3\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}-4\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\mathrm{s}}{e\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\;\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(3*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)
)*a^2/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^2}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.210 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}$$

```
[Out] -2/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)+4/3*a^4*(e*
cos(d*x+c))^(1/2)/d/e^3/(a^2-a^2*sin(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2670, 2680, 2642, 2641}

$$\frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c
 + d*x]]) + (4*a^4*Sqrt[e*Cos[c + d*x]])/(3*d*e^3*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx = \frac{a^4\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^2}\,dx}{e^4}$$
$$= \frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)} - \frac{a^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$
$$= \frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)} - \frac{\left(a^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2\sqrt{e\cos(c+dx)}}$$
$$= -\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)}$$

Mathematica [C] time = 0.07, size = 66, normalized size = 0.74

$$\frac{4\sqrt[4]{2}\,a^2(\sin(c+dx)+1)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*2^(1/4)*a^2*Hypergeometric2F1[-3/4, -1/4, 1/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(3/4))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\cos\left(dx+c\right)^2-2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\sqrt{e\cos\left(dx+c\right)}}{e^3\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^2}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(5/2), x)
```

maple [A] time = 1.04, size = 193, normalized size = 2.17

$$\frac{2\left(2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\text{El}}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)/e^2*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c
))*a^2/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^2}{(e\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.211 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=127

$$\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}-\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)}$$

```
[Out] 2/5*a^4*(e*cos(d*x+c))^(3/2)/d/e^5/(a-a*sin(d*x+c))^2+2/5*a^4*(e*cos(d*x+c)
)^(3/2)/d/e^5/(a^2-a^2*sin(d*x+c))-2/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/
d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 2681, 2683, 2640, 2639}

$$\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)}+\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}-\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
 + d*x]]) + (2*a^4*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a - a*Sin[c + d*x])^2)
 + (2*a^4*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
```

```
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx = \frac{a^4\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^2}\,dx}{e^4}$$
$$= \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} + \frac{a^3\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}\,dx}{5e^4}$$
$$= \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} + \frac{2a^3(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))} - \frac{a^2\int \sqrt{e\cos(c+dx)}\,dx}{5e^4}$$
$$= \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} + \frac{2a^3(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))} - \frac{\left(a^2\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+}}{5e^4\sqrt{\cos(c+dx)}}$$
$$= -\frac{2a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} + \frac{2a^3(e\cos(c+dx}{5de^5(a-a\sin(c+}$$

Mathematica [C] time = 0.09, size = 66, normalized size = 0.52

$$\frac{2\,2^{3/4}a^2(\sin(c+dx)+1)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*2^(3/4)*a^2*Hypergeometric2F1[-5/4, 1/4, -1/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(5/4))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2\right)\sqrt{e\cos(dx+c)}}{e^4\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e^4*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(7/2), x)
```

maple [**B**] time = 1.38, size = 305, normalized size = 2.40

$$-\frac{2\left(4\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-4*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x
+1/2*c))*a^2/d
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(7/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^2}{(e\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(7/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.212 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7de^4\sqrt{e\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}}+\frac{4\left(a^2\sin(c+dx)+a^2\right)}{7de(e\cos(c+dx))^{7/2}}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/e^3/(e*cos(d*x+c))^(3/2)+4/7*(a^2+a^2*sin(d*x+c))/d/e/
(e*cos(d*x+c))^(7/2)+2/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^4/(e*cos(d*x+c
))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2676, 2636, 2642, 2641}

$$\frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}}+\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7de^4\sqrt{e\cos(c+dx)}}+\frac{4\left(a^2\sin(c+dx)+a^2\right)}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*d*e^4*Sqrt[e*Cos[c
+ d*x]]) + (2*a^2*Sin[c + d*x])/(7*d*e^3*(e*Cos[c + d*x])^(3/2)) + (4*(a^2
+ a^2*Sin[c + d*x]))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{9/2}}\,dx = \frac{4\left(a^2+a^2\sin(c+dx)\right)}{7de(e\cos(c+dx))^{7/2}} + \frac{(3a^2)\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx}{7e^2}$$
$$= \frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}} + \frac{4\left(a^2+a^2\sin(c+dx)\right)}{7de(e\cos(c+dx))^{7/2}} + \frac{a^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{7e^4}$$
$$= \frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}} + \frac{4\left(a^2+a^2\sin(c+dx)\right)}{7de(e\cos(c+dx))^{7/2}} + \frac{\left(a^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}$$
$$= \frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7de^4\sqrt{e\cos(c+dx)}} + \frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}} + \frac{4\left(a^2+a^2\sin(c+d\right.}{7de(e\cos(c+dx))^7}$$

Mathematica [C] time = 0.12, size = 66, normalized size = 0.58

$$\frac{2\sqrt[4]{2}\,a^2(\sin(c+dx)+1)^{7/4}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*2^(1/4)*a^2*Hypergeometric2F1[-7/4, 3/4, -3/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(7/4))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2\right)\sqrt{e\cos(dx+c)}}{e^5\cos(dx+c)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e^5*cos(d*x + c)^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(9/2), x)
```

maple [B] time = 1.59, size = 375, normalized size = 3.29

$$-\frac{2\left(8\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] -2/7/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(8*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)+6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1
/2*c))*a^2/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^2}{(e\,\cos(c+d\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

3.213 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3de^6\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}}+\frac{4\left(a^2\sin(c+dx)+a^2\right)}{9de(e\cos(c+dx))^{9/2}}$$

```
[Out] 2/9*a^2*sin(d*x+c)/d/e^3/(e*cos(d*x+c))^(5/2)+4/9*(a^2+a^2*sin(d*x+c))/d/e/
(e*cos(d*x+c))^(9/2)+2/3*a^2*sin(d*x+c)/d/e^5/(e*cos(d*x+c))^(1/2)-2/3*a^2*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^6/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2676, 2636, 2640, 2639}

$$\frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}}-\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3de^6\sqrt{\cos(c+dx)}}+\frac{4\left(a^2\sin(c+dx)+a^2\right)}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (-2*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*d*e^6*Sqrt[Cos[c
 + d*x]]) + (2*a^2*Sin[c + d*x])/(9*d*e^3*(e*Cos[c + d*x])^(5/2)) + (2*a^2*
Sin[c + d*x])/(3*d*e^5*Sqrt[e*Cos[c + d*x]]) + (4*(a^2 + a^2*Sin[c + d*x]))
/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{11/2}}\,dx = \frac{4\left(a^2+a^2\sin(c+dx)\right)}{9de(e\cos(c+dx))^{9/2}} + \frac{\left(5a^2\right)\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx}{9e^2}$$
$$= \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{4\left(a^2+a^2\sin(c+dx)\right)}{9de(e\cos(c+dx))^{9/2}} + \frac{a^2\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{3e^4}$$
$$= \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}} + \frac{4\left(a^2+a^2\sin(c+dx)\right)}{9de(e\cos(c+dx))^{9/2}} - \frac{a^2\int \sqrt{e\cos}}{3}$$
$$= \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}} + \frac{4\left(a^2+a^2\sin(c+dx)\right)}{9de(e\cos(c+dx))^{9/2}} - \frac{\left(a^2\sqrt{e\cos(\right.}{}$$
$$= -\frac{2a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^6\sqrt{\cos(c+dx)}} + \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}}$$

Mathematica [C] time = 0.16, size = 66, normalized size = 0.46

$$\frac{2^{3/4}a^2(\sin(c+dx)+1)^{9/4}\,{}_2F_1\left(-\frac{9}{4},\frac{5}{4};-\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2^(3/4)*a^2*Hypergeometric2F1[-9/4, 5/4, -5/4, (1 - Sin[c + d*x])/2]*(1 +
Sin[c + d*x])^(9/4))/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2\right)\sqrt{e\cos(dx+c)}}{e^6\cos(dx+c)^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e^6*cos(d*x + c)^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(11/2), x)
```

maple [B] time = 2.44, size = 488, normalized size = 3.37

$$-\frac{2\left(48\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] -2/9/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)
^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+
e)^(1/2)/e^5*(48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*d
*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-12*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)-2*sin(1/2*d*x+1/2*c))*a^2/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^2}{(e\,\cos(c+d\,x))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(11/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

3.214 $\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=203

$$\frac{170a^3e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{170a^3e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}-\frac{34\left(a^3\sin(c\right.}{}$$

```
[Out] -34/99*a^3*(e*cos(d*x+c))^(9/2)/d/e+34/77*a^3*e*(e*cos(d*x+c))^(5/2)*sin(d*
x+c)/d-2/13*a*(e*cos(d*x+c))^(9/2)*(a+a*sin(d*x+c))^2/d/e-34/143*(e*cos(d*x
+c))^(9/2)*(a^3+a^3*sin(d*x+c))/d/e+170/231*a^3*e^4*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(e*cos(d*x+c))^(1/2)+170/231*a^3*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/2)
/d
```

Rubi [A] time = 0.21, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2678, 2669, 2635, 2642, 2641}

$$\frac{170a^3e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{170a^3e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}-\frac{34\left(a^3\sin(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-34*a^3*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (170*a^3*e^4*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (170*a^3*e^3*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (34*a^3*e*(e*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(77*d) - (2*a*(e*Cos[c + d*x])^(9/2)*(a + a*Sin[c + d*x])^2)/
(13*d*e) - (34*(e*Cos[c + d*x])^(9/2)*(a^3 + a^3*Sin[c + d*x]))/(143*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^3\,dx = -\frac{2a(e\cos(c+dx))^{9/2}(a+a\sin(c+dx))^2}{13de} + \frac{1}{13}(17a)\int (e\cos(c+$$

$$= -\frac{2a(e\cos(c+dx))^{9/2}(a+a\sin(c+dx))^2}{13de} - \frac{34(e\cos(c+dx))^{9/2}(}{143}$$

$$= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de} - \frac{2a(e\cos(c+dx))^{9/2}(a+a\sin(c+dx))^2}{13de}$$

$$= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de} + \frac{34a^3e(e\cos(c+dx))^{5/2}\sin(c+dx)}{77d} - 2$$

$$= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de} + \frac{170a^3e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{231d} +$$

$$= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de} + \frac{170a^3e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{231d} +$$

$$= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de} + \frac{170a^3e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.33

$$-\frac{64\sqrt[4]{2}\,a^3(e\cos(c+dx))^{9/2}\,{}_2F_1\left(-\frac{17}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^3,x]
[Out] (-64*2^(1/4)*a^3*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[-17/4, 9/4, 13/4,
 (1 - Sin[c + d*x])/2])/(9*d*e*(1 + Sin[c + d*x])^(9/4))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3e^3\cos\left(dx+c\right)^5-4\,a^3e^3\cos\left(dx+c\right)^3+\left(a^3e^3\cos\left(dx+c\right)^5-4\,a^3e^3\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] integral(-(3*a^3*e^3*cos(d*x + c)^5 - 4*a^3*e^3*cos(d*x + c)^3 + (a^3*e^3*c
os(d*x + c)^5 - 4*a^3*e^3*cos(d*x + c)^3)*sin(d*x + c))*sqrt(e*cos(d*x + c)
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}\,(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^3, x)

maple [A] time = 0.94, size = 321, normalized size = 1.58

$$-\frac{2a^3e^4\left(88704\left(\sin^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-157248\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-310464\left(\sin^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+393120\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x)

[Out] -2/9009/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e^4*(88704*sin(1/2*d*x+1/2*c)^15-157248*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-310464*sin(1/2*d*x+1/2*c)^13+393120*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+337568*sin(1/2*d*x+1/2*c)^11-361296*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-67760*sin(1/2*d*x+1/2*c)^9+148824*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-126280*sin(1/2*d*x+1/2*c)^7-12012*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+101948*sin(1/2*d*x+1/2*c)^5+3315*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-5694*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-30338*sin(1/2*d*x+1/2*c)^3+3311*sin(1/2*d*x+1/2*c))/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos(c+d\,x))^{7/2}\,(a+a\,\sin(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^3,x)

[Out] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))**(7/2)*(a+a*sin(d*x+c))**3,x)

[Out] Timed out

3.215 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=170

$$\frac{2a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{10a^3(e\cos(c+dx))^{7/2}}{21de}-\frac{10\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{7/2}}{33de}+\frac{2a^3e}{\cdots}$$

```
[Out] -10/21*a^3*(e*cos(d*x+c))^(7/2)/d/e+2/3*a^3*e*(e*cos(d*x+c))^(3/2)*sin(d*x+
c)/d-2/11*a*(e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2/d/e-10/33*(e*cos(d*x+c)
)^(7/2)*(a^3+a^3*sin(d*x+c))/d/e+2*a^3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/
d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 2669, 2635, 2640, 2639}

$$\frac{2a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{10a^3(e\cos(c+dx))^{7/2}}{21de}-\frac{10\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{7/2}}{33de}+\frac{2a^3e}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-10*a^3*(e*Cos[c + d*x])^(7/2))/(21*d*e) + (2*a^3*e^2*Sqrt[e*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*e*(e*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(3*d) - (2*a*(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c +
d*x])^2)/(11*d*e) - (10*(e*Cos[c + d*x])^(7/2)*(a^3 + a^3*Sin[c + d*x]))/(3
3*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3\,dx &= -\frac{2a(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}{11de}+\frac{1}{11}(15a)\int (e\cos(c+dx)\\ &= -\frac{2a(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}{11de}-\frac{10(e\cos(c+dx))^{7/2}\left(a^3\right.}{33de}\\ &= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de}-\frac{2a(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}{11de}-\\ &= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de}+\frac{2a^3e(e\cos(c+dx))^{3/2}\sin(c+dx)}{3d}-\frac{2a(e\,c}{}\\ &= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de}+\frac{2a^3e(e\cos(c+dx))^{3/2}\sin(c+dx)}{3d}-\frac{2a(e\,c}{}\\ &= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de}+\frac{2a^3e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2a}{}\end{aligned}$$

Mathematica [C] time = 0.11, size = 66, normalized size = 0.39

$$-\frac{32\,2^{3/4}a^3(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{15}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-32*2^(3/4)*a^3*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-15/4, 7/4, 11/4,
 (1 - Sin[c + d*x])/2])/(7*d*e*(1 + Sin[c + d*x])^(7/4))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3e^2\cos(dx+c)^4-4\,a^3e^2\cos(dx+c)^2+\left(a^3e^2\cos(dx+c)^4-4\,a^3e^2\cos(dx+c)^2\right)\sin(dx+c)\right)\sqrt{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*e^2*cos(d*x + c)^4 - 4*a^3*e^2*cos(d*x + c)^2 + (a^3*e^2*c
os(d*x + c)^4 - 4*a^3*e^2*cos(d*x + c)^2)*sin(d*x + c))*sqrt(e*cos(d*x + c)
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^3, x)
```

maple [A] time = 1.03, size = 264, normalized size = 1.55

$$2a^3e^3\left(1344\left(\sin^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2464\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4032\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4928\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/231/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e^3*(1344*
sin(1/2*d*x+1/2*c)^13-2464*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-4032*si
n(1/2*d*x+1/2*c)^11+4928*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+2928*sin(1
/2*d*x+1/2*c)^9-3080*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+864*sin(1/2*d*
x+1/2*c)^7+616*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-1908*sin(1/2*d*x+1/2
*c)^5+231*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+804*sin(1/2*d*x+1/2*c)^3-111*sin(1/2*d*x
+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{5/2}\,(a+a\,\sin(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.216 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=172

$$\frac{26a^3e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{26\left(a^3\sin(c+d\right.}{}$$

```
[Out] -26/35*a^3*(e*cos(d*x+c))^(5/2)/d/e-2/9*a*(e*cos(d*x+c))^(5/2)*(a+a*sin(d*x
+c))^2/d/e-26/63*(e*cos(d*x+c))^(5/2)*(a^3+a^3*sin(d*x+c))/d/e+26/21*a^3*e^
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+26/21*a^3*e*sin(d*x+c)
*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2678, 2669, 2635, 2642, 2641}

$$\frac{26a^3e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{26\left(a^3\sin(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-26*a^3*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (26*a^3*e^2*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (26*a^3*e*Sqrt[e*C
os[c + d*x]]*Sin[c + d*x])/(21*d) - (2*a*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[
c + d*x])^2)/(9*d*e) - (26*(e*Cos[c + d*x])^(5/2)*(a^3 + a^3*Sin[c + d*x]))
/(63*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3\,dx = -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}{9de}+\frac{1}{9}(13a)\int (e\cos(c+d$$

$$= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}{9de}-\frac{26(e\cos(c+dx))^{5/2}\,(}{63a}$$

$$= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}-\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}{9de}$$

$$= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d}-\frac{2a}{}$$

$$= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d}-\frac{2a}{}$$

$$= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+$$

Mathematica [C] time = 0.07, size = 66, normalized size = 0.38

$$-\frac{32\sqrt[4]{2}\,a^3(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{13}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-32*2^(1/4)*a^3*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-13/4, 5/4, 9/4,
(1 - Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(5/4))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3e\cos\left(dx+c\right)^3-4\,a^3e\cos\left(dx+c\right)+\left(a^3e\cos\left(dx+c\right)^3-4\,a^3e\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)\sqrt{e\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*e*cos(d*x + c)^3 - 4*a^3*e*cos(d*x + c) + (a^3*e*cos(d*x +
 c)^3 - 4*a^3*e*cos(d*x + c))*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sin\left(dx+c\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^3, x)
```

maple [A] time = 1.26, size = 251, normalized size = 1.46

$$-\frac{2a^3e^2\left(1120\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2800\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3240\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e^2*(1120
*sin(1/2*d*x+1/2*c)^11-2160*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2800*si
n(1/2*d*x+1/2*c)^9+3240*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+784*sin(1/2
*d*x+1/2*c)^7-840*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+1624*sin(1/2*d*x+
1/2*c)^5+195*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-120*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-1162*sin(1/2*d*x+1/2*c)^3+217*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{3/2}\,(a+a\,\sin(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.217 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=140

$$-\frac{22a^3(e\cos(c+dx))^{3/2}}{15de}-\frac{22\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{3/2}}{35de}+\frac{22a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2a(}{}$$

```
[Out] -22/15*a^3*(e*cos(d*x+c))^(3/2)/d/e-2/7*a*(e*cos(d*x+c))^(3/2)*(a+a*sin(d*x
+c))^2/d/e-22/35*(e*cos(d*x+c))^(3/2)*(a^3+a^3*sin(d*x+c))/d/e+22/5*a^3*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2678, 2669, 2640, 2639}

$$-\frac{22a^3(e\cos(c+dx))^{3/2}}{15de}-\frac{22\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{3/2}}{35de}+\frac{22a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2a(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-22*a^3*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (22*a^3*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*a*(e*Cos[c + d*x])^(
3/2)*(a + a*Sin[c + d*x])^2)/(7*d*e) - (22*(e*Cos[c + d*x])^(3/2)*(a^3 + a^
3*Sin[c + d*x]))/(35*d*e)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3\,dx = -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de} + \frac{1}{7}(11a)\int \sqrt{e\cos(c+dx)}\,($$

$$= -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de} - \frac{22(e\cos(c+dx))^{3/2}\left(a^3+\right.}{35de}$$

$$= -\frac{22a^3(e\cos(c+dx))^{3/2}}{15de} - \frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de} - $$

$$= -\frac{22a^3(e\cos(c+dx))^{3/2}}{15de} - \frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de} - $$

$$= -\frac{22a^3(e\cos(c+dx))^{3/2}}{15de} + \frac{22a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} - 2a($$

Mathematica [C] time = 0.05, size = 66, normalized size = 0.47

$$-\frac{16\,2^{3/4}a^3(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{11}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3,x]
[Out] (-16*2^(3/4)*a^3*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-11/4, 3/4, 7/4,
(1 - Sin[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(3/4))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3\cos(dx+c)^2 - 4\,a^3 + \left(a^3\cos(dx+c)^2 - 4\,a^3\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^3, x)
```

maple [A] time = 0.85, size = 214, normalized size = 1.53

$$\frac{2a^3e\left(240\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - 504\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right) - 480\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + 504\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/105/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e*(240*sin
(1/2*d*x+1/2*c)^9-504*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-480*sin(1/2*d
*x+1/2*c)^7+504*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-200*sin(1/2*d*x+1/2
*c)^5+231*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-126*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+440*sin(1/2*d*x+1/2*c)^3-125*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\ \cos(c+d\,x)}\ (a+a\ \sin(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.218 $\int \frac{(a+a\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=136

$$-\frac{6a^3\sqrt{e\cos(c+dx)}}{de}-\frac{6\left(a^3\sin(c+dx)+a^3\right)\sqrt{e\cos(c+dx)}}{5de}+\frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a(a\sin(c+dx$$

```
[Out] 6*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-6*a^3*(e*cos(d*x+c
))^(1/2)/d/e-2/5*a*(a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2)/d/e-6/5*(a^3+a^3
*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A] time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2678, 2669, 2642, 2641}

$$-\frac{6a^3\sqrt{e\cos(c+dx)}}{de}-\frac{6\left(a^3\sin(c+dx)+a^3\right)\sqrt{e\cos(c+dx)}}{5de}+\frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a(a\sin(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-6*a^3*Sqrt[e*Cos[c + d*x]])/(d*e) + (6*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*(a + a
*Sin[c + d*x])^2)/(5*d*e) - (6*Sqrt[e*Cos[c + d*x]]*(a^3 + a^3*Sin[c + d*x]
))/(5*d*e)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}dx = -\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}{5de} + \frac{1}{5}(9a)\int \frac{(a+a\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}dx$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}{5de} - \frac{6\sqrt{e\cos(c+dx)}\left(a^3+a^3\sin(c+dx)\right)}{5de} -$$

$$= -\frac{6a^3\sqrt{e\cos(c+dx)}}{de} - \frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}{5de} - \frac{6\sqrt{e\cos(c+dx)}\,(}{5}$$

$$= -\frac{6a^3\sqrt{e\cos(c+dx)}}{de} - \frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}{5de} - \frac{6\sqrt{e\cos(c+dx)}\,(}{5}$$

$$= -\frac{6a^3\sqrt{e\cos(c+dx)}}{de} + \frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}} - \frac{2a\sqrt{e\cos(c+dx)}\,(a+}{5de}$$

Mathematica [C] time = 0.04, size = 64, normalized size = 0.47

$$-\frac{16\sqrt[4]{2}\,a^3\sqrt{e\cos(c+dx)}\;{}_2F_1\left(-\frac{9}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-16*2^(1/4)*a^3*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-9/4, 1/4, 5/4, (1
- Sin[c + d*x])/2])/(d*e*(1 + Sin[c + d*x])^(1/4))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^3}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/sqrt(e*cos(d*x + c)), x)
```

maple [A] time = 0.77, size = 178, normalized size = 1.31

$$-\frac{2a^3\left(8\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticF}\left(\right.\right.}{5\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*(8*sin(1/2*
d*x+1/2*c)^7-20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*
c)^5+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
-34*sin(1/2*d*x+1/2*c)^3+19*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^3}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^3}{\sqrt{e\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.219 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2}+\frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3}-\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}$$

```
[Out] 14/3*a^3*(e*cos(d*x+c))^(3/2)/d/e^3+4*a^5*(e*cos(d*x+c))^(7/2)/d/e^5/(a-a*s
in(d*x+c))^2-14*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 2680, 2682, 2640, 2639}

$$\frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3}+\frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2}-\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (14*a^3*(e*Cos[c + d*x])^(3/2))/(3*d*e^3) - (14*a^3*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (4*a^5*(e*Cos[c + d*x
])^(7/2))/(d*e^5*(a - a*Sin[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
```

```
&& EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx = \frac{a^6\int \frac{(e\cos(c+dx))^{9/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}$$

$$= \frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2} - \frac{(7a^4)\int \frac{(e\cos(c+dx))^{5/2}}{a-a\sin(c+dx)}\,dx}{e^4}$$

$$= \frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3} + \frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2} - \frac{(7a^3)\int \sqrt{e\cos(c+dx)}\,dx}{e^2}$$

$$= \frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3} + \frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2} - \frac{\left(7a^3\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+}}{e^2\sqrt{\cos(c+dx)}}$$

$$= \frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3} - \frac{14a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}} + \frac{4a^5(e\cos(c+dx))^{7}}{de^5(a-a\sin(c+dx}$$

Mathematica [C] time = 0.06, size = 64, normalized size = 0.60

$$\frac{8\,2^{3/4}a^3\sqrt[4]{\sin(c+dx)+1}\;{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*2^(3/4)*a^3*Hypergeometric2F1[-7/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(1/4))/(d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^2\cos\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^3}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(3/2), x)
```

maple [A] time = 1.18, size = 146, normalized size = 1.38

$$-\frac{2\left(-4\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}-24\left(\sin^2\left(\right.\right.}{3e\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-4*sin(1/2*d
*x+1/2*c)^5+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-24*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+4*sin(1/2*d*x+1/2*c)^3-13*sin(1/2*d*x+1/2*c))*a^3/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^3}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.220 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2}+\frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3}-\frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}$$

```
[Out] 4/3*a^5*(e*cos(d*x+c))^(5/2)/d/e^5/(a-a*sin(d*x+c))^2-10/3*a^3*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)+10/3*a^3*(e*cos(d*x+c))^(1/2)/d/
e^3
```

Rubi [A] time = 0.20, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 2680, 2682, 2642, 2641}

$$\frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3}+\frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2}-\frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (10*a^3*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) - (10*a^3*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (4*a^5*(e*Cos[c + d*x
])^(5/2))/(3*d*e^5*(a - a*Sin[c + d*x])^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
```

```
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx = \frac{a^6\int \frac{(e\cos(c+dx))^{7/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}$$
$$= \frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2} - \frac{\left(5a^4\right)\int \frac{(e\cos(c+dx))^{3/2}}{a-a\sin(c+dx)}\,dx}{3e^4}$$
$$= \frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3} + \frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2} - \frac{\left(5a^3\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$
$$= \frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3} + \frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2} - \frac{\left(5a^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}}{3e^2\sqrt{e\cos(c+dx)}}$$
$$= \frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3} - \frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{4a^5(e\cos(c+dx))^5}{3de^5(a-a\sin(c+dx}$$

Mathematica [C] time = 0.05, size = 66, normalized size = 0.60

$$\frac{8\sqrt[4]{2}\,a^3(\sin(c+dx)+1)^{3/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
[Out] (8*2^(1/4)*a^3*Hypergeometric2F1[-5/4, -3/4, 1/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(3/4))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^3\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^3}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(5/2), x)
```

maple [A] time = 1.27, size = 219, normalized size = 1.99

$$\frac{2\left(10\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)/e^2*(10*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*sin(1/2
*d*x+1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+12*sin(1/2*d*x+1/2*c)^3-7*sin(1/2*d*x+1/2*c))*a^3/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^3}{(e\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.221 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=127

$$\frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}+\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}-\frac{6a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^3-a^3\sin(c+dx)\right)}$$

```
[Out] 4/5*a^5*(e*cos(d*x+c))^(3/2)/d/e^5/(a-a*sin(d*x+c))^2-6/5*a^6*(e*cos(d*x+c)
)^(3/2)/d/e^5/(a^3-a^3*sin(d*x+c))+6/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/
d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 2680, 2683, 2640, 2639}

$$-\frac{6a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^3-a^3\sin(c+dx)\right)}+\frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}+\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (6*a^3*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
+ d*x]]) + (4*a^5*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a - a*Sin[c + d*x])^2)
- (6*a^6*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^3 - a^3*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
```

```
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx = \frac{a^6\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}$$

$$= \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} - \frac{\left(3a^4\right)\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}\,dx}{5e^4}$$

$$= \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} - \frac{6a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))} + \frac{\left(3a^3\right)\int \sqrt{e\cos(c+dx)}\,dx}{5e^4}$$

$$= \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} - \frac{6a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))} + \frac{\left(3a^3\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c}}{5e^4\sqrt{\cos(c+dx)}}$$

$$= \frac{6a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} - \frac{6a^4(e\cos(c+dx))}{5de^5(a-a\sin(c+}$$

Mathematica [C] time = 0.09, size = 66, normalized size = 0.52

$$\frac{4\,2^{3/4}a^3(\sin(c+dx)+1)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (4*2^(3/4)*a^3*Hypergeometric2F1[-5/4, -3/4, -1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(5/4))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^4\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e^4*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(7/2), x)
```

maple [B] time = 1.88, size = 332, normalized size = 2.61

$$\frac{2\left(12\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-20
*sin(1/2*d*x+1/2*c)^5+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+2*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)+20*sin(1/2*d*x+1/2*c)^3-sin(1/2*d*x+1/2*c))*a^3/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^{3}}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{3}}{(e\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.222 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=127

$$\frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2}-\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}-\frac{2a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^3-a^3\sin(c+dx)\right)}$$

```
[Out] -2/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^4/(e*cos(d*x+c))^(1/2)+4/7*a^5*(e
*cos(d*x+c))^(1/2)/d/e^5/(a-a*sin(d*x+c))^2-2/21*a^6*(e*cos(d*x+c))^(1/2)/d
/e^5/(a^3-a^3*sin(d*x+c))
```

Rubi [A] time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 2680, 2683, 2642, 2641}

$$-\frac{2a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^3-a^3\sin(c+dx)\right)}+\frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2}-\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (-2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[
c + d*x]]) + (4*a^5*Sqrt[e*Cos[c + d*x]])/(7*d*e^5*(a - a*Sin[c + d*x])^2)
- (2*a^6*Sqrt[e*Cos[c + d*x]])/(21*d*e^5*(a^3 - a^3*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
```

```
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx = \frac{a^6\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}$$
$$= \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2} - \frac{a^4\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a-a\sin(c+dx))}\,dx}{7e^4}$$
$$= \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2} - \frac{2a^4\sqrt{e\cos(c+dx)}}{21de^5(a-a\sin(c+dx))} - \frac{a^3\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{21e^4}$$
$$= \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2} - \frac{2a^4\sqrt{e\cos(c+dx)}}{21de^5(a-a\sin(c+dx))} - \frac{\left(a^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}}}{21e^4\sqrt{e\cos(c+dx)}}$$
$$= -\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}} + \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2} - \frac{2a^4\sqrt{e\cos(c-}}{21de^5(a-a\sin(}$$

Mathematica [C] time = 0.09, size = 66, normalized size = 0.52

$$\frac{4\sqrt[4]{2}\,a^3(\sin(c+dx)+1)^{7/4}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (4*2^(1/4)*a^3*Hypergeometric2F1[-7/4, -1/4, -3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(7/4))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^5\cos\left(dx+c\right)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e^5*cos(d*x + c)^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^3}{\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(9/2), x)
```

maple [B] time = 2.00, size = 401, normalized size = 3.16

$$\frac{2\left(8\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] 2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(8*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+28*sin(
1/2*d*x+1/2*c)^5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-22*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)-28*sin(1/2*d*x+1/2*c)^3-5*sin(1/2*d*x+1/2*c))*a^3/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^3}{(e\,\cos(c+d\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

3.223 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=165

$$\frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2}-\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15de^6\sqrt{\cos(c+dx)}}+\frac{2a^6(e\cos(c+dx)}{15de^7\left(a^3-a^3\sin(c\right.}$$

```
[Out] 2/9*a^6*(e*cos(d*x+c))^(3/2)/d/e^7/(a-a*sin(d*x+c))^3+2/15*a^5*(e*cos(d*x+c
))^(3/2)/d/e^7/(a-a*sin(d*x+c))^2+2/15*a^6*(e*cos(d*x+c))^(3/2)/d/e^7/(a^3-
a^3*sin(d*x+c))-2/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^6/cos(d*x+c)^(
1/2)
```

Rubi [A] time = 0.24, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 2681, 2683, 2640, 2639}

$$\frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7\left(a^3-a^3\sin(c+dx)\right)}+\frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2}-\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c}}{15de^6\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (-2*a^3*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^6*Sqrt[Cos[
c + d*x]]) + (2*a^6*(e*Cos[c + d*x])^(3/2))/(9*d*e^7*(a - a*Sin[c + d*x])^3
) + (2*a^5*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a - a*Sin[c + d*x])^2) + (2*a
^6*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^3 - a^3*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{11/2}}\,dx = \frac{a^6\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^3}\,dx}{e^6}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{a^5\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^2}\,dx}{3e^6}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} + \frac{a^4\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}\,dx}{15e^6}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} + \frac{2a^4(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))} - \frac{a^3 \ldots}{\ldots}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} + \frac{2a^4(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))} - \frac{(\ldots}{\ldots}$$

$$= -\frac{2a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^6\sqrt{\cos(c+dx)}} + \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+d\ldots}{15de^7(a-a\sin(c\ldots}$$

Mathematica [C] time = 0.13, size = 66, normalized size = 0.40

$$\frac{2\,2^{3/4}a^3(\sin(c+dx)+1)^{9/4}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*2^(3/4)*a^3*Hypergeometric2F1[-9/4, 1/4, -5/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(9/4))/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^6\cos(dx+c)^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e^6*cos(d*x + c)^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(11/2), x)
```

maple [B] time = 2.68, size = 514, normalized size = 3.12

$$-\frac{2\left(48\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e
+e)^(1/2)/e^5*(48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+36*sin(1/2*d*x+1/2*c)^5+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-48*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-36*sin(1/2*d*x+1/2*c)^3-11*sin(1/2*d*x+1/2*c)
)*a^3/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a\sin(dx+c)+a)^{3}}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+a\ \sin(c+d\,x))^{3}}{(e\ \cos(c+d\,x))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(11/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

3.224 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=210

$$\frac{442a^4e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{442a^4(e\cos(c+dx))^{5/2}}{385de}+\frac{442a^4e\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{442\left(a^4\sin(\right.}{}$$

```
[Out] -442/385*a^4*(e*cos(d*x+c))^(5/2)/d/e-2/11*a*(e*cos(d*x+c))^(5/2)*(a+a*sin(
d*x+c))^3/d/e-34/99*(e*cos(d*x+c))^(5/2)*(a^2+a^2*sin(d*x+c))^2/d/e-442/693
*(e*cos(d*x+c))^(5/2)*(a^4+a^4*sin(d*x+c))/d/e+442/231*a^4*e^2*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+442/231*a^4*e*sin(d*x+c)*(e*cos(d*x+
c))^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 2669, 2635, 2642, 2641}

$$\frac{442a^4e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{442a^4(e\cos(c+dx))^{5/2}}{385de}+\frac{442a^4e\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{34\left(a^2\sin(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-442*a^4*(e*Cos[c + d*x])^(5/2))/(385*d*e) + (442*a^4*e^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (442*a^4*e*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) - (2*a*(e*Cos[c + d*x])^(5/2)*(a +
a*Sin[c + d*x])^3)/(11*d*e) - (34*(e*Cos[c + d*x])^(5/2)*(a^2 + a^2*Sin[c +
 d*x])^2)/(99*d*e) - (442*(e*Cos[c + d*x])^(5/2)*(a^4 + a^4*Sin[c + d*x]))/
(693*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4\,dx = -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3}{11de} + \frac{1}{11}(17a)\int (e\cos(c+$$

$$= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3}{11de} - \frac{34(e\cos(c+dx))^{5/2}\left(\right.}{99}$$

$$= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3}{11de} - \frac{34(e\cos(c+dx))^{5/2}\left(\right.}{99}$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de} - \frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx)}{11de}$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de} + \frac{442a^4e\sqrt{e\cos(c+dx)}\sin(c+dx)}{231d} -$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de} + \frac{442a^4e\sqrt{e\cos(c+dx)}\sin(c+dx)}{231d} -$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de} + \frac{442a^4e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{231d\sqrt{e\cos(c+dx)}}$$

Mathematica [C] time = 0.11, size = 66, normalized size = 0.31

$$-\frac{64\sqrt[4]{2}\,a^4(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{17}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-64*2^(1/4)*a^4*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-17/4, 5/4, 9/4,
(1 - Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(5/4))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

integral $\left(\left(a^4e\cos(dx+c)^5 - 8\,a^4e\cos(dx+c)^3 + 8\,a^4e\cos(dx+c) - 4\left(a^4e\cos(dx+c)^3 - 2\,a^4e\cos(dx+c)\right)\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*e*cos(d*x + c)^5 - 8*a^4*e*cos(d*x + c)^3 + 8*a^4*e*cos(d*x +
 c) - 4*(a^4*e*cos(d*x + c)^3 - 2*a^4*e*cos(d*x + c))*sin(d*x + c))*sqrt(e*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^4, x)
```

maple [A] time = 0.94, size = 295, normalized size = 1.40

$$-\frac{2a^4e^2\left(20160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-50400\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+49280\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/3465/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^4*e^2*(201
60*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-50400*sin(1/2*d*x+1/2*c)^10*cos
(1/2*d*x+1/2*c)+49280*sin(1/2*d*x+1/2*c)^11-6480*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^8-123200*sin(1/2*d*x+1/2*c)^9+60120*sin(1/2*d*x+1/2*c)^6*cos(1/
2*d*x+1/2*c)+78848*sin(1/2*d*x+1/2*c)^7-23100*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+4928*sin(1/2*d*x+1/2*c)^5+3315*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-150*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-17864*sin(1/2*d*x+1/2*c)^3+4004*sin(1/2
*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos(c+d\,x))^{3/2}\,(a+a\,\sin(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.225 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=178

$$-\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{22\left(a^4\sin(c+dx)+a^4\right)(e\cos(c+dx))^{3/2}}{21de}+\frac{22a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}-\frac{10}{}$$

```
[Out] -22/9*a^4*(e*cos(d*x+c))^(3/2)/d/e-2/9*a*(e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+
c))^3/d/e-10/21*(e*cos(d*x+c))^(3/2)*(a^2+a^2*sin(d*x+c))^2/d/e-22/21*(e*co
s(d*x+c))^(3/2)*(a^4+a^4*sin(d*x+c))/d/e+22/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2678, 2669, 2640, 2639}

$$-\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{10\left(a^2\sin(c+dx)+a^2\right)^2(e\cos(c+dx))^{3/2}}{21de}-\frac{22\left(a^4\sin(c+dx)+a^4\right)(e\cos(c+dx))^{3/2}}{21de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-22*a^4*(e*Cos[c + d*x])^(3/2))/(9*d*e) + (22*a^4*Sqrt[e*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2])/(3*d*Sqrt[Cos[c + d*x]]) - (2*a*(e*Cos[c + d*x])^(3
/2)*(a + a*Sin[c + d*x])^3)/(9*d*e) - (10*(e*Cos[c + d*x])^(3/2)*(a^2 + a^2
*Sin[c + d*x])^2)/(21*d*e) - (22*(e*Cos[c + d*x])^(3/2)*(a^4 + a^4*Sin[c +
d*x]))/(21*d*e)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4\,dx = -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}+\frac{1}{3}(5a)\int \sqrt{e\cos(c+dx)}\,(a$$

$$= -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-\frac{10(e\cos(c+dx))^{3/2}\left(a^2+\right.}{21de}$$

$$= -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-\frac{10(e\cos(c+dx))^{3/2}\left(a^2+\right.}{21de}$$

$$= -\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-\frac{1}{}$$

$$= -\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-\frac{1}{}$$

$$= -\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}+\frac{22a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}}-\frac{2a(}{}$$

Mathematica [C] time = 0.06, size = 66, normalized size = 0.37

$$-\frac{32\ 2^{3/4}a^4(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{15}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-32*2^(3/4)*a^4*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-15/4, 3/4, 7/4,
(1 - Sin[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(3/4))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^4, x)
```

maple [A] time = 0.98, size = 258, normalized size = 1.45

$$\frac{2a^4e\left(224\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-448\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+576\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-392\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/63/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^4*e*(224*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-448*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^8+576*sin(1/2*d*x+1/2*c)^9-392*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-
1152*sin(1/2*d*x+1/2*c)^7+616*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+192*s
in(1/2*d*x+1/2*c)^5+231*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-168*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)+384*sin(1/2*d*x+1/2*c)^3-132*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e \cos(dx+c)}\,(a \sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos(c+d\,x)}\,{\left(a+a\,\sin(c+d\,x)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.226 $\int \frac{(a+a\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}-\frac{78\left(a^4\sin(c+dx)+a^4\right)\sqrt{e\cos(c+dx)}}{35de}+\frac{78a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{26\left(a^2\sin(c\right.}{}$$

```
[Out] 78/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-78/7*a^4*(e*cos
(d*x+c))^(1/2)/d/e-2/7*a*(a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2)/d/e-26/35*
(a^2+a^2*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2)/d/e-78/35*(a^4+a^4*sin(d*x+c))*
(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A] time = 0.21, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2678, 2669, 2642, 2641}

$$-\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}-\frac{26\left(a^2\sin(c+dx)+a^2\right)^2\sqrt{e\cos(c+dx)}}{35de}-\frac{78\left(a^4\sin(c+dx)+a^4\right)\sqrt{e\cos(c+dx)}}{35de}+\frac{78a^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-78*a^4*Sqrt[e*Cos[c + d*x]])/(7*d*e) + (78*a^4*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(7*d*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*
(a + a*Sin[c + d*x])^3)/(7*d*e) - (26*Sqrt[e*Cos[c + d*x]]*(a^2 + a^2*Sin[c
 + d*x])^2)/(35*d*e) - (78*Sqrt[e*Cos[c + d*x]]*(a^4 + a^4*Sin[c + d*x]))/(
35*d*e)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}dx = -\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}{7de} + \frac{1}{7}(13a)\int \frac{(a+a\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}dx$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}{7de} - \frac{26\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c+dx)\right)^2}{35de}$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}{7de} - \frac{26\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c+dx)\right)^2}{35de}$$

$$= -\frac{78a^4\sqrt{e\cos(c+dx)}}{7de} - \frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}{7de} - \frac{26\sqrt{e\cos(c+dx)}}{}$$

$$= -\frac{78a^4\sqrt{e\cos(c+dx)}}{7de} - \frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}{7de} - \frac{26\sqrt{e\cos(c+dx)}}{}$$

$$= -\frac{78a^4\sqrt{e\cos(c+dx)}}{7de} + \frac{78a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}} - \frac{2a\sqrt{e\cos(c+dx)}(}{7d}$$

Mathematica [C] time = 0.06, size = 64, normalized size = 0.36

$$-\frac{32\sqrt[4]{2}a^4\sqrt{e\cos(c+dx)}\,{}_2F_1\left(-\frac{13}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-32*2^(1/4)*a^4*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-13/4, 1/4, 5/4, (1
 - Sin[c + d*x])/2])/(d*e*(1 + Sin[c + d*x])^(1/4))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4\cos(dx+c)^4 - 8a^4\cos(dx+c)^2 + 8a^4 - 4\left(a^4\cos(dx+c)^2 - 2a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^4}{\sqrt{e\cos(dx+c)}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/sqrt(e*cos(d*x + c)), x)
```

maple [A] time = 0.86, size = 222, normalized size = 1.25

$$-\frac{2a^4\left(80\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - 120\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right) + 224\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - 280\left(\sin^4\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^4*(80*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+224*sin(1/2*d*x+1/2*c)^7-280*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-336
*sin(1/2*d*x+1/2*c)^5+195*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+160*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-392*sin(1/2*d*x+1/2*c)^3+252*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \sin(dx+c)+a)^4}{\sqrt{e \cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^4}{\sqrt{e\,\cos(c+d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.227 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=156

$$\frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3}-\frac{154a^4\sin(c+dx)(e\cos(c+dx))^{3/2}}{15de^3}-\frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^2\sqrt{\cos(c+dx)}}+\frac{44a^8(e\,\cdots}{3de^5\left(a^4\,\cdots\right.}$$

```
[Out] -154/15*a^4*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/d/e^3+4*a^7*(e*cos(d*x+c))^(11/
2)/d/e^7/(a-a*sin(d*x+c))^3+44/3*a^8*(e*cos(d*x+c))^(7/2)/d/e^5/(a^4-a^4*si
n(d*x+c))-154/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 2680, 2635, 2640, 2639}

$$\frac{44a^8(e\cos(c+dx))^{7/2}}{3de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3}-\frac{154a^4\sin(c+dx)(e\cos(c+dx))^{3/2}}{15de^3}-\frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\,\cdots\right.}{5de^2\sqrt{\cos(c\,\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-154*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos
[c + d*x]]) - (154*a^4*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*d*e^3) + (4
*a^7*(e*Cos[c + d*x])^(11/2))/(d*e^7*(a - a*Sin[c + d*x])^3) + (44*a^8*(e*C
os[c + d*x])^(7/2))/(3*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
```

```
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx = \frac{a^8\int \frac{(e\cos(c+dx))^{13/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}$$

$$= \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} - \frac{\left(11a^6\right)\int \frac{(e\cos(c+dx))^{9/2}}{(a-a\sin(c+dx))^2}\,dx}{e^6}$$

$$= \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} + \frac{44a^6(e\cos(c+dx))^{7/2}}{3de^5\left(a^2-a^2\sin(c+dx)\right)} - \frac{\left(77a^4\right)\int (e\cos(c+dx))^{5/2}\,dx}{3e^4}$$

$$= -\frac{154a^4(e\cos(c+dx))^{3/2}\sin(c+dx)}{15de^3} + \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} + \frac{44a^6(e\cos(c+dx$$

$$= -\frac{154a^4(e\cos(c+dx))^{3/2}\sin(c+dx)}{15de^3} + \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} + \frac{44a^6(e\cos(c+dx$$

$$= -\frac{154a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}} - \frac{154a^4(e\cos(c+dx))^{3/2}\sin(c+dx)}{15de^3} + \frac{4a^7(}{de^7(}$$

Mathematica [C] time = 0.07, size = 64, normalized size = 0.41

$$\frac{16\ 2^{3/4}a^4\sqrt[4]{\sin(c+dx)+1}\ {}_2F_1\left(-\frac{11}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (16*2^(3/4)*a^4*Hypergeometric2F1[-11/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(
1 + Sin[c + d*x])^(1/4))/(d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4\cos(dx+c)^4 - 8a^4\cos(dx+c)^2 + 8a^4 - 4\left(a^4\cos(dx+c)^2 - 2a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^2\cos(dx+c)^2}, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^2*cos(d*x + c)^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(3/2), x)
```

maple [A] time = 1.15, size = 190, normalized size = 1.22

$$-\frac{2\left(-24\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+24\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-80\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+231\,\mathrm{EllipticE}\left(\right.}{15e\sqrt{-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/15/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-24*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-80*sin(1/2*d*x+1/2*c)^5+231*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-246*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+80*sin(1/2*d*x+1/2*c)^3-140*sin(1/2*d*x+1/2*c))*a^4/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^{4}}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{4}}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.228 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3}-\frac{10a^4\sin(c+dx)\sqrt{e\cos(c+dx)}}{de^3}-\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{e\cos(c+dx)}}+\frac{12a^8(e\cos(c+dx))^{5/2}}{de^5\left(a^4-a^4\sin(c+dx)\right)}$$

```
[Out] 4/3*a^7*(e*cos(d*x+c))^(9/2)/d/e^7/(a-a*sin(d*x+c))^3+12*a^8*(e*cos(d*x+c))
^(5/2)/d/e^5/(a^4-a^4*sin(d*x+c))-10*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/
(e*cos(d*x+c))^(1/2)-10*a^4*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d/e^3
```

Rubi [A] time = 0.22, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 2680, 2635, 2642, 2641}

$$\frac{12a^8(e\cos(c+dx))^{5/2}}{de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3}-\frac{10a^4\sin(c+dx)\sqrt{e\cos(c+dx)}}{de^3}-\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*e^2*Sqrt[e*Cos[c
+ d*x]]) - (10*a^4*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(d*e^3) + (4*a^7*(e*C
os[c + d*x])^(9/2))/(3*d*e^7*(a - a*Sin[c + d*x])^3) + (12*a^8*(e*Cos[c + d
*x])^(5/2))/(d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
```

```
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx = \frac{a^8\int \frac{(e\cos(c+dx))^{11/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}$$

$$= \frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3} - \frac{\left(3a^6\right)\int \frac{(e\cos(c+dx))^{7/2}}{(a-a\sin(c+dx))^2}\,dx}{e^6}$$

$$= \frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3} + \frac{12a^6(e\cos(c+dx))^{5/2}}{de^5\left(a^2-a^2\sin(c+dx)\right)} - \frac{\left(15a^4\right)\int (e\cos(c+dx))^{3/2}}{e^4}$$

$$= -\frac{10a^4\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{de^3} + \frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3} + \frac{12a^6(e\cos(c+d}{de^5\left(a^2-a^2\sin(c\right.}$$

$$= -\frac{10a^4\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{de^3} + \frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3} + \frac{12a^6(e\cos(c+d}{de^5\left(a^2-a^2\sin(c\right.}$$

$$= -\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{e\cos(c+dx)}} - \frac{10a^4\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{de^3} + \frac{4a^7(e}{3de^7(a}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.43

$$\frac{16\sqrt[4]{2}\,a^4(\sin(c+dx)+1)^{3/4}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (16*2^(1/4)*a^4*Hypergeometric2F1[-9/4, -3/4, 1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(3/4))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4\cos(dx+c)^4 - 8\,a^4\cos(dx+c)^2 + 8\,a^4 - 4\left(a^4\cos(dx+c)^2 - 2\,a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^3\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^3*cos(d*x + c)^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(5/2), x)

maple [A] time = 1.28, size = 263, normalized size = 1.73

$$2\left(-8\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+30\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x)

[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^2*(-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-48*sin(1/2*d*x+1/2*c)^5-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-18*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+48*sin(1/2*d*x+1/2*c)^3-20*sin(1/2*d*x+1/2*c))*a^4/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^{4}}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{4}}{(e\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(5/2),x)

[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(5/2),x)

[Out] Timed out

3.229 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=127

$$\frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3}+\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}-\frac{28a^8(e\cos(c+dx))^{3/2}}{5de^5\left(a^4-a^4\sin(c+dx)\right)}$$

```
[Out] 4/5*a^7*(e*cos(d*x+c))^(7/2)/d/e^7/(a-a*sin(d*x+c))^3-28/5*a^8*(e*cos(d*x+c
))^(3/2)/d/e^5/(a^4-a^4*sin(d*x+c))+42/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2
)/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2670, 2680, 2640, 2639}

$$-\frac{28a^8(e\cos(c+dx))^{3/2}}{5de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3}+\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (42*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
 + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(7/2))/(5*d*e^7*(a - a*Sin[c + d*x])^3)
 - (28*a^8*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx = \frac{a^8\int\frac{(e\cos(c+dx))^{9/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}$$
$$= \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3} - \frac{(7a^6)\int\frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^2}\,dx}{5e^6}$$
$$= \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3} - \frac{28a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)} + \frac{(21a^4)\int\sqrt{e\cos(c+dx)}\,dx}{5e^4}$$
$$= \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3} - \frac{28a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)} + \frac{\left(21a^4\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}}{5e^4\sqrt{\cos(c+dx)}}$$
$$= \frac{42a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3} - \frac{28a^6(e\cos(c+dx)}{5de^5\left(a^2-a^2\sin(\right.}$$

Mathematica [C] time = 0.10, size = 66, normalized size = 0.52

$$\frac{8\,2^{3/4}a^4(\sin(c+dx)+1)^{5/4}\,{}_2F_1\left(-\frac{7}{4},-\frac{5}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (8*2^(3/4)*a^4*Hypergeometric2F1[-7/4, -5/4, -1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(5/4))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^4\cos(dx+c)^4},x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^4*cos(d*x + c)^4),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(7/2), x)
```

maple [B] time = 1.80, size = 332, normalized size = 2.61

$$2\left(84\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-128\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(84*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-84*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
80*sin(1/2*d*x+1/2*c)^5+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+80*sin(1/2*d*x+1/2*c)^3-12*sin(1/2*d*x+1/2*c))*a^4/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.230 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=127

$$\frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3}+\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}-\frac{20a^8\sqrt{e\cos(c+dx)}}{21de^5\left(a^4-a^4\sin(c+dx)\right)}$$

```
[Out] 4/7*a^7*(e*cos(d*x+c))^(5/2)/d/e^7/(a-a*sin(d*x+c))^3+10/21*a^4*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)/d/e^4/(e*cos(d*x+c))^(1/2)-20/21*a^8*(e*cos(d*x+c))^(1/2)/
d/e^5/(a^4-a^4*sin(d*x+c))
```

Rubi [A] time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2670, 2680, 2642, 2641}

$$-\frac{20a^8\sqrt{e\cos(c+dx)}}{21de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3}+\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[
c + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(5/2))/(7*d*e^7*(a - a*Sin[c + d*x])^3
) - (20*a^8*Sqrt[e*Cos[c + d*x]])/(21*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx = \frac{a^8\int \frac{(e\cos(c+dx))^{7/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}$$
$$= \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3} - \frac{(5a^6)\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^2}\,dx}{7e^6}$$
$$= \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3} - \frac{20a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^2-a^2\sin(c+dx)\right)} + \frac{(5a^4)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{21e^4}$$
$$= \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3} - \frac{20a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^2-a^2\sin(c+dx)\right)} + \frac{\left(5a^4\sqrt{\cos(c+dx)}\right)\int \frac{}{\sqrt{c}}}{21e^4\sqrt{e\cos(c+}}$$
$$= \frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}} + \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3} - \frac{20a^6\sqrt{e\cos(}}{21de^5\left(a^2-a^2\sin\right.}$$

Mathematica [C] time = 0.11, size = 66, normalized size = 0.52

$$\frac{8\sqrt[4]{2}\,a^4(\sin(c+dx)+1)^{7/4}\,{}_2F_1\left(-\frac{7}{4},-\frac{5}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (8*2^(1/4)*a^4*Hypergeometric2F1[-7/4, -5/4, -3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(7/4))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4\cos(dx+c)^4 - 8a^4\cos(dx+c)^2 + 8a^4 - 4\left(a^4\cos(dx+c)^2 - 2a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^5\cos(dx+c)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^5*cos(d*x + c)^5),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(9/2), x)
```

maple [B] time = 2.00, size = 401, normalized size = 3.16

$$-\frac{2\left(40\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-60\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(40*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6-60*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
112*sin(1/2*d*x+1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+112*sin(1/2*d*x+1/2*c)^3-4*sin(1/2*d*x+1/2*c))*a^4/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

3.231 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=169

$$\frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15de^6\sqrt{\cos(c+dx)}}-\frac{2a^8(e\cos(c+dx))^{3/2}}{15de^7\left(a^4-a^4\sin(c+dx)\right)}-\frac{2a^8(e\cos(c+ \dots}{15de^7\left(a^2-a^2\sin \dots\right.}$$

```
[Out] 4/9*a^7*(e*cos(d*x+c))^(3/2)/d/e^7/(a-a*sin(d*x+c))^3-2/15*a^8*(e*cos(d*x+c
))^(3/2)/d/e^7/(a^2-a^2*sin(d*x+c))^2-2/15*a^8*(e*cos(d*x+c))^(3/2)/d/e^7/(
a^4-a^4*sin(d*x+c))+2/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^6/cos(d*x+
c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2670, 2680, 2681, 2683, 2640, 2639}

$$-\frac{2a^8(e\cos(c+dx))^{3/2}}{15de^7\left(a^4-a^4\sin(c+dx)\right)}-\frac{2a^8(e\cos(c+dx))^{3/2}}{15de^7\left(a^2-a^2\sin(c+dx)\right)^2}+\frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e \dots}}{15de^6\sqrt{\cos(c+ \dots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^6*Sqrt[Cos[c
 + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(3/2))/(9*d*e^7*(a - a*Sin[c + d*x])^3)
 - (2*a^8*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^2 - a^2*Sin[c + d*x])^2) - (
2*a^8*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx &= \frac{a^8\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}\\
&= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{a^6\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^2}\,dx}{3e^6}\\
&= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} - \frac{a^5\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}\,dx}{15e^6}\\
&= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} - \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))} + \frac{a^4}{\cdots}\\
&= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} - \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))} + \frac{(a}{\cdots}\\
&= \frac{2a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^6\sqrt{\cos(c+dx)}} + \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx)}{15de^7(a-a\sin(c+}
\end{aligned}$$

Mathematica [C] time = 0.14, size = 66, normalized size = 0.39

$$\frac{4\,2^{3/4}a^4(\sin(c+dx)+1)^{9/4}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{4};-\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (4*2^(3/4)*a^4*Hypergeometric2F1[-9/4, -3/4, -5/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(9/4))/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4\cos(dx+c)^4 - 8a^4\cos(dx+c)^2 + 8a^4 - 4\left(a^4\cos(dx+c)^2 - 2a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^6\cos(dx+c)^6}, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^6*cos(d*x + c)^6), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="giac")

[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(11/2), x)

maple [B] time = 2.94, size = 514, normalized size = 3.04

$$2\left(48\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x)

[Out] 2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^5*(48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-272*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+176*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-144*sin(1/2*d*x+1/2*c)^5+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+42*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+144*sin(1/2*d*x+1/2*c)^3+4*sin(1/2*d*x+1/2*c))*a^4/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")

[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(11/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(11/2),x)

```
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(11/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

3.232 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{13/2}}\,dx$

Optimal. Leaf size=169

$$\frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77de^6\sqrt{e\cos(c+dx)}}-\frac{2a^8\sqrt{e\cos(c+dx)}}{77de^7\left(a^4-a^4\sin(c+dx)\right)}-\frac{2a^8\sqrt{e\cos(c+\ldots}}{77de^7\left(a^2-a^2\sin(\ldots\right.}$$

```
[Out] -2/77*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^6/(e*cos(d*x+c))^(1/2)+4/11*a^7*(
e*cos(d*x+c))^(1/2)/d/e^7/(a-a*sin(d*x+c))^3-2/77*a^8*(e*cos(d*x+c))^(1/2)/
d/e^7/(a^2-a^2*sin(d*x+c))^2-2/77*a^8*(e*cos(d*x+c))^(1/2)/d/e^7/(a^4-a^4*s
in(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2670, 2680, 2681, 2683, 2642, 2641}

$$-\frac{2a^8\sqrt{e\cos(c+dx)}}{77de^7\left(a^4-a^4\sin(c+dx)\right)}-\frac{2a^8\sqrt{e\cos(c+dx)}}{77de^7\left(a^2-a^2\sin(c+dx)\right)^2}+\frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\ldots\right.}{77de^6\sqrt{e\cos(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (-2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*e^6*Sqrt[e*Cos[
c + d*x]]) + (4*a^7*Sqrt[e*Cos[c + d*x]])/(11*d*e^7*(a - a*Sin[c + d*x])^3)
 - (2*a^8*Sqrt[e*Cos[c + d*x]])/(77*d*e^7*(a^2 - a^2*Sin[c + d*x])^2) - (2*
a^8*Sqrt[e*Cos[c + d*x]])/(77*d*e^7*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{13/2}}\,dx &= \frac{a^8\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}\\
&= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{a^6\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a-a\sin(c+dx))^2}\,dx}{11e^6}\\
&= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{2a^6\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))^2}-\frac{\left(3a^5\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a-a\sin(c+}\\
&= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{2a^6\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))^2}-\frac{2a^5\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))}-\\
&= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{2a^6\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))^2}-\frac{2a^5\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))}-\\
&= -\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77de^6\sqrt{e\cos(c+dx)}}+\frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{2a^6\sqrt{e\cos(c+}{77de^7(a-a\sin(c+}\end{aligned}$$

Mathematica [C] time = 0.24, size = 66, normalized size = 0.39

$$\frac{4\sqrt[4]{2}\,a^4(\sin(c+dx)+1)^{11/4}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11de(e\cos(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (4*2^(1/4)*a^4*Hypergeometric2F1[-11/4, -1/4, -7/4, (1 - Sin[c + d*x])/2]*(
1 + Sin[c + d*x])^(11/4))/(11*d*e*(e*Cos[c + d*x])^(11/2))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4\cos(dx+c)^4-8a^4\cos(dx+c)^2+8a^4-4\left(a^4\cos(dx+c)^2-2a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^7\cos(dx+c)^7},x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^7*cos(d*x + c)^7),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(13/2), x)
```

maple [B] time = 3.24, size = 583, normalized size = 3.45

$$2\left(32\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-80\,\mathrm{EllipticF}\left(\mathrm{cc}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x)
```

```
[Out] 2/77/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*c
)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^6*(32*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^10-80*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8+32*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+80*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^6-64*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-40*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^4+176*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-144*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+176*sin(1/2*d*x+1/2*c)^5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-78*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-176*sin(1/2*d*x+1/2*c)^3-12*sin(1/2*d*x+1/2*c)
)*a^4/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(13/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(13/2),x)
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(13/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(13/2),x)
[Out] Timed out
```

3.233 $\int \frac{(e\cos(c+dx))^{11/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad\sqrt{e\cos(c+dx)}}+\frac{10e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{21ad}+\frac{2e^3\sin(c+dx)(e\cos(c+dx))^{5/2}}{7ad}+\frac{2e(e\text{cc}}{}$$

```
[Out] 2/9*e*(e*cos(d*x+c))^(9/2)/a/d+2/7*e^3*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/a/d+
10/21*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a/d/(e*cos(d*x+c))^(1/2)+10/21*e^5*si
n(d*x+c)*(e*cos(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2682, 2635, 2642, 2641}

$$\frac{10e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{21ad}+\frac{2e^3\sin(c+dx)(e\cos(c+dx))^{5/2}}{7ad}+\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad\sqrt{e\cos(c+dx)}}+\frac{2e(e\text{cc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*(e*Cos[c + d*x])^(9/2))/(9*a*d) + (10*e^6*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2])/(21*a*d*Sqrt[e*Cos[c + d*x]]) + (10*e^5*Sqrt[e*Cos[c + d*
x]]*Sin[c + d*x])/(21*a*d) + (2*e^3*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*a*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{a+a\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{9/2}}{9ad} + \frac{e^2\int (e\cos(c+dx))^{7/2}\,dx}{a}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9ad} + \frac{2e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{7ad} + \frac{(5e^4)\int (e\cos(c+dx))^{3/2}\,dx}{7a}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9ad} + \frac{10e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21ad} + \frac{2e^3(e\cos(c+dx))^{5/2}\sin(c+}{7ad}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9ad} + \frac{10e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21ad} + \frac{2e^3(e\cos(c+dx))^{5/2}\sin(c+}{7ad}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9ad} + \frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad\sqrt{e\cos(c+dx)}} + \frac{10e^5\sqrt{e\cos(c+dx)}\,\sin(c}{21ad}$$

Mathematica [C] time = 0.14, size = 66, normalized size = 0.50

$$-\frac{8\sqrt[4]{2}\,(e\cos(c+dx))^{13/2}\,{}_2F_1\left(-\frac{5}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13ade(\sin(c+dx)+1)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-8*2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[-5/4, 13/4, 17/4, (1
- Sin[c + d*x])/2])/(13*a*d*e*(1 + Sin[c + d*x])^(13/4))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e^5\cos(dx+c)^5}{a\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^5*cos(d*x + c)^5/(a*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.89, size = 251, normalized size = 1.90

$$-\frac{2e^6\left(224\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+144\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-560\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-216\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{d}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/63/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^6*(224*sin
(1/2*d*x+1/2*c)^11+144*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-560*sin(1/2*
d*x+1/2*c)^9-216*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+560*sin(1/2*d*x+1/
2*c)^7+168*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-280*sin(1/2*d*x+1/2*c)^5
+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)-48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*
sin(1/2*d*x+1/2*c)^3-7*sin(1/2*d*x+1/2*c))/d
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(a*sin(d*x + c) + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\cos(c+d\,x))^{11/2}}{a+a\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.234 $\int \frac{(e\cos(c+dx))^{9/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5ad\sqrt{\cos(c+dx)}}+\frac{2e^3\sin(c+dx)(e\cos(c+dx))^{3/2}}{5ad}+\frac{2e(e\cos(c+dx))^{7/2}}{7ad}$$

```
[Out] 2/7*e*(e*cos(d*x+c))^(7/2)/a/d+2/5*e^3*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+
6/5*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2682, 2635, 2640, 2639}

$$\frac{2e^3\sin(c+dx)(e\cos(c+dx))^{3/2}}{5ad}+\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5ad\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*(e*Cos[c + d*x])^(7/2))/(7*a*d) + (6*e^4*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]) + (2*e^3*(e*Cos[c + d*x])^(3/
2)*Sin[c + d*x])/(5*a*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{a+a\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{7/2}}{7ad} + \frac{e^2\int (e\cos(c+dx))^{5/2}\,dx}{a}$$
$$= \frac{2e(e\cos(c+dx))^{7/2}}{7ad} + \frac{2e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{5ad} + \frac{\left(3e^4\right)\int \sqrt{e\cos(c+dx)}\,dx}{5a}$$
$$= \frac{2e(e\cos(c+dx))^{7/2}}{7ad} + \frac{2e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{5ad} + \frac{\left(3e^4\sqrt{e\cos(c+dx)}\right)\int \sqrt{c}}{5a\sqrt{\cos(c+d}}$$
$$= \frac{2e(e\cos(c+dx))^{7/2}}{7ad} + \frac{6e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}} + \frac{2e^3(e\cos(c+dx))^{3/2}\sin}{5ad}$$

Mathematica [C] time = 0.10, size = 66, normalized size = 0.65

$$-\frac{4\,2^{3/4}(e\cos(c+dx))^{11/2}\,{}_2F_1\left(-\frac{3}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11ade(\sin(c+dx)+1)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-4*2^(3/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[-3/4, 11/4, 15/4, (1
- Sin[c + d*x])/2])/(11*a*d*e*(1 + Sin[c + d*x])^(11/4))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}\,e^4\cos\left(dx+c\right)^4}{a\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^4*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.79, size = 216, normalized size = 2.14

$$\frac{2e^5\left(80\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+56\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-160\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-56\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/35/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^5*(80*sin(1
/2*d*x+1/2*c)^9+56*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-160*sin(1/2*d*x+
```

```
1/2*c)^7-56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+120*sin(1/2*d*x+1/2*c)^
5+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)+14*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-40
*sin(1/2*d*x+1/2*c)^3+5*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{9/2}}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.235 $\int \frac{(e\cos(c+dx))^{7/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}+\frac{2e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{3ad}+\frac{2e(e\cos(c+dx))^{5/2}}{5ad}$$

```
[Out] 2/5*e*(e*cos(d*x+c))^(5/2)/a/d+2/3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a/d/(e*c
os(d*x+c))^(1/2)+2/3*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2682, 2635, 2642, 2641}

$$\frac{2e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{3ad}+\frac{2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*(e*Cos[c + d*x])^(5/2))/(5*a*d) + (2*e^4*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2])/(3*a*d*Sqrt[e*Cos[c + d*x]]) + (2*e^3*Sqrt[e*Cos[c + d*x]]
*Sin[c + d*x])/(3*a*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{a+a\sin(c+dx)}dx = \frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{e^2\int(e\cos(c+dx))^{3/2}\,dx}{a}$$
$$=\frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3ad}+\frac{e^4\int\frac{1}{\sqrt{e\cos(c+dx)}}dx}{3a}$$
$$=\frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3ad}+\frac{\left(e^4\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}}{3a\sqrt{e\cos(c+dx)}}$$
$$=\frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}+\frac{2e^3\sqrt{e\cos(c+dx)}\,\sin(c+d}{3ad}$$

Mathematica [C] time = 0.07, size = 66, normalized size = 0.65

$$-\frac{4\sqrt[4]{2}\,(e\cos(c+dx))^{9/2}\,{}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9ade(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x]),x]
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[-1/4, 9/4, 13/4, (1 - Sin[c + d*x])/2])/(9*a*d*e*(1 + Sin[c + d*x])^(9/4))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e^3\cos(dx+c)^3}{a\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*e^3*cos(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a), x)
```

maple [A] time = 1.09, size = 181, normalized size = 1.79

$$-\frac{2e^4\left(24\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-36\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticF}\left(\text{co}\right.\right.}{15a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/15/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(24*sin(
1/2*d*x+1/2*c)^7+20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-36*sin(1/2*d*x+
1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+18*sin(1/2*d*x+1/2*c)^3-3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cos(dx+c))^{\frac{7}{2}}}{a \sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\ \cos(c+d\,x))^{7/2}}{a+a\ \sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.236 $\int \frac{(e\cos(c+dx))^{5/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{ad\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{3ad}$$

```
[Out] 2/3*e*(e*cos(d*x+c))^(3/2)/a/d+2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a/d/co
s(d*x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2682, 2640, 2639}

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{ad\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*(e*Cos[c + d*x])^(3/2))/(3*a*d) + (2*e^2*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{5/2}}{a+a\sin(c+dx)}\,dx &= \frac{2e(e\cos(c+dx))^{3/2}}{3ad}+\frac{e^2\int \sqrt{e\cos(c+dx)}\,dx}{a}\\ &= \frac{2e(e\cos(c+dx))^{3/2}}{3ad}+\frac{\left(e^2\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}}\\ &= \frac{2e(e\cos(c+dx))^{3/2}}{3ad}+\frac{2e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.10, size = 66, normalized size = 0.97

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7ade(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[1/4, 7/4, 11/4, (1 - S
in[c + d*x])/2])/(7*a*d*e*(1 + Sin[c + d*x])^(7/4))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e^2\cos(dx+c)^2}{a\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^2*cos(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a), x)
```

maple [A] time = 0.71, size = 122, normalized size = 1.79

$$\frac{2e^3\left(4\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,-4\left(\sin^3\left(\frac{dx}{2}\right.\right.}{3a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/3/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(4*sin(1/2
*d*x+1/2*c)^5+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-4*sin(1/2*d*x+1/2*c)^3+sin(1/2*d*x
+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,\cos\left(c+d\,x\right)\right)^{5/2}}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.237 $\int \frac{(e\cos(c+dx))^{3/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{ad}$$

```
[Out] 2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a/d/(e*cos(d*x+c))^(1/2)+2*e*(e*cos(d*x+c
))^(1/2)/a/d
```

Rubi [A] time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2682, 2642, 2641}

$$\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*Sqrt[e*Cos[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2])/(a*d*Sqrt[e*Cos[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{3/2}}{a+a\sin(c+dx)}\,dx &= \frac{2e\sqrt{e\cos(c+dx)}}{ad}+\frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{a}\\
&= \frac{2e\sqrt{e\cos(c+dx)}}{ad}+\frac{\left(e^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a\sqrt{e\cos(c+dx)}}\\
&= \frac{2e\sqrt{e\cos(c+dx)}}{ad}+\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{e\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 1.00

$$-\frac{2\sqrt[4]{2}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5ade(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[3/4, 5/4, 9/4, (1 - Si
n[c + d*x])/2])/(5*a*d*e*(1 + Sin[c + d*x])^(5/4))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e\cos(dx+c)}{a\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\cos(dx+c))^{\frac{3}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a), x)
```

maple [A] time = 0.52, size = 110, normalized size = 1.67

$$-\frac{2e^{2}\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}+2\left(\sin^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*((sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)+2*sin(1/2*d*x+1/2*c)^3-sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\cos(dx+c))^{\frac{3}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(e\,\cos(c+d\,x))^{3/2}}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

3.238 $\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$-\frac{2(e\cos(c+dx))^{3/2}}{de(a\sin(c+dx)+a)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] -2*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1
/2)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2683, 2640, 2639}

$$-\frac{2(e\cos(c+dx))^{3/2}}{de(a\sin(c+dx)+a)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]]
) - (2*(e*Cos[c + d*x])^(3/2))/(d*e*(a + a*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx &= -\frac{2(e\cos(c+dx))^{3/2}}{de(a+a\sin(c+dx))}-\frac{\int \sqrt{e\cos(c+dx)}\,dx}{a}\\ &= -\frac{2(e\cos(c+dx))^{3/2}}{de(a+a\sin(c+dx))}-\frac{\sqrt{e\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}}\\ &= -\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{de(a+a\sin(c+dx))}\end{aligned}$$

Mathematica [C] time = 0.04, size = 66, normalized size = 0.89

$$-\frac{2^{3/4}(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3ade(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(2^(3/4)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 5/4, 7/4, (1 -
Sin[c + d*x])/2])/(a*d*e*(1 + Sin[c + d*x])^(3/4))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{a\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(dx+c)}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a), x)
```

maple [A] time = 1.14, size = 115, normalized size = 1.55

$$-\frac{2\left(\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\text{s}}{\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a*(EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))*e/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(dx+c)}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\cos{\left(c+dx \right)}}}{\sin{\left(c+dx \right)}+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c)),x)
[Out] Integral(sqrt(e*cos(c + d*x))/(sin(c + d*x) + 1), x)/a
```

3.239 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=78

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{3de(a\sin(c+dx)+a)}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a/d/(e*cos(d*x+c))^(1/2)-2/3*(e*cos(d*x+c))
^(1/2)/d/e/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2683, 2642, 2641}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{3de(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a*d*Sqrt[e*Cos[c + d*x]
]) - (2*Sqrt[e*Cos[c + d*x]])/(3*d*e*(a + a*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}\,dx &= -\frac{2\sqrt{e\cos(c+dx)}}{3de(a+a\sin(c+dx))}+\frac{\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a}\\
&= -\frac{2\sqrt{e\cos(c+dx)}}{3de(a+a\sin(c+dx))}+\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a\sqrt{e\cos(c+dx)}}\\
&= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{3de(a+a\sin(c+dx))}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 64, normalized size = 0.82

$$-\frac{\sqrt[4]{2}\sqrt{e\cos(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{ade\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])),x]
```

```
[Out] -((2^(1/4)*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 7/4, 5/4, (1 - Sin[c
 + d*x])/2])/(a*d*e*(1 + Sin[c + d*x])^(1/4)))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{ae\cos(dx+c)\sin(dx+c)+ae\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a*e*cos(d*x + c)*sin(d*x + c) + a*e*cos(d*x
+ c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)), x)
```

maple [B] time = 1.48, size = 190, normalized size = 2.44

$$-\frac{2\left(2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\text{Elli}}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)
^2*e+e)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))),x)
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\cos\left(c+dx\right)}\sin\left(c+dx\right)+\sqrt{e\cos\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*cos(c + d*x))*sin(c + d*x) + sqrt(e*cos(c + d*x))), x)/a
```

3.240 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=112

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5ade^2\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{2}{5de(a\sin(c+dx)+a)\sqrt{e\cos(c+dx)}}$$

```
[Out] 6/5*sin(d*x+c)/a/d/e/(e*cos(d*x+c))^(1/2)-2/5/d/e/(a+a*sin(d*x+c))/(e*cos(d
*x+c))^(1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2683, 2636, 2640, 2639}

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5ade^2\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{2}{5de(a\sin(c+dx)+a)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (-6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a*d*e^2*Sqrt[Cos[c +
 d*x]]) + (6*Sin[c + d*x])/(5*a*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(5*d*e*Sqrt[e
*Cos[c + d*x]]*(a + a*Sin[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))}dx = -\frac{2}{5de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))} + \frac{3\int \frac{1}{(e\cos(c+dx))^{3/2}}dx}{5a}$$
$$= \frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}} - \frac{2}{5de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))} - \frac{3}{}$$
$$= \frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}} - \frac{2}{5de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))} - \frac{(3}{}$$
$$= -\frac{6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ade^2\sqrt{\cos(c+dx)}} + \frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}} - \frac{}{5de\sqrt{}}$$

Mathematica [C] time = 0.06, size = 63, normalized size = 0.56

$$\frac{\sqrt[4]{\sin(c+dx)+1}\;{}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{\sqrt[4]{2}\,ade\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Hypergeometric2F1[-1/4, 9/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])
^(1/4))/(2^(1/4)*a*d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{ae^2\cos(dx+c)^2\sin(dx+c)+ae^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a*e^2*cos(d*x + c)^2*sin(d*x + c) + a*e^2*co
s(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)), x)
```

maple [B] time = 2.00, size = 304, normalized size = 2.71

$$-\frac{2\left(12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a/sin(1/2*d*x+1/2*c)
/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2
*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+sin(1/2
*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e \cos\left(dx + c\right)\right)^{\frac{3}{2}} \left(a \sin\left(dx + c\right) + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.241 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=112

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ade^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}}-\frac{2}{7de(a\sin(c+dx)+a)(e\cos(c+dx))^{3/2}}$$

```
[Out] 10/21*sin(d*x+c)/a/d/e/(e*cos(d*x+c))^(3/2)-2/7/d/e/(e*cos(d*x+c))^(3/2)/(a
+a*sin(d*x+c))+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a/d/e^2/(e*cos(d*x+c))^(1/
2)
```

Rubi [A] time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2683, 2636, 2642, 2641}

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ade^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}}-\frac{2}{7de(a\sin(c+dx)+a)(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a*d*e^2*Sqrt[e*Cos[c
 + d*x]]) + (10*Sin[c + d*x])/(21*a*d*e*(e*Cos[c + d*x])^(3/2)) - 2/(7*d*e*(
e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))}\,dx = -\frac{2}{7de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))} + \frac{5\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx}{7a}$$

$$= \frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}} - \frac{2}{7de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))} +$$

$$= \frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}} - \frac{2}{7de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))} +$$

$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ade^2\sqrt{e\cos(c+dx)}} + \frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}} - \frac{}{7de(e\,c}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.59

$$\frac{(\sin(c+dx)+1)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{11}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3\,2^{3/4}ade(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Hypergeometric2F1[-3/4, 11/4, 1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(3/4))/(3*2^(3/4)*a*d*e*(e*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{ae^3\cos(dx+c)^3\sin(dx+c)+ae^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a*e^3*cos(d*x + c)^3*sin(d*x + c) + a*e^3*co
s(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)), x)
```

maple [B] time = 2.25, size = 375, normalized size = 3.35

$$-\frac{2\left(40\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-60\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^2*(40*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6-60*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4+40*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*sin
(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sin\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.242 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=143

$$-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15ade^4\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}}-\frac{2}{9de(a\sin(c+dx)+a)(e\cos(c+dx))^{5/2}}$$

```
[Out] 14/45*sin(d*x+c)/a/d/e/(e*cos(d*x+c))^(5/2)-2/9/d/e/(e*cos(d*x+c))^(5/2)/(a
+a*sin(d*x+c))+14/15*sin(d*x+c)/a/d/e^3/(e*cos(d*x+c))^(1/2)-14/15*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*(e*cos(d*x+c))^(1/2)/a/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2683, 2636, 2640, 2639}

$$\frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15ade^4\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}}-\frac{2}{9de(a\sin(c+dx)+a)(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (-14*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a*d*e^4*Sqrt[Cos[c
 + d*x]]) + (14*Sin[c + d*x])/(45*a*d*e*(e*Cos[c + d*x])^(5/2)) + (14*Sin[c
 + d*x])/(15*a*d*e^3*Sqrt[e*Cos[c + d*x]]) - 2/(9*d*e*(e*Cos[c + d*x])^(5/2
)*(a + a*Sin[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))}\,dx = -\frac{2}{9de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))} + \frac{7\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx}{9a}$$
$$= \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} - \frac{2}{9de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))}$$
$$= \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}} - \frac{}{9de(e\cos(c+}$$
$$= \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}} - \frac{}{9de(e\cos(c+}$$
$$= -\frac{14\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^4\sqrt{\cos(c+dx)}} + \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} +$$

Mathematica [C] time = 0.10, size = 66, normalized size = 0.46

$$\frac{(\sin(c+dx)+1)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{13}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{10\sqrt[4]{2}\,ade(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Hypergeometric2F1[-5/4, 13/4, -1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(5/4))/(10*2^(1/4)*a*d*e*(e*Cos[c + d*x])^(5/2))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{ae^4\cos(dx+c)^4\sin(dx+c)+ae^4\cos(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a*e^4*cos(d*x + c)^4*sin(d*x + c) + a*e^4*co
s(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)), x)
```

maple [B] time = 2.72, size = 488, normalized size = 3.41

$$-\frac{2\left(336\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-672\left(\sin^{10}\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^3*(336*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-672*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-672*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^6+1344*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+504*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^4-1064*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)-168*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+392*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-66*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+5*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(e\,\cos(c+d\,x))^{7/2}\,(a+a\,\sin(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.243 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=145

$$\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}+\frac{6e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{7a^2d}+\frac{18e^3\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^2d}+\frac{4e(e\,c}{5d\,(a^2\,s}$$

```
[Out] 18/35*e^3*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/a^2/d+4/5*e*(e*cos(d*x+c))^(9/2)/
d/(a^2+a^2*sin(d*x+c))+6/7*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/(e*cos(d*x
+c))^(1/2)+6/7*e^5*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2680, 2635, 2642, 2641}

$$\frac{6e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{7a^2d}+\frac{18e^3\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^2d}+\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}+\frac{4e(e\,c}{5d\,(a^2\,s}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c
 + d*x]]) + (6*e^5*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(7*a^2*d) + (18*e^3*(e
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(35*a^2*d) + (4*e*(e*Cos[c + d*x])^(9/2)
)/(5*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^2}\,dx = \frac{4e(e\cos(c+dx))^{9/2}}{5d\left(a^2+a^2\sin(c+dx)\right)} + \frac{\left(9e^2\right)\int (e\cos(c+dx))^{7/2}\,dx}{5a^2}$$

$$= \frac{18e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^2d} + \frac{4e(e\cos(c+dx))^{9/2}}{5d\left(a^2+a^2\sin(c+dx)\right)} + \frac{\left(9e^4\right)\int (e\cos(c+dx)}{7a^2}$$

$$= \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^2d} + \frac{18e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^2d} + \frac{4e(e\cos(c+}{5d\left(a^2+a^2\sin\right.}$$

$$= \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^2d} + \frac{18e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^2d} + \frac{4e(e\cos(c+}{5d\left(a^2+a^2\sin\right.}$$

$$= \frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^2d} + \frac{18e^3(e\cos(c+}{3}$$

Mathematica [C] time = 0.18, size = 66, normalized size = 0.46

$$-\frac{4\sqrt[4]{2}\,(e\cos(c+dx))^{13/2}\,{}_2F_1\left(-\frac{1}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13a^2de(\sin(c+dx)+1)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^2,x]
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[-1/4, 13/4, 17/4, (1
 - Sin[c + d*x])/2])/(13*a^2*d*e*(1 + Sin[c + d*x])^(13/4))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e^5\cos(dx+c)^5}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] integral(-sqrt(e*cos(d*x + c))*e^5*cos(d*x + c)^5/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.92, size = 203, normalized size = 1.40

$$-\frac{2e^6\left(-80\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+112\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-168\left(\sin^5\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/35/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^6*(-80*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+112*sin(1/2*d*x+1/2*c)^7-168*sin(1/2*d*x+1/2*c)^5+15*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)-20*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+84*sin(1/2*d*x+1/2*c)
^3-14*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{11/2}}{(a+a\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.244 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$\frac{14e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^2d\sqrt{\cos(c+dx)}}+\frac{14e^3\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^2d}+\frac{4e(e\cos(c+dx))^{7/2}}{3d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] 14/15*e^3*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+4/3*e*(e*cos(d*x+c))^(7/2)/
d/(a^2+a^2*sin(d*x+c))+14/5*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^2/d/cos(d
*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2680, 2635, 2640, 2639}

$$\frac{14e^3\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^2d}+\frac{14e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^2d\sqrt{\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^{7/2}}{3d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (14*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c
 + d*x]]) + (14*e^3*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*a^2*d) + (4*e*
(e*Cos[c + d*x])^(7/2))/(3*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^2}\,dx = \frac{4e(e\cos(c+dx))^{7/2}}{3d\left(a^2+a^2\sin(c+dx)\right)} + \frac{\left(7e^2\right)\int (e\cos(c+dx))^{5/2}\,dx}{3a^2}$$

$$= \frac{14e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^2d} + \frac{4e(e\cos(c+dx))^{7/2}}{3d\left(a^2+a^2\sin(c+dx)\right)} + \frac{\left(7e^4\right)\int \sqrt{e\cos(c-}}{5a^2}$$

$$= \frac{14e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^2d} + \frac{4e(e\cos(c+dx))^{7/2}}{3d\left(a^2+a^2\sin(c+dx)\right)} + \frac{\left(7e^4\sqrt{e\cos(c+dx}\right.}{5a^2\sqrt{c}}$$

$$= \frac{14e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}} + \frac{14e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^2d} + \frac{4e(e}{3d\left(a^2\right.}$$

Mathematica [C] time = 0.10, size = 66, normalized size = 0.58

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11a^2de(\sin(c+dx)+1)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^2,x]
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[1/4, 11/4, 15/4, (1 -
 Sin[c + d*x])/2])/(11*a^2*d*e*(1 + Sin[c + d*x])^(11/4))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e^4\cos(dx+c)^4}{a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] integral(-sqrt(e*cos(d*x + c))*e^4*cos(d*x + c)^4/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.00, size = 190, normalized size = 1.67

$$\frac{2e^5\left(-24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+24\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+40\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\,\text{EllipticE}\left(\right.\right.}{15a^2\sin\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/15/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^5*(-24*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+40*sin(1/2*d*x+1/2*c)^5+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-40*sin(1/2*d*x+1/2*c)^3+10*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{9/2}}{(a+a\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.245 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}+\frac{10e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{3a^2d}+\frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] 4*e*(e*cos(d*x+c))^(5/2)/d/(a^2+a^2*sin(d*x+c))+10/3*e^4*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)/a^2/d/(e*cos(d*x+c))^(1/2)+10/3*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/
2)/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2680, 2635, 2642, 2641}

$$\frac{10e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{3a^2d}+\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[e*Cos[c
 + d*x]]) + (10*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) + (4*e*(e*
Cos[c + d*x])^(5/2))/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^2}\,dx = \frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2+a^2\sin(c+dx)\right)} + \frac{\left(5e^2\right)\int (e\cos(c+dx))^{3/2}\,dx}{a^2}$$

$$= \frac{10e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2+a^2\sin(c+dx)\right)} + \frac{\left(5e^4\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a^2}$$

$$= \frac{10e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2+a^2\sin(c+dx)\right)} + \frac{\left(5e^4\sqrt{\cos(c+dx)}\right)\int \frac{}{\sqrt{c}}}{3a^2\sqrt{e\cos(c+a}}$$

$$= \frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}} + \frac{10e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{4e(e\cos(c}{d\left(a^2+a^2\sin\right.}$$

Mathematica [C] time = 0.07, size = 66, normalized size = 0.59

$$-\frac{2\sqrt[4]{2}\,(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9a^2de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[3/4, 9/4, 13/4, (1 - S
in[c + d*x])/2])/(9*a^2*d*e*(1 + Sin[c + d*x])^(9/4))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}\,e^3\cos\left(dx+c\right)^3}{a^2\cos\left(dx+c\right)^2-2\,a^2\sin\left(dx+c\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^3*cos(d*x + c)^3/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.76, size = 155, normalized size = 1.38

$$-\frac{2e^4\left(-4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right. +}{3a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/3/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-4*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+2*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+12*sin(1/2*d*x+1/2*c)^3-6*sin(1/2*d*x+1/2*c)
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{7/2}}{(a+a\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.246 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{6e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{a^2d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] -4*e*(e*cos(d*x+c))^(3/2)/d/(a^2+a^2*sin(d*x+c))-6*e^2*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*
x+c))^(1/2)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2680, 2640, 2639}

$$-\frac{6e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{a^2d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-6*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(a^2*d*Sqrt[Cos[c +
 d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^2}\,dx &= -\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\left(3e^2\right)\int \sqrt{e\cos(c+dx)}\,dx}{a^2}\\
&= -\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\left(3e^2\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{a^2\sqrt{\cos(c+dx)}}\\
&= -\frac{6e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2+a^2\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 66, normalized size = 0.84

$$-\frac{2^{3/4}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7a^2de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/7*(2^(3/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[5/4, 7/4, 11/4, (1 -
 Sin[c + d*x])/2])/(a^2*d*e*(1 + Sin[c + d*x])^(7/4))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e^2\cos(dx+c)^2}{a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^2*cos(d*x + c)^2/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^2, x)
```

maple [A] time = 1.15, size = 120, normalized size = 1.52

$$-\frac{2\left(3\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}-4\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a^2*(3*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*
c))*e^3/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.247 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] -2/3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/(e*cos(d*x+c))^(1/2)-4/3*e*(e*co
s(d*x+c))^(1/2)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2680, 2642, 2641}

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[e*Cos[c
 + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(3*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^2}\,dx &= -\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2+a^2\sin(c+dx)\right)}-\frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a^2}\\ &= -\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\left(e^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2\sqrt{e\cos(c+dx)}}\\ &= -\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2+a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.80

$$-\frac{\sqrt[4]{2}(e\cos(c+dx))^{5/2}{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5a^2de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/5*(2^(1/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 7/4, 9/4, (1 -
Sin[c + d*x])/2])/(a^2*d*e*(1 + Sin[c + d*x])^(5/4))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e\cos(dx+c)}{a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e*cos(d*x + c)/(a^2*cos(d*x + c)^2 - 2*a^2*s
in(d*x + c) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\cos(dx+c))^{\frac{3}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^2, x)
```

maple [A] time = 1.40, size = 193, normalized size = 2.33

$$\frac{2\left(2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\text{Ellip}}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*c
))*e^2/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\cos(dx+c))^{\frac{3}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

3.248 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2\sin(c+dx)+a^2\right)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^2d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{5de(a\sin(c+dx)+a)^2}$$

```
[Out] -2/5*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^2-2/5*(e*cos(d*x+c))^(3/2)/d
/e/(a^2+a^2*sin(d*x+c))-2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c
)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2681, 2683, 2640, 2639}

$$-\frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2\sin(c+dx)+a^2\right)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^2d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{5de(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d
*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(5*d*e*(a + a*Sin[c + d*x])^2) - (2*(e*C
os[c + d*x])^(3/2))/(5*d*e*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx = -\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2} + \frac{\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx}{5a}$$
$$= -\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2} - \frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2+a^2\sin(c+dx)\right)} - \frac{\int \sqrt{e\cos(c+dx)}\,dx}{5a^2}$$
$$= -\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2} - \frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2+a^2\sin(c+dx)\right)} - \frac{\sqrt{e\cos(c+dx)}\int \sqrt{\cos(c}}{5a^2\sqrt{\cos(c+dx)}}$$
$$= -\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}} - \frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2} - \frac{2(e\cos(c+dx}{5de\left(a^2+a^2\sin(c\right.}$$

Mathematica [C] time = 0.04, size = 66, normalized size = 0.57

$$-\frac{(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3\sqrt[4]{2}\,a^2de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*((e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 9/4, 7/4, (1 - Sin[c +
d*x])/2])/(2^(1/4)*a^2*d*e*(1 + Sin[c + d*x])^(3/4))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^2-2\,a^2\sin\left(dx+c\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2
*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{\left(a\sin\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^2, x)
```

maple [B] time = 2.17, size = 303, normalized size = 2.61

$$-\frac{2\left(4\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-4*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x
+1/2*c))*e/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(dx+c)}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,\cos(c+d\,x)}}{(a+a\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.249 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2\sin(c+dx)+a^2\right)}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{7de(a\sin(c+dx)+a)^2}$$

```
[Out] 2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/(e*cos(d*x+c))^(1/2)-2/7*(e*cos(d*x+c
))^(1/2)/d/e/(a+a*sin(d*x+c))^2-2/7*(e*cos(d*x+c))^(1/2)/d/e/(a^2+a^2*sin(d
*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2681, 2683, 2642, 2641}

$$-\frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2\sin(c+dx)+a^2\right)}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{7de(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c + d*
x]]) - (2*Sqrt[e*Cos[c + d*x]])/(7*d*e*(a + a*Sin[c + d*x])^2) - (2*Sqrt[e*
Cos[c + d*x]])/(7*d*e*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2} + \frac{3\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}\,dx}{7a}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2} - \frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2+a^2\sin(c+dx)\right)} + \frac{\int \frac{1}{\sqrt{e\cos(c+dx)}}}{7a^2}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2} - \frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2+a^2\sin(c+dx)\right)} + \frac{\sqrt{\cos(c+dx)}}{7a^2\sqrt{e\,c}}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}} - \frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2} - \frac{2\sqrt{e}}{7de\left(a^2+\right.}$$

Mathematica [C] time = 0.04, size = 64, normalized size = 0.55

$$-\frac{\sqrt{e\cos(c+dx)}\;{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{3/4}a^2de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] -((Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 11/4, 5/4, (1 - Sin[c + d*x]
)/2])/(2^(3/4)*a^2*d*e*(1 + Sin[c + d*x])^(1/4)))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{a^2e\cos(dx+c)^3-2\,a^2e\cos(dx+c)\sin(dx+c)-2\,a^2e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(a^2*e*cos(d*x + c)^3 - 2*a^2*e*cos(d*x + c)
*sin(d*x + c) - 2*a^2*e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^2), x)
```

maple [B] time = 2.24, size = 372, normalized size = 3.21

$$-\frac{2\left(8\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/7/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(8*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)+6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1
/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,\cos(c+d\,x)}\,\left(a+a\,\sin(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.250 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3a^2de^2\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}}-\frac{2}{9de\left(a^2\sin(c+dx)+a^2\right)\sqrt{e\cos(c+dx)}}-\frac{\ldots}{9de(a\sin(c\ldots}$$

```
[Out] 2/3*sin(d*x+c)/a^2/d/e/(e*cos(d*x+c))^(1/2)-2/9/d/e/(a+a*sin(d*x+c))^2/(e*c
os(d*x+c))^(1/2)-2/9/d/e/(a^2+a^2*sin(d*x+c))/(e*cos(d*x+c))^(1/2)-2/3*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2))*(e*cos(d*x+c))^(1/2)/a^2/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2681, 2683, 2636, 2640, 2639}

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3a^2de^2\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}}-\frac{2}{9de\left(a^2\sin(c+dx)+a^2\right)\sqrt{e\cos(c+dx)}}-\frac{\ldots}{9de(a\sin(c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*e^2*Sqrt[Cos[c
 + d*x]]) + (2*Sin[c + d*x])/(3*a^2*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(9*d*e*Sq
rt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2) - 2/(9*d*e*Sqrt[e*Cos[c + d*x]]*
(a^2 + a^2*Sin[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}\,dx = -\frac{2}{9de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2} + \frac{5\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\,\mathrm{si}}}{9a}$$

$$= -\frac{2}{9de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2} - \frac{2}{9de\sqrt{e\cos(c+dx)}\,(a}$$

$$= \frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}} - \frac{2}{9de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2} -$$

$$= \frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}} - \frac{2}{9de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2} -$$

$$= -\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2de^2\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}} - \frac{}{9d}$$

Mathematica [C] time = 0.07, size = 66, normalized size = 0.44

$$\frac{\sqrt[4]{\sin(c+dx)+1}\;{}_2F_1\left(-\frac{1}{4},\frac{13}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{2\sqrt[4]{2}\,a^2de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (Hypergeometric2F1[-1/4, 13/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(1/4))/(2*2^(1/4)*a^2*d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{a^2e^2\cos(dx+c)^4 - 2\,a^2e^2\cos(dx+c)^2\sin(dx+c) - 2\,a^2e^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(a^2*e^2*cos(d*x + c)^4 - 2*a^2*e^2*cos(d*x
+ c)^2*sin(d*x + c) - 2*a^2*e^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}\,(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^2), x)
```

maple [B] time = 3.08, size = 488, normalized size = 3.25

$$-\frac{2\left(48\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/9/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)
^4-8*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
2*e+e)^(1/2)/e*(48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2
*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1
/2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-12*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)+2*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.251 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2de^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}}-\frac{2}{11de\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{3/2}}-\frac{}{11de}$$

```
[Out] 10/33*sin(d*x+c)/a^2/d/e/(e*cos(d*x+c))^(3/2)-2/11/d/e/(e*cos(d*x+c))^(3/2)
/(a+a*sin(d*x+c))^2-2/11/d/e/(e*cos(d*x+c))^(3/2)/(a^2+a^2*sin(d*x+c))+10/3
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/e^2/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2681, 2683, 2636, 2642, 2641}

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2de^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}}-\frac{2}{11de\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{3/2}}-\frac{}{11de}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(33*a^2*d*e^2*Sqrt[e*Cos[
c + d*x]]) + (10*Sin[c + d*x])/(33*a^2*d*e*(e*Cos[c + d*x])^(3/2)) - 2/(11*
d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2) - 2/(11*d*e*(e*Cos[c + d
*x])^(3/2)*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}\,dx = -\frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2} + \frac{7\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin \ldots}}{11a}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2} - \frac{2 \ldots}{11de(e\cos(c+dx))^{3/2} \ldots}$$

$$= \frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}} - \frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx)) \ldots}$$

$$= \frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}} - \frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx)) \ldots}$$

$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2de^2\sqrt{e\cos(c+dx)}} + \frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}} - \frac{\ldots}{11de \ldots}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.44

$$\frac{(\sin(c+dx)+1)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{15}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{6\ 2^{3/4}a^2de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (Hypergeometric2F1[-3/4, 15/4, 1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(3/4))/(6*2^(3/4)*a^2*d*e*(e*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{a^2e^3\cos(dx+c)^5 - 2\,a^2e^3\cos(dx+c)^3\sin(dx+c) - 2\,a^2e^3\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(a^2*e^3*cos(d*x + c)^5 - 2*a^2*e^3*cos(d*x
+ c)^3*sin(d*x + c) - 2*a^2*e^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^2), x)
```

maple [B] time = 3.65, size = 557, normalized size = 3.71

$$-\frac{2\left(160\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-400\,\mathrm{EllipticF}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/33/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*
c)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^2*(160*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*s
in(1/2*d*x+1/2*c)^10-400*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8+160*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+400*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1
/2*d*x+1/2*c)^6-320*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-200*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+264*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+50*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-104*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+28*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-6*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.252 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=181

$$-\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{65a^2de^4\sqrt{\cos(c+dx)}}+\frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}}+\frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}}-\frac{2}{13de\left(a^2\sin(c+dx)+a\right.}$$

```
[Out] 14/65*sin(d*x+c)/a^2/d/e/(e*cos(d*x+c))^(5/2)-2/13/d/e/(e*cos(d*x+c))^(5/2)
/(a+a*sin(d*x+c))^2-2/13/d/e/(e*cos(d*x+c))^(5/2)/(a^2+a^2*sin(d*x+c))+42/6
5*sin(d*x+c)/a^2/d/e^3/(e*cos(d*x+c))^(1/2)-42/65*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))
^(1/2)/a^2/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2681, 2683, 2636, 2640, 2639}

$$\frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}}-\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{65a^2de^4\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}}-\frac{2}{13de\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (-42*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*e^4*Sqrt[Cos
[c + d*x]]) + (14*Sin[c + d*x])/(65*a^2*d*e*(e*Cos[c + d*x])^(5/2)) + (42*S
in[c + d*x])/(65*a^2*d*e^3*Sqrt[e*Cos[c + d*x]]) - 2/(13*d*e*(e*Cos[c + d*x
])^(5/2)*(a + a*Sin[c + d*x])^2) - 2/(13*d*e*(e*Cos[c + d*x])^(5/2)*(a^2 +
a^2*Sin[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}\,dx = -\frac{2}{13de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2} + \frac{9\int \frac{1}{(e\cos(c+dx))^{7/2}(a+}}{13a}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2} - \frac{}{13de(e\cos(c+dx)}$$

$$= \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}} - \frac{2}{13de(e\cos(c+dx))^{5/2}(a+a\sin(c+d}$$

$$= \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}} + \frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}} - \frac{}{13de(e\cos}$$

$$= \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}} + \frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}} - \frac{}{13de(e\cos}$$

$$= -\frac{42\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^4\sqrt{\cos(c+dx)}} + \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}}$$

Mathematica [C] time = 0.11, size = 66, normalized size = 0.36

$$\frac{(\sin(c+dx)+1)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{17}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{20\sqrt[4]{2}\,a^2de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (Hypergeometric2F1[-5/4, 17/4, -1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(5/4))/(20*2^(1/4)*a^2*d*e*(e*Cos[c + d*x])^(5/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{a^2e^4\cos(dx+c)^6 - 2a^2e^4\cos(dx+c)^4\sin(dx+c) - 2a^2e^4\cos(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(a^2*e^4*cos(d*x + c)^6 - 2*a^2*e^4*cos(d*x
+ c)^4*sin(d*x + c) - 2*a^2*e^4*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^2), x)
```

maple [B] time = 4.59, size = 670, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x)
[Out] -2/65/(64*sin(1/2*d*x+1/2*c)^12-192*sin(1/2*d*x+1/2*c)^10+240*sin(1/2*d*x+1
/2*c)^8-160*sin(1/2*d*x+1/2*c)^6+60*sin(1/2*d*x+1/2*c)^4-12*sin(1/2*d*x+1/2
*c)^2+1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(13
44*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12-2688*sin(1/2*d*x+1/2*c)^1
4*cos(1/2*d*x+1/2*c)-4032*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+80
64*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+5040*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*s
in(1/2*d*x+1/2*c)^8-10304*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-3360*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+7168*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+1260*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-2896*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-252*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2+656*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+21*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)-86*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*sin(1/2*d*x+1/2*c))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^2),x)
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

3.253 $\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=169

$$\frac{26e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}}+\frac{26e^7\sin(c+dx)\sqrt{e\cos(c+dx)}}{21a^3d}+\frac{26e^5\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^3d}+\frac{26e^3}{}$$

```
[Out] 26/45*e^3*(e*cos(d*x+c))^(9/2)/a^3/d+26/35*e^5*(e*cos(d*x+c))^(5/2)*sin(d*x
+c)/a^3/d+4/5*e*(e*cos(d*x+c))^(13/2)/a/d/(a+a*sin(d*x+c))^2+26/21*e^8*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)/a^3/d/(e*cos(d*x+c))^(1/2)+26/21*e^7*sin(d*x+c)*(e*
cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.18, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2680, 2682, 2635, 2642, 2641}

$$\frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d}+\frac{26e^7\sin(c+dx)\sqrt{e\cos(c+dx)}}{21a^3d}+\frac{26e^5\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^3d}+\frac{26e^8\sqrt{\cos(c+dx)}}{21a^3d\sqrt{e\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (26*e^3*(e*Cos[c + d*x])^(9/2))/(45*a^3*d) + (26*e^8*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2])/(21*a^3*d*Sqrt[e*Cos[c + d*x]]) + (26*e^7*Sqrt[e*Co
s[c + d*x]]*Sin[c + d*x])/(21*a^3*d) + (26*e^5*(e*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(35*a^3*d) + (4*e*(e*Cos[c + d*x])^(13/2))/(5*a*d*(a + a*Sin[c + d
*x])^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^3}\,dx = \frac{4e(e\cos(c+dx))^{13/2}}{5ad(a+a\sin(c+dx))^2} + \frac{\left(13e^2\right)\int \frac{(e\cos(c+dx))^{11/2}}{a+a\sin(c+dx)}\,dx}{5a^2}$$

$$= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d} + \frac{4e(e\cos(c+dx))^{13/2}}{5ad(a+a\sin(c+dx))^2} + \frac{\left(13e^4\right)\int (e\cos(c+dx))^{7/2}\,dx}{5a^3}$$

$$= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d} + \frac{26e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^3d} + \frac{4e(e\cos(c+dx))^{13/2}}{5ad(a+a\sin(c+dx))^2}$$

$$= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d} + \frac{26e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21a^3d} + \frac{26e^5(e\cos(c+dx))^{5/2}\sin}{35a^3d}$$

$$= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d} + \frac{26e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21a^3d} + \frac{26e^5(e\cos(c+dx))^{5/2}\sin}{35a^3d}$$

$$= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d} + \frac{26e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}} + \frac{26e^7\sqrt{e\cos(c+dx)}\,\text{si}}{21a^3d}$$

Mathematica [C] time = 0.39, size = 66, normalized size = 0.39

$$-\frac{4\sqrt[4]{2}\,(e\cos(c+dx))^{17/2}\,{}_2F_1\left(-\frac{1}{4},\frac{17}{4};\frac{21}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{17a^3de(\sin(c+dx)+1)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(17/2)*Hypergeometric2F1[-1/4, 17/4, 21/4, (1
- Sin[c + d*x])/2])/(17*a^3*d*e*(1 + Sin[c + d*x])^(17/4))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e^7\cos(dx+c)^7}{3\,a^3\cos(dx+c)^2 - 4\,a^3 + \left(a^3\cos(dx+c)^2 - 4\,a^3\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^7*cos(d*x + c)^7/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.19, size = 251, normalized size = 1.49

$$-\frac{2e^{8}\left(-1120\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2800\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3240\left(\sin^{6}\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/315/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^8*(-112
0*sin(1/2*d*x+1/2*c)^11-2160*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+2800*s
in(1/2*d*x+1/2*c)^9+3240*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-784*sin(1/
2*d*x+1/2*c)^7-840*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-1624*sin(1/2*d*x
+1/2*c)^5+195*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-120*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+1162*sin(1/2*d*x+1/2*c)^3-217*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{15}{2}}}{(a\sin(dx+c)+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(15/2)/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{15/2}}{(a+a\,\sin(c+d\,x))^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(15/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(15/2)/(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(15/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.254 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=138

$$\frac{22e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^3d\sqrt{\cos(c+dx)}}+\frac{22e^5\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^3d}+\frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d}+\frac{4e(e\cos(c+d\ldots}{3ad(a\sin(c+d\ldots}$$

```
[Out] 22/21*e^3*(e*cos(d*x+c))^(7/2)/a^3/d+22/15*e^5*(e*cos(d*x+c))^(3/2)*sin(d*x
+c)/a^3/d+4/3*e*(e*cos(d*x+c))^(11/2)/a/d/(a+a*sin(d*x+c))^2+22/5*e^6*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*(e*cos(d*x+c))^(1/2)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2680, 2682, 2635, 2640, 2639}

$$\frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d}+\frac{22e^5\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^3d}+\frac{22e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^3d\sqrt{\cos(c+dx)}}+\frac{4e(e\cos(c+d\ldots}{3ad(a\sin(c+d\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (22*e^3*(e*Cos[c + d*x])^(7/2))/(21*a^3*d) + (22*e^6*Sqrt[e*Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]) + (22*e^5*(e*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(15*a^3*d) + (4*e*(e*Cos[c + d*x])^(11/2))/(3*a*
d*(a + a*Sin[c + d*x])^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
```

```
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^3}\,dx = \frac{4e(e\cos(c+dx))^{11/2}}{3ad(a+a\sin(c+dx))^2} + \frac{\left(11e^2\right)\int \frac{(e\cos(c+dx))^{9/2}}{a+a\sin(c+dx)}\,dx}{3a^2}$$
$$= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d} + \frac{4e(e\cos(c+dx))^{11/2}}{3ad(a+a\sin(c+dx))^2} + \frac{\left(11e^4\right)\int (e\cos(c+dx))^{5/2}\,dx}{3a^3}$$
$$= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d} + \frac{22e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^3d} + \frac{4e(e\cos(c+dx))^{11}}{3ad(a+a\sin(c+dx}$$
$$= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d} + \frac{22e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^3d} + \frac{4e(e\cos(c+dx))^{11}}{3ad(a+a\sin(c+dx}$$
$$= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d} + \frac{22e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}} + \frac{22e^5(e\cos(c+dx}{15a}$$

Mathematica [C] time = 0.24, size = 66, normalized size = 0.48

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{15/2}\,{}_2F_1\left(\frac{1}{4},\frac{15}{4};\frac{19}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{15a^3de(\sin(c+dx)+1)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(15/2)*Hypergeometric2F1[1/4, 15/4, 19/4, (1 -
 Sin[c + d*x])/2])/(15*a^3*d*e*(1 + Sin[c + d*x])^(15/4))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}\,e^6\cos\left(dx+c\right)^6}{3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^6*cos(d*x + c)^6/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.10, size = 216, normalized size = 1.57

$$2e^7\left(-240\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-504\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+480\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+504\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/105/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^7*(-240*
sin(1/2*d*x+1/2*c)^9-504*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+480*sin(1/
2*d*x+1/2*c)^7+504*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+200*sin(1/2*d*x+
1/2*c)^5+231*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-126*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-440*sin(1/2*d*x+1/2*c)^3+125*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{13}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(13/2)/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\ \cos(c+d\,x))^{13/2}}{(a+a\ \sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(13/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(13/2)/(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.255 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=132

$$\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{e\cos(c+dx)}}+\frac{6e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{a^3d}+\frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d}+\frac{4e(e\cos(c+dx))^9}{ad(a\sin(c+dx)+}$$

```
[Out] 18/5*e^3*(e*cos(d*x+c))^(5/2)/a^3/d+4*e*(e*cos(d*x+c))^(9/2)/a/d/(a+a*sin(d
*x+c))^2+6*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^3/d/(e*cos(d*x+c))^(1/2)+6*e^5
*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2680, 2682, 2635, 2642, 2641}

$$\frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d}+\frac{6e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{a^3d}+\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{e\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^9}{ad(a\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (18*e^3*(e*Cos[c + d*x])^(5/2))/(5*a^3*d) + (6*e^6*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(a^3*d*Sqrt[e*Cos[c + d*x]]) + (6*e^5*Sqrt[e*Cos[c +
d*x]]*Sin[c + d*x])/(a^3*d) + (4*e*(e*Cos[c + d*x])^(9/2))/(a*d*(a + a*Sin[
c + d*x])^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
```

```
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^3}\,dx = \frac{4e(e\cos(c+dx))^{9/2}}{ad(a+a\sin(c+dx))^2} + \frac{\left(9e^2\right)\int \frac{(e\cos(c+dx))^{7/2}}{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d} + \frac{4e(e\cos(c+dx))^{9/2}}{ad(a+a\sin(c+dx))^2} + \frac{\left(9e^4\right)\int (e\cos(c+dx))^{3/2}\,dx}{a^3}$$
$$= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d} + \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{a^3d} + \frac{4e(e\cos(c+dx))^{9/2}}{ad(a+a\sin(c+dx))^2} + \cdots$$
$$= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d} + \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{a^3d} + \frac{4e(e\cos(c+dx))^{9/2}}{ad(a+a\sin(c+dx))^2} + \cdots$$
$$= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d} + \frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{e\cos(c+dx)}} + \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(\cdots}{a^3d}$$

Mathematica [C] time = 0.16, size = 66, normalized size = 0.50

$$-\frac{2\sqrt[4]{2}\,(e\cos(c+dx))^{13/2}\,{}_2F_1\left(\frac{3}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13a^3de(\sin(c+dx)+1)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[3/4, 13/4, 17/4, (1 -
 Sin[c + d*x])/2])/(13*a^3*d*e*(1 + Sin[c + d*x])^(13/4))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e^5\cos(dx+c)^5}{3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^5*cos(d*x + c)^5/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.93, size = 181, normalized size = 1.37

$$-\frac{2e^6\left(-8\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+12\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticF}\left(\cdots\right.\right.}{5a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/5/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^6*(-8*sin
(1/2*d*x+1/2*c)^7-20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*sin(1/2*d*x
+1/2*c)^5+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)+34*sin(1/2*d*x+1/2*c)^3-19*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{11/2}}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.256 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{14e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{a^3d\sqrt{\cos(c+dx)}}-\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d}-\frac{4e(e\cos(c+dx))^{7/2}}{ad(a\sin(c+dx)+a)^2}$$

```
[Out] -14/3*e^3*(e*cos(d*x+c))^(3/2)/a^3/d-4*e*(e*cos(d*x+c))^(7/2)/a/d/(a+a*sin(
d*x+c))^2-14*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2680, 2682, 2640, 2639}

$$-\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d}-\frac{14e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{a^3d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{7/2}}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-14*e^3*(e*Cos[c + d*x])^(3/2))/(3*a^3*d) - (14*e^4*Sqrt[e*Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2])/(a^3*d*Sqrt[Cos[c + d*x]]) - (4*e*(e*Cos[c + d*x]
)^(7/2))/(a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^3}\,dx = -\frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2} - \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}}{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= -\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d} - \frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2} - \frac{(7e^4)\int\sqrt{e\cos(c+dx)}\,dx}{a^3}$$
$$= -\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d} - \frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2} - \frac{\left(7e^4\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos(c}}{a^3\sqrt{\cos(c+dx)}}$$
$$= -\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d} - \frac{14e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}} - \frac{4e(e\cos(c+dx}{ad(a+a\sin(c+}$$

Mathematica [C] time = 0.10, size = 66, normalized size = 0.64

$$-\frac{2^{3/4}(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11a^3de(\sin(c+dx)+1)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^3,x]
[Out] -1/11*(2^(3/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[5/4, 11/4, 15/4, (
1 - Sin[c + d*x])/2])/(a^3*d*e*(1 + Sin[c + d*x])^(11/4))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e^4\cos(dx+c)^4}{3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] integral(-sqrt(e*cos(d*x + c))*e^4*cos(d*x + c)^4/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.26, size = 146, normalized size = 1.42

$$-\frac{2\left(4\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}-24\left(\sin^2\left(\frac{dx}{2}\right.\right.}{3\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/3/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a^3*(4*sin(1/2*
d*x+1/2*c)^5+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-24*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-4*sin(1/2*d*x+1/2*c)^3+13*sin(1/2*d*x+1/2*c))*e^5/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{9/2}}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.257 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=107

$$-\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\sqrt{e\cos(c+dx)}}-\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d}-\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a\sin(c+dx)+a)^2}$$

```
[Out] -4/3*e*(e*cos(d*x+c))^(5/2)/a/d/(a+a*sin(d*x+c))^2-10/3*e^4*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)/a^3/d/(e*cos(d*x+c))^(1/2)-10/3*e^3*(e*cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2680, 2682, 2642, 2641}

$$-\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d}-\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-10*e^3*Sqrt[e*Cos[c + d*x]])/(3*a^3*d) - (10*e^4*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(3*a^3*d*Sqrt[e*Cos[c + d*x]]) - (4*e*(e*Cos[c + d*x]
)^(5/2))/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^3}\,dx = -\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2} - \frac{(5e^2)\int \frac{(e\cos(c+dx))^{3/2}}{a+a\sin(c+dx)}\,dx}{3a^2}$$
$$= -\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d} - \frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2} - \frac{(5e^4)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a^3}$$
$$= -\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d} - \frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2} - \frac{\left(5e^4\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^3\sqrt{e\cos(c+dx)}}$$
$$= -\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d} - \frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\sqrt{e\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.62

$$-\frac{\sqrt[4]{2}\,(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9a^3de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/9*(2^(1/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[7/4, 9/4, 13/4, (1 -
 Sin[c + d*x])/2])/(a^3*d*e*(1 + Sin[c + d*x])^(9/4))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e^3\cos(dx+c)^3}{3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^3*cos(d*x + c)^3/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.37, size = 219, normalized size = 2.05

$$\frac{2\left(10\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)*(10*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+12*sin(1/2
*d*x+1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)-12*sin(1/2*d*x+1/2*c)^3+7*sin(1/2*d*x+1/2*c))*e^4/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\cos(c+d\,x))^{7/2}}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.258 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=118

$$\frac{6e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^3d\sqrt{\cos(c+dx)}}+\frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a\sin(c+dx)+a)^2}$$

```
[Out] -4/5*e*(e*cos(d*x+c))^(3/2)/a/d/(a+a*sin(d*x+c))^2+6/5*e*(e*cos(d*x+c))^(3/
2)/d/(a^3+a^3*sin(d*x+c))+6/5*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^3/d/cos
(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2680, 2683, 2640, 2639}

$$\frac{6e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^3d\sqrt{\cos(c+dx)}}+\frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c
+ d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(5*a*d*(a + a*Sin[c + d*x])^2) + (6
*e*(e*Cos[c + d*x])^(3/2))/(5*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^3}\,dx = -\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2} - \frac{\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx}{5a^2}$$
$$= -\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2} + \frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3+a^3\sin(c+dx)\right)} + \frac{\left(3e^2\right)\int \sqrt{e\cos(c+dx)}\,dx}{5a^3}$$
$$= -\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2} + \frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3+a^3\sin(c+dx)\right)} + \frac{\left(3e^2\sqrt{e\cos(c+dx)}\right)\int \sqrt{c}}{5a^3\sqrt{\cos(c+d}}$$
$$= \frac{6e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2} + \frac{6e(e\cos(c+dx}{5d\left(a^3+a^3\sin(c\right.}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.56

$$-\frac{(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7\sqrt[4]{2}\,a^3de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/7*((e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, 9/4, 11/4, (1 - Sin[c +
 d*x])/2])/(2^(1/4)*a^3*d*e*(1 + Sin[c + d*x])^(7/4))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e^2\cos(dx+c)^2}{3a^3\cos(dx+c)^2-4a^3+\left(a^3\cos(dx+c)^2-4a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^2*cos(d*x + c)^2/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.14, size = 330, normalized size = 2.80

$$2\left(12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^3/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+20
*sin(1/2*d*x+1/2*c)^5+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+2*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-20*sin(1/2*d*x+1/2*c)^3+sin(1/2*d*x+1/2*c))*e^3/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.259 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^3}\,dx$

<u>**Optimal**</u>. Leaf size=118

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a\sin(c+dx)+a)^2}$$

```
[Out] -2/21*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^3/d/(e*cos(d*x+c))^(1/2)-4/7*e*(e*c
os(d*x+c))^(1/2)/a/d/(a+a*sin(d*x+c))^2+2/21*e*(e*cos(d*x+c))^(1/2)/d/(a^3+
a^3*sin(d*x+c))
```

<u>**Rubi**</u> **[A]** time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2680, 2683, 2642, 2641}

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a^3*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(7*a*d*(a + a*Sin[c + d*x])^2) + (2
*e*Sqrt[e*Cos[c + d*x]])/(21*d*(a^3 + a^3*Sin[c + d*x]))
```

<u>Rule 2641</u>

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

<u>Rule 2642</u>

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

<u>Rule 2680</u>

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

<u>Rule 2683</u>

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

<u>Rubi steps</u>

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^3}\,dx = -\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}\,dx}{7a^2}$$
$$= -\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} + \frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3+a^3\sin(c+dx)\right)} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{21a^3}$$
$$= -\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} + \frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3+a^3\sin(c+dx)\right)} - \frac{\left(e^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+d}}}{21a^3\sqrt{e\cos(c+dx)}}$$
$$= -\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}} - \frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} + \frac{2e\sqrt{e\cos(c+dx}}{21d\left(a^3+a^3\sin(c+\right.}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.56

$$-\frac{(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5\,2^{3/4}a^3de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/5*((e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 11/4, 9/4, (1 - Sin[c +
 d*x])/2])/(2^(3/4)*a^3*d*e*(1 + Sin[c + d*x])^(5/4))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e\cos(dx+c)}{3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e*cos(d*x + c)/(3*a^3*cos(d*x + c)^2 - 4*a^3
 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^3, x)
```

maple [B] time = 2.32, size = 401, normalized size = 3.40

$$\frac{2\left(8\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(8*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-28*sin(
1/2*d*x+1/2*c)^5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-22*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)+28*sin(1/2*d*x+1/2*c)^3+5*sin(1/2*d*x+1/2*c))*e^2/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{3/2}}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.260 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{2(e\cos(c+dx))^{3/2}}{15de\left(a^3\sin(c+dx)+a^3\right)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15a^3d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a\sin(c+dx)+a)^2}-\frac{2(e\cos(c+dx))^{3/2}}{9de(a\sin(c+dx)+a)^3}$$

```
[Out] -2/9*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^3-2/15*(e*cos(d*x+c))^(3/2)/
a/d/e/(a+a*sin(d*x+c))^2-2/15*(e*cos(d*x+c))^(3/2)/d/e/(a^3+a^3*sin(d*x+c))
-2/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2681, 2683, 2640, 2639}

$$-\frac{2(e\cos(c+dx))^{3/2}}{15de\left(a^3\sin(c+dx)+a^3\right)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15a^3d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a\sin(c+dx)+a)^2}-\frac{2(e\cos(c+dx))^{3/2}}{9de(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a^3*d*Sqrt[Cos[c +
d*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(9*d*e*(a + a*Sin[c + d*x])^3) - (2*(e*
Cos[c + d*x])^(3/2))/(15*a*d*e*(a + a*Sin[c + d*x])^2) - (2*(e*Cos[c + d*x]
)^(3/2))/(15*d*e*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^3}dx = -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3} + \frac{\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}dx}{3a}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3} - \frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2} + \frac{\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}dx}{15a^2}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3} - \frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2} - \frac{2(e\cos(c+dx))^{3/2}}{15de\left(a^3+a^3\sin(c+dx)\right.}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3} - \frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2} - \frac{2(e\cos(c+dx))^{3/2}}{15de\left(a^3+a^3\sin(c+dx)\right.}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}} - \frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3} - \frac{2(e\cos(c+dx}{15ade(a+a\sin(c}$$

Mathematica [C] time = 0.04, size = 66, normalized size = 0.43

$$-\frac{(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{13}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{6\sqrt[4]{2}\,a^3de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*((e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 13/4, 7/4, (1 - Sin[c +
 d*x])/2])/(2^(1/4)*a^3*d*e*(1 + Sin[c + d*x])^(3/4))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{3a^3\cos(dx+c)^2-4a^3+\left(a^3\cos(dx+c)^2-4a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x
 + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(dx+c)}}{(a\sin(dx+c)+a)^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^3, x)
```

maple [B] time = 3.41, size = 512, normalized size = 3.35

$$-\frac{2\left(48\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)
^2*e+e)^(1/2)*(48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)-36*sin(1/2*d*x+1/2*c)^5+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-48*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)+36*sin(1/2*d*x+1/2*c)^3+11*sin(1/2*d*x+1/2*c)
)*e/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{\left(a\sin\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.261 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{10\sqrt{e\cos(c+dx)}}{77de\left(a^3\sin(c+dx)+a^3\right)}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^3d\sqrt{e\cos(c+dx)}}-\frac{10\sqrt{e\cos(c+dx)}}{77ade(a\sin(c+dx)+a)^2}-\frac{2\sqrt{e\cos(c+dx)}}{11de(a\sin(c+dx)+a)^3}$$

```
[Out] 10/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^3/d/(e*cos(d*x+c))^(1/2)-2/11*(e*cos(d*
x+c))^(1/2)/d/e/(a+a*sin(d*x+c))^3-10/77*(e*cos(d*x+c))^(1/2)/a/d/e/(a+a*si
n(d*x+c))^2-10/77*(e*cos(d*x+c))^(1/2)/d/e/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2681, 2683, 2642, 2641}

$$-\frac{10\sqrt{e\cos(c+dx)}}{77de\left(a^3\sin(c+dx)+a^3\right)}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^3d\sqrt{e\cos(c+dx)}}-\frac{10\sqrt{e\cos(c+dx)}}{77ade(a\sin(c+dx)+a)^2}-\frac{2\sqrt{e\cos(c+dx)}}{11de(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*a^3*d*Sqrt[e*Cos[c +
d*x]]) - (2*Sqrt[e*Cos[c + d*x]])/(11*d*e*(a + a*Sin[c + d*x])^3) - (10*Sqr
t[e*Cos[c + d*x]])/(77*a*d*e*(a + a*Sin[c + d*x])^2) - (10*Sqrt[e*Cos[c + d
*x]])/(77*d*e*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3} + \frac{5\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}\,dx}{11a}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3} - \frac{10\sqrt{e\cos(c+dx)}}{77ade(a+a\sin(c+dx))^2} + \frac{15\int \frac{}{\sqrt{e\cos(c}}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3} - \frac{10\sqrt{e\cos(c+dx)}}{77ade(a+a\sin(c+dx))^2} - \frac{10\sqrt{e\,\mathrm{co}}}{77de\left(a^3+\right.}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3} - \frac{10\sqrt{e\cos(c+dx)}}{77ade(a+a\sin(c+dx))^2} - \frac{10\sqrt{e\,\mathrm{co}}}{77de\left(a^3+\right.}$$

$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^3d\sqrt{e\cos(c+dx)}} - \frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3} - \frac{10}{77ade(}$$

Mathematica [C] time = 0.05, size = 66, normalized size = 0.43

$$-\frac{\sqrt{e\cos(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{15}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{2\,2^{3/4}a^3de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] -1/2*(Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 15/4, 5/4, (1 - Sin[c + d
*x])/2])/(2^(3/4)*a^3*d*e*(1 + Sin[c + d*x])^(1/4))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{3\,a^3e\cos(dx+c)^3-4\,a^3e\cos(dx+c)+\left(a^3e\cos(dx+c)^3-4\,a^3e\cos(dx+c)\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(3*a^3*e*cos(d*x + c)^3 - 4*a^3*e*cos(d*x +
c) + (a^3*e*cos(d*x + c)^3 - 4*a^3*e*cos(d*x + c))*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^3), x)
```

maple [B] time = 3.99, size = 580, normalized size = 3.79

$$-\frac{2\left(160\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-400\,\mathrm{EllipticF}\left(\mathrm{c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/77/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*
c)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/a^3/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(160*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1
/2*d*x+1/2*c)^10-400*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8+160*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+400*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d
*x+1/2*c)^6-320*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-200*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+264*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+
50*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-104*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+72*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-44*sin(1/2*d*x+1/2*c)^3-17*sin(1/2*d*x
+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.262 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=187

$$-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{39a^3de^2\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}}-\frac{14}{117de\left(a^3\sin(c+dx)+a^3\right)\sqrt{e\cos(c+dx)}}-\frac{}{117ade}$$

```
[Out] 14/39*sin(d*x+c)/a^3/d/e/(e*cos(d*x+c))^(1/2)-2/13/d/e/(a+a*sin(d*x+c))^3/(
e*cos(d*x+c))^(1/2)-14/117/a/d/e/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2)-14
/117/d/e/(a^3+a^3*sin(d*x+c))/(e*cos(d*x+c))^(1/2)-14/39*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(
d*x+c))^(1/2)/a^3/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2681, 2683, 2636, 2640, 2639}

$$-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{39a^3de^2\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}}-\frac{14}{117de\left(a^3\sin(c+dx)+a^3\right)\sqrt{e\cos(c+dx)}}-\frac{}{117ade}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] (-14*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(39*a^3*d*e^2*Sqrt[Cos
[c + d*x]]) + (14*Sin[c + d*x])/(39*a^3*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(13*d
*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3) - 14/(117*a*d*e*Sqrt[e*Cos[
c + d*x]]*(a + a*Sin[c + d*x])^2) - 14/(117*d*e*Sqrt[e*Cos[c + d*x]]*(a^3 +
 a^3*Sin[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}\,dx = -\frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3} + \frac{7\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a}}{13a}$$

$$= -\frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3} - \frac{1}{117ade\sqrt{e\cos(c+d}}$$

$$= -\frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3} - \frac{1}{117ade\sqrt{e\cos(c+d}}$$

$$= \frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}} - \frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}$$

$$= \frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}} - \frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}$$

$$= -\frac{14\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3de^2\sqrt{\cos(c+dx)}} + \frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}} -$$

Mathematica [C] time = 0.06, size = 66, normalized size = 0.35

$$\frac{\sqrt[4]{\sin(c+dx)+1}\;{}_2F_1\left(-\frac{1}{4},\frac{17}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{4\sqrt[4]{2}\,a^3de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] (Hypergeometric2F1[-1/4, 17/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(1/4))/(4*2^(1/4)*a^3*d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{3a^3e^2\cos(dx+c)^4-4a^3e^2\cos(dx+c)^2+\left(a^3e^2\cos(dx+c)^4-4a^3e^2\cos(dx+c)^2\right)\sin(dx+c)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(3*a^3*e^2*cos(d*x + c)^4 - 4*a^3*e^2*cos(d*
x + c)^2 + (a^3*e^2*cos(d*x + c)^4 - 4*a^3*e^2*cos(d*x + c)^2)*sin(d*x + c)
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}\,(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^3), x)
```

maple [B] time = 4.73, size = 696, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/117/(64*sin(1/2*d*x+1/2*c)^12-192*sin(1/2*d*x+1/2*c)^10+240*sin(1/2*d*x+
1/2*c)^8-160*sin(1/2*d*x+1/2*c)^6+60*sin(1/2*d*x+1/2*c)^4-12*sin(1/2*d*x+1/
2*c)^2+1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e*(134
4*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12-2688*sin(1/2*d*x+1/2*c)^14
*cos(1/2*d*x+1/2*c)-4032*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+806
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+5040*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*si
n(1/2*d*x+1/2*c)^8-10304*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-3360*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+7168*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8+1260*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-2896*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-252*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1
/2*c)^2+656*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-52*sin(1/2*d*x+1/2*c)^5
+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)-138*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+52
*sin(1/2*d*x+1/2*c)^3+23*sin(1/2*d*x+1/2*c))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.263 $\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=180

$$\frac{78e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^4d\sqrt{e\cos(c+dx)}}+\frac{78e^7\sin(c+dx)\sqrt{e\cos(c+dx)}}{7a^4d}+\frac{234e^5\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^4d}+\frac{52}{5d\,(}$$

```
[Out] 234/35*e^5*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/a^4/d+4*e*(e*cos(d*x+c))^(13/2)/
a/d/(a+a*sin(d*x+c))^3+52/5*e^3*(e*cos(d*x+c))^(9/2)/d/(a^4+a^4*sin(d*x+c))
+78/7*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^4/d/(e*cos(d*x+c))^(1/2)+78/7*e^7*s
in(d*x+c)*(e*cos(d*x+c))^(1/2)/a^4/d
```

Rubi [A] time = 0.18, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2680, 2635, 2642, 2641}

$$\frac{78e^7\sin(c+dx)\sqrt{e\cos(c+dx)}}{7a^4d}+\frac{234e^5\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^4d}+\frac{52e^3(e\cos(c+dx))^{9/2}}{5d\left(a^4\sin(c+dx)+a^4\right)}+\frac{78e^8\sqrt{\cos(c+}}{7a^4d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (78*e^8*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^4*d*Sqrt[e*Cos[c
 + d*x]]) + (78*e^7*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(7*a^4*d) + (234*e^5
*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(35*a^4*d) + (4*e*(e*Cos[c + d*x])^(1
3/2))/(a*d*(a + a*Sin[c + d*x])^3) + (52*e^3*(e*Cos[c + d*x])^(9/2))/(5*d*(
a^4 + a^4*Sin[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^4}\,dx = \frac{4e(e\cos(c+dx))^{13/2}}{ad(a+a\sin(c+dx))^3} + \frac{\left(13e^2\right)\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4e(e\cos(c+dx))^{13/2}}{ad(a+a\sin(c+dx))^3} + \frac{52e^3(e\cos(c+dx))^{9/2}}{5d\left(a^4+a^4\sin(c+dx)\right)} + \frac{\left(117e^4\right)\int (e\cos(c+dx))^{7/2}\,dx}{5a^4}$$

$$= \frac{234e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^4d} + \frac{4e(e\cos(c+dx))^{13/2}}{ad(a+a\sin(c+dx))^3} + \frac{52e^3(e\cos(c+dx))^{9/}}{5d\left(a^4+a^4\sin(c+d\right.}$$

$$= \frac{78e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^4d} + \frac{234e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^4d} + \frac{4e(e\cos(c}{ad(a+a\sin}$$

$$= \frac{78e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^4d} + \frac{234e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^4d} + \frac{4e(e\cos(c}{ad(a+a\sin}$$

$$= \frac{78e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^4d\sqrt{e\cos(c+dx)}} + \frac{78e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^4d} + \frac{234e^5(e\cos}{}$$

Mathematica [C] time = 0.32, size = 66, normalized size = 0.37

$$-\frac{2\sqrt[4]{2}\,(e\cos(c+dx))^{17/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{4};\frac{21}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{17a^4de(\sin(c+dx)+1)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(17/2)*Hypergeometric2F1[3/4, 17/4, 21/4, (1 -
 Sin[c + d*x])/2])/(17*a^4*d*e*(1 + Sin[c + d*x])^(17/4))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e^7\cos(dx+c)^7}{a^4\cos(dx+c)^4 - 8a^4\cos(dx+c)^2 + 8a^4 - 4\left(a^4\cos(dx+c)^2 - 2a^4\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^7*cos(d*x + c)^7/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.08, size = 225, normalized size = 1.25

$$-\frac{2e^8\left(80\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-224\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-280\left(\sin^4\left(\frac{d}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/35/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^8*(80*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)-224*sin(1/2*d*x+1/2*c)^7-280*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+336*sin(1/2*d*x+1/2*c)^5+195*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+160*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+392*sin(1/2*d*x+1/2*c)^3-252*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{15}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(15/2)/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{15/2}}{(a+a\,\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(15/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(15/2)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(15/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.264 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=149

$$-\frac{154e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^4d\sqrt{\cos(c+dx)}}-\frac{154e^5\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^4d}-\frac{44e^3(e\cos(c+dx))^{7/2}}{3d\left(a^4\sin(c+dx)+a^4\right)}-\frac{4e(e\cos(\cdots}{ad(a\sin(c\cdots}$$

```
[Out] -154/15*e^5*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/a^4/d-4*e*(e*cos(d*x+c))^(11/2)
/a/d/(a+a*sin(d*x+c))^3-44/3*e^3*(e*cos(d*x+c))^(7/2)/d/(a^4+a^4*sin(d*x+c)
)-154/5*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^4/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2680, 2635, 2640, 2639}

$$-\frac{154e^5\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^4d}-\frac{44e^3(e\cos(c+dx))^{7/2}}{3d\left(a^4\sin(c+dx)+a^4\right)}-\frac{154e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^4d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(\cdots}{ad(a\sin(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-154*e^6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos
[c + d*x]]) - (154*e^5*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*a^4*d) - (4
*e*(e*Cos[c + d*x])^(11/2))/(a*d*(a + a*Sin[c + d*x])^3) - (44*e^3*(e*Cos[c
 + d*x])^(7/2))/(3*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^4}dx = -\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3} - \frac{(11e^2)\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^2}dx}{a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3} - \frac{44e^3(e\cos(c+dx))^{7/2}}{3d(a^4+a^4\sin(c+dx))} - \frac{(77e^4)\int(e\cos(c+dx))^{5/2}d}{3a^4}$$

$$= -\frac{154e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^4d} - \frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3} - \frac{44e^3(e\cos(c+d}{3d(a^4+a^4\sin(c}$$

$$= -\frac{154e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^4d} - \frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3} - \frac{44e^3(e\cos(c+d}{3d(a^4+a^4\sin(c}$$

$$= -\frac{154e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}} - \frac{154e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^4d} - \frac{4}{ad}$$

Mathematica [C] time = 0.19, size = 66, normalized size = 0.44

$$-\frac{2^{3/4}(e\cos(c+dx))^{15/2}\,{}_2F_1\left(\frac{5}{4},\frac{15}{4};\frac{19}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{15a^4de(\sin(c+dx)+1)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/15*(2^(3/4)*(e*Cos[c + d*x])^(15/2)*Hypergeometric2F1[5/4, 15/4, 19/4, (
1 - Sin[c + d*x])/2])/(a^4*d*e*(1 + Sin[c + d*x])^(15/4))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e^6\cos(dx+c)^6}{a^4\cos(dx+c)^4 - 8a^4\cos(dx+c)^2 + 8a^4 - 4\left(a^4\cos(dx+c)^2 - 2a^4\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^6*cos(d*x + c)^6/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.65, size = 190, normalized size = 1.28

$$-\frac{2\left(-24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+24\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+80\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+231\,\text{EllipticE}\left(\right.}{15\sqrt{-2\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/15/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a^4*(-24*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+80*sin(1/2*d*x+1/2*c)^5+231*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-246*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-80*sin(1/2*d*x+1/2*c)^3+140*sin(1/2*d*x+1/2*c))*e^
7/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{13}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(13/2)/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{13/2}}{(a+a\,\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(13/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(13/2)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.265 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$-\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^4d\sqrt{e\cos(c+dx)}}-\frac{10e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{a^4d}-\frac{12e^3(e\cos(c+dx))^{5/2}}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{4e(e\cos(c+}{3ad(a\sin(c+}$$

```
[Out] -4/3*e*(e*cos(d*x+c))^(9/2)/a/d/(a+a*sin(d*x+c))^3-12*e^3*(e*cos(d*x+c))^(5
/2)/d/(a^4+a^4*sin(d*x+c))-10*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^4/d/(e*cos(
d*x+c))^(1/2)-10*e^5*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/a^4/d
```

Rubi [A] time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2680, 2635, 2642, 2641}

$$-\frac{10e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{a^4d}-\frac{12e^3(e\cos(c+dx))^{5/2}}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^4d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+}{3ad(a\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-10*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(a^4*d*Sqrt[e*Cos[c
 + d*x]]) - (10*e^5*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(a^4*d) - (4*e*(e*Cos
[c + d*x])^(9/2))/(3*a*d*(a + a*Sin[c + d*x])^3) - (12*e^3*(e*Cos[c + d*x])
^(5/2))/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^4}dx = -\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3} - \frac{(3e^2)\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^2}dx}{a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3} - \frac{12e^3(e\cos(c+dx))^{5/2}}{d(a^4+a^4\sin(c+dx))} - \frac{(15e^4)\int (e\cos(c+dx))^{3/2}dx}{a^4}$$

$$= -\frac{10e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{a^4d} - \frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3} - \frac{12e^3(e\cos(c+dx))^{5/2}}{d(a^4+a^4\sin(c+dx)}$$

$$= -\frac{10e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{a^4d} - \frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3} - \frac{12e^3(e\cos(c+dx))^{5/2}}{d(a^4+a^4\sin(c+dx)}$$

$$= -\frac{10e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^4d\sqrt{e\cos(c+dx)}} - \frac{10e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{a^4d} - \frac{4e(e\cos(}{3ad(a+a\,s}$$

Mathematica [C] time = 0.17, size = 66, normalized size = 0.46

$$-\frac{\sqrt[4]{2}(e\cos(c+dx))^{13/2}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13a^4de(\sin(c+dx)+1)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/13*(2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[7/4, 13/4, 17/4, (
1 - Sin[c + d*x])/2])/(a^4*d*e*(1 + Sin[c + d*x])^(13/4))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e^5\cos(dx+c)^5}{a^4\cos(dx+c)^4-8a^4\cos(dx+c)^2+8a^4-4\left(a^4\cos(dx+c)^2-2a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^5*cos(d*x + c)^5/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.68, size = 263, normalized size = 1.81

$$2\left(-8\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+30\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\text{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)*(-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
48*sin(1/2*d*x+1/2*c)^5-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-18*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-48*sin(1/2*d*x+1/2*c)^3+20*sin(1/2*d*x+1/2*c))*e^6/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{11/2}}{(a+a\,\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.266 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{42e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^4d\sqrt{\cos(c+dx)}}+\frac{28e^3(e\cos(c+dx))^{3/2}}{5d\left(a^4\sin(c+dx)+a^4\right)}-\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a\sin(c+dx)+a)^3}$$

```
[Out] -4/5*e*(e*cos(d*x+c))^(7/2)/a/d/(a+a*sin(d*x+c))^3+28/5*e^3*(e*cos(d*x+c))^
(3/2)/d/(a^4+a^4*sin(d*x+c))+42/5*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^4/d
/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2680, 2640, 2639}

$$\frac{28e^3(e\cos(c+dx))^{3/2}}{5d\left(a^4\sin(c+dx)+a^4\right)}+\frac{42e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^4d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (42*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c
 + d*x]]) - (4*e*(e*Cos[c + d*x])^(7/2))/(5*a*d*(a + a*Sin[c + d*x])^3) + (
28*e^3*(e*Cos[c + d*x])^(3/2))/(5*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} - \frac{\left(7e^2\right)\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} + \frac{28e^3(e\cos(c+dx))^{3/2}}{5d\left(a^4+a^4\sin(c+dx)\right)} + \frac{\left(21e^4\right)\int \sqrt{e\cos(c+dx)}\, d}{5a^4}$$

$$= -\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} + \frac{28e^3(e\cos(c+dx))^{3/2}}{5d\left(a^4+a^4\sin(c+dx)\right)} + \frac{\left(21e^4\sqrt{e\cos(c+dx)}\right)\int \sqrt{}}{5a^4\sqrt{\cos(c+}}$$

$$= \frac{42e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} + \frac{28e^3(e\cos(c+}{5d\left(a^4+a^4\sin(\right.}$$

Mathematica [C] time = 0.09, size = 66, normalized size = 0.55

$$-\frac{(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{9}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11\sqrt[4]{2}\,a^4de(\sin(c+dx)+1)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/11*((e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[9/4, 11/4, 15/4, (1 - Sin[
c + d*x])/2])/(2^(1/4)*a^4*d*e*(1 + Sin[c + d*x])^(11/4))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}\,e^4\cos\left(dx+c\right)^4}{a^4\cos\left(dx+c\right)^4-8\,a^4\cos\left(dx+c\right)^2+8\,a^4-4\left(a^4\cos\left(dx+c\right)^2-2\,a^4\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^4*cos(d*x + c)^4/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.47, size = 332, normalized size = 2.77

$$\frac{2\left(84\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-128\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^4/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(84*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-84*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
80*sin(1/2*d*x+1/2*c)^5+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-80*sin(1/2*d*x+1/2*c)^3+12*sin(1/2*d*x+1/2*c))*e^5/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{9/2}}{(a+a\,\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**4,x)
[Out] Timed out
```

3.267 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d\sqrt{e\cos(c+dx)}}+\frac{20e^3\sqrt{e\cos(c+dx)}}{21d\left(a^4\sin(c+dx)+a^4\right)}-\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a\sin(c+dx)+a)^3}$$

```
[Out] -4/7*e*(e*cos(d*x+c))^(5/2)/a/d/(a+a*sin(d*x+c))^3+10/21*e^4*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)/a^4/d/(e*cos(d*x+c))^(1/2)+20/21*e^3*(e*cos(d*x+c))^(1/2)/d/(
a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2680, 2642, 2641}

$$\frac{20e^3\sqrt{e\cos(c+dx)}}{21d\left(a^4\sin(c+dx)+a^4\right)}+\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a^4*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*(e*Cos[c + d*x])^(5/2))/(7*a*d*(a + a*Sin[c + d*x])^3) +
(20*e^3*Sqrt[e*Cos[c + d*x]])/(21*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3} - \frac{(5e^2)\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^2}\,dx}{7a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3} + \frac{20e^3\sqrt{e\cos(c+dx)}}{21d\left(a^4+a^4\sin(c+dx)\right)} + \frac{(5e^4)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{21a^4}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3} + \frac{20e^3\sqrt{e\cos(c+dx)}}{21d\left(a^4+a^4\sin(c+dx)\right)} + \frac{\left(5e^4\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c-}}}{21a^4\sqrt{e\cos(c+dx)}}$$

$$= \frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d\sqrt{e\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3} + \frac{20e^3\sqrt{e\cos(c+d}}{21d\left(a^4+a^4\sin(c+\right.}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.55

$$-\frac{(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{9}{4},\frac{11}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9\,2^{3/4}a^4de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/9*((e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[9/4, 11/4, 13/4, (1 - Sin[c
+ d*x])/2])/(2^(3/4)*a^4*d*e*(1 + Sin[c + d*x])^(9/4))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e^3\cos(dx+c)^3}{a^4\cos(dx+c)^4 - 8a^4\cos(dx+c)^2 + 8a^4 - 4\left(a^4\cos(dx+c)^2 - 2a^4\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^3*cos(d*x + c)^3/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.69, size = 401, normalized size = 3.34

$$-\frac{2\left(40\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-60\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(40*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6-60*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
112*sin(1/2*d*x+1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-112*sin(1/2*d*x+1/2*c)^3+4*sin(1/2*d*x+1/2*c))*e^4/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\ \cos(c+d\,x))^{7/2}}{(a+a\ \sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.268 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=154

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15a^4d\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a\sin(c+dx)+a}$$

```
[Out] -4/9*e*(e*cos(d*x+c))^(3/2)/a/d/(a+a*sin(d*x+c))^3+2/15*e*(e*cos(d*x+c))^(3
/2)/d/(a^2+a^2*sin(d*x+c))^2+2/15*e*(e*cos(d*x+c))^(3/2)/d/(a^4+a^4*sin(d*x
+c))+2/15*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^4/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2680, 2681, 2683, 2640, 2639}

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15a^4d\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a\sin(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (2*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a^4*d*Sqrt[Cos[c
 + d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(9*a*d*(a + a*Sin[c + d*x])^3) + (
2*e*(e*Cos[c + d*x])^(3/2))/(15*d*(a^2 + a^2*Sin[c + d*x])^2) + (2*e*(e*Cos
[c + d*x])^(3/2))/(15*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx}{3a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx}{15a^3}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^4+a^4\sin(c+dx)\right)}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^4+a^4\sin(c+dx)\right)}$$

$$= \frac{2e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+d}{15d\left(a^2+a^2\sin(\right.}$$

Mathematica [C] time = 0.09, size = 66, normalized size = 0.43

$$-\frac{(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{14\sqrt[4]{2}\,a^4de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/14*((e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, 13/4, 11/4, (1 - Sin[c
 + d*x])/2])/(2^(1/4)*a^4*d*e*(1 + Sin[c + d*x])^(7/4))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e^2\cos(dx+c)^2}{a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^2*cos(d*x + c)^2/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 3.40, size = 514, normalized size = 3.34

$$\frac{2\left(48\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)
^4-8*sin(1/2*d*x+1/2*c)^2+1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
2*e+e)^(1/2)*(48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*d
*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^4-272*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+176*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+144*sin(1/2*d*x+1/2*c)^5+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+42*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-144*sin(1/2*d*x+1/2*c)^3-4*sin(1/2*d*x+1/2*c
))*e^3/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{(a+a\,\sin(c+d\,x))^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.269 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=154

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^4d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4e\sqrt{e\cos(c+d}}{11ad(a\sin(c+dx)}$$

```
[Out] -2/77*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^4/d/(e*cos(d*x+c))^(1/2)-4/11*e*(e*
cos(d*x+c))^(1/2)/a/d/(a+a*sin(d*x+c))^3+2/77*e*(e*cos(d*x+c))^(1/2)/d/(a^2
+a^2*sin(d*x+c))^2+2/77*e*(e*cos(d*x+c))^(1/2)/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2680, 2681, 2683, 2642, 2641}

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^4d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4e\sqrt{e\cos(c+d}}{11ad(a\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*a^4*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(11*a*d*(a + a*Sin[c + d*x])^3) + (
2*e*Sqrt[e*Cos[c + d*x]])/(77*d*(a^2 + a^2*Sin[c + d*x])^2) + (2*e*Sqrt[e*C
os[c + d*x]])/(77*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}\,dx}{11a^2}$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2} - \frac{\left(3e^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4+a^4\sin(c+dx)\right)}$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4+a^4\sin(c+dx)\right)}$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^4d\sqrt{e\cos(c+dx)}} - \frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+d}{77d\left(a^2+a^2\sin(c\right.}$$

Mathematica [C] time = 0.07, size = 66, normalized size = 0.43

$$-\frac{(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{15}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{10\ 2^{3/4}a^4de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/10*((e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 15/4, 9/4, (1 - Sin[c
+ d*x])/2])/(2^(3/4)*a^4*d*e*(1 + Sin[c + d*x])^(5/4))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e\cos(dx+c)}{a^4\cos(dx+c)^4 - 8\,a^4\cos(dx+c)^2 + 8\,a^4 - 4\left(a^4\cos(dx+c)^2 - 2\,a^4\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e*cos(d*x + c)/(a^4*cos(d*x + c)^4 - 8*a^4*co
s(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 4.37, size = 583, normalized size = 3.79

$$\frac{2\left(32\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-80\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 2/77/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*c
)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/a^4/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(32*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^10-80*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8+32*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+80*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^6-64*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-40*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^4+176*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-144*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)-176*sin(1/2*d*x+1/2*c)^5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-78*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+176*sin(1/2*d*x+1/2*c)^3+12*sin(1/2*d*x+1/2*c)
)*e^2/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(dx+c\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,\cos\left(c+d\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.270 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=191

$$-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^4\sin(c+dx)+a^4\right)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{39a^4d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{10(e\cos(c+dx)}{117ade(a\sin(c+dx}$$

```
[Out] -2/13*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^4-10/117*(e*cos(d*x+c))^(3/
2)/a/d/e/(a+a*sin(d*x+c))^3-2/39*(e*cos(d*x+c))^(3/2)/d/e/(a^2+a^2*sin(d*x+
c))^2-2/39*(e*cos(d*x+c))^(3/2)/d/e/(a^4+a^4*sin(d*x+c))-2/39*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e
*cos(d*x+c))^(1/2)/a^4/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2681, 2683, 2640, 2639}

$$-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^4\sin(c+dx)+a^4\right)}-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{39a^4d\sqrt{\cos(c+dx)}}-\frac{10(e\cos(c+dx)}{117ade(a\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(39*a^4*d*Sqrt[Cos[c +
d*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(13*d*e*(a + a*Sin[c + d*x])^4) - (10*(
e*Cos[c + d*x])^(3/2))/(117*a*d*e*(a + a*Sin[c + d*x])^3) - (2*(e*Cos[c + d
*x])^(3/2))/(39*d*e*(a^2 + a^2*Sin[c + d*x])^2) - (2*(e*Cos[c + d*x])^(3/2)
)/(39*d*e*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^4}\,dx = -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}+\frac{5\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^3}\,dx}{13a}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3}+\frac{5\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx}{39a^2}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^2+a^2\sin(c+d\right.}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^2+a^2\sin(c+d\right.}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^2+a^2\sin(c+d\right.}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+\right.}{117ade(a+a\sin}$$

Mathematica [C] time = 0.05, size = 66, normalized size = 0.35

$$-\frac{(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{12\sqrt[4]{2}\,a^4de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^4,x]
[Out] -1/12*((e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 17/4, 7/4, (1 - Sin[c
+ d*x])/2])/(2^(1/4)*a^4*d*e*(1 + Sin[c + d*x])^(3/4))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{a^4\cos(dx+c)^4-8a^4\cos(dx+c)^2+8a^4-4\left(a^4\cos(dx+c)^2-2a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 +
8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(dx+c)}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^4, x)
```

maple [B] time = 5.34, size = 694, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/117/(64*sin(1/2*d*x+1/2*c)^12-192*sin(1/2*d*x+1/2*c)^10+240*sin(1/2*d*x+
1/2*c)^8-160*sin(1/2*d*x+1/2*c)^6+60*sin(1/2*d*x+1/2*c)^4-12*sin(1/2*d*x+1/
2*c)^2+1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(192*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12-384*sin(1/2*d*x+1/2*c)^14*cos
(1/2*d*x+1/2*c)-576*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+1152*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+720*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^8-1472*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-480*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+1024*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^8+180*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-280*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)-36*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-40*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*sin(1/2*d*x+1/2*c)^5+3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)-120*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+208*sin(1/2*d*
x+1/2*c)^3+20*sin(1/2*d*x+1/2*c))*e/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{\left(a\sin\left(dx+c\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.271 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=191

$$-\frac{2\sqrt{e\cos(c+dx)}}{33de\left(a^4\sin(c+dx)+a^4\right)}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^4d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{33de\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{14\sqrt{e\cos(c+}{165ade(a\sin(c+a}$$

```
[Out] 2/33*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^4/d/(e*cos(d*x+c))^(1/2)-2/15*(e*cos(d*x
+c))^(1/2)/d/e/(a+a*sin(d*x+c))^4-14/165*(e*cos(d*x+c))^(1/2)/a/d/e/(a+a*si
n(d*x+c))^3-2/33*(e*cos(d*x+c))^(1/2)/d/e/(a^2+a^2*sin(d*x+c))^2-2/33*(e*co
s(d*x+c))^(1/2)/d/e/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.24, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2681, 2683, 2642, 2641}

$$-\frac{2\sqrt{e\cos(c+dx)}}{33de\left(a^4\sin(c+dx)+a^4\right)}-\frac{2\sqrt{e\cos(c+dx)}}{33de\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^4d\sqrt{e\cos(c+dx)}}-\frac{14\sqrt{e\cos(c+}{165ade(a\sin(c+a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(33*a^4*d*Sqrt[e*Cos[c + d
*x]]) - (2*Sqrt[e*Cos[c + d*x]])/(15*d*e*(a + a*Sin[c + d*x])^4) - (14*Sqrt
[e*Cos[c + d*x]])/(165*a*d*e*(a + a*Sin[c + d*x])^3) - (2*Sqrt[e*Cos[c + d*
x]])/(33*d*e*(a^2 + a^2*Sin[c + d*x])^2) - (2*Sqrt[e*Cos[c + d*x]])/(33*d*e
*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4} + \frac{7\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}\,dx}{15a}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4} - \frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3} + \frac{7\int \frac{}{\sqrt{e\cos(c}}}{}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4} - \frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3} - \frac{2\sqrt{e\,c}}{33de\left(a^2+\right.}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4} - \frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3} - \frac{2\sqrt{e\,c}}{33de\left(a^2+\right.}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4} - \frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3} - \frac{2\sqrt{e\,c}}{33de\left(a^2+\right.}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^4d\sqrt{e\cos(c+dx)}} - \frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4} - \frac{14}{165ade(}$$

Mathematica [C] time = 0.06, size = 66, normalized size = 0.35

$$-\frac{\sqrt{e\cos(c+dx)}\;{}_2F_1\left(\frac{1}{4},\frac{19}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{4\,2^{3/4}a^4de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] -1/4*(Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 19/4, 5/4, (1 - Sin[c + d
*x])/2])/(2^(3/4)*a^4*d*e*(1 + Sin[c + d*x])^(1/4))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{a^4e\cos(dx+c)^5 - 8\,a^4e\cos(dx+c)^3 + 8\,a^4e\cos(dx+c) - 4\left(a^4e\cos(dx+c)^3 - 2\,a^4e\cos(dx+c)\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a^4*e*cos(d*x + c)^5 - 8*a^4*e*cos(d*x + c)^
3 + 8*a^4*e*cos(d*x + c) - 4*(a^4*e*cos(d*x + c)^3 - 2*a^4*e*cos(d*x + c))*
sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^4), x)
```

maple [B] time = 5.29, size = 762, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/165/(128*sin(1/2*d*x+1/2*c)^14-448*sin(1/2*d*x+1/2*c)^12+672*sin(1/2*d*x
+1/2*c)^10-560*sin(1/2*d*x+1/2*c)^8+280*sin(1/2*d*x+1/2*c)^6-84*sin(1/2*d*x
+1/2*c)^4+14*sin(1/2*d*x+1/2*c)^2-1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*e+e)^(1/2)*(640*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^14-22
40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^12+640*sin(1/2*d*x+1/2*c)^14
*cos(1/2*d*x+1/2*c)+3360*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^10-192
0*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-2800*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*si
n(1/2*d*x+1/2*c)^8+2496*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+1400*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6-1792*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^8-420*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+616*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)+70*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2-40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+240*sin(1/2*d*x+1/2*c)^5-5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)+160*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-240*sin
(1/2*d*x+1/2*c)^3-28*sin(1/2*d*x+1/2*c))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^4),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.272 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=225

$$-\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{221a^4de^2\sqrt{\cos(c+dx)}}+\frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}}-\frac{14}{221de\left(a^4\sin(c+dx)+a^4\right)\sqrt{e\cos(c+dx)}}-\frac{}{221d}$$

```
[Out] 42/221*sin(d*x+c)/a^4/d/e/(e*cos(d*x+c))^(1/2)-2/17/d/e/(a+a*sin(d*x+c))^4/
(e*cos(d*x+c))^(1/2)-18/221/a/d/e/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2)-1
4/221/d/e/(a^2+a^2*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2)-14/221/d/e/(a^4+a^4*s
in(d*x+c))/(e*cos(d*x+c))^(1/2)-42/221*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^4/
d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2681, 2683, 2636, 2640, 2639}

$$-\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{221a^4de^2\sqrt{\cos(c+dx)}}+\frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}}-\frac{14}{221de\left(a^4\sin(c+dx)+a^4\right)\sqrt{e\cos(c+dx)}}-\frac{}{221d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] (-42*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(221*a^4*d*e^2*Sqrt[Co
s[c + d*x]]) + (42*Sin[c + d*x])/(221*a^4*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(17
*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4) - 18/(221*a*d*e*Sqrt[e*Co
s[c + d*x]]*(a + a*Sin[c + d*x])^3) - 14/(221*d*e*Sqrt[e*Cos[c + d*x]]*(a^2
 + a^2*Sin[c + d*x])^2) - 14/(221*d*e*Sqrt[e*Cos[c + d*x]]*(a^4 + a^4*Sin[c
 + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
```

```
IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4}\,dx = -\frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4} + \frac{9\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a}}{17a}$$

$$= -\frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4} - \frac{1}{221ade\sqrt{e\cos(c+d}}$$

$$= -\frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4} - \frac{1}{221ade\sqrt{e\cos(c+d}}$$

$$= -\frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4} - \frac{1}{221ade\sqrt{e\cos(c+d}}$$

$$= \frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}} - \frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx)}$$

$$= \frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}} - \frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx)}$$

$$= -\frac{42\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{221a^4de^2\sqrt{\cos(c+dx)}} + \frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}} -$$

Mathematica [C] time = 0.09, size = 66, normalized size = 0.29

$$\frac{\sqrt[4]{\sin(c+dx)+1}\;{}_2F_1\left(-\frac{1}{4},\frac{21}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{8\sqrt[4]{2}\,a^4de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] (Hypergeometric2F1[-1/4, 21/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(1/4))/(8*2^(1/4)*a^4*d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{a^4e^2\cos(dx+c)^6 - 8a^4e^2\cos(dx+c)^4 + 8a^4e^2\cos(dx+c)^2 - 4\left(a^4e^2\cos(dx+c)^4 - 2a^4e^2\cos(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a^4*e^2*cos(d*x + c)^6 - 8*a^4*e^2*cos(d*x +
 c)^4 + 8*a^4*e^2*cos(d*x + c)^2 - 4*(a^4*e^2*cos(d*x + c)^4 - 2*a^4*e^2*co
s(d*x + c)^2)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^4), x)
```

maple [B] time = 6.76, size = 878, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/221/(256*sin(1/2*d*x+1/2*c)^16-1024*sin(1/2*d*x+1/2*c)^14+1792*sin(1/2*d
*x+1/2*c)^12-1792*sin(1/2*d*x+1/2*c)^10+1120*sin(1/2*d*x+1/2*c)^8-448*sin(1
/2*d*x+1/2*c)^6+112*sin(1/2*d*x+1/2*c)^4-16*sin(1/2*d*x+1/2*c)^2+1)/a^4/sin
(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e*(5376*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^16-10752*sin(1/2*d*x+1/2*c)^18*cos(1/2*d*x+1/2
*c)-21504*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^14+43008*sin(1/2*d*x+
1/2*c)^16*cos(1/2*d*x+1/2*c)+37632*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^12-76160*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)-37632*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+77952*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^12+23520*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-50560*sin(1/2*d*x+1
/2*c)^10*cos(1/2*d*x+1/2*c)-9408*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c
)^6+21376*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+2352*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*sin(1/2*d*x+1/2*c)^4-5656*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-336*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+792*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)-272*sin(1/2*d*x+1/2*c)^5+21*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-242*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+272*sin(1/2*d*x+1/2*c)^3+36*sin(1/2*d*
x+1/2*c))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,\cos(c+d\,x))^{3/2}\,(a+a\,\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^4),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.273 $\int (e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=236

$$\frac{3e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{3e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)}}{4d(\sin(c+dx)+\cos(c}$$

```
[Out] -1/2*a*(e*cos(d*x+c))^(5/2)/d/e/(a+a*sin(d*x+c))^(1/2)+3/4*e*(e*cos(d*x+c))
^(1/2)*(a+a*sin(d*x+c))^(1/2)/d-3/4*e^(3/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^
(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+
c))+3/4*e^(3/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c
))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d
*x+c))
```

Rubi [A] time = 0.36, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2678, 2685, 2677, 2775, 203, 2833, 63, 215}

$$\frac{3e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{3e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)}}{4d(\sin(c+dx)+\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(a*(e*Cos[c + d*x])^(5/2))/(2*d*e*Sqrt[a + a*Sin[c + d*x]]) + (3*e*Sqrt[e*
Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d) - (3*e^(3/2)*ArcSinh[Sqrt[e*C
os[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d
*(1 + Cos[c + d*x] + Sin[c + d*x])) + (3*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c + d*
x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*S
qrt[a + a*Sin[c + d*x]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
```

```
EqQ[a^2 - b^2, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}}+\frac{1}{4}(3a)\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}}+\frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}}+\frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}}+\frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}}+\frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}}+\frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d}$$

Mathematica [C] time = 0.95, size = 269, normalized size = 1.14

$$-\frac{iee^{-i(c+dx)}\sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}\left(-3dxe^{2i(c+dx)}-2e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}+2ie^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}+\right.}{4d\left(e^{i(c+dx)}+i\right)\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-1/4*I)*e*Sqrt[e*Cos[c + d*x]]*((-I)*Sqrt[1 + E^((2*I)*(c + d*x))] - 2*E^
(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))] + (2*I)*E^((2*I)*(c + d*x))*Sqr
t[1 + E^((2*I)*(c + d*x))] + E^((3*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x
))] - 3*d*E^((2*I)*(c + d*x))*x + 3*E^((2*I)*(c + d*x))*ArcSinh[E^(I*(c + d
*x))] - (3*I)*E^((2*I)*(c + d*x))*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]])*S
qrt[a*(1 + Sin[c + d*x])])/(d*E^(I*(c + d*x))*(I + E^(I*(c + d*x)))*Sqrt[1
+ E^((2*I)*(c + d*x))])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}\sqrt{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*sqrt(a*sin(d*x + c) + a), x)
```

maple [A] time = 0.35, size = 241, normalized size = 1.02

$$-\frac{\left(3\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sin(dx+c)-3\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c}{2\cos(dx+c)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d*(3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-3*2^(1/2)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-4*cos(d*x+c)^3-4*cos(d*x+c)^2*sin(d*x+c
)-2*cos(d*x+c)^2+6*cos(d*x+c)*sin(d*x+c)+6*cos(d*x+c))*(e*cos(d*x+c))^(3/2)
*(a*(1+sin(d*x+c)))^(1/2)/(-1+cos(d*x+c)-sin(d*x+c))/cos(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}\sqrt{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos(c+d\,x))^{3/2}\,\sqrt{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin(c+dx)+1\right)}\,(e\cos(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(3/2), x)
```

3.274 $\int \sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a\sin(c+dx)+a}}+\frac{\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{\sqrt{e}\sqrt{\cos(c+d}}{}$$

```
[Out] -a*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^(1/2)+arcsinh((e*cos(d*x+c))^(
1/2)/e^(1/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(
d*x+c)+sin(d*x+c))+arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*
x+c))^(1/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d
*x+c)+sin(d*x+c))
```

Rubi [A] time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2678, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a\sin(c+dx)+a}}+\frac{\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{\sqrt{e}\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((a*(e*Cos[c + d*x])^(3/2))/(d*e*Sqrt[a + a*Sin[c + d*x]])) + (Sqrt[e]*Arc
Sinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
+ d*x]])/(d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (Sqrt[e]*ArcTan[(Sqrt[e]*S
in[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
+ d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{1}{2}a\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\left(ae\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right.}{2(a+a\cos(c+dx)+a\sin(c}$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\left(ae\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right.}{2d(a+a\cos(c+dx}$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+}}{d(1+\cos(c+dx)+s}$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\sqrt{e}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx}}{d(1+\cos(c+dx)+\sin(c}$$

Mathematica [C] time = 0.74, size = 195, normalized size = 1.01

$$-\frac{i\sqrt{a(\sin(c+dx)+1)}\,\sqrt{e\cos(c+dx)}\left(idxe^{i(c+dx)}+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}-i\sqrt{1+e^{2i(c+dx)}}-e^{i(c+dx)}\log\left(1+\sqrt{\right.\right.}}{d\left(e^{i(c+dx)}+i\right)\sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[e*Cos[c + d*x]]*((-I)*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(I*(c +
d*x))*Sqrt[1 + E^((2*I)*(c + d*x))] + I*d*E^(I*(c + d*x))*x + I*E^(I*(c + d
*x))*ArcSinh[E^(I*(c + d*x))] - E^(I*(c + d*x))*Log[1 + Sqrt[1 + E^((2*I)*(
```

```
c + d*x))]])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(I + E^(I*(c + d*x)))*Sqrt[1 +
E^((2*I)*(c + d*x))])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,\sqrt{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a), x)
```

maple [A] time = 0.27, size = 213, normalized size = 1.10

$$-\frac{\left(\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cdots}{2d\,(1-\cos(dx+c)+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-2*co
s(d*x+c))*(e*cos(d*x+c))^(1/2)*(a*(1+sin(d*x+c)))^(1/2)/(1-cos(d*x+c)+sin(d
*x+c))/cos(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,\sqrt{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sqrt{e\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sqrt(e*cos(c + d*x)), x)
```

3.275 $\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=161

$$\frac{2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}-\frac{2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sin}{d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx}$$

```
[Out] -2*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+
c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))/e^(1/2)+2*arctan(sin(d*x+c)*e^(1/2)/(
e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x
+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))/e^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2677, 2775, 203, 2833, 63, 215}

$$\frac{2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}-\frac{2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sin}{d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a
*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (2*ArcTan[(
Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1
 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + S
in[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\left(a\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{a+a\cos(c+dx)+a\sin(c+dx)} + \frac{\left(a\sqrt{1+\cos(c+dx)}\right.\cdots}{a+\cdots}$$

$$= -\frac{\left(a\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d(a+a\cos(c+dx)+a\sin(c+dx))} - \cdots$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d\sqrt{e}\,(1+\cos(c+dx)+\sin(c+dx))} - \frac{\left(2a\sqrt{1+\cdots\right.}{\cdots}$$

$$= -\frac{2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d\sqrt{e}\,(1+\cos(c+dx)+\sin(c+dx))} + \frac{2\tan^{-1}\left(\frac{\sqrt{e}\cdots}{\sqrt{e\cos(c+\cdots}}\right.\cdots}{\cdots}$$

Mathematica [C] time = 0.47, size = 108, normalized size = 0.67

$$\frac{\sqrt{1+e^{2i(c+dx)}}\sqrt{a(\sin(c+dx)+1)}\left(i\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-\sinh^{-1}\left(e^{i(c+dx)}\right)+dx\right)}{d\left(1-ie^{i(c+dx)}\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[1 + E^((2*I)*(c + d*x))]*(d*x - ArcSinh[E^(I*(c + d*x))] + I*Log[1 +
Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(1 - I*E^(I*
(c + d*x)))*Sqrt[e*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin(dx+c)+a}}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

maple [A] time = 0.19, size = 142, normalized size = 0.88

$$-\frac{\left(\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)-\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\right)\sqrt{a\left(1+\sin(dx+c)\right)}\,\left(-1+\cos(dx+c)+\sin(\right.}{d\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{e\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)))*(a*(1+
sin(d*x+c)))^(1/2)*(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/(e*cos(d*x+c))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin(dx+c)+a}}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin(c+d\,x)}}{\sqrt{e\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin(c+dx)+1\right)}}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/sqrt(e*cos(c + d*x)), x)
```

3.276 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{2\sqrt{a\sin(c+dx)+a}}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*(a+a*sin(d*x+c))^(1/2)/d/e/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2671}

$$\frac{2\sqrt{a\sin(c+dx)+a}}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx=\frac{2\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 0.11, size = 34, normalized size = 1.00

$$\frac{2\sqrt{a(\sin(c+dx)+1)}}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])])/(d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [A] time = 0.94, size = 38, normalized size = 1.12

$$\frac{2\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}}{de^2\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*e^2*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin(dx+c)+a}}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

maple [A] time = 0.21, size = 34, normalized size = 1.00

$$\frac{2\cos(dx+c)\sqrt{a(1+\sin(dx+c))}}{d(e\cos(dx+c))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d*cos(d*x+c)*(a*(1+sin(d*x+c)))^(1/2)/(e*cos(d*x+c))^(3/2)
```

maxima [B] time = 0.96, size = 131, normalized size = 3.85

$$\frac{2\left(\sqrt{a}\sqrt{e}-\frac{\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{\left(e^2+\frac{e^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] 2*(sqrt(a)*sqrt(e) - sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/((e^2 + e^2*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2)*d*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(-sin(d*x + c)/(
cos(d*x + c) + 1) + 1)^(3/2))
```

mupad [B] time = 5.31, size = 30, normalized size = 0.88

$$\frac{2\sqrt{a+a\,\sin(c+d\,x)}}{d\,e\,\sqrt{e\,\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] (2*(a + a*sin(c + d*x))^(1/2))/(d*e*(e*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(c+dx)+1)}}{(e\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/(e*cos(c + d*x))**(3/2), x)
```

3.277 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{4(a\sin(c+dx)+a)^{3/2}}{3ade(e\cos(c+dx))^{3/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{de(e\cos(c+dx))^{3/2}}$$

```
[Out] 4/3*(a+a*sin(d*x+c))^(3/2)/a/d/e/(e*cos(d*x+c))^(3/2)-2*(a+a*sin(d*x+c))^(1
/2)/d/e/(e*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{4(a\sin(c+dx)+a)^{3/2}}{3ade(e\cos(c+dx))^{3/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(d*e*(e*Cos[c + d*x])^(3/2)) + (4*(a + a*Sin[
c + d*x])^(3/2))/(3*a*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx &= -\frac{2\sqrt{a+a\sin(c+dx)}}{de(e\cos(c+dx))^{3/2}}+\frac{2\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{5/2}}\,dx}{a}\\ &= -\frac{2\sqrt{a+a\sin(c+dx)}}{de(e\cos(c+dx))^{3/2}}+\frac{4(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.25, size = 46, normalized size = 0.62

$$\frac{2(2\sin(c+dx)-1)\sqrt{a(\sin(c+dx)+1)}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])]*(-1 + 2*Sin[c + d*x]))/(3*d*e*(e*Cos[c + d*x]
)^(3/2))
```

fricas [A] time = 0.89, size = 48, normalized size = 0.65

$$\frac{2 \sqrt{e \cos\left(dx + c\right)} \sqrt{a \sin\left(dx + c\right) + a} \left(2 \sin\left(dx + c\right) - 1\right)}{3 \, de^{3} \cos\left(dx + c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*(2*sin(d*x + c) - 1)/(d*e
^3*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.21, size = 44, normalized size = 0.59

$$\frac{2 \left(2 \sin\left(dx + c\right) - 1\right) \sqrt{a \left(1 + \sin\left(dx + c\right)\right)}\, \cos\left(dx + c\right)}{3 d \left(e \cos\left(dx + c\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*(2*sin(d*x+c)-1)*(a*(1+sin(d*x+c)))^(1/2)*cos(d*x+c)/(e*cos(d*x+c))^(
5/2)
```

maxima [B] time = 0.97, size = 206, normalized size = 2.78

$$-\frac{2 \left(\sqrt{a} \sqrt{e} - \frac{4 \sqrt{a} \sqrt{e} \sin(dx+c)}{\cos(dx+c)+1} + \frac{4 \sqrt{a} \sqrt{e} \sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} - \frac{\sqrt{a} \sqrt{e} \sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right) \left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + 1\right)^{2}}{3 \left(e^{3} + \frac{2 e^{3} \sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{e^{3} \sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right) d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)^{\frac{3}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*(sqrt(a)*sqrt(e) - 4*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1) +
 4*sqrt(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - sqrt(a)*sqrt(e)*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
)^2/((e^3 + 2*e^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + e^3*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

mupad [B] time = 5.67, size = 61, normalized size = 0.82

$$-\frac{4 \sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(\cos\left(c+d\,x\right)-\sin\left(2\,c+2\,d\,x\right)\right)}{3\,d\,e^2\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,\sqrt{e\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] -(4*(a*(sin(c + d*x) + 1))^(1/2)*(cos(c + d*x) - sin(2*c + 2*d*x)))/(3*d*e^
2*(cos(2*c + 2*d*x) + 1)*(e*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(c+dx)+1)}}{(e\cos(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/(e*cos(c + d*x))**(5/2), x)
```

3.278 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{16(a\sin(c+dx)+a)^{5/2}}{15a^2de(e\cos(c+dx))^{5/2}}+\frac{8(a\sin(c+dx)+a)^{3/2}}{3ade(e\cos(c+dx))^{5/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{3de(e\cos(c+dx))^{5/2}}$$

```
[Out] 8/3*(a+a*sin(d*x+c))^(3/2)/a/d/e/(e*cos(d*x+c))^(5/2)-16/15*(a+a*sin(d*x+c)
)^(5/2)/a^2/d/e/(e*cos(d*x+c))^(5/2)-2/3*(a+a*sin(d*x+c))^(1/2)/d/e/(e*cos(
d*x+c))^(5/2)
```

Rubi [A] time = 0.22, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{16(a\sin(c+dx)+a)^{5/2}}{15a^2de(e\cos(c+dx))^{5/2}}+\frac{8(a\sin(c+dx)+a)^{3/2}}{3ade(e\cos(c+dx))^{5/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{3de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(3*d*e*(e*Cos[c + d*x])^(5/2)) + (8*(a + a*Si
n[c + d*x])^(3/2))/(3*a*d*e*(e*Cos[c + d*x])^(5/2)) - (16*(a + a*Sin[c + d*
x])^(5/2))/(15*a^2*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx &= -\frac{2\sqrt{a+a\sin(c+dx)}}{3de(e\cos(c+dx))^{5/2}}+\frac{4\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{7/2}}\,dx}{3a}\\
&= -\frac{2\sqrt{a+a\sin(c+dx)}}{3de(e\cos(c+dx))^{5/2}}+\frac{8(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{5/2}}-\frac{8\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx}{3a^2}\\
&= -\frac{2\sqrt{a+a\sin(c+dx)}}{3de(e\cos(c+dx))^{5/2}}+\frac{8(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{5/2}}-\frac{16(a+a\sin(c+dx))^{5/2}}{15a^2de(e\cos(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 56, normalized size = 0.49

$$\frac{2\sqrt{a(\sin(c+dx)+1)}\,(4\sin(c+dx)+4\cos(2(c+dx))+3)}{15de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])]*(3 + 4*Cos[2*(c + d*x)] + 4*Sin[c + d*x]))/(1
5*d*e*(e*Cos[c + d*x])^(5/2))
```

fricas [A] time = 1.08, size = 58, normalized size = 0.50

$$\frac{2\sqrt{e\cos\left(dx+c\right)}\left(8\cos\left(dx+c\right)^{2}+4\sin\left(dx+c\right)-1\right)\sqrt{a\sin\left(dx+c\right)+a}}{15\,de^{4}\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/15*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 + 4*sin(d*x + c) - 1)*sqrt(a*si
n(d*x + c) + a)/(d*e^4*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.21, size = 54, normalized size = 0.47

$$\frac{2\left(8\left(\cos^{2}\left(dx+c\right)\right)+4\sin\left(dx+c\right)-1\right)\sqrt{a\left(1+\sin\left(dx+c\right)\right)}\;\cos\left(dx+c\right)}{15d\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] 2/15/d*(8*cos(d*x+c)^2+4*sin(d*x+c)-1)*(a*(1+sin(d*x+c)))^(1/2)*cos(d*x+c)/
(e*cos(d*x+c))^(7/2)
```

maxima [B] time = 0.98, size = 282, normalized size = 2.45

$$\frac{2\left(7\sqrt{a}\sqrt{e}+\frac{8\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{a}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{25\sqrt{a}\sqrt{e}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{8\sqrt{a}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{7\sqrt{a}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)\left(\frac{\sin(}{(\cos(d}\right.}{15\left(e^{4}+\frac{3e^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3e^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{e^{4}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxima"
)
```

```
[Out] 2/15*(7*sqrt(a)*sqrt(e) + 8*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
 - 25*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt(a)*sqrt
(e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 8*sqrt(a)*sqrt(e)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 7*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((e^4 + 3*e^4*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 3*e^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + e^4*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

mupad [B] time = 6.06, size = 97, normalized size = 0.84

$$\frac{8\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(2\,\sin\left(c+d\,x\right)+7\,\cos\left(2\,c+2\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+2\,\sin\left(3\,c+3\,d\,x\right)+5\right)}{15\,d\,e^3\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] (8*(a*(sin(c + d*x) + 1))^(1/2)*(2*sin(c + d*x) + 7*cos(2*c + 2*d*x) + 2*co
s(4*c + 4*d*x) + 2*sin(3*c + 3*d*x) + 5))/(15*d*e^3*(e*cos(c + d*x))^(1/2)*
(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.279 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{32(a\sin(c+dx)+a)^{7/2}}{35a^3de(e\cos(c+dx))^{7/2}}+\frac{16(a\sin(c+dx)+a)^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}-\frac{12(a\sin(c+dx)+a)^{3/2}}{5ade(e\cos(c+dx))^{7/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{5de(e\cos(c+dx))^{7/2}}$$

```
[Out] -12/5*(a+a*sin(d*x+c))^(3/2)/a/d/e/(e*cos(d*x+c))^(7/2)+16/5*(a+a*sin(d*x+c
))^(5/2)/a^2/d/e/(e*cos(d*x+c))^(7/2)-32/35*(a+a*sin(d*x+c))^(7/2)/a^3/d/e/
(e*cos(d*x+c))^(7/2)-2/5*(a+a*sin(d*x+c))^(1/2)/d/e/(e*cos(d*x+c))^(7/2)
```

Rubi [A] time = 0.31, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{32(a\sin(c+dx)+a)^{7/2}}{35a^3de(e\cos(c+dx))^{7/2}}+\frac{16(a\sin(c+dx)+a)^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}-\frac{12(a\sin(c+dx)+a)^{3/2}}{5ade(e\cos(c+dx))^{7/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{5de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(5*d*e*(e*Cos[c + d*x])^(7/2)) - (12*(a + a*S
in[c + d*x])^(3/2))/(5*a*d*e*(e*Cos[c + d*x])^(7/2)) + (16*(a + a*Sin[c + d
*x])^(5/2))/(5*a^2*d*e*(e*Cos[c + d*x])^(7/2)) - (32*(a + a*Sin[c + d*x])^(
7/2))/(35*a^3*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{9/2}}\,dx &= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}}+\frac{6\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{9/2}}\,dx}{5a}\\
&= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}}-\frac{12(a+a\sin(c+dx))^{3/2}}{5ade(e\cos(c+dx))^{7/2}}+\frac{24\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx}{5a^2}\\
&= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}}-\frac{12(a+a\sin(c+dx))^{3/2}}{5ade(e\cos(c+dx))^{7/2}}+\frac{16(a+a\sin(c+dx))^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}-\frac{16\int}{}\\
&= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}}-\frac{12(a+a\sin(c+dx))^{3/2}}{5ade(e\cos(c+dx))^{7/2}}+\frac{16(a+a\sin(c+dx))^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}-\frac{32(}{35a}
\end{aligned}$$

Mathematica [A] time = 0.80, size = 74, normalized size = 0.48

$$\frac{2\sec^4(c+dx)\sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}(10\sin(c+dx)+4\sin(3(c+dx))-4\cos(2(c+dx))-5)}{35de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*Sqrt[a*(1 + Sin[c + d*x])]*(-5 - 4*C
os[2*(c + d*x)] + 10*Sin[c + d*x] + 4*Sin[3*(c + d*x)]))/(35*d*e^5)
```

fricas [A] time = 0.75, size = 70, normalized size = 0.45

$$-\frac{2\sqrt{e\cos(dx+c)}\left(8\cos(dx+c)^2-2\left(8\cos(dx+c)^2+3\right)\sin(dx+c)+1\right)\sqrt{a\sin(dx+c)+a}}{35\,de^5\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x, algorithm="fricas"
)
```

```
[Out] -2/35*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 - 2*(8*cos(d*x + c)^2 + 3)*sin
(d*x + c) + 1)*sqrt(a*sin(d*x + c) + a)/(d*e^5*cos(d*x + c)^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.23, size = 70, normalized size = 0.45

$$\frac{2\left(16\left(\cos^2(dx+c)\right)\sin(dx+c)-8\left(\cos^2(dx+c)\right)+6\sin(dx+c)-1\right)\sqrt{a(1+\sin(dx+c))}\ \cos(dx+c)}{35d\,(e\cos(dx+c))^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] 2/35/d*(16*cos(d*x+c)^2*sin(d*x+c)-8*cos(d*x+c)^2+6*sin(d*x+c)-1)*(a*(1+sin
(d*x+c)))^(1/2)*cos(d*x+c)/(e*cos(d*x+c))^(9/2)
```

maxima [B] time = 0.98, size = 357, normalized size = 2.32

$$-\frac{2\left(9\sqrt{a}\sqrt{e}-\frac{44\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{84\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{84\sqrt{a}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{14\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{44\ \cdots}{(\cdots}\right.}{35\left(e^5+\frac{4e^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6e^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4e^5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{e^5\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x, algorithm="maxima"
)
```

```
[Out] -2/35*(9*sqrt(a)*sqrt(e) - 44*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
1) - 14*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 84*sqrt(a)*sq
rt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 84*sqrt(a)*sqrt(e)*sin(d*x + c)
```

```
^5/(cos(d*x + c) + 1)^5 + 14*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + 44*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 9*sqrt(a)*
sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)^4/((e^5 + 4*e^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*e^5*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*e^5*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + e^5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2))
```

mupad [B] time = 7.24, size = 129, normalized size = 0.84

$$-\frac{16\sqrt{a\,(\sin(c+d\,x)+1)}\,\left(23\cos(c+d\,x)+11\cos(3\,c+3\,d\,x)+2\cos(5\,c+5\,d\,x)-16\sin(2\,c+2\,d\,x)-\cdots\right.}{35\,d\,e^4\sqrt{e\cos(c+d\,x)}\,\left(15\cos(2\,c+2\,d\,x)+6\cos(4\,c+4\,d\,x)+\cos(6\,c+6\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] -(16*(a*(sin(c + d*x) + 1))^(1/2)*(23*cos(c + d*x) + 11*cos(3*c + 3*d*x) +
2*cos(5*c + 5*d*x) - 16*sin(2*c + 2*d*x) - 11*sin(4*c + 4*d*x) - 2*sin(6*c
+ 6*d*x)))/(35*d*e^4*(e*cos(c + d*x))^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*
c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

3.280 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=319

$$-\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a\sin(c+dx)+a)^{3/2}}-\frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a\sin(c+dx)+a}}+\frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a\sin(c+dx)+a}}+\frac{45ae^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c}}{64d(\sin(c+d}$$

```
[Out] -15/32*a^3*(e*cos(d*x+c))^(7/2)/d/e/(a+a*sin(d*x+c))^(3/2)+15/64*a^2*e*(e*c
os(d*x+c))^(3/2)/d/(a+a*sin(d*x+c))^(1/2)-3/8*a^2*(e*cos(d*x+c))^(7/2)/d/e/
(a+a*sin(d*x+c))^(1/2)-1/4*a*(e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^(1/2)/d/
e+45/64*a*e^(5/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2
)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))+45/64*a*e^(5/2)*arctan
(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c
))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))
```

Rubi [A] time = 0.56, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2678, 2686, 2679, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a\sin(c+dx)+a}}-\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a\sin(c+dx)+a)^{3/2}}+\frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a\sin(c+dx)+a}}+\frac{45ae^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c}}{64d(\sin(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-15*a^3*(e*Cos[c + d*x])^(7/2))/(32*d*e*(a + a*Sin[c + d*x])^(3/2)) + (15*
a^2*e*(e*Cos[c + d*x])^(3/2))/(64*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a^2*(e*C
os[c + d*x])^(7/2))/(8*d*e*Sqrt[a + a*Sin[c + d*x]]) - (a*(e*Cos[c + d*x])^
(7/2)*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) + (45*a*e^(5/2)*ArcSinh[Sqrt[e*Cos[
c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d*(
1 + Cos[c + d*x] + Sin[c + d*x])) + (45*a*e^(5/2)*ArcTan[(Sqrt[e]*Sin[c + d
*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*
Sqrt[a + a*Sin[c + d*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
```

```
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2686

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1))/(f*g*(2*p - 1)*
(a + b*Sin[e + f*x])^(3/2)), x] + Dist[(2*a*(p - 2))/(2*p - 1), Int[(g*Cos[
e + f*x])^p/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 2] && IntegerQ[2*p]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}{4de} + \frac{1}{8}(9a)\int (e\cos(c+dx))^{5/}$$

$$= -\frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}{4de} +$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} - \frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+}{}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos}{8de\sqrt{a+a}}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos}{8de\sqrt{a+a}}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos}{8de\sqrt{a+a}}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos}{8de\sqrt{a+a}}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos}{8de\sqrt{a+a}}$$

Mathematica [C] time = 0.17, size = 78, normalized size = 0.24

$$-\frac{16\sqrt[4]{2}\,a\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{9}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-16*2^(1/4)*a*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-9/4, 7/4, 11/4, (1
 - Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(7*d*e*(1 + Sin[c + d*x])^(
9/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}\,(a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^(3/2), x)

maple [A] time = 0.34, size = 314, normalized size = 0.98

$$\frac{\left(32 \sin \left(d x+c\right) \left(\cos^{4}\left(d x+c\right)\right)-32 \left(\cos^{5}\left(d x+c\right)\right)+45 \sqrt{2}\, \sqrt{-\frac{2 \cos \left(d x+c\right)}{1+\cos \left(d x+c\right)}}\, \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2 \cos \left(d x+c\right)}{1+\cos \left(d x+c\right)}}\, \sin \left(d x+c\right) \sqrt{2}}{2 \cos \left(d x+c\right)}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x)

[Out] 1/128/d*(32*sin(d*x+c)*cos(d*x+c)^4-32*cos(d*x+c)^5+45*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+45*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+48*sin(d*x+c)*cos(d*x+c)^3+80*cos(d*x+c)^4-60*cos(d*x+c)^2*sin(d*x+c)+12*cos(d*x+c)^3+90*cos(d*x+c)*sin(d*x+c)+30*cos(d*x+c)^2-90*cos(d*x+c))*(e*cos(d*x+c))^(5/2)*(a*(1+sin(d*x+c)))^(3/2)/(cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-2*sin(d*x+c)+cos(d*x+c)-2)/cos(d*x+c)^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e \cos \left(d x+c\right)\right)^{\frac{5}{2}} \left(a \sin \left(d x+c\right)+a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^(3/2),x)

[Out] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**(3/2),x)

[Out] Timed out

3.281 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=278

$$-\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a\sin(c+dx)+a}}+\frac{7ae^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{8d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{7ae^{3/2}\sqrt{c\cdots}}{}$$

```
[Out] -7/12*a^2*(e*cos(d*x+c))^(5/2)/d/e/(a+a*sin(d*x+c))^(1/2)-1/3*a*(e*cos(d*x+
c))^(5/2)*(a+a*sin(d*x+c))^(1/2)/d/e+7/8*a*e*(e*cos(d*x+c))^(1/2)*(a+a*sin(
d*x+c))^(1/2)/d-7/8*a*e^(3/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*(1+cos(
d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))+7/8*a*e^(3
/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1
+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))
```

Rubi **[A]** time = 0.47, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2678, 2685, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a\sin(c+dx)+a}}+\frac{7ae^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{8d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{7ae^{3/2}\sqrt{c\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-7*a^2*(e*Cos[c + d*x])^(5/2))/(12*d*e*Sqrt[a + a*Sin[c + d*x]]) + (7*a*e*
Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d) - (a*(e*Cos[c + d*x])^
(5/2)*Sqrt[a + a*Sin[c + d*x]])/(3*d*e) - (7*a*e^(3/2)*ArcSinh[Sqrt[e*Cos[c
 + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d*(1
+ Cos[c + d*x] + Sin[c + d*x])) + (7*a*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c + d*x]
)/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*Sqr
t[a + a*Sin[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
```

```
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}{3de} + \frac{1}{6}(7a)\int (e\cos(c+dx))^{3/}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}{3de}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} + \frac{7ae\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{8d}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} + \frac{7ae\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{8d}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} + \frac{7ae\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{8d}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} + \frac{7ae\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{8d}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} + \frac{7ae\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{8d}$$

Mathematica [C] time = 0.13, size = 78, normalized size = 0.28

$$-\frac{8\,2^{3/4}a\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{7}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*2^(3/4)*a*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-7/4, 5/4, 9/4, (1 -
 Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(5*d*e*(1 + Sin[c + d*x])^(7/
4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.33, size = 288, normalized size = 1.04

$$\left(16\sin(dx+c)\left(\cos^3(dx+c)\right)+21\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)-21\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/48/d*(16*sin(d*x+c)*cos(d*x+c)^3+21*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*sin(d*x+c)-21*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-16*cos(d*x
+c)^4+28*cos(d*x+c)^2*sin(d*x+c)+44*cos(d*x+c)^3-42*cos(d*x+c)*sin(d*x+c)+1
4*cos(d*x+c)^2-42*cos(d*x+c))*(e*cos(d*x+c))^(3/2)*(a*(1+sin(d*x+c)))^(3/2)
/(cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-2*sin(d*x+c)+cos(d*x+c)-2)/cos(d*x+c)^
2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos(c+d\,x))^{3/2}\,(a+a\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.282 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=243

$$-\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a\sin(c+dx)+a}}-\frac{a\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}{2de}+\frac{5a\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-}}{4d(\sin(c+dx)+\cos(c+d}$$

```
[Out] -5/4*a^2*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^(1/2)-1/2*a*(e*cos(d*x+c
))^(3/2)*(a+a*sin(d*x+c))^(1/2)/d/e+5/4*a*arcsinh((e*cos(d*x+c))^(1/2)/e^(1
/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+si
n(d*x+c))+5/4*a*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c
))^(1/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+
c)+sin(d*x+c))
```

Rubi [A] time = 0.36, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2678, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a\sin(c+dx)+a}}-\frac{a\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}{2de}+\frac{5a\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-}}{4d(\sin(c+dx)+\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-5*a^2*(e*Cos[c + d*x])^(3/2))/(4*d*e*Sqrt[a + a*Sin[c + d*x]]) - (a*(e*Co
s[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]])/(2*d*e) + (5*a*Sqrt[e]*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (5*a*Sqrt[e]*ArcTan[(Sqrt[e]*
Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
 + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
```

```
*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de}+\frac{1}{4}(5a)\int \sqrt{e\cos(c+dx}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}}-\frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}}-\frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}}-\frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}}-\frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}}-\frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de}$$

Mathematica [C] time = 0.12, size = 77, normalized size = 0.32

$$-\frac{8\sqrt[4]{2}\,(a(\sin(c+dx)+1))^{3/2}(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*2^(1/4)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-5/4, 3/4, 7/4, (1 - S
in[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(3*d*e*(1 + Sin[c + d*x])^(9/
4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.30, size = 262, normalized size = 1.08

$$\frac{\left(5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)+5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \cdots}{2}\cdots\right.\right.}{8d\left(\cos(dx+c)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+5*2^(
1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+4*cos(d*x+c)^2*sin(d*x+c)-4*cos(d*x+c)^3
+10*cos(d*x+c)*sin(d*x+c)+14*cos(d*x+c)^2-10*cos(d*x+c))*(a*(1+sin(d*x+c)))
^(3/2)*(e*cos(d*x+c))^(1/2)/(cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-2*sin(d*x+c
)+cos(d*x+c)-2)/cos(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\,\cos(c+d\,x)}\,\left(a+a\,\sin(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sin (c + dx) + 1))^{\frac{3}{2}} \sqrt{e \cos (c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sqrt(e*cos(c + d*x)), x)
```

3.283 $\int \frac{(a+a\sin(c+dx))^{3/2}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=198

$$-\frac{a\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{de}+\frac{3a\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}-\ 3$$

```
[Out] -a*(e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e-3*a*arcsinh((e*cos(d*x+c
))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+
c)+sin(d*x+c))/e^(1/2)+3*a*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(
1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d
*x+c)+sin(d*x+c))/e^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2678, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{a\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{de}+\frac{3a\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}-\ 3$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((a*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e)) - (3*a*ArcSinh[S
qrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]
])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (3*a*ArcTan[(Sqrt[e]*Sin
[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c +
d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x]
))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
```

```
EqQ[a^2 - b^2, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de} + \frac{1}{2}(3a)\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de} + \frac{\left(3a^2\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+d\ldots\right.}{2(a+a\cos(c+dx)+a\sin\ldots}$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de} - \frac{\left(3a^2\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+d\ldots\right.}{2d(a+a\cos(c+\ldots}$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de} + \frac{3a^2\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1-\ldots}}{d\sqrt{e}\,(a+a\cos(c+dx)\ldots}$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de} - \frac{3a^2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+\ldots}}{d\sqrt{e}\,(a+a\cos(c+dx)+a\sin\ldots}$$

Mathematica [C] time = 0.11, size = 75, normalized size = 0.38

$$-\frac{4\,2^{3/4}(a(\sin(c+dx)+1))^{3/2}\sqrt{e\cos(c+dx)}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-4*2^(3/4)*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-3/4, 1/4, 5/4, (1 - Sin
[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(d*e*(1 + Sin[c + d*x])^(7/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.26, size = 228, normalized size = 1.15

$$-\frac{\left(3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin\left(dx+c\right)-3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)}{2\cos(dx+c)}\right.\right.}{2d\left(\cos\left(dx+c\right)\sin\left(dx+c\right)+\cos^{2}\left(dx+c\right)-2\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-3*2^(1/2)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-2*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-
2*cos(d*x+c))*(a*(1+sin(d*x+c)))^(3/2)/(cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-
2*sin(d*x+c)+cos(d*x+c)-2)/(e*cos(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(c+dx)+1))^{\frac{3}{2}}}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)/sqrt(e*cos(c + d*x)), x)
```

3.284 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=210

$$-\frac{2a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{de^{3/2}(a\sin(c+dx)+a\cos(c+dx)+a)}-\frac{2a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+}}{de^{3/2}(a\sin(c+dx)+a\cos(}$$

```
[Out] 4*a*(a+a*sin(d*x+c))^(1/2)/d/e/(e*cos(d*x+c))^(1/2)-2*a^2*arcsinh((e*cos(d*
x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e^(3/2)/
(a+a*cos(d*x+c)+a*sin(d*x+c))-2*a^2*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c)
)^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d
/e^(3/2)/(a+a*cos(d*x+c)+a*sin(d*x+c))
```

Rubi [A] time = 0.29, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2676, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{2a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{de^{3/2}(a\sin(c+dx)+a\cos(c+dx)+a)}-\frac{2a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+}}{de^{3/2}(a\sin(c+dx)+a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*a*Sqrt[a + a*Sin[c + d*x]])/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a^2*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(d*e^(3/2)*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) - (2*a^2*ArcTan[(Sqrt
[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + C
os[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(3/2)*(a + a*Cos[c + d*x] + a*S
in[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
```

```
gersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{a^2\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{e^2}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(a^2\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}}{e(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(a^2\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{ex}}\right.}{de(a+a\cos(c+dx)+a\sin(c+d}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{2a^2\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{}}{de^{3/2}(a+a\cos(c+dx)+a\sin(c+dx)}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{2a^2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c}}{de^{3/2}(a+a\cos(c+dx)+a\sin(c+dx))}$$

Mathematica [C] time = 0.11, size = 75, normalized size = 0.36

$$\frac{4\sqrt[4]{2}\,(a(\sin(c+dx)+1))^{3/2}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de(\sin(c+dx)+1)^{5/4}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(3/2),x]
```

[Out] (4*2^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(d*e*Sqrt[e*Cos[c + d*x]]*(1 + Sin[c + d*x])^(5/4))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.21, size = 323, normalized size = 1.54

$$-\frac{2\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{3}{2}}\left(-1+\cos\left(dx+c\right)\right)\left(\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sqrt{2}}{2}\right)\sin\left(dx+c\right)+\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{2\cdots}\right.\right.}{d\sin\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x)

[Out] -2/d*(a*(1+sin(d*x+c)))^(3/2)*(-1+cos(d*x+c))*(2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(1-cos(d*x+c)+sin(d*x+c))/(e*cos(d*x+c))^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((a*sin(d*x + c) + a)^(3/2)/(e*cos(d*x + c))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(c+dx)+1))^{\frac{3}{2}}}{(e\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)/(e*cos(c + d*x))**(3/2), x)
```

3.285 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2671}

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.10, size = 36, normalized size = 1.00

$$\frac{2(a(\sin(c+dx)+1))^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(3/2))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

fricas [A] time = 0.84, size = 45, normalized size = 1.25

$$-\frac{2\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}\,a}{3\left(de^3\sin(dx+c)-de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*a/(d*e^3*sin(d*x + c) -
d*e^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.18, size = 34, normalized size = 0.94

$$\frac{2\cos\left(dx+c\right)\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{3}{2}}}{3d\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x)
[Out] 2/3/d*cos(d*x+c)*(a*(1+sin(d*x+c)))^(3/2)/(e*cos(d*x+c))^(5/2)
```

maxima [B] time = 1.75, size = 131, normalized size = 3.64

$$\frac{2\left(a^{\frac{3}{2}}\sqrt{e}-\frac{a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{3\left(e^{3}+\frac{e^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxima"
)
[Out] 2/3*(a^(3/2)*sqrt(e) - a^(3/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 1)/((e^3 + e^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d*(-sin(d*x + c)
/(cos(d*x + c) + 1) + 1)^(5/2))
```

mupad [B] time = 5.62, size = 47, normalized size = 1.31

$$-\frac{2\,a\,\cos\left(c+d\,x\right)\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{3\,d\,e^2\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(5/2),x)
[Out] -(2*a*cos(c + d*x)*(a*(sin(c + d*x) + 1))^(1/2))/(3*d*e^2*(e*cos(c + d*x))^
(1/2)*(sin(c + d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.286 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=74

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{de(e\cos(c+dx))^{5/2}}-\frac{4(a\sin(c+dx)+a)^{5/2}}{5ade(e\cos(c+dx))^{5/2}}$$

```
[Out] 2*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(5/2)-4/5*(a+a*sin(d*x+c))^(5/2
)/a/d/e/(e*cos(d*x+c))^(5/2)
```

Rubi [A] time = 0.15, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{de(e\cos(c+dx))^{5/2}}-\frac{4(a\sin(c+dx)+a)^{5/2}}{5ade(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(d*e*(e*Cos[c + d*x])^(5/2)) - (4*(a + a*Sin
[c + d*x])^(5/2))/(5*a*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{7/2}}\,dx &= \frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{5/2}}-\frac{2\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx}{a}\\ &= \frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{5/2}}-\frac{4(a+a\sin(c+dx))^{5/2}}{5ade(e\cos(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.13, size = 72, normalized size = 0.97

$$-\frac{2a(2\sin(c+dx)-3)\sqrt{a(\sin(c+dx)+1)}}{5de^3\sqrt{e\cos(c+dx)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*a*Sqrt[a*(1 + Sin[c + d*x])]*(-3 + 2*Sin[c + d*x]))/(5*d*e^3*Sqrt[e*Cos
[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2)
```

fricas [A] time = 0.72, size = 69, normalized size = 0.93

$$\frac{2\sqrt{e\cos\left(dx+c\right)}\left(2\,a\sin\left(dx+c\right)-3\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{5\left(de^{4}\cos\left(dx+c\right)\sin\left(dx+c\right)-de^{4}\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/5*sqrt(e*cos(d*x + c))*(2*a*sin(d*x + c) - 3*a)*sqrt(a*sin(d*x + c) + a)/
(d*e^4*cos(d*x + c)*sin(d*x + c) - d*e^4*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.19, size = 44, normalized size = 0.59

$$-\frac{2\left(2\sin\left(dx+c\right)-3\right)\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)}{5d\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/d*(2*sin(d*x+c)-3)*(a*(1+sin(d*x+c)))^(3/2)*cos(d*x+c)/(e*cos(d*x+c))^
(7/2)
```

maxima [B] time = 0.97, size = 207, normalized size = 2.80

$$\frac{2\left(3\,a^{\frac{3}{2}}\sqrt{e}-\frac{4\,a^{\frac{3}{2}}\sqrt{e}\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\,a^{\frac{3}{2}}\sqrt{e}\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{3\,a^{\frac{3}{2}}\sqrt{e}\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{5\left(e^{4}+\frac{2\,e^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{e^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxima"
)
```

```
[Out] 2/5*(3*a^(3/2)*sqrt(e) - 4*a^(3/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
+ 4*a^(3/2)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*a^(3/2)*sqrt(e)
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 1)^2/((e^4 + 2*e^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + e^4*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4)*d*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(-sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

mupad [B] time = 5.98, size = 71, normalized size = 0.96

$$\frac{4\,a\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(5\,\sin\left(c+d\,x\right)+\cos\left(2\,c+2\,d\,x\right)-4\right)}{5\,d\,e^3\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(4\,\sin\left(c+d\,x\right)+\cos\left(2\,c+2\,d\,x\right)-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] (4*a*(a*(sin(c + d*x) + 1))^(1/2)*(5*sin(c + d*x) + cos(2*c + 2*d*x) - 4))/
(5*d*e^3*(e*cos(c + d*x))^(1/2)*(4*sin(c + d*x) + cos(2*c + 2*d*x) - 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.287 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{16(a\sin(c+dx)+a)^{7/2}}{21a^2de(e\cos(c+dx))^{7/2}}+\frac{8(a\sin(c+dx)+a)^{5/2}}{3ade(e\cos(c+dx))^{7/2}}-\frac{2(a\sin(c+dx)+a)^{3/2}}{de(e\cos(c+dx))^{7/2}}$$

```
[Out] -2*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(7/2)+8/3*(a+a*sin(d*x+c))^(5/
2)/a/d/e/(e*cos(d*x+c))^(7/2)-16/21*(a+a*sin(d*x+c))^(7/2)/a^2/d/e/(e*cos(d
*x+c))^(7/2)
```

Rubi [A] time = 0.23, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{16(a\sin(c+dx)+a)^{7/2}}{21a^2de(e\cos(c+dx))^{7/2}}+\frac{8(a\sin(c+dx)+a)^{5/2}}{3ade(e\cos(c+dx))^{7/2}}-\frac{2(a\sin(c+dx)+a)^{3/2}}{de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^(3/2))/(d*e*(e*Cos[c + d*x])^(7/2)) + (8*(a + a*Si
n[c + d*x])^(5/2))/(3*a*d*e*(e*Cos[c + d*x])^(7/2)) - (16*(a + a*Sin[c + d*
x])^(7/2))/(21*a^2*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{9/2}}\,dx &= -\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{7/2}}+\frac{4\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx}{a}\\ &= -\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{7/2}}+\frac{8(a+a\sin(c+dx))^{5/2}}{3ade(e\cos(c+dx))^{7/2}}-\frac{8\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{9/2}}\,dx}{3a^2}\\ &= -\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{7/2}}+\frac{8(a+a\sin(c+dx))^{5/2}}{3ade(e\cos(c+dx))^{7/2}}-\frac{16(a+a\sin(c+dx))^{7/2}}{21a^2de(e\cos(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.20, size = 105, normalized size = 0.93

$$\frac{2a\sqrt{a(\sin(c+dx)+1)}\,(12\sin(c+dx)+4\cos(2(c+dx))-5)}{21de^4\sqrt{e\cos(c+dx)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Sin[c + d*x])]*(-5 + 4*Cos[2*(c + d*x)] + 12*Sin[c + d*x])
)/(21*d*e^4*Sqrt[e*Cos[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.66, size = 84, normalized size = 0.74

$$-\frac{2\left(8\,a\cos\left(dx+c\right)^{2}+12\,a\sin\left(dx+c\right)-9\,a\right)\sqrt{e\cos\left(dx+c\right)}\,\sqrt{a\sin\left(dx+c\right)+a}}{21\left(de^{5}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-de^{5}\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x, algorithm="fricas"
)
```

```
[Out] -2/21*(8*a*cos(d*x + c)^2 + 12*a*sin(d*x + c) - 9*a)*sqrt(e*cos(d*x + c))*s
qrt(a*sin(d*x + c) + a)/(d*e^5*cos(d*x + c)^2*sin(d*x + c) - d*e^5*cos(d*x
+ c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 54, normalized size = 0.48

$$\frac{2\left(8\left(\cos^{2}\left(dx+c\right)\right)+12\sin\left(dx+c\right)-9\right)\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)}{21d\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] 2/21/d*(8*cos(d*x+c)^2+12*sin(d*x+c)-9)*(a*(1+sin(d*x+c)))^(3/2)*cos(d*x+c)
/(e*cos(d*x+c))^(9/2)
```

maxima [B] time = 1.13, size = 281, normalized size = 2.49

$$-\frac{2\left(a^{\frac{3}{2}}\sqrt{e}-\frac{24\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{33\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{33\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{24\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)}\right.}{21\left(e^{5}+\frac{3\,e^{5}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,e^{5}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{e^{5}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x, algorithm="maxima"
)
```

```
[Out] -2/21*(a^(3/2)*sqrt(e) - 24*a^(3/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
 + 33*a^(3/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 33*a^(3/2)*sqrt
(e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 24*a^(3/2)*sqrt(e)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - a^(3/2)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((e^5 + 3*e^5*sin(d*x + c)^2/
```

```
(cos(d*x + c) + 1)^2 + 3*e^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + e^5*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3
/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2))
```

mupad [B] time = 6.81, size = 116, normalized size = 1.03

$$\frac{8\,a\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(12\,\cos\left(c+d\,x\right)-10\,\cos\left(3\,c+3\,d\,x\right)-17\,\sin\left(2\,c+2\,d\,x\right)+2\,\sin\left(4\,c+4\,d\,x\right)\right)}{21\,d\,e^4\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(4\,\sin\left(c+d\,x\right)-4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+4\,\sin\left(3\,c+3\,d\,x\right)-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] (8*a*(a*(sin(c + d*x) + 1))^(1/2)*(12*cos(c + d*x) - 10*cos(3*c + 3*d*x) -
17*sin(2*c + 2*d*x) + 2*sin(4*c + 4*d*x)))/(21*d*e^4*(e*cos(c + d*x))^(1/2)
*(4*sin(c + d*x) - 4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 4*sin(3*c + 3*d*
x) - 5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

3.288 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=152

$$\frac{32(a\sin(c+dx)+a)^{9/2}}{45a^3de(e\cos(c+dx))^{9/2}}-\frac{16(a\sin(c+dx)+a)^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}}+\frac{4(a\sin(c+dx)+a)^{5/2}}{ade(e\cos(c+dx))^{9/2}}-\frac{2(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{9/2}}$$

```
[Out] -2/3*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(9/2)+4*(a+a*sin(d*x+c))^(5/
2)/a/d/e/(e*cos(d*x+c))^(9/2)-16/5*(a+a*sin(d*x+c))^(7/2)/a^2/d/e/(e*cos(d*
x+c))^(9/2)+32/45*(a+a*sin(d*x+c))^(9/2)/a^3/d/e/(e*cos(d*x+c))^(9/2)
```

Rubi [A] time = 0.31, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{32(a\sin(c+dx)+a)^{9/2}}{45a^3de(e\cos(c+dx))^{9/2}}-\frac{16(a\sin(c+dx)+a)^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}}+\frac{4(a\sin(c+dx)+a)^{5/2}}{ade(e\cos(c+dx))^{9/2}}-\frac{2(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d*x])^(9/2)) + (4*(a + a*
Sin[c + d*x])^(5/2))/(a*d*e*(e*Cos[c + d*x])^(9/2)) - (16*(a + a*Sin[c + d*
x])^(7/2))/(5*a^2*d*e*(e*Cos[c + d*x])^(9/2)) + (32*(a + a*Sin[c + d*x])^(9
/2))/(45*a^3*d*e*(e*Cos[c + d*x])^(9/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{11/2}}\,dx &= -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}}+\frac{2\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{11/2}}\,dx}{a}\\
&= -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}}+\frac{4(a+a\sin(c+dx))^{5/2}}{ade(e\cos(c+dx))^{9/2}}-\frac{8\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{11/2}}\,dx}{a^2}\\
&= -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}}+\frac{4(a+a\sin(c+dx))^{5/2}}{ade(e\cos(c+dx))^{9/2}}-\frac{16(a+a\sin(c+dx))^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}}+16\int\\
&= -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}}+\frac{4(a+a\sin(c+dx))^{5/2}}{ade(e\cos(c+dx))^{9/2}}-\frac{16(a+a\sin(c+dx))^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}}+\frac{32(}{45a}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 74, normalized size = 0.49

$$\frac{2\sec^5(c+dx)(a(\sin(c+dx)+1))^{3/2}\sqrt{e\cos(c+dx)}\,(6\sin(c+dx)-4\sin(3(c+dx))+12\cos(2(c+dx))+7)}{45de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(3/2)*(7 + 12
*Cos[2*(c + d*x)] + 6*Sin[c + d*x] - 4*Sin[3*(c + d*x)]))/(45*d*e^6)
```

fricas [A] time = 0.80, size = 98, normalized size = 0.64

$$-\frac{2\left(24\,a\cos\left(dx+c\right)^2-2\left(8\,a\cos\left(dx+c\right)^2-5\,a\right)\sin\left(dx+c\right)-5\,a\right)\sqrt{e\cos\left(dx+c\right)}\,\sqrt{a\sin\left(dx+c\right)+a}}{45\left(de^6\cos\left(dx+c\right)^3\sin\left(dx+c\right)-de^6\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x, algorithm="fricas
")
```

```
[Out] -2/45*(24*a*cos(d*x + c)^2 - 2*(8*a*cos(d*x + c)^2 - 5*a)*sin(d*x + c) - 5*
a)*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*e^6*cos(d*x + c)^3*sin(
d*x + c) - d*e^6*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 70, normalized size = 0.46

$$-\frac{2\left(16\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)-24\left(\cos^2\left(dx+c\right)\right)-10\sin\left(dx+c\right)+5\right)\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)}{45d\left(e\cos\left(dx+c\right)\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] -2/45/d*(16*cos(d*x+c)^2*sin(d*x+c)-24*cos(d*x+c)^2-10*sin(d*x+c)+5)*(a*(1+
sin(d*x+c)))^(3/2)*cos(d*x+c)/(e*cos(d*x+c))^(11/2)
```

maxima [B] time = 1.01, size = 357, normalized size = 2.35

$$\frac{2\left(19\,a^{\frac{3}{2}}\sqrt{e}-\frac{12\,a^{\frac{3}{2}}\sqrt{e}\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{58\,a^{\frac{3}{2}}\sqrt{e}\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{116\,a^{\frac{3}{2}}\sqrt{e}\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{116\,a^{\frac{3}{2}}\sqrt{e}\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{58\,a^{\frac{3}{2}}\sqrt{e}\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+12\cdots\right.}{45\left(e^6+\frac{4\,e^6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,e^6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,e^6\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{e^6\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x, algorithm="maxima
")
```

```
[Out] 2/45*(19*a^(3/2)*sqrt(e) - 12*a^(3/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
1) - 58*a^(3/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 116*a^(3/2)*s
qrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 116*a^(3/2)*sqrt(e)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 + 58*a^(3/2)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + 12*a^(3/2)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 19*a^(3/
2)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^4/((e^6 + 4*e^6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*e^6*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*e^6*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + e^6*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2))
```

mupad [B] time = 10.96, size = 261, normalized size = 1.72

$$\frac{14\,a\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+12\,a\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+24\,a\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}-}{\frac{45\,d\,e^5\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{2}+\frac{45\,d\,e^5\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{2}-\frac{45\,d\,e^5\,\sin\left(3\,c+3\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{4}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(11/2),x)
```

```
[Out] (14*a*(a + a*sin(c + d*x))^(1/2) + 12*a*sin(c + d*x)*(a + a*sin(c + d*x))^(
1/2) + 24*a*cos(2*c + 2*d*x)*(a + a*sin(c + d*x))^(1/2) - 8*a*sin(3*c + 3*d
*x)*(a + a*sin(c + d*x))^(1/2))/((45*d*e^5*((e*exp(- c*1i - d*x*1i))/2 + (e
*exp(c*1i + d*x*1i))/2)^(1/2))/2 + (45*d*e^5*cos(2*c + 2*d*x)*((e*exp(- c*1
i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/2 - (45*d*e^5*sin(3*c + 3
*d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 - (4
5*d*e^5*sin(c + d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2
)^(1/2))/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

3.289 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=323

$$-\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a\sin(c+dx)+a}}+\frac{77a^2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{64d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{77a^2}{}$$

```
[Out] -1/4*a*(e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2)/d/e-77/96*a^3*(e*cos(d*x
+c))^(5/2)/d/e/(a+a*sin(d*x+c))^(1/2)-11/24*a^2*(e*cos(d*x+c))^(5/2)*(a+a*s
in(d*x+c))^(1/2)/d/e+77/64*a^2*e*(e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2
)/d-77/64*a^2*e^(3/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^
(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))+77/64*a^2*e^(3/2)*
arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos
(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))
```

Rubi [A] time = 0.54, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2678, 2685, 2677, 2775, 203, 2833, 63, 215}

$$\frac{77a^2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{64d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{77a^2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin}}{64d(\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-77*a^3*(e*Cos[c + d*x])^(5/2))/(96*d*e*Sqrt[a + a*Sin[c + d*x]]) + (77*a^
2*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d) - (11*a^2*(e*Cos[
c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]])/(24*d*e) - (77*a^2*e^(3/2)*ArcSin
h[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d
*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (77*a^2*e^(3/2)*ArcTan[(Sq
rt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 +
 Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c +
d*x])) - (a*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2))/(4*d*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
```

```
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}}{4de} + \frac{1}{8}(11a)\int (e\cos(c+$$

$$= -\frac{11a^2(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}{24de} - \frac{a(e\cos(c+dx))^{5/2}}{}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} - \frac{11a^2(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c}}{24de}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} + \frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c}}{64d}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} + \frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c}}{64d}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} + \frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c}}{64d}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} + \frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c}}{64d}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} + \frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c}}{64d}$$

Mathematica [C] time = 0.30, size = 77, normalized size = 0.24

$$-\frac{16\,2^{3/4}(a(\sin(c+dx)+1))^{5/2}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{11}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-16*2^(3/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-11/4, 5/4, 9/4, (1 -
 Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(5/2))/(5*d*e*(1 + Sin[c + d*x])^(
15/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(5/2), x)
```

maple [A] time = 0.33, size = 344, normalized size = 1.07

$$-\frac{\left(96 \sin \left(dx+c\right) \left(\cos^{4}\left(dx+c\right)\right)+96 \left(\cos^{5}\left(dx+c\right)\right)-368 \sin \left(dx+c\right) \left(\cos^{3}\left(dx+c\right)\right)-231 \sqrt{2}\, \sqrt{-\frac{2 \cos \left(dx+c\right)}{1+\cos \left(dx+c\right)}}\ \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/384/d*(96*sin(d*x+c)*cos(d*x+c)^4+96*cos(d*x+c)^5-368*sin(d*x+c)*cos(d*x
+c)^3-231*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+231*
2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+272*cos(d*x+c)^4-308*cos(d*x+c)^2*sin
(d*x+c)-676*cos(d*x+c)^3+462*cos(d*x+c)*sin(d*x+c)-154*cos(d*x+c)^2+462*cos
(d*x+c))*(e*cos(d*x+c))^(3/2)*(a*(1+sin(d*x+c)))^(5/2)/(cos(d*x+c)^2*sin(d*
x+c)-cos(d*x+c)^3+2*cos(d*x+c)*sin(d*x+c)+3*cos(d*x+c)^2-4*sin(d*x+c)+2*cos
(d*x+c)-4)/cos(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e \cos \left(dx+c\right)\right)^{\frac{3}{2}} \left(a \sin \left(dx+c\right)+a\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.290 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=286

$$-\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a\sin(c+dx)+a}}-\frac{3a^2\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}{4de}+\frac{15a^2\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+}}{8d(\sin(c+dx)+\cos}$$

```
[Out] -1/3*a*(e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2)/d/e-15/8*a^3*(e*cos(d*x+
c))^(3/2)/d/e/(a+a*sin(d*x+c))^(1/2)-3/4*a^2*(e*cos(d*x+c))^(3/2)*(a+a*sin(
d*x+c))^(1/2)/d/e+15/8*a^2*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*e^(1/2)*(1
+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))+15/8*
a^2*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*e^
(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c
))
```

Rubi [A] time = 0.44, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2678, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a\sin(c+dx)+a}}-\frac{3a^2\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}{4de}+\frac{15a^2\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+}}{8d(\sin(c+dx)+\cos}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-15*a^3*(e*Cos[c + d*x])^(3/2))/(8*d*e*Sqrt[a + a*Sin[c + d*x]]) - (3*a^2*
(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) + (15*a^2*Sqrt[e]*
ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin
[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (15*a^2*Sqrt[e]*ArcTa
n[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqr
t[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[
c + d*x])) - (a*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2))/(3*d*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
```

```
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}{3de}+\frac{1}{2}(3a)\int \sqrt{e\cos(c+dx)}$$

$$= -\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a(e\cos(c+dx))^{3/2}(a+a}{3de}$$

$$= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}$$

$$= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}$$

$$= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}$$

$$= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}$$

$$= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}$$

Mathematica [C] time = 0.12, size = 78, normalized size = 0.27

$$-\frac{16\sqrt[4]{2}\,a(a(\sin(c+dx)+1))^{3/2}(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{9}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-16*2^(1/4)*a*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-9/4, 3/4, 7/4, (1
- Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(3*d*e*(1 + Sin[c + d*x])^
(9/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e \cos(dx + c)} (a \sin(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(5/2), x)
```

maple [A] time = 0.34, size = 318, normalized size = 1.11

$$-\frac{\left(16 \sin(dx + c) \left(\cos^3(dx + c)\right) - 45\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sin(dx + c) - 45\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\right.}{48d\left(\left(\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/48/d*(16*sin(d*x+c)*cos(d*x+c)^3-45*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c
)-45*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+16*cos(d*
x+c)^4-68*cos(d*x+c)^2*sin(d*x+c)+52*cos(d*x+c)^3-90*cos(d*x+c)*sin(d*x+c)-
158*cos(d*x+c)^2+90*cos(d*x+c))*(a*(1+sin(d*x+c)))^(5/2)*(e*cos(d*x+c))^(1/
2)/(cos(d*x+c)^2*sin(d*x+c)-cos(d*x+c)^3+2*cos(d*x+c)*sin(d*x+c)+3*cos(d*x+
c)^2-4*sin(d*x+c)+2*cos(d*x+c)-4)/cos(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e \cos(dx + c)} (a \sin(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e \cos(c + d x)} (a + a \sin(c + d x))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.291 $\int \frac{(a+a\sin(c+dx))^{5/2}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=247

$$-\frac{7a^2\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{4de}+\frac{21a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}$$

```
[Out] -1/2*a*(a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2)/d/e-7/4*a^2*(e*cos(d*x+c
))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e-21/4*a^2*arcsinh((e*cos(d*x+c))^(1/2)/e
^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x
+c))/e^(1/2)+21/4*a^2*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos
(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)
+sin(d*x+c))/e^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2678, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{7a^2\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{4de}+\frac{21a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-7*a^2*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) - (21*a^2*Ar
cSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
 + d*x]])/(4*d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (21*a^2*ArcTan[
(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[
1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*Sqrt[e]*(1 + Cos[c + d*x]
+ Sin[c + d*x])) - (a*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2))/(2*d
*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
```

```
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}{2de}+\frac{1}{4}(7a)\int \frac{(a+a\sin(c+dx))^{3/2}}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}{2de}+$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}{2de}+$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}{2de}-$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}{2de}+$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}{2de}-$$

Mathematica [C] time = 0.10, size = 76, normalized size = 0.31

$$-\frac{8\,2^{3/4}a(a(\sin(c+dx)+1))^{3/2}\sqrt{e\cos(c+dx)}\;{}_2F_1\left(-\frac{7}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-8*2^(3/4)*a*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-7/4, 1/4, 5/4, (1 - S
in[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(d*e*(1 + Sin[c + d*x])^(7/4)
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.30, size = 284, normalized size = 1.15

$$\frac{(a\,(1+\sin(dx+c)))^{\frac{5}{2}}\left(21\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)-21\sqrt{2}\,\sqrt{-\frac{2\cos(d}{1+\cos(}}}{8d\left(\left(\cos^2(dx+c)\right)\sin(dx+c)-\left(\cos^3(dx+c)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(a*(1+sin(d*x+c)))^(5/2)*(21*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
*2^(1/2))*sin(d*x+c)-21*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-4*cos(d*x+c)^
2*sin(d*x+c)-4*cos(d*x+c)^3+22*cos(d*x+c)*sin(d*x+c)-18*cos(d*x+c)^2+22*cos
(d*x+c))/(cos(d*x+c)^2*sin(d*x+c)-cos(d*x+c)^3+2*cos(d*x+c)*sin(d*x+c)+3*co
s(d*x+c)^2-4*sin(d*x+c)+2*cos(d*x+c)-4)/(e*cos(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^{\frac{5}{2}}}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{\sqrt{e\,\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.292 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=239

$$\frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a\sin(c+dx)+a}}-\frac{5a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{de^{3/2}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{5a^2\sqrt{\cos(c+}$$

```
[Out] 4*a*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(1/2)+5*a^3*(e*cos(d*x+c))^(3
/2)/d/e^3/(a+a*sin(d*x+c))^(1/2)-5*a^2*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2)
)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e^(3/2)/(1+cos(d*x+c)+sin(d
*x+c))-5*a^2*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^
(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e^(3/2)/(1+cos(d*x+c)+
sin(d*x+c))
```

Rubi [A] time = 0.36, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2676, 2678, 2684, 2775, 203, 2833, 63, 215}

$$\frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a\sin(c+dx)+a}}-\frac{5a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{de^{3/2}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{5a^2\sqrt{\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (5*a^3*(e*Cos[c + d*x])^(3/2))/(d*e^3*Sqrt[a + a*Sin[c + d*x]]) - (5*a^2*Ar
cSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
 + d*x]])/(d*e^(3/2)*(1 + Cos[c + d*x] + Sin[c + d*x])) - (5*a^2*ArcTan[(Sq
rt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 +
 Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(3/2)*(1 + Cos[c + d*x] + Sin
[c + d*x])) + (4*a*(a + a*Sin[c + d*x])^(3/2))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
```

```
gersQ[2*m, 2*p]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{(5a^2)\int \sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\,dx}{e^2}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{(5a^3)\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{2e^2}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{(5a^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+}}{2e(a+a\cos(c+}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{(5a^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+}}{2de(a}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{5a^3\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,\sqrt{1+\cos}}\right.}{de^{3/2}(a+a}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{5a^3\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1}}{de^{3/2}(a+a\cos(c}$$

Mathematica [C] time = 0.16, size = 75, normalized size = 0.31

$$\frac{8\sqrt[4]{2}\,(a(\sin(c+dx)+1))^{5/2}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de(\sin(c+dx)+1)^{9/4}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*2^(1/4)*Hypergeometric2F1[-5/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(a*(1 +
 Sin[c + d*x]))^(5/2))/(d*e*Sqrt[e*Cos[c + d*x]]*(1 + Sin[c + d*x])^(9/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.25, size = 445, normalized size = 1.86

$$-\frac{\left(5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(d}{2\cos(dx+c)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+5*2^(1/2)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-5*cos(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
-5*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)-5*arctan(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2)-5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+4*cos
(d*x+c)*sin(d*x+c)-36*cos(d*x+c))*(a*(1+sin(d*x+c)))^(5/2)/(-cos(d*x+c)^2+2
*sin(d*x+c)+2)/(e*cos(d*x+c))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^{\frac{5}{2}}}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\,\sin(c+d\,x))^{5/2}}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.293 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{2a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{de^{5/2}(\sin(c+dx)+\cos(c+dx)+1)}+\frac{2a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx}}{de^{5/2}(\sin(c+dx)+\cos}$$

```
[Out] 4/3*a*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(3/2)+2*a^2*arcsinh((e*cos(
d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e^(5/2
)/(1+cos(d*x+c)+sin(d*x+c))-2*a^2*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^
(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e
^(5/2)/(1+cos(d*x+c)+sin(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2676, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{2a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{de^{5/2}(\sin(c+dx)+\cos(c+dx)+1)}+\frac{2a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx}}{de^{5/2}(\sin(c+dx)+\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a
 + a*Sin[c + d*x]])/(d*e^(5/2)*(1 + Cos[c + d*x] + Sin[c + d*x])) - (2*a^2*A
rcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]
*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(5/2)*(1 + Cos[c + d
*x] + Sin[c + d*x])) + (4*a*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d
*x])^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} - \frac{a^2\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{e^2}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} - \frac{\left(a^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}}{e^2(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} + \frac{\left(a^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{}}\right.}{de^2(a+a\cos(c+dx)+a\sin(c+d}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} - \frac{2a^3\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a}}{de^{5/2}(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} + \frac{2a^3\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c}}{de^{5/2}(a+a\cos(c+dx)+a\sin(c+dx))}$$

Mathematica [C] time = 0.19, size = 77, normalized size = 0.38

$$\frac{4\,2^{3/4}(a(\sin(c+dx)+1))^{5/2}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{7/4}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*2^(3/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, (1 - Sin[c + d*x])/2]*(a*(1 +
 Sin[c + d*x]))^(5/2))/(3*d*e*(e*Cos[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(7/
4))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**B**] time = 0.24, size = 545, normalized size = 2.67

$$-\frac{\left(3\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sqrt{2}\,\sin(dx+c)\cos(dx+c)-3\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\,\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d*(3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*
sin(d*x+c)*cos(d*x+c)-3*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)-3*cos(d*x+c)^2*2
^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+3*cos(d*x+c
)^2*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))-4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+
c)-6*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d
*x+c)+6*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))*sin(d*x+c)-3*cos(d*x+c)*2^(1/2)*arctan(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+3*cos(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+6*2^(1/2)*arct
an(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-6*2^(1/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)))*(a*(1+
sin(d*x+c)))^(5/2)/(1+sin(d*x+c))/sin(d*x+c)/(e*cos(d*x+c))^(5/2)/(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^{\frac{5}{2}}}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)/(e*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a \sin(c + d x))^{5/2}}{(e \cos(c + d x))^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.294 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

```
[Out] 2/5*(a+a*sin(d*x+c))^(5/2)/d/e/(e*cos(d*x+c))^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2671}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(a+a\sin(c+dx))^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

Mathematica [A] time = 0.14, size = 36, normalized size = 1.00

$$\frac{2(a(\sin(c+dx)+1))^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(5/2))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

fricas [B] time = 1.21, size = 107, normalized size = 2.97

$$-\frac{2\left(a^2\cos(dx+c)+a^2\sin(dx+c)+a^2\right)\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}}{5\left(de^4\cos(dx+c)^2-de^4\cos(dx+c)-2\,de^4+\left(de^4\cos(dx+c)+2\,de^4\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fricas"
)
```

```
[Out] -2/5*(a^2*cos(d*x + c) + a^2*sin(d*x + c) + a^2)*sqrt(e*cos(d*x + c))*sqrt(
a*sin(d*x + c) + a)/(d*e^4*cos(d*x + c)^2 - d*e^4*cos(d*x + c) - 2*d*e^4 +
(d*e^4*cos(d*x + c) + 2*d*e^4)*sin(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.18, size = 34, normalized size = 0.94

$$\frac{2\cos\left(dx+c\right)\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{5}{2}}}{5d\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x)
[Out] 2/5/d*cos(d*x+c)*(a*(1+sin(d*x+c)))^(5/2)/(e*cos(d*x+c))^(7/2)
```

maxima [B] time = 1.15, size = 131, normalized size = 3.64

$$\frac{2\left(a^{\frac{5}{2}}\sqrt{e}-\frac{a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{5\left(e^4+\frac{e^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxima"
)
[Out] 2/5*(a^(5/2)*sqrt(e) - a^(5/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)/((e^4 + e^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

mupad [B] time = 6.06, size = 65, normalized size = 1.81

$$-\frac{2\,a^2\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{5\,d\,e^3\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(4\,\sin\left(c+d\,x\right)+\cos\left(2\,c+2\,d\,x\right)-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(7/2),x)
[Out] -(2*a^2*(cos(2*c + 2*d*x) + 1)*(a*(sin(c + d*x) + 1))^(1/2))/(5*d*e^3*(e*co
s(c + d*x))^(1/2)*(4*sin(c + d*x) + cos(2*c + 2*d*x) - 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.295 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=76

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{3de(e\cos(c+dx))^{7/2}}-\frac{4(a\sin(c+dx)+a)^{7/2}}{21ade(e\cos(c+dx))^{7/2}}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^(5/2)/d/e/(e*cos(d*x+c))^(7/2)-4/21*(a+a*sin(d*x+c))^(
7/2)/a/d/e/(e*cos(d*x+c))^(7/2)
```

Rubi [A] time = 0.15, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{3de(e\cos(c+dx))^{7/2}}-\frac{4(a\sin(c+dx)+a)^{7/2}}{21ade(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(3*d*e*(e*Cos[c + d*x])^(7/2)) - (4*(a + a*S
in[c + d*x])^(7/2))/(21*a*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx &= \frac{2(a+a\sin(c+dx))^{5/2}}{3de(e\cos(c+dx))^{7/2}}-\frac{2\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{9/2}}\,dx}{3a}\\ &= \frac{2(a+a\sin(c+dx))^{5/2}}{3de(e\cos(c+dx))^{7/2}}-\frac{4(a+a\sin(c+dx))^{7/2}}{21ade(e\cos(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 54, normalized size = 0.71

$$-\frac{2(2\sin(c+dx)-5)\sec^4(c+dx)(a(\sin(c+dx)+1))^{5/2}\sqrt{e\cos(c+dx)}}{21de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(9/2),x]
```

[Out] (-2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*(a*(1 + Sin[c + d*x]))^(5/2)*(-5 + 2*Sin[c + d*x]))/(21*d*e^5)

fricas [A] time = 1.16, size = 75, normalized size = 0.99

$$\frac{2\left(2\,a^{2}\sin\left(dx+c\right)-5\,a^{2}\right)\sqrt{e\cos\left(dx+c\right)}\,\sqrt{a\sin\left(dx+c\right)+a}}{21\left(de^{5}\cos\left(dx+c\right)^{2}+2\,de^{5}\sin\left(dx+c\right)-2\,de^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x, algorithm="fricas"
)
```

```
[Out] 2/21*(2*a^2*sin(d*x + c) - 5*a^2)*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c)
+ a)/(d*e^5*cos(d*x + c)^2 + 2*d*e^5*sin(d*x + c) - 2*d*e^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 44, normalized size = 0.58

$$-\frac{2\left(2\sin\left(dx+c\right)-5\right)\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)}{21d\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] -2/21/d*(2*sin(d*x+c)-5)*(a*(1+sin(d*x+c)))^(5/2)*cos(d*x+c)/(e*cos(d*x+c))
^(9/2)
```

maxima [B] time = 1.00, size = 207, normalized size = 2.72

$$\frac{2\left(5\,a^{\frac{5}{2}}\sqrt{e}-\frac{4\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{5\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{21\left(e^{5}+\frac{2\,e^{5}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{e^{5}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x, algorithm="maxima"
)
```

```
[Out] 2/21*(5*a^(5/2)*sqrt(e) - 4*a^(5/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
 + 4*a^(5/2)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*a^(5/2)*sqrt(e
)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((e^5 + 2*e^5*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + e^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d*(-sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2))
```

mupad [B] time = 6.34, size = 96, normalized size = 1.26

$$\frac{4\,a^{2}\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(\cos\left(3\,c+3\,d\,x\right)-11\,\cos\left(c+d\,x\right)+7\,\sin\left(2\,c+2\,d\,x\right)\right)}{21\,d\,e^{4}\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(15\,\sin\left(c+d\,x\right)+6\,\cos\left(2\,c+2\,d\,x\right)-\sin\left(3\,c+3\,d\,x\right)-10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] (4*a^2*(a*(sin(c + d*x) + 1))^(1/2)*(cos(3*c + 3*d*x) - 11*cos(c + d*x) + 7
*sin(2*c + 2*d*x)))/(21*d*e^4*(e*cos(c + d*x))^(1/2)*(15*sin(c + d*x) + 6*c
os(2*c + 2*d*x) - sin(3*c + 3*d*x) - 10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

3.296 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=113

$$\frac{16(a\sin(c+dx)+a)^{9/2}}{45a^2de(e\cos(c+dx))^{9/2}}-\frac{8(a\sin(c+dx)+a)^{7/2}}{5ade(e\cos(c+dx))^{9/2}}+\frac{2(a\sin(c+dx)+a)^{5/2}}{de(e\cos(c+dx))^{9/2}}$$

```
[Out] 2*(a+a*sin(d*x+c))^(5/2)/d/e/(e*cos(d*x+c))^(9/2)-8/5*(a+a*sin(d*x+c))^(7/2
)/a/d/e/(e*cos(d*x+c))^(9/2)+16/45*(a+a*sin(d*x+c))^(9/2)/a^2/d/e/(e*cos(d*
x+c))^(9/2)
```

Rubi [A] time = 0.22, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2672, 2671}

$$\frac{16(a\sin(c+dx)+a)^{9/2}}{45a^2de(e\cos(c+dx))^{9/2}}-\frac{8(a\sin(c+dx)+a)^{7/2}}{5ade(e\cos(c+dx))^{9/2}}+\frac{2(a\sin(c+dx)+a)^{5/2}}{de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(d*e*(e*Cos[c + d*x])^(9/2)) - (8*(a + a*Sin
[c + d*x])^(7/2))/(5*a*d*e*(e*Cos[c + d*x])^(9/2)) + (16*(a + a*Sin[c + d*x
])^(9/2))/(45*a^2*d*e*(e*Cos[c + d*x])^(9/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{11/2}}\,dx &= \frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{9/2}}-\frac{4\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{11/2}}\,dx}{a}\\
&= \frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{9/2}}-\frac{8(a+a\sin(c+dx))^{7/2}}{5ade(e\cos(c+dx))^{9/2}}+\frac{8\int \frac{(a+a\sin(c+dx))^{9/2}}{(e\cos(c+dx))^{11/2}}\,dx}{5a^2}\\
&= \frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{9/2}}-\frac{8(a+a\sin(c+dx))^{7/2}}{5ade(e\cos(c+dx))^{9/2}}+\frac{16(a+a\sin(c+dx))^{9/2}}{45a^2de(e\cos(c+dx))^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 64, normalized size = 0.57

$$\frac{2\left(8\sin^2(c+dx)-20\sin(c+dx)+17\right)\sec^5(c+dx)(a(\sin(c+dx)+1))^{5/2}\sqrt{e\cos(c+dx)}}{45de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(5/2)*(17 - 2
0*Sin[c + d*x] + 8*Sin[c + d*x]^2))/(45*d*e^6)
```

fricas [A] time = 0.99, size = 100, normalized size = 0.88

$$\frac{2\left(8\,a^{2}\cos\left(dx+c\right)^{2}+20\,a^{2}\sin\left(dx+c\right)-25\,a^{2}\right)\sqrt{e\cos\left(dx+c\right)}\,\sqrt{a\sin\left(dx+c\right)+a}}{45\left(de^{6}\cos\left(dx+c\right)^{3}+2\,de^{6}\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,de^{6}\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x, algorithm="fricas
")
```

```
[Out] 2/45*(8*a^2*cos(d*x + c)^2 + 20*a^2*sin(d*x + c) - 25*a^2)*sqrt(e*cos(d*x +
 c))*sqrt(a*sin(d*x + c) + a)/(d*e^6*cos(d*x + c)^3 + 2*d*e^6*cos(d*x + c)*
sin(d*x + c) - 2*d*e^6*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 54, normalized size = 0.48

$$-\frac{2\left(8\left(\cos^{2}\left(dx+c\right)\right)+20\sin\left(dx+c\right)-25\right)\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)}{45d\left(e\cos\left(dx+c\right)\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] -2/45/d*(8*cos(d*x+c)^2+20*sin(d*x+c)-25)*(a*(1+sin(d*x+c)))^(5/2)*cos(d*x+
c)/(e*cos(d*x+c))^(11/2)
```

maxima [B] time = 1.01, size = 282, normalized size = 2.50

$$\frac{2\left(17\,a^{\frac{5}{2}}\sqrt{e}-\frac{40\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{49\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{49\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{40\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{17\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)\left(\frac{\sin}{(\cos(}\right.}{45\left(e^{6}+\frac{3\,e^{6}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,e^{6}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{e^{6}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x, algorithm="maxima
")
```

```
[Out] 2/45*(17*a^(5/2)*sqrt(e) - 40*a^(5/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
1) + 49*a^(5/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 49*a^(5/2)*sq
rt(e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 40*a^(5/2)*sqrt(e)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 17*a^(5/2)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((e^6 + 3*e^6*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 3*e^6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + e^6
```

```
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*sqrt(sin(d*x + c)/(cos(d*x + c) + 1
) + 1)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2))
```

mupad [B] time = 6.69, size = 119, normalized size = 1.05

$$\frac{8\,a^2\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(2\,\cos\left(4\,c+4\,d\,x\right)-73\,\cos\left(2\,c+2\,d\,x\right)-162\,\sin\left(c+d\,x\right)+18\,\sin\left(3\,c+3\,d\,x\right)+10\right.}{45\,d\,e^5\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(\cos\left(4\,c+4\,d\,x\right)-28\,\cos\left(2\,c+2\,d\,x\right)-56\,\sin\left(c+d\,x\right)+8\,\sin\left(3\,c+3\,d\,x\right)+35\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(11/2),x)
```

```
[Out] (8*a^2*(a*(sin(c + d*x) + 1))^(1/2)*(2*cos(4*c + 4*d*x) - 73*cos(2*c + 2*d*
x) - 162*sin(c + d*x) + 18*sin(3*c + 3*d*x) + 105))/(45*d*e^5*(e*cos(c + d*
x))^(1/2)*(cos(4*c + 4*d*x) - 28*cos(2*c + 2*d*x) - 56*sin(c + d*x) + 8*sin
(3*c + 3*d*x) + 35))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

3.297 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{13/2}}\,dx$

Optimal. Leaf size=150

$$\frac{32(a\sin(c+dx)+a)^{11/2}}{77a^3de(e\cos(c+dx))^{11/2}}-\frac{16(a\sin(c+dx)+a)^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}}+\frac{4(a\sin(c+dx)+a)^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{2(a\sin(c+dx)+a)^{5/2}}{de(e\cos(c+dx))^{11/2}}$$

```
[Out] -2*(a+a*sin(d*x+c))^(5/2)/d/e/(e*cos(d*x+c))^(11/2)+4*(a+a*sin(d*x+c))^(7/2
)/a/d/e/(e*cos(d*x+c))^(11/2)-16/7*(a+a*sin(d*x+c))^(9/2)/a^2/d/e/(e*cos(d*
x+c))^(11/2)+32/77*(a+a*sin(d*x+c))^(11/2)/a^3/d/e/(e*cos(d*x+c))^(11/2)
```

Rubi [A] time = 0.31, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{32(a\sin(c+dx)+a)^{11/2}}{77a^3de(e\cos(c+dx))^{11/2}}-\frac{16(a\sin(c+dx)+a)^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}}+\frac{4(a\sin(c+dx)+a)^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{2(a\sin(c+dx)+a)^{5/2}}{de(e\cos(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^(5/2))/(d*e*(e*Cos[c + d*x])^(11/2)) + (4*(a + a*S
in[c + d*x])^(7/2))/(a*d*e*(e*Cos[c + d*x])^(11/2)) - (16*(a + a*Sin[c + d*
x])^(9/2))/(7*a^2*d*e*(e*Cos[c + d*x])^(11/2)) + (32*(a + a*Sin[c + d*x])^(
11/2))/(77*a^3*d*e*(e*Cos[c + d*x])^(11/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{13/2}}\,dx &= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}}+\frac{6\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{13/2}}\,dx}{a}\\ &= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}}+\frac{4(a+a\sin(c+dx))^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{8\int \frac{(a+a\sin(c+dx))^{9/2}}{(e\cos(c+dx))^{13/2}}\,dx}{a^2}\\ &= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}}+\frac{4(a+a\sin(c+dx))^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{16(a+a\sin(c+dx))^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}}+\\ &= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}}+\frac{4(a+a\sin(c+dx))^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{16(a+a\sin(c+dx))^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}}+\end{aligned}$$

Mathematica [A] time = 0.24, size = 74, normalized size = 0.49

$$\frac{2\left(16\sin^3(c+dx)-40\sin^2(c+dx)+26\sin(c+dx)+5\right)\sec^6(c+dx)(a(\sin(c+dx)+1))^{5/2}\sqrt{e\cos(c+dx)}}{77de^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^6*(a*(1 + Sin[c + d*x]))^(5/2)*(5 + 26
*Sin[c + d*x] - 40*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3))/(77*d*e^7)
```

fricas [A] time = 1.28, size = 120, normalized size = 0.80

$$-\frac{2\left(40\,a^2\cos\left(dx+c\right)^2-35\,a^2-2\left(8\,a^2\cos\left(dx+c\right)^2-21\,a^2\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}\,\sqrt{a\sin\left(dx+c\right)+a}}{77\left(de^7\cos\left(dx+c\right)^4+2\,de^7\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,de^7\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x, algorithm="fricas
")
```

```
[Out] -2/77*(40*a^2*cos(d*x + c)^2 - 35*a^2 - 2*(8*a^2*cos(d*x + c)^2 - 21*a^2)*s
in(d*x + c))*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*e^7*cos(d*x +
 c)^4 + 2*d*e^7*cos(d*x + c)^2*sin(d*x + c) - 2*d*e^7*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.21, size = 70, normalized size = 0.47

$$-\frac{2\left(16\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)-40\left(\cos^2\left(dx+c\right)\right)-42\sin\left(dx+c\right)+35\right)\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)}{77d\left(e\cos\left(dx+c\right)\right)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x)
```

```
[Out] -2/77/d*(16*cos(d*x+c)^2*sin(d*x+c)-40*cos(d*x+c)^2-42*sin(d*x+c)+35)*(a*(1
+sin(d*x+c)))^(5/2)*cos(d*x+c)/(e*cos(d*x+c))^(13/2)
```

maxima [B] time = 1.13, size = 357, normalized size = 2.38

$$\frac{2\left(5\,a^{\frac{5}{2}}\sqrt{e}+\frac{52\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{150\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{180\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{180\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{150\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{52\,a^{\frac{5}{2}}}{(\mathrm{co}}\right.}{77\left(e^7+\frac{4\,e^7\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,e^7\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,e^7\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{e^7\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x, algorithm="maxima
")
```

```
[Out] 2/77*(5*a^(5/2)*sqrt(e) + 52*a^(5/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1
) - 150*a^(5/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 180*a^(5/2)*s
qrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 180*a^(5/2)*sqrt(e)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 + 150*a^(5/2)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 - 52*a^(5/2)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5*a^(5/
2)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^4/((e^7 + 4*e^7*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*e^7*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*e^7*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + e^7*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2))
```

mupad [B] time = 11.17, size = 232, normalized size = 1.55

$$\frac{30\,a^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}-40\,a^2\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+8\,a^2\,\sin\left(3\,c+3\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+\cdots\right.}}{\frac{77\,d\,e^6\,\cos\left(3\,c+3\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{4}+77\,d\,e^6\,\sin\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}-\frac{385\,d\cdots}{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(13/2),x)
```

```
[Out] (30*a^2*(a + a*sin(c + d*x))^(1/2) - 40*a^2*cos(2*c + 2*d*x)*(a + a*sin(c +
 d*x))^(1/2) + 8*a^2*sin(3*c + 3*d*x)*(a + a*sin(c + d*x))^(1/2) - 76*a^2*s
in(c + d*x)*(a + a*sin(c + d*x))^(1/2))/((77*d*e^6*cos(3*c + 3*d*x)*((e*exp
(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 + 77*d*e^6*sin(2*
c + 2*d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2) -
(385*d*e^6*cos(c + d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i)
)/2)^(1/2))/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(13/2),x)
```

```
[Out] Timed out
```

3.298 $\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+}}{4d(a\sin(c+dx)+a\cos(c}$$

```
[Out] -1/2*a*(e*cos(d*x+c))^(7/2)/d/e/(a+a*sin(d*x+c))^(3/2)+1/4*e*(e*cos(d*x+c))
^(3/2)/d/(a+a*sin(d*x+c))^(1/2)+3/4*e^(5/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^
(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(a+a*cos(d*x+c)+a*sin(
d*x+c))+3/4*e^(5/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d
*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(a+a*cos(d*x+c)
+a*sin(d*x+c))
```

Rubi [A] time = 0.37, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2686, 2679, 2684, 2775, 203, 2833, 63, 215}

$$\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+}}{4d(a\sin(c+dx)+a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(a*(e*Cos[c + d*x])^(7/2))/(2*d*e*(a + a*Sin[c + d*x])^(3/2)) + (e*(e*Cos[
c + d*x])^(3/2))/(4*d*Sqrt[a + a*Sin[c + d*x]]) + (3*e^(5/2)*ArcSinh[Sqrt[e
*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4
*d*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) + (3*e^(5/2)*ArcTan[(Sqrt[e]*Sin[
c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d
*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(a + a*Cos[c + d*x] + a*Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
```

```
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2686

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1))/(f*g*(2*p - 1)*
(a + b*Sin[e + f*x])^(3/2)), x] + Dist[(2*a*(p - 2))/(2*p - 1), Int[(g*Cos[
e + f*x])^p/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 2] && IntegerQ[2*p]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}}+\frac{1}{4}a\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}}+\frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}}+\frac{1}{8}\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+d}}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}}+\frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}}+\frac{\left(3e^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+}\right.}{8(a+a\cos(c+}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}}+\frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}}+\frac{\left(3e^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+}\right.}{8d(a-}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}}+\frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}}+\frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+d}{\sqrt{e\cos(c+dx)}\sqrt{1+c}}\right.}{4d(a+a}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}}+\frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}}+\frac{3e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{}}{4d(a+a\cos(c}$$

Mathematica [C] time = 0.16, size = 77, normalized size = 0.32

$$-\frac{4\sqrt[4]{2}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(\sin(c+dx)+1)^{5/4}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-1/4, 7/4, 11/4, (1 -
Sin[c + d*x])/2])/(7*d*e*(1 + Sin[c + d*x])^(5/4)*Sqrt[a*(1 + Sin[c + d*x])
])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.28, size = 239, normalized size = 0.98

$$-\frac{\left(3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c}{2\cos(dx+c)}\right.\right.}{8d(-1+\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d*(3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+3*2^(1/2)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+4*cos(d*x+c)^3-4*cos(d*x+c)^2*sin(d*x+c
)+2*cos(d*x+c)^2+6*cos(d*x+c)*sin(d*x+c)-6*cos(d*x+c))*(e*cos(d*x+c))^(5/2)
/(-1+cos(d*x+c)+sin(d*x+c))/cos(d*x+c)^2/(a*(1+sin(d*x+c)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.299 $\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=200

$$\frac{e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{ad(\sin(c+dx)+\cos(c+dx)+1)}-\frac{e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}}{ad(\sin(c+dx)+\cos(c+}$$

```
[Out] e*(e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a/d-e^(3/2)*arcsinh((e*cos(d*
x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a/d/(1+cos
(d*x+c)+sin(d*x+c))+e^(3/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/
(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a/d/(1+co
s(d*x+c)+sin(d*x+c))
```

Rubi [A] time = 0.28, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2685, 2677, 2775, 203, 2833, 63, 215}

$$\frac{e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{ad(\sin(c+dx)+\cos(c+dx)+1)}-\frac{e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}}{ad(\sin(c+dx)+\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a*d) - (e^(3/2)*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(a*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (e^(3/2)*ArcTan[(Sqrt[e]*Sin[
c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d
*x]]*Sqrt[a + a*Sin[c + d*x]])/(a*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{ad}+\frac{e^2\int\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{2a}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{ad}+\frac{\left(e^2\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\int}{2(a+a\cos(c+dx)+a\sin(c+}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{ad}-\frac{\left(e^2\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\mathrm{S}}{2d(a+a\cos(c+dx)+}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{ad}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\mathrm{co}}}{d(a+a\cos(c+dx)+a\,\mathrm{s}}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{ad}-\frac{e^{3/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}}{d(a+a\cos(c+dx)+a\sin(c+}$$

Mathematica [C] time = 0.11, size = 77, normalized size = 0.38

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{3/4}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[1/4, 5/4, 9/4, (1 - Si
n[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(3/4)*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.24, size = 212, normalized size = 1.06

$$\frac{\left(\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)-\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right.\right.}{2d\left(-1+\cos(dx+c)+\sin(dx+c)\right)\sqrt{a\,(1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-2^(1/2)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+2*cos
(d*x+c))*(e*cos(d*x+c))^(3/2)/(-1+cos(d*x+c)+sin(d*x+c))/(a*(1+sin(d*x+c)))
^(1/2)/cos(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral((e*cos(c + d*x))**(3/2)/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.300 $\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=169

$$\frac{2\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{2\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)}\cdots}{d(a\sin(c+dx)+a\cos(c\cdots}$$

```
[Out] 2*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*s
in(d*x+c))^(1/2)/d/(a+a*cos(d*x+c)+a*sin(d*x+c))+2*arctan(sin(d*x+c)*e^(1/2
)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(
a+a*sin(d*x+c))^(1/2)/d/(a+a*cos(d*x+c)+a*sin(d*x+c))
```

Rubi [A] time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2684, 2775, 203, 2833, 63, 215}

$$\frac{2\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{2\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)}\cdots}{d(a\sin(c+dx)+a\cos(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[e]*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqr
t[a + a*Sin[c + d*x]])/(d*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) + (2*Sqrt[
e]*ArcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x
]])]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a + a*Cos[c + d*x
] + a*Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(e\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{a+a\cos(c+dx)+a\sin(c+dx)} - \frac{\left(e\sqrt{1+\cos(c+dx)}\right.}{a+}$$

$$= \frac{\left(e\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d(a+a\cos(c+dx)+a\sin(c+dx))} - \frac{(}{}$$

$$= \frac{2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d(a+a\cos(c+dx)+a\sin(c+dx))} + \frac{\left(2\sqrt{1}\right.}{}$$

$$= \frac{2\sqrt{e}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d(a+a\cos(c+dx)+a\sin(c+dx))} + \frac{2\sqrt{e}\tan^{-1}\left(\frac{}{\sqrt{e\,c}}\right.}{}$$

Mathematica [C] time = 0.08, size = 77, normalized size = 0.46

$$-\frac{2\sqrt[4]{2}(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de\sqrt[4]{\sin(c+dx)+1}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 3/4, 7/4, (1 - Si
n[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(1/4)*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(dx+c)}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e*cos(d*x + c))/sqrt(a*sin(d*x + c) + a), x)

maple [A] time = 0.18, size = 141, normalized size = 0.83

$$\frac{\sqrt{e\cos(dx+c)}\,(1-\cos(dx+c)+\sin(dx+c))\left(\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)+\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\right)}{d\sqrt{a(1+\sin(dx+c))}\,\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x)

[Out] 1/d*(e*cos(d*x+c))^(1/2)*(1-cos(d*x+c)+sin(d*x+c))*(arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)))/(a*(1+sin(d*x+c)))^(1/2)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(dx+c)}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e*cos(d*x + c))/sqrt(a*sin(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,\cos(c+d\,x)}}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^(1/2),x)

[Out] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(1/2),x)

[Out] Integral(sqrt(e*cos(c + d*x))/sqrt(a*(sin(c + d*x) + 1)), x)

3.301 $\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=34

$$-\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2*(e*cos(d*x+c))^(1/2)/d/e/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2671}

$$-\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx=-\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.07, size = 34, normalized size = 1.00

$$-\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(d*e*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.82, size = 41, normalized size = 1.21

$$-\frac{2\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}}{ade\sin(dx+c)+ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(a*d*e*sin(d*x + c) + a*d*
e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)), x)
```

maple [A] time = 0.17, size = 34, normalized size = 1.00

$$-\frac{2\cos(dx+c)}{d\sqrt{e\cos(dx+c)}\,\sqrt{a(1+\sin(dx+c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/d*cos(d*x+c)/(e*cos(d*x+c))^(1/2)/(a*(1+sin(d*x+c)))^(1/2)
```

maxima [B] time = 0.81, size = 130, normalized size = 3.82

$$-\frac{2\left(\sqrt{a}\sqrt{e}-\frac{\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{\left(ae+\frac{ae\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2*(sqrt(a)*sqrt(e) - sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/((a*e + a*e*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*sqrt(-sin(d*x
 + c)/(cos(d*x + c) + 1) + 1))
```

mupad [B] time = 5.66, size = 46, normalized size = 1.35

$$-\frac{2\cos(c+d\,x)\,\sqrt{a\,(\sin(c+d\,x)+1)}}{a\,d\,\sqrt{e\,\cos(c+d\,x)}\,\,(\sin(c+d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] -(2*cos(c + d*x)*(a*(sin(c + d*x) + 1))^(1/2))/(a*d*(e*cos(c + d*x))^(1/2)*
(sin(c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*sqrt(e*cos(c + d*x))), x)
```

3.302 $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{4\sqrt{a\sin(c+dx)+a}}{3ade\sqrt{e\cos(c+dx)}}-\frac{2}{3de\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}$$

```
[Out] -2/3/d/e/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2)+4/3*(a+a*sin(d*x+c))^(
1/2)/a/d/e/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{4\sqrt{a\sin(c+dx)+a}}{3ade\sqrt{e\cos(c+dx)}}-\frac{2}{3de\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] -2/(3*d*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (4*Sqrt[a + a*Si
n[c + d*x]])/(3*a*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{2}{3de\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}+\frac{2\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx}{3a}\\ &= -\frac{2}{3de\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}+\frac{4\sqrt{a+a\sin(c+dx)}}{3ade\sqrt{e\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 46, normalized size = 0.61

$$\frac{2(2\sin(c+dx)+1)}{3de\sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (2*(1 + 2*Sin[c + d*x]))/(3*d*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a*(1 + Sin[c + d*
x])])
```

fricas [A] time = 0.74, size = 67, normalized size = 0.88

$$\frac{2 \sqrt{e \cos(dx+c)} \sqrt{a \sin(dx+c)+a} \,(2 \sin(dx+c)+1)}{3 \left(ade^2 \cos(dx+c) \sin(dx+c)+ade^2 \cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/3*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*(2*sin(d*x + c) + 1)/(a*d
*e^2*cos(d*x + c)*sin(d*x + c) + a*d*e^2*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e \cos(dx+c))^{\frac{3}{2}} \sqrt{a \sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*sqrt(a*sin(d*x + c) + a)), x)
```

maple [A] time = 0.18, size = 44, normalized size = 0.58

$$\frac{2 (2 \sin(dx+c)+1) \cos(dx+c)}{3d (e \cos(dx+c))^{\frac{3}{2}} \sqrt{a (1+\sin(dx+c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d*(2*sin(d*x+c)+1)*cos(d*x+c)/(e*cos(d*x+c))^(3/2)/(a*(1+sin(d*x+c)))^(
1/2)
```

maxima [B] time = 1.03, size = 210, normalized size = 2.76

$$\frac{2 \left(\sqrt{a} \sqrt{e}+\frac{4 \sqrt{a} \sqrt{e} \sin(dx+c)}{\cos(dx+c)+1}-\frac{4 \sqrt{a} \sqrt{e} \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sqrt{a} \sqrt{e} \sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{3 \left(ae^2+\frac{2 ae^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{ae^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right) d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*(sqrt(a)*sqrt(e) + 4*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1) -
4*sqrt(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - sqrt(a)*sqrt(e)*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)
^2/((a*e^2 + 2*a*e^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*e^2*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-
sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

mupad [B] time = 6.01, size = 77, normalized size = 1.01

$$\frac{4 \sqrt{a}\,(\sin(c+d\,x)+1)\,\,(3 \sin(c+d\,x)-\cos(2\,c+2\,d\,x)+2)}{3\,a\,d\,e \sqrt{e \cos(c+d\,x)}\,\,(4 \sin(c+d\,x)-\cos(2\,c+2\,d\,x)+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] (4*(a*(sin(c + d*x) + 1))^(1/2)*(3*sin(c + d*x) - cos(2*c + 2*d*x) + 2))/(3
*a*d*e*(e*cos(c + d*x))^(1/2)*(4*sin(c + d*x) - cos(2*c + 2*d*x) + 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(3/2)), x)
```

3.303 $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=115

$$\frac{16(a\sin(c+dx)+a)^{3/2}}{15a^2de(e\cos(c+dx))^{3/2}}-\frac{8\sqrt{a\sin(c+dx)+a}}{5ade(e\cos(c+dx))^{3/2}}-\frac{2}{5de\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}$$

```
[Out] 16/15*(a+a*sin(d*x+c))^(3/2)/a^2/d/e/(e*cos(d*x+c))^(3/2)-2/5/d/e/(e*cos(d*
x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2)-8/5*(a+a*sin(d*x+c))^(1/2)/a/d/e/(e*cos(
d*x+c))^(3/2)
```

Rubi [A] time = 0.21, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{16(a\sin(c+dx)+a)^{3/2}}{15a^2de(e\cos(c+dx))^{3/2}}-\frac{8\sqrt{a\sin(c+dx)+a}}{5ade(e\cos(c+dx))^{3/2}}-\frac{2}{5de\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] -2/(5*d*e*(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (8*Sqrt[a + a*
Sin[c + d*x]])/(5*a*d*e*(e*Cos[c + d*x])^(3/2)) + (16*(a + a*Sin[c + d*x])^
(3/2))/(15*a^2*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{2}{5de(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}+\frac{4\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx}{5a}\\ &= -\frac{2}{5de(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}-\frac{8\sqrt{a+a\sin(c+dx)}}{5ade(e\cos(c+dx))^{3/2}}\\ &= -\frac{2}{5de(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}-\frac{8\sqrt{a+a\sin(c+dx)}}{5ade(e\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 56, normalized size = 0.49

$$\frac{2\left(8\sin^2(c+dx)+4\sin(c+dx)-7\right)}{15de\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (2*(-7 + 4*Sin[c + d*x] + 8*Sin[c + d*x]^2))/(15*d*e*(e*Cos[c + d*x])^(3/2)
*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.58, size = 81, normalized size = 0.70

$$-\frac{2\,\sqrt{e\cos\left(dx+c\right)}\left(8\,\cos\left(dx+c\right)^{2}-4\,\sin\left(dx+c\right)-1\right)\sqrt{a\sin\left(dx+c\right)+a}}{15\,\left(ade^{3}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+ade^{3}\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/15*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 - 4*sin(d*x + c) - 1)*sqrt(a*s
in(d*x + c) + a)/(a*d*e^3*cos(d*x + c)^2*sin(d*x + c) + a*d*e^3*cos(d*x + c
)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*sqrt(a*sin(d*x + c) + a)), x)
```

maple [A] time = 0.20, size = 54, normalized size = 0.47

$$\frac{2\left(-8\left(\cos^{2}\left(dx+c\right)\right)+4\sin\left(dx+c\right)+1\right)\cos\left(dx+c\right)}{15d\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d*(-8*cos(d*x+c)^2+4*sin(d*x+c)+1)*cos(d*x+c)/(e*cos(d*x+c))^(5/2)/(a*
(1+sin(d*x+c)))^(1/2)
```

maxima [B] time = 1.02, size = 287, normalized size = 2.50

$$-\frac{2\left(7\sqrt{a}\sqrt{e}-\frac{8\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{a}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{25\sqrt{a}\sqrt{e}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{8\sqrt{a}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{7\sqrt{a}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)\left(\frac{\mathrm{si}}{(\mathrm{cos}}\right.}{15\left(ae^{3}+\frac{3ae^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3ae^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{ae^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*(7*sqrt(a)*sqrt(e) - 8*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1
) - 25*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt(a)*sqr
t(e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 8*sqrt(a)*sqrt(e)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 7*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) + 1)
```

```
^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((a*e^3 + 3*a*e^3*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a*e^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
a*e^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x + c)/(cos(d*x + c) +
1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

mupad [B] time = 6.68, size = 120, normalized size = 1.04

$$-\frac{8\sqrt{a\,(\sin(c+d\,x)+1)}\,\left(8\,\cos(c+d\,x)+6\,\cos(3\,c+3\,d\,x)-\sin(2\,c+2\,d\,x)+2\,\sin(4\,c+4\,d\,x)\right)}{15\,a\,d\,e^2\,\sqrt{e\,\cos(c+d\,x)}\,\left(4\,\sin(c+d\,x)+4\,\cos(2\,c+2\,d\,x)-\cos(4\,c+4\,d\,x)+4\,\sin(3\,c+3\,d\,x)+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] -(8*(a*(sin(c + d*x) + 1))^(1/2)*(8*cos(c + d*x) + 6*cos(3*c + 3*d*x) - sin
(2*c + 2*d*x) + 2*sin(4*c + 4*d*x)))/(15*a*d*e^2*(e*cos(c + d*x))^(1/2)*(4*
sin(c + d*x) + 4*cos(2*c + 2*d*x) - cos(4*c + 4*d*x) + 4*sin(3*c + 3*d*x) +
 5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.304 $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=154

$$-\frac{32(a\sin(c+dx)+a)^{5/2}}{35a^3de(e\cos(c+dx))^{5/2}}+\frac{16(a\sin(c+dx)+a)^{3/2}}{7a^2de(e\cos(c+dx))^{5/2}}-\frac{4\sqrt{a\sin(c+dx)+a}}{7ade(e\cos(c+dx))^{5/2}}-\frac{2}{7de\sqrt{a\sin(c+dx)+a}\,(e\cos(c+}$$

```
[Out] 16/7*(a+a*sin(d*x+c))^(3/2)/a^2/d/e/(e*cos(d*x+c))^(5/2)-32/35*(a+a*sin(d*x
+c))^(5/2)/a^3/d/e/(e*cos(d*x+c))^(5/2)-2/7/d/e/(e*cos(d*x+c))^(5/2)/(a+a*s
in(d*x+c))^(1/2)-4/7*(a+a*sin(d*x+c))^(1/2)/a/d/e/(e*cos(d*x+c))^(5/2)
```

Rubi [A] time = 0.29, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2672, 2671}

$$-\frac{32(a\sin(c+dx)+a)^{5/2}}{35a^3de(e\cos(c+dx))^{5/2}}+\frac{16(a\sin(c+dx)+a)^{3/2}}{7a^2de(e\cos(c+dx))^{5/2}}-\frac{4\sqrt{a\sin(c+dx)+a}}{7ade(e\cos(c+dx))^{5/2}}-\frac{2}{7de\sqrt{a\sin(c+dx)+a}\,(e\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] -2/(7*d*e*(e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]) - (4*Sqrt[a + a*
Sin[c + d*x]])/(7*a*d*e*(e*Cos[c + d*x])^(5/2)) + (16*(a + a*Sin[c + d*x])^
(3/2))/(7*a^2*d*e*(e*Cos[c + d*x])^(5/2)) - (32*(a + a*Sin[c + d*x])^(5/2))
/(35*a^3*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}+\frac{6\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx}{7a}\\ &= -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}-\frac{4\sqrt{a+a\sin(c+dx}}{7ade(e\cos(c+dx))^{5}}\\ &= -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}-\frac{4\sqrt{a+a\sin(c+dx}}{7ade(e\cos(c+dx))^{5}}\\ &= -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}-\frac{4\sqrt{a+a\sin(c+dx}}{7ade(e\cos(c+dx))^{5}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 66, normalized size = 0.43

$$\frac{2(10\sin(c+dx)+4\sin(3(c+dx))+4\cos(2(c+dx))+5)}{35de\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (2*(5 + 4*Cos[2*(c + d*x)] + 10*Sin[c + d*x] + 4*Sin[3*(c + d*x)]))/(35*d*e
*(e*Cos[c + d*x])^(5/2)*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.90, size = 93, normalized size = 0.60

$$\frac{2\sqrt{e\cos\left(dx+c\right)}\left(8\cos\left(dx+c\right)^{2}+2\left(8\cos\left(dx+c\right)^{2}+3\right)\sin\left(dx+c\right)+1\right)\sqrt{a\sin\left(dx+c\right)+a}}{35\left(ade^{4}\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ade^{4}\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/35*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^2 + 3)*sin(
d*x + c) + 1)*sqrt(a*sin(d*x + c) + a)/(a*d*e^4*cos(d*x + c)^3*sin(d*x + c)
 + a*d*e^4*cos(d*x + c)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*sqrt(a*sin(d*x + c) + a)), x)
```

maple [A] time = 0.19, size = 70, normalized size = 0.45

$$\frac{2\left(16\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)+8\left(\cos^{2}\left(dx+c\right)\right)+6\sin\left(dx+c\right)+1\right)\cos\left(dx+c\right)}{35d\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/35/d*(16*cos(d*x+c)^2*sin(d*x+c)+8*cos(d*x+c)^2+6*sin(d*x+c)+1)*cos(d*x+c
)/(e*cos(d*x+c))^(7/2)/(a*(1+sin(d*x+c)))^(1/2)
```

maxima [B] time = 0.87, size = 363, normalized size = 2.36

$$\frac{2\left(9\sqrt{a}\sqrt{e}+\frac{44\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\sqrt{a}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{84\sqrt{a}\sqrt{e}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{84\sqrt{a}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{14\sqrt{a}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{44\sqrt{a}\ldots}{(\mathrm{co}\ldots}\right.}{35\left(ae^{4}+\frac{4ae^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6ae^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4ae^{4}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{ae^{4}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/35*(9*sqrt(a)*sqrt(e) + 44*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1
) - 14*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 84*sqrt(a)*sqr
t(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(a)*sqrt(e)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 + 14*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 - 44*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 9*sqrt(a)*s
qrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 1)^4/((a*e^4 + 4*a*e^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*e^
4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*e^4*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + a*e^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

mupad [B] time = 11.01, size = 261, normalized size = 1.69

$$\frac{20\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+10\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+8\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+8}{\frac{35\,a\,d\,e^3\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{2}+\frac{35\,a\,d\,e^3\,\sin\left(c+d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{4}+\frac{35\,a\,d\,e^3\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{2}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] (20*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2) + 10*(a + a*sin(c + d*x))^(1/2)
 + 8*cos(2*c + 2*d*x)*(a + a*sin(c + d*x))^(1/2) + 8*sin(3*c + 3*d*x)*(a +
a*sin(c + d*x))^(1/2))/((35*a*d*e^3*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*
1i + d*x*1i))/2)^(1/2))/2 + (35*a*d*e^3*sin(c + d*x)*((e*exp(- c*1i - d*x*1
i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 + (35*a*d*e^3*cos(2*c + 2*d*x)*(
(e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/2 + (35*a*d*e
^3*sin(3*c + 3*d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)
^(1/2))/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.305 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=247

$$\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4a^2d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)-}}{4a^2d(\sin(c+dx)+\cos(c}$$

```
[Out] 1/2*e*(e*cos(d*x+c))^(5/2)/a/d/(a+a*sin(d*x+c))^(1/2)+5/4*e^3*(e*cos(d*x+c)
)^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^2/d-5/4*e^(7/2)*arcsinh((e*cos(d*x+c))^(1/
2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^2/d/(1+cos(d*x+c)
+sin(d*x+c))+5/4*e^(7/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+
cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^2/d/(1+cos
(d*x+c)+sin(d*x+c))
```

Rubi [A] time = 0.37, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2679, 2685, 2677, 2775, 203, 2833, 63, 215}

$$\frac{5e^3\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{4a^2d}+\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4a^2d(\sin(c+dx)+\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (e*(e*Cos[c + d*x])^(5/2))/(2*a*d*Sqrt[a + a*Sin[c + d*x]]) + (5*e^3*Sqrt[e
*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*a^2*d) - (5*e^(7/2)*ArcSinh[Sqr
t[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])
/(4*a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (5*e^(7/2)*ArcTan[(Sqrt[e]*S
in[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
+ d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x]
))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
```

```
f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{(5e^2)\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{(5e^4)\int \frac{\sqrt{a+a\sin(}{\sqrt{e\cos(c}}}{8a^2}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{(5e^4\sqrt{1+\cos(c}}{8a(a}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} - \frac{(5e^4\sqrt{1+\cos(c}}{}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{5e^{7/2}\tan^{-1}\left(\frac{}{\sqrt{e\,c}}\right.}{}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} - \frac{5e^{7/2}\sinh^{-1}\left(\sqrt{e}\right.}{4d\,(a}$$

Mathematica [C] time = 0.13, size = 80, normalized size = 0.32

$$-\frac{2\,2^{3/4}\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9a^2de(\sin(c+dx)+1)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - S
in[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(9*a^2*d*e*(1 + Sin[c + d*x])^(
11/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.27, size = 266, normalized size = 1.08

$$-\frac{(e\cos(dx+c))^{\frac{7}{2}}\left(5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)-5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\right.}{8d\left(\cos(dx+c)\sin(dx+c)-\left(\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/8/d*(e*cos(d*x+c))^(7/2)*(5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-5*2^(1
/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+4*cos(d*x+c)^2*sin
(d*x+c)+4*cos(d*x+c)^3+10*cos(d*x+c)*sin(d*x+c)-14*cos(d*x+c)^2+10*cos(d*x+
c))/(cos(d*x+c)*sin(d*x+c)-cos(d*x+c)^2-2*sin(d*x+c)-cos(d*x+c)+2)/(a*(1+si
n(d*x+c)))^(3/2)/cos(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{7/2}}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.306 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d\left(a^2\sin(c+dx)+a^2\cos(c+dx)+a^2\right)}+\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)\cdots}}{d\left(a^2\sin(c+dx)+a^2\cos(\cdots\right.}$$

```
[Out] e*(e*cos(d*x+c))^(3/2)/a/d/(a+a*sin(d*x+c))^(1/2)+3*e^(5/2)*arcsinh((e*cos(
d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(a^2+a
^2*cos(d*x+c)+a^2*sin(d*x+c))+3*e^(5/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*
x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/
2)/d/(a^2+a^2*cos(d*x+c)+a^2*sin(d*x+c))
```

Rubi [A] time = 0.29, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2679, 2684, 2775, 203, 2833, 63, 215}

$$\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d\left(a^2\sin(c+dx)+a^2\cos(c+dx)+a^2\right)}+\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)\cdots}}{d\left(a^2\sin(c+dx)+a^2\cos(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (e*(e*Cos[c + d*x])^(3/2))/(a*d*Sqrt[a + a*Sin[c + d*x]]) + (3*e^(5/2)*ArcS
inh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c +
 d*x]])/(d*(a^2 + a^2*Cos[c + d*x] + a^2*Sin[c + d*x])) + (3*e^(5/2)*ArcTan
[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt
[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a^2 + a^2*Cos[c + d*x] + a
^2*Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
```

```
egersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+d}}{\sqrt{e\cos(c+dx}}}{2a(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{}{\sqrt{e}}\right.}{2ad(a+a\cos(c+dx)+a\sin(c+}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}}{d\left(a^2+a^2\cos(c+dx)+a^2\sin(c+dx\right.}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{3e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(}}{d\left(a^2+a^2\cos(c+dx)+a^2\sin(c+dx)\right)}$$

Mathematica [C] time = 0.15, size = 80, normalized size = 0.37

$$-\frac{2\sqrt[4]{2}\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7a^2de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[3/4, 7/4, 11/4, (1 - S
in[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(7*a^2*d*e*(1 + Sin[c + d*x])^(
9/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.24, size = 232, normalized size = 1.08

$$-\frac{(e\cos(dx+c))^{\frac{5}{2}}\left(-3\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sin(dx+c)-3\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.\right.}{2d\left(\cos(dx+c)\sin(dx+c)-\left(\cos^2(dx+c)\right)-2\sin(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*(e*cos(d*x+c))^(5/2)*(-3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-3*2^(
1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*cos(d*x+c)*sin(
d*x+c)+2*cos(d*x+c)^2-2*cos(d*x+c))/(cos(d*x+c)*sin(d*x+c)-cos(d*x+c)^2-2*s
in(d*x+c)-cos(d*x+c)+2)/(a*(1+sin(d*x+c)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.307 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{a^2d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx}}{a^2d(\sin(c+dx)+\cos(}$$

```
[Out] -2*(e*cos(d*x+c))^(5/2)/d/e/(a+a*sin(d*x+c))^(3/2)-2*e*(e*cos(d*x+c))^(1/2)
*(a+a*sin(d*x+c))^(1/2)/a^2/d+2*e^(3/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2
))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^2/d/(1+cos(d*x+c)+sin(d*x+
c))-2*e^(3/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))
^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^2/d/(1+cos(d*x+c)+sin
(d*x+c))
```

Rubi [A] time = 0.36, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2681, 2685, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{a^2d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx}}{a^2d(\sin(c+dx)+\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(5/2))/(d*e*(a + a*Sin[c + d*x])^(3/2)) - (2*e*Sqrt[e*
Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^2*d) + (2*e^(3/2)*ArcSinh[Sqrt[e
*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a
^2*d*(1 + Cos[c + d*x] + Sin[c + d*x])) - (2*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c
+ d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x
]]*Sqrt[a + a*Sin[c + d*x]])/(a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
```

```
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^2d} - \frac{e^2\int \frac{\sqrt{a+a\sin}}{\sqrt{e\cos(c}}}{a^2}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^2d} - \frac{\left(e^2\sqrt{1+\cos}\right.}{a}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^2d} + \frac{\left(e^2\sqrt{1+\cos}\right.}{}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^2d} - \frac{2e^{3/2}\tan^{-1}\left(\right.}{}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^2d} + \frac{2e^{3/2}\sinh^{-1}}{a}$$

Mathematica [C] time = 0.12, size = 80, normalized size = 0.34

$$-\frac{2^{3/4}\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5a^2de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/5*(2^(3/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 5/4, 9/4, (1 - 
Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(a^2*d*e*(1 + Sin[c + d*x])^(7
/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 321, normalized size = 1.36

$$-\frac{2\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(-1+\cos\left(dx+c\right)\right)\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin\left(dx+c\right)-\sqrt{2}\,\arctanh\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin}{2\cos(dx+}\right.\right.}{d\sin\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(e*cos(d*x+c))^(3/2)*(-1+cos(d*x+c))*(2^(1/2)*arctan(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-2^(1/2)*arctanh(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*cos(d*x+c)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2
^(1/2))-2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))-2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)/(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(a*(1+sin(d*x+c)))^(3/2)/(-1+cos(d*x+c)+s
in(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+d\,x))^{3/2}}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^{\frac{3}{2}}}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**(3/2)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.308 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{2(e\cos(c+dx))^{3/2}}{3de(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -2/3*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2671}

$$-\frac{2(e\cos(c+dx))^{3/2}}{3de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2))/(3*d*e*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2(e\cos(c+dx))^{3/2}}{3de(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.07, size = 49, normalized size = 1.36

$$-\frac{2\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{3/2}}{3a^2de(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2)*Sqrt[a*(1 + Sin[c + d*x])])/(3*a^2*d*e*(1 + Sin[
c + d*x])^2)
```

fricas [B] time = 0.99, size = 100, normalized size = 2.78

$$\frac{2\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{3\left(a^2d\cos(dx+c)^2-a^2d\cos(dx+c)-2\,a^2d-\left(a^2d\cos(dx+c)+2\,a^2d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x +
 c) + 1)/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(
d*x + c) + 2*a^2*d)*sin(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.19, size = 34, normalized size = 0.94

$$-\frac{2\sqrt{e\cos\left(dx+c\right)}\,\cos\left(dx+c\right)}{3d\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(e*cos(d*x+c))^(1/2)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(3/2)
```

maxima [B] time = 0.83, size = 131, normalized size = 3.64

$$-\frac{2\left(\sqrt{a}\sqrt{e}-\frac{\sqrt{a}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{3\left(a^{2}+\frac{a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*(sqrt(a)*sqrt(e) - sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d*(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(5/2))
```

mupad [B] time = 5.99, size = 82, normalized size = 2.28

$$-\frac{4\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(2\,\cos\left(c+d\,x\right)+\sin\left(2\,c+2\,d\,x\right)\right)}{3\,a^2\,d\,\left(15\,\sin\left(c+d\,x\right)-6\,\cos\left(2\,c+2\,d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] -(4*(e*cos(c + d*x))^(1/2)*(a*(sin(c + d*x) + 1))^(1/2)*(2*cos(c + d*x) + s
in(2*c + 2*d*x)))/(3*a^2*d*(15*sin(c + d*x) - 6*cos(2*c + 2*d*x) - sin(3*c
+ 3*d*x) + 10))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(c+dx\right)}}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.309 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{4\sqrt{e\cos(c+dx)}}{5ade\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{e\cos(c+dx)}}{5de(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -2/5*(e*cos(d*x+c))^(1/2)/d/e/(a+a*sin(d*x+c))^(3/2)-4/5*(e*cos(d*x+c))^(1/
2)/a/d/e/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{4\sqrt{e\cos(c+dx)}}{5ade\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{e\cos(c+dx)}}{5de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(5*d*e*(a + a*Sin[c + d*x])^(3/2)) - (4*Sqrt[e*Co
s[c + d*x]])/(5*a*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{2\sqrt{e\cos(c+dx)}}{5de(a+a\sin(c+dx))^{3/2}}+\frac{2\int \frac{1}{\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}\,dx}{5a}\\ &= -\frac{2\sqrt{e\cos(c+dx)}}{5de(a+a\sin(c+dx))^{3/2}}-\frac{4\sqrt{e\cos(c+dx)}}{5ade\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 59, normalized size = 0.78

$$-\frac{2(2\sin(c+dx)+3)\sqrt{a(\sin(c+dx)+1)}\,\sqrt{e\cos(c+dx)}}{5a^2de(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*Sqrt[a*(1 + Sin[c + d*x])]*(3 + 2*Sin[c + d*x]))/(
5*a^2*d*e*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.92, size = 71, normalized size = 0.93

$$\frac{2\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}\,(2\sin(dx+c)+3)}{5\left(a^2de\cos(dx+c)^2-2\,a^2de\sin(dx+c)-2\,a^2de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/5*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*(2*sin(d*x + c) + 3)/(a^2
*d*e*cos(d*x + c)^2 - 2*a^2*d*e*sin(d*x + c) - 2*a^2*d*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(3/2)), x)
```

maple [A] time = 0.19, size = 44, normalized size = 0.58

$$-\frac{2\,(2\sin(dx+c)+3)\cos(dx+c)}{5d\,(a\,(1+\sin(dx+c)))^{\frac{3}{2}}\sqrt{e\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/5/d*(2*sin(d*x+c)+3)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(3/2)/(e*cos(d*x+c))^
(1/2)
```

maxima [B] time = 1.09, size = 211, normalized size = 2.78

$$-\frac{2\left(3\sqrt{a}\sqrt{e}+\frac{4\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{5\left(a^2e+\frac{2\,a^2e\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2e\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/5*(3*sqrt(a)*sqrt(e) + 4*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
 - 4*sqrt(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sqrt(a)*sqrt(e
)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 1)^2/((a^2*e + 2*a^2*e*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*e*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/
2)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + 1))
```

mupad [B] time = 6.38, size = 95, normalized size = 1.25

$$-\frac{4\sqrt{a}\,\left(\sin(c+d\,x)+1\right)\,\left(7\cos(c+d\,x)-\cos(3\,c+3\,d\,x)+5\sin(2\,c+2\,d\,x)\right)}{5\,a^2\,d\sqrt{e\cos(c+d\,x)}\,\left(15\sin(c+d\,x)-6\cos(2\,c+2\,d\,x)-\sin(3\,c+3\,d\,x)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] -(4*(a*(sin(c + d*x) + 1))^(1/2)*(7*cos(c + d*x) - cos(3*c + 3*d*x) + 5*sin
(2*c + 2*d*x)))/(5*a^2*d*(e*cos(c + d*x))^(1/2)*(15*sin(c + d*x) - 6*cos(2*
c + 2*d*x) - sin(3*c + 3*d*x) + 10))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a(\sin(c+dx)+1))^{\frac{3}{2}}\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(c + d*x) + 1))**(3/2)*sqrt(e*cos(c + d*x))), x)
```

3.310 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{16\sqrt{a\sin(c+dx)+a}}{21a^2de\sqrt{e\cos(c+dx)}}-\frac{8}{21ade\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}-\frac{2}{7de(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}$$

```
[Out] -2/7/d/e/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2)-8/21/a/d/e/(e*cos(d*x+
c))^(1/2)/(a+a*sin(d*x+c))^(1/2)+16/21*(a+a*sin(d*x+c))^(1/2)/a^2/d/e/(e*co
s(d*x+c))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{16\sqrt{a\sin(c+dx)+a}}{21a^2de\sqrt{e\cos(c+dx)}}-\frac{8}{21ade\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}-\frac{2}{7de(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -2/(7*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)) - 8/(21*a*d*e*Sq
rt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (16*Sqrt[a + a*Sin[c + d*x]]
)/(21*a^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2}{7de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}+\frac{4\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a}\cdots}}{7a}$$

$$= -\frac{2}{7de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}-\frac{\cdots}{21ade\sqrt{e\cos(c+d\cdots}}$$

$$= -\frac{2}{7de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}-\frac{\cdots}{21ade\sqrt{e\cos(c+d\cdots}}$$

Mathematica [A] time = 0.10, size = 56, normalized size = 0.49

$$\frac{16\sin^2(c+dx)+24\sin(c+dx)+2}{21de(a(\sin(c+dx)+1))^{3/2}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (2 + 24*Sin[c + d*x] + 16*Sin[c + d*x]^2)/(21*d*e*Sqrt[e*Cos[c + d*x]]*(a*(
1 + Sin[c + d*x]))^(3/2))
```

fricas [A] time = 1.04, size = 99, normalized size = 0.86

$$\frac{2\,\sqrt{e\cos\left(dx+c\right)}\left(8\,\cos\left(dx+c\right)^{2}-12\,\sin\left(dx+c\right)-9\right)\sqrt{a\sin\left(dx+c\right)+a}}{21\,\left(a^{2}de^{2}\cos\left(dx+c\right)^{3}-2\,a^{2}de^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^{2}de^{2}\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/21*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 - 12*sin(d*x + c) - 9)*sqrt(a*s
in(d*x + c) + a)/(a^2*d*e^2*cos(d*x + c)^3 - 2*a^2*d*e^2*cos(d*x + c)*sin(d
*x + c) - 2*a^2*d*e^2*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(3/2)), x)
```

maple [A] time = 0.17, size = 54, normalized size = 0.47

$$\frac{2\left(-8\left(\cos^{2}\left(dx+c\right)\right)+12\sin\left(dx+c\right)+9\right)\cos\left(dx+c\right)}{21d\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/21/d*(-8*cos(d*x+c)^2+12*sin(d*x+c)+9)*cos(d*x+c)/(e*cos(d*x+c))^(3/2)/(a
*(1+sin(d*x+c)))^(3/2)
```

maxima [B] time = 0.89, size = 294, normalized size = 2.56

$$\frac{2\left(\sqrt{a}\sqrt{e}+\frac{24\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{33\sqrt{a}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{33\sqrt{a}\sqrt{e}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{24\sqrt{a}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{\sqrt{a}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)\left(\frac{\sin(dx+c)}{(\cos(dx+c)+\right.}{21\left(a^{2}e^{2}+\frac{3a^{2}e^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a^{2}e^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a^{2}e^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/21*(sqrt(a)*sqrt(e) + 24*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
+ 33*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 33*sqrt(a)*sqrt(
e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 24*sqrt(a)*sqrt(e)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)
```

```
*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((a^2*e^2 + 3*a^2*e^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*e^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + a^2*e^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

mupad [B] time = 6.82, size = 119, normalized size = 1.03

$$\frac{8\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(70\,\sin\left(c+d\,x\right)-41\,\cos\left(2\,c+2\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)-14\,\sin\left(3\,c+3\,d\,x\right)+41\right)}{21\,a^2\,d\,e\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(56\,\sin\left(c+d\,x\right)-28\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)-8\,\sin\left(3\,c+3\,d\,x\right)+35\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] (8*(a*(sin(c + d*x) + 1))^(1/2)*(70*sin(c + d*x) - 41*cos(2*c + 2*d*x) + 2*
cos(4*c + 4*d*x) - 14*sin(3*c + 3*d*x) + 41))/(21*a^2*d*e*(e*cos(c + d*x))^
(1/2)*(56*sin(c + d*x) - 28*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) - 8*sin(3*c
 + 3*d*x) + 35))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a*(sin(c + d*x) + 1))**(3/2)*(e*cos(c + d*x))**(3/2)), x)
```

3.311 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{32(a\sin(c+dx)+a)^{3/2}}{45a^3de(e\cos(c+dx))^{3/2}}-\frac{16\sqrt{a\sin(c+dx)+a}}{15a^2de(e\cos(c+dx))^{3/2}}-\frac{4}{15ade\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}-\frac{}{9de(a\sin(c+dx)}$$

```
[Out] -2/9/d/e/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2)+32/45*(a+a*sin(d*x+c))
^(3/2)/a^3/d/e/(e*cos(d*x+c))^(3/2)-4/15/a/d/e/(e*cos(d*x+c))^(3/2)/(a+a*si
n(d*x+c))^(1/2)-16/15*(a+a*sin(d*x+c))^(1/2)/a^2/d/e/(e*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.29, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{32(a\sin(c+dx)+a)^{3/2}}{45a^3de(e\cos(c+dx))^{3/2}}-\frac{16\sqrt{a\sin(c+dx)+a}}{15a^2de(e\cos(c+dx))^{3/2}}-\frac{4}{15ade\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}-\frac{}{9de(a\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -2/(9*d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)) - 4/(15*a*d*e*
(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (16*Sqrt[a + a*Sin[c + d
*x]])/(15*a^2*d*e*(e*Cos[c + d*x])^(3/2)) + (32*(a + a*Sin[c + d*x])^(3/2))
/(45*a^3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}+\frac{2\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+}}}{3a}\\
&= -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}-\frac{}{15ade(e\cos(c+dx)}\\
&= -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}-\frac{}{15ade(e\cos(c+dx)}\\
&= -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}-\frac{}{15ade(e\cos(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 66, normalized size = 0.43

$$-\frac{2(-6\sin(c+dx)+4\sin(3(c+dx))+12\cos(2(c+dx))+7)}{45de(a(\sin(c+dx)+1))^{3/2}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*(7 + 12*Cos[2*(c + d*x)] - 6*Sin[c + d*x] + 4*Sin[3*(c + d*x)]))/(45*d*
e*(e*Cos[c + d*x])^(3/2)*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [A] time = 0.98, size = 115, normalized size = 0.75

$$\frac{2\sqrt{e\cos(dx+c)}\left(24\cos(dx+c)^2+2\left(8\cos(dx+c)^2-5\right)\sin(dx+c)-5\right)\sqrt{a\sin(dx+c)+a}}{45\left(a^2de^3\cos(dx+c)^4-2a^2de^3\cos(dx+c)^2\sin(dx+c)-2a^2de^3\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/45*sqrt(e*cos(d*x + c))*(24*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^2 - 5)*sin
(d*x + c) - 5)*sqrt(a*sin(d*x + c) + a)/(a^2*d*e^3*cos(d*x + c)^4 - 2*a^2*d
*e^3*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*d*e^3*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^(3/2)), x)
```

maple [A] time = 0.18, size = 70, normalized size = 0.45

$$-\frac{2\left(16\left(\cos^2(dx+c)\right)\sin(dx+c)+24\left(\cos^2(dx+c)\right)-10\sin(dx+c)-5\right)\cos(dx+c)}{45d(e\cos(dx+c))^{\frac{5}{2}}(a(1+\sin(dx+c)))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/45/d*(16*cos(d*x+c)^2*sin(d*x+c)+24*cos(d*x+c)^2-10*sin(d*x+c)-5)*cos(d*
x+c)/(e*cos(d*x+c))^(5/2)/(a*(1+sin(d*x+c)))^(3/2)
```

maxima [B] time = 0.58, size = 373, normalized size = 2.42

$$-\frac{2\left(19\sqrt{a}\sqrt{e}+\frac{12\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{58\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{116\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{116\sqrt{a}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{58\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\cdots\right.}{45\left(a^2e^3+\frac{4a^2e^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2e^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^2e^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2e^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(d\cdots}{\cos(dx\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2/45*(19*sqrt(a)*sqrt(e) + 12*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
 1) - 58*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 116*sqrt(a)*
sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 116*sqrt(a)*sqrt(e)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 + 58*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 - 12*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 19*sqrt
(a)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^4/((a^2*e^3 + 4*a^2*e^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 6*a^2*e^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*e^3*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + a^2*e^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(5/2))
```

mupad [B] time = 11.10, size = 230, normalized size = 1.49

$$-\frac{14\sqrt{a+a\,\sin\left(c+d\,x\right)}-12\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+24\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+8\,\mathrm{s}}{\frac{225\,a^2\,d\,e^2\,\cos(c+d\,x)\,\sqrt{\frac{e\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}{4}-\frac{45\,a^2\,d\,e^2\,\cos(3\,c+3\,d\,x)\,\sqrt{\frac{e\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}{4}+45\,a^2\,d\,e^2\,\sin\left(2\,c+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] -(14*(a + a*sin(c + d*x))^(1/2) - 12*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2
) + 24*cos(2*c + 2*d*x)*(a + a*sin(c + d*x))^(1/2) + 8*sin(3*c + 3*d*x)*(a
+ a*sin(c + d*x))^(1/2))/((225*a^2*d*e^2*cos(c + d*x)*((e*exp(- c*1i - d*x*
1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 - (45*a^2*d*e^2*cos(3*c + 3*d*x
)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 + 45*a^2
*d*e^2*sin(2*c + 2*d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i)
)/2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.312 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{256(a\sin(c+dx)+a)^{5/2}}{385a^4de(e\cos(c+dx))^{5/2}}+\frac{128(a\sin(c+dx)+a)^{3/2}}{77a^3de(e\cos(c+dx))^{5/2}}-\frac{32\sqrt{a\sin(c+dx)+a}}{77a^2de(e\cos(c+dx))^{5/2}}-\frac{16}{77ade\sqrt{a\sin(c+dx)+a}\,(e\cos}$$

```
[Out] -2/11/d/e/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2)+128/77*(a+a*sin(d*x+c
))^(3/2)/a^3/d/e/(e*cos(d*x+c))^(5/2)-256/385*(a+a*sin(d*x+c))^(5/2)/a^4/d/
e/(e*cos(d*x+c))^(5/2)-16/77/a/d/e/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1
/2)-32/77*(a+a*sin(d*x+c))^(1/2)/a^2/d/e/(e*cos(d*x+c))^(5/2)
```

Rubi [A] time = 0.37, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{256(a\sin(c+dx)+a)^{5/2}}{385a^4de(e\cos(c+dx))^{5/2}}+\frac{128(a\sin(c+dx)+a)^{3/2}}{77a^3de(e\cos(c+dx))^{5/2}}-\frac{32\sqrt{a\sin(c+dx)+a}}{77a^2de(e\cos(c+dx))^{5/2}}-\frac{16}{77ade\sqrt{a\sin(c+dx)+a}\,(e\cos}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -2/(11*d*e*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2)) - 16/(77*a*d*
e*(e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]) - (32*Sqrt[a + a*Sin[c +
 d*x]])/(77*a^2*d*e*(e*Cos[c + d*x])^(5/2)) + (128*(a + a*Sin[c + d*x])^(3/
2))/(77*a^3*d*e*(e*Cos[c + d*x])^(5/2)) - (256*(a + a*Sin[c + d*x])^(5/2))/
(385*a^4*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} + \frac{8\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a}}}{11a}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{77ade(e\cos(c+dx}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{77ade(e\cos(c+dx}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{77ade(e\cos(c+dx}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{77ade(e\cos(c+dx}$$

Mathematica [A] time = 0.30, size = 76, normalized size = 0.39

$$\frac{2(104\sin(c+dx)+48\sin(3(c+dx))+8\cos(2(c+dx))-16\cos(4(c+dx))+45)}{385de(a(\sin(c+dx)+1))^{3/2}(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (2*(45 + 8*Cos[2*(c + d*x)] - 16*Cos[4*(c + d*x)] + 104*Sin[c + d*x] + 48*S
in[3*(c + d*x)]))/(385*d*e*(e*Cos[c + d*x])^(5/2)*(a*(1 + Sin[c + d*x]))^(3
/2))
```

fricas [A] time = 0.52, size = 125, normalized size = 0.65

$$\frac{2\left(128\cos(dx+c)^4-144\cos(dx+c)^2-8\left(24\cos(dx+c)^2+7\right)\sin(dx+c)-21\right)\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+}}{385\left(a^2de^4\cos(dx+c)^5-2a^2de^4\cos(dx+c)^3\sin(dx+c)-2a^2de^4\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/385*(128*cos(d*x + c)^4 - 144*cos(d*x + c)^2 - 8*(24*cos(d*x + c)^2 + 7)*
sin(d*x + c) - 21)*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(a^2*d*e^4
*cos(d*x + c)^5 - 2*a^2*d*e^4*cos(d*x + c)^3*sin(d*x + c) - 2*a^2*d*e^4*cos
(d*x + c)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^(3/2)), x)
```

maple [A] time = 0.22, size = 80, normalized size = 0.41

$$\frac{2\left(-128\left(\cos^4(dx+c)\right)+192\left(\cos^2(dx+c)\right)\sin(dx+c)+144\left(\cos^2(dx+c)\right)+56\sin(dx+c)+21\right)\cos(dx+}{385d(e\cos(dx+c))^{\frac{7}{2}}(a(1+\sin(dx+c)))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/385/d*(-128*cos(d*x+c)^4+192*cos(d*x+c)^2*sin(d*x+c)+144*cos(d*x+c)^2+56*
sin(d*x+c)+21)*cos(d*x+c)/(e*cos(d*x+c))^(7/2)/(a*(1+sin(d*x+c)))^(3/2)
```

maxima [B] time = 1.04, size = 451, normalized size = 2.34

$$\frac{2\left(37\sqrt{a}\sqrt{e}+\frac{496\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{559\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{544\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1526\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1526\sqrt{a}\sqrt{e}\sin(dx}{(\cos(dx+c)+1)^6}\right.}{385\left(a^2e^4+\frac{5a^2e^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2e^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^2e^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2e^4\sin}{(\cos(dx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/385*(37*sqrt(a)*sqrt(e) + 496*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c)
+ 1) + 559*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 544*sqrt(a
)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1526*sqrt(a)*sqrt(e)*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 1526*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 544*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5
59*sqrt(a)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 496*sqrt(a)*sqrt(e
)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 37*sqrt(a)*sqrt(e)*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((a^2*e^
4 + 5*a^2*e^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*e^4*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 10*a^2*e^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
5*a^2*e^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^2*e^4*sin(d*x + c)^10/(co
s(d*x + c) + 1)^10)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*
x + c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

mupad [B] time = 11.65, size = 413, normalized size = 2.14

$$\frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}\,\left(\frac{288\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}}{77\,a^2\,d\,e^3}+\frac{256\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4}}{385}\right.}{10\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\sqrt{e\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}+8\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\sin\left(c+d\,x\right)\,\sqrt{e\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}+8\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] ((a + a*sin(c + d*x))^(1/2)*((288*exp(c*4i + d*x*4i))/(77*a^2*d*e^3) + (256
*exp(c*4i + d*x*4i)*cos(2*c + 2*d*x))/(385*a^2*d*e^3) - (512*exp(c*4i + d*x
*4i)*cos(4*c + 4*d*x))/(385*a^2*d*e^3) + (1536*exp(c*4i + d*x*4i)*sin(3*c +
 3*d*x))/(385*a^2*d*e^3) + (3328*exp(c*4i + d*x*4i)*sin(c + d*x))/(385*a^2*
d*e^3)))/(10*exp(c*4i + d*x*4i)*(e*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x
*1i)/2))^(1/2) + 8*exp(c*4i + d*x*4i)*sin(c + d*x)*(e*(exp(- c*1i - d*x*1i)
/2 + exp(c*1i + d*x*1i)/2))^(1/2) + 8*exp(c*4i + d*x*4i)*cos(2*c + 2*d*x)*(
e*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2) - 2*exp(c*4i + d*x
*4i)*cos(4*c + 4*d*x)*(e*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(
1/2) + 8*exp(c*4i + d*x*4i)*sin(3*c + 3*d*x)*(e*(exp(- c*1i - d*x*1i)/2 + e
xp(c*1i + d*x*1i)/2))^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.313 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=261

$$\frac{21e^{9/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}+\frac{21e^{9/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+}}{4d\left(a^3\sin(c+dx)+a^3\right.}$$

```
[Out] 1/2*e*(e*cos(d*x+c))^(7/2)/a/d/(a+a*sin(d*x+c))^(3/2)+7/4*e^3*(e*cos(d*x+c)
)^(3/2)/a^2/d/(a+a*sin(d*x+c))^(1/2)+21/4*e^(9/2)*arcsinh((e*cos(d*x+c))^(1
/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(a^3+a^3*cos(d*x
+c)+a^3*sin(d*x+c))+21/4*e^(9/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(
1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(a
^3+a^3*cos(d*x+c)+a^3*sin(d*x+c))
```

Rubi [A] time = 0.47, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2680, 2686, 2679, 2684, 2775, 203, 2833, 63, 215}

$$\frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{21e^{9/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}+\frac{21e^{9/2}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (e*(e*Cos[c + d*x])^(7/2))/(2*a*d*(a + a*Sin[c + d*x])^(3/2)) + (7*e^3*(e*C
os[c + d*x])^(3/2))/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (21*e^(9/2)*ArcSin
h[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d
*x]])/(4*d*(a^3 + a^3*Cos[c + d*x] + a^3*Sin[c + d*x])) + (21*e^(9/2)*ArcTa
n[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqr
t[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(a^3 + a^3*Cos[c + d*x]
+ a^3*Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
```

```
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2686

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1))/(f*g*(2*p - 1)*
(a + b*Sin[e + f*x])^(3/2)), x] + Dist[(2*a*(p - 2))/(2*p - 1), Int[(g*Cos[
e + f*x])^p/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 2] && IntegerQ[2*p]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^{3/2}} + \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{(21e^4)\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{8a^2}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{(21e^5\sqrt{1+\cos(c+dx)}\,\sqrt{}}{8a^2(a+a\cos}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{(21e^5\sqrt{1+\cos(c+dx)}\,\sqrt{}}{8a^2}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{21e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}\sin}{\sqrt{e\cos(c+dx)}}\right.}{4d\left(a^3+\right.}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{21e^{9/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right.}{4d\left(a^3+a^3\,\mathrm{c}\right.}$$

Mathematica [C] time = 0.18, size = 80, normalized size = 0.31

$$-\frac{2\sqrt[4]{2}\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{3}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11a^3de(\sin(c+dx)+1)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^(5/2),x]
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[3/4, 11/4, 15/4, (1 -
 Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(11*a^3*d*e*(1 + Sin[c + d*x]
)^(13/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.28, size = 282, normalized size = 1.08

$$\frac{(e\cos(dx+c))^{\frac{9}{2}}\left(21\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+21\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\sqrt{\right.}\right.}{8d\left(\left(\cos^2(dx+c)\right)\sin(dx+c)+\cos^3(dx+c)+2\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/8/d*(e*cos(d*x+c))^(9/2)*(21*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+21*2^(
1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+4*cos(d*x+c)^2*si
n(d*x+c)-4*cos(d*x+c)^3-22*cos(d*x+c)*sin(d*x+c)-18*cos(d*x+c)^2+22*cos(d*x
+c))/(cos(d*x+c)^2*sin(d*x+c)+cos(d*x+c)^3+2*cos(d*x+c)*sin(d*x+c)-3*cos(d*
x+c)^2-4*sin(d*x+c)-2*cos(d*x+c)+4)/(a*(1+sin(d*x+c)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{9/2}}{(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.314 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{a^3d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+ \cdots}{a^3d(\sin(c+dx)+c\cdots}$$

```
[Out] -4*e*(e*cos(d*x+c))^(5/2)/a/d/(a+a*sin(d*x+c))^(3/2)-5*e^3*(e*cos(d*x+c))^(
1/2)*(a+a*sin(d*x+c))^(1/2)/a^3/d+5*e^(7/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^
(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^3/d/(1+cos(d*x+c)+sin(
d*x+c))-5*e^(7/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x
+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^3/d/(1+cos(d*x+c)
+sin(d*x+c))
```

Rubi [A] time = 0.37, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2680, 2685, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{5e^3\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{a^3d}-\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+ \cdots}}\right.}{a^3d(\sin(c+dx)+\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*e*(e*Cos[c + d*x])^(5/2))/(a*d*(a + a*Sin[c + d*x])^(3/2)) - (5*e^3*Sqr
t[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^3*d) + (5*e^(7/2)*ArcSinh[Sq
rt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]
)/(a^3*d*(1 + Cos[c + d*x] + Sin[c + d*x])) - (5*e^(7/2)*ArcTan[(Sqrt[e]*Si
n[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c +
 d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^3*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
```

```
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^3d} - \frac{\left(5e^4\right)\int \frac{\sqrt{a+a\sin}}{\sqrt{e\cos}}}{2a^3}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^3d} - \frac{\left(5e^4\sqrt{1+\cos\right.}}{2a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^3d} + \frac{\left(5e^4\sqrt{1+\cos\right.}}{}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^3d} - \frac{5e^{7/2}\tan^{-1}\left(\right.}{}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^3d} + \frac{5e^{7/2}\sinh^{-1}\left(\right.}{d\left(\right.}$$

Mathematica [C] time = 0.12, size = 80, normalized size = 0.33

$$-\frac{2^{3/4}\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9a^3de(\sin(c+dx)+1)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/9*(2^(3/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[5/4, 9/4, 13/4, (1 -
 Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(a^3*d*e*(1 + Sin[c + d*x])^(
11/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.23, size = 443, normalized size = 1.85

$$\left(5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)-5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctanh\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+}{2\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/4/d*(5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-5*2^(1/2)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+5*cos(d*x+c)*2^(1/2)*arctan(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-
5*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+5*2^(1/2)*ar
ctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)-5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+4*cos(
d*x+c)*sin(d*x+c)+36*cos(d*x+c))*(e*cos(d*x+c))^(7/2)/(cos(d*x+c)^2+2*sin(d
*x+c)-2)/(a*(1+sin(d*x+c)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.315 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{2e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}-\frac{2e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+}{d\left(a^3\sin(c+dx)+a^3\right.}$$

```
[Out] -4/3*e*(e*cos(d*x+c))^(3/2)/a/d/(a+a*sin(d*x+c))^(3/2)-2*e^(5/2)*arcsinh((e
*cos(d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(
a^3+a^3*cos(d*x+c)+a^3*sin(d*x+c))-2*e^(5/2)*arctan(sin(d*x+c)*e^(1/2)/(e*c
os(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c)
)^(1/2)/d/(a^3+a^3*cos(d*x+c)+a^3*sin(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2680, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{2e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}-\frac{2e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+}{d\left(a^3\sin(c+dx)+a^3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*e*(e*Cos[c + d*x])^(3/2))/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (2*e^(5/
2)*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*
Sin[c + d*x]])/(d*(a^3 + a^3*Cos[c + d*x] + a^3*Sin[c + d*x])) - (2*e^(5/2)
*ArcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]
)]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a^3 + a^3*Cos[c + d
*x] + a^3*Sin[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{\left(e^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+d}}{\sqrt{e\cos(c+d}}}{a^2(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{\left(e^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{}{\sqrt{e}}\right.}{a^2d(a+a\cos(c+dx)+a\sin(c+}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}}{d\left(a^3+a^3\cos(c+dx)+a^3\sin(c+d\right.}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin}}{d\left(a^3+a^3\cos(c+dx)+a^3\sin(c+dx)\right)}$$

Mathematica [C] time = 0.14, size = 80, normalized size = 0.37

$$-\frac{\sqrt[4]{2}\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7a^3de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/7*(2^(1/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, 7/4, 11/4, (1 -
 Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(a^3*d*e*(1 + Sin[c + d*x])^(
9/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.22, size = 545, normalized size = 2.50

$$-\frac{\left(3\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sqrt{2}\,\sin(dx+c)\cos(dx+c)+3\left(\cos^2(dx+c)\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d*(3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*
sin(d*x+c)*cos(d*x+c)+3*cos(d*x+c)^2*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*2^(1/2))+3*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)+3*cos(d*x+c
)^2*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))-6*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
2^(1/2))*sin(d*x+c)+3*cos(d*x+c)*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*2^(1/2))-6*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+3*cos(d*x+c)*2^(1/2)*arcta
nh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-
4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)-6*2^(1/2)*arct
an(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-6*2^(1/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)))*(e*cos
(d*x+c))^(5/2)/(sin(d*x+c)-1)/(a*(1+sin(d*x+c)))^(5/2)/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(5/2),x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.316 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{2(e\cos(c+dx))^{5/2}}{5de(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] -2/5*(e*cos(d*x+c))^(5/2)/d/e/(a+a*sin(d*x+c))^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2671}

$$-\frac{2(e\cos(c+dx))^{5/2}}{5de(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(5/2))/(5*d*e*(a + a*Sin[c + d*x])^(5/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2(e\cos(c+dx))^{5/2}}{5de(a+a\sin(c+dx))^{5/2}}$$

Mathematica [A] time = 0.12, size = 49, normalized size = 1.36

$$-\frac{2\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{5/2}}{5a^3de(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(5/2)*Sqrt[a*(1 + Sin[c + d*x])])/(5*a^3*d*e*(1 + Sin[
c + d*x])^3)
```

fricas [B] time = 0.67, size = 70, normalized size = 1.94

$$-\frac{2\sqrt{e\cos(dx+c)}\,\sqrt{a\sin(dx+c)+a}\,(e\sin(dx+c)-e)}{5\left(a^3d\cos(dx+c)^2-2\,a^3d\sin(dx+c)-2\,a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/5*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*(e*sin(d*x + c) - e)/(a^
3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [**A**] time = 0.19, size = 34, normalized size = 0.94

$$-\frac{2\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)}{5d\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x)
[Out] -2/5/d*(e*cos(d*x+c))^(3/2)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(5/2)
```

maxima [**B**] time = 0.96, size = 131, normalized size = 3.64

$$-\frac{2\left(\sqrt{a}e^{\frac{3}{2}}-\frac{\sqrt{a}e^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{5\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
[Out] -2/5*(sqrt(a)*e^(3/2) - sqrt(a)*e^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 1)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d*(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

mupad [**B**] time = 6.57, size = 102, normalized size = 2.83

$$-\frac{4\,e\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(\sin\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+2\right)}{5\,a^3\,d\,\left(56\,\sin\left(c+d\,x\right)-28\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)-8\,\sin\left(3\,c+3\,d\,x\right)+35\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^(5/2),x)
[Out] -(4*e*(e*cos(c + d*x))^(1/2)*(a*(sin(c + d*x) + 1))^(1/2)*(sin(c + d*x) + 2
*cos(2*c + 2*d*x) + sin(3*c + 3*d*x) + 2))/(5*a^3*d*(56*sin(c + d*x) - 28*c
os(2*c + 2*d*x) + cos(4*c + 4*d*x) - 8*sin(3*c + 3*d*x) + 35))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.317 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{4(e\cos(c+dx))^{3/2}}{21ade(a\sin(c+dx)+a)^{3/2}}-\frac{2(e\cos(c+dx))^{3/2}}{7de(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] -2/7*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^(5/2)-4/21*(e*cos(d*x+c))^(3
/2)/a/d/e/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{4(e\cos(c+dx))^{3/2}}{21ade(a\sin(c+dx)+a)^{3/2}}-\frac{2(e\cos(c+dx))^{3/2}}{7de(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2))/(7*d*e*(a + a*Sin[c + d*x])^(5/2)) - (4*(e*Cos[
c + d*x])^(3/2))/(21*a*d*e*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{5/2}}\,dx&=-\frac{2(e\cos(c+dx))^{3/2}}{7de(a+a\sin(c+dx))^{5/2}}+\frac{2\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{3/2}}\,dx}{7a}\\&=-\frac{2(e\cos(c+dx))^{3/2}}{7de(a+a\sin(c+dx))^{5/2}}-\frac{4(e\cos(c+dx))^{3/2}}{21ade(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 59, normalized size = 0.78

$$-\frac{2(2\sin(c+dx)+5)\sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{3/2}}{21a^3de(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(5/2),x]
```

[Out] (-2*(e*Cos[c + d*x])^(3/2)*Sqrt[a*(1 + Sin[c + d*x])]*(5 + 2*Sin[c + d*x]))
/(21*a^3*d*e*(1 + Sin[c + d*x])^3)

fricas [B] time = 0.63, size = 148, normalized size = 1.95

$$\frac{2\sqrt{e\cos(dx+c)}\left(2\cos(dx+c)^2+(2\cos(dx+c)-3)\sin(dx+c)+5\cos(dx+c)+3\right)\sqrt{a\sin(dx-}}{21\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-4a^3d+\left(a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)

[Out] 2/21*sqrt(e*cos(d*x + c))*(2*cos(d*x + c)^2 + (2*cos(d*x + c) - 3)*sin(d*x
+ c) + 5*cos(d*x + c) + 3)*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x +
c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(dx+c)}}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^(5/2), x)

maple [A] time = 0.20, size = 44, normalized size = 0.58

$$-\frac{2(2\sin(dx+c)+5)\cos(dx+c)\sqrt{e\cos(dx+c)}}{21d\,(a(1+\sin(dx+c)))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x)

[Out] -2/21/d*(2*sin(d*x+c)+5)*cos(d*x+c)*(e*cos(d*x+c))^(1/2)/(a*(1+sin(d*x+c)))
^(5/2)

maxima [B] time = 0.70, size = 207, normalized size = 2.72

$$-\frac{2\left(5\sqrt{a}\sqrt{e}+\frac{4\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{21\left(a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)

[Out] -2/21*(5*sqrt(a)*sqrt(e) + 4*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1
) - 4*sqrt(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sqrt(a)*sqrt(
e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*sqrt(-sin(d*x + c)/(cos(d*x + c) +
1) + 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((a^3 + 2*a^3*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d*(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2))

mupad [B] time = 7.05, size = 145, normalized size = 1.91

$$-\frac{8\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\sqrt{-e\,\left(2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}\,\left(-58\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+18\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2+26\,\sin\left(2\,c+\right.\right.}{21\,a^3\,d\,\left(240\sin\left(c+d\,x\right)^2+210\,\sin\left(c+d\,x\right)-20\sin\left(2\,c+2\,d\,x\right)^2-45\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] -(8*(a*(sin(c + d*x) + 1))^(1/2)*(-e*(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(2
6*sin(2*c + 2*d*x) - sin(4*c + 4*d*x) - 58*sin(c/2 + (d*x)/2)^2 + 18*sin((3
*c)/2 + (3*d*x)/2)^2 + 20))/(21*a^3*d*(210*sin(c + d*x) - 45*sin(3*c + 3*d*
x) + sin(5*c + 5*d*x) - 20*sin(2*c + 2*d*x)^2 + 240*sin(c + d*x)^2 + 16))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(c+dx\right)}}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.318 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{16\sqrt{e\cos(c+dx)}}{45a^2de\sqrt{a\sin(c+dx)+a}}-\frac{8\sqrt{e\cos(c+dx)}}{45ade(a\sin(c+dx)+a)^{3/2}}-\frac{2\sqrt{e\cos(c+dx)}}{9de(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] -2/9*(e*cos(d*x+c))^(1/2)/d/e/(a+a*sin(d*x+c))^(5/2)-8/45*(e*cos(d*x+c))^(1
/2)/a/d/e/(a+a*sin(d*x+c))^(3/2)-16/45*(e*cos(d*x+c))^(1/2)/a^2/d/e/(a+a*si
n(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{16\sqrt{e\cos(c+dx)}}{45a^2de\sqrt{a\sin(c+dx)+a}}-\frac{8\sqrt{e\cos(c+dx)}}{45ade(a\sin(c+dx)+a)^{3/2}}-\frac{2\sqrt{e\cos(c+dx)}}{9de(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(9*d*e*(a + a*Sin[c + d*x])^(5/2)) - (8*Sqrt[e*Co
s[c + d*x]])/(45*a*d*e*(a + a*Sin[c + d*x])^(3/2)) - (16*Sqrt[e*Cos[c + d*x
]])/(45*a^2*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}}\,dx &= -\frac{2\sqrt{e\cos(c+dx)}}{9de(a+a\sin(c+dx))^{5/2}}+\frac{4\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}\,dx}{9a}\\ &= -\frac{2\sqrt{e\cos(c+dx)}}{9de(a+a\sin(c+dx))^{5/2}}-\frac{8\sqrt{e\cos(c+dx)}}{45ade(a+a\sin(c+dx))^{3/2}}+\frac{8\int \frac{1}{\sqrt{e\cos}}\\ &= -\frac{2\sqrt{e\cos(c+dx)}}{9de(a+a\sin(c+dx))^{5/2}}-\frac{8\sqrt{e\cos(c+dx)}}{45ade(a+a\sin(c+dx))^{3/2}}-\frac{16\sqrt{}}{45a^2de\sqrt{}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 69, normalized size = 0.60

$$-\frac{2\left(8\sin^2(c+dx)+20\sin(c+dx)+17\right)\sqrt{a(\sin(c+dx)+1)}\,\sqrt{e\cos(c+dx)}}{45a^3de(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*Sqrt[a*(1 + Sin[c + d*x])]*(17 + 20*Sin[c + d*x] +
 8*Sin[c + d*x]^2))/(45*a^3*d*e*(1 + Sin[c + d*x])^3)
```

fricas [A] time = 0.88, size = 98, normalized size = 0.85

$$-\frac{2\sqrt{e\cos\left(dx+c\right)}\left(8\cos\left(dx+c\right)^{2}-20\sin\left(dx+c\right)-25\right)\sqrt{a\sin\left(dx+c\right)+a}}{45\left(3\,a^{3}de\cos\left(dx+c\right)^{2}-4\,a^{3}de+\left(a^{3}de\cos\left(dx+c\right)^{2}-4\,a^{3}de\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/45*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 - 20*sin(d*x + c) - 25)*sqrt(a
*sin(d*x + c) + a)/(3*a^3*d*e*cos(d*x + c)^2 - 4*a^3*d*e + (a^3*d*e*cos(d*x
 + c)^2 - 4*a^3*d*e)*sin(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(5/2)), x)
```

maple [A] time = 0.19, size = 54, normalized size = 0.47

$$-\frac{2\left(-8\left(\cos^{2}\left(dx+c\right)\right)+20\sin\left(dx+c\right)+25\right)\cos\left(dx+c\right)}{45d\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{5}{2}}\sqrt{e\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/45/d*(-8*cos(d*x+c)^2+20*sin(d*x+c)+25)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(5
/2)/(e*cos(d*x+c))^(1/2)
```

maxima [B] time = 0.53, size = 287, normalized size = 2.50

$$-\frac{2\left(17\sqrt{a}\sqrt{e}+\frac{40\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{49\sqrt{a}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{49\sqrt{a}\sqrt{e}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{40\sqrt{a}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{17\sqrt{a}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)(}{45\left(a^{3}e+\frac{3\,a^{3}e\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,a^{3}e\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a^{3}e\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/45*(17*sqrt(a)*sqrt(e) + 40*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
 1) + 49*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 49*sqrt(a)*s
qrt(e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 40*sqrt(a)*sqrt(e)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 17*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c)
```

```
+ 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((a^3*e + 3*a^3*e*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*e*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + a^3*e*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)^(11/2)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + 1))
```

mupad [B] time = 7.66, size = 137, normalized size = 1.19

$$-\frac{8\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(137\,\cos\left(c+d\,x\right)-71\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(5\,c+5\,d\,x\right)+144\,\sin\left(2\,c+2\,d\,x\right)-}{45\,a^3\,d\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(210\,\sin\left(c+d\,x\right)-120\,\cos\left(2\,c+2\,d\,x\right)+10\,\cos\left(4\,c+4\,d\,x\right)-45\,\sin\left(3\,c+3\,d\,x\right)+s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] -(8*(a*(sin(c + d*x) + 1))^(1/2)*(137*cos(c + d*x) - 71*cos(3*c + 3*d*x) +
2*cos(5*c + 5*d*x) + 144*sin(2*c + 2*d*x) - 18*sin(4*c + 4*d*x)))/(45*a^3*d
*(e*cos(c + d*x))^(1/2)*(210*sin(c + d*x) - 120*cos(2*c + 2*d*x) + 10*cos(4
*c + 4*d*x) - 45*sin(3*c + 3*d*x) + sin(5*c + 5*d*x) + 126))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.319 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{32\sqrt{a\sin(c+dx)+a}}{77a^3de\sqrt{e\cos(c+dx)}}-\frac{16}{77a^2de\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}-\frac{12}{77ade(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}$$

```
[Out] -2/11/d/e/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2)-12/77/a/d/e/(a+a*sin(
d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2)-16/77/a^2/d/e/(e*cos(d*x+c))^(1/2)/(a+a*
sin(d*x+c))^(1/2)+32/77*(a+a*sin(d*x+c))^(1/2)/a^3/d/e/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{32\sqrt{a\sin(c+dx)+a}}{77a^3de\sqrt{e\cos(c+dx)}}-\frac{16}{77a^2de\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}-\frac{12}{77ade(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -2/(11*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2)) - 12/(77*a*d*e*
Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)) - 16/(77*a^2*d*e*Sqrt[e*Co
s[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (32*Sqrt[a + a*Sin[c + d*x]])/(77*a
^3*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2}{11de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}}+\frac{6\int \frac{1}{(e\cos(c+dx))^{3/2}(a}}{11}$$

$$= -\frac{2}{11de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}}-\frac{}{77ade\sqrt{e\cos(c+}}$$

$$= -\frac{2}{11de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}}-\frac{}{77ade\sqrt{e\cos(c+}}$$

$$= -\frac{2}{11de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}}-\frac{}{77ade\sqrt{e\cos(c+}}$$

Mathematica [A] time = 0.15, size = 66, normalized size = 0.43

$$\frac{32\sin^3(c+dx)+80\sin^2(c+dx)+52\sin(c+dx)-10}{77de(a(\sin(c+dx)+1))^{5/2}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-10 + 52*Sin[c + d*x] + 80*Sin[c + d*x]^2 + 32*Sin[c + d*x]^3)/(77*d*e*Sqr
t[e*Cos[c + d*x]]*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [A] time = 0.54, size = 130, normalized size = 0.84

$$\frac{2\sqrt{e\cos(dx+c)}\left(40\cos(dx+c)^2+2\left(8\cos(dx+c)^2-21\right)\sin(dx+c)-35\right)\sqrt{a\sin(dx+c)+a}}{77\left(3a^3de^2\cos(dx+c)^3-4a^3de^2\cos(dx+c)+\left(a^3de^2\cos(dx+c)^3-4a^3de^2\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/77*sqrt(e*cos(d*x + c))*(40*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^2 - 21)*si
n(d*x + c) - 35)*sqrt(a*sin(d*x + c) + a)/(3*a^3*d*e^2*cos(d*x + c)^3 - 4*a
^3*d*e^2*cos(d*x + c) + (a^3*d*e^2*cos(d*x + c)^3 - 4*a^3*d*e^2*cos(d*x + c
))*sin(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(5/2)), x)
```

maple [A] time = 0.19, size = 70, normalized size = 0.45

$$-\frac{2\left(16\left(\cos^2(dx+c)\right)\sin(dx+c)+40\left(\cos^2(dx+c)\right)-42\sin(dx+c)-35\right)\cos(dx+c)}{77d(e\cos(dx+c))^{\frac{3}{2}}(a(1+\sin(dx+c)))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/77/d*(16*cos(d*x+c)^2*sin(d*x+c)+40*cos(d*x+c)^2-42*sin(d*x+c)-35)*cos(d
*x+c)/(e*cos(d*x+c))^(3/2)/(a*(1+sin(d*x+c)))^(5/2)
```

maxima [B] time = 0.54, size = 373, normalized size = 2.42

$$-\frac{2\left(5\sqrt{a}\sqrt{e}-\frac{52\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{150\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{180\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{180\sqrt{a}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{150\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\cdots\right.}{77\left(a^3e^2+\frac{4a^3e^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3e^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^3e^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3e^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/77*(5*sqrt(a)*sqrt(e) - 52*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
1) - 150*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 180*sqrt(a)*
sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 180*sqrt(a)*sqrt(e)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 + 150*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 52*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5*sqrt
(a)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^4/((a^3*e^2 + 4*a^3*e^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 6*a^3*e^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*e^2*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + a^3*e^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(3/2))
```

mupad [B] time = 11.17, size = 261, normalized size = 1.69

$$\frac{76\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+30\,\sqrt{a+a\,\sin\left(c+d\,x\right)}-40\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}-}{\frac{385\,a^3\,d\,e\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}{2}+\frac{1155\,a^3\,d\,e\,\sin\left(c+d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}{4}-\frac{231\,a^3\,d\,e\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1\mathrm{i}+d\,x}}{2}}}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] (76*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2) + 30*(a + a*sin(c + d*x))^(1/2)
 - 40*cos(2*c + 2*d*x)*(a + a*sin(c + d*x))^(1/2) - 8*sin(3*c + 3*d*x)*(a +
 a*sin(c + d*x))^(1/2))/((385*a^3*d*e*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(
c*1i + d*x*1i))/2)^(1/2))/2 + (1155*a^3*d*e*sin(c + d*x)*((e*exp(- c*1i - d
*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 - (231*a^3*d*e*cos(2*c + 2*d
*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/2 - (77*
a^3*d*e*sin(3*c + 3*d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i
))/2)^(1/2))/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.320 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=193

$$\frac{256(a\sin(c+dx)+a)^{3/2}}{585a^4de(e\cos(c+dx))^{3/2}}-\frac{128\sqrt{a\sin(c+dx)+a}}{195a^3de(e\cos(c+dx))^{3/2}}-\frac{32}{195a^2de\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}-\frac{}{117ade(a\sin}$$

```
[Out] -2/13/d/e/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2)-16/117/a/d/e/(e*cos(d
*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2)+256/585*(a+a*sin(d*x+c))^(3/2)/a^4/d/e/
(e*cos(d*x+c))^(3/2)-32/195/a^2/d/e/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(
1/2)-128/195*(a+a*sin(d*x+c))^(1/2)/a^3/d/e/(e*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.38, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{256(a\sin(c+dx)+a)^{3/2}}{585a^4de(e\cos(c+dx))^{3/2}}-\frac{128\sqrt{a\sin(c+dx)+a}}{195a^3de(e\cos(c+dx))^{3/2}}-\frac{32}{195a^2de\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}-\frac{}{117ade(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -2/(13*d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2)) - 16/(117*a*d
*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)) - 32/(195*a^2*d*e*(e*
Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (128*Sqrt[a + a*Sin[c + d*x
]])/(195*a^3*d*e*(e*Cos[c + d*x])^(3/2)) + (256*(a + a*Sin[c + d*x])^(3/2))
/(585*a^4*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} + \frac{8\int \frac{}{(e\cos(c+dx))^{5/}}}{1}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{}{117ade(e\cos(c}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{}{117ade(e\cos(c}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{}{117ade(e\cos(c}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{}{117ade(e\cos(c}$$

Mathematica [A] time = 0.28, size = 76, normalized size = 0.39

$$-\frac{2(-40\sin(c+dx)+80\sin(3(c+dx))+136\cos(2(c+dx))-16\cos(4(c+dx))+77)}{585de(a(\sin(c+dx)+1))^{5/2}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(77 + 136*Cos[2*(c + d*x)] - 16*Cos[4*(c + d*x)] - 40*Sin[c + d*x] + 80
*Sin[3*(c + d*x)]))/(585*d*e*(e*Cos[c + d*x])^(3/2)*(a*(1 + Sin[c + d*x]))^
(5/2))
```

fricas [A] time = 0.47, size = 144, normalized size = 0.75

$$-\frac{2\left(128\cos(dx+c)^4-400\cos(dx+c)^2-40\left(8\cos(dx+c)^2-3\right)\sin(dx+c)+75\right)\sqrt{e\cos(dx+c)}\sqrt{a\sin(}}{585\left(3a^3de^3\cos(dx+c)^4-4a^3de^3\cos(dx+c)^2+\left(a^3de^3\cos(dx+c)^4-4a^3de^3\cos(dx+c)^2\right)\sin(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] -2/585*(128*cos(d*x + c)^4 - 400*cos(d*x + c)^2 - 40*(8*cos(d*x + c)^2 - 3)
*sin(d*x + c) + 75)*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(3*a^3*d*
e^3*cos(d*x + c)^4 - 4*a^3*d*e^3*cos(d*x + c)^2 + (a^3*d*e^3*cos(d*x + c)^4
 - 4*a^3*d*e^3*cos(d*x + c)^2)*sin(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^(5/2)), x)
```

maple [A] time = 0.20, size = 80, normalized size = 0.41

$$-\frac{2\left(-128\left(\cos^4(dx+c)\right)+320\left(\cos^2(dx+c)\right)\sin(dx+c)+400\left(\cos^2(dx+c)\right)-120\sin(dx+c)-75\right)\cos(}{585d\,(e\cos(dx+c))^{\frac{5}{2}}\,(a\,(1+\sin(dx+c)))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/585/d*(-128*cos(d*x+c)^4+320*cos(d*x+c)^2*sin(d*x+c)+400*cos(d*x+c)^2-12
0*sin(d*x+c)-75)*cos(d*x+c)/(e*cos(d*x+c))^(5/2)/(a*(1+sin(d*x+c)))^(5/2)
```

maxima [B] time = 0.79, size = 451, normalized size = 2.34

$$-\frac{2\left(197\sqrt{a}\sqrt{e}+\frac{400\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{15\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1760\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2230\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2230\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\cdots\right.}{585\left(a^3e^3+\frac{5a^3e^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3e^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^3e^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^3e^3\sin\cdots}{(\cos(dx\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/585*(197*sqrt(a)*sqrt(e) + 400*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c
) + 1) + 15*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1760*sqrt
(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2230*sqrt(a)*sqrt(e)*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 + 2230*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1760*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
- 15*sqrt(a)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 400*sqrt(a)*sqrt
(e)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 197*sqrt(a)*sqrt(e)*sin(d*x + c)^
10/(cos(d*x + c) + 1)^10)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((a^3
*e^3 + 5*a^3*e^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*e^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*e^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + 5*a^3*e^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*e^3*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

mupad [B] time = 11.54, size = 379, normalized size = 1.96

$$-\frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}\,\left(\frac{{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,2464{}\mathrm{i}}{585\,a^3\,d\,e^2}+\frac{{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\cos\left(2\,c+2\,d\,x\right)\,4352{}\mathrm{i}}{585\,a^3\,d\,e^2}-\cdots\right.}{\cos\left(c+d\,x\right)\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\sqrt{e\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,28{}\mathrm{i}-{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\cos\left(3\,c+3\,d\,x\right)\,\sqrt{e\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}\,12{}\mathrm{i}-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] -((a + a*sin(c + d*x))^(1/2)*((exp(c*4i + d*x*4i)*2464i)/(585*a^3*d*e^2) +
(exp(c*4i + d*x*4i)*cos(2*c + 2*d*x)*4352i)/(585*a^3*d*e^2) - (exp(c*4i + d
*x*4i)*cos(4*c + 4*d*x)*512i)/(585*a^3*d*e^2) + (exp(c*4i + d*x*4i)*sin(3*c
 + 3*d*x)*512i)/(117*a^3*d*e^2) - (exp(c*4i + d*x*4i)*sin(c + d*x)*256i)/(1
17*a^3*d*e^2)))/(cos(c + d*x)*exp(c*4i + d*x*4i)*(e*(exp(- c*1i - d*x*1i)/2
 + exp(c*1i + d*x*1i)/2))^(1/2)*28i - exp(c*4i + d*x*4i)*cos(3*c + 3*d*x)*(
e*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*12i + exp(c*4i + d
*x*4i)*sin(2*c + 2*d*x)*(e*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))
^(1/2)*28i - exp(c*4i + d*x*4i)*sin(4*c + 4*d*x)*(e*(exp(- c*1i - d*x*1i)/2
 + exp(c*1i + d*x*1i)/2))^(1/2)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.321 $\int \frac{(e\cos(c+dx))^{7/3}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3\sqrt[6]{2}\,a(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[6]{\sin(c+dx)+1}\,(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -3/5*2^(1/6)*a*(e*cos(d*x+c))^(10/3)*hypergeom([-1/6, 5/3],[8/3],1/2-1/2*si
n(d*x+c))/d/e/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2689, 70, 69}

$$-\frac{3\sqrt[6]{2}\,a(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[6]{\sin(c+dx)+1}\,(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*2^(1/6)*a*(e*Cos[c + d*x])^(10/3)*Hypergeometric2F1[-1/6, 5/3, 8/3, (1
- Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(3
/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/3}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{10/3}\right)\text{Subst}\left(\int (a-ax)^{2/3}\sqrt[6]{a+ax}\,dx,x,\sin(c+dx)\right)}{de(a-a\sin(c+dx))^{5/3}(a+a\sin(c+dx))^{5/3}}$$
$$= \frac{\left(\sqrt[6]{2}\,a^2(e\cos(c+dx))^{10/3}\right)\text{Subst}\left(\int \sqrt[6]{\frac{1}{2}+\frac{x}{2}}\,(a-ax)^{2/3}\,dx,x,\sin(c+dx)\right)}{de(a-a\sin(c+dx))^{5/3}(a+a\sin(c+dx))^{3/2}\sqrt[6]{\frac{a+a\sin(c+dx)}{a}}}$$
$$= -\frac{3\sqrt[6]{2}\,a(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[6]{1+\sin(c+dx)}\,(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.16, size = 77, normalized size = 0.99

$$-\frac{3\sqrt[6]{2}\,(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{7/6}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*2^(1/6)*(e*Cos[c + d*x])^(10/3)*Hypergeometric2F1[-1/6, 5/3, 8/3, (1 -
Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(7/6)*Sqrt[a*(1 + Sin[c + d*x])
])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\cos(dx+c))^{\frac{1}{3}}\,e^2\cos(dx+c)^2}{\sqrt{a\sin(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*cos(d*x + c))^(1/3)*e^2*cos(d*x + c)^2/sqrt(a*sin(d*x + c) + a)
, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{3}}}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(7/3)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{7/3}}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/3)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(7/3)/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.322 $\int \frac{(e\cos(c+dx))^{5/3}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3a\sqrt[6]{\sin(c+dx)+1}\,(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{4\sqrt[6]{2}\,de(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -3/8*a*(e*cos(d*x+c))^(8/3)*hypergeom([1/6, 4/3],[7/3],1/2-1/2*sin(d*x+c))*
(1+sin(d*x+c))^(1/6)*2^(5/6)/d/e/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$-\frac{3a\sqrt[6]{\sin(c+dx)+1}\,(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{4\sqrt[6]{2}\,de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*a*(e*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/6, 4/3, 7/3, (1 - Sin[c +
d*x])/2]*(1 + Sin[c + d*x])^(1/6))/(4*2^(1/6)*d*e*(a + a*Sin[c + d*x])^(3/2
))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/3}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{8/3}\right)\text{Subst}\left(\int \frac{\sqrt[3]{a-ax}}{\sqrt[6]{a+ax}}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{4/3}(a+a\sin(c+dx))^{4/3}}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{8/3}\sqrt[6]{\frac{a+a\sin(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{\sqrt[3]{a-ax}}{\sqrt[6]{\frac{1}{2}+\frac{x}{2}}}\,dx, x, \sin(c+dx)\right)}{\sqrt[6]{2}\,de(a-a\sin(c+dx))^{4/3}(a+a\sin(c+dx))^{3/2}}$$

$$= -\frac{3a(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[6]{1+\sin(c+dx)}}{4\sqrt[6]{2}\,de(a+a\sin(c+dx))^{3/2}}$$

Mathematica [**A**] time = 0.11, size = 77, normalized size = 0.99

$$-\frac{3(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{4\sqrt[6]{2}\,de(\sin(c+dx)+1)^{5/6}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*(e*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/6, 4/3, 7/3, (1 - Sin[c + d*
x])/2])/(4*2^(1/6)*d*e*(1 + Sin[c + d*x])^(5/6)*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [**F**] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\cos(dx+c))^{\frac{2}{3}}\,e\cos(dx+c)}{\sqrt{a\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*cos(d*x + c))^(2/3)*e*cos(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**F**] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{3}}}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/3)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{5/3}}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/3)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(5/3)/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.323 $\int \frac{(e\cos(c+dx))^{2/3}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3\sqrt[3]{2}\,a(\sin(c+dx)+1)^{2/3}(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -3/5*2^(1/3)*a*(e*cos(d*x+c))^(5/3)*hypergeom([2/3, 5/6],[11/6],1/2-1/2*sin
(d*x+c))*(1+sin(d*x+c))^(2/3)/d/e/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2689, 70, 69}

$$-\frac{3\sqrt[3]{2}\,a(\sin(c+dx)+1)^{2/3}(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*2^(1/3)*a*(e*Cos[c + d*x])^(5/3)*Hypergeometric2F1[2/3, 5/6, 11/6, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(2/3))/(5*d*e*(a + a*Sin[c + d*x])^(3/
2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{2/3}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{a-ax}\,(a+ax)^{2/3}}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{5/6}(a+a\sin(c+dx))^{5/6}}$$
$$= \frac{\left(a^2(e\cos(c+dx))^{5/3}\left(\frac{a+a\sin(c+dx)}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{2/3}\sqrt[6]{a-ax}}\,dx, x, \sin(c+dx)\right)}{2^{2/3}de(a-a\sin(c+dx))^{5/6}(a+a\sin(c+dx))^{3/2}}$$
$$= -\frac{3\sqrt[3]{2}\,a(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{2/3}}{5de(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.99

$$-\frac{3\sqrt[3]{2}\,(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[3]{\sin(c+dx)+1}\,\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(2/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*2^(1/3)*(e*Cos[c + d*x])^(5/3)*Hypergeometric2F1[2/3, 5/6, 11/6, (1 - S
in[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(1/3)*Sqrt[a*(1 + Sin[c + d*x])]
)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\cos(dx+c))^{\frac{2}{3}}}{\sqrt{a\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*cos(d*x + c))^(2/3)/sqrt(a*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{2}{3}}}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{2}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(2/3)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{2/3}}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(2/3)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(2/3)/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^{\frac{2}{3}}}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**(2/3)/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.324 $\int \frac{\sqrt[3]{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3a(\sin(c+dx)+1)^{5/6}(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\,2^{5/6}de(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -3/4*a*(e*cos(d*x+c))^(4/3)*hypergeom([2/3, 5/6],[5/3],1/2-1/2*sin(d*x+c))*
(1+sin(d*x+c))^(5/6)*2^(1/6)/d/e/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{2689, 70, 69\}$

$$-\frac{3a(\sin(c+dx)+1)^{5/6}(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\,2^{5/6}de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*a*(e*Cos[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 5/6, 5/3, (1 - Sin[c +
d*x])/2]*(1 + Sin[c + d*x])^(5/6))/(2*2^(5/6)*d*e*(a + a*Sin[c + d*x])^(3/2
))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[3]{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a-ax}\,(a+ax)^{5/6}}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{2/3}(a+a\sin(c+dx))^{2/3}}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{4/3}\left(\frac{a+a\sin(c+dx)}{a}\right)^{5/6}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{5/6}\sqrt[3]{a-ax}}\,dx, x, \sin(c+dx)\right)}{2^{5/6}de(a-a\sin(c+dx))^{2/3}(a+a\sin(c+dx))^{3/2}}$$

$$= -\frac{3a(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{5/6}}{2\,2^{5/6}de(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.99

$$-\frac{3(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\,2^{5/6}de\sqrt[6]{\sin(c+dx)+1}\,\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(1/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*(e*Cos[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 5/6, 5/3, (1 - Sin[c + d*
x])/2])/(2*2^(5/6)*d*e*(1 + Sin[c + d*x])^(1/6)*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\cos(dx+c))^{\frac{1}{3}}}{\sqrt{a\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*cos(d*x + c))^(1/3)/sqrt(a*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{1}{3}}}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{1}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(1/3)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{1/3}}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/3)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(1/3)/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{e\cos(c+dx)}}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**(1/3)/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.325 $\int \frac{1}{\sqrt[3]{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{3\sqrt[6]{\sin(c+dx)+1}\,(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\sqrt[6]{2}\,de\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -3/4*(e*cos(d*x+c))^(2/3)*hypergeom([1/3, 7/6],[4/3],1/2-1/2*sin(d*x+c))*(1
+sin(d*x+c))^(1/6)*2^(5/6)/d/e/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$-\frac{3\sqrt[6]{\sin(c+dx)+1}\,(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\sqrt[6]{2}\,de\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(1/3)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (-3*(e*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - Sin[c + d*
x])/2]*(1 + Sin[c + d*x])^(1/6))/(2*2^(1/6)*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{(a-ax)^{2/3}(a+ax)^{7/6}}\,dx, x, \sin(c+dx)\right)}{de\sqrt[3]{a-a\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$

$$= \frac{\left(a(e\cos(c+dx))^{2/3}\sqrt[6]{\frac{a+a\sin(c+dx)}{a}}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{7/6}(a-ax)^{2/3}}\,dx, x, \sin(\right.}{2\sqrt[6]{2}\,de\sqrt[3]{a-a\sin(c+dx)}\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{3(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[6]{1+\sin(c+dx)}}{2\sqrt[6]{2}\,de\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 1.00

$$-\frac{3\sqrt[6]{\sin(c+dx)+1}\,(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\sqrt[6]{2}\,de\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(1/3)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (-3*(e*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - Sin[c + d*
x])/2]*(1 + Sin[c + d*x])^(1/6))/(2*2^(1/6)*d*e*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(e\cos(dx+c))^{\frac{2}{3}}\sqrt{a\sin(dx+c)+a}}{ae\cos(dx+c)\sin(dx+c)+ae\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((e*cos(d*x + c))^(2/3)*sqrt(a*sin(d*x + c) + a)/(a*e*cos(d*x + c)*
sin(d*x + c) + a*e*cos(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{1}{3}}\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{1}{3}}\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(1/3)*sqrt(a*sin(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(e\,\cos(c+d\,x))^{1/3}\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/3)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/3)*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sin(c+dx)+1)}\,\sqrt[3]{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(1/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(1/3)), x)
```

3.326 $\int \frac{1}{(e\cos(c+dx))^{4/3}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{3(\sin(c+dx)+1)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{2/3}de\sqrt{a\sin(c+dx)+a}\,\sqrt[3]{e\cos(c+dx)}}$$

```
[Out] 3/2*hypergeom([-1/6, 5/3],[5/6],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(2/3)*2^
(1/3)/d/e/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{2689, 70, 69\}$

$$\frac{3(\sin(c+dx)+1)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{2/3}de\sqrt{a\sin(c+dx)+a}\,\sqrt[3]{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(4/3)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(2/3))/(2^(2/3)*d*e*(e*Cos[c + d*x])^(1/3)*Sqrt[a + a*Sin[c + d*x]])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{4/3}\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2\sqrt[6]{a-a\sin(c+dx)}\,\sqrt[6]{a+a\sin(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{(a-ax)^{7/6}(a+ax}\right.}{de\sqrt[3]{e\cos(c+dx)}}$$
$$= \frac{\left(a\sqrt[6]{a-a\sin(c+dx)}\left(\frac{a+a\sin(c+dx)}{a}\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{5/3}(a-ax)^{7/6}}\right.}{2\,2^{2/3}de\sqrt[3]{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}$$
$$= \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{2/3}}{2^{2/3}de\sqrt[3]{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 1.00

$$\frac{3(\sin(c+dx)+1)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{2/3}de\sqrt{a(\sin(c+dx)+1)}\sqrt[3]{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(4/3)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(2/3))/(2^(2/3)*d*e*(e*Cos[c + d*x])^(1/3)*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(e\cos(dx+c))^{\frac{2}{3}}\sqrt{a\sin(dx+c)+a}}{ae^2\cos(dx+c)^2\sin(dx+c)+ae^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((e*cos(d*x + c))^(2/3)*sqrt(a*sin(d*x + c) + a)/(a*e^2*cos(d*x + c
)^2*sin(d*x + c) + a*e^2*cos(d*x + c)^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{4}{3}}\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(4/3)*sqrt(a*sin(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(4/3)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(4/3)*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\left(e\cos\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(4/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(4/3)), x)
```

3.327 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=95

$$-\frac{a^8 2^{\frac{p}{2}+\frac{17}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-15),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

```
[Out] -2^(17/2+1/2*p)*a^8*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, -15/2-1/2*p]
,[3/2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/d/e/(1+p)
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2688, 69}

$$-\frac{a^8 2^{\frac{p}{2}+\frac{17}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-15),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^(17/2 + p/2)*a^8*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-15 - p)/
2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)
/2))/(d*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^8\,dx = \frac{\left(a^8(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right.}{de}$$

$$= -\frac{2^{\frac{17}{2}+\frac{p}{2}}a^8(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-15-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+\right.}{de(1+p)}$$

Mathematica [A] time = 0.20, size = 94, normalized size = 0.99

$$-\frac{a^8 2^{\frac{p+17}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{1}{2}(-p-15),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^((17 + p)/2)*a^8*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(-1
5 - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((
-1 - p)/2))/(d*(1 + p)))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^8 \cos(dx+c)^8 - 32\,a^8 \cos(dx+c)^6 + 160\,a^8 \cos(dx+c)^4 - 256\,a^8 \cos(dx+c)^2 + 128\,a^8 - 8\left(a^8 \cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] integral((a^8*cos(d*x + c)^8 - 32*a^8*cos(d*x + c)^6 + 160*a^8*cos(d*x + c)
^4 - 256*a^8*cos(d*x + c)^2 + 128*a^8 - 8*(a^8*cos(d*x + c)^6 - 10*a^8*cos(
d*x + c)^4 + 24*a^8*cos(d*x + c)^2 - 16*a^8)*sin(d*x + c))*(e*cos(d*x + c))
^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^8 (e\cos(dx+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^8*(e*cos(d*x + c))^p, x)
```

maple [F] time = 14.57, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p (a+a\sin(dx+c))^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^8 (e\cos(dx+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^8*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^8 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^8,x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.328 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=95

$$-\frac{a^3 2^{\frac{p}{2}+\frac{7}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-5),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

```
[Out] -2^(7/2+1/2*p)*a^3*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, -5/2-1/2*p],[
3/2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/d/e/(1+p)
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{a^3 2^{\frac{p}{2}+\frac{7}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-5),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2^(7/2 + p/2)*a^3*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-5 - p)/2,
 (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2
))/(d*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^3\,dx = \frac{\left(a^3(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)}{de}$$

$$= -\frac{2^{\frac{7}{2}+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-5-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx)\right)}{de(1+p)}$$

Mathematica [A] time = 0.11, size = 94, normalized size = 0.99

$$-\frac{a^3 2^{\frac{p+7}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{1}{2}(-p-5),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2^((7 + p)/2)*a^3*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(-5
- p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1
 - p)/2))/(d*(1 + p)))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\left(e\cos\left(dx+c\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*(e*cos(d*x + c))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^3\left(e\cos\left(dx+c\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*(e*cos(d*x + c))^p, x)
```

maple [F] time = 5.36, size = 0, normalized size = 0.00

$$\int\left(e\cos\left(dx+c\right)\right)^p\left(a+a\sin\left(dx+c\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^3\left(e\cos\left(dx+c\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.329 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=95

$$-\frac{a^2 2^{\frac{p}{2}+\frac{5}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-3),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

```
[Out] -2^(5/2+1/2*p)*a^2*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, -3/2-1/2*p],[
3/2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/d/e/(1+p)
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{a^2 2^{\frac{p}{2}+\frac{5}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-3),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^(5/2 + p/2)*a^2*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-3 - p)/2,
 (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2
))/(d*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^2\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)}{de}$$

$$= -\frac{2^{\frac{5}{2}+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-3-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx)\right)}{de(1+p)}$$

Mathematica [A] time = 0.11, size = 94, normalized size = 0.99

$$-\frac{a^2 2^{\frac{p+5}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{1}{2}(-p-3),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^((5 + p)/2)*a^2*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(-3
- p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1
 - p)/2))/(d*(1 + p)))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos\left(dx+c\right)^2-2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\left(e\cos\left(dx+c\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*(e*cos(d*x + c)
)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^2\left(e\cos\left(dx+c\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*(e*cos(d*x + c))^p, x)
```

maple [F] time = 4.30, size = 0, normalized size = 0.00

$$\int\left(e\cos\left(dx+c\right)\right)^p\left(a+a\sin\left(dx+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^2\left(e\cos\left(dx+c\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\left(e\cos\left(c+dx\right)\right)^p\,dx+\int 2\left(e\cos\left(c+dx\right)\right)^p\sin\left(c+dx\right)dx+\int\left(e\cos\left(c+dx\right)\right)^p\sin^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral((e*cos(c + d*x))**p, x) + Integral(2*(e*cos(c + d*x))**p*sin
(c + d*x), x) + Integral((e*cos(c + d*x))**p*sin(c + d*x)**2, x))
```

3.330 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=93

$$-\frac{a2^{\frac{p}{2}+\frac{3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

```
[Out] -2^(3/2+1/2*p)*a*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, -1/2-1/2*p],[3/
2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/d/e/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2688, 69}

$$-\frac{a2^{\frac{p}{2}+\frac{3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((2^(3/2 + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-1 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))
/(d*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))\,dx = \frac{\left(a(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Su}}{de}$$

$$= -\frac{2^{\frac{3}{2}+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-1-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(}{de(1+p)}$$

Mathematica [C] time = 1.40, size = 245, normalized size = 2.63

$$-\frac{ia2^{-p-1}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{p+1}(\sin(c+dx)+1)\left((p+1)e^{i(c+dx)}\left(ipe^{i(c+dx)}\,{}_2F_1\left(1,\frac{p+3}{2};\frac{3-p}{2};-e^{2i(c+dx)}\right)-2(p-\right.\right.}{d(p-1)p(p+1)\left(\sin\left(\frac{1}{2}(c+dx)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-I)*2^(-1 - p)*a*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^(1 + p)*(e*C
os[c + d*x])^p*((-I)*(-1 + p)*p*Hypergeometric2F1[1, (1 + p)/2, (1 - p)/2,
-E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(1 + p)*(-2*(-1 + p)*Hypergeometric
2F1[1, (2 + p)/2, 1 - p/2, -E^((2*I)*(c + d*x))] + I*E^(I*(c + d*x))*p*Hype
rgeometric2F1[1, (3 + p)/2, (3 - p)/2, -E^((2*I)*(c + d*x))]))*(1 + Sin[c +
 d*x]))/(d*(-1 + p)*p*(1 + p)*Cos[c + d*x]^p*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(dx+c\right)+a\right)\left(e\cos\left(dx+c\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)\left(e\cos\left(dx+c\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

maple [F] time = 1.49, size = 0, normalized size = 0.00

$$\int\left(e\cos\left(dx+c\right)\right)^{p}\left(a+a\sin\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)\left(e\cos\left(dx+c\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,\left(a+a\,\sin\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (e\cos(c+dx))^p\,dx+\int (e\cos(c+dx))^p\sin(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral((e*cos(c + d*x))**p, x) + Integral((e*cos(c + d*x))**p*sin(c +
d*x), x))
```

3.331 $\int \frac{(e\cos(c+dx))^p}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{2^{\frac{p}{2}-\frac{1}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{3-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ade(p+1)}$$

```
[Out] -2^(-1/2+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, 3/2-1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/a/d/e/(1+p)
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{2^{\frac{p}{2}-\frac{1}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{3-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ade(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((2^(-1/2 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(3 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a*d*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{a+a\sin(c+dx)}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(1-x)^{\frac{1}{2}\cdots}\right.}{ade}$$

$$= -\frac{2^{-\frac{1}{2}+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{3-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-\cdots}}{ade(1+p)}$$

Mathematica [A] time = 0.16, size = 94, normalized size = 0.99

$$-\frac{2^{\frac{p-1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{3-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((2^((-1 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(3 - p)
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p
)/2))/(a*d*(1 + p)))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\cos(dx+c))^p}{a\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p/(a*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e \cos (c+dx))^p}{\sin (c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(sin(c + d*x) + 1), x)/a
```

3.332 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{p-3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{5-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^2de(p+1)}$$

```
[Out] -2^(-3/2+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, 5/2-1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/a^2/d/e/(1+p)
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{2^{\frac{p-3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{5-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^2de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^((-3 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(5 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a^2*d*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^2}\,dx &= \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(1-x)^{\frac{1}{2}}\right.}{a^2de}\\ &= -\frac{2^{\frac{1}{2}(-3+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{5-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}}}{a^2de(1+p)}\end{aligned}$$

Mathematica [A] time = 0.17, size = 94, normalized size = 1.01

$$-\frac{2^{\frac{p-3}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{5-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^((-3 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(5 - p)
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p
)/2))/(a^2*d*(1 + p)))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(e\cos(dx+c))^p}{a^2\cos(dx+c)^2 - 2\,a^2\sin(dx+c) - 2\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(e*cos(d*x + c))^p/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^2, x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+a\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\cos(c+dx))^p}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**
2
```

3.333 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{p-5}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{7-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^3de(p+1)}$$

```
[Out] -2^(-5/2+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, 7/2-1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/a^3/d/e/(1+p)
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{2^{\frac{p-5}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{7-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^3de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2^((-5 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(7 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a^3*d*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^3}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(1-\right.}{a^3de}$$

$$= -\frac{2^{\frac{1}{2}(-5+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{7-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx}{a^3de(1+p)}$$

Mathematica [A] time = 0.16, size = 94, normalized size = 1.01

$$-\frac{2^{\frac{p-5}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{7-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^3d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2^((-5 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(7 - p)
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p
)/2))/(a^3*d*(1 + p)))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(e\cos(dx+c))^p}{3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(e*cos(d*x + c))^p/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x +
 c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^3, x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.334 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{p-15}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{17-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^8de(p+1)}$$

```
[Out] -2^(-15/2+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, 17/2-1/2*p],[3/
2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/a^8/d/e/(1+p)
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{2^{\frac{p-15}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{17-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^8de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^((-15 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(17 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))
/(a^8*d*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^8}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int (1-x)^{\frac{1}{2}}\right.}{a^8de}$$

$$= -\frac{2^{\frac{1}{2}(-15+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{17-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))}{a^8de(1+p)}$$

Mathematica [A] time = 0.21, size = 94, normalized size = 1.01

$$-\frac{2^{\frac{p-15}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{17-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^8d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^((-15 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(17 -
p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 -
 p)/2))/(a^8*d*(1 + p)))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\cos(dx+c))^p}{a^8\cos(dx+c)^8 - 32\,a^8\cos(dx+c)^6 + 160\,a^8\cos(dx+c)^4 - 256\,a^8\cos(dx+c)^2 + 128\,a^8 - 8\left(a^8\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p/(a^8*cos(d*x + c)^8 - 32*a^8*cos(d*x + c)^6 + 1
60*a^8*cos(d*x + c)^4 - 256*a^8*cos(d*x + c)^2 + 128*a^8 - 8*(a^8*cos(d*x +
 c)^6 - 10*a^8*cos(d*x + c)^4 + 24*a^8*cos(d*x + c)^2 - 16*a^8)*sin(d*x + c
)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^8, x)
```

maple [F] time = 3.31, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+a\,\sin(c+d\,x))^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^8,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.335 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^4 2^{\frac{p}{2}+4}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-6),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2^(4+1/2*p)*a^4*(e*cos(d*x+c))^(1+p)*hypergeom([-3-1/2*p, 1/2+1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))/d/e/(1+p)/((1+sin(d*x+c))^(1/2*p))/(a+a*sin(d*x+
c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2689, 70, 69}

$$-\frac{a^4 2^{\frac{p}{2}+4}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-6),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -((2^(4 + p/2)*a^4*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-6 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{7/2}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1}\right.}{de}$$

$$= \frac{\left(2^{3+\frac{p}{2}}a^5(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx\right.}{}$$

$$= -\frac{2^{4+\frac{p}{2}}a^4(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-6-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx\right.}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.22, size = 102, normalized size = 0.99

$$-\frac{a^4 2^{\frac{p}{2}+4}\cos(c+dx)(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^p\,{}_2F_1\left(-\frac{p}{2}-3,\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -((2^(4 + p/2)*a^4*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[-3 - p
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*(1 + p)*(1 + Sin[c + d*
x])^(p/2)*Sqrt[a*(1 + Sin[c + d*x])]))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)\right)\sqrt{a\sin(dx+c)+a}\,\left(e\cos(dx+c)\right)^p,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(cos
((d*x+c)/2-pi/4))]Simplification assuming c near 0Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Evaluation t
ime: 0.53Unable to divide, perhaps due to rounding error%%%{64*i,[0,2,0,2,2
,1,3,1,1]%%%} / %%%{128*i,[0,2,0,2,2,0,0,0,0]%%%} Error: Bad Argument Value
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sin(dx+c)+a)^{\frac{7}{2}} (e\cos(dx+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e \cos(c+d\,x))^p \, (a+a\sin(c+d\,x))^{7/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(7/2),x)
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

3.336 $\int (e\cos(c+dx))^p (a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^3 2^{\frac{p}{2}+3}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-4),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2^(3+1/2*p)*a^3*(e*cos(d*x+c))^(1+p)*hypergeom([-2-1/2*p, 1/2+1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))/d/e/(1+p)/((1+sin(d*x+c))^(1/2*p))/(a+a*sin(d*x+
c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = $\{2689, 70, 69\}$

$$-\frac{a^3 2^{\frac{p}{2}+3}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-4),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2^(3 + p/2)*a^3*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-4 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{5/2}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}}\right.}{}$$

$$= \frac{\left(2^{2+\frac{p}{2}}a^4(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+\right.}{}$$

$$= -\frac{2^{3+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-4-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+\right.}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.18, size = 102, normalized size = 0.99

$$-\frac{a^3 2^{\frac{p}{2}+3}\cos(c+dx)(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^p\,{}_2F_1\left(-\frac{p}{2}-2,\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2^(3 + p/2)*a^3*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[-2 - p
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*(1 + p)*(1 + Sin[c + d*
x])^(p/2)*Sqrt[a*(1 + Sin[c + d*x])]))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2\right)\sqrt{a\sin(dx+c)+a}\,\left(e\cos(dx+c)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(a*sin(d*x
+ c) + a)*(e*cos(d*x + c))^p, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(cos
((d*x+c)/2-pi/4))]Simplification assuming c near 0Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to di
vide, perhaps due to rounding error%%%{-64,[0,2,0,2,2,1,2,1,1]%%%} / %%%{12
8*i,[0,2,0,2,2,0,0,0,0]%%%} Error: Bad Argument Value
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(e\cos(dx+c)\right)^p\left(a+a\sin(dx+c)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+d\,x))^p\,(a+a\,\sin(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(5/2),x)
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.337 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^2 2^{\frac{p}{2}+2}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-2),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2^(2+1/2*p)*a^2*(e*cos(d*x+c))^(1+p)*hypergeom([-1-1/2*p, 1/2+1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))/d/e/(1+p)/((1+sin(d*x+c))^(1/2*p))/(a+a*sin(d*x+
c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2689, 70, 69}

$$-\frac{a^2 2^{\frac{p}{2}+2}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-2),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(2 + p/2)*a^2*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-2 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{3/2}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1}\right.}{de}$$

$$= \frac{\left(2^{1+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx\right.}{}$$

$$= -\frac{2^{2+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-2-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx\right.}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.19, size = 101, normalized size = 0.98

$$-\frac{2^{\frac{p}{2}+2}\cos(c+dx)(a(\sin(c+dx)+1))^{3/2}(\sin(c+dx)+1)^{-\frac{p}{2}-2}(e\cos(c+dx))^p\,{}_2F_1\left(-\frac{p}{2}-1,\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+\right.}{d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(2 + p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[-1 - p/2,
(1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-2 - p/2)*(
a*(1 + Sin[c + d*x]))^(3/2))/(d*(1 + p)))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(cos
((d*x+c)/2-pi/4))]Simplification assuming c near 0Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to di
vide, perhaps due to rounding error%%%{-64*i,[0,2,0,2,2,1,1,1,1]%%%} / %%%{
128*i,[0,2,0,2,2,2,0,0,0,0]%%%} Error: Bad Argument Value
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p(a+a\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+d\,x))^p\,(a+a\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(3/2),x)
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{3}{2}}(e\cos(c+dx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*(e*cos(c + d*x))**p, x)
```

3.338 $\int (e\cos(c+dx))^p\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{a2^{\frac{p}{2}+1}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2^(1+1/2*p)*a*(e*cos(d*x+c))^(1+p)*hypergeom([-1/2*p, 1/2+1/2*p],[3/2+1/2*
p],1/2-1/2*sin(d*x+c))/d/e/(1+p)/((1+sin(d*x+c))^(1/2*p))/(a+a*sin(d*x+c))^
(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2689, 70, 69}

$$-\frac{a2^{\frac{p}{2}+1}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((2^(1 + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[-p/2, (1 + p)/2
, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])^(p/2)*S
qrt[a + a*Sin[c + d*x]]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p\sqrt{a+a\sin(c+dx)}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(\cdots}\right.}{de\cdots}$$

$$= \frac{\left(2^{p/2}a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx\cdots\right.}{\cdots}$$

$$= -\frac{2^{1+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(-\frac{p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\cdots}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 3.89, size = 310, normalized size = 3.20

$$\frac{(1+i)2^{-p}e^{-\frac{1}{2}idx}\sqrt{a(\sin(c+dx)+1)}\,\cos^{-p}(c+dx)(e\cos(c+dx))^p\left(e^{-idx}\left(i\sin(c)\left(-1+e^{2idx}\right)+\cos(c)\left(1+e^{2idx\cdots}\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((1 + I)*(e*Cos[c + d*x])^p*(E^(I*d*x)*(1 + 2*p)*Hypergeometric2F1[(1 - 2*p
)/4, -p, (5 - 2*p)/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[c/2] + I
*Sin[c/2]) + (-1 + 2*p)*Hypergeometric2F1[(-1 - 2*p)/4, -p, (3 - 2*p)/4, -(
E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(I*Cos[c/2] + Sin[c/2]))*(((1 + E^((2
*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^p*Sqrt[a*(1 +
Sin[c + d*x])])/(2^p*d*E^((I/2)*d*x)*(-1 + 2*p)*(1 + 2*p)*Cos[c + d*x]^p*(1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c])^p*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sin\left(dx+c\right)+a}\,\left(e\cos\left(dx+c\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(cos
((d*x+c)/2-pi/4))]Simplification assuming c near 0Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to di
vide, perhaps due to rounding error%%%{64,[0,2,0,2,2,1,1,1]%%%} / %%%{128*i
,[0,2,0,2,2,0,0,0]%%%} Error: Bad Argument Value
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^p\sqrt{a+a\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin(dx + c) + a}\, (e \cos(dx + c))^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e \cos(c + d x))^{p} \sqrt{a + a \sin(c + d x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a (\sin(c + dx) + 1)}\, (e \cos(c + dx))^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**p, x)
```

3.339 $\int \frac{(e\cos(c+dx))^p}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=101

$$-\frac{a2^{p/2}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{2-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -2^(1/2*p)*a*(e*cos(d*x+c))^(1+p)*hypergeom([1-1/2*p, 1/2+1/2*p],[3/2+1/2*p
],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(1-1/2*p)/d/e/(1+p)/(a+a*sin(d*x+c))^(
3/2)
```

Rubi [A] time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2689, 70, 69}

$$-\frac{a2^{p/2}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{2-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((2^(p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(2 - p)/2, (1 + p)/
2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(d*e*(1 +
 p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(a\right.}{de}$$

$$= \frac{\left(2^{-1+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{-1+\frac{1}{2}(-1-p)+\frac{p}{2}}\right.}{de}$$

$$= -\frac{2^{p/2}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{2-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{1-\frac{p}{2}}}{de(1+p)(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.10, size = 97, normalized size = 0.96

$$-\frac{2^{p/2}\cos(c+dx)(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^p\,{}_2F_1\left(1-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/Sqrt[a + a*Sin[c + d*x]],x]
[Out] -((2^(p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[1 - p/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*(1 + p)*(1 + Sin[c + d*x])^(p/2)
*Sqrt[a*(1 + Sin[c + d*x])]))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\cos(dx+c))^p}{\sqrt{a\sin(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((e*cos(d*x + c))^p/sqrt(a*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^p/sqrt(a*sin(d*x + c) + a), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x)
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^p/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral((e*cos(c + d*x))**p/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.340 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{2^{\frac{p}{2}-1}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{4-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -2^(-1+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([2-1/2*p, 1/2+1/2*p],[3/2+1/2*
p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(1-1/2*p)/d/e/(1+p)/(a+a*sin(d*x+c))^
(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2689, 70, 69}

$$-\frac{2^{\frac{p}{2}-1}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{4-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(-1 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(4 - p)/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(d*e*(
1 + p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\Big(}{de}$$

$$= \frac{\Big(2^{-2+\frac{p}{2}}a(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{-1+\frac{1}{2}(-1-p)+}}{}$$

$$= -\frac{2^{-1+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{4-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx)}{de(1+p)(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.17, size = 101, normalized size = 0.99

$$-\frac{2^{\frac{p}{2}-1}\cos(c+dx)(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^p\,{}_2F_1\left(2-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(-1 + p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[2 - p/2,
(1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(
d*(1 + p)*(a*(1 + Sin[c + d*x]))^(3/2)))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin(dx+c)+a}\,(e\cos(dx+c))^p}{a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p/(a^2*cos(d*x + c)^2 -
 2*a^2*sin(d*x + c) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(3/2),x)
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral((e*cos(c + d*x))**p/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.341 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{2^{\frac{p}{2}-2}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{6-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ade(p+1)(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -2^(-2+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([3-1/2*p, 1/2+1/2*p],[3/2+1/2*
p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(1-1/2*p)/a/d/e/(1+p)/(a+a*sin(d*x+c)
)^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2689, 70, 69}

$$-\frac{2^{\frac{p}{2}-2}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{6-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ade(p+1)(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2^(-2 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(6 - p)/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(a*d*e
*(1 + p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(\right.}{de}$$
$$= \frac{\left(2^{-3+\frac{p}{2}}(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{-1+\frac{1}{2}(-1-p)+\frac{p}{2}}\left(\right.\right.}{de}$$
$$= -\frac{2^{-2+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{6-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{1-}}{ade(1+p)(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.14, size = 102, normalized size = 0.97

$$-\frac{2^{\frac{p}{2}-2}\cos(c+dx)(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^p\,{}_2F_1\left(3-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^2d(p+1)\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2^(-2 + p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[3 - p/2,
(1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(a^2*d*(1 + p)*(1 + Sin[c + d*
x])^(p/2)*Sqrt[a*(1 + Sin[c + d*x])]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin(dx+c)+a}\,(e\cos(dx+c))^p}{3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p/(3*a^3*cos(d*x + c)^2
 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(5/2),x)
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**(5/2),x)
[Out] Integral((e*cos(c + d*x))**p/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.342 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=114

$$-\frac{a2^{m+\frac{p}{2}+\frac{1}{2}}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{p+1}(\sin(c+dx)+1)^{\frac{1}{2}(-2m-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2m-p+1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\right.}{de(p+1)}$$

```
[Out] -2^(1/2+m+1/2*p)*a*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, 1/2-m-1/2*p],
[3/2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(1/2-m-1/2*p)*(a+a*sin(d*x+c
))^(-1+m)/d/e/(1+p)
```

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{p}{2}+\frac{1}{2}}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{p+1}(\sin(c+dx)+1)^{\frac{1}{2}(-2m-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2m-p+1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\right.}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((2^(1/2 + m + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(1 - 2*m
- p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((1
- 2*m - p)/2)*(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}}\right.}{a}$$

$$= \frac{\left(2^{-\frac{1}{2}+m+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+m+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(1-2m-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin\right.}{de(1+}$$

Mathematica [A] time = 0.20, size = 112, normalized size = 0.98

$$-\frac{2^{\frac{1}{2}(2m+p+1)}\cos(c+dx)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^p(\sin(c+dx)+1)^{\frac{1}{2}(-2m-p-1)}\,{}_2F_1\left(\frac{1}{2}(-2m-p+1),\right.}{d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((2^((1 + 2*m + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(1
 - 2*m - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^((-1 - 2*m - p)/2)*(a*(1 + Sin[c + d*x]))^m)/(d*(1 + p)))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^p\,(a\sin(dx+c)+a)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p*(a*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 1.34, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos (c+d\,x))^p\,(a+a\,\sin (c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^m,x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (c+dx)+1))^m (e\cos (c+dx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**p, x)
```

3.343 $\int \cos^7(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=109

$$-\frac{(a\sin(c+dx)+a)^{m+7}}{a^7d(m+7)}+\frac{6(a\sin(c+dx)+a)^{m+6}}{a^6d(m+6)}-\frac{12(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}+\frac{8(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}$$

```
[Out] 8*(a+a*sin(d*x+c))^(4+m)/a^4/d/(4+m)-12*(a+a*sin(d*x+c))^(5+m)/a^5/d/(5+m)+
6*(a+a*sin(d*x+c))^(6+m)/a^6/d/(6+m)-(a+a*sin(d*x+c))^(7+m)/a^7/d/(7+m)
```

Rubi [A] time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2667, 43}

$$\frac{8(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}-\frac{12(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}+\frac{6(a\sin(c+dx)+a)^{m+6}}{a^6d(m+6)}-\frac{(a\sin(c+dx)+a)^{m+7}}{a^7d(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(4 + m))/(a^4*d*(4 + m)) - (12*(a + a*Sin[c + d*x])
^(5 + m))/(a^5*d*(5 + m)) + (6*(a + a*Sin[c + d*x])^(6 + m))/(a^6*d*(6 + m)
) - (a + a*Sin[c + d*x])^(7 + m)/(a^7*d*(7 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \cos^7(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{3+m}\,dx,x,a\sin(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{3+m}-12a^2(a+x)^{4+m}+6a(a+x)^{5+m}-(a+x)^6\right.\right.}{a^7d}\\
&= \frac{8(a+a\sin(c+dx))^{4+m}}{a^4d(4+m)}-\frac{12(a+a\sin(c+dx))^{5+m}}{a^5d(5+m)}+\frac{6(a+a\sin(c+\right.}{a^6d(6+}
\end{aligned}$$

Mathematica [A] time = 0.70, size = 89, normalized size = 0.82

$$\frac{(a(\sin(c+dx)+1))^{m+4}\left(\frac{6a^3(\sin(c+dx)+1)^2}{m+6}-\frac{12a^3(\sin(c+dx)+1)}{m+5}+\frac{8a^3}{m+4}-\frac{(a\sin(c+dx)+a)^3}{m+7}\right)}{a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4 + m)*((8*a^3)/(4 + m) - (12*a^3*(1 + Sin[c + d*x
]))/(5 + m) + (6*a^3*(1 + Sin[c + d*x])^2)/(6 + m) - (a + a*Sin[c + d*x])^3
/(7 + m)))/(a^7*d)
```

fricas [A] time = 0.49, size = 153, normalized size = 1.40

$$\frac{\left(\left(m^{3}+9\,m^{2}+20\,m\right)\cos\left(dx+c\right)^{6}+12\left(m^{2}+3\,m\right)\cos\left(dx+c\right)^{4}+96\,m\cos\left(dx+c\right)^{2}+\left(\left(m^{3}+15\,m^{2}+74\,m+1\right.\right.}{dm^{4}+22\,dm^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^3 + 9*m^2 + 20*m)*cos(d*x + c)^6 + 12*(m^2 + 3*m)*cos(d*x + c)^4 + 96*m
*cos(d*x + c)^2 + ((m^3 + 15*m^2 + 74*m + 120)*cos(d*x + c)^6 + 12*(m^2 + 7
*m + 12)*cos(d*x + c)^4 + 96*(m + 2)*cos(d*x + c)^2 + 384)*sin(d*x + c) + 3
84)*(a*sin(d*x + c) + a)^m/(d*m^4 + 22*d*m^3 + 179*d*m^2 + 638*d*m + 840*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 6.77, size = 0, normalized size = 0.00

$$\int \left(\cos^{7}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x)
```

maxima [B] time = 0.71, size = 520, normalized size = 4.77

$$-\frac{\left(\left(m^{6}+21\,m^{5}+175\,m^{4}+735\,m^{3}+1624\,m^{2}+1764\,m+720\right)a^{m}\sin(dx+c)^{7}+\left(m^{6}+15\,m^{5}+85\,m^{4}+225\,m^{3}+274\,m^{2}+120\,m\right)a^{m}\sin(dx+c)^{6}-6\left(m^{5}+10\,m^{4}+35\right.\right.}{m^{7}+28\,m^{6}+32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] -(((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*a^m*sin(d*x
 + c)^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*a^m*sin(d*x +
 c)^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*a^m*sin(d*x + c)^5 + 30*(
m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(d*x + c)^4 - 120*(m^3 + 3*m^2 + 2*m)*a^
m*sin(d*x + c)^3 + 360*(m^2 + m)*a^m*sin(d*x + c)^2 - 720*a^m*m*sin(d*x + c
) + 720*a^m)*(sin(d*x + c) + 1)^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769
*m^3 + 13132*m^2 + 13068*m + 5040) - 3*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
*a^m*sin(d*x + c)^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(d*x + c)^4 - 4*(
m^3 + 3*m^2 + 2*m)*a^m*sin(d*x + c)^3 + 12*(m^2 + m)*a^m*sin(d*x + c)^2 - 2
4*a^m*m*sin(d*x + c) + 24*a^m)*(sin(d*x + c) + 1)^m/(m^5 + 15*m^4 + 85*m^3
+ 225*m^2 + 274*m + 120) + 3*((m^2 + 3*m + 2)*a^m*sin(d*x + c)^3 + (m^2 + m
)*a^m*sin(d*x + c)^2 - 2*a^m*m*sin(d*x + c) + 2*a^m)*(sin(d*x + c) + 1)^m/(
m^3 + 6*m^2 + 11*m + 6) - (a*sin(d*x + c) + a)^(m + 1)/(a*(m + 1)))/d
```

mupad [B] time = 10.48, size = 555, normalized size = 5.09

$$e^{-c\,7i-d\,x\,7i}\,(a+a\,\sin(c+d\,x))^m\left(\frac{e^{c\,7i+d\,x\,7i}\,\left(m^3\,40i+m^2\,936i+m\,8672i+49152i\right)}{128\,d\,\left(m^4\,1i+m^3\,22i+m^2\,179i+m\,638i+840i\right)}+\frac{e^{c\,7i+d\,x\,7i}\,\cos(2\,c+2\,d}{64\,d\,\left(m^4\,1i+m^3\,22\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^m,x)
```

```
[Out] exp(- c*7i - d*x*7i)*(a + a*sin(c + d*x))^m*((exp(c*7i + d*x*7i)*(m*8672i +
 m^2*936i + m^3*40i + 49152i))/(128*d*(m*638i + m^2*179i + m^3*22i + m^4*1i
 + 840i)) + (exp(c*7i + d*x*7i)*cos(2*c + 2*d*x)*(m*4824i + m^2*654i + m^3*
30i))/(64*d*(m*638i + m^2*179i + m^3*22i + m^4*1i + 840i)) + (exp(c*7i + d*
x*7i)*sin(5*c + 5*d*x)*(706*m + 123*m^2 + 5*m^3 + 1176)*1i)/(64*d*(m*638i +
 m^2*179i + m^3*22i + m^4*1i + 840i)) + (exp(c*7i + d*x*7i)*sin(3*c + 3*d*x
)*(3210*m + 279*m^2 + 9*m^3 + 5880)*1i)/(64*d*(m*638i + m^2*179i + m^3*22i
+ m^4*1i + 840i)) + (exp(c*7i + d*x*7i)*sin(7*c + 7*d*x)*(74*m + 15*m^2 + m
^3 + 120)*1i)/(64*d*(m*638i + m^2*179i + m^3*22i + m^4*1i + 840i)) + (exp(c
*7i + d*x*7i)*sin(c + d*x)*(2578*m + 171*m^2 + 5*m^3 + 29400)*1i)/(64*d*(m*
638i + m^2*179i + m^3*22i + m^4*1i + 840i)) + (m*exp(c*7i + d*x*7i)*cos(6*c
 + 6*d*x)*(m*9i + m^2*1i + 20i))/(32*d*(m*638i + m^2*179i + m^3*22i + m^4*1
i + 840i)) + (3*m*exp(c*7i + d*x*7i)*cos(4*c + 4*d*x)*(m*17i + m^2*1i + 44i
))/(16*d*(m*638i + m^2*179i + m^3*22i + m^4*1i + 840i)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.344 $\int \cos^5(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=81

$$\frac{(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}-\frac{4(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}+\frac{4(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}$$

```
[Out] 4*(a+a*sin(d*x+c))^(3+m)/a^3/d/(3+m)-4*(a+a*sin(d*x+c))^(4+m)/a^4/d/(4+m)+(
a+a*sin(d*x+c))^(5+m)/a^5/d/(5+m)
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}-\frac{4(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}+\frac{(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) - (4*(a + a*Sin[c + d*x])^
(4 + m))/(a^4*d*(4 + m)) + (a + a*Sin[c + d*x])^(5 + m)/(a^5*d*(5 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^m\,dx=\frac{\text{Subst}\left(\int (a-x)^2(a+x)^{2+m}\,dx,x,a\sin(c+dx)\right)}{a^5d}$$

$$=\frac{\text{Subst}\left(\int \left(4a^2(a+x)^{2+m}-4a(a+x)^{3+m}+(a+x)^{4+m}\right)dx,x,a\sin(c+dx)\right)}{a^5d}$$

$$=\frac{4(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}-\frac{4(a+a\sin(c+dx))^{4+m}}{a^4d(4+m)}+\frac{(a+a\sin(c+dx))^{5+m}}{a^5d(5+m)}$$

Mathematica [A] time = 0.32, size = 68, normalized size = 0.84

$$\frac{(a(\sin(c+dx)+1))^{m+3}\left(-\frac{4a^2(\sin(c+dx)+1)}{m+4}+\frac{4a^2}{m+3}+\frac{(a\sin(c+dx)+a)^2}{m+5}\right)}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3 + m)*((4*a^2)/(3 + m) - (4*a^2*(1 + Sin[c + d*x]
))/(4 + m) + (a + a*Sin[c + d*x])^2/(5 + m)))/(a^5*d)
```

fricas [A] time = 0.47, size = 102, normalized size = 1.26

$$\frac{\left(\left(m^2+3\,m\right)\cos\left(dx+c\right)^4+8\,m\cos\left(dx+c\right)^2+\left(\left(m^2+7\,m+12\right)\cos\left(dx+c\right)^4+8\left(m+2\right)\cos\left(dx+c\right)^2+32\right.\right.}{dm^3+12\,dm^2+47\,dm+60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^2 + 3*m)*cos(d*x + c)^4 + 8*m*cos(d*x + c)^2 + ((m^2 + 7*m + 12)*cos(d*
x + c)^4 + 8*(m + 2)*cos(d*x + c)^2 + 32)*sin(d*x + c) + 32)*(a*sin(d*x + c
) + a)^m/(d*m^3 + 12*d*m^2 + 47*d*m + 60*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 3.22, size = 0, normalized size = 0.00

$$\int \left(\cos^5\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

maxima [B] time = 0.82, size = 266, normalized size = 3.28

$$\frac{\left(\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)a^m\sin(dx+c)^5+\left(m^4+6\,m^3+11\,m^2+6\,m\right)a^m\sin(dx+c)^4-4\left(m^3+3\,m^2+2\,m\right)a^m\sin(dx+c)^3+12\left(m^2+m\right)a^m\sin(dx+c)^2\right.}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*a^m*sin(d*x + c)^5 + (m^4 + 6*m^3 + 1
1*m^2 + 6*m)*a^m*sin(d*x + c)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^m*sin(d*x + c)^3
+ 12*(m^2 + m)*a^m*sin(d*x + c)^2 - 24*a^m*m*sin(d*x + c) + 24*a^m)*(sin(d*
x + c) + 1)^m/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120) - 2*((m^2 + 3
*m + 2)*a^m*sin(d*x + c)^3 + (m^2 + m)*a^m*sin(d*x + c)^2 - 2*a^m*m*sin(d*x
 + c) + 2*a^m)*(sin(d*x + c) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) + (a*sin(d*x +
 c) + a)^(m + 1)/(a*(m + 1)))/d
```

mupad [B] time = 1.97, size = 195, normalized size = 2.41

$$\frac{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)^m\,\left(82\,m+600\,\sin\left(c+d\,x\right)+100\,\sin\left(3\,c+3\,d\,x\right)+12\,\sin\left(5\,c+5\,d\,x\right)+46\,m\,\sin\left(c+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^m,x)
```

[Out] ((a*(sin(c + d*x) + 1))^m*(82*m + 600*sin(c + d*x) + 100*sin(3*c + 3*d*x) + 12*sin(5*c + 5*d*x) + 46*m*sin(c + d*x) + 88*m*cos(2*c + 2*d*x) + 6*m*cos(4*c + 4*d*x) + 53*m*sin(3*c + 3*d*x) + 7*m*sin(5*c + 5*d*x) + 2*m^2*sin(c + d*x) + 6*m^2 + 8*m^2*cos(2*c + 2*d*x) + 2*m^2*cos(4*c + 4*d*x) + 3*m^2*sin(3*c + 3*d*x) + m^2*sin(5*c + 5*d*x) + 512))/(16*d*(47*m + 12*m^2 + m^3 + 60))

sympy [A] time = 172.27, size = 5534, normalized size = 68.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**m,x)

[Out] Piecewise((x*(a*sin(c) + a)**m*cos(c)**5, Eq(d, 0)), (12*log(sin(c + d*x) + 1)*sin(c + d*x)**4/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 48*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 72*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 48*log(sin(c + d*x) + 1)*sin(c + d*x)/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 12*log(sin(c + d*x) + 1)/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 20*sin(c + d*x)**3/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 6*sin(c + d*x)**2*cos(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 56*sin(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 8*sin(c + d*x)*cos(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 52*sin(c + d*x)/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) - 3*cos(c + d*x)**4/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 2*cos(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 16/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d), Eq(m, -5)), (-12*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x) + 1)*sin(c + d*x)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 12*log(sin(c + d*x) + 1)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 8*sin(c + d*x)**4/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 4*sin(c + d*x)**2*cos(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 52*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 6*sin(c + d*x)*cos(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 72*sin(c + d*x)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - cos(c + d*x)**4/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 2*cos(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 28/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d), Eq(m, -4)), (8*log(tan(c/2 + d*x/2) + 1)*tan(c/2

```
+ d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a*
*3*d) + 16*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 +
d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 8*log(tan(c/2 + d*x/2)
 + 1)/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d)
- 4*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/
2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 8*log(tan(c/2 + d*x/2)**2
+ 1)*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d
*x/2)**2 + a**3*d) - 4*log(tan(c/2 + d*x/2)**2 + 1)/(a**3*d*tan(c/2 + d*x/2
)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 6*tan(c/2 + d*x/2)**3/(a**3
*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 2*tan(c/2
 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a
**3*d) - 6*tan(c/2 + d*x/2)/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2
+ d*x/2)**2 + a**3*d), Eq(m, -3)), (6*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2
 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2
+ 3*a**2*d) - 12*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2
*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 20*tan(
c/2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**
4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 12*tan(c/2 + d*x/2)**2/(3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2
+ d*x/2)**2 + 3*a**2*d) + 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**6
+ 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d),
Eq(m, -2)), (6*tan(c/2 + d*x/2)**7/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(
c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 +
 3*a*d) - 6*tan(c/2 + d*x/2)**6/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2
 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*
a*d) + 10*tan(c/2 + d*x/2)**5/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 +
 d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*
d) + 10*tan(c/2 + d*x/2)**3/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d
*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d)
 - 6*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d*x/
2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) +
6*tan(c/2 + d*x/2)/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d*x/2)**6
+ 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d), Eq(m, -
1)), (m**4*(a*sin(c + d*x) + a)**m*sin(c + d*x)*cos(c + d*x)**4/(d*m**5 + 1
5*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + m**4*(a*sin(c + d*x)
 + a)**m*cos(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274
*d*m + 120*d) + 4*m**3*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3*cos(c + d*x)
**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 8*m**
3*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2*cos(c + d*x)**2/(d*m**5 + 15*d*m*
*4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 14*m**3*(a*sin(c + d*x) +
a)**m*sin(c + d*x)*cos(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*
m**2 + 274*d*m + 120*d) + 4*m**3*(a*sin(c + d*x) + a)**m*sin(c + d*x)*cos(c
 + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d)
+ 14*m**3*(a*sin(c + d*x) + a)**m*cos(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*
d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 8*m**2*(a*sin(c + d*x) + a)**m*sin
(c + d*x)**5/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d
) + 24*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**5 + 15*d*m**4 + 8
5*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 44*m**2*(a*sin(c + d*x) + a)**m*
sin(c + d*x)**3*cos(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**
2 + 274*d*m + 120*d) + 24*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m
**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 84*m**2*(a*si
n(c + d*x) + a)**m*sin(c + d*x)**2*cos(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85
*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 8*m**2*(a*sin(c + d*x) + a)**m*si
n(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*
d) + 71*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)*cos(c + d*x)**4/(d*m**5 +
 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 36*m**2*(a*sin(c +
 d*x) + a)**m*sin(c + d*x)*cos(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**3
+ 225*d*m**2 + 274*d*m + 120*d) + 71*m**2*(a*sin(c + d*x) + a)**m*cos(c + d
*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) - 4*
```

```
m**2*(a*sin(c + d*x) + a)**m*cos(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**
3 + 225*d*m**2 + 274*d*m + 120*d) + 48*m*(a*sin(c + d*x) + a)**m*sin(c + d*
x)**5/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 120
*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3
+ 225*d*m**2 + 274*d*m + 120*d) + 152*m*(a*sin(c + d*x) + a)**m*sin(c + d*x
)**3*cos(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m
 + 120*d) + 72*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**5 + 15*d*m**
4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 268*m*(a*sin(c + d*x) + a)*
*m*sin(c + d*x)**2*cos(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*
m**2 + 274*d*m + 120*d) - 24*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2/(d*m
**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 154*m*(a*sin(
c + d*x) + a)**m*sin(c + d*x)*cos(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m*
*3 + 225*d*m**2 + 274*d*m + 120*d) + 80*m*(a*sin(c + d*x) + a)**m*sin(c + d
*x)*cos(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m
+ 120*d) - 24*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)/(d*m**5 + 15*d*m**4 +
85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 154*m*(a*sin(c + d*x) + a)**m*c
os(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120
*d) - 36*m*(a*sin(c + d*x) + a)**m*cos(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85
*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 64*(a*sin(c + d*x) + a)**m*sin(c
+ d*x)**5/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) +
 120*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**
3 + 225*d*m**2 + 274*d*m + 120*d) + 160*(a*sin(c + d*x) + a)**m*sin(c + d*x
)**3*cos(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m
 + 120*d) + 240*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2*cos(c + d*x)**2/(d*
m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) - 80*(a*sin(c
+ d*x) + a)**m*sin(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2
 + 274*d*m + 120*d) + 120*(a*sin(c + d*x) + a)**m*sin(c + d*x)*cos(c + d*x)
**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 120*(
a*sin(c + d*x) + a)**m*cos(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 22
5*d*m**2 + 274*d*m + 120*d) - 80*(a*sin(c + d*x) + a)**m*cos(c + d*x)**2/(d
*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 24*(a*sin(c
 + d*x) + a)**m/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 12
0*d), True))
```

3.345 $\int \cos^3(c+dx)(a+a\sin(c+dx))^m \, dx$

Optimal. Leaf size=55

$$\frac{2(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)} - \frac{(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}$$

```
[Out] 2*(a+a*sin(d*x+c))^(2+m)/a^2/d/(2+m)-(a+a*sin(d*x+c))^(3+m)/a^3/d/(3+m)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)} - \frac{(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*(2 + m)) - (a + a*Sin[c + d*x])^(3
+ m)/(a^3*d*(3 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+a\sin(c+dx))^m \, dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^{1+m} \, dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(2a(a+x)^{1+m}-(a+x)^{2+m}\right) dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{2(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)} - \frac{(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 52, normalized size = 0.95

$$-\frac{(\sin(c+dx)+1)^2((m+2)\sin(c+dx)-m-4)(a(\sin(c+dx)+1))^m}{d(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(((1 + Sin[c + d*x])^2*(a*(1 + Sin[c + d*x]))^m*(-4 - m + (2 + m)*Sin[c +
d*x]))/(d*(2 + m)*(3 + m)))
```

fricas [A] time = 0.58, size = 61, normalized size = 1.11

$$\frac{\left(m\cos\left(dx+c\right)^{2}+\left((m+2)\cos\left(dx+c\right)^{2}+4\right)\sin\left(dx+c\right)+4\right)\left(a\sin\left(dx+c\right)+a\right)^{m}}{dm^{2}+5\,dm+6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (m*cos(d*x + c)^2 + ((m + 2)*cos(d*x + c)^2 + 4)*sin(d*x + c) + 4)*(a*sin(d
*x + c) + a)^m/(d*m^2 + 5*d*m + 6*d)
```

giac [B] time = 1.47, size = 152, normalized size = 2.76

$$-\frac{\left(a\sin\left(dx+c\right)+a\right)^{m}m\sin\left(dx+c\right)^{3}+\left(a\sin\left(dx+c\right)+a\right)^{m}m\sin\left(dx+c\right)^{2}+2\left(a\sin\left(dx+c\right)+a\right)^{m}\sin\left(dx+c\right)^{3}}{(n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] -((a*sin(d*x + c) + a)^m*m*sin(d*x + c)^3 + (a*sin(d*x + c) + a)^m*m*sin(d*
x + c)^2 + 2*(a*sin(d*x + c) + a)^m*sin(d*x + c)^3 - (a*sin(d*x + c) + a)^m
*m*sin(d*x + c) - (a*sin(d*x + c) + a)^m*m - 6*(a*sin(d*x + c) + a)^m*sin(d
*x + c) - 4*(a*sin(d*x + c) + a)^m)/((m^2 + 5*m + 6)*d)
```

maple [F] time = 1.74, size = 0, normalized size = 0.00

$$\int \left(\cos^{3}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

maxima [B] time = 0.93, size = 111, normalized size = 2.02

$$-\frac{\frac{\left(\left(m^{2}+3\,m+2\right)a^{m}\sin(dx+c)^{3}+\left(m^{2}+m\right)a^{m}\sin(dx+c)^{2}-2\,a^{m}m\sin(dx+c)+2\,a^{m}\right)\left(\sin(dx+c)+1\right)^{m}}{m^{3}+6\,m^{2}+11\,m+6}-\frac{\left(a\sin(dx+c)+a\right)^{m+1}}{a(m+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] -((((m^2 + 3*m + 2)*a^m*sin(d*x + c)^3 + (m^2 + m)*a^m*sin(d*x + c)^2 - 2*a^
m*m*sin(d*x + c) + 2*a^m)*(sin(d*x + c) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) - (
a*sin(d*x + c) + a)^(m + 1)/(a*(m + 1)))/d
```

mupad [B] time = 0.73, size = 85, normalized size = 1.55

$$\frac{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)^{m}\,\left(2\,m+18\,\sin\left(c+d\,x\right)+2\,\sin\left(3\,c+3\,d\,x\right)+m\,\sin\left(c+d\,x\right)-2\,m\,\left(2\sin\left(c+d\,x\right)^{2}-1\right)\right.}{4\,d\,\left(m^{2}+5\,m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(2*m + 18*sin(c + d*x) + 2*sin(3*c + 3*d*x) + m*s
in(c + d*x) - 2*m*(2*sin(c + d*x)^2 - 1) + m*sin(3*c + 3*d*x) + 16))/(4*d*(
5*m + m^2 + 6))
```

sympy [A] time = 21.47, size = 1114, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*cos(c)**3, Eq(d, 0)), (-2*log(sin(c + d*x) +
 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a
**3*d) - 4*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4
*a**3*d*sin(c + d*x) + 2*a**3*d) - 2*log(sin(c + d*x) + 1)/(2*a**3*d*sin(c
+ d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 2*sin(c + d*x)/(2*a**3*d*si
n(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - cos(c + d*x)**2/(2*a**3
*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 2/(2*a**3*d*sin(c
+ d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Eq(m, -3)), (2*log(sin(c + d
*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) + 2*log(sin(c + d*x) +
 1)/(a**2*d*sin(c + d*x) + a**2*d) - 2*sin(c + d*x)**2/(a**2*d*sin(c + d*x)
 + a**2*d) - cos(c + d*x)**2/(a**2*d*sin(c + d*x) + a**2*d) + 2/(a**2*d*sin
(c + d*x) + a**2*d), Eq(m, -2)), (2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/
2)**4 + 2*a*d*tan(c/2 + d*x/2)**2 + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c
/2 + d*x/2)**4 + 2*a*d*tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d
*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2 + d*x/2)**2 + a*d), Eq(m, -1)), (m**2*
(a*sin(c + d*x) + a)**m*sin(c + d*x)*cos(c + d*x)**2/(d*m**3 + 6*d*m**2 + 1
1*d*m + 6*d) + m**2*(a*sin(c + d*x) + a)**m*cos(c + d*x)**2/(d*m**3 + 6*d*m
**2 + 11*d*m + 6*d) + 2*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**3 +
 6*d*m**2 + 11*d*m + 6*d) + 4*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2/(d*
m**3 + 6*d*m**2 + 11*d*m + 6*d) + 5*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)*
cos(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 2*m*(a*sin(c + d*x) +
a)**m*sin(c + d*x)/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 5*m*(a*sin(c + d*x)
 + a)**m*cos(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 4*(a*sin(c +
d*x) + a)**m*sin(c + d*x)**3/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 6*(a*sin(
c + d*x) + a)**m*sin(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 6*(a*
sin(c + d*x) + a)**m*sin(c + d*x)*cos(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*d
*m + 6*d) + 6*(a*sin(c + d*x) + a)**m*cos(c + d*x)**2/(d*m**3 + 6*d*m**2 +
11*d*m + 6*d) - 2*(a*sin(c + d*x) + a)**m/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d
), True))
```

3.346 $\int \cos(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=26

$$\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

[Out] (a+a*sin(d*x+c))^(1+m)/a/d/(1+m)

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2667, 32}

$$\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^m,x]

[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 1.00

$$\frac{(a(\sin(c+dx)+1))^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^m,x]

[Out] (a*(1 + Sin[c + d*x]))^(1 + m)/(a*d*(1 + m))

fricas [A] time = 0.46, size = 28, normalized size = 1.08

$$\frac{(a\sin(dx+c)+a)^m(\sin(dx+c)+1)}{dm+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (a*sin(d*x + c) + a)^m*(sin(d*x + c) + 1)/(d*m + d)
```

giac [A] time = 0.92, size = 26, normalized size = 1.00

$$\frac{(a\sin(dx+c)+a)^{m+1}}{ad(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] (a*sin(d*x + c) + a)^(m + 1)/(a*d*(m + 1))
```

maple [A] time = 0.02, size = 27, normalized size = 1.04

$$\frac{(a+a\sin(dx+c))^{1+m}}{ad(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] (a+a*sin(d*x+c))^(1+m)/a/d/(1+m)
```

maxima [A] time = 0.54, size = 26, normalized size = 1.00

$$\frac{(a\sin(dx+c)+a)^{m+1}}{ad(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (a*sin(d*x + c) + a)^(m + 1)/(a*d*(m + 1))
```

mupad [B] time = 0.22, size = 29, normalized size = 1.12

$$\frac{(a\ (\sin(c+d\,x)+1))^m\ (\sin(c+d\,x)+1)}{d\ (m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(sin(c + d*x) + 1))/(d*(m + 1))
```

sympy [A] time = 2.47, size = 80, normalized size = 3.08

$$\begin{cases} \frac{x\cos(c)}{a\sin(c)+a} & \text{for}\ d=0 \wedge m=-1 \\ x(a\sin(c)+a)^m\cos(c) & \text{for}\ d=0 \\ \frac{\log(\sin(c+dx)+1)}{ad} & \text{for}\ m=-1 \\ \frac{(a\sin(c+dx)+a)^m\sin(c+dx)}{dm+d}+\frac{(a\sin(c+dx)+a)^m}{dm+d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*cos(c)/(a*sin(c) + a), Eq(d, 0) & Eq(m, -1)), (x*(a*sin(c) + a
)**m*cos(c), Eq(d, 0)), (log(sin(c + d*x) + 1)/(a*d), Eq(m, -1)), ((a*sin(c
 + d*x) + a)**m*sin(c + d*x)/(d*m + d) + (a*sin(c + d*x) + a)**m/(d*m + d),
 True))
```

3.347 $\int \sec(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=40

$$\frac{(a\sin(c+dx)+a)^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(\sin(c+dx)+1)\right)}{2dm}$$

```
[Out] 1/2*hypergeom([1, m],[1+m],1/2+1/2*sin(d*x+c))*(a+a*sin(d*x+c))^m/d/m
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2667, 68}

$$\frac{(a\sin(c+dx)+a)^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(\sin(c+dx)+1)\right)}{2dm}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, m, 1 + m, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^
m)/(2*d*m)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^m}{2dm}\end{aligned}$$

Mathematica [A] time = 0.10, size = 63, normalized size = 1.58

$$\frac{(a(\sin(c+dx)+1))^m\left(m(\sin(c+dx)+1)\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(c+dx)+1)\right)+2(m+1)\right)}{4dm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^m*(2*(1 + m) + m*Hypergeometric2F1[1, 1 + m, 2 + m,
 (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x])))/(4*d*m*(1 + m))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^m\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/cos(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^m/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*sec(c + d*x), x)
```

3.348 $\int \sec^3(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=47

$$-\frac{a(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(2,m-1;m;\frac{1}{2}(\sin(c+dx)+1)\right)}{4d(1-m)}$$

```
[Out] -1/4*a*hypergeom([2, -1+m],[m],1/2+1/2*sin(d*x+c))*(a+a*sin(d*x+c))^(-1+m)/
d/(1-m)
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 68}

$$-\frac{a(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(2,m-1;m;\frac{1}{2}(\sin(c+dx)+1)\right)}{4d(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(a*Hypergeometric2F1[2, -1 + m, m, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*
x])^(-1 + m))/(4*d*(1 - m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{a^3\,\text{Subst}\left(\int \frac{(a+x)^{-2+m}}{(a-x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a\,{}_2F_1\left(2,-1+m;m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^{-1+m}}{4d(1-m)}\end{aligned}$$

Mathematica [B] time = 0.36, size = 111, normalized size = 2.36

$$\frac{(a(\sin(c+dx)+1))^m\left(\frac{2(\sin(c+dx)+1)\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(c+dx)+1)\right)}{m+1}+\frac{(\sin(c+dx)+1)\,{}_2F_1\left(2,m+1;m+2;\frac{1}{2}(\sin(c+dx)+1)\right)}{m+1}+4\left(\frac{}{(m-1)(\mathrm{si}}\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^m*((2*Hypergeometric2F1[1, 1 + m, 2 + m, (1 + Sin[c
 + d*x])/2]*(1 + Sin[c + d*x]))/(1 + m) + (Hypergeometric2F1[2, 1 + m, 2 +
m, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]))/(1 + m) + 4*(m^(-1) + 1/((-1 +
 m)*(1 + Sin[c + d*x])))))/(16*d)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^{m}\sec(dx+c)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{m}\sec(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}(dx+c)\right)(a+a\sin(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{m}\sec(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\,\sin(c+d\,x))^{m}}{\cos(c+d\,x)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^m/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.349 $\int \sec^5(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=51

$$-\frac{a^2(a\sin(c+dx)+a)^{m-2}\,{}_2F_1\left(3,m-2;m-1;\frac{1}{2}(\sin(c+dx)+1)\right)}{8d(2-m)}$$

```
[Out] -1/8*a^2*hypergeom([3, -2+m],[-1+m],1/2+1/2*sin(d*x+c))*(a+a*sin(d*x+c))^(-2+m)/d/(2-m)
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 68}

$$-\frac{a^2(a\sin(c+dx)+a)^{m-2}\,{}_2F_1\left(3,m-2;m-1;\frac{1}{2}(\sin(c+dx)+1)\right)}{8d(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(a^2*Hypergeometric2F1[3, -2 + m, -1 + m, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^(-2 + m))/(8*d*(2 - m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2])
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{a^5\,\text{Subst}\left(\int \frac{(a+x)^{-3+m}}{(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a^2\,{}_2F_1\left(3,-2+m;-1+m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^{-2+m}}{8d(2-m)}\end{aligned}$$

Mathematica [B] time = 0.74, size = 163, normalized size = 3.20

$$\frac{(a(\sin(c+dx)+1))^m\left(\frac{6(\sin(c+dx)+1)\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(c+dx)+1)\right)}{m+1}+\frac{3(\sin(c+dx)+1)\,{}_2F_1\left(2,m+1;m+2;\frac{1}{2}(\sin(c+dx)+1)\right)}{m+1}+\frac{(\sin(c+dx)\cdots}{}\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^m*(12/m + 8/((-2 + m)*(1 + Sin[c + d*x])^2) + 12/((
-1 + m)*(1 + Sin[c + d*x])) + (6*Hypergeometric2F1[1, 1 + m, 2 + m, (1 + Si
n[c + d*x])/2]*(1 + Sin[c + d*x]))/(1 + m) + (3*Hypergeometric2F1[2, 1 + m,
 2 + m, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]))/(1 + m) + (Hypergeometric
2F1[3, 1 + m, 2 + m, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]))/(1 + m)))/(6
4*d)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^m\sec(dx+c)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sin(dx+c)+a)^m\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int\left(\sec^5(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sin(dx+c)+a)^m\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+a\,\sin(c+d\,x))^m}{\cos(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/cos(c + d*x)^5,x)
```

```
[Out] int((a + a*sin(c + d*x))^m/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.350 $\int \cos^4(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{a^2 2^{m+\frac{5}{2}}\cos^5(c+dx)(\sin(c+dx)+1)^{-m-\frac{1}{2}}(a\sin(c+dx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d}$$

```
[Out] -1/5*2^(5/2+m)*a^2*cos(d*x+c)^5*hypergeom([5/2, -3/2-m],[7/2],1/2-1/2*sin(d
*x+c))*(1+sin(d*x+c))^(-1/2-m)*(a+a*sin(d*x+c))^(-2+m)/d
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2689, 70, 69}

$$-\frac{a^2 2^{m+\frac{5}{2}}\cos^5(c+dx)(\sin(c+dx)+1)^{-m-\frac{1}{2}}(a\sin(c+dx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(5/2 + m)*a^2*Cos[c + d*x]^5*Hypergeometric2F1[5/2, -3/2 - m, 7/2, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - m)*(a + a*Sin[c + d*x])^(-2 +
m))/(5*d)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\cos^5(c+dx)\right)\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{\frac{3}{2}+m}\,dx, x, \sin(c+dx)\right)}{d(a-a\sin(c+dx))^{5/2}(a+a\sin(c+dx))^{5/2}}$$
$$= \frac{\left(2^{\frac{3}{2}+m}a^3\cos^5(c+dx)(a+a\sin(c+dx))^{-2+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{-\frac{1}{2}-m}\right)\text{Subst}}{d(a-a\sin(c+dx))^{5/2}}$$
$$= -\frac{2^{\frac{5}{2}+m}a^2\cos^5(c+dx)\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+}{5d}$$

Mathematica [A] time = 0.12, size = 78, normalized size = 0.94

$$-\frac{2^{m+\frac{5}{2}}\cos^5(c+dx)(\sin(c+dx)+1)^{-m-\frac{5}{2}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
[Out] -1/5*(2^(5/2 + m)*Cos[c + d*x]^5*Hypergeometric2F1[5/2, -3/2 - m, 7/2, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-5/2 - m)*(a*(1 + Sin[c + d*x]))^m)/d
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^m\cos(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

maple [F] time = 2.44, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^m,x)
[Out] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(c + d x \right)} + 1\right)\right)^{m} \cos^{4}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*cos(c + d*x)**4, x)
```

3.351 $\int \cos^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{a2^{m+\frac{3}{2}}\cos^3(c+dx)(\sin(c+dx)+1)^{-m-\frac{1}{2}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3d}$$

```
[Out] -1/3*2^(3/2+m)*a*cos(d*x+c)^3*hypergeom([3/2, -1/2-m],[5/2],1/2-1/2*sin(d*x
+c))*(1+sin(d*x+c))^(-1/2-m)*(a+a*sin(d*x+c))^(-1+m)/d
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2689, 70, 69\}$

$$-\frac{a2^{m+\frac{3}{2}}\cos^3(c+dx)(\sin(c+dx)+1)^{-m-\frac{1}{2}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(3/2 + m)*a*Cos[c + d*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 - S
in[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - m)*(a + a*Sin[c + d*x])^(-1 + m)
)/(3*d)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\cos^3(c+dx)\right)\text{Subst}\left(\int \sqrt{a-ax}\,(a+ax)^{\frac{1}{2}+m}\,dx, x, \sin(c+dx)\right)}{d(a-a\sin(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^2\cos^3(c+dx)(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{-\frac{1}{2}-m}\right)\text{Sub}}{d(a-a\sin(c+dx))^{3/2}}$$

$$= -\frac{2^{\frac{3}{2}+m}a\cos^3(c+dx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(}{3d}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.96

$$-\frac{2^{m+\frac{3}{2}}\cos^3(c+dx)(\sin(c+dx)+1)^{-m-\frac{3}{2}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
[Out] -1/3*(2^(3/2 + m)*Cos[c + d*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-3/2 - m)*(a*(1 + Sin[c + d*x]))^m)/d
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^m\cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

maple [F] time = 1.34, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^m,x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(c + d x \right)} + 1\right)\right)^{m} \cos^{2}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*cos(c + d*x)**2, x)
```

3.352 $\int \sec^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=73

$$\frac{2^{m-\frac{1}{2}}\sec(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

```
[Out] 2^(-1/2+m)*hypergeom([-1/2, 3/2-m],[1/2],1/2-1/2*sin(d*x+c))*sec(d*x+c)*(1+
sin(d*x+c))^(1/2-m)*(a+a*sin(d*x+c))^m/d
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2689, 70, 69}

$$\frac{2^{m-\frac{1}{2}}\sec(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(-1/2 + m)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[c + d*x])/2]*S
ec[c + d*x]*(1 + Sin[c + d*x])^(1/2 - m)*(a + a*Sin[c + d*x])^m)/d
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\sec(c+dx)\sqrt{a-a\sin(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{3}{2}}}{(a-ax)^{3/}}\right.}{d}$$

$$= \frac{\left(2^{-\frac{3}{2}+m}a\sec(c+dx)\sqrt{a-a\sin(c+dx)}\,(a+a\sin(c+dx))^m\left(\frac{a+a\sin(c+dx)}{a}\right.\right.}{d}$$

$$= \frac{2^{-\frac{1}{2}+m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec(c+dx)(1+\sin(c+dx)}{d}$$

<u>Mathematica</u> [C] time = 16.12, size = 3917, normalized size = 53.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/4*((Cos[(-c + Pi/2 - d*x)/4]^2)^(2*m)*Cot[(-c + Pi/2 - d*x)/4]*(a + a*Si
n[c + d*x])^m*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-c + Pi/2 - d*x)/4]^2,
 -Tan[(-c + Pi/2 - d*x)/4]^2]*(Sec[(-c + Pi/2 - d*x)/4]^2)^(2*m)) + (3*Appe
llF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x
)/4]^2]*Tan[(-c + Pi/2 - d*x)/4]^2*(1 - Tan[(-c + Pi/2 - d*x)/4]^2)^(2*m))/
(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/
2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x
)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Ta
n[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2])*Tan[(-c + Pi/2 - d*
x)/4]^2)))/(d*(Cos[Pi/4 + (c - Pi/2 + d*x)/2] - Sin[Pi/4 + (c - Pi/2 + d*x)
/2])^2*(-1/2*(m*(Cos[(-c + Pi/2 - d*x)/4]^2)^(2*m)*(-(AppellF1[-1/2, -2*m,
2*m, 1/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*(Sec[(-c
 + Pi/2 - d*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi
/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Tan[(-c + Pi/2 - d*x)/4]^2*(1
- Tan[(-c + Pi/2 - d*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(
-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1
- 2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] +
 AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + P
i/2 - d*x)/4]^2])*Tan[(-c + Pi/2 - d*x)/4]^2))) - ((Cos[(-c + Pi/2 - d*x)/4
]^2)^(2*m)*Csc[(-c + Pi/2 - d*x)/4]^2*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan
[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*(Sec[(-c + Pi/2 - d*x
)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^
2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Tan[(-c + Pi/2 - d*x)/4]^2*(1 - Tan[(-c + P
i/2 - d*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d
*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5
/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + AppellF1[3/2
, -2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]
^2])*Tan[(-c + Pi/2 - d*x)/4]^2)))/8 + ((Cos[(-c + Pi/2 - d*x)/4]^2)^(2*m)*
Cot[(-c + Pi/2 - d*x)/4]*(-(m*AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-c + Pi/2
 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*(Sec[(-c + Pi/2 - d*x)/4]^2)^(2*
m)*Tan[(-c + Pi/2 - d*x)/4]) - (Sec[(-c + Pi/2 - d*x)/4]^2)^(2*m)*(m*Appell
F1[1/2, 1 - 2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*
x)/4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4] + m*AppellF1[1
/2, -2*m, 1 + 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/
4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]) + (3*AppellF1[1/
2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]
*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]*(1 - Tan[(-c + Pi/2 -
```

```
d*x)/4]^2)^(2*m))/(2*(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)
/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2,
 Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + AppellF1[3/2, -
2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]
)*Tan[(-c + Pi/2 - d*x)/4]^2)) + (3*Tan[(-c + Pi/2 - d*x)/4]^2*(-1/3*(m*App
ellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 -
 d*x)/4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]) - (m*Appel
lF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 -
d*x)/4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4])/3)*(1 - Tan
[(-c + Pi/2 - d*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c +
Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m
, 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + Appe
llF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 -
 d*x)/4]^2])*Tan[(-c + Pi/2 - d*x)/4]^2) - (3*m*AppellF1[1/2, -2*m, 2*m, 3/
2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec[(-c + Pi/2
- d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]^3*(1 - Tan[(-c + Pi/2 - d*x)/4]^2)^(-1
 + 2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[
(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-c + P
i/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m
, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2])*Tan[(-c +
Pi/2 - d*x)/4]^2) - (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/
4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Tan[(-c + Pi/2 - d*x)/4]^2*(1 - Tan[(-c
+ Pi/2 - d*x)/4]^2)^(2*m)*(-2*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-c +
 Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2
*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2])*Sec[(-c
+ Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4] + 3*(-1/3*(m*AppellF1[3/2, 1 -
2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec
[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]) - (m*AppellF1[3/2, -2*m,
1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec[
(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4])/3) - 4*m*Tan[(-c + Pi/2 -
d*x)/4]^2*((-6*m*AppellF1[5/2, 1 - 2*m, 1 + 2*m, 7/2, Tan[(-c + Pi/2 - d*x)
/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi
/2 - d*x)/4])/5 + (3*(1 - 2*m)*AppellF1[5/2, 2 - 2*m, 2*m, 7/2, Tan[(-c + P
i/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Ta
n[(-c + Pi/2 - d*x)/4])/10 - (3*(1 + 2*m)*AppellF1[5/2, -2*m, 2 + 2*m, 7/2,
 Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec[(-c + Pi/2 -
d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4])/10)))/(3*AppellF1[1/2, -2*m, 2*m, 3/2,
Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/
2, 1 - 2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]
^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-
c + Pi/2 - d*x)/4]^2])*Tan[(-c + Pi/2 - d*x)/4]^2)^2))/2)) + (Hypergeometri
c2F1[1/2, (-1 + 2*m)/2, (1 + 2*m)/2, Cos[(-c + Pi/2 - d*x)/2]^2]*(a + a*Sin
[c + d*x])^m*Tan[(-c + Pi/2 - d*x)/2])/(2*d*(-1 + 2*m)*Sqrt[Sin[(-c + Pi/2
- d*x)/2]^2])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^m\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(a+a\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m \sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{\cos(c+d\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/cos(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^m/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m \sec^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*sec(c + d*x)**2, x)
```

3.353 $\int \sec^4(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=83

$$\frac{2^{m-\frac{3}{2}}\sec^3(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}-m}(a\sin(c+dx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3ad}$$

```
[Out] 1/3*2^(-3/2+m)*hypergeom([-3/2, 5/2-m],[-1/2],1/2-1/2*sin(d*x+c))*sec(d*x+c
)^3*(1+sin(d*x+c))^(1/2-m)*(a+a*sin(d*x+c))^(1+m)/a/d
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2689, 70, 69}

$$\frac{2^{m-\frac{3}{2}}\sec^3(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}-m}(a\sin(c+dx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(-3/2 + m)*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, (1 - Sin[c + d*x])/2]*
Sec[c + d*x]^3*(1 + Sin[c + d*x])^(1/2 - m)*(a + a*Sin[c + d*x])^(1 + m))/(
3*a*d)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\sec^3(c+dx)(a-a\sin(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+}{(a}\right.}{d}$$

$$= \frac{\left(2^{-\frac{5}{2}+m}\sec^3(c+dx)(a-a\sin(c+dx))^{3/2}(a+a\sin(c+dx))^{1+m}\left(\frac{a+a\sin(c}{a}\right.\right.}{d}$$

$$= \frac{2^{-\frac{3}{2}+m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec^3(c+dx)(1+\sin(c+}{3ad}$$

Mathematica [C] time = 21.28, size = 9400, normalized size = 113.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
[Out] Result too large to show
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^m\sec(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\sec^4(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^m\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin(c + d\,x))^m}{\cos(c + d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/cos(c + d*x)^4,x)
```

```
[Out] int((a + a*sin(c + d*x))^m/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.354 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{a2^{m+\frac{11}{4}}(e\cos(c+dx))^{7/2}(\sin(c+dx)+1)^{-m-\frac{3}{4}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{7}{4},-m-\frac{3}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de}$$

```
[Out] -1/7*2^(11/4+m)*a*(e*cos(d*x+c))^(7/2)*hypergeom([7/4, -3/4-m],[11/4],1/2-1
/2*sin(d*x+c))*(1+sin(d*x+c))^(-3/4-m)*(a+a*sin(d*x+c))^(-1+m)/d/e
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = $\{2689, 70, 69\}$

$$-\frac{a2^{m+\frac{11}{4}}(e\cos(c+dx))^{7/2}(\sin(c+dx)+1)^{-m-\frac{3}{4}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{7}{4},-m-\frac{3}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(11/4 + m)*a*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, -3/4 - m, 11
/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-3/4 - m)*(a + a*Sin[c + d*x]
)^(-1 + m))/(7*d*e)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{7/2}\right)\text{Subst}\left(\int (a-ax)^{3/4}(a+ax)^{\frac{3}{4}+m}\,dx, x, \sin(c\right.}{de(a-a\sin(c+dx))^{7/4}(a+a\sin(c+dx))^{7/4}}$$

$$= \frac{\left(2^{\frac{3}{4}+m}a^2(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{-\frac{3}{4}}\right.}{de(a-a\sin(c+}$$

$$= -\frac{2^{\frac{11}{4}+m}a(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},-\frac{3}{4}-m;\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de}$$

Mathematica [A] time = 0.21, size = 85, normalized size = 0.97

$$-\frac{2^{m+\frac{11}{4}}(e\cos(c+dx))^{7/2}(\sin(c+dx)+1)^{-m-\frac{7}{4}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{7}{4},-m-\frac{3}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^m,x]
[Out] -1/7*(2^(11/4 + m)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, -3/4 - m,
11/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-7/4 - m)*(a*(1 + Sin[c + d
*x]))^m)/(d*e)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^m e^2\cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m*e^2*cos(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.355 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{a2^{m+\frac{9}{4}}(e\cos(c+dx))^{5/2}(\sin(c+dx)+1)^{-m-\frac{1}{4}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de}$$

```
[Out] -1/5*2^(9/4+m)*a*(e*cos(d*x+c))^(5/2)*hypergeom([5/4, -1/4-m],[9/4],1/2-1/2
*sin(d*x+c))*(1+sin(d*x+c))^(-1/4-m)*(a+a*sin(d*x+c))^(-1+m)/d/e
```

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{9}{4}}(e\cos(c+dx))^{5/2}(\sin(c+dx)+1)^{-m-\frac{1}{4}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(9/4 + m)*a*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9/4
, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/4 - m)*(a + a*Sin[c + d*x])^
(-1 + m))/(5*d*e)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{5/2}\right)\text{Subst}\left(\int \sqrt[4]{a-ax}\,(a+ax)^{\frac{1}{4}+m}\,dx,x,\sin(c+d\right.}{de(a-a\sin(c+dx))^{5/4}(a+a\sin(c+dx))^{5/4}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{-\frac{1}{4}-m}\right.}{de(a-a\sin(c+dx)}$$

$$= -\frac{2^{\frac{9}{4}+m}a(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-m;\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+}{5de}$$

Mathematica [A] time = 0.15, size = 85, normalized size = 0.97

$$-\frac{2^{m+\frac{9}{4}}(e\cos(c+dx))^{5/2}(\sin(c+dx)+1)^{-m-\frac{5}{4}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^m,x]
[Out] -1/5*(2^(9/4 + m)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9
/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-5/4 - m)*(a*(1 + Sin[c + d*x
]))^m)/(d*e)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^m e\cos(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m*e*cos(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+dx))^{3/2}\,(a+a\sin(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**m,x)
[Out] Timed out
```

3.356 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{a2^{m+\frac{7}{4}}(e\cos(c+dx))^{3/2}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de}$$

```
[Out] -1/3*2^(7/4+m)*a*(e*cos(d*x+c))^(3/2)*hypergeom([3/4, 1/4-m],[7/4],1/2-1/2*
sin(d*x+c))*(1+sin(d*x+c))^(1/4-m)*(a+a*sin(d*x+c))^(-1+m)/d/e
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = $\{2689, 70, 69\}$

$$-\frac{a2^{m+\frac{7}{4}}(e\cos(c+dx))^{3/2}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(7/4 + m)*a*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 1/4 - m, 7/4,
 (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1/4 - m)*(a + a*Sin[c + d*x])^(-
1 + m))/(3*d*e)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{4}+m}}{\sqrt[4]{a-ax}}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{3/4}(a+a\sin(c+dx))^{3/4}}$$

$$= \frac{\left(2^{-\frac{1}{4}+m}a^2(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{1}{4}-}\right.}{de(a-a\sin(c+dx))^{3/4}}$$

$$= -\frac{2^{\frac{7}{4}+m}a(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+}{3de}$$

Mathematica [A] time = 0.08, size = 85, normalized size = 0.97

$$-\frac{2^{m+\frac{7}{4}}(e\cos(c+dx))^{3/2}(\sin(c+dx)+1)^{-m-\frac{3}{4}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/3*(2^(7/4 + m)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 1/4 - m, 7/
4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-3/4 - m)*(a*(1 + Sin[c + d*x]
))^m)/(d*e)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,\,(a+a\sin(dx+c))^m\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m\sqrt{e\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*sqrt(e*cos(c + d*x)), x)
```

3.357 $\int \frac{(a+a\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=86

$$-\frac{a2^{m+\frac{5}{4}}\sqrt{e\cos(c+dx)}\,(\sin(c+dx)+1)^{\frac{3}{4}-m}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de}$$

```
[Out] -2^(5/4+m)*a*hypergeom([1/4, 3/4-m],[5/4],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c)
)^(3/4-m)*(a+a*sin(d*x+c))^(-1+m)*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{5}{4}}\sqrt{e\cos(c+dx)}\,(\sin(c+dx)+1)^{\frac{3}{4}-m}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((2^(5/4 + m)*a*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 3/4 - m, 5/4,
(1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(3/4 - m)*(a + a*Sin[c + d*x])^(-1
 + m))/(d*e))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\left(a^2\sqrt{e\cos(c+dx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{3}{4}+m}}{(a-ax)^{3/4}}\,dx, x, \sin(c+dx)\right)}{d e\sqrt[4]{a-a\sin(c+dx)}\,\sqrt[4]{a+a\sin(c+dx)}}$$

$$= \frac{\left(2^{-\frac{3}{4}+m}a^2\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{3}{4}-m}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{x}{2}\right)^{-}}{(a-ax)}\right.}{de\sqrt[4]{a-a\sin(c+dx)}}$$

$$= -\frac{2^{\frac{5}{4}+m}a\sqrt{e\cos(c+dx)}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{3}{4}-m}(a}{de}$$

Mathematica [A] time = 0.08, size = 83, normalized size = 0.97

$$-\frac{2^{m+\frac{5}{4}}\sqrt{e\cos(c+dx)}\,(\sin(c+dx)+1)^{-m-\frac{1}{4}}(a(\sin(c+dx)+1))^m\;{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((2^(5/4 + m)*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 3/4 - m, 5/4, (1
 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/4 - m)*(a*(1 + Sin[c + d*x]))^m)
/(d*e))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^m}{e\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m/(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^m}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/sqrt(e*cos(d*x + c)), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(dx+c))^m}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^m}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(c+dx)+1))^m}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m/sqrt(e*cos(c + d*x)), x)
```

3.358 $\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{2^{m+\frac{3}{4}}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2^(3/4+m)*hypergeom([-1/4, 5/4-m],[3/4],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^
(1/4-m)*(a+a*sin(d*x+c))^m/d/e/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2689, 70, 69}

$$\frac{2^{m+\frac{3}{4}}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2^(3/4 + m)*Hypergeometric2F1[-1/4, 5/4 - m, 3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(1/4 - m)*(a + a*Sin[c + d*x])^m)/(d*e*Sqrt[e*Cos[c + d*x]
])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx = \frac{\left(a^2\sqrt[4]{a-a\sin(c+dx)}\,\sqrt[4]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{5}{4}+m}}{(a-ax)^{5/4}}\,dx, x, \sin(c+dx)\right)}{de\sqrt{e\cos(c+dx)}}$$

$$= \frac{\left(2^{-\frac{5}{4}+m}a\sqrt[4]{a-a\sin(c+dx)}\,(a+a\sin(c+dx))^m\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{1}{4}-m}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{x}{2}\right.}{(a-}\right.}{de\sqrt{e\cos(c+dx)}}$$

$$= \frac{2^{\frac{3}{4}+m}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{4}-m}(a+a\sin(c+dx}{de\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 0.10, size = 82, normalized size = 1.00

$$\frac{2^{m+\frac{3}{4}}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
[Out] (2^(3/4 + m)*Hypergeometric2F1[-1/4, 5/4 - m, 3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(1/4 - m)*(a*(1 + Sin[c + d*x]))^m)/(d*e*Sqrt[e*Cos[c + d*
x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^m}{e^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m/(e^2*cos(d*x + c)^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(3/2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(dx+c))^m}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(c+dx)+1))^m}{(e\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/(e*cos(d*x+c))**(3/2),x)
[Out] Integral((a*(sin(c + d*x) + 1))**m/(e*cos(c + d*x))**(3/2), x)
```

3.359 $\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2^{m+\frac{1}{4}}(\sin(c+dx)+1)^{\frac{3}{4}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] 1/3*2^(1/4+m)*hypergeom([-3/4, 7/4-m],[1/4],1/2-1/2*sin(d*x+c))*(1+sin(d*x+
c))^(3/4-m)*(a+a*sin(d*x+c))^m/d/e/(e*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2689, 70, 69}

$$\frac{2^{m+\frac{1}{4}}(\sin(c+dx)+1)^{\frac{3}{4}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2^(1/4 + m)*Hypergeometric2F1[-3/4, 7/4 - m, 1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(3/4 - m)*(a + a*Sin[c + d*x])^m)/(3*d*e*(e*Cos[c + d*x])^
(3/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\left(a^2(a-a\sin(c+dx))^{3/4}(a+a\sin(c+dx))^{3/4}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{7}{4}+m}}{(a-ax)^{7/4}}\,dx, x, \sin(c+dx)\right)}{de(e\cos(c+dx))^{3/2}}$$

$$= \frac{\left(2^{-\frac{7}{4}+m}a(a-a\sin(c+dx))^{3/4}(a+a\sin(c+dx))^m\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{3}{4}-m}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{x}{2}\right)}{(a-a}\right.}{de(e\cos(c+dx))^{3/2}}$$

$$= \frac{2^{\frac{1}{4}+m}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{3}{4}-m}(a+a\sin(c+dx))^m}{3de(e\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 1.00

$$\frac{2^{m+\frac{1}{4}}(\sin(c+dx)+1)^{\frac{3}{4}-m}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2^(1/4 + m)*Hypergeometric2F1[-3/4, 7/4 - m, 1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(3/4 - m)*(a*(1 + Sin[c + d*x]))^m)/(3*d*e*(e*Cos[c + d*x]
)^(3/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,(a\sin(dx+c)+a)^m}{e^3\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m/(e^3*cos(d*x + c)^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(5/2), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(dx+c))^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/(e*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.360 $\int (e\cos(c+dx))^{-4-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=201

$$-\frac{6(a\sin(c+dx)+a)^{m+3}(e\cos(c+dx))^{-m-3}}{a^3de\left(m^4-10m^2+9\right)}+\frac{6(a\sin(c+dx)+a)^{m+2}(e\cos(c+dx))^{-m-3}}{a^2de(3-m)\left(1-m^2\right)}-\frac{(a\sin(c+dx)+a)^m(e}{de(3-}$$

```
[Out] -(e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m/d/e/(3-m)-3*(e*cos(d*x+c))^(-3-m)
*(a+a*sin(d*x+c))^(1+m)/a/d/e/(1-m)/(3-m)+6*(e*cos(d*x+c))^(-3-m)*(a+a*sin(
d*x+c))^(2+m)/a^2/d/e/(3-m)/(-m^2+1)-6*(e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c
))^(3+m)/a^3/d/e/(m^4-10*m^2+9)
```

Rubi [A] time = 0.32, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2672, 2671}

$$\frac{6(a\sin(c+dx)+a)^{m+2}(e\cos(c+dx))^{-m-3}}{a^2de(3-m)\left(1-m^2\right)}-\frac{6(a\sin(c+dx)+a)^{m+3}(e\cos(c+dx))^{-m-3}}{a^3de\left(m^4-10m^2+9\right)}-\frac{(a\sin(c+dx)+a)^m(e\,c}{de(3-m}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-4 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(3 - m))) - (3*(e
*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(1 - m)*(3 - m
)) + (6*(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*e*(3
 - m)*(1 - m^2)) - (6*(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(3 + m
))/(a^3*d*e*(9 - 10*m^2 + m^4))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{-4-m}(a+a\sin(c+dx))^m\,dx &= -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)}+\frac{3\int (e\cos(c+dx))^{-4-}}{a(}\\ &= -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)}-\frac{3(e\cos(c+dx))^{-3-m}(}{ade(1-m}\\ &= -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)}-\frac{3(e\cos(c+dx))^{-3-m}(}{ade(1-m}\\ &= -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)}-\frac{3(e\cos(c+dx))^{-3-m}(}{ade(1-m}\end{aligned}$$

Mathematica [A] time = 0.19, size = 101, normalized size = 0.50

$$\frac{\sec^3(c+dx)\left(-3\left(m^2-3\right)\sin(c+dx)+6m\sin^2(c+dx)-6\sin^3(c+dx)+m\left(m^2-7\right)\right)(a(\sin(c+dx)+1))^m(}{de^4(m-3)(m-1)(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-4 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Sec[c + d*x]^3*(a*(1 + Sin[c + d*x]))^m*(m*(-7 + m^2) - 3*(-3 + m^2)*Sin[c
 + d*x] + 6*m*Sin[c + d*x]^2 - 6*Sin[c + d*x]^3))/(d*e^4*(-3 + m)*(-1 + m)*
(1 + m)*(3 + m)*(e*Cos[c + d*x])^m)
```

fricas [A] time = 0.48, size = 104, normalized size = 0.52

$$-\frac{\left(6\,m\cos\left(dx+c\right)^3-\left(m^3-m\right)\cos\left(dx+c\right)-3\left(2\,\cos\left(dx+c\right)^3-\left(m^2-1\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)\left(e\cos\left(\right.\right.}{dm^4-10\,dm^2+9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -(6*m*cos(d*x + c)^3 - (m^3 - m)*cos(d*x + c) - 3*(2*cos(d*x + c)^3 - (m^2
- 1)*cos(d*x + c))*sin(d*x + c))*(e*cos(d*x + c))^(-m - 4)*(a*sin(d*x + c)
+ a)^m/(d*m^4 - 10*d*m^2 + 9*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(e\cos\left(dx+c\right)\right)^{-m-4}\left(a\sin\left(dx+c\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 4)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int\left(e\cos\left(dx+c\right)\right)^{-4-m}\left(a+a\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(e\cos\left(dx+c\right)\right)^{-m-4}\left(a\sin\left(dx+c\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 4)*(a*sin(d*x + c) + a)^m, x)
```

mupad [B] time = 6.83, size = 137, normalized size = 0.68

$$\frac{2\left(a\left(\sin\left(c+d\,x\right)+1\right)\right)^m\left(12\,\sin\left(2\,c+2\,d\,x\right)+3\,\sin\left(4\,c+4\,d\,x\right)-22\,m\,\cos\left(c+d\,x\right)-6\,m\,\cos\left(3\,c+3\,d\,x\right)-\right.}{d\,e^4\left(e\,\cos\left(c+d\,x\right)\right)^m\,\left(4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+3\right)\,\left(m^4-10\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 4),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^m*(12*sin(2*c + 2*d*x) + 3*sin(4*c + 4*d*x) - 22*
m*cos(c + d*x) - 6*m*cos(3*c + 3*d*x) + 4*m^3*cos(c + d*x) - 6*m^2*sin(2*c
+ 2*d*x)))/(d*e^4*(e*cos(c + d*x))^m*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x)
 + 3)*(m^4 - 10*m^2 + 9))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-4-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.361 $\int (e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=142

$$-\frac{2(a\sin(c+dx)+a)^{m+2}(e\cos(c+dx))^{-m-2}}{a^2dem\left(4-m^2\right)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-2}}{de(2-m)}+\frac{2(a\sin(c+dx)+a)^{m+}}{ade(2-}$$

```
[Out] -(e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m/d/e/(2-m)+2*(e*cos(d*x+c))^(-2-m)
*(a+a*sin(d*x+c))^(1+m)/a/d/e/(2-m)/m-2*(e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+
c))^(2+m)/a^2/d/e/m/(-m^2+4)
```

Rubi [A] time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{2(a\sin(c+dx)+a)^{m+2}(e\cos(c+dx))^{-m-2}}{a^2dem\left(4-m^2\right)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-2}}{de(2-m)}+\frac{2(a\sin(c+dx)+a)^{m+}}{ade(2-}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(2 - m))) + (2*(e
*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(2 - m)*m) - (
2*(e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*e*m*(4 - m
^2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m\,dx &= -\frac{(e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m}{de(2-m)}+\frac{2\int (e\cos(c+dx))}{}\\ &= -\frac{(e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m}{de(2-m)}+\frac{2(e\cos(c+dx))^{-2-}}{ade}\\ &= -\frac{(e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m}{de(2-m)}+\frac{2(e\cos(c+dx))^{-2-}}{ade}\end{aligned}$$

Mathematica [A] time = 0.13, size = 76, normalized size = 0.54

$$\frac{\sec^2(c+dx)\left(-2m\sin(c+dx)+2\sin^2(c+dx)+m^2-2\right)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-m}}{de^3(m-2)m(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Sec[c + d*x]^2*(a*(1 + Sin[c + d*x]))^m*(-2 + m^2 - 2*m*Sin[c + d*x] + 2*S
in[c + d*x]^2))/(d*e^3*(-2 + m)*m*(2 + m)*(e*Cos[c + d*x])^m)
```

fricas [A] time = 0.46, size = 75, normalized size = 0.53

$$\frac{\left(m^2\cos\left(dx+c\right)-2\cos\left(dx+c\right)^3-2\,m\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\left(e\cos\left(dx+c\right)\right)^{-m-3}\left(a\sin\left(dx+c\right)+a\right)^m}{dm^3-4\,dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (m^2*cos(d*x + c) - 2*cos(d*x + c)^3 - 2*m*cos(d*x + c)*sin(d*x + c))*(e*co
s(d*x + c))^(-m - 3)*(a*sin(d*x + c) + a)^m/(d*m^3 - 4*d*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(e\cos\left(dx+c\right)\right)^{-m-3}\left(a\sin\left(dx+c\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 3)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int\left(e\cos\left(dx+c\right)\right)^{-3-m}\left(a+a\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(e\cos\left(dx+c\right)\right)^{-m-3}\left(a\sin\left(dx+c\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 3)*(a*sin(d*x + c) + a)^m, x)
```

mupad [B] time = 6.13, size = 103, normalized size = 0.73

$$-\frac{2\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)^m\,\left(-2\,\cos\left(c+d\,x\right)\,m^2+2\,\sin\left(2\,c+2\,d\,x\right)\,m+3\,\cos\left(c+d\,x\right)+\cos\left(3\,c+3\,d\,x\right)\right)}{d\,e^3\,m\,\left(e\,\cos\left(c+d\,x\right)\right)^m\,\left(m^2-4\right)\,\left(3\,\cos\left(c+d\,x\right)+\cos\left(3\,c+3\,d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 3),x)
```

```
[Out] -(2*(a*(sin(c + d*x) + 1))^m*(3*cos(c + d*x) + cos(3*c + 3*d*x) - 2*m^2*cos
(c + d*x) + 2*m*sin(2*c + 2*d*x)))/(d*e^3*m*(e*cos(c + d*x))^m*(m^2 - 4)*(3
*cos(c + d*x) + cos(3*c + 3*d*x)))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-3-m)*(a+a*sin(d*x+c))**m,x)
[Out] Timed out
```

3.362 $\int (e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=89

$$\frac{(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-m-1}}{ade\left(1-m^2\right)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-1}}{de(1-m)}$$

```
[Out] -(e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m/d/e/(1-m)+(e*cos(d*x+c))^(-1-m)*(
a+a*sin(d*x+c))^(1+m)/a/d/e/(-m^2+1)
```

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2672, 2671}

$$\frac{(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-m-1}}{ade\left(1-m^2\right)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-1}}{de(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(1 - m))) + ((e*C
os[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(1 - m^2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m\,dx &= -\frac{(e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m}{de(1-m)}+\frac{\int (e\cos(c+dx))^{-2-m}}{a(1}\\ &= -\frac{(e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m}{de(1-m)}+\frac{(e\cos(c+dx))^{-1-m}(a}{ade\left(1\right.}\end{aligned}$$

Mathematica [A] time = 0.12, size = 53, normalized size = 0.60

$$\frac{(m-\sin(c+dx))(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-m-1}}{de(m-1)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((e*Cos[c + d*x])^(-1 - m)*(m - Sin[c + d*x])*(a*(1 + Sin[c + d*x]))^m)/(d*
e*(-1 + m)*(1 + m))
```

fricas [A] time = 0.47, size = 61, normalized size = 0.69

$$\frac{(m\cos(dx+c) - \cos(dx+c)\sin(dx+c))\,(e\cos(dx+c))^{-m-2}\,(a\sin(dx+c)+a)^m}{dm^2-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (m*cos(d*x + c) - cos(d*x + c)*sin(d*x + c))*(e*cos(d*x + c))^(-m - 2)*(a*s
in(d*x + c) + a)^m/(d*m^2 - d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-2}\,(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 2)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2-m}\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-2}\,(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 2)*(a*sin(d*x + c) + a)^m, x)
```

mupad [B] time = 5.60, size = 71, normalized size = 0.80

$$-\frac{(\sin(2c+2dx) - 2m\cos(c+dx))\,(a\,(\sin(c+dx)+1))^m}{d\,e^2\,(\cos(2c+2dx)+1)\,(e\,\cos(c+dx))^m\,\left(m^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 2),x)
```

```
[Out] -((sin(2*c + 2*d*x) - 2*m*cos(c + d*x))*(a*(sin(c + d*x) + 1))^m)/(d*e^2*(c
os(2*c + 2*d*x) + 1)*(e*cos(c + d*x))^m*(m^2 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-2-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.363 $\int (e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=34

$$\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m}}{dem}$$

```
[Out] (a+a*sin(d*x+c))^m/d/e/m/((e*cos(d*x+c))^m)
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2671}

$$\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m}}{dem}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^m,x]
[Out] (a + a*Sin[c + d*x])^m/(d*e*m*(e*Cos[c + d*x])^m)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m\,dx = \frac{(e\cos(c+dx))^{-m}(a+a\sin(c+dx))^m}{dem}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 1.00

$$\frac{(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-m}}{dem}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^m,x]
[Out] (a*(1 + Sin[c + d*x]))^m/(d*e*m*(e*Cos[c + d*x])^m)
```

fricas [A] time = 0.46, size = 39, normalized size = 1.15

$$\frac{(e\cos(dx+c))^{-m-1}(a\sin(dx+c)+a)^m\cos(dx+c)}{dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] (e*cos(d*x + c))^(-m - 1)*(a*sin(d*x + c) + a)^m*cos(d*x + c)/(d*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-1}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 1)*(a*sin(d*x + c) + a)^m, x)
```

maple [**F**] time = 0.30, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-1-m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x)
```

maxima [**A**] time = 0.46, size = 65, normalized size = 1.91

$$\frac{a^m e^{-m-1} e^{\left(m\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)-m\log\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)\right)}}{dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] a^m*e^(-m - 1)*e^(m*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1) - m*log(-sin(d
*x + c)/(cos(d*x + c) + 1) + 1))/(d*m)
```

mupad [**B**] time = 0.29, size = 34, normalized size = 1.00

$$\frac{(a\;(\sin(c+d\,x)+1))^m}{d\,e\,m\,(e\;\cos(c+d\,x))^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 1),x)
```

```
[Out] (a*(sin(c + d*x) + 1))^m/(d*e*m*(e*cos(c + d*x))^m)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-1-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.364 $\int (e\cos(c+dx))^{-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=115

$$-\frac{a2^{\frac{m}{2}+\frac{1}{2}}(\sin(c+dx)+1)^{\frac{1-m}{2}}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{1-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(1-m)}$$

```
[Out] -2^(1/2+1/2*m)*a*(e*cos(d*x+c))^(1-m)*hypergeom([1/2-1/2*m, 1/2-1/2*m],[3/2
-1/2*m],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(1/2-1/2*m)*(a+a*sin(d*x+c))^(-1
+m)/d/e/(1-m)
```

Rubi [A] time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2689, 70, 69}

$$-\frac{a2^{\frac{m}{2}+\frac{1}{2}}(\sin(c+dx)+1)^{\frac{1-m}{2}}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{1-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
```

```
[Out] -((2^(1/2 + m/2)*a*(e*Cos[c + d*x])^(1 - m)*Hypergeometric2F1[(1 - m)/2, (1
 - m)/2, (3 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((1 - m)/2)*(a
 + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 - m)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{1-m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+m)}(a+a\sin(c+dx)\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{m}{2}}a^2(e\cos(c+dx))^{1-m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+m)}(a+a\sin(\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+\frac{m}{2}}a(e\cos(c+dx))^{1-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c+dx)\right.}{de(1-m)}$$

Mathematica [A] time = 0.12, size = 108, normalized size = 0.94

$$\frac{2^{\frac{m+1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-m-1)}(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c\right.}{d(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
[Out] (2^((1 + m)/2)*Cos[c + d*x]*Hypergeometric2F1[(1 - m)/2, (1 - m)/2, (3 - m)
/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - m)/2)*(a*(1 + Sin[c + d
*x]))^m)/(d*(-1 + m)*(e*Cos[c + d*x])^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int (a+a\sin(dx+c))^m\,(e\cos(dx+c))^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
[Out] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\ \sin(c + d\,x))^m}{(e\ \cos(c + d\,x))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^m,x)
```

```
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c + dx) + 1))^m (e\cos(c + dx))^{-m}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/((e*cos(d*x+c))**m),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(-m), x)
```

3.365 $\int (e\cos(c+dx))^{1-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=97

$$\frac{2^{1-\frac{m}{2}}(1-\sin(c+dx))^{\frac{m}{2}-1}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{2-m}\,{}_2F_1\left(\frac{m}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de(m+2)}$$

```
[Out] 2^(1-1/2*m)*(e*cos(d*x+c))^(2-m)*hypergeom([1/2*m, 1+1/2*m],[2+1/2*m],1/2+1
/2*sin(d*x+c))*(1-sin(d*x+c))^(-1+1/2*m)*(a+a*sin(d*x+c))^m/d/e/(2+m)
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$\frac{2^{1-\frac{m}{2}}(1-\sin(c+dx))^{\frac{m}{2}-1}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{2-m}\,{}_2F_1\left(\frac{m}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(1 - m/2)*(e*Cos[c + d*x])^(2 - m)*Hypergeometric2F1[m/2, (2 + m)/2, (4
+ m)/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-1 + m/2)*(a + a*Sin[c +
d*x])^m)/(d*e*(2 + m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{1-m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{2-m}(a-a\sin(c+dx))^{\frac{1}{2}(-2+m)}(a+a\sin(c+dx))^{\frac{1}{2}}\right.}{de}$$

$$= \frac{\left(2^{-m/2}a^2(e\cos(c+dx))^{2-m}(a-a\sin(c+dx))^{\frac{1}{2}(-2+m)-\frac{m}{2}}\left(\frac{a-a\sin(c+dx}{a}\right.\right.}{}$$

$$= \frac{2^{1-\frac{m}{2}}(e\cos(c+dx))^{2-m}\,{}_2F_1\left(\frac{m}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1-s}{de(2+m)}$$

Mathematica [A] time = 0.26, size = 97, normalized size = 1.00

$$\frac{2^{\frac{m}{2}+1}(\sin(c+dx)+1)^{-\frac{m}{2}-1}(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{2-m}\,{}_2F_1\left(1-\frac{m}{2},-\frac{m}{2};2-\frac{m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(m-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(1 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(1 + m/2)*(e*Cos[c + d*x])^(2 - m)*Hypergeometric2F1[1 - m/2, -1/2*m, 2
- m/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1 - m/2)*(a*(1 + Sin[c +
d*x]))^m)/(d*e*(-2 + m))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-m+1}(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m + 1)*(a*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m+1}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 1)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{1-m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m+1}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 1)*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+d\,x))^{1-m}\,(a+a\sin(c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1 - m)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] int((e*cos(c + d*x))^(1 - m)*(a + a*sin(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m\,(e\cos(c+dx))^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(1 - m), x)
```

3.366 $\int (e\cos(c+dx))^{2-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=115

$$-\frac{a2^{\frac{m}{2}+\frac{3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-m-1)}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{3-m}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(3-m)}$$

```
[Out] -2^(3/2+1/2*m)*a*(e*cos(d*x+c))^(3-m)*hypergeom([3/2-1/2*m, -1/2-1/2*m],[5/
2-1/2*m],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*m)*(a+a*sin(d*x+c))^(
-1+m)/d/e/(3-m)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{2689, 70, 69\}$

$$-\frac{a2^{\frac{m}{2}+\frac{3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-m-1)}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{3-m}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((2^(3/2 + m/2)*a*(e*Cos[c + d*x])^(3 - m)*Hypergeometric2F1[(-1 - m)/2, (
3 - m)/2, (5 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - m)/2)*
(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(3 - m)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{2-m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{3-m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+m)}(a+a\sin(c+dx\right.}{c}$$

$$= \frac{\left(2^{\frac{1}{2}+\frac{m}{2}}a^2(e\cos(c+dx))^{3-m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+m)}(a+a\sin(\right.}{}$$

$$= -\frac{2^{\frac{3}{2}+\frac{m}{2}}a(e\cos(c+dx))^{3-m}\,{}_2F_1\left(\frac{1}{2}(-1-m),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\sin(\right.}{de(3-}$$

Mathematica [A] time = 0.24, size = 113, normalized size = 0.98

$$\frac{e^2 2^{\frac{m+3}{2}}\cos^3(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-m-3)}(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-m}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}\right.}{d(m-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^((3 + m)/2)*e^2*Cos[c + d*x]^3*Hypergeometric2F1[(-1 - m)/2, (3 - m)/2,
(5 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-3 - m)/2)*(a*(1 + Si
n[c + d*x]))^m)/(d*(-3 + m)*(e*Cos[c + d*x])^m)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-m+2}(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m + 2)*(a*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m+2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 2)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{2-m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m+2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 2)*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+dx))^{2-m}\,(a+a\sin(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(2 - m)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] int((e*cos(c + d*x))^(2 - m)*(a + a*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.367 $\int (e\cos(c+dx))^{5-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=150

$$-\frac{8a^3(a\sin(c+dx)+a)^{m-3}(e\cos(c+dx))^{6-2m}}{de(5-m)\left(m^2-7m+12\right)}-\frac{4a^2(a\sin(c+dx)+a)^{m-2}(e\cos(c+dx))^{6-2m}}{de\left(m^2-9m+20\right)}-\frac{a(a\sin(c+dx)}{}$$

```
[Out] -8*a^3*(e*cos(d*x+c))^(6-2*m)*(a+a*sin(d*x+c))^(-3+m)/d/e/(-m^3+12*m^2-47*m
+60)-4*a^2*(e*cos(d*x+c))^(6-2*m)*(a+a*sin(d*x+c))^(-2+m)/d/e/(4-m)/(5-m)-a
*(e*cos(d*x+c))^(6-2*m)*(a+a*sin(d*x+c))^(-1+m)/d/e/(5-m)
```

Rubi [A] time = 0.24, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2674, 2673}

$$-\frac{8a^3(a\sin(c+dx)+a)^{m-3}(e\cos(c+dx))^{6-2m}}{de(5-m)\left(m^2-7m+12\right)}-\frac{4a^2(a\sin(c+dx)+a)^{m-2}(e\cos(c+dx))^{6-2m}}{de\left(m^2-9m+20\right)}-\frac{a(a\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (-8*a^3*(e*Cos[c + d*x])^(6 - 2*m)*(a + a*Sin[c + d*x])^(-3 + m))/(d*e*(5 -
 m)*(12 - 7*m + m^2)) - (4*a^2*(e*Cos[c + d*x])^(6 - 2*m)*(a + a*Sin[c + d*
x])^(-2 + m))/(d*e*(20 - 9*m + m^2)) - (a*(e*Cos[c + d*x])^(6 - 2*m)*(a + a
*Sin[c + d*x])^(-1 + m))/(d*e*(5 - m))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{5-2m}(a+a\sin(c+dx))^m\,dx &= -\frac{a(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-1+m}}{de(5-m)}+\frac{(4a)\int (e\cos(c}{}\\ &= -\frac{4a^2(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-2+m}}{de\left(20-9m+m^2\right)}-\frac{a(e\cos(c+}{}\\ &= -\frac{8a^3(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-3+m}}{de(3-m)\left(20-9m+m^2\right)}-\frac{4a^2(e\cos(c}{}\end{aligned}$$

Mathematica [A] time = 0.39, size = 105, normalized size = 0.70

$$\frac{e^5\cos^6(c+dx)\left(\left(m^2-7m+12\right)\sin^2(c+dx)+2\left(m^2-9m+18\right)\sin(c+dx)+m^2-11m+32\right)(a(\sin(c+dx)}{d(m-5)(m-4)(m-3)(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (e^5*Cos[c + d*x]^6*(a*(1 + Sin[c + d*x]))^m*(32 - 11*m + m^2 + 2*(18 - 9*m
 + m^2)*Sin[c + d*x] + (12 - 7*m + m^2)*Sin[c + d*x]^2))/(d*(-5 + m)*(-4 +
m)*(-3 + m)*(e*Cos[c + d*x])^(2*m)*(1 + Sin[c + d*x])^3)
```

fricas [B] time = 0.48, size = 314, normalized size = 2.09

$$-\frac{\left(\left(m^2-7\,m+12\right)\cos\left(dx+c\right)^3-\left(m^2-11\,m+24\right)\cos\left(dx+c\right)\right.}{4\,dm^3-\left(dm^3-12\,dm^2+47\,dm-60\,d\right)\cos\left(dx+c\right)^3-48\,dm^2-3\left(dm^3-12\,dm^2+47\,dm-60\,d\right)\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -((m^2 - 7*m + 12)*cos(d*x + c)^3 - (m^2 - 11*m + 24)*cos(d*x + c)^2 - 2*(m
^2 - 9*m + 22)*cos(d*x + c) - ((m^2 - 7*m + 12)*cos(d*x + c)^2 + 2*(m^2 - 9
*m + 18)*cos(d*x + c) - 8)*sin(d*x + c) - 8)*(e*cos(d*x + c))^(-2*m + 5)*(a
*sin(d*x + c) + a)^m/(4*d*m^3 - (d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x
+ c)^3 - 48*d*m^2 - 3*(d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c)^2 + 1
88*d*m + 2*(d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c) + (4*d*m^3 - 48*
d*m^2 - (d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c)^2 + 188*d*m + 2*(d*
m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c) - 240*d)*sin(d*x + c) - 240*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{-2\,m+5}\left(a\sin\left(dx+c\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 5)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 1.68, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{5-2m}\left(a+a\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x)
```

maxima [B] time = 1.39, size = 624, normalized size = 4.16

$$\frac{\left(\left(m^2-11\,m+32\right)a^me^5-\frac{2\left(m^2-15\,m+60\right)a^me^5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(3\,m^2-m-160\right)a^me^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\left(m^2-7\,m-20\right)a^me^5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2\left(m^2+5\right.}{}\right.}{\left(\left(m^3-12\,m^2+47\,m-60\right)e^{2\,m}+\frac{5\left(m^3-12\right.}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((m^2 - 11*m + 32)*a^m*e^5 - 2*(m^2 - 15*m + 60)*a^m*e^5*sin(d*x + c)/(cos(
d*x + c) + 1) - (3*m^2 - m - 160)*a^m*e^5*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 8*(m^2 - 7*m - 20)*a^m*e^5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(m^
2 + 5*m + 160)*a^m*e^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(3*m^2 - 13*
```

```
m + 116)*a^m*e^5*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*(m^2 + 5*m + 160)*
a^m*e^5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*(m^2 - 7*m - 20)*a^m*e^5*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 - (3*m^2 - m - 160)*a^m*e^5*sin(d*x + c)^
8/(cos(d*x + c) + 1)^8 - 2*(m^2 - 15*m + 60)*a^m*e^5*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 + (m^2 - 11*m + 32)*a^m*e^5*sin(d*x + c)^10/(cos(d*x + c) + 1
)^10)*e^(-2*m*log(-sin(d*x + c)/(cos(d*x + c) + 1) + 1) + m*log(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1))/(((m^3 - 12*m^2 + 47*m - 60)*e^(2*m) + 5*(m^
3 - 12*m^2 + 47*m - 60)*e^(2*m)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*(m
^3 - 12*m^2 + 47*m - 60)*e^(2*m)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*(
m^3 - 12*m^2 + 47*m - 60)*e^(2*m)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*(
m^3 - 12*m^2 + 47*m - 60)*e^(2*m)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + (m^
3 - 12*m^2 + 47*m - 60)*e^(2*m)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

mupad [B] time = 13.48, size = 601, normalized size = 4.01

$$-\frac{(a+a\,\sin(c+d\,x))^m\left(-\frac{(e\,\cos(c+d\,x))^{5-2\,m}\left(m^2-7\,m+12\right)}{d\left(m^3-12\,m^2+47\,m-60\right)}+\frac{(e\,\cos(c+d\,x))^{5-2\,m}\,(\cos(c+d\,x)+\sin(c+d\,x)\,1\mathrm{i})\left(m^2\,3\mathrm{i}-m\,29\mathrm{i}+60\mathrm{i}\right)}{d\left(m^3-12\,m^2+47\,m-60\right)}-\frac{(e\,\mathrm{c}}{}\right.}{5\,\cos(c+d\,x)+\sin(c+d\,x)\,5\mathrm{i}-10\,\cos(3\,c+3\,d\,x)+\cos(5\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5 - 2*m)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] -((a + a*sin(c + d*x))^m*(((e*cos(c + d*x))^(5 - 2*m)*(cos(c + d*x) + sin(c
 + d*x)*1i)*(m^2*3i - m*29i + 60i))/(d*(47*m - 12*m^2 + m^3 - 60)) - ((e*co
s(c + d*x))^(5 - 2*m)*(m^2 - 7*m + 12))/(d*(47*m - 12*m^2 + m^3 - 60)) - ((
e*cos(c + d*x))^(5 - 2*m)*(cos(5*c + 5*d*x) + sin(5*c + 5*d*x)*1i)*(m^2*1i
- m*7i + 12i))/(d*(47*m - 12*m^2 + m^3 - 60)) + ((e*cos(c + d*x))^(5 - 2*m)
*(cos(4*c + 4*d*x) + sin(4*c + 4*d*x)*1i)*(3*m^2 - 29*m + 60))/(d*(47*m - 1
2*m^2 + m^3 - 60)) + ((e*cos(c + d*x))^(5 - 2*m)*(cos(2*c + 2*d*x) + sin(2*
c + 2*d*x)*1i)*(2*m^2 - 22*m + 80))/(d*(47*m - 12*m^2 + m^3 - 60)) + ((e*co
s(c + d*x))^(5 - 2*m)*(cos(3*c + 3*d*x) + sin(3*c + 3*d*x)*1i)*(m^2*2i - m*
22i + 80i))/(d*(47*m - 12*m^2 + m^3 - 60))))/(5*cos(c + d*x) + sin(c + d*x)
*5i - 10*cos(3*c + 3*d*x) + cos(5*c + 5*d*x) - sin(3*c + 3*d*x)*10i + sin(5
*c + 5*d*x)*1i + (m*47i - m^2*12i + m^3*1i - 60i)/(47*m - 12*m^2 + m^3 - 60
) - (10*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i)*(m*47i - m^2*12i + m^3*1i
- 60i))/(47*m - 12*m^2 + m^3 - 60) + (5*(cos(4*c + 4*d*x) + sin(4*c + 4*d*x
)*1i)*(m*47i - m^2*12i + m^3*1i - 60i))/(47*m - 12*m^2 + m^3 - 60))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.368 $\int (e\cos(c+dx))^{3-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=94

$$-\frac{2a^2(a\sin(c+dx)+a)^{m-2}(e\cos(c+dx))^{4-2m}}{de\left(m^2-5m+6\right)}-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{4-2m}}{de(3-m)}$$

```
[Out] -2*a^2*(e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^(-2+m)/d/e/(2-m)/(3-m)-a*(e*
cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^(-1+m)/d/e/(3-m)
```

Rubi [A] time = 0.14, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2674, 2673}

$$-\frac{2a^2(a\sin(c+dx)+a)^{m-2}(e\cos(c+dx))^{4-2m}}{de\left(m^2-5m+6\right)}-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{4-2m}}{de(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (-2*a^2*(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^(-2 + m))/(d*e*(6 -
 5*m + m^2)) - (a*(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^(-1 + m))
/(d*e*(3 - m))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{3-2m}(a+a\sin(c+dx))^m\,dx &= -\frac{a(e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^{-1+m}}{de(3-m)}+\frac{(2a)\int (e\cos(c+\end{aligned}$$

$$= -\frac{2a^2(e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^{-2+m}}{de\left(6-5m+m^2\right)}-\frac{a(e\cos(c+dx)}{}$$

Mathematica [A] time = 0.22, size = 72, normalized size = 0.77

$$\frac{e^3\cos^4(c+dx)((m-2)\sin(c+dx)+m-4)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}}{d(m-3)(m-2)(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (e^3*Cos[c + d*x]^4*(a*(1 + Sin[c + d*x]))^m*(-4 + m + (-2 + m)*Sin[c + d*x
]))/(d*(-3 + m)*(-2 + m)*(e*Cos[c + d*x])^(2*m)*(1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.46, size = 170, normalized size = 1.81

$$\frac{\left((m-2)\cos(dx+c)^2+(m-4)\cos(dx+c)+((m-2)\cos(dx+c)+2)\sin(dx+c)-2\right)(e\cos\cdots}{2\,dm^2-\left(dm^2-5\,dm+6\,d\right)\cos(dx+c)^2-10\,dm+\left(dm^2-5\,dm+6\,d\right)\cos(dx+c)+\left(2\,dm^2-10\,dm+\left(dm^{\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m - 2)*cos(d*x + c)^2 + (m - 4)*cos(d*x + c) + ((m - 2)*cos(d*x + c) + 2)
*sin(d*x + c) - 2)*(e*cos(d*x + c))^(-2*m + 3)*(a*sin(d*x + c) + a)^m/(2*d*
m^2 - (d*m^2 - 5*d*m + 6*d)*cos(d*x + c)^2 - 10*d*m + (d*m^2 - 5*d*m + 6*d)
*cos(d*x + c) + (2*d*m^2 - 10*d*m + (d*m^2 - 5*d*m + 6*d)*cos(d*x + c) + 12
*d)*sin(d*x + c) + 12*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2\,m+3}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 3)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 1.57, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{3-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x)
```

maxima [B] time = 1.38, size = 351, normalized size = 3.73

$$\frac{\left(a^m e^3(m-4)-\frac{2\,a^m e^3(m-6)\sin(dx+c)}{\cos(dx+c)+1}-\frac{a^m e^3(m+12)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4\,a^m e^3(m+2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a^m e^3(m+12)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2\,a^m e^3(m-6)\cdots}{(\cos(dx+\cdots}\right.}{\left(\left(m^2-5\,m+6\right)e^{2\,m}+\frac{3\left(m^2-5\,m+6\right)e^{2\,m}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(m^2-5\,m+6\right)e^{2\,m}\sin(dx+\cdots}{(\cos(dx+c)+1)^4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (a^m*e^3*(m - 4) - 2*a^m*e^3*(m - 6)*sin(d*x + c)/(cos(d*x + c) + 1) - a^m*
e^3*(m + 12)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^m*e^3*(m + 2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 - a^m*e^3*(m + 12)*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 - 2*a^m*e^3*(m - 6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^m*e^3
*(m - 4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*e^(-2*m*log(-sin(d*x + c)/(co
s(d*x + c) + 1) + 1) + m*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1))/(((m
^2 - 5*m + 6)*e^(2*m) + 3*(m^2 - 5*m + 6)*e^(2*m)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 3*(m^2 - 5*m + 6)*e^(2*m)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
+ (m^2 - 5*m + 6)*e^(2*m)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d)
```

mupad [B] time = 8.77, size = 241, normalized size = 2.56

$$\frac{e^3 (a\ (\sin(c+d\,x)+1))^m\ \left(14\,m-24\ \sin(c+d\,x)-36\ \sin(3\,c+3\,d\,x)-12\ \sin(5\,c+5\,d\,x)+24\sin(2\,c+2\,d\,x)\right.}{8\,d\left(-e\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3 - 2*m)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] (e^3*(a*(sin(c + d*x) + 1))^m*(14*m - 24*sin(c + d*x) - 36*sin(3*c + 3*d*x)
 - 12*sin(5*c + 5*d*x) + 24*sin(2*c + 2*d*x)^2 - 4*sin(3*c + 3*d*x)^2 + 8*m
*sin(c + d*x) - 17*m*(2*sin(c + d*x)^2 - 1) + 12*m*sin(3*c + 3*d*x) + 4*m*s
in(5*c + 5*d*x) - 2*m*(2*sin(2*c + 2*d*x)^2 - 1) + m*(2*sin(3*c + 3*d*x)^2
- 1) + 132*sin(c + d*x)^2 - 128))/(8*d*(-e*(2*sin(c/2 + (d*x)/2)^2 - 1))^(2
*m)*(m^2 - 5*m + 6)*(15*sin(c + d*x) - sin(3*c + 3*d*x) + 12*sin(c + d*x)^2
 + 4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.369 $\int (e\cos(c+dx))^{1-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=44

$$-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{2-2m}}{de(1-m)}$$

```
[Out] -a*(e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^(-1+m)/d/e/(1-m)
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2673}

$$-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{2-2m}}{de(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((a*(e*Cos[c + d*x])^(2 - 2*m)*(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 - m)
))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (e\cos(c+dx))^{1-2m}(a+a\sin(c+dx))^m\,dx=-\frac{a(e\cos(c+dx))^{2-2m}(a+a\sin(c+dx))^{-1+m}}{de(1-m)}$$

Mathematica [A] time = 0.15, size = 43, normalized size = 0.98

$$-\frac{e(\sin(c+dx)-1)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}}{d(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((e*(-1 + Sin[c + d*x])*(a*(1 + Sin[c + d*x]))^m)/(d*(-1 + m)*(e*Cos[c + d
*x])^(2*m)))
```

fricas [A] time = 0.49, size = 80, normalized size = 1.82

$$\frac{(e\cos(dx+c))^{-2m+1}(a\sin(dx+c)+a)^m(\cos(dx+c)-\sin(dx+c)+1)}{dm+(dm-d)\cos(dx+c)+(dm-d)\sin(dx+c)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (e*cos(d*x + c))^(-2*m + 1)*(a*sin(d*x + c) + a)^m*(cos(d*x + c) - sin(d*x
+ c) + 1)/(d*m + (d*m - d)*cos(d*x + c) + (d*m - d)*sin(d*x + c) - d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m+1}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(-2*m + 1)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 2.20, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{1-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x)
```

maxima [B] time = 0.95, size = 144, normalized size = 3.27

$$\frac{\left(a^m e - \frac{2\,a^m e\sin(dx+c)}{\cos(dx+c)+1} + \frac{a^m e\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)e^{\left(-2\,m\log\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)+m\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)\right)}}{\left(e^{2\,m}(m-1) + \frac{e^{2\,m}(m-1)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] (a^m*e - 2*a^m*e*sin(d*x + c)/(cos(d*x + c) + 1) + a^m*e*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2)*e^(-2*m*log(-sin(d*x + c)/(cos(d*x + c) + 1) + 1) + m*lo
g(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1))/((e^(2*m)*(m - 1) + e^(2*m)*(m
- 1)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d)
```

mupad [B] time = 5.58, size = 58, normalized size = 1.32

$$\frac{e\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)}^m}{2\,d\,{\left(e\,\cos\left(c+d\,x\right)\right)}^{2\,m}\,\left(m-1\right)\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1 - 2*m)*(a + a*sin(c + d*x))^m,x)
[Out] (e*(cos(2*c + 2*d*x) + 1)*(a*(sin(c + d*x) + 1))^m)/(2*d*(e*cos(c + d*x))^(
2*m)*(m - 1)*(sin(c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m(e\cos(c+dx))^{1-2m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1-2*m)*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(1 - 2*m), x)
```

3.370 $\int (e\cos(c+dx))^{-1-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=61

$$\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)}{2dem}$$

```
[Out] 1/2*hypergeom([1, -m],[1-m],1/2-1/2*sin(d*x+c))*(a+a*sin(d*x+c))^m/d/e/m/((
e*cos(d*x+c))^(2*m))
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 7, 68}

$$\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)}{2dem}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, -m, 1 - m, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])
^m)/(2*d*e*m*(e*Cos[c + d*x])^(2*m))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-1-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{-2m}(a-a\sin(c+dx))^m(a+a\sin(c+dx))^m\right.}{}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{-2m}(a-a\sin(c+dx))^m(a+a\sin(c+dx))^m\right.}{de}$$

$$= \frac{(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)(a+}{2dem}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 1.00

$$\frac{(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)}{2dem}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, -m, 1 - m, (1 - Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]
))^m)/(2*d*e*m*(e*Cos[c + d*x])^(2*m))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-2m-1}(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-2*m - 1)*(a*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m-1}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 1)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 1.33, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-1-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m-1}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 1)*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{2\,m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 1),x)
```

```
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-1-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.371 $\int (e\cos(c+dx))^{-3-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=70

$$\frac{(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-2(m+1)}\,{}_2F_1\left(2,-m-1;-m;\frac{1}{2}(1-\sin(c+dx))\right)}{4ade(m+1)}$$

```
[Out] 1/4*hypergeom([2, -1-m],[-m],1/2-1/2*sin(d*x+c))*(a+a*sin(d*x+c))^(1+m)/a/d
/e/(1+m)/((e*cos(d*x+c))^(2+2*m))
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 7, 68}

$$\frac{(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-2(m+1)}\,{}_2F_1\left(2,-m-1;-m;\frac{1}{2}(1-\sin(c+dx))\right)}{4ade(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-3 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, -1 - m, -m, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x]
)^(1 + m))/(4*a*d*e*(1 + m)*(e*Cos[c + d*x])^(2*(1 + m)))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 && !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-3-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{-2-2m}(a-a\sin(c+dx))^{\frac{1}{2}(2+2m)}(a+a\sin(c+d\right.}{}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{-2-2m}(a-a\sin(c+dx))^{\frac{1}{2}(2+2m)}(a+a\sin(c+d\right.}{de}$$

$$= \frac{(e\cos(c+dx))^{-2(1+m)}\,{}_2F_1\left(2,-1-m;-m;\frac{1}{2}(1-\sin(c+dx))\right)(a}{4ade(1+m)}$$

Mathematica [A] time = 0.13, size = 76, normalized size = 1.09

$$\frac{\sec^2(c+dx)(a(\sin(c+dx)+1))^{m+1}(e\cos(c+dx))^{-2m}\,{}_2F_1\left(2,-m-1;-m;\frac{1}{2}(1-\sin(c+dx))\right)}{4ade^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-3 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, -1 - m, -m, (1 - Sin[c + d*x])/2]*Sec[c + d*x]^2*(a*(
1 + Sin[c + d*x]))^(1 + m))/(4*a*d*e^3*(1 + m)*(e*Cos[c + d*x])^(2*m))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((e\cos(dx+c))^{-2m-3}(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-2*m - 3)*(a*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m-3}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 3)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 1.37, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-3-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m-3}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 3)*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{2\,m+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 3),x)
```

```
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-3-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.372 $\int (e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=89

$$\frac{2^{\frac{5}{2}-m}(1-\sin(c+dx))^{m-\frac{5}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{5-2m}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(2m-3);\frac{7}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{5de}$$

```
[Out] 1/5*2^(5/2-m)*(e*cos(d*x+c))^(5-2*m)*hypergeom([5/2, -3/2+m],[7/2],1/2+1/2*
sin(d*x+c))*(1-sin(d*x+c))^(-5/2+m)*(a+a*sin(d*x+c))^m/d/e
```

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = $\{2689, 7, 70, 69\}$

$$\frac{2^{\frac{5}{2}-m}(1-\sin(c+dx))^{m-\frac{5}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{5-2m}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(2m-3);\frac{7}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(5/2 - m)*(e*Cos[c + d*x])^(5 - 2*m)*Hypergeometric2F1[5/2, (-3 + 2*m)/2
, 7/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-5/2 + m)*(a + a*Sin[c + d
*x])^m)/(5*d*e)
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{5-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-5+2m)}(a+a\sin(c+dx\right.}{}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{5-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-5+2m)}(a+a\sin(c+dx\right.}{d}$$

$$= \frac{\left(2^{\frac{3}{2}-m}a^3(e\cos(c+dx))^{5-2m}(a-a\sin(c+dx))^{\frac{1}{2}-m+\frac{1}{2}(-5+2m)}\left(\frac{a-a\sin}{}\right.\right.}{}$$

$$= \frac{2^{\frac{5}{2}-m}(e\cos(c+dx))^{5-2m}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(-3+2m);\frac{7}{2};\frac{1}{2}(1+\sin(c+dx))\right.}{5de}$$

Mathematica [A] time = 0.20, size = 96, normalized size = 1.08

$$\frac{4\sqrt{2}e^4\cos^5(c+dx)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;\frac{7}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)}{d(2m-5)(\sin(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (4*Sqrt[2]*e^4*Cos[c + d*x]^5*Hypergeometric2F1[-3/2, 5/2 - m, 7/2 - m, (1
- Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^m)/(d*(-5 + 2*m)*(e*Cos[c + d*x])
^(2*m)*(1 + Sin[c + d*x])^(5/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-2m+4}(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-2*m + 4)*(a*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m+4}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 4)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 1.84, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{4-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m+4}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(-2*m + 4)*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+d\,x))^{4-2\,m}\,(a+a\,\sin(c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(4 - 2*m)*(a + a*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(4 - 2*m)*(a + a*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(4-2*m)*(a+a*sin(d*x+c))**m,x)
[Out] Timed out
```

3.373 $\int (e\cos(c+dx))^{2-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=89

$$\frac{2^{\frac{3}{2}-m}(1-\sin(c+dx))^{m-\frac{3}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{3-2m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(2m-1);\frac{5}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{3de}$$

```
[Out] 1/3*2^(3/2-m)*(e*cos(d*x+c))^(3-2*m)*hypergeom([3/2, -1/2+m],[5/2],1/2+1/2*
sin(d*x+c))*(1-sin(d*x+c))^(-3/2+m)*(a+a*sin(d*x+c))^m/d/e
```

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2689, 7, 70, 69}

$$\frac{2^{\frac{3}{2}-m}(1-\sin(c+dx))^{m-\frac{3}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{3-2m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(2m-1);\frac{5}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(3/2 - m)*(e*Cos[c + d*x])^(3 - 2*m)*Hypergeometric2F1[3/2, (-1 + 2*m)/2
, 5/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-3/2 + m)*(a + a*Sin[c + d
*x])^m)/(3*d*e)
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 && !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{2-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{3-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+2m)}(a+a\sin(c+\right.}{}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{3-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+2m)}(a+a\sin(c+\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}-m}a^2(e\cos(c+dx))^{3-2m}(a-a\sin(c+dx))^{\frac{1}{2}-m+\frac{1}{2}(-3+2m)}\left(\frac{a-}{}\right.\right.}{}$$

$$= \frac{2^{\frac{3}{2}-m}(e\cos(c+dx))^{3-2m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1+2m);\frac{5}{2};\frac{1}{2}(1+\sin(c+d\right.}{3de}$$

Mathematica [A] time = 0.19, size = 96, normalized size = 1.08

$$\frac{2\sqrt{2}e^2\cos^3(c+dx)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{5}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)}{d(2m-3)(\sin(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2*Sqrt[2]*e^2*Cos[c + d*x]^3*Hypergeometric2F1[-1/2, 3/2 - m, 5/2 - m, (1
- Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^m)/(d*(-3 + 2*m)*(e*Cos[c + d*x])
^(2*m)*(1 + Sin[c + d*x])^(3/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-2m+2}(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-2*m + 2)*(a*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m+2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 2)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 1.71, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{2-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m+2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 2)*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{2-2\,m}\,(a+a\,\sin(c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(2 - 2*m)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] int((e*cos(c + d*x))^(2 - 2*m)*(a + a*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.374 $\int (e\cos(c+dx))^{-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=86

$$\frac{2^{\frac{1}{2}-m}(1-\sin(c+dx))^{m-\frac{1}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{1-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2m+1);\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de}$$

```
[Out] 2^(1/2-m)*(e*cos(d*x+c))^(1-2*m)*hypergeom([1/2, 1/2+m],[3/2],1/2+1/2*sin(d
*x+c))*(1-sin(d*x+c))^(-1/2+m)*(a+a*sin(d*x+c))^m/d/e
```

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2689, 7, 70, 69}

$$\frac{2^{\frac{1}{2}-m}(1-\sin(c+dx))^{m-\frac{1}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{1-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2m+1);\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(2*m),x]
```

```
[Out] (2^(1/2 - m)*(e*Cos[c + d*x])^(1 - 2*m)*Hypergeometric2F1[1/2, (1 + 2*m)/2,
 3/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-1/2 + m)*(a + a*Sin[c + d*
x])^m)/(d*e)
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 && !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+2m)}(a+a\sin(c+dx)\right.}{}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+2m)}(a+a\sin(c+dx)\right.}{de}$$

$$= \frac{\left(2^{-\frac{1}{2}-m}a^2(e\cos(c+dx))^{1-2m}(a-a\sin(c+dx))^{-\frac{1}{2}-m+\frac{1}{2}(-1+2m)}\left(\frac{a-a\sin}{}\right.\right.}{}$$

$$= \frac{2^{\frac{1}{2}-m}(e\cos(c+dx))^{1-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2m);\frac{3}{2};\frac{1}{2}(1+\sin(c+dx))\right)(}{de}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 1.05

$$\frac{\sqrt{2}\cos(c+dx)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)}{d(2m-1)\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(2*m),x]
[Out] (Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2 - m, (1 - Sin[c +
 d*x])/2]*(a*(1 + Sin[c + d*x]))^m)/(d*(-1 + 2*m)*(e*Cos[c + d*x])^(2*m)*Sq
rt[1 + Sin[c + d*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^{2m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(2*m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^{2m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(2*m), x)
```

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int (a+a\sin(dx+c))^m\,(e\cos(dx+c))^{-2m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^{2m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(2*m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{2m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m),x)
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m (e\cos(c+dx))^{-2m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/((e*cos(d*x+c))**(2*m)),x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(-2*m), x)
```

3.375 $\int (e\cos(c+dx))^{-2-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=87

$$-\frac{2^{-m-\frac{1}{2}}(1-\sin(c+dx))^{m+\frac{1}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-2m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(2m+3);\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de}$$

```
[Out] -2^(-1/2-m)*(e*cos(d*x+c))^(-1-2*m)*hypergeom([-1/2, 3/2+m],[1/2],1/2+1/2*s
in(d*x+c))*(1-sin(d*x+c))^(1/2+m)*(a+a*sin(d*x+c))^m/d/e
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2689, 7, 70, 69}

$$-\frac{2^{-m-\frac{1}{2}}(1-\sin(c+dx))^{m+\frac{1}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-2m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(2m+3);\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((2^(-1/2 - m)*(e*Cos[c + d*x])^(-1 - 2*m)*Hypergeometric2F1[-1/2, (3 + 2*
m)/2, 1/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(1/2 + m)*(a + a*Sin[c
+ d*x])^m)/(d*e))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-2-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{-1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(1+2m)}(a+a\sin(c\right.}{}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{-1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(1+2m)}(a+a\sin(c\right.}{de}$$

$$= \frac{\left(2^{-\frac{3}{2}-m}a(e\cos(c+dx))^{-1-2m}(a-a\sin(c+dx))^{-\frac{1}{2}-m+\frac{1}{2}(1+2m)}\right.}{}$$

$$= -\frac{2^{-\frac{1}{2}-m}(e\cos(c+dx))^{-1-2m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(3+2m);\frac{1}{2};\frac{1}{2}(1+\sin(\right.}{de}$$

Mathematica [A] time = 0.22, size = 87, normalized size = 1.00

$$\frac{\sqrt{\sin(c+dx)+1}\,(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{1}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)}{\sqrt{2}\,e(2dm+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((e*Cos[c + d*x])^(-1 - 2*m)*Hypergeometric2F1[3/2, -1/2 - m, 1/2 - m, (1 -
 Sin[c + d*x])/2]*Sqrt[1 + Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^m)/(Sqrt[2]
*e*(d + 2*d*m))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-2m-2}(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-2*m - 2)*(a*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m-2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 2)*(a*sin(d*x + c) + a)^m, x)
```

maple [F] time = 1.29, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2m-2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(-2*m - 2)*(a*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{2\,m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 2),x)
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-2-2*m)*(a+a*sin(d*x+c))**m,x)
[Out] Timed out
```

3.376 $\int \cos^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^6(c+dx)}{6d}$$

```
[Out] -1/6*b*cos(d*x+c)^6/d+a*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^
5/d
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 641, 194}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^6)/(6*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\left(b^2-x^2\right)^2\,dx,x,b\sin(c+dx)\right)}{b^5d}\\
&= -\frac{b\cos^6(c+dx)}{6d}+\frac{a\,\text{Subst}\left(\int \left(b^2-x^2\right)^2\,dx,x,b\sin(c+dx)\right)}{b^5d}\\
&= -\frac{b\cos^6(c+dx)}{6d}+\frac{a\,\text{Subst}\left(\int \left(b^4-2b^2x^2+x^4\right)\,dx,x,b\sin(c+dx)\right)}{b^5d}\\
&= -\frac{b\cos^6(c+dx)}{6d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.00

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(b*Cos[c + d*x]^6)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.46, size = 51, normalized size = 0.85

$$-\frac{5\,b\cos\left(dx+c\right)^{6}-2\,\left(3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(5*b*cos(d*x + c)^6 - 2*(3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*
a)*sin(d*x + c))/d
```

giac [A] time = 0.92, size = 88, normalized size = 1.47

$$-\frac{b\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{b\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,b\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{a\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*b*cos(6*d*x + 6*c)/d - 1/32*b*cos(4*d*x + 4*c)/d - 5/64*b*cos(2*d*x
+ 2*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 5/8*a*si
n(d*x + c)/d
```

maple [A] time = 0.15, size = 46, normalized size = 0.77

$$\frac{-\frac{b\left(\cos^{6}\left(dx+c\right)\right)}{6}+\frac{a\left(\frac{8}{3}+\cos^{4}\left(dx+c\right)+\frac{4\left(\cos^{2}\left(dx+c\right)\right)}{3}\right)\sin\left(dx+c\right)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/6*b*cos(d*x+c)^6+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+
c))
```

maxima [A] time = 0.34, size = 70, normalized size = 1.17

$$\frac{5\,b\sin\left(dx+c\right)^{6}+6\,a\sin\left(dx+c\right)^{5}-15\,b\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}+15\,b\sin\left(dx+c\right)^{2}+30\,a\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*b*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 - 15*b*sin(d*x + c)^4 - 20*a*
sin(d*x + c)^3 + 15*b*sin(d*x + c)^2 + 30*a*sin(d*x + c))/d
```

mupad [B] time = 0.07, size = 68, normalized size = 1.13

$$\frac{\frac{b\sin(c+d\,x)^6}{6}+\frac{a\sin(c+d\,x)^5}{5}-\frac{b\sin(c+d\,x)^4}{2}-\frac{2\,a\sin(c+d\,x)^3}{3}+\frac{b\sin(c+d\,x)^2}{2}+a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) - (2*a*sin(c + d*x)^3)/3 + (a*sin(c + d*x)^5)/5 + (b*sin(c
+ d*x)^2)/2 - (b*sin(c + d*x)^4)/2 + (b*sin(c + d*x)^6)/6)/d
```

sympy [A] time = 4.42, size = 83, normalized size = 1.38

$$\begin{cases} \frac{8a\sin^{5}(c+dx)}{15d} + \frac{4a\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^{4}(c+dx)}{d} - \frac{b\cos^{6}(c+dx)}{6d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\cos^{5}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d - b*cos(c + d*x)**6/(6*d), Ne(d,
0)), (x*(a + b*sin(c))*cos(c)**5, True))
```

3.377 $\int \cos^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}$$

```
[Out] -1/4*b*cos(d*x+c)^4/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 641}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^4)/(4*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= -\frac{b\cos^4(c+dx)}{4d}+\frac{a\,\text{Subst}\left(\int \left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= -\frac{b\cos^4(c+dx)}{4d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(b*Cos[c + d*x]^4)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.47, size = 39, normalized size = 0.89

$$-\frac{3\,b\cos\left(dx+c\right)^4-4\left(a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b*cos(d*x + c)^4 - 4*(a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

giac [A] time = 0.44, size = 48, normalized size = 1.09

$$-\frac{3\,b\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-6\,b\sin\left(dx+c\right)^{2}-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*b*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

maple [A] time = 0.14, size = 36, normalized size = 0.82

$$\frac{-\frac{\left(\cos^{4}(dx+c)\right)b}{4}+\frac{a\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/4*cos(d*x+c)^4*b+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 48, normalized size = 1.09

$$-\frac{3\,b\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-6\,b\sin\left(dx+c\right)^{2}-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*b*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

mupad [B] time = 0.06, size = 46, normalized size = 1.05

$$\frac{-\frac{b\sin(c+d\,x)^4}{4}-\frac{a\sin(c+d\,x)^3}{3}+\frac{b\sin(c+d\,x)^2}{2}+a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) - (a*sin(c + d*x)^3)/3 + (b*sin(c + d*x)^2)/2 - (b*sin(c +
d*x)^4)/4)/d
```

sympy [A] time = 1.23, size = 60, normalized size = 1.36

$$\begin{cases} \frac{2a\sin^{3}(c+dx)}{3d}+\frac{a\sin(c+dx)\cos^{2}(c+dx)}{d}-\frac{b\cos^{4}(c+dx)}{4d} & \text{for } d\neq 0\\ x\left(a+b\sin\left(c\right)\right)\cos^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d - b
*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*sin(c))*cos(c)**3, True))
```

3.378 $\int \cos(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^2}{2bd}$$

```
[Out] 1/2*(a+b*sin(d*x+c))^2/b/d
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2668}

$$\frac{a\sin(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x]),x]
[Out] (a*Sin[c + d*x])/d + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{a\sin(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.77

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}-\frac{b\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x]),x]
[Out] -1/2*(b*Cos[c + d*x]^2)/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

fricas [A] time = 0.45, size = 25, normalized size = 1.14

$$-\frac{b\cos\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(b*cos(d*x + c)^2 - 2*a*sin(d*x + c))/d
```

giac [A] time = 0.40, size = 25, normalized size = 1.14

$$\frac{b\sin\left(dx+c\right)^2+2\,a\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*(b*sin(d*x + c)^2 + 2*a*sin(d*x + c))/d
```

maple [A] time = 0.05, size = 25, normalized size = 1.14

$$\frac{\frac{\left(\sin^2(dx+c)\right)b}{2} + a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c)),x)
[Out] 1/d*(1/2*sin(d*x+c)^2*b+a*sin(d*x+c))
```

maxima [A] time = 0.35, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(b*sin(d*x + c) + a)^2/(b*d)
```

mupad [B] time = 0.04, size = 23, normalized size = 1.05

$$\frac{\sin(c+d\,x)\,\left(2\,a+b\,\sin(c+d\,x)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x)),x)
[Out] (sin(c + d*x)*(2*a + b*sin(c + d*x)))/(2*d)
```

sympy [A] time = 0.25, size = 34, normalized size = 1.55

$$\begin{cases} \frac{a\sin(c+dx)}{d} + \frac{b\sin^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\,(a+b\sin(c))\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x)
[Out] Piecewise((a*sin(c + d*x)/d + b*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b
*sin(c))*cos(c), True))
```

3.379 $\int \sec(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{(a-b)\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)\log(1-\sin(c+dx))}{2d}$$

```
[Out] -1/2*(a+b)*ln(1-sin(d*x+c))/d+1/2*(a-b)*ln(1+sin(d*x+c))/d
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2668, 633, 31}

$$\frac{(a-b)\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)\log(1-\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a + b)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)*Log[1 + Sin[c + d*x]])/(2
*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sin(c+dx))\,dx &= \frac{b\,\mathrm{Subst}\left(\int \frac{a+x}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= -\frac{(a-b)\,\mathrm{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2d}+\frac{(a+b)\,\mathrm{Subst}\left(\int \frac{1}{b-x}\,dx, x, b\right.}{2d}\\ &= -\frac{(a+b)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)\log(1+\sin(c+dx))}{2d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.60

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (b*Log[Cos[c + d*x]])/d
```

fricas [A] time = 0.46, size = 37, normalized size = 0.86

$$\frac{(a-b)\log(\sin(dx+c)+1)-(a+b)\log(-\sin(dx+c)+1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*((a - b)*log(sin(d*x + c) + 1) - (a + b)*log(-sin(d*x + c) + 1))/d
```

giac [A] time = 0.75, size = 37, normalized size = 0.86

$$\frac{(a-b)\log(|\sin(dx+c)+1|)-(a+b)\log(|\sin(dx+c)-1|)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*((a - b)*log(abs(sin(d*x + c) + 1)) - (a + b)*log(abs(sin(d*x + c) - 1)
))/d
```

maple [A] time = 0.10, size = 34, normalized size = 0.79

$$-\frac{b\ln(\cos(dx+c))}{d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c)),x)
[Out] -1/d*b*ln(cos(d*x+c))+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.36, size = 35, normalized size = 0.81

$$\frac{(a-b)\log(\sin(dx+c)+1)-(a+b)\log(\sin(dx+c)-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((a - b)*log(sin(d*x + c) + 1) - (a + b)*log(sin(d*x + c) - 1))/d
```

mupad [B] time = 0.07, size = 54, normalized size = 1.26

$$-\frac{\frac{a\ln(\sin(c+d\,x)-1)}{2}-\frac{a\ln(\sin(c+d\,x)+1)}{2}+\frac{b\ln(\sin(c+d\,x)-1)}{2}+\frac{b\ln(\sin(c+d\,x)+1)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x),x)
[Out] -((a*log(sin(c + d*x) - 1))/2 - (a*log(sin(c + d*x) + 1))/2 + (b*log(sin(c
+ d*x) - 1))/2 + (b*log(sin(c + d*x) + 1))/2)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x)
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x), x)
```

3.380 $\int \sec^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$\frac{\sec^2(c+dx)(a\sin(c+dx)+b)}{2d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+1/2*sec(d*x+c)^2*(b+a*sin(d*x+c))/d
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 639, 206}

$$\frac{\sec^2(c+dx)(a\sin(c+dx)+b)}{2d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c + d*x]))/(2*
d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+b\sin(c+dx))\,dx &= \frac{b^3\,\text{Subst}\left(\int \frac{a+x}{\left(b^2-x^2\right)^2}\,dx,x,b\sin(c+dx)\right)}{d}\\ &= \frac{\sec^2(c+dx)(b+a\sin(c+dx))}{2d}+\frac{(ab)\,\text{Subst}\left(\int \frac{1}{b^2-x^2}\,dx,x,b\sin(c+dx)\right)}{2d}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)(b+a\sin(c+dx))}{2d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.27

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^2)/(2*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

fricas [A] time = 0.48, size = 67, normalized size = 1.63

$$\frac{a\cos(dx+c)^2\log(\sin(dx+c)+1) - a\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2a\sin(dx+c) + 2b}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*a*sin(d*x + c) + 2*b)/(d*cos(d*x + c)^2)
```

giac [A] time = 0.66, size = 55, normalized size = 1.34

$$\frac{a\log(|\sin(dx+c)+1|) - a\log(|\sin(dx+c)-1|) - \frac{2(a\sin(dx+c)+b)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(a*log(abs(sin(d*x + c) + 1)) - a*log(abs(sin(d*x + c) - 1)) - 2*(a*sin
(d*x + c) + b)/(sin(d*x + c)^2 - 1))/d
```

maple [A] time = 0.17, size = 54, normalized size = 1.32

$$\frac{a\sec(dx+c)\tan(dx+c)}{2d} + \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{b}{2d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*b/cos
(d*x+c)^2
```

maxima [A] time = 0.32, size = 53, normalized size = 1.29

$$\frac{a\log(\sin(dx+c)+1) - a\log(\sin(dx+c)-1) - \frac{2(a\sin(dx+c)+b)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c)
+ b)/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 0.07, size = 44, normalized size = 1.07

$$\frac{a\,\mathrm{atanh}(\sin(c+d\,x))}{2\,d} - \frac{\frac{b}{2} + \frac{a\,\sin(c+d\,x)}{2}}{d\,\left(\sin(c+d\,x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(sin(c + d*x)))/(2*d) - (b/2 + (a*sin(c + d*x))/2)/(d*(sin(c + d*x)
^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

3.381 $\int \sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{\sec^4(c+dx)(a\sin(c+dx)+b)}{4d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(b+a*sin(d*x+c))/d+3/8*a*sec(d
*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2668, 639, 199, 206}

$$\frac{\sec^4(c+dx)(a\sin(c+dx)+b)}{4d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x]))/(
4*d) + (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{a+x}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))}{4d} + \frac{\left(3ab^3\right)\text{Subst}\left(\int \frac{1}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+\right.}{4d}$$
$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))}{4d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{(3ab)\,\text{Sub}}{}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(b+a\sin(c+dx))}{4d} + \frac{3a\sec(c+dx}{8}$$

Mathematica [A] time = 0.15, size = 68, normalized size = 1.11

$$\frac{a\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\sec(c+dx)\right)}{8d} + \frac{b\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

fricas [A] time = 0.46, size = 82, normalized size = 1.34

$$\frac{3\,a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right) - 3\,a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(3\,a\cos\left(dx+c\right)^2 + 2\,a\right)\sin\left(\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(3*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(3*a*cos(d*x + c)^2 + 2*a)*sin(d*x + c) + 4*b)/(d*cos(d
*x + c)^4)
```

giac [A] time = 0.58, size = 70, normalized size = 1.15

$$\frac{3\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right) - 3\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right) - \frac{2\left(3\,a\sin\left(dx+c\right)^3 - 5\,a\sin\left(dx+c\right) - 2\,b\right)}{\left(\sin\left(dx+c\right)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(3*a*log(abs(sin(d*x + c) + 1)) - 3*a*log(abs(sin(d*x + c) - 1)) - 2*(
3*a*sin(d*x + c)^3 - 5*a*sin(d*x + c) - 2*b)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.17, size = 74, normalized size = 1.21

$$\frac{a\tan\left(dx+c\right)\left(\sec^3\left(dx+c\right)\right)}{4d} + \frac{3a\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d} + \frac{3a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d} + \frac{b}{4d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*tan(d*x+c)*sec(d*x+c)^3+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(se
c(d*x+c)+tan(d*x+c))+1/4/d*b/cos(d*x+c)^4
```

maxima [A] time = 0.33, size = 78, normalized size = 1.28

$$\frac{3\,a\log\left(\sin\left(dx+c\right)+1\right)-3\,a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,a\sin\left(dx+c\right)^{3}-5\,a\sin\left(dx+c\right)-2\,b\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*a*log(sin(d*x + c) + 1) - 3*a*log(sin(d*x + c) - 1) - 2*(3*a*sin(d*
x + c)^3 - 5*a*sin(d*x + c) - 2*b)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))
/d
```

mupad [B] time = 5.14, size = 64, normalized size = 1.05

$$\frac{3\,a\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{8\,d}+\frac{-\frac{3\,a\,\sin\left(c+d\,x\right)^{3}}{8}+\frac{5\,a\,\sin\left(c+d\,x\right)}{8}+\frac{b}{4}}{d\,\left(\sin\left(c+d\,x\right)^{4}-2\,\sin\left(c+d\,x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(sin(c + d*x)))/(8*d) + (b/4 + (5*a*sin(c + d*x))/8 - (3*a*sin(c
+ d*x)^3)/8)/(d*(sin(c + d*x)^4 - 2*sin(c + d*x)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\sec^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**5, x)
```

3.382 $\int \cos^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^5(c+dx)}{5d}$$

```
[Out] 3/8*a*x-1/5*b*cos(d*x+c)^5/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 2635, 8}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^5)/(5*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sin(c+dx))\,dx &= -\frac{b\cos^5(c+dx)}{5d}+a\int \cos^4(c+dx)\,dx\\ &= -\frac{b\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3a)\int \cos^2(c+dx)\,dx\\ &= -\frac{b\cos^5(c+dx)}{5d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}\\ &= \frac{3ax}{8}-\frac{b\cos^5(c+dx)}{5d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}\end{aligned}$$

Mathematica [A] time = 0.11, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^5)/(5*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.46, size = 51, normalized size = 0.78

$$-\frac{8\,b\cos\left(dx+c\right)^{5}-15\,adx-5\left(2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40*(8*b*cos(d*x + c)^5 - 15*a*d*x - 5*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x
+ c))*sin(d*x + c))/d
```

giac [A] time = 2.02, size = 77, normalized size = 1.18

$$\frac{3}{8}\,ax-\frac{b\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{b\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x - 1/80*b*cos(5*d*x + 5*c)/d - 1/16*b*cos(3*d*x + 3*c)/d - 1/8*b*cos
(d*x + c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.15, size = 52, normalized size = 0.80

$$\frac{-\frac{b\left(\cos^{5}\left(dx+c\right)\right)}{5}+a\left(\frac{\left(\cos^{3}\left(dx+c\right)+\frac{3\cos\left(dx+c\right)}{2}\right)\sin\left(dx+c\right)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/5*b*cos(d*x+c)^5+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
8*d*x+3/8*c))
```

maxima [A] time = 0.37, size = 48, normalized size = 0.74

$$-\frac{32\,b\cos\left(dx+c\right)^{5}-5\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/160*(32*b*cos(d*x + c)^5 - 5*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2
*d*x + 2*c))*a)/d
```

mupad [B] time = 8.69, size = 111, normalized size = 1.71

$$\frac{3\,a\,x}{8}-\frac{\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4}+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2}+4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{2}-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{2\,b}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/8 - ((2*b)/5 - (5*a*tan(c/2 + (d*x)/2))/4 - (a*tan(c/2 + (d*x)/2)^3
)/2 + (a*tan(c/2 + (d*x)/2)^7)/2 + (5*a*tan(c/2 + (d*x)/2)^9)/4 + 4*b*tan(c
/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5
)
```

sympy [A] time = 2.34, size = 124, normalized size = 1.91

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{b\cos^5(c+dx)}{5d} & \text{fo} \\ x(a+b\sin(c))\cos^4(c) & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) - b*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x
*(a + b*sin(c))*cos(c)**4, True))
```

3.383 $\int \cos^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\cos^3(c+dx)}{3d}$$

```
[Out] 1/2*a*x-1/3*b*cos(d*x+c)^3/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 2635, 8}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/2 - (b*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sin(c+dx))\,dx &= -\frac{b\cos^3(c+dx)}{3d}+a\int \cos^2(c+dx)\,dx\\ &= -\frac{b\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2}-\frac{b\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 46, normalized size = 1.07

$$\frac{a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^3)/(3*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.43, size = 37, normalized size = 0.86

$$-\frac{2\,b\cos\left(dx+c\right)^{3}-3\,adx-3\,a\cos\left(dx+c\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*cos(d*x + c)^3 - 3*a*d*x - 3*a*cos(d*x + c)*sin(d*x + c))/d
```

giac [A] time = 0.35, size = 47, normalized size = 1.09

$$\frac{1}{2}\,ax-\frac{b\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{b\cos\left(dx+c\right)}{4\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x - 1/12*b*cos(3*d*x + 3*c)/d - 1/4*b*cos(d*x + c)/d + 1/4*a*sin(2*d*
x + 2*c)/d
```

maple [A] time = 0.09, size = 41, normalized size = 0.95

$$\frac{-\frac{b\left(\cos^{3}\left(dx+c\right)\right)}{3}+a\left(\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/3*b*cos(d*x+c)^3+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.33, size = 37, normalized size = 0.86

$$-\frac{4\,b\cos\left(dx+c\right)^{3}-3\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 7.38, size = 68, normalized size = 1.58

$$\frac{a\,x}{2}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{2\,b}{3}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*x)/2 - ((2*b)/3 - a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^5 + 2*b*ta
n(c/2 + (d*x)/2)^4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

sympy [A] time = 0.64, size = 71, normalized size = 1.65

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} + \frac{a\sin(c+dx)\cos(c+dx)}{2d} - \frac{b\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) - b*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sin(c))*c
os(c)**2, True))
```

3.384 $\int \sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=23

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] b*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sin(c+dx))\,dx &= \frac{b\sec(c+dx)}{d}+a\int \sec^2(c+dx)\,dx\\ &= \frac{b\sec(c+dx)}{d}-\frac{a\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

fricas [A] time = 0.43, size = 22, normalized size = 0.96

$$\frac{a\sin\left(dx+c\right)+b}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] (a*sin(d*x + c) + b)/(d*cos(d*x + c))
```

giac [A] time = 0.82, size = 33, normalized size = 1.43

$$-\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -2*(a*tan(1/2*d*x + 1/2*c) + b)/((tan(1/2*d*x + 1/2*c)^2 - 1)*d)
```

maple [A] time = 0.15, size = 24, normalized size = 1.04

$$\frac{\tan\left(dx+c\right)a+\frac{b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c)),x)
[Out] 1/d*(tan(d*x+c)*a+b/cos(d*x+c))
```

maxima [A] time = 0.33, size = 23, normalized size = 1.00

$$\frac{a\tan\left(dx+c\right)+\frac{b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] (a*tan(d*x + c) + b/cos(d*x + c))/d
```

mupad [B] time = 5.13, size = 22, normalized size = 0.96

$$\frac{b+a\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x)^2,x)
[Out] (b + a*sin(c + d*x))/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**2, x)
```

3.385 $\int \sec^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}$$

```
[Out] 1/3*b*sec(d*x+c)^3/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2669, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\sin(c+dx))\,dx &= \frac{b\sec^3(c+dx)}{3d}+a\int \sec^4(c+dx)\,dx\\ &= \frac{b\sec^3(c+dx)}{3d}-\frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{b\sec^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.93

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^3)/(3*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.43, size = 35, normalized size = 0.80

$$\frac{\left(2\,a\cos\left(dx+c\right)^2+a\right)\sin\left(dx+c\right)+b}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*((2*a*cos(d*x + c)^2 + a)*sin(d*x + c) + b)/(d*cos(d*x + c)^3)
```

giac [A] time = 0.41, size = 76, normalized size = 1.73

$$-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*tan(1/2*d*x + 1/2*c)^5 + 3*b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2
*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c) + b)/((tan(1/2*d*x + 1/2*c)^2 -
1)^3*d)
```

maple [A] time = 0.16, size = 38, normalized size = 0.86

$$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan\left(dx+c\right)+\frac{b}{3\cos(dx+c)^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/3*b/cos(d*x+c)^3)
```

maxima [A] time = 0.33, size = 35, normalized size = 0.80

$$\frac{\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a+\frac{b}{\cos(dx+c)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a + b/cos(d*x + c)^3)/d
```

mupad [B] time = 5.27, size = 42, normalized size = 0.95

$$\frac{\frac{2\,a\,\sin(c+d\,x)\cos(c+d\,x)^{2}}{3}+\frac{b}{3}+\frac{a\,\sin(c+d\,x)}{3}}{d\cos\left(c+d\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (b/3 + (a*sin(c + d*x))/3 + (2*a*cos(c + d*x)^2*sin(c + d*x))/3)/(d*cos(c +
 d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin\left(c+dx\right))\sec^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**4, x)
```

3.386 $\int \sec^6(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^5(c+dx)}{5d}$$

```
[Out] 1/5*b*sec(d*x+c)^5/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5
/d
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2669, 3767}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+b\sin(c+dx))\,dx &= \frac{b\sec^5(c+dx)}{5d}+a\int \sec^6(c+dx)\,dx\\ &= \frac{b\sec^5(c+dx)}{5d}-\frac{a\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{b\sec^5(c+dx)}{5d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.18, size = 53, normalized size = 0.88

$$\frac{a\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
+ d*x]^5/5))/d
```

fricas [A] time = 0.44, size = 50, normalized size = 0.83

$$\frac{\left(8\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+3\,a\right)\sin\left(dx+c\right)+3\,b}{15\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*((8*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 3*a)*sin(d*x + c) + 3*b)/(
d*cos(d*x + c)^5)
```

giac [B] time = 1.27, size = 120, normalized size = 2.00

$$-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-20\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+58\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+30\,b\tan\right.}{15\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/15*(15*a*tan(1/2*d*x + 1/2*c)^9 + 15*b*tan(1/2*d*x + 1/2*c)^8 - 20*a*tan
(1/2*d*x + 1/2*c)^7 + 58*a*tan(1/2*d*x + 1/2*c)^5 + 30*b*tan(1/2*d*x + 1/2*
c)^4 - 20*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*d*x + 1/2*c) + 3*b)/((tan
(1/2*d*x + 1/2*c)^2 - 1)^5*d)
```

maple [A] time = 0.16, size = 48, normalized size = 0.80

$$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan\left(dx+c\right)+\frac{b}{5\cos(dx+c)^{5}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/5*b/cos(d*x
+c)^5)
```

maxima [A] time = 0.33, size = 48, normalized size = 0.80

$$\frac{\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a+\frac{3\,b}{\cos(dx+c)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a + 3*b/cos(
d*x + c)^5)/d
```

mupad [B] time = 5.30, size = 75, normalized size = 1.25

$$\frac{b}{5\,d\cos\left(c+d\,x\right)^{5}}+\frac{8\,a\,\sin\left(c+d\,x\right)}{15\,d\,\cos\left(c+d\,x\right)}+\frac{4\,a\,\sin\left(c+d\,x\right)}{15\,d\cos\left(c+d\,x\right)^{3}}+\frac{a\,\sin\left(c+d\,x\right)}{5\,d\cos\left(c+d\,x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x)^6,x)
```

```
[Out] b/(5*d*cos(c + d*x)^5) + (8*a*sin(c + d*x))/(15*d*cos(c + d*x)) + (4*a*sin(
c + d*x))/(15*d*cos(c + d*x)^3) + (a*sin(c + d*x))/(5*d*cos(c + d*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.387 $\int \cos^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=99

$$\frac{(a^2-2b^2)\sin^5(c+dx)}{5d}-\frac{(2a^2-b^2)\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{ab\cos^6(c+dx)}{3d}+\frac{b^2\sin^7(c+dx)}{7d}$$

```
[Out] -1/3*a*b*cos(d*x+c)^6/d+a^2*sin(d*x+c)/d-1/3*(2*a^2-b^2)*sin(d*x+c)^3/d+1/5
*(a^2-2*b^2)*sin(d*x+c)^5/d+1/7*b^2*sin(d*x+c)^7/d
```

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 696, 1810}

$$\frac{(a^2-2b^2)\sin^5(c+dx)}{5d}-\frac{(2a^2-b^2)\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{ab\cos^6(c+dx)}{3d}+\frac{b^2\sin^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*b*Cos[c + d*x]^6)/(3*d) + (a^2*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sin[c +
 d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^5)/(5*d) + (b^2*Sin[c + d*x]^7
)/(7*d)
```

Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\operatorname{Subst}\left(\int (a+x)^2\left(b^2-x^2\right)^2\,dx,x,b\sin(c+dx)\right)}{b^5d}\\
&= -\frac{ab\cos^6(c+dx)}{3d}+\frac{\operatorname{Subst}\left(\int \left(b^2-x^2\right)^2\left(-2ax+(a+x)^2\right)\,dx,x,b\sin\right.}{b^5d}\\
&= -\frac{ab\cos^6(c+dx)}{3d}+\frac{\operatorname{Subst}\left(\int \left(a^2b^4+b^2\left(-2a^2+b^2\right)x^2+\left(a^2-2b^2\right)\right.\right.}{b^5d}\\
&= -\frac{ab\cos^6(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{(2a^2-b^2)\sin^3(c+dx)}{3d}+\frac{(a^2-}{}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 104, normalized size = 1.05

$$\frac{\sin(c+dx)\left(21\left(a^2-2b^2\right)\sin^4(c+dx)+35\left(b^2-2a^2\right)\sin^2(c+dx)+105a^2+35ab\sin^5(c+dx)-105ab\sin^3(c+dx)\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]*(105*a^2 + 105*a*b*Sin[c + d*x] + 35*(-2*a^2 + b^2)*Sin[c + d
*x]^2 - 105*a*b*Sin[c + d*x]^3 + 21*(a^2 - 2*b^2)*Sin[c + d*x]^4 + 35*a*b*S
in[c + d*x]^5 + 15*b^2*Sin[c + d*x]^6))/(105*d)
```

fricas [A] time = 0.48, size = 87, normalized size = 0.88

$$-\frac{35\,ab\cos\left(dx+c\right)^6+\left(15\,b^2\cos\left(dx+c\right)^6-3\left(7\,a^2+b^2\right)\cos\left(dx+c\right)^4-4\left(7\,a^2+b^2\right)\cos\left(dx+c\right)^2-56\,a^2-8\,b^2\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(35*a*b*cos(d*x + c)^6 + (15*b^2*cos(d*x + c)^6 - 3*(7*a^2 + b^2)*co
s(d*x + c)^4 - 4*(7*a^2 + b^2)*cos(d*x + c)^2 - 56*a^2 - 8*b^2)*sin(d*x + c
))/d
```

giac [A] time = 0.75, size = 136, normalized size = 1.37

$$-\frac{ab\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{ab\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{5\,ab\cos\left(2\,dx+2\,c\right)}{32\,d}-\frac{b^2\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{\left(4\,a^2-3\,b^2\right)\sin\left(5\,dx+5\,c\right)}{320\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*a*b*cos(6*d*x + 6*c)/d - 1/16*a*b*cos(4*d*x + 4*c)/d - 5/32*a*b*cos(2
*d*x + 2*c)/d - 1/448*b^2*sin(7*d*x + 7*c)/d + 1/320*(4*a^2 - 3*b^2)*sin(5*
d*x + 5*c)/d + 1/192*(20*a^2 - b^2)*sin(3*d*x + 3*c)/d + 5/64*(8*a^2 + b^2)
*sin(d*x + c)/d
```

maple [A] time = 0.20, size = 98, normalized size = 0.99

$$\frac{b^2\left(-\frac{\left(\cos^6(dx+c)\right)\sin(dx+c)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{ab\left(\cos^6(dx+c)\right)}{3}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/7*cos(d*x+c)^6*sin(d*x+c)+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/3*a*b*cos(d*x+c)^6+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 106, normalized size = 1.07

$$\frac{15\,b^2\sin\left(dx+c\right)^7+35\,ab\sin\left(dx+c\right)^6-105\,ab\sin\left(dx+c\right)^4+21\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^5+105\,ab\sin\left(dx+c\right)^2}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*sin(d*x + c)^7 + 35*a*b*sin(d*x + c)^6 - 105*a*b*sin(d*x + c)
^4 + 21*(a^2 - 2*b^2)*sin(d*x + c)^5 + 105*a*b*sin(d*x + c)^2 - 35*(2*a^2 -
 b^2)*sin(d*x + c)^3 + 105*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.07, size = 104, normalized size = 1.05

$$\frac{a^2\,\sin\left(c+d\,x\right)-{\sin\left(c+d\,x\right)}^3\,\left(\frac{2\,a^2}{3}-\frac{b^2}{3}\right)+{\sin\left(c+d\,x\right)}^5\,\left(\frac{a^2}{5}-\frac{2\,b^2}{5}\right)+\frac{b^2\,{\sin\left(c+d\,x\right)}^7}{7}+a\,b\,{\sin\left(c+d\,x\right)}^2-a\,b\,\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x) - sin(c + d*x)^3*((2*a^2)/3 - b^2/3) + sin(c + d*x)^5*(a^
2/5 - (2*b^2)/5) + (b^2*sin(c + d*x)^7)/7 + a*b*sin(c + d*x)^2 - a*b*sin(c
+ d*x)^4 + (a*b*sin(c + d*x)^6)/3)/d
```

sympy [A] time = 7.45, size = 158, normalized size = 1.60

$$\begin{cases} \frac{8a^2\sin^5(c+dx)}{15d} + \frac{4a^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^2\sin(c+dx)\cos^4(c+dx)}{d} - \frac{ab\cos^6(c+dx)}{3d} + \frac{8b^2\sin^7(c+dx)}{105d} + \frac{4b^2\sin^5(c+dx)\cos^2(c+dx)}{15d} \\ x\left(a+b\sin(c)\right)^2\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**5/(15*d) + 4*a**2*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**4/d - a*b*cos(c + d*x)**6/(3
*d) + 8*b**2*sin(c + d*x)**7/(105*d) + 4*b**2*sin(c + d*x)**5*cos(c + d*x)*
*2/(15*d) + b**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d), Ne(d, 0)), (x*(a +
b*sin(c))**2*cos(c)**5, True))
```

3.388 $\int \cos^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=77

$$-\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^3}{3b^3d}-\frac{(a+b\sin(c+dx))^5}{5b^3d}+\frac{a(a+b\sin(c+dx))^4}{2b^3d}$$

```
[Out] -1/3*(a^2-b^2)*(a+b*sin(d*x+c))^3/b^3/d+1/2*a*(a+b*sin(d*x+c))^4/b^3/d-1/5*
(a+b*sin(d*x+c))^5/b^3/d
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2668, 697}

$$-\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^3}{3b^3d}-\frac{(a+b\sin(c+dx))^5}{5b^3d}+\frac{a(a+b\sin(c+dx))^4}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2 - b^2)*(a + b*Sin[c + d*x])^3)/(3*b^3*d) + (a*(a + b*Sin[c + d*x])^4
)/(2*b^3*d) - (a + b*Sin[c + d*x])^5/(5*b^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^2+2a(a+x)^3-(a+x)^4\right)\,dx,x,b\sin(c+d\right.}{b^3d}\\ &= -\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^3}{3b^3d}+\frac{a(a+b\sin(c+dx))^4}{2b^3d}-\frac{(a+b\sin(c+}{5b^3d}\end{aligned}$$

Mathematica [A] time = 0.12, size = 56, normalized size = 0.73

$$\frac{(a+b\sin(c+dx))^3\left(-a^2+3ab\sin(c+dx)+3b^2\cos(2(c+dx))+7b^2\right)}{30b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a + b*Sin[c + d*x])^3*(-a^2 + 7*b^2 + 3*b^2*Cos[2*(c + d*x)] + 3*a*b*Sin[
c + d*x]))/(30*b^3*d)
```

fricas [A] time = 0.48, size = 69, normalized size = 0.90

$$-\frac{15\,ab\cos\left(dx+c\right)^{4}+2\left(3\,b^{2}\cos\left(dx+c\right)^{4}-\left(5\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}-10\,a^{2}-2\,b^{2}\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(15*a*b*cos(d*x + c)^4 + 2*(3*b^2*cos(d*x + c)^4 - (5*a^2 + b^2)*cos(
d*x + c)^2 - 10*a^2 - 2*b^2)*sin(d*x + c))/d
```

giac [A] time = 0.63, size = 80, normalized size = 1.04

$$-\frac{6\,b^{2}\sin\left(dx+c\right)^{5}+15\,ab\sin\left(dx+c\right)^{4}+10\,a^{2}\sin\left(dx+c\right)^{3}-10\,b^{2}\sin\left(dx+c\right)^{3}-30\,ab\sin\left(dx+c\right)^{2}-30\,a^{2}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3
 - 10*b^2*sin(d*x + c)^3 - 30*a*b*sin(d*x + c)^2 - 30*a^2*sin(d*x + c))/d
```

maple [A] time = 0.20, size = 78, normalized size = 1.01

$$\frac{b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{5}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{ab\left(\cos^{4}(dx+c)\right)}{2}+\frac{a^{2}\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
2*a*b*cos(d*x+c)^4+1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 73, normalized size = 0.95

$$-\frac{6\,b^{2}\sin\left(dx+c\right)^{5}+15\,ab\sin\left(dx+c\right)^{4}-30\,ab\sin\left(dx+c\right)^{2}+10\left(a^{2}-b^{2}\right)\sin\left(dx+c\right)^{3}-30\,a^{2}\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 - 30*a*b*sin(d*x + c)^2
 + 10*(a^2 - b^2)*sin(d*x + c)^3 - 30*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.05, size = 74, normalized size = 0.96

$$-\frac{\sin\left(c+d\,x\right)^{3}\,\left(\frac{a^{2}}{3}-\frac{b^{2}}{3}\right)-a^{2}\,\sin\left(c+d\,x\right)+\frac{b^{2}\sin(c+d\,x)^{5}}{5}-a\,b\sin\left(c+d\,x\right)^{2}+\frac{a\,b\sin(c+d\,x)^{4}}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] -(sin(c + d*x)^3*(a^2/3 - b^2/3) - a^2*sin(c + d*x) + (b^2*sin(c + d*x)^5)/
5 - a*b*sin(c + d*x)^2 + (a*b*sin(c + d*x)^4)/2)/d
```

sympy [A] time = 3.16, size = 107, normalized size = 1.39

$$\begin{cases} \frac{2a^2\sin^3(c+dx)}{3d} + \frac{a^2\sin(c+dx)\cos^2(c+dx)}{d} - \frac{ab\cos^4(c+dx)}{2d} + \frac{2b^2\sin^5(c+dx)}{15d} + \frac{b^2\sin^3(c+dx)\cos^2(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a+b\sin(c))^2\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2
/d - a*b*cos(c + d*x)**4/(2*d) + 2*b**2*sin(c + d*x)**5/(15*d) + b**2*sin(c
 + d*x)**3*cos(c + d*x)**2/(3*d), Ne(d, 0)), (x*(a + b*sin(c))**2*cos(c)**3
, True))
```

3.389 $\int \cos(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^3}{3bd}$$

```
[Out] 1/3*(a+b*sin(d*x+c))^3/b/d
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2668, 32\}$

$$\frac{(a+b\sin(c+dx))^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a + b*Sin[c + d*x])^3/(3*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^3}{3bd}\end{aligned}$$

Mathematica [B] time = 0.01, size = 46, normalized size = 2.09

$$\frac{a^2\sin(c+dx)}{d}+\frac{ab\sin^2(c+dx)}{d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d + (a*b*Sin[c + d*x]^2)/d + (b^2*Sin[c + d*x]^3)/(3*d)
```

fricas [B] time = 0.49, size = 48, normalized size = 2.18

$$-\frac{3\,ab\cos\left(dx+c\right)^2+\left(b^2\cos\left(dx+c\right)^2-3\,a^2-b^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*b*cos(d*x + c)^2 + (b^2*cos(d*x + c)^2 - 3*a^2 - b^2)*sin(d*x + c
))/d
```

giac [A] time = 0.66, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*sin(d*x + c) + a)^3/(b*d)
```

maple [A] time = 0.08, size = 21, normalized size = 0.95

$$\frac{(a+b\sin(dx+c))^3}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/3*(a+b*sin(d*x+c))^3/b/d
```

maxima [A] time = 0.33, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*sin(d*x + c) + a)^3/(b*d)
```

mupad [B] time = 0.06, size = 39, normalized size = 1.77

$$\frac{a^2\,\sin(c+d\,x)+a\,b\,\sin(c+d\,x)^2+\frac{b^2\,\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x) + (b^2*sin(c + d*x)^3)/3 + a*b*sin(c + d*x)^2)/d
```

sympy [A] time = 0.77, size = 53, normalized size = 2.41

$$\begin{cases} \frac{a^2\sin(c+dx)}{d} + \frac{ab\sin^2(c+dx)}{d} + \frac{b^2\sin^3(c+dx)}{3d} & \text{for}\ d \neq 0 \\ x\,(a+b\sin(c))^2\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)/d + a*b*sin(c + d*x)**2/d + b**2*sin(c + d*x)*
*3/(3*d), Ne(d, 0)), (x*(a + b*sin(c))**2*cos(c), True))
```

3.390 $\int \sec(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=61

$$\frac{(a-b)^2\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{b^2\sin(c+dx)}{d}$$

```
[Out] -1/2*(a+b)^2*ln(1-sin(d*x+c))/d+1/2*(a-b)^2*ln(1+sin(d*x+c))/d-b^2*sin(d*x+
c)/d
```

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2668, 702, 633, 31}

$$\frac{(a-b)^2\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{b^2\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a + b)^2*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^2*Log[1 + Sin[c + d*x]]
)/(2*d) - (b^2*Sin[c + d*x])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(-1+\frac{a^2+b^2+2ax}{b^2-x^2}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b^2\sin(c+dx)}{d} + \frac{b\,\text{Subst}\left(\int \frac{a^2+b^2+2ax}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b^2\sin(c+dx)}{d} - \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2d} + \frac{(a+b)^2\,\text{S}}{}$$
$$= -\frac{(a+b)^2\log(1-\sin(c+dx))}{2d} + \frac{(a-b)^2\log(1+\sin(c+dx))}{2d} - \frac{b^2\sin(c}{d}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 0.89

$$\frac{(a-b)^2\log(\sin(c+dx)+1)-(a+b)^2\log(1-\sin(c+dx))-2b^2\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-((a + b)^2*Log[1 - Sin[c + d*x]]) + (a - b)^2*Log[1 + Sin[c + d*x]] - 2*b
^2*Sin[c + d*x])/(2*d)
```

fricas [A] time = 0.49, size = 62, normalized size = 1.02

$$-\frac{2\,b^2\sin(dx+c)-\left(a^2-2\,ab+b^2\right)\log(\sin(dx+c)+1)+\left(a^2+2\,ab+b^2\right)\log(-\sin(dx+c)+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin(d*x + c) + 1) + (a^2
 + 2*a*b + b^2)*log(-sin(d*x + c) + 1))/d
```

giac [A] time = 0.51, size = 62, normalized size = 1.02

$$-\frac{2\,b^2\sin(dx+c)-\left(a^2-2\,ab+b^2\right)\log(|\sin(dx+c)+1|)+\left(a^2+2\,ab+b^2\right)\log(|\sin(dx+c)-1|)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(abs(sin(d*x + c) + 1)) +
 (a^2 + 2*a*b + b^2)*log(abs(sin(d*x + c) - 1)))/d
```

maple [A] time = 0.14, size = 72, normalized size = 1.18

$$-\frac{b^2\sin(dx+c)}{d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{2ab\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -b^2*sin(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2*ln(sec(d*x+c)+t
an(d*x+c))-2/d*a*b*ln(cos(d*x+c))
```

maxima [A] time = 0.33, size = 60, normalized size = 0.98

$$-\frac{2\,b^2\sin\left(dx+c\right)-\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2+2\,ab+b^2\right)\log\left(\sin\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin(d*x + c) + 1) + (a^2
 + 2*a*b + b^2)*log(sin(d*x + c) - 1))/d
```

mupad [B] time = 5.16, size = 50, normalized size = 0.82

$$-\frac{\frac{\ln(\sin(c+d\,x)-1)\,(a+b)^2}{2}-\frac{\ln(\sin(c+d\,x)+1)\,(a-b)^2}{2}+b^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] -((log(sin(c + d*x) - 1)*(a + b)^2)/2 - (log(sin(c + d*x) + 1)*(a - b)^2)/2
 + b^2*sin(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x), x)
```

3.391 $\int \sec^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=59

$$\frac{(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{2d}$$

```
[Out] 1/2*(a^2-b^2)*arctanh(sin(d*x+c))/d+1/2*sec(d*x+c)^2*(b+a*sin(d*x+c))*(a+b*
sin(d*x+c))/d
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 723, 206}

$$\frac{(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x]))/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^2\,dx=\frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^2}{\left(b^2-x^2\right)^2}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$=\frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{2d}+\frac{\left(b\left(a^2-b^2\right)\right)\text{Subst}\left(\right.}{}$$

$$=\frac{(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin}{2d}$$

Mathematica [A] time = 0.92, size = 113, normalized size = 1.92

$$\frac{-2\left(a^4-b^4\right)\tan(c+dx)\sec(c+dx)+\left(4ab^3-6a^3b\right)\tan^2(c+dx)+2a^3b\sec^2(c+dx)+\left(a^2-b^2\right)^2(\log(1-\sin}{4d\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)^2*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^3*b*Se
c[c + d*x]^2 - 2*(a^4 - b^4)*Sec[c + d*x]*Tan[c + d*x] + (-6*a^3*b + 4*a*b^
3)*Tan[c + d*x]^2)/(4*(-a^2 + b^2)*d)
```

fricas [A] time = 0.47, size = 90, normalized size = 1.53

$$\frac{\left(a^2-b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+4\,ab+2\left(a^2+b\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^2 - b^2)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 4*a*b + 2*(a^2 + b^2)*sin(d*x + c))/(d*cos
(d*x + c)^2)
```

giac [A] time = 1.15, size = 86, normalized size = 1.46

$$\frac{\left(a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(a^2\sin(dx+c)+b^2\sin(dx+c)+2\,ab\right)}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*((a^2 - b^2)*log(abs(sin(d*x + c) + 1)) - (a^2 - b^2)*log(abs(sin(d*x +
 c) - 1)) - 2*(a^2*sin(d*x + c) + b^2*sin(d*x + c) + 2*a*b)/(sin(d*x + c)^2
 - 1))/d
```

maple [B] time = 0.25, size = 118, normalized size = 2.00

$$\frac{a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{ab}{d\cos\left(dx+c\right)^2}+\frac{b^2\left(\sin^3\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^2}+\frac{b^2\sin\left(dx\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d*a^2*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*b
/cos(d*x+c)^2+1/2/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b^2*sin(d*x+c)/d-1/2/
d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.32, size = 78, normalized size = 1.32

$$\frac{\left(a^2-b^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(2\,ab+\left(a^2+b^2\right)\sin(dx+c)\right)}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

[Out] 1/4*((a^2 - b^2)*log(sin(d*x + c) + 1) - (a^2 - b^2)*log(sin(d*x + c) - 1)
- 2*(2*a*b + (a^2 + b^2)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d

mupad [B] time = 0.11, size = 62, normalized size = 1.05

$$\frac{\operatorname{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)}{d}-\frac{a\,b+\sin\left(c+d\,x\right)\,\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{d\,\left({\sin\left(c+d\,x\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^3,x)

[Out] (atanh(sin(c + d*x))*(a^2/2 - b^2/2))/d - (a*b + sin(c + d*x)*(a^2/2 + b^2/
2))/(d*(sin(c + d*x)^2 - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^2 \sec^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**2,x)

[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x)**3, x)

3.392 $\int \sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=99

$$\frac{(3a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^2(c+dx)\left((3a^2-b^2)\sin(c+dx)+2ab\right)}{8d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+b)(}{4d}$$

```
[Out] 1/8*(3*a^2-b^2)*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(b+a*sin(d*x+c))*(a+
b*sin(d*x+c))/d+1/8*sec(d*x+c)^2*(2*a*b+(3*a^2-b^2)*sin(d*x+c))/d
```

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2668, 739, 639, 206}

$$\frac{(3a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^2(c+dx)\left((3a^2-b^2)\sin(c+dx)+2ab\right)}{8d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+b)(}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*(2*a*b + (3*a^2 - b^2
)*Sin[c + d*x]))/(8*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{4d} - \frac{b^3\,\text{Subst}\left(\int \frac{-3a^2+b^2-}{\left(b^2-x^2\right.}\right.}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{4d} + \frac{\sec^2(c+dx)\left(2ab+\right.}{}$$

$$= \frac{\left(3a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\,\text{s}}{4d}$$

Mathematica [A] time = 0.74, size = 166, normalized size = 1.68

$$\frac{4\left(b^2-a^2\right)\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^3+\left(b^2-3a^2\right)\left(-2\left(a^4-b^4\right)\tan(c+dx)\sec(c+dx)+\right.}{16d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*(-a^2 + b^2)*Sec[c + d*x]^4*(b - a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3
+ (-3*a^2 + b^2)*((a^2 - b^2)^2*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*
x]]) + 2*a^3*b*Sec[c + d*x]^2 - 2*(a^4 - b^4)*Sec[c + d*x]*Tan[c + d*x] + (
-6*a^3*b + 4*a*b^3)*Tan[c + d*x]^2))/(16*(a^2 - b^2)^2*d)
```

fricas [A] time = 0.48, size = 118, normalized size = 1.19

$$\frac{\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+8\,ab+2\left(\left(3\,a^2\right.\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*((3*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a^2 - b^2)*co
s(d*x + c)^4*log(-sin(d*x + c) + 1) + 8*a*b + 2*((3*a^2 - b^2)*cos(d*x + c)
^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.97, size = 118, normalized size = 1.19

$$\frac{\left(3\,a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(3\,a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,a^2\sin(dx+c)^3-b^2\sin(dx+c)^3-5\,a^2\sin(dx+c)-b^2\,\text{si}\right.}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*((3*a^2 - b^2)*log(abs(sin(d*x + c) + 1)) - (3*a^2 - b^2)*log(abs(sin(
d*x + c) - 1)) - 2*(3*a^2*sin(d*x + c)^3 - b^2*sin(d*x + c)^3 - 5*a^2*sin(d
*x + c) - b^2*sin(d*x + c) - 4*a*b)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.25, size = 165, normalized size = 1.67

$$\frac{a^2\tan\left(dx+c\right)\left(\sec^3\left(dx+c\right)\right)}{4d}+\frac{3a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{ab}{2d\cos\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^2*sec(d*x+c)*tan(d*x+c)+3/8/d*a^2
*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*b/cos(d*x+c)^4+1/4/d*b^2*sin(d*x+c)^3/co
s(d*x+c)^4+1/8/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8*b^2*sin(d*x+c)/d-1/8/d*b
^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.32, size = 115, normalized size = 1.16

$$\frac{\left(3\,a^{2}-b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,a^{2}-b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,\left(\left(3\,a^{2}-b^{2}\right)\sin\left(dx+c\right)^{3}-4\,ab-\left(5\,a^{2}+b^{2}\right)\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*((3*a^2 - b^2)*log(sin(d*x + c) + 1) - (3*a^2 - b^2)*log(sin(d*x + c)
- 1) - 2*((3*a^2 - b^2)*sin(d*x + c)^3 - 4*a*b - (5*a^2 + b^2)*sin(d*x + c)
)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 5.10, size = 93, normalized size = 0.94

$$\frac{\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(\frac{3\,a^2}{8}-\frac{b^2}{8}\right)}{d}+\frac{\left(\frac{b^2}{8}-\frac{3\,a^2}{8}\right)\,\sin\left(c+d\,x\right)^3+\left(\frac{5\,a^2}{8}+\frac{b^2}{8}\right)\,\sin\left(c+d\,x\right)+\frac{a\,b}{2}}{d\,\left({\sin\left(c+d\,x\right)}^4-2\,{\sin\left(c+d\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^5,x)
```

```
[Out] (atanh(sin(c + d*x))*((3*a^2)/8 - b^2/8))/d + ((a*b)/2 + sin(c + d*x)*((5*a
^2)/8 + b^2/8) - sin(c + d*x)^3*((3*a^2)/8 - b^2/8))/(d*(sin(c + d*x)^4 - 2
*sin(c + d*x)^2 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.393 $\int \cos^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=146

$$\frac{(8a^2+b^2)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5(8a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5(8a^2+b^2)\sin(c+dx)\cos(c+dx)}{128d}+$$

```
[Out] 5/128*(8*a^2+b^2)*x-9/56*a*b*cos(d*x+c)^7/d+5/128*(8*a^2+b^2)*cos(d*x+c)*si
n(d*x+c)/d+5/192*(8*a^2+b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/48*(8*a^2+b^2)*cos
(d*x+c)^5*sin(d*x+c)/d-1/8*b*cos(d*x+c)^7*(a+b*sin(d*x+c))/d
```

Rubi [A] time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2692, 2669, 2635, 8}

$$\frac{(8a^2+b^2)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5(8a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5(8a^2+b^2)\sin(c+dx)\cos(c+dx)}{128d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(8*a^2 + b^2)*x)/128 - (9*a*b*Cos[c + d*x]^7)/(56*d) + (5*(8*a^2 + b^2)*
Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*(8*a^2 + b^2)*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + ((8*a^2 + b^2)*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (b*C
os[c + d*x]^7*(a + b*Sin[c + d*x]))/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{b\cos^7(c+dx)(a+b\sin(c+dx))}{8d} + \frac{1}{8}\int \cos^6(c+dx)\left(8a^2+b^2+9a\right.$$
$$= -\frac{9ab\cos^7(c+dx)}{56d} - \frac{b\cos^7(c+dx)(a+b\sin(c+dx))}{8d} + \frac{1}{8}\left(8a^2+b^2\right)$$
$$= -\frac{9ab\cos^7(c+dx)}{56d} + \frac{\left(8a^2+b^2\right)\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{b\cos^7(c+}{}$$
$$= -\frac{9ab\cos^7(c+dx)}{56d} + \frac{5\left(8a^2+b^2\right)\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{\left(8a^2+b\right.}{}$$
$$= -\frac{9ab\cos^7(c+dx)}{56d} + \frac{5\left(8a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{128d} + \frac{5\left(8a^2+\right.}{}$$
$$= \frac{5}{128}\left(8a^2+b^2\right)x - \frac{9ab\cos^7(c+dx)}{56d} + \frac{5\left(8a^2+b^2\right)\cos(c+dx)\sin(c}{128d}$$

Mathematica [A] time = 0.35, size = 141, normalized size = 0.97

$$\frac{840\left(8a^2+b^2\right)(c+dx)+336\left(15a^2+b^2\right)\sin(2(c+dx))+168\left(6a^2-b^2\right)\sin(4(c+dx))+112(a-b)(a+b)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (840*(8*a^2 + b^2)*(c + d*x) - 3360*a*b*Cos[c + d*x] - 2016*a*b*Cos[3*(c +
d*x)] - 672*a*b*Cos[5*(c + d*x)] - 96*a*b*Cos[7*(c + d*x)] + 336*(15*a^2 +
b^2)*Sin[2*(c + d*x)] + 168*(6*a^2 - b^2)*Sin[4*(c + d*x)] + 112*(a - b)*(a
 + b)*Sin[6*(c + d*x)] - 21*b^2*Sin[8*(c + d*x)])/(21504*d)
```

fricas [A] time = 0.49, size = 108, normalized size = 0.74

$$-\frac{768\,ab\cos\left(dx+c\right)^7 - 105\left(8\,a^2+b^2\right)dx + 7\left(48\,b^2\cos\left(dx+c\right)^7 - 8\left(8\,a^2+b^2\right)\cos\left(dx+c\right)^5 - 10\left(8\,a^2+b^2\right)\right.}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2688*(768*a*b*cos(d*x + c)^7 - 105*(8*a^2 + b^2)*d*x + 7*(48*b^2*cos(d*x
 + c)^7 - 8*(8*a^2 + b^2)*cos(d*x + c)^5 - 10*(8*a^2 + b^2)*cos(d*x + c)^3
- 15*(8*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.62, size = 162, normalized size = 1.11

$$\frac{5}{128}\left(8\,a^2+b^2\right)x - \frac{ab\cos\left(7\,dx+7\,c\right)}{224\,d} - \frac{ab\cos\left(5\,dx+5\,c\right)}{32\,d} - \frac{3\,ab\cos\left(3\,dx+3\,c\right)}{32\,d} - \frac{5\,ab\cos\left(dx+c\right)}{32\,d} - \frac{b^2\sin\left(8\,dx\right.}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/128*(8*a^2 + b^2)*x - 1/224*a*b*cos(7*d*x + 7*c)/d - 1/32*a*b*cos(5*d*x +
 5*c)/d - 3/32*a*b*cos(3*d*x + 3*c)/d - 5/32*a*b*cos(d*x + c)/d - 1/1024*b^
2*sin(8*d*x + 8*c)/d + 1/192*(a^2 - b^2)*sin(6*d*x + 6*c)/d + 1/128*(6*a^2
- b^2)*sin(4*d*x + 4*c)/d + 1/64*(15*a^2 + b^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.23, size = 128, normalized size = 0.88

$$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8} + \frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48} + \frac{5dx}{128} + \frac{5c}{128}\right) - \frac{2ab\left(\cos^7(dx+c)\right)}{7} + a^2\left(\frac{\left(\cos^5(dx+c)+\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-2/7*a*b*cos(d*x+c)^7+a^2*(1/
6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*
c))
```

maxima [A] time = 0.33, size = 114, normalized size = 0.78

$$-\frac{6144\,ab\cos\left(dx+c\right)^{7}+112\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^{2}-7\left(6\right.}{21504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/21504*(6144*a*b*cos(d*x + c)^7 + 112*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60
*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 - 7*(64*sin(2*d*x + 2*c)
^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*b^2)/d
```

mupad [B] time = 5.47, size = 178, normalized size = 1.22

$$\frac{5\,a^2\,x}{16}+\frac{5\,b^2\,x}{128}+\frac{5\,a^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{24\,d}+\frac{a^2\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{6\,d}+\frac{5\,b^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{192\,d}+\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (5*a^2*x)/16 + (5*b^2*x)/128 + (5*a^2*cos(c + d*x)^3*sin(c + d*x))/(24*d) +
 (a^2*cos(c + d*x)^5*sin(c + d*x))/(6*d) + (5*b^2*cos(c + d*x)^3*sin(c + d*
x))/(192*d) + (b^2*cos(c + d*x)^5*sin(c + d*x))/(48*d) - (b^2*cos(c + d*x)^
7*sin(c + d*x))/(8*d) - (2*a*b*cos(c + d*x)^7)/(7*d) + (5*a^2*cos(c + d*x)*
sin(c + d*x))/(16*d) + (5*b^2*cos(c + d*x)*sin(c + d*x))/(128*d)
```

sympy [A] time = 13.09, size = 398, normalized size = 2.73

$$\begin{cases} \frac{5a^2x\sin^6(c+dx)}{16}+\frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15a^2x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5a^2x\cos^6(c+dx)}{16}+\frac{5a^2\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{5a^2\sin^3}{} \\ x\left(a+b\sin\left(c\right)\right)^2\cos^6\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**2*x*cos(c
+ d*x)**6/16 + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**2*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - 2*a*b*cos(c + d*x)**7/(7*d) + 5*b**2*x*sin(c + d*x)**8/128 + 5*b**2*x*si
n(c + d*x)**6*cos(c + d*x)**2/32 + 15*b**2*x*sin(c + d*x)**4*cos(c + d*x)**
4/64 + 5*b**2*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 5*b**2*x*cos(c + d*x)*
*8/128 + 5*b**2*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*b**2*sin(c + d*x)
**5*cos(c + d*x)**3/(384*d) + 73*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*
d) - 5*b**2*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sin(
c))**2*cos(c)**6, True))
```

3.394 $\int \cos^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{(6a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6a^2+b^2)-\frac{7ab\cos^5(c+dx)}{30d}-\frac{b}{}$$

```
[Out] 1/16*(6*a^2+b^2)*x-7/30*a*b*cos(d*x+c)^5/d+1/16*(6*a^2+b^2)*cos(d*x+c)*sin(
d*x+c)/d+1/24*(6*a^2+b^2)*cos(d*x+c)^3*sin(d*x+c)/d-1/6*b*cos(d*x+c)^5*(a+b
*sin(d*x+c))/d
```

Rubi [A] time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2692, 2669, 2635, 8}

$$\frac{(6a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6a^2+b^2)-\frac{7ab\cos^5(c+dx)}{30d}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((6*a^2 + b^2)*x)/16 - (7*a*b*Cos[c + d*x]^5)/(30*d) + ((6*a^2 + b^2)*Cos[c
 + d*x]*Sin[c + d*x])/(16*d) + ((6*a^2 + b^2)*Cos[c + d*x]^3*Sin[c + d*x])/
(24*d) - (b*Cos[c + d*x]^5*(a + b*Sin[c + d*x]))/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{b\cos^5(c+dx)(a+b\sin(c+dx))}{6d} + \frac{1}{6}\int \cos^4(c+dx)\left(6a^2+b^2+7ab\,\mathrm{s}\right.$$

$$= -\frac{7ab\cos^5(c+dx)}{30d} - \frac{b\cos^5(c+dx)(a+b\sin(c+dx))}{6d} + \frac{1}{6}\left(6a^2+b^2\right)\int$$

$$= -\frac{7ab\cos^5(c+dx)}{30d} + \frac{\left(6a^2+b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{b\cos^5(c+d\cdot}{}$$

$$= -\frac{7ab\cos^5(c+dx)}{30d} + \frac{\left(6a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{\left(6a^2+b^2\right)\mathrm{co}}{}$$

$$= \frac{1}{16}\left(6a^2+b^2\right)x - \frac{7ab\cos^5(c+dx)}{30d} + \frac{\left(6a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{16d}$$

Mathematica [A] time = 0.19, size = 133, normalized size = 1.15

$$\frac{240a^2\sin(2(c+dx))+30a^2\sin(4(c+dx))+360a^2c+360a^2dx-240ab\cos(c+dx)-120ab\cos(3(c+dx))-24}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (360*a^2*c + 60*b^2*c + 360*a^2*d*x + 60*b^2*d*x - 240*a*b*Cos[c + d*x] - 1
20*a*b*Cos[3*(c + d*x)] - 24*a*b*Cos[5*(c + d*x)] + 240*a^2*Sin[2*(c + d*x)
] + 15*b^2*Sin[2*(c + d*x)] + 30*a^2*Sin[4*(c + d*x)] - 15*b^2*Sin[4*(c + d
*x)] - 5*b^2*Sin[6*(c + d*x)])/(960*d)
```

fricas [A] time = 0.46, size = 89, normalized size = 0.77

$$-\frac{96\,ab\cos\left(dx+c\right)^5-15\left(6\,a^2+b^2\right)dx+5\left(8\,b^2\cos\left(dx+c\right)^5-2\left(6\,a^2+b^2\right)\cos\left(dx+c\right)^3-3\left(6\,a^2+b^2\right)\cos\left(dx\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(96*a*b*cos(d*x + c)^5 - 15*(6*a^2 + b^2)*d*x + 5*(8*b^2*cos(d*x + c
)^5 - 2*(6*a^2 + b^2)*cos(d*x + c)^3 - 3*(6*a^2 + b^2)*cos(d*x + c))*sin(d*
x + c))/d
```

giac [A] time = 0.94, size = 123, normalized size = 1.06

$$\frac{1}{16}\left(6\,a^2+b^2\right)x-\frac{ab\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{ab\cos\left(3\,dx+3\,c\right)}{8\,d}-\frac{ab\cos\left(dx+c\right)}{4\,d}-\frac{b^2\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{\left(2\,a^2-b^2\right)\sin\left(4\,dx\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(6*a^2 + b^2)*x - 1/40*a*b*cos(5*d*x + 5*c)/d - 1/8*a*b*cos(3*d*x + 3*
c)/d - 1/4*a*b*cos(d*x + c)/d - 1/192*b^2*sin(6*d*x + 6*c)/d + 1/64*(2*a^2
- b^2)*sin(4*d*x + 4*c)/d + 1/64*(16*a^2 + b^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.22, size = 108, normalized size = 0.93

$$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-2/5*a*b*cos(d*x+c)^5+a^2*(1/4*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.32, size = 88, normalized size = 0.76

$$-\frac{384\,ab\cos\left(dx+c\right)^{5}-30\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a^{2}-5\left(4\,\sin\left(2\,dx+2\,c\right)^{3}+12\,c\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/960*(384*a*b*cos(d*x + c)^5 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*a^2 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*
x + 4*c))*b^2)/d
```

mupad [B] time = 5.31, size = 134, normalized size = 1.16

$$\frac{3\,a^2\,x}{8}+\frac{b^2\,x}{16}+\frac{a^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{b^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{24\,d}-\frac{b^2\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{6\,d}-\frac{2\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/8 + (b^2*x)/16 + (a^2*cos(c + d*x)^3*sin(c + d*x))/(4*d) + (b^2*c
os(c + d*x)^3*sin(c + d*x))/(24*d) - (b^2*cos(c + d*x)^5*sin(c + d*x))/(6*d
) - (2*a*b*cos(c + d*x)^5)/(5*d) + (3*a^2*cos(c + d*x)*sin(c + d*x))/(8*d)
+ (b^2*cos(c + d*x)*sin(c + d*x))/(16*d)
```

sympy [A] time = 4.69, size = 287, normalized size = 2.47

$$\begin{cases}\frac{3a^2x\sin^4(c+dx)}{8}+\frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^2x\cos^4(c+dx)}{8}+\frac{3a^2\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{2ab\cos^5}{5d} \\ x\left(a+b\sin\left(c\right)\right)^2\cos^4\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a*b*cos(c + d*x)**5/(
5*d) + b**2*x*sin(c + d*x)**6/16 + 3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2
/16 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b**2*x*cos(c + d*x)**6/
16 + b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + b**2*sin(c + d*x)**3*cos(c
+ d*x)**3/(6*d) - b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(
a + b*sin(c))**2*cos(c)**4, True))
```

3.395 $\int \cos^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=86

$$\frac{(4a^2+b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a^2+b^2)-\frac{5ab\cos^3(c+dx)}{12d}-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d}$$

```
[Out] 1/8*(4*a^2+b^2)*x-5/12*a*b*cos(d*x+c)^3/d+1/8*(4*a^2+b^2)*cos(d*x+c)*sin(d*
x+c)/d-1/4*b*cos(d*x+c)^3*(a+b*sin(d*x+c))/d
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2692, 2669, 2635, 8}

$$\frac{(4a^2+b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a^2+b^2)-\frac{5ab\cos^3(c+dx)}{12d}-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((4*a^2 + b^2)*x)/8 - (5*a*b*Cos[c + d*x]^3)/(12*d) + ((4*a^2 + b^2)*Cos[c
+ d*x]*Sin[c + d*x])/(8*d) - (b*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d} + \frac{1}{4}\int \cos^2(c+dx)\left(4a^2+b^2+5a\right.$$

$$= -\frac{5ab\cos^3(c+dx)}{12d} - \frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d} + \frac{1}{4}\left(4a^2+b^2\right)$$

$$= -\frac{5ab\cos^3(c+dx)}{12d} + \frac{\left(4a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\cos^3(c+}{}$$

$$= \frac{1}{8}\left(4a^2+b^2\right)x - \frac{5ab\cos^3(c+dx)}{12d} + \frac{\left(4a^2+b^2\right)\cos(c+dx)\sin(c+d}{8d}$$

Mathematica [A] time = 0.23, size = 85, normalized size = 0.99

$$\frac{3\left(8a^2\sin(2(c+dx))+16a^2c+16a^2dx-b^2\sin(4(c+dx))+4b^2c+4b^2dx\right)-48ab\cos(c+dx)-16ab\cos(3(c}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-48*a*b*Cos[c + d*x] - 16*a*b*Cos[3*(c + d*x)] + 3*(16*a^2*c + 4*b^2*c + 1
6*a^2*d*x + 4*b^2*d*x + 8*a^2*Sin[2*(c + d*x)] - b^2*Sin[4*(c + d*x)]))/(96
*d)
```

fricas [A] time = 0.45, size = 70, normalized size = 0.81

$$-\frac{16\,ab\cos\left(dx+c\right)^3-3\left(4\,a^2+b^2\right)dx+3\left(2\,b^2\cos\left(dx+c\right)^3-\left(4\,a^2+b^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(16*a*b*cos(d*x + c)^3 - 3*(4*a^2 + b^2)*d*x + 3*(2*b^2*cos(d*x + c)^
3 - (4*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 1.01, size = 76, normalized size = 0.88

$$\frac{1}{8}\left(4\,a^2+b^2\right)x-\frac{ab\cos\left(3\,dx+3\,c\right)}{6\,d}-\frac{ab\cos\left(dx+c\right)}{2\,d}-\frac{b^2\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2 + b^2)*x - 1/6*a*b*cos(3*d*x + 3*c)/d - 1/2*a*b*cos(d*x + c)/d -
 1/32*b^2*sin(4*d*x + 4*c)/d + 1/4*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.15, size = 86, normalized size = 1.00

$$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-2/3*a*b*cos(d*x+c)^3+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.33, size = 64, normalized size = 0.74

$$-\frac{64\,ab\cos\left(dx+c\right)^{3}-24\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}-3\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)b^{2}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(64*a*b*cos(d*x + c)^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 3*
(4*d*x + 4*c - sin(4*d*x + 4*c))*b^2)/d
```

mupad [B] time = 5.38, size = 71, normalized size = 0.83

$$\frac{6\,a^2\,\sin\left(2\,c+2\,d\,x\right)-\frac{3\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{4}-12\,a\,b\,\cos\left(c+d\,x\right)-4\,a\,b\,\cos\left(3\,c+3\,d\,x\right)+12\,a^2\,d\,x+3\,b^2\,d\,x}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (6*a^2*sin(2*c + 2*d*x) - (3*b^2*sin(4*c + 4*d*x))/4 - 12*a*b*cos(c + d*x)
- 4*a*b*cos(3*c + 3*d*x) + 12*a^2*d*x + 3*b^2*d*x)/(24*d)
```

sympy [A] time = 1.41, size = 180, normalized size = 2.09

$$\begin{cases} \frac{a^{2}x\sin^{2}\left(c+dx\right)}{2}+\frac{a^{2}x\cos^{2}\left(c+dx\right)}{2}+\frac{a^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{2ab\cos^{3}\left(c+dx\right)}{3d}+\frac{b^{2}x\sin^{4}\left(c+dx\right)}{8}+\frac{b^{2}x\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{b^{2}x\cos^{4}\left(c\right.}{8} \\ x\left(a+b\sin\left(c\right)\right)^{2}\cos^{2}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) - 2*a*b*cos(c + d*x)**3/(3*d) + b**2*x*sin(c + d
*x)**4/8 + b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + b**2*x*cos(c + d*x)**
4/8 + b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - b**2*sin(c + d*x)*cos(c + d
*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sin(c))**2*cos(c)**2, True))
```

3.396 $\int \sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=49

$$\frac{ab\cos(c+dx)}{d}+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{d}+b^2(-x)$$

```
[Out] -b^2*x+a*b*cos(d*x+c)/d+sec(d*x+c)*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2691, 2638}

$$\frac{ab\cos(c+dx)}{d}+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{d}+b^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(b^2*x) + (a*b*Cos[c + d*x])/d + (Sec[c + d*x]*(b + a*Sin[c + d*x])*(a + b
*Sin[c + d*x]))/d
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{d}-\int \left(b^2+ab\sin(c+\right.\\ &= -b^2x+\frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{d}-(ab)\int \sin\\ &= -b^2x+\frac{ab\cos(c+dx)}{d}+\frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+}{d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 1.12

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\tan^{-1}(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b^2*ArcTan[Tan[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d + (a^2*Tan[c + d*x
])/d + (b^2*Tan[c + d*x])/d
```

fricas [A] time = 0.46, size = 45, normalized size = 0.92

$$-\frac{b^2 dx\cos(dx+c)-2\,ab-\left(a^2+b^2\right)\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b^2*d*x*cos(d*x + c) - 2*a*b - (a^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.78, size = 63, normalized size = 1.29

$$-\frac{(dx+c)b^2+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*b^2 + 2*(a^2*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c) +
2*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

maple [A] time = 0.19, size = 46, normalized size = 0.94

$$\frac{a^2\tan(dx+c)+\frac{2ab}{\cos(dx+c)}+b^2\left(\tan(dx+c)-dx-c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*tan(d*x+c)+2*a*b/cos(d*x+c)+b^2*(tan(d*x+c)-d*x-c))
```

maxima [A] time = 0.48, size = 46, normalized size = 0.94

$$-\frac{(dx+c-\tan(dx+c))b^2-a^2\tan(dx+c)-\frac{2\,ab}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*b^2 - a^2*tan(d*x + c) - 2*a*b/cos(d*x + c))/d
```

mupad [B] time = 5.21, size = 53, normalized size = 1.08

$$-b^2\,x-\frac{4\,a\,b+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+2\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] - b^2*x - (4*a*b + tan(c/2 + (d*x)/2)*(2*a^2 + 2*b^2))/(d*(tan(c/2 + (d*x)/
2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x)**2, x)
```

3.397 $\int \sec^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=75

$$\frac{(2a^2-b^2)\tan(c+dx)}{3d}+\frac{ab\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{3d}$$

```
[Out] 1/3*a*b*sec(d*x+c)/d+1/3*sec(d*x+c)^3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d+1
/3*(2*a^2-b^2)*tan(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2691, 2669, 3767, 8}

$$\frac{(2a^2-b^2)\tan(c+dx)}{3d}+\frac{ab\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[
c + d*x]))/(3*d) + ((2*a^2 - b^2)*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d} - \frac{1}{3}\int \sec^2(c+dx$$
$$= \frac{ab\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d} -$$
$$= \frac{ab\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d} -$$
$$= \frac{ab\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d} +$$

Mathematica [A] time = 0.33, size = 105, normalized size = 1.40

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(3\left(2a^2+b^2\right)\sin(c+dx)+\left(2a^2-b^2\right)\sin(3(c+dx))+8ab\right)}{12d(\sin(c+dx)-1)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(8*a*b + 3*(2*a^2 + b^2)*Sin[c + d*x
] + (2*a^2 - b^2)*Sin[3*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^3*(-1 + Sin[c + d*x])^2)
```

fricas [A] time = 0.43, size = 52, normalized size = 0.69

$$\frac{2\,ab+\left(\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2+a^2+b^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*b + ((2*a^2 - b^2)*cos(d*x + c)^2 + a^2 + b^2)*sin(d*x + c))/(d*co
s(d*x + c)^3)
```

giac [A] time = 0.51, size = 102, normalized size = 1.36

$$-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan\left(\right.\right.}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*a*b*tan(1/2*d*x + 1/2*c)^4 - 2*a^2*t
an(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x +
1/2*c) + 2*a*b)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*d)
```

maple [A] time = 0.28, size = 62, normalized size = 0.83

$$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan\left(dx+c\right)+\frac{2ab}{3\cos(dx+c)^3}+\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2/3*a*b/cos(d*x+c)^3+1/3*b^2*s
in(d*x+c)^3/cos(d*x+c)^3)
```

maxima [A] time = 0.33, size = 51, normalized size = 0.68

$$\frac{b^2 \tan\left(dx+c\right)^3 + \left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)a^2 + \frac{2\,ab}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 2*a*b/cos
(d*x + c)^3)/d
```

mupad [B] time = 5.26, size = 71, normalized size = 0.95

$$\frac{\frac{2\,a\,b}{3} + \frac{a^2\,\sin(c+d\,x)}{3} + \frac{b^2\,\sin(c+d\,x)}{3} + \cos\left(c+d\,x\right)^2\,\left(\frac{2\,a^2\,\sin(c+d\,x)}{3} - \frac{b^2\,\sin(c+d\,x)}{3}\right)}{d\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] ((2*a*b)/3 + (a^2*sin(c + d*x))/3 + (b^2*sin(c + d*x))/3 + cos(c + d*x)^2*(
(2*a^2*sin(c + d*x))/3 - (b^2*sin(c + d*x))/3))/(d*cos(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(c + dx\right)\right)^2 \sec^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x)**4, x)
```

3.398 $\int \sec^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{(4a^2-b^2)\tan^3(c+dx)}{15d}+\frac{(4a^2-b^2)\tan(c+dx)}{5d}+\frac{ab\sec^3(c+dx)}{5d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+b)(a+b\sin(c$$

```
[Out] 1/5*a*b*sec(d*x+c)^3/d+1/5*sec(d*x+c)^5*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d
+1/5*(4*a^2-b^2)*tan(d*x+c)/d+1/15*(4*a^2-b^2)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2691, 2669, 3767}

$$\frac{(4a^2-b^2)\tan^3(c+dx)}{15d}+\frac{(4a^2-b^2)\tan(c+dx)}{5d}+\frac{ab\sec^3(c+dx)}{5d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+b)(a+b\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^3)/(5*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Si
n[c + d*x]))/(5*d) + ((4*a^2 - b^2)*Tan[c + d*x])/(5*d) + ((4*a^2 - b^2)*Ta
n[c + d*x]^3)/(15*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d}-\frac{1}{5}\int \sec^4(c+dx\\ &= \frac{ab\sec^3(c+dx)}{5d}+\frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d}-\\ &= \frac{ab\sec^3(c+dx)}{5d}+\frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d}-\\ &= \frac{ab\sec^3(c+dx)}{5d}+\frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d}-\end{aligned}$$

Mathematica [A] time = 0.43, size = 84, normalized size = 0.82

$$\frac{\sec^5(c+dx)\left(20\left(2a^2+b^2\right)\sin(c+dx)+5\left(4a^2-b^2\right)\sin(3(c+dx))+4a^2\sin(5(c+dx))+48ab-b^2\sin(5(c+d\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^5*(48*a*b + 20*(2*a^2 + b^2)*Sin[c + d*x] + 5*(4*a^2 - b^2)*S
in[3*(c + d*x)] + 4*a^2*Sin[5*(c + d*x)] - b^2*Sin[5*(c + d*x)]))/(120*d)
```

fricas [A] time = 0.46, size = 77, normalized size = 0.75

$$\frac{6\,ab+\left(2\left(4\,a^2-b^2\right)\cos\left(dx+c\right)^4+\left(4\,a^2-b^2\right)\cos\left(dx+c\right)^2+3\,a^2+3\,b^2\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(6*a*b + (2*(4*a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 - b^2)*cos(d*x + c)^
2 + 3*a^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [A] time = 0.54, size = 181, normalized size = 1.76

$$-\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-20\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+58\,a^2\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*tan(1/2*d*x + 1/2*c)^9 + 30*a*b*tan(1/2*d*x + 1/2*c)^8 - 20*a
^2*tan(1/2*d*x + 1/2*c)^7 + 20*b^2*tan(1/2*d*x + 1/2*c)^7 + 58*a^2*tan(1/2*
d*x + 1/2*c)^5 + 8*b^2*tan(1/2*d*x + 1/2*c)^5 + 60*a*b*tan(1/2*d*x + 1/2*c)
^4 - 20*a^2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 + 15*a^2
*tan(1/2*d*x + 1/2*c) + 6*a*b)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*d)
```

maple [A] time = 0.25, size = 92, normalized size = 0.89

$$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan\left(dx+c\right)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+2/5*a*b/cos
(d*x+c)^5+b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3
))
```

maxima [A] time = 0.37, size = 76, normalized size = 0.74

$$\frac{\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)a^2+\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)b^2+\frac{6\,ab}{\cos(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + (3*tan
(d*x + c)^5 + 5*tan(d*x + c)^3)*b^2 + 6*a*b/cos(d*x + c)^5)/d
```

mupad [B] time = 5.36, size = 103, normalized size = 1.00

$$\frac{\frac{2\,a\,b}{5}+\frac{a^2\,\sin(c+d\,x)}{5}+\frac{b^2\,\sin(c+d\,x)}{5}+\cos\left(c+d\,x\right)^2\,\left(\frac{4\,a^2\,\sin(c+d\,x)}{15}-\frac{b^2\,\sin(c+d\,x)}{15}\right)+\cos\left(c+d\,x\right)^4\,\left(\frac{8\,a^2\,\sin(c+d\,x)}{15}-\frac{2\,b^2}{}\right.}{d\,\cos\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^6,x)
```

```
[Out] ((2*a*b)/5 + (a^2*sin(c + d*x))/5 + (b^2*sin(c + d*x))/5 + cos(c + d*x)^2*(
(4*a^2*sin(c + d*x))/15 - (b^2*sin(c + d*x))/15) + cos(c + d*x)^4*((8*a^2*s
in(c + d*x))/15 - (2*b^2*sin(c + d*x))/15))/(d*cos(c + d*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.399 $\int \sec^8(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{(6a^2-b^2)\tan^5(c+dx)}{35d}+\frac{2(6a^2-b^2)\tan^3(c+dx)}{21d}+\frac{(6a^2-b^2)\tan(c+dx)}{7d}+\frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(a\sin(\cdots}{\cdots}$$

```
[Out] 1/7*a*b*sec(d*x+c)^5/d+1/7*sec(d*x+c)^7*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d
+1/7*(6*a^2-b^2)*tan(d*x+c)/d+2/21*(6*a^2-b^2)*tan(d*x+c)^3/d+1/35*(6*a^2-b
^2)*tan(d*x+c)^5/d
```

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2691, 2669, 3767}

$$\frac{(6a^2-b^2)\tan^5(c+dx)}{35d}+\frac{2(6a^2-b^2)\tan^3(c+dx)}{21d}+\frac{(6a^2-b^2)\tan(c+dx)}{7d}+\frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(a\sin(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^5)/(7*d) + (Sec[c + d*x]^7*(b + a*Sin[c + d*x])*(a + b*Si
n[c + d*x]))/(7*d) + ((6*a^2 - b^2)*Tan[c + d*x])/(7*d) + (2*(6*a^2 - b^2)*
Tan[c + d*x]^3)/(21*d) + ((6*a^2 - b^2)*Tan[c + d*x]^5)/(35*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}-\frac{1}{7}\int \sec^6(c+dx)\left(-\cdots\right.\\ &= \frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}-\frac{1}{7}\cdots\\ &= \frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}-\frac{(\cdots}{\cdots}\\ &= \frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}+\frac{(\cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.81, size = 110, normalized size = 0.85

$$\frac{\sec^7(c+dx)\left(105\left(2a^2+b^2\right)\sin(c+dx)+21\left(6a^2-b^2\right)\sin(3(c+dx))+42a^2\sin(5(c+dx))+6a^2\sin(7(c+d\right.}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^7*(240*a*b + 105*(2*a^2 + b^2)*Sin[c + d*x] + 21*(6*a^2 - b^2
)*Sin[3*(c + d*x)] + 42*a^2*Sin[5*(c + d*x)] - 7*b^2*Sin[5*(c + d*x)] + 6*a
^2*Sin[7*(c + d*x)] - b^2*Sin[7*(c + d*x)]))/(840*d)
```

fricas [A] time = 0.45, size = 99, normalized size = 0.77

$$\frac{30\,ab+\left(8\left(6\,a^2-b^2\right)\cos\left(dx+c\right)^6+4\left(6\,a^2-b^2\right)\cos\left(dx+c\right)^4+3\left(6\,a^2-b^2\right)\cos\left(dx+c\right)^2+15\,a^2+15\,b^2\right)\mathrm{s}}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(30*a*b + (8*(6*a^2 - b^2)*cos(d*x + c)^6 + 4*(6*a^2 - b^2)*cos(d*x +
 c)^4 + 3*(6*a^2 - b^2)*cos(d*x + c)^2 + 15*a^2 + 15*b^2)*sin(d*x + c))/(d*
cos(d*x + c)^7)
```

giac [B] time = 0.40, size = 260, normalized size = 2.02

$$-\frac{2\left(105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+210\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-210\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+140\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/105*(105*a^2*tan(1/2*d*x + 1/2*c)^13 + 210*a*b*tan(1/2*d*x + 1/2*c)^12 -
 210*a^2*tan(1/2*d*x + 1/2*c)^11 + 140*b^2*tan(1/2*d*x + 1/2*c)^11 + 903*a^
2*tan(1/2*d*x + 1/2*c)^9 + 112*b^2*tan(1/2*d*x + 1/2*c)^9 + 1050*a*b*tan(1/
2*d*x + 1/2*c)^8 - 636*a^2*tan(1/2*d*x + 1/2*c)^7 + 456*b^2*tan(1/2*d*x + 1
/2*c)^7 + 903*a^2*tan(1/2*d*x + 1/2*c)^5 + 112*b^2*tan(1/2*d*x + 1/2*c)^5 +
 630*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^2*tan(1/2*d*x + 1/2*c)^3 + 140*b^2*
tan(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c) + 30*a*b)/((tan(1/2*d
*x + 1/2*c)^2 - 1)^7*d)
```

maple [A] time = 0.30, size = 120, normalized size = 0.93

$$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan\left(dx+c\right)+\frac{2ab}{7\cos(dx+c)^7}+b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+2/7*a*b/cos(d*x+c)^7+b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*
x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

maxima [A] time = 0.32, size = 97, normalized size = 0.75

$$\frac{3\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+35\,\tan\left(dx+c\right)\right)a^{2}+\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+\right.\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan
(d*x + c))*a^2 + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3
)*b^2 + 30*a*b/cos(d*x + c)^7)/d
```

mupad [B] time = 5.55, size = 135, normalized size = 1.05

$$\frac{\frac{2\,a\,b}{7}+\frac{a^2\,\sin(c+d\,x)}{7}+\frac{b^2\,\sin(c+d\,x)}{7}+\cos\left(c+d\,x\right)^2\,\left(\frac{6\,a^2\,\sin(c+d\,x)}{35}-\frac{b^2\,\sin(c+d\,x)}{35}\right)+\cos\left(c+d\,x\right)^4\,\left(\frac{8\,a^2\,\sin(c+d\,x)}{35}-\frac{4\,b^2\,\sin(c+d\,x)}{10\ldots}\right.}{d\,\cos\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^8,x)
```

```
[Out] ((2*a*b)/7 + (a^2*sin(c + d*x))/7 + (b^2*sin(c + d*x))/7 + cos(c + d*x)^2*(
(6*a^2*sin(c + d*x))/35 - (b^2*sin(c + d*x))/35) + cos(c + d*x)^4*((8*a^2*s
in(c + d*x))/35 - (4*b^2*sin(c + d*x))/105) + cos(c + d*x)^6*((16*a^2*sin(c
 + d*x))/35 - (8*b^2*sin(c + d*x))/105))/(d*cos(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.400 $\int \cos^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=144

$$\frac{(3a^2-b^2)(a+b\sin(c+dx))^6}{3b^5d}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^5}{5b^5d}+\frac{(a^2-b^2)^2(a+b\sin(c+dx))^4}{4b^5d}+\frac{(a+b\sin(c}{8b^5d}$$

```
[Out] 1/4*(a^2-b^2)^2*(a+b*sin(d*x+c))^4/b^5/d-4/5*a*(a^2-b^2)*(a+b*sin(d*x+c))^5
/b^5/d+1/3*(3*a^2-b^2)*(a+b*sin(d*x+c))^6/b^5/d-4/7*a*(a+b*sin(d*x+c))^7/b^
5/d+1/8*(a+b*sin(d*x+c))^8/b^5/d
```

Rubi [A] time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{(3a^2-b^2)(a+b\sin(c+dx))^6}{3b^5d}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^5}{5b^5d}+\frac{(a^2-b^2)^2(a+b\sin(c+dx))^4}{4b^5d}+\frac{(a+b\sin(c}{8b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^4)/(4*b^5*d) - (4*a*(a^2 - b^2)*(a + b*
Sin[c + d*x])^5)/(5*b^5*d) + ((3*a^2 - b^2)*(a + b*Sin[c + d*x])^6)/(3*b^5*
d) - (4*a*(a + b*Sin[c + d*x])^7)/(7*b^5*d) + (a + b*Sin[c + d*x])^8/(8*b^5
*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\left(b^2-x^2\right)^2\,dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^3-4\left(a^3-ab^2\right)(a+x)^4+2\left(3a^2-b^2\right)(a\right.\right.}{b^5d}\\ &= \frac{(a^2-b^2)^2(a+b\sin(c+dx))^4}{4b^5d}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^5}{5b^5d}+\end{aligned}$$

Mathematica [A] time = 0.52, size = 120, normalized size = 0.83

$$\frac{\frac{1}{3}(3a^2-b^2)(a+b\sin(c+dx))^6+\frac{1}{4}(a^2-b^2)^2(a+b\sin(c+dx))^4+\frac{1}{8}(a+b\sin(c+dx))^8-\frac{4}{7}a(a+b\sin(c+}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((a^2 - b^2)^2*(a + b*Sin[c + d*x])^4)/4 - (4*a*(a - b)*(a + b)*(a + b*Sin
[c + d*x])^5)/5 + ((3*a^2 - b^2)*(a + b*Sin[c + d*x])^6)/3 - (4*a*(a + b*Si
n[c + d*x])^7)/7 + (a + b*Sin[c + d*x])^8/8)/(b^5*d)
```

fricas [A] time = 0.49, size = 117, normalized size = 0.81

$$\frac{105\,b^3\cos\left(dx+c\right)^8-140\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^6-8\left(45\,ab^2\cos\left(dx+c\right)^6-3\left(7\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^4-56\cdots}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^3*cos(d*x + c)^8 - 140*(3*a^2*b + b^3)*cos(d*x + c)^6 - 8*(45*
a*b^2*cos(d*x + c)^6 - 3*(7*a^3 + 3*a*b^2)*cos(d*x + c)^4 - 56*a^3 - 24*a*b
^2 - 4*(7*a^3 + 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [A] time = 1.00, size = 185, normalized size = 1.28

$$\frac{b^3\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{3\,ab^2\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{\left(6\,a^2b-b^3\right)\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{\left(24\,a^2b+b^3\right)\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{3\left(10\,a^2b\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1024*b^3*cos(8*d*x + 8*c)/d - 3/448*a*b^2*sin(7*d*x + 7*c)/d - 1/384*(6*a
^2*b - b^3)*cos(6*d*x + 6*c)/d - 1/256*(24*a^2*b + b^3)*cos(4*d*x + 4*c)/d
- 3/128*(10*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/320*(4*a^3 - 9*a*b^2)*sin(5
*d*x + 5*c)/d + 1/192*(20*a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 5/64*(8*a^3 +
 3*a*b^2)*sin(d*x + c)/d
```

maple [A] time = 0.23, size = 135, normalized size = 0.94

$$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+3a\,b^2\left(-\frac{\left(\cos^6(dx+c)\right)\sin(dx+c)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{a^2b\left(\cos^6\cdots\right.}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+3*a*b^2*(-1/7*c
os(d*x+c)^6*sin(d*x+c)+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
-1/2*a^2*b*cos(d*x+c)^6+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))
```

maxima [A] time = 0.34, size = 144, normalized size = 1.00

$$\frac{105\,b^3\sin\left(dx+c\right)^8+360\,ab^2\sin\left(dx+c\right)^7+140\left(3\,a^2b-2\,b^3\right)\sin\left(dx+c\right)^6+168\left(a^3-6\,ab^2\right)\sin\left(dx+c\right)^5+12\cdots}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(105*b^3*sin(d*x + c)^8 + 360*a*b^2*sin(d*x + c)^7 + 140*(3*a^2*b - 2
*b^3)*sin(d*x + c)^6 + 168*(a^3 - 6*a*b^2)*sin(d*x + c)^5 + 1260*a^2*b*sin(
d*x + c)^2 - 210*(6*a^2*b - b^3)*sin(d*x + c)^4 + 840*a^3*sin(d*x + c) - 28
0*(2*a^3 - 3*a*b^2)*sin(d*x + c)^3)/d
```

mupad [B] time = 0.09, size = 141, normalized size = 0.98

$$\frac{\sin(c+d\,x)^3\,\left(a\,b^2-\frac{2\,a^3}{3}\right)-\sin(c+d\,x)^5\,\left(\frac{6\,a\,b^2}{5}-\frac{a^3}{5}\right)+\sin(c+d\,x)^6\,\left(\frac{a^2\,b}{2}-\frac{b^3}{3}\right)-\sin(c+d\,x)^4\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{4}\right)\cdot}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (sin(c + d*x)^3*(a*b^2 - (2*a^3)/3) - sin(c + d*x)^5*((6*a*b^2)/5 - a^3/5)
+ sin(c + d*x)^6*((a^2*b)/2 - b^3/3) - sin(c + d*x)^4*((3*a^2*b)/2 - b^3/4)
 + a^3*sin(c + d*x) + (b^3*sin(c + d*x)^8)/8 + (3*a^2*b*sin(c + d*x)^2)/2 +
 (3*a*b^2*sin(c + d*x)^7)/7)/d
```

sympy [A] time = 12.92, size = 202, normalized size = 1.40

$$\begin{cases} \frac{8a^3\sin^5(c+dx)}{15d}+\frac{4a^3\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^4(c+dx)}{d}-\frac{a^2b\cos^6(c+dx)}{2d}+\frac{8ab^2\sin^7(c+dx)}{35d}+\frac{4ab^2\sin^5(c+dx)\cos^2(c}{5d} \\ x\,(a+b\sin(c))^3\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**4/d - a**2*b*cos(c + d*x)**6
/(2*d) + 8*a*b**2*sin(c + d*x)**7/(35*d) + 4*a*b**2*sin(c + d*x)**5*cos(c +
 d*x)**2/(5*d) + a*b**2*sin(c + d*x)**3*cos(c + d*x)**4/d - b**3*sin(c + d*
x)**2*cos(c + d*x)**6/(6*d) - b**3*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a
 + b*sin(c))**3*cos(c)**5, True))
```

3.401 $\int \cos^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=77

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^4}{4b^3d}-\frac{(a+b\sin(c+dx))^6}{6b^3d}+\frac{2a(a+b\sin(c+dx))^5}{5b^3d}$$

```
[Out] -1/4*(a^2-b^2)*(a+b*sin(d*x+c))^4/b^3/d+2/5*a*(a+b*sin(d*x+c))^5/b^3/d-1/6*
(a+b*sin(d*x+c))^6/b^3/d
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^4}{4b^3d}-\frac{(a+b\sin(c+dx))^6}{6b^3d}+\frac{2a(a+b\sin(c+dx))^5}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a^2 - b^2)*(a + b*Sin[c + d*x])^4)/(4*b^3*d) + (2*a*(a + b*Sin[c + d*x])
^5)/(5*b^3*d) - (a + b*Sin[c + d*x])^6/(6*b^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^3+2a(a+x)^4-(a+x)^5\right)\,dx,x,b\sin(c+d\right.}{b^3d}\\ &= -\frac{(a^2-b^2)(a+b\sin(c+dx))^4}{4b^3d}+\frac{2a(a+b\sin(c+dx))^5}{5b^3d}-\frac{(a+b\sin(c+}{6b^3d}\end{aligned}$$

Mathematica [A] time = 0.14, size = 56, normalized size = 0.73

$$\frac{(a+b\sin(c+dx))^4\left(-a^2+4ab\sin(c+dx)+5b^2\cos(2(c+dx))+10b^2\right)}{60b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a + b*Sin[c + d*x])^4*(-a^2 + 10*b^2 + 5*b^2*Cos[2*(c + d*x)] + 4*a*b*Sin
[c + d*x]))/(60*b^3*d)
```

fricas [A] time = 0.44, size = 95, normalized size = 1.23

$$\frac{10\,b^3\cos(dx+c)^6 - 15\left(3\,a^2b+b^3\right)\cos(dx+c)^4 - 4\left(9\,ab^2\cos(dx+c)^4 - 10\,a^3 - 6\,ab^2 - \left(5\,a^3+3\,ab^2\right)\cos(\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^3*cos(d*x + c)^6 - 15*(3*a^2*b + b^3)*cos(d*x + c)^4 - 4*(9*a*b^
2*cos(d*x + c)^4 - 10*a^3 - 6*a*b^2 - (5*a^3 + 3*a*b^2)*cos(d*x + c)^2)*sin
(d*x + c))/d
```

giac [A] time = 0.50, size = 112, normalized size = 1.45

$$-\frac{10\,b^3\sin(dx+c)^6 + 36\,ab^2\sin(dx+c)^5 + 45\,a^2b\sin(dx+c)^4 - 15\,b^3\sin(dx+c)^4 + 20\,a^3\sin(dx+c)^3 - 6}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^3*sin(d*x + c)^6 + 36*a*b^2*sin(d*x + c)^5 + 45*a^2*b*sin(d*x +
 c)^4 - 15*b^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3 - 60*a*b^2*sin(d*x +
c)^3 - 90*a^2*b*sin(d*x + c)^2 - 60*a^3*sin(d*x + c))/d
```

maple [A] time = 0.23, size = 115, normalized size = 1.49

$$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6} - \frac{\left(\cos^4(dx+c)\right)}{12}\right) + 3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5} + \frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right) - \frac{3a^2b\left(\cos^4(dx+c)\right)}{4} +}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/6*cos(d*x+c)^4*sin(d*x+c)^2-1/12*cos(d*x+c)^4)+3*a*b^2*(-1/5*s
in(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/4*a^2*b*cos(d*x+
c)^4+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 100, normalized size = 1.30

$$-\frac{10\,b^3\sin(dx+c)^6 + 36\,ab^2\sin(dx+c)^5 - 90\,a^2b\sin(dx+c)^2 + 15\left(3\,a^2b-b^3\right)\sin(dx+c)^4 - 60\,a^3\sin(dx}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(10*b^3*sin(d*x + c)^6 + 36*a*b^2*sin(d*x + c)^5 - 90*a^2*b*sin(d*x +
 c)^2 + 15*(3*a^2*b - b^3)*sin(d*x + c)^4 - 60*a^3*sin(d*x + c) + 20*(a^3 -
 3*a*b^2)*sin(d*x + c)^3)/d
```

mupad [B] time = 5.13, size = 98, normalized size = 1.27

$$\frac{\sin(c+d\,x)^3\left(a\,b^2-\frac{a^3}{3}\right) - \sin(c+d\,x)^4\left(\frac{3\,a^2\,b}{4}-\frac{b^3}{4}\right) + a^3\,\sin(c+d\,x) - \frac{b^3\sin(c+d\,x)^6}{6} + \frac{3\,a^2\,b\sin(c+d\,x)^2}{2} - \frac{3\,a\,b^2\sin}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (sin(c + d*x)^3*(a*b^2 - a^3/3) - sin(c + d*x)^4*((3*a^2*b)/4 - b^3/4) + a^
3*sin(c + d*x) - (b^3*sin(c + d*x)^6)/6 + (3*a^2*b*sin(c + d*x)^2)/2 - (3*a
*b^2*sin(c + d*x)^5)/5)/d
```

sympy [A] time = 4.87, size = 151, normalized size = 1.96

$$\begin{cases} \frac{2a^3\sin^3(c+dx)}{3d} + \frac{a^3\sin(c+dx)\cos^2(c+dx)}{d} - \frac{3a^2b\cos^4(c+dx)}{4d} + \frac{2ab^2\sin^5(c+dx)}{5d} + \frac{ab^2\sin^3(c+dx)\cos^2(c+dx)}{d} - \frac{b^3\sin^2(c+dx)\cos^4(c+dx)}{4d} \\ x(a+b\sin(c))^3\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d - 3*a**2*b*cos(c + d*x)**4/(4*d) + 2*a*b**2*sin(c + d*x)**5/(5*d) + a*b*
*2*sin(c + d*x)**3*cos(c + d*x)**2/d - b**3*sin(c + d*x)**2*cos(c + d*x)**4
/(4*d) - b**3*cos(c + d*x)**6/(12*d), Ne(d, 0)), (x*(a + b*sin(c))**3*cos(c
)**3, True))
```

3.402 $\int \cos(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^4}{4bd}$$

```
[Out] 1/4*(a+b*sin(d*x+c))^4/b/d
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2668, 32\}$

$$\frac{(a+b\sin(c+dx))^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a + b*Sin[c + d*x])^4/(4*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^4}{4bd}\end{aligned}$$

Mathematica [B] time = 0.06, size = 57, normalized size = 2.59

$$\frac{\sin(c+dx)\left(4a^3+6a^2b\sin(c+dx)+4ab^2\sin^2(c+dx)+b^3\sin^3(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]*(4*a^3 + 6*a^2*b*Sin[c + d*x] + 4*a*b^2*Sin[c + d*x]^2 + b^3*
Sin[c + d*x]^3))/(4*d)
```

fricas [B] time = 0.43, size = 71, normalized size = 3.23

$$\frac{b^3\cos(dx+c)^4 - 2\left(3a^2b+b^3\right)\cos(dx+c)^2 - 4\left(ab^2\cos(dx+c)^2 - a^3 - ab^2\right)\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*cos(d*x + c)^4 - 2*(3*a^2*b + b^3)*cos(d*x + c)^2 - 4*(a*b^2*cos(d
*x + c)^2 - a^3 - a*b^2)*sin(d*x + c))/d
```

giac [A] time = 0.47, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^4}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*sin(d*x + c) + a)^4/(b*d)
```

maple [A] time = 0.10, size = 21, normalized size = 0.95

$$\frac{(a+b\sin(dx+c))^4}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4*(a+b*sin(d*x+c))^4/b/d
```

maxima [A] time = 0.31, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^4}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*sin(d*x + c) + a)^4/(b*d)
```

mupad [B] time = 0.06, size = 55, normalized size = 2.50

$$\frac{a^3\,\sin(c+d\,x)+\frac{3\,a^2\,b\,\sin(c+d\,x)^2}{2}+a\,b^2\,\sin(c+d\,x)^3+\frac{b^3\,\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x) + (b^3*sin(c + d*x)^4)/4 + (3*a^2*b*sin(c + d*x)^2)/2 + a
*b^2*sin(c + d*x)^3)/d
```

sympy [A] time = 1.27, size = 73, normalized size = 3.32

$$\begin{cases} \frac{a^3\sin(c+dx)}{d}+\frac{3a^2b\sin^2(c+dx)}{2d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{b^3\sin^4(c+dx)}{4d} & \text{for } d\neq 0 \\ x\,(a+b\sin(c))^3\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)/d + 3*a**2*b*sin(c + d*x)**2/(2*d) + a*b**2*si
n(c + d*x)**3/d + b**3*sin(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*sin(c))*
*3*cos(c), True))
```

3.403 $\int \sec(c + dx)(a + b\sin(c + dx))^3\, dx$

Optimal. Leaf size=80

$$-\frac{3ab^2\sin(c+dx)}{d}+\frac{(a-b)^3\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{b^3\sin^2(c+dx)}{2d}$$

```
[Out] -1/2*(a+b)^3*ln(1-sin(d*x+c))/d+1/2*(a-b)^3*ln(1+sin(d*x+c))/d-3*a*b^2*sin(
d*x+c)/d-1/2*b^3*sin(d*x+c)^2/d
```

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2668, 702, 633, 31}

$$-\frac{3ab^2\sin(c+dx)}{d}+\frac{(a-b)^3\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{b^3\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a + b)^3*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^3*Log[1 + Sin[c + d*x]]
)/(2*d) - (3*a*b^2*Sin[c + d*x])/d - (b^3*Sin[c + d*x]^2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^3}{b^2-x^2}\,dx,x,b\sin(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(-3a-x+\frac{a^3+3ab^2+(3a^2+b^2)x}{b^2-x^2}\right)dx,x,b\sin(c+dx)\right)}{d}$$
$$= -\frac{3ab^2\sin(c+dx)}{d} - \frac{b^3\sin^2(c+dx)}{2d} + \frac{b\,\text{Subst}\left(\int \frac{a^3+3ab^2+(3a^2+b^2)x}{b^2-x^2}\,dx,x,b\right.}{d}$$
$$= -\frac{3ab^2\sin(c+dx)}{d} - \frac{b^3\sin^2(c+dx)}{2d} - \frac{(a-b)^3\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx,x,b\sin(\right.}{2d}$$
$$= -\frac{(a+b)^3\log(1-\sin(c+dx))}{2d} + \frac{(a-b)^3\log(1+\sin(c+dx))}{2d} - \frac{3ab^2\sin(}{d}$$

Mathematica [A] time = 0.12, size = 67, normalized size = 0.84

$$-\frac{6ab^2\sin(c+dx)+(a-b)^3(-\log(\sin(c+dx)+1))+(a+b)^3\log(1-\sin(c+dx))+b^3\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*((a + b)^3*Log[1 - Sin[c + d*x]] - (a - b)^3*Log[1 + Sin[c + d*x]] + 6
*a*b^2*Sin[c + d*x] + b^3*Sin[c + d*x]^2)/d
```

fricas [A] time = 0.48, size = 93, normalized size = 1.16

$$\frac{b^3\cos(dx+c)^2 - 6\,ab^2\sin(dx+c) + \left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log(\sin(dx+c)+1) - \left(a^3+3\,a^2b+3\,ab^2+b^3\right)\text{l}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*cos(d*x + c)^2 - 6*a*b^2*sin(d*x + c) + (a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*log(sin(d*x + c) + 1) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(-sin(d*x
+ c) + 1))/d
```

giac [A] time = 0.53, size = 93, normalized size = 1.16

$$-\frac{b^3\sin(dx+c)^2 + 6\,ab^2\sin(dx+c) - \left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log(|\sin(dx+c)+1|) + \left(a^3+3\,a^2b+3\,ab^2+b^3\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*sin(d*x + c)^2 + 6*a*b^2*sin(d*x + c) - (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*log(abs(sin(d*x + c) + 1)) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs
(sin(d*x + c) - 1)))/d
```

maple [A] time = 0.18, size = 108, normalized size = 1.35

$$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{3a^2b\ln(\cos(dx+c))}{d} - \frac{3a\,b^2\sin(dx+c)}{d} + \frac{3a\,b^2\ln(\sec(dx+c)+\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))-3/d*a^2*b*ln(cos(d*x+c))-3*a*b^2*sin(d*x+
c)/d+3/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))-1/2*b^3*sin(d*x+c)^2/d-1/d*b^3*ln(
cos(d*x+c))
```

maxima [A] time = 0.33, size = 91, normalized size = 1.14

$$-\frac{b^3\sin(dx+c)^2+6\,ab^2\sin(dx+c)-\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log(\sin(dx+c)+1)+\left(a^3+3\,a^2b+3\,ab^2+\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*sin(d*x + c)^2 + 6*a*b^2*sin(d*x + c) - (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*log(sin(d*x + c) + 1) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(sin(d*x
+ c) - 1))/d
```

mupad [B] time = 5.13, size = 65, normalized size = 0.81

$$-\frac{\frac{\ln(\sin(c+d\,x)-1)\,(a+b)^3}{2}-\frac{\ln(\sin(c+d\,x)+1)\,(a-b)^3}{2}+\frac{b^3\sin(c+d\,x)^2}{2}+3\,a\,b^2\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] -((log(sin(c + d*x) - 1)*(a + b)^3)/2 - (log(sin(c + d*x) + 1)*(a - b)^3)/2
 + (b^3*sin(c + d*x)^2)/2 + 3*a*b^2*sin(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^3\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sec(c + d*x), x)
```

3.404 $\int \sec^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=111

$$\frac{ab^2\sin(c+dx)}{2d}+\frac{(a+2b)(a-b)^2\log(\sin(c+dx)+1)}{4d}-\frac{(a-2b)(a+b)^2\log(1-\sin(c+dx))}{4d}+\frac{\sec^2(c+dx)(a\sin(}{}$$

```
[Out] -1/4*(a-2*b)*(a+b)^2*ln(1-sin(d*x+c))/d+1/4*(a-b)^2*(a+2*b)*ln(1+sin(d*x+c)
)/d+1/2*a*b^2*sin(d*x+c)/d+1/2*sec(d*x+c)^2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c
))^2/d
```

Rubi [A] time = 0.13, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2668, 739, 774, 633, 31}

$$\frac{ab^2\sin(c+dx)}{2d}+\frac{(a+2b)(a-b)^2\log(\sin(c+dx)+1)}{4d}-\frac{(a-2b)(a+b)^2\log(1-\sin(c+dx))}{4d}+\frac{\sec^2(c+dx)(a\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a - 2*b)*(a + b)^2*Log[1 - Sin[c + d*x]])/(4*d) + ((a - b)^2*(a + 2*b)*L
og[1 + Sin[c + d*x]])/(4*d) + (a*b^2*Sin[c + d*x])/(2*d) + (Sec[c + d*x]^2*
(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^3 \operatorname{Subst}\left(\int \frac{(a+x)^3}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{2d} - \frac{b\operatorname{Subst}\left(\int \frac{(a+x)}{}\right.}{}$$

$$= \frac{ab^2\sin(c+dx)}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{2d}$$

$$= \frac{ab^2\sin(c+dx)}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{2d}$$

$$= -\frac{(a-2b)(a+b)^2\log(1-\sin(c+dx))}{4d} + \frac{(a-b)^2(a+2b)\log(1+\sin(}{4d}$$

Mathematica [A] time = 1.34, size = 176, normalized size = 1.59

$$\frac{2a^4b\sec^2(c+dx) + \left(a^2-b^2\right)\left((a-2b)(a+b)^2\log(1-\sin(c+dx)) - (a-b)^2(a+2b)\log(\sin(c+dx)+1)\right) - a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a^2 - b^2)*((a - 2*b)*(a + b)^2*Log[1 - Sin[c + d*x]] - (a - b)^2*(a + 2*
b)*Log[1 + Sin[c + d*x]]) + 2*a^4*b*Sec[c + d*x]^2 - a*(2*a^4 + 4*a^2*b^2 -
 7*b^4 + b^4*Cos[2*(c + d*x)])*Sec[c + d*x]*Tan[c + d*x] + (-8*a^4*b + 4*a^
2*b^3 + 2*b^5 - 2*a*b^4*Sin[c + d*x])*Tan[c + d*x]^2)/(4*(-a^2 + b^2)*d)
```

fricas [A] time = 0.47, size = 112, normalized size = 1.01

$$\frac{\left(a^3-3\,ab^2+2\,b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - \left(a^3-3\,ab^2-2\,b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^3 -
3*a*b^2 - 2*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 6*a^2*b + 2*b^3 +
2*(a^3 + 3*a*b^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 1.13, size = 114, normalized size = 1.03

$$\frac{\left(a^3-3\,ab^2+2\,b^3\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right) - \left(a^3-3\,ab^2-2\,b^3\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right) - \frac{2\left(b^3\sin(dx+c)^2+a^3\sin(dx+c)\right.}{\sin(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*log(abs(sin(d*x + c) + 1)) - (a^3 - 3*a*b^2 -
2*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(b^3*sin(d*x + c)^2 + a^3*sin(d*x + c
) + 3*a*b^2*sin(d*x + c) + 3*a^2*b)/(sin(d*x + c)^2 - 1))/d
```

maple [A] time = 0.28, size = 154, normalized size = 1.39

$$\frac{a^3 \sec(dx+c)\tan(dx+c)}{2d} + \frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{3a^2 b}{2d\cos(dx+c)^2} + \frac{3a\,b^2\left(\sin^3(dx+c)\right)}{2d\cos(dx+c)^2} + \frac{3a\,b^2 \sin}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d*a^3*sec(d*x+c)*tan(d*x+c)+1/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a
^2*b/cos(d*x+c)^2+3/2/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^2+3/2*a*b^2*sin(d*x+c
)/d-3/2/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*b^3*tan(d*x+c)^2+1/d*b^3*ln
(cos(d*x+c))
```

maxima [A] time = 0.33, size = 98, normalized size = 0.88

$$\frac{\left(a^3 - 3\,ab^2 + 2\,b^3\right)\log\left(\sin\left(dx+c\right)+1\right) - \left(a^3 - 3\,ab^2 - 2\,b^3\right)\log\left(\sin\left(dx+c\right)-1\right) - \frac{2\left(3\,a^2b+b^3+\left(a^3+3\,ab^2\right)\sin(dx+c)\right)}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*log(sin(d*x + c) + 1) - (a^3 - 3*a*b^2 - 2*b^3
)*log(sin(d*x + c) - 1) - 2*(3*a^2*b + b^3 + (a^3 + 3*a*b^2)*sin(d*x + c))/
(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 5.21, size = 99, normalized size = 0.89

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(a-b\right)^2\,\left(a+2\,b\right)}{4\,d} - \frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(a+b\right)^2\,\left(a-2\,b\right)}{4\,d} - \frac{\frac{3\,a^2\,b}{2}+\frac{b^3}{2}+\sin\left(c+d\,x\right)\,\left(\frac{a^3}{2}+\frac{3\,a}{}\right.}{d\,\left(\sin\left(c+d\,x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (log(sin(c + d*x) + 1)*(a - b)^2*(a + 2*b))/(4*d) - (log(sin(c + d*x) - 1)*
(a + b)^2*(a - 2*b))/(4*d) - ((3*a^2*b)/2 + b^3/2 + sin(c + d*x)*((3*a*b^2)
/2 + a^3/2))/(d*(sin(c + d*x)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(c+dx\right)\right)^3 \sec^3\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sec(c + d*x)**3, x)
```

3.405 $\int \sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=94

$$\frac{3a\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{8d}+\frac{\tan(c+dx)\sec^3(c+\ldots}{4\ldots}$$

[Out] 3/8*a*(a^2-b^2)*arctanh(sin(d*x+c))/d+3/8*a*sec(d*x+c)^2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d+1/4*sec(d*x+c)^3*(a+b*sin(d*x+c))^3*tan(d*x+c)/d

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2668, 729, 723, 206}

$$\frac{3a\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{8d}+\frac{\tan(c+dx)\sec^3(c+\ldots}{4\ldots}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]

[Out] (3*a*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (3*a*Sec[c + d*x]^2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3*Tan[c + d*x])/(4*d)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 729

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(2*c*x)*(a + c*x^2)^(p + 1))/(4*a*c*(p + 1)), x] - Dist[(m*(2*c*d))/(4*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5 \operatorname{Subst}\left(\int \frac{(a+x)^3}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)}{4d} + \frac{(3ab^3)\operatorname{Subst}\left(\int \frac{(a+x)^2}{(b^2-x^2)^2}\,d\right.}{4d}$$

$$= \frac{3a\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\sec^3(c+dx)(a+}{}$$

$$= \frac{3a(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3a\sec^2(c+dx)(b+a\sin(c+dx))(a}{8d}$$

Mathematica [B] time = 4.04, size = 318, normalized size = 3.38

$$\frac{-6a(a^2-b^2)^3(\log(1-\sin(c+dx))-\log(\sin(c+dx)+1))+16a^4b(3a^2-2b^2)\tan^2(c+dx)+16a^2b\sec^2(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-6*a*(a^2 - b^2)^3*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + a*b*S
ec[c + d*x]^4*(-8*a^5 + 8*a^3*b^2 + (18*a^4*b - 11*a^2*b^3 + 5*b^5)*Sin[3*(
c + d*x)]) + a*(8*a^6 - 22*a^4*b^2 + 29*a^2*b^4 - 3*b^6)*Sec[c + d*x]^3*Tan
[c + d*x] + 16*a^4*b*(3*a^2 - 2*b^2)*Tan[c + d*x]^2 + 8*b^3*(4*a^4 - 5*a^2*
b^2 + b^4)*Tan[c + d*x]^4 + 4*a*Sec[c + d*x]*Tan[c + d*x]*(3*(a^6 - 5*a^4*b
^2) + 4*b^2*(3*a^4 - 5*a^2*b^2 + 2*b^4)*Tan[c + d*x]^2) + 16*a^2*b*Sec[c +
d*x]^2*(-a^4 + (2*a^4 - 5*a^2*b^2 + 3*b^4)*Tan[c + d*x]^2))/(32*(a^2 - b^2)
^2*d)
```

fricas [A] time = 0.48, size = 138, normalized size = 1.47

$$\frac{3(a^3-ab^2)\cos(dx+c)^4\log(\sin(dx+c)+1)-3(a^3-ab^2)\cos(dx+c)^4\log(-\sin(dx+c)+1)-8b^3\cos(dx}{16\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(a^3 - a*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^3 - a*b^2
)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 8*b^3*cos(d*x + c)^2 + 12*a^2*b +
 4*b^3 + 2*(2*a^3 + 6*a*b^2 + 3*(a^3 - a*b^2)*cos(d*x + c)^2)*sin(d*x + c))
/(d*cos(d*x + c)^4)
```

giac [A] time = 0.79, size = 139, normalized size = 1.48

$$\frac{3(a^3-ab^2)\log(|\sin(dx+c)+1|)-3(a^3-ab^2)\log(|\sin(dx+c)-1|)-\frac{2(3a^3\sin(dx+c)^3-3ab^2\sin(dx+c)^3-4b^3\sin(dx+c)}{(\sin(dx+c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*(a^3 - a*b^2)*log(abs(sin(d*x + c) + 1)) - 3*(a^3 - a*b^2)*log(abs(
sin(d*x + c) - 1)) - 2*(3*a^3*sin(d*x + c)^3 - 3*a*b^2*sin(d*x + c)^3 - 4*b
```

```
^3*sin(d*x + c)^2 - 5*a^3*sin(d*x + c) - 3*a*b^2*sin(d*x + c) - 6*a^2*b + 2
*b^3)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [B] time = 0.27, size = 195, normalized size = 2.07

$$\frac{a^3 \tan(dx+c)\left(\sec^3(dx+c)\right)}{4d}+\frac{3a^3 \sec(dx+c)\tan(dx+c)}{8d}+\frac{3a^3 \ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{3a^2b}{4d\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*sec(d*x+c)*tan(d*x+c)+3/8/d*a^3
*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^2*b/cos(d*x+c)^4+3/4/d*a*b^2*sin(d*x+c)^
3/cos(d*x+c)^4+3/8/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^2+3/8*a*b^2*sin(d*x+c)/d
-3/8/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b^3*sin(d*x+c)^4/cos(d*x+c)^4
```

maxima [A] time = 0.32, size = 136, normalized size = 1.45

$$\frac{3\left(a^3-ab^2\right)\log(\sin(dx+c)+1)-3\left(a^3-ab^2\right)\log(\sin(dx+c)-1)+\frac{2\left(4b^3\sin(dx+c)^2-3\left(a^3-ab^2\right)\sin(dx+c)^3+6a^2b-2}{\sin(dx+c)^4-2\sin(dx+c)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(a^3 - a*b^2)*log(sin(d*x + c) + 1) - 3*(a^3 - a*b^2)*log(sin(d*x +
 c) - 1) + 2*(4*b^3*sin(d*x + c)^2 - 3*(a^3 - a*b^2)*sin(d*x + c)^3 + 6*a^2
*b - 2*b^3 + (5*a^3 + 3*a*b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x +
c)^2 + 1))/d
```

mupad [B] time = 5.17, size = 114, normalized size = 1.21

$$\frac{\sin(c+d\,x)^3\,\left(\frac{3\,a\,b^2}{8}-\frac{3\,a^3}{8}\right)+\frac{3\,a^2\,b}{4}-\frac{b^3}{4}+\sin(c+d\,x)\,\left(\frac{5\,a^3}{8}+\frac{3\,a\,b^2}{8}\right)+\frac{b^3\sin(c+d\,x)^2}{2}}{d\,\left(\sin(c+d\,x)^4-2\sin(c+d\,x)^2+1\right)}+\frac{3\,a\,\mathrm{atanh}(\sin(c+d\,x))\,(a}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^5,x)
```

```
[Out] (sin(c + d*x)^3*((3*a*b^2)/8 - (3*a^3)/8) + (3*a^2*b)/4 - b^3/4 + sin(c + d
*x)*((3*a*b^2)/8 + (5*a^3)/8) + (b^3*sin(c + d*x)^2)/2)/(d*(sin(c + d*x)^4
- 2*sin(c + d*x)^2 + 1)) + (3*a*atanh(sin(c + d*x))*(a^2 - b^2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.406 $\int \cos^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=158

$$-\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}+\frac{a\left(2a^2+b^2\right)\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3a\left(2a^2+b^2\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{3}{16}a$$

```
[Out] 3/16*a*(2*a^2+b^2)*x-1/70*b*(17*a^2+4*b^2)*cos(d*x+c)^5/d+3/16*a*(2*a^2+b^2
)*cos(d*x+c)*sin(d*x+c)/d+1/8*a*(2*a^2+b^2)*cos(d*x+c)^3*sin(d*x+c)/d-3/14*
a*b*cos(d*x+c)^5*(a+b*sin(d*x+c))/d-1/7*b*cos(d*x+c)^5*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.22, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2692, 2862, 2669, 2635, 8}

$$-\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}+\frac{a\left(2a^2+b^2\right)\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3a\left(2a^2+b^2\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{3}{16}a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*(2*a^2 + b^2)*x)/16 - (b*(17*a^2 + 4*b^2)*Cos[c + d*x]^5)/(70*d) + (3*
a*(2*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(2*a^2 + b^2)*Cos[c
+ d*x]^3*Sin[c + d*x])/(8*d) - (3*a*b*Cos[c + d*x]^5*(a + b*Sin[c + d*x]))/
(14*d) - (b*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
```

```
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{b\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}+\frac{1}{7}\int \cos^4(c+dx)(a+b\sin(c+$$
$$= -\frac{3ab\cos^5(c+dx)(a+b\sin(c+dx))}{14d}-\frac{b\cos^5(c+dx)(a+b\sin(c+d}{7d}$$
$$= -\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}-\frac{3ab\cos^5(c+dx)(a+b\sin(c+dx))}{14d}-$$
$$= -\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}+\frac{a\left(2a^2+b^2\right)\cos^3(c+dx)\sin(c+dx)}{8d}$$
$$= -\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}+\frac{3a\left(2a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{16d}$$
$$= \frac{3}{16}a\left(2a^2+b^2\right)x-\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}+\frac{3a\left(2a^2+b^2\right)\cos(c}{16}$$

Mathematica [A] time = 0.38, size = 182, normalized size = 1.15

$$\frac{560a^3\sin(2(c+dx))+70a^3\sin(4(c+dx))+840a^3c+840a^3dx-35\left(12a^2b+b^3\right)\cos(3(c+dx))-105b\left(8a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (840*a^3*c + 420*a*b^2*c + 840*a^3*d*x + 420*a*b^2*d*x - 105*b*(8*a^2 + b^2
)*Cos[c + d*x] - 35*(12*a^2*b + b^3)*Cos[3*(c + d*x)] - 84*a^2*b*Cos[5*(c +
 d*x)] + 7*b^3*Cos[5*(c + d*x)] + 5*b^3*Cos[7*(c + d*x)] + 560*a^3*Sin[2*(c
 + d*x)] + 105*a*b^2*Sin[2*(c + d*x)] + 70*a^3*Sin[4*(c + d*x)] - 105*a*b^2
*Sin[4*(c + d*x)] - 35*a*b^2*Sin[6*(c + d*x)])/(2240*d)
```

fricas [A] time = 0.47, size = 117, normalized size = 0.74

$$\frac{80\,b^3\cos\left(dx+c\right)^7-112\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^5+105\left(2\,a^3+ab^2\right)dx-35\left(8\,ab^2\cos\left(dx+c\right)^5-2\left(2\,a^3+a\right.\right.}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(80*b^3*cos(d*x + c)^7 - 112*(3*a^2*b + b^3)*cos(d*x + c)^5 + 105*(2*
a^3 + a*b^2)*d*x - 35*(8*a*b^2*cos(d*x + c)^5 - 2*(2*a^3 + a*b^2)*cos(d*x +
 c)^3 - 3*(2*a^3 + a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 1.26, size = 173, normalized size = 1.09

$$\frac{b^3\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{ab^2\sin\left(6\,dx+6\,c\right)}{64\,d}+\frac{3}{16}\left(2\,a^3+ab^2\right)x-\frac{\left(12\,a^2b-b^3\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{\left(12\,a^2b+b^3\right)\cos\left(3\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/448*b^3*cos(7*d*x + 7*c)/d - 1/64*a*b^2*sin(6*d*x + 6*c)/d + 3/16*(2*a^3
+ a*b^2)*x - 1/320*(12*a^2*b - b^3)*cos(5*d*x + 5*c)/d - 1/64*(12*a^2*b + b
^3)*cos(3*d*x + 3*c)/d - 3/64*(8*a^2*b + b^3)*cos(d*x + c)/d + 1/64*(2*a^3
- 3*a*b^2)*sin(4*d*x + 4*c)/d + 1/64*(16*a^3 + 3*a*b^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.28, size = 145, normalized size = 0.92

$$\frac{b^3\left(-\frac{(\sin^2(dx+c))(\cos^5(dx+c))}{7}-\frac{2(\cos^5(dx+c))}{35}\right)+3a\,b^2\left(-\frac{\sin(dx+c)(\cos^5(dx+c))}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a*b^2*(-1/6*s
in(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d
*x+1/16*c)-3/5*a^2*b*cos(d*x+c)^5+a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*si
n(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.42, size = 117, normalized size = 0.74

$$-\frac{1344\,a^2b\cos(dx+c)^5-70\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^3-35\left(4\,\sin(2\,dx+2\,c)^3+12\,\right.}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2240*(1344*a^2*b*cos(d*x + c)^5 - 70*(12*d*x + 12*c + sin(4*d*x + 4*c) +
 8*sin(2*d*x + 2*c))*a^3 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin
(4*d*x + 4*c))*a*b^2 - 64*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^3)/d
```

mupad [B] time = 6.93, size = 474, normalized size = 3.00

$$\frac{3\,a\,\mathrm{atan}\left(\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(2\,a^2+b^2)}{8\left(\frac{3\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)}\right)\left(2\,a^2+b^2\right)}{8\,d}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a\,b^2}{8}-\frac{5\,a^3}{4}\right)+\frac{6\,a^2\,b}{5}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(3\,a^3+\frac{11\,a\,b^2}{2}\right)+\mathrm{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (3*a*atan((3*a*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/(8*((3*a*b^2)/8 + (3*a^3)/
4)))*(2*a^2 + b^2))/(8*d) - (tan(c/2 + (d*x)/2)*((3*a*b^2)/8 - (5*a^3)/4) +
 (6*a^2*b)/5 - tan(c/2 + (d*x)/2)^3*((11*a*b^2)/2 + 3*a^3) + tan(c/2 + (d*x
)/2)^11*((11*a*b^2)/2 + 3*a^3) - tan(c/2 + (d*x)/2)^13*((3*a*b^2)/8 - (5*a^
3)/4) + tan(c/2 + (d*x)/2)^5*((31*a*b^2)/8 - (9*a^3)/4) - tan(c/2 + (d*x)/2
)^9*((31*a*b^2)/8 - (9*a^3)/4) + tan(c/2 + (d*x)/2)^10*(12*a^2*b + 4*b^3) +
 tan(c/2 + (d*x)/2)^2*((12*a^2*b)/5 + (4*b^3)/5) + tan(c/2 + (d*x)/2)^8*(18
*a^2*b - 4*b^3) + tan(c/2 + (d*x)/2)^6*(24*a^2*b + 8*b^3) + tan(c/2 + (d*x)
/2)^4*((66*a^2*b)/5 - (8*b^3)/5) + (4*b^3)/35 + 6*a^2*b*tan(c/2 + (d*x)/2)^
12)/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*
x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 +
(d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1)) - (3*a*(2*a^2 + b^2)*(atan(tan(c/
2 + (d*x)/2)) - (d*x)/2))/(8*d)
```

sympy [A] time = 8.67, size = 348, normalized size = 2.20

$$\begin{cases} \frac{3a^3x\sin^4(c+dx)}{8} + \frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^3x\cos^4(c+dx)}{8} + \frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{3a^2b\cos^5}{5d} \\ x(a+b\sin(c))^3\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 3*a**2*b*cos(c + d*x)**
5/(5*d) + 3*a*b**2*x*sin(c + d*x)**6/16 + 9*a*b**2*x*sin(c + d*x)**4*cos(c
+ d*x)**2/16 + 9*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*a*b**2*x*c
os(c + d*x)**6/16 + 3*a*b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + a*b**2*s
in(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a*b**2*sin(c + d*x)*cos(c + d*x)**
5/(16*d) - b**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*b**3*cos(c + d*x)
**7/(35*d), Ne(d, 0)), (x*(a + b*sin(c))**3*cos(c)**4, True))
```

3.407 $\int \cos^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$-\frac{b\left(27a^2+8b^2\right)\cos^3(c+dx)}{60d}+\frac{a\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax\left(4a^2+3b^2\right)-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}$$

[Out] 1/8*a*(4*a^2+3*b^2)*x-1/60*b*(27*a^2+8*b^2)*cos(d*x+c)^3/d+1/8*a*(4*a^2+3*b^2)*cos(d*x+c)*sin(d*x+c)/d-7/20*a*b*cos(d*x+c)^3*(a+b*sin(d*x+c))/d-1/5*b*cos(d*x+c)^3*(a+b*sin(d*x+c))^2/d

Rubi [A] time = 0.19, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2692, 2862, 2669, 2635, 8}

$$-\frac{b\left(27a^2+8b^2\right)\cos^3(c+dx)}{60d}+\frac{a\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax\left(4a^2+3b^2\right)-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]

[Out] (a*(4*a^2 + 3*b^2)*x)/8 - (b*(27*a^2 + 8*b^2)*Cos[c + d*x]^3)/(60*d) + (a*(4*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (7*a*b*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(20*d) - (b*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(5*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sin(c+dx))^3\,dx &= -\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}+\frac{1}{5}\int \cos^2(c+dx)(a+b\sin(c+\\ &= -\frac{7ab\cos^3(c+dx)(a+b\sin(c+dx))}{20d}-\frac{b\cos^3(c+dx)(a+b\sin(c+d}{5d}\\ &= -\frac{b\left(27a^2+8b^2\right)\cos^3(c+dx)}{60d}-\frac{7ab\cos^3(c+dx)(a+b\sin(c+dx))}{20d}-\\ &= -\frac{b\left(27a^2+8b^2\right)\cos^3(c+dx)}{60d}+\frac{a\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8d}\\ &= \frac{1}{8}a\left(4a^2+3b^2\right)x-\frac{b\left(27a^2+8b^2\right)\cos^3(c+dx)}{60d}+\frac{a\left(4a^2+3b^2\right)\cos(c}{8a}\end{aligned}$$

Mathematica [A] time = 0.51, size = 107, normalized size = 0.82

$$\frac{-10\left(12a^2b+b^3\right)\cos(3(c+dx))+15a\left(4\left(4a^2+3b^2\right)(c+dx)+8a^2\sin(2(c+dx))-3b^2\sin(4(c+dx))\right)-60b}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-60*b*(6*a^2 + b^2)*Cos[c + d*x] - 10*(12*a^2*b + b^3)*Cos[3*(c + d*x)] +
6*b^3*Cos[5*(c + d*x)] + 15*a*(4*(4*a^2 + 3*b^2)*(c + d*x) + 8*a^2*Sin[2*(c
 + d*x)] - 3*b^2*Sin[4*(c + d*x)]))/(480*d)
```

fricas [A] time = 0.50, size = 98, normalized size = 0.75

$$\frac{24\,b^3\cos(dx+c)^5-40\left(3\,a^2b+b^3\right)\cos(dx+c)^3+15\left(4\,a^3+3\,ab^2\right)dx-15\left(6\,ab^2\cos(dx+c)^3-\left(4\,a^3+3\,ab\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(24*b^3*cos(d*x + c)^5 - 40*(3*a^2*b + b^3)*cos(d*x + c)^3 + 15*(4*a^
3 + 3*a*b^2)*d*x - 15*(6*a*b^2*cos(d*x + c)^3 - (4*a^3 + 3*a*b^2)*cos(d*x +
 c))*sin(d*x + c))/d
```

giac [A] time = 0.62, size = 113, normalized size = 0.86

$$\frac{b^3\cos(5\,dx+5\,c)}{80\,d}-\frac{3\,ab^2\sin(4\,dx+4\,c)}{32\,d}+\frac{a^3\sin(2\,dx+2\,c)}{4\,d}+\frac{1}{8}\left(4\,a^3+3\,ab^2\right)x-\frac{\left(12\,a^2b+b^3\right)\cos(3\,dx+3\,c)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/80*b^3*cos(5*d*x + 5*c)/d - 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/4*a^3*sin(2
*d*x + 2*c)/d + 1/8*(4*a^3 + 3*a*b^2)*x - 1/48*(12*a^2*b + b^3)*cos(3*d*x +
 3*c)/d - 1/8*(6*a^2*b + b^3)*cos(d*x + c)/d
```

maple [A] time = 0.20, size = 123, normalized size = 0.94

$$\frac{b^3\left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5}-\frac{2(\cos^3(dx+c))}{15}\right)+3a\,b^2\left(-\frac{\sin(dx+c)(\cos^3(dx+c))}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-a^2b\left(\cos^3(d\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a*b^2*(-1/4*s
in(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-a^2*b*cos(d
*x+c)^3+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 1.02, size = 93, normalized size = 0.71

$$-\frac{480\,a^2b\cos(dx+c)^3-120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3-45\,(4\,dx+4\,c-\sin(4\,dx+4\,c))ab^2-32\left(3\,\cos(d\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/480*(480*a^2*b*cos(d*x + c)^3 - 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3
 - 45*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b^2 - 32*(3*cos(d*x + c)^5 - 5*cos
(d*x + c)^3)*b^3)/d
```

mupad [B] time = 6.63, size = 356, normalized size = 2.72

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(4\,a^2+3\,b^2\right)}{4\left(a^3+\frac{3\,a\,b^2}{4}\right)}\right)\left(4\,a^2+3\,b^2\right)}{4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a\,b^2}{4}-a^3\right)+2\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(2\,a^3+\frac{9\,a\,b^2}{2}\right)-\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(4*a^2 + 3*b^2))/(4*((3*a*b^2)/4 + a^3)))*(4*
a^2 + 3*b^2))/(4*d) - (tan(c/2 + (d*x)/2)*((3*a*b^2)/4 - a^3) + 2*a^2*b - t
an(c/2 + (d*x)/2)^3*((9*a*b^2)/2 + 2*a^3) - tan(c/2 + (d*x)/2)^9*((3*a*b^2)
/4 - a^3) + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/2 + 2*a^3) + tan(c/2 + (d*x)/2)
^2*(4*a^2*b + (4*b^3)/3) + tan(c/2 + (d*x)/2)^4*(8*a^2*b - (4*b^3)/3) + tan
(c/2 + (d*x)/2)^6*(12*a^2*b + 4*b^3) + (4*b^3)/15 + 6*a^2*b*tan(c/2 + (d*x)
/2)^8)/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
(d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) - (a*(4*a
^2 + 3*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d)
```

sympy [A] time = 3.16, size = 236, normalized size = 1.80

$$\begin{cases}\frac{a^3x\sin^2(c+dx)}{2}+\frac{a^3x\cos^2(c+dx)}{2}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^2b\cos^3(c+dx)}{d}+\frac{3ab^2x\sin^4(c+dx)}{8}+\frac{3ab^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3ab^2x}{} \\ x\,(a+b\sin(c))^3\cos^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**2/2 + a**3*x*cos(c + d*x)**2/2 + a**3*sin(c
 + d*x)*cos(c + d*x)/(2*d) - a**2*b*cos(c + d*x)**3/d + 3*a*b**2*x*sin(c +
```

```
d*x)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*b**2*x*cos(c
 + d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*b**2*sin(c
 + d*x)*cos(c + d*x)**3/(8*d) - b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d)
- 2*b**3*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sin(c))**3*cos(c)**2,
 True))
```

3.408 $\int \sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=79

$$\frac{2b\left(a^2+b^2\right)\cos(c+dx)}{d}+\frac{ab^2\sin(c+dx)\cos(c+dx)}{d}-3ab^2x+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{d}$$

```
[Out] -3*a*b^2*x+2*b*(a^2+b^2)*cos(d*x+c)/d+a*b^2*cos(d*x+c)*sin(d*x+c)/d+sec(d*x
+c)*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^2/d
```

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2691, 2734}

$$\frac{2b\left(a^2+b^2\right)\cos(c+dx)}{d}+\frac{ab^2\sin(c+dx)\cos(c+dx)}{d}-3ab^2x+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -3*a*b^2*x + (2*b*(a^2 + b^2)*Cos[c + d*x])/d + (a*b^2*Cos[c + d*x]*Sin[c +
 d*x])/d + (Sec[c + d*x]*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/d
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{d}-\int (a+b\sin(c+dx)\\ &= -3ab^2x+\frac{2b\left(a^2+b^2\right)\cos(c+dx)}{d}+\frac{ab^2\cos(c+dx)\sin(c+dx)}{d}+\sec(\end{aligned}$$

Mathematica [A] time = 0.31, size = 68, normalized size = 0.86

$$\frac{\sec(c+dx)\left(6a^2b+b^3\cos(2(c+dx))+3b^3\right)+2a\left(a^2+3b^2\right)\tan(c+dx)-6ab^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-6*a*b^2*(c + d*x) + (6*a^2*b + 3*b^3 + b^3*Cos[2*(c + d*x)])*Sec[c + d*x]
 + 2*a*(a^2 + 3*b^2)*Tan[c + d*x])/(2*d)
```

fricas [A] time = 0.45, size = 70, normalized size = 0.89

$$-\frac{3\,ab^2dx\cos\left(dx+c\right)-b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3-\left(a^3+3\,ab^2\right)\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*a*b^2*d*x*cos(d*x + c) - b^3*cos(d*x + c)^2 - 3*a^2*b - b^3 - (a^3 + 3*
a*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.73, size = 123, normalized size = 1.56

$$-\frac{3\,(dx+c)ab^2+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2b+2\,b^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)*a*b^2 + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) + 3*a
*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b + 2*b^3)/(tan(1/2*d*x + 1/2*c)^4 - 1))/
d
```

maple [A] time = 0.40, size = 89, normalized size = 1.13

$$\frac{a^3\tan\left(dx+c\right)+\frac{3a^2b}{\cos(dx+c)}+3a\,b^2\left(\tan\left(dx+c\right)-dx-c\right)+b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2\left(dx+c\right)\right)\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*tan(d*x+c)+3*a^2*b/cos(d*x+c)+3*a*b^2*(tan(d*x+c)-d*x-c)+b^3*(sin(
d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.51, size = 70, normalized size = 0.89

$$-\frac{3\,(dx+c-\tan\left(dx+c\right))ab^2-b^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)-a^3\tan\left(dx+c\right)-\frac{3\,a^2b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*(d*x + c - tan(d*x + c))*a*b^2 - b^3*(1/cos(d*x + c) + cos(d*x + c)) -
a^3*tan(d*x + c) - 3*a^2*b/cos(d*x + c))/d
```

mupad [B] time = 5.81, size = 103, normalized size = 1.30

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^3+6\,a\,b^2\right)+6\,a^2\,b+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^3+6\,a\,b^2\right)+4\,b^3+6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}-3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] - (tan(c/2 + (d*x)/2)*(6*a*b^2 + 2*a^3) + 6*a^2*b + tan(c/2 + (d*x)/2)^3*(6
*a*b^2 + 2*a^3) + 4*b^3 + 6*a^2*b*tan(c/2 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)
/2)^4 - 1)) - 3*a*b^2*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^3 \sec^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sec(c + d*x)**2, x)
```

3.409 $\int \sec^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=84

$$\frac{2a\left(a^2-b^2\right)\tan(c+dx)}{3d}+\frac{2b\left(a^2-b^2\right)\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{3d}$$

```
[Out] 2/3*b*(a^2-b^2)*sec(d*x+c)/d+1/3*sec(d*x+c)^3*(b+a*sin(d*x+c))*(a+b*sin(d*x
+c))^2/d+2/3*a*(a^2-b^2)*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2691, 12, 2669, 3767, 8}

$$\frac{2a\left(a^2-b^2\right)\tan(c+dx)}{3d}+\frac{2b\left(a^2-b^2\right)\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(a^2 - b^2)*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^2)/(3*d) + (2*a*(a^2 - b^2)*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3d} - \frac{1}{3}\int \left(-2a^2+2b^2\right) s$$
$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3d} + \frac{1}{3}\left(2\left(a^2-b^2\right)\right)\int s$$
$$= \frac{2b\left(a^2-b^2\right)\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+$$
$$= \frac{2b\left(a^2-b^2\right)\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+$$
$$= \frac{2b\left(a^2-b^2\right)\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+$$

Mathematica [A] time = 0.42, size = 136, normalized size = 1.62

$$\frac{\sec^3(c+dx)\left(12a^3\sin(c+dx)+4a^3\sin(3(c+dx))+\left(15b^3-9a^2b\right)\cos(c+dx)-3a^2b\cos(3(c+dx))+24a^2b+\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(24*a^2*b - 4*b^3 + (-9*a^2*b + 15*b^3)*Cos[c + d*x] - 12*b
^3*Cos[2*(c + d*x)] - 3*a^2*b*Cos[3*(c + d*x)] + 5*b^3*Cos[3*(c + d*x)] + 1
2*a^3*Sin[c + d*x] + 18*a*b^2*Sin[c + d*x] + 4*a^3*Sin[3*(c + d*x)] - 6*a*b
^2*Sin[3*(c + d*x)]))/(24*d)
```

fricas [A] time = 0.47, size = 77, normalized size = 0.92

$$-\frac{3\,b^3\cos(dx+c)^2-3\,a^2b-b^3-\left(a^3+3\,ab^2+\left(2\,a^3-3\,ab^2\right)\cos(dx+c)^2\right)\sin(dx+c)}{3\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^3*cos(d*x + c)^2 - 3*a^2*b - b^3 - (a^3 + 3*a*b^2 + (2*a^3 - 3*a*
b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 1.92, size = 128, normalized size = 1.52

$$-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,b^3\tan\left(\right.\right.}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 2*a^3
*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*b^3*tan(1/2*d
*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 3*a^2*b - 2*b^3)/((tan(1/2*d*x
 + 1/2*c)^2 - 1)^3*d)
```

maple [A] time = 0.40, size = 122, normalized size = 1.45

$$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{a^2b}{\cos(dx+c)^3}+\frac{a\,b^2\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+b^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^2*b/cos(d*x+c)^3+a*b^2*sin(d
*x+c)^3/cos(d*x+c)^3+b^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/co
s(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.33, size = 80, normalized size = 0.95

$$\frac{3\,ab^2\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3-\frac{\left(3\,\cos(dx+c)^2-1\right)b^3}{\cos(dx+c)^3}+\frac{3\,a^2b}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a*b^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - (3*co
s(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3 + 3*a^2*b/cos(d*x + c)^3)/d
```

mupad [B] time = 5.25, size = 81, normalized size = 0.96

$$\frac{a^2\,b+\frac{a^3\,\sin(c+d\,x)}{3}+\frac{b^3}{3}-\cos\left(c+d\,x\right)^2\,\left(-\frac{2\,\sin(c+d\,x)\,a^3}{3}+\sin\left(c+d\,x\right)\,a\,b^2+b^3\right)+a\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^4,x)
```

```
[Out] (a^2*b + (a^3*sin(c + d*x))/3 + b^3/3 - cos(c + d*x)^2*(b^3 - (2*a^3*sin(c
+ d*x))/3 + a*b^2*sin(c + d*x)) + a*b^2*sin(c + d*x))/(d*cos(c + d*x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.410 $\int \sec^6(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=135

$$\frac{2a\left(4a^2-3b^2\right)\tan(c+dx)}{15d}+\frac{2b\left(2a^2-b^2\right)\sec(c+dx)}{15d}+\frac{2\sec^3(c+dx)(a+b\sin(c+dx))\left(\left(2a^2-b^2\right)\sin(c+dx)+\right.}{15d}$$

```
[Out] 2/15*b*(2*a^2-b^2)*sec(d*x+c)/d+1/5*sec(d*x+c)^5*(b+a*sin(d*x+c))*(a+b*sin(
d*x+c))^2/d+2/15*sec(d*x+c)^3*(a+b*sin(d*x+c))*(a*b+(2*a^2-b^2)*sin(d*x+c))
/d+2/15*a*(4*a^2-3*b^2)*tan(d*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2691, 2861, 2669, 3767, 8}

$$\frac{2a\left(4a^2-3b^2\right)\tan(c+dx)}{15d}+\frac{2b\left(2a^2-b^2\right)\sec(c+dx)}{15d}+\frac{2\sec^3(c+dx)(a+b\sin(c+dx))\left(\left(2a^2-b^2\right)\sin(c+dx)+\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(2*a^2 - b^2)*Sec[c + d*x])/(15*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d*
x])*(a + b*Sin[c + d*x])^2)/(5*d) + (2*Sec[c + d*x]^3*(a + b*Sin[c + d*x])*
(a*b + (2*a^2 - b^2)*Sin[c + d*x]))/(15*d) + (2*a*(4*a^2 - 3*b^2)*Tan[c + d
*x])/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5d} - \frac{1}{5}\int \sec^4(c+dx$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5d} + \frac{2\sec^3(c+dx)(a}{}$$

$$= \frac{2b\left(2a^2-b^2\right)\sec(c+dx)}{15d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(}{5d}$$

$$= \frac{2b\left(2a^2-b^2\right)\sec(c+dx)}{15d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(}{5d}$$

$$= \frac{2b\left(2a^2-b^2\right)\sec(c+dx)}{15d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(}{5d}$$

Mathematica [A] time = 0.54, size = 190, normalized size = 1.41

$$\frac{\sec^5(c+dx)\left(640a^3\sin(c+dx)+320a^3\sin(3(c+dx))+64a^3\sin(5(c+dx))+\left(110b^3-270a^2b\right)\cos(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^5*(1152*a^2*b + 64*b^3 + (-270*a^2*b + 110*b^3)*Cos[c + d*x]
- 320*b^3*Cos[2*(c + d*x)] - 135*a^2*b*Cos[3*(c + d*x)] + 55*b^3*Cos[3*(c +
 d*x)] - 27*a^2*b*Cos[5*(c + d*x)] + 11*b^3*Cos[5*(c + d*x)] + 640*a^3*Sin[
c + d*x] + 960*a*b^2*Sin[c + d*x] + 320*a^3*Sin[3*(c + d*x)] - 240*a*b^2*Si
n[3*(c + d*x)] + 64*a^3*Sin[5*(c + d*x)] - 48*a*b^2*Sin[5*(c + d*x)]))/(192
0*d)
```

fricas [A] time = 0.43, size = 101, normalized size = 0.75

$$-\frac{5\,b^3\cos(dx+c)^2 - 9\,a^2b - 3\,b^3 - \left(2\left(4\,a^3-3\,ab^2\right)\cos(dx+c)^4 + 3\,a^3 + 9\,ab^2 + \left(4\,a^3-3\,ab^2\right)\cos(dx+c)^2\right)}{15\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^3*cos(d*x + c)^2 - 9*a^2*b - 3*b^3 - (2*(4*a^3 - 3*a*b^2)*cos(d*
x + c)^4 + 3*a^3 + 9*a*b^2 + (4*a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c)
)/(d*cos(d*x + c)^5)
```

giac [A] time = 1.41, size = 243, normalized size = 1.80

$$-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 45\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 - 20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 60\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 30\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 20
*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 30*b^3*tan(
1/2*d*x + 1/2*c)^6 + 58*a^3*tan(1/2*d*x + 1/2*c)^5 + 24*a*b^2*tan(1/2*d*x +
 1/2*c)^5 + 90*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 10*b^3*tan(1/2*d*x + 1/2*c)^4
 - 20*a^3*tan(1/2*d*x + 1/2*c)^3 + 60*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 10*b^3
*tan(1/2*d*x + 1/2*c)^2 + 15*a^3*tan(1/2*d*x + 1/2*c) + 9*a^2*b - 2*b^3)/((
tan(1/2*d*x + 1/2*c)^2 - 1)^5*d)
```

maple [A] time = 0.32, size = 173, normalized size = 1.28

$$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{3a^2b}{5\cos(dx+c)^5}+3a\,b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+3/5*a^2*b/c
os(d*x+c)^5+3*a*b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*
x+c)^3)+b^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1
/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.33, size = 105, normalized size = 0.78

$$\frac{\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)a^3+3\left(3\,\tan(dx+c)^5+5\,\tan(dx+c)^3\right)ab^2-\frac{\left(5\,\cos(dx+c)^2-\right.}{\cos(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 3*(3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5 + 9*a^2*b/cos(d*x + c)^5)/d
```

mupad [B] time = 5.41, size = 119, normalized size = 0.88

$$\frac{\cos(c+d\,x)^4\left(\frac{8\,a^3\,\sin(c+d\,x)}{15}-\frac{2\,a\,b^2\,\sin(c+d\,x)}{5}\right)-\cos(c+d\,x)^2\left(-\frac{4\,\sin(c+d\,x)\,a^3}{15}+\frac{\sin(c+d\,x)\,a\,b^2}{5}+\frac{b^3}{3}\right)+\frac{3\,a^2\,b}{5}+\frac{a^3\,\sin(c+d}{5}}{d\cos(c+d\,x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^6,x)
```

```
[Out] (cos(c + d*x)^4*((8*a^3*sin(c + d*x))/15 - (2*a*b^2*sin(c + d*x))/5) - cos(
c + d*x)^2*(b^3/3 - (4*a^3*sin(c + d*x))/15 + (a*b^2*sin(c + d*x))/5) + (3*
a^2*b)/5 + (a^3*sin(c + d*x))/5 + b^3/5 + (3*a*b^2*sin(c + d*x))/5)/(d*cos(
c + d*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.411 $\int \sec^8(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=165

$$\frac{4a\left(2a^2-b^2\right)\tan^3(c+dx)}{35d}+\frac{12a\left(2a^2-b^2\right)\tan(c+dx)}{35d}+\frac{2b\left(3a^2-b^2\right)\sec^3(c+dx)}{35d}+\frac{2\sec^5(c+dx)(a+b\sin(c}{}$$

```
[Out] 2/35*b*(3*a^2-b^2)*sec(d*x+c)^3/d+1/7*sec(d*x+c)^7*(b+a*sin(d*x+c))*(a+b*si
n(d*x+c))^2/d+2/35*sec(d*x+c)^5*(a+b*sin(d*x+c))*(2*a*b+(3*a^2-b^2)*sin(d*x
+c))/d+12/35*a*(2*a^2-b^2)*tan(d*x+c)/d+4/35*a*(2*a^2-b^2)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.21, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2691, 2861, 2669, 3767}

$$\frac{4a\left(2a^2-b^2\right)\tan^3(c+dx)}{35d}+\frac{12a\left(2a^2-b^2\right)\tan(c+dx)}{35d}+\frac{2b\left(3a^2-b^2\right)\sec^3(c+dx)}{35d}+\frac{2\sec^5(c+dx)(a+b\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(3*a^2 - b^2)*Sec[c + d*x]^3)/(35*d) + (Sec[c + d*x]^7*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x])^2)/(7*d) + (2*Sec[c + d*x]^5*(a + b*Sin[c + d*x]
)*(2*a*b + (3*a^2 - b^2)*Sin[c + d*x]))/(35*d) + (12*a*(2*a^2 - b^2)*Tan[c
+ d*x])/(35*d) + (4*a*(2*a^2 - b^2)*Tan[c + d*x]^3)/(35*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7d} - \frac{1}{7}\int \sec^6(c+dx)(a$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7d} + \frac{2\sec^5(c+dx)(a+$$

$$= \frac{2b\left(3a^2-b^2\right)\sec^3(c+dx)}{35d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c$$

$$= \frac{2b\left(3a^2-b^2\right)\sec^3(c+dx)}{35d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c$$

$$= \frac{2b\left(3a^2-b^2\right)\sec^3(c+dx)}{35d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c$$

Mathematica [A] time = 0.90, size = 245, normalized size = 1.48

$$\sec^7(c+dx)\left(8960a^3\sin(c+dx)+5376a^3\sin(3(c+dx))+1792a^3\sin(5(c+dx))+256a^3\sin(7(c+dx))+35b\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^7*(15360*a^2*b + 1536*b^3 + 35*b*(-75*a^2 + 17*b^2)*Cos[c + d
*x] - 3584*b^3*Cos[2*(c + d*x)] - 1575*a^2*b*Cos[3*(c + d*x)] + 357*b^3*Cos
[3*(c + d*x)] - 525*a^2*b*Cos[5*(c + d*x)] + 119*b^3*Cos[5*(c + d*x)] - 75*
a^2*b*Cos[7*(c + d*x)] + 17*b^3*Cos[7*(c + d*x)] + 8960*a^3*Sin[c + d*x] +
13440*a*b^2*Sin[c + d*x] + 5376*a^3*Sin[3*(c + d*x)] - 2688*a*b^2*Sin[3*(c
+ d*x)] + 1792*a^3*Sin[5*(c + d*x)] - 896*a*b^2*Sin[5*(c + d*x)] + 256*a^3*
Sin[7*(c + d*x)] - 128*a*b^2*Sin[7*(c + d*x)]))/(35840*d)
```

fricas [A] time = 0.45, size = 124, normalized size = 0.75

$$-\frac{7\,b^3\cos(dx+c)^2-15\,a^2b-5\,b^3-\left(8\left(2\,a^3-ab^2\right)\cos(dx+c)^6+4\left(2\,a^3-ab^2\right)\cos(dx+c)^4+5\,a^3+15\,ab^2+3\right.}{35\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b^3*cos(d*x + c)^2 - 15*a^2*b - 5*b^3 - (8*(2*a^3 - a*b^2)*cos(d*x
 + c)^6 + 4*(2*a^3 - a*b^2)*cos(d*x + c)^4 + 5*a^3 + 15*a*b^2 + 3*(2*a^3 -
a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

giac [B] time = 1.88, size = 358, normalized size = 2.17

$$-\frac{2\left(35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+105\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-70\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+140\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^3*tan(1/2*d*x + 1/2*c)^13 + 105*a^2*b*tan(1/2*d*x + 1/2*c)^12 -
 70*a^3*tan(1/2*d*x + 1/2*c)^11 + 140*a*b^2*tan(1/2*d*x + 1/2*c)^11 + 70*b^
```

```
3*tan(1/2*d*x + 1/2*c)^10 + 301*a^3*tan(1/2*d*x + 1/2*c)^9 + 112*a*b^2*tan(
1/2*d*x + 1/2*c)^9 + 525*a^2*b*tan(1/2*d*x + 1/2*c)^8 + 70*b^3*tan(1/2*d*x
+ 1/2*c)^8 - 212*a^3*tan(1/2*d*x + 1/2*c)^7 + 456*a*b^2*tan(1/2*d*x + 1/2*c
)^7 + 140*b^3*tan(1/2*d*x + 1/2*c)^6 + 301*a^3*tan(1/2*d*x + 1/2*c)^5 + 112
*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 315*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 28*b^3*t
an(1/2*d*x + 1/2*c)^4 - 70*a^3*tan(1/2*d*x + 1/2*c)^3 + 140*a*b^2*tan(1/2*d
*x + 1/2*c)^3 + 14*b^3*tan(1/2*d*x + 1/2*c)^2 + 35*a^3*tan(1/2*d*x + 1/2*c)
 + 15*a^2*b - 2*b^3)/((tan(1/2*d*x + 1/2*c)^2 - 1)^7*d)
```

maple [A] time = 0.37, size = 219, normalized size = 1.33

$$\frac{-a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{3a^2b}{7\cos(dx+c)^7}+3ab^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+3/7*a^2*b/cos(d*x+c)^7+3*a*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*
sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/7*sin(d*x
+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x
+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 0.33, size = 124, normalized size = 0.75

$$\frac{\left(5\tan(dx+c)^7+21\tan(dx+c)^5+35\tan(dx+c)^3+35\tan(dx+c)\right)a^3+\left(15\tan(dx+c)^7+42\tan(dx+\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/35*((5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*
x + c))*a^3 + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a
*b^2 - (7*cos(d*x + c)^2 - 5)*b^3/cos(d*x + c)^7 + 15*a^2*b/cos(d*x + c)^7)
/d
```

mupad [B] time = 5.60, size = 152, normalized size = 0.92

$$\frac{\cos(c+d\,x)^4\left(\frac{8\,a^3\,\sin(c+d\,x)}{35}-\frac{4\,a\,b^2\,\sin(c+d\,x)}{35}\right)+\cos(c+d\,x)^6\left(\frac{16\,a^3\,\sin(c+d\,x)}{35}-\frac{8\,a\,b^2\,\sin(c+d\,x)}{35}\right)-\cos(c+d\,x)^2\left(-\right.}{d\cos(c+d\,x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^8,x)
```

```
[Out] (cos(c + d*x)^4*((8*a^3*sin(c + d*x))/35 - (4*a*b^2*sin(c + d*x))/35) + cos
(c + d*x)^6*((16*a^3*sin(c + d*x))/35 - (8*a*b^2*sin(c + d*x))/35) - cos(c
+ d*x)^2*(b^3/5 - (6*a^3*sin(c + d*x))/35 + (3*a*b^2*sin(c + d*x))/35) + (3
*a^2*b)/7 + (a^3*sin(c + d*x))/7 + b^3/7 + (3*a*b^2*sin(c + d*x))/7)/(d*cos
(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.412 $\int \sec^{10}(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=192

$$\frac{2a(8a^2-3b^2)\tan^5(c+dx)}{105d}+\frac{4a(8a^2-3b^2)\tan^3(c+dx)}{63d}+\frac{2a(8a^2-3b^2)\tan(c+dx)}{21d}+\frac{2b(4a^2-b^2)\sec^5(c+dx)}{63d}$$

```
[Out] 2/63*b*(4*a^2-b^2)*sec(d*x+c)^5/d+1/9*sec(d*x+c)^9*(b+a*sin(d*x+c))*(a+b*si
n(d*x+c))^2/d+2/63*sec(d*x+c)^7*(a+b*sin(d*x+c))*(3*a*b+(4*a^2-b^2)*sin(d*x
+c))/d+2/21*a*(8*a^2-3*b^2)*tan(d*x+c)/d+4/63*a*(8*a^2-3*b^2)*tan(d*x+c)^3/
d+2/105*a*(8*a^2-3*b^2)*tan(d*x+c)^5/d
```

Rubi [A] time = 0.22, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2691, 2861, 2669, 3767}

$$\frac{2a(8a^2-3b^2)\tan^5(c+dx)}{105d}+\frac{4a(8a^2-3b^2)\tan^3(c+dx)}{63d}+\frac{2a(8a^2-3b^2)\tan(c+dx)}{21d}+\frac{2b(4a^2-b^2)\sec^5(c+dx)}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(4*a^2 - b^2)*Sec[c + d*x]^5)/(63*d) + (Sec[c + d*x]^9*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x])^2)/(9*d) + (2*Sec[c + d*x]^7*(a + b*Sin[c + d*x]
)*(3*a*b + (4*a^2 - b^2)*Sin[c + d*x]))/(63*d) + (2*a*(8*a^2 - 3*b^2)*Tan[c
 + d*x])/(21*d) + (4*a*(8*a^2 - 3*b^2)*Tan[c + d*x]^3)/(63*d) + (2*a*(8*a^2
 - 3*b^2)*Tan[c + d*x]^5)/(105*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\int \sec^{10}(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{9d} - \frac{1}{9}\int \sec^8(c+d$$
$$= \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{9d} + \frac{2\sec^7(c+dx)($$
$$= \frac{2b\left(4a^2-b^2\right)\sec^5(c+dx)}{63d} + \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin}{9d}$$
$$= \frac{2b\left(4a^2-b^2\right)\sec^5(c+dx)}{63d} + \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin}{9d}$$
$$= \frac{2b\left(4a^2-b^2\right)\sec^5(c+dx)}{63d} + \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin}{9d}$$

Mathematica [A] time = 1.52, size = 299, normalized size = 1.56

$$\frac{\sec^9(c+dx)\left(2064384a^3\sin(c+dx)+1376256a^3\sin(3(c+dx))+589824a^3\sin(5(c+dx))+147456a^3\sin(7(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^9*(3440640*a^2*b + 409600*b^3 + 3150*b*(-147*a^2 + 23*b^2)*Co
s[c + d*x] - 737280*b^3*Cos[2*(c + d*x)] - 308700*a^2*b*Cos[3*(c + d*x)] +
48300*b^3*Cos[3*(c + d*x)] - 132300*a^2*b*Cos[5*(c + d*x)] + 20700*b^3*Cos[
5*(c + d*x)] - 33075*a^2*b*Cos[7*(c + d*x)] + 5175*b^3*Cos[7*(c + d*x)] - 3
675*a^2*b*Cos[9*(c + d*x)] + 575*b^3*Cos[9*(c + d*x)] + 2064384*a^3*Sin[c +
 d*x] + 3096576*a*b^2*Sin[c + d*x] + 1376256*a^3*Sin[3*(c + d*x)] - 516096*
a*b^2*Sin[3*(c + d*x)] + 589824*a^3*Sin[5*(c + d*x)] - 221184*a*b^2*Sin[5*(
c + d*x)] + 147456*a^3*Sin[7*(c + d*x)] - 55296*a*b^2*Sin[7*(c + d*x)] + 16
384*a^3*Sin[9*(c + d*x)] - 6144*a*b^2*Sin[9*(c + d*x)]))/(10321920*d)
```

fricas [A] time = 0.46, size = 146, normalized size = 0.76

$$-\frac{45\,b^3\cos\left(dx+c\right)^2-105\,a^2b-35\,b^3-\left(16\left(8\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^8+8\left(8\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^6+6\left(8\,a\right.\right.}{315\,d\cos\left(dx+c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/315*(45*b^3*cos(d*x + c)^2 - 105*a^2*b - 35*b^3 - (16*(8*a^3 - 3*a*b^2)*
cos(d*x + c)^8 + 8*(8*a^3 - 3*a*b^2)*cos(d*x + c)^6 + 6*(8*a^3 - 3*a*b^2)*c
os(d*x + c)^4 + 35*a^3 + 105*a*b^2 + 5*(8*a^3 - 3*a*b^2)*cos(d*x + c)^2)*si
n(d*x + c))/(d*cos(d*x + c)^9)
```

giac [B] time = 1.60, size = 473, normalized size = 2.46

$$-\frac{2\left(315\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+945\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}-840\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+1260\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/315*(315*a^3*tan(1/2*d*x + 1/2*c)^17 + 945*a^2*b*tan(1/2*d*x + 1/2*c)^16
 - 840*a^3*tan(1/2*d*x + 1/2*c)^15 + 1260*a*b^2*tan(1/2*d*x + 1/2*c)^15 + 6
30*b^3*tan(1/2*d*x + 1/2*c)^14 + 4788*a^3*tan(1/2*d*x + 1/2*c)^13 + 1512*a*
b^2*tan(1/2*d*x + 1/2*c)^13 + 8820*a^2*b*tan(1/2*d*x + 1/2*c)^12 + 1050*b^3
*tan(1/2*d*x + 1/2*c)^12 - 5112*a^3*tan(1/2*d*x + 1/2*c)^11 + 8532*a*b^2*ta
n(1/2*d*x + 1/2*c)^11 + 3150*b^3*tan(1/2*d*x + 1/2*c)^10 + 10658*a^3*tan(1/
2*d*x + 1/2*c)^9 + 4272*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 13230*a^2*b*tan(1/2*
d*x + 1/2*c)^8 + 1890*b^3*tan(1/2*d*x + 1/2*c)^8 - 5112*a^3*tan(1/2*d*x + 1
/2*c)^7 + 8532*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 1890*b^3*tan(1/2*d*x + 1/2*c)
^6 + 4788*a^3*tan(1/2*d*x + 1/2*c)^5 + 1512*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
3780*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 270*b^3*tan(1/2*d*x + 1/2*c)^4 - 840*a^
3*tan(1/2*d*x + 1/2*c)^3 + 1260*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 90*b^3*tan(1
/2*d*x + 1/2*c)^2 + 315*a^3*tan(1/2*d*x + 1/2*c) + 105*a^2*b - 10*b^3)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)^9*d)
```

maple [A] time = 0.38, size = 265, normalized size = 1.38

$$\frac{-a^3\left(-\frac{128}{315}-\frac{\left(\sec^8(dx+c)\right)}{9}-\frac{8\left(\sec^6(dx+c)\right)}{63}-\frac{16\left(\sec^4(dx+c)\right)}{105}-\frac{64\left(\sec^2(dx+c)\right)}{315}\right)\tan(dx+c)+\frac{a^2b}{3\cos(dx+c)^9}+3ab^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^4-
64/315*sec(d*x+c)^2)*tan(d*x+c)+1/3*a^2*b/cos(d*x+c)^9+3*a*b^2*(1/9*sin(d*x
+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*
x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/9*sin(d*x+c)^4/cos(d*x+c)^9
+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1/63*sin(d*x
+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

maxima [A] time = 0.34, size = 145, normalized size = 0.76

$$\frac{\left(35\tan(dx+c)^9+180\tan(dx+c)^7+378\tan(dx+c)^5+420\tan(dx+c)^3+315\tan(dx+c)\right)a^3+3\left(35\tan\right.}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*a^3 + 3*(35*tan(d*x + c)^9 + 135*tan(d*x
+ c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*a*b^2 - 5*(9*cos(d*x + c)
^2 - 7)*b^3/cos(d*x + c)^9 + 105*a^2*b/cos(d*x + c)^9)/d
```

mupad [B] time = 6.13, size = 275, normalized size = 1.43

$$\frac{b^3}{9d\cos(c+dx)^9}-\frac{b^3}{7d\cos(c+dx)^7}+\frac{a^2b}{3d\cos(c+dx)^9}+\frac{128a^3\sin(c+dx)}{315d\cos(c+dx)}+\frac{64a^3\sin(c+dx)}{315d\cos(c+dx)^3}+\frac{16a^3\sin(c+}{105d\cos(c+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^10,x)
```

```
[Out] b^3/(9*d*cos(c + d*x)^9) - b^3/(7*d*cos(c + d*x)^7) + (a^2*b)/(3*d*cos(c +
d*x)^9) + (128*a^3*sin(c + d*x))/(315*d*cos(c + d*x)) + (64*a^3*sin(c + d*x
))/(315*d*cos(c + d*x)^3) + (16*a^3*sin(c + d*x))/(105*d*cos(c + d*x)^5) +
```

```
(8*a^3*sin(c + d*x))/(63*d*cos(c + d*x)^7) + (a^3*sin(c + d*x))/(9*d*cos(c
+ d*x)^9) - (16*a*b^2*sin(c + d*x))/(105*d*cos(c + d*x)) - (8*a*b^2*sin(c +
 d*x))/(105*d*cos(c + d*x)^3) - (2*a*b^2*sin(c + d*x))/(35*d*cos(c + d*x)^5
) - (a*b^2*sin(c + d*x))/(21*d*cos(c + d*x)^7) + (a*b^2*sin(c + d*x))/(3*d*
cos(c + d*x)^9)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.413 $\int \cos^5(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=144

$$\frac{2(3a^2-b^2)(a+b\sin(c+dx))^{11}}{11b^5d}-\frac{2a(a^2-b^2)(a+b\sin(c+dx))^{10}}{5b^5d}+\frac{(a^2-b^2)^2(a+b\sin(c+dx))^9}{9b^5d}+\frac{(a+b\sin(c$$

```
[Out] 1/9*(a^2-b^2)^2*(a+b*sin(d*x+c))^9/b^5/d-2/5*a*(a^2-b^2)*(a+b*sin(d*x+c))^1
0/b^5/d+2/11*(3*a^2-b^2)*(a+b*sin(d*x+c))^11/b^5/d-1/3*a*(a+b*sin(d*x+c))^1
2/b^5/d+1/13*(a+b*sin(d*x+c))^13/b^5/d
```

Rubi [A] time = 0.22, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2668, 697}

$$\frac{2(3a^2-b^2)(a+b\sin(c+dx))^{11}}{11b^5d}-\frac{2a(a^2-b^2)(a+b\sin(c+dx))^{10}}{5b^5d}+\frac{(a^2-b^2)^2(a+b\sin(c+dx))^9}{9b^5d}+\frac{(a+b\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^9)/(9*b^5*d) - (2*a*(a^2 - b^2)*(a + b*
Sin[c + d*x])^10)/(5*b^5*d) + (2*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^11)/(11
*b^5*d) - (a*(a + b*Sin[c + d*x])^12)/(3*b^5*d) + (a + b*Sin[c + d*x])^13/(
13*b^5*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a+x)^8\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^8-4\left(a^3-ab^2\right)(a+x)^9+2\left(3a^2-b^2\right)(a+\right.\right.}{b^5d}\\ &= \frac{(a^2-b^2)^2(a+b\sin(c+dx))^9}{9b^5d}-\frac{2a(a^2-b^2)(a+b\sin(c+dx))^{10}}{5b^5d}+\frac{2(}{}\end{aligned}$$

Mathematica [A] time = 1.99, size = 120, normalized size = 0.83

$$\frac{\frac{2}{11}(3a^2-b^2)(a+b\sin(c+dx))^{11}+\frac{1}{9}(a^2-b^2)^2(a+b\sin(c+dx))^9+\frac{1}{13}(a+b\sin(c+dx))^{13}-\frac{1}{3}a(a+b\sin(c+}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (((a^2 - b^2)^2*(a + b*Sin[c + d*x])^9)/9 - (2*a*(a - b)*(a + b)*(a + b*Sin
[c + d*x])^10)/5 + (2*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^11)/11 - (a*(a + b
*Sin[c + d*x])^12)/3 + (a + b*Sin[c + d*x])^13/13)/(b^5*d)
```

fricas [B] time = 0.56, size = 356, normalized size = 2.47

$$4290\,ab^7\cos(dx+c)^{12} - 5148\left(7\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^{10} + 6435\left(7\,a^5b^3 + 14\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^8 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/6435*(4290*a*b^7*cos(d*x + c)^12 - 5148*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c
)^10 + 6435*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^8 - 8580*(a^7*b
 + 7*a^5*b^3 + 7*a^3*b^5 + a*b^7)*cos(d*x + c)^6 + (495*b^8*cos(d*x + c)^12
 - 180*(91*a^2*b^6 + 10*b^8)*cos(d*x + c)^10 + 10*(5005*a^4*b^4 + 4186*a^2*
b^6 + 229*b^8)*cos(d*x + c)^8 + 3432*a^8 + 13728*a^6*b^2 + 11440*a^4*b^4 +
2080*a^2*b^6 + 40*b^8 - 20*(1287*a^6*b^2 + 3575*a^4*b^4 + 1469*a^2*b^6 + 53
*b^8)*cos(d*x + c)^6 + 3*(429*a^8 + 1716*a^6*b^2 + 1430*a^4*b^4 + 260*a^2*b
^6 + 5*b^8)*cos(d*x + c)^4 + 4*(429*a^8 + 1716*a^6*b^2 + 1430*a^4*b^4 + 260
*a^2*b^6 + 5*b^8)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [B] time = 3.94, size = 464, normalized size = 3.22

$$\frac{ab^7\cos(12\,dx+12\,c)}{3072\,d} + \frac{b^8\sin(13\,dx+13\,c)}{53248\,d} - \frac{\left(14\,a^3b^5 + ab^7\right)\cos(10\,dx+10\,c)}{1280\,d} + \frac{\left(28\,a^5b^3 - ab^7\right)\cos(8\,dx+8}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3072*a*b^7*cos(12*d*x + 12*c)/d + 1/53248*b^8*sin(13*d*x + 13*c)/d - 1/12
80*(14*a^3*b^5 + a*b^7)*cos(10*d*x + 10*c)/d + 1/512*(28*a^5*b^3 - a*b^7)*c
os(8*d*x + 8*c)/d - 1/768*(32*a^7*b - 112*a^5*b^3 - 70*a^3*b^5 - 5*a*b^7)*c
os(6*d*x + 6*c)/d - 1/1024*(256*a^7*b + 224*a^5*b^3 - 5*a*b^7)*cos(4*d*x +
4*c)/d - 1/128*(80*a^7*b + 168*a^5*b^3 + 70*a^3*b^5 + 5*a*b^7)*cos(2*d*x +
2*c)/d - 1/45056*(112*a^2*b^6 + 3*b^8)*sin(11*d*x + 11*c)/d + 1/18432*(560*
a^4*b^4 + 56*a^2*b^6 - b^8)*sin(9*d*x + 9*c)/d - 1/2048*(128*a^6*b^2 - 80*a
^4*b^4 - 40*a^2*b^6 - b^8)*sin(7*d*x + 7*c)/d + 1/20480*(256*a^8 - 5376*a^6
*b^2 - 4480*a^4*b^4 - 560*a^2*b^6 - 5*b^8)*sin(5*d*x + 5*c)/d + 1/12288*(12
80*a^8 - 1792*a^6*b^2 - 4480*a^4*b^4 - 1120*a^2*b^6 - 25*b^8)*sin(3*d*x + 3
*c)/d + 5/1024*(128*a^8 + 448*a^6*b^2 + 336*a^4*b^4 + 56*a^2*b^6 + b^8)*sin
(d*x + c)/d
```

maple [B] time = 0.27, size = 530, normalized size = 3.68

$$b^8\left(-\frac{\left(\sin^7(dx+c)\right)\left(\cos^6(dx+c)\right)}{13} - \frac{7\left(\sin^5(dx+c)\right)\left(\cos^6(dx+c)\right)}{143} - \frac{35\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{1287} - \frac{5\left(\cos^6(dx+c)\right)\sin(dx+c)}{429} + \frac{\left(\frac{8}{3}+\cos^4(dx+c)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(b^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-3
5/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*cos(d*x+c)^6*sin(d*x+c)+1/429*(8/3+c
os(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+8*a*b^7*(-1/12*sin(d*x+c)^6*cos(d
*x+c)^6-1/20*sin(d*x+c)^4*cos(d*x+c)^6-1/40*sin(d*x+c)^2*cos(d*x+c)^6-1/120
```

```
*cos(d*x+c)^6)+28*a^2*b^6*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^
3*cos(d*x+c)^6-5/231*cos(d*x+c)^6*sin(d*x+c)+1/231*(8/3+cos(d*x+c)^4+4/3*co
s(d*x+c)^2)*sin(d*x+c))+56*a^3*b^5*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*si
n(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6)+70*a^4*b^4*(-1/9*sin(d*x+c)^3*co
s(d*x+c)^6-1/21*cos(d*x+c)^6*sin(d*x+c)+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))+56*a^5*b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+
c)^6)+28*a^6*b^2*(-1/7*cos(d*x+c)^6*sin(d*x+c)+1/35*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c))-4/3*a^7*b*cos(d*x+c)^6+1/5*a^8*(8/3+cos(d*x+c)^4+4
/3*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [B] time = 0.33, size = 311, normalized size = 2.16

$$\frac{495\,b^8\sin(dx+c)^{13}+4290\,ab^7\sin(dx+c)^{12}+1170\left(14\,a^2b^6-b^8\right)\sin(dx+c)^{11}+5148\left(7\,a^3b^5-2\,ab^7\right)\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/6435*(495*b^8*sin(d*x + c)^13 + 4290*a*b^7*sin(d*x + c)^12 + 1170*(14*a^2
*b^6 - b^8)*sin(d*x + c)^11 + 5148*(7*a^3*b^5 - 2*a*b^7)*sin(d*x + c)^10 +
25740*a^7*b*sin(d*x + c)^2 + 715*(70*a^4*b^4 - 56*a^2*b^6 + b^8)*sin(d*x +
c)^9 + 6435*a^8*sin(d*x + c) + 6435*(7*a^5*b^3 - 14*a^3*b^5 + a*b^7)*sin(d*
x + c)^8 + 25740*(a^6*b^2 - 5*a^4*b^4 + a^2*b^6)*sin(d*x + c)^7 + 8580*(a^7
*b - 14*a^5*b^3 + 7*a^3*b^5)*sin(d*x + c)^6 + 1287*(a^8 - 56*a^6*b^2 + 70*a
^4*b^4)*sin(d*x + c)^5 - 12870*(2*a^7*b - 7*a^5*b^3)*sin(d*x + c)^4 - 4290*
(a^8 - 14*a^6*b^2)*sin(d*x + c)^3)/d
```

mupad [B] time = 5.45, size = 306, normalized size = 2.12

$$\frac{\sin(c+d\,x)^5\left(\frac{a^8}{5}-\frac{56\,a^6\,b^2}{5}+14\,a^4\,b^4\right)+\sin(c+d\,x)^9\left(\frac{70\,a^4\,b^4}{9}-\frac{56\,a^2\,b^6}{9}+\frac{b^8}{9}\right)+a^8\,\sin(c+d\,x)+\frac{b^8\sin(c+d\,x)^{13}}{13}-\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^8,x)
```

```
[Out] (sin(c + d*x)^5*(a^8/5 + 14*a^4*b^4 - (56*a^6*b^2)/5) + sin(c + d*x)^9*(b^8
/9 - (56*a^2*b^6)/9 + (70*a^4*b^4)/9) + a^8*sin(c + d*x) + (b^8*sin(c + d*x
)^13)/13 - sin(c + d*x)^4*(4*a^7*b - 14*a^5*b^3) - sin(c + d*x)^10*((8*a*b^
7)/5 - (28*a^3*b^5)/5) - (2*a^6*sin(c + d*x)^3*(a^2 - 14*b^2))/3 + 4*a^7*b*
sin(c + d*x)^2 + (2*a*b^7*sin(c + d*x)^12)/3 + (2*b^6*sin(c + d*x)^11*(14*a
^2 - b^2))/11 + (4*a^3*b*sin(c + d*x)^6*(a^4 + 7*b^4 - 14*a^2*b^2))/3 + a*b
^3*sin(c + d*x)^8*(7*a^4 + b^4 - 14*a^2*b^2) + 4*a^2*b^2*sin(c + d*x)^7*(a^
4 + b^4 - 5*a^2*b^2))/d
```

sympy [A] time = 119.11, size = 614, normalized size = 4.26

$$\begin{cases} \frac{8a^8\sin^5(c+dx)}{15d}+\frac{4a^8\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a^8\sin(c+dx)\cos^4(c+dx)}{d}-\frac{4a^7b\cos^6(c+dx)}{3d}+\frac{32a^6b^2\sin^7(c+dx)}{15d}+\frac{112a^6b^2\sin^5(c+dx)\cos^2}{15d} \\ x\left(a+b\sin(c)\right)^8\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((8*a**8*sin(c + d*x)**5/(15*d) + 4*a**8*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**8*sin(c + d*x)*cos(c + d*x)**4/d - 4*a**7*b*cos(c + d*x)*
*6/(3*d) + 32*a**6*b**2*sin(c + d*x)**7/(15*d) + 112*a**6*b**2*sin(c + d*x)
**5*cos(c + d*x)**2/(15*d) + 28*a**6*b**2*sin(c + d*x)**3*cos(c + d*x)**4/(
```

```
3*d) - 28*a**5*b**3*sin(c + d*x)**2*cos(c + d*x)**6/(3*d) - 7*a**5*b**3*cos
(c + d*x)**8/(3*d) + 16*a**4*b**4*sin(c + d*x)**9/(9*d) + 8*a**4*b**4*sin(c
 + d*x)**7*cos(c + d*x)**2/d + 14*a**4*b**4*sin(c + d*x)**5*cos(c + d*x)**4
/d - 28*a**3*b**5*sin(c + d*x)**4*cos(c + d*x)**6/(3*d) - 14*a**3*b**5*sin(
c + d*x)**2*cos(c + d*x)**8/(3*d) - 14*a**3*b**5*cos(c + d*x)**10/(15*d) +
32*a**2*b**6*sin(c + d*x)**11/(99*d) + 16*a**2*b**6*sin(c + d*x)**9*cos(c +
 d*x)**2/(9*d) + 4*a**2*b**6*sin(c + d*x)**7*cos(c + d*x)**4/d - 4*a*b**7*s
in(c + d*x)**6*cos(c + d*x)**6/(3*d) - a*b**7*sin(c + d*x)**4*cos(c + d*x)*
*8/d - 2*a*b**7*sin(c + d*x)**2*cos(c + d*x)**10/(5*d) - a*b**7*cos(c + d*x
)**12/(15*d) + 8*b**8*sin(c + d*x)**13/(1287*d) + 4*b**8*sin(c + d*x)**11*c
os(c + d*x)**2/(99*d) + b**8*sin(c + d*x)**9*cos(c + d*x)**4/(9*d), Ne(d, 0
)), (x*(a + b*sin(c))**8*cos(c)**5, True))
```

3.414 $\int \cos^3(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=77

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^9}{9b^3d}-\frac{(a+b\sin(c+dx))^{11}}{11b^3d}+\frac{a(a+b\sin(c+dx))^{10}}{5b^3d}$$

```
[Out] -1/9*(a^2-b^2)*(a+b*sin(d*x+c))^9/b^3/d+1/5*a*(a+b*sin(d*x+c))^10/b^3/d-1/1
1*(a+b*sin(d*x+c))^11/b^3/d
```

Rubi [A] time = 0.15, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^9}{9b^3d}-\frac{(a+b\sin(c+dx))^{11}}{11b^3d}+\frac{a(a+b\sin(c+dx))^{10}}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -((a^2 - b^2)*(a + b*Sin[c + d*x])^9)/(9*b^3*d) + (a*(a + b*Sin[c + d*x])^1
0)/(5*b^3*d) - (a + b*Sin[c + d*x])^11/(11*b^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a+x)^8\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^8+2a(a+x)^9-(a+x)^{10}\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= -\frac{(a^2-b^2)(a+b\sin(c+dx))^9}{9b^3d}+\frac{a(a+b\sin(c+dx))^{10}}{5b^3d}-\frac{(a+b\sin(c+dx))^{11}}{11b^3d}\end{aligned}$$

Mathematica [A] time = 0.85, size = 56, normalized size = 0.73

$$\frac{(a+b\sin(c+dx))^9\left(-2a^2+18ab\sin(c+dx)+45b^2\cos(2(c+dx))+65b^2\right)}{990b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((a + b*Sin[c + d*x])^9*(-2*a^2 + 65*b^2 + 45*b^2*Cos[2*(c + d*x)] + 18*a*b
*Sin[c + d*x]))/(990*b^3*d)
```

fricas [B] time = 0.52, size = 310, normalized size = 4.03

$$\frac{396\,ab^7\cos(dx+c)^{10} - 495\left(7\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^8 + 660\left(7\,a^5b^3 + 14\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^6 - 990}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/495*(396*a*b^7*cos(d*x + c)^10 - 495*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^8
 + 660*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^6 - 990*(a^7*b + 7*a
^5*b^3 + 7*a^3*b^5 + a*b^7)*cos(d*x + c)^4 + (45*b^8*cos(d*x + c)^10 - 10*(
154*a^2*b^6 + 17*b^8)*cos(d*x + c)^8 + 330*a^8 + 1848*a^6*b^2 + 1980*a^4*b^
4 + 440*a^2*b^6 + 10*b^8 + 10*(495*a^4*b^4 + 418*a^2*b^6 + 23*b^8)*cos(d*x
+ c)^6 - 12*(231*a^6*b^2 + 660*a^4*b^4 + 275*a^2*b^6 + 10*b^8)*cos(d*x + c)
^4 + (165*a^8 + 924*a^6*b^2 + 990*a^4*b^4 + 220*a^2*b^6 + 5*b^8)*cos(d*x +
c)^2)*sin(d*x + c))/d
```

giac [B] time = 1.97, size = 272, normalized size = 3.53

$$-\frac{45\,b^8\sin(dx+c)^{11} + 396\,ab^7\sin(dx+c)^{10} + 1540\,a^2b^6\sin(dx+c)^9 - 55\,b^8\sin(dx+c)^9 + 3465\,a^3b^5\sin(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/495*(45*b^8*sin(d*x + c)^11 + 396*a*b^7*sin(d*x + c)^10 + 1540*a^2*b^6*s
in(d*x + c)^9 - 55*b^8*sin(d*x + c)^9 + 3465*a^3*b^5*sin(d*x + c)^8 - 495*a
*b^7*sin(d*x + c)^8 + 4950*a^4*b^4*sin(d*x + c)^7 - 1980*a^2*b^6*sin(d*x +
c)^7 + 4620*a^5*b^3*sin(d*x + c)^6 - 4620*a^3*b^5*sin(d*x + c)^6 + 2772*a^6
*b^2*sin(d*x + c)^5 - 6930*a^4*b^4*sin(d*x + c)^5 + 990*a^7*b*sin(d*x + c)^
4 - 6930*a^5*b^3*sin(d*x + c)^4 + 165*a^8*sin(d*x + c)^3 - 4620*a^6*b^2*sin
(d*x + c)^3 - 1980*a^7*b*sin(d*x + c)^2 - 495*a^8*sin(d*x + c))/d
```

maple [B] time = 0.26, size = 480, normalized size = 6.23

$$\frac{b^8\left(-\frac{\left(\sin^7(dx+c)\right)\left(\cos^4(dx+c)\right)}{11} - \frac{7\left(\sin^5(dx+c)\right)\left(\cos^4(dx+c)\right)}{99} - \frac{5\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{99} - \frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{33} + \frac{\left(2+\cos^2(dx+c)\right)\sin}{99}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(b^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*sin(d*x+c)^5*cos(d*x+c)^4-5/
99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*sin(d*x+c)*cos(d*x+c)^4+1/99*(2+cos(d*x+c
)^2)*sin(d*x+c))+8*a*b^7*(-1/10*sin(d*x+c)^6*cos(d*x+c)^4-3/40*sin(d*x+c)^4
*cos(d*x+c)^4-1/20*cos(d*x+c)^4*sin(d*x+c)^2-1/40*cos(d*x+c)^4)+28*a^2*b^6*
(-1/9*sin(d*x+c)^5*cos(d*x+c)^4-5/63*sin(d*x+c)^3*cos(d*x+c)^4-1/21*sin(d*x
+c)*cos(d*x+c)^4+1/63*(2+cos(d*x+c)^2)*sin(d*x+c))+56*a^3*b^5*(-1/8*sin(d*x
+c)^4*cos(d*x+c)^4-1/12*cos(d*x+c)^4*sin(d*x+c)^2-1/24*cos(d*x+c)^4)+70*a^4
*b^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+c
os(d*x+c)^2)*sin(d*x+c))+56*a^5*b^3*(-1/6*cos(d*x+c)^4*sin(d*x+c)^2-1/12*co
s(d*x+c)^4)+28*a^6*b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*
sin(d*x+c))-2*a^7*b*cos(d*x+c)^4+1/3*a^8*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [B] time = 0.33, size = 233, normalized size = 3.03

$$-\frac{45\,b^8\sin(dx+c)^{11} + 396\,ab^7\sin(dx+c)^{10} - 1980\,a^7b\sin(dx+c)^2 + 55\left(28\,a^2b^6 - b^8\right)\sin(dx+c)^9 - 495\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/495*(45*b^8*sin(d*x + c)^11 + 396*a*b^7*sin(d*x + c)^10 - 1980*a^7*b*sin
(d*x + c)^2 + 55*(28*a^2*b^6 - b^8)*sin(d*x + c)^9 - 495*a^8*sin(d*x + c) +
 495*(7*a^3*b^5 - a*b^7)*sin(d*x + c)^8 + 990*(5*a^4*b^4 - 2*a^2*b^6)*sin(d
*x + c)^7 + 4620*(a^5*b^3 - a^3*b^5)*sin(d*x + c)^6 + 1386*(2*a^6*b^2 - 5*a
^4*b^4)*sin(d*x + c)^5 + 990*(a^7*b - 7*a^5*b^3)*sin(d*x + c)^4 + 165*(a^8
- 28*a^6*b^2)*sin(d*x + c)^3)/d
```

mupad [B] time = 5.37, size = 231, normalized size = 3.00

$$-\frac{\sin(c+d\,x)^3\,\left(\frac{a^8}{3}-\frac{28\,a^6\,b^2}{3}\right)-\sin(c+d\,x)^5\,\left(14\,a^4\,b^4-\frac{28\,a^6\,b^2}{5}\right)-\sin(c+d\,x)^7\,\left(4\,a^2\,b^6-10\,a^4\,b^4\right)-a^8\,\sin(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^8,x)
```

```
[Out] -(sin(c + d*x)^3*(a^8/3 - (28*a^6*b^2)/3) - sin(c + d*x)^5*(14*a^4*b^4 - (2
8*a^6*b^2)/5) - sin(c + d*x)^7*(4*a^2*b^6 - 10*a^4*b^4) - a^8*sin(c + d*x)
- sin(c + d*x)^9*(b^8/9 - (28*a^2*b^6)/9) + (b^8*sin(c + d*x)^11)/11 - 4*a^
7*b*sin(c + d*x)^2 + (4*a*b^7*sin(c + d*x)^10)/5 + 2*a^5*b*sin(c + d*x)^4*(
a^2 - 7*b^2) + a*b^5*sin(c + d*x)^8*(7*a^2 - b^2) + (28*a^3*b^3*sin(c + d*x
)^6*(a^2 - b^2))/3)/d
```

sympy [A] time = 54.46, size = 468, normalized size = 6.08

$$\begin{cases} \frac{2a^8\sin^3(c+dx)}{3d} + \frac{a^8\sin(c+dx)\cos^2(c+dx)}{d} - \frac{2a^7b\cos^4(c+dx)}{d} + \frac{56a^6b^2\sin^5(c+dx)}{15d} + \frac{28a^6b^2\sin^3(c+dx)\cos^2(c+dx)}{3d} - \frac{14a^5b^3\sin^2(c+dx)\cos}{d} \\ x\,(a+b\sin(c))^8\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((2*a**8*sin(c + d*x)**3/(3*d) + a**8*sin(c + d*x)*cos(c + d*x)**2
/d - 2*a**7*b*cos(c + d*x)**4/d + 56*a**6*b**2*sin(c + d*x)**5/(15*d) + 28*
a**6*b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - 14*a**5*b**3*sin(c + d*x)
**2*cos(c + d*x)**4/d - 14*a**5*b**3*cos(c + d*x)**6/(3*d) + 4*a**4*b**4*si
n(c + d*x)**7/d + 14*a**4*b**4*sin(c + d*x)**5*cos(c + d*x)**2/d - 14*a**3*
b**5*sin(c + d*x)**4*cos(c + d*x)**4/d - 28*a**3*b**5*sin(c + d*x)**2*cos(c
 + d*x)**6/(3*d) - 7*a**3*b**5*cos(c + d*x)**8/(3*d) + 8*a**2*b**6*sin(c +
d*x)**9/(9*d) + 4*a**2*b**6*sin(c + d*x)**7*cos(c + d*x)**2/d - 2*a*b**7*si
n(c + d*x)**6*cos(c + d*x)**4/d - 2*a*b**7*sin(c + d*x)**4*cos(c + d*x)**6/
d - a*b**7*sin(c + d*x)**2*cos(c + d*x)**8/d - a*b**7*cos(c + d*x)**10/(5*d
) + 2*b**8*sin(c + d*x)**11/(99*d) + b**8*sin(c + d*x)**9*cos(c + d*x)**2/(
9*d), Ne(d, 0)), (x*(a + b*sin(c))**8*cos(c)**3, True))
```

3.415 $\int \cos(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^9}{9bd}$$

```
[Out] 1/9*(a+b*sin(d*x+c))^9/b/d
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 32}

$$\frac{(a+b\sin(c+dx))^9}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a + b*Sin[c + d*x])^9/(9*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a+x)^8\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^9}{9bd}\end{aligned}$$

Mathematica [B] time = 0.36, size = 137, normalized size = 6.23

$$\frac{\sin(c+dx)\left(9a^8+36a^7b\sin(c+dx)+84a^6b^2\sin^2(c+dx)+126a^5b^3\sin^3(c+dx)+126a^4b^4\sin^4(c+dx)+8\right.}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sin[c + d*x]*(9*a^8 + 36*a^7*b*Sin[c + d*x] + 84*a^6*b^2*Sin[c + d*x]^2 + 126*a^5*b^3*Sin[c + d*x]^3 + 126*a^4*b^4*Sin[c + d*x]^4 + 84*a^3*b^5*Sin[c + d*x]^5 + 36*a^2*b^6*Sin[c + d*x]^6 + 9*a*b^7*Sin[c + d*x]^7 + b^8*Sin[c + d*x]^8))/(9*d)
```

fricas [B] time = 0.51, size = 257, normalized size = 11.68

$$\frac{9\,ab^7\cos(dx+c)^8 - 12\left(7\,a^3b^5+3\,ab^7\right)\cos(dx+c)^6 + 18\left(7\,a^5b^3+14\,a^3b^5+3\,ab^7\right)\cos(dx+c)^4 - 36\left(a^7b+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*b^7*cos(d*x + c)^8 - 12*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^6 + 18*
(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 - 36*(a^7*b + 7*a^5*b^3 +
 7*a^3*b^5 + a*b^7)*cos(d*x + c)^2 + (b^8*cos(d*x + c)^8 + 9*a^8 + 84*a^6*b
^2 + 126*a^4*b^4 + 36*a^2*b^6 + b^8 - 4*(9*a^2*b^6 + b^8)*cos(d*x + c)^6 +
6*(21*a^4*b^4 + 18*a^2*b^6 + b^8)*cos(d*x + c)^4 - 4*(21*a^6*b^2 + 63*a^4*b
^4 + 27*a^2*b^6 + b^8)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [A] time = 2.93, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^9}{9\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/9*(b*sin(d*x + c) + a)^9/(b*d)
```

maple [A] time = 0.11, size = 21, normalized size = 0.95

$$\frac{(a+b\sin(dx+c))^9}{9bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/9*(a+b*sin(d*x+c))^9/b/d
```

maxima [A] time = 0.34, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^9}{9\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/9*(b*sin(d*x + c) + a)^9/(b*d)
```

mupad [B] time = 5.27, size = 135, normalized size = 6.14

$$\frac{a^8\sin(c+d\,x)+4\,a^7\,b\sin(c+d\,x)^2+\frac{28\,a^6\,b^2\sin(c+d\,x)^3}{3}+14\,a^5\,b^3\sin(c+d\,x)^4+14\,a^4\,b^4\sin(c+d\,x)^5+\frac{28\,a^3\,b^5\,\mathrm{si}}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^8,x)
```

```
[Out] (a^8*sin(c + d*x) + (b^8*sin(c + d*x)^9)/9 + 4*a^7*b*sin(c + d*x)^2 + a*b^7
*sin(c + d*x)^8 + (28*a^6*b^2*sin(c + d*x)^3)/3 + 14*a^5*b^3*sin(c + d*x)^4
 + 14*a^4*b^4*sin(c + d*x)^5 + (28*a^3*b^5*sin(c + d*x)^6)/3 + 4*a^2*b^6*si
n(c + d*x)^7)/d
```

sympy [A] time = 20.96, size = 168, normalized size = 7.64

$$\begin{cases} \frac{a^8\sin(c+dx)}{d}+\frac{4a^7b\sin^2(c+dx)}{d}+\frac{28a^6b^2\sin^3(c+dx)}{3d}+\frac{14a^5b^3\sin^4(c+dx)}{d}+\frac{14a^4b^4\sin^5(c+dx)}{d}+\frac{28a^3b^5\sin^6(c+dx)}{3d}+\frac{4a^2b^6\sin^7(c+dx)}{d} \\ x(a+b\sin(c))^8\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((a**8*sin(c + d*x)/d + 4*a**7*b*sin(c + d*x)**2/d + 28*a**6*b**2*
sin(c + d*x)**3/(3*d) + 14*a**5*b**3*sin(c + d*x)**4/d + 14*a**4*b**4*sin(c
 + d*x)**5/d + 28*a**3*b**5*sin(c + d*x)**6/(3*d) + 4*a**2*b**6*sin(c + d*x
)**7/d + a*b**7*sin(c + d*x)**8/d + b**8*sin(c + d*x)**9/(9*d), Ne(d, 0)),
(x*(a + b*sin(c))**8*cos(c), True))
```

3.416 $\int \sec(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=245

$$-\frac{b^6\left(28a^2+b^2\right)\sin^5(c+dx)}{5d}-\frac{2ab^5\left(7a^2+b^2\right)\sin^4(c+dx)}{d}-\frac{b^4\left(70a^4+28a^2b^2+b^4\right)\sin^3(c+dx)}{3d}-\frac{4ab^3\left(7a^4+7\right.}{}$$

```
[Out] -1/2*(a+b)^8*ln(1-sin(d*x+c))/d+1/2*(a-b)^8*ln(1+sin(d*x+c))/d-b^2*(28*a^6+
70*a^4*b^2+28*a^2*b^4+b^6)*sin(d*x+c)/d-4*a*b^3*(7*a^4+7*a^2*b^2+b^4)*sin(d
*x+c)^2/d-1/3*b^4*(70*a^4+28*a^2*b^2+b^4)*sin(d*x+c)^3/d-2*a*b^5*(7*a^2+b^2
)*sin(d*x+c)^4/d-1/5*b^6*(28*a^2+b^2)*sin(d*x+c)^5/d-4/3*a*b^7*sin(d*x+c)^6
/d-1/7*b^8*sin(d*x+c)^7/d
```

Rubi [A] time = 0.18, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2668, 702, 633, 31}

$$-\frac{b^6\left(28a^2+b^2\right)\sin^5(c+dx)}{5d}-\frac{2ab^5\left(7a^2+b^2\right)\sin^4(c+dx)}{d}-\frac{b^4\left(28a^2b^2+70a^4+b^4\right)\sin^3(c+dx)}{3d}-\frac{4ab^3\left(7a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -((a + b)^8*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^8*Log[1 + Sin[c + d*x]]
)/(2*d) - (b^2*(28*a^6 + 70*a^4*b^2 + 28*a^2*b^4 + b^6)*Sin[c + d*x])/d - (
4*a*b^3*(7*a^4 + 7*a^2*b^2 + b^4)*Sin[c + d*x]^2)/d - (b^4*(70*a^4 + 28*a^2
*b^2 + b^4)*Sin[c + d*x]^3)/(3*d) - (2*a*b^5*(7*a^2 + b^2)*Sin[c + d*x]^4)/
d - (b^6*(28*a^2 + b^2)*Sin[c + d*x]^5)/(5*d) - (4*a*b^7*Sin[c + d*x]^6)/(3
*d) - (b^8*Sin[c + d*x]^7)/(7*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^8}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(-28a^6-70a^4b^2-28a^2b^4-b^6-8a\left(7a^4+7a^2b^2+b^4\right)x-\right.\right.}{}$$

$$= -\frac{b^2\left(28a^6+70a^4b^2+28a^2b^4+b^6\right)\sin(c+dx)}{d} - \frac{4ab^3\left(7a^4+7a^2b^2+\right.}{d}$$

$$= -\frac{b^2\left(28a^6+70a^4b^2+28a^2b^4+b^6\right)\sin(c+dx)}{d} - \frac{4ab^3\left(7a^4+7a^2b^2+\right.}{d}$$

$$= -\frac{(a+b)^8\log(1-\sin(c+dx))}{2d} + \frac{(a-b)^8\log(1+\sin(c+dx))}{2d} - \frac{b^2\left(28\right.}{}$$

Mathematica [**A**] time = 0.21, size = 227, normalized size = 0.93

$$\frac{b\left(-\frac{1}{5}b^5\left(28a^2+b^2\right)\sin^5(c+dx)-2ab^4\left(7a^2+b^2\right)\sin^4(c+dx)-4ab^2\left(7a^4+7a^2b^2+b^4\right)\sin^2(c+dx)-\frac{1}{3}b^3\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (b*(-1/2*((a + b)^8*Log[1 - Sin[c + d*x]])/b + ((a - b)^8*Log[1 + Sin[c + d
*x]])/(2*b) - b*(28*a^6 + 70*a^4*b^2 + 28*a^2*b^4 + b^6)*Sin[c + d*x] - 4*a
*b^2*(7*a^4 + 7*a^2*b^2 + b^4)*Sin[c + d*x]^2 - (b^3*(70*a^4 + 28*a^2*b^2 +
 b^4)*Sin[c + d*x]^3)/3 - 2*a*b^4*(7*a^2 + b^2)*Sin[c + d*x]^4 - (b^5*(28*a
^2 + b^2)*Sin[c + d*x]^5)/5 - (4*a*b^6*Sin[c + d*x]^6)/3 - (b^7*Sin[c + d*x
]^7)/7))/d
```

fricas [**A**] time = 0.52, size = 327, normalized size = 1.33

$$\frac{280\,ab^7\cos(dx+c)^6-420\left(7\,a^3b^5+3\,ab^7\right)\cos(dx+c)^4+840\left(7\,a^5b^3+14\,a^3b^5+3\,ab^7\right)\cos(dx+c)^2+105}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/210*(280*a*b^7*cos(d*x + c)^6 - 420*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4
+ 840*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 105*(a^8 - 8*a^7*
b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*a^2*b^6 - 8*a*b^
7 + b^8)*log(sin(d*x + c) + 1) - 105*(a^8 + 8*a^7*b + 28*a^6*b^2 + 56*a^5*b
^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 + b^8)*log(-sin(d*x + c
) + 1) + 2*(15*b^8*cos(d*x + c)^6 - 2940*a^6*b^2 - 9800*a^4*b^4 - 4508*a^2*
b^6 - 176*b^8 - 6*(98*a^2*b^6 + 11*b^8)*cos(d*x + c)^4 + 2*(1225*a^4*b^4 +
1078*a^2*b^6 + 61*b^8)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [**A**] time = 1.99, size = 378, normalized size = 1.54

$$-\frac{30\,b^8\sin(dx+c)^7+280\,ab^7\sin(dx+c)^6+1176\,a^2b^6\sin(dx+c)^5+42\,b^8\sin(dx+c)^5+2940\,a^3b^5\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/210*(30*b^8*sin(d*x + c)^7 + 280*a*b^7*sin(d*x + c)^6 + 1176*a^2*b^6*sin
(d*x + c)^5 + 42*b^8*sin(d*x + c)^5 + 2940*a^3*b^5*sin(d*x + c)^4 + 420*a*b
^7*sin(d*x + c)^4 + 4900*a^4*b^4*sin(d*x + c)^3 + 1960*a^2*b^6*sin(d*x + c)
^3 + 70*b^8*sin(d*x + c)^3 + 5880*a^5*b^3*sin(d*x + c)^2 + 5880*a^3*b^5*sin
(d*x + c)^2 + 840*a*b^7*sin(d*x + c)^2 + 5880*a^6*b^2*sin(d*x + c) + 14700*
a^4*b^4*sin(d*x + c) + 5880*a^2*b^6*sin(d*x + c) + 210*b^8*sin(d*x + c) - 1
05*(a^8 - 8*a^7*b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*
a^2*b^6 - 8*a*b^7 + b^8)*log(abs(sin(d*x + c) + 1)) + 105*(a^8 + 8*a^7*b +
28*a^6*b^2 + 56*a^5*b^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 +
b^8)*log(abs(sin(d*x + c) - 1)))/d
```

maple [A] time = 0.23, size = 465, normalized size = 1.90

$$-\frac{b^8\left(\sin^7(dx+c)\right)}{7d}-\frac{28a^2b^6\left(\sin^5(dx+c)\right)}{5d}-\frac{28a^2b^6\left(\sin^3(dx+c)\right)}{3d}-\frac{2a\,b^7\left(\sin^4(dx+c)\right)}{d}-\frac{4a\,b^7\left(\sin^2(dx+c)\right)}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^8,x)
```

```
[Out] -4/3*a*b^7*sin(d*x+c)^6/d-1/7*b^8*sin(d*x+c)^7/d-1/d*sin(d*x+c)*b^8-1/5/d*b
^8*sin(d*x+c)^5-1/3/d*b^8*sin(d*x+c)^3+1/d*b^8*ln(sec(d*x+c)+tan(d*x+c))+1/
d*a^8*ln(sec(d*x+c)+tan(d*x+c))-28/5/d*a^2*b^6*sin(d*x+c)^5-28/3/d*a^2*b^6*
sin(d*x+c)^3-28/d*a^2*b^6*sin(d*x+c)+28/d*a^2*b^6*ln(sec(d*x+c)+tan(d*x+c))
-2/d*a*b^7*sin(d*x+c)^4-4/d*a*b^7*sin(d*x+c)^2-8/d*a*b^7*ln(cos(d*x+c))-14/
d*a^3*b^5*sin(d*x+c)^4-28/d*a^3*b^5*sin(d*x+c)^2-56/d*a^3*b^5*ln(cos(d*x+c)
)-70/3/d*a^4*b^4*sin(d*x+c)^3-70/d*a^4*b^4*sin(d*x+c)+70/d*a^4*b^4*ln(sec(d
*x+c)+tan(d*x+c))-8/d*a^7*b*ln(cos(d*x+c))-28/d*a^6*b^2*sin(d*x+c)+28/d*a^6
*b^2*ln(sec(d*x+c)+tan(d*x+c))-28/d*a^5*b^3*sin(d*x+c)^2-56/d*a^5*b^3*ln(co
s(d*x+c))
```

maxima [A] time = 0.33, size = 317, normalized size = 1.29

$$-\frac{30\,b^8\sin(dx+c)^7+280\,ab^7\sin(dx+c)^6+42\left(28\,a^2b^6+b^8\right)\sin(dx+c)^5+420\left(7\,a^3b^5+ab^7\right)\sin(dx+c)^4+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/210*(30*b^8*sin(d*x + c)^7 + 280*a*b^7*sin(d*x + c)^6 + 42*(28*a^2*b^6 +
 b^8)*sin(d*x + c)^5 + 420*(7*a^3*b^5 + a*b^7)*sin(d*x + c)^4 + 70*(70*a^4*
b^4 + 28*a^2*b^6 + b^8)*sin(d*x + c)^3 + 840*(7*a^5*b^3 + 7*a^3*b^5 + a*b^7
)*sin(d*x + c)^2 - 105*(a^8 - 8*a^7*b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^
4 - 56*a^3*b^5 + 28*a^2*b^6 - 8*a*b^7 + b^8)*log(sin(d*x + c) + 1) + 105*(a
^8 + 8*a^7*b + 28*a^6*b^2 + 56*a^5*b^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b
^6 + 8*a*b^7 + b^8)*log(sin(d*x + c) - 1) + 210*(28*a^6*b^2 + 70*a^4*b^4 +
28*a^2*b^6 + b^8)*sin(d*x + c))/d
```

mupad [B] time = 5.36, size = 212, normalized size = 0.87

$$-\frac{\frac{\ln(\sin(c+d\,x)-1)\,(a+b)^8}{2}+\sin(c+d\,x)^3\left(\frac{70\,a^4\,b^4}{3}+\frac{28\,a^2\,b^6}{3}+\frac{b^8}{3}\right)-\frac{\ln(\sin(c+d\,x)+1)\,(a-b)^8}{2}+\sin(c+d\,x)^5\left(\frac{28\,a^2\,b^6}{5}+\frac{b^8}{5}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x),x)
```

```
[Out] -((log(sin(c + d*x) - 1)*(a + b)^8)/2 + sin(c + d*x)^3*(b^8/3 + (28*a^2*b^6
)/3 + (70*a^4*b^4)/3) - (log(sin(c + d*x) + 1)*(a - b)^8)/2 + sin(c + d*x)^
5*(b^8/5 + (28*a^2*b^6)/5) + sin(c + d*x)*(b^8 + 28*a^2*b^6 + 70*a^4*b^4 +
```

```
28*a^6*b^2) + sin(c + d*x)^2*(4*a*b^7 + 28*a^3*b^5 + 28*a^5*b^3) + (b^8*sin
(c + d*x)^7)/7 + sin(c + d*x)^4*(2*a*b^7 + 14*a^3*b^5) + (4*a*b^7*sin(c + d
*x)^6)/3)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.417 $\int \sec^3(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=284

$$\frac{7b^6\left(5a^2+b^2\right)\sin^5(c+dx)}{10d}+\frac{3ab^5\left(7a^2+4b^2\right)\sin^4(c+dx)}{2d}+\frac{7b^4\left(15a^4+20a^2b^2+b^4\right)\sin^3(c+dx)}{6d}+\frac{ab^3\left(35a^4+\right.}{}$$

```
[Out] -1/4*(a-7*b)*(a+b)^7*ln(1-sin(d*x+c))/d+1/4*(a-b)^7*(a+7*b)*ln(1+sin(d*x+c)
)/d+7/2*b^2*(3*a^6+30*a^4*b^2+20*a^2*b^4+b^6)*sin(d*x+c)/d+1/2*a*b^3*(35*a^
4+112*a^2*b^2+24*b^4)*sin(d*x+c)^2/d+7/6*b^4*(15*a^4+20*a^2*b^2+b^4)*sin(d*
x+c)^3/d+3/2*a*b^5*(7*a^2+4*b^2)*sin(d*x+c)^4/d+7/10*b^6*(5*a^2+b^2)*sin(d*
x+c)^5/d+1/2*a*b^7*sin(d*x+c)^6/d+1/2*sec(d*x+c)^2*(b+a*sin(d*x+c))*(a+b*si
n(d*x+c))^7/d
```

Rubi [A] time = 0.24, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2668, 739, 801, 633, 31}

$$\frac{7b^6\left(5a^2+b^2\right)\sin^5(c+dx)}{10d}+\frac{3ab^5\left(7a^2+4b^2\right)\sin^4(c+dx)}{2d}+\frac{7b^4\left(20a^2b^2+15a^4+b^4\right)\sin^3(c+dx)}{6d}+\frac{ab^3\left(112a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -((a - 7*b)*(a + b)^7*Log[1 - Sin[c + d*x]])/(4*d) + ((a - b)^7*(a + 7*b)*L
og[1 + Sin[c + d*x]])/(4*d) + (7*b^2*(3*a^6 + 30*a^4*b^2 + 20*a^2*b^4 + b^6
)*Sin[c + d*x])/(2*d) + (a*b^3*(35*a^4 + 112*a^2*b^2 + 24*b^4)*Sin[c + d*x]
^2)/(2*d) + (7*b^4*(15*a^4 + 20*a^2*b^2 + b^4)*Sin[c + d*x]^3)/(6*d) + (3*a
*b^5*(7*a^2 + 4*b^2)*Sin[c + d*x]^4)/(2*d) + (7*b^6*(5*a^2 + b^2)*Sin[c + d
*x]^5)/(10*d) + (a*b^7*Sin[c + d*x]^6)/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c
 + d*x])*(a + b*Sin[c + d*x])^7)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^8}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{2d} - \frac{b\,\text{Subst}\left(\int \frac{(a+x)}{\cdots}\right.}{\cdots}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{2d} - \frac{b\,\text{Subst}\left(\int \left(-7\cdots\right.\right.}{\cdots}$$

$$= \frac{7b^2\left(3a^6+30a^4b^2+20a^2b^4+b^6\right)\sin(c+dx)}{2d} + \frac{ab^3\left(35a^4+112a^2b^2\cdots\right.}{2\cdots}$$

$$= \frac{7b^2\left(3a^6+30a^4b^2+20a^2b^4+b^6\right)\sin(c+dx)}{2d} + \frac{ab^3\left(35a^4+112a^2b^2\cdots\right.}{2\cdots}$$

$$= -\frac{(a-7b)(a+b)^7\log(1-\sin(c+dx))}{4d} + \frac{(a-b)^7(a+7b)\log(1+\sin(\cdots}{4d}$$

Mathematica [A] time = 2.35, size = 366, normalized size = 1.29

$$\frac{\frac{1}{2}b\left(a^2-b^2\right)\left((a-7b)(a+b)^7\log(1-\sin(c+dx))-(a-b)^7(a+7b)\log(\sin(c+dx)+1)\right)+b^9\left(b^2-9a^2\right)\sin^7\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((b*(a^2 - b^2)*((a - 7*b)*(a + b)^7*Log[1 - Sin[c + d*x]] - (a - b)^7*(a +
 7*b)*Log[1 + Sin[c + d*x]]))/2 + b^3*(-36*a^8 - 182*a^6*b^2 + 70*a^4*b^4 +
 133*a^2*b^6 + 7*b^8)*Sin[c + d*x] - 4*a*b^4*(21*a^6 + 14*a^4*b^2 - 22*a^2*
b^4 - 6*b^6)*Sin[c + d*x]^2 + (7*b^5*(-54*a^6 + 10*a^4*b^2 + 19*a^2*b^4 + b
^6)*Sin[c + d*x]^3)/3 - 2*a*b^6*(63*a^4 - 22*a^2*b^2 - 6*b^4)*Sin[c + d*x]^
4 + (7*b^7*(-60*a^4 + 19*a^2*b^2 + b^4)*Sin[c + d*x]^5)/5 - 4*a*b^8*(9*a^2
- 2*b^2)*Sin[c + d*x]^6 + b^9*(-9*a^2 + b^2)*Sin[c + d*x]^7 - a*b^10*Sin[c
+ d*x]^8 + b*Sec[c + d*x]^2*(b - a*Sin[c + d*x])*(a + b*Sin[c + d*x])^9)/(2
*b*(-a^2 + b^2)*d)
```

fricas [A] time = 0.54, size = 368, normalized size = 1.30

$$\frac{120\,ab^7\cos(dx+c)^6+240\,a^7b+1680\,a^5b^3+1680\,a^3b^5+240\,ab^7-240\left(7\,a^3b^5+3\,ab^7\right)\cos(dx+c)^4+15\left(a\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/60*(120*a*b^7*cos(d*x + c)^6 + 240*a^7*b + 1680*a^5*b^3 + 1680*a^3*b^5 +
240*a*b^7 - 240*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 + 15*(a^8 - 28*a^6*b^2
 + 112*a^5*b^3 - 210*a^4*b^4 + 224*a^3*b^5 - 140*a^2*b^6 + 48*a*b^7 - 7*b^8
)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 15*(a^8 - 28*a^6*b^2 - 112*a^5*b^3
 - 210*a^4*b^4 - 224*a^3*b^5 - 140*a^2*b^6 - 48*a*b^7 - 7*b^8)*cos(d*x + c)
^2*log(-sin(d*x + c) + 1) + 105*(8*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 2*(6
*b^8*cos(d*x + c)^6 + 15*a^8 + 420*a^6*b^2 + 1050*a^4*b^4 + 420*a^2*b^6 + 1
5*b^8 - 8*(35*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + 4*(525*a^4*b^4 + 490*a^2*b^
6 + 29*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.60, size = 408, normalized size = 1.44

$$\frac{12\,b^8 \sin\left(dx+c\right)^5 + 120\,ab^7 \sin\left(dx+c\right)^4 + 560\,a^2b^6 \sin\left(dx+c\right)^3 + 40\,b^8 \sin\left(dx+c\right)^3 + 1680\,a^3b^5 \sin\left(dx+c\right)^2 + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/60*(12*b^8*sin(d*x + c)^5 + 120*a*b^7*sin(d*x + c)^4 + 560*a^2*b^6*sin(d*
x + c)^3 + 40*b^8*sin(d*x + c)^3 + 1680*a^3*b^5*sin(d*x + c)^2 + 480*a*b^7*
sin(d*x + c)^2 + 4200*a^4*b^4*sin(d*x + c) + 3360*a^2*b^6*sin(d*x + c) + 18
0*b^8*sin(d*x + c) + 15*(a^8 - 28*a^6*b^2 + 112*a^5*b^3 - 210*a^4*b^4 + 224
*a^3*b^5 - 140*a^2*b^6 + 48*a*b^7 - 7*b^8)*log(abs(sin(d*x + c) + 1)) - 15*
(a^8 - 28*a^6*b^2 - 112*a^5*b^3 - 210*a^4*b^4 - 224*a^3*b^5 - 140*a^2*b^6 -
 48*a*b^7 - 7*b^8)*log(abs(sin(d*x + c) - 1)) - 30*(56*a^5*b^3*sin(d*x + c)
^2 + 112*a^3*b^5*sin(d*x + c)^2 + 24*a*b^7*sin(d*x + c)^2 + a^8*sin(d*x + c
) + 28*a^6*b^2*sin(d*x + c) + 70*a^4*b^4*sin(d*x + c) + 28*a^2*b^6*sin(d*x
+ c) + b^8*sin(d*x + c) + 8*a^7*b - 56*a^3*b^5 - 16*a*b^7)/(sin(d*x + c)^2
- 1))/d
```

maple [B] time = 0.33, size = 645, normalized size = 2.27

$$\frac{b^8 \left(\sin^7\left(dx+c\right)\right)}{2d} + \frac{14a^2b^6 \left(\sin^5\left(dx+c\right)\right)}{d} + \frac{70a^2b^6 \left(\sin^3\left(dx+c\right)\right)}{3d} + \frac{6a\,b^7 \left(\sin^4\left(dx+c\right)\right)}{d} + \frac{12a\,b^7 \left(\sin^2\left(dx+c\right)\right)}{d} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 4*a*b^7*sin(d*x+c)^6/d+1/2*b^8*sin(d*x+c)^7/d+7/2/d*sin(d*x+c)*b^8+7/10/d*b
^8*sin(d*x+c)^5+7/6/d*b^8*sin(d*x+c)^3-7/2/d*b^8*ln(sec(d*x+c)+tan(d*x+c))+
1/2/d*a^8*ln(sec(d*x+c)+tan(d*x+c))+14/d*a^2*b^6*sin(d*x+c)^5+70/3/d*a^2*b^
6*sin(d*x+c)^3+70/d*a^2*b^6*sin(d*x+c)-70/d*a^2*b^6*ln(sec(d*x+c)+tan(d*x+c
))+6/d*a*b^7*sin(d*x+c)^4+12/d*a*b^7*sin(d*x+c)^2+24/d*a*b^7*ln(cos(d*x+c))
+28/d*a^3*b^5*sin(d*x+c)^4+56/d*a^3*b^5*sin(d*x+c)^2+112/d*a^3*b^5*ln(cos(d
*x+c))+35/d*a^4*b^4*sin(d*x+c)^3+105/d*a^4*b^4*sin(d*x+c)-105/d*a^4*b^4*ln(
sec(d*x+c)+tan(d*x+c))+14/d*a^6*b^2*sin(d*x+c)-14/d*a^6*b^2*ln(sec(d*x+c)+t
an(d*x+c))+56/d*a^5*b^3*ln(cos(d*x+c))+14/d*a^6*b^2*sin(d*x+c)^3/cos(d*x+c)
^2+35/d*a^4*b^4*sin(d*x+c)^5/cos(d*x+c)^2+28/d*a^3*b^5*sin(d*x+c)^6/cos(d*x
+c)^2+14/d*a^2*b^6*sin(d*x+c)^7/cos(d*x+c)^2+4/d*a*b^7*sin(d*x+c)^8/cos(d*x
+c)^2+1/2/d*b^8*sin(d*x+c)^9/cos(d*x+c)^2+1/2/d*a^8*sec(d*x+c)*tan(d*x+c)+4
/d*a^7*b/cos(d*x+c)^2+28/d*a^5*b^3*tan(d*x+c)^2
```

maxima [A] time = 0.33, size = 323, normalized size = 1.14

$$\frac{12\,b^8 \sin\left(dx+c\right)^5 + 120\,ab^7 \sin\left(dx+c\right)^4 + 40\left(14\,a^2b^6 + b^8\right)\sin\left(dx+c\right)^3 + 240\left(7\,a^3b^5 + 2\,ab^7\right)\sin\left(dx+c\right)^2 + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^8*sin(d*x + c)^5 + 120*a*b^7*sin(d*x + c)^4 + 40*(14*a^2*b^6 + b
^8)*sin(d*x + c)^3 + 240*(7*a^3*b^5 + 2*a*b^7)*sin(d*x + c)^2 + 15*(a^8 - 2
8*a^6*b^2 + 112*a^5*b^3 - 210*a^4*b^4 + 224*a^3*b^5 - 140*a^2*b^6 + 48*a*b^
7 - 7*b^8)*log(sin(d*x + c) + 1) - 15*(a^8 - 28*a^6*b^2 - 112*a^5*b^3 - 210
*a^4*b^4 - 224*a^3*b^5 - 140*a^2*b^6 - 48*a*b^7 - 7*b^8)*log(sin(d*x + c) -
 1) + 60*(70*a^4*b^4 + 56*a^2*b^6 + 3*b^8)*sin(d*x + c) - 30*(8*a^7*b + 56*
a^5*b^3 + 56*a^3*b^5 + 8*a*b^7 + (a^8 + 28*a^6*b^2 + 70*a^4*b^4 + 28*a^2*b^
6 + b^8)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 5.39, size = 257, normalized size = 0.90

$$\frac{\sin(c+dx)^3\left(\frac{28a^2b^6}{3}+\frac{2b^8}{3}\right)}{d}+\frac{b^8\sin(c+dx)^5}{5d}+\frac{\sin(c+dx)^2\left(28a^3b^5+8ab^7\right)}{d}+\frac{\sin(c+dx)\left(70a^4b^4+56\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^3,x)
```

```
[Out] (sin(c + d*x)^3*((2*b^8)/3 + (28*a^2*b^6)/3))/d + (b^8*sin(c + d*x)^5)/(5*d
) + (sin(c + d*x)^2*(8*a*b^7 + 28*a^3*b^5))/d + (sin(c + d*x)*(3*b^8 + 56*a
^2*b^6 + 70*a^4*b^4))/d - (sin(c + d*x)*(a^8/2 + b^8/2 + 14*a^2*b^6 + 35*a^
4*b^4 + 14*a^6*b^2) + 4*a*b^7 + 4*a^7*b + 28*a^3*b^5 + 28*a^5*b^3)/(d*(sin(
c + d*x)^2 - 1)) + (2*a*b^7*sin(c + d*x)^4)/d - (log(sin(c + d*x) - 1)*(a +
 b)^7*(a - 7*b))/(4*d) + (log(sin(c + d*x) + 1)*(a - b)^7*(a + 7*b))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.418 $\int \sec^5(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=320

$$-\frac{(a+b)^6\left(3a^2-18ab+35b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{(a-b)^6\left(3a^2+18ab+35b^2\right)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+$$

```
[Out] -1/16*(a+b)^6*(3*a^2-18*a*b+35*b^2)*ln(1-sin(d*x+c))/d+1/16*(a-b)^6*(3*a^2+
18*a*b+35*b^2)*ln(1+sin(d*x+c))/d+5/8*b^2*(6*a^6-35*a^4*b^2-84*a^2*b^4-7*b^
6)*sin(d*x+c)/d+1/4*a*b^3*(15*a^4-77*a^2*b^2-48*b^4)*sin(d*x+c)^2/d+5/24*b^
4*(9*a^4-42*a^2*b^2-7*b^4)*sin(d*x+c)^3/d-1/8*a*(13-3/b^2*a^2)*b^7*sin(d*x+
c)^4/d+1/4*sec(d*x+c)^4*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^7/d-1/8*sec(d*x+c
)^2*(a+b*sin(d*x+c))^5*(b*(a^2+7*b^2)-a*(3*a^2-11*b^2)*sin(d*x+c))/d
```

Rubi [A] time = 0.30, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2668, 739, 819, 801, 633, 31}

$$-\frac{ab^7\left(13-\frac{3a^2}{b^2}\right)\sin^4(c+dx)}{8d}+\frac{5b^4\left(-42a^2b^2+9a^4-7b^4\right)\sin^3(c+dx)}{24d}+\frac{ab^3\left(-77a^2b^2+15a^4-48b^4\right)\sin^2(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -((a + b)^6*(3*a^2 - 18*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + ((a -
 b)^6*(3*a^2 + 18*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) + (5*b^2*(6*a
^6 - 35*a^4*b^2 - 84*a^2*b^4 - 7*b^6)*Sin[c + d*x])/(8*d) + (a*b^3*(15*a^4
- 77*a^2*b^2 - 48*b^4)*Sin[c + d*x]^2)/(4*d) + (5*b^4*(9*a^4 - 42*a^2*b^2 -
 7*b^4)*Sin[c + d*x]^3)/(24*d) - (a*(13 - (3*a^2)/b^2)*b^7*Sin[c + d*x]^4)/
(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^7)/(4*d)
- (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^5*(b*(a^2 + 7*b^2) - a*(3*a^2 - 11*b
^2)*Sin[c + d*x]))/(8*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{(a+x)^8}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{4d} - \frac{b^3\,\text{Subst}\left(\int \frac{(a+x}{\ }\right.}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{4d} - \frac{\sec^2(c+dx)(a+}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{4d} - \frac{\sec^2(c+dx)(a+}{}$$

$$= \frac{5b^2\left(6a^6-35a^4b^2-84a^2b^4-7b^6\right)\sin(c+dx)}{8d} + \frac{ab^3\left(15a^4-77a^2b^2\right.}{4}$$

$$= \frac{5b^2\left(6a^6-35a^4b^2-84a^2b^4-7b^6\right)\sin(c+dx)}{8d} + \frac{ab^3\left(15a^4-77a^2b^2\right.}{4}$$

$$= -\frac{(a+b)^6\left(3a^2-18ab+35b^2\right)\log(1-\sin(c+dx))}{16d} + \frac{(a-b)^6\left(3a^2+\right.}{}$$

Mathematica [A] time = 4.12, size = 514, normalized size = 1.61

$$-\frac{3\left(a^2-b^2\right)^2\left((a+b)^6\left(3a^2-18ab+35b^2\right)\log(1-\sin(c+dx))-(a-b)^6\left(3a^2+18ab+35b^2\right)\log(\sin(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/48*(3*(a^2 - b^2)^2*((a + b)^6*(3*a^2 - 18*a*b + 35*b^2)*Log[1 - Sin[c +
 d*x]] - (a - b)^6*(3*a^2 + 18*a*b + 35*b^2)*Log[1 + Sin[c + d*x]]) + 6*b^2
```

```
*(-108*a^10 + 234*a^8*b^2 - 28*a^6*b^4 - 595*a^4*b^6 + 350*a^2*b^8 + 35*b^1
0)*Sin[c + d*x] - 24*a*b^3*(63*a^8 - 21*a^6*b^2 + 88*a^4*b^4 - 8*a^2*b^6 -
24*b^8)*Sin[c + d*x]^2 + 14*b^4*(-162*a^8 - 144*a^6*b^2 - 85*a^4*b^4 + 50*a
^2*b^6 + 5*b^8)*Sin[c + d*x]^3 - 12*a*b^5*(189*a^6 + 333*a^4*b^2 - 8*a^2*b^
4 - 24*b^6)*Sin[c + d*x]^4 + 42*b^6*(-36*a^6 - 87*a^4*b^2 + 10*a^2*b^4 + b^
6)*Sin[c + d*x]^5 - 24*a*b^7*(27*a^4 + 79*a^2*b^2 - 8*b^4)*Sin[c + d*x]^6 +
 6*b^8*(-27*a^4 - 90*a^2*b^2 + 5*b^4)*Sin[c + d*x]^7 - 6*a*b^9*(3*a^2 + 11*
b^2)*Sin[c + d*x]^8 + 12*(a^2 - b^2)*Sec[c + d*x]^4*(b - a*Sin[c + d*x])*(a
 + b*Sin[c + d*x])^9 + 6*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^9*(9*a^2*b + 5
*b^3 - a*(3*a^2 + 11*b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

fricas [A] time = 0.54, size = 366, normalized size = 1.14

$$\frac{192\,ab^7\cos(dx+c)^6 - 96\,ab^7\cos(dx+c)^4 + 96\,a^7b + 672\,a^5b^3 + 672\,a^3b^5 + 96\,ab^7 + 3\left(3\,a^8 - 28\,a^6b^2 + 210\,a^4b \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/48*(192*a*b^7*cos(d*x + c)^6 - 96*a*b^7*cos(d*x + c)^4 + 96*a^7*b + 672*a
^5*b^3 + 672*a^3*b^5 + 96*a*b^7 + 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 - 448
*a^3*b^5 + 420*a^2*b^6 - 192*a*b^7 + 35*b^8)*cos(d*x + c)^4*log(sin(d*x + c
) + 1) - 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 + 448*a^3*b^5 + 420*a^2*b^6 +
192*a*b^7 + 35*b^8)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 192*(7*a^5*b^3
+ 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 2*(8*b^8*cos(d*x + c)^6 + 6*a^8 +
168*a^6*b^2 + 420*a^4*b^4 + 168*a^2*b^6 + 6*b^8 - 16*(42*a^2*b^6 + 5*b^8)*c
os(d*x + c)^4 + 3*(3*a^8 - 28*a^6*b^2 - 350*a^4*b^4 - 252*a^2*b^6 - 13*b^8)
*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 1.00, size = 429, normalized size = 1.34

$$-\frac{16\,b^8\sin(dx+c)^3 + 192\,ab^7\sin(dx+c)^2 + 1344\,a^2b^6\sin(dx+c) + 144\,b^8\sin(dx+c) - 3\left(3\,a^8 - 28\,a^6b^2 + 21 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/48*(16*b^8*sin(d*x + c)^3 + 192*a*b^7*sin(d*x + c)^2 + 1344*a^2*b^6*sin(
d*x + c) + 144*b^8*sin(d*x + c) - 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 - 448
*a^3*b^5 + 420*a^2*b^6 - 192*a*b^7 + 35*b^8)*log(abs(sin(d*x + c) + 1)) + 3
*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 + 448*a^3*b^5 + 420*a^2*b^6 + 192*a*b^7
+ 35*b^8)*log(abs(sin(d*x + c) - 1)) - 6*(336*a^3*b^5*sin(d*x + c)^4 + 144*
a*b^7*sin(d*x + c)^4 - 3*a^8*sin(d*x + c)^3 + 28*a^6*b^2*sin(d*x + c)^3 + 3
50*a^4*b^4*sin(d*x + c)^3 + 252*a^2*b^6*sin(d*x + c)^3 + 13*b^8*sin(d*x + c
)^3 + 224*a^5*b^3*sin(d*x + c)^2 - 224*a^3*b^5*sin(d*x + c)^2 - 192*a*b^7*s
in(d*x + c)^2 + 5*a^8*sin(d*x + c) + 28*a^6*b^2*sin(d*x + c) - 210*a^4*b^4*
sin(d*x + c) - 196*a^2*b^6*sin(d*x + c) - 11*b^8*sin(d*x + c) + 16*a^7*b -
112*a^5*b^3 + 64*a*b^7)/(sin(d*x + c)^2 - 1)^2)/d
```

maple [B] time = 0.35, size = 760, normalized size = 2.38

$$-\frac{5b^8\left(\sin^7(dx+c)\right)}{8d}-\frac{21a^2b^6\left(\sin^5(dx+c)\right)}{2d}-\frac{35a^2b^6\left(\sin^3(dx+c)\right)}{2d}-\frac{6a\,b^7\left(\sin^4(dx+c)\right)}{d}-\frac{12a\,b^7\left(\sin^2(dx+c)\right)}{d} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x)
```

```
[Out] -4*a*b^7*sin(d*x+c)^6/d-5/8*b^8*sin(d*x+c)^7/d-35/8/d*sin(d*x+c)*b^8-7/8/d*
b^8*sin(d*x+c)^5-35/24/d*b^8*sin(d*x+c)^3+35/8/d*b^8*ln(sec(d*x+c)+tan(d*x+
c))+3/8/d*a^8*ln(sec(d*x+c)+tan(d*x+c))+14/d*a^5*b^3*sin(d*x+c)^4/cos(d*x+c
)^4+7/d*a^6*b^2*sin(d*x+c)^3/cos(d*x+c)^4+35/2/d*a^4*b^4*sin(d*x+c)^5/cos(d
*x+c)^4+7/d*a^2*b^6*sin(d*x+c)^7/cos(d*x+c)^4+2/d*a*b^7*sin(d*x+c)^8/cos(d*
x+c)^4-21/2/d*a^2*b^6*sin(d*x+c)^5-35/2/d*a^2*b^6*sin(d*x+c)^3-105/2/d*a^2*
b^6*sin(d*x+c)+105/2/d*a^2*b^6*ln(sec(d*x+c)+tan(d*x+c))-6/d*a*b^7*sin(d*x+
c)^4-12/d*a*b^7*sin(d*x+c)^2-24/d*a*b^7*ln(cos(d*x+c))-56/d*a^3*b^5*ln(cos(
d*x+c))-35/4/d*a^4*b^4*sin(d*x+c)^3-105/4/d*a^4*b^4*sin(d*x+c)+105/4/d*a^4*
b^4*ln(sec(d*x+c)+tan(d*x+c))+7/2/d*a^6*b^2*sin(d*x+c)-7/2/d*a^6*b^2*ln(sec
(d*x+c)+tan(d*x+c))+7/2/d*a^6*b^2*sin(d*x+c)^3/cos(d*x+c)^2-35/4/d*a^4*b^4*
sin(d*x+c)^5/cos(d*x+c)^2-21/2/d*a^2*b^6*sin(d*x+c)^7/cos(d*x+c)^2-4/d*a*b^
7*sin(d*x+c)^8/cos(d*x+c)^2+14/d*a^3*b^5*tan(d*x+c)^4-28/d*a^3*b^5*tan(d*x+
c)^2+1/4/d*a^8*tan(d*x+c)*sec(d*x+c)^3+2/d*a^7*b/cos(d*x+c)^4+1/4/d*b^8*sin
(d*x+c)^9/cos(d*x+c)^4-5/8/d*b^8*sin(d*x+c)^9/cos(d*x+c)^2+3/8/d*a^8*sec(d*
x+c)*tan(d*x+c)
```

maxima [A] time = 0.33, size = 348, normalized size = 1.09

$$-\frac{16\, b^8 \sin\left(dx+c\right)^3 + 192\, ab^7 \sin\left(dx+c\right)^2 - 3\left(3\, a^8 - 28\, a^6b^2 + 210\, a^4b^4 - 448\, a^3b^5 + 420\, a^2b^6 - 192\, ab^7 + 35\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/48*(16*b^8*sin(d*x + c)^3 + 192*a*b^7*sin(d*x + c)^2 - 3*(3*a^8 - 28*a^6
*b^2 + 210*a^4*b^4 - 448*a^3*b^5 + 420*a^2*b^6 - 192*a*b^7 + 35*b^8)*log(si
n(d*x + c) + 1) + 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 + 448*a^3*b^5 + 420*a
^2*b^6 + 192*a*b^7 + 35*b^8)*log(sin(d*x + c) - 1) + 48*(28*a^2*b^6 + 3*b^8
)*sin(d*x + c) - 6*(16*a^7*b - 112*a^5*b^3 - 336*a^3*b^5 - 80*a*b^7 - (3*a^
8 - 28*a^6*b^2 - 350*a^4*b^4 - 252*a^2*b^6 - 13*b^8)*sin(d*x + c)^3 + 32*(7
*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*sin(d*x + c)^2 + (5*a^8 + 28*a^6*b^2 - 210
*a^4*b^4 - 196*a^2*b^6 - 11*b^8)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1))/d
```

mupad [B] time = 5.48, size = 305, normalized size = 0.95

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,{\left(a-b\right)}^6\,\left(3\,a^2+18\,a\,b+35\,b^2\right)}{16\,d}-\frac{b^8\,{\sin\left(c+d\,x\right)}^3}{3\,d}-\frac{\sin\left(c+d\,x\right)\,\left(28\,a^2\,b^6+3\,b^8\right)}{d}-\frac{\sin\left(c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^5,x)
```

```
[Out] (log(sin(c + d*x) + 1)*(a - b)^6*(18*a*b + 3*a^2 + 35*b^2))/(16*d) - (b^8*s
in(c + d*x)^3)/(3*d) - (sin(c + d*x)*(3*b^8 + 28*a^2*b^6))/d - (sin(c + d*x
)*((11*b^8)/8 - (5*a^8)/8 + (49*a^2*b^6)/2 + (105*a^4*b^4)/4 - (7*a^6*b^2)/
2) - sin(c + d*x)^3*((13*b^8)/8 - (3*a^8)/8 + (63*a^2*b^6)/2 + (175*a^4*b^4
)/4 + (7*a^6*b^2)/2) + 10*a*b^7 - 2*a^7*b - sin(c + d*x)^2*(12*a*b^7 + 56*a
^3*b^5 + 28*a^5*b^3) + 42*a^3*b^5 + 14*a^5*b^3)/(d*(sin(c + d*x)^4 - 2*sin(
c + d*x)^2 + 1)) - (4*a*b^7*sin(c + d*x)^2)/d - (log(sin(c + d*x) - 1)*(a +
 b)^6*(3*a^2 - 18*a*b + 35*b^2))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.419 $\int \cos^2(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=423

$$-\frac{b\left(64a^2+21b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))^5}{240d}-\frac{ab\left(112a^2+109b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))^4}{336d}-\frac{b\left(78\right.}{}$$

```
[Out] 1/256*(128*a^8+896*a^6*b^2+1120*a^4*b^4+280*a^2*b^6+7*b^8)*x-11/40320*a*b*(
1792*a^6+10536*a^4*b^2+9588*a^2*b^4+1289*b^6)*cos(d*x+c)^3/d+1/256*(128*a^8
+896*a^6*b^2+1120*a^4*b^4+280*a^2*b^6+7*b^8)*cos(d*x+c)*sin(d*x+c)/d-1/1344
0*b*(6272*a^6+28088*a^4*b^2+15956*a^2*b^4+735*b^6)*cos(d*x+c)^3*(a+b*sin(d*
x+c))/d-13/3360*a*b*(112*a^4+348*a^2*b^2+101*b^4)*cos(d*x+c)^3*(a+b*sin(d*x
+c))^2/d-1/2016*b*(784*a^4+1500*a^2*b^2+147*b^4)*cos(d*x+c)^3*(a+b*sin(d*x+
c))^3/d-1/336*a*b*(112*a^2+109*b^2)*cos(d*x+c)^3*(a+b*sin(d*x+c))^4/d-1/240
*b*(64*a^2+21*b^2)*cos(d*x+c)^3*(a+b*sin(d*x+c))^5/d-17/90*a*b*cos(d*x+c)^3
*(a+b*sin(d*x+c))^6/d-1/10*b*cos(d*x+c)^3*(a+b*sin(d*x+c))^7/d
```

Rubi [A] time = 1.22, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2692, 2862, 2669, 2635, 8}

$$-\frac{11ab\left(10536a^4b^2+9588a^2b^4+1792a^6+1289b^6\right)\cos^3(c+dx)}{40320d}-\frac{b\left(64a^2+21b^2\right)\cos^3(c+dx)(a+b\sin(c+d}{240d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*x)/256 - (11*
a*b*(1792*a^6 + 10536*a^4*b^2 + 9588*a^2*b^4 + 1289*b^6)*Cos[c + d*x]^3)/(4
0320*d) + ((128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*Cos
[c + d*x]*Sin[c + d*x])/(256*d) - (b*(6272*a^6 + 28088*a^4*b^2 + 15956*a^2*
b^4 + 735*b^6)*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(13440*d) - (13*a*b*(11
2*a^4 + 348*a^2*b^2 + 101*b^4)*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(3360
*d) - (b*(784*a^4 + 1500*a^2*b^2 + 147*b^4)*Cos[c + d*x]^3*(a + b*Sin[c + d
*x])^3)/(2016*d) - (a*b*(112*a^2 + 109*b^2)*Cos[c + d*x]^3*(a + b*Sin[c + d
*x])^4)/(336*d) - (b*(64*a^2 + 21*b^2)*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^
5)/(240*d) - (17*a*b*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^6)/(90*d) - (b*Cos
[c + d*x]^3*(a + b*Sin[c + d*x])^7)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^8\,dx = -\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^7}{10d} + \frac{1}{10}\int \cos^2(c+dx)(a+b\sin(c+$$
$$= -\frac{17ab\cos^3(c+dx)(a+b\sin(c+dx))^6}{90d} - \frac{b\cos^3(c+dx)(a+b\sin(c+dx}{10d}$$
$$= -\frac{b\left(64a^2+21b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))^5}{240d} - \frac{17ab\cos^3(c+dx)(a}{90}$$
$$= -\frac{ab\left(112a^2+109b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))^4}{336d} - \frac{b\left(64a^2+21b^2\right)}{}$$
$$= -\frac{b\left(784a^4+1500a^2b^2+147b^4\right)\cos^3(c+dx)(a+b\sin(c+dx))^3}{2016d} - \frac{ab(1}{}$$
$$= -\frac{13ab\left(112a^4+348a^2b^2+101b^4\right)\cos^3(c+dx)(a+b\sin(c+dx))^2}{3360d} - \frac{b(}{}$$
$$= -\frac{b\left(6272a^6+28088a^4b^2+15956a^2b^4+735b^6\right)\cos^3(c+dx)(a+b\sin(c}{13440d}$$
$$= -\frac{11ab\left(1792a^6+10536a^4b^2+9588a^2b^4+1289b^6\right)\cos^3(c+dx)}{40320d} - \frac{b\left(62\right.}{}$$
$$= -\frac{11ab\left(1792a^6+10536a^4b^2+9588a^2b^4+1289b^6\right)\cos^3(c+dx)}{40320d} + \frac{(128}{}$$
$$= \frac{1}{256}\left(128a^8+896a^6b^2+1120a^4b^4+280a^2b^6+7b^8\right)x - \frac{11ab\left(1792a^6+\right.}{}$$

Mathematica [A] time = 1.01, size = 457, normalized size = 1.08

$$\frac{161280a^8\sin(2(c+dx)) + 322560a^8c + 322560a^8dx - 564480a^6b^2\sin(4(c+dx)) + 2257920a^6b^2c + 2257920a^6b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (322560*a^8*c + 2257920*a^6*b^2*c + 2822400*a^4*b^4*c + 705600*a^2*b^6*c +
17640*b^8*c + 322560*a^8*d*x + 2257920*a^6*b^2*d*x + 2822400*a^4*b^4*d*x +
705600*a^2*b^6*d*x + 17640*b^8*d*x - 40320*a*b*(32*a^6 + 112*a^4*b^2 + 70*a
```

```
^2*b^4 + 7*b^6)*Cos[c + d*x] - 26880*(16*a^7*b + 28*a^5*b^3 + 7*a^3*b^5)*Co
s[3*(c + d*x)] + 451584*a^5*b^3*Cos[5*(c + d*x)] + 338688*a^3*b^5*Cos[5*(c
+ d*x)] + 32256*a*b^7*Cos[5*(c + d*x)] - 80640*a^3*b^5*Cos[7*(c + d*x)] - 1
4400*a*b^7*Cos[7*(c + d*x)] + 2240*a*b^7*Cos[9*(c + d*x)] + 161280*a^8*Sin[
2*(c + d*x)] - 705600*a^4*b^4*Sin[2*(c + d*x)] - 282240*a^2*b^6*Sin[2*(c +
d*x)] - 8820*b^8*Sin[2*(c + d*x)] - 564480*a^6*b^2*Sin[4*(c + d*x)] - 70560
0*a^4*b^4*Sin[4*(c + d*x)] - 141120*a^2*b^6*Sin[4*(c + d*x)] - 2520*b^8*Sin
[4*(c + d*x)] + 235200*a^4*b^4*Sin[6*(c + d*x)] + 94080*a^2*b^6*Sin[6*(c +
d*x)] + 2730*b^8*Sin[6*(c + d*x)] - 17640*a^2*b^6*Sin[8*(c + d*x)] - 945*b^
8*Sin[8*(c + d*x)] + 126*b^8*Sin[10*(c + d*x)])/(645120*d)
```

fricas [A] time = 0.52, size = 315, normalized size = 0.74

$$\frac{71680\,ab^7\cos(dx+c)^9 - 92160\left(7\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^7 + 129024\left(7\,a^5b^3 + 14\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/80640*(71680*a*b^7*cos(d*x + c)^9 - 92160*(7*a^3*b^5 + 3*a*b^7)*cos(d*x +
 c)^7 + 129024*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^5 - 215040*(
a^7*b + 7*a^5*b^3 + 7*a^3*b^5 + a*b^7)*cos(d*x + c)^3 + 315*(128*a^8 + 896*
a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*d*x + 21*(384*b^8*cos(d*x + c
)^9 - 48*(280*a^2*b^6 + 31*b^8)*cos(d*x + c)^7 + 8*(5600*a^4*b^4 + 4760*a^2
*b^6 + 263*b^8)*cos(d*x + c)^5 - 10*(2688*a^6*b^2 + 7840*a^4*b^4 + 3304*a^2
*b^6 + 121*b^8)*cos(d*x + c)^3 + 15*(128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 +
 280*a^2*b^6 + 7*b^8)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 3.19, size = 364, normalized size = 0.86

$$\frac{ab^7\cos(9\,dx+9\,c)}{288\,d} + \frac{b^8\sin(10\,dx+10\,c)}{5120\,d} + \frac{1}{256}\left(128\,a^8 + 896\,a^6b^2 + 1120\,a^4b^4 + 280\,a^2b^6 + 7\,b^8\right)x - \frac{\left(28\,a^3b^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/288*a*b^7*cos(9*d*x + 9*c)/d + 1/5120*b^8*sin(10*d*x + 10*c)/d + 1/256*(1
28*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*x - 1/224*(28*a^
3*b^5 + 5*a*b^7)*cos(7*d*x + 7*c)/d + 1/40*(28*a^5*b^3 + 21*a^3*b^5 + 2*a*b
^7)*cos(5*d*x + 5*c)/d - 1/24*(16*a^7*b + 28*a^5*b^3 + 7*a^3*b^5)*cos(3*d*x
 + 3*c)/d - 1/16*(32*a^7*b + 112*a^5*b^3 + 70*a^3*b^5 + 7*a*b^7)*cos(d*x +
c)/d - 1/2048*(56*a^2*b^6 + 3*b^8)*sin(8*d*x + 8*c)/d + 1/3072*(1120*a^4*b^
4 + 448*a^2*b^6 + 13*b^8)*sin(6*d*x + 6*c)/d - 1/256*(224*a^6*b^2 + 280*a^4
*b^4 + 56*a^2*b^6 + b^8)*sin(4*d*x + 4*c)/d + 1/512*(128*a^8 - 560*a^4*b^4
- 224*a^2*b^6 - 7*b^8)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.20, size = 497, normalized size = 1.17

$$\frac{b^8\left(-\frac{\left(\sin^7(dx+c)\right)\left(\cos^3(dx+c)\right)}{10} - \frac{7\left(\sin^5(dx+c)\right)\left(\cos^3(dx+c)\right)}{80} - \frac{7\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{96} - \frac{7\sin(dx+c)\left(\cos^3(dx+c)\right)}{128} + \frac{7\cos(dx+c)\sin(d}{256}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(b^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*sin(d*x+c)^5*cos(d*x+c)^3-7/
96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*sin(d*x+c)*cos(d*x+c)^3+7/256*cos(d*x+c)
*sin(d*x+c)+7/256*d*x+7/256*c)+8*a*b^7*(-1/9*sin(d*x+c)^6*cos(d*x+c)^3-2/21
*sin(d*x+c)^4*cos(d*x+c)^3-8/105*sin(d*x+c)^2*cos(d*x+c)^3-16/315*cos(d*x+c
```

```
)^3)+28*a^2*b^6*(-1/8*sin(d*x+c)^5*cos(d*x+c)^3-5/48*sin(d*x+c)^3*cos(d*x+c
)^3-5/64*sin(d*x+c)*cos(d*x+c)^3+5/128*cos(d*x+c)*sin(d*x+c)+5/128*d*x+5/12
8*c)+56*a^3*b^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c
)^3-8/105*cos(d*x+c)^3)+70*a^4*b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(
d*x+c)*cos(d*x+c)^3+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+56*a^5*b^3*
(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+28*a^6*b^2*(-1/4*sin(d*x
+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-8/3*a^7*b*cos(d*x
+c)^3+a^8*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.34, size = 336, normalized size = 0.79

$$-\frac{1720320\,a^7 b\cos\left(dx+c\right)^3 - 161280\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^8 - 564480\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^6b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/645120*(1720320*a^7*b*cos(d*x + c)^3 - 161280*(2*d*x + 2*c + sin(2*d*x +
 2*c))*a^8 - 564480*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^6*b^2 - 2408448*(3*c
os(d*x + c)^5 - 5*cos(d*x + c)^3)*a^5*b^3 + 235200*(4*sin(2*d*x + 2*c)^3 -
12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^4*b^4 + 344064*(15*cos(d*x + c)^7 - 4
2*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^3*b^5 + 5880*(64*sin(2*d*x + 2*c)^3
 - 120*d*x - 120*c + 3*sin(8*d*x + 8*c) + 24*sin(4*d*x + 4*c))*a^2*b^6 - 16
384*(35*cos(d*x + c)^9 - 135*cos(d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(
d*x + c)^3)*a*b^7 - 21*(96*sin(2*d*x + 2*c)^5 - 640*sin(2*d*x + 2*c)^3 + 84
0*d*x + 840*c - 45*sin(8*d*x + 8*c) - 120*sin(4*d*x + 4*c))*b^8)/d
```

mupad [B] time = 7.34, size = 467, normalized size = 1.10

$$-\frac{\frac{2205\,b^8\,\sin\left(2\,c+2\,d\,x\right)}{2} - 20160\,a^8\,\sin\left(2\,c+2\,d\,x\right) + 315\,b^8\,\sin\left(4\,c+4\,d\,x\right) - \frac{1365\,b^8\,\sin\left(6\,c+6\,d\,x\right)}{4} + \frac{945\,b^8\,\sin\left(8\,c+8\,d\,x\right)}{8} - \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^8,x)
```

```
[Out] -((2205*b^8*sin(2*c + 2*d*x))/2 - 20160*a^8*sin(2*c + 2*d*x) + 315*b^8*sin(
4*c + 4*d*x) - (1365*b^8*sin(6*c + 6*d*x))/4 + (945*b^8*sin(8*c + 8*d*x))/8
 - (63*b^8*sin(10*c + 10*d*x))/4 + 53760*a^7*b*cos(3*c + 3*d*x) - 4032*a*b^
7*cos(5*c + 5*d*x) + 1800*a*b^7*cos(7*c + 7*d*x) - 280*a*b^7*cos(9*c + 9*d*
x) + 352800*a^3*b^5*cos(c + d*x) + 564480*a^5*b^3*cos(c + d*x) + 23520*a^3*
b^5*cos(3*c + 3*d*x) + 94080*a^5*b^3*cos(3*c + 3*d*x) - 42336*a^3*b^5*cos(5
*c + 5*d*x) - 56448*a^5*b^3*cos(5*c + 5*d*x) + 10080*a^3*b^5*cos(7*c + 7*d*
x) + 35280*a^2*b^6*sin(2*c + 2*d*x) + 88200*a^4*b^4*sin(2*c + 2*d*x) + 1764
0*a^2*b^6*sin(4*c + 4*d*x) + 88200*a^4*b^4*sin(4*c + 4*d*x) + 70560*a^6*b^2
*sin(4*c + 4*d*x) - 11760*a^2*b^6*sin(6*c + 6*d*x) - 29400*a^4*b^4*sin(6*c
+ 6*d*x) + 2205*a^2*b^6*sin(8*c + 8*d*x) + 35280*a*b^7*cos(c + d*x) + 16128
0*a^7*b*cos(c + d*x) - 40320*a^8*d*x - 2205*b^8*d*x - 88200*a^2*b^6*d*x - 3
52800*a^4*b^4*d*x - 282240*a^6*b^2*d*x)/(80640*d)
```

sympy [A] time = 39.90, size = 1115, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((a**8*x*sin(c + d*x)**2/2 + a**8*x*cos(c + d*x)**2/2 + a**8*sin(c
 + d*x)*cos(c + d*x)/(2*d) - 8*a**7*b*cos(c + d*x)**3/(3*d) + 7*a**6*b**2*x
*sin(c + d*x)**4/2 + 7*a**6*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2 + 7*a**6
```

```
*b**2*x*cos(c + d*x)**4/2 + 7*a**6*b**2*sin(c + d*x)**3*cos(c + d*x)/(2*d)
- 7*a**6*b**2*sin(c + d*x)*cos(c + d*x)**3/(2*d) - 56*a**5*b**3*sin(c + d*x
)**2*cos(c + d*x)**3/(3*d) - 112*a**5*b**3*cos(c + d*x)**5/(15*d) + 35*a**4
*b**4*x*sin(c + d*x)**6/8 + 105*a**4*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2
/8 + 105*a**4*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + 35*a**4*b**4*x*cos
(c + d*x)**6/8 + 35*a**4*b**4*sin(c + d*x)**5*cos(c + d*x)/(8*d) - 35*a**4*
b**4*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 35*a**4*b**4*sin(c + d*x)*cos(
c + d*x)**5/(8*d) - 56*a**3*b**5*sin(c + d*x)**4*cos(c + d*x)**3/(3*d) - 22
4*a**3*b**5*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 64*a**3*b**5*cos(c + d
*x)**7/(15*d) + 35*a**2*b**6*x*sin(c + d*x)**8/32 + 35*a**2*b**6*x*sin(c +
d*x)**6*cos(c + d*x)**2/8 + 105*a**2*b**6*x*sin(c + d*x)**4*cos(c + d*x)**4
/16 + 35*a**2*b**6*x*sin(c + d*x)**2*cos(c + d*x)**6/8 + 35*a**2*b**6*x*cos
(c + d*x)**8/32 + 35*a**2*b**6*sin(c + d*x)**7*cos(c + d*x)/(32*d) - 511*a*
*2*b**6*sin(c + d*x)**5*cos(c + d*x)**3/(96*d) - 385*a**2*b**6*sin(c + d*x)
**3*cos(c + d*x)**5/(96*d) - 35*a**2*b**6*sin(c + d*x)*cos(c + d*x)**7/(32*
d) - 8*a*b**7*sin(c + d*x)**6*cos(c + d*x)**3/(3*d) - 16*a*b**7*sin(c + d*x
)**4*cos(c + d*x)**5/(5*d) - 64*a*b**7*sin(c + d*x)**2*cos(c + d*x)**7/(35*
d) - 128*a*b**7*cos(c + d*x)**9/(315*d) + 7*b**8*x*sin(c + d*x)**10/256 + 3
5*b**8*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 35*b**8*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 35*b**8*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 35*b**8
*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 7*b**8*x*cos(c + d*x)**10/256 + 7*
b**8*sin(c + d*x)**9*cos(c + d*x)/(256*d) - 79*b**8*sin(c + d*x)**7*cos(c +
 d*x)**3/(384*d) - 7*b**8*sin(c + d*x)**5*cos(c + d*x)**5/(30*d) - 49*b**8*
sin(c + d*x)**3*cos(c + d*x)**7/(384*d) - 7*b**8*sin(c + d*x)*cos(c + d*x)*
*9/(256*d), Ne(d, 0)), (x*(a + b*sin(c))**8*cos(c)**2, True))
```

3.420 $\int \sec^2(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=349

$$\frac{b\left(6a^2+7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{6d}+\frac{ab\left(30a^2+113b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{30d}+\frac{b\left(120a^4+992\right.}{}$$

```
[Out] -7/16*b^2*(64*a^6+240*a^4*b^2+120*a^2*b^4+5*b^6)*x+1/20*a*b*(40*a^6+1664*a^
4*b^2+2789*a^2*b^4+512*b^6)*cos(d*x+c)/d+1/80*b^2*(80*a^6+2248*a^4*b^2+2502
*a^2*b^4+175*b^6)*cos(d*x+c)*sin(d*x+c)/d+1/40*a*b*(40*a^4+624*a^2*b^2+337*
b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^2/d+1/120*b*(120*a^4+992*a^2*b^2+175*b^4)*
cos(d*x+c)*(a+b*sin(d*x+c))^3/d+1/30*a*b*(30*a^2+113*b^2)*cos(d*x+c)*(a+b*s
in(d*x+c))^4/d+1/6*b*(6*a^2+7*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^5/d+a*b*cos(
d*x+c)*(a+b*sin(d*x+c))^6/d+sec(d*x+c)*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^7/
d
```

Rubi [A] time = 0.56, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2691, 2753, 2734}

$$\frac{ab\left(1664a^4b^2+2789a^2b^4+40a^6+512b^6\right)\cos(c+dx)}{20d}+\frac{b\left(6a^2+7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{6d}+\frac{ab\left(30a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (-7*b^2*(64*a^6 + 240*a^4*b^2 + 120*a^2*b^4 + 5*b^6)*x)/16 + (a*b*(40*a^6 +
 1664*a^4*b^2 + 2789*a^2*b^4 + 512*b^6)*Cos[c + d*x])/(20*d) + (b^2*(80*a^6
 + 2248*a^4*b^2 + 2502*a^2*b^4 + 175*b^6)*Cos[c + d*x]*Sin[c + d*x])/(80*d)
 + (a*b*(40*a^4 + 624*a^2*b^2 + 337*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^
2)/(40*d) + (b*(120*a^4 + 992*a^2*b^2 + 175*b^4)*Cos[c + d*x]*(a + b*Sin[c
+ d*x])^3)/(120*d) + (a*b*(30*a^2 + 113*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*
x])^4)/(30*d) + (b*(6*a^2 + 7*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^5)/(6*
d) + (a*b*Cos[c + d*x]*(a + b*Sin[c + d*x])^6)/d + (Sec[c + d*x]*(b + a*Sin
[c + d*x])*(a + b*Sin[c + d*x])^7)/d
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{d} - \int (a+b\sin(c+$$

$$= \frac{ab\cos(c+dx)(a+b\sin(c+dx))^6}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx))(a}{d}$$

$$= \frac{b\left(6a^2+7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{6d} + \frac{ab\cos(c+dx)(a+b\,s}{d}$$

$$= \frac{ab\left(30a^2+113b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{30d} + \frac{b\left(6a^2+7b^2\right)\cos}{}$$

$$= \frac{b\left(120a^4+992a^2b^2+175b^4\right)\cos(c+dx)(a+b\sin(c+dx))^3}{120d} + \frac{ab\left(30\right.}{}$$

$$= \frac{ab\left(40a^4+624a^2b^2+337b^4\right)\cos(c+dx)(a+b\sin(c+dx))^2}{40d} + \frac{b\left(120\right.}{}$$

$$= -\frac{7}{16}b^2\left(64a^6+240a^4b^2+120a^2b^4+5b^6\right)x + \frac{ab\left(40a^6+1664a^4b^2+\right.}{}$$

Mathematica [A] time = 1.09, size = 313, normalized size = 0.90

$$\frac{\sec(c+dx)\left(1920a^8\sin(c+dx)+15360a^7b+53760a^6b^2\sin(c+dx)+161280a^5b^3+151200a^4b^4\sin(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]*(15360*a^7*b + 161280*a^5*b^3 + 201600*a^3*b^5 + 33600*a*b^7
- 840*b^2*(64*a^6 + 240*a^4*b^2 + 120*a^2*b^4 + 5*b^6)*(c + d*x)*Cos[c + d*
x] + 1120*(48*a^5*b^3 + 80*a^3*b^5 + 15*a*b^7)*Cos[2*(c + d*x)] - 4480*a^3*
b^5*Cos[4*(c + d*x)] - 1344*a*b^7*Cos[4*(c + d*x)] + 96*a*b^7*Cos[6*(c + d*
x)] + 1920*a^8*Sin[c + d*x] + 53760*a^6*b^2*Sin[c + d*x] + 151200*a^4*b^4*S
in[c + d*x] + 67200*a^2*b^6*Sin[c + d*x] + 2625*b^8*Sin[c + d*x] + 16800*a^
4*b^4*Sin[3*(c + d*x)] + 12600*a^2*b^6*Sin[3*(c + d*x)] + 630*b^8*Sin[3*(c
+ d*x)] - 840*a^2*b^6*Sin[5*(c + d*x)] - 70*b^8*Sin[5*(c + d*x)] + 5*b^8*Si
n[7*(c + d*x)]))/(1920*d)
```

fricas [A] time = 0.52, size = 266, normalized size = 0.76

$$\frac{384\,ab^7\cos(dx+c)^6+1920\,a^7b+13440\,a^5b^3+13440\,a^3b^5+1920\,ab^7-640\left(7\,a^3b^5+3\,ab^7\right)\cos(dx+c)^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(384*a*b^7*cos(d*x + c)^6 + 1920*a^7*b + 13440*a^5*b^3 + 13440*a^3*b^
5 + 1920*a*b^7 - 640*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 - 105*(64*a^6*b^2
 + 240*a^4*b^4 + 120*a^2*b^6 + 5*b^8)*d*x*cos(d*x + c) + 1920*(7*a^5*b^3 +
14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 5*(8*b^8*cos(d*x + c)^6 + 48*a^8 + 1
344*a^6*b^2 + 3360*a^4*b^4 + 1344*a^2*b^6 + 48*b^8 - 2*(168*a^2*b^6 + 19*b^
8)*cos(d*x + c)^4 + 3*(560*a^4*b^4 + 504*a^2*b^6 + 29*b^8)*cos(d*x + c)^2)*
sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 0.78, size = 799, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/240*(105*(64*a^6*b^2 + 240*a^4*b^4 + 120*a^2*b^6 + 5*b^8)*(d*x + c) + 48
0*(a^8*tan(1/2*d*x + 1/2*c) + 28*a^6*b^2*tan(1/2*d*x + 1/2*c) + 70*a^4*b^4*
tan(1/2*d*x + 1/2*c) + 28*a^2*b^6*tan(1/2*d*x + 1/2*c) + b^8*tan(1/2*d*x +
1/2*c) + 8*a^7*b + 56*a^5*b^3 + 56*a^3*b^5 + 8*a*b^7)/(tan(1/2*d*x + 1/2*c)
^2 - 1) + 2*(8400*a^4*b^4*tan(1/2*d*x + 1/2*c)^11 + 5880*a^2*b^6*tan(1/2*d*
x + 1/2*c)^11 + 285*b^8*tan(1/2*d*x + 1/2*c)^11 - 13440*a^5*b^3*tan(1/2*d*x
 + 1/2*c)^10 - 13440*a^3*b^5*tan(1/2*d*x + 1/2*c)^10 - 1920*a*b^7*tan(1/2*d
*x + 1/2*c)^10 + 25200*a^4*b^4*tan(1/2*d*x + 1/2*c)^9 + 24360*a^2*b^6*tan(1
/2*d*x + 1/2*c)^9 + 1295*b^8*tan(1/2*d*x + 1/2*c)^9 - 67200*a^5*b^3*tan(1/2
*d*x + 1/2*c)^8 - 94080*a^3*b^5*tan(1/2*d*x + 1/2*c)^8 - 13440*a*b^7*tan(1/
2*d*x + 1/2*c)^8 + 16800*a^4*b^4*tan(1/2*d*x + 1/2*c)^7 + 18480*a^2*b^6*tan
(1/2*d*x + 1/2*c)^7 + 1650*b^8*tan(1/2*d*x + 1/2*c)^7 - 134400*a^5*b^3*tan(
1/2*d*x + 1/2*c)^6 - 224000*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 - 42240*a*b^7*ta
n(1/2*d*x + 1/2*c)^6 - 16800*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 18480*a^2*b^6
*tan(1/2*d*x + 1/2*c)^5 - 1650*b^8*tan(1/2*d*x + 1/2*c)^5 - 134400*a^5*b^3*
tan(1/2*d*x + 1/2*c)^4 - 241920*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 49920*a*b^
7*tan(1/2*d*x + 1/2*c)^4 - 25200*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 24360*a^2
*b^6*tan(1/2*d*x + 1/2*c)^3 - 1295*b^8*tan(1/2*d*x + 1/2*c)^3 - 67200*a^5*b
^3*tan(1/2*d*x + 1/2*c)^2 - 120960*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 - 23424*a
*b^7*tan(1/2*d*x + 1/2*c)^2 - 8400*a^4*b^4*tan(1/2*d*x + 1/2*c) - 5880*a^2*
b^6*tan(1/2*d*x + 1/2*c) - 285*b^8*tan(1/2*d*x + 1/2*c) - 13440*a^5*b^3 - 2
2400*a^3*b^5 - 4224*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.42, size = 406, normalized size = 1.16

$$a^8 \tan(dx+c) + \frac{8a^7b}{\cos(dx+c)} + 28a^6b^2(\tan(dx+c) - dx - c) + 56a^5b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)} + \left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*tan(d*x+c)+8*a^7*b/cos(d*x+c)+28*a^6*b^2*(tan(d*x+c)-d*x-c)+56*a^5
*b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+70*a^4*b^4*(sin(
d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)
+56*a^3*b^5*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*co
s(d*x+c))+28*a^2*b^6*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^
3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+8*a*b^7*(sin(d*x+c)^8/cos(d*
x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*
(sin(d*x+c)^9/cos(d*x+c)+(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+
35/16*sin(d*x+c))*cos(d*x+c)-35/16*d*x-35/16*c))
```

maxima [A] time = 0.43, size = 348, normalized size = 1.00

$$-\frac{6720\,(dx+c-\tan(dx+c))a^6b^2 + 8400\left(3\,dx+3\,c - \frac{\tan(dx+c)}{\tan(dx+c)^2+1} - 2\,\tan(dx+c)\right)a^4b^4 + 4480\left(\cos(dx+c)^3 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/240*(6720*(d*x + c - tan(d*x + c))*a^6*b^2 + 8400*(3*d*x + 3*c - tan(d*x
 + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^4*b^4 + 4480*(cos(d*x + c)^3
```

```
 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3*b^5 + 840*(15*d*x + 15*c - (9*tan(d
*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 8*tan
(d*x + c))*a^2*b^6 - 384*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/cos(d*x + c
) + 15*cos(d*x + c))*a*b^7 + 5*(105*d*x + 105*c - (87*tan(d*x + c)^5 + 136*
tan(d*x + c)^3 + 57*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*ta
n(d*x + c)^2 + 1) - 48*tan(d*x + c))*b^8 - 13440*a^5*b^3*(1/cos(d*x + c) +
cos(d*x + c)) - 240*a^8*tan(d*x + c) - 1920*a^7*b/cos(d*x + c))/d
```

mupad [B] time = 7.73, size = 767, normalized size = 2.20

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(240\,a^7\,b+1120\,a^5\,b^3+\frac{1792\,a^3\,b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(96\,a^7\,b+224\,a^5\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^8*(240*a^7*b + (1792*a^3*b^5)/3 + 1120*a^5*b^3) + tan(c
/2 + (d*x)/2)^10*(96*a^7*b + 224*a^5*b^3) + tan(c/2 + (d*x)/2)*(2*a^8 + (35
*b^8)/8 + 105*a^2*b^6 + 210*a^4*b^4 + 56*a^6*b^2) + (256*a*b^7)/5 + 16*a^7*
b + tan(c/2 + (d*x)/2)^2*(256*a*b^7 + 96*a^7*b + (4480*a^3*b^5)/3 + 1120*a^
5*b^3) + tan(c/2 + (d*x)/2)^4*((2304*a*b^7)/5 + 240*a^7*b + 2688*a^3*b^5 +
2240*a^5*b^3) + tan(c/2 + (d*x)/2)^6*(256*a*b^7 + 320*a^7*b + (6272*a^3*b^5
)/3 + 2240*a^5*b^3) + tan(c/2 + (d*x)/2)^13*(2*a^8 + (35*b^8)/8 + 105*a^2*b
^6 + 210*a^4*b^4 + 56*a^6*b^2) + tan(c/2 + (d*x)/2)^3*(12*a^8 + (245*b^8)/1
2 + 490*a^2*b^6 + 980*a^4*b^4 + 336*a^6*b^2) + tan(c/2 + (d*x)/2)^11*(12*a^
8 + (245*b^8)/12 + 490*a^2*b^6 + 980*a^4*b^4 + 336*a^6*b^2) + tan(c/2 + (d*
x)/2)^5*(30*a^8 + (791*b^8)/24 + 791*a^2*b^6 + 2030*a^4*b^4 + 840*a^6*b^2)
+ tan(c/2 + (d*x)/2)^9*(30*a^8 + (791*b^8)/24 + 791*a^2*b^6 + 2030*a^4*b^4
+ 840*a^6*b^2) + tan(c/2 + (d*x)/2)^7*(40*a^8 + (25*b^8)/2 + 812*a^2*b^6 +
2520*a^4*b^4 + 1120*a^6*b^2) + (896*a^3*b^5)/3 + 224*a^5*b^3 + 16*a^7*b*tan
(c/2 + (d*x)/2)^12)/(d*(5*tan(c/2 + (d*x)/2)^2 + 9*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 - 9*tan(c/2 + (d*x)/2)^10 -
5*tan(c/2 + (d*x)/2)^12 - tan(c/2 + (d*x)/2)^14 + 1)) - (7*b^2*atan((7*b^2*
tan(c/2 + (d*x)/2)*(64*a^6 + 5*b^6 + 120*a^2*b^4 + 240*a^4*b^2))/(35*b^8 +
840*a^2*b^6 + 1680*a^4*b^4 + 448*a^6*b^2))*(64*a^6 + 5*b^6 + 120*a^2*b^4 +
240*a^4*b^2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.421 $\int \sec^4(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=369

$$\frac{b\left(2a^2-7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{3d}+\frac{ab\left(2a^2-13b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{3d}-\frac{\sec(c+dx)\left(5ab\right.}{}$$

```
[Out] 35/8*b^4*(16*a^4+16*a^2*b^2+b^4)*x+1/6*a*b*(8*a^6-104*a^4*b^2-803*a^2*b^4-2
56*b^6)*cos(d*x+c)/d+1/24*b^2*(16*a^6-200*a^4*b^2-866*a^2*b^4-105*b^6)*cos(
d*x+c)*sin(d*x+c)/d+1/12*a*b*(8*a^4-88*a^2*b^2-151*b^4)*cos(d*x+c)*(a+b*sin
(d*x+c))^2/d+1/12*b*(8*a^4-72*a^2*b^2-35*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^3
/d+1/3*a*b*(2*a^2-13*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^4/d+1/3*b*(2*a^2-7*b^
2)*cos(d*x+c)*(a+b*sin(d*x+c))^5/d+1/3*sec(d*x+c)^3*(b+a*sin(d*x+c))*(a+b*s
in(d*x+c))^7/d-1/3*sec(d*x+c)*(a+b*sin(d*x+c))^6*(5*a*b-(2*a^2-7*b^2)*sin(d
*x+c))/d
```

Rubi [A] time = 0.65, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2691, 2861, 2753, 2734}

$$\frac{ab\left(-104a^4b^2-803a^2b^4+8a^6-256b^6\right)\cos(c+dx)}{6d}+\frac{b\left(2a^2-7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{3d}+\frac{ab\left(2a^2-13\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (35*b^4*(16*a^4 + 16*a^2*b^2 + b^4)*x)/8 + (a*b*(8*a^6 - 104*a^4*b^2 - 803*
a^2*b^4 - 256*b^6)*Cos[c + d*x])/(6*d) + (b^2*(16*a^6 - 200*a^4*b^2 - 866*a
^2*b^4 - 105*b^6)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (a*b*(8*a^4 - 88*a^2*
b^2 - 151*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(12*d) + (b*(8*a^4 - 72
*a^2*b^2 - 35*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(12*d) + (a*b*(2*a^
2 - 13*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(3*d) + (b*(2*a^2 - 7*b^2)
*Cos[c + d*x]*(a + b*Sin[c + d*x])^5)/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^7)/(3*d) - (Sec[c + d*x]*(a + b*Sin[c + d*x])^
6*(5*a*b - (2*a^2 - 7*b^2)*Sin[c + d*x]))/(3*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{3d} - \frac{1}{3}\int \sec^2(c+dx$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{3d} - \frac{\sec(c+dx)(a+}{}$$

$$= \frac{b\left(2a^2-7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{3d} + \frac{\sec^3(c+dx)(b+a\sin}{}$$

$$= \frac{ab\left(2a^2-13b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{3d} + \frac{b\left(2a^2-7b^2\right)\cos(c}{}$$

$$= \frac{b\left(8a^4-72a^2b^2-35b^4\right)\cos(c+dx)(a+b\sin(c+dx))^3}{12d} + \frac{ab\left(2a^2-1\right.}{}$$

$$= \frac{ab\left(8a^4-88a^2b^2-151b^4\right)\cos(c+dx)(a+b\sin(c+dx))^2}{12d} + \frac{b\left(8a^4-\right.}{}$$

$$= \frac{35}{8}b^4\left(16a^4+16a^2b^2+b^4\right)x + \frac{ab\left(8a^6-104a^4b^2-803a^2b^4-256b^6\right)}{6d}$$

Mathematica [A] time = 1.11, size = 414, normalized size = 1.12

$$\frac{\sec^3(c+dx)\left(384a^8\sin(c+dx)+128a^8\sin(3(c+dx))+2048a^7b+5376a^6b^2\sin(c+dx)-1792a^6b^2\sin(3(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^3*(2048*a^7*b - 7168*a^5*b^3 - 44800*a^3*b^5 - 13440*a*b^7 +
40320*a^4*b^4*(c + d*x)*Cos[c + d*x] + 40320*a^2*b^6*(c + d*x)*Cos[c + d*x]
 + 2520*b^8*(c + d*x)*Cos[c + d*x] - 21504*a^5*b^3*Cos[2*(c + d*x)] - 64512
*a^3*b^5*Cos[2*(c + d*x)] - 17472*a*b^7*Cos[2*(c + d*x)] + 13440*a^4*b^4*(c
 + d*x)*Cos[3*(c + d*x)] + 13440*a^2*b^6*(c + d*x)*Cos[3*(c + d*x)] + 840*b
^8*(c + d*x)*Cos[3*(c + d*x)] - 5376*a^3*b^5*Cos[4*(c + d*x)] - 1920*a*b^7*
Cos[4*(c + d*x)] + 64*a*b^7*Cos[6*(c + d*x)] + 384*a^8*Sin[c + d*x] + 5376*
a^6*b^2*Sin[c + d*x] - 6720*a^2*b^6*Sin[c + d*x] - 525*b^8*Sin[c + d*x] + 1
28*a^8*Sin[3*(c + d*x)] - 1792*a^6*b^2*Sin[3*(c + d*x)] - 17920*a^4*b^4*Sin
[3*(c + d*x)] - 14560*a^2*b^6*Sin[3*(c + d*x)] - 847*b^8*Sin[3*(c + d*x)] -
 672*a^2*b^6*Sin[5*(c + d*x)] - 63*b^8*Sin[5*(c + d*x)] + 3*b^8*Sin[7*(c +
d*x)]))/(768*d)
```

fricas [A] time = 0.50, size = 268, normalized size = 0.73

$$64\,ab^{7}\cos\left(dx+c\right)^{6}+64\,a^{7}b+448\,a^{5}b^{3}+448\,a^{3}b^{5}+64\,ab^{7}+105\left(16\,a^{4}b^{4}+16\,a^{2}b^{6}+b^{8}\right)dx\cos\left(dx+c\right)^{3}-192$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/24*(64*a*b^7*cos(d*x + c)^6 + 64*a^7*b + 448*a^5*b^3 + 448*a^3*b^5 + 64*a
*b^7 + 105*(16*a^4*b^4 + 16*a^2*b^6 + b^8)*d*x*cos(d*x + c)^3 - 192*(7*a^3*
b^5 + 3*a*b^7)*cos(d*x + c)^4 - 192*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(
d*x + c)^2 + (6*b^8*cos(d*x + c)^6 + 8*a^8 + 224*a^6*b^2 + 560*a^4*b^4 + 22
4*a^2*b^6 + 8*b^8 - 3*(112*a^2*b^6 + 13*b^8)*cos(d*x + c)^4 + 16*(a^8 - 14*
a^6*b^2 - 140*a^4*b^4 - 98*a^2*b^6 - 5*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(
d*cos(d*x + c)^3)
```

giac [A] time = 0.76, size = 684, normalized size = 1.85

$$105\left(16\,a^{4}b^{4}+16\,a^{2}b^{6}+b^{8}\right)(dx+c)-\frac{16\left(3\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-210\,a^{4}b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-168\,a^{2}b^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,b^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/24*(105*(16*a^4*b^4 + 16*a^2*b^6 + b^8)*(d*x + c) - 16*(3*a^8*tan(1/2*d*x
 + 1/2*c)^5 - 210*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 168*a^2*b^6*tan(1/2*d*x
+ 1/2*c)^5 - 9*b^8*tan(1/2*d*x + 1/2*c)^5 + 24*a^7*b*tan(1/2*d*x + 1/2*c)^4
 - 168*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 48*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 2
*a^8*tan(1/2*d*x + 1/2*c)^3 + 112*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 700*a^4*
b^4*tan(1/2*d*x + 1/2*c)^3 + 448*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 22*b^8*ta
n(1/2*d*x + 1/2*c)^3 + 336*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 672*a^3*b^5*tan
(1/2*d*x + 1/2*c)^2 + 144*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 3*a^8*tan(1/2*d*x
+ 1/2*c) - 210*a^4*b^4*tan(1/2*d*x + 1/2*c) - 168*a^2*b^6*tan(1/2*d*x + 1/2
*c) - 9*b^8*tan(1/2*d*x + 1/2*c) + 8*a^7*b - 112*a^5*b^3 - 280*a^3*b^5 - 64
*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3 + 2*(336*a^2*b^6*tan(1/2*d*x + 1/2*c
)^7 + 33*b^8*tan(1/2*d*x + 1/2*c)^7 - 1344*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 -
 384*a*b^7*tan(1/2*d*x + 1/2*c)^6 + 336*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 57
*b^8*tan(1/2*d*x + 1/2*c)^5 - 4032*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 1536*a*
b^7*tan(1/2*d*x + 1/2*c)^4 - 336*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 57*b^8*ta
n(1/2*d*x + 1/2*c)^3 - 4032*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 - 1664*a*b^7*tan
(1/2*d*x + 1/2*c)^2 - 336*a^2*b^6*tan(1/2*d*x + 1/2*c) - 33*b^8*tan(1/2*d*x
 + 1/2*c) - 1344*a^3*b^5 - 512*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.46, size = 495, normalized size = 1.34

$$-a^{8}\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan\left(dx+c\right)+\frac{8a^{7}b}{3\cos(dx+c)^{3}}+\frac{28a^{6}b^{2}\left(\sin^{3}(dx+c)\right)}{3\cos(dx+c)^{3}}+56a^{5}b^{3}\left(\frac{\sin^{4}(dx+c)}{3\cos(dx+c)^{3}}-\frac{\sin^{4}(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^{2}(dx+c)\right)}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-a^8*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+8/3*a^7*b/cos(d*x+c)^3+28/3*a^
6*b^2*sin(d*x+c)^3/cos(d*x+c)^3+56*a^5*b^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1
```

```
/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+70*a^4*b^4*(1/3
*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+56*a^3*b^5*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-s
in(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+28*a
^2*b^6*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(
d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+8*a*b^
7*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+sin(
d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*(1/3*sin(d*x+c)
^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+
35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/8*d*x+35/8*c))
```

maxima [A] time = 0.43, size = 328, normalized size = 0.89

$$\frac{224\,a^6b^2\tan\left(dx+c\right)^3+8\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^8+560\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/24*(224*a^6*b^2*tan(d*x + c)^3 + 8*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^8
+ 560*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^4*b^4 + 112*(2*tan(
d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d
*x + c))*a^2*b^6 + 64*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)
^3 - 9*cos(d*x + c))*a*b^7 + (8*tan(d*x + c)^3 + 105*d*x + 105*c - 3*(13*ta
n(d*x + c)^3 + 11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 7
2*tan(d*x + c))*b^8 - 448*a^3*b^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 +
3*cos(d*x + c)) - 448*(3*cos(d*x + c)^2 - 1)*a^5*b^3/cos(d*x + c)^3 + 64*a^
7*b/cos(d*x + c)^3)/d
```

mupad [B] time = 7.82, size = 726, normalized size = 1.97

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(\frac{304\,a^7\,b}{3}+\frac{2464\,a^5\,b^3}{3}+\frac{1792\,a^3\,b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(64\,a^7\,b+224\,a^5\,b^3\right)-\frac{256\,a\,b^7}{3}+\frac{16\,a^7\,b}{3}-\tan\left(\frac{c}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^8*((304*a^7*b)/3 + (1792*a^3*b^5)/3 + (2464*a^5*b^3)/3)
 + tan(c/2 + (d*x)/2)^10*(64*a^7*b + 224*a^5*b^3) - (256*a*b^7)/3 + (16*a^7
*b)/3 - tan(c/2 + (d*x)/2)*((35*b^8)/4 - 2*a^8 + 140*a^2*b^6 + 140*a^4*b^4)
 - tan(c/2 + (d*x)/2)^2*((256*a*b^7)/3 - (64*a^7*b)/3 + (896*a^3*b^5)/3 + (
224*a^5*b^3)/3) + tan(c/2 + (d*x)/2)^4*(256*a*b^7 + 48*a^7*b + 896*a^3*b^5
+ 448*a^5*b^3) + tan(c/2 + (d*x)/2)^6*(256*a*b^7 + (256*a^7*b)/3 + (4480*a^
3*b^5)/3 + (3136*a^5*b^3)/3) - tan(c/2 + (d*x)/2)^3*((35*b^8)/6 - (20*a^8)/
3 + (280*a^2*b^6)/3 + (280*a^4*b^4)/3 - (224*a^6*b^2)/3) - tan(c/2 + (d*x)/
2)^11*((35*b^8)/6 - (20*a^8)/3 + (280*a^2*b^6)/3 + (280*a^4*b^4)/3 - (224*a
^6*b^2)/3) + tan(c/2 + (d*x)/2)^7*(8*a^8 + 17*b^8 + 784*a^2*b^6 + 1680*a^4*
b^4 + 448*a^6*b^2) + tan(c/2 + (d*x)/2)^5*((26*a^8)/3 + (329*b^8)/12 + (131
6*a^2*b^6)/3 + (2660*a^4*b^4)/3 + (896*a^6*b^2)/3) + tan(c/2 + (d*x)/2)^9*(
(26*a^8)/3 + (329*b^8)/12 + (1316*a^2*b^6)/3 + (2660*a^4*b^4)/3 + (896*a^6*
b^2)/3) - (896*a^3*b^5)/3 - (224*a^5*b^3)/3 - tan(c/2 + (d*x)/2)^13*((35*b^
8)/4 - 2*a^8 + 140*a^2*b^6 + 140*a^4*b^4) + 16*a^7*b*tan(c/2 + (d*x)/2)^12)
/(d*(tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^6
 + 3*tan(c/2 + (d*x)/2)^8 + 3*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^12
 - tan(c/2 + (d*x)/2)^14 + 1)) + (35*b^4*atan((35*b^4*tan(c/2 + (d*x)/2)*(1
6*a^4 + b^4 + 16*a^2*b^2))/(35*b^8 + 560*a^2*b^6 + 560*a^4*b^4))*(16*a^4 +
b^4 + 16*a^2*b^2))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.422 $\int \sec^6(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=381

$$\frac{4ab\left(2a^2+b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{15d}-\frac{\sec^3(c+dx)\left(3ab-\left(4a^2-7b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx)}{15d}$$

```
[Out] -7/2*b^6*(8*a^2+b^2)*x+2/15*a*b*(8*a^6-48*a^4*b^2+163*a^2*b^4+192*b^6)*cos(
d*x+c)/d+1/30*b^2*(16*a^6-88*a^4*b^2+282*a^2*b^4+105*b^6)*cos(d*x+c)*sin(d*
x+c)/d+1/15*a*b*(8*a^4-32*a^2*b^2+87*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^2/d+1
/15*b*(8*a^4-16*a^2*b^2+35*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^3/d+4/15*a*b*(2
*a^2+b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^4/d+1/5*sec(d*x+c)^5*(b+a*sin(d*x+c))
*(a+b*sin(d*x+c))^7/d-1/15*sec(d*x+c)^3*(a+b*sin(d*x+c))^6*(3*a*b-(4*a^2-7*
b^2)*sin(d*x+c))/d-4/15*sec(d*x+c)*(a+b*sin(d*x+c))^5*(b*(4*a^2-7*b^2)-a*(2
*a^2+b^2)*sin(d*x+c))/d
```

Rubi [A] time = 0.72, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2691, 2861, 2753, 2734}

$$\frac{2ab\left(-48a^4b^2+163a^2b^4+8a^6+192b^6\right)\cos(c+dx)}{15d}+\frac{4ab\left(2a^2+b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{15d}+\frac{b\left(-16a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (-7*b^6*(8*a^2 + b^2)*x)/2 + (2*a*b*(8*a^6 - 48*a^4*b^2 + 163*a^2*b^4 + 192
*b^6)*Cos[c + d*x])/(15*d) + (b^2*(16*a^6 - 88*a^4*b^2 + 282*a^2*b^4 + 105*
b^6)*Cos[c + d*x]*Sin[c + d*x])/(30*d) + (a*b*(8*a^4 - 32*a^2*b^2 + 87*b^4)
*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(15*d) + (b*(8*a^4 - 16*a^2*b^2 + 35*
b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(15*d) + (4*a*b*(2*a^2 + b^2)*Cos
[c + d*x]*(a + b*Sin[c + d*x])^4)/(15*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d
*x])*(a + b*Sin[c + d*x])^7)/(5*d) - (Sec[c + d*x]^3*(a + b*Sin[c + d*x])^6
*(3*a*b - (4*a^2 - 7*b^2)*Sin[c + d*x]))/(15*d) - (4*Sec[c + d*x]*(a + b*Si
n[c + d*x])^5*(b*(4*a^2 - 7*b^2) - a*(2*a^2 + b^2)*Sin[c + d*x]))/(15*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{5d} - \frac{1}{5}\int \sec^4(c+dx)(a$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{5d} - \frac{\sec^3(c+dx)(a+b}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{5d} - \frac{\sec^3(c+dx)(a+b}{}$$

$$= \frac{4ab\left(2a^2+b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{15d} + \frac{\sec^5(c+dx)(b+a\sin(}{}$$

$$= \frac{b\left(8a^4-16a^2b^2+35b^4\right)\cos(c+dx)(a+b\sin(c+dx))^3}{15d} + \frac{4ab\left(2a^2+b^2\right.}{}$$

$$= \frac{ab\left(8a^4-32a^2b^2+87b^4\right)\cos(c+dx)(a+b\sin(c+dx))^2}{15d} + \frac{b\left(8a^4-16a\right.}{}$$

$$= -\frac{7}{2}b^6\left(8a^2+b^2\right)x + \frac{2ab\left(8a^6-48a^4b^2+163a^2b^4+192b^6\right)\cos(c+dx)}{15d}$$

Mathematica [A] time = 1.33, size = 472, normalized size = 1.24

$$\frac{\sec^5(c+dx)\left(640a^8\sin(c+dx)+320a^8\sin(3(c+dx))+64a^8\sin(5(c+dx))+3072a^7b+8960a^6b^2\sin(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^5*(3072*a^7*b + 3584*a^5*b^3 + 25984*a^3*b^5 + 17472*a*b^7 -
33600*a^2*b^6*(c + d*x)*Cos[c + d*x] - 4200*b^8*(c + d*x)*Cos[c + d*x] - 17
920*a^5*b^3*Cos[2*(c + d*x)] + 17920*a^3*b^5*Cos[2*(c + d*x)] + 22560*a*b^7
*Cos[2*(c + d*x)] - 16800*a^2*b^6*(c + d*x)*Cos[3*(c + d*x)] - 2100*b^8*(c
+ d*x)*Cos[3*(c + d*x)] + 13440*a^3*b^5*Cos[4*(c + d*x)] + 8640*a*b^7*Cos[4
*(c + d*x)] - 3360*a^2*b^6*(c + d*x)*Cos[5*(c + d*x)] - 420*b^8*(c + d*x)*C
os[5*(c + d*x)] + 480*a*b^7*Cos[6*(c + d*x)] + 640*a^8*Sin[c + d*x] + 8960*
a^6*b^2*Sin[c + d*x] + 16800*a^4*b^4*Sin[c + d*x] + 11200*a^2*b^6*Sin[c + d
*x] + 875*b^8*Sin[c + d*x] + 320*a^8*Sin[3*(c + d*x)] - 2240*a^6*b^2*Sin[3*
(c + d*x)] - 8400*a^4*b^4*Sin[3*(c + d*x)] + 5600*a^2*b^6*Sin[3*(c + d*x)]
+ 1015*b^8*Sin[3*(c + d*x)] + 64*a^8*Sin[5*(c + d*x)] - 448*a^6*b^2*Sin[5*(
c + d*x)] + 1680*a^4*b^4*Sin[5*(c + d*x)] + 5152*a^2*b^6*Sin[5*(c + d*x)] +
 539*b^8*Sin[5*(c + d*x)] + 15*b^8*Sin[7*(c + d*x)]))/(1920*d)
```

fricas [A] time = 0.50, size = 281, normalized size = 0.74

$$\frac{240\, ab^7 \cos\left(dx+c\right)^6 + 48\, a^7 b + 336\, a^5 b^3 + 336\, a^3 b^5 + 48\, ab^7 - 105\left(8\, a^2 b^6 + b^8\right) dx \cos\left(dx+c\right)^5 + 240\left(7\, a^3 b \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/30*(240*a*b^7*cos(d*x + c)^6 + 48*a^7*b + 336*a^5*b^3 + 336*a^3*b^5 + 48*
a*b^7 - 105*(8*a^2*b^6 + b^8)*d*x*cos(d*x + c)^5 + 240*(7*a^3*b^5 + 3*a*b^7
)*cos(d*x + c)^4 - 80*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + (
15*b^8*cos(d*x + c)^6 + 6*a^8 + 168*a^6*b^2 + 420*a^4*b^4 + 168*a^2*b^6 + 6
*b^8 + 4*(4*a^8 - 28*a^6*b^2 + 105*a^4*b^4 + 322*a^2*b^6 + 29*b^8)*cos(d*x
+ c)^4 + 8*(a^8 - 7*a^6*b^2 - 105*a^4*b^4 - 77*a^2*b^6 - 4*b^8)*cos(d*x + c
)^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [A] time = 1.19, size = 663, normalized size = 1.74

$$-\frac{105\left(8\, a^2 b^6 + b^8\right)\left(dx+c\right) + \frac{30\left(b^8 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^3 - 16\, ab^7 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^2 - b^8 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right) - 16\, ab^7\right)}{\left(\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^2 + 1\right)^2} + \frac{4\left(15\, a^8 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^9 + 420 \cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/30*(105*(8*a^2*b^6 + b^8)*(d*x + c) + 30*(b^8*tan(1/2*d*x + 1/2*c)^3 - 1
6*a*b^7*tan(1/2*d*x + 1/2*c)^2 - b^8*tan(1/2*d*x + 1/2*c) - 16*a*b^7)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2 + 4*(15*a^8*tan(1/2*d*x + 1/2*c)^9 + 420*a^2*b^6*
tan(1/2*d*x + 1/2*c)^9 + 45*b^8*tan(1/2*d*x + 1/2*c)^9 + 120*a^7*b*tan(1/2*
d*x + 1/2*c)^8 + 120*a*b^7*tan(1/2*d*x + 1/2*c)^8 - 20*a^8*tan(1/2*d*x + 1/
2*c)^7 + 560*a^6*b^2*tan(1/2*d*x + 1/2*c)^7 - 2240*a^2*b^6*tan(1/2*d*x + 1/
2*c)^7 - 220*b^8*tan(1/2*d*x + 1/2*c)^7 + 1680*a^5*b^3*tan(1/2*d*x + 1/2*c)
^6 - 720*a*b^7*tan(1/2*d*x + 1/2*c)^6 + 58*a^8*tan(1/2*d*x + 1/2*c)^5 + 224
*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 3360*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 498
4*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 398*b^8*tan(1/2*d*x + 1/2*c)^5 + 240*a^7
*b*tan(1/2*d*x + 1/2*c)^4 + 560*a^5*b^3*tan(1/2*d*x + 1/2*c)^4 + 4480*a^3*b
^5*tan(1/2*d*x + 1/2*c)^4 + 1920*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 20*a^8*tan(
1/2*d*x + 1/2*c)^3 + 560*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 2240*a^2*b^6*tan(
1/2*d*x + 1/2*c)^3 - 220*b^8*tan(1/2*d*x + 1/2*c)^3 + 560*a^5*b^3*tan(1/2*d
*x + 1/2*c)^2 - 2240*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 - 1200*a*b^7*tan(1/2*d*
x + 1/2*c)^2 + 15*a^8*tan(1/2*d*x + 1/2*c) + 420*a^2*b^6*tan(1/2*d*x + 1/2*
c) + 45*b^8*tan(1/2*d*x + 1/2*c) + 24*a^7*b - 112*a^5*b^3 + 448*a^3*b^5 + 2
64*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.52, size = 544, normalized size = 1.43

$$\frac{-a^8\left(-\frac{8}{15} - \frac{\left(\sec^4\left(dx+c\right)\right)}{5} - \frac{4\left(\sec^2\left(dx+c\right)\right)}{15}\right)\tan\left(dx+c\right) + \frac{8a^7 b}{5\cos\left(dx+c\right)^5} + 28a^6 b^2\left(\frac{\sin^3\left(dx+c\right)}{5\cos\left(dx+c\right)^5} + \frac{2\left(\sin^3\left(dx+c\right)\right)}{15\cos\left(dx+c\right)^3}\right) + 56a^5 b^3 \left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-a^8*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+8/5*a^7*b/c
os(d*x+c)^5+28*a^6*b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos
```

```
(d*x+c)^3)+56*a^5*b^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(
d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+14*
a^4*b^4*sin(d*x+c)^5/cos(d*x+c)^5+56*a^3*b^5*(1/5*sin(d*x+c)^6/cos(d*x+c)^5
-1/15*sin(d*x+c)^6/cos(d*x+c)^3+1/5*sin(d*x+c)^6/cos(d*x+c)+1/5*(8/3+sin(d*
x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+28*a^2*b^6*(1/5*tan(d*x+c)^5-1/3*tan(d
*x+c)^3+tan(d*x+c)-d*x-c)+8*a*b^7*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*
x+c)^8/cos(d*x+c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+
c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*(1/5*sin(d*x+c)^9/cos(d*x+c)^5-4/15*
sin(d*x+c)^9/cos(d*x+c)^3+8/5*sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d*x+c)^7+7/6
*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-7/2*d*x-7/2*c
))
```

maxima [A] time = 0.44, size = 315, normalized size = 0.83

$$\frac{420\,a^4b^4\tan(dx+c)^5 + 2\left(3\,\tan(dx+c)^5 + 10\,\tan(dx+c)^3 + 15\,\tan(dx+c)\right)a^8 + 56\left(3\,\tan(dx+c)^5 + 5\,\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/30*(420*a^4*b^4*tan(d*x + c)^5 + 2*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3
+ 15*tan(d*x + c))*a^8 + 56*(3*tan(d*x + c)^5 + 5*tan(d*x + c)^3)*a^6*b^2 +
 56*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))
*a^2*b^6 + (6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan
(d*x + c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*b^8 + 48*a*b^7*((15*cos(d
*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*cos(d*x + c)) - 112*(5
*cos(d*x + c)^2 - 3)*a^5*b^3/cos(d*x + c)^5 + 112*(15*cos(d*x + c)^4 - 10*c
os(d*x + c)^2 + 3)*a^3*b^5/cos(d*x + c)^5 + 48*a^7*b/cos(d*x + c)^5)/d
```

mupad [B] time = 7.60, size = 665, normalized size = 1.75

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\left(2\,a^8+56\,a^2\,b^6+7\,b^8\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(48\,a^7\,b+\frac{1568\,a^5\,b^3}{3}+\frac{1792\,a^3\,b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(32\,a^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^6,x)
```

```
[Out] - (tan(c/2 + (d*x)/2)^13*(2*a^8 + 7*b^8 + 56*a^2*b^6) + tan(c/2 + (d*x)/2)^
8*(48*a^7*b + (1792*a^3*b^5)/3 + (1568*a^5*b^3)/3) + tan(c/2 + (d*x)/2)^10*
(32*a^7*b + 224*a^5*b^3) + (256*a*b^7)/5 + (16*a^7*b)/5 + tan(c/2 + (d*x)/2
)*(2*a^8 + 7*b^8 + 56*a^2*b^6) + tan(c/2 + (d*x)/2)^6*(256*a*b^7 + 64*a^7*b
 + 896*a^3*b^5 + 448*a^5*b^3) - tan(c/2 + (d*x)/2)^2*((768*a*b^7)/5 - (32*a
^7*b)/5 + (896*a^3*b^5)/5 - (224*a^5*b^3)/5) + tan(c/2 + (d*x)/2)^4*((256*a
*b^7)/5 + (176*a^7*b)/5 + (896*a^3*b^5)/15 + (3136*a^5*b^3)/15) + tan(c/2 +
 (d*x)/2)^5*((22*a^8)/5 + (77*b^8)/5 + (616*a^2*b^6)/5 + 448*a^4*b^4 + (896
*a^6*b^2)/5) + tan(c/2 + (d*x)/2)^9*((22*a^8)/5 + (77*b^8)/5 + (616*a^2*b^6
)/5 + 448*a^4*b^4 + (896*a^6*b^2)/5) + tan(c/2 + (d*x)/2)^7*((152*a^8)/15 +
 (412*b^8)/15 + (10976*a^2*b^6)/15 + 896*a^4*b^4 + (3136*a^6*b^2)/15) + (89
6*a^3*b^5)/15 - (224*a^5*b^3)/15 + tan(c/2 + (d*x)/2)^3*((4*a^8)/3 - (70*b^
8)/3 - (560*a^2*b^6)/3 + (224*a^6*b^2)/3) + tan(c/2 + (d*x)/2)^11*((4*a^8)/
3 - (70*b^8)/3 - (560*a^2*b^6)/3 + (224*a^6*b^2)/3) + 16*a^7*b*tan(c/2 + (d
*x)/2)^12)/(d*(3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - 5*tan(c/2 +
(d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 3*tan(c/2 + (
d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)) - (7*b^6*atan((7*b^6*tan(c/2 + (d*
x)/2)*(8*a^2 + b^2))/(7*b^8 + 56*a^2*b^6))*(8*a^2 + b^2))/d
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.423 $\int \sec^8(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=404

$$-\frac{\sec^5(c+dx)(a+b\sin(c+dx))^6\left(ab-\left(6a^2-7b^2\right)\sin(c+dx)\right)}{35d}-\frac{2\sec^3(c+dx)(a+b\sin(c+dx))^5\left(b\left(6a^2-7b^2\right.\right.}{105d}$$

```
[Out] b^8*x+4/105*a*b*(24*a^6-88*a^4*b^2+125*a^2*b^4-96*b^6)*cos(d*x+c)/d+1/105*b
^2*(48*a^6-152*a^4*b^2+174*a^2*b^4-105*b^6)*cos(d*x+c)*sin(d*x+c)/d+2/105*a
*b*(24*a^4-40*a^2*b^2+9*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^2/d+2/105*b*(24*a^
4+8*a^2*b^2-35*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^3/d+1/7*sec(d*x+c)^7*(b+a*s
in(d*x+c))*(a+b*sin(d*x+c))^7/d-2/105*sec(d*x+c)^3*(a+b*sin(d*x+c))^5*(b*(6
*a^2-7*b^2)-a*(12*a^2-11*b^2)*sin(d*x+c))/d-1/35*sec(d*x+c)^5*(a+b*sin(d*x+
c))^6*(a*b-(6*a^2-7*b^2)*sin(d*x+c))/d-2/105*sec(d*x+c)*(a+b*sin(d*x+c))^4*
(3*a*b*(12*a^2-11*b^2)-(24*a^4+8*a^2*b^2-35*b^4)*sin(d*x+c))/d
```

Rubi [A] time = 0.82, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2691, 2861, 2753, 2734}

$$\frac{4ab\left(-88a^4b^2+125a^2b^4+24a^6-96b^6\right)\cos(c+dx)}{105d}+\frac{b^2\left(-152a^4b^2+174a^2b^4+48a^6-105b^6\right)\sin(c+dx)\cos(c}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] b^8*x + (4*a*b*(24*a^6 - 88*a^4*b^2 + 125*a^2*b^4 - 96*b^6)*Cos[c + d*x])/(
105*d) + (b^2*(48*a^6 - 152*a^4*b^2 + 174*a^2*b^4 - 105*b^6)*Cos[c + d*x]*S
in[c + d*x])/(105*d) + (2*a*b*(24*a^4 - 40*a^2*b^2 + 9*b^4)*Cos[c + d*x]*(a
 + b*Sin[c + d*x])^2)/(105*d) + (2*b*(24*a^4 + 8*a^2*b^2 - 35*b^4)*Cos[c +
d*x]*(a + b*Sin[c + d*x])^3)/(105*d) + (Sec[c + d*x]^7*(b + a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^7)/(7*d) - (2*Sec[c + d*x]^3*(a + b*Sin[c + d*x])^5*(
b*(6*a^2 - 7*b^2) - a*(12*a^2 - 11*b^2)*Sin[c + d*x]))/(105*d) - (Sec[c + d
*x]^5*(a + b*Sin[c + d*x])^6*(a*b - (6*a^2 - 7*b^2)*Sin[c + d*x]))/(35*d) -
 (2*Sec[c + d*x]*(a + b*Sin[c + d*x])^4*(3*a*b*(12*a^2 - 11*b^2) - (24*a^4
+ 8*a^2*b^2 - 35*b^4)*Sin[c + d*x]))/(105*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
```

```
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{1}{7}\int \sec^6(c+dx$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{\sec^5(c+dx)(a+}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{2\sec^3(c+dx)(a}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{2\sec^3(c+dx)(a}{}$$

$$= \frac{2b\left(24a^4+8a^2b^2-35b^4\right)\cos(c+dx)(a+b\sin(c+dx))^3}{105d} + \frac{\sec^7(c+}{}$$

$$= \frac{2ab\left(24a^4-40a^2b^2+9b^4\right)\cos(c+dx)(a+b\sin(c+dx))^2}{105d} + \frac{2b\left(24a^4\right.}{}$$

$$= b^8x + \frac{4ab\left(24a^6-88a^4b^2+125a^2b^4-96b^6\right)\cos(c+dx)}{105d} + \frac{b^2\left(48a^6\right.}{}$$

Mathematica [A] time = 1.43, size = 479, normalized size = 1.19

$$\frac{\sec^7(c+dx)\left(1680a^8\sin(c+dx)+1008a^8\sin(3(c+dx))+336a^8\sin(5(c+dx))+48a^8\sin(7(c+dx))+7680\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^7*(7680*a^7*b + 16128*a^5*b^3 + 25536*a^3*b^5 - 5088*a*b^7 +
3675*b^8*(c + d*x)*Cos[c + d*x] - 37632*a^5*b^3*Cos[2*(c + d*x)] - 12544*a^
3*b^5*Cos[2*(c + d*x)] - 14448*a*b^7*Cos[2*(c + d*x)] + 2205*b^8*(c + d*x)*
Cos[3*(c + d*x)] + 15680*a^3*b^5*Cos[4*(c + d*x)] - 3360*a*b^7*Cos[4*(c + d
*x)] + 735*b^8*(c + d*x)*Cos[5*(c + d*x)] - 1680*a*b^7*Cos[6*(c + d*x)] + 1
05*b^8*(c + d*x)*Cos[7*(c + d*x)] + 1680*a^8*Sin[c + d*x] + 23520*a^6*b^2*S
in[c + d*x] + 44100*a^4*b^4*Sin[c + d*x] + 14700*a^2*b^6*Sin[c + d*x] + 100
8*a^8*Sin[3*(c + d*x)] - 4704*a^6*b^2*Sin[3*(c + d*x)] - 20580*a^4*b^4*Sin[
3*(c + d*x)] - 8820*a^2*b^6*Sin[3*(c + d*x)] - 1176*b^8*Sin[3*(c + d*x)] +
336*a^8*Sin[5*(c + d*x)] - 1568*a^6*b^2*Sin[5*(c + d*x)] + 2940*a^4*b^4*Sin
[5*(c + d*x)] + 2940*a^2*b^6*Sin[5*(c + d*x)] - 392*b^8*Sin[5*(c + d*x)] +
48*a^8*Sin[7*(c + d*x)] - 224*a^6*b^2*Sin[7*(c + d*x)] + 420*a^4*b^4*Sin[7*
(c + d*x)] - 420*a^2*b^6*Sin[7*(c + d*x)] - 176*b^8*Sin[7*(c + d*x)]))/(672
0*d)
```

fricas [A] time = 0.53, size = 306, normalized size = 0.76

$$\frac{105\, b^8 dx \cos(dx+c)^7 - 840\, ab^7 \cos(dx+c)^6 + 120\, a^7 b + 840\, a^5 b^3 + 840\, a^3 b^5 + 120\, ab^7 + 280\left(7\, a^3 b^5 + 3\, ab^7\right) \mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*b^8*d*x*cos(d*x + c)^7 - 840*a*b^7*cos(d*x + c)^6 + 120*a^7*b +
840*a^5*b^3 + 840*a^3*b^5 + 120*a*b^7 + 280*(7*a^3*b^5 + 3*a*b^7)*cos(d*x +
 c)^4 - 168*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + (15*a^8 + 4
20*a^6*b^2 + 1050*a^4*b^4 + 420*a^2*b^6 + 15*b^8 + 4*(12*a^8 - 56*a^6*b^2 +
 105*a^4*b^4 - 105*a^2*b^6 - 44*b^8)*cos(d*x + c)^6 + 2*(12*a^8 - 56*a^6*b^
2 + 105*a^4*b^4 + 630*a^2*b^6 + 61*b^8)*cos(d*x + c)^4 + 6*(3*a^8 - 14*a^6*
b^2 - 280*a^4*b^4 - 210*a^2*b^6 - 11*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(d*
cos(d*x + c)^7)
```

giac [A] time = 0.70, size = 726, normalized size = 1.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(105*(d*x + c)*b^8 - 2*(105*a^8*tan(1/2*d*x + 1/2*c)^13 - 105*b^8*tan
(1/2*d*x + 1/2*c)^13 + 840*a^7*b*tan(1/2*d*x + 1/2*c)^12 - 210*a^8*tan(1/2*
d*x + 1/2*c)^11 + 3920*a^6*b^2*tan(1/2*d*x + 1/2*c)^11 + 770*b^8*tan(1/2*d*
x + 1/2*c)^11 + 11760*a^5*b^3*tan(1/2*d*x + 1/2*c)^10 + 903*a^8*tan(1/2*d*x
 + 1/2*c)^9 + 3136*a^6*b^2*tan(1/2*d*x + 1/2*c)^9 + 23520*a^4*b^4*tan(1/2*d
*x + 1/2*c)^9 - 2471*b^8*tan(1/2*d*x + 1/2*c)^9 + 4200*a^7*b*tan(1/2*d*x +
1/2*c)^8 + 11760*a^5*b^3*tan(1/2*d*x + 1/2*c)^8 + 31360*a^3*b^5*tan(1/2*d*x
 + 1/2*c)^8 - 636*a^8*tan(1/2*d*x + 1/2*c)^7 + 12768*a^6*b^2*tan(1/2*d*x +
1/2*c)^7 + 20160*a^4*b^4*tan(1/2*d*x + 1/2*c)^7 + 26880*a^2*b^6*tan(1/2*d*x
 + 1/2*c)^7 + 4572*b^8*tan(1/2*d*x + 1/2*c)^7 + 23520*a^5*b^3*tan(1/2*d*x +
 1/2*c)^6 + 15680*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 + 13440*a*b^7*tan(1/2*d*x
+ 1/2*c)^6 + 903*a^8*tan(1/2*d*x + 1/2*c)^5 + 3136*a^6*b^2*tan(1/2*d*x + 1/
2*c)^5 + 23520*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 2471*b^8*tan(1/2*d*x + 1/2*
c)^5 + 2520*a^7*b*tan(1/2*d*x + 1/2*c)^4 + 4704*a^5*b^3*tan(1/2*d*x + 1/2*c
)^4 + 9408*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 8064*a*b^7*tan(1/2*d*x + 1/2*c)
^4 - 210*a^8*tan(1/2*d*x + 1/2*c)^3 + 3920*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 +
 770*b^8*tan(1/2*d*x + 1/2*c)^3 + 2352*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 313
6*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 + 2688*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 105*
a^8*tan(1/2*d*x + 1/2*c) - 105*b^8*tan(1/2*d*x + 1/2*c) + 120*a^7*b - 336*a
^5*b^3 + 448*a^3*b^5 - 384*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

maple [A] time = 0.41, size = 567, normalized size = 1.40

$$\frac{-a^8\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{8a^7 b}{7\cos(dx+c)^7}+28a^6b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-a^8*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+8/7*a^7*b/cos(d*x+c)^7+28*a^6*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/
```

```
35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+56*a^5*b^3*(1
/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)
^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+
c))+70*a^4*b^4*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^
5)+56*a^3*b^5*(1/7*sin(d*x+c)^6/cos(d*x+c)^7+1/35*sin(d*x+c)^6/cos(d*x+c)^5
-1/105*sin(d*x+c)^6/cos(d*x+c)^3+1/35*sin(d*x+c)^6/cos(d*x+c)+1/35*(8/3+sin
(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+4*a^2*b^6*sin(d*x+c)^7/cos(d*x+c)^7
+8*a*b^7*(1/7*sin(d*x+c)^8/cos(d*x+c)^7-1/35*sin(d*x+c)^8/cos(d*x+c)^5+1/35
*sin(d*x+c)^8/cos(d*x+c)^3-1/7*sin(d*x+c)^8/cos(d*x+c)-1/7*(16/5+sin(d*x+c)
^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*(1/7*tan(d*x+c)^7-1/5
*tan(d*x+c)^5+1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

maxima [A] time = 0.43, size = 310, normalized size = 0.77

$$\frac{420\,a^2b^6\tan(dx+c)^7+3\left(5\,\tan(dx+c)^7+21\,\tan(dx+c)^5+35\,\tan(dx+c)^3+35\,\tan(dx+c)\right)a^8+28\,(15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/105*(420*a^2*b^6*tan(d*x + c)^7 + 3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5
 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*a^8 + 28*(15*tan(d*x + c)^7 + 42*ta
n(d*x + c)^5 + 35*tan(d*x + c)^3)*a^6*b^2 + 210*(5*tan(d*x + c)^7 + 7*tan(d
*x + c)^5)*a^4*b^4 + (15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x +
c)^3 + 105*d*x + 105*c - 105*tan(d*x + c))*b^8 - 168*(7*cos(d*x + c)^2 - 5)
*a^5*b^3/cos(d*x + c)^7 + 56*(35*cos(d*x + c)^4 - 42*cos(d*x + c)^2 + 15)*a
^3*b^5/cos(d*x + c)^7 - 24*(35*cos(d*x + c)^6 - 35*cos(d*x + c)^4 + 21*cos(
d*x + c)^2 - 5)*a*b^7/cos(d*x + c)^7 + 120*a^7*b/cos(d*x + c)^7)/d
```

mupad [B] time = 8.85, size = 546, normalized size = 1.35

$$b^8\,x-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(-4\,a^8+\frac{224\,a^6\,b^2}{3}+\frac{44\,b^8}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(-4\,a^8+\frac{224\,a^6\,b^2}{3}+\frac{44\,b^8}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\left(2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^8,x)
```

```
[Out] b^8*x - (tan(c/2 + (d*x)/2)^3*((44*b^8)/3 - 4*a^8 + (224*a^6*b^2)/3) + tan(
c/2 + (d*x)/2)^11*((44*b^8)/3 - 4*a^8 + (224*a^6*b^2)/3) + tan(c/2 + (d*x)/
2)^13*(2*a^8 - 2*b^8) + tan(c/2 + (d*x)/2)^2*((256*a*b^7)/5 - (896*a^3*b^5)
/15 + (224*a^5*b^3)/5) + tan(c/2 + (d*x)/2)^6*(256*a*b^7 + (896*a^3*b^5)/3
+ 448*a^5*b^3) + tan(c/2 + (d*x)/2)^8*(80*a^7*b + (1792*a^3*b^5)/3 + 224*a^
5*b^3) - (256*a*b^7)/35 + (16*a^7*b)/7 + tan(c/2 + (d*x)/2)^4*(48*a^7*b - (
768*a*b^7)/5 + (896*a^3*b^5)/5 + (448*a^5*b^3)/5) + tan(c/2 + (d*x)/2)*(2*a
^8 - 2*b^8) + tan(c/2 + (d*x)/2)^7*((3048*b^8)/35 - (424*a^8)/35 + 512*a^2*
b^6 + 384*a^4*b^4 + (1216*a^6*b^2)/5) + (128*a^3*b^5)/15 - (32*a^5*b^3)/5 +
 tan(c/2 + (d*x)/2)^5*((86*a^8)/5 - (706*b^8)/15 + 448*a^4*b^4 + (896*a^6*b
^2)/15) + tan(c/2 + (d*x)/2)^9*((86*a^8)/5 - (706*b^8)/15 + 448*a^4*b^4 + (
896*a^6*b^2)/15) + 224*a^5*b^3*tan(c/2 + (d*x)/2)^10 + 16*a^7*b*tan(c/2 + (
d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c
/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*ta
n(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.424 $\int \sec^{10}(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=236

$$\frac{128a^2\left(a^2-b^2\right)^3\tan(c+dx)}{315d}+\frac{128ab\left(a^2-b^2\right)^3\sec(c+dx)}{315d}+\frac{\sec^7(c+dx)(a+b\sin(c+dx))^6\left(\left(8a^2-7b^2\right)\sin(\right.}{63d}$$

```
[Out] 128/315*a*b*(a^2-b^2)^3*sec(d*x+c)/d+64/315*a*(a^2-b^2)^2*sec(d*x+c)^3*(b+a
*sin(d*x+c))*(a+b*sin(d*x+c))^2/d+16/105*a*(a^2-b^2)*sec(d*x+c)^5*(b+a*sin(
d*x+c))*(a+b*sin(d*x+c))^4/d+1/9*sec(d*x+c)^9*(b+a*sin(d*x+c))*(a+b*sin(d*x
+c))^7/d+1/63*sec(d*x+c)^7*(a+b*sin(d*x+c))^6*(a*b+(8*a^2-7*b^2)*sin(d*x+c)
)/d+128/315*a^2*(a^2-b^2)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.38, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2691, 2861, 12, 2669, 3767, 8}

$$\frac{128a^2\left(a^2-b^2\right)^3\tan(c+dx)}{315d}+\frac{128ab\left(a^2-b^2\right)^3\sec(c+dx)}{315d}+\frac{\sec^7(c+dx)(a+b\sin(c+dx))^6\left(\left(8a^2-7b^2\right)\sin(\right.}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (128*a*b*(a^2 - b^2)^3*Sec[c + d*x])/(315*d) + (64*a*(a^2 - b^2)^2*Sec[c +
d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(315*d) + (16*a*(a^2 -
b^2)*Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^4)/(105*d) +
(Sec[c + d*x]^9*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^7)/(9*d) + (Sec[c
 + d*x]^7*(a + b*Sin[c + d*x])^6*(a*b + (8*a^2 - 7*b^2)*Sin[c + d*x]))/(63*
d) + (128*a^2*(a^2 - b^2)^3*Tan[c + d*x])/(315*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
```

```
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{9d} - \frac{1}{9}\int \sec^8(c+dx)($$

$$= \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{9d} + \frac{\sec^7(c+dx)(a+b}{}$$

$$= \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{9d} + \frac{\sec^7(c+dx)(a+b}{}$$

$$= \frac{16a\left(a^2-b^2\right)\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^4}{105d} + \frac{\sec^9}{}$$

$$= \frac{16a\left(a^2-b^2\right)\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^4}{105d} + \frac{\sec^9}{}$$

$$= \frac{64a\left(a^2-b^2\right)^2\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{315d} + \frac{16a}{}$$

$$= \frac{64a\left(a^2-b^2\right)^2\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{315d} + \frac{16a}{}$$

$$= \frac{128ab\left(a^2-b^2\right)^3\sec(c+dx)}{315d} + \frac{64a\left(a^2-b^2\right)^2\sec^3(c+dx)(b+a\sin(c+}{315d}$$

$$= \frac{128ab\left(a^2-b^2\right)^3\sec(c+dx)}{315d} + \frac{64a\left(a^2-b^2\right)^2\sec^3(c+dx)(b+a\sin(c+}{315d}$$

$$= \frac{128ab\left(a^2-b^2\right)^3\sec(c+dx)}{315d} + \frac{64a\left(a^2-b^2\right)^2\sec^3(c+dx)(b+a\sin(c+}{315d}$$

Mathematica [A] time = 4.46, size = 313, normalized size = 1.33

$$\cos(c+dx)\left(\frac{a\Big(8(a-b)(1-\sin(c+dx))\Big((a-b)(1-\sin(c+dx))\Big(2(a-b)(1-\sin(c+dx))\Big((a-b)(1-\sin(c+dx))\Big(35(a+b\sin(c+dx))^4-4(a-b)(1-\sin(c+dx))\Big((a+}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Cos[c + d*x]*(-(Sec[c + d*x]^10*(a + b*Sin[c + d*x])^9) + (a*(35*(a + b*Si
n[c + d*x])^8 + 8*(a - b)*(1 - Sin[c + d*x])*(5*(a + b*Sin[c + d*x])^7 + (a
 - b)*(1 - Sin[c + d*x])*(7*(a + b*Sin[c + d*x])^6 + 2*(a - b)*(1 - Sin[c +
 d*x])*(7*(a + b*Sin[c + d*x])^5 + (a - b)*(1 - Sin[c + d*x])*(35*(a + b*Si
n[c + d*x])^4 - 4*(a - b)*(1 - Sin[c + d*x])*(5*(a + b*Sin[c + d*x])^3 + (a
```

```
 + b)*(1 + Sin[c + d*x])*(7*a^2 + 6*a*b + 2*b^2 + 6*(a^2 + 3*a*b + b^2)*Sin
[c + d*x] + (2*a^2 + 6*a*b + 7*b^2)*Sin[c + d*x]^2))))))))/(35*(1 - Sin[c +
 d*x])^5*(1 + Sin[c + d*x])^4)))/(9*(a - b)*d)
```

fricas [A] time = 0.51, size = 336, normalized size = 1.42

$$-\frac{840\,ab^7\cos(dx+c)^6 - 280\,a^7b - 1960\,a^5b^3 - 1960\,a^3b^5 - 280\,ab^7 - 504\left(7\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^4 + 360}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/315*(840*a*b^7*cos(d*x + c)^6 - 280*a^7*b - 1960*a^5*b^3 - 1960*a^3*b^5
- 280*a*b^7 - 504*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 + 360*(7*a^5*b^3 + 1
4*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 - ((128*a^8 - 448*a^6*b^2 + 560*a^4*b^4
 - 280*a^2*b^6 + 35*b^8)*cos(d*x + c)^8 + 35*a^8 + 980*a^6*b^2 + 2450*a^4*b
^4 + 980*a^2*b^6 + 35*b^8 + 4*(16*a^8 - 56*a^6*b^2 + 70*a^4*b^4 - 35*a^2*b^
6 - 35*b^8)*cos(d*x + c)^6 + 6*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 + 350*a^2*b
^6 + 35*b^8)*cos(d*x + c)^4 + 20*(2*a^8 - 7*a^6*b^2 - 175*a^4*b^4 - 133*a^2
*b^6 - 7*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

giac [B] time = 1.92, size = 892, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/315*(315*a^8*tan(1/2*d*x + 1/2*c)^17 + 2520*a^7*b*tan(1/2*d*x + 1/2*c)^1
6 - 840*a^8*tan(1/2*d*x + 1/2*c)^15 + 11760*a^6*b^2*tan(1/2*d*x + 1/2*c)^15
 + 35280*a^5*b^3*tan(1/2*d*x + 1/2*c)^14 + 4788*a^8*tan(1/2*d*x + 1/2*c)^13
 + 14112*a^6*b^2*tan(1/2*d*x + 1/2*c)^13 + 70560*a^4*b^4*tan(1/2*d*x + 1/2*
c)^13 + 23520*a^7*b*tan(1/2*d*x + 1/2*c)^12 + 58800*a^5*b^3*tan(1/2*d*x + 1
/2*c)^12 + 94080*a^3*b^5*tan(1/2*d*x + 1/2*c)^12 - 5112*a^8*tan(1/2*d*x + 1
/2*c)^11 + 79632*a^6*b^2*tan(1/2*d*x + 1/2*c)^11 + 120960*a^4*b^4*tan(1/2*d
*x + 1/2*c)^11 + 80640*a^2*b^6*tan(1/2*d*x + 1/2*c)^11 + 176400*a^5*b^3*tan
(1/2*d*x + 1/2*c)^10 + 141120*a^3*b^5*tan(1/2*d*x + 1/2*c)^10 + 40320*a*b^7
*tan(1/2*d*x + 1/2*c)^10 + 10658*a^8*tan(1/2*d*x + 1/2*c)^9 + 39872*a^6*b^2
*tan(1/2*d*x + 1/2*c)^9 + 244160*a^4*b^4*tan(1/2*d*x + 1/2*c)^9 + 89600*a^2
*b^6*tan(1/2*d*x + 1/2*c)^9 + 8960*b^8*tan(1/2*d*x + 1/2*c)^9 + 35280*a^7*b
*tan(1/2*d*x + 1/2*c)^8 + 105840*a^5*b^3*tan(1/2*d*x + 1/2*c)^8 + 197568*a^
3*b^5*tan(1/2*d*x + 1/2*c)^8 + 24192*a*b^7*tan(1/2*d*x + 1/2*c)^8 - 5112*a^
8*tan(1/2*d*x + 1/2*c)^7 + 79632*a^6*b^2*tan(1/2*d*x + 1/2*c)^7 + 120960*a^
4*b^4*tan(1/2*d*x + 1/2*c)^7 + 80640*a^2*b^6*tan(1/2*d*x + 1/2*c)^7 + 10584
0*a^5*b^3*tan(1/2*d*x + 1/2*c)^6 + 56448*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 + 1
0752*a*b^7*tan(1/2*d*x + 1/2*c)^6 + 4788*a^8*tan(1/2*d*x + 1/2*c)^5 + 14112
*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 70560*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 10
080*a^7*b*tan(1/2*d*x + 1/2*c)^4 + 15120*a^5*b^3*tan(1/2*d*x + 1/2*c)^4 + 1
6128*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 4608*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 8
40*a^8*tan(1/2*d*x + 1/2*c)^3 + 11760*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 5040
*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 4032*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 + 115
2*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 315*a^8*tan(1/2*d*x + 1/2*c) + 280*a^7*b -
 560*a^5*b^3 + 448*a^3*b^5 - 128*a*b^7)/((tan(1/2*d*x + 1/2*c)^2 - 1)^9*d)
```

maple [B] time = 0.40, size = 662, normalized size = 2.81

$$\frac{-a^8\left(-\frac{128}{315} - \frac{(\sec^8(dx+c))}{9} - \frac{8(\sec^6(dx+c))}{63} - \frac{16(\sec^4(dx+c))}{105} - \frac{64(\sec^2(dx+c))}{315}\right)\tan(dx+c) + \frac{8a^7b}{9\cos(dx+c)^9} + 28a^6b^2\left(\frac{\sin^3(dx}{9\cos(dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-a^8*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^4-
64/315*sec(d*x+c)^2)*tan(d*x+c)+8/9*a^7*b/cos(d*x+c)^9+28*a^6*b^2*(1/9*sin(
d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos
(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+56*a^5*b^3*(1/9*sin(d*x+c)^4/co
s(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1/
63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+c
)^2)*cos(d*x+c))+70*a^4*b^4*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^
5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)+56*a^3*b^5*(1/9*sin(d*x+c)^
6/cos(d*x+c)^9+1/21*sin(d*x+c)^6/cos(d*x+c)^7+1/105*sin(d*x+c)^6/cos(d*x+c)
^5-1/315*sin(d*x+c)^6/cos(d*x+c)^3+1/105*sin(d*x+c)^6/cos(d*x+c)+1/105*(8/3
+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+28*a^2*b^6*(1/9*sin(d*x+c)^7/co
s(d*x+c)^9+2/63*sin(d*x+c)^7/cos(d*x+c)^7)+8*a*b^7*(1/9*sin(d*x+c)^8/cos(d*
x+c)^9+1/63*sin(d*x+c)^8/cos(d*x+c)^7-1/315*sin(d*x+c)^8/cos(d*x+c)^5+1/315
*sin(d*x+c)^8/cos(d*x+c)^3-1/63*sin(d*x+c)^8/cos(d*x+c)-1/63*(16/5+sin(d*x+
c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+1/9*b^8*sin(d*x+c)^9/co
s(d*x+c)^9)
```

maxima [A] time = 0.33, size = 315, normalized size = 1.33

$$35\,b^8\tan\left(dx+c\right)^9+\left(35\,\tan\left(dx+c\right)^9+180\,\tan\left(dx+c\right)^7+378\,\tan\left(dx+c\right)^5+420\,\tan\left(dx+c\right)^3+315\,\tan\left(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^8*tan(d*x + c)^9 + (35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 37
8*tan(d*x + c)^5 + 420*tan(d*x + c)^3 + 315*tan(d*x + c))*a^8 + 28*(35*tan(
d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*
a^6*b^2 + 70*(35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*a^
4*b^4 + 140*(7*tan(d*x + c)^9 + 9*tan(d*x + c)^7)*a^2*b^6 - 280*(9*cos(d*x
+ c)^2 - 7)*a^5*b^3/cos(d*x + c)^9 + 56*(63*cos(d*x + c)^4 - 90*cos(d*x + c
)^2 + 35)*a^3*b^5/cos(d*x + c)^9 - 8*(105*cos(d*x + c)^6 - 189*cos(d*x + c)
^4 + 135*cos(d*x + c)^2 - 35)*a*b^7/cos(d*x + c)^9 + 280*a^7*b/cos(d*x + c)
^9)/d
```

mupad [B] time = 6.68, size = 659, normalized size = 2.79

$$\frac{(a-b)^8}{2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^8}-\frac{(a+b)^8}{9\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^9}-\frac{(a+b)^8}{2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^8}-\frac{(a-b)^8}{9\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^9}-\frac{(a+b)^7\,(3}{28\,d\left(\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^10,x)
```

```
[Out] (a - b)^8/(2*d*(tan(c/2 + (d*x)/2) + 1)^8) - (a + b)^8/(9*d*(tan(c/2 + (d*x
)/2) - 1)^9) - (a + b)^8/(2*d*(tan(c/2 + (d*x)/2) - 1)^8) - (a - b)^8/(9*d*
(tan(c/2 + (d*x)/2) + 1)^9) - ((a + b)^7*(37*a + 21*b))/(28*d*(tan(c/2 + (d
*x)/2) - 1)^7) - ((a + b)^7*(55*a + 7*b))/(24*d*(tan(c/2 + (d*x)/2) - 1)^6)
 + ((a - b)^5*(65*a*b^2 + 191*a^2*b + 187*a^3 + 5*b^3))/(128*d*(tan(c/2 + (
d*x)/2) + 1)^2) + ((a - b)^5*(67*a*b^2 - 67*a^2*b - 463*a^3 + 15*b^3))/(192
*d*(tan(c/2 + (d*x)/2) + 1)^3) + ((a - b)^6*(18*a*b + 95*a^2 - b^2))/(32*d*
(tan(c/2 + (d*x)/2) + 1)^4) + ((a - b)^6*(114*a*b - 241*a^2 + 15*b^2))/(80*
d*(tan(c/2 + (d*x)/2) + 1)^5) - ((a - b)^7*(37*a - 21*b))/(28*d*(tan(c/2 +
(d*x)/2) + 1)^7) + ((a - b)^7*(55*a - 7*b))/(24*d*(tan(c/2 + (d*x)/2) + 1)^
6) + ((a + b)^6*(18*a*b - 95*a^2 + b^2))/(32*d*(tan(c/2 + (d*x)/2) - 1)^4)
```

```
- ((a + b)^5*(65*a*b^2 - 191*a^2*b + 187*a^3 - 5*b^3))/(128*d*(tan(c/2 + (d
*x)/2) - 1)^2) + ((a + b)^5*(67*a*b^2 + 67*a^2*b - 463*a^3 - 15*b^3))/(192*
d*(tan(c/2 + (d*x)/2) - 1)^3) - ((a + b)^6*(114*a*b + 241*a^2 - 15*b^2))/(8
0*d*(tan(c/2 + (d*x)/2) - 1)^5) - (a*(a + b)^4*(20*a*b^2 - 29*a^2*b + 16*a^
3 - 5*b^3))/(16*d*(tan(c/2 + (d*x)/2) - 1)) - (a*(a - b)^4*(20*a*b^2 + 29*a
^2*b + 16*a^3 + 5*b^3))/(16*d*(tan(c/2 + (d*x)/2) + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.425 $\int \frac{\cos^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^5d}-\frac{a(a^2-2b^2)\sin(c+dx)}{b^4d}+\frac{(a^2-2b^2)\sin^2(c+dx)}{2b^3d}-\frac{a\sin^3(c+dx)}{3b^2d}+\frac{\sin^4(c+dx)}{4bd}$$

```
[Out] (a^2-b^2)^2*ln(a+b*sin(d*x+c))/b^5/d-a*(a^2-2*b^2)*sin(d*x+c)/b^4/d+1/2*(a^
2-2*b^2)*sin(d*x+c)^2/b^3/d-1/3*a*sin(d*x+c)^3/b^2/d+1/4*sin(d*x+c)^4/b/d
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{(a^2-2b^2)\sin^2(c+dx)}{2b^3d}-\frac{a(a^2-2b^2)\sin(c+dx)}{b^4d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^5d}-\frac{a\sin^3(c+dx)}{3b^2d}+\frac{\sin^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^5*d) - (a*(a^2 - 2*b^2)*Sin[c +
d*x])/(b^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^3*d) - (a*Sin[c + d*x]^
3)/(3*b^2*d) + Sin[c + d*x]^4/(4*b*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{a+x}\,dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(-a^3\left(1-\frac{2b^2}{a^2}\right)+(a^2-2b^2)x-ax^2+x^3+\frac{(a^2-b^2)^2}{a+x}\right)dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= \frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^5d}-\frac{a(a^2-2b^2)\sin(c+dx)}{b^4d}+\frac{(a^2-2b^2)\sin^2(c+dx)}{2b^3d}-\end{aligned}$$

Mathematica [A] time = 0.23, size = 103, normalized size = 0.87

$$\frac{6b^2(a^2-b^2)\sin^2(c+dx)-12ab(a^2-2b^2)\sin(c+dx)+12(a^2-b^2)^2\log(a+b\sin(c+dx))-4ab^3\sin^3(c+dx)}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b^4*Cos[c + d*x]^4 + 12*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]] - 12*a*b*(
a^2 - 2*b^2)*Sin[c + d*x] + 6*b^2*(a^2 - b^2)*Sin[c + d*x]^2 - 4*a*b^3*Sin[
c + d*x]^3)/(12*b^5*d)
```

fricas [A] time = 0.49, size = 107, normalized size = 0.91

$$\frac{3\,b^4\cos\left(dx+c\right)^4-6\left(a^2b^2-b^4\right)\cos\left(dx+c\right)^2+12\left(a^4-2\,a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)+4\left(ab^3\cos\left(dx\right.\right.}{12\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*cos(d*x + c)^4 - 6*(a^2*b^2 - b^4)*cos(d*x + c)^2 + 12*(a^4 - 2
*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a) + 4*(a*b^3*cos(d*x + c)^2 - 3*a^3*b
 + 5*a*b^3)*sin(d*x + c))/(b^5*d)
```

giac [A] time = 1.09, size = 120, normalized size = 1.02

$$\frac{\frac{3\,b^3\sin(dx+c)^4-4\,ab^2\sin(dx+c)^3+6\,a^2b\sin(dx+c)^2-12\,b^3\sin(dx+c)^2-12\,a^3\sin(dx+c)+24\,ab^2\sin(dx+c)}{b^4}+\frac{12\left(a^4-2\,a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*a^2*b*sin(d*x + c)
^2 - 12*b^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c) + 24*a*b^2*sin(d*x + c))/b
^4 + 12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/b^5)/d
```

maple [A] time = 0.15, size = 163, normalized size = 1.38

$$\frac{\sin^4\left(dx+c\right)}{4bd}-\frac{a\left(\sin^3\left(dx+c\right)\right)}{3b^2d}+\frac{\left(\sin^2\left(dx+c\right)\right)a^2}{2d\,b^3}-\frac{\sin^2\left(dx+c\right)}{bd}-\frac{a^3\sin\left(dx+c\right)}{d\,b^4}+\frac{2a\sin\left(dx+c\right)}{b^2d}+\frac{\ln\left(a+b\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4*sin(d*x+c)^4/b/d-1/3*a*sin(d*x+c)^3/b^2/d+1/2/d/b^3*sin(d*x+c)^2*a^2-si
n(d*x+c)^2/b/d-1/d/b^4*a^3*sin(d*x+c)+2*a*sin(d*x+c)/b^2/d+1/d/b^5*ln(a+b*s
in(d*x+c))*a^4-2/d/b^3*ln(a+b*sin(d*x+c))*a^2+ln(a+b*sin(d*x+c))/b/d
```

maxima [A] time = 0.31, size = 108, normalized size = 0.92

$$\frac{\frac{3\,b^3\sin(dx+c)^4-4\,ab^2\sin(dx+c)^3+6\left(a^2b-2\,b^3\right)\sin(dx+c)^2-12\left(a^3-2\,ab^2\right)\sin(dx+c)}{b^4}+\frac{12\left(a^4-2\,a^2b^2+b^4\right)\log(b\sin(dx+c)+a)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*(a^2*b - 2*b^3)*si
n(d*x + c)^2 - 12*(a^3 - 2*a*b^2)*sin(d*x + c))/b^4 + 12*(a^4 - 2*a^2*b^2 +
 b^4)*log(b*sin(d*x + c) + a)/b^5)/d
```

mupad [B] time = 5.07, size = 109, normalized size = 0.92

$$\frac{\frac{\sin(c+d\,x)^4}{4\,b}-\sin\left(c+d\,x\right)^2\left(\frac{1}{b}-\frac{a^2}{2\,b^3}\right)+\frac{\ln(a+b\,\sin(c+d\,x))\left(a^4-2\,a^2\,b^2+b^4\right)}{b^5}-\frac{a\sin(c+d\,x)^3}{3\,b^2}+\frac{a\,\sin(c+d\,x)\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^4/(4*b) - sin(c + d*x)^2*(1/b - a^2/(2*b^3)) + (log(a + b*sin
(c + d*x))*(a^4 + b^4 - 2*a^2*b^2))/b^5 - (a*sin(c + d*x)^3)/(3*b^2) + (a*s
in(c + d*x)*(2/b - a^2/b^3))/b)/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.426 $\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=61

$$-\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] -(a^2-b^2)*ln(a+b*sin(d*x+c))/b^3/d+a*sin(d*x+c)/b^2/d-1/2*sin(d*x+c)^2/b/d
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^3*d)) + (a*Sin[c + d*x])/(b^2*d)
 - Sin[c + d*x]^2/(2*b*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= -\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}\end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 0.89

$$\frac{-(a^2-b^2)\log(a+b\sin(c+dx))+ab\sin(c+dx)-\frac{1}{2}b^2\sin^2(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((a^2 - b^2)*Log[a + b*Sin[c + d*x]]) + a*b*Sin[c + d*x] - (b^2*Sin[c + d
*x]^2)/2)/(b^3*d)
```

fricas [A] time = 0.47, size = 53, normalized size = 0.87

$$\frac{b^2\cos{(dx+c)}^2+2\,ab\sin{(dx+c)}-2\left(a^2-b^2\right)\log{(b\sin{(dx+c)}+a)}}{2\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) - 2*(a^2 - b^2)*log(b*sin(d*x
+ c) + a))/(b^3*d)
```

giac [A] time = 0.80, size = 56, normalized size = 0.92

$$-\frac{\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}+\frac{2\left(a^2-b^2\right)\log(|b\sin(dx+c)+a|)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(abs(b*s
in(d*x + c) + a))/b^3)/d
```

maple [A] time = 0.14, size = 72, normalized size = 1.18

$$-\frac{\sin^2{(dx+c)}}{2bd}+\frac{a\sin{(dx+c)}}{b^2d}-\frac{\ln{(a+b\sin{(dx+c)})}\,a^2}{d\,b^3}+\frac{\ln{(a+b\sin{(dx+c)})}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2*sin(d*x+c)^2/b/d+a*sin(d*x+c)/b^2/d-1/d/b^3*ln(a+b*sin(d*x+c))*a^2+ln(
a+b*sin(d*x+c))/b/d
```

maxima [A] time = 0.31, size = 55, normalized size = 0.90

$$-\frac{\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}+\frac{2\left(a^2-b^2\right)\log(b\sin(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(b*sin(d
*x + c) + a)/b^3)/d
```

mupad [B] time = 0.08, size = 55, normalized size = 0.90

$$-\frac{\frac{\sin(c+d\,x)^2}{2\,b}+\frac{\ln(a+b\,\sin(c+d\,x))\left(a^2-b^2\right)}{b^3}-\frac{a\,\sin(c+d\,x)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^2/(2*b) + (log(a + b*sin(c + d*x))*(a^2 - b^2))/b^3 - (a*sin
(c + d*x))/b^2)/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.427 $\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

```
[Out] ln(a+b*sin(d*x+c))/b/d
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2668, 31}

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\sin(c+dx))}{bd}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

fricas [A] time = 0.47, size = 18, normalized size = 1.00

$$\frac{\log\left(b\sin\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] log(b*sin(d*x + c) + a)/(b*d)
```

giac [A] time = 0.37, size = 19, normalized size = 1.06

$$\frac{\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*sin(d*x + c) + a))/(b*d)
```

maple [A] time = 0.08, size = 19, normalized size = 1.06

$$\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] ln(a+b*sin(d*x+c))/b/d
```

maxima [A] time = 0.33, size = 18, normalized size = 1.00

$$\frac{\log\left(b\sin\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(b*sin(d*x + c) + a)/(b*d)
```

mupad [B] time = 5.09, size = 18, normalized size = 1.00

$$\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x)),x)
```

```
[Out] log(a + b*sin(c + d*x))/(b*d)
```

sympy [A] time = 1.06, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x\cos\left(c\right)}{a} & \text{for}\: b=0 \wedge d=0 \\ \frac{\sin\left(c+dx\right)}{ad} & \text{for}\: b=0 \\ \frac{x\cos\left(c\right)}{a+b\sin\left(c\right)} & \text{for}\: d=0 \\ \frac{\log\left(\frac{a}{b}+\sin\left(c+dx\right)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a*d), Eq(b, 0))
, (x*cos(c)/(a + b*sin(c)), Eq(d, 0)), (log(a/b + sin(c + d*x))/(b*d), True
))
```

3.428 $\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d+1/2*ln(1+sin(d*x+c))/(a-b)/d-b*ln(a+b*sin(d*x
+c))/(a^2-b^2)/d
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2668, 706, 31, 633}

$$-\frac{b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
- (b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{b\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d} - \frac{b\,\text{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{b\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)d} - \frac{\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2(a-b)d} + \frac{\text{Subst}\left(\int \frac{1}{b-x}\,dx, x\right.}{2(a+}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.85

$$\frac{(b-a)\log(1-\sin(c+dx)) + (a+b)\log(\sin(c+dx)+1) - 2b\log(a+b\sin(c+dx))}{2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a + b)*Log[1 - Sin[c + d*x]] + (a + b)*Log[1 + Sin[c + d*x]] - 2*b*Log[a
 + b*Sin[c + d*x]])/(2*(a - b)*(a + b)*d)
```

fricas [A] time = 0.47, size = 62, normalized size = 0.83

$$-\frac{2\,b\log\left(b\sin\left(dx+c\right)+a\right) - (a+b)\log\left(\sin\left(dx+c\right)+1\right) + (a-b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) + (a - b)
*log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

giac [A] time = 0.41, size = 71, normalized size = 0.95

$$-\frac{\frac{2\,b^2\log(|b\sin(dx+c)+a|)}{a^2b-b^3} - \frac{\log(|\sin(dx+c)+1|)}{a-b} + \frac{\log(|\sin(dx+c)-1|)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x +
c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b))/d
```

maple [A] time = 0.15, size = 76, normalized size = 1.01

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{d\left(2a+2b\right)} - \frac{b\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)\left(a-b\right)} + \frac{\ln\left(1+\sin\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(2*a+2*b)*ln(sin(d*x+c)-1)-1/d*b/(a+b)/(a-b)*ln(a+b*sin(d*x+c))+1/d/(2
*a-2*b)*ln(1+sin(d*x+c))
```

maxima [A] time = 0.33, size = 64, normalized size = 0.85

$$-\frac{\frac{2\,b\log\left(b\sin\left(dx+c\right)+a\right)}{a^2-b^2}-\frac{\log\left(\sin\left(dx+c\right)+1\right)}{a-b}+\frac{\log\left(\sin\left(dx+c\right)-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a -
b) + log(sin(d*x + c) - 1)/(a + b))/d
```

mupad [B] time = 5.13, size = 69, normalized size = 0.92

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)}-\frac{b\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(sin(c + d*x) + 1)/(2*d*(a - b)) - log(sin(c + d*x) - 1)/(2*d*(a + b)) -
 (b*log(a + b*sin(c + d*x)))/(d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.429 $\int \frac{\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=123

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{b^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{(a+2b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(a-2b)\log(\sin(c}{4d(a-b)}$$

```
[Out] -1/4*(a+2*b)*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(a-2*b)*ln(1+sin(d*x+c))/(a-b)^
2/d+b^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d-1/2*sec(d*x+c)^2*(b-a*sin(d*x+c))/
d/(a^2-b^2)
```

Rubi [A] time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 741, 801}

$$\frac{b^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)}-\frac{(a+2b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(a-2b)\log(\sin(c+}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a + 2*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((a - 2*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^2*d) + (b^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*
d) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{b\,\text{Subst}\left(\int \frac{a^2-2b^2+ax}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{b\,\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right.\right.}{2\left(a^2-b^2\right)d}$$

$$= -\frac{(a+2b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-2b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b^3\log(a+b\sin(c+d}{\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 0.58, size = 170, normalized size = 1.38

$$\frac{\frac{4b^3\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2} + \frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{1}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{2(a+2b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.}{(a+b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(a + 2*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^2 + (2*(a
- 2*b)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^2 + (4*b^3*Log[a +
 b*Sin[c + d*x]])/(a^2 - b^2)^2 + 1/((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2) - 1/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(4*d)
```

fricas [A] time = 0.58, size = 153, normalized size = 1.24

$$\frac{4\,b^3\cos(dx+c)^2\log(b\sin(dx+c)+a) + \left(a^3-3\,ab^2-2\,b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1) - \left(a^3-3\,ab^2+\right.}{4\left(a^4-2\,a^2b^2+b^4\right)d\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*cos(d*x + c)^2*log(b*sin(d*x + c) + a) + (a^3 - 3*a*b^2 - 2*b^3)
*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^3 - 3*a*b^2 + 2*b^3)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c)
)/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

giac [A] time = 0.43, size = 177, normalized size = 1.44

$$\frac{\frac{4\,b^4\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5} + \frac{(a-2\,b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2} - \frac{(a+2\,b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2} + \frac{2\left(b^3\sin(dx+c)^2-a^3\sin(dx+c)+ab^2\sin(dx+c)+a^2b-2\,b^3\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(4*b^4*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) + (a - 2*
b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (a + 2*b)*log(abs(sin(d
*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(b^3*sin(d*x + c)^2 - a^3*sin(d*x + c
```

```
) + a*b^2*sin(d*x + c) + a^2*b - 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x +
 c)^2 - 1)))/d
```

maple [A] time = 0.18, size = 164, normalized size = 1.33

$$-\frac{1}{d\left(4a+4b\right)\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{4d\left(a+b\right)^{2}}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)b}{2d\left(a+b\right)^{2}}+\frac{b^{3}\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)^{2}\left(a-b\right)^{2}}-\frac{}{d\left(4a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)-1/4/d/(a+b)^2*ln(sin(d*x+c)-1)*a-1/2/d/(a+b)^
2*ln(sin(d*x+c)-1)*b+1/d*b^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))-1/d/(4*a-4*
b)/(1+sin(d*x+c))+1/4/d/(a-b)^2*ln(1+sin(d*x+c))*a-1/2/d/(a-b)^2*ln(1+sin(d
*x+c))*b
```

maxima [A] time = 0.33, size = 139, normalized size = 1.13

$$\frac{\frac{4\,b^{3}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{4}-2\,a^{2}b^{2}+b^{4}}+\frac{\left(a-2\,b\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{2}-2\,ab+b^{2}}-\frac{\left(a+2\,b\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{2}+2\,ab+b^{2}}-\frac{2\,\left(a\sin\left(dx+c\right)-b\right)}{\left(a^{2}-b^{2}\right)\sin\left(dx+c\right)^{2}-a^{2}+b^{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^3*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) + (a - 2*b)*log(
sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (a + 2*b)*log(sin(d*x + c) - 1)/(a^
2 + 2*a*b + b^2) - 2*(a*sin(d*x + c) - b)/((a^2 - b^2)*sin(d*x + c)^2 - a^2
 + b^2))/d
```

mupad [B] time = 5.39, size = 148, normalized size = 1.20

$$\frac{\frac{b}{2\left(a^{2}-b^{2}\right)}-\frac{a\,\sin\left(c+d\,x\right)}{2\left(a^{2}-b^{2}\right)}}{d\,\left(\sin\left(c+d\,x\right)^{2}-1\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{b}{4\,\left(a+b\right)^{2}}+\frac{1}{4\,\left(a+b\right)}\right)}{d}+\frac{b^{3}\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^{4}-2\,a^{2}\,b^{2}+b^{4}\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right.}{4\,d\,\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b/(2*(a^2 - b^2)) - (a*sin(c + d*x))/(2*(a^2 - b^2)))/(d*(sin(c + d*x)^2 -
 1)) - (log(sin(c + d*x) - 1)*(b/(4*(a + b)^2) + 1/(4*(a + b))))/d + (b^3*l
og(a + b*sin(c + d*x)))/(d*(a^4 + b^4 - 2*a^2*b^2)) + (log(sin(c + d*x) + 1
)*(a - 2*b))/(4*d*(a - b)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.430 $\int \frac{\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=195

$$-\frac{(3a^2+9ab+8b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{(3a^2-9ab+8b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx)}{4d(a^2-b^2)}$$

```
[Out] -1/16*(3*a^2+9*a*b+8*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*(3*a^2-9*a*b+8*b^
2)*ln(1+sin(d*x+c))/(a-b)^3/d-b^5*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/4*sec(
d*x+c)^4*(b-a*sin(d*x+c))/d/(a^2-b^2)+1/8*sec(d*x+c)^2*(4*b^3+a*(3*a^2-7*b^
2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.25, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2668, 741, 823, 801}

$$-\frac{b^5\log(a+b\sin(c+dx))}{d(a^2-b^2)^3}-\frac{(3a^2+9ab+8b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{(3a^2-9ab+8b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\sec$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((3*a^2
 - 9*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (b^5*Log[a + b*
Sin[c + d*x]])/((a^2 - b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4
*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(4*b^3 + a*(3*a^2 - 7*b^2)*Sin[c + d*x]))
/(8*(a^2 - b^2)^2*d)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{b^3\,\mathrm{Subst}\left(\int \frac{3a^2-4b^2+3ax}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\left(4b^3+a\left(3a^2-7b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} -$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\left(4b^3+a\left(3a^2-7b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} -$$

$$= -\frac{\left(3a^2+9ab+8b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3 d} + \frac{\left(3a^2-9ab+8b^2\right)\log(1+\sin(c+dx))}{16(a-b)^3 d} -$$

Mathematica [A] time = 0.95, size = 266, normalized size = 1.36

$$-\frac{2\left(3a^2+9ab+8b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^3} + \frac{2\left(3a^2-9ab+8b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^3} + \frac{16b^5\log(a+b\sin(c+dx))}{\left(b^2-a^2\right)^3} + \frac{\quad}{(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(3*a^2 + 9*a*b + 8*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a +
 b)^3 + (2*(3*a^2 - 9*a*b + 8*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
)/(a - b)^3 + (16*b^5*Log[a + b*Sin[c + d*x]])/(-a^2 + b^2)^3 + 1/((a + b)*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + (3*a + 5*b)/((a + b)^2*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])^2) - 1/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^4) + (-3*a + 5*b)/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
2))/(16*d)
```

fricas [A] time = 0.64, size = 253, normalized size = 1.30

$$-\frac{16\,b^5\cos(dx+c)^4\log(b\sin(dx+c)+a) - \left(3\,a^5-10\,a^3b^2+15\,ab^4+8\,b^5\right)\cos(dx+c)^4\log(\sin(dx+c)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*b^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (3*a^5 - 10*a^3*b^2
+ 15*a*b^4 + 8*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (3*a^5 - 10*a^3*
b^2 + 15*a*b^4 - 8*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^4*b - 8
*a^2*b^3 + 4*b^5 - 8*(a^2*b^3 - b^5)*cos(d*x + c)^2 - 2*(2*a^5 - 4*a^3*b^2
```

```
+ 2*a*b^4 + (3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

giac [A] time = 0.47, size = 332, normalized size = 1.70

$$-\frac{\frac{16\,b^6\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(3\,a^2-9\,ab+8\,b^2\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,a^2+9\,ab+8\,b^2\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,b^5\sin(dx+c)^4+3\,a^5\sin(dx+c)^3-10\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*b^6*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7) - (3*a^2 - 9*a*b + 8*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b +
 3*a*b^2 - b^3) + (3*a^2 + 9*a*b + 8*b^2)*log(abs(sin(d*x + c) - 1))/(a^3 +
 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*b^5*sin(d*x + c)^4 + 3*a^5*sin(d*x + c)^3
- 10*a^3*b^2*sin(d*x + c)^3 + 7*a*b^4*sin(d*x + c)^3 + 4*a^2*b^3*sin(d*x +
c)^2 - 16*b^5*sin(d*x + c)^2 - 5*a^5*sin(d*x + c) + 14*a^3*b^2*sin(d*x + c)
 - 9*a*b^4*sin(d*x + c) + 2*a^4*b - 8*a^2*b^3 + 12*b^5)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.18, size = 305, normalized size = 1.56

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{3a}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{5b}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{3\ln\left(\sin\left(dx+c\right)\right.}{16d\left(a+b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2-3/16/d/(a+b)^2/(sin(d*x+c)-1)*a-5/16/d/(a+
b)^2/(sin(d*x+c)-1)*b-3/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-9/16/d/(a+b)^3*ln
(sin(d*x+c)-1)*a*b-1/2/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2-1/d*b^5/(a+b)^3/(a-b)
^3*ln(a+b*sin(d*x+c))-1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2-3/16/d/(a-b)^2/(1+si
n(d*x+c))*a+5/16/d/(a-b)^2/(1+sin(d*x+c))*b+3/16/d/(a-b)^3*ln(1+sin(d*x+c))
*a^2-9/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b+1/2/d/(a-b)^3*ln(1+sin(d*x+c))*b^2
```

maxima [A] time = 0.34, size = 278, normalized size = 1.43

$$-\frac{\frac{16\,b^5\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(3\,a^2-9\,ab+8\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,a^2+9\,ab+8\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(4\,b^3\sin(dx+c)^2+\left(3\,a^3-7\,ab^2\right)\sin(dx+c\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2\,a^2b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*b^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
 (3*a^2 - 9*a*b + 8*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b
^3) + (3*a^2 + 9*a*b + 8*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^
2 + b^3) + 2*(4*b^3*sin(d*x + c)^2 + (3*a^3 - 7*a*b^2)*sin(d*x + c)^3 + 2*a
^2*b - 6*b^3 - (5*a^3 - 9*a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin
(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c
)^2))/d
```

mupad [B] time = 0.59, size = 322, normalized size = 1.65

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{b^2}{8\,{\left(a-b\right)}^3}-\frac{3\,b}{16\,{\left(a-b\right)}^2}+\frac{3}{16\,\left(a-b\right)}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{3\,b}{16\,{\left(a+b\right)}^2}+\frac{3}{16\,\left(a+b\right)}+\frac{b^2}{8\,{\left(a+b\right)}^3}\right)}{d}-\frac{\frac{a^2\,b-3\,b^3}{4\left(a^4-2\,a^2\,b^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(b^2/(8*(a - b)^3) - (3*b)/(16*(a - b)^2) + 3/(16*(a
 - b))))/d - (log(sin(c + d*x) - 1)*((3*b)/(16*(a + b)^2) + 3/(16*(a + b))
+ b^2/(8*(a + b)^3)))/d - ((a^2*b - 3*b^3)/(4*(a^4 + b^4 - 2*a^2*b^2)) + (b
^3*sin(c + d*x)^2)/(2*(a^4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^3*(7*a*b^2 -
 3*a^3))/(8*(a^4 + b^4 - 2*a^2*b^2)) + (sin(c + d*x)*(9*a*b^2 - 5*a^3))/(8*
(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c + d*x)^2 - sin(c + d*x)^2 + sin(c + d*x
)^4)) - (b^5*log(a + b*sin(c + d*x)))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

3.431 $\int \frac{\cos^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=188

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{\cos(c+dx)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\sin(c+dx)\right)}{8b^5d}-\frac{\cos^3(c+dx)\left(4\left(\right.\right.}{}$$

```
[Out] 1/8*a*(8*a^4-20*a^2*b^2+15*b^4)*x/b^6-2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2
*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^6/d+1/5*cos(d*x+c)^5/b/d-1/12*cos(d*x+c)^3*
(4*a^2-4*b^2-3*a*b*sin(d*x+c))/b^3/d+1/8*cos(d*x+c)*(8*(a^2-b^2)^2-a*b*(4*a
^2-7*b^2)*sin(d*x+c))/b^5/d
```

Rubi [A] time = 0.46, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2695, 2865, 2735, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{12b^3d}+\frac{\cos(c+dx)\left(8\left(a^2-b^2\right)^2-\right.}{8b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x)/(8*b^6) - (2*(a^2 - b^2)^(5/2)*ArcTan[(
b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + Cos[c + d*x]^5/(5*b*d)
- (Cos[c + d*x]^3*(4*(a^2 - b^2) - 3*a*b*Sin[c + d*x]))/(12*b^3*d) + (Cos[c
 + d*x]*(8*(a^2 - b^2)^2 - a*b*(4*a^2 - 7*b^2)*Sin[c + d*x]))/(8*b^5*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos^5(c+dx)}{5bd} + \frac{\int \frac{\cos^4(c+dx)(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{12b^3d} + \frac{\int \frac{\cos^2(c+dx)\left(-b\left(a^2-4b^2\right)-\right.}{a+b\sin(}}{4b}$$

$$= \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{12b^3d} + \frac{\cos(c+dx)\left(8\left(a^2-b\right.\right.}{}$$

$$= \frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+\right.}{12b^3d}$$

$$= \frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+\right.}{12b^3d}$$

$$= \frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+\right.}{12b^3d}$$

$$= \frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} - \frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} + \frac{\cos^5(c+dx)}{5bd} -$$

Mathematica [B] time = 6.31, size = 2827, normalized size = 15.04

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^5*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/
2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((5/(16*(1 + ((a - b)*(-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(8*(1 + ((a - b)*(-(b/(a - b))
 - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*
Sin[c + d*x])/(a - b)))/(2*b))^(-1))/2 - (15*b^3*(((a - b)*(-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/
(a - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a
```

```
 - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Si
n[c + d*x])/(a - b)))/(2*b)])))/(64*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b))^3*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*
b))^3)))/(5*(a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))
/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*S
in[c + d*x])/(a - b))^(3/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 +
 ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((3*(5/(8
*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(6*
(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + (
(a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/8 + (15*b^
2*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[a
- b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(S
qrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sq
rt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(64*(a
 - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a -
 b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(3*(a + b)^2*Sqrt[((a + b)*(b
/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b
))*((8*Sqrt[2]*b*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a +
b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x
])/(a - b)))/(2*b))^(7/2)*((5*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(8*Sqrt[2]*Sqrt[a - b]*S
qrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)) + (15/(8*(1 + ((a - b)*(-(b/(a - b
)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(4*(1 + ((a - b)*(-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b
*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/6))/((a + b)^2*Sqrt[((a + b)*(b/(a +
b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*(-(
((-((a*b)/(a + b)) - b^2/(a + b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*((2*
Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a -
b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[a +
 b]) - (2*Sqrt[-((a*b)/(a + b)) - b^2/(a + b)]*ArcTanh[(Sqrt[-((a*b)/(a + b
)) - b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[-((a
*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(
b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)])))/b) + (2*Sqrt[2]*(a - b)*Sqrt[-(b/
(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a +
 b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)*
((Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)
])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c +
d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*
b))^(3/2)) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/
(2*b)))))/(b*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/
b])))/b) + (4*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]
*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (
b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqr
t[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(4*Sqrt[2]*
Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(
b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)
*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/
(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/4))/((a + b)^2*Sqrt[((a
+ b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b])))/b))/b))/b))/(d*(1 - (a +
 b*Sin[c + d*x])/(a - b))^(5/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/2))
```

fricas [A] time = 0.50, size = 483, normalized size = 2.57

$$\left[\frac{24\,b^5\cos\left(dx+c\right)^5 - 40\left(a^2b^3-b^5\right)\cos\left(dx+c\right)^3 + 15\left(8\,a^5-20\,a^3b^2+15\,ab^4\right)dx + 60\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/120*(24*b^5*cos(d*x + c)^5 - 40*(a^2*b^3 - b^5)*cos(d*x + c)^3 + 15*(8*a
^5 - 20*a^3*b^2 + 15*a*b^4)*d*x + 60*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^
2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 120*(a^4*b - 2*a^2*b^3 + b^5)
*cos(d*x + c) + 15*(2*a*b^4*cos(d*x + c)^3 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x
+ c))*sin(d*x + c))/(b^6*d), 1/120*(24*b^5*cos(d*x + c)^5 - 40*(a^2*b^3 - b
^5)*cos(d*x + c)^3 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*d*x + 120*(a^4 - 2*
a^2*b^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))) + 120*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c) + 15*(2*a*b^4
*cos(d*x + c)^3 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d)
]
```

giac [B] time = 0.49, size = 496, normalized size = 2.64

$$\frac{15\left(8a^5-20a^3b^2+15ab^4\right)(dx+c)}{b^6} - \frac{240\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6} + \frac{2\left(60a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-135\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*(d*x + c)/b^6 - 240*(a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(60*
a^3*b*tan(1/2*d*x + 1/2*c)^9 - 135*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*t
an(1/2*d*x + 1/2*c)^8 - 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 360*b^4*tan(1/
2*d*x + 1/2*c)^8 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 150*a*b^3*tan(1/2*d*x
 + 1/2*c)^7 + 480*a^4*tan(1/2*d*x + 1/2*c)^6 - 1200*a^2*b^2*tan(1/2*d*x + 1
/2*c)^6 + 720*b^4*tan(1/2*d*x + 1/2*c)^6 + 720*a^4*tan(1/2*d*x + 1/2*c)^4 -
 1600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 1120*b^4*tan(1/2*d*x + 1/2*c)^4 - 12
0*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 150*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*a^4
*tan(1/2*d*x + 1/2*c)^2 - 1040*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 560*b^4*tan
(1/2*d*x + 1/2*c)^2 - 60*a^3*b*tan(1/2*d*x + 1/2*c) + 135*a*b^3*tan(1/2*d*x
 + 1/2*c) + 120*a^4 - 280*a^2*b^2 + 184*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^
5*b^5))/d
```

maple [B] time = 0.16, size = 1055, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] 15/4/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))+2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+12/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)^4*a^4-80/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d
*x+1/2*c)^4*a^2-2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a^3
+5/2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a+8/d/b^5/(1+tan
(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^4-52/3/d/b^3/(1+tan(1/2*d*x+1/2
*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^2-1/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/
2*d*x+1/2*c)*a^3+9/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a+
1/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9*a^3-9/4/d/b^2/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9*a+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^
```

```
2)^5*tan(1/2*d*x+1/2*c)^8*a^4+8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*
x+1/2*c)^6*a^4-6/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^2+
2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a^3-5/2/d/b^2/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a-20/d/b^3/(1+tan(1/2*d*x+1/2*c)
^2)^5*tan(1/2*d*x+1/2*c)^6*a^2-2/d/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^6+6/d/b^4/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^4+12/d/b/(1+tan(1/2*d*x+1/
2*c)^2)^5*tan(1/2*d*x+1/2*c)^6+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*a^4+2/d/b
^6*arctan(tan(1/2*d*x+1/2*c))*a^5+28/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1
/2*d*x+1/2*c)^2-5/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3-14/3/d/b^3/(1+tan(1/
2*d*x+1/2*c)^2)^5*a^2+56/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c
)^4-6/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))*a^2+46/15/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5+6/d/b/(1+tan(1/2*d*x+1/2
*c)^2)^5*tan(1/2*d*x+1/2*c)^8
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.66, size = 3075, normalized size = 16.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x)),x)
```

```
[Out] ((2*(15*a^4 + 23*b^4 - 35*a^2*b^2))/(15*b^5) + (tan(c/2 + (d*x)/2)^3*(5*a*b
^2 - 4*a^3))/(2*b^4) - (tan(c/2 + (d*x)/2)^7*(5*a*b^2 - 4*a^3))/(2*b^4) - (
tan(c/2 + (d*x)/2)^9*(9*a*b^2 - 4*a^3))/(4*b^4) + (2*tan(c/2 + (d*x)/2)^8*(
a^4 + 3*b^4 - 3*a^2*b^2))/b^5 + (4*tan(c/2 + (d*x)/2)^6*(2*a^4 + 3*b^4 - 5*
a^2*b^2))/b^5 + (4*tan(c/2 + (d*x)/2)^2*(6*a^4 + 7*b^4 - 13*a^2*b^2))/(3*b^
5) + (4*tan(c/2 + (d*x)/2)^4*(9*a^4 + 14*b^4 - 20*a^2*b^2))/(3*b^5) + (tan(
c/2 + (d*x)/2)*(9*a*b^2 - 4*a^3))/(4*b^4))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*
tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + t
an(c/2 + (d*x)/2)^10 + 1)) + (atan((((-(a + b)^5*(a - b)^5)^(1/2)*(((225*a^
4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9 - 160*a^10*b^7 + 32*a^12*b^5)/b^14 -
 (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*a^3*b^15 + 2385*a^5*b^13 - 3160*a^7*b
^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) + ((-(a + b)^5*
(a - b)^5)^(1/2)*((28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 + ((-(a +
b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a
^3*b^17))/(2*b^15)))/b^6 - (tan(c/2 + (d*x)/2)*(128*a*b^18 - 384*a^3*b^16 +
 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15)))/b^6)*1i)/b^6 + ((-(a + b)^5*(a -
b)^5)^(1/2)*(((225*a^4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9 - 160*a^10*b^7
+ 32*a^12*b^5)/b^14 - (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*a^3*b^15 + 2385*
a^5*b^13 - 3160*a^7*b^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 128*a^13*b^5))/(2*
b^15) + ((-(a + b)^5*(a - b)^5)^(1/2)*(((-(a + b)^5*(a - b)^5)^(1/2)*(32*a^
2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^15)))/b^6 - (
28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 + (tan(c/2 + (d*x)/2)*(128*a*
b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15)))/b^6)*1i)/b^6
)/((32*a^16 - 120*a^2*b^14 + 655*a^4*b^12 - 1549*a^6*b^10 + 2069*a^8*b^8 -
1695*a^10*b^6 + 856*a^12*b^4 - 248*a^14*b^2)/b^14 + ((-(a + b)^5*(a - b)^5)
^(1/2)*(((225*a^4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9 - 160*a^10*b^7 + 32*
```

```
a^12*b^5)/b^14 - (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*a^3*b^15 + 2385*a^5*b
^13 - 3160*a^7*b^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15)
 + ((-(a + b)^5*(a - b)^5)^(1/2)*((28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)
/b^14 + ((-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(19
2*a*b^19 - 128*a^3*b^17))/(2*b^15)))/b^6 - (tan(c/2 + (d*x)/2)*(128*a*b^18
- 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15)))/b^6))/b^6 - ((-(a
 + b)^5*(a - b)^5)^(1/2)*(((225*a^4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9 -
160*a^10*b^7 + 32*a^12*b^5)/b^14 - (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*a^3
*b^15 + 2385*a^5*b^13 - 3160*a^7*b^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 128*a
^13*b^5))/(2*b^15) + ((-(a + b)^5*(a - b)^5)^(1/2)*(((-(a + b)^5*(a - b)^5)
^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^
15)))/b^6 - (28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 + (tan(c/2 + (d*
x)/2)*(128*a*b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15)))
/b^6))/b^6 + (tan(c/2 + (d*x)/2)*(128*a^17 - 450*a^3*b^14 + 2550*a^5*b^12 -
 6230*a^7*b^10 + 8530*a^9*b^8 - 7088*a^11*b^6 + 3584*a^13*b^4 - 1024*a^15*b
^2))/b^15))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(b^6*d) + (a*atan(((a*(((225*a
^4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9 - 160*a^10*b^7 + 32*a^12*b^5)/b^14
- (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*a^3*b^15 + 2385*a^5*b^13 - 3160*a^7*
b^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) + (a*(8*a^4 +
15*b^4 - 20*a^2*b^2)*((28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 - (tan
(c/2 + (d*x)/2)*(128*a*b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/
(2*b^15) + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17)
)/(2*b^15))*(8*a^4 + 15*b^4 - 20*a^2*b^2)*1i)/(8*b^6))*1i)/(8*b^6))*(8*a^4
+ 15*b^4 - 20*a^2*b^2))/(8*b^6) + (a*(((225*a^4*b^13)/2 - 300*a^6*b^11 + 32
0*a^8*b^9 - 160*a^10*b^7 + 32*a^12*b^5)/b^14 - (tan(c/2 + (d*x)/2)*(64*a*b^
17 - 834*a^3*b^15 + 2385*a^5*b^13 - 3160*a^7*b^11 + 2240*a^9*b^9 - 832*a^11
*b^7 + 128*a^13*b^5))/(2*b^15) + (a*(8*a^4 + 15*b^4 - 20*a^2*b^2)*((tan(c/2
 + (d*x)/2)*(128*a*b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b
^15) - (28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 + (a*(32*a^2*b^3 + (t
an(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^15))*(8*a^4 + 15*b^4 -
20*a^2*b^2)*1i)/(8*b^6))*1i)/(8*b^6))*(8*a^4 + 15*b^4 - 20*a^2*b^2))/(8*b^6
))/((32*a^16 - 120*a^2*b^14 + 655*a^4*b^12 - 1549*a^6*b^10 + 2069*a^8*b^8 -
 1695*a^10*b^6 + 856*a^12*b^4 - 248*a^14*b^2)/b^14 + (tan(c/2 + (d*x)/2)*(1
28*a^17 - 450*a^3*b^14 + 2550*a^5*b^12 - 6230*a^7*b^10 + 8530*a^9*b^8 - 708
8*a^11*b^6 + 3584*a^13*b^4 - 1024*a^15*b^2))/b^15 + (a*(((225*a^4*b^13)/2 -
 300*a^6*b^11 + 320*a^8*b^9 - 160*a^10*b^7 + 32*a^12*b^5)/b^14 - (tan(c/2 +
 (d*x)/2)*(64*a*b^17 - 834*a^3*b^15 + 2385*a^5*b^13 - 3160*a^7*b^11 + 2240*
a^9*b^9 - 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) + (a*(8*a^4 + 15*b^4 - 20*
a^2*b^2)*((28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 - (tan(c/2 + (d*x)
/2)*(128*a*b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15) + (
a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^15))*
(8*a^4 + 15*b^4 - 20*a^2*b^2)*1i)/(8*b^6))*1i)/(8*b^6))*(8*a^4 + 15*b^4 - 2
0*a^2*b^2)*1i)/(8*b^6) - (a*(((225*a^4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9
 - 160*a^10*b^7 + 32*a^12*b^5)/b^14 - (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*
a^3*b^15 + 2385*a^5*b^13 - 3160*a^7*b^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 12
8*a^13*b^5))/(2*b^15) + (a*(8*a^4 + 15*b^4 - 20*a^2*b^2)*((tan(c/2 + (d*x)/
2)*(128*a*b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15) - (2
8*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 + (a*(32*a^2*b^3 + (tan(c/2 +
(d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^15))*(8*a^4 + 15*b^4 - 20*a^2*b^
2)*1i)/(8*b^6))*1i)/(8*b^6))*(8*a^4 + 15*b^4 - 20*a^2*b^2)*1i)/(8*b^6)))*(8
*a^4 + 15*b^4 - 20*a^2*b^2))/(4*b^6*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.432 $\int \frac{\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}-\frac{ax\left(2a^2-3b^2\right)}{2b^4}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d}+\frac{\cos^3(c+dx)}{3bd}$$

```
[Out] -1/2*a*(2*a^2-3*b^2)*x/b^4+2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c)
)/(a^2-b^2)^(1/2))/b^4/d+1/3*cos(d*x+c)^3/b/d-1/2*cos(d*x+c)*(2*a^2-2*b^2-a
*b*sin(d*x+c))/b^3/d
```

Rubi [A] time = 0.25, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2695, 2865, 2735, 2660, 618, 204}

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d}-\frac{ax\left(2a^2-3b^2\right)}{2b^4}+\frac{\cos^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*(2*a^2 - 3*b^2)*x)/(2*b^4) + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + Cos[c + d*x]^3/(3*b*d) - (Cos[c + d*
x]*(2*(a^2 - b^2) - a*b*Sin[c + d*x]))/(2*b^3*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3bd} + \frac{\int \frac{\cos^2(c+dx)(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d} + \frac{\int \frac{-b\left(a^2-2b^2\right)-a\left(2a^2-3b^2\right)\sin}{a+b\sin(c+dx)}}{2b^3}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d} + \frac{\left(a^2\right.}{}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d} + \frac{\left(2\left(\right.\right.}{}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d} - \frac{\left(4\left(\right.\right.}{}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+d}{}$$

Mathematica [B] time = 4.51, size = 428, normalized size = 3.37

$$\frac{\cos^3(c+dx)\left(\sqrt{a+b}\left(\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\left(\sqrt{a-b}\sqrt{1-\sin(c+dx)}\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}\left(6a^2-3ab\sin(c+dx)+2b^2\mathrm{s}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^3*(12*(a - b)^2*(a + b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + S
in[c + d*x]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b
))])]*Sqrt[1 - Sin[c + d*x]] + Sqrt[a + b]*(-12*Sqrt[a - b]*(a^2 - b^2)*Arc
Tanh[Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]/Sqrt[-((b*(-1 + Sin[c + d*x]))/(
a + b))]]*Sqrt[1 - Sin[c + d*x]] + Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]
*(-6*Sqrt[b]*(-2*a^2 + a*b + 2*b^2)*ArcSinh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin
[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])] + Sqrt[a - b]*Sqrt[1 - Sin[c + d*
x]]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(6*a^2 - 8*b^2 - 3*a*b*Sin[c + d*
```

```
x] + 2*b^2*Sin[c + d*x]^2)))))/(6*(a - b)^(3/2)*b^2*Sqrt[a + b]*d*(1 - Sin[
c + d*x])^(3/2)*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*(-((b*(1 + Sin[c +
 d*x]))/(a - b)))^(3/2))
```

fricas [A] time = 0.50, size = 332, normalized size = 2.61

$$\left[\frac{2\,b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)-3\left(2\,a^3-3\,ab^2\right)dx-3\left(a^2-b^2\right)\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx}{}\right.}{6\,b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sin(d*x + c) - 3*(2*a^3 -
 3*a*b^2)*d*x - 3*(a^2 - b^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b
*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2)) - 6*(a^2*b - b^3)*cos(d*x + c))/(b^4*d), 1/6*(2*b^3*cos(d*x +
c)^3 + 3*a*b^2*cos(d*x + c)*sin(d*x + c) - 3*(2*a^3 - 3*a*b^2)*d*x - 6*(a^2
 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))
- 6*(a^2*b - b^3)*cos(d*x + c))/(b^4*d)]
```

giac [A] time = 2.81, size = 226, normalized size = 1.78

$$-\frac{\frac{3\left(2\,a^3-3\,ab^2\right)(dx+c)}{b^4}-\frac{12\left(a^4-2\,a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,b^2\,\mathrm{ta}}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 - 3*a*b^2)*(d*x + c)/b^4 - 12*(a^4 - 2*a^2*b^2 + b^4)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sq
rt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^4) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6
*a^2*tan(1/2*d*x + 1/2*c)^4 - 12*b^2*tan(1/2*d*x + 1/2*c)^4 + 12*a^2*tan(1/
2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*
c) + 6*a^2 - 8*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

maple [B] time = 0.15, size = 450, normalized size = 3.54

$$-\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4}{db\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5-2/d/b^3/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2+4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^
3*tan(1/2*d*x+1/2*c)^4-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c
)^2*a^2+4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2+1/d/b^2/(1+ta
n(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)-2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)
^3*a^2+8/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3-2/d/b^4*arctan(tan(1/2*d*x+1/2*c)
)*a^3+3/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))+2/d/b^4/(a^2-b^2)^(1/2)*arctan(1
```

```
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^4-4/d/b^2/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+2/d/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.12, size = 364, normalized size = 2.87

$$\frac{\frac{5\cos(c+d\,x)}{4}+\frac{\cos(3\,c+3\,d\,x)}{12}}{b\,d}+\frac{3\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{a\,\sin(2\,c+2\,d\,x)}{4}}{b^2\,d}-\frac{a^2\,\cos\left(c+d\,x\right)}{b^3\,d}-\frac{2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^4\,d}+\frac{2\,\mathrm{atanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x)),x)
```

```
[Out] ((5*cos(c + d*x))/4 + cos(3*c + 3*d*x)/12)/(b*d) + (3*a*atan(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2)) + (a*sin(2*c + 2*d*x))/4)/(b^2*d) - (a^2*cos(c + d
*x))/(b^3*d) - (2*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^4*d)
+ (2*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1
/2) - a^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*
b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/
2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*
b*sin(c/2 + (d*x)/2) - 2*a^3*b^2*cos(c/2 + (d*x)/2) - 4*a^2*b^3*sin(c/2 + (
d*x)/2)))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.433 $\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}$$

```
[Out] a*x/b^2+cos(d*x+c)/b/d-2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(
a^2-b^2)^(1/2)/b^2/d
```

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2695, 2735, 2660, 618, 204}

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^2*d) + Cos[c + d*x]/(b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos(c+dx)}{bd}+\frac{\int \frac{b+a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$
$$= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}-\frac{\left(a^2-b^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2}$$
$$= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}-\frac{\left(2\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}$$
$$= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}+\frac{\left(4\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}$$
$$= \frac{ax}{b^2}-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{\cos(c+dx)}{bd}$$

Mathematica [B] time = 1.41, size = 361, normalized size = 5.16

$$\frac{\cos(c+dx)\left(2(a-b)\sqrt{1-\sin(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}}{\sqrt{a+b}\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}}\right)+\sqrt{a+b}\left(\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\left(\sqrt{a-b}\sqrt{1-\sin\right.\right.}{bd\sqrt{a-b}\sqrt{a+b}\sqrt{1-\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(2*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[c + d*x]))
/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))])]*Sqrt[1
- Sin[c + d*x]] + Sqrt[a + b]*(-2*Sqrt[a - b]*ArcTanh[Sqrt[(b*(1 + Sin[c +
d*x]))/(-a + b)]/Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]]*Sqrt[1 - Sin[c +
 d*x]] + Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*(2*Sqrt[b]*ArcSinh[(Sqrt[
a - b]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])] + Sqrt[a
 - b]*Sqrt[1 - Sin[c + d*x]]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]))))/(Sqr
t[a - b]*b*Sqrt[a + b]*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[-((b*(-1 + Sin[c + d*x
]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])
```

fricas [A] time = 0.50, size = 214, normalized size = 3.06

$$\left[\frac{2\,adx+2\,b\cos(dx+c)+\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right.}{2\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x + 2*b*cos(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)))/(b^2*d), (a*d*x + b*cos(d*x + c) + sqrt(a^2 - b^2)*arcta
n(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/(b^2*d)]
```

giac [A] time = 0.97, size = 95, normalized size = 1.36

$$\frac{\frac{(dx+c)a}{b^2} - \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2} + \frac{2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a/b^2 - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2 + 2/((tan(1/
2*d*x + 1/2*c)^2 + 1)*b))/d
```

maple [B] time = 0.14, size = 142, normalized size = 2.03

$$-\frac{2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)a^2}{d\,b^2\sqrt{a^2-b^2}}+\frac{2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{2}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^
(1/2))*a^2+2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2
-b^2)^(1/2))+2/d/b/(1+tan(1/2*d*x+1/2*c)^2)+2/d/b^2*a*arctan(tan(1/2*d*x+1/
2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 5.37, size = 318, normalized size = 4.54

$$\frac{2}{b\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^2-\frac{64\,a^4}{b^2}}+\frac{64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4-64\,a^2\,b^2}\right)}{b^2\,d}+\frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\,\sqrt{b^2-a^2}}{64\,a^2\,b-\frac{64\,a^4}{b}-128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\tan\dots}\right.}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

```
[Out] 2/(b*d*(tan(c/2 + (d*x)/2)^2 + 1)) + (2*a*atan((64*a^2*tan(c/2 + (d*x)/2))/
(64*a^2 - (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 - 64*a^2*b^2)
))/(b^2*d) + (2*atanh((64*a^2*(b^2 - a^2)^(1/2))/(64*a^2*b - (64*a^4)/b - 1
28*a^3*tan(c/2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2
```

```
+ (d*x)/2)*(b^2 - a^2)^(1/2))/(64*a^2 - (64*a^4)/b^2 - (128*a^3*tan(c/2 + (
d*x)/2))/b + 128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2))/(64*a^4 - 64*a^2*b^2 - 128*a*b^3*tan(c/2 + (d*x)/2) + 128*a^3
*b*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2))/(b^2*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.434 $\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)}$$

```
[Out] -2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-s
ec(d*x+c)*(b-a*sin(d*x+c))/(a^2-b^2)/d
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2696, 12, 2660, 618, 204}

$$-\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (Sec[c + d*x]*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
```

```
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d} + \frac{\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$
$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d} - \frac{b^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$
$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d} - \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}$$
$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d} + \frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c\right.\right.}{\left(a^2-b^2\right)d}$$
$$= -\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.32, size = 152, normalized size = 1.81

$$\frac{\sqrt{a^2-b^2}\,(-a\sin(c+dx)+b(-\cos(c+dx))+b)+2b^2\cos(c+dx)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(b-a)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(b - b*Cos[c + d*x] - a*Sin[c + d*x]))/((-a + b)*(a + b)*Sqrt[a
^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]))
```

fricas [A] time = 0.50, size = 305, normalized size = 3.63

$$\left[\frac{\sqrt{-a^2+b^2}\,b^2\cos(dx+c)\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^{}}{2\left(a^4-2a^2b^2+b^4\right)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*b^2*arctan(-(a*sin(d*x + c) + b)/(s
```

```
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

giac [A] time = 1.17, size = 107, normalized size = 1.27

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*b^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [A] time = 0.16, size = 117, normalized size = 1.39

$$-\frac{2}{d\,(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{d\,(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\,(a-b)\,(a+b)\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)-2/d/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)-
2/d*b^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 5.26, size = 149, normalized size = 1.77

$$\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\frac{b^2\,\left(2\,a^2\,b-2\,b^3\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^2-b^2\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}}{2\,b^2}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*b)/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2))/(a^2 - b^2))/(d*(tan(c/2 + (d
*x)/2)^2 - 1)) - (2*b^2*atan(((b^2*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a - b
```

```
)^(3/2)) + (2*a*b^2*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a - b)^
(3/2)))/(2*b^2)))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.435 $\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3d\left(a^2-b^2\right)}+\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3d\left(a^2-b^2\right)^2}$$

```
[Out] 2*b^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d-1/
3*sec(d*x+c)^3*(b-a*sin(d*x+c))/d/(a^2-b^2)+1/3*sec(d*x+c)*(3*b^3+a*(2*a^2-
5*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.25, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2696, 2866, 12, 2660, 618, 204}

$$\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3d\left(a^2-b^2\right)}+\frac{\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) - (Sec[c + d*x]^3*(b - a*Sin[c + d*x]))/(3*(a^2 - b^2)*d) + (Sec[c + d*
x]*(3*b^3 + a*(2*a^2 - 5*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
```

```
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^2(c+dx)\left(-2a^2+3b^2-2ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} + \frac{\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \cdots$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} + \frac{\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \cdots$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} + \frac{\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \cdots$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} + \frac{\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} - \cdots$$

$$= \frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} - \frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} + \frac{\sec(c+dx)\left(3b^3+a\cdots\right.}{3\left(a^2\cdots\right.}$$

Mathematica [A] time = 1.34, size = 202, normalized size = 1.47

$$\frac{\frac{24b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{\sec^3(c+dx)\left(6a^3\sin(c+dx)+2a^3\sin(3(c+dx))+\frac{3}{2}b\left(a^2-7b^2\right)\cos(c+dx)+\frac{1}{2}a^2b\cos(3(c+dx))-4a^2b-9ab^2\sin(c+dx)-5\cdots\right.}{(a-b)^2(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((24*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
) + (Sec[c + d*x]^3*(-4*a^2*b + 10*b^3 + (3*b*(a^2 - 7*b^2)*Cos[c + d*x])/2
 + 6*b^3*Cos[2*(c + d*x)] + (a^2*b*Cos[3*(c + d*x)])/2 - (7*b^3*Cos[3*(c +
d*x)])/2 + 6*a^3*Sin[c + d*x] - 9*a*b^2*Sin[c + d*x] + 2*a^3*Sin[3*(c + d*x
)] - 5*a*b^2*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(12*d)
```

fricas [A] time = 0.51, size = 466, normalized size = 3.40

$$\left[-\frac{3\sqrt{-a^2+b^2}\,b^4\cos(dx+c)^3\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2}{6\left(a^6-3a^4b^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*b^4*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(a^2*b^3 - b^5)*cos(d*x + c)^2
 - 2*(a^5 - 2*a^3*b^2 + a*b^4 + (2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^
2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3), -1
/3*(3*sqrt(a^2 - b^2)*b^4*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos
(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(a^2*b^3 - b^5)*co
s(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 + (2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3)]
```

giac [B] time = 0.53, size = 273, normalized size = 1.99

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^4}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a}{\left(a^4-2a\right.}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*b^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
- (3*a^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*
tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 2*a^3*tan(1/2*d*x +
 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*b^3*tan(1/2*d*x + 1/2*c)^2 +
 3*a^3*tan(1/2*d*x + 1/2*c) - 6*a*b^2*tan(1/2*d*x + 1/2*c) - a^2*b + 4*b^3)
/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

maple [B] time = 0.19, size = 270, normalized size = 1.97

$$-\frac{2}{3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(2a+2b)}-\frac{1}{d(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a}{d(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3}{2d(a+b)^2\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(2*a+2*b)-1/d/(2*a+2*b)/(tan(1/2*d*x+1/2*c)
-1)^2-1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-3/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c
)-1)*b+2/d*b^4/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+
1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/3/d/(tan(1/2*d*x+1/2*c)+1)^3/(2*a-2*b)+1/d/(
2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)^2-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a+3/2/
d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.95, size = 387, normalized size = 2.82

$$\frac{\frac{2\left(a^2\,b-4\,b^3\right)}{3\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a\,b^2-a^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4-2\,a^2\,b^2+b^4}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(4\,a\,b^2-a^3\right)}{3\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(2\,a\,b^2-a^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{2\,b\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{a^4-2\,a}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*(a^2*b - 4*b^3))/(3*(a^4 + b^4 - 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)*(2
*a*b^2 - a^3))/(a^4 + b^4 - 2*a^2*b^2) + (4*b^3*tan(c/2 + (d*x)/2)^2)/(a^4
+ b^4 - 2*a^2*b^2) - (4*tan(c/2 + (d*x)/2)^3*(4*a*b^2 - a^3))/(3*(a^4 + b^4
 - 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(2*a*b^2 - a^3))/(a^4 + b^4 - 2*a^
2*b^2) + (2*b*tan(c/2 + (d*x)/2)^4*(a^2 - 2*b^2))/(a^4 + b^4 - 2*a^2*b^2))/
(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6
- 1)) + (2*b^4*atan(((b^4*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a
- b)^(5/2)) + (2*a*b^4*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)
^(5/2)*(a - b)^(5/2)))/(2*b^4)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

3.436 $\int \frac{\sec^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$-\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5d\left(a^2-b^2\right)}-\frac{2b^6\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{\sec^3(c+dx)\left(a\left(4a^2-9b^2\right)\sin(c+dx)+5b^3\right)}{15d\left(a^2-b^2\right)^2}-\text{se}$$

```
[Out] -2*b^6*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d-1
/5*sec(d*x+c)^5*(b-a*sin(d*x+c))/d/(a^2-b^2)+1/15*sec(d*x+c)^3*(5*b^3+a*(4*
a^2-9*b^2)*sin(d*x+c))/(a^2-b^2)^2/d-1/15*sec(d*x+c)*(15*b^5-a*(8*a^4-26*a^
2*b^2+33*b^4)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A] time = 0.50, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2696, 2866, 12, 2660, 618, 204}

$$-\frac{2b^6\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5d\left(a^2-b^2\right)}+\frac{\sec^3(c+dx)\left(a\left(4a^2-9b^2\right)\sin(c+dx)+5b^3\right)}{15d\left(a^2-b^2\right)^2}-\text{se}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2
)*d) - (Sec[c + d*x]^5*(b - a*Sin[c + d*x]))/(5*(a^2 - b^2)*d) + (Sec[c + d
*x]^3*(5*b^3 + a*(4*a^2 - 9*b^2)*Sin[c + d*x]))/(15*(a^2 - b^2)^2*d) - (Sec
[c + d*x]*(15*b^5 - a*(8*a^4 - 26*a^2*b^2 + 33*b^4)*Sin[c + d*x]))/(15*(a^2
 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^4(c+dx)\left(-4a^2+5b^2-4ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{5\left(a^2-b^2\right)}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d} +$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d} -$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d} -$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d} -$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d} -$$

$$= -\frac{2b^6\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+\right.}{15\left(\right.}$$

Mathematica [A] time = 2.53, size = 370, normalized size = 1.88

$$\frac{\sec^5(c+dx)\left(640a^5\sin(c+dx)+320a^5\sin(3(c+dx))+64a^5\sin(5(c+dx))+45a^4b\cos(3(c+dx))+9a^4b\,\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2
)*d) + (Sec[c + d*x]^5*(-384*a^4*b + 1088*a^2*b^3 - 1424*b^5 + 10*b*(9*a^4
- 38*a^2*b^2 + 149*b^4)*Cos[c + d*x] + 320*b^3*(a^2 - 4*b^2)*Cos[2*(c + d*x
)] + 45*a^4*b*Cos[3*(c + d*x)] - 190*a^2*b^3*Cos[3*(c + d*x)] + 745*b^5*Cos
[3*(c + d*x)] - 240*b^5*Cos[4*(c + d*x)] + 9*a^4*b*Cos[5*(c + d*x)] - 38*a^
2*b^3*Cos[5*(c + d*x)] + 149*b^5*Cos[5*(c + d*x)] + 640*a^5*Sin[c + d*x] -
1600*a^3*b^2*Sin[c + d*x] + 1200*a*b^4*Sin[c + d*x] + 320*a^5*Sin[3*(c + d*
x)] - 1040*a^3*b^2*Sin[3*(c + d*x)] + 1080*a*b^4*Sin[3*(c + d*x)] + 64*a^5*
Sin[5*(c + d*x)] - 208*a^3*b^2*Sin[5*(c + d*x)] + 264*a*b^4*Sin[5*(c + d*x)
]))/(1920*(a - b)^3*(a + b)^3*d)
```

fricas [A] time = 0.52, size = 666, normalized size = 3.38

$$\left[\frac{15\,\sqrt{-a^2+b^2}\,b^6\cos\left(dx+c\right)^5\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)-6\,a \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(-a^2 + b^2)*b^6*cos(d*x + c)^5*log(((2*a^2 - b^2)*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*
cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
a^2 - b^2)) - 6*a^6*b + 18*a^4*b^3 - 18*a^2*b^5 + 6*b^7 - 30*(a^2*b^5 - b^7
)*cos(d*x + c)^4 + 10*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2 + 2*(3*a^7
 - 9*a^5*b^2 + 9*a^3*b^4 - 3*a*b^6 + (8*a^7 - 34*a^5*b^2 + 59*a^3*b^4 - 33*
a*b^6)*cos(d*x + c)^4 + (4*a^7 - 17*a^5*b^2 + 22*a^3*b^4 - 9*a*b^6)*cos(d*x
 + c)^2)*sin(d*x + c))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*c
os(d*x + c)^5), 1/15*(15*sqrt(a^2 - b^2)*b^6*arctan(-(a*sin(d*x + c) + b)/(
sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)^5 - 3*a^6*b + 9*a^4*b^3 - 9*a^2
*b^5 + 3*b^7 - 15*(a^2*b^5 - b^7)*cos(d*x + c)^4 + 5*(a^4*b^3 - 2*a^2*b^5 +
 b^7)*cos(d*x + c)^2 + (3*a^7 - 9*a^5*b^2 + 9*a^3*b^4 - 3*a*b^6 + (8*a^7 -
34*a^5*b^2 + 59*a^3*b^4 - 33*a*b^6)*cos(d*x + c)^4 + (4*a^7 - 17*a^5*b^2 +
22*a^3*b^4 - 9*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x + c)^5)]
```

giac [B] time = 1.23, size = 584, normalized size = 2.96

$$-\frac{2\left(\frac{15\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^6}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{15\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-45\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+45\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-15\,a^4b\tan\left(\frac{1}{2}\,dx \cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/15*(15*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))*b^6/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sq
rt(a^2 - b^2)) + (15*a^5*tan(1/2*d*x + 1/2*c)^9 - 45*a^3*b^2*tan(1/2*d*x +
1/2*c)^9 + 45*a*b^4*tan(1/2*d*x + 1/2*c)^9 - 15*a^4*b*tan(1/2*d*x + 1/2*c)^
8 + 45*a^2*b^3*tan(1/2*d*x + 1/2*c)^8 - 45*b^5*tan(1/2*d*x + 1/2*c)^8 - 20*
a^5*tan(1/2*d*x + 1/2*c)^7 + 80*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*a*b^4*
tan(1/2*d*x + 1/2*c)^7 - 30*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 + 90*b^5*tan(1/2
*d*x + 1/2*c)^6 + 58*a^5*tan(1/2*d*x + 1/2*c)^5 - 166*a^3*b^2*tan(1/2*d*x +
 1/2*c)^5 + 198*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 30*a^4*b*tan(1/2*d*x + 1/2*c
```

```
)^4 + 80*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 140*b^5*tan(1/2*d*x + 1/2*c)^4 -
20*a^5*tan(1/2*d*x + 1/2*c)^3 + 80*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b
^4*tan(1/2*d*x + 1/2*c)^3 - 10*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 70*b^5*tan(
1/2*d*x + 1/2*c)^2 + 15*a^5*tan(1/2*d*x + 1/2*c) - 45*a^3*b^2*tan(1/2*d*x +
 1/2*c) + 45*a*b^4*tan(1/2*d*x + 1/2*c) - 3*a^4*b + 11*a^2*b^3 - 23*b^5)/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)^5))/d
```

maple [B] time = 0.18, size = 525, normalized size = 2.66

$$-\frac{2}{5d\left(2a+2b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}-\frac{1}{2d\left(a+b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{7a}{8d\left(a+b\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{}{8d\left(a+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/5/d/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)^5-1/2/d/(a+b)/(tan(1/2*d*x+1/2*c)-1
)^4-7/8/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^2*a-9/8/d/(a+b)^2/(tan(1/2*d*x+1/2
*c)-1)^2*b-11/12/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3*a-13/12/d/(a+b)^2/(tan(
1/2*d*x+1/2*c)-1)^3*b-1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)*a^2-21/8/d/(a+b)^3
/(tan(1/2*d*x+1/2*c)-1)*a*b-15/8/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)*b^2-2/d*b
^6/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))-2/5/d/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)^5+1/2/d/(a-b)/(tan(
1/2*d*x+1/2*c)+1)^4+7/8/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2*a-9/8/d/(a-b)^2/
(tan(1/2*d*x+1/2*c)+1)^2*b-11/12/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3*a+13/12
/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3*b-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)*a^
2+21/8/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)*a*b-15/8/d/(a-b)^3/(tan(1/2*d*x+1/2
*c)+1)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.06, size = 774, normalized size = 3.93

$$\frac{\frac{2\left(3a^4b-11a^2b^3+23b^5\right)}{15\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^5-3a^3b^2+3ab^4\right)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(7b^5-a^2b^3\right)}{3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(3b^5-a^2b^3\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3\left(a^6-3a^4\right.}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*(3*a^4*b + 23*b^5 - 11*a^2*b^3))/(15*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
)) - (2*tan(c/2 + (d*x)/2)*(3*a*b^4 + a^5 - 3*a^3*b^2))/(a^6 - b^6 + 3*a^2*
b^4 - 3*a^4*b^2) - (4*tan(c/2 + (d*x)/2)^2*(7*b^5 - a^2*b^3))/(3*(a^6 - b^6
 + 3*a^2*b^4 - 3*a^4*b^2)) - (4*tan(c/2 + (d*x)/2)^6*(3*b^5 - a^2*b^3))/(a^
```

```
6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (8*tan(c/2 + (d*x)/2)^3*(6*a*b^4 + a^5 -
 4*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2
)^9*(3*a*b^4 + a^5 - 3*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (8*t
an(c/2 + (d*x)/2)^7*(6*a*b^4 + a^5 - 4*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4
- 3*a^4*b^2)) - (4*tan(c/2 + (d*x)/2)^5*(99*a*b^4 + 29*a^5 - 83*a^3*b^2))/(
15*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^8*(a^4*b +
3*b^5 - 3*a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (4*tan(c/2 + (d*x
)/2)^4*(3*a^4*b + 14*b^5 - 8*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2)))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d
*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (2*b^6*at
an(((b^6*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^3))/((a + b)^(7/2)*(a - b)^
(7/2)) + (2*a*b^6*tan(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(
(a + b)^(7/2)*(a - b)^(7/2)))/(2*b^6)))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*sin(c + d*x)), x)
```

3.437 $\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=184

$$\frac{\left(a^2-b^2\right)^3}{b^7d(a+b\sin(c+dx))}+\frac{6a\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^7d}+\frac{a\left(2a^2-3b^2\right)\sin^2(c+dx)}{b^5d}-\frac{\left(a^2-b^2\right)\sin^3(c+dx}{b^4d}$$

```
[Out] 6*a*(a^2-b^2)^2*ln(a+b*sin(d*x+c))/b^7/d-(5*a^4-9*a^2*b^2+3*b^4)*sin(d*x+c)
/b^6/d+a*(2*a^2-3*b^2)*sin(d*x+c)^2/b^5/d-(a^2-b^2)*sin(d*x+c)^3/b^4/d+1/2*
a*sin(d*x+c)^4/b^3/d-1/5*sin(d*x+c)^5/b^2/d+(a^2-b^2)^3/b^7/d/(a+b*sin(d*x+
c))
```

Rubi [A] time = 0.17, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{\left(a^2-b^2\right)\sin^3(c+dx)}{b^4d}+\frac{a\left(2a^2-3b^2\right)\sin^2(c+dx)}{b^5d}-\frac{\left(-9a^2b^2+5a^4+3b^4\right)\sin(c+dx)}{b^6d}+\frac{\left(a^2-b^2\right)^3}{b^7d(a+b\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6*a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^7*d) - ((5*a^4 - 9*a^2*b^2 +
 3*b^4)*Sin[c + d*x])/(b^6*d) + (a*(2*a^2 - 3*b^2)*Sin[c + d*x]^2)/(b^5*d)
- ((a^2 - b^2)*Sin[c + d*x]^3)/(b^4*d) + (a*Sin[c + d*x]^4)/(2*b^3*d) - Sin
[c + d*x]^5/(5*b^2*d) + (a^2 - b^2)^3/(b^7*d*(a + b*Sin[c + d*x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{(a+x)^2}\,dx,x,b\sin(c+dx)\right)}{b^7d}\\ &= \frac{\text{Subst}\left(\int \left(-5a^4\left(1+\frac{3b^2\left(-3a^2+b^2\right)}{5a^4}\right)+2a\left(2a^2-3b^2\right)x-3\left(a^2-b^2\right)x^2+2ax^3-x^4\right.\right.}{b^7d}\\ &= \frac{6a\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^7d}-\frac{\left(5a^4-9a^2b^2+3b^4\right)\sin(c+dx)}{b^6d}+\frac{a\left(2a^2-\right.}{}\end{aligned}$$

Mathematica [A] time = 0.48, size = 235, normalized size = 1.28

$$\frac{-4a^2b^4\sin^4(c+dx)+4\left(a^2-b^2\right)^2\left(15a^2\log(a+b\sin(c+dx))+4a^2-4b^2\right)+b^4\cos^4(c+dx)\left(-a^2+3ab\sin(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b^6*Cos[c + d*x]^6 + 4*(a^2 - b^2)^2*(4*a^2 - 4*b^2 + 15*a^2*Log[a + b*S
in[c + d*x]]) + 4*a*b*(-11*a^4 + 18*a^2*b^2 - 4*b^4 + 15*(a^2 - b^2)^2*Log[
a + b*Sin[c + d*x]])*Sin[c + d*x] - 2*b^2*(15*a^4 - 29*a^2*b^2 + 8*b^4)*Sin
[c + d*x]^2 + 2*a*b^3*(5*a^2 - 7*b^2)*Sin[c + d*x]^3 - 4*a^2*b^4*Sin[c + d*
x]^4 + b^4*Cos[c + d*x]^4*(-a^2 + 4*b^2 + 3*a*b*Sin[c + d*x]))/(10*b^7*d*(a
 + b*Sin[c + d*x]))
```

fricas [A] time = 0.54, size = 243, normalized size = 1.32

$$\frac{16\,b^6\cos(dx+c)^6+80\,a^6-560\,a^4b^2+785\,a^2b^4-256\,b^6-8\left(5\,a^2b^4-4\,b^6\right)\cos(dx+c)^4+16\left(15\,a^4b^2-25\,a^2b^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*b^6*cos(d*x + c)^6 + 80*a^6 - 560*a^4*b^2 + 785*a^2*b^4 - 256*b^6
- 8*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 + 16*(15*a^4*b^2 - 25*a^2*b^4 + 8*b^
6)*cos(d*x + c)^2 + 480*(a^6 - 2*a^4*b^2 + a^2*b^4 + (a^5*b - 2*a^3*b^3 + a
*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) + (24*a*b^5*cos(d*x + c)^4 - 40
0*a^5*b + 720*a^3*b^3 - 271*a*b^5 - 16*(5*a^3*b^3 - 7*a*b^5)*cos(d*x + c)^2
)*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)
```

giac [A] time = 0.98, size = 251, normalized size = 1.36

$$\frac{\frac{60\left(a^5-2\,a^3b^2+ab^4\right)\log(|b\sin(dx+c)+a|)}{b^7}-\frac{10\left(6\,a^5b\sin(dx+c)-12\,a^3b^3\sin(dx+c)+6\,ab^5\sin(dx+c)+5\,a^6-9\,a^4b^2+3\,a^2b^4+b^6\right)}{(b\sin(dx+c)+a)b^7}-\frac{2\,b^8\sin(dx+c)^5-5\,ab^7}{}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/10*(60*(a^5 - 2*a^3*b^2 + a*b^4)*log(abs(b*sin(d*x + c) + a))/b^7 - 10*(6
*a^5*b*sin(d*x + c) - 12*a^3*b^3*sin(d*x + c) + 6*a*b^5*sin(d*x + c) + 5*a^
6 - 9*a^4*b^2 + 3*a^2*b^4 + b^6)/((b*sin(d*x + c) + a)*b^7) - (2*b^8*sin(d*
x + c)^5 - 5*a*b^7*sin(d*x + c)^4 + 10*a^2*b^6*sin(d*x + c)^3 - 10*b^8*sin(
d*x + c)^3 - 20*a^3*b^5*sin(d*x + c)^2 + 30*a*b^7*sin(d*x + c)^2 + 50*a^4*b
^4*sin(d*x + c) - 90*a^2*b^6*sin(d*x + c) + 30*b^8*sin(d*x + c))/b^10)/d
```

maple [A] time = 0.25, size = 305, normalized size = 1.66

$$-\frac{\sin^5(dx+c)}{5b^2d}+\frac{a\left(\sin^4(dx+c)\right)}{2b^3d}-\frac{\left(\sin^3(dx+c)\right)a^2}{d\,b^4}+\frac{\sin^3(dx+c)}{b^2d}+\frac{2\left(\sin^2(dx+c)\right)a^3}{d\,b^5}-\frac{3a\left(\sin^2(dx+c)\right)}{b^3d}-\frac{5a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5*sin(d*x+c)^5/b^2/d+1/2*a*sin(d*x+c)^4/b^3/d-1/d/b^4*sin(d*x+c)^3*a^2+s
in(d*x+c)^3/b^2/d+2/d/b^5*sin(d*x+c)^2*a^3-3*a*sin(d*x+c)^2/b^3/d-5/d/b^6*a
^4*sin(d*x+c)+9/d/b^4*a^2*sin(d*x+c)-3*sin(d*x+c)/b^2/d+6/d*a^5/b^7*ln(a+b*
sin(d*x+c))-12/d*a^3/b^5*ln(a+b*sin(d*x+c))+6*a*ln(a+b*sin(d*x+c))/b^3/d+1/
d/b^7/(a+b*sin(d*x+c))*a^6-3/d/b^5/(a+b*sin(d*x+c))*a^4+3/d/b^3/(a+b*sin(d*
x+c))*a^2-1/b/d/(a+b*sin(d*x+c))
```

maxima [A] time = 0.31, size = 190, normalized size = 1.03

$$\frac{\frac{10\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)}{b^8\sin(dx+c)+ab^7}-\frac{2\,b^4\sin(dx+c)^5-5\,ab^3\sin(dx+c)^4+10\left(a^2b^2-b^4\right)\sin(dx+c)^3-10\left(2\,a^3b-3\,ab^3\right)\sin(dx+c)^2+10\left(5\,a^4-9\,a^2b^2+3\,b^4\right)\sin(dx+}{b^6}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/10*(10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)/(b^8*sin(d*x + c) + a*b^7) - (
2*b^4*sin(d*x + c)^5 - 5*a*b^3*sin(d*x + c)^4 + 10*(a^2*b^2 - b^4)*sin(d*x
+ c)^3 - 10*(2*a^3*b - 3*a*b^3)*sin(d*x + c)^2 + 10*(5*a^4 - 9*a^2*b^2 + 3*
b^4)*sin(d*x + c))/b^6 + 60*(a^5 - 2*a^3*b^2 + a*b^4)*log(b*sin(d*x + c) +
a)/b^7)/d
```

mupad [B] time = 0.12, size = 259, normalized size = 1.41

$$\frac{\sin(c+d\,x)^3\,\left(\frac{1}{b^2}-\frac{a^2}{b^4}\right)}{d}-\frac{\sin(c+d\,x)^5}{5\,b^2\,d}-\frac{\sin(c+d\,x)^2\,\left(\frac{a^3}{b^5}+\frac{a\,\left(\frac{3}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)}{d}-\frac{\sin(c+d\,x)\,\left(\frac{3}{b^2}+\frac{a^2\,\left(\frac{3}{b^2}-\frac{3\,a^2}{b^4}\right)}{b^2}-\frac{2\,a\,\left(\frac{2\,a^3}{b^5}\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^3*(1/b^2 - a^2/b^4))/d - sin(c + d*x)^5/(5*b^2*d) - (sin(c +
d*x)^2*(a^3/b^5 + (a*(3/b^2 - (3*a^2)/b^4))/b))/d - (sin(c + d*x)*(3/b^2 +
(a^2*(3/b^2 - (3*a^2)/b^4))/b^2 - (2*a*((2*a^3)/b^5 + (2*a*(3/b^2 - (3*a^2)
/b^4))/b))/b))/d + (a*sin(c + d*x)^4)/(2*b^3*d) + (log(a + b*sin(c + d*x))*
(6*a*b^4 + 6*a^5 - 12*a^3*b^2))/(b^7*d) + (a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2)/(b*d*(a*b^6 + b^7*sin(c + d*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.438 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$-\frac{\left(a^2-b^2\right)^2}{b^5d(a+b\sin(c+dx))}-\frac{4a\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^5d}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^4d}-\frac{a\sin^2(c+dx)}{b^3d}+\frac{\sin^3(c+d}{3b^2d}$$

```
[Out] -4*a*(a^2-b^2)*ln(a+b*sin(d*x+c))/b^5/d+(3*a^2-2*b^2)*sin(d*x+c)/b^4/d-a*si
n(d*x+c)^2/b^3/d+1/3*sin(d*x+c)^3/b^2/d-(a^2-b^2)^2/b^5/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^4d}-\frac{\left(a^2-b^2\right)^2}{b^5d(a+b\sin(c+dx))}-\frac{4a\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^5d}-\frac{a\sin^2(c+dx)}{b^3d}+\frac{\sin^3(c+dx}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d) + ((3*a^2 - 2*b^2)*Sin[c
 + d*x])/(b^4*d) - (a*Sin[c + d*x]^2)/(b^3*d) + Sin[c + d*x]^3/(3*b^2*d) -
(a^2 - b^2)^2/(b^5*d*(a + b*Sin[c + d*x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5d}\\
&= \frac{\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-2ax+x^2+\frac{\left(a^2-b^2\right)^2}{(a+x)^2}-\frac{4\left(a^3-ab^2\right)}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^5d}\\
&= -\frac{4a\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^5d}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^4d}-\frac{a\sin^2(c+dx)}{b^3d}+\text{si}
\end{aligned}$$

Mathematica [A] time = 0.63, size = 127, normalized size = 1.06

$$\frac{\left(8a^2b-4b^3\right)\sin(c+dx)+\frac{b^4\cos^4(c+dx)-4\left(a^2-b^2\right)\left(3a^2\log(a+b\sin(c+dx))+a^2+3ab\sin(c+dx)\log(a+b\sin(c+dx))-b^2\right)}{a+b\sin(c+dx)}-2ab^2\sin^2(c+}{3b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((8*a^2*b - 4*b^3)*Sin[c + d*x] - 2*a*b^2*Sin[c + d*x]^2 + (b^4*Cos[c + d*x
]^4 - 4*(a^2 - b^2)*(a^2 - b^2 + 3*a^2*Log[a + b*Sin[c + d*x]] + 3*a*b*Log[
a + b*Sin[c + d*x]]*Sin[c + d*x]))/(a + b*Sin[c + d*x]))/(3*b^5*d)
```

fricas [A] time = 0.49, size = 156, normalized size = 1.30

$$\frac{2\,b^4\cos\left(dx+c\right)^4-6\,a^4+27\,a^2b^2-16\,b^4-4\left(3\,a^2b^2-2\,b^4\right)\cos\left(dx+c\right)^2-24\left(a^4-a^2b^2+\left(a^3b-ab^3\right)\sin\left(d\right.\right.}{6\left(b^6d\sin\left(dx+c\right)+ab^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^4*cos(d*x + c)^4 - 6*a^4 + 27*a^2*b^2 - 16*b^4 - 4*(3*a^2*b^2 - 2*
b^4)*cos(d*x + c)^2 - 24*(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d*x + c))*log
(b*sin(d*x + c) + a) + (4*a*b^3*cos(d*x + c)^2 + 18*a^3*b - 13*a*b^3)*sin(d
*x + c))/(b^6*d*sin(d*x + c) + a*b^5*d)
```

giac [A] time = 0.84, size = 150, normalized size = 1.25

$$-\frac{\frac{12\left(a^3-ab^2\right)\log\left(\left|b\sin(dx+c)+a\right|\right)}{b^5}-\frac{b^4\sin(dx+c)^3-3\,ab^3\sin(dx+c)^2+9\,a^2b^2\sin(dx+c)-6\,b^4\sin(dx+c)}{b^6}-\frac{3\left(4\,a^3b\sin(dx+c)-4\,ab^3\sin(dx+c)+3\,a\right.}{(b\sin(dx+c)+a)b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3 - a*b^2)*log(abs(b*sin(d*x + c) + a))/b^5 - (b^4*sin(d*x + c)
^3 - 3*a*b^3*sin(d*x + c)^2 + 9*a^2*b^2*sin(d*x + c) - 6*b^4*sin(d*x + c))/
b^6 - 3*(4*a^3*b*sin(d*x + c) - 4*a*b^3*sin(d*x + c) + 3*a^4 - 2*a^2*b^2 -
b^4)/((b*sin(d*x + c) + a)*b^5))/d
```

maple [A] time = 0.25, size = 174, normalized size = 1.45

$$\frac{\sin^3\left(dx+c\right)}{3b^2d}-\frac{a\left(\sin^2\left(dx+c\right)\right)}{b^3d}+\frac{3a^2\sin\left(dx+c\right)}{d\,b^4}-\frac{2\sin\left(dx+c\right)}{b^2d}-\frac{4a^3\ln\left(a+b\sin\left(dx+c\right)\right)}{d\,b^5}+\frac{4a\ln\left(a+b\sin\left(\right.\right.}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/3*sin(d*x+c)^3/b^2/d-a*sin(d*x+c)^2/b^3/d+3/d/b^4*a^2*sin(d*x+c)-2*sin(d*
x+c)/b^2/d-4/d*a^3/b^5*ln(a+b*sin(d*x+c))+4*a*ln(a+b*sin(d*x+c))/b^3/d-1/d/
b^5/(a+b*sin(d*x+c))*a^4+2/d/b^3/(a+b*sin(d*x+c))*a^2-1/b/d/(a+b*sin(d*x+c)
)
```

maxima [A] time = 0.31, size = 116, normalized size = 0.97

$$-\frac{\frac{3\left(a^4-2\,a^2b^2+b^4\right)}{b^6\sin(dx+c)+ab^5}-\frac{b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+3\left(3\,a^2-2\,b^2\right)\sin(dx+c)}{b^4}+\frac{12\left(a^3-ab^2\right)\log(b\sin(dx+c)+a)}{b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 - 2*a^2*b^2 + b^4)/(b^6*sin(d*x + c) + a*b^5) - (b^2*sin(d*x +
 c)^3 - 3*a*b*sin(d*x + c)^2 + 3*(3*a^2 - 2*b^2)*sin(d*x + c))/b^4 + 12*(a^
3 - a*b^2)*log(b*sin(d*x + c) + a)/b^5)/d
```

mupad [B] time = 0.08, size = 118, normalized size = 0.98

$$-\frac{\sin\left(c+d\,x\right)\,\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)-\frac{{\sin\left(c+d\,x\right)}^3}{3\,b^2}+\frac{a\,{\sin\left(c+d\,x\right)}^2}{b^3}-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(4\,a\,b^2-4\,a^3\right)}{b^5}+\frac{a^4-2\,a^2\,b^2+b^4}{b\,\left(\sin\left(c+d\,x\right)\,b^5+a\,b^4\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^2,x)
```

```
[Out] -(sin(c + d*x)*(2/b^2 - (3*a^2)/b^4) - sin(c + d*x)^3/(3*b^2) + (a*sin(c +
d*x)^2)/b^3 - (log(a + b*sin(c + d*x))*(4*a*b^2 - 4*a^3))/b^5 + (a^4 + b^4
- 2*a^2*b^2)/(b*(a*b^4 + b^5*sin(c + d*x))))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.439 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=63

$$\frac{a^2-b^2}{b^3d(a+b\sin(c+dx))}+\frac{2a\log(a+b\sin(c+dx))}{b^3d}-\frac{\sin(c+dx)}{b^2d}$$

```
[Out] 2*a*ln(a+b*sin(d*x+c))/b^3/d-sin(d*x+c)/b^2/d+(a^2-b^2)/b^3/d/(a+b*sin(d*x+
c))
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{a^2-b^2}{b^3d(a+b\sin(c+dx))}+\frac{2a\log(a+b\sin(c+dx))}{b^3d}-\frac{\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*Log[a + b*Sin[c + d*x]])/(b^3*d) - Sin[c + d*x]/(b^2*d) + (a^2 - b^2)/
(b^3*d*(a + b*Sin[c + d*x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{-a^2+b^2}{(a+x)^2}+\frac{2a}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^3d}\\ &= \frac{2a\log(a+b\sin(c+dx))}{b^3d}-\frac{\sin(c+dx)}{b^2d}+\frac{a^2-b^2}{b^3d(a+b\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.83

$$\frac{\frac{(a-b)(a+b)}{a+b\sin(c+dx)}+2a\log(a+b\sin(c+dx))-b\sin(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*Log[a + b*Sin[c + d*x]] - b*Sin[c + d*x] + ((a - b)*(a + b))/(a + b*Si
n[c + d*x]))/(b^3*d)
```

fricas [A] time = 0.46, size = 78, normalized size = 1.24

$$\frac{b^2\cos(dx+c)^2 - ab\sin(dx+c) + a^2 - 2b^2 + 2\left(ab\sin(dx+c) + a^2\right)\log(b\sin(dx+c) + a)}{b^4d\sin(dx+c) + ab^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (b^2*cos(d*x + c)^2 - a*b*sin(d*x + c) + a^2 - 2*b^2 + 2*(a*b*sin(d*x + c)
+ a^2)*log(b*sin(d*x + c) + a))/(b^4*d*sin(d*x + c) + a*b^3*d)
```

giac [A] time = 1.69, size = 91, normalized size = 1.44

$$-\frac{\frac{2a\log\left(\frac{|b\sin(dx+c)+a|}{(b\sin(dx+c)+a)^2|b|}\right)}{b^3} + \frac{b\sin(dx+c)+a}{b^3} - \frac{a^2}{(b\sin(dx+c)+a)b^3} + \frac{1}{(b\sin(dx+c)+a)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*sin(d*x + c) + a)/((b*sin(d*x + c) + a)^2*abs(b)))/b^3 + (b
*sin(d*x + c) + a)/b^3 - a^2/((b*sin(d*x + c) + a)*b^3) + 1/((b*sin(d*x + c
) + a)*b))/d
```

maple [A] time = 0.24, size = 78, normalized size = 1.24

$$-\frac{\sin(dx+c)}{b^2d} + \frac{2a\ln(a+b\sin(dx+c))}{b^3d} + \frac{a^2}{d\,b^3\,(a+b\sin(dx+c))} - \frac{1}{bd\,(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -sin(d*x+c)/b^2/d+2*a*ln(a+b*sin(d*x+c))/b^3/d+1/d/b^3/(a+b*sin(d*x+c))*a^2
-1/b/d/(a+b*sin(d*x+c))
```

maxima [A] time = 0.31, size = 61, normalized size = 0.97

$$\frac{\frac{a^2-b^2}{b^4\sin(dx+c)+ab^3} + \frac{2a\log(b\sin(dx+c)+a)}{b^3} - \frac{\sin(dx+c)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] ((a^2 - b^2)/(b^4*sin(d*x + c) + a*b^3) + 2*a*log(b*sin(d*x + c) + a)/b^3 -
 sin(d*x + c)/b^2)/d
```

mupad [B] time = 0.08, size = 69, normalized size = 1.10

$$\frac{2\,a\,\ln(a+b\,\sin(c+d\,x))}{b^3\,d} - \frac{\sin(c+d\,x)}{b^2\,d} + \frac{a^2-b^2}{b\,d\,\left(\sin(c+d\,x)\,b^3+a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (2*a*log(a + b*sin(c + d*x)))/(b^3*d) - sin(c + d*x)/(b^2*d) + (a^2 - b^2)/
(b*d*(a*b^2 + b^3*sin(c + d*x)))
```

sympy [A] time = 1.88, size = 221, normalized size = 3.51

$$\begin{cases} \frac{x\cos^3(c)}{a^2} & \text{for } b \\ \frac{\frac{2\sin^3(c+dx)}{3d}+\frac{\sin(c+dx)\cos^2(c+dx)}{d}}{a^2} & \text{for } b \\ \frac{x\cos^3(c)}{(a+b\sin(c))^2} & \text{for } a \\ \frac{2a^2\log\left(\frac{a}{b}+\sin(c+dx)\right)}{ab^3d+b^4d\sin(c+dx)}+\frac{2a^2}{ab^3d+b^4d\sin(c+dx)}+\frac{2ab\log\left(\frac{a}{b}+\sin(c+dx)\right)\sin(c+dx)}{ab^3d+b^4d\sin(c+dx)}-\frac{2b^2\sin^2(c+dx)}{ab^3d+b^4d\sin(c+dx)}-\frac{b^2\cos^2(c+dx)}{ab^3d+b^4d\sin(c+dx)} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*cos(c)**3/a**2, Eq(b, 0) & Eq(d, 0)), ((2*sin(c + d*x)**3/(3*d
) + sin(c + d*x)*cos(c + d*x)**2/d)/a**2, Eq(b, 0)), (x*cos(c)**3/(a + b*si
n(c))**2, Eq(d, 0)), (2*a**2*log(a/b + sin(c + d*x))/(a*b**3*d + b**4*d*sin
(c + d*x)) + 2*a**2/(a*b**3*d + b**4*d*sin(c + d*x)) + 2*a*b*log(a/b + sin(
c + d*x))*sin(c + d*x)/(a*b**3*d + b**4*d*sin(c + d*x)) - 2*b**2*sin(c + d*
x)**2/(a*b**3*d + b**4*d*sin(c + d*x)) - b**2*cos(c + d*x)**2/(a*b**3*d + b
**4*d*sin(c + d*x)), True))
```

3.440 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{bd(a+b\sin(c+dx))}$$

[Out] -1/b/d/(a+b*sin(d*x+c))

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2668, 32\}$

$$-\frac{1}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^2,x]

[Out] -(1/(b*d*(a + b*Sin[c + d*x])))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{1}{bd(a+b\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{1}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^2,x]

[Out] -(1/(b*d*(a + b*Sin[c + d*x])))

fricas [A] time = 0.45, size = 20, normalized size = 1.00

$$-\frac{1}{b^2 d\sin(dx+c)+abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*d*sin(d*x + c) + a*b*d)
```

giac [A] time = 0.36, size = 20, normalized size = 1.00

$$-\frac{1}{(b\sin(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((b*sin(d*x + c) + a)*b*d)
```

maple [A] time = 0.13, size = 21, normalized size = 1.05

$$-\frac{1}{bd\,(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/b/d/(a+b*sin(d*x+c))
```

maxima [A] time = 0.31, size = 20, normalized size = 1.00

$$-\frac{1}{(b\sin(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*sin(d*x + c) + a)*b*d)
```

mupad [B] time = 5.07, size = 20, normalized size = 1.00

$$-\frac{1}{b\,d\,\left(a+b\,\sin(c+d\,x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] -1/(b*d*(a + b*sin(c + d*x)))
```

sympy [A] time = 1.24, size = 51, normalized size = 2.55

$$\begin{cases} \frac{x\cos(c)}{a^2} & \text{for } b=0 \wedge d=0 \\ \frac{\sin(c+dx)}{a^2d} & \text{for } b=0 \\ \frac{x\cos(c)}{(a+b\sin(c))^2} & \text{for } d=0 \\ -\frac{1}{abd+b^2d\sin(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*cos(c)/a**2, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**2*d), Eq(
b, 0)), (x*cos(c)/(a + b*sin(c))**2, Eq(d, 0)), (-1/(a*b*d + b**2*d*sin(c +
 d*x)), True))
```

3.441 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^2} \, dx$

Optimal. Leaf size=104

$$\frac{b}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{2ab\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^2/d+1/2*ln(1+sin(d*x+c))/(a-b)^2/d-2*a*b*ln(a+b
*sin(d*x+c))/(a^2-b^2)^2/d+b/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 710, 801}

$$\frac{b}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{2ab\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2
*d) - (2*a*b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) + b/((a^2 - b^2)*d*
(a + b*Sin[c + d*x]))
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)^2(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{b\,\text{Subst}\left(\int \frac{a-x}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{(a^2-b^2)\,d}$$

$$= \frac{b}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{b\,\text{Subst}\left(\int \left(\frac{a-b}{2b(a+b)(b-x)} - \frac{2a}{(a-b)(a+b)(a+x)} + \frac{a+b}{2(a-b)b(b}\right.\right.}{(a^2-b^2)\,d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{2ab\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d} + \frac{}{(a^2}$$

Mathematica [A] time = 0.21, size = 102, normalized size = 0.98

$$\frac{b\left(\frac{1}{(a^2-b^2)(a+b\sin(c+dx))} - \frac{\log(1-\sin(c+dx))}{2b(a+b)^2} + \frac{\log(\sin(c+dx)+1)}{2b(a-b)^2} - \frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (b*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a
- b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 -
 b^2)*(a + b*Sin[c + d*x]))))/d
```

fricas [A] time = 0.54, size = 188, normalized size = 1.81

$$\frac{2\,a^2b - 2\,b^3 - 4\left(ab^2\sin(dx+c) + a^2b\right)\log(b\sin(dx+c)+a) + \left(a^3 + 2\,a^2b + ab^2 + \left(a^2b + 2\,ab^2 + b^3\right)\sin(d}{2\left(\left(a^4b - 2\,a^2b^3 + b^5\right)d\sin(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*b - 2*b^3 - 4*(a*b^2*sin(d*x + c) + a^2*b)*log(b*sin(d*x + c) +
a) + (a^3 + 2*a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*sin(d*x + c))*log(sin
(d*x + c) + 1) - (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*sin(d*x +
 c))*log(-sin(d*x + c) + 1))/((a^4*b - 2*a^2*b^3 + b^5)*d*sin(d*x + c) + (a
^5 - 2*a^3*b^2 + a*b^4)*d)
```

giac [A] time = 1.28, size = 147, normalized size = 1.41

$$-\frac{\frac{4\,ab^2\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5} - \frac{\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2} + \frac{\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2} - \frac{2\left(2\,ab^2\sin(dx+c)+3\,a^2b-b^3\right)}{(a^4-2\,a^2b^2+b^4)(b\sin(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b^2*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - log(
abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + log(abs(sin(d*x + c) - 1))/(a^
2 + 2*a*b + b^2) - 2*(2*a*b^2*sin(d*x + c) + 3*a^2*b - b^3)/((a^4 - 2*a^2*b
^2 + b^4)*(b*sin(d*x + c) + a)))/d
```

maple [A] time = 0.26, size = 101, normalized size = 0.97

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{2d\left(a+b\right)^{2}}+\frac{b}{d\left(a+b\right)\left(a-b\right)\left(a+b\sin\left(dx+c\right)\right)}-\frac{2ab\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)^{2}\left(a-b\right)^{2}}+\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{2\left(a-b\right)^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d/(a+b)^2*ln(sin(d*x+c)-1)+1/d*b/(a+b)/(a-b)/(a+b*sin(d*x+c))-2/d*a*b/
(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))+1/2*ln(1+sin(d*x+c))/(a-b)^2/d
```

maxima [A] time = 0.32, size = 118, normalized size = 1.13

$$-\frac{\frac{4\,ab\log(b\sin(dx+c)+a)}{a^{4}-2\,a^{2}b^{2}+b^{4}}-\frac{2\,b}{a^{3}-ab^{2}+\left(a^{2}b-b^{3}\right)\sin(dx+c)}-\frac{\log(\sin(dx+c)+1)}{a^{2}-2\,ab+b^{2}}+\frac{\log(\sin(dx+c)-1)}{a^{2}+2\,ab+b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*b*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - 2*b/(a^3 - a*
b^2 + (a^2*b - b^3)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^
2) + log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

mupad [B] time = 0.21, size = 98, normalized size = 0.94

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)^{2}}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)^{2}}+\frac{b}{d\,\left(a^{2}-b^{2}\right)\,\left(a+b\,\sin\left(c+d\,x\right)\right)}-\frac{2\,a\,b\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,{\left(a^{2}-b^{2}\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] log(sin(c + d*x) + 1)/(2*d*(a - b)^2) - log(sin(c + d*x) - 1)/(2*d*(a + b)^
2) + b/(d*(a^2 - b^2)*(a + b*sin(c + d*x))) - (2*a*b*log(a + b*sin(c + d*x)
))/(d*(a^2 - b^2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

3.442 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^2} dx$

Optimal. Leaf size=177

$$-\frac{b\left(a^2+3b^2\right)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{4ab^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{(a+3b)\log(}{4d(}$$

```
[Out] -1/4*(a+3*b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/4*(a-3*b)*ln(1+sin(d*x+c))/(a-b)^
3/d+4*a*b^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/2*b*(a^2+3*b^2)/(a^2-b^2)^2/
d/(a+b*sin(d*x+c))-1/2*sec(d*x+c)^2*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d
*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 741, 801}

$$-\frac{b\left(a^2+3b^2\right)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{4ab^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{(a+3b)\log(}{4d(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a + 3*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3*d) + ((a - 3*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^3*d) + (4*a*b^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2
)^3*d) - (b*(a^2 + 3*b^2))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[
c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{b\,\text{Subst}\left(\int \frac{a^2-3b^2+2ax}{(a+x)^2\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{b\,\text{Subst}\left(\int \left(\frac{(a-b)(a+3b)}{2b(a+b)^2(b-x)} + \frac{a^2+3b^2}{(a-b)(a+b)(a+x)^2} + \frac{}{(a-b)^2(}\right.\right.}{2\left(a^2-b\right.}$$

$$= -\frac{(a+3b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-3b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{4ab^3\log(a+b\sin(}{\left(a^2-b^2\right)^3 d}$$

Mathematica [A] time = 1.75, size = 222, normalized size = 1.25

$$\frac{-b\left(-a^2-3b^2\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))} - \frac{\log(1-\sin(c+dx))}{2b(a+b)^2} + \frac{\log(\sin(c+dx)+1)}{2b(a-b)^2} - \frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}\right) + \frac{a((a-b)\log(1-\sin(c+dx))-(a}{}}{2d\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a*((a - b)*Log[1 - Sin[c + d*x]] - (a + b)*Log[1 + Sin[c + d*x]] + 2*b*Lo
g[a + b*Sin[c + d*x]]))/((a - b)*(a + b)) + (Sec[c + d*x]^2*(b - a*Sin[c +
d*x]))/(a + b*Sin[c + d*x]) - b*(-a^2 - 3*b^2)*(-1/2*Log[1 - Sin[c + d*x]]/
(b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[
c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x]))))/(
2*(-a^2 + b^2)*d)
```

fricas [B] time = 0.58, size = 381, normalized size = 2.15

$$-\frac{2\,a^4b-4\,a^2b^3+2\,b^5+2\left(a^4b+2\,a^2b^3-3\,b^5\right)\cos\left(dx+c\right)^2-16\left(ab^4\cos\left(dx+c\right)^2\sin\left(dx+c\right)+a^2b^3\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(a^4*b + 2*a^2*b^3 - 3*b^5)*cos(d*x +
 c)^2 - 16*(a*b^4*cos(d*x + c)^2*sin(d*x + c) + a^2*b^3*cos(d*x + c)^2)*log
(b*sin(d*x + c) + a) - ((a^4*b - 6*a^2*b^3 - 8*a*b^4 - 3*b^5)*cos(d*x + c)^
2*sin(d*x + c) + (a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^2)*lo
g(sin(d*x + c) + 1) + ((a^4*b - 6*a^2*b^3 + 8*a*b^4 - 3*b^5)*cos(d*x + c)^2
*sin(d*x + c) + (a^5 - 6*a^3*b^2 + 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^2)*log
(-sin(d*x + c) + 1) - 2*(a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)^2*sin(d*x + c) + (a^7 - 3*a^5*b^
2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2)
```

giac [A] time = 0.99, size = 244, normalized size = 1.38

$$\frac{\frac{16\,ab^4\log\left(|b\sin(dx+c)+a|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7} + \frac{(a-3\,b)\log\left(|\sin(dx+c)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{(a+3\,b)\log\left(|\sin(dx+c)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\left(a^2b\sin(dx+c)^2+3\,b^3\sin(dx+c)^2+a^3\sin(dx+c)-ab^2\sin(a\right.}{\left(a^4-2\,a^2b^2+b^4\right)\left(b\sin(dx+c)^3+a\sin(dx+c)^2-b\sin(\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(16*a*b^4*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7) + (a - 3*b)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3
) - (a + 3*b)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) -
2*(a^2*b*sin(d*x + c)^2 + 3*b^3*sin(d*x + c)^2 + a^3*sin(d*x + c) - a*b^2*s
in(d*x + c) - 2*a^2*b - 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)^3 +
 a*sin(d*x + c)^2 - b*sin(d*x + c) - a)))/d
```

maple [A] time = 0.32, size = 192, normalized size = 1.08

$$-\frac{1}{4d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{4d\left(a+b\right)^{3}}-\frac{3\ln\left(\sin\left(dx+c\right)-1\right)b}{4d\left(a+b\right)^{3}}-\frac{b^{3}}{d\left(a+b\right)^{2}\left(a-b\right)^{2}\left(a+b\sin\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/(a+b)^2/(sin(d*x+c)-1)-1/4/d/(a+b)^3*ln(sin(d*x+c)-1)*a-3/4/d/(a+b)^
3*ln(sin(d*x+c)-1)*b-1/d*b^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))+4/d*b^3*a/(a+
b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-1/4/d/(a-b)^2/(1+sin(d*x+c))+1/4/d/(a-b)^3*
ln(1+sin(d*x+c))*a-3/4/d/(a-b)^3*ln(1+sin(d*x+c))*b
```

maxima [A] time = 0.34, size = 275, normalized size = 1.55

$$\frac{\frac{16\,ab^{3}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}+\frac{\left(a-3\,b\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{\left(a+3\,b\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{2\left(2\,a^{2}b+2\,b^{3}-\left(a^{2}b+3\,b^{3}\right)\sin\left(dx+c\right.\right.}{a^{5}-2\,a^{3}b^{2}+ab^{4}-\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\sin\left(dx+c\right)^{3}-\left(a^{5}-2\,a^{3}b^{2}\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(16*a*b^3*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) +
 (a - 3*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (a + 3*b
)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(2*a^2*b + 2*b^
3 - (a^2*b + 3*b^3)*sin(d*x + c)^2 - (a^3 - a*b^2)*sin(d*x + c))/(a^5 - 2*a
^3*b^2 + a*b^4 - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)^3 - (a^5 - 2*a^3*b^
2 + a*b^4)*sin(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)))/d
```

mupad [B] time = 5.47, size = 227, normalized size = 1.28

$$\frac{\frac{\sin\left(c+d\,x\right)^{2}\left(a^{2}\,b+3\,b^{3}\right)}{2\left(a^{4}-2\,a^{2}\,b^{2}+b^{4}\right)}-\frac{a^{2}\,b+b^{3}}{\left(a^{2}-b^{2}\right)^{2}}+\frac{a\,\sin\left(c+d\,x\right)}{2\left(a^{2}-b^{2}\right)}}{d\left(-b\sin\left(c+d\,x\right)^{3}-a\sin\left(c+d\,x\right)^{2}+b\,\sin\left(c+d\,x\right)+a\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\left(\frac{b}{2\left(a+b\right)^{3}}+\frac{1}{4\left(a+b\right)^{2}}\right)}{d}+\frac{\ln\left(\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((sin(c + d*x)^2*(a^2*b + 3*b^3))/(2*(a^4 + b^4 - 2*a^2*b^2)) - (a^2*b + b^
3)/(a^2 - b^2)^2 + (a*sin(c + d*x))/(2*(a^2 - b^2)))/(d*(a + b*sin(c + d*x)
 - a*sin(c + d*x)^2 - b*sin(c + d*x)^3)) - (log(sin(c + d*x) - 1)*(b/(2*(a
+ b)^3) + 1/(4*(a + b)^2)))/d + (log(sin(c + d*x) + 1)*(a - 3*b))/(4*d*(a -
 b)^3) + (4*a*b^3*log(a + b*sin(c + d*x)))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

3.443 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=269

$$-\frac{3\left(a^2+4ab+5b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^4}+\frac{3\left(a^2-4ab+5b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^4}-\frac{\sec^4(c+dx)(b-a\sin(c+}{4d\left(a^2-b^2\right)(a+b\sin(c+}$$

```
[Out] -3/16*(a^2+4*a*b+5*b^2)*ln(1-sin(d*x+c))/(a+b)^4/d+3/16*(a^2-4*a*b+5*b^2)*l
n(1+sin(d*x+c))/(a-b)^4/d-6*a*b^5*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d-3/8*b*(a
^4-4*a^2*b^2-5*b^4)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-1/4*sec(d*x+c)^4*(b-a*si
n(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))+1/8*sec(d*x+c)^2*(b*(a^2+5*b^2)+3*a*
(a^2-3*b^2)*sin(d*x+c))/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.32, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2668, 741, 823, 801}

$$-\frac{3b\left(-4a^2b^2+a^4-5b^4\right)}{8d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{6ab^5\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{3\left(a^2+4ab+5b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^4}+\frac{3\left(a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*(a^2 + 4*a*b + 5*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^4*d) + (3*(a^2
 - 4*a*b + 5*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^4*d) - (6*a*b^5*Log[a
+ b*Sin[c + d*x]])/((a^2 - b^2)^4*d) - (3*b*(a^4 - 4*a^2*b^2 - 5*b^4))/(8*(
a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x])
)/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(b*(a^2 + 5*b^2)
 + 3*a*(a^2 - 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])
)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{b^3\,\text{Subst}\left(\int \frac{3a^2-5b^2+4ax}{(a+x)^2\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\left(b\left(a^2+5b^2\right)+3a\left(a^2-3b^2\right)\sin(c-\right.}{8\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\left(b\left(a^2+5b^2\right)+3a\left(a^2-3b^2\right)\sin(c-\right.}{8\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))}$$

$$= -\frac{3\left(a^2+4ab+5b^2\right)\log(1-\sin(c+dx))}{16(a+b)^4 d} + \frac{3\left(a^2-4ab+5b^2\right)\log(1+\sin(c+dx))}{16(a-b)^4 d}$$

Mathematica [A] time = 6.11, size = 406, normalized size = 1.51

$$\frac{b^5\left(\frac{\sec^4(c+dx)\left(b^2-ab\sin(c+dx)\right)}{4b^6\left(b^2-a^2\right)(a+b\sin(c+dx))} - \frac{\frac{\left(6a^2\left(a^2-3b^2\right)-3\left(a^4-2a^2b^2+5b^4\right)\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2b(a-b)^2}-\frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}\right)-6a\left(a^2-\right.}{2b^2\left(b^2-a^2\right)}}{4b^2(}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (b^5*((Sec[c + d*x]^4*(b^2 - a*b*Sin[c + d*x]))/(4*b^6*(-a^2 + b^2)*(a + b*
Sin[c + d*x])) - (-1/2*(Sec[c + d*x]^2*(4*a^2*b^2 - b^2*(3*a^2 - 5*b^2) - b
*(4*a*b^2 - a*(3*a^2 - 5*b^2))*Sin[c + d*x]))/(b^4*(-a^2 + b^2)*(a + b*Sin[
c + d*x])) + (-6*a*(a^2 - 3*b^2)*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)) +
Log[1 + Sin[c + d*x]]/(2*(a - b)*b) - Log[a + b*Sin[c + d*x]]/(a^2 - b^2))
+ (6*a^2*(a^2 - 3*b^2) - 3*(a^4 - 2*a^2*b^2 + 5*b^4))*(-1/2*Log[1 - Sin[c +
 d*x]]/(b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a +
 b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x
]))))/(2*b^2*(-a^2 + b^2)))/(4*b^2*(-a^2 + b^2))))/d
```

fricas [B] time = 0.77, size = 527, normalized size = 1.96

$$-\frac{4a^6b-12a^4b^3+12a^2b^5-4b^7+6\left(a^6b-5a^4b^3-a^2b^5+5b^7\right)\cos(dx+c)^4-2\left(a^6b+3a^4b^3-9a^2b^5+5b^7\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 6*(a^6*b - 5*a^4*b^3 - a
^2*b^5 + 5*b^7)*cos(d*x + c)^4 - 2*(a^6*b + 3*a^4*b^3 - 9*a^2*b^5 + 5*b^7)*
cos(d*x + c)^2 + 96*(a*b^6*cos(d*x + c)^4*sin(d*x + c) + a^2*b^5*cos(d*x +
c)^4)*log(b*sin(d*x + c) + a) - 3*((a^6*b - 5*a^4*b^3 + 15*a^2*b^5 + 16*a*b
^6 + 5*b^7)*cos(d*x + c)^4*sin(d*x + c) + (a^7 - 5*a^5*b^2 + 15*a^3*b^4 + 1
6*a^2*b^5 + 5*a*b^6)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) + 3*((a^6*b - 5*
a^4*b^3 + 15*a^2*b^5 - 16*a*b^6 + 5*b^7)*cos(d*x + c)^4*sin(d*x + c) + (a^7
 - 5*a^5*b^2 + 15*a^3*b^4 - 16*a^2*b^5 + 5*a*b^6)*cos(d*x + c)^4)*log(-sin(
d*x + c) + 1) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + 3*(a^7 - 5*a^5
*b^2 + 7*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b
^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^4*sin(d*x + c) + (a^9 - 4*
a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^4)
```

giac [A] time = 0.44, size = 460, normalized size = 1.71

$$-\frac{\frac{96\,ab^6\log(|b\sin(dx+c)+a|)}{a^8b-4\,a^6b^3+6\,a^4b^5-4\,a^2b^7+b^9}-\frac{3\left(a^2-4\,ab+5\,b^2\right)\log(|\sin(dx+c)+1|)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{3\left(a^2+4\,ab+5\,b^2\right)\log(|\sin(dx+c)-1|)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{16\left(6\,ab^6\sin(dx+c)+7\cdots\right.}{\left(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(96*a*b^6*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3 + 6*a^4*b^5
 - 4*a^2*b^7 + b^9) - 3*(a^2 - 4*a*b + 5*b^2)*log(abs(sin(d*x + c) + 1))/(a
^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 3*(a^2 + 4*a*b + 5*b^2)*log(abs
(sin(d*x + c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(6*a*b
^6*sin(d*x + c) + 7*a^2*b^5 - b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*(b*sin(d*x + c) + a)) + 2*(36*a*b^5*sin(d*x + c)^4 + 3*a^6*sin(d*x
 + c)^3 - 15*a^4*b^2*sin(d*x + c)^3 + 5*a^2*b^4*sin(d*x + c)^3 + 7*b^6*sin(
d*x + c)^3 + 16*a^3*b^3*sin(d*x + c)^2 - 88*a*b^5*sin(d*x + c)^2 - 5*a^6*si
n(d*x + c) + 17*a^4*b^2*sin(d*x + c) - 3*a^2*b^4*sin(d*x + c) - 9*b^6*sin(d
*x + c) + 4*a^5*b - 24*a^3*b^3 + 56*a*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 -
4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.31, size = 331, normalized size = 1.23

$$\frac{1}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{7b}{16d\left(a+b\right)^{3}\left(\sin\left(dx+c\right)-1\right)}-\frac{3a}{16d\left(a+b\right)^{3}\left(\sin\left(dx+c\right)-1\right)}-\frac{3\ln\left(\sin\left(dx+\cdots\right.\right.}{16d\left(a+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/16/d/(a+b)^2/(sin(d*x+c)-1)^2-7/16/d/(a+b)^3/(sin(d*x+c)-1)*b-3/16/d/(a+b
)^3/(sin(d*x+c)-1)*a-3/16/d/(a+b)^4*ln(sin(d*x+c)-1)*a^2-3/4/d/(a+b)^4*ln(s
in(d*x+c)-1)*a*b-15/16/d/(a+b)^4*ln(sin(d*x+c)-1)*b^2+1/d*b^5/(a+b)^3/(a-b)
^3/(a+b*sin(d*x+c))-6/d*b^5*a/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))-1/16/d/(a-
b)^2/(1+sin(d*x+c))^2+7/16/d/(a-b)^3/(1+sin(d*x+c))*b-3/16/d/(a-b)^3/(1+sin
(d*x+c))*a+3/16/d/(a-b)^4*ln(1+sin(d*x+c))*a^2-3/4/d/(a-b)^4*ln(1+sin(d*x+c
))*a*b+15/16/d/(a-b)^4*ln(1+sin(d*x+c))*b^2
```

maxima [A] time = 0.34, size = 505, normalized size = 1.88

$$-\frac{\frac{96\,ab^5\log(b\sin(dx+c)+a)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{3\left(a^2-4\,ab+5\,b^2\right)\log(\sin(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{3\left(a^2+4\,ab+5\,b^2\right)\log(\sin(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(4\,a^{\cdots}\right.}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6+\left(a^6b-3\,a\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/16*(96*a*b^5*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^
2*b^6 + b^8) - 3*(a^2 - 4*a*b + 5*b^2)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b
 + 6*a^2*b^2 - 4*a*b^3 + b^4) + 3*(a^2 + 4*a*b + 5*b^2)*log(sin(d*x + c) -
1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(4*a^4*b - 20*a^2*b^3 -
8*b^5 + 3*(a^4*b - 4*a^2*b^3 - 5*b^5)*sin(d*x + c)^4 + 3*(a^5 - 4*a^3*b^2 +
 3*a*b^4)*sin(d*x + c)^3 - (5*a^4*b - 28*a^2*b^3 - 25*b^5)*sin(d*x + c)^2 -
 (5*a^5 - 16*a^3*b^2 + 11*a*b^4)*sin(d*x + c))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4
 - a*b^6 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)^5 + (a^7 - 3*
a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^4 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*
b^5 - b^7)*sin(d*x + c)^3 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x
 + c)^2 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)))/d
```

mupad [**B**] time = 5.94, size = 449, normalized size = 1.67

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{3\,b^2}{8\,{\left(a-b\right)}^4}-\frac{3\,b}{8\,{\left(a-b\right)}^3}+\frac{3}{16\,{\left(a-b\right)}^2}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{3\,b}{8\,{\left(a+b\right)}^3}+\frac{3}{16\,{\left(a+b\right)}^2}+\frac{3\,b^2}{8\,{\left(a+b\right)}^4}\right)}{d}+\frac{-a^4\,b+5\,a\cdots}{2\,\left(a^2-b^2\right)\,\left(a^4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (log(sin(c + d*x) + 1)*((3*b^2)/(8*(a - b)^4) - (3*b)/(8*(a - b)^3) + 3/(16
*(a - b)^2)))/d - (log(sin(c + d*x) - 1)*((3*b)/(8*(a + b)^3) + 3/(16*(a +
b)^2) + (3*b^2)/(8*(a + b)^4)))/d + ((2*b^5 - a^4*b + 5*a^2*b^3)/(2*(a^2 -
b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (3*sin(c + d*x)^3*(3*a*b^2 - a^3))/(8*(a^4
+ b^4 - 2*a^2*b^2)) + (3*sin(c + d*x)^4*(5*b^5 - a^4*b + 4*a^2*b^3))/(8*(a^
6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)*(11*a*b^2 - 5*a^3))/(8*(a
^4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^2*(25*b^5 - 5*a^4*b + 28*a^2*b^3))/(
8*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(a + b*sin(c + d*x) - 2*a*sin(c
+ d*x)^2 + a*sin(c + d*x)^4 - 2*b*sin(c + d*x)^3 + b*sin(c + d*x)^5)) - (6*
a*b^5*log(a + b*sin(c + d*x)))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^
6*b^2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

3.444 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=187

$$\frac{10a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{5\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(4a^2-3b^2\right)\sin(c+dx)\right)}{8b^5d}-\frac{5x\left(8a^4-12a\right.}{8b}$$

```
[Out] -5/8*(8*a^4-12*a^2*b^2+3*b^4)*x/b^6+10*a*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/
2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^6/d+5/12*cos(d*x+c)^3*(4*a-3*b*sin(d*x+c))
/b^3/d-cos(d*x+c)^5/b/d/(a+b*sin(d*x+c))-5/8*cos(d*x+c)*(8*a*(a^2-b^2)-b*(4
*a^2-3*b^2)*sin(d*x+c))/b^5/d
```

Rubi [A] time = 0.37, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2693, 2865, 2735, 2660, 618, 204}

$$\frac{10a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{5\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(4a^2-3b^2\right)\sin(c+dx)\right)}{8b^5d}-\frac{5x\left(-12a^2b^2\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-5*(8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^6) + (10*a*(a^2 - b^2)^(3/2)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + (5*Cos[c + d*x]^3*(4
*a - 3*b*Sin[c + d*x]))/(12*b^3*d) - Cos[c + d*x]^5/(b*d*(a + b*Sin[c + d*x
])) - (5*Cos[c + d*x]*(8*a*(a^2 - b^2) - b*(4*a^2 - 3*b^2)*Sin[c + d*x]))/(
8*b^5*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} - \frac{5\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} - \frac{5\int \frac{\cos^2(c+dx)\left(-ab-\left(4a^2-3\right.\right.}{a+b\sin(c+dx}}{4b^3}$$

$$= \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} - \frac{5\cos(c+dx)\left(8a\left(a^2-\right.\right.}{}$$

$$= -\frac{5\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} + \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+}$$

$$= -\frac{5\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} + \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+}$$

$$= -\frac{5\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} + \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+}$$

$$= -\frac{5\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} + \frac{10a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} + \frac{5\cos^3(c+dx}{}$$

Mathematica [B] time = 6.52, size = 3679, normalized size = 19.67

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]^5*(-((b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*(b/(a
 + b) - (b*Sin[c + d*x])/(a + b))^(7/2))/(((a*b)/(a - b) - b^2/(a - b))*((a
*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x]))) - ((48*Sqrt[2]*(a - b)*b^
3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*Sqrt[b/(a + b) - (b*Sin[c
 + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/
(2*b))^(7/2)*((7*(3/(16*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
```

```
 b)))/(2*b))^3) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(
2*b))^(-1)))/12 + (35*b^4*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b
)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) + (
2*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3)/(15*b^3) - (Sqrt[2
]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(S
qrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])
))/(128*(a - b)^4*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^4*(1 + ((a - b)
*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(7*(a + b)^2*(a^2 -
 b^2)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) + (5*a*b^2*
((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*Sqrt[b/(a + b
) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x]
)/(a - b)))/(2*b))^(7/2)*((5/(16*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)))/(2*b))^3) + 5/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
 - b)))/(2*b))^(-1))/2 - (15*b^3*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b
^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
 b)))/(2*b)])))/(64*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3*(
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(5*(a +
 b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)
/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b))^(3/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(
a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((3*(5/(8*(1 + ((a - b)*(
-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(6*(1 + ((a - b)*(-
(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/8 + (15*b^2*(((a - b)*(-(b
/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sq
rt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]
*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*
(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(64*(a - b)^2*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c
+ d*x])/(a - b)))/(2*b))^3)))/(3*(a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Si
n[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b
*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c +
d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*
b))^(7/2)*((5*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(8*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b))
 - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)))/(2*b))^(7/2)) + (15/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c +
d*x])/(a - b)))/(2*b))^3) + 5/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(
a - b)))/(2*b))^(-1))/6))/((a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c +
d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a + b
)) - b^2/(a + b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*((2*Sqrt[a - b]*ArcT
anh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[a + b
]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[a + b]) - (2*Sqrt[-
((a*b)/(a + b)) - b^2/(a + b)]*ArcTanh[(Sqrt[-((a*b)/(a + b)) - b^2/(a + b)
]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[-((a*b)/(a - b)) + b
^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[-((a*b)/(
a - b)) + b^2/(a - b)])))/b) + (2*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Si
n[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a -
b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)*((Sqrt[b]*ArcSin
h[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt
[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)) + 1/(
```

```
2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)))))/(b*(a
+ b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b])))/b) + (4*Sq
rt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b)
- (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/
(a - b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(4*Sqrt[2]*Sqrt[a - b]*Sqrt
[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*
Sin[c + d*x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)*(-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Si
n[c + d*x])/(a - b)))/(2*b))^(-1))/4))/((a + b)^2*Sqrt[((a + b)*(b/(a + b)
- (b*Sin[c + d*x])/(a + b)))/b])))/b))/b))/b))/(a^2 - b^2))/(((a*b)/(a - b)
 - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))))))/(d*(1 - (a + b*Sin[c + d*x
])/(a - b))^(5/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/2))
```

fricas [A] time = 0.54, size = 599, normalized size = 3.20

$$\left[\frac{6\,b^5\cos(dx+c)^5 - 5\left(4\,a^2b^3 - 3\,b^5\right)\cos(dx+c)^3 - 15\left(8\,a^5 - 12\,a^3b^2 + 3\,ab^4\right)dx - 60\left(a^4 - a^2b^2 + \left(a^3b - ab^3\right)s\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(6*b^5*cos(d*x + c)^5 - 5*(4*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 15*(8*
a^5 - 12*a^3*b^2 + 3*a*b^4)*d*x - 60*(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d
*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-
a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 15*(8*
a^4*b - 12*a^2*b^3 + 3*b^5)*cos(d*x + c) + 5*(2*a*b^4*cos(d*x + c)^3 - 3*(8
*a^4*b - 12*a^2*b^3 + 3*b^5)*d*x - 3*(4*a^3*b^2 - 5*a*b^4)*cos(d*x + c))*si
n(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d), 1/24*(6*b^5*cos(d*x + c)^5 - 5*
(4*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 15*(8*a^5 - 12*a^3*b^2 + 3*a*b^4)*d*x
- 120*(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 15*(8*a^4*b - 12*a
^2*b^3 + 3*b^5)*cos(d*x + c) + 5*(2*a*b^4*cos(d*x + c)^3 - 3*(8*a^4*b - 12*
a^2*b^3 + 3*b^5)*d*x - 3*(4*a^3*b^2 - 5*a*b^4)*cos(d*x + c))*sin(d*x + c))/
(b^7*d*sin(d*x + c) + a*b^6*d)]
```

giac [B] time = 2.50, size = 469, normalized size = 2.51

$$-\frac{\frac{15\left(8\,a^4-12\,a^2b^2+3\,b^4\right)(dx+c)}{b^6} - \frac{240\left(a^5-2\,a^3b^2+ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6} + \frac{48\left(a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2b^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(15*(8*a^4 - 12*a^2*b^2 + 3*b^4)*(d*x + c)/b^6 - 240*(a^5 - 2*a^3*b^2
 + a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 48*(a^4*b*tan(1/2*d
*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a
^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*
c) + a)*a*b^5) + 2*(36*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 27*b^3*tan(1/2*d*x +
1/2*c)^7 + 96*a^3*tan(1/2*d*x + 1/2*c)^6 - 144*a*b^2*tan(1/2*d*x + 1/2*c)^6
 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 288*a^3
*tan(1/2*d*x + 1/2*c)^4 - 336*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1
```

```
/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^3 + 288*a^3*tan(1/2*d*x + 1/
2*c)^2 - 304*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 36*a^2*b*tan(1/2*d*x + 1/2*c) +
 27*b^3*tan(1/2*d*x + 1/2*c) + 96*a^3 - 112*a*b^2)/((tan(1/2*d*x + 1/2*c)^2
 + 1)^4*b^5))/d
```

maple [B] time = 0.25, size = 1021, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^2-2/d/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan(1/2*d*x+1/2*c)+9/4/d/b^2/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+1/4/d/b^2/(1+tan(1/2*d*x+1/2*c
)^2)^4*tan(1/2*d*x+1/2*c)^5-1/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*
x+1/2*c)^3-9/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-8/d/b^5/
(1+tan(1/2*d*x+1/2*c)^2)^4*a^3+28/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*a-10/d
/b^6*arctan(tan(1/2*d*x+1/2*c))*a^4+15/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^2
-2/d/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^4-15/4/d/b^2*a
rctan(tan(1/2*d*x+1/2*c))-2/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c
)*b+a)+4/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*
d*x+1/2*c)+10/d/b^6*a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2))-20/d/b^4*a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+10/d/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^7*a^2-8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^6*a^3+12/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a-3/d/
b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2-24/d/b^5/(1+tan(1/2
*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3+28/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)
^4*tan(1/2*d*x+1/2*c)^4*a+3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^3*a^2-24/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a^3+76/
3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a+3/d/b^4/(1+tan(1/
2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a^2-2/d/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)*a^3*tan(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.58, size = 2530, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^2,x)
```

```
[Out] - ((2*(15*a^4 + 3*b^4 - 20*a^2*b^2))/(3*b^5) - (5*tan(c/2 + (d*x)/2)^8*(b^4
 - 4*a^4 + 4*a^2*b^2))/(2*b^5) + (5*tan(c/2 + (d*x)/2)^6*(16*a^4 + 3*b^4 -
20*a^2*b^2))/(2*b^5) + (5*tan(c/2 + (d*x)/2)^2*(48*a^4 + 15*b^4 - 68*a^2*b^
2))/(6*b^5) + (5*tan(c/2 + (d*x)/2)^4*(72*a^4 + 15*b^4 - 100*a^2*b^2))/(6*b
```

^5) + (tan(c/2 + (d*x)/2)*(180*a^4 + 24*b^4 - 245*a^2*b^2))/(12*a*b^4) + (4
*tan(c/2 + (d*x)/2)^5*(15*a^4 + 3*b^4 - 20*a^2*b^2))/(a*b^4) + (tan(c/2 + (
d*x)/2)^9*(20*a^4 + 8*b^4 - 25*a^2*b^2))/(4*a*b^4) + (tan(c/2 + (d*x)/2)^7*
(60*a^4 + 16*b^4 - 85*a^2*b^2))/(2*a*b^4) + (tan(c/2 + (d*x)/2)^3*(300*a^4
+ 48*b^4 - 385*a^2*b^2))/(6*a*b^4))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 5*a*ta
n(c/2 + (d*x)/2)^2 + 10*a*tan(c/2 + (d*x)/2)^4 + 10*a*tan(c/2 + (d*x)/2)^6
+ 5*a*tan(c/2 + (d*x)/2)^8 + a*tan(c/2 + (d*x)/2)^10 + 8*b*tan(c/2 + (d*x)/
2)^3 + 12*b*tan(c/2 + (d*x)/2)^5 + 8*b*tan(c/2 + (d*x)/2)^7 + 2*b*tan(c/2 +
(d*x)/2)^9)) - (atan((((a^4*8i + b^4*3i - a^2*b^2*12i)*(((225*a^2*b^13)/2
- 900*a^4*b^11 + 2400*a^6*b^9 - 2400*a^8*b^7 + 800*a^10*b^5)/b^14 + (tan(c/
2 + (d*x)/2)*(450*a*b^15 - 5425*a^3*b^13 + 17800*a^5*b^11 - 24000*a^7*b^9 +
14400*a^9*b^7 - 3200*a^11*b^5))/(2*b^15) - (5*(a^4*8i + b^4*3i - a^2*b^2*1
2i)*((60*a*b^16 - 140*a^3*b^14 + 80*a^5*b^12)/b^14 - (5*(32*a^2*b^3 + (tan(
c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^15))*(a^4*8i + b^4*3i - a^
2*b^2*12i))/(8*b^6) + (tan(c/2 + (d*x)/2)*(640*a^2*b^16 - 1280*a^4*b^14 + 6
40*a^6*b^12))/(2*b^15)))/(8*b^6))*5i)/(8*b^6) + ((a^4*8i + b^4*3i - a^2*b^2
12i)(((225*a^2*b^13)/2 - 900*a^4*b^11 + 2400*a^6*b^9 - 2400*a^8*b^7 + 800
*a^10*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(450*a*b^15 - 5425*a^3*b^13 + 17800*a
^5*b^11 - 24000*a^7*b^9 + 14400*a^9*b^7 - 3200*a^11*b^5))/(2*b^15) + (5*(a^
4*8i + b^4*3i - a^2*b^2*12i)*((60*a*b^16 - 140*a^3*b^14 + 80*a^5*b^12)/b^14
+ (5*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^1
5))*(a^4*8i + b^4*3i - a^2*b^2*12i))/(8*b^6) + (tan(c/2 + (d*x)/2)*(640*a^2
*b^16 - 1280*a^4*b^14 + 640*a^6*b^12))/(2*b^15)))/(8*b^6))*5i)/(8*b^6))/((4
000*a^13 - 1875*a^3*b^10 + 12750*a^5*b^8 - 30875*a^7*b^6 + 35000*a^9*b^4 -
19000*a^11*b^2)/b^14 - (5*(a^4*8i + b^4*3i - a^2*b^2*12i)*(((225*a^2*b^13)/
2 - 900*a^4*b^11 + 2400*a^6*b^9 - 2400*a^8*b^7 + 800*a^10*b^5)/b^14 + (tan(
c/2 + (d*x)/2)*(450*a*b^15 - 5425*a^3*b^13 + 17800*a^5*b^11 - 24000*a^7*b^9
+ 14400*a^9*b^7 - 3200*a^11*b^5))/(2*b^15) - (5*(a^4*8i + b^4*3i - a^2*b^2
12i)((60*a*b^16 - 140*a^3*b^14 + 80*a^5*b^12)/b^14 - (5*(32*a^2*b^3 + (ta
n(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^15))*(a^4*8i + b^4*3i -
a^2*b^2*12i))/(8*b^6) + (tan(c/2 + (d*x)/2)*(640*a^2*b^16 - 1280*a^4*b^14 +
640*a^6*b^12))/(2*b^15)))/(8*b^6)))/(8*b^6) + (5*(a^4*8i + b^4*3i - a^2*b^
2*12i)*(((225*a^2*b^13)/2 - 900*a^4*b^11 + 2400*a^6*b^9 - 2400*a^8*b^7 + 80
0*a^10*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(450*a*b^15 - 5425*a^3*b^13 + 17800*
a^5*b^11 - 24000*a^7*b^9 + 14400*a^9*b^7 - 3200*a^11*b^5))/(2*b^15) + (5*(a
^4*8i + b^4*3i - a^2*b^2*12i)*((60*a*b^16 - 140*a^3*b^14 + 80*a^5*b^12)/b^1
4 + (5*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^
15))*(a^4*8i + b^4*3i - a^2*b^2*12i))/(8*b^6) + (tan(c/2 + (d*x)/2)*(640*a^
2*b^16 - 1280*a^4*b^14 + 640*a^6*b^12))/(2*b^15)))/(8*b^6)))/(8*b^6) + (tan
(c/2 + (d*x)/2)*(16000*a^14 + 2250*a^2*b^12 - 22500*a^4*b^10 + 86250*a^6*b^
8 - 162000*a^8*b^6 + 160000*a^10*b^4 - 80000*a^12*b^2))/b^15))*(a^4*8i + b^
4*3i - a^2*b^2*12i)*5i)/(4*b^6*d) - (10*a*atanh((1125*a^3*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2))/(3250*a^5*b - 1125*a^3*b^3 - (3125*a^7)/b + (1000
*a^9)/b^3 - 6250*a^6*tan(c/2 + (d*x)/2) - 2250*a^2*b^4*tan(c/2 + (d*x)/2) +
6500*a^4*b^2*tan(c/2 + (d*x)/2) + (2000*a^8*tan(c/2 + (d*x)/2))/b^2) + (10
00*a^5*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(3125*a^7*b + 1125*a^3*b^
5 - 3250*a^5*b^3 - (1000*a^9)/b - 2000*a^8*tan(c/2 + (d*x)/2) + 2250*a^2*b^
6*tan(c/2 + (d*x)/2) - 6500*a^4*b^4*tan(c/2 + (d*x)/2) + 6250*a^6*b^2*tan(c
/2 + (d*x)/2)) + (2250*a^2*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^
4*b^2)^(1/2))/(3250*a^5 - 1125*a^3*b^2 - (3125*a^7)/b^2 + (1000*a^9)/b^4 +
6500*a^4*b*tan(c/2 + (d*x)/2) - 2250*a^2*b^3*tan(c/2 + (d*x)/2) - (6250*a^6
*tan(c/2 + (d*x)/2))/b + (2000*a^8*tan(c/2 + (d*x)/2))/b^3) + (3125*a^4*tan
(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(3125*a^7 + 1125
*a^3*b^4 - 3250*a^5*b^2 - (1000*a^9)/b^2 + 6250*a^6*b*tan(c/2 + (d*x)/2) +
2250*a^2*b^5*tan(c/2 + (d*x)/2) - 6500*a^4*b^3*tan(c/2 + (d*x)/2) - (2000*a
^8*tan(c/2 + (d*x)/2))/b) + (1000*a^6*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2
*b^4 + 3*a^4*b^2)^(1/2))/(1000*a^9 - 1125*a^3*b^6 + 3250*a^5*b^4 - 3125*a^7
*b^2 + 2000*a^8*b*tan(c/2 + (d*x)/2) - 2250*a^2*b^7*tan(c/2 + (d*x)/2) + 65
00*a^4*b^5*tan(c/2 + (d*x)/2) - 6250*a^6*b^3*tan(c/2 + (d*x)/2)))*(-(a + b)

```
^3*(a - b)^3)^(1/2))/(b^6*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
[Out] Timed out
```

3.445 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=128

$$-\frac{6a\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{3x\left(2a^2-b^2\right)}{2b^4}+\frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d}-\frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))}$$

```
[Out] 3/2*(2*a^2-b^2)*x/b^4+3/2*cos(d*x+c)*(2*a-b*sin(d*x+c))/b^3/d-cos(d*x+c)^3/
b/d/(a+b*sin(d*x+c))-6*a*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(
a^2-b^2)^(1/2)/b^4/d
```

Rubi [A] time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2693, 2865, 2735, 2660, 618, 204}

$$-\frac{6a\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{3x\left(2a^2-b^2\right)}{2b^4}+\frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d}-\frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*(2*a^2 - b^2)*x)/(2*b^4) - (6*a*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + (3*Cos[c + d*x]*(2*a - b*Sin[c + d*x]))
/(2*b^3*d) - Cos[c + d*x]^3/(b*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{3\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{3\int \frac{-ab-\left(2a^2-b^2\right)\sin(c+d}{a+b\sin(c+dx)}}{2b^3}$$

$$= \frac{3\left(2a^2-b^2\right)x}{2b^4} + \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\left(3a\left(\right.\right.}{}$$

$$= \frac{3\left(2a^2-b^2\right)x}{2b^4} + \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\left(6a\left(\right.\right.}{}$$

$$= \frac{3\left(2a^2-b^2\right)x}{2b^4} + \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} + \frac{\left(12a\right.}{}$$

$$= \frac{3\left(2a^2-b^2\right)x}{2b^4} - \frac{6a\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d} + \frac{3\cos(c+dx)(2a-b\sin(c}{2b^3d}$$

Mathematica [B] time = 5.11, size = 448, normalized size = 3.50

$$\frac{\cos^3(c+dx)\left(\sqrt{a+b}\left(\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\left(\sqrt{a-b}\sqrt{1-\sin(c+dx)}\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}\left(-6a^2-3ab\sin(c+dx)+b^2\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]^3*(-12*a*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[c +
d*x]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))])]*S
qrt[1 - Sin[c + d*x]]*(a + b*Sin[c + d*x]) + Sqrt[a + b]*(12*a*Sqrt[a - b]*
ArcTanh[Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]/Sqrt[-((b*(-1 + Sin[c + d*x])
)/(a + b))]]*Sqrt[1 - Sin[c + d*x]]*(a + b*Sin[c + d*x]) + Sqrt[-((b*(-1 +
Sin[c + d*x]))/(a + b))]*(6*Sqrt[b]*(-2*a + b)*ArcSinh[(Sqrt[a - b]*Sqrt[-(
(b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*(a + b*Sin[c + d*x]) +
```

```
 Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(
-6*a^2 + 2*b^2 - 3*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)))))/(2*(a - b)^(3
/2)*b^2*Sqrt[a + b]*d*(1 - Sin[c + d*x])^(3/2)*Sqrt[-((b*(-1 + Sin[c + d*x]
))/(a + b))]*(-((b*(1 + Sin[c + d*x]))/(a - b)))^(3/2)*(a + b*Sin[c + d*x])
)
```

fricas [A] time = 0.48, size = 411, normalized size = 3.21

$$\left[\frac{b^3\cos\left(dx+c\right)^3+3\left(2\,a^3-ab^2\right)dx+3\left(ab\sin\left(dx+c\right)+a^2\right)\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2}{b^2\cos(dx+c)^2-2\,a}\right.}{2\left(b^5d\sin\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(b^3*cos(d*x + c)^3 + 3*(2*a^3 - a*b^2)*d*x + 3*(a*b*sin(d*x + c) + a^
2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 +
b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 3*(2*a^2*b -
 b^3)*cos(d*x + c) + 3*(a*b^2*cos(d*x + c) + (2*a^2*b - b^3)*d*x)*sin(d*x +
 c))/(b^5*d*sin(d*x + c) + a*b^4*d), 1/2*(b^3*cos(d*x + c)^3 + 3*(2*a^3 - a
*b^2)*d*x + 6*(a*b*sin(d*x + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
 c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 3*(2*a^2*b - b^3)*cos(d*x + c) +
 3*(a*b^2*cos(d*x + c) + (2*a^2*b - b^3)*d*x)*sin(d*x + c))/(b^5*d*sin(d*x
+ c) + a*b^4*d)]
```

giac [A] time = 1.40, size = 235, normalized size = 1.84

$$\frac{\frac{3\left(2\,a^2-b^2\right)(dx+c)}{b^4}-\frac{12\left(a^3-ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*a^2 - b^2)*(d*x + c)/b^4 - 12*(a^3 - a*b^2)*(pi*floor(1/2*(d*x +
c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))
/(sqrt(a^2 - b^2)*b^4) + 2*(b*tan(1/2*d*x + 1/2*c)^3 + 4*a*tan(1/2*d*x + 1/
2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 4*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^3)
 + 4*(a^2*b*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c) + a^3 - a*b^2)/
((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a*b^3))/d
```

maple [B] time = 0.24, size = 385, normalized size = 3.01

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4a}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{6\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+4/d/b^3/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*a+6/d/b^4*arctan(tan(1
```

```
/2*d*x+1/2*c))*a^2-3/d/b^2*arctan(tan(1/2*d*x+1/2*c))+2/d/b^2/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*d*x+1/2*c)-2/d/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan(1/2*d*x+1/2*c)+2/d/b^3/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^2-2/d/b/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)-6/d/b^4*a*(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.40, size = 601, normalized size = 4.70

$$\frac{\frac{2\left(3a^2-b^2\right)}{b^3}+\frac{6a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{b^3}+\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2-b^2\right)}{b^3}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(9a^2-2b^2\right)}{ab^2}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(3a^2-b^2\right)}{ab^2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(3\cdots\right.}{ab^2}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*(3*a^2 - b^2))/b^3 + (6*a^2*tan(c/2 + (d*x)/2)^4)/b^3 + (6*tan(c/2 + (d
*x)/2)^2*(2*a^2 - b^2))/b^3 + (tan(c/2 + (d*x)/2)*(9*a^2 - 2*b^2))/(a*b^2)
+ (4*tan(c/2 + (d*x)/2)^3*(3*a^2 - b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^5*(3
*a^2 - 2*b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 3*a*tan(c/2 + (d*x
)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6 + 4*b*tan(c/2 +
(d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) - (atan((432*a^5*tan(c/2 + (d*x)/2)
)/(216*a*b^4 + 432*a^5 - 648*a^3*b^2) - (648*a^3*tan(c/2 + (d*x)/2))/(216*a
*b^2 - 648*a^3 + (432*a^5)/b^2) + (216*a*tan(c/2 + (d*x)/2))/(216*a - (648*
a^3)/b^2 + (432*a^5)/b^4))*(a^2*2i - b^2*1i)*3i)/(b^4*d) + (6*a*atanh((432*
a^3*(b^2 - a^2)^(1/2))/(432*a^3*b - (432*a^5)/b - 864*a^4*tan(c/2 + (d*x)/2
) + 864*a^2*b^2*tan(c/2 + (d*x)/2)) + (864*a^2*tan(c/2 + (d*x)/2)*(b^2 - a^
2)^(1/2))/(432*a^3 - (432*a^5)/b^2 + 864*a^2*b*tan(c/2 + (d*x)/2) - (864*a^
4*tan(c/2 + (d*x)/2))/b) + (432*a^4*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(
432*a^5 - 432*a^3*b^2 + 864*a^4*b*tan(c/2 + (d*x)/2) - 864*a^2*b^3*tan(c/2
+ (d*x)/2)))*(b^2 - a^2)^(1/2))/(b^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.446 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=84

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}-\frac{x}{b^2}$$

```
[Out] -x/b^2-cos(d*x+c)/b/d/(a+b*sin(d*x+c))+2*a*arctan((b+a*tan(1/2*d*x+1/2*c))/
(a^2-b^2)^(1/2))/b^2/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2693, 2735, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}-\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(x/b^2) + (2*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt
[a^2 - b^2]*d) - Cos[c + d*x]/(b*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= -\frac{x}{b^2} - \frac{\cos(c+dx)}{bd(a+b\sin(c+dx))} + \frac{a\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= -\frac{x}{b^2} - \frac{\cos(c+dx)}{bd(a+b\sin(c+dx))} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}$$

$$= -\frac{x}{b^2} - \frac{\cos(c+dx)}{bd(a+b\sin(c+dx))} - \frac{(4a)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+\right.\right.}{b^2d}$$

$$= -\frac{x}{b^2} + \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d} - \frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}$$

Mathematica [B] time = 2.69, size = 414, normalized size = 4.93

$$\frac{\cos(c+dx)\left(\sqrt{a+b}\left((b-a)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\left(\sqrt{a-b}\,(a+b)\sqrt{1-\sin(c+dx)}\,\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}} + 2\sqrt{b}\,(a+b\sin\right.\right.\right.}{bd(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]*(-2*a*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[c + d*x
]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))])]*Sqrt
[1 - Sin[c + d*x]]*(a + b*Sin[c + d*x]) + Sqrt[a + b]*(2*a*Sqrt[a - b]*ArcT
anh[Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]/Sqrt[-((b*(-1 + Sin[c + d*x]))/(a
 + b))]]*Sqrt[1 - Sin[c + d*x]]*(a + b*Sin[c + d*x]) + (-a + b)*Sqrt[-((b*(
-1 + Sin[c + d*x]))/(a + b))]*(Sqrt[a - b]*(a + b)*Sqrt[1 - Sin[c + d*x]]*S
qrt[-((b*(1 + Sin[c + d*x]))/(a - b))] + 2*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqr
t[-((b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*(a + b*Sin[c + d*x
])))))/((a - b)^(3/2)*b*(a + b)^(3/2)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[-((b*(-
1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))]*(a + b
*Sin[c + d*x]))
```

fricas [A] time = 0.50, size = 388, normalized size = 4.62

$$\left[-\frac{2\left(a^2b-b^3\right)dx\sin(dx+c)+2\left(a^3-ab^2\right)dx+\left(ab\sin(dx+c)+a^2\right)\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(}{b^2}\right.}{2\left(\left(a^2b^3-b^5\right)d\sin(dx+c)+\left(a^3b^2-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b - b^3)*d*x*sin(d*x + c) + 2*(a^3 - a*b^2)*d*x + (a*b*sin(d*
x + c) + a^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*si
```

```
n(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*s
qrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2
*(a^2*b - b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d*sin(d*x + c) + (a^3*b^2 - a
*b^4)*d), -((a^2*b - b^3)*d*x*sin(d*x + c) + (a^3 - a*b^2)*d*x + (a*b*sin(d
*x + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))) + (a^2*b - b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d*sin(d*x +
 c) + (a^3*b^2 - a*b^4)*d)]
```

giac [A] time = 1.38, size = 126, normalized size = 1.50

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}\,b^2}-\frac{dx+c}{b^2}-\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a/(sqrt(a^2 - b^2)*b^2) - (d*x + c)/b^2 - 2*(b*tan
(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c
) + a)*a*b))/d
```

maple [A] time = 0.23, size = 153, normalized size = 1.82

$$-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)a}-\frac{2}{db\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2/d/b^2*arctan(tan(1/2*d*x+1/2*c))-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)/a*tan(1/2*d*x+1/2*c)-2/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)+2/d/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 5.56, size = 329, normalized size = 3.92

$$-\frac{b^2\,\sin\left(c+d\,x\right)+\frac{\left(2\,a^3\,\mathrm{atan}\left(\frac{\left(-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2\right)\,1{}\mathrm{i}}{\sqrt{b^2-a^2}\,\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)-a^2\,\sqrt{b^2-a^2}\,\left(c+d\,x\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{\sqrt{b^2-a^2}}-\frac{b\left(a\,\sqrt{b^2-a^2}\,1{}\mathrm{i}+a\,\cos\left(c+d\,x\right)\,\sqrt{b^2\right.}{}}{a\,b^2\,d\,\left(a+b\,\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^2,x)
```

```
[Out] -(b^2*sin(c + d*x) + ((2*a^3*atan(((2*b^2*sin(c/2 + (d*x)/2) - a^2*sin(c/2
+ (d*x)/2) + a*b*cos(c/2 + (d*x)/2))*1i)/((b^2 - a^2)^(1/2)*(a*cos(c/2 + (d
*x)/2) + 2*b*sin(c/2 + (d*x)/2)))) - a^2*(b^2 - a^2)^(1/2)*(c + d*x)*1i)*1i
)/(b^2 - a^2)^(1/2) - (b*(a*(b^2 - a^2)^(1/2)*1i + a*cos(c + d*x)*(b^2 - a^
2)^(1/2)*1i - 2*a^2*atan(((2*b^2*sin(c/2 + (d*x)/2) - a^2*sin(c/2 + (d*x)/2
) + a*b*cos(c/2 + (d*x)/2))*1i)/((b^2 - a^2)^(1/2)*(a*cos(c/2 + (d*x)/2) +
2*b*sin(c/2 + (d*x)/2))))*sin(c + d*x) + a*sin(c + d*x)*(b^2 - a^2)^(1/2)*(
c + d*x)*1i)*1i)/(b^2 - a^2)^(1/2))/(a*b^2*d*(a + b*sin(c + d*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.447 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=130

$$-\frac{6ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\sec(c+dx)\left(3ab-\left(a^2+2b^2\right)\sin(c+dx)\right)}{d\left(a^2-b^2\right)^2}$$

```
[Out] -6*a*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d
+b*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))-sec(d*x+c)*(3*a*b-(a^2+2*b^2)*si
n(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.21, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2694, 2866, 12, 2660, 618, 204}

$$-\frac{6ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\sec(c+dx)\left(3ab-\left(a^2+2b^2\right)\sin(c+dx)\right)}{d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-6*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (b*Sec[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (Sec[c + d
*x]*(3*a*b - (a^2 + 2*b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
```

```
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+dx)(-a+2b\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-(a^2+2b^2)\sin(c+dx)\right)}{(a^2-b^2)^2 d} + \frac{\int -$$

$$= \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-(a^2+2b^2)\sin(c+dx)\right)}{(a^2-b^2)^2 d} - \frac{(3a$$

$$= \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-(a^2+2b^2)\sin(c+dx)\right)}{(a^2-b^2)^2 d} - \frac{(6a$$

$$= \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-(a^2+2b^2)\sin(c+dx)\right)}{(a^2-b^2)^2 d} + \frac{(12$$

$$= -\frac{6ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2} d} + \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab$$

Mathematica [A] time = 1.15, size = 162, normalized size = 1.25

$$\frac{-\frac{6ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{b^3\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{(a+b)^2\left(\cos\left(\frac{1}{2}\right.\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-6*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5
/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
 + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (b^3*Cos[c + d*x]
)/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

fricas [A] time = 0.50, size = 538, normalized size = 4.14

$$\left[-\frac{2\,a^4b-4\,a^2b^3+2\,b^5+2\left(a^4b+a^2b^3-2\,b^5\right)\cos\left(dx+c\right)^2+3\left(ab^3\cos\left(dx+c\right)\sin\left(dx+c\right)+a^2b^2\cos\left(dx+c\right)\right.}{2\left(\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)d\cos\left(dx+c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x +
c)^2 + 3*(a*b^3*cos(d*x + c)*sin(d*x + c) + a^2*b^2*cos(d*x + c))*sqrt(-a^2
 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^5 - 2*a^3*b^2 + a*
b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*si
n(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)), -(a^4*b
 - 2*a^2*b^3 + b^5 + (a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^2 - 3*(a*b^3*co
s(d*x + c)*sin(d*x + c) + a^2*b^2*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^
4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(
d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c))]
```

giac [B] time = 2.55, size = 271, normalized size = 2.08

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab^2}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3\,ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a^4}{\left(a^5-2\,a^3b^2+ab^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2\,b\tan\left(\frac{1}{2}dx\right.\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/
2*c) + b)/sqrt(a^2 - b^2)))*a*b^2/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
 + (a^4*tan(1/2*d*x + 1/2*c)^3 + a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1
/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 + a^4*tan(1/2*d*x + 1/2*
c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) - b^4*tan(1/2*d*x + 1/2*c) - 2*a^3*b -
a*b^3)/((a^5 - 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d
*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/d
```

maple [A] time = 0.22, size = 222, normalized size = 1.71

$$-\frac{1}{d\left(a+b\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2b^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a-b\right)^2\left(a+b\right)^2\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)a}-\frac{}{d\left(a-b\right)^2\left(a+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-2/d*b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan(1/2*d*x+1/2*c)-2/d*b^3/(a-b)^2/(a+
b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-6/d*b^2/(a-b)^2/(a+b
)^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2))-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.40, size = 303, normalized size = 2.33

$$-\frac{\frac{2\left(2a^2b+b^3\right)}{\left(a^2-b^2\right)^2}-\frac{6b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4-2a^2b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-a^4+3a^2b^2+b^4\right)}{a\left(a^2-b^2\right)^2}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(a^4+a^2b^2+b^4\right)}{a\left(a^2-b^2\right)^2}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{6ab^2\operatorname{atan}\left(\frac{\frac{3ab^2\left(2a^4b-4a^2b^3+2b\cdots\right.}{(a+b)^{5/2}(a-b)^{5/2}}\cdots}{\cdots}\right.}{d\,(a+b)^{5/\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] - ((2*(2*a^2*b + b^3))/(a^2 - b^2)^2 - (6*b^3*tan(c/2 + (d*x)/2)^2)/(a^4 +
b^4 - 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(b^4 - a^4 + 3*a^2*b^2))/(a*(a^2 -
 b^2)^2) - (2*tan(c/2 + (d*x)/2)^3*(a^4 + b^4 + a^2*b^2))/(a*(a^2 - b^2)^2)
)/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (
d*x)/2)^3)) - (6*a*b^2*atan(((3*a*b^2*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a +
b)^(5/2)*(a - b)^(5/2)) + (6*a^2*b^2*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*
b^2))/((a + b)^(5/2)*(a - b)^(5/2)))/(6*a*b^2)))/(d*(a + b)^(5/2)*(a - b)^(
5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.448 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=193

$$\frac{b\sec^3(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{10ab^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{\sec(c$$

```
[Out] 10*a*b^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d
+b*sec(d*x+c)^3/(a^2-b^2)/d/(a+b*sin(d*x+c))-1/3*sec(d*x+c)^3*(5*a*b-(a^2+4
*b^2)*sin(d*x+c))/(a^2-b^2)^2/d+1/3*sec(d*x+c)*(15*a*b^3+(2*a^4-9*a^2*b^2-8
*b^4)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A] time = 0.37, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2694, 2866, 12, 2660, 618, 204}

$$\frac{10ab^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{b\sec^3(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{\sec(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (10*a*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7
/2)*d) + (b*Sec[c + d*x]^3)/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (Sec[c +
 d*x]^3*(5*a*b - (a^2 + 4*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d) + (Sec[c
+ d*x]*(15*a*b^3 + (2*a^4 - 9*a^2*b^2 - 8*b^4)*Sin[c + d*x]))/(3*(a^2 - b^2
)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^4(c+dx)(-a+4b\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\int$$

$$= \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\mathrm{se}$$

$$= \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\mathrm{se}$$

$$= \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\mathrm{se}$$

$$= \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\mathrm{se}$$

$$= \frac{10ab^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2} d} + \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab\right.}{3\left(\right.}$$

Mathematica [A] time = 1.86, size = 336, normalized size = 1.74

$$\frac{\frac{120ab^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{12b^5\cos(c+dx)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))} + \frac{4(2a+5b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((120*a*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
7/2) + 1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (2*Sin[(c +
d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (4*(2*a + 5*
b)*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2
*Sin[(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - 1/
((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(2*a - 5*b)*Sin[(c
 + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12*b^5*Cos
[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x]))))/(12*d)
```

fricas [A] time = 0.53, size = 782, normalized size = 4.05

$$\left[-\frac{2\,a^6b-6\,a^4b^3+6\,a^2b^5-2\,b^7+2\left(2\,a^6b-11\,a^4b^3+a^2b^5+8\,b^7\right)\cos\left(dx+c\right)^4-2\left(a^6b+2\,a^4b^3-7\,a^2b^5+4\,b^7\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + 2*(2*a^6*b - 11*a^4*b^3 +
a^2*b^5 + 8*b^7)*cos(d*x + c)^4 - 2*(a^6*b + 2*a^4*b^3 - 7*a^2*b^5 + 4*b^7)
*cos(d*x + c)^2 - 15*(a*b^5*cos(d*x + c)^3*sin(d*x + c) + a^2*b^4*cos(d*x +
 c)^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + (2*a^7 - 11*a^5*b^2 + 16*a^3*b^4 - 7*a*b^
6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^
7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a
^3*b^6 + a*b^8)*d*cos(d*x + c)^3), -1/3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7 + (2*a^6*b - 11*a^4*b^3 + a^2*b^5 + 8*b^7)*cos(d*x + c)^4 - (a^6*b + 2*a^
4*b^3 - 7*a^2*b^5 + 4*b^7)*cos(d*x + c)^2 + 15*(a*b^5*cos(d*x + c)^3*sin(d*
x + c) + a^2*b^4*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 +
 (2*a^7 - 11*a^5*b^2 + 16*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/
((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x
 + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3)
]
```

giac [B] time = 0.67, size = 427, normalized size = 2.21

$$\frac{2\left(\frac{15\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab^4}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{3\left(b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab^5\right)}{\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}-\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(15*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a*b^4/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sq
rt(a^2 - b^2)) + 3*(b^6*tan(1/2*d*x + 1/2*c) + a*b^5)/((a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)
) - (3*a^4*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*b^
4*tan(1/2*d*x + 1/2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 18*a*b^3*tan(1/
2*d*x + 1/2*c)^4 - 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 18*a^2*b^2*tan(1/2*d*x +
1/2*c)^3 + 8*b^4*tan(1/2*d*x + 1/2*c)^3 - 24*a*b^3*tan(1/2*d*x + 1/2*c)^2 +
```

```
3*a^4*tan(1/2*d*x + 1/2*c) - 9*a^2*b^2*tan(1/2*d*x + 1/2*c) - 6*b^4*tan(1/
2*d*x + 1/2*c) - 2*a^3*b + 14*a*b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(
tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

maple [B] time = 0.32, size = 370, normalized size = 1.92

$$-\frac{1}{3d\,(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2d\,(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a}{d\,(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{}{d\,(a+b)^3\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1
)^2-1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)*a-2/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)
*b+2/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a
)/a*tan(1/2*d*x+1/2*c)+2/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)+10/d*b^4/(a-b)^3/(a+b)^3*a/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/3/d/(a-b)^2/(tan(1/2*d*x+1
/2*c)+1)^3+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2-1/d/(a-b)^3/(tan(1/2*d*x+
1/2*c)+1)*a+2/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.51, size = 727, normalized size = 3.77

$$\frac{\frac{2\left(-2a^4b+14a^2b^3+3b^5\right)}{3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}-\frac{10b^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-4a^4b+28a^2b^3+21b^5\right)}{3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3a^6-13a^4b^2+22a^2b^4+3b^6\right)}{3a\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}-}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((2*(3*b^5 - 2*a^4*b + 14*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
 - (10*b^5*tan(c/2 + (d*x)/2)^6)/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*t
an(c/2 + (d*x)/2)^2*(21*b^5 - 4*a^4*b + 28*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*
b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)*(3*a^6 + 3*b^6 + 22*a^2*b^4 - 13*
a^4*b^2))/(3*a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)
^7*(b^6 - a^6 + 2*a^2*b^4 + 3*a^4*b^2))/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b
^2)) + (2*tan(c/2 + (d*x)/2)^5*(a^6 + 9*b^6 + 38*a^2*b^4 - 3*a^4*b^2))/(3*a
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^3*(a^6 - 9*b^
6 - 46*a^2*b^4 + 9*a^4*b^2))/(3*a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (1
0*b*tan(c/2 + (d*x)/2)^4*(5*b^4 - 2*a^4 + 6*a^2*b^2))/(3*(a^6 - b^6 + 3*a^2
*b^4 - 3*a^4*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 2*a*tan(c/2 + (d*x)/2)
```

```
^2 + 2*a*tan(c/2 + (d*x)/2)^6 - a*tan(c/2 + (d*x)/2)^8 - 6*b*tan(c/2 + (d*x
)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 - 2*b*tan(c/2 + (d*x)/2)^7)) + (10*a*b^4*
atan(((5*a*b^4*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^3))/((a + b)^(7/2)*(a
 - b)^(7/2)) + (10*a^2*b^4*tan(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/(10*a*b^4)))/(d*(a + b)^(7/2)*(a - b
)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

3.449 $\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=190

$$-\frac{6a\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))}+\frac{\left(a^2-b^2\right)^3}{2b^7d(a+b\sin(c+dx))^2}+\frac{a\left(10a^2-9b^2\right)\sin(c+dx)}{b^6d}-\frac{3\left(2a^2-b^2\right)\sin^2(c+dx)}{2b^5d}-\frac{3(5$$

```
[Out] -3*(5*a^4-6*a^2*b^2+b^4)*ln(a+b*sin(d*x+c))/b^7/d+a*(10*a^2-9*b^2)*sin(d*x+
c)/b^6/d-3/2*(2*a^2-b^2)*sin(d*x+c)^2/b^5/d+a*sin(d*x+c)^3/b^4/d-1/4*sin(d*
x+c)^4/b^3/d+1/2*(a^2-b^2)^3/b^7/d/(a+b*sin(d*x+c))^2-6*a*(a^2-b^2)^2/b^7/d
/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2668, 697}

$$-\frac{3\left(2a^2-b^2\right)\sin^2(c+dx)}{2b^5d}+\frac{a\left(10a^2-9b^2\right)\sin(c+dx)}{b^6d}-\frac{6a\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))}+\frac{\left(a^2-b^2\right)^3}{2b^7d(a+b\sin(c+dx))^2}-\frac{3(-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*(5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^7*d) + (a*(10*a^2
 - 9*b^2)*Sin[c + d*x])/(b^6*d) - (3*(2*a^2 - b^2)*Sin[c + d*x]^2)/(2*b^5*d
) + (a*Sin[c + d*x]^3)/(b^4*d) - Sin[c + d*x]^4/(4*b^3*d) + (a^2 - b^2)^3/(
2*b^7*d*(a + b*Sin[c + d*x])^2) - (6*a*(a^2 - b^2)^2)/(b^7*d*(a + b*Sin[c +
 d*x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{b^7d}\\ &= \frac{\text{Subst}\left(\int \left(10a^3\left(1-\frac{9b^2}{10a^2}\right)-3\left(2a^2-b^2\right)x+3ax^2-x^3-\frac{\left(a^2-b^2\right)^3}{(a+x)^3}+\frac{6a\left(a^2-b^2\right)^2}{(a+x)^2}-\frac{3(5\right.\right.}{b^7d}\\ &= -\frac{3\left(5a^4-6a^2b^2+b^4\right)\log(a+b\sin(c+dx))}{b^7d}+\frac{a\left(10a^2-9b^2\right)\sin(c+dx)}{b^6d}-\frac{3(2a\ }{}\end{aligned}$$

Mathematica [A] time = 0.64, size = 282, normalized size = 1.48

$$\frac{b^4\cos^4(c+dx)\left(-a^2+2ab\sin(c+dx)+3b^2\right)-2\left(2a^2b^4\sin^4(c+dx)-10ab^3\left(a^2-b^2\right)\sin^3(c+dx)+2b^2\sin^2(c$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b^6*Cos[c + d*x]^6 + b^4*Cos[c + d*x]^4*(-a^2 + 3*b^2 + 2*a*b*Sin[c + d*x]
) - 2*((a^2 - b^2)*(19*a^4 - 16*a^2*b^2 - 3*b^4 + 6*a^2*(5*a^2 - b^2)*Log[a
 + b*Sin[c + d*x]]) + 2*a*b*(4*a^4 - 17*a^2*b^2 + 11*b^4 + 6*(5*a^4 - 6*a^2
*b^2 + b^4)*Log[a + b*Sin[c + d*x]])*Sin[c + d*x] + 2*b^2*(-13*a^4 + 10*a^2
*b^2 + 3*(5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])*Sin[c + d*x]^2
- 10*a*b^3*(a^2 - b^2)*Sin[c + d*x]^3 + 2*a^2*b^4*Sin[c + d*x]^4))/(4*b^7*d
*(a + b*Sin[c + d*x])^2)
```

fricas [A] time = 0.55, size = 304, normalized size = 1.60

$$-\frac{8\,b^6\cos(dx+c)^6-176\,a^6+928\,a^4b^2-685\,a^2b^4+3\,b^6-8\left(5\,a^2b^4-3\,b^6\right)\cos(dx+c)^4-\left(544\,a^4b^2-560\,a^2b^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*b^6*cos(d*x + c)^6 - 176*a^6 + 928*a^4*b^2 - 685*a^2*b^4 + 3*b^6 -
 8*(5*a^2*b^4 - 3*b^6)*cos(d*x + c)^4 - (544*a^4*b^2 - 560*a^2*b^4 + 51*b^6
)*cos(d*x + c)^2 - 96*(5*a^6 - a^4*b^2 - 5*a^2*b^4 + b^6 - (5*a^4*b^2 - 6*a
^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(5*a^5*b - 6*a^3*b^3 + a*b^5)*sin(d*x + c)
)*log(b*sin(d*x + c) + a) + 2*(8*a*b^5*cos(d*x + c)^4 + 64*a^5*b + 176*a^3*
b^3 - 205*a*b^5 - 80*(a^3*b^3 - a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^9*d
*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x + c) - (a^2*b^7 + b^9)*d)
```

giac [A] time = 0.47, size = 245, normalized size = 1.29

$$-\frac{\frac{12\left(5\,a^4-6\,a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{b^7}-\frac{2\left(45\,a^4b^2\sin(dx+c)^2-54\,a^2b^4\sin(dx+c)^2+9\,b^6\sin(dx+c)^2+78\,a^5b\sin(dx+c)-84\,a^3b^3\sin(dx+c)+6\,ab^5\,\mathrm{s}}{(b\sin(dx+c)+a)^2b^7}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(12*(5*a^4 - 6*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/b^7 - 2*(45
*a^4*b^2*sin(d*x + c)^2 - 54*a^2*b^4*sin(d*x + c)^2 + 9*b^6*sin(d*x + c)^2
+ 78*a^5*b*sin(d*x + c) - 84*a^3*b^3*sin(d*x + c) + 6*a*b^5*sin(d*x + c) +
34*a^6 - 33*a^4*b^2 - b^6)/((b*sin(d*x + c) + a)^2*b^7) + (b^9*sin(d*x + c)
^4 - 4*a*b^8*sin(d*x + c)^3 + 12*a^2*b^7*sin(d*x + c)^2 - 6*b^9*sin(d*x + c
)^2 - 40*a^3*b^6*sin(d*x + c) + 36*a*b^8*sin(d*x + c))/b^12)/d
```

maple [A] time = 0.28, size = 320, normalized size = 1.68

$$-\frac{\sin^4(dx+c)}{4b^3d}+\frac{a\left(\sin^3(dx+c)\right)}{b^4d}-\frac{3\left(\sin^2(dx+c)\right)a^2}{d\,b^5}+\frac{3\left(\sin^2(dx+c)\right)}{2b^3d}+\frac{10\sin(dx+c)\,a^3}{d\,b^6}-\frac{9a\sin(dx+c)}{b^4d}-\frac{15\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/4*sin(d*x+c)^4/b^3/d+a*sin(d*x+c)^3/b^4/d-3/d/b^5*sin(d*x+c)^2*a^2+3/2*s
in(d*x+c)^2/b^3/d+10/d/b^6*sin(d*x+c)*a^3-9*a*sin(d*x+c)/b^4/d-15/d/b^7*ln(
```

```
a+b*sin(d*x+c))*a^4+18/d/b^5*ln(a+b*sin(d*x+c))*a^2-3*ln(a+b*sin(d*x+c))/b^
3/d-6/d*a^5/b^7/(a+b*sin(d*x+c))+12/d*a^3/b^5/(a+b*sin(d*x+c))-6*a/b^3/d/(a
+b*sin(d*x+c))+1/2/d/b^7/(a+b*sin(d*x+c))^2*a^6-3/2/d/b^5/(a+b*sin(d*x+c))^
2*a^4+3/2/d/b^3/(a+b*sin(d*x+c))^2*a^2-1/2/b/d/(a+b*sin(d*x+c))^2
```

maxima [A] time = 0.33, size = 200, normalized size = 1.05

$$-\frac{\frac{2\left(11\,a^6-21\,a^4b^2+9\,a^2b^4+b^6+12\left(a^5b-2\,a^3b^3+ab^5\right)\sin(dx+c)\right)}{b^9\sin(dx+c)^2+2\,ab^8\sin(dx+c)+a^2b^7}+\frac{b^3\sin(dx+c)^4-4\,ab^2\sin(dx+c)^3+6\left(2\,a^2b-b^3\right)\sin(dx+c)^2-4\left(10\,a^3-9\,ab^2\right)\sin(dx+c)}{b^6}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(11*a^6 - 21*a^4*b^2 + 9*a^2*b^4 + b^6 + 12*(a^5*b - 2*a^3*b^3 + a*
b^5)*sin(d*x + c))/(b^9*sin(d*x + c)^2 + 2*a*b^8*sin(d*x + c) + a^2*b^7) +
(b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*(2*a^2*b - b^3)*sin(d*x +
c)^2 - 4*(10*a^3 - 9*a*b^2)*sin(d*x + c))/b^6 + 12*(5*a^4 - 6*a^2*b^2 + b^4
)*log(b*sin(d*x + c) + a)/b^7)/d
```

mupad [B] time = 0.12, size = 234, normalized size = 1.23

$$\frac{\sin(c+d\,x)^2\,\left(\frac{3}{2\,b^3}-\frac{3\,a^2}{b^5}\right)}{d}-\frac{\sin(c+d\,x)^4}{4\,b^3\,d}-\frac{\sin(c+d\,x)\,\left(\frac{8\,a^3}{b^6}+\frac{3\,a\,\left(\frac{3}{b^3}-\frac{6\,a^2}{b^5}\right)}{b}\right)}{d}-\frac{\frac{11\,a^6-21\,a^4\,b^2+9\,a^2\,b^4+b^6}{2\,b}+\sin(c+d\,x)\ldots}{d\,\left(a^2\,b^6+2\,a\,b^7\,\sin(c+d\,x)\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (sin(c + d*x)^2*(3/(2*b^3) - (3*a^2)/b^5))/d - sin(c + d*x)^4/(4*b^3*d) - (
sin(c + d*x)*((8*a^3)/b^6 + (3*a*(3/b^3 - (6*a^2)/b^5))/b))/d - ((11*a^6 +
b^6 + 9*a^2*b^4 - 21*a^4*b^2)/(2*b) + sin(c + d*x)*(6*a*b^4 + 6*a^5 - 12*a^
3*b^2))/(d*(a^2*b^6 + b^8*sin(c + d*x)^2 + 2*a*b^7*sin(c + d*x))) + (a*sin(
c + d*x)^3)/(b^4*d) - (log(a + b*sin(c + d*x))*(15*a^4 + 3*b^4 - 18*a^2*b^2
))/(b^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.450 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(a^2-b^2)^2}{2b^5d(a+b\sin(c+dx))^2}+\frac{4a(a^2-b^2)}{b^5d(a+b\sin(c+dx))}+\frac{2(3a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}-\frac{3a\sin(c+dx)}{b^4d}+\frac{\sin^2(c+}{2b^3d}$$

```
[Out] 2*(3*a^2-b^2)*ln(a+b*sin(d*x+c))/b^5/d-3*a*sin(d*x+c)/b^4/d+1/2*sin(d*x+c)^
2/b^3/d-1/2*(a^2-b^2)^2/b^5/d/(a+b*sin(d*x+c))^2+4*a*(a^2-b^2)/b^5/d/(a+b*s
in(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{(a^2-b^2)^2}{2b^5d(a+b\sin(c+dx))^2}+\frac{4a(a^2-b^2)}{b^5d(a+b\sin(c+dx))}+\frac{2(3a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}-\frac{3a\sin(c+dx)}{b^4d}+\frac{\sin^2(c+}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*(3*a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d) - (3*a*Sin[c + d*x])/(b^4
*d) + Sin[c + d*x]^2/(2*b^3*d) - (a^2 - b^2)^2/(2*b^5*d*(a + b*Sin[c + d*x]
)^2) + (4*a*(a^2 - b^2))/(b^5*d*(a + b*Sin[c + d*x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(-3a+x+\frac{(a^2-b^2)^2}{(a+x)^3}-\frac{4(a^3-ab^2)}{(a+x)^2}+\frac{2(3a^2-b^2)}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^5d}\\ &= \frac{2(3a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}-\frac{3a\sin(c+dx)}{b^4d}+\frac{\sin^2(c+dx)}{2b^3d}-\frac{(a^2-b}{2b^5d(a+b\sin}\end{aligned}$$

Mathematica [A] time = 0.98, size = 143, normalized size = 1.13

$$\frac{2(b^2-a^2)\left(-\frac{3a^2+4ab\sin(c+dx)+b^2}{2(a+b\sin(c+dx))^2}-\log(a+b\sin(c+dx))\right)+\frac{b^4\cos^4(c+dx)}{2(a+b\sin(c+dx))^2}+2a\left(\frac{(a-b)(a+b)}{a+b\sin(c+dx)}+2a\log(a+b\sin(c\right.}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((b^4*Cos[c + d*x]^4)/(2*(a + b*Sin[c + d*x])^2) + 2*a*(2*a*Log[a + b*Sin[c
 + d*x]] - b*Sin[c + d*x] + ((a - b)*(a + b))/(a + b*Sin[c + d*x])) + 2*(-a
^2 + b^2)*(-Log[a + b*Sin[c + d*x]] - (3*a^2 + b^2 + 4*a*b*Sin[c + d*x])/(2
*(a + b*Sin[c + d*x])^2)))/(b^5*d)
```

fricas [A] time = 0.48, size = 212, normalized size = 1.67

$$-\frac{2\,b^4\cos\left(dx+c\right)^4+14\,a^4-35\,a^2b^2-b^4+\left(22\,a^2b^2-3\,b^4\right)\cos\left(dx+c\right)^2+8\left(3\,a^4+2\,a^2b^2-b^4-\left(3\,a^2b^2-b^4\right.\right.}{4\left(b^7d\cos\left(dx+c\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^4*cos(d*x + c)^4 + 14*a^4 - 35*a^2*b^2 - b^4 + (22*a^2*b^2 - 3*b^
4)*cos(d*x + c)^2 + 8*(3*a^4 + 2*a^2*b^2 - b^4 - (3*a^2*b^2 - b^4)*cos(d*x
+ c)^2 + 2*(3*a^3*b - a*b^3)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 2*(4*a
*b^3*cos(d*x + c)^2 + 2*a^3*b - 13*a*b^3)*sin(d*x + c))/(b^7*d*cos(d*x + c)
^2 - 2*a*b^6*d*sin(d*x + c) - (a^2*b^5 + b^7)*d)
```

giac [A] time = 0.51, size = 142, normalized size = 1.12

$$\frac{\frac{4\left(3\,a^2-b^2\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{b^5}+\frac{b^3\sin\left(dx+c\right)^2-6\,ab^2\sin\left(dx+c\right)}{b^6}-\frac{18\,a^2b^2\sin\left(dx+c\right)^2-6\,b^4\sin\left(dx+c\right)^2+28\,a^3b\sin\left(dx+c\right)-4\,ab^3\sin\left(dx+c\right)+11\,a}{\left(b\sin\left(dx+c\right)+a\right)^2b^5}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*(3*a^2 - b^2)*log(abs(b*sin(d*x + c) + a))/b^5 + (b^3*sin(d*x + c)^2
 - 6*a*b^2*sin(d*x + c))/b^6 - (18*a^2*b^2*sin(d*x + c)^2 - 6*b^4*sin(d*x +
 c)^2 + 28*a^3*b*sin(d*x + c) - 4*a*b^3*sin(d*x + c) + 11*a^4 + b^4)/((b*si
n(d*x + c) + a)^2*b^5))/d
```

maple [A] time = 0.29, size = 183, normalized size = 1.44

$$\frac{\sin^2\left(dx+c\right)}{2b^3d}-\frac{3a\sin\left(dx+c\right)}{b^4d}+\frac{6\ln\left(a+b\sin\left(dx+c\right)\right)a^2}{d\,b^5}-\frac{2\ln\left(a+b\sin\left(dx+c\right)\right)}{b^3d}+\frac{4a^3}{d\,b^5\left(a+b\sin\left(dx+c\right)\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2*sin(d*x+c)^2/b^3/d-3*a*sin(d*x+c)/b^4/d+6/d/b^5*ln(a+b*sin(d*x+c))*a^2-
2*ln(a+b*sin(d*x+c))/b^3/d+4/d*a^3/b^5/(a+b*sin(d*x+c))-4*a/b^3/d/(a+b*sin(
d*x+c))-1/2/d/b^5/(a+b*sin(d*x+c))^2*a^4+1/d/b^3/(a+b*sin(d*x+c))^2*a^2-1/2
/b/d/(a+b*sin(d*x+c))^2
```

maxima [A] time = 0.32, size = 131, normalized size = 1.03

$$\frac{\frac{7\,a^4-6\,a^2b^2-b^4+8\left(a^3b-ab^3\right)\sin\left(dx+c\right)}{b^7\sin\left(dx+c\right)^2+2\,ab^6\sin\left(dx+c\right)+a^2b^5}+\frac{b\sin\left(dx+c\right)^2-6\,a\sin\left(dx+c\right)}{b^4}+\frac{4\left(3\,a^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)}{b^5}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((7*a^4 - 6*a^2*b^2 - b^4 + 8*(a^3*b - a*b^3)*sin(d*x + c))/(b^7*sin(d*
x + c)^2 + 2*a*b^6*sin(d*x + c) + a^2*b^5) + (b*sin(d*x + c)^2 - 6*a*sin(d*
x + c))/b^4 + 4*(3*a^2 - b^2)*log(b*sin(d*x + c) + a)/b^5)/d
```

mupad [B] time = 5.14, size = 142, normalized size = 1.12

$$\frac{\sin(c+d\,x)^2}{2\,b^3\,d}-\frac{\frac{-7\,a^4+6\,a^2\,b^2+b^4}{2\,b}+\sin(c+d\,x)\,\left(4\,a\,b^2-4\,a^3\right)}{d\,\left(a^2\,b^4+2\,a\,b^5\,\sin(c+d\,x)+b^6\,\sin(c+d\,x)^2\right)}-\frac{3\,a\,\sin(c+d\,x)}{b^4\,d}+\frac{\ln(a+b\,\sin(c+d\,x))\,\left(6\,a^2-\right.}{b^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^3,x)
```

```
[Out] sin(c + d*x)^2/(2*b^3*d) - ((b^4 - 7*a^4 + 6*a^2*b^2)/(2*b) + sin(c + d*x)*
(4*a*b^2 - 4*a^3))/(d*(a^2*b^4 + b^6*sin(c + d*x)^2 + 2*a*b^5*sin(c + d*x))
) - (3*a*sin(c + d*x))/(b^4*d) + (log(a + b*sin(c + d*x))*(6*a^2 - 2*b^2))/
(b^5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.451 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=72

$$\frac{a^2-b^2}{2b^3d(a+b\sin(c+dx))^2}-\frac{2a}{b^3d(a+b\sin(c+dx))}-\frac{\log(a+b\sin(c+dx))}{b^3d}$$

```
[Out] -ln(a+b*sin(d*x+c))/b^3/d+1/2*(a^2-b^2)/b^3/d/(a+b*sin(d*x+c))^2-2*a/b^3/d/
(a+b*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{a^2-b^2}{2b^3d(a+b\sin(c+dx))^2}-\frac{2a}{b^3d(a+b\sin(c+dx))}-\frac{\log(a+b\sin(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(Log[a + b*Sin[c + d*x]]/(b^3*d)) + (a^2 - b^2)/(2*b^3*d*(a + b*Sin[c + d*
x])^2) - (2*a)/(b^3*d*(a + b*Sin[c + d*x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{-a^2+b^2}{(a+x)^3}+\frac{2a}{(a+x)^2}\right)dx, x, b\sin(c+dx)\right)}{b^3d}\\ &= -\frac{\log(a+b\sin(c+dx))}{b^3d}+\frac{a^2-b^2}{2b^3d(a+b\sin(c+dx))^2}-\frac{2a}{b^3d(a+b\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 0.76

$$-\frac{\frac{3a^2+4ab\sin(c+dx)+b^2}{2(a+b\sin(c+dx))^2}+\log(a+b\sin(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((Log[a + b*Sin[c + d*x]] + (3*a^2 + b^2 + 4*a*b*Sin[c + d*x])/(2*(a + b*S
in[c + d*x])^2))/(b^3*d))
```

fricas [A] time = 0.45, size = 110, normalized size = 1.53

$$\frac{4\,ab\sin\left(dx+c\right)+3\,a^2+b^2-2\left(b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)}{2\left(b^5d\cos\left(dx+c\right)^2-2\,ab^4d\sin\left(dx+c\right)-\left(a^2b^3+b^5\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*sin(d*x + c) + 3*a^2 + b^2 - 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2)*log(b*sin(d*x + c) + a))/(b^5*d*cos(d*x + c)^2 - 2*a*b
^4*d*sin(d*x + c) - (a^2*b^3 + b^5)*d)
```

giac [A] time = 1.02, size = 62, normalized size = 0.86

$$-\frac{\frac{2\,\log\left(|b\sin\left(dx+c\right)+a|\right)}{b^3}+\frac{4\,a\sin\left(dx+c\right)+\frac{3\,a^2+b^2}{b}}{\left(b\sin\left(dx+c\right)+a\right)^2b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(b*sin(d*x + c) + a))/b^3 + (4*a*sin(d*x + c) + (3*a^2 + b^2
)/b)/((b*sin(d*x + c) + a)^2*b^2))/d
```

maple [A] time = 0.26, size = 85, normalized size = 1.18

$$-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{b^3d}-\frac{2a}{b^3d\left(a+b\sin\left(dx+c\right)\right)}+\frac{a^2}{2d\,b^3\left(a+b\sin\left(dx+c\right)\right)^2}-\frac{1}{2bd\left(a+b\sin\left(dx+c\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -ln(a+b*sin(d*x+c))/b^3/d-2*a/b^3/d/(a+b*sin(d*x+c))+1/2/d/b^3/(a+b*sin(d*x
+c))^2*a^2-1/2/b/d/(a+b*sin(d*x+c))^2
```

maxima [A] time = 0.32, size = 76, normalized size = 1.06

$$-\frac{\frac{4\,ab\sin\left(dx+c\right)+3\,a^2+b^2}{b^5\sin\left(dx+c\right)^2+2\,ab^4\sin\left(dx+c\right)+a^2b^3}+\frac{2\,\log\left(b\sin\left(dx+c\right)+a\right)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*b*sin(d*x + c) + 3*a^2 + b^2)/(b^5*sin(d*x + c)^2 + 2*a*b^4*sin(
d*x + c) + a^2*b^3) + 2*log(b*sin(d*x + c) + a)/b^3)/d
```

mupad [B] time = 0.09, size = 80, normalized size = 1.11

$$-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{b^3\,d}-\frac{\frac{3\,a^2+b^2}{2\,b^3}+\frac{2\,a\,\sin\left(c+d\,x\right)}{b^2}}{d\,\left(a^2+2\,a\,b\,\sin\left(c+d\,x\right)+b^2\,{\sin\left(c+d\,x\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^3,x)
```

```
[Out] - log(a + b*sin(c + d*x))/(b^3*d) - ((3*a^2 + b^2)/(2*b^3) + (2*a*sin(c + d
*x))/b^2)/(d*(a^2 + b^2*sin(c + d*x)^2 + 2*a*b*sin(c + d*x)))
```

sympy [A] time = 2.39, size = 398, normalized size = 5.53

$$\begin{cases} \frac{x\cos^3(c)}{a^3} \\ \frac{\frac{2\sin^3(c+dx)}{3d}+\frac{\sin(c+dx)\cos^2(c+dx)}{d}}{a^3} \\ \frac{x\cos^3(c)}{(a+b\sin(c))^3} \\ -\frac{2a^2\log\left(\frac{a}{b}+\sin(c+dx)\right)}{2a^2b^3d+4ab^4d\sin(c+dx)+2b^5d\sin^2(c+dx)}-\frac{2a^2}{2a^2b^3d+4ab^4d\sin(c+dx)+2b^5d\sin^2(c+dx)}-\frac{4ab\log\left(\frac{a}{b}+\sin(c+dx)\right)\sin(c+dx)}{2a^2b^3d+4ab^4d\sin(c+dx)+2b^5d\sin^2(c+dx)}-\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((x*cos(c)**3/a**3, Eq(b, 0) & Eq(d, 0)), ((2*sin(c + d*x)**3/(3*d
) + sin(c + d*x)*cos(c + d*x)**2/d)/a**3, Eq(b, 0)), (x*cos(c)**3/(a + b*si
n(c))**3, Eq(d, 0)), (-2*a**2*log(a/b + sin(c + d*x))/(2*a**2*b**3*d + 4*a*
b**4*d*sin(c + d*x) + 2*b**5*d*sin(c + d*x)**2) - 2*a**2/(2*a**2*b**3*d + 4
*a*b**4*d*sin(c + d*x) + 2*b**5*d*sin(c + d*x)**2) - 4*a*b*log(a/b + sin(c
+ d*x))*sin(c + d*x)/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d*x) + 2*b**5*d*si
n(c + d*x)**2) - 2*a*b*sin(c + d*x)/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d*x
) + 2*b**5*d*sin(c + d*x)**2) - 2*b**2*log(a/b + sin(c + d*x))*sin(c + d*x)
**2/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d*x) + 2*b**5*d*sin(c + d*x)**2) -
b**2*cos(c + d*x)**2/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d*x) + 2*b**5*d*si
n(c + d*x)**2), True))
```

3.452 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2bd(a+b\sin(c+dx))^2}$$

```
[Out] -1/2/b/d/(a+b*sin(d*x+c))^2
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2668, 32}

$$-\frac{1}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/(2*b*d*(a + b*Sin[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{1}{2bd(a+b\sin(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$-\frac{1}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*1/(b*d*(a + b*Sin[c + d*x])^2)
```

fricas [B] time = 0.42, size = 43, normalized size = 1.95

$$\frac{1}{2\left(b^3d\cos(dx+c)^2 - 2ab^2d\sin(dx+c) - \left(a^2b+b^3\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] 1/2/(b^3*d*cos(d*x + c)^2 - 2*a*b^2*d*sin(d*x + c) - (a^2*b + b^3)*d)
```

giac [A] time = 0.41, size = 20, normalized size = 0.91

$$-\frac{1}{2\,(b\sin\left(dx+c\right)+a)^{2}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] -1/2/((b*sin(d*x + c) + a)^2*b*d)
```

maple [A] time = 0.11, size = 21, normalized size = 0.95

$$-\frac{1}{2bd\left(a+b\sin\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
[Out] -1/2/b/d/(a+b*sin(d*x+c))^2
```

maxima [A] time = 0.31, size = 20, normalized size = 0.91

$$-\frac{1}{2\,(b\sin\left(dx+c\right)+a)^{2}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2/((b*sin(d*x + c) + a)^2*b*d)
```

mupad [B] time = 0.06, size = 39, normalized size = 1.77

$$-\frac{1}{d\,\left(2\,a^2\,b+4\,a\,b^2\,\sin\left(c+d\,x\right)+2\,b^3\,{\sin\left(c+d\,x\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^3,x)
[Out] -1/(d*(2*a^2*b + 2*b^3*sin(c + d*x)^2 + 4*a*b^2*sin(c + d*x)))
```

sympy [A] time = 1.97, size = 73, normalized size = 3.32

$$\begin{cases} \frac{x\cos\left(c\right)}{a^{3}} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{\sin\left(c+dx\right)}{a^{3}d} & \text{for}\: b = 0 \\ \frac{x\cos\left(c\right)}{\left(a+b\sin\left(c\right)\right)^{3}} & \text{for}\: d = 0 \\ -\frac{1}{2a^{2}bd+4ab^{2}d\sin\left(c+dx\right)+2b^{3}d\sin^{2}\left(c+dx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
[Out] Piecewise((x*cos(c)/a**3, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**3*d), Eq(
b, 0)), (x*cos(c)/(a + b*sin(c))**3, Eq(d, 0)), (-1/(2*a**2*b*d + 4*a*b**2*
d*sin(c + d*x) + 2*b**3*d*sin(c + d*x)**2), True))
```

3.453 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{2ab}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{b\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\log(1-\sin(c+dx))}{2d(a}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^3/d+1/2*ln(1+sin(d*x+c))/(a-b)^3/d-b*(3*a^2+b^2
)*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/2*b/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+2*a
*b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 710, 801}

$$\frac{2ab}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{b\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\log(1-\sin(c+dx))}{2d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^3*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^3
*d) - (b*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) + b/(2*(a
^2 - b^2)*d*(a + b*Sin[c + d*x])^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Sin[c
 + d*x]))
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)^3(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b\,\text{Subst}\left(\int \frac{a-x}{(a+x)^2(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d}$$

$$= \frac{b}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b\,\text{Subst}\left(\int\left(\frac{a-b}{2b(a+b)^2(b-x)} - \frac{2a}{(a-b)(a+b)(a+x)^2} + \frac{}{(a-}\right.\right.}{\left(a^2-\right.}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^3d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^3d} - \frac{b\left(3a^2+b^2\right)\log(a+b\sin(c+dx}{\left(a^2-b^2\right)^3d}$$

Mathematica [A] time = 0.59, size = 135, normalized size = 0.93

$$\frac{b\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))^2} - \frac{2\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{4a}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} - \frac{\log(1-\sin(c+dx))}{b(a+b)^3} + \frac{\log(\sin(c+dx)+1)}{b(a-b)^3}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(-(Log[1 - Sin[c + d*x]]/(b*(a + b)^3)) + Log[1 + Sin[c + d*x]]/((a - b)
^3*b) - (2*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a + b)^3) + 1
/((a^2 - b^2)*(a + b*Sin[c + d*x])^2) + (4*a)/((a - b)^2*(a + b)^2*(a + b*S
in[c + d*x]))))/(2*d)
```

fricas [B] time = 0.53, size = 462, normalized size = 3.19

$$-\frac{5\,a^4b - 6\,a^2b^3 + b^5 - 2\left(3\,a^4b + 4\,a^2b^3 + b^5 - \left(3\,a^2b^3 + b^5\right)\cos\left(dx+c\right)^2 + 2\left(3\,a^3b^2 + ab^4\right)\sin\left(dx+c\right)\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(5*a^4*b - 6*a^2*b^3 + b^5 - 2*(3*a^4*b + 4*a^2*b^3 + b^5 - (3*a^2*b^3
 + b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 + a*b^4)*sin(d*x + c))*log(b*sin(d*x
+ c) + a) + (a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b
^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a
^2*b^3 + a*b^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - (a^5 - 3*a^4*b + 4*a^
3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*c
os(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x + c))*log
(-sin(d*x + c) + 1) + 4*(a^3*b^2 - a*b^4)*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*sin(d*x + c) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d)
```

giac [A] time = 0.69, size = 242, normalized size = 1.67

$$-\frac{\frac{2\left(3\,a^2b^2+b^4\right)\log\left(|b\sin(dx+c)+a|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7} - \frac{\log\left(|\sin(dx+c)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{\log\left(|\sin(dx+c)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{9\,a^2b^3\sin(dx+c)^2+3\,b^5\sin(dx+c)^2+22\,a^3b^2\sin(dx+c)+2\,ab}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)(b\sin(dx+c}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 +
 3*a^2*b^5 - b^7) - log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b
^3) + log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (9*a^2*b
^3*sin(d*x + c)^2 + 3*b^5*sin(d*x + c)^2 + 22*a^3*b^2*sin(d*x + c) + 2*a*b^
4*sin(d*x + c) + 14*a^4*b - 3*a^2*b^3 + b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*(b*sin(d*x + c) + a)^2))/d
```

maple [A] time = 0.29, size = 166, normalized size = 1.14

$$-\frac{\ln(\sin(dx+c)-1)}{2d(a+b)^3}+\frac{b}{2d(a+b)(a-b)(a+b\sin(dx+c))^2}+\frac{2ab}{d(a+b)^2(a-b)^2(a+b\sin(dx+c))}-\frac{3b\ln(a+b}{d(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d/(a+b)^3*ln(sin(d*x+c)-1)+1/2/d*b/(a+b)/(a-b)/(a+b*sin(d*x+c))^2+2/d*
a*b/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))-3/d*b/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c
))*a^2-1/d*b^3/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2*ln(1+sin(d*x+c))/(a-b
)^3/d
```

maxima [A] time = 0.34, size = 223, normalized size = 1.54

$$-\frac{\frac{2\left(3a^2b+b^3\right)\log(b\sin(dx+c)+a)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{4ab^2\sin(dx+c)+5a^2b-b^3}{a^6-2a^4b^2+a^2b^4+\left(a^4b^2-2a^2b^4+b^6\right)\sin(dx+c)^2+2\left(a^5b-2a^3b^3+ab^5\right)\sin(dx+c)}-\frac{\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}+\frac{\log(s}{a^3+3a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(3*a^2*b + b^3)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - (4*a*b^2*sin(d*x + c) + 5*a^2*b - b^3)/(a^6 - 2*a^4*b^2 + a^2*b^
4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b
^5)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) +
 log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d
```

mupad [B] time = 5.40, size = 169, normalized size = 1.17

$$\frac{\ln(a+b\,\sin(c+d\,x))\left(\frac{1}{2\,(a+b)^3}-\frac{1}{2\,(a-b)^3}\right)}{d}+\frac{\frac{5\,a^2\,b-b^3}{2\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\,a\,b^2\,\sin(c+d\,x)}{a^4-2\,a^2\,b^2+b^4}}{d\,\left(a^2+2\,a\,b\,\sin(c+d\,x)+b^2\,\sin(c+d\,x)^2\right)}+\frac{\ln(\sin(c+d\,x)+1)}{2\,d\,(a-b)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (log(a + b*sin(c + d*x))*(1/(2*(a + b)^3) - 1/(2*(a - b)^3)))/d + ((5*a^2*b
 - b^3)/(2*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*b^2*sin(c + d*x))/(a^4 + b^4 - 2
*a^2*b^2))/(d*(a^2 + b^2*sin(c + d*x)^2 + 2*a*b*sin(c + d*x))) + log(sin(c
+ d*x) + 1)/(2*d*(a - b)^3) - log(sin(c + d*x) - 1)/(2*d*(a + b)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

3.454 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=226

$$-\frac{ab\left(a^2+11b^2\right)}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{b\left(a^2+2b^2\right)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}+\frac{2b^3\left(5\right.}{}$$

```
[Out] -1/4*(a+4*b)*ln(1-sin(d*x+c))/(a+b)^4/d+1/4*(a-4*b)*ln(1+sin(d*x+c))/(a-b)^
4/d+2*b^3*(5*a^2+b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d-1/2*b*(a^2+2*b^2)/(a
^2-b^2)^2/d/(a+b*sin(d*x+c))^2-1/2*sec(d*x+c)^2*(b-a*sin(d*x+c))/(a^2-b^2)/
d/(a+b*sin(d*x+c))^2-1/2*a*b*(a^2+11*b^2)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.28, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 741, 801}

$$-\frac{ab\left(a^2+11b^2\right)}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{b\left(a^2+2b^2\right)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{2b^3\left(5a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{\mathrm{s}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a + 4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^4*d) + ((a - 4*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^4*d) + (2*b^3*(5*a^2 + b^2)*Log[a + b*Sin[c + d*x]])
/((a^2 - b^2)^4*d) - (b*(a^2 + 2*b^2))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*
x])^2) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[
c + d*x])^2) - (a*b*(a^2 + 11*b^2))/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])
)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+x)^3\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b\,\text{Subst}\left(\int \frac{a^2-4b^2+3ax}{(a+x)^3\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b\,\text{Subst}\left(\int\left(\frac{(a-b)(a+4b)}{2b(a+b)^3(b-x)} + \frac{2\left(a^2+2b^2\right)}{(a-b)(a+b)(a+x)^3} + \ldots\right.\right.}{\ldots}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^4 d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^4 d} + \frac{2b^3\left(5a^2+b^2\right)\log(\ldots}{\left(a^2-\ldots\right.}$$

Mathematica [A] time = 3.95, size = 283, normalized size = 1.25

$$\frac{b\left(a^2+2b^2\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))^2} - \frac{2\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{4a}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} - \frac{\log(1-\sin(c+dx))}{b(a+b)^3} + \frac{\log(\sin(c+dx)\ldots}{b(a-b)^3}\right.}{2d\left(b^2-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2 + b*(a^2 + 2*
b^2)*(-(Log[1 - Sin[c + d*x]]/(b*(a + b)^3)) + Log[1 + Sin[c + d*x]]/((a -
b)^3*b) - (2*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a + b)^3) +
 1/((a^2 - b^2)*(a + b*Sin[c + d*x])^2) + (4*a)/((a - b)^2*(a + b)^2*(a + b
*Sin[c + d*x]))) + (3*a*(Log[1 - Sin[c + d*x]]/(a + b)^2 - Log[1 + Sin[c +
d*x]]/(a - b)^2 + (4*a*b*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + (
2*b)/((-a^2 + b^2)*(a + b*Sin[c + d*x]))))/2)/(2*(-a^2 + b^2)*d)
```

fricas [B] time = 0.74, size = 707, normalized size = 3.13

$$\frac{2a^6b - 6a^4b^3 + 6a^2b^5 - 2b^7 + 4\left(a^6b + 5a^4b^3 - 7a^2b^5 + b^7\right)\cos(dx+c)^2 + 8\left(\left(5a^2b^5 + b^7\right)\cos(dx+c)^4 - 2\left(5\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + 4*(a^6*b + 5*a^4*b^3 - 7*a^2
*b^5 + b^7)*cos(d*x + c)^2 + 8*((5*a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(5*a^3
*b^4 + a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (5*a^4*b^3 + 6*a^2*b^5 + b^7)*c
os(d*x + c)^2)*log(b*sin(d*x + c) + a) + ((a^5*b^2 - 10*a^3*b^4 - 20*a^2*b^
5 - 15*a*b^6 - 4*b^7)*cos(d*x + c)^4 - 2*(a^6*b - 10*a^4*b^3 - 20*a^3*b^4 -
 15*a^2*b^5 - 4*a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (a^7 - 9*a^5*b^2 - 20*
a^4*b^3 - 25*a^3*b^4 - 24*a^2*b^5 - 15*a*b^6 - 4*b^7)*cos(d*x + c)^2)*log(s
in(d*x + c) + 1) - ((a^5*b^2 - 10*a^3*b^4 + 20*a^2*b^5 - 15*a*b^6 + 4*b^7)*
cos(d*x + c)^4 - 2*(a^6*b - 10*a^4*b^3 + 20*a^3*b^4 - 15*a^2*b^5 + 4*a*b^6)
*cos(d*x + c)^2*sin(d*x + c) - (a^7 - 9*a^5*b^2 + 20*a^4*b^3 - 25*a^3*b^4 +
 24*a^2*b^5 - 15*a*b^6 + 4*b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*
(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (a^5*b^2 + 10*a^3*b^4 - 11*a*b^6)*co
s(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
 b^10)*d*cos(d*x + c)^4 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*
```

b^9)*d*cos(d*x + c)^2*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^2)

giac [A] time = 1.38, size = 413, normalized size = 1.83

$$\frac{\frac{8\left(5a^2b^4+b^6\right)\log(|b\sin(dx+c)+a|)}{a^8b-4a^6b^3+6a^4b^5-4a^2b^7+b^9}+\frac{(a-4b)\log(|\sin(dx+c)+1|)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{(a+4b)\log(|\sin(dx+c)-1|)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{2\left(10a^2b^3\sin(dx+c)^2+2b^5\sin(dx+c)^2-a^5\sin\right.}{\left(a^8-4a^6b^2+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(8*(5*a^2*b^4 + b^6)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3 +
6*a^4*b^5 - 4*a^2*b^7 + b^9) + (a - 4*b)*log(abs(sin(d*x + c) + 1))/(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a + 4*b)*log(abs(sin(d*x + c) - 1))
/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(10*a^2*b^3*sin(d*x + c)^2
 + 2*b^5*sin(d*x + c)^2 - a^5*sin(d*x + c) - 2*a^3*b^2*sin(d*x + c) + 3*a*b
^4*sin(d*x + c) + 3*a^4*b - 12*a^2*b^3 - 3*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)) - 2*(30*a^2*b^5*sin(d*x + c)^2
+ 6*b^7*sin(d*x + c)^2 + 68*a^3*b^4*sin(d*x + c) + 4*a*b^6*sin(d*x + c) + 3
9*a^4*b^3 - 4*a^2*b^5 + b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^
8)*(b*sin(d*x + c) + a)^2))/d
```

maple [A] time = 0.32, size = 258, normalized size = 1.14

$$-\frac{1}{4d\left(a+b\right)^3\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{4d\left(a+b\right)^4}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)b}{d\left(a+b\right)^4}-\frac{b^3}{2d\left(a+b\right)^2\left(a-b\right)^2\left(a+b\sin\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d/(a+b)^3/(sin(d*x+c)-1)-1/4/d/(a+b)^4*ln(sin(d*x+c)-1)*a-1/d/(a+b)^4*
ln(sin(d*x+c)-1)*b-1/2/d*b^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))^2-4/d*a*b^3/(
a+b)^3/(a-b)^3/(a+b*sin(d*x+c))+10/d*b^3/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))
*a^2+2/d*b^5/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))-1/4/d/(a-b)^3/(1+sin(d*x+c)
)+1/4/d/(a-b)^4*ln(1+sin(d*x+c))*a-1/d/(a-b)^4*ln(1+sin(d*x+c))*b
```

maxima [B] time = 0.36, size = 438, normalized size = 1.94

$$\frac{\frac{8\left(5a^2b^3+b^5\right)\log(b\sin(dx+c)+a)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{(a-4b)\log(\sin(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{(a+4b)\log(\sin(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{2\left(3a^4b+\right.}{a^8-3a^6b^2+3a^4b^4-a^2b^6-\left(a^6b^2-3a^4b^4+3a^2b^6\right.}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*(5*a^2*b^3 + b^5)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8) + (a - 4*b)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b + 6*
a^2*b^2 - 4*a*b^3 + b^4) - (a + 4*b)*log(sin(d*x + c) - 1)/(a^4 + 4*a^3*b +
 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(3*a^4*b + 10*a^2*b^3 - b^5 - (a^3*b^2 + 11
*a*b^4)*sin(d*x + c)^3 - 2*(a^4*b + 6*a^2*b^3 - b^5)*sin(d*x + c)^2 - (a^5
- 3*a^3*b^2 - 10*a*b^4)*sin(d*x + c))/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*sin(d*x + c)^4 - 2*(a^7*b - 3*a
^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 -
 4*a^2*b^6 + b^8)*sin(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7
)*sin(d*x + c)))/d
```

mupad [B] time = 5.78, size = 388, normalized size = 1.72

$$\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(\frac{3\,b}{4\,{\left(a+b\right)}^4}+\frac{1}{4\,{\left(a+b\right)}^3}+\frac{3\,b}{4\,{\left(a-b\right)}^4}-\frac{1}{4\,{\left(a-b\right)}^3}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{3\,b}{4\,{\left(a+b\right)}^4}+\frac{1}{4\,{\left(a+b\right)}^3}\right)}{d}+\frac{\frac{3\,a^4\,b+1}{2\,\left(a^2-b^2\right)\,\left(a}}{d\,\left(\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (log(a + b*sin(c + d*x))*((3*b)/(4*(a + b)^4) + 1/(4*(a + b)^3) + (3*b)/(4*
(a - b)^4) - 1/(4*(a - b)^3)))/d - (log(sin(c + d*x) - 1)*((3*b)/(4*(a + b)
^4) + 1/(4*(a + b)^3)))/d + ((3*a^4*b - b^5 + 10*a^2*b^3)/(2*(a^2 - b^2)*(a
^4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^3*(11*a*b^4 + a^3*b^2))/(2*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)^2*(a^4*b - b^5 + 6*a^2*b^3))/((
a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (a*sin(c + d*x)*(10*b^4 - a^4 + 3*a^2
*b^2))/(2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(sin(c + d*x)^2*(a^2 - b
^2) - a^2 + b^2*sin(c + d*x)^4 - 2*a*b*sin(c + d*x) + 2*a*b*sin(c + d*x)^3)
) + (log(sin(c + d*x) + 1)*(a - 4*b))/(4*d*(a - b)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

3.455 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=328

$$-\frac{3\left(a^2+5ab+8b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^5}+\frac{3\left(a^2-5ab+8b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^5}-\frac{\sec^4(c+dx)(b-a\sin(c+}{4d\left(a^2-b^2\right)(a+b\sin(c+}$$

```
[Out] -3/16*(a^2+5*a*b+8*b^2)*ln(1-sin(d*x+c))/(a+b)^5/d+3/16*(a^2-5*a*b+8*b^2)*l
n(1+sin(d*x+c))/(a-b)^5/d-3*b^5*(7*a^2+b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^5/
d-3/8*b*(a^4-5*a^2*b^2-4*b^4)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^2-1/4*sec(d*x+
c)^4*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))^2-3/8*a*b*(a^4-6*a^2*b^2
-27*b^4)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))+1/8*sec(d*x+c)^2*(2*b*(a^2+3*b^2)+a
*(3*a^2-11*b^2)*sin(d*x+c))/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^2
```

Rubi [A] time = 0.42, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2668, 741, 823, 801}

$$-\frac{3ab\left(-6a^2b^2+a^4-27b^4\right)}{8d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}-\frac{3b\left(-5a^2b^2+a^4-4b^4\right)}{8d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2}-\frac{3b^5\left(7a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^5}-\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*(a^2 + 5*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^5*d) + (3*(a^2
 - 5*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) - (3*b^5*(7*a^2 +
 b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) - (3*b*(a^4 - 5*a^2*b^2 -
4*b^4))/(8*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) - (Sec[c + d*x]^4*(b - a
*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (3*a*b*(a^4 - 6*
a^2*b^2 - 27*b^4))/(8*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]
^2*(2*b*(a^2 + 3*b^2) + a*(3*a^2 - 11*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*
d*(a + b*Sin[c + d*x])^2)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{1}{(a+x)^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b^3\,\text{Subst}\left(\int \frac{3\left(a^2-2b^2\right)+5ax}{(a+x)^3\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\sec^2(c+dx)\left(2b\left(a^2+3b^2\right)+a\left(3a^2-11b^2\right)\sin\right.}{8\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^2}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\sec^2(c+dx)\left(2b\left(a^2+3b^2\right)+a\left(3a^2-11b^2\right)\sin\right.}{8\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^2}$$

$$= -\frac{3\left(a^2+5ab+8b^2\right)\log(1-\sin(c+dx))}{16(a+b)^5 d} + \frac{3\left(a^2-5ab+8b^2\right)\log(1+\sin(c+dx))}{16(a-b)^5 d}$$

Mathematica [A] time = 2.74, size = 388, normalized size = 1.18

$$\frac{\frac{\sec^2(c+dx)\left(a\left(3a^2-11b^2\right)\sin(c+dx)+2b\left(a^2+3b^2\right)\right)}{\left(b^2-a^2\right)(a+b\sin(c+dx))^2} - \frac{b\left(3\left(a^4-5a^2b^2-4b^4\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{2\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{4a}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}\right.}{}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2 + (Sec[c +
d*x]^2*(2*b*(a^2 + 3*b^2) + a*(3*a^2 - 11*b^2)*Sin[c + d*x]))/((-a^2 + b^2)
*(a + b*Sin[c + d*x])^2) - (b*(3*(a^4 - 5*a^2*b^2 - 4*b^4)*(-(Log[1 - Sin[c
 + d*x]]/(b*(a + b)^3)) + Log[1 + Sin[c + d*x]]/((a - b)^3*b) - (2*(3*a^2 +
 b^2)*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a + b)^3) + 1/((a^2 - b^2)*(a +
b*Sin[c + d*x])^2) + (4*a)/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))) - 3*
a*(3*a^2 - 11*b^2)*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^2) + Log[1 + Sin[
c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b
)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x])))))/(-a^2 + b^2))/(8*(-a^2 + b^2
)*d)
```

fricas [B] time = 1.44, size = 895, normalized size = 2.73

$$\frac{4a^8b - 16a^6b^3 + 24a^4b^5 - 16a^2b^7 + 4b^9 + 12\left(a^8b - 7a^6b^3 - 7a^4b^5 + 15a^2b^7 - 2b^9\right)\cos(dx+c)^4 - 4\left(a^8b - 6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 + 12*(a^8*b -
7*a^6*b^3 - 7*a^4*b^5 + 15*a^2*b^7 - 2*b^9)*cos(d*x + c)^4 - 4*(a^8*b - 6*a
^4*b^5 + 8*a^2*b^7 - 3*b^9)*cos(d*x + c)^2 - 48*((7*a^2*b^7 + b^9)*cos(d*x
+ c)^6 - 2*(7*a^3*b^6 + a*b^8)*cos(d*x + c)^4*sin(d*x + c) - (7*a^4*b^5 + 8
*a^2*b^7 + b^9)*cos(d*x + c)^4)*log(b*sin(d*x + c) + a) + 3*((a^7*b^2 - 7*a
^5*b^4 + 35*a^3*b^6 + 56*a^2*b^7 + 35*a*b^8 + 8*b^9)*cos(d*x + c)^6 - 2*(a^
8*b - 7*a^6*b^3 + 35*a^4*b^5 + 56*a^3*b^6 + 35*a^2*b^7 + 8*a*b^8)*cos(d*x +
 c)^4*sin(d*x + c) - (a^9 - 6*a^7*b^2 + 28*a^5*b^4 + 56*a^4*b^5 + 70*a^3*b^
6 + 64*a^2*b^7 + 35*a*b^8 + 8*b^9)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) -
3*((a^7*b^2 - 7*a^5*b^4 + 35*a^3*b^6 - 56*a^2*b^7 + 35*a*b^8 - 8*b^9)*cos(d
*x + c)^6 - 2*(a^8*b - 7*a^6*b^3 + 35*a^4*b^5 - 56*a^3*b^6 + 35*a^2*b^7 - 8
*a*b^8)*cos(d*x + c)^4*sin(d*x + c) - (a^9 - 6*a^7*b^2 + 28*a^5*b^4 - 56*a^
4*b^5 + 70*a^3*b^6 - 64*a^2*b^7 + 35*a*b^8 - 8*b^9)*cos(d*x + c)^4)*log(-si
n(d*x + c) + 1) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 -
 3*(a^7*b^2 - 7*a^5*b^4 - 21*a^3*b^6 + 27*a*b^8)*cos(d*x + c)^4 + (3*a^9 -
20*a^7*b^2 + 42*a^5*b^4 - 36*a^3*b^6 + 11*a*b^8)*cos(d*x + c)^2)*sin(d*x +
c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d
*cos(d*x + c)^6 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b
^9 - a*b^11)*d*cos(d*x + c)^4*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4
 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^4)
```

giac [A] time = 0.73, size = 575, normalized size = 1.75

$$-\frac{\frac{48\left(7a^2b^6+b^8\right)\log(|b\sin(dx+c)+a|)}{a^{10}b-5a^8b^3+10a^6b^5-10a^4b^7+5a^2b^9-b^{11}}-\frac{3\left(a^2-5ab+8b^2\right)\log(|\sin(dx+c)+1|)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5}+\frac{3\left(a^2+5ab+8b^2\right)\log(|\sin(dx+c)-1|)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5}+\frac{2\left(3a^5b^2\sin(d\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(48*(7*a^2*b^6 + b^8)*log(abs(b*sin(d*x + c) + a))/(a^10*b - 5*a^8*b^
3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) - 3*(a^2 - 5*a*b + 8*b^2)*l
og(abs(sin(d*x + c) + 1))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^
4 - b^5) + 3*(a^2 + 5*a*b + 8*b^2)*log(abs(sin(d*x + c) - 1))/(a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) + 2*(3*a^5*b^2*sin(d*x + c)^5
- 18*a^3*b^4*sin(d*x + c)^5 - 81*a*b^6*sin(d*x + c)^5 + 6*a^6*b*sin(d*x + c
)^4 - 36*a^4*b^3*sin(d*x + c)^4 - 78*a^2*b^5*sin(d*x + c)^4 + 12*b^7*sin(d*
x + c)^4 + 3*a^7*sin(d*x + c)^3 - 23*a^5*b^2*sin(d*x + c)^3 + 61*a^3*b^4*si
n(d*x + c)^3 + 151*a*b^6*sin(d*x + c)^3 - 10*a^6*b*sin(d*x + c)^2 + 74*a^4*
b^3*sin(d*x + c)^2 + 146*a^2*b^5*sin(d*x + c)^2 - 18*b^7*sin(d*x + c)^2 - 5
*a^7*sin(d*x + c) + 26*a^5*b^2*sin(d*x + c) - 49*a^3*b^4*sin(d*x + c) - 68*
a*b^6*sin(d*x + c) + 6*a^6*b - 44*a^4*b^3 - 62*a^2*b^5 + 4*b^7)/((a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c)^3 + a*sin(d*x + c)^2
- b*sin(d*x + c) - a)^2))/d
```

maple [A] time = 0.34, size = 398, normalized size = 1.21

$$\frac{1}{16d\left(a+b\right)^{3}\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{3a}{16d\left(a+b\right)^{4}\left(\sin\left(dx+c\right)-1\right)}-\frac{9b}{16d\left(a+b\right)^{4}\left(\sin\left(dx+c\right)-1\right)}-\frac{3\ln\left(\sin\left(dx+\right.\right.}{16d\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/16/d/(a+b)^3/(sin(d*x+c)-1)^2-3/16/d/(a+b)^4/(sin(d*x+c)-1)*a-9/16/d/(a+b
)^4/(sin(d*x+c)-1)*b-3/16/d/(a+b)^5*ln(sin(d*x+c)-1)*a^2-15/16/d/(a+b)^5*ln
```

```
(sin(d*x+c)-1)*a*b-3/2/d/(a+b)^5*ln(sin(d*x+c)-1)*b^2+1/2/d*b^5/(a+b)^3/(a-
b)^3/(a+b*sin(d*x+c))^2+6/d*b^5*a/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))-21/d*b^5
/(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c))*a^2-3/d*b^7/(a+b)^5/(a-b)^5*ln(a+b*sin(
d*x+c))-1/16/d/(a-b)^3/(1+sin(d*x+c))^2-3/16/d/(a-b)^4/(1+sin(d*x+c))*a+9/1
6/d/(a-b)^4/(1+sin(d*x+c))*b+3/16/d/(a-b)^5*ln(1+sin(d*x+c))*a^2-15/16/d/(a
-b)^5*ln(1+sin(d*x+c))*a*b+3/2/d/(a-b)^5*ln(1+sin(d*x+c))*b^2
```

maxima [B] time = 0.37, size = 725, normalized size = 2.21

$$-\frac{\frac{48\left(7a^2b^5+b^7\right)\log(b\sin(dx+c)+a)}{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}} - \frac{3\left(a^2-5ab+8b^2\right)\log(\sin(dx+c)+1)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5} + \frac{3\left(a^2+5ab+8b^2\right)\log(\sin(dx+c)-1)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5} + \frac{\cdots}{a^{10}-4a^8b^2+6a^6b^4-\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/16*(48*(7*a^2*b^5 + b^7)*log(b*sin(d*x + c) + a)/(a^10 - 5*a^8*b^2 + 10*
a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(a^2 - 5*a*b + 8*b^2)*log(sin(
d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) + 3
*(a^2 + 5*a*b + 8*b^2)*log(sin(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 +
10*a^2*b^3 + 5*a*b^4 + b^5) + 2*(6*a^6*b - 44*a^4*b^3 - 62*a^2*b^5 + 4*b^7
+ 3*(a^5*b^2 - 6*a^3*b^4 - 27*a*b^6)*sin(d*x + c)^5 + 6*(a^6*b - 6*a^4*b^3
- 13*a^2*b^5 + 2*b^7)*sin(d*x + c)^4 + (3*a^7 - 23*a^5*b^2 + 61*a^3*b^4 + 1
51*a*b^6)*sin(d*x + c)^3 - 2*(5*a^6*b - 37*a^4*b^3 - 73*a^2*b^5 + 9*b^7)*si
n(d*x + c)^2 - (5*a^7 - 26*a^5*b^2 + 49*a^3*b^4 + 68*a*b^6)*sin(d*x + c))/(
a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8 + (a^8*b^2 - 4*a^6*b^4 +
 6*a^4*b^6 - 4*a^2*b^8 + b^10)*sin(d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^
5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)^5 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4
- 16*a^4*b^6 + 9*a^2*b^8 - 2*b^10)*sin(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 +
6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)^3 - (2*a^10 - 9*a^8*b^2 + 16*a^
6*b^4 - 14*a^4*b^6 + 6*a^2*b^8 - b^10)*sin(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^
3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)))/d
```

mupad [B] time = 6.56, size = 688, normalized size = 2.10

$$\frac{\ln(\sin(c+d\,x)+1)\left(\frac{3\,b^2}{4\,(a-b)^5}-\frac{9\,b}{16\,(a-b)^4}+\frac{3}{16\,(a-b)^3}\right)}{d}-\frac{\ln(\sin(c+d\,x)-1)\left(\frac{9\,b}{16\,(a+b)^4}+\frac{3}{16\,(a+b)^3}+\frac{3\,b^2}{4\,(a+b)^5}\right)}{d}-\frac{\frac{3\,a^6\,b-\cdots}{4\,(a^8-4\,a^6\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (log(sin(c + d*x) + 1)*((3*b^2)/(4*(a - b)^5) - (9*b)/(16*(a - b)^4) + 3/(1
6*(a - b)^3)))/d - (log(sin(c + d*x) - 1)*((9*b)/(16*(a + b)^4) + 3/(16*(a
+ b)^3) + (3*b^2)/(4*(a + b)^5)))/d - ((3*a^6*b + 2*b^7 - 31*a^2*b^5 - 22*a
^4*b^3)/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (sin(c + d*x)
*(68*a*b^6 + 5*a^7 + 49*a^3*b^4 - 26*a^5*b^2))/(8*(a^8 + b^8 - 4*a^2*b^6 +
6*a^4*b^4 - 4*a^6*b^2)) - (3*sin(c + d*x)^5*(27*a*b^6 + 6*a^3*b^4 - a^5*b^2
))/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (sin(c + d*x)^3*(1
51*a*b^6 + 3*a^7 + 61*a^3*b^4 - 23*a^5*b^2))/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*
a^4*b^4 - 4*a^6*b^2)) + (3*sin(c + d*x)^4*(a^6*b + 2*b^7 - 13*a^2*b^5 - 6*a
^4*b^3))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (sin(c + d*x
)^2*(5*a^6*b + 9*b^7 - 73*a^2*b^5 - 37*a^4*b^3))/(4*(a^8 + b^8 - 4*a^2*b^6
+ 6*a^4*b^4 - 4*a^6*b^2)))/(d*(sin(c + d*x)^4*(a^2 - 2*b^2) + a^2 - sin(c +
 d*x)^2*(2*a^2 - b^2) + b^2*sin(c + d*x)^6 + 2*a*b*sin(c + d*x) - 4*a*b*sin
(c + d*x)^3 + 2*a*b*sin(c + d*x)^5)) - (log(a + b*sin(c + d*x))*(3*b^7 + 21
*a^2*b^5))/(d*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x))**3, x)
```

3.456 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=197

$$\frac{5ax\left(4a^2-3b^2\right)}{2b^6}+\frac{5\cos(c+dx)\left(4a^2-2ab\sin(c+dx)-b^2\right)}{2b^5d}-\frac{5\left(4a^4-5a^2b^2+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}-\frac{5\cos^3}{6}$$

```
[Out] 5/2*a*(4*a^2-3*b^2)*x/b^6-1/2*cos(d*x+c)^5/b/d/(a+b*sin(d*x+c))^2-5/6*cos(d
*x+c)^3*(4*a+b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))+5/2*cos(d*x+c)*(4*a^2-b^2
-2*a*b*sin(d*x+c))/b^5/d-5*(4*a^4-5*a^2*b^2+b^4)*arctan((b+a*tan(1/2*d*x+1/
2*c))/(a^2-b^2)^(1/2))/b^6/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2693, 2863, 2865, 2735, 2660, 618, 204}

$$-\frac{5\left(-5a^2b^2+4a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}+\frac{5\cos(c+dx)\left(4a^2-2ab\sin(c+dx)-b^2\right)}{2b^5d}+\frac{5ax\left(4a^2-3b^2\right)}{2b^6}-5c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a*(4*a^2 - 3*b^2)*x)/(2*b^6) - (5*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d) - Cos[c + d*x
]^5/(2*b*d*(a + b*Sin[c + d*x])^2) - (5*Cos[c + d*x]^3*(4*a + b*Sin[c + d*x
]))/(6*b^3*d*(a + b*Sin[c + d*x])) + (5*Cos[c + d*x]*(4*a^2 - b^2 - 2*a*b*S
in[c + d*x]))/(2*b^5*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{2b}$$

$$= -\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} + \frac{5\int \frac{\cos^2(c+dx)(-b-}{a+b\sin(}}{2b^3}$$

$$= -\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} + \frac{5\cos(c+dx)\left(4\right.}{}$$

$$= \frac{5a\left(4a^2-3b^2\right)x}{2b^6} - \frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} +$$

$$= \frac{5a\left(4a^2-3b^2\right)x}{2b^6} - \frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} +$$

$$= \frac{5a\left(4a^2-3b^2\right)x}{2b^6} - \frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} +$$

$$= \frac{5a\left(4a^2-3b^2\right)x}{2b^6} - \frac{5\left(4a^4-5a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d} - \frac{\cos^5(c+d}{2bd(a+b\sin(c}$$

Mathematica [B] time = 6.57, size = 3889, normalized size = 19.74

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^5*(-1/2*(b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*(b
/(a + b) - (b*Sin[c + d*x])/(a + b))^(7/2))/(((a*b)/(a - b) - b^2/(a - b))*
((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x])^2) - ((-3*a*b^3*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*(b/(a + b) - (b*Sin[c + d*x])/(a +
b))^(7/2))/((a^2 - b^2)*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/
(a + b))*(a + b*Sin[c + d*x])) - ((144*Sqrt[2]*a*b^5*(-(b/(a - b)) - (b*Sin
[c + d*x])/(a - b))^(7/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + (
(a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((7*(3/(16*
(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 1/(2*(
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((
a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/12 + (35*b^
4*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-(b/
(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) + (2*(a - b)^3*(-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b))^3)/(15*b^3) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(S
qrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])
]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)
*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(128*(a - b)^4*(-(b/(
a - b)) - (b*Sin[c + d*x])/(a - b))^4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b)))/(2*b))^3)))/(7*(a - b)*(a + b)^4*Sqrt[((a + b)*(b/(a +
b) - (b*Sin[c + d*x])/(a + b)))/b]) + (((18*a^2*b^5)/((a - b)^2*(a + b)^2)
+ (b^5*(2*a^2 - 5*b^2))/((a - b)^2*(a + b)^2))*((8*Sqrt[2]*b*(-(b/(a - b))
- (b*Sin[c + d*x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)
]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((5
/(16*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5
/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1
 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/2 - (15
*b^3*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-
(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*A
rcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]
*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 +
 ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(64*(a - b)^
3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3*(1 + ((a - b)*(-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(5*(a + b)^2*Sqrt[((a + b)*(b/(a +
b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*((8
*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(3/2)*Sqrt[b/(a + b) -
 (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(
a - b)))/(2*b))^(7/2)*((3*(5/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b)))/(2*b))^3) + 5/(6*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x
])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b)))/(2*b))^(-1)))/8 + (15*b^2*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)))/b - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b))
- (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)))/(2*b)])))/(64*(a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3))
)/(3*(a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) -
((-((a*b)/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b*Sqrt[-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)
*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((5*Sqrt[b]*ArcSin
h[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt
[b])])/(8*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]
*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)) + (
15/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) +
5/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/6))/((
a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a
```

```
*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*(-(((-((a*
b)/(a + b)) - b^2/(a + b))*((2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b/(a
 - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin[c
+ d*x])/(a + b)])])/(b*Sqrt[a + b]) - (2*Sqrt[-((a*b)/(a + b)) - b^2/(a + b
)]*ArcTanh[(Sqrt[-((a*b)/(a + b)) - b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin
[c + d*x])/(a - b)])/(Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) -
 (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)])))/b)
 + (2*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/
(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c
+ d*x])/(a - b)))/(2*b))^(3/2)*((Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a -
 b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*
Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)) + 1/(2*(1 + ((a - b)*(-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)))/(2*b)))))/(b*(a + b)*Sqrt[((a + b)*(b/(a + b
) - (b*Sin[c + d*x])/(a + b)))/b])))/b) + (4*Sqrt[2]*(a - b)*Sqrt[-(b/(a -
b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*
(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((3*S
qrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/
(Sqrt[2]*Sqrt[b])])/(4*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b
))^(5/2)) + (3/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/
(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))
^(-1))/4))/((a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))
/b])))/b))/b))/b))/b)/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(
a + b))))/(2*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b)))))
/(d*(1 - (a + b*Sin[c + d*x])/(a - b))^(5/2)*(1 - (a + b*Sin[c + d*x])/(a +
 b))^(5/2))
```

fricas [A] time = 0.88, size = 752, normalized size = 3.82

$$\left[-\frac{4\,b^5\cos(dx+c)^5 - 30\left(4\,a^3b^2 - 3\,ab^4\right)dx\cos(dx+c)^2 - 20\left(2\,a^2b^3 - b^5\right)\cos(dx+c)^3 + 30\left(4\,a^5 + a^3b^2 - 3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*b^5*cos(d*x + c)^5 - 30*(4*a^3*b^2 - 3*a*b^4)*d*x*cos(d*x + c)^2
- 20*(2*a^2*b^3 - b^5)*cos(d*x + c)^3 + 30*(4*a^5 + a^3*b^2 - 3*a*b^4)*d*x
- 15*(4*a^4 + 3*a^2*b^2 - b^4 - (4*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(4*a^3
*b - a*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*co
s(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2)) + 30*(4*a^4*b - a^2*b^3 - b^5)*cos(d*x + c) + 10*(a*b^4*cos(d*x +
 c)^3 + 6*(4*a^4*b - 3*a^2*b^3)*d*x + 6*(3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))
*sin(d*x + c))/(b^8*d*cos(d*x + c)^2 - 2*a*b^7*d*sin(d*x + c) - (a^2*b^6 +
b^8)*d), -1/6*(2*b^5*cos(d*x + c)^5 - 15*(4*a^3*b^2 - 3*a*b^4)*d*x*cos(d*x
+ c)^2 - 10*(2*a^2*b^3 - b^5)*cos(d*x + c)^3 + 15*(4*a^5 + a^3*b^2 - 3*a*b^
4)*d*x + 15*(4*a^4 + 3*a^2*b^2 - b^4 - (4*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2
*(4*a^3*b - a*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 15*(4*a^4*b - a^2*b^3 - b^5)*cos(d*x +
 c) + 5*(a*b^4*cos(d*x + c)^3 + 6*(4*a^4*b - 3*a^2*b^3)*d*x + 6*(3*a^3*b^2
- 2*a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^8*d*cos(d*x + c)^2 - 2*a*b^7*d*si
n(d*x + c) - (a^2*b^6 + b^8)*d)]
```

giac [B] time = 0.70, size = 457, normalized size = 2.32

$$\frac{15\left(4a^3-3ab^2\right)(dx+c)}{b^6}-\frac{30\left(4a^4-5a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{2\left(9ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+36a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-18b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*a^3 - 3*a*b^2)*(d*x + c)/b^6 - 30*(4*a^4 - 5*a^2*b^2 + b^4)*(pi*
floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/
sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 +
 36*a^2*tan(1/2*d*x + 1/2*c)^4 - 18*b^2*tan(1/2*d*x + 1/2*c)^4 + 72*a^2*tan
(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1
/2*c) + 36*a^2 - 14*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^5) + 6*(7*a^5*b*
tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2
*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d*x + 1/2*c)^2 + 9*a^4*b^2*tan(1/2*d*x + 1/
2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 2*b^6*tan(1/2*d*x + 1/2*c)^2 +
 25*a^5*b*tan(1/2*d*x + 1/2*c) - 23*a^3*b^3*tan(1/2*d*x + 1/2*c) - 2*a*b^5*
tan(1/2*d*x + 1/2*c) + 8*a^6 - 7*a^4*b^2 - a^2*b^4)/((a*tan(1/2*d*x + 1/2*c
)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^2*b^5))/d
```

maple [B] time = 0.31, size = 1060, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -6/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4-8/d/b^3/(1+tan(1/2
*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2+12/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^3*a
^2+20/d/b^6*arctan(tan(1/2*d*x+1/2*c))*a^3-15/d/b^4*arctan(tan(1/2*d*x+1/2*
c))*a-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x
+1/2*c)^3-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2
*d*x+1/2*c)-15/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(
1/2*d*x+1/2*c)^2+8/d/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^
2*a^4-7/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2-5/d/b
^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
+7/d/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^3*tan(1/2*d*
x+1/2*c)^3-5/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*ta
n(1/2*d*x+1/2*c)^3+8/d/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a
)^2*a^4*tan(1/2*d*x+1/2*c)^2+9/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)^2*a^2*tan(1/2*d*x+1/2*c)^2-2/d*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2/a^2*tan(1/2*d*x+1/2*c)^2+25/d/b^4/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^3*tan(1/2*d*x+1/2*c)-23/d/b^2/(tan(1/2*d
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*c)-20/d/b^6/(a^
2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^4+2
5/d/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))*a^2+3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5+12/d/b^
5/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2+24/d/b^5/(1+tan(1/2*d
*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2*a^2-3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*
a*tan(1/2*d*x+1/2*c)-14/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3-1/d/b/(tan(1/2*d
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.58, size = 1226, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (atanh((1000*a^2*(b^2 - a^2)^(1/2))/(1000*a^2*b - (5000*a^4)/b + (4000*a^6)
/b^3 - 10000*a^3*tan(c/2 + (d*x)/2) + 2000*a*b^2*tan(c/2 + (d*x)/2) + (8000
*a^5*tan(c/2 + (d*x)/2))/b^2) - (4000*a^4*(b^2 - a^2)^(1/2))/(1000*a^2*b^3
- 5000*a^4*b + (4000*a^6)/b + 8000*a^5*tan(c/2 + (d*x)/2) + 2000*a*b^4*tan(
c/2 + (d*x)/2) - 10000*a^3*b^2*tan(c/2 + (d*x)/2)) + (2000*a*tan(c/2 + (d*x
)/2)*(b^2 - a^2)^(1/2))/(1000*a^2 - (5000*a^4)/b^2 + (4000*a^6)/b^4 - (1000
0*a^3*tan(c/2 + (d*x)/2))/b + (8000*a^5*tan(c/2 + (d*x)/2))/b^3 + 2000*a*b*
tan(c/2 + (d*x)/2)) - (9000*a^3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(1000
*a^2*b^2 - 5000*a^4 + (4000*a^6)/b^2 + 2000*a*b^3*tan(c/2 + (d*x)/2) - 1000
0*a^3*b*tan(c/2 + (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/b) + (4000*a^5*t
an(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(4000*a^6 + 1000*a^2*b^4 - 5000*a^4*b^
2 + 2000*a*b^5*tan(c/2 + (d*x)/2) + 8000*a^5*b*tan(c/2 + (d*x)/2) - 10000*a
^3*b^3*tan(c/2 + (d*x)/2)))*(20*a^2*(b^2 - a^2)^(1/2) - 5*b^2*(b^2 - a^2)^(
1/2)))/(b^6*d) - ((3*b^4 - 60*a^4 + 35*a^2*b^2)/(3*b^5) + (tan(c/2 + (d*x)/
2)*(6*b^4 - 210*a^4 + 125*a^2*b^2))/(3*a*b^4) - (tan(c/2 + (d*x)/2)^8*(20*a
^6 - 2*b^6 - 15*a^2*b^4 + 15*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^6*
(40*a^6 - 3*b^6 - 35*a^2*b^4 + 30*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/
2)^2*(120*a^6 - 3*b^6 - 55*a^2*b^4 + 10*a^4*b^2))/(3*a^2*b^5) - (2*tan(c/2
+ (d*x)/2)^4*(180*a^6 - 9*b^6 - 120*a^2*b^4 + 95*a^4*b^2))/(3*a^2*b^5) + (t
an(c/2 + (d*x)/2)^9*(2*b^4 - 10*a^4 + 5*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d
*x)/2)^7*(4*b^4 - 50*a^4 + 25*a^2*b^2))/(a*b^4) + (4*tan(c/2 + (d*x)/2)^5*(
3*b^4 - 60*a^4 + 35*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^3*(12*b^4 - 3
30*a^4 + 205*a^2*b^2))/(3*a*b^4))/(d*(tan(c/2 + (d*x)/2)^2*(5*a^2 + 4*b^2)
+ tan(c/2 + (d*x)/2)^8*(5*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 + 12*
b^2) + tan(c/2 + (d*x)/2)^6*(10*a^2 + 12*b^2) + a^2*tan(c/2 + (d*x)/2)^10 +
 a^2 + 16*a*b*tan(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 + 16*a*b*t
an(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2)
)) + (5*a*atan((3000*a^2*tan(c/2 + (d*x)/2))/(3000*a^2 - (7000*a^4)/b^2 + (
4000*a^6)/b^4) - (7000*a^4*tan(c/2 + (d*x)/2))/(3000*a^2*b^2 - 7000*a^4 + (
4000*a^6)/b^2) + (4000*a^6*tan(c/2 + (d*x)/2))/(4000*a^6 + 3000*a^2*b^4 - 7
000*a^4*b^2))*(4*a^2 - 3*b^2))/(b^6*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.457 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=139

$$\frac{3\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}}-\frac{3ax}{b^4}-\frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))}-\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

```
[Out] -3*a*x/b^4-1/2*cos(d*x+c)^3/b/d/(a+b*sin(d*x+c))^2-3/2*cos(d*x+c)*(2*a+b*si
n(d*x+c))/b^3/d/(a+b*sin(d*x+c))+3*(2*a^2-b^2)*arctan((b+a*tan(1/2*d*x+1/2*
c))/(a^2-b^2)^(1/2))/b^4/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2693, 2863, 2735, 2660, 618, 204}

$$\frac{3\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}}-\frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))}-\frac{3ax}{b^4}-\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*x)/b^4 + (3*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(b^4*Sqrt[a^2 - b^2]*d) - Cos[c + d*x]^3/(2*b*d*(a + b*Sin[c + d*x])
^2) - (3*Cos[c + d*x]*(2*a + b*Sin[c + d*x]))/(2*b^3*d*(a + b*Sin[c + d*x])
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{2b}$$

$$= -\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} + \frac{3\int \frac{-b-2a\sin(c+dx)}{a+b\sin(c+dx)}}{2b^3}$$

$$= -\frac{3ax}{b^4} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} + \frac{\left(3\left(2a^2-b\right.\right.}{}$$

$$= -\frac{3ax}{b^4} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} + \frac{\left(3\left(2a^2-b\right.\right.}{}$$

$$= -\frac{3ax}{b^4} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} - \frac{\left(6\left(2a^2-b\right.\right.}{}$$

$$= -\frac{3ax}{b^4} + \frac{3\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\,d} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+}{2b^3}$$

Mathematica [B] time = 6.28, size = 2641, normalized size = 19.00

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^3*(-1/2*(b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*(b
/(a + b) - (b*Sin[c + d*x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*
((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x])^2) - (-((a*b^3*(-(b/(a -
 b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*(b/(a + b) - (b*Sin[c + d*x])/(a + b
))^(5/2))/((a^2 - b^2)*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(
a + b))*(a + b*Sin[c + d*x]))) - ((16*Sqrt[2]*a*b^4*(-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((
a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((5*(1/(2*(1
 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a
 - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/8 - (15*b^3*
(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-(b/(a
 - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSin
```

```
h[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt
[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a
- b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)]))))/(32*(a - b)^3*(-(
b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3*(1 + ((a - b)*(-(b/(a - b)) - (b*S
in[c + d*x])/(a - b)))/(2*b))^2)))/(5*(a - b)*(a + b)^3*Sqrt[((a + b)*(b/(a
 + b) - (b*Sin[c + d*x])/(a + b)))/b]) + (((4*a^2*b^5)/((a - b)^2*(a + b)^2
) + (b^5*(2*a^2 - 3*b^2))/((a - b)^2*(a + b)^2))*((4*Sqrt[2]*(-(b/(a - b))
- (b*Sin[c + d*x])/(a - b))^(3/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)
]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((3
/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1
 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/2 + (3*
b^2*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[
a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/
(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*
Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)]))))/(8*(
a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2)))/(3*(a + b)*Sqrt[((a + b)*(b/
(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b)
)*(-(((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))
*((2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/
(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])]))/(b*Sqr
t[a + b]) - (2*Sqrt[-((a*b)/(a + b)) - b^2/(a + b)]*ArcTanh[(Sqrt[-((a*b)/(
a + b)) - b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt
[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]
)])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)])))/b) + (2*Sqrt[2]*(a - b)*Sqrt
[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])
/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(
3/2)*((Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a
 - b)])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin
[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))
)/(2*b))^(3/2)) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
b)))/(2*b)))))/(b*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a +
b)))/b])))/b) + (4*Sqrt[2]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sq
rt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*S
in[c + d*x])/(a - b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-
(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(4*Sqrt[2]*Sqr
t[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(
a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)*(-
(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/4))/((a + b)*Sqrt[((a + b)*
(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b])))/b))/b)/((((a*b)/(a - b) - b^2/
(a - b))*((a*b)/(a + b) + b^2/(a + b)))))/(2*((a*b)/(a - b) - b^2/(a - b))*(
(a*b)/(a + b) + b^2/(a + b)))))/(d*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/2)
*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/2))
```

fricas [B] time = 0.78, size = 716, normalized size = 5.15

$$\left[-\frac{12\left(a^3b^2-ab^4\right)dx\cos\left(dx+c\right)^2+4\left(a^2b^3-b^5\right)\cos\left(dx+c\right)^3-12\left(a^5-ab^4\right)dx+3\left(2a^4+a^2b^2-b^4-\left(2a^2b^2-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(12*(a^3*b^2 - a*b^4)*d*x*cos(d*x + c)^2 + 4*(a^2*b^3 - b^5)*cos(d*x
+ c)^3 - 12*(a^5 - a*b^4)*d*x + 3*(2*a^4 + a^2*b^2 - b^4 - (2*a^2*b^2 - b^4
)*cos(d*x + c)^2 + 2*(2*a^3*b - a*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
```

```
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c) - 6*(4*(a^4*b - a^2*b^3)*d*x + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d
*x + c))/((a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^3*b^5 - a*b^7)*d*sin(d*x
+ c) - (a^4*b^4 - b^8)*d), -1/2*(6*(a^3*b^2 - a*b^4)*d*x*cos(d*x + c)^2 + 2
*(a^2*b^3 - b^5)*cos(d*x + c)^3 - 6*(a^5 - a*b^4)*d*x - 3*(2*a^4 + a^2*b^2
- b^4 - (2*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(2*a^3*b - a*b^3)*sin(d*x + c)
)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c
))) - 3*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x + c) - 3*(4*(a^4*b - a^2*b^3)*d*x
 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^6 - b^8)*d*cos(d
*x + c)^2 - 2*(a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^4*b^4 - b^8)*d)]
```

giac [B] time = 0.42, size = 272, normalized size = 1.96

$$-\frac{\frac{3\,(dx+c)a}{b^4}-\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(2\,a^2-b^2\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b^3}+\frac{3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)*a/b^4 - 3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((
a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*(2*a^2 - b^2)/(sqrt(a^2 - b^2
)*b^4) + 2/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^3) + (3*a^3*b*tan(1/2*d*x + 1/2*
c)^3 + 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan(1/2*d*x + 1/2*c)^2 + 9*a^
2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2*c)^2 + 13*a^3*b*tan(
1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c) + 4*a^4 + a^2*b^2)/((a*tan(
1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^2*b^3))/d
```

maple [B] time = 0.31, size = 560, normalized size = 4.03

$$-\frac{2}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^4}-\frac{3a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^2}-\frac{}{d\left(\left(\mathrm{ta}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)-6/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a-3/d/
b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*c
)^3-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1
/2*c)^3-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2*tan
(1/2*d*x+1/2*c)^2-9/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
*tan(1/2*d*x+1/2*c)^2-2/d*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2/a^2*tan(1/2*d*x+1/2*c)^2-13/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*c)-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)^2*a^2-1/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2+6/d/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))*a^2-3/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.55, size = 1360, normalized size = 9.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^3,x)
```

```
[Out] - ((6*a^2 + b^2)/b^3 + (2*tan(c/2 + (d*x)/2)^2*(6*a^4 + b^4 + 9*a^2*b^2))/(
a^2*b^3) + (tan(c/2 + (d*x)/2)*(21*a^2 + 2*b^2))/(a*b^2) + (4*tan(c/2 + (d*
x)/2)^3*(6*a^2 + b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^4*(6*a^4 + 2*b^4 + 9*a
^2*b^2))/(a^2*b^3) + (tan(c/2 + (d*x)/2)^5*(3*a^2 + 2*b^2))/(a*b^2))/(d*(ta
n(c/2 + (d*x)/2)^2*(3*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(3*a^2 + 4*b^2) +
 a^2*tan(c/2 + (d*x)/2)^6 + a^2 + 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) - (3*a*x)/b^4 - (atan((((-(a +
b)*(a - b))^(1/2)*(2*a^2 - b^2)*((288*a^4)/b^5 - (8*tan(c/2 + (d*x)/2)*(9*a
*b^7 - 108*a^3*b^5 + 72*a^5*b^3))/b^9 + (3*(-(a + b)*(a - b))^(1/2)*(2*a^2
- b^2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 - 24*a^3*b^8))/b^9 - 48*a^2 + (3*(
-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(
12*a*b^13 - 8*a^3*b^11))/b^9))/(2*(b^6 - a^2*b^4))))/(2*(b^6 - a^2*b^4)))*3
i)/(2*(b^6 - a^2*b^4)) + ((-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*((288*a^4)
/b^5 - (8*tan(c/2 + (d*x)/2)*(9*a*b^7 - 108*a^3*b^5 + 72*a^5*b^3))/b^9 + (3
*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*(48*a^2 - (8*tan(c/2 + (d*x)/2)*(12
*a*b^10 - 24*a^3*b^8))/b^9 + (3*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*(32*
a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(2*(b^6 - a
^2*b^4))))/(2*(b^6 - a^2*b^4)))*3i)/(2*(b^6 - a^2*b^4)))/((16*(54*a^4 - 27*
a^2*b^2))/b^8 + (16*tan(c/2 + (d*x)/2)*(216*a^5 - 108*a^3*b^2))/b^9 - (3*(-
(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*((288*a^4)/b^5 - (8*tan(c/2 + (d*x)/2)
*(9*a*b^7 - 108*a^3*b^5 + 72*a^5*b^3))/b^9 + (3*(-(a + b)*(a - b))^(1/2)*(2
*a^2 - b^2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 - 24*a^3*b^8))/b^9 - 48*a^2 +
 (3*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)
/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(2*(b^6 - a^2*b^4))))/(2*(b^6 - a^2*b^4
))))/(2*(b^6 - a^2*b^4)) + (3*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*((288*
a^4)/b^5 - (8*tan(c/2 + (d*x)/2)*(9*a*b^7 - 108*a^3*b^5 + 72*a^5*b^3))/b^9
+ (3*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*(48*a^2 - (8*tan(c/2 + (d*x)/2)
*(12*a*b^10 - 24*a^3*b^8))/b^9 + (3*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*
(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(2*(b^6
 - a^2*b^4))))/(2*(b^6 - a^2*b^4))))/(2*(b^6 - a^2*b^4))))*(-(a + b)*(a - b
))^(1/2)*(2*a^2 - b^2)*3i)/(d*(b^6 - a^2*b^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.458 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=115

$$\frac{\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

```
[Out] arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-1/2*cos(
d*x+c)/b/d/(a+b*sin(d*x+c))^2+1/2*a*cos(d*x+c)/b/(a^2-b^2)/d/(a+b*sin(d*x+c
))
```

Rubi [A] time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2693, 2754, 12, 2660, 618, 204}

$$\frac{\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(3/2)*d) - Co
s[c + d*x]/(2*b*d*(a + b*Sin[c + d*x])^2) + (a*Cos[c + d*x])/(2*b*(a^2 - b^
2)*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\int \frac{\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{2b}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{b}{a+b\sin(c+dx)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{1}{a+b\sin(c+dx)}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,d\right.}{\left(a^2-\right.}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{2\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.25, size = 93, normalized size = 0.81

$$\frac{\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{\cos(c+dx)(a\sin(c+dx)+b)}{(a+b\sin(c+dx))^2}}{2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (Co
s[c + d*x]*(b + a*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2)/(2*(a - b)*(a + b)
*d)
```

fricas [A] time = 0.64, size = 501, normalized size = 4.36

$$\left[-\frac{2\left(a^3-ab^2\right)\cos(dx+c)\sin(dx+c) - \left(b^2\cos(dx+c)^2 - 2ab\sin(dx+c) - a^2 - b^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)}{}\right.}{4\left(\left(a^4b^2 - 2a^2b^4 + b^6\right)d\cos(dx+c)^2 - 2\left(a^5b - 2a^3b^3 + ab^5\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^3 - a*b^2)*cos(d*x + c)*sin(d*x + c) - (b^2*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x + c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*
d*cos(d*x + c)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*sin(d*x + c) - (a^6 - a^
4*b^2 - a^2*b^4 + b^6)*d), -1/2*((a^3 - a*b^2)*cos(d*x + c)*sin(d*x + c) +
(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(a^2 - b^2)*arcta
n(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (a^2*b - b^3)*cos
(d*x + c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(d*x + c)^2 - 2*(a^5*b - 2*a^3
*b^3 + a*b^5)*d*sin(d*x + c) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*d)]
```

giac [A] time = 0.55, size = 207, normalized size = 1.80

$$\frac{\frac{\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a^3\tan\left(\frac{1}{2}dx\right.}{\left(a^4-a^2b^2\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
+ b)/sqrt(a^2 - b^2)))/(a^2 - b^2)^(3/2) - (a^3*tan(1/2*d*x + 1/2*c)^3 - 2*
a*b^2*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*d*x + 1/2*c)^2 - 2*b^3*tan(1/2
*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 2*a*b^2*tan(1/2*d*x + 1/2*c) -
 a^2*b)/((a^4 - a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*
c) + a)^2))/d
```

maple [B] time = 0.30, size = 443, normalized size = 3.85

$$-\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^2\left(a^2-b^2\right)}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{d\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^2a\left(a^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a/(a^2-b^2)*tan(1/
2*d*x+1/2*c)^3+2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/(a
^2-b^2)*tan(1/2*d*x+1/2*c)^3*b^2+1/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)^2*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2+2/d/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)/a^2*tan(1/2*d*x+1/2*c)^2+1/d/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a/(a^2-b^2)*tan(1/2*d*x+1/2
*c)+2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/(a^2-b^2)*tan
(1/2*d*x+1/2*c)*b^2+1/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
*b/(a^2-b^2)+1/d/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a
^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.37, size = 282, normalized size = 2.45

$$\frac{\frac{b}{a^2-b^2}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(a^2-2b^2\right)}{a\left(a^2-b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^2+2b^2\right)}{a\left(a^2-b^2\right)}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(a^2+2b^2\right)}{a^2\left(a^2-b^2\right)}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2+4b^2\right)+a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+a^2+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\mathrm{atan}\left(\left(a^2-b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (b/(a^2 - b^2) - (tan(c/2 + (d*x)/2)^3*(a^2 - 2*b^2))/(a*(a^2 - b^2)) + (ta
n(c/2 + (d*x)/2)*(a^2 + 2*b^2))/(a*(a^2 - b^2)) + (b*tan(c/2 + (d*x)/2)^2*(
a^2 + 2*b^2))/(a^2*(a^2 - b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2) +
 a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2))) + atan((a^2 - b^2)*((a^2*b - b^3)/((a + b)^(3/2)*(a^2 - b^2)
*(a - b)^(3/2)) + (a*tan(c/2 + (d*x)/2))/((a + b)^(3/2)*(a - b)^(3/2))))/(d
*(a + b)^(3/2)*(a - b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.459 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=192

$$-\frac{3b^2\left(4a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{5ab\sec(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sec(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\text{se}$$

```
[Out] -3*b^2*(4*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^
2)^(7/2)/d+1/2*b*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+5/2*a*b*sec(d*x+
c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-1/2*sec(d*x+c)*(3*b*(4*a^2+b^2)-a*(2*a^2+
13*b^2)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A] time = 0.39, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2694, 2864, 2866, 12, 2660, 618, 204}

$$-\frac{3b^2\left(4a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{5ab\sec(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sec(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\text{se}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b^2*(4*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(7/2)*d) + (b*Sec[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
^2) + (5*a*b*Sec[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[
c + d*x]*(3*b*(4*a^2 + b^2) - a*(2*a^2 + 13*b^2)*Sin[c + d*x]))/(2*(a^2 - b
^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(-2a+3b\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+dx)\left(2a^2 \cdots\right.}{a+b \cdots}}{2\left(\cdots\right.}$$

$$= \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3 \cdots\right.}{\cdots}$$

$$= \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3 \cdots\right.}{\cdots}$$

$$= \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3 \cdots\right.}{\cdots}$$

$$= \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3 \cdots\right.}{\cdots}$$

$$= -\frac{3b^2\left(4a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\cdots}{2\left(a^2- \cdots\right.}$$

Mathematica [A] time = 3.06, size = 193, normalized size = 1.01

$$\frac{-\frac{6b^2(4a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\frac{b^3\cos(c+dx)\left(-8a^2-7ab\sin(c+dx)+b^2\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a-b)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*b^2*(4*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (b^3*
Cos[c + d*x]*(-8*a^2 + b^2 - 7*a*b*Sin[c + d*x]))/((a - b)^3*(a + b)^3*(a +
 b*Sin[c + d*x])^2))/(2*d)
```

fricas [B] time = 0.74, size = 894, normalized size = 4.66

$$\left[\frac{4\,a^6b-12\,a^4b^3+12\,a^2b^5-4\,b^7+2\left(4\,a^6b+10\,a^4b^3-17\,a^2b^5+3\,b^7\right)\cos\left(dx+c\right)^2+3\left(\left(4\,a^2b^4+b^6\right)\cos\left(dx\right.\right.}{4\left(\left(a^8b^2-4\,a^6b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(4*a^6*b + 10*a^4*b^3 -
 17*a^2*b^5 + 3*b^7)*cos(d*x + c)^2 + 3*((4*a^2*b^4 + b^6)*cos(d*x + c)^3 -
 2*(4*a^3*b^3 + a*b^5)*cos(d*x + c)*sin(d*x + c) - (4*a^4*b^2 + 5*a^2*b^4 +
 b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
 c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - (2*a^5*b^2 + 11*a^3*b^4 -
 13*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6
- 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4
*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b
^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), 1/2*(2*a^6*b - 6*a^4*b^
3 + 6*a^2*b^5 - 2*b^7 + (4*a^6*b + 10*a^4*b^3 - 17*a^2*b^5 + 3*b^7)*cos(d*x
 + c)^2 + 3*((4*a^2*b^4 + b^6)*cos(d*x + c)^3 - 2*(4*a^3*b^3 + a*b^5)*cos(d
*x + c)*sin(d*x + c) - (4*a^4*b^2 + 5*a^2*b^4 + b^6)*cos(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a
^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6)*
cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8
 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*
b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

giac [B] time = 4.78, size = 385, normalized size = 2.01

$$-\frac{\frac{3\left(4\,a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b-b^3\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{9\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(4*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*
tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(1/2*d*x
+ 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x
 + 1/2*c)^2 - 1)) + (9*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^6*tan(1/2*d*x
 + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*b^5*tan(1/2*d*x + 1
/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 + 23*a^3*b^4*tan(1/2*d*x + 1/2*c) -
2*a*b^6*tan(1/2*d*x + 1/2*c) + 8*a^4*b^3 - a^2*b^5)/((a^8 - 3*a^6*b^2 + 3*a
^4*b^4 - a^2*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)
^2))/d
```

maple [B] time = 0.28, size = 705, normalized size = 3.67

$$-\frac{1}{d\left(a+b\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{9b^{4}a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a-b\right)^{3}\left(a+b\right)^{3}\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^{2}}+\frac{\cdots}{d\left(a-b\right)^{3}\left(a+b\right)^{3}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-9/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*c)^3+2/d*b^6/(a-b)^3
/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+
1/2*c)^3-8/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^2*a^2*tan(1/2*d*x+1/2*c)^2-15/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+2/d*b^7/(a-b)^3/(
a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a^2*tan(1/2*d*x+
1/2*c)^2-23/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*a*tan(1/2*d*x+1/2*c)+2/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)-8/d*b^3/(a-b)^3/(a+b)
^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2+1/d*b^5/(a-b)^3/
(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-12/d*b^2/(a-b)^
3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))*a^2-3/d*b^4/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.82, size = 650, normalized size = 3.39

$$-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(2a^{5}-2a^{3}b^{2}+15ab^{4}\right)}{a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}}-\frac{6a^{4}b+10a^{2}b^{3}-b^{5}}{a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\left(2a^{6}+6a^{4}b^{2}+9a^{2}b^{4}-2b^{6}\right)}{a\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}\right)}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(2a^{6}b+2a^{4}b^{3}+12a^{2}\cdots\right)}{a^{2}\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}\right)}\cdots}{d\left(a^{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-a^{2}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(a^{2}+4b^{2}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(a^{2}+4b\cdots\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] - ((2*tan(c/2 + (d*x)/2)^3*(15*a*b^4 + 2*a^5 - 2*a^3*b^2))/(a^6 - b^6 + 3*a
^2*b^4 - 3*a^4*b^2) - (6*a^4*b - b^5 + 10*a^2*b^3)/(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^5*(2*a^6 - 2*b^6 + 9*a^2*b^4 + 6*a^4*b^2)
)/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^2*(2*a^6*
b - b^7 + 12*a^2*b^5 + 2*a^4*b^3))/(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)
) + (tan(c/2 + (d*x)/2)^4*(2*a^6*b - 2*b^7 + 15*a^2*b^5 + 30*a^4*b^3))/(a^2
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)*(2*a^6 + 2*b^6
- 31*a^2*b^4 - 18*a^4*b^2))/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)))/(d*(a^
2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)^2*(a^2 + 4*b^2) + tan(c/2
 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c/2 +
(d*x)/2))) - (3*b^2*atan(((3*b^2*(4*a^2 + b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5
 - 6*a^4*b^3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (3*a*b^2*tan(c/2 + (d*x)/2
)*(4*a^2 + b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)
^(7/2)))/(3*b^4 + 12*a^2*b^2))*(4*a^2 + b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c + dx)}{(a + b\sin(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

3.460 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=264

$$\frac{7ab\sec^3(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sec^3(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{\sec^3(c+dx)\left(5b\left(6a^2+b^2\right)-a\left(2a^2+33b^2\right)\right.}{6d\left(a^2-b^2\right)^3}$$

```
[Out] 5*b^4*(6*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2
)^(9/2)/d+1/2*b*sec(d*x+c)^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+7/2*a*b*sec(d*x
+c)^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-1/6*sec(d*x+c)^3*(5*b*(6*a^2+b^2)-a*(2
*a^2+33*b^2)*sin(d*x+c))/(a^2-b^2)^3/d+1/6*sec(d*x+c)*(15*b^3*(6*a^2+b^2)+a
*(4*a^4-28*a^2*b^2-81*b^4)*sin(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.64, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2694, 2864, 2866, 12, 2660, 618, 204}

$$\frac{5b^4\left(6a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{7ab\sec^3(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sec^3(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{\sec^3(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*b^4*(6*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^
2 - b^2)^(9/2)*d) + (b*Sec[c + d*x]^3)/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]
)^2) + (7*a*b*Sec[c + d*x]^3)/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (S
ec[c + d*x]^3*(5*b*(6*a^2 + b^2) - a*(2*a^2 + 33*b^2)*Sin[c + d*x]))/(6*(a^
2 - b^2)^3*d) + (Sec[c + d*x]*(15*b^3*(6*a^2 + b^2) + a*(4*a^4 - 28*a^2*b^2
 - 81*b^4)*Sin[c + d*x]))/(6*(a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\int \frac{\sec^4(c+dx)(-2a+5b\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^4(c+dx)\left(2a^2\right.}{a+b} }{2\left(\right.}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(\right.}{}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(\right.}{}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(\right.}{}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(\right.}{}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(\right.}{}$$

$$= \frac{5b^4\left(6a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d} + \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7}{2\left(a^2-b\right.}$$

Mathematica [A] time = 2.81, size = 380, normalized size = 1.44

$$\frac{\frac{60b^4\left(6a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}} + \frac{66ab^5\cos(c+dx)}{(a-b)^4(a+b)^4(a+b\sin(c+dx))} + \frac{6b^5\cos(c+dx)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2} + \frac{2(4a+13b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^4\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + }{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((60*b^4*(6*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(9/2) + 1/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
2*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (
2*(4*a + 13*b)*Sin[(c + d*x)/2])/((a + b)^4*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^3) - 1/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*(4*a -
 13*b)*Sin[(c + d*x)/2])/((a - b)^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
+ (6*b^5*Cos[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2) + (66*a
*b^5*Cos[c + d*x])/((a - b)^4*(a + b)^4*(a + b*Sin[c + d*x])))/(12*d)
```

fricas [B] time = 0.88, size = 1200, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 + 2*(8*a^8*b
- 64*a^6*b^3 - 16*a^4*b^5 + 87*a^2*b^7 - 15*b^9)*cos(d*x + c)^4 - 4*(2*a^8*
b - a^6*b^3 - 9*a^4*b^5 + 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^2 - 15*((6*a^2*b
^6 + b^8)*cos(d*x + c)^5 - 2*(6*a^3*b^5 + a*b^7)*cos(d*x + c)^3*sin(d*x + c
) - (6*a^4*b^4 + 7*a^2*b^6 + b^8)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(((2*
a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x +
 c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^
3*b^6 + 2*a*b^8 - (4*a^7*b^2 - 32*a^5*b^4 - 53*a^3*b^6 + 81*a*b^8)*cos(d*x
+ c)^4 + 2*(2*a^9 - 15*a^7*b^2 + 33*a^5*b^4 - 29*a^3*b^6 + 9*a*b^8)*cos(d*x
 + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5
*a^2*b^10 - b^12)*d*cos(d*x + c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 1
0*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3*sin(d*x + c) - (a^12 - 4*a
^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^3), 1/6
*(2*a^8*b - 8*a^6*b^3 + 12*a^4*b^5 - 8*a^2*b^7 + 2*b^9 + (8*a^8*b - 64*a^6*
b^3 - 16*a^4*b^5 + 87*a^2*b^7 - 15*b^9)*cos(d*x + c)^4 - 2*(2*a^8*b - a^6*b
^3 - 9*a^4*b^5 + 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^2 - 15*((6*a^2*b^6 + b^8)
*cos(d*x + c)^5 - 2*(6*a^3*b^5 + a*b^7)*cos(d*x + c)^3*sin(d*x + c) - (6*a^
4*b^4 + 7*a^2*b^6 + b^8)*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^9 - 8*a^7*b^2 + 12*a^5*b^
4 - 8*a^3*b^6 + 2*a*b^8 - (4*a^7*b^2 - 32*a^5*b^4 - 53*a^3*b^6 + 81*a*b^8)*
cos(d*x + c)^4 + 2*(2*a^9 - 15*a^7*b^2 + 33*a^5*b^4 - 29*a^3*b^6 + 9*a*b^8)
*cos(d*x + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4
*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x + c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7
*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3*sin(d*x + c) - (a^
12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)
^3)]
```

giac [B] time = 9.09, size = 622, normalized size = 2.36

$$\frac{\frac{15\left(6a^2b^4+b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)\sqrt{a^2-b^2}}+\frac{3\left(13a^3b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12a^4b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+23a\cdots\right.}{\left(a^{10}-4a^8b^2+6a^6b^4-4a^4b^6\cdots\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(15*(6*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*(13*a^3*b^6*tan(1/2*d*x + 1/2*c)
^3 - 2*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*tan(1/2*d*x + 1/2*c)^2 + 2
3*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2*b^9*tan(1/2*d*x + 1/2*c)^2 + 35*a^3*b^
6*tan(1/2*d*x + 1/2*c) - 2*a*b^8*tan(1/2*d*x + 1/2*c) + 12*a^4*b^5 - a^2*b^
7)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(a*tan(1/2*d*x + 1
/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - 2*(3*a^5*tan(1/2*d*x + 1/2*c)^
5 - 12*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 9
*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 + 9*b^5*t
an(1/2*d*x + 1/2*c)^4 - 2*a^5*tan(1/2*d*x + 1/2*c)^3 + 32*a^3*b^2*tan(1/2*d
*x + 1/2*c)^3 + 42*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 60*a^2*b^3*tan(1/2*d*x +
1/2*c)^2 - 12*b^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1/2*d*x + 1/2*c) - 12*
a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*a*b^4*tan(1/2*d*x + 1/2*c) - 3*a^4*b + 32
*a^2*b^3 + 7*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(tan(1/2
*d*x + 1/2*c)^2 - 1)^3))/d
```

maple [B] time = 0.34, size = 854, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1
)^2-1/d/(a+b)^4/(tan(1/2*d*x+1/2*c)-1)*a-5/2/d/(a+b)^4/(tan(1/2*d*x+1/2*c)-
1)*b+13/d*b^6/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*a*tan(1/2*d*x+1/2*c)^3-2/d*b^8/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^3+12/d*b^5/(a-b)^4/(a+b
)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2*tan(1/2*d*x+1/2
*c)^2+23/d*b^7/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2*tan(1/2*d*x+1/2*c)^2-2/d*b^9/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a^2*tan(1/2*d*x+1/2*c)^2+35/d*b^6/(a-b)^4/(a+
b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*
c)-2/d*b^8/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a
)^2/a*tan(1/2*d*x+1/2*c)+12/d*b^5/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^2*a^2-1/d*b^7/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+30/d*b^4/(a-b)^4/(a+b)^4/(a^2-b^2)^(1/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+5/d*b^6/(a-b)^4/
(a+b)^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))-1/3/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/(a-b)^3/(tan(1/2*d*x+1/2
*c)+1)^2-1/d/(a-b)^4/(tan(1/2*d*x+1/2*c)+1)*a+5/2/d/(a-b)^4/(tan(1/2*d*x+1/
2*c)+1)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.37, size = 1167, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (5*b^4*atan(((5*b^4*(6*a^2 + b^2)*(2*a^8*b + 2*b^9 - 8*a^2*b^7 + 12*a^4*b^5
 - 8*a^6*b^3))/(2*(a + b)^(9/2)*(a - b)^(9/2)) + (5*a*b^4*tan(c/2 + (d*x)/2
)*(6*a^2 + b^2)*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))/((a + b)^(
9/2)*(a - b)^(9/2)))/(5*b^6 + 30*a^2*b^4))*(6*a^2 + b^2))/(d*(a + b)^(9/2)*
(a - b)^(9/2)) - ((2*tan(c/2 + (d*x)/2)^5*(255*a*b^6 + 2*a^7 + 62*a^3*b^4 -
 4*a^5*b^2))/(3*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (6*a^6*b
 + 3*b^7 - 50*a^2*b^5 - 64*a^4*b^3)/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^7*(2*a^6 + 3*b^6 + 36*a^2*b^4 - 6*a^4*
b^2))/(3*a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(
6*a^6*b + 3*b^7 - 64*a^2*b^5 - 50*a^4*b^3))/(3*a^2*(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^9*(13*a^2*b^6 - 2*b^8 - 2*a^8 + 18*a^4*b
^4 + 8*a^6*b^2))/(a*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (tan
(c/2 + (d*x)/2)^8*(23*a^2*b^7 - 2*b^9 - 2*a^8*b + 78*a^4*b^5 + 8*a^6*b^3))/
(a^2*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)
*(6*a^8 - 6*b^8 + 161*a^2*b^6 + 202*a^4*b^4 - 48*a^6*b^2))/(3*a*(a^2 - b^2)
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^3*(2*a^8 + 3*
b^8 - 133*a^2*b^6 - 86*a^4*b^4 + 4*a^6*b^2))/(3*a*(a^2 - b^2)*(a^6 - b^6 +
```

```
3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^4*(8*a^8*b - 9*b^9 + 156*a^
2*b^7 + 188*a^4*b^5 - 28*a^6*b^3))/(3*a^2*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^6*(141*a^2*b^7 - 9*b^9 - 14*a^8*b +
 246*a^4*b^5 + 56*a^6*b^3))/(3*a^2*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2)))/(d*(tan(c/2 + (d*x)/2)^4*(2*a^2 + 12*b^2) - tan(c/2 + (d*x)/2)^6*
(2*a^2 + 12*b^2) + a^2*tan(c/2 + (d*x)/2)^10 - a^2 + tan(c/2 + (d*x)/2)^2*(
a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^8*(a^2 - 4*b^2) + 8*a*b*tan(c/2 + (d*x)/2
)^3 - 8*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 - 4*a*b*tan(c
/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

3.461 $\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=207

$$\frac{\left(a^2-b^2\right)^3}{7b^7d(a+b\sin(c+dx))^7}-\frac{a\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))^6}+\frac{5a^2-b^2}{b^7d(a+b\sin(c+dx))^3}-\frac{a\left(5a^2-3b^2\right)}{b^7d(a+b\sin(c+dx))^4}+\frac{3\left(5a^4-6\right.}{5b^7d(a+b\,\text{s}}$$

```
[Out] 1/7*(a^2-b^2)^3/b^7/d/(a+b*sin(d*x+c))^7-a*(a^2-b^2)^2/b^7/d/(a+b*sin(d*x+c
))^6+3/5*(5*a^4-6*a^2*b^2+b^4)/b^7/d/(a+b*sin(d*x+c))^5-a*(5*a^2-3*b^2)/b^7
/d/(a+b*sin(d*x+c))^4+(5*a^2-b^2)/b^7/d/(a+b*sin(d*x+c))^3-3*a/b^7/d/(a+b*s
in(d*x+c))^2+1/b^7/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{\left(a^2-b^2\right)^3}{7b^7d(a+b\sin(c+dx))^7}-\frac{a\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))^6}+\frac{5a^2-b^2}{b^7d(a+b\sin(c+dx))^3}-\frac{a\left(5a^2-3b^2\right)}{b^7d(a+b\sin(c+dx))^4}+\frac{3\left(-6a^2b^2\right.}{5b^7d(a+b\,\text{s}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a^2 - b^2)^3/(7*b^7*d*(a + b*Sin[c + d*x])^7) - (a*(a^2 - b^2)^2)/(b^7*d*(
a + b*Sin[c + d*x])^6) + (3*(5*a^4 - 6*a^2*b^2 + b^4))/(5*b^7*d*(a + b*Sin[
c + d*x])^5) - (a*(5*a^2 - 3*b^2))/(b^7*d*(a + b*Sin[c + d*x])^4) + (5*a^2
- b^2)/(b^7*d*(a + b*Sin[c + d*x])^3) - (3*a)/(b^7*d*(a + b*Sin[c + d*x])^2
) + 1/(b^7*d*(a + b*Sin[c + d*x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{(a+x)^8}\,dx,x,b\sin(c+dx)\right)}{b^7d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\left(a^2-b^2\right)^3}{(a+x)^8}+\frac{6a\left(a^2-b^2\right)^2}{(a+x)^7}-\frac{3\left(5a^4-6a^2b^2+b^4\right)}{(a+x)^6}+\frac{4\left(5a^3-3ab^2\right)}{(a+x)^5}-\frac{3\left(5a^2-b^2\right)}{(a+x)^4}+\frac{6a}{(a+x)^3}-\frac{}{(a}\right.\right.}{b^7d}\\
&= \frac{\left(a^2-b^2\right)^3}{7b^7d(a+b\sin(c+dx))^7}-\frac{a\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))^6}+\frac{3\left(5a^4-6a^2b^2+b^4\right)}{5b^7d(a+b\sin(c+dx))^5}-\frac{}{b^7d}
\end{aligned}$$

Mathematica [A] time = 1.14, size = 171, normalized size = 0.83

$$\frac{\frac{(a^2-b^2)^3}{7(a+b\sin(c+dx))^7}-\frac{a(a^2-b^2)^2}{(a+b\sin(c+dx))^6}+\frac{5a^2-b^2}{(a+b\sin(c+dx))^3}-\frac{a(5a^2-3b^2)}{(a+b\sin(c+dx))^4}+\frac{3(5a^4-6a^2b^2+b^4)}{5(a+b\sin(c+dx))^5}+\frac{1}{a+b\sin(c+dx)}-\frac{3a}{(a+b\sin(c+dx))^2}}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((a^2 - b^2)^3/(7*(a + b*Sin[c + d*x])^7) - (a*(a^2 - b^2)^2)/(a + b*Sin[c
+ d*x])^6 + (3*(5*a^4 - 6*a^2*b^2 + b^4))/(5*(a + b*Sin[c + d*x])^5) - (a*(
5*a^2 - 3*b^2))/(a + b*Sin[c + d*x])^4 + (5*a^2 - b^2)/(a + b*Sin[c + d*x])
^3 - (3*a)/(a + b*Sin[c + d*x])^2 + (a + b*Sin[c + d*x])^(-1))/(b^7*d)
```

fricas [A] time = 0.84, size = 382, normalized size = 1.85

$$\frac{35\,b^6\cos(dx+c)^6-5\,a^6-104\,a^4b^2-155\,a^2b^4-16\,b^6-35\left(5\,a^2b^4+2\,b\right.}{35\left(7\,ab^{13}d\cos(dx+c)^6-7\left(5\,a^3b^{11}+3\,ab^{13}\right)d\cos(dx+c)^4+7\left(3\,a^5b^9+10\,a^3b^{11}+3\,ab^{13}\right)d\cos(dx+c)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^6*cos(d*x + c)^6 - 5*a^6 - 104*a^4*b^2 - 155*a^2*b^4 - 16*b^6 -
35*(5*a^2*b^4 + 2*b^6)*cos(d*x + c)^4 + 7*(15*a^4*b^2 + 47*a^2*b^4 + 8*b^6)
*cos(d*x + c)^2 - 7*(15*a*b^5*cos(d*x + c)^4 + 5*a^5*b + 24*a^3*b^3 + 11*a*
b^5 - 25*(a^3*b^3 + a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(7*a*b^13*d*cos(d*
x + c)^6 - 7*(5*a^3*b^11 + 3*a*b^13)*d*cos(d*x + c)^4 + 7*(3*a^5*b^9 + 10*a
^3*b^11 + 3*a*b^13)*d*cos(d*x + c)^2 - (a^7*b^7 + 21*a^5*b^9 + 35*a^3*b^11
+ 7*a*b^13)*d + (b^14*d*cos(d*x + c)^6 - 3*(7*a^2*b^12 + b^14)*d*cos(d*x +
c)^4 + (35*a^4*b^10 + 42*a^2*b^12 + 3*b^14)*d*cos(d*x + c)^2 - (7*a^6*b^8 +
 35*a^4*b^10 + 21*a^2*b^12 + b^14)*d)*sin(d*x + c))
```

giac [A] time = 3.88, size = 215, normalized size = 1.04

$$\frac{35\,b^6\sin(dx+c)^6+105\,ab^5\sin(dx+c)^5+175\,a^2b^4\sin(dx+c)^4-35\,b^6\sin(dx+c)^4+175\,a^3b^3\sin(dx+c)^{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/35*(35*b^6*sin(d*x + c)^6 + 105*a*b^5*sin(d*x + c)^5 + 175*a^2*b^4*sin(d*
x + c)^4 - 35*b^6*sin(d*x + c)^4 + 175*a^3*b^3*sin(d*x + c)^3 - 35*a*b^5*si
n(d*x + c)^3 + 105*a^4*b^2*sin(d*x + c)^2 - 21*a^2*b^4*sin(d*x + c)^2 + 21*
b^6*sin(d*x + c)^2 + 35*a^5*b*sin(d*x + c) - 7*a^3*b^3*sin(d*x + c) + 7*a*b
^5*sin(d*x + c) + 5*a^6 - a^4*b^2 + a^2*b^4 - 5*b^6)/((b*sin(d*x + c) + a)^
7*b^7*d)
```

maple [A] time = 0.36, size = 208, normalized size = 1.00

$$\frac{-\frac{a(5a^2-3b^2)}{b^7(a+b\sin(dx+c))^4}-\frac{-a^6+3a^4b^2-3a^2b^4+b^6}{7b^7(a+b\sin(dx+c))^7}+\frac{1}{b^7(a+b\sin(dx+c))}-\frac{-15a^4+18a^2b^2-3b^4}{5b^7(a+b\sin(dx+c))^5}-\frac{a(a^4-2a^2b^2+b^4)}{b^7(a+b\sin(dx+c))^6}-\frac{3a}{b^7(a+b\sin(dx+c))^2}-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-a*(5*a^2-3*b^2)/b^7/(a+b*sin(d*x+c))^4-1/7*(-a^6+3*a^4*b^2-3*a^2*b^4+
b^6)/b^7/(a+b*sin(d*x+c))^7+1/b^7/(a+b*sin(d*x+c))-1/5*(-15*a^4+18*a^2*b^2-
```

3*b^4)/b^7/(a+b*sin(d*x+c))^5-a*(a^4-2*a^2*b^2+b^4)/b^7/(a+b*sin(d*x+c))^6-3*a/b^7/(a+b*sin(d*x+c))^2-1/3*(-15*a^2+3*b^2)/b^7/(a+b*sin(d*x+c))^3)

maxima [A] time = 0.33, size = 279, normalized size = 1.35

$$\frac{35\,b^{6}\sin\left(dx+c\right)^{6}+105\,ab^{5}\sin\left(dx+c\right)^{5}+5\,a^{6}-a^{4}b^{2}+a^{2}b^{4}-5\,b^{6}+35\left(5\,a^{2}b^{4}-b^{6}\right)\sin\left(dx+c\right)^{4}+35\left(5\,a^{3}b^{3}\right.}{35\left(b^{14}\sin\left(dx+c\right)^{7}+7\,ab^{13}\sin\left(dx+c\right)^{6}+21\,a^{2}b^{12}\sin\left(dx+c\right)^{5}+35\,a^{3}b^{11}\sin\left(dx+c\right)^{4}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/35*(35*b^6*sin(d*x + c)^6 + 105*a*b^5*sin(d*x + c)^5 + 5*a^6 - a^4*b^2 +
a^2*b^4 - 5*b^6 + 35*(5*a^2*b^4 - b^6)*sin(d*x + c)^4 + 35*(5*a^3*b^3 - a*b
^5)*sin(d*x + c)^3 + 21*(5*a^4*b^2 - a^2*b^4 + b^6)*sin(d*x + c)^2 + 7*(5*a
^5*b - a^3*b^3 + a*b^5)*sin(d*x + c))/((b^14*sin(d*x + c)^7 + 7*a*b^13*sin(
d*x + c)^6 + 21*a^2*b^12*sin(d*x + c)^5 + 35*a^3*b^11*sin(d*x + c)^4 + 35*a
^4*b^10*sin(d*x + c)^3 + 21*a^5*b^9*sin(d*x + c)^2 + 7*a^6*b^8*sin(d*x + c)
 + a^7*b^7)*d)
```

mupad [B] time = 0.24, size = 276, normalized size = 1.33

$$\frac{\frac{5\,a^6-a^4\,b^2+a^2\,b^4-5\,b^6}{35\,b^7}+\frac{\sin(c+d\,x)^6}{b}+\frac{3\,\sin(c+d\,x)^2\,\left(5\,a^4-a^2\,b^2+b^4\right)}{5\,b^5}+\frac{3\,a\,\sin(c+d\,x)^5}{b^2}+\frac{\sin(c+d\,x)^4\,\left(5\,a^2-b^2\right)}{b^3}+\frac{a\,\sin(c+d}{}}{d\,\left(a^7+7\,a^6\,b\,\sin\left(c+d\,x\right)+21\,a^5\,b^2\,\sin\left(c+d\,x\right)^2+35\,a^4\,b^3\,\sin\left(c+d\,x\right)^3+35\,a^3\,b^4\,\sin\left(c+d\,x\right)^4+21\,a^2\,b^5\,\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + b*sin(c + d*x))^8,x)
```

```
[Out] ((5*a^6 - 5*b^6 + a^2*b^4 - a^4*b^2)/(35*b^7) + sin(c + d*x)^6/b + (3*sin(c
 + d*x)^2*(5*a^4 + b^4 - a^2*b^2))/(5*b^5) + (3*a*sin(c + d*x)^5)/b^2 + (si
n(c + d*x)^4*(5*a^2 - b^2))/b^3 + (a*sin(c + d*x)*(5*a^4 + b^4 - a^2*b^2))/
(5*b^6) + (a*sin(c + d*x)^3*(5*a^2 - b^2))/b^4)/(d*(a^7 + b^7*sin(c + d*x)^
7 + 7*a*b^6*sin(c + d*x)^6 + 21*a^5*b^2*sin(c + d*x)^2 + 35*a^4*b^3*sin(c +
 d*x)^3 + 35*a^3*b^4*sin(c + d*x)^4 + 21*a^2*b^5*sin(c + d*x)^5 + 7*a^6*b*s
in(c + d*x)))
```

sympy [A] time = 47.44, size = 2530, normalized size = 12.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((x*cos(c)**7/a**8, Eq(b, 0) & Eq(d, 0)), ((16*sin(c + d*x)**7/(35
*d) + 8*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*sin(c + d*x)**3*cos(c + d
*x)**4/d + sin(c + d*x)*cos(c + d*x)**6/d)/a**8, Eq(b, 0)), (x*cos(c)**7/(a
 + b*sin(c))**8, Eq(d, 0)), (5*a**6/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c
 + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)*
*3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 +
 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) + 35*a**5*b*si
n(c + d*x)/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d
*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*si
n(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*
x)**6 + 35*b**14*d*sin(c + d*x)**7) + 104*a**4*b**2*sin(c + d*x)**2/(35*a**
7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 +
 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 73
5*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d
*sin(c + d*x)**7) - a**4*b**2*cos(c + d*x)**2/(35*a**7*b**7*d + 245*a**6*b*
```

```
*8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin
(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c +
 d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) + 16
8*a**3*b**3*sin(c + d*x)**3/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x)
+ 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 122
5*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b
**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) - 7*a**3*b**3*sin(c +
d*x)*cos(c + d*x)**2/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a
**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*
b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*
sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) + 155*a**2*b**4*sin(c + d*x)*
*4/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c +
 d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*
x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 +
35*b**14*d*sin(c + d*x)**7) - 19*a**2*b**4*sin(c + d*x)**2*cos(c + d*x)**2/
(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*
x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)*
*4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*
b**14*d*sin(c + d*x)**7) + a**2*b**4*cos(c + d*x)**4/(35*a**7*b**7*d + 245*
a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**1
0*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*
sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**
7) + 77*a*b**5*sin(c + d*x)**5/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*
x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 +
1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*
a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) - 21*a*b**5*sin(c +
 d*x)**3*cos(c + d*x)**2/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 7
35*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a
**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**1
3*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) + 7*a*b**5*sin(c + d*x)*c
os(c + d*x)**4/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b*
*9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*
d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c
+ d*x)**6 + 35*b**14*d*sin(c + d*x)**7) + 16*b**6*sin(c + d*x)**6/(35*a**7*
b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1
225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*
a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*s
in(c + d*x)**7) - 8*b**6*sin(c + d*x)**4*cos(c + d*x)**2/(35*a**7*b**7*d +
245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*
b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**1
2*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*
x)**7) + 6*b**6*sin(c + d*x)**2*cos(c + d*x)**4/(35*a**7*b**7*d + 245*a**6*
b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*s
in(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c
 + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) -
5*b**6*cos(c + d*x)**6/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735
*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**
3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*
d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7), True))
```

3.462 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=141

$$-\frac{\left(a^2-b^2\right)^2}{7b^5d(a+b\sin(c+dx))^7}+\frac{2a\left(a^2-b^2\right)}{3b^5d(a+b\sin(c+dx))^6}-\frac{2\left(3a^2-b^2\right)}{5b^5d(a+b\sin(c+dx))^5}-\frac{1}{3b^5d(a+b\sin(c+dx))^3}+\frac{}{b^5d(a}$$

```
[Out] -1/7*(a^2-b^2)^2/b^5/d/(a+b*sin(d*x+c))^7+2/3*a*(a^2-b^2)/b^5/d/(a+b*sin(d*
x+c))^6-2/5*(3*a^2-b^2)/b^5/d/(a+b*sin(d*x+c))^5+a/b^5/d/(a+b*sin(d*x+c))^4
-1/3/b^5/d/(a+b*sin(d*x+c))^3
```

Rubi [A] time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{\left(a^2-b^2\right)^2}{7b^5d(a+b\sin(c+dx))^7}+\frac{2a\left(a^2-b^2\right)}{3b^5d(a+b\sin(c+dx))^6}-\frac{2\left(3a^2-b^2\right)}{5b^5d(a+b\sin(c+dx))^5}-\frac{1}{3b^5d(a+b\sin(c+dx))^3}+\frac{}{b^5d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -(a^2 - b^2)^2/(7*b^5*d*(a + b*Sin[c + d*x])^7) + (2*a*(a^2 - b^2))/(3*b^5*
d*(a + b*Sin[c + d*x])^6) - (2*(3*a^2 - b^2))/(5*b^5*d*(a + b*Sin[c + d*x])
^5) + a/(b^5*d*(a + b*Sin[c + d*x])^4) - 1/(3*b^5*d*(a + b*Sin[c + d*x])^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{(a+x)^8}\,dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{(a+x)^8}-\frac{4\left(a^3-ab^2\right)}{(a+x)^7}+\frac{2\left(3a^2-b^2\right)}{(a+x)^6}-\frac{4a}{(a+x)^5}+\frac{1}{(a+x)^4}\right)dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= -\frac{\left(a^2-b^2\right)^2}{7b^5d(a+b\sin(c+dx))^7}+\frac{2a\left(a^2-b^2\right)}{3b^5d(a+b\sin(c+dx))^6}-\frac{2\left(3a^2-b^2\right)}{5b^5d(a+b\sin(c+dx))^5}+\frac{}{b}\end{aligned}$$

Mathematica [A] time = 0.27, size = 107, normalized size = 0.76

$$-\frac{a^4+21b^2\left(a^2-2b^2\right)\sin^2(c+dx)+7ab\left(a^2-2b^2\right)\sin(c+dx)-2a^2b^2+35ab^3\sin^3(c+dx)+35b^4\sin^4(c+dx)}{105b^5d(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/105*(a^4 - 2*a^2*b^2 + 15*b^4 + 7*a*b*(a^2 - 2*b^2)*Sin[c + d*x] + 21*b^
2*(a^2 - 2*b^2)*Sin[c + d*x]^2 + 35*a*b^3*Sin[c + d*x]^3 + 35*b^4*Sin[c + d
*x]^4)/(b^5*d*(a + b*Sin[c + d*x])^7)
```

fricas [B] time = 0.73, size = 309, normalized size = 2.19

$$\frac{35\,b^4\cos\left(dx+c\right)^4+\cdots}{105\left(7\,ab^{11}d\cos\left(dx+c\right)^6-7\left(5\,a^3b^9+3\,ab^{11}\right)d\cos\left(dx+c\right)^4+7\left(3\,a^5b^7+10\,a^3b^9+3\,ab^{11}\right)d\cos\left(dx+c\right)^2-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(35*b^4*cos(d*x + c)^4 + a^4 + 19*a^2*b^2 + 8*b^4 - 7*(3*a^2*b^2 + 4*
b^4)*cos(d*x + c)^2 - 7*(5*a*b^3*cos(d*x + c)^2 - a^3*b - 3*a*b^3)*sin(d*x
+ c))/(7*a*b^11*d*cos(d*x + c)^6 - 7*(5*a^3*b^9 + 3*a*b^11)*d*cos(d*x + c)^
4 + 7*(3*a^5*b^7 + 10*a^3*b^9 + 3*a*b^11)*d*cos(d*x + c)^2 - (a^7*b^5 + 21*
a^5*b^7 + 35*a^3*b^9 + 7*a*b^11)*d + (b^12*d*cos(d*x + c)^6 - 3*(7*a^2*b^10
 + b^12)*d*cos(d*x + c)^4 + (35*a^4*b^8 + 42*a^2*b^10 + 3*b^12)*d*cos(d*x +
 c)^2 - (7*a^6*b^6 + 35*a^4*b^8 + 21*a^2*b^10 + b^12)*d)*sin(d*x + c))
```

giac [A] time = 6.19, size = 117, normalized size = 0.83

$$-\frac{35\,b^4\sin\left(dx+c\right)^4+35\,ab^3\sin\left(dx+c\right)^3+21\,a^2b^2\sin\left(dx+c\right)^2-42\,b^4\sin\left(dx+c\right)^2+7\,a^3b\sin\left(dx+c\right)-14\cdots}{105\left(b\sin\left(dx+c\right)+a\right)^7b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*sin(d*x + c)^4 + 35*a*b^3*sin(d*x + c)^3 + 21*a^2*b^2*sin(d*
x + c)^2 - 42*b^4*sin(d*x + c)^2 + 7*a^3*b*sin(d*x + c) - 14*a*b^3*sin(d*x
+ c) + a^4 - 2*a^2*b^2 + 15*b^4)/((b*sin(d*x + c) + a)^7*b^5*d)
```

maple [A] time = 0.34, size = 127, normalized size = 0.90

$$\frac{-\frac{a^4-2a^2b^2+b^4}{7b^5(a+b\sin(dx+c))^7}-\frac{1}{3b^5(a+b\sin(dx+c))^3}-\frac{6a^2-2b^2}{5b^5(a+b\sin(dx+c))^5}+\frac{2a\left(a^2-b^2\right)}{3b^5(a+b\sin(dx+c))^6}+\frac{a}{b^5(a+b\sin(dx+c))^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-1/7*(a^4-2*a^2*b^2+b^4)/b^5/(a+b*sin(d*x+c))^7-1/3/b^5/(a+b*sin(d*x+c
))^3-1/5*(6*a^2-2*b^2)/b^5/(a+b*sin(d*x+c))^5+2/3*a*(a^2-b^2)/b^5/(a+b*sin(
d*x+c))^6+a/b^5/(a+b*sin(d*x+c))^4)
```

maxima [A] time = 0.33, size = 206, normalized size = 1.46

$$-\frac{35\,b^4\sin\left(dx+c\right)^4+35\,ab^3\sin\left(dx+c\right)^3+a^4-2\,a^2b^2+15\,b^4+21\left(a^2b^2-2\,b^4\right)\sin\left(d\cdots\right.}{105\left(b^{12}\sin\left(dx+c\right)^7+7\,ab^{11}\sin\left(dx+c\right)^6+21\,a^2b^{10}\sin\left(dx+c\right)^5+35\,a^3b^9\sin\left(dx+c\right)^4+35\,a^4b^8\sin\left(dx\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^4*sin(d*x + c)^4 + 35*a*b^3*sin(d*x + c)^3 + a^4 - 2*a^2*b^2 +
 15*b^4 + 21*(a^2*b^2 - 2*b^4)*sin(d*x + c)^2 + 7*(a^3*b - 2*a*b^3)*sin(d*x
```

```
 + c))/((b^12*sin(d*x + c)^7 + 7*a*b^11*sin(d*x + c)^6 + 21*a^2*b^10*sin(d*
x + c)^5 + 35*a^3*b^9*sin(d*x + c)^4 + 35*a^4*b^8*sin(d*x + c)^3 + 21*a^5*b
^7*sin(d*x + c)^2 + 7*a^6*b^6*sin(d*x + c) + a^7*b^5)*d)
```

mupad [B] time = 0.14, size = 206, normalized size = 1.46

$$-\frac{\frac{a^4-2\,a^2\,b^2+15\,b^4}{105\,b^5}+\frac{\sin(c+d\,x)^4}{3\,b}+\frac{\sin(c+d\,x)^2\,\left(a^2-2\,b^2\right)}{5\,b^3}+\frac{a\sin(c+d\,x)^3}{3\,b^2}+\frac{a\,\sin(c+d\,x}{15}}{d\,\left(a^7+7\,a^6\,b\,\sin\left(c+d\,x\right)+21\,a^5\,b^2\,\sin\left(c+d\,x\right)^2+35\,a^4\,b^3\,\sin\left(c+d\,x\right)^3+35\,a^3\,b^4\,\sin\left(c+d\,x\right)^4+21\,a^2\,b^5\,\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^8,x)
```

```
[Out] -((a^4 + 15*b^4 - 2*a^2*b^2)/(105*b^5) + sin(c + d*x)^4/(3*b) + (sin(c + d*
x)^2*(a^2 - 2*b^2))/(5*b^3) + (a*sin(c + d*x)^3)/(3*b^2) + (a*sin(c + d*x)*
(a^2 - 2*b^2))/(15*b^4))/(d*(a^7 + b^7*sin(c + d*x)^7 + 7*a*b^6*sin(c + d*x
)^6 + 21*a^5*b^2*sin(c + d*x)^2 + 35*a^4*b^3*sin(c + d*x)^3 + 35*a^3*b^4*si
n(c + d*x)^4 + 21*a^2*b^5*sin(c + d*x)^5 + 7*a^6*b*sin(c + d*x)))
```

sympy [A] time = 43.30, size = 1425, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((x*cos(c)**5/a**8, Eq(b, 0) & Eq(d, 0)), ((8*sin(c + d*x)**5/(15*
d) + 4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + sin(c + d*x)*cos(c + d*x)**4
/d)/a**8, Eq(b, 0)), (x*cos(c)**5/(a + b*sin(c))**8, Eq(d, 0)), (-a**4/(105
*a**7*b**5*d + 735*a**6*b**6*d*sin(c + d*x) + 2205*a**5*b**7*d*sin(c + d*x)
**2 + 3675*a**4*b**8*d*sin(c + d*x)**3 + 3675*a**3*b**9*d*sin(c + d*x)**4 +
 2205*a**2*b**10*d*sin(c + d*x)**5 + 735*a*b**11*d*sin(c + d*x)**6 + 105*b*
*12*d*sin(c + d*x)**7) - 7*a**3*b*sin(c + d*x)/(105*a**7*b**5*d + 735*a**6*
b**6*d*sin(c + d*x) + 2205*a**5*b**7*d*sin(c + d*x)**2 + 3675*a**4*b**8*d*s
in(c + d*x)**3 + 3675*a**3*b**9*d*sin(c + d*x)**4 + 2205*a**2*b**10*d*sin(c
 + d*x)**5 + 735*a*b**11*d*sin(c + d*x)**6 + 105*b**12*d*sin(c + d*x)**7) -
 19*a**2*b**2*sin(c + d*x)**2/(105*a**7*b**5*d + 735*a**6*b**6*d*sin(c + d*
x) + 2205*a**5*b**7*d*sin(c + d*x)**2 + 3675*a**4*b**8*d*sin(c + d*x)**3 +
3675*a**3*b**9*d*sin(c + d*x)**4 + 2205*a**2*b**10*d*sin(c + d*x)**5 + 735*
a*b**11*d*sin(c + d*x)**6 + 105*b**12*d*sin(c + d*x)**7) + 2*a**2*b**2*cos(
c + d*x)**2/(105*a**7*b**5*d + 735*a**6*b**6*d*sin(c + d*x) + 2205*a**5*b**
7*d*sin(c + d*x)**2 + 3675*a**4*b**8*d*sin(c + d*x)**3 + 3675*a**3*b**9*d*s
in(c + d*x)**4 + 2205*a**2*b**10*d*sin(c + d*x)**5 + 735*a*b**11*d*sin(c +
d*x)**6 + 105*b**12*d*sin(c + d*x)**7) - 21*a*b**3*sin(c + d*x)**3/(105*a**
7*b**5*d + 735*a**6*b**6*d*sin(c + d*x) + 2205*a**5*b**7*d*sin(c + d*x)**2
+ 3675*a**4*b**8*d*sin(c + d*x)**3 + 3675*a**3*b**9*d*sin(c + d*x)**4 + 220
5*a**2*b**10*d*sin(c + d*x)**5 + 735*a*b**11*d*sin(c + d*x)**6 + 105*b**12*
d*sin(c + d*x)**7) + 14*a*b**3*sin(c + d*x)*cos(c + d*x)**2/(105*a**7*b**5*
d + 735*a**6*b**6*d*sin(c + d*x) + 2205*a**5*b**7*d*sin(c + d*x)**2 + 3675*
a**4*b**8*d*sin(c + d*x)**3 + 3675*a**3*b**9*d*sin(c + d*x)**4 + 2205*a**2*
b**10*d*sin(c + d*x)**5 + 735*a*b**11*d*sin(c + d*x)**6 + 105*b**12*d*sin(c
 + d*x)**7) - 8*b**4*sin(c + d*x)**4/(105*a**7*b**5*d + 735*a**6*b**6*d*sin
(c + d*x) + 2205*a**5*b**7*d*sin(c + d*x)**2 + 3675*a**4*b**8*d*sin(c + d*x
)**3 + 3675*a**3*b**9*d*sin(c + d*x)**4 + 2205*a**2*b**10*d*sin(c + d*x)**5
 + 735*a*b**11*d*sin(c + d*x)**6 + 105*b**12*d*sin(c + d*x)**7) + 12*b**4*s
in(c + d*x)**2*cos(c + d*x)**2/(105*a**7*b**5*d + 735*a**6*b**6*d*sin(c + d
*x) + 2205*a**5*b**7*d*sin(c + d*x)**2 + 3675*a**4*b**8*d*sin(c + d*x)**3 +
 3675*a**3*b**9*d*sin(c + d*x)**4 + 2205*a**2*b**10*d*sin(c + d*x)**5 + 735
*a*b**11*d*sin(c + d*x)**6 + 105*b**12*d*sin(c + d*x)**7) - 15*b**4*cos(c +
```

```
 d*x)**4/(105*a**7*b**5*d + 735*a**6*b**6*d*sin(c + d*x) + 2205*a**5*b**7*d
*sin(c + d*x)**2 + 3675*a**4*b**8*d*sin(c + d*x)**3 + 3675*a**3*b**9*d*sin(
c + d*x)**4 + 2205*a**2*b**10*d*sin(c + d*x)**5 + 735*a*b**11*d*sin(c + d*x
)**6 + 105*b**12*d*sin(c + d*x)**7), True))
```

3.463 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=77

$$\frac{a^2-b^2}{7b^3d(a+b\sin(c+dx))^7}-\frac{a}{3b^3d(a+b\sin(c+dx))^6}+\frac{1}{5b^3d(a+b\sin(c+dx))^5}$$

```
[Out] 1/7*(a^2-b^2)/b^3/d/(a+b*sin(d*x+c))^7-1/3*a/b^3/d/(a+b*sin(d*x+c))^6+1/5/b
^3/d/(a+b*sin(d*x+c))^5
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{a^2-b^2}{7b^3d(a+b\sin(c+dx))^7}-\frac{a}{3b^3d(a+b\sin(c+dx))^6}+\frac{1}{5b^3d(a+b\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a^2 - b^2)/(7*b^3*d*(a + b*Sin[c + d*x])^7) - a/(3*b^3*d*(a + b*Sin[c + d*
x])^6) + 1/(5*b^3*d*(a + b*Sin[c + d*x])^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx&=\frac{\operatorname{Subst}\left(\int \frac{b^2-x^2}{(a+x)^8}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\&=\frac{\operatorname{Subst}\left(\int \left(\frac{-a^2+b^2}{(a+x)^8}+\frac{2a}{(a+x)^7}-\frac{1}{(a+x)^6}\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\&=\frac{a^2-b^2}{7b^3d(a+b\sin(c+dx))^7}-\frac{a}{3b^3d(a+b\sin(c+dx))^6}+\frac{1}{5b^3d(a+b\sin(c+dx))^5}\end{aligned}$$

Mathematica [A] time = 0.19, size = 54, normalized size = 0.70

$$\frac{a^2+7ab\sin(c+dx)+21b^2\sin^2(c+dx)-15b^2}{105b^3d(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a^2 - 15*b^2 + 7*a*b*Sin[c + d*x] + 21*b^2*Sin[c + d*x]^2)/(105*b^3*d*(a +
 b*Sin[c + d*x])^7)
```

fricas [B] time = 0.71, size = 254, normalized size = 3.30

$$\overline{105\left(7\,ab^{9}d\cos\left(dx+c\right)^{6}-7\left(5\,a^{3}b^{7}+3\,ab^{9}\right)d\cos\left(dx+c\right)^{4}+7\left(3\,a^{5}b^{5}+10\,a^{3}b^{7}+3\,ab^{9}\right)d\cos\left(dx+c\right)^{2}-\left(a^{7}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(21*b^2*cos(d*x + c)^2 - 7*a*b*sin(d*x + c) - a^2 - 6*b^2)/(7*a*b^9*d
*cos(d*x + c)^6 - 7*(5*a^3*b^7 + 3*a*b^9)*d*cos(d*x + c)^4 + 7*(3*a^5*b^5 +
 10*a^3*b^7 + 3*a*b^9)*d*cos(d*x + c)^2 - (a^7*b^3 + 21*a^5*b^5 + 35*a^3*b^
7 + 7*a*b^9)*d + (b^10*d*cos(d*x + c)^6 - 3*(7*a^2*b^8 + b^10)*d*cos(d*x +
c)^4 + (35*a^4*b^6 + 42*a^2*b^8 + 3*b^10)*d*cos(d*x + c)^2 - (7*a^6*b^4 + 3
5*a^4*b^6 + 21*a^2*b^8 + b^10)*d)*sin(d*x + c))
```

giac [A] time = 4.46, size = 52, normalized size = 0.68

$$\frac{21\,b^{2}\sin\left(dx+c\right)^{2}+7\,ab\sin\left(dx+c\right)+a^{2}-15\,b^{2}}{105\,\left(b\sin\left(dx+c\right)+a\right)^{7}b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(21*b^2*sin(d*x + c)^2 + 7*a*b*sin(d*x + c) + a^2 - 15*b^2)/((b*sin(d
*x + c) + a)^7*b^3*d)
```

maple [A] time = 0.34, size = 67, normalized size = 0.87

$$\frac{-\frac{-a^{2}+b^{2}}{7b^{3}\left(a+b\sin\left(dx+c\right)\right)^{7}}+\frac{1}{5b^{3}\left(a+b\sin\left(dx+c\right)\right)^{5}}-\frac{a}{3b^{3}\left(a+b\sin\left(dx+c\right)\right)^{6}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-1/7*(-a^2+b^2)/b^3/(a+b*sin(d*x+c))^7+1/5/b^3/(a+b*sin(d*x+c))^5-1/3*
a/b^3/(a+b*sin(d*x+c))^6)
```

maxima [B] time = 1.52, size = 151, normalized size = 1.96

$$\frac{21\,b^{2}\sin\left(dx+c\right)^{2}+7\,ab\sin\left(dx+c\right)+a^{2}-15\,b^{2}}{105\left(b^{10}\sin\left(dx+c\right)^{7}+7\,ab^{9}\sin\left(dx+c\right)^{6}+21\,a^{2}b^{8}\sin\left(dx+c\right)^{5}+35\,a^{3}b^{7}\sin\left(dx+c\right)^{4}+35\,a^{4}b^{6}\sin\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/105*(21*b^2*sin(d*x + c)^2 + 7*a*b*sin(d*x + c) + a^2 - 15*b^2)/((b^10*si
n(d*x + c)^7 + 7*a*b^9*sin(d*x + c)^6 + 21*a^2*b^8*sin(d*x + c)^5 + 35*a^3*
b^7*sin(d*x + c)^4 + 35*a^4*b^6*sin(d*x + c)^3 + 21*a^5*b^5*sin(d*x + c)^2
+ 7*a^6*b^4*sin(d*x + c) + a^7*b^3)*d)
```

mupad [B] time = 5.22, size = 152, normalized size = 1.97

$$\frac{\frac{a^{2}-15\,b^{2}}{105\,b^{3}}+\frac{\sin(c+d\,x)^{2}}{5\,b}+\frac{a\,\sin(c+d\,x)}{15\,b^{2}}}{d\,\left(a^{7}+7\,a^{6}\,b\,\sin\left(c+d\,x\right)+21\,a^{5}\,b^{2}\sin\left(c+d\,x\right)^{2}+35\,a^{4}\,b^{3}\sin\left(c+d\,x\right)^{3}+35\,a^{3}\,b^{4}\sin\left(c+d\,x\right)^{4}+21\,a^{2}\,b^{5}\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^8,x)
```

```
[Out] ((a^2 - 15*b^2)/(105*b^3) + sin(c + d*x)^2/(5*b) + (a*sin(c + d*x))/(15*b^2
))/(d*(a^7 + b^7*sin(c + d*x)^7 + 7*a*b^6*sin(c + d*x)^6 + 21*a^5*b^2*sin(c
 + d*x)^2 + 35*a^4*b^3*sin(c + d*x)^3 + 35*a^3*b^4*sin(c + d*x)^4 + 21*a^2*
b^5*sin(c + d*x)^5 + 7*a^6*b*sin(c + d*x)))
```

sympy [A] time = 41.33, size = 636, normalized size = 8.26

$$\begin{cases} \frac{x\cos^3(c)}{a^8} \\ \frac{\frac{2\sin^3(c+dx)}{3d}+\frac{\sin(c+dx)\cos^2(c+dx)}{d}}{a^8} \\ \frac{x\cos^3(c)}{(a+b\sin(c))^8} \\ \frac{a^2}{105a^7b^3d+735a^6b^4d\sin(c+dx)+2205a^5b^5d\sin^2(c+dx)+3675a^4b^6d\sin^3(c+dx)+3675a^3b^7d\sin^4(c+dx)+2205a^2b^8d\sin^5(c+dx)+735ab^9d\sin^6(c+dx)\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((x*cos(c)**3/a**8, Eq(b, 0) & Eq(d, 0)), ((2*sin(c + d*x)**3/(3*d
) + sin(c + d*x)*cos(c + d*x)**2/d)/a**8, Eq(b, 0)), (x*cos(c)**3/(a + b*si
n(c))**8, Eq(d, 0)), (a**2/(105*a**7*b**3*d + 735*a**6*b**4*d*sin(c + d*x)
+ 2205*a**5*b**5*d*sin(c + d*x)**2 + 3675*a**4*b**6*d*sin(c + d*x)**3 + 367
5*a**3*b**7*d*sin(c + d*x)**4 + 2205*a**2*b**8*d*sin(c + d*x)**5 + 735*a*b*
*9*d*sin(c + d*x)**6 + 105*b**10*d*sin(c + d*x)**7) + 7*a*b*sin(c + d*x)/(1
05*a**7*b**3*d + 735*a**6*b**4*d*sin(c + d*x) + 2205*a**5*b**5*d*sin(c + d*
x)**2 + 3675*a**4*b**6*d*sin(c + d*x)**3 + 3675*a**3*b**7*d*sin(c + d*x)**4
 + 2205*a**2*b**8*d*sin(c + d*x)**5 + 735*a*b**9*d*sin(c + d*x)**6 + 105*b*
*10*d*sin(c + d*x)**7) + 6*b**2*sin(c + d*x)**2/(105*a**7*b**3*d + 735*a**6
*b**4*d*sin(c + d*x) + 2205*a**5*b**5*d*sin(c + d*x)**2 + 3675*a**4*b**6*d*
sin(c + d*x)**3 + 3675*a**3*b**7*d*sin(c + d*x)**4 + 2205*a**2*b**8*d*sin(c
 + d*x)**5 + 735*a*b**9*d*sin(c + d*x)**6 + 105*b**10*d*sin(c + d*x)**7) -
15*b**2*cos(c + d*x)**2/(105*a**7*b**3*d + 735*a**6*b**4*d*sin(c + d*x) + 2
205*a**5*b**5*d*sin(c + d*x)**2 + 3675*a**4*b**6*d*sin(c + d*x)**3 + 3675*a
**3*b**7*d*sin(c + d*x)**4 + 2205*a**2*b**8*d*sin(c + d*x)**5 + 735*a*b**9*
d*sin(c + d*x)**6 + 105*b**10*d*sin(c + d*x)**7), True))
```

3.464 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{7bd(a+b\sin(c+dx))^7}$$

```
[Out] -1/7/b/d/(a+b*sin(d*x+c))^7
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 32}

$$-\frac{1}{7bd(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/(7*b*d*(a + b*Sin[c + d*x])^7)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^8}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{1}{7bd(a+b\sin(c+dx))^7}\end{aligned}$$

Mathematica [A] time = 0.08, size = 22, normalized size = 1.00

$$-\frac{1}{7bd(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/7*1/(b*d*(a + b*Sin[c + d*x])^7)
```

fricas [B] time = 0.76, size = 218, normalized size = 9.91

$$\overline{7\left(7\,ab^7d\cos\left(dx+c\right)^6-7\left(5\,a^3b^5+3\,ab^7\right)d\cos\left(dx+c\right)^4+7\left(3\,a^5b^3+10\,a^3b^5+3\,ab^7\right)d\cos\left(dx+c\right)^2-\left(a^7b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/7/(7*a*b^7*d*cos(d*x + c)^6 - 7*(5*a^3*b^5 + 3*a*b^7)*d*cos(d*x + c)^4 +
7*(3*a^5*b^3 + 10*a^3*b^5 + 3*a*b^7)*d*cos(d*x + c)^2 - (a^7*b + 21*a^5*b^3
 + 35*a^3*b^5 + 7*a*b^7)*d + (b^8*d*cos(d*x + c)^6 - 3*(7*a^2*b^6 + b^8)*d*
cos(d*x + c)^4 + (35*a^4*b^4 + 42*a^2*b^6 + 3*b^8)*d*cos(d*x + c)^2 - (7*a^
6*b^2 + 35*a^4*b^4 + 21*a^2*b^6 + b^8)*d)*sin(d*x + c))
```

giac [A] time = 2.57, size = 20, normalized size = 0.91

$$-\frac{1}{7\,(b\sin\left(dx+c\right)+a)^{7}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/7/((b*sin(d*x + c) + a)^7*b*d)
```

maple [A] time = 0.14, size = 21, normalized size = 0.95

$$-\frac{1}{7bd\left(a+b\sin\left(dx+c\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^8,x)
```

```
[Out] -1/7/b/d/(a+b*sin(d*x+c))^7
```

maxima [A] time = 0.31, size = 20, normalized size = 0.91

$$-\frac{1}{7\,(b\sin\left(dx+c\right)+a)^{7}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/7/((b*sin(d*x + c) + a)^7*b*d)
```

mupad [B] time = 5.20, size = 119, normalized size = 5.41

$$-\frac{1}{d\,\left(7\,a^7\,b+49\,a^6\,b^2\,\sin\left(c+d\,x\right)+147\,a^5\,b^3\,{\sin\left(c+d\,x\right)}^2+245\,a^4\,b^4\,{\sin\left(c+d\,x\right)}^3+245\,a^3\,b^5\,{\sin\left(c+d\,x\right)}^4+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^8,x)
```

```
[Out] -1/(d*(7*a^7*b + 7*b^8*sin(c + d*x)^7 + 49*a^6*b^2*sin(c + d*x) + 49*a*b^7*
sin(c + d*x)^6 + 147*a^5*b^3*sin(c + d*x)^2 + 245*a^4*b^4*sin(c + d*x)^3 +
245*a^3*b^5*sin(c + d*x)^4 + 147*a^2*b^6*sin(c + d*x)^5))
```

sympy [A] time = 40.65, size = 167, normalized size = 7.59

$$\begin{cases} \frac{x\cos\left(c\right)}{a^{8}} \\ \frac{\sin\left(c+dx\right)}{a^{8}d} \\ \frac{x\cos\left(c\right)}{\left(a+b\sin\left(c\right)\right)^{8}} \\ -\frac{1}{7a^{7}bd+49a^{6}b^{2}d\sin\left(c+dx\right)+147a^{5}b^{3}d\sin^{2}\left(c+dx\right)+245a^{4}b^{4}d\sin^{3}\left(c+dx\right)+245a^{3}b^{5}d\sin^{4}\left(c+dx\right)+147a^{2}b^{6}d\sin^{5}\left(c+dx\right)+49ab^{7}d\sin^{6}\left(c+dx\right)+7b^{8}d\sin\right.} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((x*cos(c)/a**8, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**8*d), Eq(
b, 0)), (x*cos(c)/(a + b*sin(c))**8, Eq(d, 0)), (-1/(7*a**7*b*d + 49*a**6*b
**2*d*sin(c + d*x) + 147*a**5*b**3*d*sin(c + d*x)**2 + 245*a**4*b**4*d*sin(
c + d*x)**3 + 245*a**3*b**5*d*sin(c + d*x)**4 + 147*a**2*b**6*d*sin(c + d*x
)**5 + 49*a*b**7*d*sin(c + d*x)**6 + 7*b**8*d*sin(c + d*x)**7), True))
```

3.465 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=385

$$\frac{ab\left(3a^2+b^2\right)\left(a^2+3b^2\right)}{d\left(a^2-b^2\right)^6(a+b\sin(c+dx))^2}+\frac{ab\left(a^2+b^2\right)}{d\left(a^2-b^2\right)^4(a+b\sin(c+dx))^4}+\frac{b\left(3a^2+b^2\right)}{5d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^5}+\frac{}{3d\left(a^2-b^2\right.}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^8/d+1/2*ln(1+sin(d*x+c))/(a-b)^8/d-8*a*b*(a^2+b
^2)*(a^4+6*a^2*b^2+b^4)*ln(a+b*sin(d*x+c))/(a^2-b^2)^8/d+1/7*b/(a^2-b^2)/d/
(a+b*sin(d*x+c))^7+1/3*a*b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^6+1/5*b*(3*a^2+b^
2)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^5+a*b*(a^2+b^2)/(a^2-b^2)^4/d/(a+b*sin(d*
x+c))^4+1/3*b*(5*a^4+10*a^2*b^2+b^4)/(a^2-b^2)^5/d/(a+b*sin(d*x+c))^3+a*b*(
3*a^4+10*a^2*b^2+3*b^4)/(a^2-b^2)^6/d/(a+b*sin(d*x+c))^2+b*(7*a^6+35*a^4*b^
2+21*a^2*b^4+b^6)/(a^2-b^2)^7/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.53, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 710, 801}

$$\frac{b\left(35a^4b^2+21a^2b^4+7a^6+b^6\right)}{d\left(a^2-b^2\right)^7(a+b\sin(c+dx))}+\frac{ab\left(3a^2+b^2\right)\left(a^2+3b^2\right)}{d\left(a^2-b^2\right)^6(a+b\sin(c+dx))^2}+\frac{b\left(10a^2b^2+5a^4+b^4\right)}{3d\left(a^2-b^2\right)^5(a+b\sin(c+dx))^3}+\frac{a}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^8*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^8
*d) - (8*a*b*(a^2 + b^2)*(a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(
(a^2 - b^2)^8*d) + b/(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (a*b)/(3*(a
^2 - b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(3*a^2 + b^2))/(5*(a^2 - b^2)^3*
d*(a + b*Sin[c + d*x])^5) + (a*b*(a^2 + b^2))/((a^2 - b^2)^4*d*(a + b*Sin[c
 + d*x])^4) + (b*(5*a^4 + 10*a^2*b^2 + b^4))/(3*(a^2 - b^2)^5*d*(a + b*Sin[
c + d*x])^3) + (a*b*(3*a^2 + b^2)*(a^2 + 3*b^2))/((a^2 - b^2)^6*d*(a + b*Si
n[c + d*x])^2) + (b*(7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6))/((a^2 - b^2)^7
*d*(a + b*Sin[c + d*x]))
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)^8(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{b\,\text{Subst}\left(\int \frac{a-x}{(a+x)^7(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{(a^2-b^2)\,d}$$

$$= \frac{b}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{b\,\text{Subst}\left(\int \left(\frac{a-b}{2b(a+b)^7(b-x)} - \frac{2a}{(a-b)(a+b)(a+x)^7} + \frac{}{(a-}\right.\right.}{}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^8 d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^8 d} - \frac{8ab(a^2+b^2)(a^4+6a^2b^2+b^4)\,\text{l}}{(a^2-b^2)^8 d}$$

Mathematica [A] time = 2.69, size = 365, normalized size = 0.95

$$b\left(\frac{a(3a^2+b^2)(a^2+3b^2)}{(a-b)^6(a+b)^6(a+b\sin(c+dx))^2} + \frac{a(a^2+b^2)}{(a-b)^4(a+b)^4(a+b\sin(c+dx))^4} + \frac{3a^2+b^2}{5(a-b)^3(a+b)^3(a+b\sin(c+dx))^5} + \frac{1}{7(a^2-b^2)(a+b\sin(c+dx))^7} + \frac{}{3(a-b)^5}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (b*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^8) + Log[1 + Sin[c + d*x]]/(2*(a
- b)^8*b) - (8*a*(a^2 + b^2)*(a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]
])/((a - b)^8*(a + b)^8) + 1/(7*(a^2 - b^2)*(a + b*Sin[c + d*x])^7) + a/(3*
(a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])^6) + (3*a^2 + b^2)/(5*(a - b)^3*(a
 + b)^3*(a + b*Sin[c + d*x])^5) + (a*(a^2 + b^2))/((a - b)^4*(a + b)^4*(a +
 b*Sin[c + d*x])^4) + (5*a^4 + 10*a^2*b^2 + b^4)/(3*(a - b)^5*(a + b)^5*(a
+ b*Sin[c + d*x])^3) + (a*(3*a^2 + b^2)*(a^2 + 3*b^2))/((a - b)^6*(a + b)^6
*(a + b*Sin[c + d*x])^2) + (7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6)/((a - b)
^7*(a + b)^7*(a + b*Sin[c + d*x]))))/d
```

fricas [B] time = 4.68, size = 3165, normalized size = 8.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/210*(2886*a^14*b + 35728*a^12*b^3 + 113862*a^10*b^5 + 11760*a^8*b^7 - 97
230*a^6*b^9 - 62496*a^4*b^11 - 4158*a^2*b^13 - 352*b^15 - 210*(7*a^8*b^7 +
28*a^6*b^9 - 14*a^4*b^11 - 20*a^2*b^13 - b^15)*cos(d*x + c)^6 + 70*(365*a^1
0*b^5 + 1378*a^8*b^7 - 602*a^6*b^9 - 944*a^4*b^11 - 187*a^2*b^13 - 10*b^15)
*cos(d*x + c)^4 - 14*(2229*a^12*b^3 + 10223*a^10*b^5 + 7960*a^8*b^7 - 10490
*a^6*b^9 - 8915*a^4*b^11 - 949*a^2*b^13 - 58*b^15)*cos(d*x + c)^2 - 1680*(a
^14*b + 28*a^12*b^3 + 189*a^10*b^5 + 400*a^8*b^7 + 315*a^6*b^9 + 84*a^4*b^1
1 + 7*a^2*b^13 - 7*(a^8*b^7 + 7*a^6*b^9 + 7*a^4*b^11 + a^2*b^13)*cos(d*x +
c)^6 + 7*(5*a^10*b^5 + 38*a^8*b^7 + 56*a^6*b^9 + 26*a^4*b^11 + 3*a^2*b^13)*
cos(d*x + c)^4 - 7*(3*a^12*b^3 + 31*a^10*b^5 + 94*a^8*b^7 + 94*a^6*b^9 + 31
*a^4*b^11 + 3*a^2*b^13)*cos(d*x + c)^2 + (7*a^13*b^2 + 84*a^11*b^4 + 315*a^
9*b^6 + 400*a^7*b^8 + 189*a^5*b^10 + 28*a^3*b^12 + a*b^14 - (a^7*b^8 + 7*a^
5*b^10 + 7*a^3*b^12 + a*b^14)*cos(d*x + c)^6 + 3*(7*a^9*b^6 + 50*a^7*b^8 +
56*a^5*b^10 + 14*a^3*b^12 + a*b^14)*cos(d*x + c)^4 - (35*a^11*b^4 + 287*a^9
*b^6 + 542*a^7*b^8 + 350*a^5*b^10 + 63*a^3*b^12 + 3*a*b^14)*cos(d*x + c)^2)
```

```
*sin(d*x + c))*log(b*sin(d*x + c) + a) + 105*(a^15 + 8*a^14*b + 49*a^13*b^2
 + 224*a^12*b^3 + 693*a^11*b^4 + 1512*a^10*b^5 + 2485*a^9*b^6 + 3200*a^8*b^
7 + 3235*a^7*b^8 + 2520*a^6*b^9 + 1491*a^5*b^10 + 672*a^4*b^11 + 231*a^3*b^
12 + 56*a^2*b^13 + 7*a*b^14 - 7*(a^9*b^6 + 8*a^8*b^7 + 28*a^7*b^8 + 56*a^6*
b^9 + 70*a^5*b^10 + 56*a^4*b^11 + 28*a^3*b^12 + 8*a^2*b^13 + a*b^14)*cos(d*
x + c)^6 + 7*(5*a^11*b^4 + 40*a^10*b^5 + 143*a^9*b^6 + 304*a^8*b^7 + 434*a^
7*b^8 + 448*a^6*b^9 + 350*a^5*b^10 + 208*a^4*b^11 + 89*a^3*b^12 + 24*a^2*b^
13 + 3*a*b^14)*cos(d*x + c)^4 - 7*(3*a^13*b^2 + 24*a^12*b^3 + 94*a^11*b^4 +
 248*a^10*b^5 + 493*a^9*b^6 + 752*a^8*b^7 + 868*a^7*b^8 + 752*a^6*b^9 + 493
*a^5*b^10 + 248*a^4*b^11 + 94*a^3*b^12 + 24*a^2*b^13 + 3*a*b^14)*cos(d*x +
c)^2 + (7*a^14*b + 56*a^13*b^2 + 231*a^12*b^3 + 672*a^11*b^4 + 1491*a^10*b^
5 + 2520*a^9*b^6 + 3235*a^8*b^7 + 3200*a^7*b^8 + 2485*a^6*b^9 + 1512*a^5*b^
10 + 693*a^4*b^11 + 224*a^3*b^12 + 49*a^2*b^13 + 8*a*b^14 + b^15 - (a^8*b^7
 + 8*a^7*b^8 + 28*a^6*b^9 + 56*a^5*b^10 + 70*a^4*b^11 + 56*a^3*b^12 + 28*a^
2*b^13 + 8*a*b^14 + b^15)*cos(d*x + c)^6 + 3*(7*a^10*b^5 + 56*a^9*b^6 + 197
*a^8*b^7 + 400*a^7*b^8 + 518*a^6*b^9 + 448*a^5*b^10 + 266*a^4*b^11 + 112*a^
3*b^12 + 35*a^2*b^13 + 8*a*b^14 + b^15)*cos(d*x + c)^4 - (35*a^12*b^3 + 280
*a^11*b^4 + 1022*a^10*b^5 + 2296*a^9*b^6 + 3629*a^8*b^7 + 4336*a^7*b^8 + 40
04*a^6*b^9 + 2800*a^5*b^10 + 1421*a^4*b^11 + 504*a^3*b^12 + 126*a^2*b^13 +
24*a*b^14 + 3*b^15)*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 1
05*(a^15 - 8*a^14*b + 49*a^13*b^2 - 224*a^12*b^3 + 693*a^11*b^4 - 1512*a^10
*b^5 + 2485*a^9*b^6 - 3200*a^8*b^7 + 3235*a^7*b^8 - 2520*a^6*b^9 + 1491*a^5
*b^10 - 672*a^4*b^11 + 231*a^3*b^12 - 56*a^2*b^13 + 7*a*b^14 - 7*(a^9*b^6 -
 8*a^8*b^7 + 28*a^7*b^8 - 56*a^6*b^9 + 70*a^5*b^10 - 56*a^4*b^11 + 28*a^3*b
^12 - 8*a^2*b^13 + a*b^14)*cos(d*x + c)^6 + 7*(5*a^11*b^4 - 40*a^10*b^5 + 1
43*a^9*b^6 - 304*a^8*b^7 + 434*a^7*b^8 - 448*a^6*b^9 + 350*a^5*b^10 - 208*a
^4*b^11 + 89*a^3*b^12 - 24*a^2*b^13 + 3*a*b^14)*cos(d*x + c)^4 - 7*(3*a^13*
b^2 - 24*a^12*b^3 + 94*a^11*b^4 - 248*a^10*b^5 + 493*a^9*b^6 - 752*a^8*b^7
 + 868*a^7*b^8 - 752*a^6*b^9 + 493*a^5*b^10 - 248*a^4*b^11 + 94*a^3*b^12 - 2
4*a^2*b^13 + 3*a*b^14)*cos(d*x + c)^2 + (7*a^14*b - 56*a^13*b^2 + 231*a^12*
b^3 - 672*a^11*b^4 + 1491*a^10*b^5 - 2520*a^9*b^6 + 3235*a^8*b^7 - 3200*a^7
*b^8 + 2485*a^6*b^9 - 1512*a^5*b^10 + 693*a^4*b^11 - 224*a^3*b^12 + 49*a^2*
b^13 - 8*a*b^14 + b^15 - (a^8*b^7 - 8*a^7*b^8 + 28*a^6*b^9 - 56*a^5*b^10 +
70*a^4*b^11 - 56*a^3*b^12 + 28*a^2*b^13 - 8*a*b^14 + b^15)*cos(d*x + c)^6 +
 3*(7*a^10*b^5 - 56*a^9*b^6 + 197*a^8*b^7 - 400*a^7*b^8 + 518*a^6*b^9 - 448
*a^5*b^10 + 266*a^4*b^11 - 112*a^3*b^12 + 35*a^2*b^13 - 8*a*b^14 + b^15)*co
s(d*x + c)^4 - (35*a^12*b^3 - 280*a^11*b^4 + 1022*a^10*b^5 - 2296*a^9*b^6 +
 3629*a^8*b^7 - 4336*a^7*b^8 + 4004*a^6*b^9 - 2800*a^5*b^10 + 1421*a^4*b^11
 - 504*a^3*b^12 + 126*a^2*b^13 - 24*a*b^14 + 3*b^15)*cos(d*x + c)^2)*sin(d*
x + c))*log(-sin(d*x + c) + 1) + 14*(1023*a^13*b^2 + 5136*a^11*b^4 + 7255*a
^9*b^6 - 5160*a^7*b^8 - 6435*a^5*b^10 - 1768*a^3*b^12 - 51*a*b^14 + 15*(45*
a^9*b^6 + 172*a^7*b^8 - 98*a^5*b^10 - 116*a^3*b^12 - 3*a*b^14)*cos(d*x + c)
^4 - 5*(533*a^11*b^4 + 2041*a^9*b^6 - 278*a^7*b^8 - 1574*a^5*b^10 - 703*a^3
*b^12 - 19*a*b^14)*cos(d*x + c)^2)*sin(d*x + c))/(7*(a^17*b^6 - 8*a^15*b^8
+ 28*a^13*b^10 - 56*a^11*b^12 + 70*a^9*b^14 - 56*a^7*b^16 + 28*a^5*b^18 - 8
*a^3*b^20 + a*b^22)*d*cos(d*x + c)^6 - 7*(5*a^19*b^4 - 37*a^17*b^6 + 116*a^
15*b^8 - 196*a^13*b^10 + 182*a^11*b^12 - 70*a^9*b^14 - 28*a^7*b^16 + 44*a^5
*b^18 - 19*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^4 + 7*(3*a^21*b^2 - 14*a^19*
b^4 + 7*a^17*b^6 + 88*a^15*b^8 - 266*a^13*b^10 + 364*a^11*b^12 - 266*a^9*b^
14 + 88*a^7*b^16 + 7*a^5*b^18 - 14*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^2 -
(a^23 + 13*a^21*b^2 - 105*a^19*b^4 + 259*a^17*b^6 - 182*a^15*b^8 - 350*a^13
*b^10 + 910*a^11*b^12 - 890*a^9*b^14 + 421*a^7*b^16 - 63*a^5*b^18 - 21*a^3*
b^20 + 7*a*b^22)*d + ((a^16*b^7 - 8*a^14*b^9 + 28*a^12*b^11 - 56*a^10*b^13
+ 70*a^8*b^15 - 56*a^6*b^17 + 28*a^4*b^19 - 8*a^2*b^21 + b^23)*d*cos(d*x +
c)^6 - 3*(7*a^18*b^5 - 55*a^16*b^7 + 188*a^14*b^9 - 364*a^12*b^11 + 434*a^1
0*b^13 - 322*a^8*b^15 + 140*a^6*b^17 - 28*a^4*b^19 - a^2*b^21 + b^23)*d*cos
(d*x + c)^4 + (35*a^20*b^3 - 238*a^18*b^5 + 647*a^16*b^7 - 808*a^14*b^9 + 1
82*a^12*b^11 + 812*a^10*b^13 - 1162*a^8*b^15 + 728*a^6*b^17 - 217*a^4*b^19
+ 18*a^2*b^21 + 3*b^23)*d*cos(d*x + c)^2 - (7*a^22*b - 21*a^20*b^3 - 63*a^1
```

```
8*b^5 + 421*a^16*b^7 - 890*a^14*b^9 + 910*a^12*b^11 - 350*a^10*b^13 - 182*a
^8*b^15 + 259*a^6*b^17 - 105*a^4*b^19 + 13*a^2*b^21 + b^23)*d)*sin(d*x + c)
)
```

giac [B] time = 2.07, size = 1010, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/210*(1680*(a^7*b^2 + 7*a^5*b^4 + 7*a^3*b^6 + a*b^8)*log(abs(b*sin(d*x +
c) + a))/(a^16*b - 8*a^14*b^3 + 28*a^12*b^5 - 56*a^10*b^7 + 70*a^8*b^9 - 56
*a^6*b^11 + 28*a^4*b^13 - 8*a^2*b^15 + b^17) - 105*log(abs(sin(d*x + c) + 1
))/(a^8 - 8*a^7*b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*
a^2*b^6 - 8*a*b^7 + b^8) + 105*log(abs(sin(d*x + c) - 1))/(a^8 + 8*a^7*b +
28*a^6*b^2 + 56*a^5*b^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 +
b^8) - 2*(2178*a^7*b^8*sin(d*x + c)^7 + 15246*a^5*b^10*sin(d*x + c)^7 + 152
46*a^3*b^12*sin(d*x + c)^7 + 2178*a*b^14*sin(d*x + c)^7 + 15981*a^8*b^7*sin
(d*x + c)^6 + 109662*a^6*b^9*sin(d*x + c)^6 + 105252*a^4*b^11*sin(d*x + c)^
6 + 13146*a^2*b^13*sin(d*x + c)^6 - 105*b^15*sin(d*x + c)^6 + 50463*a^9*b^6
*sin(d*x + c)^5 + 338226*a^7*b^8*sin(d*x + c)^5 + 309876*a^5*b^10*sin(d*x +
 c)^5 + 33558*a^3*b^12*sin(d*x + c)^5 - 315*a*b^14*sin(d*x + c)^5 + 89005*a
^10*b^5*sin(d*x + c)^4 + 579635*a^8*b^7*sin(d*x + c)^4 + 503720*a^6*b^9*sin
(d*x + c)^4 + 47600*a^4*b^11*sin(d*x + c)^4 - 245*a^2*b^13*sin(d*x + c)^4 -
 35*b^15*sin(d*x + c)^4 + 94885*a^11*b^4*sin(d*x + c)^3 + 595595*a^9*b^6*si
n(d*x + c)^3 + 487760*a^7*b^8*sin(d*x + c)^3 + 41720*a^5*b^10*sin(d*x + c)^
3 - 245*a^3*b^12*sin(d*x + c)^3 - 35*a*b^14*sin(d*x + c)^3 + 61341*a^12*b^3
*sin(d*x + c)^2 + 366177*a^10*b^5*sin(d*x + c)^2 + 281631*a^8*b^7*sin(d*x +
 c)^2 + 23268*a^6*b^9*sin(d*x + c)^2 - 735*a^4*b^11*sin(d*x + c)^2 + 147*a^
2*b^13*sin(d*x + c)^2 - 21*b^15*sin(d*x + c)^2 + 22407*a^13*b^2*sin(d*x + c
) + 124019*a^11*b^4*sin(d*x + c) + 90797*a^9*b^6*sin(d*x + c) + 6916*a^7*b^
8*sin(d*x + c) - 245*a^5*b^10*sin(d*x + c) + 49*a^3*b^12*sin(d*x + c) - 7*a
*b^14*sin(d*x + c) + 3621*a^14*b + 17507*a^12*b^3 + 13391*a^10*b^5 - 167*a^
8*b^7 + 805*a^6*b^9 - 413*a^4*b^11 + 119*a^2*b^13 - 15*b^15)/((a^16 - 8*a^1
4*b^2 + 28*a^12*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 56*a^6*b^10 + 28*a^4*b^12
- 8*a^2*b^14 + b^16)*(b*sin(d*x + c) + a)^7))/d
```

maple [A] time = 0.44, size = 699, normalized size = 1.82

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{2d\left(a+b\right)^{8}}+\frac{b}{7d\left(a+b\right)\left(a-b\right)\left(a+b\sin\left(dx+c\right)\right)^{7}}+\frac{ab}{3d\left(a+b\right)^{2}\left(a-b\right)^{2}\left(a+b\sin\left(dx+c\right)\right)^{6}}+\frac{}{5d\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^8,x)
```

```
[Out] -1/2/d/(a+b)^8*ln(sin(d*x+c)-1)+1/7/d*b/(a+b)/(a-b)/(a+b*sin(d*x+c))^7+1/3/
d*a*b/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))^6+3/5/d*b/(a+b)^3/(a-b)^3/(a+b*sin(d
*x+c))^5*a^2+1/5/d*b^3/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))^5+5/3/d*b/(a+b)^5/(
a-b)^5/(a+b*sin(d*x+c))^3*a^4+10/3/d*b^3/(a+b)^5/(a-b)^5/(a+b*sin(d*x+c))^3
*a^2+1/3/d*b^5/(a+b)^5/(a-b)^5/(a+b*sin(d*x+c))^3+7/d*b/(a+b)^7/(a-b)^7/(a+
b*sin(d*x+c))*a^6+35/d*b^3/(a+b)^7/(a-b)^7/(a+b*sin(d*x+c))*a^4+21/d*b^5/(a
+b)^7/(a-b)^7/(a+b*sin(d*x+c))*a^2+1/d*b^7/(a+b)^7/(a-b)^7/(a+b*sin(d*x+c))
+1/d*b*a^3/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))^4+1/d*b^3*a/(a+b)^4/(a-b)^4/(a+
b*sin(d*x+c))^4+3/d*b*a^5/(a+b)^6/(a-b)^6/(a+b*sin(d*x+c))^2+10/d*b^3*a^3/(
a+b)^6/(a-b)^6/(a+b*sin(d*x+c))^2+3/d*b^5*a/(a+b)^6/(a-b)^6/(a+b*sin(d*x+c)
)^2-8/d*b*a^7/(a+b)^8/(a-b)^8*ln(a+b*sin(d*x+c))-56/d*b^3*a^5/(a+b)^8/(a-b)
^8*ln(a+b*sin(d*x+c))-56/d*b^5*a^3/(a+b)^8/(a-b)^8*ln(a+b*sin(d*x+c))-8/d*b
^7*a/(a+b)^8/(a-b)^8*ln(a+b*sin(d*x+c))+1/2*ln(1+sin(d*x+c))/(a-b)^8/d
```

maxima [B] time = 0.40, size = 1160, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/210*(1680*(a^7*b + 7*a^5*b^3 + 7*a^3*b^5 + a*b^7)*log(b*sin(d*x + c) + a
)/(a^16 - 8*a^14*b^2 + 28*a^12*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 56*a^6*b^10
 + 28*a^4*b^12 - 8*a^2*b^14 + b^16) - 2*(1443*a^12*b + 3704*a^10*b^3 + 1849
*a^8*b^5 - 496*a^6*b^7 + 309*a^4*b^9 - 104*a^2*b^11 + 15*b^13 + 105*(7*a^6*
b^7 + 35*a^4*b^9 + 21*a^2*b^11 + b^13)*sin(d*x + c)^6 + 105*(45*a^7*b^6 + 2
17*a^5*b^8 + 119*a^3*b^10 + 3*a*b^12)*sin(d*x + c)^5 + 35*(365*a^8*b^5 + 16
80*a^6*b^7 + 826*a^4*b^9 + 8*a^2*b^11 + b^13)*sin(d*x + c)^4 + 35*(533*a^9*
b^4 + 2304*a^7*b^6 + 994*a^5*b^8 + 8*a^3*b^10 + a*b^12)*sin(d*x + c)^3 + 21
*(743*a^10*b^3 + 2934*a^8*b^5 + 1099*a^6*b^7 + 29*a^4*b^9 - 6*a^2*b^11 + b^
13)*sin(d*x + c)^2 + 7*(1023*a^11*b^2 + 3494*a^9*b^4 + 1219*a^7*b^6 + 29*a^
5*b^8 - 6*a^3*b^10 + a*b^12)*sin(d*x + c))/(a^21 - 7*a^19*b^2 + 21*a^17*b^4
 - 35*a^15*b^6 + 35*a^13*b^8 - 21*a^11*b^10 + 7*a^9*b^12 - a^7*b^14 + (a^14
*b^7 - 7*a^12*b^9 + 21*a^10*b^11 - 35*a^8*b^13 + 35*a^6*b^15 - 21*a^4*b^17
+ 7*a^2*b^19 - b^21)*sin(d*x + c)^7 + 7*(a^15*b^6 - 7*a^13*b^8 + 21*a^11*b^
10 - 35*a^9*b^12 + 35*a^7*b^14 - 21*a^5*b^16 + 7*a^3*b^18 - a*b^20)*sin(d*x
 + c)^6 + 21*(a^16*b^5 - 7*a^14*b^7 + 21*a^12*b^9 - 35*a^10*b^11 + 35*a^8*b
^13 - 21*a^6*b^15 + 7*a^4*b^17 - a^2*b^19)*sin(d*x + c)^5 + 35*(a^17*b^4 -
7*a^15*b^6 + 21*a^13*b^8 - 35*a^11*b^10 + 35*a^9*b^12 - 21*a^7*b^14 + 7*a^5
*b^16 - a^3*b^18)*sin(d*x + c)^4 + 35*(a^18*b^3 - 7*a^16*b^5 + 21*a^14*b^7
- 35*a^12*b^9 + 35*a^10*b^11 - 21*a^8*b^13 + 7*a^6*b^15 - a^4*b^17)*sin(d*x
 + c)^3 + 21*(a^19*b^2 - 7*a^17*b^4 + 21*a^15*b^6 - 35*a^13*b^8 + 35*a^11*b
^10 - 21*a^9*b^12 + 7*a^7*b^14 - a^5*b^16)*sin(d*x + c)^2 + 7*(a^20*b - 7*a
^18*b^3 + 21*a^16*b^5 - 35*a^14*b^7 + 35*a^12*b^9 - 21*a^10*b^11 + 7*a^8*b^
13 - a^6*b^15)*sin(d*x + c)) - 105*log(sin(d*x + c) + 1)/(a^8 - 8*a^7*b + 2
8*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*a^2*b^6 - 8*a*b^7 + b
^8) + 105*log(sin(d*x + c) - 1)/(a^8 + 8*a^7*b + 28*a^6*b^2 + 56*a^5*b^3 +
70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 + b^8))/d
```

mupad [B] time = 7.64, size = 937, normalized size = 2.43

$$\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(\frac{1}{2\,{\left(a+b\right)}^8}-\frac{1}{2\,{\left(a-b\right)}^8}\right)}{d}+\frac{\frac{1443\,a^{12}\,b+3704\,a^{10}\,b^3+1849\,a^8\,b^5-496\,a^6\,b^7+309\,a^4\,b^9-104\,a^2\,b^{11}+15\,b^{13}}{105\,\left(a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}\right)}+\frac{\sin\left(c+d\,x\right)\,\left(1\dots\right.}{15\,\left(a^{14}-7\,a^1\dots\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^8),x)
```

```
[Out] (log(a + b*sin(c + d*x))*(1/(2*(a + b)^8) - 1/(2*(a - b)^8)))/d + ((1443*a^
12*b + 15*b^13 - 104*a^2*b^11 + 309*a^4*b^9 - 496*a^6*b^7 + 1849*a^8*b^5 +
3704*a^10*b^3)/(105*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 -
35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)) + (sin(c + d*x)*(a*b^12 - 6*a^3*b^1
0 + 29*a^5*b^8 + 1219*a^7*b^6 + 3494*a^9*b^4 + 1023*a^11*b^2))/(15*(a^14 -
b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7
*a^12*b^2)) + (sin(c + d*x)^3*(a*b^12 + 8*a^3*b^10 + 994*a^5*b^8 + 2304*a^7
*b^6 + 533*a^9*b^4))/(3*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^
8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)) + (sin(c + d*x)^5*(3*a*b^12 + 1
19*a^3*b^10 + 217*a^5*b^8 + 45*a^7*b^6))/(a^14 - b^14 + 7*a^2*b^12 - 21*a^4
*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2) + (sin(c + d*x)
^2*(b^13 - 6*a^2*b^11 + 29*a^4*b^9 + 1099*a^6*b^7 + 2934*a^8*b^5 + 743*a^10
*b^3))/(5*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6
 + 21*a^10*b^4 - 7*a^12*b^2)) + (sin(c + d*x)^4*(b^13 + 8*a^2*b^11 + 826*a^
```

```
4*b^9 + 1680*a^6*b^7 + 365*a^8*b^5))/(3*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*
b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)) + (sin(c + d*x)
^6*(b^13 + 21*a^2*b^11 + 35*a^4*b^9 + 7*a^6*b^7))/(a^14 - b^14 + 7*a^2*b^12
 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2))/(d*(a
^7 + b^7*sin(c + d*x)^7 + 7*a*b^6*sin(c + d*x)^6 + 21*a^5*b^2*sin(c + d*x)^
2 + 35*a^4*b^3*sin(c + d*x)^3 + 35*a^3*b^4*sin(c + d*x)^4 + 21*a^2*b^5*sin(
c + d*x)^5 + 7*a^6*b*sin(c + d*x))) + log(sin(c + d*x) + 1)/(2*d*(a - b)^8)
 - log(sin(c + d*x) - 1)/(2*d*(a + b)^8)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**8,x)
[Out] Timed out
```

3.466 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=527

$$-\frac{ab\left(3a^2+13b^2\right)}{6d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^6}-\frac{b\left(7a^2+9b^2\right)}{14d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^7}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^7}-\frac{ab}{2d\left(a^2\right.}$$

```
[Out] -1/4*(a+9*b)*ln(1-sin(d*x+c))/(a+b)^9/d+1/4*(a-9*b)*ln(1+sin(d*x+c))/(a-b)^
9/d+8*a*b^3*(15*a^6+63*a^4*b^2+45*a^2*b^4+5*b^6)*ln(a+b*sin(d*x+c))/(a^2-b^
2)^9/d-1/14*b*(7*a^2+9*b^2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^7-1/2*sec(d*x+c)
^2*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))^7-1/6*a*b*(3*a^2+13*b^2)/(
a^2-b^2)^3/d/(a+b*sin(d*x+c))^6-1/10*b*(5*a^4+50*a^2*b^2+9*b^4)/(a^2-b^2)^4
/d/(a+b*sin(d*x+c))^5-1/2*a*b*(a^4+20*a^2*b^2+11*b^4)/(a^2-b^2)^5/d/(a+b*si
n(d*x+c))^4-1/6*b*(3*a^6+115*a^4*b^2+129*a^2*b^4+9*b^6)/(a^2-b^2)^6/d/(a+b*
sin(d*x+c))^3-1/2*a*b*(a^6+77*a^4*b^2+147*a^2*b^4+31*b^6)/(a^2-b^2)^7/d/(a+
b*sin(d*x+c))^2-1/2*b*(a^8+196*a^6*b^2+574*a^4*b^4+244*a^2*b^6+9*b^8)/(a^2-
b^2)^8/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.74, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 741, 801}

$$-\frac{b\left(196a^6b^2+574a^4b^4+244a^2b^6+a^8+9b^8\right)}{2d\left(a^2-b^2\right)^8(a+b\sin(c+dx))}-\frac{ab\left(77a^4b^2+147a^2b^4+a^6+31b^6\right)}{2d\left(a^2-b^2\right)^7(a+b\sin(c+dx))^2}-\frac{b\left(115a^4b^2+129a^2b^4+3a^6\right.}{6d\left(a^2-b^2\right)^6(a+b\sin(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -((a + 9*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^9*d) + ((a - 9*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^9*d) + (8*a*b^3*(15*a^6 + 63*a^4*b^2 + 45*a^2*b^4 +
5*b^6)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^9*d) - (b*(7*a^2 + 9*b^2))/(14
*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^7) - (Sec[c + d*x]^2*(b - a*Sin[c + d
*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) - (a*b*(3*a^2 + 13*b^2))/(6*
(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^6) - (b*(5*a^4 + 50*a^2*b^2 + 9*b^4))/
(10*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^5) - (a*b*(a^4 + 20*a^2*b^2 + 11*b
^4))/(2*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^4) - (b*(3*a^6 + 115*a^4*b^2 +
 129*a^2*b^4 + 9*b^6))/(6*(a^2 - b^2)^6*d*(a + b*Sin[c + d*x])^3) - (a*b*(a
^6 + 77*a^4*b^2 + 147*a^2*b^4 + 31*b^6))/(2*(a^2 - b^2)^7*d*(a + b*Sin[c +
d*x])^2) - (b*(a^8 + 196*a^6*b^2 + 574*a^4*b^4 + 244*a^2*b^6 + 9*b^8))/(2*(
a^2 - b^2)^8*d*(a + b*Sin[c + d*x]))
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{(a+x)^8\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{b\operatorname{Subst}\left(\int \frac{a^2-9b^2+8ax}{(a+x)^8\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{b\operatorname{Subst}\left(\int \left(\frac{(a-b)(a+9b)}{2b(a+b)^8(b-x)} + \frac{7a^2+9b^2}{(a-b)(a+b)(a+x)^8} + \frac{}{(a}\right.\right.}{}$$

$$= -\frac{(a+9b)\log(1-\sin(c+dx))}{4(a+b)^9d} + \frac{(a-9b)\log(1+\sin(c+dx))}{4(a-b)^9d} + \frac{8ab^3\left(15a^6+63\right.}{}$$

Mathematica [A] time = 6.75, size = 770, normalized size = 1.46

$$b^3\left(\frac{\sec^2(c+dx)\left(b^2-ab\sin(c+dx)\right)}{2b^4\left(b^2-a^2\right)(a+b\sin(c+dx))^7} - \frac{8a\left(\frac{2a\left(3a^2+b^2\right)\left(a^2+3b^2\right)}{(a-b)^6(a+b)^6(a+b\sin(c+dx))} + \frac{4a\left(a^2+b^2\right)}{3(a-b)^4(a+b)^4(a+b\sin(c+dx))^3} + \frac{3a^2+b^2}{4(a-b)^3(a+b)^3(a+b\sin(c+dx))^4} + \frac{1}{6\left(a^2-b^2\right)(a+b\sin(c+dx))}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (b^3*((Sec[c + d*x]^2*(b^2 - a*b*Sin[c + d*x]))/(2*b^4*(-a^2 + b^2)*(a + b*
Sin[c + d*x])^7) - (8*a*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^7) + Log[1 +
 Sin[c + d*x]]/(2*(a - b)^7*b) - ((7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6)*L
og[a + b*Sin[c + d*x]])/((a - b)^7*(a + b)^7) + 1/(6*(a^2 - b^2)*(a + b*Sin
[c + d*x])^6) + (2*a)/(5*(a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])^5) + (3*a
^2 + b^2)/(4*(a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^4) + (4*a*(a^2 + b^2)
)/(3*(a - b)^4*(a + b)^4*(a + b*Sin[c + d*x])^3) + (5*a^4 + 10*a^2*b^2 + b^
4)/(2*(a - b)^5*(a + b)^5*(a + b*Sin[c + d*x])^2) + (2*a*(3*a^2 + b^2)*(a^2
 + 3*b^2))/((a - b)^6*(a + b)^6*(a + b*Sin[c + d*x]))) + (-7*a^2 - 9*b^2)*(
-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^8) + Log[1 + Sin[c + d*x]]/(2*(a - b)
^8*b) - (8*a*(a^2 + b^2)*(a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(
(a - b)^8*(a + b)^8) + 1/(7*(a^2 - b^2)*(a + b*Sin[c + d*x])^7) + a/(3*(a -
 b)^2*(a + b)^2*(a + b*Sin[c + d*x])^6) + (3*a^2 + b^2)/(5*(a - b)^3*(a + b
)^3*(a + b*Sin[c + d*x])^5) + (a*(a^2 + b^2))/((a - b)^4*(a + b)^4*(a + b*S
in[c + d*x])^4) + (5*a^4 + 10*a^2*b^2 + b^4)/(3*(a - b)^5*(a + b)^5*(a + b*
Sin[c + d*x])^3) + (a*(3*a^2 + b^2)*(a^2 + 3*b^2))/((a - b)^6*(a + b)^6*(a
+ b*Sin[c + d*x])^2) + (7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6)/((a - b)^7*(
a + b)^7*(a + b*Sin[c + d*x])))))/(2*b^2*(-a^2 + b^2))))/d
```

fricas [B] time = 8.07, size = 3678, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="fricas")

[Out] 1/420*(210*a^16*b - 1680*a^14*b^3 + 5880*a^12*b^5 - 11760*a^10*b^7 + 14700*
a^8*b^9 - 11760*a^6*b^11 + 5880*a^4*b^13 - 1680*a^2*b^15 + 210*b^17 - 210*(
a^10*b^7 + 195*a^8*b^9 + 378*a^6*b^11 - 330*a^4*b^13 - 235*a^2*b^15 - 9*b^1
7)*cos(d*x + c)^8 + 70*(63*a^12*b^5 + 10015*a^10*b^7 + 18468*a^8*b^9 - 1427
4*a^6*b^11 - 12025*a^4*b^13 - 2157*a^2*b^15 - 90*b^17)*cos(d*x + c)^6 - 14*
(525*a^14*b^3 + 59730*a^12*b^5 + 174637*a^10*b^7 + 77130*a^8*b^9 - 194265*a
^6*b^11 - 106450*a^4*b^13 - 10785*a^2*b^15 - 522*b^17)*cos(d*x + c)^4 + 2*(
735*a^16*b + 37165*a^14*b^3 + 437199*a^12*b^5 + 836549*a^10*b^7 - 111195*a^
8*b^9 - 812385*a^6*b^11 - 362915*a^4*b^13 - 23569*a^2*b^15 - 1584*b^17)*cos
(d*x + c)^2 + 3360*(7*(15*a^8*b^9 + 63*a^6*b^11 + 45*a^4*b^13 + 5*a^2*b^15)
*cos(d*x + c)^8 - 7*(75*a^10*b^7 + 360*a^8*b^9 + 414*a^6*b^11 + 160*a^4*b^1
3 + 15*a^2*b^15)*cos(d*x + c)^6 + 7*(45*a^12*b^5 + 339*a^10*b^7 + 810*a^8*b
^9 + 654*a^6*b^11 + 185*a^4*b^13 + 15*a^2*b^15)*cos(d*x + c)^4 - (15*a^14*b
^3 + 378*a^12*b^5 + 1893*a^10*b^7 + 3260*a^8*b^9 + 2121*a^6*b^11 + 490*a^4*
b^13 + 35*a^2*b^15)*cos(d*x + c)^2 + ((15*a^7*b^10 + 63*a^5*b^12 + 45*a^3*b
^14 + 5*a*b^16)*cos(d*x + c)^8 - 3*(105*a^9*b^8 + 456*a^7*b^10 + 378*a^5*b^
12 + 80*a^3*b^14 + 5*a*b^16)*cos(d*x + c)^6 + (525*a^11*b^6 + 2835*a^9*b^8
+ 4266*a^7*b^10 + 2254*a^5*b^12 + 345*a^3*b^14 + 15*a*b^16)*cos(d*x + c)^4
- (105*a^13*b^4 + 966*a^11*b^6 + 2835*a^9*b^8 + 2948*a^7*b^10 + 1183*a^5*b^
12 + 150*a^3*b^14 + 5*a*b^16)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x +
 c) + a) + 105*(7*(a^11*b^6 - 45*a^9*b^8 - 240*a^8*b^9 - 630*a^7*b^10 - 100
8*a^6*b^11 - 1050*a^5*b^12 - 720*a^4*b^13 - 315*a^3*b^14 - 80*a^2*b^15 - 9*
a*b^16)*cos(d*x + c)^8 - 7*(5*a^13*b^4 - 222*a^11*b^6 - 1200*a^10*b^7 - 328
5*a^9*b^8 - 5760*a^8*b^9 - 7140*a^7*b^10 - 6624*a^6*b^11 - 4725*a^5*b^12 -
2560*a^4*b^13 - 990*a^3*b^14 - 240*a^2*b^15 - 27*a*b^16)*cos(d*x + c)^6 + 7
*(3*a^15*b^2 - 125*a^13*b^4 - 720*a^12*b^5 - 2337*a^11*b^6 - 5424*a^10*b^7
- 9585*a^9*b^8 - 12960*a^8*b^9 - 13335*a^7*b^10 - 10464*a^6*b^11 - 6327*a^5
*b^12 - 2960*a^4*b^13 - 1035*a^3*b^14 - 240*a^2*b^15 - 27*a*b^16)*cos(d*x +
 c)^4 - (a^17 - 24*a^15*b^2 - 240*a^14*b^3 - 1540*a^13*b^4 - 6048*a^12*b^5
- 15848*a^11*b^6 - 30288*a^10*b^7 - 44730*a^9*b^8 - 52160*a^8*b^9 - 47784*a
^7*b^10 - 33936*a^6*b^11 - 18564*a^5*b^12 - 7840*a^4*b^13 - 2520*a^3*b^14 -
 560*a^2*b^15 - 63*a*b^16)*cos(d*x + c)^2 + ((a^10*b^7 - 45*a^8*b^9 - 240*a
^7*b^10 - 630*a^6*b^11 - 1008*a^5*b^12 - 1050*a^4*b^13 - 720*a^3*b^14 - 315
*a^2*b^15 - 80*a*b^16 - 9*b^17)*cos(d*x + c)^8 - 3*(7*a^12*b^5 - 314*a^10*b
^7 - 1680*a^9*b^8 - 4455*a^8*b^9 - 7296*a^7*b^10 - 7980*a^6*b^11 - 6048*a^5
*b^12 - 3255*a^4*b^13 - 1280*a^3*b^14 - 378*a^2*b^15 - 80*a*b^16 - 9*b^17)*
cos(d*x + c)^6 + (35*a^14*b^3 - 1533*a^12*b^5 - 8400*a^11*b^6 - 23937*a^10*
b^7 - 45360*a^9*b^8 - 63345*a^8*b^9 - 68256*a^7*b^10 - 57015*a^6*b^11 - 360
64*a^5*b^12 - 16695*a^4*b^13 - 5520*a^3*b^14 - 1323*a^2*b^15 - 240*a*b^16 -
 27*b^17)*cos(d*x + c)^4 - (7*a^16*b - 280*a^14*b^3 - 1680*a^13*b^4 - 5964*
a^12*b^5 - 15456*a^11*b^6 - 30344*a^10*b^7 - 45360*a^9*b^8 - 52230*a^8*b^9
- 47168*a^7*b^10 - 33768*a^6*b^11 - 18928*a^5*b^12 - 7980*a^4*b^13 - 2400*a
^3*b^14 - 504*a^2*b^15 - 80*a*b^16 - 9*b^17)*cos(d*x + c)^2)*sin(d*x + c))*
log(sin(d*x + c) + 1) - 105*(7*(a^11*b^6 - 45*a^9*b^8 + 240*a^8*b^9 - 630*a
^7*b^10 + 1008*a^6*b^11 - 1050*a^5*b^12 + 720*a^4*b^13 - 315*a^3*b^14 + 80*
a^2*b^15 - 9*a*b^16)*cos(d*x + c)^8 - 7*(5*a^13*b^4 - 222*a^11*b^6 + 1200*a
^10*b^7 - 3285*a^9*b^8 + 5760*a^8*b^9 - 7140*a^7*b^10 + 6624*a^6*b^11 - 472
5*a^5*b^12 + 2560*a^4*b^13 - 990*a^3*b^14 + 240*a^2*b^15 - 27*a*b^16)*cos(d
*x + c)^6 + 7*(3*a^15*b^2 - 125*a^13*b^4 + 720*a^12*b^5 - 2337*a^11*b^6 + 5
424*a^10*b^7 - 9585*a^9*b^8 + 12960*a^8*b^9 - 13335*a^7*b^10 + 10464*a^6*b^
11 - 6327*a^5*b^12 + 2960*a^4*b^13 - 1035*a^3*b^14 + 240*a^2*b^15 - 27*a*b^
16)*cos(d*x + c)^4 - (a^17 - 24*a^15*b^2 + 240*a^14*b^3 - 1540*a^13*b^4 + 6
048*a^12*b^5 - 15848*a^11*b^6 + 30288*a^10*b^7 - 44730*a^9*b^8 + 52160*a^8*
b^9 - 47784*a^7*b^10 + 33936*a^6*b^11 - 18564*a^5*b^12 + 7840*a^4*b^13 - 25
20*a^3*b^14 + 560*a^2*b^15 - 63*a*b^16)*cos(d*x + c)^2 + ((a^10*b^7 - 45*a^
8*b^9 + 240*a^7*b^10 - 630*a^6*b^11 + 1008*a^5*b^12 - 1050*a^4*b^13 + 720*a
^3*b^14 - 315*a^2*b^15 + 80*a*b^16 - 9*b^17)*cos(d*x + c)^8 - 3*(7*a^12*b^5
 - 314*a^10*b^7 + 1680*a^9*b^8 - 4455*a^8*b^9 + 7296*a^7*b^10 - 7980*a^6*b^
```

```
11 + 6048*a^5*b^12 - 3255*a^4*b^13 + 1280*a^3*b^14 - 378*a^2*b^15 + 80*a*b^
16 - 9*b^17)*cos(d*x + c)^6 + (35*a^14*b^3 - 1533*a^12*b^5 + 8400*a^11*b^6
- 23937*a^10*b^7 + 45360*a^9*b^8 - 63345*a^8*b^9 + 68256*a^7*b^10 - 57015*a
^6*b^11 + 36064*a^5*b^12 - 16695*a^4*b^13 + 5520*a^3*b^14 - 1323*a^2*b^15 +
 240*a*b^16 - 27*b^17)*cos(d*x + c)^4 - (7*a^16*b - 280*a^14*b^3 + 1680*a^1
3*b^4 - 5964*a^12*b^5 + 15456*a^11*b^6 - 30344*a^10*b^7 + 45360*a^9*b^8 - 5
2230*a^8*b^9 + 47168*a^7*b^10 - 33768*a^6*b^11 + 18928*a^5*b^12 - 7980*a^4*
b^13 + 2400*a^3*b^14 - 504*a^2*b^15 + 80*a*b^16 - 9*b^17)*cos(d*x + c)^2)*s
in(d*x + c))*log(-sin(d*x + c) + 1) - 14*(15*a^17 - 120*a^15*b^2 + 420*a^13
*b^4 - 840*a^11*b^6 + 1050*a^9*b^8 - 840*a^7*b^10 + 420*a^5*b^12 - 120*a^3*
b^14 + 15*a*b^16 - 15*(7*a^11*b^6 + 1245*a^9*b^8 + 2262*a^7*b^10 - 2166*a^5
*b^12 - 1325*a^3*b^14 - 23*a*b^16)*cos(d*x + c)^6 + 5*(105*a^13*b^4 + 14464
*a^11*b^6 + 28953*a^9*b^8 - 11736*a^7*b^10 - 23605*a^5*b^12 - 8040*a^3*b^14
 - 141*a*b^16)*cos(d*x + c)^4 - (315*a^15*b^2 + 26665*a^13*b^4 + 97499*a^11
*b^6 + 88065*a^9*b^8 - 106455*a^7*b^10 - 85325*a^5*b^12 - 20415*a^3*b^14 -
349*a*b^16)*cos(d*x + c)^2)*sin(d*x + c))/(7*(a^19*b^6 - 9*a^17*b^8 + 36*a^
15*b^10 - 84*a^13*b^12 + 126*a^11*b^14 - 126*a^9*b^16 + 84*a^7*b^18 - 36*a^
5*b^20 + 9*a^3*b^22 - a*b^24)*d*cos(d*x + c)^8 - 7*(5*a^21*b^4 - 42*a^19*b^
6 + 153*a^17*b^8 - 312*a^15*b^10 + 378*a^13*b^12 - 252*a^11*b^14 + 42*a^9*b
^16 + 72*a^7*b^18 - 63*a^5*b^20 + 22*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^6
+ 7*(3*a^23*b^2 - 17*a^21*b^4 + 21*a^19*b^6 + 81*a^17*b^8 - 354*a^15*b^10 +
 630*a^13*b^12 - 630*a^11*b^14 + 354*a^9*b^16 - 81*a^7*b^18 - 21*a^5*b^20 +
 17*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^4 - (a^25 + 12*a^23*b^2 - 118*a^21*
b^4 + 364*a^19*b^6 - 441*a^17*b^8 - 168*a^15*b^10 + 1260*a^13*b^12 - 1800*a
^11*b^14 + 1311*a^9*b^16 - 484*a^7*b^18 + 42*a^5*b^20 + 28*a^3*b^22 - 7*a*b
^24)*d*cos(d*x + c)^2 + ((a^18*b^7 - 9*a^16*b^9 + 36*a^14*b^11 - 84*a^12*b^
13 + 126*a^10*b^15 - 126*a^8*b^17 + 84*a^6*b^19 - 36*a^4*b^21 + 9*a^2*b^23
- b^25)*d*cos(d*x + c)^8 - 3*(7*a^20*b^5 - 62*a^18*b^7 + 243*a^16*b^9 - 552
*a^14*b^11 + 798*a^12*b^13 - 756*a^10*b^15 + 462*a^8*b^17 - 168*a^6*b^19 +
27*a^4*b^21 + 2*a^2*b^23 - b^25)*d*cos(d*x + c)^6 + (35*a^22*b^3 - 273*a^20
*b^5 + 885*a^18*b^7 - 1455*a^16*b^9 + 990*a^14*b^11 + 630*a^12*b^13 - 1974*
a^10*b^15 + 1890*a^8*b^17 - 945*a^6*b^19 + 235*a^4*b^21 - 15*a^2*b^23 - 3*b
^25)*d*cos(d*x + c)^4 - (7*a^24*b - 28*a^22*b^3 - 42*a^20*b^5 + 484*a^18*b^
7 - 1311*a^16*b^9 + 1800*a^14*b^11 - 1260*a^12*b^13 + 168*a^10*b^15 + 441*a
^8*b^17 - 364*a^6*b^19 + 118*a^4*b^21 - 12*a^2*b^23 - b^25)*d*cos(d*x + c)^
2)*sin(d*x + c))
```

giac [B] time = 4.38, size = 1327, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/420*(3360*(15*a^7*b^4 + 63*a^5*b^6 + 45*a^3*b^8 + 5*a*b^10)*log(abs(b*sin
(d*x + c) + a))/(a^18*b - 9*a^16*b^3 + 36*a^14*b^5 - 84*a^12*b^7 + 126*a^10
*b^9 - 126*a^8*b^11 + 84*a^6*b^13 - 36*a^4*b^15 + 9*a^2*b^17 - b^19) + 105*
(a - 9*b)*log(abs(sin(d*x + c) + 1))/(a^9 - 9*a^8*b + 36*a^7*b^2 - 84*a^6*b
^3 + 126*a^5*b^4 - 126*a^4*b^5 + 84*a^3*b^6 - 36*a^2*b^7 + 9*a*b^8 - b^9) -
 105*(a + 9*b)*log(abs(sin(d*x + c) - 1))/(a^9 + 9*a^8*b + 36*a^7*b^2 + 84*
a^6*b^3 + 126*a^5*b^4 + 126*a^4*b^5 + 84*a^3*b^6 + 36*a^2*b^7 + 9*a*b^8 + b
^9) + 210*(120*a^7*b^3*sin(d*x + c)^2 + 504*a^5*b^5*sin(d*x + c)^2 + 360*a^
3*b^7*sin(d*x + c)^2 + 40*a*b^9*sin(d*x + c)^2 - a^10*sin(d*x + c) - 27*a^8
*b^2*sin(d*x + c) - 42*a^6*b^4*sin(d*x + c) + 42*a^4*b^6*sin(d*x + c) + 27*
a^2*b^8*sin(d*x + c) + b^10*sin(d*x + c) + 8*a^9*b - 72*a^7*b^3 - 504*a^5*b
^5 - 408*a^3*b^7 - 48*a*b^9)/((a^18 - 9*a^16*b^2 + 36*a^14*b^4 - 84*a^12*b^
6 + 126*a^10*b^8 - 126*a^8*b^10 + 84*a^6*b^12 - 36*a^4*b^14 + 9*a^2*b^16 -
b^18)*(sin(d*x + c)^2 - 1)) - 4*(32670*a^7*b^10*sin(d*x + c)^7 + 137214*a^5
*b^12*sin(d*x + c)^7 + 98010*a^3*b^14*sin(d*x + c)^7 + 10890*a*b^16*sin(d*x
 + c)^7 + 237510*a^8*b^9*sin(d*x + c)^6 + 978138*a^6*b^11*sin(d*x + c)^6 +
```

```
670950*a^4*b^13*sin(d*x + c)^6 + 65310*a^2*b^15*sin(d*x + c)^6 - 420*b^17*s
in(d*x + c)^6 + 741930*a^9*b^8*sin(d*x + c)^5 + 2987334*a^7*b^10*sin(d*x +
c)^5 + 1959930*a^5*b^12*sin(d*x + c)^5 + 166530*a^3*b^14*sin(d*x + c)^5 - 1
260*a*b^16*sin(d*x + c)^5 + 1291675*a^10*b^7*sin(d*x + c)^4 + 5064885*a^8*b
^9*sin(d*x + c)^4 + 3165120*a^6*b^11*sin(d*x + c)^4 + 237020*a^4*b^13*sin(d
*x + c)^4 - 1155*a^2*b^15*sin(d*x + c)^4 - 105*b^17*sin(d*x + c)^4 + 135467
5*a^11*b^6*sin(d*x + c)^3 + 5144685*a^9*b^8*sin(d*x + c)^3 + 3051720*a^7*b^
10*sin(d*x + c)^3 + 207620*a^5*b^12*sin(d*x + c)^3 - 1155*a^3*b^14*sin(d*x
+ c)^3 - 105*a*b^16*sin(d*x + c)^3 + 856905*a^12*b^5*sin(d*x + c)^2 + 31265
01*a^10*b^7*sin(d*x + c)^2 + 1759590*a^8*b^9*sin(d*x + c)^2 + 113400*a^6*b^
11*sin(d*x + c)^2 - 2205*a^4*b^13*sin(d*x + c)^2 + 315*a^2*b^15*sin(d*x + c
)^2 - 42*b^17*sin(d*x + c)^2 + 303275*a^13*b^4*sin(d*x + c) + 1049727*a^11*
b^6*sin(d*x + c) + 565530*a^9*b^8*sin(d*x + c) + 33600*a^7*b^10*sin(d*x + c
) - 735*a^5*b^12*sin(d*x + c) + 105*a^3*b^14*sin(d*x + c) - 14*a*b^16*sin(d
*x + c) + 46475*a^14*b^3 + 149331*a^12*b^5 + 79845*a^10*b^7 + 2385*a^8*b^9
+ 1155*a^6*b^11 - 525*a^4*b^13 + 133*a^2*b^15 - 15*b^17)/((a^18 - 9*a^16*b^
2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 - 126*a^8*b^10 + 84*a^6*b^12 -
 36*a^4*b^14 + 9*a^2*b^16 - b^18)*(b*sin(d*x + c) + a)^7))/d
```

maple [A] time = 0.48, size = 804, normalized size = 1.53

$$-\frac{b^7}{d\,(a+b)^6\,(a-b)^6\,(a+b\sin(dx+c))^3}-\frac{\ln(\sin(dx+c)-1)\,a}{4d\,(a+b)^9}-\frac{9\ln(\sin(dx+c)-1)\,b}{4d\,(a+b)^9}+\frac{\ln(1+\sin(dx+c))\,a}{4d\,(a-b)^9}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x)
```

```
[Out] -1/d*b^7/(a+b)^6/(a-b)^6/(a+b*sin(d*x+c))^3-1/4/d/(a+b)^9*ln(sin(d*x+c)-1)*
a-9/4/d/(a+b)^9*ln(sin(d*x+c)-1)*b+1/4/d/(a-b)^9*ln(1+sin(d*x+c))*a-9/4/d/(
a-b)^9*ln(1+sin(d*x+c))*b-2/5/d*b^5/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))^5-4/d*
b^9/(a+b)^8/(a-b)^8/(a+b*sin(d*x+c))-1/7/d*b^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x
+c))^7-5/d*b^3*a^3/(a+b)^5/(a-b)^5/(a+b*sin(d*x+c))^4-3/d*b^5*a/(a+b)^5/(a-
b)^5/(a+b*sin(d*x+c))^4-28/d*b^3*a^5/(a+b)^7/(a-b)^7/(a+b*sin(d*x+c))^2-56/
d*b^5*a^3/(a+b)^7/(a-b)^7/(a+b*sin(d*x+c))^2-12/d*b^7*a/(a+b)^7/(a-b)^7/(a+
b*sin(d*x+c))^2+120/d*b^3*a^7/(a+b)^9/(a-b)^9*ln(a+b*sin(d*x+c))+504/d*b^5*
a^5/(a+b)^9/(a-b)^9*ln(a+b*sin(d*x+c))+360/d*b^7*a^3/(a+b)^9/(a-b)^9*ln(a+b
*sin(d*x+c))-1/4/d/(a+b)^8/(sin(d*x+c)-1)-1/4/d/(a-b)^8/(1+sin(d*x+c))+40/d
*b^9*a/(a+b)^9/(a-b)^9*ln(a+b*sin(d*x+c))-35/3/d*b^3/(a+b)^6/(a-b)^6/(a+b*s
in(d*x+c))^3*a^4-14/d*b^5/(a+b)^6/(a-b)^6/(a+b*sin(d*x+c))^3*a^2-2/3/d*a*b^
3/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))^6-2/d*b^3/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c
))^5*a^2-84/d*b^3/(a+b)^8/(a-b)^8/(a+b*sin(d*x+c))*a^6-252/d*b^5/(a+b)^8/(a
-b)^8/(a+b*sin(d*x+c))*a^4-108/d*b^7/(a+b)^8/(a-b)^8/(a+b*sin(d*x+c))*a^2
```

maxima [B] time = 0.44, size = 1670, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/420*(3360*(15*a^7*b^3 + 63*a^5*b^5 + 45*a^3*b^7 + 5*a*b^9)*log(b*sin(d*x
+ c) + a)/(a^18 - 9*a^16*b^2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 - 1
26*a^8*b^10 + 84*a^6*b^12 - 36*a^4*b^14 + 9*a^2*b^16 - b^18) + 105*(a - 9*b
)*log(sin(d*x + c) + 1)/(a^9 - 9*a^8*b + 36*a^7*b^2 - 84*a^6*b^3 + 126*a^5*
b^4 - 126*a^4*b^5 + 84*a^3*b^6 - 36*a^2*b^7 + 9*a*b^8 - b^9) - 105*(a + 9*b
)*log(sin(d*x + c) - 1)/(a^9 + 9*a^8*b + 36*a^7*b^2 + 84*a^6*b^3 + 126*a^5*
b^4 + 126*a^4*b^5 + 84*a^3*b^6 + 36*a^2*b^7 + 9*a*b^8 + b^9) - 2*(840*a^14*
b + 33490*a^12*b^3 + 57724*a^10*b^5 + 16354*a^8*b^7 - 1496*a^6*b^9 + 814*a^
4*b^11 - 236*a^2*b^13 + 30*b^15 - 105*(a^8*b^7 + 196*a^6*b^9 + 574*a^4*b^11
```

```
 + 244*a^2*b^13 + 9*b^15)*sin(d*x + c)^8 - 105*(7*a^9*b^6 + 1252*a^7*b^8 +
3514*a^5*b^10 + 1348*a^3*b^12 + 23*a*b^14)*sin(d*x + c)^7 - 35*(63*a^10*b^5
 + 10066*a^8*b^7 + 26194*a^6*b^9 + 7384*a^4*b^11 - 681*a^2*b^13 - 18*b^15)*
sin(d*x + c)^6 - 35*(105*a^11*b^4 + 14506*a^9*b^6 + 32254*a^7*b^8 + 160*a^5
*b^10 - 3951*a^3*b^12 - 66*a*b^14)*sin(d*x + c)^5 - 7*(525*a^12*b^3 + 59310
*a^10*b^5 + 83812*a^8*b^7 - 98528*a^6*b^9 - 44663*a^4*b^11 - 438*a^2*b^13 -
 18*b^15)*sin(d*x + c)^4 - 7*(315*a^13*b^2 + 25930*a^11*b^4 - 20896*a^9*b^6
 - 166336*a^7*b^8 - 53641*a^5*b^10 - 386*a^3*b^12 - 26*a*b^14)*sin(d*x + c)
^3 - (735*a^14*b + 30550*a^12*b^3 - 361856*a^10*b^5 - 919070*a^8*b^7 - 2528
45*a^6*b^9 - 3050*a^4*b^11 + 310*a^2*b^13 - 54*b^15)*sin(d*x + c)^2 - 7*(15
*a^15 - 420*a^13*b^2 - 26140*a^11*b^4 - 52264*a^9*b^6 - 13189*a^7*b^8 - 184
*a^5*b^10 + 26*a^3*b^12 - 4*a*b^14)*sin(d*x + c))/(a^23 - 8*a^21*b^2 + 28*a
^19*b^4 - 56*a^17*b^6 + 70*a^15*b^8 - 56*a^13*b^10 + 28*a^11*b^12 - 8*a^9*b
^14 + a^7*b^16 - (a^16*b^7 - 8*a^14*b^9 + 28*a^12*b^11 - 56*a^10*b^13 + 70*
a^8*b^15 - 56*a^6*b^17 + 28*a^4*b^19 - 8*a^2*b^21 + b^23)*sin(d*x + c)^9 -
7*(a^17*b^6 - 8*a^15*b^8 + 28*a^13*b^10 - 56*a^11*b^12 + 70*a^9*b^14 - 56*a
^7*b^16 + 28*a^5*b^18 - 8*a^3*b^20 + a*b^22)*sin(d*x + c)^8 - (21*a^18*b^5
- 169*a^16*b^7 + 596*a^14*b^9 - 1204*a^12*b^11 + 1526*a^10*b^13 - 1246*a^8*
b^15 + 644*a^6*b^17 - 196*a^4*b^19 + 29*a^2*b^21 - b^23)*sin(d*x + c)^7 - 7
*(5*a^19*b^4 - 41*a^17*b^6 + 148*a^15*b^8 - 308*a^13*b^10 + 406*a^11*b^12 -
 350*a^9*b^14 + 196*a^7*b^16 - 68*a^5*b^18 + 13*a^3*b^20 - a*b^22)*sin(d*x
+ c)^6 - 7*(5*a^20*b^3 - 43*a^18*b^5 + 164*a^16*b^7 - 364*a^14*b^9 + 518*a^
12*b^11 - 490*a^10*b^13 + 308*a^8*b^15 - 124*a^6*b^17 + 29*a^4*b^19 - 3*a^2
*b^21)*sin(d*x + c)^5 - 7*(3*a^21*b^2 - 29*a^19*b^4 + 124*a^17*b^6 - 308*a^
15*b^8 + 490*a^13*b^10 - 518*a^11*b^12 + 364*a^9*b^14 - 164*a^7*b^16 + 43*a
^5*b^18 - 5*a^3*b^20)*sin(d*x + c)^4 - 7*(a^22*b - 13*a^20*b^3 + 68*a^18*b^
5 - 196*a^16*b^7 + 350*a^14*b^9 - 406*a^12*b^11 + 308*a^10*b^13 - 148*a^8*b
^15 + 41*a^6*b^17 - 5*a^4*b^19)*sin(d*x + c)^3 - (a^23 - 29*a^21*b^2 + 196*
a^19*b^4 - 644*a^17*b^6 + 1246*a^15*b^8 - 1526*a^13*b^10 + 1204*a^11*b^12 -
 596*a^9*b^14 + 169*a^7*b^16 - 21*a^5*b^18)*sin(d*x + c)^2 + 7*(a^22*b - 8*
a^20*b^3 + 28*a^18*b^5 - 56*a^16*b^7 + 70*a^14*b^9 - 56*a^12*b^11 + 28*a^10
*b^13 - 8*a^8*b^15 + a^6*b^17)*sin(d*x + c)))/d
```

mupad [B] time = 9.89, size = 1443, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^8),x)
```

```
[Out] ((sin(c + d*x)^7*(23*a*b^14 + 1348*a^3*b^12 + 3514*a^5*b^10 + 1252*a^7*b^8
+ 7*a^9*b^6))/(2*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70
*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) - (420*a^14*b + 15*b^15
 - 118*a^2*b^13 + 407*a^4*b^11 - 748*a^6*b^9 + 8177*a^8*b^7 + 28862*a^10*b^
5 + 16745*a^12*b^3)/(105*(a^2 - b^2)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^1
0 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)) + (sin(c + d*x)^6*
(7384*a^4*b^11 - 681*a^2*b^13 - 18*b^15 + 26194*a^6*b^9 + 10066*a^8*b^7 + 6
3*a^10*b^5))/(6*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*
a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) + (sin(c + d*x)^8*(9*b^1
5 + 244*a^2*b^13 + 574*a^4*b^11 + 196*a^6*b^9 + a^8*b^7))/(2*(a^16 + b^16 -
 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^1
2*b^4 - 8*a^14*b^2)) + (sin(c + d*x)^5*(160*a^5*b^10 - 3951*a^3*b^12 - 66*a
*b^14 + 32254*a^7*b^8 + 14506*a^9*b^6 + 105*a^11*b^4))/(6*(a^16 + b^16 - 8*
a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b
^4 - 8*a^14*b^2)) + (sin(c + d*x)^4*(18*b^13 + 456*a^2*b^11 + 45119*a^4*b^9
 + 143647*a^6*b^7 + 59835*a^8*b^5 + 525*a^10*b^3))/(30*(a^14 - b^14 + 7*a^2
*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2))
- (sin(c + d*x)^2*(54*b^15 - 735*a^14*b - 310*a^2*b^13 + 3050*a^4*b^11 + 25
2845*a^6*b^9 + 919070*a^8*b^7 + 361856*a^10*b^5 - 30550*a^12*b^3))/(210*(a^
2 - b^2)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6
```

```
+ 21*a^10*b^4 - 7*a^12*b^2)) - (sin(c + d*x)^3*(26*a*b^14 + 386*a^3*b^12 +
53641*a^5*b^10 + 166336*a^7*b^8 + 20896*a^9*b^6 - 25930*a^11*b^4 - 315*a^13
*b^2))/(30*(a^2 - b^2)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8
 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)) - (a*sin(c + d*x)*(4*b^14 - 15*a
^14 - 26*a^2*b^12 + 184*a^4*b^10 + 13189*a^6*b^8 + 52264*a^8*b^6 + 26140*a^
10*b^4 + 420*a^12*b^2))/(30*(a^2 - b^2)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*
b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)))/(d*(sin(c + d*
x)^7*(b^7 - 21*a^2*b^5) - sin(c + d*x)^2*(a^7 - 21*a^5*b^2) + sin(c + d*x)^
4*(35*a^3*b^4 - 21*a^5*b^2) + sin(c + d*x)^5*(21*a^2*b^5 - 35*a^4*b^3) + a^
7 - b^7*sin(c + d*x)^9 - sin(c + d*x)^3*(7*a^6*b - 35*a^4*b^3) + sin(c + d*
x)^6*(7*a*b^6 - 35*a^3*b^4) - 7*a*b^6*sin(c + d*x)^8 + 7*a^6*b*sin(c + d*x)
)) - (log(sin(c + d*x) - 1)*((2*b)/(a + b)^9 + 1/(4*(a + b)^8)))/d + (log(a
 + b*sin(c + d*x))*((2*b)/(a + b)^9 + 1/(4*(a + b)^8) + (2*b)/(a - b)^9 - 1
/(4*(a - b)^8)))/d + (log(sin(c + d*x) + 1)*(a - 9*b))/(4*d*(a - b)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.467 $\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=491

$$\frac{a\cos^7(c+dx)}{6bd\left(a^2-b^2\right)(a+b\sin(c+dx))^6}-\frac{\cos^3(c+dx)\left(ab\left(6a^2-11b^2\right)\sin(c+dx)+8\left(a^2-b^2\right)^2\right)}{24b^5d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^3}+\frac{\cos^5(c+dx)\left(6\left(\right.\right.}{30b^3d\left(a^2-\right.}$$

```
[Out] x/b^8-1/8*a*(16*a^6-56*a^4*b^2+70*a^2*b^4-35*b^6)*arctan((b+a*tan(1/2*d*x+1
/2*c))/(a^2-b^2)^(1/2))/b^8/(a^2-b^2)^(7/2)/d-1/7*cos(d*x+c)^7/b/d/(a+b*sin
(d*x+c))^7+1/6*a*cos(d*x+c)^7/b/(a^2-b^2)/d/(a+b*sin(d*x+c))^6-1/24*a*(6*a^
2-11*b^2)*cos(d*x+c)^5/b^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^4+1/16*a*(8*a^4-2
2*a^2*b^2+19*b^4)*cos(d*x+c)^3/b^5/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^2+1/30*co
s(d*x+c)^5*(6*a^2-6*b^2+5*a*b*sin(d*x+c))/b^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^
5-1/24*cos(d*x+c)^3*(8*(a^2-b^2)^2+a*b*(6*a^2-11*b^2)*sin(d*x+c))/b^5/(a^2-
b^2)^2/d/(a+b*sin(d*x+c))^3+1/16*cos(d*x+c)*(16*(a^2-b^2)^3+a*b*(8*a^4-22*a
^2*b^2+19*b^4)*sin(d*x+c))/b^7/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 1.27, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2693, 2864, 2863, 2735, 2660, 618, 204}

$$-\frac{a\left(-56a^4b^2+70a^2b^4+16a^6-35b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8b^8d\left(a^2-b^2\right)^{7/2}}+\frac{a\cos^7(c+dx)}{6bd\left(a^2-b^2\right)(a+b\sin(c+dx))^6}+\frac{\cos^5(c+dx}{30b^3d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] x/b^8 - (a*(16*a^6 - 56*a^4*b^2 + 70*a^2*b^4 - 35*b^6)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(8*b^8*(a^2 - b^2)^(7/2)*d) - Cos[c + d*x]^7/
(7*b*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]^7)/(6*b*(a^2 - b^2)*d*(a +
 b*Sin[c + d*x])^6) - (a*(6*a^2 - 11*b^2)*Cos[c + d*x]^5)/(24*b^3*(a^2 - b^
2)^2*d*(a + b*Sin[c + d*x])^4) + (a*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Cos[c + d
*x]^3)/(16*b^5*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (Cos[c + d*x]^5*(6
*(a^2 - b^2) + 5*a*b*Sin[c + d*x]))/(30*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*
x])^5) - (Cos[c + d*x]^3*(8*(a^2 - b^2)^2 + a*b*(6*a^2 - 11*b^2)*Sin[c + d*
x]))/(24*b^5*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^3) + (Cos[c + d*x]*(16*(a
^2 - b^2)^3 + a*b*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Sin[c + d*x]))/(16*b^7*(a^2
 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{b}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\int \frac{\cos^6(c+dx)(6b+a\sin}{(a+b\sin(c+dx}}{6b\left(a^2-b\right.}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\cos^5(c+dx)\left(6\left(\right.\right.}{30b^3\left(a^2-b^2\right.}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right.}{24b^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right.}{24b^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right.}{24b^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right.}{24b^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{x}{b^8} - \frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-\right.}{24b^3\left(a^2-b^2\right.}$$

$$= \frac{x}{b^8} - \frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-\right.}{24b^3\left(a^2-b^2\right.}$$

$$= \frac{x}{b^8} - \frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-\right.}{24b^3\left(a^2-b^2\right.}$$

$$= \frac{x}{b^8} - \frac{a\left(16a^6-56a^4b^2+70a^2b^4-35b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8b^8\left(a^2-b^2\right)^{7/2}d} - \frac{\cos^7(c+a}{7bd(a+b\sin(c}$$

Mathematica [**B**] time = 8.50, size = 6570, normalized size = 13.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^8,x]
[Out] Result too large to show
```

fricas [**B**] time = 1.87, size = 3721, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/3360*(23520*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*x
*cos(d*x + c)^6 + 2*(4356*a^8*b^7 - 16864*a^6*b^9 + 24001*a^4*b^11 - 14309*
a^2*b^13 + 2816*b^15)*cos(d*x + c)^7 - 23520*(5*a^11*b^4 - 17*a^9*b^6 + 18*
a^7*b^8 - 2*a^5*b^10 - 7*a^3*b^12 + 3*a*b^14)*d*x*cos(d*x + c)^4 - 28*(2754
*a^10*b^5 - 9717*a^8*b^7 + 11528*a^6*b^9 - 3782*a^4*b^11 - 1247*a^2*b^13 +
464*b^15)*cos(d*x + c)^5 + 23520*(3*a^13*b^2 - 2*a^11*b^4 - 19*a^9*b^6 + 36
*a^7*b^8 - 19*a^5*b^10 - 2*a^3*b^12 + 3*a*b^14)*d*x*cos(d*x + c)^2 + 70*(85
6*a^12*b^3 - 1090*a^10*b^5 - 3477*a^8*b^7 + 7907*a^6*b^9 - 4423*a^4*b^11 +
67*a^2*b^13 + 160*b^15)*cos(d*x + c)^3 - 3360*(a^15 + 17*a^13*b^2 - 43*a^11
*b^4 - 11*a^9*b^6 + 99*a^7*b^8 - 77*a^5*b^10 + 7*a^3*b^12 + 7*a*b^14)*d*x +
 105*(16*a^14 + 280*a^12*b^2 - 546*a^10*b^4 - 413*a^8*b^6 + 1323*a^6*b^8 -
735*a^4*b^10 - 245*a^2*b^12 - 7*(16*a^8*b^6 - 56*a^6*b^8 + 70*a^4*b^10 - 35
*a^2*b^12)*cos(d*x + c)^6 + 7*(80*a^10*b^4 - 232*a^8*b^6 + 182*a^6*b^8 + 35
*a^4*b^10 - 105*a^2*b^12)*cos(d*x + c)^4 - 7*(48*a^12*b^2 - 8*a^10*b^4 - 30
2*a^8*b^6 + 427*a^6*b^8 - 140*a^4*b^10 - 105*a^2*b^12)*cos(d*x + c)^2 + (11
2*a^13*b + 168*a^11*b^3 - 1134*a^9*b^5 + 1045*a^7*b^7 + 189*a^5*b^9 - 665*a
^3*b^11 - 35*a*b^13 - (16*a^7*b^7 - 56*a^5*b^9 + 70*a^3*b^11 - 35*a*b^13)*c
os(d*x + c)^6 + 3*(112*a^9*b^5 - 376*a^7*b^7 + 434*a^5*b^9 - 175*a^3*b^11 -
 35*a*b^13)*cos(d*x + c)^4 - (560*a^11*b^3 - 1288*a^9*b^5 + 146*a^7*b^7 + 1
547*a^5*b^9 - 1260*a^3*b^11 - 105*a*b^13)*cos(d*x + c)^2)*sin(d*x + c))*sqr
t(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))
/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 420*(8*a^14*b + 1
12*a^12*b^3 - 322*a^10*b^5 + 63*a^8*b^7 + 479*a^6*b^9 - 379*a^4*b^11 + 31*a
^2*b^13 + 8*b^15)*cos(d*x + c) + 14*(240*(a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11
 - 4*a^2*b^13 + b^15)*d*x*cos(d*x + c)^6 - 720*(7*a^10*b^5 - 27*a^8*b^7 + 38
*a^6*b^9 - 22*a^4*b^11 + 3*a^2*b^13 + b^15)*d*x*cos(d*x + c)^4 - (2676*a^9*
b^6 - 10264*a^7*b^8 + 14371*a^5*b^10 - 8204*a^3*b^12 + 1421*a*b^14)*cos(d*x
 + c)^5 + 240*(35*a^12*b^3 - 98*a^10*b^5 + 45*a^8*b^7 + 100*a^6*b^9 - 115*a
^4*b^11 + 30*a^2*b^13 + 3*b^15)*d*x*cos(d*x + c)^2 + 10*(638*a^11*b^4 - 192
5*a^9*b^6 + 1427*a^7*b^8 + 861*a^5*b^10 - 1253*a^3*b^12 + 252*a*b^14)*cos(d
*x + c)^3 - 240*(7*a^14*b + 7*a^12*b^3 - 77*a^10*b^5 + 99*a^8*b^7 - 11*a^6*
b^9 - 43*a^4*b^11 + 17*a^2*b^13 + b^15)*d*x - 15*(104*a^13*b^2 + 26*a^11*b^
4 - 897*a^9*b^6 + 1306*a^7*b^8 - 308*a^5*b^10 - 308*a^3*b^12 + 77*a*b^14)*c
os(d*x + c))*sin(d*x + c))/(7*(a^9*b^14 - 4*a^7*b^16 + 6*a^5*b^18 - 4*a^3*b
^20 + a*b^22)*d*cos(d*x + c)^6 - 7*(5*a^11*b^12 - 17*a^9*b^14 + 18*a^7*b^16
 - 2*a^5*b^18 - 7*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^4 + 7*(3*a^13*b^10 -
2*a^11*b^12 - 19*a^9*b^14 + 36*a^7*b^16 - 19*a^5*b^18 - 2*a^3*b^20 + 3*a*b^
22)*d*cos(d*x + c)^2 - (a^15*b^8 + 17*a^13*b^10 - 43*a^11*b^12 - 11*a^9*b^1
4 + 99*a^7*b^16 - 77*a^5*b^18 + 7*a^3*b^20 + 7*a*b^22)*d + ((a^8*b^15 - 4*a
^6*b^17 + 6*a^4*b^19 - 4*a^2*b^21 + b^23)*d*cos(d*x + c)^6 - 3*(7*a^10*b^13
 - 27*a^8*b^15 + 38*a^6*b^17 - 22*a^4*b^19 + 3*a^2*b^21 + b^23)*d*cos(d*x +
 c)^4 + (35*a^12*b^11 - 98*a^10*b^13 + 45*a^8*b^15 + 100*a^6*b^17 - 115*a^4
*b^19 + 30*a^2*b^21 + 3*b^23)*d*cos(d*x + c)^2 - (7*a^14*b^9 + 7*a^12*b^11
 - 77*a^10*b^13 + 99*a^8*b^15 - 11*a^6*b^17 - 43*a^4*b^19 + 17*a^2*b^21 + b^
23)*d)*sin(d*x + c)), 1/1680*(11760*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a
^3*b^12 + a*b^14)*d*x*cos(d*x + c)^6 + (4356*a^8*b^7 - 16864*a^6*b^9 + 2400
1*a^4*b^11 - 14309*a^2*b^13 + 2816*b^15)*cos(d*x + c)^7 - 11760*(5*a^11*b^4
 - 17*a^9*b^6 + 18*a^7*b^8 - 2*a^5*b^10 - 7*a^3*b^12 + 3*a*b^14)*d*x*cos(d*
x + c)^4 - 14*(2754*a^10*b^5 - 9717*a^8*b^7 + 11528*a^6*b^9 - 3782*a^4*b^11
 - 1247*a^2*b^13 + 464*b^15)*cos(d*x + c)^5 + 11760*(3*a^13*b^2 - 2*a^11*b^
4 - 19*a^9*b^6 + 36*a^7*b^8 - 19*a^5*b^10 - 2*a^3*b^12 + 3*a*b^14)*d*x*cos(
d*x + c)^2 + 35*(856*a^12*b^3 - 1090*a^10*b^5 - 3477*a^8*b^7 + 7907*a^6*b^9
 - 4423*a^4*b^11 + 67*a^2*b^13 + 160*b^15)*cos(d*x + c)^3 - 1680*(a^15 + 17
*a^13*b^2 - 43*a^11*b^4 - 11*a^9*b^6 + 99*a^7*b^8 - 77*a^5*b^10 + 7*a^3*b^1
2 + 7*a*b^14)*d*x - 105*(16*a^14 + 280*a^12*b^2 - 546*a^10*b^4 - 413*a^8*b^
6 + 1323*a^6*b^8 - 735*a^4*b^10 - 245*a^2*b^12 - 7*(16*a^8*b^6 - 56*a^6*b^8
 + 70*a^4*b^10 - 35*a^2*b^12)*cos(d*x + c)^6 + 7*(80*a^10*b^4 - 232*a^8*b^6
```

```
 + 182*a^6*b^8 + 35*a^4*b^10 - 105*a^2*b^12)*cos(d*x + c)^4 - 7*(48*a^12*b^
2 - 8*a^10*b^4 - 302*a^8*b^6 + 427*a^6*b^8 - 140*a^4*b^10 - 105*a^2*b^12)*c
os(d*x + c)^2 + (112*a^13*b + 168*a^11*b^3 - 1134*a^9*b^5 + 1045*a^7*b^7 +
189*a^5*b^9 - 665*a^3*b^11 - 35*a*b^13 - (16*a^7*b^7 - 56*a^5*b^9 + 70*a^3*
b^11 - 35*a*b^13)*cos(d*x + c)^6 + 3*(112*a^9*b^5 - 376*a^7*b^7 + 434*a^5*b
^9 - 175*a^3*b^11 - 35*a*b^13)*cos(d*x + c)^4 - (560*a^11*b^3 - 1288*a^9*b^
5 + 146*a^7*b^7 + 1547*a^5*b^9 - 1260*a^3*b^11 - 105*a*b^13)*cos(d*x + c)^2
)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^
2)*cos(d*x + c))) - 210*(8*a^14*b + 112*a^12*b^3 - 322*a^10*b^5 + 63*a^8*b^
7 + 479*a^6*b^9 - 379*a^4*b^11 + 31*a^2*b^13 + 8*b^15)*cos(d*x + c) + 7*(24
0*(a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*x*cos(d*x + c)^6
 - 720*(7*a^10*b^5 - 27*a^8*b^7 + 38*a^6*b^9 - 22*a^4*b^11 + 3*a^2*b^13 + b
^15)*d*x*cos(d*x + c)^4 - (2676*a^9*b^6 - 10264*a^7*b^8 + 14371*a^5*b^10 -
8204*a^3*b^12 + 1421*a*b^14)*cos(d*x + c)^5 + 240*(35*a^12*b^3 - 98*a^10*b^
5 + 45*a^8*b^7 + 100*a^6*b^9 - 115*a^4*b^11 + 30*a^2*b^13 + 3*b^15)*d*x*cos
(d*x + c)^2 + 10*(638*a^11*b^4 - 1925*a^9*b^6 + 1427*a^7*b^8 + 861*a^5*b^10
 - 1253*a^3*b^12 + 252*a*b^14)*cos(d*x + c)^3 - 240*(7*a^14*b + 7*a^12*b^3
- 77*a^10*b^5 + 99*a^8*b^7 - 11*a^6*b^9 - 43*a^4*b^11 + 17*a^2*b^13 + b^15)
*d*x - 15*(104*a^13*b^2 + 26*a^11*b^4 - 897*a^9*b^6 + 1306*a^7*b^8 - 308*a^
5*b^10 - 308*a^3*b^12 + 77*a*b^14)*cos(d*x + c))*sin(d*x + c))/(7*(a^9*b^14
 - 4*a^7*b^16 + 6*a^5*b^18 - 4*a^3*b^20 + a*b^22)*d*cos(d*x + c)^6 - 7*(5*a
^11*b^12 - 17*a^9*b^14 + 18*a^7*b^16 - 2*a^5*b^18 - 7*a^3*b^20 + 3*a*b^22)*
d*cos(d*x + c)^4 + 7*(3*a^13*b^10 - 2*a^11*b^12 - 19*a^9*b^14 + 36*a^7*b^16
 - 19*a^5*b^18 - 2*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^2 - (a^15*b^8 + 17*a
^13*b^10 - 43*a^11*b^12 - 11*a^9*b^14 + 99*a^7*b^16 - 77*a^5*b^18 + 7*a^3*b
^20 + 7*a*b^22)*d + ((a^8*b^15 - 4*a^6*b^17 + 6*a^4*b^19 - 4*a^2*b^21 + b^2
3)*d*cos(d*x + c)^6 - 3*(7*a^10*b^13 - 27*a^8*b^15 + 38*a^6*b^17 - 22*a^4*b
^19 + 3*a^2*b^21 + b^23)*d*cos(d*x + c)^4 + (35*a^12*b^11 - 98*a^10*b^13 +
45*a^8*b^15 + 100*a^6*b^17 - 115*a^4*b^19 + 30*a^2*b^21 + 3*b^23)*d*cos(d*x
 + c)^2 - (7*a^14*b^9 + 7*a^12*b^11 - 77*a^10*b^13 + 99*a^8*b^15 - 11*a^6*b
^17 - 43*a^4*b^19 + 17*a^2*b^21 + b^23)*d)*sin(d*x + c))]
```

giac [B] time = 9.88, size = 2326, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/840*(105*(16*a^7 - 56*a^5*b^2 + 70*a^3*b^4 - 35*a*b^6)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^
2)))/((a^6*b^8 - 3*a^4*b^10 + 3*a^2*b^12 - b^14)*sqrt(a^2 - b^2)) - (840*a^
18*b*tan(1/2*d*x + 1/2*c)^13 - 2310*a^16*b^3*tan(1/2*d*x + 1/2*c)^13 + 1995
*a^14*b^5*tan(1/2*d*x + 1/2*c)^13 - 1680*a^12*b^7*tan(1/2*d*x + 1/2*c)^13 +
 5040*a^10*b^9*tan(1/2*d*x + 1/2*c)^13 - 5040*a^8*b^11*tan(1/2*d*x + 1/2*c)
^13 + 1680*a^6*b^13*tan(1/2*d*x + 1/2*c)^13 + 1680*a^19*tan(1/2*d*x + 1/2*c
)^12 + 5880*a^17*b^2*tan(1/2*d*x + 1/2*c)^12 - 24990*a^15*b^4*tan(1/2*d*x +
 1/2*c)^12 + 24255*a^13*b^6*tan(1/2*d*x + 1/2*c)^12 - 10080*a^11*b^8*tan(1/
2*d*x + 1/2*c)^12 + 30240*a^9*b^10*tan(1/2*d*x + 1/2*c)^12 - 30240*a^7*b^12
*tan(1/2*d*x + 1/2*c)^12 + 10080*a^5*b^14*tan(1/2*d*x + 1/2*c)^12 + 26880*a
^18*b*tan(1/2*d*x + 1/2*c)^11 - 19320*a^16*b^3*tan(1/2*d*x + 1/2*c)^11 - 87
640*a^14*b^5*tan(1/2*d*x + 1/2*c)^11 + 118790*a^12*b^7*tan(1/2*d*x + 1/2*c)
^11 - 26880*a^10*b^9*tan(1/2*d*x + 1/2*c)^11 + 94080*a^8*b^11*tan(1/2*d*x +
 1/2*c)^11 - 98560*a^6*b^13*tan(1/2*d*x + 1/2*c)^11 + 33600*a^4*b^15*tan(1/
2*d*x + 1/2*c)^11 + 10080*a^19*tan(1/2*d*x + 1/2*c)^10 + 144480*a^17*b^2*ta
n(1/2*d*x + 1/2*c)^10 - 299880*a^15*b^4*tan(1/2*d*x + 1/2*c)^10 - 15680*a^1
3*b^6*tan(1/2*d*x + 1/2*c)^10 + 276430*a^11*b^8*tan(1/2*d*x + 1/2*c)^10 + 3
6960*a^9*b^10*tan(1/2*d*x + 1/2*c)^10 + 97440*a^7*b^12*tan(1/2*d*x + 1/2*c)
^10 - 166880*a^5*b^14*tan(1/2*d*x + 1/2*c)^10 + 67200*a^3*b^16*tan(1/2*d*x
+ 1/2*c)^10 + 121800*a^18*b*tan(1/2*d*x + 1/2*c)^9 + 238770*a^16*b^3*tan(1/
```

```
2*d*x + 1/2*c)^9 - 1067605*a^14*b^5*tan(1/2*d*x + 1/2*c)^9 + 656390*a^12*b^
7*tan(1/2*d*x + 1/2*c)^9 + 345156*a^10*b^9*tan(1/2*d*x + 1/2*c)^9 + 214032*
a^8*b^11*tan(1/2*d*x + 1/2*c)^9 - 87472*a^6*b^13*tan(1/2*d*x + 1/2*c)^9 - 1
26336*a^4*b^15*tan(1/2*d*x + 1/2*c)^9 + 80640*a^2*b^17*tan(1/2*d*x + 1/2*c)
^9 + 25200*a^19*tan(1/2*d*x + 1/2*c)^8 + 514360*a^17*b^2*tan(1/2*d*x + 1/2*
c)^8 - 490350*a^15*b^4*tan(1/2*d*x + 1/2*c)^8 - 1389885*a^13*b^6*tan(1/2*d*
x + 1/2*c)^8 + 1764630*a^11*b^8*tan(1/2*d*x + 1/2*c)^8 + 201544*a^9*b^10*ta
n(1/2*d*x + 1/2*c)^8 + 305088*a^7*b^12*tan(1/2*d*x + 1/2*c)^8 - 336448*a^5*
b^14*tan(1/2*d*x + 1/2*c)^8 + 27776*a^3*b^16*tan(1/2*d*x + 1/2*c)^8 + 53760
*a*b^18*tan(1/2*d*x + 1/2*c)^8 + 235200*a^18*b*tan(1/2*d*x + 1/2*c)^7 + 744
800*a^16*b^3*tan(1/2*d*x + 1/2*c)^7 - 2263800*a^14*b^5*tan(1/2*d*x + 1/2*c)
^7 + 382620*a^12*b^7*tan(1/2*d*x + 1/2*c)^7 + 1776432*a^10*b^9*tan(1/2*d*x
+ 1/2*c)^7 + 204848*a^8*b^11*tan(1/2*d*x + 1/2*c)^7 - 47616*a^6*b^13*tan(1/
2*d*x + 1/2*c)^7 - 258560*a^4*b^15*tan(1/2*d*x + 1/2*c)^7 + 111616*a^2*b^17
*tan(1/2*d*x + 1/2*c)^7 + 15360*b^19*tan(1/2*d*x + 1/2*c)^7 + 33600*a^19*ta
n(1/2*d*x + 1/2*c)^6 + 730240*a^17*b^2*tan(1/2*d*x + 1/2*c)^6 - 534240*a^15
*b^4*tan(1/2*d*x + 1/2*c)^6 - 2260440*a^13*b^6*tan(1/2*d*x + 1/2*c)^6 + 244
3980*a^11*b^8*tan(1/2*d*x + 1/2*c)^6 + 593824*a^9*b^10*tan(1/2*d*x + 1/2*c)
^6 + 148848*a^7*b^12*tan(1/2*d*x + 1/2*c)^6 - 336448*a^5*b^14*tan(1/2*d*x +
 1/2*c)^6 + 27776*a^3*b^16*tan(1/2*d*x + 1/2*c)^6 + 53760*a*b^18*tan(1/2*d*
x + 1/2*c)^6 + 231000*a^18*b*tan(1/2*d*x + 1/2*c)^5 + 643230*a^16*b^3*tan(1
/2*d*x + 1/2*c)^5 - 2226175*a^14*b^5*tan(1/2*d*x + 1/2*c)^5 + 749980*a^12*b
^7*tan(1/2*d*x + 1/2*c)^5 + 1482936*a^10*b^9*tan(1/2*d*x + 1/2*c)^5 - 72128
*a^8*b^11*tan(1/2*d*x + 1/2*c)^5 - 87472*a^6*b^13*tan(1/2*d*x + 1/2*c)^5 -
126336*a^4*b^15*tan(1/2*d*x + 1/2*c)^5 + 80640*a^2*b^17*tan(1/2*d*x + 1/2*c
)^5 + 25200*a^19*tan(1/2*d*x + 1/2*c)^4 + 461160*a^17*b^2*tan(1/2*d*x + 1/2
*c)^4 - 667674*a^15*b^4*tan(1/2*d*x + 1/2*c)^4 - 857003*a^13*b^6*tan(1/2*d*
x + 1/2*c)^4 + 1686188*a^11*b^8*tan(1/2*d*x + 1/2*c)^4 - 290976*a^9*b^10*ta
n(1/2*d*x + 1/2*c)^4 + 118160*a^7*b^12*tan(1/2*d*x + 1/2*c)^4 - 166880*a^5*
b^14*tan(1/2*d*x + 1/2*c)^4 + 67200*a^3*b^16*tan(1/2*d*x + 1/2*c)^4 + 11424
0*a^18*b*tan(1/2*d*x + 1/2*c)^3 + 89880*a^16*b^3*tan(1/2*d*x + 1/2*c)^3 - 8
81776*a^14*b^5*tan(1/2*d*x + 1/2*c)^3 + 996478*a^12*b^7*tan(1/2*d*x + 1/2*c
)^3 - 212688*a^10*b^9*tan(1/2*d*x + 1/2*c)^3 + 108976*a^8*b^11*tan(1/2*d*x
+ 1/2*c)^3 - 98560*a^6*b^13*tan(1/2*d*x + 1/2*c)^3 + 33600*a^4*b^15*tan(1/2
*d*x + 1/2*c)^3 + 10080*a^19*tan(1/2*d*x + 1/2*c)^2 + 101920*a^17*b^2*tan(1
/2*d*x + 1/2*c)^2 - 344568*a^15*b^4*tan(1/2*d*x + 1/2*c)^2 + 331128*a^13*b^
6*tan(1/2*d*x + 1/2*c)^2 - 79226*a^11*b^8*tan(1/2*d*x + 1/2*c)^2 + 44800*a^
9*b^10*tan(1/2*d*x + 1/2*c)^2 - 33264*a^7*b^12*tan(1/2*d*x + 1/2*c)^2 + 100
80*a^5*b^14*tan(1/2*d*x + 1/2*c)^2 + 22680*a^18*b*tan(1/2*d*x + 1/2*c) - 64
330*a^16*b^3*tan(1/2*d*x + 1/2*c) + 58569*a^14*b^5*tan(1/2*d*x + 1/2*c) - 1
4322*a^12*b^7*tan(1/2*d*x + 1/2*c) + 8372*a^10*b^9*tan(1/2*d*x + 1/2*c) - 5
824*a^8*b^11*tan(1/2*d*x + 1/2*c) + 1680*a^6*b^13*tan(1/2*d*x + 1/2*c) + 16
80*a^19 - 4760*a^17*b^2 + 4326*a^15*b^4 - 1143*a^13*b^6 + 958*a^11*b^8 - 77
6*a^9*b^10 + 240*a^7*b^12)/((a^13*b^7 - 3*a^11*b^9 + 3*a^9*b^11 - a^7*b^13)
*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7) - 840*(d*x +
c)/b^8)/d
```

maple [B] time = 0.42, size = 9454, normalized size = 19.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 32.22, size = 9647, normalized size = 19.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + b*sin(c + d*x))^8,x)
```

```
[Out] (2*atan(((((((((32*a^2*b^35 - 192*a^4*b^33 + 480*a^6*b^31 - 640*a^8*b^29 + 4
80*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b^23)*1i)/(b^32 - 6*a^2*b^30 + 15*a^
4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 +
(d*x)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 - 23040*a^7*b^31 + 21
760*a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^15*b^23)*1i)/(8*(b^
33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a
^12*b^21)))*1i)/b^8 + ((32*a*b^28 - 154*a^3*b^26 + 322*a^5*b^24 - 378*a^7*b
^22 + 262*a^9*b^20 - 100*a^11*b^18 + 16*a^13*b^16)*1i)/(b^32 - 6*a^2*b^30 +
 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(
c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a^4*b^26 + 11872*a^6*b^24 - 13728*a^8*
b^22 + 9152*a^10*b^20 - 3328*a^12*b^18 + 512*a^14*b^16)*1i)/(8*(b^33 - 6*a^
2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)
))/b^8 + (32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 640*a^8*b^13 + 480*a^
10*b^11 - 192*a^12*b^9 + 32*a^14*b^7)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20
*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x)/2)*(5
12*a*b^21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^7*b^15 + 21760*a^9*b^1
3 - 12288*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7))/(8*(b^33 - 6*a^2*b^31
+ 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))/b^8
+ ((32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 640*a^8*b^13 + 480*a^10*b^1
1 - 192*a^12*b^9 + 32*a^14*b^7)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b
^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) - (((32*a*b^28 - 154*a^3*b^26
+ 322*a^5*b^24 - 378*a^7*b^22 + 262*a^9*b^20 - 100*a^11*b^18 + 16*a^13*b^16
)*1i)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10
*b^22 + a^12*b^20) - ((((32*a^2*b^35 - 192*a^4*b^33 + 480*a^6*b^31 - 640*a^
8*b^29 + 480*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b^23)*1i)/(b^32 - 6*a^2*b^
30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (
tan(c/2 + (d*x)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 - 23040*a^7
*b^31 + 21760*a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^15*b^23)*
1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^1
0*b^23 + a^12*b^21)))*1i)/b^8 + (tan(c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a
^4*b^26 + 11872*a^6*b^24 - 13728*a^8*b^22 + 9152*a^10*b^20 - 3328*a^12*b^18
 + 512*a^14*b^16)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 1
5*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))/b^8 + (tan(c/2 + (d*x)/2)*(512*a*b^
21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^7*b^15 + 21760*a^9*b^13 - 122
88*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7))/(8*(b^33 - 6*a^2*b^31 + 15*a^
4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))/b^8)/((32*a
^13 - (665*a^3*b^10)/4 + 525*a^5*b^8 - 721*a^7*b^6 + 524*a^9*b^4 - 200*a^11
*b^2)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10
*b^22 + a^12*b^20) - (((32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 640*a^8
*b^13 + 480*a^10*b^11 - 192*a^12*b^9 + 32*a^14*b^7)*1i)/(b^32 - 6*a^2*b^30
+ 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) - ((((
32*a*b^28 - 154*a^3*b^26 + 322*a^5*b^24 - 378*a^7*b^22 + 262*a^9*b^20 - 100
*a^11*b^18 + 16*a^13*b^16)*1i)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^
26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) - ((((32*a^2*b^35 - 192*a^4*b^3
3 + 480*a^6*b^31 - 640*a^8*b^29 + 480*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b
^23)*1i)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a
```

^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 - 23040*a^7*b^31 + 21760*a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^15*b^23)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))*1i)/b^8 + (tan(c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a^4*b^26 + 11872*a^6*b^24 - 13728*a^8*b^22 + 9152*a^10*b^20 - 3328*a^12*b^18 + 512*a^14*b^16)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))*1i)/b^8 + (tan(c/2 + (d*x)/2)*(512*a*b^21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^7*b^15 + 21760*a^9*b^13 - 12288*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))/b^8 + ((((((32*a^2*b^35 - 192*a^4*b^33 + 480*a^6*b^31 - 640*a^8*b^29 + 480*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b^23)*1i)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 - 23040*a^7*b^31 + 21760*a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^15*b^23)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))*1i)/b^8 + ((32*a*b^28 - 154*a^3*b^26 + 322*a^5*b^24 - 378*a^7*b^22 + 262*a^9*b^20 - 100*a^11*b^18 + 16*a^13*b^16)*1i)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a^4*b^26 + 11872*a^6*b^24 - 13728*a^8*b^22 + 9152*a^10*b^20 - 3328*a^12*b^18 + 512*a^14*b^16)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))*1i)/b^8 + ((32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 640*a^8*b^13 + 480*a^10*b^11 - 192*a^12*b^9 + 32*a^14*b^7)*1i)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x)/2)*(512*a*b^21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^7*b^15 + 21760*a^9*b^13 - 12288*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))/b^8 + (tan(c/2 + (d*x)/2)*(512*a^14 + 1120*a^2*b^12 - 5600*a^4*b^10 + 11872*a^6*b^8 - 13728*a^8*b^6 + 9152*a^10*b^4 - 3328*a^12*b^2))/(4*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))))/(b^8*d) + ((1680*a^12 + 240*b^12 - 776*a^2*b^10 + 958*a^4*b^8 - 1143*a^6*b^6 + 4326*a^8*b^4 - 4760*a^10*b^2)/(840*b^7*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)*(3240*a^12 + 240*b^12 - 832*a^2*b^10 + 1196*a^4*b^8 - 2046*a^6*b^6 + 8367*a^8*b^4 - 9190*a^10*b^2))/(120*a*b^6*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^6*(1200*a^18 + 1920*b^18 + 992*a^2*b^16 - 12016*a^4*b^14 + 5316*a^6*b^12 + 21208*a^8*b^10 + 87285*a^10*b^8 - 80730*a^12*b^6 - 19080*a^14*b^4 + 26080*a^16*b^2))/(30*a^6*b^7*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^8*(3600*a^18 + 7680*b^18 + 3968*a^2*b^16 - 48064*a^4*b^14 + 43584*a^6*b^12 + 28792*a^8*b^10 + 252090*a^10*b^8 - 198555*a^12*b^6 - 70050*a^14*b^4 + 73480*a^16*b^2))/(120*a^6*b^7*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^10*(144*a^16 + 960*b^16 - 2384*a^2*b^14 + 1392*a^4*b^12 + 528*a^6*b^10 + 3949*a^8*b^8 - 224*a^10*b^6 - 4284*a^12*b^4 + 2064*a^14*b^2))/(12*a^4*b^7*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^9*(17400*a^16 + 11520*b^16 - 18048*a^2*b^14 - 12496*a^4*b^12 + 30576*a^6*b^10 + 49308*a^8*b^8 + 93770*a^10*b^6 - 152515*a^12*b^4 + 34110*a^14*b^2))/(120*a^5*b^6*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^4*(3600*a^16 + 9600*b^16 - 23840*a^2*b^14 + 16880*a^4*b^12 - 41568*a^6*b^10 + 240884*a^8*b^8 - 122429*a^10*b^6 - 95382*a^12*b^4 + 65880*a^14*b^2))/(120*a^4*b^7*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^5*(33000*a^16 + 11520*b^16 - 18048*a^2*b^14 - 12496*a^4*b^12 - 10304*a^6*b^10 + 211848*a^8*b^8 + 107140*a^10*b^6 - 318025*a^12*b^4 + 91890*a^14*b^2))/(120*a^5*b^6*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^12*(16*a^14 + 96*b^14 - 288*a^2*b^12 + 288*a^4*b^10 - 96*a^6*b^8 + 231*a^8*b^6 - 238*a^10*b^4 + 56*a^12*b^2))/(8*a^2*b^7*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^11*(384*a^14 + 480*b^14 - 1408*a^2*b^12 + 1344*a^4*b^10 - 384*a^6*b^8 + 1697*a^8*b^6 - 1252*a^10*b^4 - 276*a^12*b^2))/(12*a^3*b^6*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^2*(720*a^14 + 720*b

^14 - 2376*a^2*b^12 + 3200*a^4*b^10 - 5659*a^6*b^8 + 23652*a^8*b^6 - 24612*
a^10*b^4 + 7280*a^12*b^2))/(60*a^2*b^7*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
 + (tan(c/2 + (d*x)/2)^3*(8160*a^14 + 2400*b^14 - 7040*a^2*b^12 + 7784*a^4*
b^10 - 15192*a^6*b^8 + 71177*a^8*b^6 - 62984*a^10*b^4 + 6420*a^12*b^2))/(60
*a^3*b^6*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^13*(8*a
^12 + 16*b^12 - 48*a^2*b^10 + 48*a^4*b^8 - 16*a^6*b^6 + 19*a^8*b^4 - 22*a^1
0*b^2))/(8*a*b^6*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)
^7*(35*a^6 + 16*b^6 + 168*a^2*b^4 + 210*a^4*b^2)*(1680*a^12 + 240*b^12 - 77
6*a^2*b^10 + 958*a^4*b^8 - 1143*a^6*b^6 + 4326*a^8*b^4 - 4760*a^10*b^2))/(2
10*a^7*b^6*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)))/(d*(tan(c/2 + (d*x)/2)^5*(
210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + tan(c/2 + (d*x)/2)^9*(210*a^6*b +
 672*a^2*b^5 + 1120*a^4*b^3) + a^7*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/
2)^3*(84*a^6*b + 280*a^4*b^3) + tan(c/2 + (d*x)/2)^11*(84*a^6*b + 280*a^4*b
^3) + tan(c/2 + (d*x)/2)^6*(448*a*b^6 + 35*a^7 + 1680*a^3*b^4 + 840*a^5*b^2
) + tan(c/2 + (d*x)/2)^8*(448*a*b^6 + 35*a^7 + 1680*a^3*b^4 + 840*a^5*b^2)
+ tan(c/2 + (d*x)/2)^7*(280*a^6*b + 128*b^7 + 1344*a^2*b^5 + 1680*a^4*b^3)
+ a^7 + tan(c/2 + (d*x)/2)^4*(21*a^7 + 560*a^3*b^4 + 420*a^5*b^2) + tan(c/2
 + (d*x)/2)^10*(21*a^7 + 560*a^3*b^4 + 420*a^5*b^2) + tan(c/2 + (d*x)/2)^2*
(7*a^7 + 84*a^5*b^2) + tan(c/2 + (d*x)/2)^12*(7*a^7 + 84*a^5*b^2) + 14*a^6*
b*tan(c/2 + (d*x)/2) + 14*a^6*b*tan(c/2 + (d*x)/2)^13)) + (a*atan(((a*(-(a
+ b)^7*(a - b)^7)^(1/2)*((32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 640*a
^8*b^13 + 480*a^10*b^11 - 192*a^12*b^9 + 32*a^14*b^7)/(b^32 - 6*a^2*b^30 +
15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c
/2 + (d*x)/2)*(512*a*b^21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^7*b^15
 + 21760*a^9*b^13 - 12288*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7))/(8*(b^
33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a
^12*b^21)) - (a*(-(a + b)^7*(a - b)^7)^(1/2)*((32*a*b^28 - 154*a^3*b^26 + 3
22*a^5*b^24 - 378*a^7*b^22 + 262*a^9*b^20 - 100*a^11*b^18 + 16*a^13*b^16)/(
b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 +
 a^12*b^20) + (tan(c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a^4*b^26 + 11872*a^
6*b^24 - 13728*a^8*b^22 + 9152*a^10*b^20 - 3328*a^12*b^18 + 512*a^14*b^16))
/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b
^23 + a^12*b^21)) - (a*((32*a^2*b^35 - 192*a^4*b^33 + 480*a^6*b^31 - 640*a^
8*b^29 + 480*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b^23)/(b^32 - 6*a^2*b^30 +
 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(
c/2 + (d*x)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 - 23040*a^7*b^3
1 + 21760*a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^15*b^23))/(8*
(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23
+ a^12*b^21)))*(-(a + b)^7*(a - b)^7)^(1/2)*(16*a^6 - 35*b^6 + 70*a^2*b^4 -
 56*a^4*b^2))/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 + 35*a^8*b
^14 - 21*a^10*b^12 + 7*a^12*b^10 - a^14*b^8)))*(16*a^6 - 35*b^6 + 70*a^2*b^
4 - 56*a^4*b^2))/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 + 35*a^
8*b^14 - 21*a^10*b^12 + 7*a^12*b^10 - a^14*b^8)))*(16*a^6 - 35*b^6 + 70*a^2
*b^4 - 56*a^4*b^2)*1i)/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 +
 35*a^8*b^14 - 21*a^10*b^12 + 7*a^12*b^10 - a^14*b^8)) + (a*(-(a + b)^7*(a
- b)^7)^(1/2)*((32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 640*a^8*b^13 +
480*a^10*b^11 - 192*a^12*b^9 + 32*a^14*b^7)/(b^32 - 6*a^2*b^30 + 15*a^4*b^2
8 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x)
/2)*(512*a*b^21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^7*b^15 + 21760*a
^9*b^13 - 12288*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7))/(8*(b^33 - 6*a^2
*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21))
 + (a*(-(a + b)^7*(a - b)^7)^(1/2)*((32*a*b^28 - 154*a^3*b^26 + 322*a^5*b^2
4 - 378*a^7*b^22 + 262*a^9*b^20 - 100*a^11*b^18 + 16*a^13*b^16)/(b^32 - 6*a
^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20
) + (tan(c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a^4*b^26 + 11872*a^6*b^24 - 1
3728*a^8*b^22 + 9152*a^10*b^20 - 3328*a^12*b^18 + 512*a^14*b^16))/(8*(b^33
- 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12
b^21)) + (a((32*a^2*b^35 - 192*a^4*b^33 + 480*a^6*b^31 - 640*a^8*b^29 + 4
80*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b^23)/(b^32 - 6*a^2*b^30 + 15*a^4*b^

28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x
)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 - 23040*a^7*b^31 + 21760*
a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^15*b^23))/(8*(b^33 - 6*
a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^2
1)))*(-(a + b)^7*(a - b)^7)^(1/2)*(16*a^6 - 35*b^6 + 70*a^2*b^4 - 56*a^4*b^
2))/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 + 35*a^8*b^14 - 21*a
^10*b^12 + 7*a^12*b^10 - a^14*b^8)))*(16*a^6 - 35*b^6 + 70*a^2*b^4 - 56*a^4
b^2))/(16(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 + 35*a^8*b^14 - 2
1*a^10*b^12 + 7*a^12*b^10 - a^14*b^8)))*(16*a^6 - 35*b^6 + 70*a^2*b^4 - 56*
a^4*b^2)*1i)/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 + 35*a^8*b^
14 - 21*a^10*b^12 + 7*a^12*b^10 - a^14*b^8)))/((32*a^13 - (665*a^3*b^10)/4
+ 525*a^5*b^8 - 721*a^7*b^6 + 524*a^9*b^4 - 200*a^11*b^2)/(b^32 - 6*a^2*b^3
0 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (t
an(c/2 + (d*x)/2)*(512*a^14 + 1120*a^2*b^12 - 5600*a^4*b^10 + 11872*a^6*b^8
 - 13728*a^8*b^6 + 9152*a^10*b^4 - 3328*a^12*b^2))/(4*(b^33 - 6*a^2*b^31 +
15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)) - (a*(-
(a + b)^7*(a - b)^7)^(1/2)*((32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 64
0*a^8*b^13 + 480*a^10*b^11 - 192*a^12*b^9 + 32*a^14*b^7)/(b^32 - 6*a^2*b^30
 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (ta
n(c/2 + (d*x)/2)*(512*a*b^21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^7*b
^15 + 21760*a^9*b^13 - 12288*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7))/(8*
(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23
+ a^12*b^21)) - (a*(-(a + b)^7*(a - b)^7)^(1/2)*((32*a*b^28 - 154*a^3*b^26
+ 322*a^5*b^24 - 378*a^7*b^22 + 262*a^9*b^20 - 100*a^11*b^18 + 16*a^13*b^16
)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^2
2 + a^12*b^20) + (tan(c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a^4*b^26 + 11872
*a^6*b^24 - 13728*a^8*b^22 + 9152*a^10*b^20 - 3328*a^12*b^18 + 512*a^14*b^1
6))/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^1
0*b^23 + a^12*b^21)) - (a*((32*a^2*b^35 - 192*a^4*b^33 + 480*a^6*b^31 - 640
*a^8*b^29 + 480*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b^23)/(b^32 - 6*a^2*b^3
0 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (t
an(c/2 + (d*x)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 - 23040*a^7*
b^31 + 21760*a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^15*b^23))/
(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^
23 + a^12*b^21)))*(-(a + b)^7*(a - b)^7)^(1/2)*(16*a^6 - 35*b^6 + 70*a^2*b^
4 - 56*a^4*b^2))/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 + 35*a^
8*b^14 - 21*a^10*b^12 + 7*a^12*b^10 - a^14*b^8)))*(16*a^6 - 35*b^6 + 70*a^2
*b^4 - 56*a^4*b^2))/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 + 35
*a^8*b^14 - 21*a^10*b^12 + 7*a^12*b^10 - a^14*b^8)))*(16*a^6 - 35*b^6 + 70*
a^2*b^4 - 56*a^4*b^2))/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 +
 35*a^8*b^14 - 21*a^10*b^12 + 7*a^12*b^10 - a^14*b^8)) + (a*(-(a + b)^7*(a
- b)^7)^(1/2)*((32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 640*a^8*b^13 +
480*a^10*b^11 - 192*a^12*b^9 + 32*a^14*b^7)/(b^32 - 6*a^2*b^30 + 15*a^4*b^2
8 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x)
/2)*(512*a*b^21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^7*b^15 + 21760*a
^9*b^13 - 12288*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7))/(8*(b^33 - 6*a^2
*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21))
 + (a*(-(a + b)^7*(a - b)^7)^(1/2)*((32*a*b^28 - 154*a^3*b^26 + 322*a^5*b^2
4 - 378*a^7*b^22 + 262*a^9*b^20 - 100*a^11*b^18 + 16*a^13*b^16)/(b^32 - 6*a
^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20
) + (tan(c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a^4*b^26 + 11872*a^6*b^24 - 1
3728*a^8*b^22 + 9152*a^10*b^20 - 3328*a^12*b^18 + 512*a^14*b^16))/(8*(b^33
- 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12
b^21)) + (a((32*a^2*b^35 - 192*a^4*b^33 + 480*a^6*b^31 - 640*a^8*b^29 + 4
80*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b^23)/(b^32 - 6*a^2*b^30 + 15*a^4*b^
28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x
)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 - 23040*a^7*b^31 + 21760*
a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^15*b^23))/(8*(b^33 - 6*
a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^2

```
1)))*(-(a + b)^7*(a - b)^7)^(1/2)*(16*a^6 - 35*b^6 + 70*a^2*b^4 - 56*a^4*b^
2))/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 + 35*a^8*b^14 - 21*a
^10*b^12 + 7*a^12*b^10 - a^14*b^8)))*(16*a^6 - 35*b^6 + 70*a^2*b^4 - 56*a^4
*b^2))/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 + 35*a^8*b^14 - 2
1*a^10*b^12 + 7*a^12*b^10 - a^14*b^8)))*(16*a^6 - 35*b^6 + 70*a^2*b^4 - 56*
a^4*b^2))/(16*(b^22 - 7*a^2*b^20 + 21*a^4*b^18 - 35*a^6*b^16 + 35*a^8*b^14
- 21*a^10*b^12 + 7*a^12*b^10 - a^14*b^8))))*(-(a + b)^7*(a - b)^7)^(1/2)*(1
6*a^6 - 35*b^6 + 70*a^2*b^4 - 56*a^4*b^2)*1i)/(8*d*(b^22 - 7*a^2*b^20 + 21*
a^4*b^18 - 35*a^6*b^16 + 35*a^8*b^14 - 21*a^10*b^12 + 7*a^12*b^10 - a^14*b^
8))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.468 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=407

$$\frac{5a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{9/2}}-\frac{\cos(c+dx)\left(4a^2+10ab\sin(c+dx)+9b^2\right)}{42b^5d(a+b\sin(c+dx))^5}+\frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5d\left(a^2-b^2\right)(a+b\sin(c+dx))^4}+\frac{a}{33}$$

```
[Out] 5/8*a*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(9/2)/d-1/
7*cos(d*x+c)^5/b/d/(a+b*sin(d*x+c))^7+1/168*a*(4*a^2-b^2)*cos(d*x+c)/b^5/(a
^2-b^2)/d/(a+b*sin(d*x+c))^4+1/168*(4*a^4-9*a^2*b^2+12*b^4)*cos(d*x+c)/b^5/
(a^2-b^2)^2/d/(a+b*sin(d*x+c))^3+1/336*a*(8*a^4-30*a^2*b^2+57*b^4)*cos(d*x+
c)/b^5/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^2+1/336*(8*a^6-38*a^4*b^2+87*a^2*b^4+
48*b^6)*cos(d*x+c)/b^5/(a^2-b^2)^4/d/(a+b*sin(d*x+c))+5/42*cos(d*x+c)^3*(2*
a+3*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))^6-1/42*cos(d*x+c)*(4*a^2+9*b^2+10*
a*b*sin(d*x+c))/b^5/d/(a+b*sin(d*x+c))^5
```

Rubi [A] time = 0.79, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2693, 2863, 2754, 12, 2660, 618, 204}

$$\frac{5a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{9/2}}-\frac{\cos(c+dx)\left(4a^2+10ab\sin(c+dx)+9b^2\right)}{42b^5d(a+b\sin(c+dx))^5}+\frac{\left(-38a^4b^2+87a^2b^4+8a^6+48b^6\right)\cos(c}{336b^5d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (5*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(9/2)
*d) - Cos[c + d*x]^5/(7*b*d*(a + b*Sin[c + d*x])^7) + (a*(4*a^2 - b^2)*Cos[
c + d*x])/(168*b^5*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^4) + ((4*a^4 - 9*a^2*
b^2 + 12*b^4)*Cos[c + d*x])/(168*b^5*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^3
) + (a*(8*a^4 - 30*a^2*b^2 + 57*b^4)*Cos[c + d*x])/(336*b^5*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])^2) + ((8*a^6 - 38*a^4*b^2 + 87*a^2*b^4 + 48*b^6)*Cos[
c + d*x])/(336*b^5*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (5*Cos[c + d*x]^
3*(2*a + 3*b*Sin[c + d*x]))/(42*b^3*d*(a + b*Sin[c + d*x])^6) - (Cos[c + d*
x]*(4*a^2 + 9*b^2 + 10*a*b*Sin[c + d*x]))/(42*b^5*d*(a + b*Sin[c + d*x])^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{5\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^7}dx}{7b}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{5\cos^3(c+dx)(2a+3b\sin(c+dx))}{42b^3d(a+b\sin(c+dx))^6} - \frac{5\int \frac{\cos^2(c+dx)(-6b-4}{(a+b\sin(c+}}{28b^3}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{5\cos^3(c+dx)(2a+3b\sin(c+dx))}{42b^3d(a+b\sin(c+dx))^6} - \frac{\cos(c+dx)\left(4a^2+\right.}{42b^5d(a-}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{5\cos^3(c+dx)(2}{42b^3d(a+b}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+\right.}{168b^5\left(a^2-b^2\right)^2}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+\right.}{168b^5\left(a^2-b^2\right)^2}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+\right.}{168b^5\left(a^2-b^2\right)^2}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+\right.}{168b^5\left(a^2-b^2\right)^2}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+\right.}{168b^5\left(a^2-b^2\right)^2}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+\right.}{168b^5\left(a^2-b^2\right)^2}$$

$$= \frac{5a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{9/2}d} - \frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx}{168b^5\left(a^2-b^2\right)d(a+b\sin(c}$$

Mathematica [A] time = 6.00, size = 386, normalized size = 0.95

$$\cos(c+dx)\left(\frac{35a(\sin(c+dx)+1)}{8(b-a)^3(a+b)^3(a+b\sin(c+dx))^2} + \frac{7a(\sin(c+dx)+1)^2}{4(b-a)^3(a+b)^2(a+b\sin(c+dx))^3} + \frac{3a(\sin(c+dx)+1)^3}{4(b-a)^3(a+b)(a+b\sin(c+dx))^4} + \frac{3a(\sin(c+dx)+1)^4}{(a-b)^3(a+b\sin(c+dx))^5} - \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Cos[c + d*x]*((6*Cos[c + d*x]^6)/(a + b*Sin[c + d*x])^7 + (6*a*(-1 + Sin[c
 + d*x])^2*(1 + Sin[c + d*x])^4)/((a - b)*(a + b*Sin[c + d*x])^7) - (5*a*(-
1 + Sin[c + d*x])*(1 + Sin[c + d*x])^4)/((a - b)^2*(a + b*Sin[c + d*x])^6)
+ (3*a*(1 + Sin[c + d*x])^4)/((a - b)^3*(a + b*Sin[c + d*x])^5) + (3*a*(1 +
 Sin[c + d*x])^3)/(4*(-a + b)^3*(a + b)*(a + b*Sin[c + d*x])^4) + (7*a*(1 +
 Sin[c + d*x])^2)/(4*(-a + b)^3*(a + b)^2*(a + b*Sin[c + d*x])^3) + (35*a*(
```

```
1 + Sin[c + d*x]))/(8*(-a + b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2) + (105*a
*((2*ArcTanh[(Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[-a - b]*Sqrt[1 + Si
n[c + d*x]])])/((-a - b)^(9/2)*(a - b)^(7/2)*Sqrt[Cos[c + d*x]^2]) - 1/((a
- b)^3*(a + b)^4*(a + b*Sin[c + d*x]))))/8))/(42*(a - b)*d)
```

fricas [B] time = 1.44, size = 2250, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/672*(2*(8*a^8*b - 46*a^6*b^3 + 125*a^4*b^5 - 39*a^2*b^7 - 48*b^9)*cos(d*
x + c)^7 + 28*(7*a^8*b - 56*a^6*b^3 - 44*a^4*b^5 + 93*a^2*b^7)*cos(d*x + c)
^5 + 70*(7*a^8*b + 83*a^6*b^3 - 43*a^4*b^5 - 47*a^2*b^7)*cos(d*x + c)^3 - 1
05*(7*a^2*b^6*cos(d*x + c)^6 - a^8 - 21*a^6*b^2 - 35*a^4*b^4 - 7*a^2*b^6 -
7*(5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^4 + 7*(3*a^6*b^2 + 10*a^4*b^4 + 3*a^
2*b^6)*cos(d*x + c)^2 + (a*b^7*cos(d*x + c)^6 - 7*a^7*b - 35*a^5*b^3 - 21*a
^3*b^5 - a*b^7 - 3*(7*a^3*b^5 + a*b^7)*cos(d*x + c)^4 + (35*a^5*b^3 + 42*a^
3*b^5 + 3*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)
*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2)) - 420*(3*a^8*b + 7*a^6*b^3 - 7*a^4*b^5 - 3*a^
2*b^7)*cos(d*x + c) - 14*((8*a^9 - 46*a^7*b^2 + 125*a^5*b^4 - 54*a^3*b^6 -
33*a*b^8)*cos(d*x + c)^5 + 10*(a^9 - 11*a^7*b^2 - 25*a^5*b^4 + 31*a^3*b^6 +
 4*a*b^8)*cos(d*x + c)^3 + 15*(a^9 + 14*a^7*b^2 - 14*a^3*b^6 - a*b^8)*cos(d
*x + c))*sin(d*x + c))/(7*(a^11*b^6 - 5*a^9*b^8 + 10*a^7*b^10 - 10*a^5*b^12
 + 5*a^3*b^14 - a*b^16)*d*cos(d*x + c)^6 - 7*(5*a^13*b^4 - 22*a^11*b^6 + 35
*a^9*b^8 - 20*a^7*b^10 - 5*a^5*b^12 + 10*a^3*b^14 - 3*a*b^16)*d*cos(d*x + c
)^4 + 7*(3*a^15*b^2 - 5*a^13*b^4 - 17*a^11*b^6 + 55*a^9*b^8 - 55*a^7*b^10 +
 17*a^5*b^12 + 5*a^3*b^14 - 3*a*b^16)*d*cos(d*x + c)^2 - (a^17 + 16*a^15*b^
2 - 60*a^13*b^4 + 32*a^11*b^6 + 110*a^9*b^8 - 176*a^7*b^10 + 84*a^5*b^12 -
7*a*b^16)*d + ((a^10*b^7 - 5*a^8*b^9 + 10*a^6*b^11 - 10*a^4*b^13 + 5*a^2*b^
15 - b^17)*d*cos(d*x + c)^6 - 3*(7*a^12*b^5 - 34*a^10*b^7 + 65*a^8*b^9 - 60
*a^6*b^11 + 25*a^4*b^13 - 2*a^2*b^15 - b^17)*d*cos(d*x + c)^4 + (35*a^14*b^
3 - 133*a^12*b^5 + 143*a^10*b^7 + 55*a^8*b^9 - 215*a^6*b^11 + 145*a^4*b^13
- 27*a^2*b^15 - 3*b^17)*d*cos(d*x + c)^2 - (7*a^16*b - 84*a^12*b^5 + 176*a^
10*b^7 - 110*a^8*b^9 - 32*a^6*b^11 + 60*a^4*b^13 - 16*a^2*b^15 - b^17)*d)*s
in(d*x + c)), 1/336*((8*a^8*b - 46*a^6*b^3 + 125*a^4*b^5 - 39*a^2*b^7 - 48*
b^9)*cos(d*x + c)^7 + 14*(7*a^8*b - 56*a^6*b^3 - 44*a^4*b^5 + 93*a^2*b^7)*c
os(d*x + c)^5 + 35*(7*a^8*b + 83*a^6*b^3 - 43*a^4*b^5 - 47*a^2*b^7)*cos(d*x
 + c)^3 - 105*(7*a^2*b^6*cos(d*x + c)^6 - a^8 - 21*a^6*b^2 - 35*a^4*b^4 - 7
*a^2*b^6 - 7*(5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^4 + 7*(3*a^6*b^2 + 10*a^4
*b^4 + 3*a^2*b^6)*cos(d*x + c)^2 + (a*b^7*cos(d*x + c)^6 - 7*a^7*b - 35*a^5
*b^3 - 21*a^3*b^5 - a*b^7 - 3*(7*a^3*b^5 + a*b^7)*cos(d*x + c)^4 + (35*a^5*
b^3 + 42*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 210*(3*a^8*b
+ 7*a^6*b^3 - 7*a^4*b^5 - 3*a^2*b^7)*cos(d*x + c) - 7*((8*a^9 - 46*a^7*b^2
+ 125*a^5*b^4 - 54*a^3*b^6 - 33*a*b^8)*cos(d*x + c)^5 + 10*(a^9 - 11*a^7*b^
2 - 25*a^5*b^4 + 31*a^3*b^6 + 4*a*b^8)*cos(d*x + c)^3 + 15*(a^9 + 14*a^7*b^
2 - 14*a^3*b^6 - a*b^8)*cos(d*x + c))*sin(d*x + c))/(7*(a^11*b^6 - 5*a^9*b^
8 + 10*a^7*b^10 - 10*a^5*b^12 + 5*a^3*b^14 - a*b^16)*d*cos(d*x + c)^6 - 7*(
5*a^13*b^4 - 22*a^11*b^6 + 35*a^9*b^8 - 20*a^7*b^10 - 5*a^5*b^12 + 10*a^3*b
^14 - 3*a*b^16)*d*cos(d*x + c)^4 + 7*(3*a^15*b^2 - 5*a^13*b^4 - 17*a^11*b^6
 + 55*a^9*b^8 - 55*a^7*b^10 + 17*a^5*b^12 + 5*a^3*b^14 - 3*a*b^16)*d*cos(d*
x + c)^2 - (a^17 + 16*a^15*b^2 - 60*a^13*b^4 + 32*a^11*b^6 + 110*a^9*b^8 -
176*a^7*b^10 + 84*a^5*b^12 - 7*a*b^16)*d + ((a^10*b^7 - 5*a^8*b^9 + 10*a^6*
b^11 - 10*a^4*b^13 + 5*a^2*b^15 - b^17)*d*cos(d*x + c)^6 - 3*(7*a^12*b^5 -
34*a^10*b^7 + 65*a^8*b^9 - 60*a^6*b^11 + 25*a^4*b^13 - 2*a^2*b^15 - b^17)*d
*cos(d*x + c)^4 + (35*a^14*b^3 - 133*a^12*b^5 + 143*a^10*b^7 + 55*a^8*b^9 -
```

```
 215*a^6*b^11 + 145*a^4*b^13 - 27*a^2*b^15 - 3*b^17)*d*cos(d*x + c)^2 - (7*
a^16*b - 84*a^12*b^5 + 176*a^10*b^7 - 110*a^8*b^9 - 32*a^6*b^11 + 60*a^4*b^
13 - 16*a^2*b^15 - b^17)*d)*sin(d*x + c))]
```

giac [B] time = 9.95, size = 1650, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/168*(105*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
 + 1/2*c) + b)/sqrt(a^2 - b^2)))*a/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*sqrt(a^2 - b^2)) - (231*a^14*tan(1/2*d*x + 1/2*c)^13 - 1344*a^12*b
^2*tan(1/2*d*x + 1/2*c)^13 + 2016*a^10*b^4*tan(1/2*d*x + 1/2*c)^13 - 1344*a
^8*b^6*tan(1/2*d*x + 1/2*c)^13 + 336*a^6*b^8*tan(1/2*d*x + 1/2*c)^13 + 651*
a^13*b*tan(1/2*d*x + 1/2*c)^12 - 8064*a^11*b^3*tan(1/2*d*x + 1/2*c)^12 + 12
096*a^9*b^5*tan(1/2*d*x + 1/2*c)^12 - 8064*a^7*b^7*tan(1/2*d*x + 1/2*c)^12
+ 2016*a^5*b^9*tan(1/2*d*x + 1/2*c)^12 + 196*a^14*tan(1/2*d*x + 1/2*c)^11 -
 4354*a^12*b^2*tan(1/2*d*x + 1/2*c)^11 - 21504*a^10*b^4*tan(1/2*d*x + 1/2*c
)^11 + 36736*a^8*b^6*tan(1/2*d*x + 1/2*c)^11 - 25984*a^6*b^8*tan(1/2*d*x +
1/2*c)^11 + 6720*a^4*b^10*tan(1/2*d*x + 1/2*c)^11 + 140*a^13*b*tan(1/2*d*x
+ 1/2*c)^10 - 40250*a^11*b^3*tan(1/2*d*x + 1/2*c)^10 - 6720*a^9*b^5*tan(1/2
*d*x + 1/2*c)^10 + 49280*a^7*b^7*tan(1/2*d*x + 1/2*c)^10 - 45920*a^5*b^9*ta
n(1/2*d*x + 1/2*c)^10 + 13440*a^3*b^11*tan(1/2*d*x + 1/2*c)^10 + 595*a^14*t
an(1/2*d*x + 1/2*c)^9 - 20650*a^12*b^2*tan(1/2*d*x + 1/2*c)^9 - 103740*a^10
*b^4*tan(1/2*d*x + 1/2*c)^9 + 70336*a^8*b^6*tan(1/2*d*x + 1/2*c)^9 + 2576*a
^6*b^8*tan(1/2*d*x + 1/2*c)^9 - 40320*a^4*b^10*tan(1/2*d*x + 1/2*c)^9 + 161
28*a^2*b^12*tan(1/2*d*x + 1/2*c)^9 - 3045*a^13*b*tan(1/2*d*x + 1/2*c)^8 - 1
00450*a^11*b^3*tan(1/2*d*x + 1/2*c)^8 - 92120*a^9*b^5*tan(1/2*d*x + 1/2*c)^
8 + 129024*a^7*b^7*tan(1/2*d*x + 1/2*c)^8 - 74816*a^5*b^9*tan(1/2*d*x + 1/2
*c)^8 - 4480*a^3*b^11*tan(1/2*d*x + 1/2*c)^8 + 10752*a*b^13*tan(1/2*d*x + 1
/2*c)^8 - 39060*a^12*b^2*tan(1/2*d*x + 1/2*c)^7 - 188720*a^10*b^4*tan(1/2*d
*x + 1/2*c)^7 + 58352*a^8*b^6*tan(1/2*d*x + 1/2*c)^7 + 39936*a^6*b^8*tan(1/
2*d*x + 1/2*c)^7 - 73216*a^4*b^10*tan(1/2*d*x + 1/2*c)^7 + 19456*a^2*b^12*t
an(1/2*d*x + 1/2*c)^7 + 3072*b^14*tan(1/2*d*x + 1/2*c)^7 - 6720*a^13*b*tan(
1/2*d*x + 1/2*c)^6 - 122500*a^11*b^3*tan(1/2*d*x + 1/2*c)^6 - 109760*a^9*b^
5*tan(1/2*d*x + 1/2*c)^6 + 127344*a^7*b^7*tan(1/2*d*x + 1/2*c)^6 - 74816*a^
5*b^9*tan(1/2*d*x + 1/2*c)^6 - 4480*a^3*b^11*tan(1/2*d*x + 1/2*c)^6 + 10752
*a*b^13*tan(1/2*d*x + 1/2*c)^6 - 595*a^14*tan(1/2*d*x + 1/2*c)^5 - 37940*a^
12*b^2*tan(1/2*d*x + 1/2*c)^5 - 140280*a^10*b^4*tan(1/2*d*x + 1/2*c)^5 + 65
296*a^8*b^6*tan(1/2*d*x + 1/2*c)^5 + 2576*a^6*b^8*tan(1/2*d*x + 1/2*c)^5 -
40320*a^4*b^10*tan(1/2*d*x + 1/2*c)^5 + 16128*a^2*b^12*tan(1/2*d*x + 1/2*c)
^5 - 5999*a^13*b*tan(1/2*d*x + 1/2*c)^4 - 70084*a^11*b^3*tan(1/2*d*x + 1/2*
c)^4 - 16800*a^9*b^5*tan(1/2*d*x + 1/2*c)^4 + 50288*a^7*b^7*tan(1/2*d*x + 1
/2*c)^4 - 45920*a^5*b^9*tan(1/2*d*x + 1/2*c)^4 + 13440*a^3*b^11*tan(1/2*d*x
 + 1/2*c)^4 - 196*a^14*tan(1/2*d*x + 1/2*c)^3 - 19082*a^12*b^2*tan(1/2*d*x
+ 1/2*c)^3 - 29232*a^10*b^4*tan(1/2*d*x + 1/2*c)^3 + 37744*a^8*b^6*tan(1/2*
d*x + 1/2*c)^3 - 25984*a^6*b^8*tan(1/2*d*x + 1/2*c)^3 + 6720*a^4*b^10*tan(1
/2*d*x + 1/2*c)^3 - 2604*a^13*b*tan(1/2*d*x + 1/2*c)^2 - 13090*a^11*b^3*tan
(1/2*d*x + 1/2*c)^2 + 13888*a^9*b^5*tan(1/2*d*x + 1/2*c)^2 - 8400*a^7*b^7*t
an(1/2*d*x + 1/2*c)^2 + 2016*a^5*b^9*tan(1/2*d*x + 1/2*c)^2 - 231*a^14*tan(
1/2*d*x + 1/2*c) - 2562*a^12*b^2*tan(1/2*d*x + 1/2*c) + 2548*a^10*b^4*tan(1
/2*d*x + 1/2*c) - 1456*a^8*b^6*tan(1/2*d*x + 1/2*c) + 336*a^6*b^8*tan(1/2*d
*x + 1/2*c) - 279*a^13*b + 326*a^11*b^3 - 200*a^9*b^5 + 48*a^7*b^7)/((a^15
- 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*(a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d
```

maple [B] time = 0.41, size = 6933, normalized size = 17.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.47, size = 1868, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^8,x)
```

```
[Out] ((279*a^6*b - 48*b^7 + 200*a^2*b^5 - 326*a^4*b^3)/(168*(a^8 + b^8 - 4*a^2*b
^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)*(33*a^8 - 48*b^8 + 208*a
^2*b^6 - 364*a^4*b^4 + 366*a^6*b^2))/(24*a*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b
^4 - 4*a^6*b^2)) - (tan(c/2 + (d*x)/2)^9*(85*a^12 + 2304*b^12 - 5760*a^2*b^
10 + 368*a^4*b^8 + 10048*a^6*b^6 - 14820*a^8*b^4 - 2950*a^10*b^2))/(24*a^5*
(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^5*(8
5*a^12 - 2304*b^12 + 5760*a^2*b^10 - 368*a^4*b^8 - 9328*a^6*b^6 + 20040*a^8
*b^4 + 5420*a^10*b^2))/(24*a^5*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b
^2)) - (tan(c/2 + (d*x)/2)^11*(14*a^10 + 480*b^10 - 1856*a^2*b^8 + 2624*a^4
*b^6 - 1536*a^6*b^4 - 311*a^8*b^2))/(12*a^3*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*
b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^3*(14*a^10 - 480*b^10 + 1856*a^2*b^
8 - 2696*a^4*b^6 + 2088*a^6*b^4 + 1363*a^8*b^2))/(12*a^3*(a^8 + b^8 - 4*a^2
*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (tan(c/2 + (d*x)/2)^13*(11*a^8 + 16*b^8 -
64*a^2*b^6 + 96*a^4*b^4 - 64*a^6*b^2))/(8*a*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*
b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^6*(240*a^12*b - 384*b^13 + 160*a^2*
b^11 + 2672*a^4*b^9 - 4548*a^6*b^7 + 3920*a^8*b^5 + 4375*a^10*b^3))/(6*a^6*
(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^8*(4
35*a^12*b - 1536*b^13 + 640*a^2*b^11 + 10688*a^4*b^9 - 18432*a^6*b^7 + 1316
0*a^8*b^5 + 14350*a^10*b^3))/(24*a^6*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4
*a^6*b^2)) - (5*tan(c/2 + (d*x)/2)^10*(2*a^10*b + 192*b^11 - 656*a^2*b^9 +
704*a^4*b^7 - 96*a^6*b^5 - 575*a^8*b^3))/(12*a^4*(a^8 + b^8 - 4*a^2*b^6 + 6
*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^4*(857*a^10*b - 1920*b^11 + 65
60*a^2*b^9 - 7184*a^4*b^7 + 2400*a^6*b^5 + 10012*a^8*b^3))/(24*a^4*(a^8 + b
^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (tan(c/2 + (d*x)/2)^12*(31*a^8*b
 + 96*b^9 - 384*a^2*b^7 + 576*a^4*b^5 - 384*a^6*b^3))/(8*a^2*(a^8 + b^8 - 4
*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^2*(186*a^8*b - 144
*b^9 + 600*a^2*b^7 - 992*a^4*b^5 + 935*a^6*b^3))/(12*a^2*(a^8 + b^8 - 4*a^2
*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (b*tan(c/2 + (d*x)/2)^7*(35*a^6 + 16*b^6 +
 168*a^2*b^4 + 210*a^4*b^2)*(279*a^6*b - 48*b^7 + 200*a^2*b^5 - 326*a^4*b^3
))/(42*a^7*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)))/(d*(tan(c/2 +
(d*x)/2)^5*(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + tan(c/2 + (d*x)/2)^9*
(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + a^7*tan(c/2 + (d*x)/2)^14 + tan(
c/2 + (d*x)/2)^3*(84*a^6*b + 280*a^4*b^3) + tan(c/2 + (d*x)/2)^11*(84*a^6*b
 + 280*a^4*b^3) + tan(c/2 + (d*x)/2)^6*(448*a*b^6 + 35*a^7 + 1680*a^3*b^4 +
 840*a^5*b^2) + tan(c/2 + (d*x)/2)^8*(448*a*b^6 + 35*a^7 + 1680*a^3*b^4 + 8
```

```
40*a^5*b^2) + tan(c/2 + (d*x)/2)^7*(280*a^6*b + 128*b^7 + 1344*a^2*b^5 + 16
80*a^4*b^3) + a^7 + tan(c/2 + (d*x)/2)^4*(21*a^7 + 560*a^3*b^4 + 420*a^5*b^
2) + tan(c/2 + (d*x)/2)^10*(21*a^7 + 560*a^3*b^4 + 420*a^5*b^2) + tan(c/2 +
 (d*x)/2)^2*(7*a^7 + 84*a^5*b^2) + tan(c/2 + (d*x)/2)^12*(7*a^7 + 84*a^5*b^
2) + 14*a^6*b*tan(c/2 + (d*x)/2) + 14*a^6*b*tan(c/2 + (d*x)/2)^13)) + (5*a*
atan((8*((5*a^2*tan(c/2 + (d*x)/2))/(8*(a + b)^(9/2)*(a - b)^(9/2)) + (5*a*
(16*a^8*b + 16*b^9 - 64*a^2*b^7 + 96*a^4*b^5 - 64*a^6*b^3))/(128*(a + b)^(9
/2)*(a - b)^(9/2)*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)))*(a^8 +
b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))/(5*a)))/(8*d*(a + b)^(9/2)*(a - b
)^(9/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.469 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^8} dx$

Optimal. Leaf size=411

$$\frac{3a\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{11/2}}-\frac{a\left(2a^2-11b^2\right)\cos(c+dx)}{280b^3d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^4}-\frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3d\left(a^2-b^2\right)(a+b\sin(c+d$$

```
[Out] 3/8*a*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2
)^(11/2)/d-1/7*cos(d*x+c)^3/b/d/(a+b*sin(d*x+c))^7-1/140*(a^2-3*b^2)*cos(d*
x+c)/b^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^5-1/280*a*(2*a^2-11*b^2)*cos(d*x+c)/b
^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^4-1/280*(2*a^4-15*a^2*b^2-8*b^4)*cos(d*x+
c)/b^3/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^3-1/560*a*(4*a^4-36*a^2*b^2-73*b^4)*c
os(d*x+c)/b^3/(a^2-b^2)^4/d/(a+b*sin(d*x+c))^2-1/560*(4*a^6-40*a^4*b^2-247*
a^2*b^4-32*b^6)*cos(d*x+c)/b^3/(a^2-b^2)^5/d/(a+b*sin(d*x+c))+1/28*cos(d*x+
c)*(a+3*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))^6
```

Rubi [A] time = 0.79, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2693, 2863, 2754, 12, 2660, 618, 204}

$$\frac{3a\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{11/2}}-\frac{\left(-40a^4b^2-247a^2b^4+4a^6-32b^6\right)\cos(c+dx)}{560b^3d\left(a^2-b^2\right)^5(a+b\sin(c+dx))}-\frac{a\left(-36a^2b^2+4a^4-73b\right.}{560b^3d\left(a^2-b^2\right)^4(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (3*a*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^
2 - b^2)^(11/2)*d) - Cos[c + d*x]^3/(7*b*d*(a + b*Sin[c + d*x])^7) - ((a^2
- 3*b^2)*Cos[c + d*x])/(140*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^5) - (a*
(2*a^2 - 11*b^2)*Cos[c + d*x])/(280*b^3*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]
)^4) - ((2*a^4 - 15*a^2*b^2 - 8*b^4)*Cos[c + d*x])/(280*b^3*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])^3) - (a*(4*a^4 - 36*a^2*b^2 - 73*b^4)*Cos[c + d*x])/(
560*b^3*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^2) - ((4*a^6 - 40*a^4*b^2 - 24
7*a^2*b^4 - 32*b^6)*Cos[c + d*x])/(560*b^3*(a^2 - b^2)^5*d*(a + b*Sin[c + d
*x])) + (Cos[c + d*x]*(a + 3*b*Sin[c + d*x]))/(28*b^3*d*(a + b*Sin[c + d*x]
)^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{3\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{7b}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{\cos(c+dx)(a+3b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))^6} - \frac{\int \frac{-6b-2a\sin(c+dx)}{(a+b\sin(c+dx))^6}\,dx}{56b^3}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} + \frac{\cos(c+dx)(a}{28b^3d(a+}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-\right.}{280b^3\left(a^2-b^2\right.}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-\right.}{280b^3\left(a^2-b^2\right.}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-\right.}{280b^3\left(a^2-b^2\right.}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-\right.}{280b^3\left(a^2-b^2\right.}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-\right.}{280b^3\left(a^2-b^2\right.}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-\right.}{280b^3\left(a^2-b^2\right.}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-\right.}{280b^3\left(a^2-b^2\right.}$$

$$= \frac{3a\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{11/2}d} - \frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right.}{140b^3\left(a^2-b^2\right)}$$

<u>**Mathematica**</u> [**B**] time = 6.08, size = 1167, normalized size = 2.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Cos[c + d*x]^5/(5*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-1/7
*(b*(1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])^(7/2))/((-a + b)*(a + b)*(a
 + b*Sin[c + d*x])^7) - (-1/6*((a*b + (7*a - b)*b)*(1 - Sin[c + d*x])^(5/2)
*(1 + Sin[c + d*x])^(7/2))/((-a + b)*(a + b)*(a + b*Sin[c + d*x])^6) - (7*(
6*a^2 - 2*a*b + b^2)*(-1/5*((1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(7/
2))/((-a + b)*(a + b*Sin[c + d*x])^5) - (3*(-1/4*(Sqrt[1 - Sin[c + d*x]]*(1
 + Sin[c + d*x])^(7/2))/((-a + b)*(a + b*Sin[c + d*x])^4) - (-1/3*(Sqrt[1 -
 Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/((a + b)*(a + b*Sin[c + d*x])^3) +
 (5*(-1/2*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(3/2))/((a + b)*(a + b
*Sin[c + d*x])^2) + (3*((-2*ArcTanh[(Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]])/(S
qrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a - b)^(3/2)*Sqrt[a - b]) + (Sqrt[
1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[c + d*x]))))
/(2*(a + b))))/(3*(a + b)))/(4*(-a + b))))/(5*(-a + b))))/(6*(-a + b)*(a +
b)))/(7*(-a + b)*(a + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[
c + d*x]]) + (2*b*(Cos[c + d*x]^7/(7*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a
*Cos[c + d*x]*(-1/7*((1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])^(9/2))/((-
a + b)*(a + b*Sin[c + d*x])^7) - (5*(-1/6*((1 - Sin[c + d*x])^(3/2)*(1 + Si
n[c + d*x])^(9/2))/((-a + b)*(a + b*Sin[c + d*x])^6) - (-1/5*(Sqrt[1 - Sin[
c + d*x]]*(1 + Sin[c + d*x])^(9/2))/((-a + b)*(a + b*Sin[c + d*x])^5) - (-1
/4*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(7/2))/((a + b)*(a + b*Sin[c
+ d*x])^4) + (7*(-1/3*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/((a
 + b)*(a + b*Sin[c + d*x])^3) + (5*(-1/2*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c
 + d*x])^(3/2))/((a + b)*(a + b*Sin[c + d*x])^2) + (3*((-2*ArcTanh[(Sqrt[a
- b]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a -
 b)^(3/2)*Sqrt[a - b]) + (Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((
-a - b)*(a + b*Sin[c + d*x]))))/(2*(a + b))))/(3*(a + b))))/(4*(a + b)))/(5
*(-a + b)))/(2*(-a + b))))/(7*(-a + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]
*Sqrt[1 + Sin[c + d*x]])))/(5*(a - b))
```

fricas [B] time = 1.38, size = 2657, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [-1/1120*(2*(4*a^8*b^3 - 44*a^6*b^5 - 207*a^4*b^7 + 215*a^2*b^9 + 32*b^11)*
cos(d*x + c)^7 - 28*(6*a^10*b - 65*a^8*b^3 - 224*a^6*b^5 + 222*a^4*b^7 + 53
*a^2*b^9 + 8*b^11)*cos(d*x + c)^5 - 70*(14*a^10*b + 173*a^8*b^3 - 3*a^6*b^5
 - 137*a^4*b^7 - 47*a^2*b^9)*cos(d*x + c)^3 + 105*(2*a^10 + 43*a^8*b^2 + 91
*a^6*b^4 + 49*a^4*b^6 + 7*a^2*b^8 - 7*(2*a^4*b^6 + a^2*b^8)*cos(d*x + c)^6
+ 7*(10*a^6*b^4 + 11*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^4 - 7*(6*a^8*b^2 + 2
3*a^6*b^4 + 16*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^2 + (14*a^9*b + 77*a^7*b^3
 + 77*a^5*b^5 + 23*a^3*b^7 + a*b^9 - (2*a^3*b^7 + a*b^9)*cos(d*x + c)^6 + 3
*(14*a^5*b^5 + 9*a^3*b^7 + a*b^9)*cos(d*x + c)^4 - (70*a^7*b^3 + 119*a^5*b^
5 + 48*a^3*b^7 + 3*a*b^9)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*lo
g(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 420*(6*a^10*b + 17*a^8*b^3 - 7*a^
6*b^5 - 13*a^4*b^7 - 3*a^2*b^9)*cos(d*x + c) - 14*((4*a^9*b^2 - 44*a^7*b^4
- 177*a^5*b^6 + 200*a^3*b^8 + 17*a*b^10)*cos(d*x + c)^5 - 10*(2*a^11 - 21*a
^9*b^2 - 61*a^7*b^4 + 37*a^5*b^6 + 39*a^3*b^8 + 4*a*b^10)*cos(d*x + c)^3 -
15*(2*a^11 + 29*a^9*b^2 + 14*a^7*b^4 - 28*a^5*b^6 - 16*a^3*b^8 - a*b^10)*co
s(d*x + c))*sin(d*x + c))/(7*(a^13*b^6 - 6*a^11*b^8 + 15*a^9*b^10 - 20*a^7*
b^12 + 15*a^5*b^14 - 6*a^3*b^16 + a*b^18)*d*cos(d*x + c)^6 - 7*(5*a^15*b^4
- 27*a^13*b^6 + 57*a^11*b^8 - 55*a^9*b^10 + 15*a^7*b^12 + 15*a^5*b^14 - 13*
```

```
a^3*b^16 + 3*a*b^18)*d*cos(d*x + c)^4 + 7*(3*a^17*b^2 - 8*a^15*b^4 - 12*a^1
3*b^6 + 72*a^11*b^8 - 110*a^9*b^10 + 72*a^7*b^12 - 12*a^5*b^14 - 8*a^3*b^16
 + 3*a*b^18)*d*cos(d*x + c)^2 - (a^19 + 15*a^17*b^2 - 76*a^15*b^4 + 92*a^13
*b^6 + 78*a^11*b^8 - 286*a^9*b^10 + 260*a^7*b^12 - 84*a^5*b^14 - 7*a^3*b^16
 + 7*a*b^18)*d + ((a^12*b^7 - 6*a^10*b^9 + 15*a^8*b^11 - 20*a^6*b^13 + 15*a
^4*b^15 - 6*a^2*b^17 + b^19)*d*cos(d*x + c)^6 - 3*(7*a^14*b^5 - 41*a^12*b^7
 + 99*a^10*b^9 - 125*a^8*b^11 + 85*a^6*b^13 - 27*a^4*b^15 + a^2*b^17 + b^19
)*d*cos(d*x + c)^4 + (35*a^16*b^3 - 168*a^14*b^5 + 276*a^12*b^7 - 88*a^10*b
^9 - 270*a^8*b^11 + 360*a^6*b^13 - 172*a^4*b^15 + 24*a^2*b^17 + 3*b^19)*d*c
os(d*x + c)^2 - (7*a^18*b - 7*a^16*b^3 - 84*a^14*b^5 + 260*a^12*b^7 - 286*a
^10*b^9 + 78*a^8*b^11 + 92*a^6*b^13 - 76*a^4*b^15 + 15*a^2*b^17 + b^19)*d)*
sin(d*x + c)), -1/560*((4*a^8*b^3 - 44*a^6*b^5 - 207*a^4*b^7 + 215*a^2*b^9
+ 32*b^11)*cos(d*x + c)^7 - 14*(6*a^10*b - 65*a^8*b^3 - 224*a^6*b^5 + 222*a
^4*b^7 + 53*a^2*b^9 + 8*b^11)*cos(d*x + c)^5 - 35*(14*a^10*b + 173*a^8*b^3
- 3*a^6*b^5 - 137*a^4*b^7 - 47*a^2*b^9)*cos(d*x + c)^3 - 105*(2*a^10 + 43*a
^8*b^2 + 91*a^6*b^4 + 49*a^4*b^6 + 7*a^2*b^8 - 7*(2*a^4*b^6 + a^2*b^8)*cos(
d*x + c)^6 + 7*(10*a^6*b^4 + 11*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^4 - 7*(6*
a^8*b^2 + 23*a^6*b^4 + 16*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^2 + (14*a^9*b +
 77*a^7*b^3 + 77*a^5*b^5 + 23*a^3*b^7 + a*b^9 - (2*a^3*b^7 + a*b^9)*cos(d*x
 + c)^6 + 3*(14*a^5*b^5 + 9*a^3*b^7 + a*b^9)*cos(d*x + c)^4 - (70*a^7*b^3 +
 119*a^5*b^5 + 48*a^3*b^7 + 3*a*b^9)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 210*
(6*a^10*b + 17*a^8*b^3 - 7*a^6*b^5 - 13*a^4*b^7 - 3*a^2*b^9)*cos(d*x + c) -
 7*((4*a^9*b^2 - 44*a^7*b^4 - 177*a^5*b^6 + 200*a^3*b^8 + 17*a*b^10)*cos(d*
x + c)^5 - 10*(2*a^11 - 21*a^9*b^2 - 61*a^7*b^4 + 37*a^5*b^6 + 39*a^3*b^8 +
 4*a*b^10)*cos(d*x + c)^3 - 15*(2*a^11 + 29*a^9*b^2 + 14*a^7*b^4 - 28*a^5*b
^6 - 16*a^3*b^8 - a*b^10)*cos(d*x + c))*sin(d*x + c))/(7*(a^13*b^6 - 6*a^11
*b^8 + 15*a^9*b^10 - 20*a^7*b^12 + 15*a^5*b^14 - 6*a^3*b^16 + a*b^18)*d*cos
(d*x + c)^6 - 7*(5*a^15*b^4 - 27*a^13*b^6 + 57*a^11*b^8 - 55*a^9*b^10 + 15*
a^7*b^12 + 15*a^5*b^14 - 13*a^3*b^16 + 3*a*b^18)*d*cos(d*x + c)^4 + 7*(3*a^
17*b^2 - 8*a^15*b^4 - 12*a^13*b^6 + 72*a^11*b^8 - 110*a^9*b^10 + 72*a^7*b^1
2 - 12*a^5*b^14 - 8*a^3*b^16 + 3*a*b^18)*d*cos(d*x + c)^2 - (a^19 + 15*a^17
*b^2 - 76*a^15*b^4 + 92*a^13*b^6 + 78*a^11*b^8 - 286*a^9*b^10 + 260*a^7*b^1
2 - 84*a^5*b^14 - 7*a^3*b^16 + 7*a*b^18)*d + ((a^12*b^7 - 6*a^10*b^9 + 15*a
^8*b^11 - 20*a^6*b^13 + 15*a^4*b^15 - 6*a^2*b^17 + b^19)*d*cos(d*x + c)^6 -
 3*(7*a^14*b^5 - 41*a^12*b^7 + 99*a^10*b^9 - 125*a^8*b^11 + 85*a^6*b^13 - 2
7*a^4*b^15 + a^2*b^17 + b^19)*d*cos(d*x + c)^4 + (35*a^16*b^3 - 168*a^14*b^
5 + 276*a^12*b^7 - 88*a^10*b^9 - 270*a^8*b^11 + 360*a^6*b^13 - 172*a^4*b^15
 + 24*a^2*b^17 + 3*b^19)*d*cos(d*x + c)^2 - (7*a^18*b - 7*a^16*b^3 - 84*a^1
4*b^5 + 260*a^12*b^7 - 286*a^10*b^9 + 78*a^8*b^11 + 92*a^6*b^13 - 76*a^4*b^
15 + 15*a^2*b^17 + b^19)*d)*sin(d*x + c))]
```

giac [B] time = 5.09, size = 1932, normalized size = 4.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/280*(105*(2*a^3 + a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arcta
n((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^10 - 5*a^8*b^2 + 10*a^
6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*sqrt(a^2 - b^2)) - (350*a^16*tan(1/2
*d*x + 1/2*c)^13 - 2905*a^14*b^2*tan(1/2*d*x + 1/2*c)^13 + 5600*a^12*b^4*ta
n(1/2*d*x + 1/2*c)^13 - 5600*a^10*b^6*tan(1/2*d*x + 1/2*c)^13 + 2800*a^8*b^
8*tan(1/2*d*x + 1/2*c)^13 - 560*a^6*b^10*tan(1/2*d*x + 1/2*c)^13 + 630*a^15
*b*tan(1/2*d*x + 1/2*c)^12 - 18165*a^13*b^3*tan(1/2*d*x + 1/2*c)^12 + 33600
*a^11*b^5*tan(1/2*d*x + 1/2*c)^12 - 33600*a^9*b^7*tan(1/2*d*x + 1/2*c)^12 +
 16800*a^7*b^9*tan(1/2*d*x + 1/2*c)^12 - 3360*a^5*b^11*tan(1/2*d*x + 1/2*c)
^12 + 840*a^16*tan(1/2*d*x + 1/2*c)^11 - 15680*a^14*b^2*tan(1/2*d*x + 1/2*c
)^11 - 41090*a^12*b^4*tan(1/2*d*x + 1/2*c)^11 + 89600*a^10*b^6*tan(1/2*d*x
```

```
+ 1/2*c)^11 - 100800*a^8*b^8*tan(1/2*d*x + 1/2*c)^11 + 53760*a^6*b^10*tan(1
/2*d*x + 1/2*c)^11 - 11200*a^4*b^12*tan(1/2*d*x + 1/2*c)^11 - 840*a^15*b*ta
n(1/2*d*x + 1/2*c)^10 - 102760*a^13*b^3*tan(1/2*d*x + 1/2*c)^10 + 11270*a^1
1*b^5*tan(1/2*d*x + 1/2*c)^10 + 78400*a^9*b^7*tan(1/2*d*x + 1/2*c)^10 - 151
200*a^7*b^9*tan(1/2*d*x + 1/2*c)^10 + 97440*a^5*b^11*tan(1/2*d*x + 1/2*c)^1
0 - 22400*a^3*b^13*tan(1/2*d*x + 1/2*c)^10 + 630*a^16*tan(1/2*d*x + 1/2*c)^
9 - 51905*a^14*b^2*tan(1/2*d*x + 1/2*c)^9 - 249410*a^12*b^4*tan(1/2*d*x + 1
/2*c)^9 + 202244*a^10*b^6*tan(1/2*d*x + 1/2*c)^9 - 129360*a^8*b^8*tan(1/2*d
*x + 1/2*c)^9 - 62832*a^6*b^10*tan(1/2*d*x + 1/2*c)^9 + 92288*a^4*b^12*tan(
1/2*d*x + 1/2*c)^9 - 26880*a^2*b^14*tan(1/2*d*x + 1/2*c)^9 - 8330*a^15*b*ta
n(1/2*d*x + 1/2*c)^8 - 248745*a^13*b^3*tan(1/2*d*x + 1/2*c)^8 - 190610*a^11
*b^5*tan(1/2*d*x + 1/2*c)^8 + 253736*a^9*b^7*tan(1/2*d*x + 1/2*c)^8 - 33824
0*a^7*b^9*tan(1/2*d*x + 1/2*c)^8 + 120512*a^5*b^11*tan(1/2*d*x + 1/2*c)^8 +
 24192*a^3*b^13*tan(1/2*d*x + 1/2*c)^8 - 17920*a*b^15*tan(1/2*d*x + 1/2*c)^
8 - 96040*a^14*b^2*tan(1/2*d*x + 1/2*c)^7 - 452340*a^12*b^4*tan(1/2*d*x + 1
/2*c)^7 + 164528*a^10*b^6*tan(1/2*d*x + 1/2*c)^7 - 99344*a^8*b^8*tan(1/2*d*
x + 1/2*c)^7 - 177664*a^6*b^10*tan(1/2*d*x + 1/2*c)^7 + 153088*a^4*b^12*tan
(1/2*d*x + 1/2*c)^7 - 27648*a^2*b^14*tan(1/2*d*x + 1/2*c)^7 - 5120*b^16*tan
(1/2*d*x + 1/2*c)^7 - 15680*a^15*b*tan(1/2*d*x + 1/2*c)^6 - 296520*a^13*b^3
*tan(1/2*d*x + 1/2*c)^6 - 247940*a^11*b^5*tan(1/2*d*x + 1/2*c)^6 + 232736*a
^9*b^7*tan(1/2*d*x + 1/2*c)^6 - 339920*a^7*b^9*tan(1/2*d*x + 1/2*c)^6 + 120
512*a^5*b^11*tan(1/2*d*x + 1/2*c)^6 + 24192*a^3*b^13*tan(1/2*d*x + 1/2*c)^6
 - 17920*a*b^15*tan(1/2*d*x + 1/2*c)^6 - 630*a^16*tan(1/2*d*x + 1/2*c)^5 -
92155*a^14*b^2*tan(1/2*d*x + 1/2*c)^5 - 333060*a^12*b^4*tan(1/2*d*x + 1/2*c
)^5 + 151144*a^10*b^6*tan(1/2*d*x + 1/2*c)^5 - 133280*a^8*b^8*tan(1/2*d*x +
 1/2*c)^5 - 62832*a^6*b^10*tan(1/2*d*x + 1/2*c)^5 + 92288*a^4*b^12*tan(1/2*
d*x + 1/2*c)^5 - 26880*a^2*b^14*tan(1/2*d*x + 1/2*c)^5 - 13566*a^15*b*tan(1
/2*d*x + 1/2*c)^4 - 166775*a^13*b^3*tan(1/2*d*x + 1/2*c)^4 - 41412*a^11*b^5
*tan(1/2*d*x + 1/2*c)^4 + 72128*a^9*b^7*tan(1/2*d*x + 1/2*c)^4 - 150640*a^7
*b^9*tan(1/2*d*x + 1/2*c)^4 + 97440*a^5*b^11*tan(1/2*d*x + 1/2*c)^4 - 22400
*a^3*b^13*tan(1/2*d*x + 1/2*c)^4 - 840*a^16*tan(1/2*d*x + 1/2*c)^3 - 41944*
a^14*b^2*tan(1/2*d*x + 1/2*c)^3 - 76650*a^12*b^4*tan(1/2*d*x + 1/2*c)^3 + 8
7472*a^10*b^6*tan(1/2*d*x + 1/2*c)^3 - 100688*a^8*b^8*tan(1/2*d*x + 1/2*c)^
3 + 53760*a^6*b^10*tan(1/2*d*x + 1/2*c)^3 - 11200*a^4*b^12*tan(1/2*d*x + 1/
2*c)^3 - 5432*a^15*b*tan(1/2*d*x + 1/2*c)^2 - 33264*a^13*b^3*tan(1/2*d*x +
1/2*c)^2 + 34846*a^11*b^5*tan(1/2*d*x + 1/2*c)^2 - 34272*a^9*b^7*tan(1/2*d*
x + 1/2*c)^2 + 16912*a^7*b^9*tan(1/2*d*x + 1/2*c)^2 - 3360*a^5*b^11*tan(1/2
*d*x + 1/2*c)^2 - 350*a^16*tan(1/2*d*x + 1/2*c) - 6699*a^14*b^2*tan(1/2*d*x
 + 1/2*c) + 6790*a^12*b^4*tan(1/2*d*x + 1/2*c) - 6188*a^10*b^6*tan(1/2*d*x
+ 1/2*c) + 2912*a^8*b^8*tan(1/2*d*x + 1/2*c) - 560*a^6*b^10*tan(1/2*d*x + 1
/2*c) - 686*a^15*b + 885*a^13*b^3 - 842*a^11*b^5 + 408*a^9*b^7 - 80*a^7*b^9
)/((a^17 - 5*a^15*b^2 + 10*a^13*b^4 - 10*a^11*b^6 + 5*a^9*b^8 - a^7*b^10)*(
a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d
```

maple [B] time = 0.39, size = 9171, normalized size = 22.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.85, size = 2184, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^8,x)
```

```
[Out] ((686*a^8*b + 80*b^9 - 408*a^2*b^7 + 842*a^4*b^5 - 885*a^6*b^3)/(280*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (tan(c/2 + (d*
x)/2)*(50*a^10 + 80*b^10 - 416*a^2*b^8 + 884*a^4*b^6 - 970*a^6*b^4 + 957*a^
8*b^2))/(40*a*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^
2)) + (tan(c/2 + (d*x)/2)^9*(3840*b^14 - 90*a^14 - 13184*a^2*b^12 + 8976*a^
4*b^10 + 18480*a^6*b^8 - 28892*a^8*b^6 + 35630*a^10*b^4 + 7415*a^12*b^2))/(
40*a^5*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (
tan(c/2 + (d*x)/2)^5*(90*a^14 + 3840*b^14 - 13184*a^2*b^12 + 8976*a^4*b^10
+ 19040*a^6*b^8 - 21592*a^8*b^6 + 47580*a^10*b^4 + 13165*a^12*b^2))/(40*a^5
*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (tan(c/
2 + (d*x)/2)^11*(160*b^12 - 12*a^12 - 768*a^2*b^10 + 1440*a^4*b^8 - 1280*a^
6*b^6 + 587*a^8*b^4 + 224*a^10*b^2))/(4*a^3*(a^10 - b^10 + 5*a^2*b^8 - 10*a
^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (tan(c/2 + (d*x)/2)^3*(60*a^12 + 800*b^
12 - 3840*a^2*b^10 + 7192*a^4*b^8 - 6248*a^6*b^6 + 5475*a^8*b^4 + 2996*a^10
*b^2))/(20*a^3*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b
^2)) - (tan(c/2 + (d*x)/2)^13*(10*a^10 - 16*b^10 + 80*a^2*b^8 - 160*a^4*b^6
 + 160*a^6*b^4 - 83*a^8*b^2))/(8*a*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 +
10*a^6*b^4 - 5*a^8*b^2)) + (tan(c/2 + (d*x)/2)^6*(560*a^14*b + 640*b^15 - 8
64*a^2*b^13 - 4304*a^4*b^11 + 12140*a^6*b^9 - 8312*a^8*b^7 + 8855*a^10*b^5
+ 10590*a^12*b^3))/(10*a^6*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b
^4 - 5*a^8*b^2)) + (tan(c/2 + (d*x)/2)^8*(1190*a^14*b + 2560*b^15 - 3456*a^
2*b^13 - 17216*a^4*b^11 + 48320*a^6*b^9 - 36248*a^8*b^7 + 27230*a^10*b^5 +
35535*a^12*b^3))/(40*a^6*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4
 - 5*a^8*b^2)) + (tan(c/2 + (d*x)/2)^10*(12*a^12*b + 320*b^13 - 1392*a^2*b^
11 + 2160*a^4*b^9 - 1120*a^6*b^7 - 161*a^8*b^5 + 1468*a^10*b^3))/(4*a^4*(a^
10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (tan(c/2 +
(d*x)/2)^4*(1938*a^12*b + 3200*b^13 - 13920*a^2*b^11 + 21520*a^4*b^9 - 1030
4*a^6*b^7 + 5916*a^8*b^5 + 23825*a^10*b^3))/(40*a^4*(a^10 - b^10 + 5*a^2*b^
8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) - (3*tan(c/2 + (d*x)/2)^12*(6*a^1
0*b - 32*b^11 + 160*a^2*b^9 - 320*a^4*b^7 + 320*a^6*b^5 - 173*a^8*b^3))/(8*
a^2*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (tan
(c/2 + (d*x)/2)^2*(388*a^10*b + 240*b^11 - 1208*a^2*b^9 + 2448*a^4*b^7 - 24
89*a^6*b^5 + 2376*a^8*b^3))/(20*a^2*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 +
 10*a^6*b^4 - 5*a^8*b^2)) + (b*tan(c/2 + (d*x)/2)^7*(35*a^6 + 16*b^6 + 168*
a^2*b^4 + 210*a^4*b^2)*(686*a^8*b + 80*b^9 - 408*a^2*b^7 + 842*a^4*b^5 - 88
5*a^6*b^3))/(70*a^7*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*
a^8*b^2)))/(d*(tan(c/2 + (d*x)/2)^5*(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3
) + tan(c/2 + (d*x)/2)^9*(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + a^7*tan
(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^3*(84*a^6*b + 280*a^4*b^3) + tan(c/
2 + (d*x)/2)^11*(84*a^6*b + 280*a^4*b^3) + tan(c/2 + (d*x)/2)^6*(448*a*b^6
+ 35*a^7 + 1680*a^3*b^4 + 840*a^5*b^2) + tan(c/2 + (d*x)/2)^8*(448*a*b^6 +
35*a^7 + 1680*a^3*b^4 + 840*a^5*b^2) + tan(c/2 + (d*x)/2)^7*(280*a^6*b + 12
8*b^7 + 1344*a^2*b^5 + 1680*a^4*b^3) + a^7 + tan(c/2 + (d*x)/2)^4*(21*a^7 +
 560*a^3*b^4 + 420*a^5*b^2) + tan(c/2 + (d*x)/2)^10*(21*a^7 + 560*a^3*b^4 +
 420*a^5*b^2) + tan(c/2 + (d*x)/2)^2*(7*a^7 + 84*a^5*b^2) + tan(c/2 + (d*x)
/2)^12*(7*a^7 + 84*a^5*b^2) + 14*a^6*b*tan(c/2 + (d*x)/2) + 14*a^6*b*tan(c/
2 + (d*x)/2)^13)) + (3*a*atan((8*((3*a^2*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/
(8*(a + b)^(11/2)*(a - b)^(11/2)) + (3*a*(2*a^2 + b^2)*(16*a^10*b - 16*b^11
```

```
 + 80*a^2*b^9 - 160*a^4*b^7 + 160*a^6*b^5 - 80*a^8*b^3))/(128*(a + b)^(11/2
)*(a - b)^(11/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8
*b^2)))*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2))/(3
*a*b^2 + 6*a^3))*(2*a^2 + b^2))/(8*d*(a + b)^(11/2)*(a - b)^(11/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.470 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=422

$$\frac{a\left(20a^2+79b^2\right)\cos(c+dx)}{840bd\left(a^2-b^2\right)^3(a+b\sin(c+dx))^4}+\frac{\left(5a^2+6b^2\right)\cos(c+dx)}{210bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))^5}+\frac{a\cos(c+dx)}{42bd\left(a^2-b^2\right)(a+b\sin(c+dx))^6}+$$

```
[Out] 1/8*a*(8*a^4+20*a^2*b^2+5*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))/(a^2-b^2)^(13/2)/d-1/7*cos(d*x+c)/b/d/(a+b*sin(d*x+c))^7+1/42*a*cos(d*
x+c)/b/(a^2-b^2)/d/(a+b*sin(d*x+c))^6+1/210*(5*a^2+6*b^2)*cos(d*x+c)/b/(a^2
-b^2)^2/d/(a+b*sin(d*x+c))^5+1/840*a*(20*a^2+79*b^2)*cos(d*x+c)/b/(a^2-b^2)
^3/d/(a+b*sin(d*x+c))^4+1/840*(20*a^4+179*a^2*b^2+32*b^4)*cos(d*x+c)/b/(a^2
-b^2)^4/d/(a+b*sin(d*x+c))^3+1/1680*a*(40*a^4+718*a^2*b^2+397*b^4)*cos(d*x+
c)/b/(a^2-b^2)^5/d/(a+b*sin(d*x+c))^2+1/1680*(40*a^6+1518*a^4*b^2+1779*a^2*
b^4+128*b^6)*cos(d*x+c)/b/(a^2-b^2)^6/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.75, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2693, 2754, 12, 2660, 618, 204}

$$\frac{a\left(20a^2b^2+8a^4+5b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{13/2}}+\frac{\left(1518a^4b^2+1779a^2b^4+40a^6+128b^6\right)\cos(c+dx)}{1680bd\left(a^2-b^2\right)^6(a+b\sin(c+dx))}+\frac{a\left(718a^2b^2\right.}{1680bd\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a*(8*a^4 + 20*a^2*b^2 + 5*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(8*(a^2 - b^2)^(13/2)*d) - Cos[c + d*x]/(7*b*d*(a + b*Sin[c + d*x])^
7) + (a*Cos[c + d*x])/(42*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^6) + ((5*a^2
 + 6*b^2)*Cos[c + d*x])/(210*b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^5) + (a
*(20*a^2 + 79*b^2)*Cos[c + d*x])/(840*b*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]
)^4) + ((20*a^4 + 179*a^2*b^2 + 32*b^4)*Cos[c + d*x])/(840*b*(a^2 - b^2)^4*
d*(a + b*Sin[c + d*x])^3) + (a*(40*a^4 + 718*a^2*b^2 + 397*b^4)*Cos[c + d*x
])/(1680*b*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^2) + ((40*a^6 + 1518*a^4*b^
2 + 1779*a^2*b^4 + 128*b^6)*Cos[c + d*x])/(1680*b*(a^2 - b^2)^6*d*(a + b*Si
n[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\int \frac{\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{7b}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\int \frac{6b-5a\sin(c+dx)}{(a+b\sin(c+dx))^6}\,dx}{42b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cdots}{210b\left(a^2-b^2\right)^2 d(a\cdots}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cdots}{210b\left(a^2-b^2\right)^2 d(a\cdots}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cdots}{210b\left(a^2-b^2\right)^2 d(a\cdots}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cdots}{210b\left(a^2-b^2\right)^2 d(a\cdots}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cdots}{210b\left(a^2-b^2\right)^2 d(a\cdots}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cdots}{210b\left(a^2-b^2\right)^2 d(a\cdots}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cdots}{210b\left(a^2-b^2\right)^2 d(a\cdots}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cdots}{210b\left(a^2-b^2\right)^2 d(a\cdots}$$

$$= \frac{a\left(8a^4+20a^2b^2+5b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{13/2}d} - \frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{\cdots}{42b\left(a^2-\cdots\right.}$$

Mathematica [B] time = 6.20, size = 1896, normalized size = 4.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Cos[c + d*x]^3/(3*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-1/7
*(b*(1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(5/2))/((-a + b)*(a + b)*(a
 + b*Sin[c + d*x])^7) - (-1/6*((3*a*b + (7*a - b)*b)*(1 - Sin[c + d*x])^(3/
2)*(1 + Sin[c + d*x])^(5/2))/((-a + b)*(a + b)*(a + b*Sin[c + d*x])^6) - (-
1/5*((2*a*(10*a - b)*b + b*(42*a^2 - 16*a*b + 19*b^2))*(1 - Sin[c + d*x])^(
3/2)*(1 + Sin[c + d*x])^(5/2))/((-a + b)*(a + b)*(a + b*Sin[c + d*x])^5) -
(-1/4*((a*b*(62*a^2 - 18*a*b + 19*b^2) + b*(210*a^3 - 142*a^2*b + 213*a*b^2
 - 29*b^3))*(1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(5/2))/((-a + b)*(a
```

```
+ b)*(a + b*Sin[c + d*x])^4) - (105*(8*a^4 - 8*a^3*b + 12*a^2*b^2 - 4*a*b^
3 + b^4)*(-1/3*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/((-a + b)*
(a + b*Sin[c + d*x])^3) - (-1/2*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^
(3/2))/((a + b)*(a + b*Sin[c + d*x])^2) + (3*((-2*ArcTanh[(Sqrt[a - b]*Sqrt
[1 - Sin[c + d*x]])/(Sqrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a - b)^(3/2)
*Sqrt[a - b]) + (Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((-a - b)*(
a + b*Sin[c + d*x]))))/(2*(a + b)))/(3*(-a + b))))/(4*(-a + b)*(a + b)))/(5
*(-a + b)*(a + b)))/(6*(-a + b)*(a + b)))/(7*(-a + b)*(a + b))))/((a - b)*d
*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]]) + (4*b*(Cos[c + d*x]^5/(5*(
a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-1/7*(b*(1 - Sin[c + d*
x])^(5/2)*(1 + Sin[c + d*x])^(7/2))/((-a + b)*(a + b)*(a + b*Sin[c + d*x])^
7) - (-1/6*((a*b + (7*a - b)*b)*(1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])
^(7/2))/((-a + b)*(a + b)*(a + b*Sin[c + d*x])^6) - (7*(6*a^2 - 2*a*b + b^2
)*(-1/5*((1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(7/2))/((-a + b)*(a +
b*Sin[c + d*x])^5) - (3*(-1/4*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(7
/2))/((-a + b)*(a + b*Sin[c + d*x])^4) - (-1/3*(Sqrt[1 - Sin[c + d*x]]*(1 +
 Sin[c + d*x])^(5/2))/((a + b)*(a + b*Sin[c + d*x])^3) + (5*(-1/2*(Sqrt[1 -
 Sin[c + d*x]]*(1 + Sin[c + d*x])^(3/2))/((a + b)*(a + b*Sin[c + d*x])^2) +
 (3*((-2*ArcTanh[(Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[-a - b]*Sqrt[1
+ Sin[c + d*x]])])/((-a - b)^(3/2)*Sqrt[a - b]) + (Sqrt[1 - Sin[c + d*x]]*S
qrt[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[c + d*x]))))/(2*(a + b))))/(3*(
a + b)))/(4*(-a + b))))/(5*(-a + b))))/(6*(-a + b)*(a + b)))/(7*(-a + b)*(a
 + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]]) + (2*b*(
Cos[c + d*x]^7/(7*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-1/7
*((1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])^(9/2))/((-a + b)*(a + b*Sin[c
 + d*x])^7) - (5*(-1/6*((1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(9/2))/
((-a + b)*(a + b*Sin[c + d*x])^6) - (-1/5*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[
c + d*x])^(9/2))/((-a + b)*(a + b*Sin[c + d*x])^5) - (-1/4*(Sqrt[1 - Sin[c
+ d*x]]*(1 + Sin[c + d*x])^(7/2))/((a + b)*(a + b*Sin[c + d*x])^4) + (7*(-1
/3*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/((a + b)*(a + b*Sin[c
+ d*x])^3) + (5*(-1/2*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(3/2))/((a
 + b)*(a + b*Sin[c + d*x])^2) + (3*((-2*ArcTanh[(Sqrt[a - b]*Sqrt[1 - Sin[c
 + d*x]])/(Sqrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a - b)^(3/2)*Sqrt[a -
b]) + (Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[
c + d*x]))))/(2*(a + b))))/(3*(a + b))))/(4*(a + b)))/(5*(-a + b)))/(2*(-a
+ b))))/(7*(-a + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d
*x]]))))/(5*(a - b))))/(3*(a - b))
```

fricas [B] time = 1.52, size = 2972, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/3360*(2*(40*a^8*b^5 + 1478*a^6*b^7 + 261*a^4*b^9 - 1651*a^2*b^11 - 128*b
^13)*cos(d*x + c)^7 - 28*(60*a^10*b^3 + 1837*a^8*b^5 + 176*a^6*b^7 - 1680*a
^4*b^9 - 361*a^2*b^11 - 32*b^13)*cos(d*x + c)^5 + 70*(40*a^12*b + 900*a^10*
b^3 + 1111*a^8*b^5 - 501*a^6*b^7 - 1395*a^4*b^9 - 139*a^2*b^11 - 16*b^13)*c
os(d*x + c)^3 + 105*(8*a^12 + 188*a^10*b^2 + 705*a^8*b^4 + 861*a^6*b^6 + 31
5*a^4*b^8 + 35*a^2*b^10 - 7*(8*a^6*b^6 + 20*a^4*b^8 + 5*a^2*b^10)*cos(d*x +
 c)^6 + 7*(40*a^8*b^4 + 124*a^6*b^6 + 85*a^4*b^8 + 15*a^2*b^10)*cos(d*x + c
)^4 - 7*(24*a^10*b^2 + 140*a^8*b^4 + 239*a^6*b^6 + 110*a^4*b^8 + 15*a^2*b^1
0)*cos(d*x + c)^2 + (56*a^11*b + 420*a^9*b^3 + 903*a^7*b^5 + 603*a^5*b^7 +
125*a^3*b^9 + 5*a*b^11 - (8*a^5*b^7 + 20*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^6
 + 3*(56*a^7*b^5 + 148*a^5*b^7 + 55*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^4 - (2
80*a^9*b^3 + 1036*a^7*b^5 + 1039*a^5*b^7 + 270*a^3*b^9 + 15*a*b^11)*cos(d*x
 + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
```

```
b^2)) - 420*(24*a^12*b + 116*a^10*b^3 + 99*a^8*b^5 - 129*a^6*b^7 - 95*a^4*b
^9 - 15*a^2*b^11)*cos(d*x + c) - 14*((40*a^9*b^4 + 1358*a^7*b^6 + 81*a^5*b^
8 - 1426*a^3*b^10 - 53*a*b^12)*cos(d*x + c)^5 - 10*(20*a^11*b^2 + 535*a^9*b
^4 + 147*a^7*b^6 - 407*a^5*b^8 - 283*a^3*b^10 - 12*a*b^12)*cos(d*x + c)^3 +
 15*(8*a^13 + 132*a^11*b^2 + 285*a^9*b^4 - 42*a^7*b^6 - 288*a^5*b^8 - 90*a^
3*b^10 - 5*a*b^12)*cos(d*x + c))*sin(d*x + c))/(7*(a^15*b^6 - 7*a^13*b^8 +
21*a^11*b^10 - 35*a^9*b^12 + 35*a^7*b^14 - 21*a^5*b^16 + 7*a^3*b^18 - a*b^2
0)*d*cos(d*x + c)^6 - 7*(5*a^17*b^4 - 32*a^15*b^6 + 84*a^13*b^8 - 112*a^11*
b^10 + 70*a^9*b^12 - 28*a^5*b^16 + 16*a^3*b^18 - 3*a*b^20)*d*cos(d*x + c)^4
 + 7*(3*a^19*b^2 - 11*a^17*b^4 - 4*a^15*b^6 + 84*a^13*b^8 - 182*a^11*b^10 +
 182*a^9*b^12 - 84*a^7*b^14 + 4*a^5*b^16 + 11*a^3*b^18 - 3*a*b^20)*d*cos(d*
x + c)^2 - (a^21 + 14*a^19*b^2 - 91*a^17*b^4 + 168*a^15*b^6 - 14*a^13*b^8 -
 364*a^11*b^10 + 546*a^9*b^12 - 344*a^7*b^14 + 77*a^5*b^16 + 14*a^3*b^18 -
7*a*b^20)*d + ((a^14*b^7 - 7*a^12*b^9 + 21*a^10*b^11 - 35*a^8*b^13 + 35*a^6
*b^15 - 21*a^4*b^17 + 7*a^2*b^19 - b^21)*d*cos(d*x + c)^6 - 3*(7*a^16*b^5 -
 48*a^14*b^7 + 140*a^12*b^9 - 224*a^10*b^11 + 210*a^8*b^13 - 112*a^6*b^15 +
 28*a^4*b^17 - b^21)*d*cos(d*x + c)^4 + (35*a^18*b^3 - 203*a^16*b^5 + 444*a
^14*b^7 - 364*a^12*b^9 - 182*a^10*b^11 + 630*a^8*b^13 - 532*a^6*b^15 + 196*
a^4*b^17 - 21*a^2*b^19 - 3*b^21)*d*cos(d*x + c)^2 - (7*a^20*b - 14*a^18*b^3
 - 77*a^16*b^5 + 344*a^14*b^7 - 546*a^12*b^9 + 364*a^10*b^11 + 14*a^8*b^13
- 168*a^6*b^15 + 91*a^4*b^17 - 14*a^2*b^19 - b^21)*d)*sin(d*x + c)), 1/1680
*((40*a^8*b^5 + 1478*a^6*b^7 + 261*a^4*b^9 - 1651*a^2*b^11 - 128*b^13)*cos(
d*x + c)^7 - 14*(60*a^10*b^3 + 1837*a^8*b^5 + 176*a^6*b^7 - 1680*a^4*b^9 -
361*a^2*b^11 - 32*b^13)*cos(d*x + c)^5 + 35*(40*a^12*b + 900*a^10*b^3 + 111
1*a^8*b^5 - 501*a^6*b^7 - 1395*a^4*b^9 - 139*a^2*b^11 - 16*b^13)*cos(d*x +
c)^3 + 105*(8*a^12 + 188*a^10*b^2 + 705*a^8*b^4 + 861*a^6*b^6 + 315*a^4*b^8
 + 35*a^2*b^10 - 7*(8*a^6*b^6 + 20*a^4*b^8 + 5*a^2*b^10)*cos(d*x + c)^6 + 7
*(40*a^8*b^4 + 124*a^6*b^6 + 85*a^4*b^8 + 15*a^2*b^10)*cos(d*x + c)^4 - 7*(
24*a^10*b^2 + 140*a^8*b^4 + 239*a^6*b^6 + 110*a^4*b^8 + 15*a^2*b^10)*cos(d*
x + c)^2 + (56*a^11*b + 420*a^9*b^3 + 903*a^7*b^5 + 603*a^5*b^7 + 125*a^3*b
^9 + 5*a*b^11 - (8*a^5*b^7 + 20*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^6 + 3*(56*
a^7*b^5 + 148*a^5*b^7 + 55*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^4 - (280*a^9*b^
3 + 1036*a^7*b^5 + 1039*a^5*b^7 + 270*a^3*b^9 + 15*a*b^11)*cos(d*x + c)^2)*
sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)
*cos(d*x + c))) - 210*(24*a^12*b + 116*a^10*b^3 + 99*a^8*b^5 - 129*a^6*b^7
- 95*a^4*b^9 - 15*a^2*b^11)*cos(d*x + c) - 7*((40*a^9*b^4 + 1358*a^7*b^6 +
81*a^5*b^8 - 1426*a^3*b^10 - 53*a*b^12)*cos(d*x + c)^5 - 10*(20*a^11*b^2 +
535*a^9*b^4 + 147*a^7*b^6 - 407*a^5*b^8 - 283*a^3*b^10 - 12*a*b^12)*cos(d*x
 + c)^3 + 15*(8*a^13 + 132*a^11*b^2 + 285*a^9*b^4 - 42*a^7*b^6 - 288*a^5*b^
8 - 90*a^3*b^10 - 5*a*b^12)*cos(d*x + c))*sin(d*x + c))/(7*(a^15*b^6 - 7*a^
13*b^8 + 21*a^11*b^10 - 35*a^9*b^12 + 35*a^7*b^14 - 21*a^5*b^16 + 7*a^3*b^1
8 - a*b^20)*d*cos(d*x + c)^6 - 7*(5*a^17*b^4 - 32*a^15*b^6 + 84*a^13*b^8 -
112*a^11*b^10 + 70*a^9*b^12 - 28*a^5*b^16 + 16*a^3*b^18 - 3*a*b^20)*d*cos(d
*x + c)^4 + 7*(3*a^19*b^2 - 11*a^17*b^4 - 4*a^15*b^6 + 84*a^13*b^8 - 182*a^
11*b^10 + 182*a^9*b^12 - 84*a^7*b^14 + 4*a^5*b^16 + 11*a^3*b^18 - 3*a*b^20)
*d*cos(d*x + c)^2 - (a^21 + 14*a^19*b^2 - 91*a^17*b^4 + 168*a^15*b^6 - 14*a
^13*b^8 - 364*a^11*b^10 + 546*a^9*b^12 - 344*a^7*b^14 + 77*a^5*b^16 + 14*a^
3*b^18 - 7*a*b^20)*d + ((a^14*b^7 - 7*a^12*b^9 + 21*a^10*b^11 - 35*a^8*b^13
 + 35*a^6*b^15 - 21*a^4*b^17 + 7*a^2*b^19 - b^21)*d*cos(d*x + c)^6 - 3*(7*a
^16*b^5 - 48*a^14*b^7 + 140*a^12*b^9 - 224*a^10*b^11 + 210*a^8*b^13 - 112*a
^6*b^15 + 28*a^4*b^17 - b^21)*d*cos(d*x + c)^4 + (35*a^18*b^3 - 203*a^16*b^
5 + 444*a^14*b^7 - 364*a^12*b^9 - 182*a^10*b^11 + 630*a^8*b^13 - 532*a^6*b^
15 + 196*a^4*b^17 - 21*a^2*b^19 - 3*b^21)*d*cos(d*x + c)^2 - (7*a^20*b - 14
*a^18*b^3 - 77*a^16*b^5 + 344*a^14*b^7 - 546*a^12*b^9 + 364*a^10*b^11 + 14*
a^8*b^13 - 168*a^6*b^15 + 91*a^4*b^17 - 14*a^2*b^19 - b^21)*d)*sin(d*x + c)
)]
```

giac [B] time = 3.82, size = 2207, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/840*(105*(8*a^5 + 20*a^3*b^2 + 5*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^12 - 6*
a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*b^10 + b^12)*sqrt(a
^2 - b^2)) - (840*a^18*tan(1/2*d*x + 1/2*c)^13 - 12180*a^16*b^2*tan(1/2*d*x
 + 1/2*c)^13 + 24675*a^14*b^4*tan(1/2*d*x + 1/2*c)^13 - 33600*a^12*b^6*tan(
1/2*d*x + 1/2*c)^13 + 25200*a^10*b^8*tan(1/2*d*x + 1/2*c)^13 - 10080*a^8*b^
10*tan(1/2*d*x + 1/2*c)^13 + 1680*a^6*b^12*tan(1/2*d*x + 1/2*c)^13 - 840*a^
17*b*tan(1/2*d*x + 1/2*c)^12 - 87780*a^15*b^3*tan(1/2*d*x + 1/2*c)^12 + 144
375*a^13*b^5*tan(1/2*d*x + 1/2*c)^12 - 201600*a^11*b^7*tan(1/2*d*x + 1/2*c)
^12 + 151200*a^9*b^9*tan(1/2*d*x + 1/2*c)^12 - 60480*a^7*b^11*tan(1/2*d*x +
 1/2*c)^12 + 10080*a^5*b^13*tan(1/2*d*x + 1/2*c)^12 + 3360*a^18*tan(1/2*d*x
 + 1/2*c)^11 - 94080*a^16*b^2*tan(1/2*d*x + 1/2*c)^11 - 220500*a^14*b^4*tan
(1/2*d*x + 1/2*c)^11 + 287350*a^12*b^6*tan(1/2*d*x + 1/2*c)^11 - 537600*a^1
0*b^8*tan(1/2*d*x + 1/2*c)^11 + 450240*a^8*b^10*tan(1/2*d*x + 1/2*c)^11 - 1
92640*a^6*b^12*tan(1/2*d*x + 1/2*c)^11 + 33600*a^4*b^14*tan(1/2*d*x + 1/2*c
)^11 - 13440*a^17*b*tan(1/2*d*x + 1/2*c)^10 - 554400*a^15*b^3*tan(1/2*d*x +
 1/2*c)^10 - 165900*a^13*b^5*tan(1/2*d*x + 1/2*c)^10 - 66850*a^11*b^7*tan(1
/2*d*x + 1/2*c)^10 - 621600*a^9*b^9*tan(1/2*d*x + 1/2*c)^10 + 719040*a^7*b^
11*tan(1/2*d*x + 1/2*c)^10 - 355040*a^5*b^13*tan(1/2*d*x + 1/2*c)^10 + 6720
0*a^3*b^15*tan(1/2*d*x + 1/2*c)^10 + 4200*a^18*tan(1/2*d*x + 1/2*c)^9 - 304
500*a^16*b^2*tan(1/2*d*x + 1/2*c)^9 - 1418025*a^14*b^4*tan(1/2*d*x + 1/2*c)
^9 + 147070*a^12*b^6*tan(1/2*d*x + 1/2*c)^9 - 1316700*a^10*b^8*tan(1/2*d*x
+ 1/2*c)^9 + 242592*a^8*b^10*tan(1/2*d*x + 1/2*c)^9 + 439376*a^6*b^12*tan(1
/2*d*x + 1/2*c)^9 - 352128*a^4*b^14*tan(1/2*d*x + 1/2*c)^9 + 80640*a^2*b^16
*tan(1/2*d*x + 1/2*c)^9 - 49000*a^17*b*tan(1/2*d*x + 1/2*c)^8 - 1357300*a^1
5*b^3*tan(1/2*d*x + 1/2*c)^8 - 1726305*a^13*b^5*tan(1/2*d*x + 1/2*c)^8 - 34
6570*a^11*b^7*tan(1/2*d*x + 1/2*c)^8 - 1972600*a^9*b^9*tan(1/2*d*x + 1/2*c)
^8 + 1360128*a^7*b^11*tan(1/2*d*x + 1/2*c)^8 - 298816*a^5*b^13*tan(1/2*d*x
+ 1/2*c)^8 - 122752*a^3*b^15*tan(1/2*d*x + 1/2*c)^8 + 53760*a*b^17*tan(1/2*
d*x + 1/2*c)^8 - 509600*a^16*b^2*tan(1/2*d*x + 1/2*c)^7 - 2685200*a^14*b^4*
tan(1/2*d*x + 1/2*c)^7 - 900900*a^12*b^6*tan(1/2*d*x + 1/2*c)^7 - 2070320*a
^10*b^8*tan(1/2*d*x + 1/2*c)^7 - 278096*a^8*b^10*tan(1/2*d*x + 1/2*c)^7 + 9
52320*a^6*b^12*tan(1/2*d*x + 1/2*c)^7 - 538112*a^4*b^14*tan(1/2*d*x + 1/2*c
)^7 + 68608*a^2*b^16*tan(1/2*d*x + 1/2*c)^7 + 15360*b^18*tan(1/2*d*x + 1/2*
c)^7 - 78400*a^17*b*tan(1/2*d*x + 1/2*c)^6 - 1607200*a^15*b^3*tan(1/2*d*x +
 1/2*c)^6 - 2326800*a^13*b^5*tan(1/2*d*x + 1/2*c)^6 - 823060*a^11*b^7*tan(1
/2*d*x + 1/2*c)^6 - 2094400*a^9*b^9*tan(1/2*d*x + 1/2*c)^6 + 1351728*a^7*b^
11*tan(1/2*d*x + 1/2*c)^6 - 298816*a^5*b^13*tan(1/2*d*x + 1/2*c)^6 - 122752
*a^3*b^15*tan(1/2*d*x + 1/2*c)^6 + 53760*a*b^17*tan(1/2*d*x + 1/2*c)^6 - 42
00*a^18*tan(1/2*d*x + 1/2*c)^5 - 459900*a^16*b^2*tan(1/2*d*x + 1/2*c)^5 - 2
100175*a^14*b^4*tan(1/2*d*x + 1/2*c)^5 - 647780*a^12*b^6*tan(1/2*d*x + 1/2*
c)^5 - 1643880*a^10*b^8*tan(1/2*d*x + 1/2*c)^5 + 228592*a^8*b^10*tan(1/2*d*
x + 1/2*c)^5 + 439376*a^6*b^12*tan(1/2*d*x + 1/2*c)^5 - 352128*a^4*b^14*tan
(1/2*d*x + 1/2*c)^5 + 80640*a^2*b^16*tan(1/2*d*x + 1/2*c)^5 - 63000*a^17*b*
tan(1/2*d*x + 1/2*c)^4 - 918540*a^15*b^3*tan(1/2*d*x + 1/2*c)^4 - 858683*a^
13*b^5*tan(1/2*d*x + 1/2*c)^4 - 434644*a^11*b^7*tan(1/2*d*x + 1/2*c)^4 - 63
4368*a^9*b^9*tan(1/2*d*x + 1/2*c)^4 + 719600*a^7*b^11*tan(1/2*d*x + 1/2*c)^
4 - 355040*a^5*b^13*tan(1/2*d*x + 1/2*c)^4 + 67200*a^3*b^15*tan(1/2*d*x + 1
/2*c)^4 - 3360*a^18*tan(1/2*d*x + 1/2*c)^3 - 211680*a^16*b^2*tan(1/2*d*x +
1/2*c)^3 - 575260*a^14*b^4*tan(1/2*d*x + 1/2*c)^3 + 43918*a^12*b^6*tan(1/2*
d*x + 1/2*c)^3 - 534576*a^10*b^8*tan(1/2*d*x + 1/2*c)^3 + 449008*a^8*b^10*t
an(1/2*d*x + 1/2*c)^3 - 192640*a^6*b^12*tan(1/2*d*x + 1/2*c)^3 + 33600*a^4*
b^14*tan(1/2*d*x + 1/2*c)^3 - 24640*a^17*b*tan(1/2*d*x + 1/2*c)^2 - 199360*
a^15*b^3*tan(1/2*d*x + 1/2*c)^2 + 44604*a^13*b^5*tan(1/2*d*x + 1/2*c)^2 - 1
86410*a^11*b^7*tan(1/2*d*x + 1/2*c)^2 + 144928*a^9*b^9*tan(1/2*d*x + 1/2*c)
^2 - 59472*a^7*b^11*tan(1/2*d*x + 1/2*c)^2 + 10080*a^5*b^13*tan(1/2*d*x + 1
```

```
/2*c)^2 - 840*a^18*tan(1/2*d*x + 1/2*c) - 38780*a^16*b^2*tan(1/2*d*x + 1/2*
c) + 12565*a^14*b^4*tan(1/2*d*x + 1/2*c) - 35322*a^12*b^6*tan(1/2*d*x + 1/2
*c) + 25844*a^10*b^8*tan(1/2*d*x + 1/2*c) - 10192*a^8*b^10*tan(1/2*d*x + 1/
2*c) + 1680*a^6*b^12*tan(1/2*d*x + 1/2*c) - 3640*a^17*b + 2660*a^15*b^3 - 4
923*a^13*b^5 + 3646*a^11*b^7 - 1448*a^9*b^9 + 240*a^7*b^11)/((a^19 - 6*a^17
*b^2 + 15*a^15*b^4 - 20*a^13*b^6 + 15*a^11*b^8 - 6*a^9*b^10 + a^7*b^12)*(a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d
```

maple [B] time = 0.41, size = 11250, normalized size = 26.66

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.34, size = 2440, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^8,x)
```

```
[Out] ((3640*a^10*b - 240*b^11 + 1448*a^2*b^9 - 3646*a^4*b^7 + 4923*a^6*b^5 - 266
0*a^8*b^3)/(840*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^
8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^6*(2800*a^16*b - 1920*b^17 + 438
4*a^2*b^15 + 10672*a^4*b^13 - 48276*a^6*b^11 + 74800*a^8*b^9 + 29395*a^10*b
^7 + 83100*a^12*b^5 + 57400*a^14*b^3))/(30*a^6*(a^12 + b^12 - 6*a^2*b^10 +
15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^8
*(7000*a^16*b - 7680*b^17 + 17536*a^2*b^15 + 42688*a^4*b^13 - 194304*a^6*b^
11 + 281800*a^8*b^9 + 49510*a^10*b^7 + 246615*a^12*b^5 + 193900*a^14*b^3))/
(120*a^6*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 -
 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^10*(192*a^14*b - 960*b^15 + 5072*a^2*b^
13 - 10272*a^4*b^11 + 8880*a^6*b^9 + 955*a^8*b^7 + 2370*a^10*b^5 + 7920*a^1
2*b^3))/(12*a^4*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^
8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^4*(9000*a^14*b - 9600*b^15 + 507
20*a^2*b^13 - 102800*a^4*b^11 + 90624*a^6*b^9 + 62092*a^8*b^7 + 122669*a^10
*b^5 + 131220*a^12*b^3))/(120*a^4*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 -
20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^12*(8*a^12*b -
 96*b^13 + 576*a^2*b^11 - 1440*a^4*b^9 + 1920*a^6*b^7 - 1375*a^8*b^5 + 836*
a^10*b^3))/(8*a^2*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*
a^8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^2*(1760*a^12*b - 720*b^13 + 42
48*a^2*b^11 - 10352*a^4*b^9 + 13315*a^6*b^7 - 3186*a^8*b^5 + 14240*a^10*b^3
))/(60*a^2*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4
 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)*(120*a^12 - 240*b^12 + 1456*a^2*b^10
- 3692*a^4*b^8 + 5046*a^6*b^6 - 1795*a^8*b^4 + 5540*a^10*b^2))/(120*a*(a^12
```

```
 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2))
- (tan(c/2 + (d*x)/2)^9*(600*a^16 + 11520*b^16 - 50304*a^2*b^14 + 62768*a^4
*b^12 + 34656*a^6*b^10 - 188100*a^8*b^8 + 21010*a^10*b^6 - 202575*a^12*b^4
- 43500*a^14*b^2))/(120*a^5*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6
*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^5*(600*a^16 - 11520*
b^16 + 50304*a^2*b^14 - 62768*a^4*b^12 - 32656*a^6*b^10 + 234840*a^8*b^8 +
92540*a^10*b^6 + 300025*a^12*b^4 + 65700*a^14*b^2))/(120*a^5*(a^12 + b^12 -
 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) - (tan(c/
2 + (d*x)/2)^11*(48*a^14 + 480*b^14 - 2752*a^2*b^12 + 6432*a^4*b^10 - 7680*
a^6*b^8 + 4105*a^8*b^6 - 3150*a^10*b^4 - 1344*a^12*b^2))/(12*a^3*(a^12 + b^
12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) + (ta
n(c/2 + (d*x)/2)^3*(240*a^14 - 2400*b^14 + 13760*a^2*b^12 - 32072*a^4*b^10
+ 38184*a^6*b^8 - 3137*a^8*b^6 + 41090*a^10*b^4 + 15120*a^12*b^2))/(60*a^3*
(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b
^2)) - (tan(c/2 + (d*x)/2)^13*(8*a^12 + 16*b^12 - 96*a^2*b^10 + 240*a^4*b^8
 - 320*a^6*b^6 + 235*a^8*b^4 - 116*a^10*b^2))/(8*a*(a^12 + b^12 - 6*a^2*b^1
0 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) + (b*tan(c/2 + (d*x
)/2)^7*(35*a^6 + 16*b^6 + 168*a^2*b^4 + 210*a^4*b^2)*(3640*a^10*b - 240*b^1
1 + 1448*a^2*b^9 - 3646*a^4*b^7 + 4923*a^6*b^5 - 2660*a^8*b^3))/(210*a^7*(a
^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2
)))/(d*(tan(c/2 + (d*x)/2)^5*(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + tan
(c/2 + (d*x)/2)^9*(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + a^7*tan(c/2 +
(d*x)/2)^14 + tan(c/2 + (d*x)/2)^3*(84*a^6*b + 280*a^4*b^3) + tan(c/2 + (d*
x)/2)^11*(84*a^6*b + 280*a^4*b^3) + tan(c/2 + (d*x)/2)^6*(448*a*b^6 + 35*a^
7 + 1680*a^3*b^4 + 840*a^5*b^2) + tan(c/2 + (d*x)/2)^8*(448*a*b^6 + 35*a^7
+ 1680*a^3*b^4 + 840*a^5*b^2) + tan(c/2 + (d*x)/2)^7*(280*a^6*b + 128*b^7 +
 1344*a^2*b^5 + 1680*a^4*b^3) + a^7 + tan(c/2 + (d*x)/2)^4*(21*a^7 + 560*a^
3*b^4 + 420*a^5*b^2) + tan(c/2 + (d*x)/2)^10*(21*a^7 + 560*a^3*b^4 + 420*a^
5*b^2) + tan(c/2 + (d*x)/2)^2*(7*a^7 + 84*a^5*b^2) + tan(c/2 + (d*x)/2)^12*
(7*a^7 + 84*a^5*b^2) + 14*a^6*b*tan(c/2 + (d*x)/2) + 14*a^6*b*tan(c/2 + (d*
x)/2)^13)) + (a*atan((8*((a^2*tan(c/2 + (d*x)/2)*(8*a^4 + 5*b^4 + 20*a^2*b^
2))/(8*(a + b)^(13/2)*(a - b)^(13/2)) + (a*(8*a^4 + 5*b^4 + 20*a^2*b^2)*(16
*a^12*b + 16*b^13 - 96*a^2*b^11 + 240*a^4*b^9 - 320*a^6*b^7 + 240*a^8*b^5 -
 96*a^10*b^3))/(128*(a + b)^(13/2)*(a - b)^(13/2)*(a^12 + b^12 - 6*a^2*b^10
 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)))*(a^12 + b^12 - 6*a^
2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2))/(5*a*b^4 + 8*a
^5 + 20*a^3*b^2))*(8*a^4 + 5*b^4 + 20*a^2*b^2))/(8*d*(a + b)^(13/2)*(a - b)
^(13/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.471 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=529

$$\frac{13ab\left(28a^2+27b^2\right)\sec(c+dx)}{280d\left(a^2-b^2\right)^4(a+b\sin(c+dx))^4}+\frac{b\left(49a^2+16b^2\right)\sec(c+dx)}{70d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^5}+\frac{5ab\sec(c+dx)}{14d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^6}+\frac{}{7d}$$

```
[Out] -9/8*a*b^2*(64*a^6+336*a^4*b^2+280*a^2*b^4+35*b^6)*arctan((b+a*tan(1/2*d*x+
1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(17/2)/d+1/7*b*sec(d*x+c)/(a^2-b^2)/d/(a
+b*sin(d*x+c))^7+5/14*a*b*sec(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^6+1/70*
b*(49*a^2+16*b^2)*sec(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^5+13/280*a*b*(2
8*a^2+27*b^2)*sec(d*x+c)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))^4+1/280*b*(700*a^4+
1317*a^2*b^2+128*b^4)*sec(d*x+c)/(a^2-b^2)^5/d/(a+b*sin(d*x+c))^3+11/560*a*
b*(280*a^4+844*a^2*b^2+241*b^4)*sec(d*x+c)/(a^2-b^2)^6/d/(a+b*sin(d*x+c))^2
+1/560*b*(9800*a^6+41484*a^4*b^2+22767*a^2*b^4+1024*b^6)*sec(d*x+c)/(a^2-b^
2)^7/d/(a+b*sin(d*x+c))-1/560*sec(d*x+c)*(315*a*b*(64*a^6+336*a^4*b^2+280*a
^2*b^4+35*b^6)-(560*a^8+42472*a^6*b^2+125634*a^4*b^4+54511*a^2*b^6+2048*b^8
)*sin(d*x+c))/(a^2-b^2)^8/d
```

Rubi [A] time = 1.77, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2694, 2864, 2866, 12, 2660, 618, 204}

$$-\frac{9ab^2\left(336a^4b^2+280a^2b^4+64a^6+35b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{17/2}}+\frac{b\left(41484a^4b^2+22767a^2b^4+9800a^6+1024b\right.}{560d\left(a^2-b^2\right)^7(a+b\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (-9*a*b^2*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6)*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(17/2)*d) + (b*Sec[c + d*x])/
(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (5*a*b*Sec[c + d*x])/(14*(a^2 -
b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(49*a^2 + 16*b^2)*Sec[c + d*x])/(70*(
a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^5) + (13*a*b*(28*a^2 + 27*b^2)*Sec[c +
d*x])/(280*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^4) + (b*(700*a^4 + 1317*a^2
*b^2 + 128*b^4)*Sec[c + d*x])/(280*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^3)
+ (11*a*b*(280*a^4 + 844*a^2*b^2 + 241*b^4)*Sec[c + d*x])/(560*(a^2 - b^2)^
6*d*(a + b*Sin[c + d*x])^2) + (b*(9800*a^6 + 41484*a^4*b^2 + 22767*a^2*b^4
+ 1024*b^6)*Sec[c + d*x])/(560*(a^2 - b^2)^7*d*(a + b*Sin[c + d*x])) - (Sec
[c + d*x]*(315*a*b*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6) - (560*a^8
 + 42472*a^6*b^2 + 125634*a^4*b^4 + 54511*a^2*b^6 + 2048*b^8)*Sin[c + d*x])
)/(560*(a^2 - b^2)^8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} - \frac{\int \frac{\sec^2(c+dx)(-7a+8b\sin(c+dx))}{(a+b\sin(c+dx))^7}dx}{7\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{\int \frac{\sec^2(c+dx)(}{}}{}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2\right.}{70\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2\right.}{70\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2\right.}{70\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2\right.}{70\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2\right.}{70\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2\right.}{70\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2\right.}{70\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2\right.}{70\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2\right.}{70\left(a^2-b^2\right)}$$

$$= -\frac{9ab^2\left(64a^6+336a^4b^2+280a^2b^4+35b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{17/2}d} + \frac{b\sec(}{7\left(a^2-b^2\right)d(a}$$

Mathematica [A] time = 5.25, size = 494, normalized size = 0.93

$$-\frac{\frac{2ab^3\left(1216a^2+739b^2\right)\cos(c+dx)}{\left(a^2-b^2\right)^5(a+b\sin(c+dx))^4} + \frac{8b^3\left(129a^2+26b^2\right)\cos(c+dx)}{\left(a^2-b^2\right)^4(a+b\sin(c+dx))^5} + \frac{360ab^3\cos(c+dx)}{\left(a^2-b^2\right)^3(a+b\sin(c+dx))^6} + \frac{80b^3\cos(c+dx)}{\left(a^2-b^2\right)^2(a+b\sin(c+dx))^7} + \frac{ab^3\left(11112a^4+2306\right.}{\left(a^2-b^2\right)^7(}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/560*((630*a*b^2*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6)*ArcTan[(b
+ a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(17/2) + (80*b^3*Cos[c
+ d*x])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^7) + (360*a*b^3*Cos[c + d*x])/(
(a^2 - b^2)^3*(a + b*Sin[c + d*x])^6) + (8*b^3*(129*a^2 + 26*b^2)*Cos[c + d
*x])/((a^2 - b^2)^4*(a + b*Sin[c + d*x])^5) + (2*a*b^3*(1216*a^2 + 739*b^2)
*Cos[c + d*x])/((a^2 - b^2)^5*(a + b*Sin[c + d*x])^4) + (2*b^3*(2616*a^4 +
3207*a^2*b^2 + 232*b^4)*Cos[c + d*x])/((a^2 - b^2)^6*(a + b*Sin[c + d*x])^3
) + (a*b^3*(11112*a^4 + 23066*a^2*b^2 + 5057*b^4)*Cos[c + d*x])/((a^2 - b^2
)^7*(a + b*Sin[c + d*x])^2) + (b^3*(26792*a^6 + 86434*a^4*b^2 + 38831*a^2*b
^4 + 1488*b^6)*Cos[c + d*x])/((a^2 - b^2)^8*(a + b*Sin[c + d*x])) - (560*Se
c[c + d*x]*(-8*a*b*(a^6 + 7*a^4*b^2 + 7*a^2*b^4 + b^6) + (a^8 + 28*a^6*b^2
+ 70*a^4*b^4 + 28*a^2*b^6 + b^8)*Sin[c + d*x]))/(a^2 - b^2)^8)/d
```

fricas [B] time = 1.71, size = 3882, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/1120*(1120*a^16*b - 8960*a^14*b^3 + 31360*a^12*b^5 - 62720*a^10*b^7 + 78
400*a^8*b^9 - 62720*a^6*b^11 + 31360*a^4*b^13 - 8960*a^2*b^15 + 1120*b^17 -
 2*(560*a^10*b^7 + 41912*a^8*b^9 + 83162*a^6*b^11 - 71123*a^4*b^13 - 52463*
a^2*b^15 - 2048*b^17)*cos(d*x + c)^8 + 28*(840*a^12*b^5 + 53648*a^10*b^7 +
95441*a^8*b^9 - 77704*a^6*b^11 - 60644*a^4*b^13 - 11069*a^2*b^15 - 512*b^17
)*cos(d*x + c)^6 - 70*(560*a^14*b^3 + 27440*a^12*b^5 + 71064*a^10*b^7 + 299
27*a^8*b^9 - 81421*a^6*b^11 - 43131*a^4*b^13 - 4183*a^2*b^15 - 256*b^17)*co
s(d*x + c)^4 + 140*(56*a^16*b + 1400*a^14*b^3 + 13832*a^12*b^5 + 24080*a^10
*b^7 - 4591*a^8*b^9 - 23443*a^6*b^11 - 10717*a^4*b^13 - 553*a^2*b^15 - 64*b
^17)*cos(d*x + c)^2 - 315*(7*(64*a^8*b^8 + 336*a^6*b^10 + 280*a^4*b^12 + 35
*a^2*b^14)*cos(d*x + c)^7 - 7*(320*a^10*b^6 + 1872*a^8*b^8 + 2408*a^6*b^10
+ 1015*a^4*b^12 + 105*a^2*b^14)*cos(d*x + c)^5 + 7*(192*a^12*b^4 + 1648*a^1
0*b^6 + 4392*a^8*b^8 + 3913*a^6*b^10 + 1190*a^4*b^12 + 105*a^2*b^14)*cos(d*
x + c)^3 - (64*a^14*b^2 + 1680*a^12*b^4 + 9576*a^10*b^6 + 18123*a^8*b^8 + 1
2887*a^6*b^10 + 3185*a^4*b^12 + 245*a^2*b^14)*cos(d*x + c) + ((64*a^7*b^9 +
 336*a^5*b^11 + 280*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^7 - 3*(448*a^9*b^7 +
 2416*a^7*b^9 + 2296*a^5*b^11 + 525*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^5 +
(2240*a^11*b^5 + 14448*a^9*b^7 + 24104*a^7*b^9 + 13993*a^5*b^11 + 2310*a^3*
b^13 + 105*a*b^15)*cos(d*x + c)^3 - (448*a^13*b^3 + 4592*a^11*b^5 + 15064*a
^9*b^7 + 17165*a^7*b^9 + 7441*a^5*b^11 + 1015*a^3*b^13 + 35*a*b^15)*cos(d*x
 + c))*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) - 14*(80*a^17 - 640*a^15*b^2 + 2240*a^13*b^4 - 4480*a^11*b^6 + 5600*a^
9*b^8 - 4480*a^7*b^10 + 2240*a^5*b^12 - 640*a^3*b^14 + 80*a*b^16 - (560*a^1
1*b^6 + 39032*a^9*b^8 + 70922*a^7*b^10 - 68603*a^5*b^12 - 41438*a^3*b^14 -
473*a*b^16)*cos(d*x + c)^6 + 10*(280*a^13*b^4 + 15960*a^11*b^6 + 29463*a^9*
b^8 - 13541*a^7*b^10 - 23679*a^5*b^12 - 8391*a^3*b^14 - 92*a*b^16)*cos(d*x
+ c)^4 - 15*(112*a^15*b^2 + 4256*a^13*b^4 + 13272*a^11*b^6 + 11977*a^9*b^8
- 15634*a^7*b^10 - 11088*a^5*b^12 - 2870*a^3*b^14 - 25*a*b^16)*cos(d*x + c)
^2)*sin(d*x + c))/(7*(a^19*b^6 - 9*a^17*b^8 + 36*a^15*b^10 - 84*a^13*b^12 +
 126*a^11*b^14 - 126*a^9*b^16 + 84*a^7*b^18 - 36*a^5*b^20 + 9*a^3*b^22 - a*
b^24)*d*cos(d*x + c)^7 - 7*(5*a^21*b^4 - 42*a^19*b^6 + 153*a^17*b^8 - 312*a
^15*b^10 + 378*a^13*b^12 - 252*a^11*b^14 + 42*a^9*b^16 + 72*a^7*b^18 - 63*a
^5*b^20 + 22*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^5 + 7*(3*a^23*b^2 - 17*a^2
1*b^4 + 21*a^19*b^6 + 81*a^17*b^8 - 354*a^15*b^10 + 630*a^13*b^12 - 630*a^1
1*b^14 + 354*a^9*b^16 - 81*a^7*b^18 - 21*a^5*b^20 + 17*a^3*b^22 - 3*a*b^24)
*d*cos(d*x + c)^3 - (a^25 + 12*a^23*b^2 - 118*a^21*b^4 + 364*a^19*b^6 - 441
*a^17*b^8 - 168*a^15*b^10 + 1260*a^13*b^12 - 1800*a^11*b^14 + 1311*a^9*b^16
 - 484*a^7*b^18 + 42*a^5*b^20 + 28*a^3*b^22 - 7*a*b^24)*d*cos(d*x + c) + ((
```

a^18*b^7 - 9*a^16*b^9 + 36*a^14*b^11 - 84*a^12*b^13 + 126*a^10*b^15 - 126*a
^8*b^17 + 84*a^6*b^19 - 36*a^4*b^21 + 9*a^2*b^23 - b^25)*d*cos(d*x + c)^7 -
3*(7*a^20*b^5 - 62*a^18*b^7 + 243*a^16*b^9 - 552*a^14*b^11 + 798*a^12*b^13
- 756*a^10*b^15 + 462*a^8*b^17 - 168*a^6*b^19 + 27*a^4*b^21 + 2*a^2*b^23 -
b^25)*d*cos(d*x + c)^5 + (35*a^22*b^3 - 273*a^20*b^5 + 885*a^18*b^7 - 1455
*a^16*b^9 + 990*a^14*b^11 + 630*a^12*b^13 - 1974*a^10*b^15 + 1890*a^8*b^17
- 945*a^6*b^19 + 235*a^4*b^21 - 15*a^2*b^23 - 3*b^25)*d*cos(d*x + c)^3 - (7
*a^24*b - 28*a^22*b^3 - 42*a^20*b^5 + 484*a^18*b^7 - 1311*a^16*b^9 + 1800*a
^14*b^11 - 1260*a^12*b^13 + 168*a^10*b^15 + 441*a^8*b^17 - 364*a^6*b^19 + 1
18*a^4*b^21 - 12*a^2*b^23 - b^25)*d*cos(d*x + c))*sin(d*x + c)), 1/560*(560
*a^16*b - 4480*a^14*b^3 + 15680*a^12*b^5 - 31360*a^10*b^7 + 39200*a^8*b^9 -
31360*a^6*b^11 + 15680*a^4*b^13 - 4480*a^2*b^15 + 560*b^17 - (560*a^10*b^7
+ 41912*a^8*b^9 + 83162*a^6*b^11 - 71123*a^4*b^13 - 52463*a^2*b^15 - 2048*
b^17)*cos(d*x + c)^8 + 14*(840*a^12*b^5 + 53648*a^10*b^7 + 95441*a^8*b^9 -
77704*a^6*b^11 - 60644*a^4*b^13 - 11069*a^2*b^15 - 512*b^17)*cos(d*x + c)^6
- 35*(560*a^14*b^3 + 27440*a^12*b^5 + 71064*a^10*b^7 + 29927*a^8*b^9 - 814
21*a^6*b^11 - 43131*a^4*b^13 - 4183*a^2*b^15 - 256*b^17)*cos(d*x + c)^4 + 7
0*(56*a^16*b + 1400*a^14*b^3 + 13832*a^12*b^5 + 24080*a^10*b^7 - 4591*a^8*b
^9 - 23443*a^6*b^11 - 10717*a^4*b^13 - 553*a^2*b^15 - 64*b^17)*cos(d*x + c)
^2 + 315*(7*(64*a^8*b^8 + 336*a^6*b^10 + 280*a^4*b^12 + 35*a^2*b^14)*cos(d*
x + c)^7 - 7*(320*a^10*b^6 + 1872*a^8*b^8 + 2408*a^6*b^10 + 1015*a^4*b^12 +
105*a^2*b^14)*cos(d*x + c)^5 + 7*(192*a^12*b^4 + 1648*a^10*b^6 + 4392*a^8*
b^8 + 3913*a^6*b^10 + 1190*a^4*b^12 + 105*a^2*b^14)*cos(d*x + c)^3 - (64*a^
14*b^2 + 1680*a^12*b^4 + 9576*a^10*b^6 + 18123*a^8*b^8 + 12887*a^6*b^10 + 3
185*a^4*b^12 + 245*a^2*b^14)*cos(d*x + c) + ((64*a^7*b^9 + 336*a^5*b^11 + 2
80*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^7 - 3*(448*a^9*b^7 + 2416*a^7*b^9 + 2
296*a^5*b^11 + 525*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^5 + (2240*a^11*b^5 +
14448*a^9*b^7 + 24104*a^7*b^9 + 13993*a^5*b^11 + 2310*a^3*b^13 + 105*a*b^15
)*cos(d*x + c)^3 - (448*a^13*b^3 + 4592*a^11*b^5 + 15064*a^9*b^7 + 17165*a^
7*b^9 + 7441*a^5*b^11 + 1015*a^3*b^13 + 35*a*b^15)*cos(d*x + c))*sin(d*x +
c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x +
c))) - 7*(80*a^17 - 640*a^15*b^2 + 2240*a^13*b^4 - 4480*a^11*b^6 + 5600*a^
9*b^8 - 4480*a^7*b^10 + 2240*a^5*b^12 - 640*a^3*b^14 + 80*a*b^16 - (560*a^1
1*b^6 + 39032*a^9*b^8 + 70922*a^7*b^10 - 68603*a^5*b^12 - 41438*a^3*b^14 -
473*a*b^16)*cos(d*x + c)^6 + 10*(280*a^13*b^4 + 15960*a^11*b^6 + 29463*a^9*
b^8 - 13541*a^7*b^10 - 23679*a^5*b^12 - 8391*a^3*b^14 - 92*a*b^16)*cos(d*x
+ c)^4 - 15*(112*a^15*b^2 + 4256*a^13*b^4 + 13272*a^11*b^6 + 11977*a^9*b^8
- 15634*a^7*b^10 - 11088*a^5*b^12 - 2870*a^3*b^14 - 25*a*b^16)*cos(d*x + c)
^2)*sin(d*x + c))/(7*(a^19*b^6 - 9*a^17*b^8 + 36*a^15*b^10 - 84*a^13*b^12 +
126*a^11*b^14 - 126*a^9*b^16 + 84*a^7*b^18 - 36*a^5*b^20 + 9*a^3*b^22 - a*
b^24)*d*cos(d*x + c)^7 - 7*(5*a^21*b^4 - 42*a^19*b^6 + 153*a^17*b^8 - 312*a
^15*b^10 + 378*a^13*b^12 - 252*a^11*b^14 + 42*a^9*b^16 + 72*a^7*b^18 - 63*a
^5*b^20 + 22*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^5 + 7*(3*a^23*b^2 - 17*a^2
1*b^4 + 21*a^19*b^6 + 81*a^17*b^8 - 354*a^15*b^10 + 630*a^13*b^12 - 630*a^1
1*b^14 + 354*a^9*b^16 - 81*a^7*b^18 - 21*a^5*b^20 + 17*a^3*b^22 - 3*a*b^24)
*d*cos(d*x + c)^3 - (a^25 + 12*a^23*b^2 - 118*a^21*b^4 + 364*a^19*b^6 - 441
*a^17*b^8 - 168*a^15*b^10 + 1260*a^13*b^12 - 1800*a^11*b^14 + 1311*a^9*b^16
- 484*a^7*b^18 + 42*a^5*b^20 + 28*a^3*b^22 - 7*a*b^24)*d*cos(d*x + c) + ((
a^18*b^7 - 9*a^16*b^9 + 36*a^14*b^11 - 84*a^12*b^13 + 126*a^10*b^15 - 126*a
^8*b^17 + 84*a^6*b^19 - 36*a^4*b^21 + 9*a^2*b^23 - b^25)*d*cos(d*x + c)^7 -
3*(7*a^20*b^5 - 62*a^18*b^7 + 243*a^16*b^9 - 552*a^14*b^11 + 798*a^12*b^13
- 756*a^10*b^15 + 462*a^8*b^17 - 168*a^6*b^19 + 27*a^4*b^21 + 2*a^2*b^23 -
b^25)*d*cos(d*x + c)^5 + (35*a^22*b^3 - 273*a^20*b^5 + 885*a^18*b^7 - 1455
*a^16*b^9 + 990*a^14*b^11 + 630*a^12*b^13 - 1974*a^10*b^15 + 1890*a^8*b^17
- 945*a^6*b^19 + 235*a^4*b^21 - 15*a^2*b^23 - 3*b^25)*d*cos(d*x + c)^3 - (7
*a^24*b - 28*a^22*b^3 - 42*a^20*b^5 + 484*a^18*b^7 - 1311*a^16*b^9 + 1800*a
^14*b^11 - 1260*a^12*b^13 + 168*a^10*b^15 + 441*a^8*b^17 - 364*a^6*b^19 + 1
18*a^4*b^21 - 12*a^2*b^23 - b^25)*d*cos(d*x + c))*sin(d*x + c))]

giac [B] time = 7.97, size = 2610, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/280*(315*(64*a^7*b^2 + 336*a^5*b^4 + 280*a^3*b^6 + 35*a*b^8)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^
2 - b^2)))/((a^16 - 8*a^14*b^2 + 28*a^12*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 5
6*a^6*b^10 + 28*a^4*b^12 - 8*a^2*b^14 + b^16)*sqrt(a^2 - b^2)) + 560*(a^8*t
an(1/2*d*x + 1/2*c) + 28*a^6*b^2*tan(1/2*d*x + 1/2*c) + 70*a^4*b^4*tan(1/2*
d*x + 1/2*c) + 28*a^2*b^6*tan(1/2*d*x + 1/2*c) + b^8*tan(1/2*d*x + 1/2*c) -
 8*a^7*b - 56*a^5*b^3 - 56*a^3*b^5 - 8*a*b^7)/((a^16 - 8*a^14*b^2 + 28*a^12
*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 56*a^6*b^10 + 28*a^4*b^12 - 8*a^2*b^14 +
b^16)*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (82320*a^18*b^4*tan(1/2*d*x + 1/2*c)^
13 + 41160*a^16*b^6*tan(1/2*d*x + 1/2*c)^13 + 49665*a^14*b^8*tan(1/2*d*x +
1/2*c)^13 - 31360*a^12*b^10*tan(1/2*d*x + 1/2*c)^13 + 15680*a^10*b^12*tan(1
/2*d*x + 1/2*c)^13 - 4480*a^8*b^14*tan(1/2*d*x + 1/2*c)^13 + 560*a^6*b^16*t
an(1/2*d*x + 1/2*c)^13 + 47040*a^19*b^3*tan(1/2*d*x + 1/2*c)^12 + 952560*a^
17*b^5*tan(1/2*d*x + 1/2*c)^12 + 743400*a^15*b^7*tan(1/2*d*x + 1/2*c)^12 +
370685*a^13*b^9*tan(1/2*d*x + 1/2*c)^12 - 188160*a^11*b^11*tan(1/2*d*x + 1/
2*c)^12 + 94080*a^9*b^13*tan(1/2*d*x + 1/2*c)^12 - 26880*a^7*b^15*tan(1/2*d
*x + 1/2*c)^12 + 3360*a^5*b^17*tan(1/2*d*x + 1/2*c)^12 + 987840*a^18*b^4*ta
n(1/2*d*x + 1/2*c)^11 + 5221440*a^16*b^6*tan(1/2*d*x + 1/2*c)^11 + 4792620*
a^14*b^8*tan(1/2*d*x + 1/2*c)^11 + 1272530*a^12*b^10*tan(1/2*d*x + 1/2*c)^1
1 - 501760*a^10*b^12*tan(1/2*d*x + 1/2*c)^11 + 277760*a^8*b^14*tan(1/2*d*x
+ 1/2*c)^11 - 85120*a^6*b^16*tan(1/2*d*x + 1/2*c)^11 + 11200*a^4*b^18*tan(1
/2*d*x + 1/2*c)^11 + 282240*a^19*b^3*tan(1/2*d*x + 1/2*c)^10 + 7056000*a^17
*b^5*tan(1/2*d*x + 1/2*c)^10 + 18695040*a^15*b^7*tan(1/2*d*x + 1/2*c)^10 +
15575140*a^13*b^9*tan(1/2*d*x + 1/2*c)^10 + 2689610*a^11*b^11*tan(1/2*d*x +
 1/2*c)^10 - 721280*a^9*b^13*tan(1/2*d*x + 1/2*c)^10 + 474880*a^7*b^15*tan(
1/2*d*x + 1/2*c)^10 - 160160*a^5*b^17*tan(1/2*d*x + 1/2*c)^10 + 22400*a^3*b
^19*tan(1/2*d*x + 1/2*c)^10 + 3704400*a^18*b^4*tan(1/2*d*x + 1/2*c)^9 + 269
48040*a^16*b^6*tan(1/2*d*x + 1/2*c)^9 + 46663365*a^14*b^8*tan(1/2*d*x + 1/2
*c)^9 + 29114330*a^12*b^10*tan(1/2*d*x + 1/2*c)^9 + 3411772*a^10*b^12*tan(1
/2*d*x + 1/2*c)^9 - 305536*a^8*b^14*tan(1/2*d*x + 1/2*c)^9 + 388976*a^6*b^1
6*tan(1/2*d*x + 1/2*c)^9 - 167552*a^4*b^18*tan(1/2*d*x + 1/2*c)^9 + 26880*a
^2*b^20*tan(1/2*d*x + 1/2*c)^9 + 705600*a^19*b^3*tan(1/2*d*x + 1/2*c)^8 + 1
8780720*a^17*b^5*tan(1/2*d*x + 1/2*c)^8 + 65305800*a^15*b^7*tan(1/2*d*x + 1
/2*c)^8 + 77673085*a^13*b^9*tan(1/2*d*x + 1/2*c)^8 + 32483570*a^11*b^11*tan
(1/2*d*x + 1/2*c)^8 + 2139928*a^9*b^13*tan(1/2*d*x + 1/2*c)^8 + 587776*a^7*
b^15*tan(1/2*d*x + 1/2*c)^8 - 7616*a^5*b^17*tan(1/2*d*x + 1/2*c)^8 - 74368*
a^3*b^19*tan(1/2*d*x + 1/2*c)^8 + 17920*a*b^21*tan(1/2*d*x + 1/2*c)^8 + 658
5600*a^18*b^4*tan(1/2*d*x + 1/2*c)^7 + 51038400*a^16*b^6*tan(1/2*d*x + 1/2*
c)^7 + 104499360*a^14*b^8*tan(1/2*d*x + 1/2*c)^7 + 80185140*a^12*b^10*tan(1
/2*d*x + 1/2*c)^7 + 20029744*a^10*b^12*tan(1/2*d*x + 1/2*c)^7 + 661136*a^8*
b^14*tan(1/2*d*x + 1/2*c)^7 + 683008*a^6*b^16*tan(1/2*d*x + 1/2*c)^7 - 2176
00*a^4*b^18*tan(1/2*d*x + 1/2*c)^7 + 13312*a^2*b^20*tan(1/2*d*x + 1/2*c)^7
+ 5120*b^22*tan(1/2*d*x + 1/2*c)^7 + 940800*a^19*b^3*tan(1/2*d*x + 1/2*c)^6
 + 23614080*a^17*b^5*tan(1/2*d*x + 1/2*c)^6 + 83805120*a^15*b^7*tan(1/2*d*x
 + 1/2*c)^6 + 103990880*a^13*b^9*tan(1/2*d*x + 1/2*c)^6 + 45853220*a^11*b^1
1*tan(1/2*d*x + 1/2*c)^6 + 4650688*a^9*b^13*tan(1/2*d*x + 1/2*c)^6 + 692496
*a^7*b^15*tan(1/2*d*x + 1/2*c)^6 - 7616*a^5*b^17*tan(1/2*d*x + 1/2*c)^6 - 7
4368*a^3*b^19*tan(1/2*d*x + 1/2*c)^6 + 17920*a*b^21*tan(1/2*d*x + 1/2*c)^6
+ 6174000*a^18*b^4*tan(1/2*d*x + 1/2*c)^5 + 43023960*a^16*b^6*tan(1/2*d*x +
 1/2*c)^5 + 82755435*a^14*b^8*tan(1/2*d*x + 1/2*c)^5 + 55248340*a^12*b^10*t
an(1/2*d*x + 1/2*c)^5 + 10337432*a^10*b^12*tan(1/2*d*x + 1/2*c)^5 - 175056*
a^8*b^14*tan(1/2*d*x + 1/2*c)^5 + 388976*a^6*b^16*tan(1/2*d*x + 1/2*c)^5 -
```

```
167552*a^4*b^18*tan(1/2*d*x + 1/2*c)^5 + 26880*a^2*b^20*tan(1/2*d*x + 1/2*c
)^5 + 705600*a^19*b^3*tan(1/2*d*x + 1/2*c)^4 + 14429520*a^17*b^5*tan(1/2*d*
x + 1/2*c)^4 + 42782712*a^15*b^7*tan(1/2*d*x + 1/2*c)^4 + 41655719*a^13*b^9
*tan(1/2*d*x + 1/2*c)^4 + 10567396*a^11*b^11*tan(1/2*d*x + 1/2*c)^4 - 70403
2*a^9*b^13*tan(1/2*d*x + 1/2*c)^4 + 485520*a^7*b^15*tan(1/2*d*x + 1/2*c)^4
- 160160*a^5*b^17*tan(1/2*d*x + 1/2*c)^4 + 22400*a^3*b^19*tan(1/2*d*x + 1/2
*c)^4 + 2963520*a^18*b^4*tan(1/2*d*x + 1/2*c)^3 + 14864640*a^16*b^6*tan(1/2
*d*x + 1/2*c)^3 + 20500788*a^14*b^8*tan(1/2*d*x + 1/2*c)^3 + 5857306*a^12*b
^10*tan(1/2*d*x + 1/2*c)^3 - 479696*a^10*b^12*tan(1/2*d*x + 1/2*c)^3 + 2812
32*a^8*b^14*tan(1/2*d*x + 1/2*c)^3 - 85120*a^6*b^16*tan(1/2*d*x + 1/2*c)^3
+ 11200*a^4*b^18*tan(1/2*d*x + 1/2*c)^3 + 282240*a^19*b^3*tan(1/2*d*x + 1/2
*c)^2 + 3575040*a^17*b^5*tan(1/2*d*x + 1/2*c)^2 + 6358464*a^15*b^7*tan(1/2*
d*x + 1/2*c)^2 + 1843996*a^13*b^9*tan(1/2*d*x + 1/2*c)^2 - 146062*a^11*b^11
*tan(1/2*d*x + 1/2*c)^2 + 85120*a^9*b^13*tan(1/2*d*x + 1/2*c)^2 - 25648*a^7
*b^15*tan(1/2*d*x + 1/2*c)^2 + 3360*a^5*b^17*tan(1/2*d*x + 1/2*c)^2 + 57624
0*a^18*b^4*tan(1/2*d*x + 1/2*c) + 1111320*a^16*b^6*tan(1/2*d*x + 1/2*c) + 3
24303*a^14*b^8*tan(1/2*d*x + 1/2*c) - 26894*a^12*b^10*tan(1/2*d*x + 1/2*c)
+ 14924*a^10*b^12*tan(1/2*d*x + 1/2*c) - 4368*a^8*b^14*tan(1/2*d*x + 1/2*c)
 + 560*a^6*b^16*tan(1/2*d*x + 1/2*c) + 47040*a^19*b^3 + 82320*a^17*b^5 + 26
712*a^15*b^7 - 4161*a^13*b^9 + 2186*a^11*b^11 - 632*a^9*b^13 + 80*a^7*b^15)
/((a^23 - 8*a^21*b^2 + 28*a^19*b^4 - 56*a^17*b^6 + 70*a^15*b^8 - 56*a^13*b^
10 + 28*a^11*b^12 - 8*a^9*b^14 + a^7*b^16)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*
tan(1/2*d*x + 1/2*c) + a)^7))/d
```

maple [B] time = 0.38, size = 7675, normalized size = 14.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 53.32, size = 3273, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^8),x)
```

```
[Out] - ((4480*a^14*b + 80*b^15 - 632*a^2*b^13 + 2186*a^4*b^11 - 4161*a^6*b^9 + 3
1192*a^8*b^7 + 113680*a^10*b^5 + 78400*a^12*b^3)/(280*(a^16 + b^16 - 8*a^2*
b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 -
 8*a^14*b^2)) - (9*tan(c/2 + (d*x)/2)^8*(560*b^15 + 10360*a^2*b^13 + 59766*
a^4*b^11 + 117497*a^6*b^9 + 91112*a^8*b^7 + 25200*a^10*b^5 + 2240*a^12*b^3)
)/(8*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 5
6*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) + (tan(c/2 + (d*x)/2)*(80*b^16 - 80
```

*a^16 - 624*a^2*b^14 + 2132*a^4*b^12 - 3842*a^6*b^10 + 55209*a^8*b^8 + 2192
40*a^10*b^6 + 139440*a^12*b^4 + 6720*a^14*b^2))/(40*a*(a^16 + b^16 - 8*a^2*
b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 -
 8*a^14*b^2)) + (tan(c/2 + (d*x)/2)^7*(5120*b^22 - 19600*a^22 - 13568*a^2*b
^20 - 50048*a^4*b^18 + 294032*a^6*b^16 + 1158752*a^8*b^14 + 11762072*a^10*b
^12 + 34250720*a^12*b^10 + 32332965*a^14*b^8 + 15431080*a^16*b^6 + 1234800*
a^18*b^4 + 235200*a^20*b^2))/(280*a^7*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^
12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) -
(tan(c/2 + (d*x)/2)^9*(19600*a^22 + 5120*b^22 - 13568*a^2*b^20 - 50048*a^4*
b^18 + 294032*a^6*b^16 + 1217552*a^8*b^14 + 21572852*a^10*b^12 + 69353690*a
^12*b^10 + 86769515*a^14*b^8 + 39441080*a^16*b^6 + 6762000*a^18*b^4 + 78400
*a^20*b^2))/(280*a^7*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10
+ 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) - (3*tan(c/2 + (d*x
)/2)^11*(560*a^20 + 1280*b^20 - 8512*a^2*b^18 + 22576*a^4*b^16 - 27776*a^6*
b^14 + 201292*a^8*b^12 + 1695400*a^10*b^10 + 2917285*a^12*b^8 + 1708840*a^1
4*b^6 + 311920*a^16*b^4 + 8960*a^18*b^2))/(40*a^5*(a^16 + b^16 - 8*a^2*b^14
 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a
^14*b^2)) + (3*tan(c/2 + (d*x)/2)^5*(1280*b^20 - 560*a^20 - 8512*a^2*b^18 +
 22576*a^4*b^16 - 21728*a^6*b^14 + 643528*a^8*b^12 + 3165074*a^10*b^10 + 43
25867*a^12*b^8 + 2252600*a^14*b^6 + 337680*a^16*b^4 + 17920*a^18*b^2))/(40*
a^5*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56
*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) - (tan(c/2 + (d*x)/2)^13*(112*a^18 +
 320*b^18 - 2448*a^2*b^16 + 8064*a^4*b^14 - 14784*a^6*b^12 + 38598*a^8*b^10
 + 171465*a^10*b^8 + 232680*a^12*b^6 + 58800*a^14*b^4 + 2688*a^16*b^2))/(8*
a^3*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56
*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) + (tan(c/2 + (d*x)/2)^3*(320*b^18 -
112*a^18 - 2448*a^2*b^16 + 8160*a^4*b^14 - 14132*a^6*b^12 + 202616*a^8*b^10
 + 800359*a^10*b^8 + 621880*a^12*b^6 + 133840*a^14*b^4 + 6272*a^16*b^2))/(8
a^3(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 5
6*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) - (tan(c/2 + (d*x)/2)^15*(16*a^16 +
 16*b^16 - 128*a^2*b^14 + 448*a^4*b^12 - 896*a^6*b^10 + 1435*a^8*b^8 + 1624
*a^10*b^6 + 3472*a^12*b^4 + 448*a^14*b^2))/(8*a*(a^16 + b^16 - 8*a^2*b^14 +
 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^1
4*b^2)) + (tan(c/2 + (d*x)/2)^6*(5600*a^20*b + 2560*b^21 - 13824*a^2*b^19 +
 21792*a^4*b^17 + 29568*a^6*b^15 + 997920*a^8*b^13 + 6528192*a^10*b^11 + 12
687263*a^12*b^9 + 9211384*a^14*b^7 + 2568720*a^16*b^5 + 168000*a^18*b^3))/(
40*a^6*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 -
 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) - (tan(c/2 + (d*x)/2)^10*(3360*a^
20*b + 2560*b^21 - 13824*a^2*b^19 + 21792*a^4*b^17 + 16128*a^6*b^15 + 46250
4*a^8*b^13 + 5492760*a^10*b^11 + 12382335*a^12*b^9 + 10502520*a^14*b^7 + 30
79440*a^16*b^5 + 257600*a^18*b^3))/(40*a^6*(a^16 + b^16 - 8*a^2*b^14 + 28*a
^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2
)) - (tan(c/2 + (d*x)/2)^12*(448*a^18*b + 640*b^19 - 4672*a^2*b^17 + 14336*
a^4*b^15 - 23296*a^6*b^13 + 86702*a^8*b^11 + 550445*a^10*b^9 + 787976*a^12*
b^7 + 312368*a^14*b^5 + 31808*a^16*b^3))/(8*a^4*(a^16 + b^16 - 8*a^2*b^14 +
 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^1
4*b^2)) + (tan(c/2 + (d*x)/2)^4*(6720*a^18*b + 3200*b^19 - 23360*a^2*b^17 +
 73024*a^4*b^15 - 112736*a^6*b^13 + 1866494*a^8*b^11 + 7831069*a^10*b^9 + 7
851144*a^12*b^7 + 2787120*a^14*b^5 + 212800*a^16*b^3))/(40*a^4*(a^16 + b^16
 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a
^12*b^4 - 8*a^14*b^2)) - (3*tan(c/2 + (d*x)/2)^14*(32*a^16*b + 32*b^17 - 25
6*a^2*b^15 + 896*a^4*b^13 - 1792*a^6*b^11 + 3605*a^8*b^9 + 9128*a^10*b^7 +
14000*a^12*b^5 + 2240*a^14*b^3))/(8*a^2*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*
b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2))
+ (3*tan(c/2 + (d*x)/2)^2*(7840*a^16*b + 1120*b^17 - 8576*a^2*b^15 + 28584*
a^4*b^13 - 49416*a^6*b^11 + 738879*a^8*b^9 + 2925944*a^10*b^7 + 1932560*a^1
2*b^5 + 203840*a^14*b^3))/(280*a^2*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12
- 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)))/(d*(
tan(c/2 + (d*x)/2)^5*(126*a^6*b + 672*a^2*b^5 + 840*a^4*b^3) - tan(c/2 + (d

```
*x)/2)^11*(126*a^6*b + 672*a^2*b^5 + 840*a^4*b^3) - a^7*tan(c/2 + (d*x)/2)^
16 + tan(c/2 + (d*x)/2)^3*(70*a^6*b + 280*a^4*b^3) - tan(c/2 + (d*x)/2)^13*
(70*a^6*b + 280*a^4*b^3) + tan(c/2 + (d*x)/2)^6*(448*a*b^6 + 14*a^7 + 1120*
a^3*b^4 + 420*a^5*b^2) - tan(c/2 + (d*x)/2)^10*(448*a*b^6 + 14*a^7 + 1120*a
^3*b^4 + 420*a^5*b^2) + tan(c/2 + (d*x)/2)^7*(70*a^6*b + 128*b^7 + 672*a^2*
b^5 + 560*a^4*b^3) - tan(c/2 + (d*x)/2)^9*(70*a^6*b + 128*b^7 + 672*a^2*b^5
 + 560*a^4*b^3) + a^7 + tan(c/2 + (d*x)/2)^4*(14*a^7 + 560*a^3*b^4 + 336*a^
5*b^2) - tan(c/2 + (d*x)/2)^12*(14*a^7 + 560*a^3*b^4 + 336*a^5*b^2) + tan(c
/2 + (d*x)/2)^2*(6*a^7 + 84*a^5*b^2) - tan(c/2 + (d*x)/2)^14*(6*a^7 + 84*a^
5*b^2) + 14*a^6*b*tan(c/2 + (d*x)/2) - 14*a^6*b*tan(c/2 + (d*x)/2)^15)) - (
9*a*b^2*atan(((9*a*b^2*(64*a^6 + 35*b^6 + 280*a^2*b^4 + 336*a^4*b^2)*(16*a^
16*b + 16*b^17 - 128*a^2*b^15 + 448*a^4*b^13 - 896*a^6*b^11 + 1120*a^8*b^9
- 896*a^10*b^7 + 448*a^12*b^5 - 128*a^14*b^3))/(16*(a + b)^(17/2)*(a - b)^(
17/2)) + (9*a^2*b^2*tan(c/2 + (d*x)/2)*(64*a^6 + 35*b^6 + 280*a^2*b^4 + 336
*a^4*b^2)*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^
8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2))/((a + b)^(17/2)*(a - b)^(17/2)
))/(315*a*b^8 + 2520*a^3*b^6 + 3024*a^5*b^4 + 576*a^7*b^2))*(64*a^6 + 35*b^
6 + 280*a^2*b^4 + 336*a^4*b^2))/(8*d*(a + b)^(17/2)*(a - b)^(17/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.472 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=653

$$\frac{ab\left(118a^2+103b^2\right)\sec^3(c+dx)}{56d\left(a^2-b^2\right)^4(a+b\sin(c+dx))^4}+\frac{b\left(13a^2+4b^2\right)\sec^3(c+dx)}{14d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^5}+\frac{17ab\sec^3(c+dx)}{42d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^6}+\frac{7}{}$$

```
[Out] 165/8*a*b^4*(32*a^6+112*a^4*b^2+70*a^2*b^4+7*b^6)*arctan((b+a*tan(1/2*d*x+1
/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(19/2)/d+1/7*b*sec(d*x+c)^3/(a^2-b^2)/d/(
a+b*sin(d*x+c))^7+17/42*a*b*sec(d*x+c)^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^6+1
/14*b*(13*a^2+4*b^2)*sec(d*x+c)^3/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^5+1/56*a*b
*(118*a^2+103*b^2)*sec(d*x+c)^3/(a^2-b^2)^4/d/(a+b*sin(d*x+c))^4+1/168*b*(8
82*a^4+1421*a^2*b^2+128*b^4)*sec(d*x+c)^3/(a^2-b^2)^5/d/(a+b*sin(d*x+c))^3+
13/112*a*b*(140*a^4+336*a^2*b^2+85*b^4)*sec(d*x+c)^3/(a^2-b^2)^6/d/(a+b*sin
(d*x+c))^2+1/112*b*(9212*a^6+28420*a^4*b^2+12907*a^2*b^4+512*b^6)*sec(d*x+c
)^3/(a^2-b^2)^7/d/(a+b*sin(d*x+c))-1/336*sec(d*x+c)^3*(1155*a*b*(32*a^6+112
*a^4*b^2+70*a^2*b^4+7*b^6)-(112*a^8+52528*a^6*b^2+142902*a^4*b^4+57665*a^2*
b^6+2048*b^8)*sin(d*x+c))/(a^2-b^2)^8/d+1/336*sec(d*x+c)*(3465*a*b^3*(32*a^
6+112*a^4*b^2+70*a^2*b^4+7*b^6)+(224*a^10-6048*a^8*b^2-207332*a^6*b^4-41302
4*a^4*b^6-135489*a^2*b^8-4096*b^10)*sin(d*x+c))/(a^2-b^2)^9/d
```

Rubi [A] time = 2.14, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2694, 2864, 2866, 12, 2660, 618, 204}

$$\frac{165ab^4\left(112a^4b^2+70a^2b^4+32a^6+7b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{19/2}}+\frac{b\left(28420a^4b^2+12907a^2b^4+9212a^6+512b^6\right.}{112d\left(a^2-b^2\right)^7(a+b\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (165*a*b^4*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(19/2)*d) + (b*Sec[c + d*x]^3)
/(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (17*a*b*Sec[c + d*x]^3)/(42*(a^
2 - b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(13*a^2 + 4*b^2)*Sec[c + d*x]^3)/
(14*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^5) + (a*b*(118*a^2 + 103*b^2)*Sec[
c + d*x]^3)/(56*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^4) + (b*(882*a^4 + 142
1*a^2*b^2 + 128*b^4)*Sec[c + d*x]^3)/(168*(a^2 - b^2)^5*d*(a + b*Sin[c + d*
x])^3) + (13*a*b*(140*a^4 + 336*a^2*b^2 + 85*b^4)*Sec[c + d*x]^3)/(112*(a^2
 - b^2)^6*d*(a + b*Sin[c + d*x])^2) + (b*(9212*a^6 + 28420*a^4*b^2 + 12907*
a^2*b^4 + 512*b^6)*Sec[c + d*x]^3)/(112*(a^2 - b^2)^7*d*(a + b*Sin[c + d*x]
)) - (Sec[c + d*x]^3*(1155*a*b*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)
- (112*a^8 + 52528*a^6*b^2 + 142902*a^4*b^4 + 57665*a^2*b^6 + 2048*b^8)*Sin
[c + d*x]))/(336*(a^2 - b^2)^8*d) + (Sec[c + d*x]*(3465*a*b^3*(32*a^6 + 112
*a^4*b^2 + 70*a^2*b^4 + 7*b^6) + (224*a^10 - 6048*a^8*b^2 - 207332*a^6*b^4
- 413024*a^4*b^6 - 135489*a^2*b^8 - 4096*b^10)*Sin[c + d*x]))/(336*(a^2 - b
^2)^9*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} - \frac{\int \frac{\sec^4(c+dx)(-7a+10b\sin(c+dx))}{(a+b\sin(c+dx))^7}dx}{7(a^2-b^2)}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \int \frac{\sec^4(c+ \ldots}{\ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13 \ldots}{14(a^2- \ldots}$$

$$= \frac{165ab^4(32a^6+112a^4b^2+70a^2b^4+7b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8(a^2-b^2)^{19/2}\,d} + \frac{b\sec^3 \ldots}{7(a^2-b^2)\,d(a \ldots}$$

Mathematica [A] time = 6.00, size = 597, normalized size = 0.91

$$\frac{\frac{2ab^5(2138a^2+925b^2)\cos(c+dx)}{(a^2-b^2)^6(a+b\sin(c+dx))^4} + \frac{8b^5(167a^2+24b^2)\cos(c+dx)}{(a^2-b^2)^5(a+b\sin(c+dx))^5} + \frac{328ab^5\cos(c+dx)}{(a^2-b^2)^4(a+b\sin(c+dx))^6} + \frac{48b^5\cos(c+dx)}{(a^2-b^2)^3(a+b\sin(c+dx))^7} + \frac{ab^5(33284a^4+488 \ldots}{(a^2-b^2) \ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((6930*a*b^4*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(19/2) + (48*b^5*Cos[c + d*x])/
((a^2 - b^2)^3*(a + b*Sin[c + d*x])^7) + (328*a*b^5*Cos[c + d*x])/((a^2 - b
^2)^4*(a + b*Sin[c + d*x])^6) + (8*b^5*(167*a^2 + 24*b^2)*Cos[c + d*x])/((a
^2 - b^2)^5*(a + b*Sin[c + d*x])^5) + (2*a*b^5*(2138*a^2 + 925*b^2)*Cos[c +
 d*x])/((a^2 - b^2)^6*(a + b*Sin[c + d*x])^4) + (2*b^5*(6058*a^4 + 5273*a^2
*b^2 + 296*b^4)*Cos[c + d*x])/((a^2 - b^2)^7*(a + b*Sin[c + d*x])^3) + (a*b
^5*(33284*a^4 + 48820*a^2*b^2 + 8287*b^4)*Cos[c + d*x])/((a^2 - b^2)^8*(a +
 b*Sin[c + d*x])^2) + (b^5*(103844*a^6 + 234272*a^4*b^2 + 81057*a^2*b^4 + 2
528*b^6)*Cos[c + d*x])/((a^2 - b^2)^9*(a + b*Sin[c + d*x])) + (112*Sec[c +
d*x]^3*(-8*a*b*(a^6 + 7*a^4*b^2 + 7*a^2*b^4 + b^6) + (a^8 + 28*a^6*b^2 + 70
*a^4*b^4 + 28*a^2*b^6 + b^8)*Sin[c + d*x]))/(a^2 - b^2)^8 + (224*Sec[c + d*
x]*(12*(15*a^7*b^3 + 63*a^5*b^5 + 45*a^3*b^7 + 5*a*b^9) + (a^10 - 27*a^8*b^
2 - 462*a^6*b^4 - 798*a^4*b^6 - 243*a^2*b^8 - 7*b^10)*Sin[c + d*x]))/(a^2 -
 b^2)^9)/(336*d)
```

fricas [B] time = 2.23, size = 4500, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/672*(224*a^18*b - 2016*a^16*b^3 + 8064*a^14*b^5 - 18816*a^12*b^7 + 28224
*a^10*b^9 - 28224*a^8*b^11 + 18816*a^6*b^13 - 8064*a^4*b^15 + 2016*a^2*b^17
 - 224*b^19 - 2*(224*a^12*b^7 - 6272*a^10*b^9 - 201284*a^8*b^11 - 205692*a^
6*b^13 + 277535*a^4*b^15 + 131393*a^2*b^17 + 4096*b^19)*cos(d*x + c)^10 + 2
8*(336*a^14*b^5 - 9352*a^12*b^7 - 252014*a^10*b^9 - 230159*a^8*b^11 + 29731
2*a^6*b^13 + 165122*a^4*b^15 + 27731*a^2*b^17 + 1024*b^19)*cos(d*x + c)^8 -
 70*(224*a^16*b^3 - 5936*a^14*b^5 - 126448*a^12*b^7 - 243082*a^10*b^9 - 297
47*a^8*b^11 + 284285*a^6*b^13 + 109607*a^4*b^15 + 10585*a^2*b^17 + 512*b^19
)*cos(d*x + c)^6 + 28*(112*a^18*b - 2296*a^16*b^3 - 35224*a^14*b^5 - 308392
*a^12*b^7 - 337750*a^10*b^9 + 149783*a^8*b^11 + 394751*a^6*b^13 + 130949*a^
4*b^15 + 7427*a^2*b^17 + 640*b^19)*cos(d*x + c)^4 - 224*(7*a^18*b - 46*a^16
*b^3 + 116*a^14*b^5 - 112*a^12*b^7 - 70*a^10*b^9 + 308*a^8*b^11 - 364*a^6*b
^13 + 224*a^4*b^15 - 73*a^2*b^17 + 10*b^19)*cos(d*x + c)^2 + 3465*(7*(32*a^
8*b^10 + 112*a^6*b^12 + 70*a^4*b^14 + 7*a^2*b^16)*cos(d*x + c)^9 - 7*(160*a
^10*b^8 + 656*a^8*b^10 + 686*a^6*b^12 + 245*a^4*b^14 + 21*a^2*b^16)*cos(d*x
 + c)^7 + 7*(96*a^12*b^6 + 656*a^10*b^8 + 1426*a^8*b^10 + 1057*a^6*b^12 + 2
80*a^4*b^14 + 21*a^2*b^16)*cos(d*x + c)^5 - (32*a^14*b^4 + 784*a^12*b^6 + 3
542*a^10*b^8 + 5621*a^8*b^10 + 3381*a^6*b^12 + 735*a^4*b^14 + 49*a^2*b^16)*
cos(d*x + c)^3 + ((32*a^7*b^11 + 112*a^5*b^13 + 70*a^3*b^15 + 7*a*b^17)*cos
(d*x + c)^9 - 3*(224*a^9*b^9 + 816*a^7*b^11 + 602*a^5*b^13 + 119*a^3*b^15 +
 7*a*b^17)*cos(d*x + c)^7 + (1120*a^11*b^7 + 5264*a^9*b^9 + 7250*a^7*b^11 +
 3521*a^5*b^13 + 504*a^3*b^15 + 21*a*b^17)*cos(d*x + c)^5 - (224*a^13*b^5 +
 1904*a^11*b^7 + 5082*a^9*b^9 + 4883*a^7*b^11 + 1827*a^5*b^13 + 217*a^3*b^1
5 + 7*a*b^17)*cos(d*x + c)^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*s
in(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2)) - 14*(16*a^19 - 144*a^17*b^2 + 576*a^15*b^4 - 1
344*a^13*b^6 + 2016*a^11*b^8 - 2016*a^9*b^10 + 1344*a^7*b^12 - 576*a^5*b^14
 + 144*a^3*b^16 - 16*a*b^18 - (224*a^13*b^6 - 6272*a^11*b^8 - 185444*a^9*b^
10 - 166092*a^7*b^12 + 256745*a^5*b^14 + 100208*a^3*b^16 + 631*a*b^18)*cos(
d*x + c)^8 + 10*(112*a^15*b^4 - 3080*a^13*b^6 - 73962*a^11*b^8 - 78323*a^9*
b^10 + 60829*a^7*b^12 + 73923*a^5*b^14 + 20401*a^3*b^16 + 100*a*b^18)*cos(d
*x + c)^6 - 3*(224*a^17*b^2 - 5712*a^15*b^4 - 95648*a^13*b^6 - 254254*a^11*
b^8 - 120855*a^9*b^10 + 282886*a^7*b^12 + 157892*a^5*b^14 + 35448*a^3*b^16
+ 19*a*b^18)*cos(d*x + c)^4 + 16*(2*a^19 - 35*a^17*b^2 + 208*a^15*b^4 - 644
```

*a^13*b^6 + 1204*a^11*b^8 - 1442*a^9*b^10 + 1120*a^7*b^12 - 548*a^5*b^14 +
154*a^3*b^16 - 19*a*b^18)*cos(d*x + c)^2)*sin(d*x + c))/(7*(a^21*b^6 - 10*a
^19*b^8 + 45*a^17*b^10 - 120*a^15*b^12 + 210*a^13*b^14 - 252*a^11*b^16 + 21
0*a^9*b^18 - 120*a^7*b^20 + 45*a^5*b^22 - 10*a^3*b^24 + a*b^26)*d*cos(d*x +
c)^9 - 7*(5*a^23*b^4 - 47*a^21*b^6 + 195*a^19*b^8 - 465*a^17*b^10 + 690*a^
15*b^12 - 630*a^13*b^14 + 294*a^11*b^16 + 30*a^9*b^18 - 135*a^7*b^20 + 85*a
^5*b^22 - 25*a^3*b^24 + 3*a*b^26)*d*cos(d*x + c)^7 + 7*(3*a^25*b^2 - 20*a^2
3*b^4 + 38*a^21*b^6 + 60*a^19*b^8 - 435*a^17*b^10 + 984*a^15*b^12 - 1260*a^
13*b^14 + 984*a^11*b^16 - 435*a^9*b^18 + 60*a^7*b^20 + 38*a^5*b^22 - 20*a^3
*b^24 + 3*a*b^26)*d*cos(d*x + c)^5 - (a^27 + 11*a^25*b^2 - 130*a^23*b^4 + 4
82*a^21*b^6 - 805*a^19*b^8 + 273*a^17*b^10 + 1428*a^15*b^12 - 3060*a^13*b^1
4 + 3111*a^11*b^16 - 1795*a^9*b^18 + 526*a^7*b^20 - 14*a^5*b^22 - 35*a^3*b^
24 + 7*a*b^26)*d*cos(d*x + c)^3 + ((a^20*b^7 - 10*a^18*b^9 + 45*a^16*b^11 -
120*a^14*b^13 + 210*a^12*b^15 - 252*a^10*b^17 + 210*a^8*b^19 - 120*a^6*b^2
1 + 45*a^4*b^23 - 10*a^2*b^25 + b^27)*d*cos(d*x + c)^9 - 3*(7*a^22*b^5 - 69
*a^20*b^7 + 305*a^18*b^9 - 795*a^16*b^11 + 1350*a^14*b^13 - 1554*a^12*b^15
+ 1218*a^10*b^17 - 630*a^8*b^19 + 195*a^6*b^21 - 25*a^4*b^23 - 3*a^2*b^25 +
b^27)*d*cos(d*x + c)^7 + (35*a^24*b^3 - 308*a^22*b^5 + 1158*a^20*b^7 - 234
0*a^18*b^9 + 2445*a^16*b^11 - 360*a^14*b^13 - 2604*a^12*b^15 + 3864*a^10*b^
17 - 2835*a^8*b^19 + 1180*a^6*b^21 - 250*a^4*b^23 + 12*a^2*b^25 + 3*b^27)*d
*cos(d*x + c)^5 - (7*a^26*b - 35*a^24*b^3 - 14*a^22*b^5 + 526*a^20*b^7 - 17
95*a^18*b^9 + 3111*a^16*b^11 - 3060*a^14*b^13 + 1428*a^12*b^15 + 273*a^10*b
^17 - 805*a^8*b^19 + 482*a^6*b^21 - 130*a^4*b^23 + 11*a^2*b^25 + b^27)*d*co
s(d*x + c)^3)*sin(d*x + c)), 1/336*(112*a^18*b - 1008*a^16*b^3 + 4032*a^14*
b^5 - 9408*a^12*b^7 + 14112*a^10*b^9 - 14112*a^8*b^11 + 9408*a^6*b^13 - 403
2*a^4*b^15 + 1008*a^2*b^17 - 112*b^19 - (224*a^12*b^7 - 6272*a^10*b^9 - 201
284*a^8*b^11 - 205692*a^6*b^13 + 277535*a^4*b^15 + 131393*a^2*b^17 + 4096*b
^19)*cos(d*x + c)^10 + 14*(336*a^14*b^5 - 9352*a^12*b^7 - 252014*a^10*b^9 -
230159*a^8*b^11 + 297312*a^6*b^13 + 165122*a^4*b^15 + 27731*a^2*b^17 + 102
4*b^19)*cos(d*x + c)^8 - 35*(224*a^16*b^3 - 5936*a^14*b^5 - 126448*a^12*b^7
- 243082*a^10*b^9 - 29747*a^8*b^11 + 284285*a^6*b^13 + 109607*a^4*b^15 + 1
0585*a^2*b^17 + 512*b^19)*cos(d*x + c)^6 + 14*(112*a^18*b - 2296*a^16*b^3 -
35224*a^14*b^5 - 308392*a^12*b^7 - 337750*a^10*b^9 + 149783*a^8*b^11 + 394
751*a^6*b^13 + 130949*a^4*b^15 + 7427*a^2*b^17 + 640*b^19)*cos(d*x + c)^4 -
112*(7*a^18*b - 46*a^16*b^3 + 116*a^14*b^5 - 112*a^12*b^7 - 70*a^10*b^9 +
308*a^8*b^11 - 364*a^6*b^13 + 224*a^4*b^15 - 73*a^2*b^17 + 10*b^19)*cos(d*x
+ c)^2 - 3465*(7*(32*a^8*b^10 + 112*a^6*b^12 + 70*a^4*b^14 + 7*a^2*b^16)*c
os(d*x + c)^9 - 7*(160*a^10*b^8 + 656*a^8*b^10 + 686*a^6*b^12 + 245*a^4*b^1
4 + 21*a^2*b^16)*cos(d*x + c)^7 + 7*(96*a^12*b^6 + 656*a^10*b^8 + 1426*a^8*
b^10 + 1057*a^6*b^12 + 280*a^4*b^14 + 21*a^2*b^16)*cos(d*x + c)^5 - (32*a^1
4*b^4 + 784*a^12*b^6 + 3542*a^10*b^8 + 5621*a^8*b^10 + 3381*a^6*b^12 + 735*
a^4*b^14 + 49*a^2*b^16)*cos(d*x + c)^3 + ((32*a^7*b^11 + 112*a^5*b^13 + 70*
a^3*b^15 + 7*a*b^17)*cos(d*x + c)^9 - 3*(224*a^9*b^9 + 816*a^7*b^11 + 602*a
^5*b^13 + 119*a^3*b^15 + 7*a*b^17)*cos(d*x + c)^7 + (1120*a^11*b^7 + 5264*a
^9*b^9 + 7250*a^7*b^11 + 3521*a^5*b^13 + 504*a^3*b^15 + 21*a*b^17)*cos(d*x
+ c)^5 - (224*a^13*b^5 + 1904*a^11*b^7 + 5082*a^9*b^9 + 4883*a^7*b^11 + 182
7*a^5*b^13 + 217*a^3*b^15 + 7*a*b^17)*cos(d*x + c)^3)*sin(d*x + c))*sqrt(a^
2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 7*(
16*a^19 - 144*a^17*b^2 + 576*a^15*b^4 - 1344*a^13*b^6 + 2016*a^11*b^8 - 201
6*a^9*b^10 + 1344*a^7*b^12 - 576*a^5*b^14 + 144*a^3*b^16 - 16*a*b^18 - (224
*a^13*b^6 - 6272*a^11*b^8 - 185444*a^9*b^10 - 166092*a^7*b^12 + 256745*a^5*
b^14 + 100208*a^3*b^16 + 631*a*b^18)*cos(d*x + c)^8 + 10*(112*a^15*b^4 - 30
80*a^13*b^6 - 73962*a^11*b^8 - 78323*a^9*b^10 + 60829*a^7*b^12 + 73923*a^5*
b^14 + 20401*a^3*b^16 + 100*a*b^18)*cos(d*x + c)^6 - 3*(224*a^17*b^2 - 5712
*a^15*b^4 - 95648*a^13*b^6 - 254254*a^11*b^8 - 120855*a^9*b^10 + 282886*a^7
*b^12 + 157892*a^5*b^14 + 35448*a^3*b^16 + 19*a*b^18)*cos(d*x + c)^4 + 16*(
2*a^19 - 35*a^17*b^2 + 208*a^15*b^4 - 644*a^13*b^6 + 1204*a^11*b^8 - 1442*a
^9*b^10 + 1120*a^7*b^12 - 548*a^5*b^14 + 154*a^3*b^16 - 19*a*b^18)*cos(d*x
+ c)^2)*sin(d*x + c))/(7*(a^21*b^6 - 10*a^19*b^8 + 45*a^17*b^10 - 120*a^15*

```
b^12 + 210*a^13*b^14 - 252*a^11*b^16 + 210*a^9*b^18 - 120*a^7*b^20 + 45*a^5
*b^22 - 10*a^3*b^24 + a*b^26)*d*cos(d*x + c)^9 - 7*(5*a^23*b^4 - 47*a^21*b^
6 + 195*a^19*b^8 - 465*a^17*b^10 + 690*a^15*b^12 - 630*a^13*b^14 + 294*a^11
*b^16 + 30*a^9*b^18 - 135*a^7*b^20 + 85*a^5*b^22 - 25*a^3*b^24 + 3*a*b^26)*
d*cos(d*x + c)^7 + 7*(3*a^25*b^2 - 20*a^23*b^4 + 38*a^21*b^6 + 60*a^19*b^8
- 435*a^17*b^10 + 984*a^15*b^12 - 1260*a^13*b^14 + 984*a^11*b^16 - 435*a^9*
b^18 + 60*a^7*b^20 + 38*a^5*b^22 - 20*a^3*b^24 + 3*a*b^26)*d*cos(d*x + c)^5
 - (a^27 + 11*a^25*b^2 - 130*a^23*b^4 + 482*a^21*b^6 - 805*a^19*b^8 + 273*a
^17*b^10 + 1428*a^15*b^12 - 3060*a^13*b^14 + 3111*a^11*b^16 - 1795*a^9*b^18
 + 526*a^7*b^20 - 14*a^5*b^22 - 35*a^3*b^24 + 7*a*b^26)*d*cos(d*x + c)^3 +
((a^20*b^7 - 10*a^18*b^9 + 45*a^16*b^11 - 120*a^14*b^13 + 210*a^12*b^15 - 2
52*a^10*b^17 + 210*a^8*b^19 - 120*a^6*b^21 + 45*a^4*b^23 - 10*a^2*b^25 + b^
27)*d*cos(d*x + c)^9 - 3*(7*a^22*b^5 - 69*a^20*b^7 + 305*a^18*b^9 - 795*a^1
6*b^11 + 1350*a^14*b^13 - 1554*a^12*b^15 + 1218*a^10*b^17 - 630*a^8*b^19 +
195*a^6*b^21 - 25*a^4*b^23 - 3*a^2*b^25 + b^27)*d*cos(d*x + c)^7 + (35*a^24
*b^3 - 308*a^22*b^5 + 1158*a^20*b^7 - 2340*a^18*b^9 + 2445*a^16*b^11 - 360*
a^14*b^13 - 2604*a^12*b^15 + 3864*a^10*b^17 - 2835*a^8*b^19 + 1180*a^6*b^21
 - 250*a^4*b^23 + 12*a^2*b^25 + 3*b^27)*d*cos(d*x + c)^5 - (7*a^26*b - 35*a
^24*b^3 - 14*a^22*b^5 + 526*a^20*b^7 - 1795*a^18*b^9 + 3111*a^16*b^11 - 306
0*a^14*b^13 + 1428*a^12*b^15 + 273*a^10*b^17 - 805*a^8*b^19 + 482*a^6*b^21
- 130*a^4*b^23 + 11*a^2*b^25 + b^27)*d*cos(d*x + c)^3)*sin(d*x + c))]
```

giac [B] time = 20.51, size = 3047, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/168*(3465*(32*a^7*b^4 + 112*a^5*b^6 + 70*a^3*b^8 + 7*a*b^10)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2
 - b^2)))/((a^18 - 9*a^16*b^2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 -
126*a^8*b^10 + 84*a^6*b^12 - 36*a^4*b^14 + 9*a^2*b^16 - b^18)*sqrt(a^2 - b^
2)) - 112*(3*a^10*tan(1/2*d*x + 1/2*c)^5 - 27*a^8*b^2*tan(1/2*d*x + 1/2*c)^
5 - 882*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 1638*a^4*b^6*tan(1/2*d*x + 1/2*c)^
5 - 513*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 15*b^10*tan(1/2*d*x + 1/2*c)^5 - 2
4*a^9*b*tan(1/2*d*x + 1/2*c)^4 + 216*a^7*b^3*tan(1/2*d*x + 1/2*c)^4 + 1512*
a^5*b^5*tan(1/2*d*x + 1/2*c)^4 + 1224*a^3*b^7*tan(1/2*d*x + 1/2*c)^4 + 144*
a*b^9*tan(1/2*d*x + 1/2*c)^4 - 2*a^10*tan(1/2*d*x + 1/2*c)^3 + 162*a^8*b^2*
tan(1/2*d*x + 1/2*c)^3 + 1932*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 + 3108*a^4*b^6
*tan(1/2*d*x + 1/2*c)^3 + 918*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 + 26*b^10*tan(
1/2*d*x + 1/2*c)^3 - 720*a^7*b^3*tan(1/2*d*x + 1/2*c)^2 - 3024*a^5*b^5*tan(
1/2*d*x + 1/2*c)^2 - 2160*a^3*b^7*tan(1/2*d*x + 1/2*c)^2 - 240*a*b^9*tan(1/
2*d*x + 1/2*c)^2 + 3*a^10*tan(1/2*d*x + 1/2*c) - 27*a^8*b^2*tan(1/2*d*x + 1
/2*c) - 882*a^6*b^4*tan(1/2*d*x + 1/2*c) - 1638*a^4*b^6*tan(1/2*d*x + 1/2*c
) - 513*a^2*b^8*tan(1/2*d*x + 1/2*c) - 15*b^10*tan(1/2*d*x + 1/2*c) - 8*a^9
*b + 312*a^7*b^3 + 1512*a^5*b^5 + 1128*a^3*b^7 + 128*a*b^9)/((a^18 - 9*a^16
*b^2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 - 126*a^8*b^10 + 84*a^6*b^1
2 - 36*a^4*b^14 + 9*a^2*b^16 - b^18)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3) + (232
848*a^18*b^6*tan(1/2*d*x + 1/2*c)^13 + 142758*a^16*b^8*tan(1/2*d*x + 1/2*c)
^13 + 64911*a^14*b^10*tan(1/2*d*x + 1/2*c)^13 - 28224*a^12*b^12*tan(1/2*d*x
 + 1/2*c)^13 + 12096*a^10*b^14*tan(1/2*d*x + 1/2*c)^13 - 3024*a^8*b^16*tan(
1/2*d*x + 1/2*c)^13 + 336*a^6*b^18*tan(1/2*d*x + 1/2*c)^13 + 155232*a^19*b^
5*tan(1/2*d*x + 1/2*c)^12 + 2783088*a^17*b^7*tan(1/2*d*x + 1/2*c)^12 + 2110
878*a^15*b^9*tan(1/2*d*x + 1/2*c)^12 + 545811*a^13*b^11*tan(1/2*d*x + 1/2*c
)^12 - 169344*a^11*b^13*tan(1/2*d*x + 1/2*c)^12 + 72576*a^9*b^15*tan(1/2*d*
x + 1/2*c)^12 - 18144*a^7*b^17*tan(1/2*d*x + 1/2*c)^12 + 2016*a^5*b^19*tan(
1/2*d*x + 1/2*c)^12 + 3104640*a^18*b^6*tan(1/2*d*x + 1/2*c)^11 + 15506568*a
^16*b^8*tan(1/2*d*x + 1/2*c)^11 + 12397616*a^14*b^10*tan(1/2*d*x + 1/2*c)^1
1 + 2172366*a^12*b^12*tan(1/2*d*x + 1/2*c)^11 - 451584*a^10*b^14*tan(1/2*d*
```

```
x + 1/2*c)^11 + 213696*a^8*b^16*tan(1/2*d*x + 1/2*c)^11 - 57344*a^6*b^18*ta
n(1/2*d*x + 1/2*c)^11 + 6720*a^4*b^20*tan(1/2*d*x + 1/2*c)^11 + 931392*a^19
*b^5*tan(1/2*d*x + 1/2*c)^10 + 22042944*a^17*b^7*tan(1/2*d*x + 1/2*c)^10 +
54377400*a^15*b^9*tan(1/2*d*x + 1/2*c)^10 + 38316040*a^13*b^11*tan(1/2*d*x
+ 1/2*c)^10 + 5346390*a^11*b^13*tan(1/2*d*x + 1/2*c)^10 - 685440*a^9*b^15*t
an(1/2*d*x + 1/2*c)^10 + 372960*a^7*b^17*tan(1/2*d*x + 1/2*c)^10 - 108640*a
^5*b^19*tan(1/2*d*x + 1/2*c)^10 + 13440*a^3*b^21*tan(1/2*d*x + 1/2*c)^10 +
12030480*a^18*b^6*tan(1/2*d*x + 1/2*c)^9 + 83208510*a^16*b^8*tan(1/2*d*x +
1/2*c)^9 + 129442775*a^14*b^10*tan(1/2*d*x + 1/2*c)^9 + 68997390*a^12*b^12*
tan(1/2*d*x + 1/2*c)^9 + 8026116*a^10*b^14*tan(1/2*d*x + 1/2*c)^9 - 418320*
a^8*b^16*tan(1/2*d*x + 1/2*c)^9 + 328720*a^6*b^18*tan(1/2*d*x + 1/2*c)^9 -
115584*a^4*b^20*tan(1/2*d*x + 1/2*c)^9 + 16128*a^2*b^22*tan(1/2*d*x + 1/2*c
)^9 + 2328480*a^19*b^5*tan(1/2*d*x + 1/2*c)^8 + 60558960*a^17*b^7*tan(1/2*d
*x + 1/2*c)^8 + 194655230*a^15*b^9*tan(1/2*d*x + 1/2*c)^8 + 204067311*a^13*
b^11*tan(1/2*d*x + 1/2*c)^8 + 74359166*a^11*b^13*tan(1/2*d*x + 1/2*c)^8 + 6
423144*a^9*b^15*tan(1/2*d*x + 1/2*c)^8 + 342720*a^7*b^17*tan(1/2*d*x + 1/2*
c)^8 + 38080*a^5*b^19*tan(1/2*d*x + 1/2*c)^8 - 54656*a^3*b^21*tan(1/2*d*x +
 1/2*c)^8 + 10752*a*b^23*tan(1/2*d*x + 1/2*c)^8 + 21732480*a^18*b^6*tan(1/2
*d*x + 1/2*c)^7 + 160923840*a^16*b^8*tan(1/2*d*x + 1/2*c)^7 + 294582904*a^1
4*b^10*tan(1/2*d*x + 1/2*c)^7 + 198535596*a^12*b^12*tan(1/2*d*x + 1/2*c)^7
+ 45251248*a^10*b^14*tan(1/2*d*x + 1/2*c)^7 + 2197104*a^8*b^16*tan(1/2*d*x
+ 1/2*c)^7 + 545280*a^6*b^18*tan(1/2*d*x + 1/2*c)^7 - 137728*a^4*b^20*tan(1
/2*d*x + 1/2*c)^7 + 5120*a^2*b^22*tan(1/2*d*x + 1/2*c)^7 + 3072*b^24*tan(1/
2*d*x + 1/2*c)^7 + 3104640*a^19*b^5*tan(1/2*d*x + 1/2*c)^6 + 77468160*a^17*
b^7*tan(1/2*d*x + 1/2*c)^6 + 251081600*a^15*b^9*tan(1/2*d*x + 1/2*c)^6 + 27
4259160*a^13*b^11*tan(1/2*d*x + 1/2*c)^6 + 105524636*a^11*b^13*tan(1/2*d*x
+ 1/2*c)^6 + 11690784*a^9*b^15*tan(1/2*d*x + 1/2*c)^6 + 515760*a^7*b^17*tan
(1/2*d*x + 1/2*c)^6 + 38080*a^5*b^19*tan(1/2*d*x + 1/2*c)^6 - 54656*a^3*b^2
1*tan(1/2*d*x + 1/2*c)^6 + 10752*a*b^23*tan(1/2*d*x + 1/2*c)^6 + 20568240*a
^18*b^6*tan(1/2*d*x + 1/2*c)^5 + 136444770*a^16*b^8*tan(1/2*d*x + 1/2*c)^5
+ 229744669*a^14*b^10*tan(1/2*d*x + 1/2*c)^5 + 133540988*a^12*b^12*tan(1/2*
d*x + 1/2*c)^5 + 22390536*a^10*b^14*tan(1/2*d*x + 1/2*c)^5 - 189280*a^8*b^1
6*tan(1/2*d*x + 1/2*c)^5 + 328720*a^6*b^18*tan(1/2*d*x + 1/2*c)^5 - 115584*
a^4*b^20*tan(1/2*d*x + 1/2*c)^5 + 16128*a^2*b^22*tan(1/2*d*x + 1/2*c)^5 + 2
328480*a^19*b^5*tan(1/2*d*x + 1/2*c)^4 + 47733840*a^17*b^7*tan(1/2*d*x + 1/
2*c)^4 + 125203386*a^15*b^9*tan(1/2*d*x + 1/2*c)^4 + 105004865*a^13*b^11*ta
n(1/2*d*x + 1/2*c)^4 + 21568540*a^11*b^13*tan(1/2*d*x + 1/2*c)^4 - 612864*a
^9*b^15*tan(1/2*d*x + 1/2*c)^4 + 385168*a^7*b^17*tan(1/2*d*x + 1/2*c)^4 - 1
08640*a^5*b^19*tan(1/2*d*x + 1/2*c)^4 + 13440*a^3*b^21*tan(1/2*d*x + 1/2*c)
^4 + 9934848*a^18*b^6*tan(1/2*d*x + 1/2*c)^3 + 46275768*a^16*b^8*tan(1/2*d*
x + 1/2*c)^3 + 52916248*a^14*b^10*tan(1/2*d*x + 1/2*c)^3 + 11715494*a^12*b^
12*tan(1/2*d*x + 1/2*c)^3 - 403536*a^10*b^14*tan(1/2*d*x + 1/2*c)^3 + 21828
8*a^8*b^16*tan(1/2*d*x + 1/2*c)^3 - 57344*a^6*b^18*tan(1/2*d*x + 1/2*c)^3 +
 6720*a^4*b^20*tan(1/2*d*x + 1/2*c)^3 + 931392*a^19*b^5*tan(1/2*d*x + 1/2*c
)^2 + 11782848*a^17*b^7*tan(1/2*d*x + 1/2*c)^2 + 16561160*a^15*b^9*tan(1/2*
d*x + 1/2*c)^2 + 3685248*a^13*b^11*tan(1/2*d*x + 1/2*c)^2 - 117586*a^11*b^1
3*tan(1/2*d*x + 1/2*c)^2 + 64736*a^9*b^15*tan(1/2*d*x + 1/2*c)^2 - 17136*a^
7*b^17*tan(1/2*d*x + 1/2*c)^2 + 2016*a^5*b^19*tan(1/2*d*x + 1/2*c)^2 + 1940
400*a^18*b^6*tan(1/2*d*x + 1/2*c) + 2910138*a^16*b^8*tan(1/2*d*x + 1/2*c) +
 644413*a^14*b^10*tan(1/2*d*x + 1/2*c) - 21546*a^12*b^12*tan(1/2*d*x + 1/2*
c) + 11284*a^10*b^14*tan(1/2*d*x + 1/2*c) - 2912*a^8*b^16*tan(1/2*d*x + 1/2
*c) + 336*a^6*b^18*tan(1/2*d*x + 1/2*c) + 155232*a^19*b^5 + 218064*a^17*b^7
 + 50666*a^15*b^9 - 3555*a^13*b^11 + 1670*a^11*b^13 - 424*a^9*b^15 + 48*a^7
*b^17)/((a^25 - 9*a^23*b^2 + 36*a^21*b^4 - 84*a^19*b^6 + 126*a^17*b^8 - 126
*a^15*b^10 + 84*a^13*b^12 - 36*a^11*b^14 + 9*a^9*b^16 - a^7*b^18)*(a*tan(1/
2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d
```

maple [B] time = 0.59, size = 7823, normalized size = 11.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^8),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.473 $\int \cos^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{2(a\cdots}{}$$

```
[Out] 2/3*(a^2-b^2)^2*(a+b*sin(d*x+c))^(3/2)/b^5/d-8/5*a*(a^2-b^2)*(a+b*sin(d*x+c
))^(5/2)/b^5/d+4/7*(3*a^2-b^2)*(a+b*sin(d*x+c))^(7/2)/b^5/d-8/9*a*(a+b*sin(
d*x+c))^(9/2)/b^5/d+2/11*(a+b*sin(d*x+c))^(11/2)/b^5/d
```

Rubi [A] time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{2(a\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(3/2))/(3*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(7/2))/(7*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(9/2))/(9*b^5*d) + (2*(a + b
*Sin[c + d*x])^(11/2))/(11*b^5*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+x}\left(b^2-x^2\right)^2\,dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2\sqrt{a+x}-4\left(a^3-ab^2\right)(a+x)^{3/2}+2\left(3a^2-b^2\right)(\cdots\right.\right.}{b^5d}\\ &= \frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}{3b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^5d}\cdots\end{aligned}$$

Mathematica [A] time = 0.34, size = 117, normalized size = 0.76

$$\frac{2(a+b\sin(c+dx))^{3/2}\left(8\left(16a^4+\left(99ab^3-24a^3b\right)\sin(c+dx)+15b^2\left(2a^2-3b^2\right)\sin^2(c+dx)-66a^2b^2-35ab\cdots\right.\right.}{3465b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(3/2)*(315*b^4*Cos[c + d*x]^4 + 8*(16*a^4 - 66*a^2*
b^2 + 105*b^4 + (-24*a^3*b + 99*a*b^3)*Sin[c + d*x] + 15*b^2*(2*a^2 - 3*b^2
)*Sin[c + d*x]^2 - 35*a*b^3*Sin[c + d*x]^3)))/(3465*b^5*d)
```

fricas [A] time = 0.78, size = 142, normalized size = 0.92

$$\frac{2\left(35\,ab^{4}\cos\left(dx+c\right)^{4}+128\,a^{5}-480\,a^{3}b^{2}+992\,ab^{4}-16\left(3\,a^{3}b^{2}-8\,ab^{4}\right)\cos\left(dx+c\right)^{2}+\left(315\,b^{5}\cos\left(dx+c\right)^{4}-\right.}{3465\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(35*a*b^4*cos(d*x + c)^4 + 128*a^5 - 480*a^3*b^2 + 992*a*b^4 - 16*(3
*a^3*b^2 - 8*a*b^4)*cos(d*x + c)^2 + (315*b^5*cos(d*x + c)^4 - 64*a^4*b + 2
24*a^2*b^3 + 480*b^5 + 40*(a^2*b^3 + 9*b^5)*cos(d*x + c)^2)*sin(d*x + c))*s
qrt(b*sin(d*x + c) + a)/(b^5*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^5, x)
```

maple [A] time = 0.60, size = 126, normalized size = 0.82

$$\frac{2\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}\left(315b^{4}\left(\cos^{4}\left(dx+c\right)\right)+280a\,b^{3}\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-240a^{2}b^{2}\left(\cos^{2}\left(dx+c\right)\right)+36\right.}{3465b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3465/b^5*(a+b*sin(d*x+c))^(3/2)*(315*b^4*cos(d*x+c)^4+280*a*b^3*cos(d*x+c
)^2*sin(d*x+c)-240*a^2*b^2*cos(d*x+c)^2+360*b^4*cos(d*x+c)^2-192*a^3*b*sin(
d*x+c)+512*a*b^3*sin(d*x+c)+128*a^4-288*a^2*b^2+480*b^4)/d
```

maxima [A] time = 0.32, size = 116, normalized size = 0.75

$$\frac{2\left(315\left(b\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-1540\left(b\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a+990\left(3\,a^{2}-b^{2}\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}-2772\left(a^{3}-ab^{2}\right)\right.}{3465\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(b*sin(d*x + c) + a)^(11/2) - 1540*(b*sin(d*x + c) + a)^(9/2)*a
 + 990*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(7/2) - 2772*(a^3 - a*b^2)*(b*sin
(d*x + c) + a)^(5/2) + 1155*(a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c) + a)^(3
/2))/(b^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^5\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.474 $\int \cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=83

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^3d}-\frac{2(a+b\sin(c+dx))^{7/2}}{7b^3d}+\frac{4a(a+b\sin(c+dx))^{5/2}}{5b^3d}$$

```
[Out] -2/3*(a^2-b^2)*(a+b*sin(d*x+c))^(3/2)/b^3/d+4/5*a*(a+b*sin(d*x+c))^(5/2)/b^
3/d-2/7*(a+b*sin(d*x+c))^(7/2)/b^3/d
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^3d}-\frac{2(a+b\sin(c+dx))^{7/2}}{7b^3d}+\frac{4a(a+b\sin(c+dx))^{5/2}}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(3/2))/(3*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(5/2))/(5*b^3*d) - (2*(a + b*Sin[c + d*x])^(7/2))/(7*b^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+x}\left(b^2-x^2\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)\sqrt{a+x}+2a(a+x)^{3/2}-(a+x)^{5/2}\right)dx,x,b\sin(c+\right.}{b^3d}\\ &= -\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^3d}+\frac{4a(a+b\sin(c+dx))^{5/2}}{5b^3d}-\frac{2(a+b\sin}{7}\end{aligned}$$

Mathematica [A] time = 0.13, size = 58, normalized size = 0.70

$$\frac{(a+b\sin(c+dx))^{3/2}\left(-16a^2+24ab\sin(c+dx)+15b^2\cos(2(c+dx))+55b^2\right)}{105b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((a + b*Sin[c + d*x])^(3/2)*(-16*a^2 + 55*b^2 + 15*b^2*Cos[2*(c + d*x)] + 2
4*a*b*Sin[c + d*x]))/(105*b^3*d)
```

fricas [A] time = 0.92, size = 78, normalized size = 0.94

$$\frac{2\left(3\,ab^{2}\cos\left(dx+c\right)^{2}-8\,a^{3}+32\,ab^{2}+\left(15\,b^{3}\cos\left(dx+c\right)^{2}+4\,a^{2}b+20\,b^{3}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{105\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(3*a*b^2*cos(d*x + c)^2 - 8*a^3 + 32*a*b^2 + (15*b^3*cos(d*x + c)^2 +
 4*a^2*b + 20*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^3*d)
```

giac [A] time = 2.17, size = 78, normalized size = 0.94

$$-\frac{2\left(\frac{15\,(b\sin(dx+c)+a)^{\frac{7}{2}}}{b^{3}}-\frac{42\,(b\sin(dx+c)+a)^{\frac{5}{2}}a}{b^{3}}+\frac{35\,(b\sin(dx+c)+a)^{\frac{3}{2}}a^{2}}{b^{3}}-\frac{35\,(b\sin(dx+c)+a)^{\frac{3}{2}}}{b}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/105*(15*(b*sin(d*x + c) + a)^(7/2)/b^3 - 42*(b*sin(d*x + c) + a)^(5/2)*a
/b^3 + 35*(b*sin(d*x + c) + a)^(3/2)*a^2/b^3 - 35*(b*sin(d*x + c) + a)^(3/2
)/b)/d
```

maple [A] time = 0.38, size = 55, normalized size = 0.66

$$-\frac{2\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}\left(-15b^{2}\left(\cos^{2}\left(dx+c\right)\right)-12ab\sin\left(dx+c\right)+8a^{2}-20b^{2}\right)}{105b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/105/b^3*(a+b*sin(d*x+c))^(3/2)*(-15*b^2*cos(d*x+c)^2-12*a*b*sin(d*x+c)+8
*a^2-20*b^2)/d
```

maxima [A] time = 0.32, size = 61, normalized size = 0.73

$$-\frac{2\left(15\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}-42\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a+35\left(a^{2}-b^{2}\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\right)}{105\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/105*(15*(b*sin(d*x + c) + a)^(7/2) - 42*(b*sin(d*x + c) + a)^(5/2)*a + 3
5*(a^2 - b^2)*(b*sin(d*x + c) + a)^(3/2))/(b^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{3}\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.475 $\int \cos(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+b\sin(c+dx))^{3/2}}{3bd}$$

```
[Out] 2/3*(a+b*sin(d*x+c))^(3/2)/b/d
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 32}

$$\frac{2(a+b\sin(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(3/2))/(3*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \cos(c+dx)\sqrt{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+x}\,dx, x, b\sin(c+dx)\right)}{bd} \\ &= \frac{2(a+b\sin(c+dx))^{3/2}}{3bd} \end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{2(a+b\sin(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(3/2))/(3*b*d)
```

fricas [A] time = 1.02, size = 20, normalized size = 0.83

$$\frac{2\,(b\sin(dx+c)+a)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*sin(d*x + c) + a)^(3/2)/(b*d)
```

giac [A] time = 1.45, size = 20, normalized size = 0.83

$$\frac{2\,(b\sin\left(dx+c\right)+a)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2/3*(b*sin(d*x + c) + a)^(3/2)/(b*d)
```

maple [A] time = 0.04, size = 21, normalized size = 0.88

$$\frac{2\,(a+b\sin\left(dx+c\right))^{\frac{3}{2}}}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x)
[Out] 2/3*(a+b*sin(d*x+c))^(3/2)/b/d
```

maxima [A] time = 0.33, size = 20, normalized size = 0.83

$$\frac{2\,(b\sin\left(dx+c\right)+a)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2/3*(b*sin(d*x + c) + a)^(3/2)/(b*d)
```

mupad [B] time = 5.20, size = 20, normalized size = 0.83

$$\frac{2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^(1/2),x)
[Out] (2*(a + b*sin(c + d*x))^(3/2))/(3*b*d)
```

sympy [A] time = 0.55, size = 83, normalized size = 3.46

$$\begin{cases} \sqrt{a}\,x\cos\left(c\right) & \text{for}\ b=0 \wedge d=0 \\ \frac{\sqrt{a}\,\sin\left(c+dx\right)}{d} & \text{for}\ b=0 \\ x\sqrt{a+b\sin\left(c\right)}\,\cos\left(c\right) & \text{for}\ d=0 \\ \frac{2a\sqrt{a+b\sin\left(c+dx\right)}}{3bd}+\frac{2\sqrt{a+b\sin\left(c+dx\right)}\,\sin\left(c+dx\right)}{3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
[Out] Piecewise((sqrt(a)*x*cos(c), Eq(b, 0) & Eq(d, 0)), (sqrt(a)*sin(c + d*x)/d,
 Eq(b, 0)), (x*sqrt(a + b*sin(c))*cos(c), Eq(d, 0)), (2*a*sqrt(a + b*sin(c
+ d*x))/(3*b*d) + 2*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(3*d), True))
```

3.476 $\int \sec(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

```
[Out] -arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d+arctanh((a+b*sin
(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2668, 700, 1130, 206}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
+ b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sec(c+dx)\sqrt{a+b\sin(c+dx)}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d} \\
&= \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d} \\
&= -\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d} + \frac{(a+b)\,\text{Subst}\left(\int \frac{}{a+}\right.}{} \\
&= -\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 74, normalized size = 1.00

$$
\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d} - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
+ b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d
```

fricas [B] time = 1.19, size = 1729, normalized size = 23.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b
^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2
+ 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)
^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(
b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14
*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*
cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + sqrt(a - b)*lo
g((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*
b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2
*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x +
c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*
sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*
b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(
cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d, -1/8*(2*sqrt(-a - b)*arctan(-1/4
*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x +
c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3
- (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c))) -
 sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 -
256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(
16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 -
(b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin
(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4
- (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d, -1/8*(2*sqrt(-a +
 b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b
```

```
^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b +
2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*si
n(d*x + c))) - sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b +
320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*
x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos
(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x +
c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a
*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x +
c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d, -1/
4*(sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*
(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3
- 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b
^2 + b^3)*sin(d*x + c))) + sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8
*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c)
+ a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x
+ c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c))))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c), x)
```

maple [A] time = 0.39, size = 63, normalized size = 0.85

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin\left(dx+c\right)}}{\sqrt{a+b}}\right)\sqrt{a+b}}{d}-\frac{\sqrt{-a+b}\,\arctan\left(\frac{\sqrt{a+b\sin\left(dx+c\right)}}{\sqrt{-a+b}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d-1/d*(-a+b)^(1/2)*
arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sin\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin(c + dx)}\,\sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x), x)
```

3.477 $\int \sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d\sqrt{a+b}}+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\sin(c+dx)}}{2d}$$

```
[Out] -1/4*(2*a-b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+1/4*
(2*a+b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+1/2*sec(d
*x+c)*(a+b*sin(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2668, 737, 827, 1166, 206}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d\sqrt{a+b}}+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\sin(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d
) + ((2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*Sqrt[a + b
]*d) + (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{2d} - \frac{b\,\text{Subst}\left(\int \frac{-a-\frac{x}{2}}{\sqrt{a+x}\left(b^2-x^2\right)}\,dx, x\right.}{2d}$$

$$= \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{2d} - \frac{b\,\text{Subst}\left(\int \frac{-\frac{a}{2}-\frac{x^2}{2}}{-a^2+b^2+2ax^2-x^4}\,d\right.}{d}$$

$$= \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{2d} - \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx\right.}{4d}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}\,d} + \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4\sqrt{a+b}\,d} + \text{se}$$

Mathematica [A] time = 0.71, size = 143, normalized size = 1.15

$$\frac{(a-b)\left(\sqrt{a+b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right) + 2(a+b)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sin(c+dx)}\right) - \sqrt{a-b}\,(}{4d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-(Sqrt[a - b]*(2*a^2 + a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a
- b]]) + (a - b)*(Sqrt[a + b]*(2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sq
rt[a + b]] + 2*(a + b)*Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x]))
/(4*(a^2 - b^2)*d)
```

fricas [B] time = 1.32, size = 2101, normalized size = 16.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((2*a^2 - a*b - b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)
^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2
 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b
^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 2
8*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64
*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2
)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)
*sin(d*x + c) + 8)) - (2*a^2 + a*b - b^2)*sqrt(a - b)*cos(d*x + c)^2*log((b
^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4
- 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 - 24*a^2*b +
```

```
 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2
 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt
(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)
*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(
d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a^2 - b^2)*sqrt(b*sin(d*x + c) + a
)*sin(d*x + c))/((a^2 - b^2)*d*cos(d*x + c)^2), -1/32*(2*(2*a^2 - a*b - b^2
)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*
(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3
+ 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b
^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^2 + a*b - b^2)*sqrt(a - b)*c
os(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 -
 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*
(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 -
 (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*si
n(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4
 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(
d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(a^2 - b^2)*sqr
t(b*sin(d*x + c) + a)*sin(d*x + c))/((a^2 - b^2)*d*cos(d*x + c)^2), -1/32*(
2*(2*a^2 + a*b - b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 +
 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*s
qrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2
 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - (2*a^2 - a*b -
 b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^
3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*
cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^
3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(
d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2
+ 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(
d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) -
 16*(a^2 - b^2)*sqrt(b*sin(d*x + c) + a)*sin(d*x + c))/((a^2 - b^2)*d*cos(d
*x + c)^2), -1/16*((2*a^2 + a*b - b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x
 + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*si
n(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^
3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2
 + (2*a^2 - a*b - b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2
 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)
*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)
^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - 8*(a^2 - b^2
)*sqrt(b*sin(d*x + c) + a)*sin(d*x + c))/((a^2 - b^2)*d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^3, x)
```

maple [A] time = 0.61, size = 185, normalized size = 1.49

$$\frac{2\sqrt{a+b\sin\left(dx+c\right)}\,\sqrt{-a+b}\,\sqrt{a+b}\,\sin\left(dx+c\right)-\left(-2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a\sqrt{-a+b}\,-\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{s}}}{\sqrt{}}\right.\right.}{4\sqrt{-a+b}\,\sqrt{a+b}\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/4*(2*(a+b*sin(d*x+c))^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(d*x+c)-(-2*arcta
nh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)-arctanh((a+b*sin(d*x+
c))^(1/2)/(a+b)^(1/2))*b*(-a+b)^(1/2)-2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b
)^(1/2))*a*(a+b)^(1/2)+arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*b*(a+b)^
(1/2))*cos(d*x+c)^2)/(-a+b)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^2/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin\left(c+dx\right)}\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

3.478 $\int \sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=207

$$-\frac{(12a^2-18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{3/2}}+\frac{(12a^2+18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{3/2}}-\frac{\sec^2(c+dx)\sqrt{a+}}{}$$

```
[Out] -1/32*(12*a^2-18*a*b+5*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-
b)^(3/2)/d+1/32*(12*a^2+18*a*b+5*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^
(1/2))/(a+b)^(3/2)/d-1/16*sec(d*x+c)^2*(a*b-(6*a^2-5*b^2)*sin(d*x+c))*(a+b*
sin(d*x+c))^(1/2)/d/(a^2-b^2)+1/4*sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)*tan(d
*x+c)/d
```

Rubi [A] time = 0.32, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2668, 737, 823, 827, 1166, 206}

$$-\frac{(12a^2-18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{3/2}}+\frac{(12a^2+18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{3/2}}-\frac{\sec^2(c+dx)\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((12*a^2 - 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/
(32*(a - b)^(3/2)*d) + ((12*a^2 + 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[c
+ d*x]]/Sqrt[a + b]])/(32*(a + b)^(3/2)*d) - (Sec[c + d*x]^2*Sqrt[a + b*Sin
[c + d*x]]*(a*b - (6*a^2 - 5*b^2)*Sin[c + d*x]))/(16*(a^2 - b^2)*d) + (Sec[
c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{4d} - \frac{b^3\,\text{Subst}\left(\int \frac{-3a-\frac{5x}{2}}{\sqrt{a+x}\left(b^2-x^2\right)^2}\,d\right.}{4d}$$

$$= -\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(ab-\left(6a^2-5b^2\right)\sin(c+dx)\right)}{16\left(a^2-b^2\right)d} + \frac{\sec^3(c}{}$$

$$= -\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(ab-\left(6a^2-5b^2\right)\sin(c+dx)\right)}{16\left(a^2-b^2\right)d} + \frac{\sec^3(c}{}$$

$$= -\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(ab-\left(6a^2-5b^2\right)\sin(c+dx)\right)}{16\left(a^2-b^2\right)d} + \frac{\sec^3(c}{}$$

$$= -\frac{\left(12a^2-18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{3/2}d} + \frac{\left(12a^2+18ab+5b^2\right)\tanh}{32(a+b)}$$

Mathematica [A] time = 1.52, size = 224, normalized size = 1.08

$$\frac{-\sqrt{a-b}\,(a+b)^2\left(12a^2-18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+(a-b)^2\sqrt{a+b}\,\left(12a^2+18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-(Sqrt[a - b]*(a + b)^2*(12*a^2 - 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[c
 + d*x]]/Sqrt[a - b]]) + (a - b)^2*Sqrt[a + b]*(12*a^2 + 18*a*b + 5*b^2)*Ar
```

```
cTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + ((a^2 - b^2)*Sec[c + d*x]^4*S
qrt[a + b*Sin[c + d*x]]*(-2*a*b - 2*a*b*Cos[2*(c + d*x)] + (22*a^2 - 21*b^2
)*Sin[c + d*x] + 6*a^2*Sin[3*(c + d*x)] - 5*b^2*Sin[3*(c + d*x)]))/2)/(32*(
a^2 - b^2)^2*d)
```

fricas [F] time = 1.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5, x)
```

maple [B] time = 0.88, size = 509, normalized size = 2.46

$$-\frac{3b\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}a}{16d\left(b\sin\left(dx+c\right)+b\right)^{2}\left(a-b\right)}+\frac{5b^{2}\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}}{32d\left(b\sin\left(dx+c\right)+b\right)^{2}\left(a-b\right)}+\frac{3b\sqrt{a+b\sin\left(dx+c\right)}\,a}{16d\left(b\sin\left(dx+c\right)+b\right)^{2}}-\frac{7b^{2}\sqrt{a+b\sin\left(\right.}}{32d\left(b\sin\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -3/16/d/(b*sin(d*x+c)+b)^2*b/(a-b)*(a+b*sin(d*x+c))^(3/2)*a+5/32/d/(b*sin(d
*x+c)+b)^2*b^2/(a-b)*(a+b*sin(d*x+c))^(3/2)+3/16/d/(b*sin(d*x+c)+b)^2*b*(a+
b*sin(d*x+c))^(1/2)*a-7/32/d/(b*sin(d*x+c)+b)^2*b^2*(a+b*sin(d*x+c))^(1/2)+
3/8/d/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-9/
16/d/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*b+5/3
2/d/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*b^2-3/16
/d/(b*sin(d*x+c)-b)^2*b/(a+b)*(a+b*sin(d*x+c))^(3/2)*a-5/32/d/(b*sin(d*x+c)
-b)^2*b^2/(a+b)*(a+b*sin(d*x+c))^(3/2)+3/16/d/(b*sin(d*x+c)-b)^2*b*(a+b*sin
(d*x+c))^(1/2)*a+7/32/d/(b*sin(d*x+c)-b)^2*b^2*(a+b*sin(d*x+c))^(1/2)+3/8/d
/(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+9/16/d/(a+b)^(
3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a*b+5/32/d/(a+b)^(3/2)*arc
tanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b \sin(c + d x)}}{\cos(c + d x)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^5,x)
[Out] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.479 $\int \cos^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=298

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a\left(a^2-3b^2\right)-3b\left(a^2+7b^2\right)\sin(c+dx)\right)}{315b^3d}+\frac{32a\left(a^4-4a^2b^2+3b^4\right)\sqrt{\frac{a+b\sin}{a-}}}{315b^4d\sqrt{a+b}}$$

```
[Out] 2/9*cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/b/d-4/21*a*cos(d*x+c)^3*(a+b*sin(d*
x+c))^(1/2)/b/d-4/315*cos(d*x+c)*(4*a*(a^2-3*b^2)-3*b*(a^2+7*b^2)*sin(d*x+c
))*(a+b*sin(d*x+c))^(1/2)/b^3/d+8/315*(4*a^4-15*a^2*b^2-21*b^4)*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*sin
(d*x+c))/(a+b))^(1/2)-32/315*a*(a^4-4*a^2*b^2+3*b^4)*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*sin(d*x
+c))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2695, 2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a\left(a^2-3b^2\right)-3b\left(a^2+7b^2\right)\sin(c+dx)\right)}{315b^3d}+\frac{32a\left(-4a^2b^2+a^4+3b^4\right)\sqrt{\frac{a+b}{}}}{315b^4d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-4*a*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(21*b*d) + (2*Cos[c + d*x]^3
*(a + b*Sin[c + d*x])^(3/2))/(9*b*d) - (8*(4*a^4 - 15*a^2*b^2 - 21*b^4)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^
4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*a*(a^4 - 4*a^2*b^2 + 3*b^4)*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(315*b^4*d*Sqrt[a + b*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[
c + d*x]]*(4*a*(a^2 - 3*b^2) - 3*b*(a^2 + 7*b^2)*Sin[c + d*x]))/(315*b^3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{2\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9bd} + \frac{2\int \cos^2(c+dx)(b+a\sin(c+\ldots}{3b}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c+d\ldots}{9bd}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c+d\ldots}{9bd}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c+d\ldots}{9bd}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c+d\ldots}{9bd}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c+d\ldots}{9bd}$$

Mathematica [A] time = 0.91, size = 233, normalized size = 0.78

$$\frac{32\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\left(ab^2\left(a^2-33b^2\right)F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(4a^4-15a^2b^2-21b^4\right)\left((a+b)E\left(\frac{1}{4}(-2c-2dx+\ldots\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (32*(a*b^2*(a^2 - 33*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] +
 (4*a^4 - 15*a^2*b^2 - 21*b^4)*((a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]))*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)] + 2*b*Cos[c + d*x]*(a + b*Sin[c + d*x])*(-32*a^3
 + 106*a*b^2 + 10*a*b^2*Cos[2*(c + d*x)] + b*(24*a^2 + 203*b^2)*Sin[c + d*x
] + 35*b^3*Sin[3*(c + d*x)]))/(1260*b^4*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sin(dx+c)+a}\,\cos(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin(dx+c)+a}\,\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

maple [B] time = 0.69, size = 1189, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/315*(-35*b^6*sin(d*x+c)^6-40*a*b^5*sin(d*x+c)^5+16*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-12*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^4*b^2-64*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^3-72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+48*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5+84*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6
-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^6+76*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^4*b^2+24*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-84*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6+a^2*b^4*si
n(d*x+c)^4+112*b^6*sin(d*x+c)^4-2*a^3*b^3*sin(d*x+c)^3+146*a*b^5*sin(d*x+c)
^3-8*a^4*b^2*sin(d*x+c)^2+28*a^2*b^4*sin(d*x+c)^2-77*b^6*sin(d*x+c)^2+2*a^3
*b^3*sin(d*x+c)-106*a*b^5*sin(d*x+c)+8*a^4*b^2-29*a^2*b^4)/b^5/cos(d*x+c)/(
a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin\left(dx + c\right) + a} \cos\left(dx + c\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(c + d\,x\right)}^4\,\sqrt{a + b\,\sin\left(c + d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin{\left(c + dx \right)}} \cos^{4}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)**4, x)
```

3.480 $\int \cos^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=215

$$-\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4\left(a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\,\text{co}}{}$$

```
[Out] 2/5*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b/d-4/15*a*cos(d*x+c)*(a+b*sin(d*x+c)
)^(1/2)/b/d-4/15*(a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2)
)*(a+b*sin(d*x+c))^(1/2)/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+4/15*a*(a^2-b
^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b)
)^(1/2)/b^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2695, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4\left(a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\,\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-4*a*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*b*d) + (2*Cos[c + d*x]*(a
+ b*Sin[c + d*x])^(3/2))/(5*b*d) + (4*(a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)]) - (4*a*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(
a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^2*d*Sqrt[a + b*Sin[c + d*
x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd} + \frac{2\int(b+a\sin(c+dx))\sqrt{a+b\sin}}{5b}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+dx)}{5bd}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+dx)}{5bd}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+dx)}{5bd}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+dx)}{5bd}$$

Mathematica [A] time = 0.83, size = 185, normalized size = 0.86

$$\frac{b\cos(c+dx)\left(2a^2+8ab\sin(c+dx)-3b^2\cos(2(c+dx))+3b^2\right)+4a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\right.}{15b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-4*(a^3 + a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 4*a*(a^2 - b^2)*EllipticF[(-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*C
os[c + d*x]*(2*a^2 + 3*b^2 - 3*b^2*Cos[2*(c + d*x)] + 8*a*b*Sin[c + d*x]))/
(15*b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

maple [B] time = 0.72, size = 792, normalized size = 3.68

$$\frac{\frac{4\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\operatorname{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^3b}{15}+\frac{4a^2\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\mathrm{E}}{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/15*(2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^3*b+6*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2-2*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^3-6*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4-2*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a^4-4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^2*b^2+6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^4-3*b^4*sin(d*x+c)^4-4*a*b^3*sin(d*x+c)^3-a
^2*b^2*sin(d*x+c)^2+3*b^4*sin(d*x+c)^2+4*a*b^3*sin(d*x+c)+a^2*b^2)/b^3/cos(
d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c + d\,x)^2\,\sqrt{a + b\,\sin (c + d\,x)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin{\left(c + dx \right)}}\cos^{2}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)**2, x)
```

3.481 $\int \sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{\tan(c+dx)\sqrt{a+b\sin(c+dx)}}{d}+\frac{a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] (sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((
a+b*sin(d*x+c))/(a+b))^(1/2)-a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/
2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)+(a+b*sin(d*x+c)
)^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.17, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2690, 12, 2752, 2663, 2661, 2655, 2653}

$$\frac{\tan(c+dx)\sqrt{a+b\sin(c+dx)}}{d}+\frac{a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(
d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])) + (a*EllipticF[(c - Pi/2 + d*x)/2, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x
]]) + (Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2690

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^m*Sin[e + f*x])/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1)*(a*(p + 2) + b*(m + p + 2)*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[0
, m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx &= \frac{\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{d}-\int \frac{b\sin(c+dx)}{2\sqrt{a+b\sin(c+dx)}}\,dx\\ &= \frac{\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{d}-\frac{1}{2}b\int \frac{\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx\\ &= \frac{\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{d}-\frac{1}{2}\int \sqrt{a+b\sin(c+dx)}\,dx+\frac{1}{2}a\int\\ &= \frac{\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{d}-\frac{\sqrt{a+b\sin(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin(c}{a+}}}{2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}\\ &= -\frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+l}\right.}{d\sqrt{a+b\sin(c}}\end{aligned}$$

Mathematica [A] time = 2.81, size = 127, normalized size = 0.85

$$\frac{\tan(c+dx)(a+b\sin(c+dx))-a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+(a+b)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}E\left(\frac{1}{4}(-2c-2a\right.}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)] + (a + b*Sin[c + d*x])*Tan[c + d*x])/(d*Sqrt[a
 + b*Sin[c + d*x]])
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin(dx+c)+a}\,\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

maple [B] time = 1.01, size = 614, normalized size = 4.12

$$\frac{\sqrt{\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)b+\left(\cos^{2}\left(dx+c\right)\right)a}\;\left(\text{EllipticE}\left(\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\;\sqrt{\frac{b}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x)
[Out] 1/b*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*(EllipticE((b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b)
)^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^
(1/2)*a^2-EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2
))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^2-(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b
)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a*b+(-b
/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*Ellip
ticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*si
n(d*x+c)-b/(a-b))^(1/2)*b^2-b^2*cos(d*x+c)^2+a*b*sin(d*x+c)+b^2)/(-(a+b*sin
(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/cos(d*x+c)/(a+b*sin(d*x+c))^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^2,x)
[Out] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin\left(c+dx\right)}\,\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x)**2, x)
```

3.482 $\int \sec^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=248

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{6d\left(a^2-b^2\right)}-\frac{\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\right)}{6d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -1/6*sec(d*x+c)*(a*b-(4*a^2-3*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d/(a^
2-b^2)+1/6*(4*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a
+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-2/3*a*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d
/(a+b*sin(d*x+c))^(1/2)+1/3*sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)*tan(d*x+c)/
d
```

Rubi [A] time = 0.37, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2690, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{6d\left(a^2-b^2\right)}-\frac{\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\right)}{6d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(6*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2*a*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(3*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]
]*(a*b - (4*a^2 - 3*b^2)*Sin[c + d*x]))/(6*(a^2 - b^2)*d) + (Sec[c + d*x]^2
*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2690

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^m*Sin[e + f*x])/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1)*(a*(p + 2) + b*(m + p + 2)*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[0
, m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{3d} - \frac{1}{3}\int \frac{\sec^2(c+dx)\left(-2a \right.}{\sqrt{a+b\sin}}$$

$$= -\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{6\left(a^2-b^2\right)d} + \sec$$

$$= -\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{6\left(a^2-b^2\right)d} + \sec$$

$$= -\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{6\left(a^2-b^2\right)d} + \sec$$

$$= -\frac{\left(4a^2-3b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{6\left(a^2-b^2\right)d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{2aF\left(\frac{1}{2}\left(c-\right.\right.}{3}$$

Mathematica [A] time = 3.36, size = 270, normalized size = 1.09

$$-4a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(4a^3+4a^2b-3ab^2-3b^3\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,E\left(\frac{1}{4}(-2c-2a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 4*a*(a^2 - b^2)*EllipticF[(-
2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + (S
ec[c + d*x]^3*(8*a^2*b - 11*b^3 + (-12*a^2*b + 8*b^3)*Cos[2*(c + d*x)] + (-
4*a^2*b + 3*b^3)*Cos[4*(c + d*x)] + 24*a^3*Sin[c + d*x] - 24*a*b^2*Sin[c +
d*x] + 8*a^3*Sin[3*(c + d*x)] - 8*a*b^2*Sin[3*(c + d*x)]))/8)/(6*(a - b)*(a
 + b)*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

maple [B] time = 0.85, size = 1259, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/6*(-4*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b*(a^2-b^2)*sin(
d*x+c)*cos(d*x+c)^2-2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b*
(a^2-b^2)*sin(d*x+c)+(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*b^2*(
4*a^2-3*b^2)*cos(d*x+c)^4+(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*
(4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^3*b-3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^2*b^2-4*
(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*El
lipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*a*b^3+3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(
a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b^4-4*(-b/(a-
b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE
((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*
x+c)+b/(a+b))^(1/2)*a^4+7*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-
b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^2*b^2-3*(-b/(a-b)*si
n(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(
a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+
b/(a+b))^(1/2)*b^4-a^2*b^2+b^4)*cos(d*x+c)^2-2*(cos(d*x+c)^2*sin(d*x+c)*b+c
```

```
os(d*x+c)^2*a)^(1/2)*a^2*b^2+2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(
1/2)*b^4)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(a+b)/(si
n(d*x+c)-1)/(a-b)/(1+sin(d*x+c))/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin\left(dx + c\right) + a} \sec\left(dx + c\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin{\left(c + dx \right)}} \sec^{4}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x)**4, x)
```

3.483 $\int \cos^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=154

$$\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{9/2}}{9b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{2(a+b$$

```
[Out] 2/5*(a^2-b^2)^2*(a+b*sin(d*x+c))^(5/2)/b^5/d-8/7*a*(a^2-b^2)*(a+b*sin(d*x+c
))^(7/2)/b^5/d+4/9*(3*a^2-b^2)*(a+b*sin(d*x+c))^(9/2)/b^5/d-8/11*a*(a+b*sin
(d*x+c))^(11/2)/b^5/d+2/13*(a+b*sin(d*x+c))^(13/2)/b^5/d
```

Rubi [A] time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{2668, 697\}$

$$\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{9/2}}{9b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{2(a+b$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(9/2))/(9*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(11/2))/(11*b^5*d) + (2*(a +
 b*Sin[c + d*x])^(13/2))/(13*b^5*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{3/2}\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^{3/2}-4\left(a^3-ab^2\right)(a+x)^{5/2}+2\left(3a^2-b^2\right)(\right.\right.}{b^5d}\\ &= \frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{5/2}}{5b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{7/2}}{7b^5d}-\end{aligned}$$

Mathematica [A] time = 0.72, size = 131, normalized size = 0.85

$$\frac{2\left(\frac{2}{9}(3a^2-b^2)(a+b\sin(c+dx))^{9/2}+\frac{1}{5}(a^2-b^2)^2(a+b\sin(c+dx))^{5/2}+\frac{1}{13}(a+b\sin(c+dx))^{13/2}-\frac{4}{11}a(a+b\sin\right.}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(((a^2 - b^2)^2*(a + b*Sin[c + d*x])^(5/2))/5 - (4*a*(a - b)*(a + b)*(a
+ b*Sin[c + d*x])^(7/2))/7 + (2*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^(9/2))/9
 - (4*a*(a + b*Sin[c + d*x])^(11/2))/11 + (a + b*Sin[c + d*x])^(13/2)/13))/
(b^5*d)
```

fricas [A] time = 0.78, size = 184, normalized size = 1.19

$$-\frac{2\left(3465\,b^{6}\cos\left(dx+c\right)^{6}-384\,a^{6}+2144\,a^{4}b^{2}-8256\,a^{2}b^{4}-2464\,b^{6}-35\left(3\,a^{2}b^{4}+11\,b^{6}\right)\cos\left(dx+c\right)^{4}+8\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45045*(3465*b^6*cos(d*x + c)^6 - 384*a^6 + 2144*a^4*b^2 - 8256*a^2*b^4 -
 2464*b^6 - 35*(3*a^2*b^4 + 11*b^6)*cos(d*x + c)^4 + 8*(18*a^4*b^2 - 81*a^2
*b^4 - 77*b^6)*cos(d*x + c)^2 - 2*(2205*a*b^5*cos(d*x + c)^4 - 96*a^5*b + 5
12*a^3*b^3 + 4064*a*b^5 + 20*(3*a^3*b^3 + 137*a*b^5)*cos(d*x + c)^2)*sin(d*
x + c))*sqrt(b*sin(d*x + c) + a)/(b^5*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^5, x)
```

maple [A] time = 0.70, size = 126, normalized size = 0.82

$$\frac{2\left(a+b\sin\left(dx+c\right)\right)^{\frac{5}{2}}\left(3465b^{4}\left(\cos^{4}\left(dx+c\right)\right)+2520a\,b^{3}\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-1680a^{2}b^{2}\left(\cos^{2}\left(dx+c\right.\right.}{45045b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/45045/b^5*(a+b*sin(d*x+c))^(5/2)*(3465*b^4*cos(d*x+c)^4+2520*a*b^3*cos(d*
x+c)^2*sin(d*x+c)-1680*a^2*b^2*cos(d*x+c)^2+3080*b^4*cos(d*x+c)^2-960*a^3*b
*sin(d*x+c)+3200*a*b^3*sin(d*x+c)+384*a^4-608*a^2*b^2+2464*b^4)/d
```

maxima [A] time = 0.35, size = 116, normalized size = 0.75

$$\frac{2\left(3465\left(b\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}-16380\left(b\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}a+10010\left(3\,a^{2}-b^{2}\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-25740\right.}{45045\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3465*(b*sin(d*x + c) + a)^(13/2) - 16380*(b*sin(d*x + c) + a)^(11/
2)*a + 10010*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(9/2) - 25740*(a^3 - a*b^2)
*(b*sin(d*x + c) + a)^(7/2) + 9009*(a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)
+ a)^(5/2))/(b^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^5\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.484 $\int \cos^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=83

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^3d}-\frac{2(a+b\sin(c+dx))^{9/2}}{9b^3d}+\frac{4a(a+b\sin(c+dx))^{7/2}}{7b^3d}$$

```
[Out] -2/5*(a^2-b^2)*(a+b*sin(d*x+c))^(5/2)/b^3/d+4/7*a*(a+b*sin(d*x+c))^(7/2)/b^
3/d-2/9*(a+b*sin(d*x+c))^(9/2)/b^3/d
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^3d}-\frac{2(a+b\sin(c+dx))^{9/2}}{9b^3d}+\frac{4a(a+b\sin(c+dx))^{7/2}}{7b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(5/2))/(5*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(7/2))/(7*b^3*d) - (2*(a + b*Sin[c + d*x])^(9/2))/(9*b^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx=\frac{\text{Subst}\left(\int (a+x)^{3/2}\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}$$

$$=\frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^{3/2}+2a(a+x)^{5/2}-(a+x)^{7/2}\right)\,dx,x,b\,\text{s}\right.}{b^3d}$$

$$=-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^3d}+\frac{4a(a+b\sin(c+dx))^{7/2}}{7b^3d}-\frac{2(a}{}$$

Mathematica [A] time = 0.18, size = 58, normalized size = 0.70

$$\frac{(a+b\sin(c+dx))^{5/2}\left(-16a^2+40ab\sin(c+dx)+35b^2\cos(2(c+dx))+91b^2\right)}{315b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a + b*Sin[c + d*x])^(5/2)*(-16*a^2 + 91*b^2 + 35*b^2*Cos[2*(c + d*x)] + 4
0*a*b*Sin[c + d*x]))/(315*b^3*d)
```

fricas [A] time = 1.01, size = 111, normalized size = 1.34

$$-\frac{2\left(35\,b^4\cos\left(dx+c\right)^4+8\,a^4-60\,a^2b^2-28\,b^4-\left(3\,a^2b^2+7\,b^4\right)\cos\left(dx+c\right)^2-2\left(25\,ab^3\cos\left(dx+c\right)^2+2\,a^3b+\right.\right.}{315\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -2/315*(35*b^4*cos(d*x + c)^4 + 8*a^4 - 60*a^2*b^2 - 28*b^4 - (3*a^2*b^2 +
7*b^4)*cos(d*x + c)^2 - 2*(25*a*b^3*cos(d*x + c)^2 + 2*a^3*b + 38*a*b^3)*si
n(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

maple [A] time = 0.33, size = 55, normalized size = 0.66

$$-\frac{2\left(a+b\sin\left(dx+c\right)\right)^{\frac{5}{2}}\left(-35b^2\left(\cos^2\left(dx+c\right)\right)-20ab\sin\left(dx+c\right)+8a^2-28b^2\right)}{315b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x)
[Out] -2/315/b^3*(a+b*sin(d*x+c))^(5/2)*(-35*b^2*cos(d*x+c)^2-20*a*b*sin(d*x+c)+8
*a^2-28*b^2)/d
```

maxima [A] time = 0.32, size = 61, normalized size = 0.73

$$-\frac{2\left(35\left(b\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-90\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a+63\left(a^2-b^2\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\right)}{315\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/315*(35*(b*sin(d*x + c) + a)^(9/2) - 90*(b*sin(d*x + c) + a)^(7/2)*a + 6
3*(a^2 - b^2)*(b*sin(d*x + c) + a)^(5/2))/(b^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^3\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(3/2), x)
```

sympy [A] time = 121.57, size = 314, normalized size = 3.78

$$\begin{cases} a^{\frac{3}{2}}x\cos^3(c) \\ a^{\frac{3}{2}}\left(\frac{2\sin^3(c+dx)}{3d}+\frac{\sin(c+dx)\cos^2(c+dx)}{d}\right) \\ x(a+b\sin(c))^{\frac{3}{2}}\cos^3(c) \\ -\frac{16a^4\sqrt{a+b\sin(c+dx)}}{315b^3d}+\frac{8a^3\sqrt{a+b\sin(c+dx)}\sin(c+dx)}{315b^2d}+\frac{8a^2\sqrt{a+b\sin(c+dx)}\sin^2(c+dx)}{21bd}+\frac{2a^2\sqrt{a+b\sin(c+dx)}\cos^2(c+dx)}{5bd}+\frac{152a\sqrt{a}}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((a**(3/2)*x*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (a**(3/2)*(2*sin(c +
 d*x)**3/(3*d) + sin(c + d*x)*cos(c + d*x)**2/d), Eq(b, 0)), (x*(a + b*sin(
c))**(3/2)*cos(c)**3, Eq(d, 0)), (-16*a**4*sqrt(a + b*sin(c + d*x))/(315*b*
*3*d) + 8*a**3*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(315*b**2*d) + 8*a**2*
sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2/(21*b*d) + 2*a**2*sqrt(a + b*sin(c
 + d*x))*cos(c + d*x)**2/(5*b*d) + 152*a*sqrt(a + b*sin(c + d*x))*sin(c + d
*x)**3/(315*d) + 4*a*sqrt(a + b*sin(c + d*x))*sin(c + d*x)*cos(c + d*x)**2/
(5*d) + 8*b*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**4/(45*d) + 2*b*sqrt(a +
b*sin(c + d*x))*sin(c + d*x)**2*cos(c + d*x)**2/(5*d), True))
```

3.485 $\int \cos(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+b\sin(c+dx))^{5/2}}{5bd}$$

```
[Out] 2/5*(a+b*sin(d*x+c))^(5/2)/b/d
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 32}

$$\frac{2(a+b\sin(c+dx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(5/2))/(5*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{3/2}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{2(a+b\sin(c+dx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{2(a+b\sin(c+dx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(5/2))/(5*b*d)
```

fricas [B] time = 0.94, size = 53, normalized size = 2.21

$$-\frac{2\left(b^2\cos(dx+c)^2 - 2ab\sin(dx+c) - a^2 - b^2\right)\sqrt{b\sin(dx+c)+a}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(b*sin(d*x +
 c) + a)/(b*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

maple [A] time = 0.04, size = 21, normalized size = 0.88

$$\frac{2(a+b\sin(dx+c))^{\frac{5}{2}}}{5bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5*(a+b*sin(d*x+c))^(5/2)/b/d
```

maxima [A] time = 0.32, size = 20, normalized size = 0.83

$$\frac{2(b\sin(dx+c)+a)^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*sin(d*x + c) + a)^(5/2)/(b*d)
```

mupad [B] time = 5.41, size = 20, normalized size = 0.83

$$\frac{2\,(a+b\,\sin(c+d\,x))^{5/2}}{5\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] (2*(a + b*sin(c + d*x))^(5/2))/(5*b*d)
```

sympy [A] time = 26.32, size = 116, normalized size = 4.83

$$\begin{cases} a^{\frac{3}{2}}x\cos(c) & \text{for}\ b=0\wedge d=0 \\ \frac{a^{\frac{3}{2}}\sin(c+dx)}{d} & \text{for}\ b=0 \\ x(a+b\sin(c))^{\frac{3}{2}}\cos(c) & \text{for}\ d=0 \\ \frac{2a^2\sqrt{a+b\sin(c+dx)}}{5bd}+\frac{4a\sqrt{a+b\sin(c+dx)}\sin(c+dx)}{5d}+\frac{2b\sqrt{a+b\sin(c+dx)}\sin^2(c+dx)}{5d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((a**(3/2)*x*cos(c), Eq(b, 0) & Eq(d, 0)), (a**(3/2)*sin(c + d*x)/
d, Eq(b, 0)), (x*(a + b*sin(c))**(3/2)*cos(c), Eq(d, 0)), (2*a**2*sqrt(a +
b*sin(c + d*x))/(5*b*d) + 4*a*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(5*d) +
 2*b*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2/(5*d), True))
```

3.486 $\int \sec(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=94

$$-\frac{2b\sqrt{a+b\sin(c+dx)}}{d}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] -(a-b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d+(a+b)^(3/2)*arct
anh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/d-2*b*(a+b*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2668, 704, 827, 1166, 206}

$$-\frac{2b\sqrt{a+b\sin(c+dx)}}{d}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + ((a +
b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d - (2*b*Sqrt[a + b
*Sin[c + d*x]])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 704

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*
d^2 - a*e^2 + 2*c*d*e*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^{3/2}}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{2b\sqrt{a+b\sin(c+dx)}}{d} - \frac{b\,\text{Subst}\left(\int \frac{-a^2-b^2-2ax}{\sqrt{a+x}\,(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{2b\sqrt{a+b\sin(c+dx)}}{d} - \frac{(2b)\,\text{Subst}\left(\int \frac{a^2-b^2-2ax^2}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}$$

$$= -\frac{2b\sqrt{a+b\sin(c+dx)}}{d} - \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}$$

$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Mathematica [A] time = 0.10, size = 89, normalized size = 0.95

$$\frac{-2b\sqrt{a+b\sin(c+dx)} + (a-b)^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)\right) + (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]) + (a + b)^(
3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] - 2*b*Sqrt[a + b*Sin[c +
 d*x]])/d
```

fricas [F] time = 1.90, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)\sin(dx+c) + a\sec(dx+c))\sqrt{b\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)*sin(d*x + c) + a*sec(d*x + c))*sqrt(b*sin(d*x + c)
 + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.48, size = 218, normalized size = 2.32

$$-\frac{2b\sqrt{a+b\sin(dx+c)}}{d} + \frac{\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)a^2}{d\sqrt{-a+b}} - \frac{2b\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)a}{d\sqrt{-a+b}} + \frac{b^2\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)}{d\sqrt{-a+b}} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2*b*(a+b*sin(d*x+c))^(1/2)/d+1/d/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2
)/(-a+b)^(1/2))*a^2-2/d*b/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)
^(1/2))*a+1/d*b^2/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+
1/d/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+2/d*b/(a+b)
^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+1/d*b^2/(a+b)^(1/2)*ar
ctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \sin (c + d x))^{3/2}}{\cos (c + d x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x),x)
```

```
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.487 $\int \sec^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+\dots}{2d}$$

```
[Out] -1/4*(2*a+b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d+1/4*
(2*a-b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d+1/2*sec(d
*x+c)^2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2668, 739, 827, 1166, 206}

$$-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+\dots}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(Sqrt[a - b]*(2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*d
) + ((2*a - b)*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(
4*d) + (Sec[c + d*x]^2*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^{3/2}}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d} - \frac{b\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-2a^2 \ldots\right.}{\sqrt{a+x}\,(\ldots}\right.}{\ldots}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d} - \frac{b\,\text{Subst}\left(\int \frac{\frac{a^2}{2}+\frac{1}{2}(\ldots}{-a^2+b\ldots}\right.}{\ldots}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d} + \frac{((2a-b)(a+b))\,\text{S}\ldots}{\ldots}$$

$$= -\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d} + \frac{(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+\ldots}}{\ldots}\right.}{4d}$$

Mathematica [A] time = 0.70, size = 121, normalized size = 0.93

$$\frac{-\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2\sec^2(c+dx)(a\sin(c+dx)+\ldots}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-(Sqrt[a - b]*(2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]) + (
2*a - b)*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + 2*Sec[
c + d*x]^2*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(4*d)
```

fricas [B] time = 0.98, size = 1969, normalized size = 15.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((2*a - b)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*
a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b
^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*
a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 -
 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b +
112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x
 + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x
+ c) + 8)) - (2*a + b)*sqrt(a - b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 +
 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 2
8*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 -
 (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*
b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3
*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*si
n(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin
```

```
(d*x + c) + 8)) - 16*(a*sin(d*x + c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(
d*x + c)^2), -1/32*(2*(2*a - b)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^
2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x
+ c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos
(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - (2*
a + b)*sqrt(a - b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a
^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)
*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b
^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin
(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2
 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos
(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))
- 16*(a*sin(d*x + c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1/
32*(2*(2*a + b)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b
 - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a
 + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*
a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a - b)*sqrt(a + b
)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^
2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 -
 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^
2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b
*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*
b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*c
os(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(a*sin(d*x +
 c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1/16*((2*a + b)*sqr
t(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b
 - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^
2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b
^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a - b)*sqrt(-a - b)*arctan(-1/4*(b^2
*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*s
qrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*
b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*
x + c)^2 - 8*(a*sin(d*x + c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)
^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.68, size = 279, normalized size = 2.15

$$2a\sqrt{a+b\sin(dx+c)}\ \sqrt{-a+b}\ \sqrt{a+b}\ \sin(dx+c)-\left(-2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a^2\sqrt{-a+b}\ -b\,\mathrm{arctanh}\left(\frac{\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x)
[Out] 1/4*(2*a*(a+b*sin(d*x+c))^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(d*x+c)-(-2*arc
tanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2*(-a+b)^(1/2)-b*arctanh((a+b*si
n(d*x+c))^(1/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)+b^2*arctanh((a+b*sin(d*x+c))^(1
/2)/(a+b)^(1/2))*(-a+b)^(1/2)-2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))
*a^2*(a+b)^(1/2)+b*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2
)+b^2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*(a+b)^(1/2))*cos(d*x+c)^2
```

```
+2*(a+b*sin(d*x+c))^(1/2)*b*(-a+b)^(1/2)*(a+b)^(1/2))/(-a+b)^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^2/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.488 $\int \sec^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=188

$$-\frac{3\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d\sqrt{a-b}}+\frac{3\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d\sqrt{a+b}}+\frac{\sec^4(c+dx)(a\sin(c+$$

```
[Out] -3/32*(4*a^2-2*a*b-b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)
^(1/2)+3/32*(4*a^2+2*a*b-b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/d
/(a+b)^(1/2)-1/16*sec(d*x+c)^2*(b-6*a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+
1/4*sec(d*x+c)^4*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2668, 739, 823, 827, 1166, 206}

$$-\frac{3\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d\sqrt{a-b}}+\frac{3\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d\sqrt{a+b}}+\frac{\sec^4(c+dx)(a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*(4*a^2 - 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(3
2*Sqrt[a - b]*d) + (3*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]
]/Sqrt[a + b]])/(32*Sqrt[a + b]*d) - (Sec[c + d*x]^2*(b - 6*a*Sin[c + d*x])
*Sqrt[a + b*Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*Sq
rt[a + b*Sin[c + d*x]])/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
```

```
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{(a+x)^{3/2}}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4d} - \frac{b^3\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-6a^2\right.}{\sqrt{a+x}}\right.}{}$$

$$= -\frac{\sec^2(c+dx)(b-6a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{16d} + \frac{\sec^4(c+dx)(b}{}$$

$$= -\frac{\sec^2(c+dx)(b-6a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{16d} + \frac{\sec^4(c+dx)(b}{}$$

$$= -\frac{\sec^2(c+dx)(b-6a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{16d} + \frac{\sec^4(c+dx)(b}{}$$

$$= -\frac{3\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32\sqrt{a-b}\,d} + \frac{3\left(4a^2+2ab-b^2\right)\tanh^{-}}{32\sqrt{a+b}}$$

Mathematica [A] time = 2.60, size = 297, normalized size = 1.58

$$-\frac{8\left(b^2-a^2\right)\sec^4(c+dx)(a\sin(c+dx)-b)(a+b\sin(c+dx))^{5/2}+3\sqrt{a-b}\,(a+b)^2\left(4a^3-6a^2b+ab^2+b^3\right)\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/32*(3*Sqrt[a - b]*(a + b)^2*(4*a^3 - 6*a^2*b + a*b^2 + b^3)*ArcTanh[Sqrt
[a + b*Sin[c + d*x]]/Sqrt[a - b]] - 3*(a - b)^2*Sqrt[a + b]*(4*a^3 + 6*a^2*
b + a*b^2 - b^3)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + 8*(-a^2 +
b^2)*Sec[c + d*x]^4*(-b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(5/2) + 2*Se
c[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2)*(5*a^2*b - 3*b^3 + (-6*a^3 + 4*a*b^
```

```
2)*Sin[c + d*x]) - 2*b*Sqrt[a + b*Sin[c + d*x]]*(12*a^4 - 13*a^2*b^2 + 3*b^
4 + (6*a^3*b - 4*a*b^3)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(dx+c\right)^{5}\sin\left(dx+c\right)+a\sec\left(dx+c\right)^{5}\right)\sqrt{b\sin\left(dx+c\right)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^5*sin(d*x + c) + a*sec(d*x + c)^5)*sqrt(b*sin(d*x
+ c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.02, size = 409, normalized size = 2.18

$$\frac{4\sqrt{a+b\sin\left(dx+c\right)}\,\sqrt{-a+b}\,\sqrt{a+b}\,b\left(b\left(\cos^{2}\left(dx+c\right)\right)+8a\sin\left(dx+c\right)-b\right)+3b\left(4\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/32*(4*(a+b*sin(d*x+c))^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*b*(b*cos(d*x+c)^2+8
*a*sin(d*x+c)-b)+3*b*(4*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2*(-a
+b)^(1/2)+2*b*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)-b^
2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(-a+b)^(1/2)+4*arctan((a+b*si
n(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2*(a+b)^(1/2)-2*b*arctan((a+b*sin(d*x+c))^(
1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)-b^2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(
1/2))*(a+b)^(1/2))*cos(d*x+c)^4+6*(a+b*sin(d*x+c))^(1/2)*(-a+b)^(1/2)*(a+b)
^(1/2)*b*(2*a*sin(d*x+c)-b)*cos(d*x+c)^2-24*(a+b*sin(d*x+c))^(3/2)*a*(-a+b)
^(1/2)*(a+b)^(1/2)+24*(a+b*sin(d*x+c))^(1/2)*a^2*(-a+b)^(1/2)*(a+b)^(1/2)+1
2*(a+b*sin(d*x+c))^(1/2)*b^2*(-a+b)^(1/2)*(a+b)^(1/2))/(-a+b)^(1/2)/(a+b)^(
1/2)/b/cos(d*x+c)^4/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.489 $\int \cos^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=329

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(a^2+28ab\sin(c+dx)+3b^2\right)}{231bd}-\frac{32a\left(a^4-6a^2b^2-27b^4\right)\sqrt{a+b\sin(c+dx)}\,E}{1155b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -2/11*b*cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2)/d+2/231*cos(d*x+c)^3*(a^2+3*b^2
+28*a*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b/d-4/1155*cos(d*x+c)*(4*a^4-21*
a^2*b^2-15*b^4-3*a*b*(a^2+31*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^3/d+
32/1155*a*(a^4-6*a^2*b^2-27*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/1155*(4
*a^6-25*a^4*b^2+6*a^2*b^4+15*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^
(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2692, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(a^2+28ab\sin(c+dx)+3b^2\right)}{231bd}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(-3ab\left(a^2+\right.\right.}{1155b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(11*d) - (32*a*(a^4 - 6*a^2*
b^2 - 27*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(1155*b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^6 - 25*
a^4*b^2 + 6*a^2*b^4 + 15*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1155*b^4*d*Sqrt[a + b*Sin[c + d*x]]) +
 (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(a^2 + 3*b^2 + 28*a*b*Sin[c + d
*x]))/(231*b*d) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(4*a^4 - 21*a^2*
b^2 - 15*b^4 - 3*a*b*(a^2 + 31*b^2)*Sin[c + d*x]))/(1155*b^3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2}{11}\int \frac{\cos^4(c+dx)\left(\frac{11a^2}{2}+\frac{b^2}{2}+\right.}{\sqrt{a+b\sin(c\,-}}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx}}{23}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx}}{23}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx}}{23}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx}}{23}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{32a\left(a^4-6a^2b^2-27b^4\right)E\left(\frac{1}{2}\left(c\right.\right.}{1155b^4d\sqrt{}}$$

Mathematica [A] time = 1.09, size = 278, normalized size = 0.84

$$\frac{64\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\left(4\left(a^5-6a^3b^2-27ab^4\right)\left((a+b)E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)\right)+b^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (64*(b^2*(a^4 - 114*a^2*b^2 - 15*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)] + 4*(a^5 - 6*a^3*b^2 - 27*a*b^4)*((a + b)*EllipticE[(-2*c + Pi -
 2*d*x)/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b
)]))*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*(a + b*Sin[c + d*x])*(2*(64*a^4
 - 366*a^2*b^2 + 195*b^4)*Cos[c + d*x] + 5*b^2*(-4*a^2 + 93*b^2)*Cos[3*(c +
 d*x)] + 105*b^4*Cos[5*(c + d*x)] - 16*a*b*(3*a^2 + 128*b^2)*Sin[2*(c + d*x
)] - 280*a*b^3*Sin[4*(c + d*x)]))/(9240*b^4*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4\sin\left(dx+c\right)+a\cos\left(dx+c\right)^4\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4*sin(d*x + c) + a*cos(d*x + c)^4)*sqrt(b*sin(d*x
 + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4, x)
```

maple [B] time = 0.80, size = 1355, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/1155*(16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^6*b-581*a*b^6*sin(d*x+c)^2+2*a^4*b^3*sin(d*x+c)-373*a^
2*b^5*sin(d*x+c)-245*a*b^6*sin(d*x+c)^6-145*a^2*b^5*sin(d*x+c)^5+a^3*b^4*si
n(d*x+c)^4+766*a*b^6*sin(d*x+c)^4-2*a^4*b^3*sin(d*x+c)^3+518*a^2*b^5*sin(d*
x+c)^3-8*a^5*b^2*sin(d*x+c)^2+46*a^3*b^4*sin(d*x+c)^2+300*b^7*sin(d*x+c)^5-
255*b^7*sin(d*x+c)^3+60*b^7*sin(d*x+c)-105*b^7*sin(d*x+c)^7+8*a^5*b^2-47*a^
3*b^4+60*a*b^6+60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*b^7-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7-12*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-100*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
```

```
b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4
*b^3-360*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a^3*b^4+24*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5+372*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6+112*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b
^2+336*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^3*b^4-432*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6)/b^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^4\,(a+b\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.490 $\int \cos^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=247

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3a^2+24ab\sin(c+dx)+5b^2\right)}{105bd}+\frac{4a\left(3a^2+29b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+\right.\right.}{105b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -2/7*b*cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/d+2/105*cos(d*x+c)*(3*a^2+5*b^2+
24*a*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b/d-4/105*a*(3*a^2+29*b^2)*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c
+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/d/((a+
b*sin(d*x+c))/(a+b))^(1/2)+4/105*(3*a^4+2*a^2*b^2-5*b^4)*(sin(1/2*c+1/4*Pi+
1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*
d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*sin
(d*x+c))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2692, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3a^2+24ab\sin(c+dx)+5b^2\right)}{105bd}-\frac{4\left(2a^2b^2+3a^4-5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+\right.\right.}{105b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(7*d) + (4*a*(3*a^2 + 29*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(1
05*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (4*(3*a^4 + 2*a^2*b^2 - 5*b^
4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(105*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(3*a^2 + 5*b^2 + 24*a*b*Sin[c + d*x]))/(105*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2}{7}\int \frac{\cos^2(c+dx)\left(\frac{7a^2}{2}+\frac{b^2}{2}+4\right.}{\sqrt{a+b\sin(c+}}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{10}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{10}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{10}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{4a\left(29+\frac{3a^2}{b^2}\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\right.}{105d\sqrt{\frac{a+b\sin}{a-}}}$$

Mathematica [A] time = 1.04, size = 222, normalized size = 0.90

$$\frac{8\left(3a^4+2a^2b^2-5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+b\cos(c+dx)\left(12a^3+b\left(108a^2+5b^2\right)\sin(c+dx)\right.}{210b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a*(3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 8*(3*a^4 + 2*a^2*b^2
- 5*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)] + b*Cos[c + d*x]*(12*a^3 + 38*a*b^2 - 78*a*b^2*Cos[2*(c +
d*x)] + b*(108*a^2 + 5*b^2)*Sin[c + d*x] - 15*b^3*Sin[3*(c + d*x)]))/(210*b
^2*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+a\cos\left(dx+c\right)^{2}\right)\sqrt{b\sin\left(dx+c\right)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2*sin(d*x + c) + a*cos(d*x + c)^2)*sqrt(b*sin(d*x
+ c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

maple [B] time = 0.72, size = 943, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/105*(-15*b^5*sin(d*x+c)^5+6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b+48*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+4*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b
^3-48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a*b^4-10*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*b^5-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5-52*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+58*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
*b^4-39*a*b^4*sin(d*x+c)^4-27*a^2*b^3*sin(d*x+c)^3+25*b^5*sin(d*x+c)^3-3*a^
3*b^2*sin(d*x+c)^2+49*a*b^4*sin(d*x+c)^2+27*a^2*b^3*sin(d*x+c)-10*b^5*sin(d
*x+c)+3*a^3*b^2-10*a*b^4)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sin{\left(c + d x \right)}\right)^{\frac{3}{2}} \cos^{2}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**(3/2)*cos(c + d*x)**2, x)
```

3.491 $\int \sec^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=168

$$\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}+\frac{\sec(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{d}-\frac{a\sqrt{a+b\sin\cdots}}{\cdots}$$

```
[Out] sec(d*x+c)*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+a*(sin(1/2*c+1/4*Pi+1/
2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*
x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b
))^(1/2)-(a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*si
n(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2691, 2752, 2663, 2661, 2655, 2653}

$$\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}+\frac{\sec(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{d}-\frac{a\sqrt{a+b\sin\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/d - (a*Ellipti
cE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]) + ((a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]
])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \int \frac{\frac{b^2}{2}+\frac{1}{2}ab\sin(c+}{\sqrt{a+b\sin(c+}}$$

$$= \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \frac{1}{2}a\int \sqrt{a+b\sin(c}$$

$$= \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \frac{\left(a\sqrt{a+b\sin(c+d}\right.}{2\sqrt{}}$$

$$= \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \frac{aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\right.}{d\sqrt{\frac{a}{}}}$$

Mathematica [A] time = 0.66, size = 163, normalized size = 0.97

$$\frac{-\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+a^2\tan(c+dx)+ab\sec(c+dx)+ab\sin(c+dx)\tan(c+dx)}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*b*Sec[c + d*x] + a*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b
)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - (a^2 - b^2)*EllipticF[(-2*c + Pi -
2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + a^2*Tan[c + d
*x] + b^2*Tan[c + d*x] + a*b*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[a + b*Sin[c
 + d*x]])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^2\sin(dx+c)+a\sec(dx+c)^2\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

[Out] integral((b*sec(d*x + c)^2*sin(d*x + c) + a*sec(d*x + c)^2)*sqrt(b*sin(d*x + c) + a), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.79, size = 635, normalized size = 3.78

$$-\frac{\sqrt{\left(\cos^2(dx+c)\right)\sin(dx+c)\,b+\left(\cos^2(dx+c)\right)a}\,\left(\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\,\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\,\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x)

[Out] -1/b*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^3-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3+(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2+a*b^2*cos(d*x+c)^2-a^2*b*sin(d*x+c)-b^3*sin(d*x+c)-2*a*b^2)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*sin(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^{3/2}}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^2,x)

[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)

[Out] Timed out

3.492 $\int \sec^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=218

$$\frac{(4a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{6d\sqrt{a+b\sin(c+dx)}}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{3d}-\frac{\sec(c+dx)(}{}$$

[Out] -1/6*sec(d*x+c)*(b-4*a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+1/3*sec(d*x+c)^3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/6*(4*a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A] time = 0.44, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2691, 2866, 2752, 2663, 2661, 2655, 2653}

$$\frac{(4a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{6d\sqrt{a+b\sin(c+dx)}}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{3d}-\frac{\sec(c+dx)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(Sec[c + d*x]*(b - 4*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(6*d) + (Sec[c + d*x]^3*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(3*d) - (2*a*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((4*a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3d} - \frac{1}{3}\int \frac{\sec^2(c+\ldots}{\ldots}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+dx)(\ldots}{\ldots}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+dx)(\ldots}{\ldots}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+dx)(\ldots}{\ldots}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+dx)(\ldots}{\ldots}$$

Mathematica [A] time = 2.55, size = 211, normalized size = 0.97

$$\frac{-4\left(4a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\sec^3(c+dx)\left(12a^2\sin(c+dx)+4a^2\sin(3(c+dx))-\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)] - 4*(4*a^2 - b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + Sec[c + d*x]^3*(12*a*b -
6*a*b*Cos[2*(c + d*x)] - 2*a*b*Cos[4*(c + d*x)] + 12*a^2*Sin[c + d*x] + 7*b
^2*Sin[c + d*x] + 4*a^2*Sin[3*(c + d*x)] - b^2*Sin[3*(c + d*x)]))/(24*d*Sqr
t[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(dx+c\right)^{4}\sin\left(dx+c\right)+a\sec\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^4*sin(d*x + c) + a*sec(d*x + c)^4)*sqrt(b*sin(d*x
+ c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.20, size = 938, normalized size = 4.30

$$\frac{-\sqrt{\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)b+\left(\cos^2\left(dx+c\right)\right)a}\ b\left(4a^2-b^2\right)\sin\left(dx+c\right)\left(\cos^2\left(dx+c\right)\right)-2\sqrt{\left(\cos^2\left(dx+c\right)\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/6*(-(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*b*(4*a^2-b^2)*sin(d*
x+c)*cos(d*x+c)^2-2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*b*(a^2
+b^2)*sin(d*x+c)+4*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b^2*c
os(d*x+c)^4+(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*(4*(b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b
)/(a+b))^(1/2))*a^2*b-3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-
b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b
/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b^2-(b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)
-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b)
)^(1/2))*b^3-4*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/
(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3+4*(b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))
^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*
a*b^2-a*b^2)*cos(d*x+c)^2-4*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2
)*a*b^2)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(sin(d*x+c
)-1)/(1+sin(d*x+c))/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \sin(c + d x))^{3/2}}{\cos(c + d x)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^4,x)
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.493 $\int \sec^6(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=330

$$\frac{(32a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{60d\sqrt{a+b\sin(c+dx)}}-\frac{a(32a^2-29b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{60d(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\ldots$$

```
[Out] -1/30*sec(d*x+c)^3*(b-8*a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+1/5*sec(d*x+
c)^5*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d-1/60*sec(d*x+c)*(b*(8*a^4-13
*a^2*b^2+5*b^4)-a*(32*a^4-61*a^2*b^2+29*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^(
1/2)/(a^2-b^2)^2/d+1/60*a*(32*a^2-29*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*sin(d*x+c))/(a+b))^
(1/2)-1/60*(32*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(
(a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2691, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(-13a^2b^2+8a^4+5b^4\right)-a\left(-61a^2b^2+32a^4+29b^4\right)\sin(c+dx)\right)}{60d(a^2-b^2)^2}+\frac{(32a^2-5b^2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(Sec[c + d*x]^3*(b - 8*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(30*d) +
(Sec[c + d*x]^5*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(5*d) - (a*(
32*a^2 - 29*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Si
n[c + d*x]])/(60*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((32*a
^2 - 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(60*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a +
b*Sin[c + d*x]]*(b*(8*a^4 - 13*a^2*b^2 + 5*b^4) - a*(32*a^4 - 61*a^2*b^2 +
29*b^4)*Sin[c + d*x]))/(60*(a^2 - b^2)^2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5d} - \frac{1}{5}\int \frac{\sec^4(c+}{}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)}{}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)}{}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)}{}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)}{}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)}{}$$

Mathematica [A] time = 6.28, size = 364, normalized size = 1.10

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\sec(c+dx)\left(32a^3\sin(c+dx)-8a^2b-29ab^2\sin(c+dx)+5b^3\right)}{60\left(a^2-b^2\right)}+\frac{1}{5}\sec^5(c+dx)(a\sin(c+dx)+b)+\frac{1}{30}\sec^3(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*((Sec[c + d*x]^5*(b + a*Sin[c + d*x]))/5 + (Sec[c
 + d*x]^3*(-b + 8*a*Sin[c + d*x]))/30 + (Sec[c + d*x]*(-8*a^2*b + 5*b^3 + 3
2*a^3*Sin[c + d*x] - 29*a*b^2*Sin[c + d*x]))/(60*(a^2 - b^2))))/d - (b*((-2
*(8*a^2*b - 5*b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((32*a^3 - 29*a*b^2)*(
(2*(a + b)*EllipticE[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*a*EllipticF[(-c + Pi/2 - d*
x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c +
 d*x]]))/b))/(120*(a - b)*(a + b)*d)
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(dx+c\right)^6\sin\left(dx+c\right)+a\sec\left(dx+c\right)^6\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^6*sin(d*x + c) + a*sec(d*x + c)^6)*sqrt(b*sin(d*x
+ c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.04, size = 1519, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/120*(-2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*b*(32*a^4-37*a^
2*b^2+5*b^4)*sin(d*x+c)*cos(d*x+c)^4-4*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c
)^2*a)^(1/2)*b*(8*a^4-9*a^2*b^2+b^4)*cos(d*x+c)^2*sin(d*x+c)-24*(cos(d*x+c)
^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*b*(a^4-b^4)*sin(d*x+c)+2*(cos(d*x+c)^
2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b^2*(32*a^2-29*b^2)*cos(d*x+c)^6+2*(
cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*(32*(-b/(a-b)*sin(d*x+c)-b/
(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2)*a^4*b-24*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+
b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2
```

```
))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b^2-37*(-b/(a-b)*sin(d*x+c)-b/(
a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2)*a^2*b^3+24*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/
2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b^4+5*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)
+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
*b^5-32*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(
1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(
a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5+61*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/
2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^
2-29*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b
)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-8*a^3*b^2+8*a*b^4)*cos(d*x+c)^4-4*(co
s(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b^2*(a^2-b^2)*cos(d*x+c)^2-
48*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a^3*b^2+48*(cos(d*x+c)^
2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b^4)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-
1)*(1+sin(d*x+c)))^(1/2)/(a+b)/(a-b)/(1+sin(d*x+c))^2/(sin(d*x+c)-1)^2/b/co
s(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*sec(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^{3/2}}{\cos(c+d\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^6,x)
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.494 $\int \cos^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=154

$$\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{11/2}}{11b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{9/2}}{9b^5d}+\frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{2(a+\ldots$$

```
[Out] 2/7*(a^2-b^2)^2*(a+b*sin(d*x+c))^(7/2)/b^5/d-8/9*a*(a^2-b^2)*(a+b*sin(d*x+c
))^(9/2)/b^5/d+4/11*(3*a^2-b^2)*(a+b*sin(d*x+c))^(11/2)/b^5/d-8/13*a*(a+b*s
in(d*x+c))^(13/2)/b^5/d+2/15*(a+b*sin(d*x+c))^(15/2)/b^5/d
```

Rubi [A] time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{11/2}}{11b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{9/2}}{9b^5d}+\frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{2(a+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(9/2))/(9*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(11/2))/(11*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(13/2))/(13*b^5*d) + (2*(a
 + b*Sin[c + d*x])^(15/2))/(15*b^5*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{5/2}(b^2-x^2)^2\,dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left((a^2-b^2)^2(a+x)^{5/2}-4(a^3-ab^2)(a+x)^{7/2}+2(3a^2-b^2)(\ldots\right.\right.}{b^5d}\\ &= \frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{7/2}}{7b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{9/2}}{9b^5d}-\ldots\end{aligned}$$

Mathematica [A] time = 0.58, size = 113, normalized size = 0.73

$$\frac{2(a+b\sin(c+dx))^{7/2}\left(8190(3a^2-b^2)(a+b\sin(c+dx))^2+6435(a^2-b^2)^2+3003(a+b\sin(c+dx))^4-13860\ldots\right.}{45045b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(7/2)*(6435*(a^2 - b^2)^2 - 20020*a*(a - b)*(a + b)
*(a + b*Sin[c + d*x]) + 8190*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^2 - 13860*a
*(a + b*Sin[c + d*x])^3 + 3003*(a + b*Sin[c + d*x])^4))/(45045*b^5*d)
```

fricas [A] time = 1.00, size = 224, normalized size = 1.45

$$-\frac{2\left(7161\,ab^6\cos\left(dx+c\right)^6-128\,a^7+992\,a^5b^2-6080\,a^3b^4-5536\,ab^6-7\left(5\,a^3b^4+79\,ab^6\right)\cos\left(dx+c\right)^4+16\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/45045*(7161*a*b^6*cos(d*x + c)^6 - 128*a^7 + 992*a^5*b^2 - 6080*a^3*b^4
- 5536*a*b^6 - 7*(5*a^3*b^4 + 79*a*b^6)*cos(d*x + c)^4 + 16*(3*a^5*b^2 - 20
*a^3*b^4 - 67*a*b^6)*cos(d*x + c)^2 + (3003*b^7*cos(d*x + c)^6 + 64*a^6*b -
 480*a^4*b^3 - 9088*a^2*b^5 - 1248*b^7 - 63*(71*a^2*b^5 + 13*b^7)*cos(d*x +
 c)^4 - 8*(5*a^4*b^3 + 718*a^2*b^5 + 117*b^7)*cos(d*x + c)^2)*sin(d*x + c))
*sqrt(b*sin(d*x + c) + a)/(b^5*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^5, x)
```

maple [A] time = 0.64, size = 126, normalized size = 0.82

$$\frac{2\left(a+b\sin\left(dx+c\right)\right)^{\frac{7}{2}}\left(3003b^4\left(\cos^4\left(dx+c\right)\right)+1848a\,b^3\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)-1008a^2b^2\left(\cos^2\left(dx+c\right.\right.\right.}{45045b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/45045/b^5*(a+b*sin(d*x+c))^(7/2)*(3003*b^4*cos(d*x+c)^4+1848*a*b^3*cos(d*
x+c)^2*sin(d*x+c)-1008*a^2*b^2*cos(d*x+c)^2+2184*b^4*cos(d*x+c)^2-448*a^3*b
*sin(d*x+c)+1792*a*b^3*sin(d*x+c)+128*a^4-32*a^2*b^2+1248*b^4)/d
```

maxima [A] time = 0.33, size = 116, normalized size = 0.75

$$\frac{2\left(3003\left(b\sin\left(dx+c\right)+a\right)^{\frac{15}{2}}-13860\left(b\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}a+8190\left(3\,a^2-b^2\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-20020\left(\right.\right.}{45045\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(b*sin(d*x + c) + a)^(15/2) - 13860*(b*sin(d*x + c) + a)^(13/
2)*a + 8190*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(11/2) - 20020*(a^3 - a*b^2)
*(b*sin(d*x + c) + a)^(9/2) + 6435*(a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)
+ a)^(7/2))/(b^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.495 $\int \cos^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=83

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^3d}-\frac{2(a+b\sin(c+dx))^{11/2}}{11b^3d}+\frac{4a(a+b\sin(c+dx))^{9/2}}{9b^3d}$$

```
[Out] -2/7*(a^2-b^2)*(a+b*sin(d*x+c))^(7/2)/b^3/d+4/9*a*(a+b*sin(d*x+c))^(9/2)/b^
3/d-2/11*(a+b*sin(d*x+c))^(11/2)/b^3/d
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^3d}-\frac{2(a+b\sin(c+dx))^{11/2}}{11b^3d}+\frac{4a(a+b\sin(c+dx))^{9/2}}{9b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(7/2))/(7*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(9/2))/(9*b^3*d) - (2*(a + b*Sin[c + d*x])^(11/2))/(11*b^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{5/2}\left(b^2-x^2\right)\,dx, x, b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^{5/2}+2a(a+x)^{7/2}-(a+x)^{9/2}\right)\,dx, x, b\sin(c+dx)\right)}{b^3d}\\ &= -\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^3d}+\frac{4a(a+b\sin(c+dx))^{9/2}}{9b^3d}-\frac{2(a+b\sin(c+dx))^{11/2}}{11b^3d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 58, normalized size = 0.70

$$-\frac{2(a+b\sin(c+dx))^{7/2}\left(8a^2-28ab\sin(c+dx)+63b^2\sin^2(c+dx)-99b^2\right)}{693b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a + b*Sin[c + d*x])^(7/2)*(8*a^2 - 99*b^2 - 28*a*b*Sin[c + d*x] + 63*b
^2*Sin[c + d*x]^2))/(693*b^3*d)
```

fricas [B] time = 1.08, size = 143, normalized size = 1.72

$$-\frac{2\left(161\,ab^{4}\cos\left(dx+c\right)^{4}+8\,a^{5}-96\,a^{3}b^{2}-136\,ab^{4}-\left(3\,a^{3}b^{2}+25\,ab^{4}\right)\cos\left(dx+c\right)^{2}+\left(63\,b^{5}\cos\left(dx+c\right)^{4}-4\,a^{4}\right.}{693\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/693*(161*a*b^4*cos(d*x + c)^4 + 8*a^5 - 96*a^3*b^2 - 136*a*b^4 - (3*a^3*
b^2 + 25*a*b^4)*cos(d*x + c)^2 + (63*b^5*cos(d*x + c)^4 - 4*a^4*b - 184*a^2
*b^3 - 36*b^5 - (113*a^2*b^3 + 27*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(b
*sin(d*x + c) + a)/(b^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

maple [A] time = 0.48, size = 55, normalized size = 0.66

$$-\frac{2\left(a+b\sin\left(dx+c\right)\right)^{\frac{7}{2}}\left(-63b^{2}\left(\cos^{2}\left(dx+c\right)\right)-28ab\sin\left(dx+c\right)+8a^{2}-36b^{2}\right)}{693b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/693/b^3*(a+b*sin(d*x+c))^(7/2)*(-63*b^2*cos(d*x+c)^2-28*a*b*sin(d*x+c)+8
*a^2-36*b^2)/d
```

maxima [A] time = 0.36, size = 61, normalized size = 0.73

$$-\frac{2\left(63\left(b\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-154\left(b\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a+99\left(a^{2}-b^{2}\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}\right)}{693\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/693*(63*(b*sin(d*x + c) + a)^(11/2) - 154*(b*sin(d*x + c) + a)^(9/2)*a +
 99*(a^2 - b^2)*(b*sin(d*x + c) + a)^(7/2))/(b^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\left(a+b\,\sin\left(c+d\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.496 $\int \cos(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+b\sin(c+dx))^{7/2}}{7bd}$$

```
[Out] 2/7*(a+b*sin(d*x+c))^(7/2)/b/d
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2668, 32}

$$\frac{2(a+b\sin(c+dx))^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(7/2))/(7*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{5/2}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{2(a+b\sin(c+dx))^{7/2}}{7bd}\end{aligned}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 1.00

$$\frac{2(a+b\sin(c+dx))^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(7/2))/(7*b*d)
```

fricas [B] time = 0.76, size = 77, normalized size = 3.21

$$-\frac{2\left(3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

[Out] -2/7*(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*
b - b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b*d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c), x)

maple [A] time = 0.04, size = 21, normalized size = 0.88

$$\frac{2(a+b\sin(dx+c))^{\frac{7}{2}}}{7bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x)

[Out] 2/7*(a+b*sin(d*x+c))^(7/2)/b/d

maxima [A] time = 0.34, size = 20, normalized size = 0.83

$$\frac{2(b\sin(dx+c)+a)^{\frac{7}{2}}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 2/7*(b*sin(d*x + c) + a)^(7/2)/(b*d)

mupad [B] time = 5.57, size = 20, normalized size = 0.83

$$\frac{2\,{\left(a+b\,\sin(c+d\,x)\right)}^{7/2}}{7\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^(5/2),x)

[Out] (2*(a + b*sin(c + d*x))^(7/2))/(7*b*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)

[Out] Timed out

3.497 $\int \sec(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=117

$$-\frac{2b(a+b\sin(c+dx))^{3/2}}{3d}-\frac{4ab\sqrt{a+b\sin(c+dx)}}{d}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c}}{\sqrt{a+b}}\right.}{d}$$

```
[Out] -(a-b)^(5/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d+(a+b)^(5/2)*arct
anh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/d-2/3*b*(a+b*sin(d*x+c))^(3/2)/d-4*
a*b*(a+b*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2668, 704, 825, 827, 1166, 206}

$$-\frac{2b(a+b\sin(c+dx))^{3/2}}{3d}-\frac{4ab\sqrt{a+b\sin(c+dx)}}{d}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c}}{\sqrt{a+b}}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(((a - b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + ((a +
b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d - (4*a*b*Sqrt[a +
 b*Sin[c + d*x]])/d - (2*b*(a + b*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 704

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*
d^2 - a*e^2 + 2*c*d*e*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 1]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^{5/2}}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{2b(a+b\sin(c+dx))^{3/2}}{3d} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a+x}\left(-a^2-b^2-2ax\right)}{b^2-x^2}\,dx, x, b\sin(c$$

$$= -\frac{4ab\sqrt{a+b\sin(c+dx)}}{d} - \frac{2b(a+b\sin(c+dx))^{3/2}}{3d} + \frac{b\,\text{Subst}\left(\int \frac{a(a^2$$

$$= -\frac{4ab\sqrt{a+b\sin(c+dx)}}{d} - \frac{2b(a+b\sin(c+dx))^{3/2}}{3d} + \frac{(2b)\,\text{Subst}\left(\int\right.$$

$$= -\frac{4ab\sqrt{a+b\sin(c+dx)}}{d} - \frac{2b(a+b\sin(c+dx))^{3/2}}{3d} - \frac{(a-b)^3\,\text{Subst}}{}$$

$$= -\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Mathematica [A] time = 0.16, size = 105, normalized size = 0.90

$$\frac{-2b\sqrt{a+b\sin(c+dx)}\,(7a+b\sin(c+dx)) - 3(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right) + 3(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c}}{\sqrt{a+b}}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-3*(a - b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]] + 3*(a + b)
^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] - 2*b*Sqrt[a + b*Sin[c
 + d*x]]*(7*a + b*Sin[c + d*x]))/(3*d)
```

fricas [B] time = 3.10, size = 1937, normalized size = 16.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2 + 2*a*b + b^2)*sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4
+ 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 +
 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^
2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b
^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*
a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c
```

```
))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
 + 8)) + 3*(a^2 - 2*a*b + b^2)*sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^
4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3
 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*
b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8
*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 11
2*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x +
c) + 8)) - 16*(b^2*sin(d*x + c) + 7*a*b)*sqrt(b*sin(d*x + c) + a))/d, -1/24
*(6*(a^2 + 2*a*b + b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^
2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a
)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c
)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c))) - 3*(a^2 - 2*a*b + b^2)*sqrt
(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a
*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^
3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*
cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x
+ c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*
a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(b^2*sin(d*x + c) + 7
*a*b)*sqrt(b*sin(d*x + c) + a))/d, -1/24*(6*(a^2 - 2*a*b + b^2)*sqrt(-a + b
)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2
)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*
a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(
d*x + c))) - 3*(a^2 + 2*a*b + b^2)*sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 12
8*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a
*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (1
0*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2
 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b
+ 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d
*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*
x + c) + 8)) + 16*(b^2*sin(d*x + c) + 7*a*b)*sqrt(b*sin(d*x + c) + a))/d, -
1/12*(3*(a^2 - 2*a*b + b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8
*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c)
+ a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x
+ c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c))) + 3*(a^2 + 2*a*b + b^2)*s
qrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*
a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3
*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2
+ b^3)*sin(d*x + c))) + 8*(b^2*sin(d*x + c) + 7*a*b)*sqrt(b*sin(d*x + c) +
a))/d]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.52, size = 312, normalized size = 2.67

$$-\frac{2b\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}}{3d}-\frac{4ab\sqrt{a+b\sin\left(dx+c\right)}}{d}+\frac{\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)a^{3}}{d\sqrt{-a+b}}-\frac{3b\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)a^{2}}{d\sqrt{-a+b}}+\frac{3b^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*b*(a+b*sin(d*x+c))^(3/2)/d-4*a*b*(a+b*sin(d*x+c))^(1/2)/d+1/d/(-a+b)^(
1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^3-3/d*b/(-a+b)^(1/2)*arc
tan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2+3/d*b^2/(-a+b)^(1/2)*arctan((a
+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a-1/d*b^3/(-a+b)^(1/2)*arctan((a+b*sin(d
*x+c))^(1/2)/(-a+b)^(1/2))+1/d/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(
a+b)^(1/2))*a^3+3/d*b/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2
))*a^2+3/d*b^2/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+1/
d*b^3/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^{5/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x),x)
```

```
[Out] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.498 $\int \sec^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=155

$$\frac{ab\sqrt{a+b\sin(c+dx)}}{2d}-\frac{(a-b)^{3/2}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-3b)(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\text{se}$$

```
[Out] -1/4*(a-b)^(3/2)*(2*a+3*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d+1/
4*(2*a-3*b)*(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/d+1/2*s
ec(d*x+c)^2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d+1/2*a*b*(a+b*sin(d*x+
c))^(1/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2668, 739, 825, 827, 1166, 206}

$$\frac{ab\sqrt{a+b\sin(c+dx)}}{2d}-\frac{(a-b)^{3/2}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-3b)(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\text{se}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a - b)^(3/2)*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/
(4*d) + ((2*a - 3*b)*(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a
+ b]])/(4*d) + (a*b*Sqrt[a + b*Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b +
a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^{5/2}}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{2d} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{}}{-}\right.}{}$$

$$= \frac{ab\sqrt{a+b\sin(c+dx)}}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c}{2d}$$

$$= \frac{ab\sqrt{a+b\sin(c+dx)}}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c}{2d}$$

$$= \frac{ab\sqrt{a+b\sin(c+dx)}}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c}{2d}$$

$$= -\frac{(a-b)^{3/2}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d} + \frac{(2a-3b)(a+b)^{3/2}\tan}{4d}$$

Mathematica [A] time = 0.89, size = 147, normalized size = 0.95

$$\frac{-\sqrt{a-b}\left(2a^2+ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+\sqrt{a+b}\left(2a^2-ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2\sec^2(c}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-(Sqrt[a - b]*(2*a^2 + a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[
a - b]]) + Sqrt[a + b]*(2*a^2 - a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x
]]/Sqrt[a + b]] + 2*Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]]*(2*a*b + (a^2 +
 b^2)*Sin[c + d*x]))/(4*d)
```

fricas [B] time = 1.74, size = 2071, normalized size = 13.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((2*a^2 - a*b - 3*b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x +
 c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*
b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 +
8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b
 - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*
(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 -
 2)*sin(d*x + c) + 8)) + (2*a^2 + a*b - 3*b^2)*sqrt(a - b)*cos(d*x + c)^2*l
og((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72
*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 - 24*a^
2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x +
 c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)
*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7
*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*
(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(2*a*b + (a^2 + b^2)*sin(d*x +
 c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1/32*(2*(2*a^2 - a*b -
3*b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2
 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2
*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b +
4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^2 + a*b - 3*b^2)*sqrt(a
 - b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^
2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)
^2 + 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x +
 c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sq
rt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 -
 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 -
 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(2*a*b +
 (a^2 + b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1
/32*(2*(2*a^2 + a*b - 3*b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 -
8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c)
 + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x
 + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^2
- a*b - 3*b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4
 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3
+ 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b
^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*
b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112
*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x +
c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c
) + 8)) - 16*(2*a*b + (a^2 + b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(
d*cos(d*x + c)^2), -1/16*((2*a^2 + a*b - 3*b^2)*sqrt(-a + b)*arctan(1/4*(b^
2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*
sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a
*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d
*x + c)^2 + (2*a^2 - a*b - 3*b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c
)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*
x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*c
os(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - 8
*(2*a*b + (a^2 + b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x +
c)^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.67, size = 356, normalized size = 2.30

$$2\sin(dx+c)\sqrt{-a+b}\,\sqrt{a+b}\,\sqrt{a+b\sin(dx+c)}\,\left(a^2+b^2\right)-\left(-2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a^3\sqrt{-a+b}\,-b\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/4*(2*sin(d*x+c)*(-a+b)^(1/2)*(a+b)^(1/2)*(a+b*sin(d*x+c))^(1/2)*(a^2+b^2)
-(-2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^3*(-a+b)^(1/2)-b*arctanh
((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2*(-a+b)^(1/2)+4*b^2*arctanh((a+b*si
n(d*x+c))^(1/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)+3*b^3*arctanh((a+b*sin(d*x+c))^
(1/2)/(a+b)^(1/2))*(-a+b)^(1/2)-2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2
))*a^3*(a+b)^(1/2)+b*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2*(a+b)^
(1/2)+4*b^2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)-3*b^3
*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*(a+b)^(1/2))*cos(d*x+c)^2+4*(a
+b*sin(d*x+c))^(1/2)*b*a*(-a+b)^(1/2)*(a+b)^(1/2))/(-a+b)^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^2/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^{5/2}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.499 $\int \sec^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=199

$$-\frac{3\sqrt{a-b}\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d}+\frac{3\sqrt{a+b}\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d}+\frac{3\sec^2(c+$$

```
[Out] 1/4*sec(d*x+c)^4*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d-3/32*(4*a^2+2*a*
b-b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d+3/32*(4*a^
2-2*a*b-b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d+3/16
*sec(d*x+c)^2*(a*b+(2*a^2-b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.28, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2668, 739, 821, 827, 1166, 206}

$$-\frac{3\sqrt{a-b}\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d}+\frac{3\sqrt{a+b}\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d}+\frac{3\sec^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-3*Sqrt[a - b]*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt
[a - b]])/(32*d) + (3*Sqrt[a + b]*(4*a^2 - 2*a*b - b^2)*ArcTanh[Sqrt[a + b*
Sin[c + d*x]]/Sqrt[a + b]])/(32*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*(
a + b*Sin[c + d*x])^(3/2))/(4*d) + (3*Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x
]]*(a*b + (2*a^2 - b^2)*Sin[c + d*x]))/(16*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b^5 \operatorname{Subst}\left(\int \frac{(a+x)^{5/2}}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} - \frac{b^3 \operatorname{Subst}\left(\int \right.}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} + \frac{3\sec^2(c+dx}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} + \frac{3\sec^2(c+dx}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} + \frac{3\sec^2(c+dx}{}$$

$$= -\frac{3\sqrt{a-b}\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d} + \frac{3\sqrt{a+b}\left(4a^2\right.}{}$$

Mathematica [A] time = 3.38, size = 307, normalized size = 1.54

$$-\frac{3\sqrt{a-b}\left(a^2-b^2\right)^2\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right) - 3\sqrt{a+b}\left(a^2-b^2\right)^2\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/32*(3*Sqrt[a - b]*(a^2 - b^2)^2*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b
*Sin[c + d*x]]/Sqrt[a - b]] - 3*Sqrt[a + b]*(a^2 - b^2)^2*(4*a^2 - 2*a*b -
b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + 8*(-a^2 + b^2)*Sec[c +
 d*x]^4*(-b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(7/2) - 2*Sec[c + d*x]^2
*(-7*a^2*b + b^3 + 6*a^3*Sin[c + d*x])*(a + b*Sin[c + d*x])^(7/2) - 2*b*Sqr
t[a + b*Sin[c + d*x]]*(18*a^5 - 16*a^3*b^2 + 7*a*b^4 - 3*a^3*b^2*Cos[2*(c +
 d*x)] + b*(18*a^4 - 7*a^2*b^2 + b^4)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

fricas [B] time = 1.53, size = 2229, normalized size = 11.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/256*(3*(4*a^2 - 2*a*b - b^2)*sqrt(a + b)*cos(d*x + c)^4*log((b^4*cos(d*
x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a
^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2
 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2
*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) +
 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x
+ c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 - 2)*sin(d*x + c) + 8)) + 3*(4*a^2 + 2*a*b - b^2)*sqrt(a - b)*cos(d*x + c
)^4*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3
 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 -
24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(
d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c)
 + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^
3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*b*cos(d*x + c)^2 - 8*a
*b - (3*(2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*x + c))*sqrt(b*
sin(d*x + c) + a))/(d*cos(d*x + c)^4), -1/256*(6*(4*a^2 - 2*a*b - b^2)*sqrt
(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b
 + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^
2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b
^3)*sin(d*x + c)))*cos(d*x + c)^4 + 3*(4*a^2 + 2*a*b - b^2)*sqrt(a - b)*cos
(d*x + c)^4*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 2
56*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(1
6*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (
b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(
d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 -
 (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*b*cos(d*x + c)
^2 - 8*a*b - (3*(2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*x + c))
*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^4), -1/256*(6*(4*a^2 + 2*a*b - b
^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2
*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3
 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*
b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^4 + 3*(4*a^2 - 2*a*b - b^2)*sqrt(a +
 b)*cos(d*x + c)^4*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*
b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2
 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c
)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt
(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 1
4*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8
*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*b*cos(d
*x + c)^2 - 8*a*b - (3*(2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*
x + c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^4), -1/128*(3*(4*a^2 + 2*
a*b - b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*
b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)
/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b
 - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^4 + 3*(4*a^2 - 2*a*b - b^2)*s
qrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*
a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3
*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2
+ b^3)*sin(d*x + c)))*cos(d*x + c)^4 + 8*(a*b*cos(d*x + c)^2 - 8*a*b - (3*(
2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*x + c))*sqrt(b*sin(d*x +
 c) + a))/(d*cos(d*x + c)^4)]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.79, size = 538, normalized size = 2.70

$$\frac{4\sqrt{-a+b}\,\sqrt{a+b}\,\sqrt{a+b\sin(dx+c)}\,b\left(3ab\left(\cos^2(dx+c)\right)+8a^2\sin(dx+c)-b^2\sin(dx+c)-3ab\right)+3b\left(4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/32*(4*(-a+b)^(1/2)*(a+b)^(1/2)*(a+b*sin(d*x+c))^(1/2)*b*(3*a*b*cos(d*x+c)
^2+8*a^2*sin(d*x+c)-b^2*sin(d*x+c)-3*a*b)+3*b*(4*arctanh((a+b*sin(d*x+c))^(
1/2)/(a+b)^(1/2))*a^3*(-a+b)^(1/2)+2*b*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)
^(1/2))*a^2*(-a+b)^(1/2)-3*b^2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*
a*(-a+b)^(1/2)-b^3*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(-a+b)^(1/2)
+4*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^3*(a+b)^(1/2)-2*b*arctan((
a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2*(a+b)^(1/2)-3*b^2*arctan((a+b*sin(d
*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)+b^3*arctan((a+b*sin(d*x+c))^(1/2)/
(-a+b)^(1/2))*(a+b)^(1/2))*cos(d*x+c)^4+2*(-a+b)^(1/2)*(a+b)^(1/2)*(a+b*sin
(d*x+c))^(1/2)*b*(6*a^2*sin(d*x+c)-3*b^2*sin(d*x+c)-7*a*b)*cos(d*x+c)^2-24*
(a+b*sin(d*x+c))^(3/2)*a^2*(-a+b)^(1/2)*(a+b)^(1/2)+12*(a+b*sin(d*x+c))^(3/
2)*b^2*(-a+b)^(1/2)*(a+b)^(1/2)+24*(a+b*sin(d*x+c))^(1/2)*a^3*(-a+b)^(1/2)*
(a+b)^(1/2)+16*a*(a+b*sin(d*x+c))^(1/2)*b^2*(-a+b)^(1/2)*(a+b)^(1/2))/(-a+b
)^(1/2)/(a+b)^(1/2)/b/cos(d*x+c)^4/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.500 $\int \cos^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=398

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(7b\left(53a^2+11b^2\right)\sin(c+dx)+a\left(5a^2+59b^2\right)\right)}{3003bd}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+d}}{}$$

```
[Out] -2/13*b*cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2)/d-32/143*a*b*cos(d*x+c)^5*(a+b*
sin(d*x+c))^(1/2)/d+2/3003*cos(d*x+c)^3*(a*(5*a^2+59*b^2)+7*b*(53*a^2+11*b^
2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b/d-4/15015*cos(d*x+c)*(4*a*(5*a^4-40
*a^2*b^2-93*b^4)-3*b*(5*a^4+430*a^2*b^2+77*b^4)*sin(d*x+c))*(a+b*sin(d*x+c)
)^(1/2)/b^3/d+8/15015*(20*a^6-175*a^4*b^2-1662*a^2*b^4-231*b^6)*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*sin
(d*x+c))/(a+b))^(1/2)-32/15015*a*(5*a^6-45*a^4*b^2-53*a^2*b^4+93*b^6)*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b
^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2692, 2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(7b\left(53a^2+11b^2\right)\sin(c+dx)+a\left(5a^2+59b^2\right)\right)}{3003bd}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32*a*b*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(143*d) - (2*b*Cos[c + d*
x]^5*(a + b*Sin[c + d*x])^(3/2))/(13*d) - (8*(20*a^6 - 175*a^4*b^2 - 1662*a
^2*b^4 - 231*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(15015*b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*a*(5*a
^6 - 45*a^4*b^2 - 53*a^2*b^4 + 93*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15015*b^4*d*Sqrt[a + b*Sin[c
+ d*x]]) + (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(a*(5*a^2 + 59*b^2) +
 7*b*(53*a^2 + 11*b^2)*Sin[c + d*x]))/(3003*b*d) - (4*Cos[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]]*(4*a*(5*a^4 - 40*a^2*b^2 - 93*b^4) - 3*b*(5*a^4 + 430*a^2*
b^2 + 77*b^4)*Sin[c + d*x]))/(15015*b^3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d} + \frac{2}{13}\int \cos^4(c+dx)\sqrt{a+b\sin}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+}{13d}$$

Mathematica [A] time = 1.30, size = 321, normalized size = 0.81

$$128\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\left(b\left(5a^5b-1450a^3b^3-603ab^5\right)F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(20a^6-175a^4b^2-1662a^2b^4-231b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (128*(b*(5*a^5*b - 1450*a^3*b^3 - 603*a*b^5)*EllipticF[(-2*c + Pi - 2*d*x)/
4, (2*b)/(a + b)] + (20*a^6 - 175*a^4*b^2 - 1662*a^2*b^4 - 231*b^6)*((a + b
)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi
- 2*d*x)/4, (2*b)/(a + b)]))*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*(a + b*
Sin[c + d*x])*(4*a*(320*a^4 - 2710*a^2*b^2 + 6453*b^4)*Cos[c + d*x] - 10*a*
b^2*(20*a^2 - 2599*b^2)*Cos[3*(c + d*x)] + 5670*a*b^4*Cos[5*(c + d*x)] - b*
(480*a^4 + 56120*a^2*b^2 + 4697*b^4)*Sin[2*(c + d*x)] + 140*b^3*(-53*a^2 +
22*b^2)*Sin[4*(c + d*x)] + 1155*b^5*Sin[6*(c + d*x)]))/(240240*b^4*d*Sqrt[a
 + b*Sin[c + d*x]])
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos(dx+c)^6-2\,ab\cos(dx+c)^4\sin(dx+c)-\left(a^2+b^2\right)\cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^6 - 2*a*b*cos(d*x + c)^4*sin(d*x + c) - (a^2 +
b^2)*cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

maple [B] time = 0.96, size = 1619, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/15015*(40*a^6*b^2-3990*a*b^7*sin(d*x+c)^7+80*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-11606*a^2*b^6*
sin(d*x+c)^2+10*a^5*b^3*sin(d*x+c)-4780*a^3*b^5*sin(d*x+c)+2104*a*b^7*sin(d
*x+c)-4690*a^2*b^6*sin(d*x+c)^6-1880*a^3*b^5*sin(d*x+c)^5+11290*a*b^7*sin(d
*x+c)^5+5*a^4*b^4*sin(d*x+c)^4+14500*a^2*b^6*sin(d*x+c)^4-10*a^5*b^3*sin(d*
x+c)^3+6660*a^3*b^5*sin(d*x+c)^3-9404*a*b^7*sin(d*x+c)^3-40*a^6*b^2*sin(d*x
+c)^2+340*a^4*b^4*sin(d*x+c)^2-924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8-1155*b^8*sin(d*x+c)^8+3080*b^
8*sin(d*x+c)^6-2233*b^8*sin(d*x+c)^4+308*b^8*sin(d*x+c)^2-345*a^4*b^4+1796*
a^2*b^6+924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*b^8-80*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^8-60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-720*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^3-5
100*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^4*b^4-848*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^5+4236*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6+1488*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^7
+780*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^6*b^2+5948*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4-5724*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6)/b^5/cos(d*x+c)
/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^4\,\left(a+b\,\sin(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.501 $\int \cos^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=299

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3b\left(25a^2+7b^2\right)\sin(c+dx)+a\left(5a^2+27b^2\right)\right)}{315bd}-\frac{4a\left(5a^4+22a^2b^2-27b^4\right)\sqrt{\cdots}}{315b^2d\sqrt{a\cdots}}$$

```
[Out] -2/9*b*cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/d-8/21*a*b*cos(d*x+c)^3*(a+b*sin
(d*x+c))^(1/2)/d+2/315*cos(d*x+c)*(a*(5*a^2+27*b^2)+3*b*(25*a^2+7*b^2)*sin(
d*x+c))*(a+b*sin(d*x+c))^(1/2)/b/d-4/315*(5*a^4+102*a^2*b^2+21*b^4)*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+
1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/d/((a+b
*sin(d*x+c))/(a+b))^(1/2)+4/315*a*(5*a^4+22*a^2*b^2-27*b^4)*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*
sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2692, 2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3b\left(25a^2+7b^2\right)\sin(c+dx)+a\left(5a^2+27b^2\right)\right)}{315bd}-\frac{4a\left(22a^2b^2+5a^4-27b^4\right)\sqrt{\cdots}}{315b^2d\sqrt{a\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-8*a*b*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(21*d) - (2*b*Cos[c + d*x]
^3*(a + b*Sin[c + d*x])^(3/2))/(9*d) + (4*(5*a^4 + 102*a^2*b^2 + 21*b^4)*El
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b
^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (4*a*(5*a^4 + 22*a^2*b^2 - 27*b^
4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(315*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(a*(5*a^2 + 27*b^2) + 3*b*(25*a^2 + 7*b^2)*Sin[c + d*x]))/(31
5*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2b\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9d} + \frac{2}{9}\int \cos^2(c+dx)\sqrt{a+b\,\mathrm{si}}$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin(c}{9d}$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin(c}{9d}$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin(c}{9d}$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin(c}{9d}$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin(c}{9d}$$

Mathematica [A] time = 1.04, size = 239, normalized size = 0.80

$$\frac{b(a+b\sin(c+dx))\left(\left(40a^3-354ab^2\right)\cos(c+dx)+2b\left(\sin(2(c+dx))\left(150a^2-35b^2\cos(2(c+dx))+7b^2\right)-9\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-16*(16*b*(5*a^3*b + 3*a*b^3)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a +
b)] + (5*a^4 + 102*a^2*b^2 + 21*b^4)*((a + b)*EllipticE[(-2*c + Pi - 2*d*x)
/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]))*Sq
rt[(a + b*Sin[c + d*x])/(a + b)] + b*(a + b*Sin[c + d*x])*((40*a^3 - 354*a*
b^2)*Cos[c + d*x] + 2*b*(-95*a*b*Cos[3*(c + d*x)] + (150*a^2 + 7*b^2 - 35*b
^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])))/(1260*b^2*d*Sqrt[a + b*Sin[c + d*x
]])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(b^2\cos(dx+c)^4-2\,ab\cos(dx+c)^2\sin(dx+c)-\left(a^2+b^2\right)\cos(dx+c)^2\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2*sin(d*x + c) - (a^2 +
b^2)*cos(d*x + c)^2)*sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

maple [B] time = 0.89, size = 1190, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/315*(-35*b^6*sin(d*x+c)^6-130*a*b^5*sin(d*x+c)^5+10*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b+150*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^4*b^2+44*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^3*b^3-108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-54*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-42*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b
^6-10*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^6-194*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+162*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+42*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6-170*a^
2*b^4*sin(d*x+c)^4+49*b^6*sin(d*x+c)^4-80*a^3*b^3*sin(d*x+c)^3+212*a*b^5*si
n(d*x+c)^3-5*a^4*b^2*sin(d*x+c)^2+238*a^2*b^4*sin(d*x+c)^2-14*b^6*sin(d*x+c
)^2+80*a^3*b^3*sin(d*x+c)-82*a*b^5*sin(d*x+c)+5*a^4*b^2-68*a^2*b^4)/b^3/cos
(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\,(a+b\,\sin(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.502 $\int \sec^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=203

$$\frac{a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{\left(a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{ab\cos(c$$

```
[Out] sec(d*x+c)*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d+a*b*cos(d*x+c)*(a+b*si
n(d*x+c))^(1/2)/d+(a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2
))*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-a*(a^2-b^2)*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/
d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2691, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{\left(a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{ab\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a*b*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/d + (Sec[c + d*x]*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^(3/2))/d - ((a^2 + 3*b^2)*EllipticE[(c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a + b*Sin[c + d
*x])/(a + b)]) + (a*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{d} - \int \sqrt{a+b\sin(}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx))(}{d}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx))(}{d}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx))(}{d}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx))(}{d}$$

Mathematica [A] time = 0.90, size = 203, normalized size = 1.00

$$\frac{a^3\tan(c+dx) - a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right) + 2a^2b\sec(c+dx) + a^2b\sin(c+dx)\tan(c}{d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*b*Sec[c + d*x] + (a^3 + a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[(-2*c + P
i - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - a*(a^2 -
b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*
```

```
x])/(a + b)] + a^3*Tan[c + d*x] + 3*a*b^2*Tan[c + d*x] + a^2*b*Sin[c + d*x]
*Tan[c + d*x] + b^3*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\,ab\sec\left(dx+c\right)^{2}\sin\left(dx+c\right)-\left(b^{2}\cos\left(dx+c\right)^{2}-a^{2}-b^{2}\right)\sec\left(dx+c\right)^{2}\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*sec(d*x + c)^2*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^2)*sqrt(b*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.88, size = 1042, normalized size = 5.13

$$-\frac{\sqrt{\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)b+\left(\cos^{2}\left(dx+c\right)\right)a}\;\left(\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\;\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\;\text{EllipticF}\left(\sqrt{\frac{b\sin(dx+c}{a-b}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/b*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*((-b/(a-b)*sin(d*x+c)
-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))
^(1/2)*a^3*b+3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a
-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^2*b^2-(-b/(a-b)*sin(d*x+c)-b/(a
-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*a*b^3-3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b^4-(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2
)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b
)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^4-2*(
-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*Ell
ipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*
sin(d*x+c)+b/(a+b))^(1/2)*a^2*b^2+3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/
(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b^4+a^2*b^2*
cos(d*x+c)^2+b^4*cos(d*x+c)^2-a^3*b*sin(d*x+c)-3*a*b^3*sin(d*x+c)-3*a^2*b^2
-b^4)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/cos(d*x+c)/(a
+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^{5/2}}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^2,x)
[Out] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.503 $\int \sec^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=238

$$\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(4a^2-3b^2\right)\sin(c+dx)+ab\right)}{6d}+\frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\sin(c+dx)}}-$$

```
[Out] 1/3*sec(d*x+c)^3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d+1/6*sec(d*x+c)*(
a*b+(4*a^2-3*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+1/6*(4*a^2-3*b^2)*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1
/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((a+
b*sin(d*x+c))/(a+b))^(1/2)-2/3*a*(a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2691, 2861, 2752, 2663, 2661, 2655, 2653}

$$\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(4a^2-3b^2\right)\sin(c+dx)+ab\right)}{6d}+\frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\sin(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(3*d) - ((
4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2*a*(a^2 - b^2)*Elli
pticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
)/(3*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(
a*b + (4*a^2 - 3*b^2)*Sin[c + d*x]))/(6*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} - \frac{1}{3}\int \sec^2(c+$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} + \frac{\sec(c+dx)\sqrt{}}{}$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} + \frac{\sec(c+dx)\sqrt{}}{}$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} + \frac{\sec(c+dx)\sqrt{}}{}$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} - \frac{\left(4a^2-3b^2\right)E}{}$$

Mathematica [A] time = 3.52, size = 259, normalized size = 1.09

$$-4a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(4a^3+4a^2b-3ab^2-3b^3\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,E\left(\frac{1}{4}(-2c-2a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 4*a*(a^2 - b^2)*EllipticF[(-
2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + (S
ec[c + d*x]^3*(40*a^2*b + 5*b^3 - 4*(3*a^2*b + 2*b^3)*Cos[2*(c + d*x)] + (-
4*a^2*b + 3*b^3)*Cos[4*(c + d*x)] + 24*a^3*Sin[c + d*x] + 40*a*b^2*Sin[c +
d*x] + 8*a^3*Sin[3*(c + d*x)] - 8*a*b^2*Sin[3*(c + d*x)]))/8)/(6*d*Sqrt[a +
 b*Sin[c + d*x]])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\,ab\sec\left(dx+c\right)^{4}\sin\left(dx+c\right)-\left(b^{2}\cos\left(dx+c\right)^{2}-a^{2}-b^{2}\right)\sec\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*sec(d*x + c)^4*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^4)*sqrt(b*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.91, size = 1249, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/6*(-4*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b*(a^2-b^2)*sin(
d*x+c)*cos(d*x+c)^2-2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b*
(a^2+3*b^2)*sin(d*x+c)+(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*b^2
*(4*a^2-3*b^2)*cos(d*x+c)^4-(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2
)*(4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/
(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^4-7*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^2*b^2+3*
(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*El
lipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*b^4-4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^3*b+3*(-b/(a-
b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF
((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*
x+c)+b/(a+b))^(1/2)*a^2*b^2+4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),
((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a*b^3-3*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c
)+b/(a+b))^(1/2)*b^4+a^2*b^2-5*b^4)*cos(d*x+c)^2-6*(cos(d*x+c)^2*sin(d*x+c)
*b+cos(d*x+c)^2*a)^(1/2)*a^2*b^2-2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*
```

```
a)^(1/2)*b^4)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(sin(
d*x+c)-1)/(1+sin(d*x+c))/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^{5/2}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^4,x)
[Out] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.504 $\int \sec^6(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=322

$$\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(8a^2-3b^2\right)\sin(c+dx)+5ab\right)}{30d}+\frac{a\left(32a^2-17b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\right.}{60d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] 1/5*sec(d*x+c)^5*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d+1/30*sec(d*x+c)^
3*(5*a*b+(8*a^2-3*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d-1/60*sec(d*x+c)
*(8*a*b*(a^2-b^2)-(32*a^4-41*a^2*b^2+9*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^(1
/2)/d/(a^2-b^2)+1/60*(32*a^2-9*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b)
)^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/60*a*(32
*a^2-17*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+
c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2691, 2861, 2866, 2752, 2663, 2661, 2655, 2653}

$$\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(8a^2-3b^2\right)\sin(c+dx)+5ab\right)}{30d}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab\left(a^2-b^2\right)-\right.}{60d\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(5*d) - ((
32*a^2 - 9*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin
[c + d*x]])/(60*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (a*(32*a^2 - 17*b^2
)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(60*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d*x]^3*Sqrt[a + b*Sin[c
+ d*x]]*(5*a*b + (8*a^2 - 3*b^2)*Sin[c + d*x]))/(30*d) - (Sec[c + d*x]*Sqrt
[a + b*Sin[c + d*x]]*(8*a*b*(a^2 - b^2) - (32*a^4 - 41*a^2*b^2 + 9*b^4)*Sin
[c + d*x]))/(60*(a^2 - b^2)*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} - \frac{1}{5}\int \sec^4(c+dx$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx)\sqrt{a}}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx)\sqrt{a}}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx)\sqrt{a}}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx)\sqrt{a}}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} - \frac{(32a^2-9b^2)\,E(}{}$$

Mathematica [A] time = 6.27, size = 351, normalized size = 1.09

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{1}{5}\sec^5(c+dx)\left(a^2\sin(c+dx)+2ab+b^2\sin(c+dx)\right)+\frac{1}{30}\sec^3(c+dx)\left(8a^2\sin(c+dx)-ab\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*((Sec[c + d*x]*(-8*a*b + 32*a^2*Sin[c + d*x] - 9*
b^2*Sin[c + d*x]))/60 + (Sec[c + d*x]^3*(-(a*b) + 8*a^2*Sin[c + d*x] - 3*b^
2*Sin[c + d*x]))/30 + (Sec[c + d*x]^5*(2*a*b + a^2*Sin[c + d*x] + b^2*Sin[c
 + d*x]))/5))/d - (b*((-16*a*b*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((32*a^2 -
 9*b^2)*((2*(a + b)*EllipticE[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
 b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*a*EllipticF[(-c +
Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a +
b*Sin[c + d*x]]))/b))/(120*d)
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(2\,ab\sec\left(dx+c\right)^6\sin\left(dx+c\right)-\left(b^2\cos\left(dx+c\right)^2-a^2-b^2\right)\sec\left(dx+c\right)^6\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*sec(d*x + c)^6*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^6)*sqrt(b*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.94, size = 1360, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x)

```
[Out] 1/60*((cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b*(32*a^2-17*b^2)*
sin(d*x+c)*cos(d*x+c)^4+8*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*
a*b*(2*a^2-b^2)*cos(d*x+c)^2*sin(d*x+c)+12*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d
*x+c)^2*a)^(1/2)*a*b*(a^2+3*b^2)*sin(d*x+c)-(cos(d*x+c)^2*sin(d*x+c)*b+cos(
d*x+c)^2*a)^(1/2)*b^2*(32*a^2-9*b^2)*cos(d*x+c)^6+(cos(d*x+c)^2*sin(d*x+c)*
b+cos(d*x+c)^2*a)^(1/2)*(32*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((
a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^4-41*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b
/(a+b))^(1/2)*a^2*b^2+9*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)
/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b^4-32*(-b/(a-b)*sin(d*x
+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+
b))^(1/2)*a^3*b+24*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b
))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^2*b^2+17*(-b/(a-b)*sin(d*x+
c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b
))^(1/2)*a*b^3-9*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))
^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b^4+8*a^2*b^2-3*b^4)*cos(d*x+c)
^4+2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*b^2*(a^2-9*b^2)*cos(d
*x+c)^2+36*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a^2*b^2+12*(cos
(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*b^4)/(-(a+b*sin(d*x+c))*(sin(d
*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(sin(d*x+c)-1)^2/(1+sin(d*x+c))^2/b/cos(d*x+
c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^6, x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^6,x)

[Out] \text{Hanged}

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.505 $\int \sec^8(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=439

$$\frac{3\sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(4a^2-b^2\right)\sin(c+dx)+3ab\right)}{70d}+\frac{2a\left(8a^2-3b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\right.\right.}{35d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] 1/7*sec(d*x+c)^7*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d+3/70*sec(d*x+c)^
5*(3*a*b+(4*a^2-b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d-1/140*sec(d*x+c)^
3*(4*a*b*(a^2-b^2)-(32*a^4-39*a^2*b^2+7*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^(
1/2)/d/(a^2-b^2)-1/280*sec(d*x+c)*(a*b*(32*a^4-59*a^2*b^2+27*b^4)-(128*a^6-
272*a^4*b^2+165*a^2*b^4-21*b^6)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2
)^2/d+1/280*(128*a^4-144*a^2*b^2+21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/
(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*sin(d*x+c))/(a+b))^(
1/2)-2/35*a*(8*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(
(a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2691, 2861, 2866, 2752, 2663, 2661, 2655, 2653}

$$\frac{3\sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(4a^2-b^2\right)\sin(c+dx)+3ab\right)}{70d}-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab\left(a^2-b\right.\right.}{140d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^7*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(7*d) - ((
128*a^4 - 144*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[a + b*Sin[c + d*x]])/(280*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)]) + (2*a*(8*a^2 - 3*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(35*d*Sqrt[a + b*Sin[c + d*x]]) + (3*S
ec[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]]*(3*a*b + (4*a^2 - b^2)*Sin[c + d*x])
)/(70*d) - (Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(4*a*b*(a^2 - b^2) - (3
2*a^4 - 39*a^2*b^2 + 7*b^4)*Sin[c + d*x]))/(140*(a^2 - b^2)*d) - (Sec[c + d
*x]*Sqrt[a + b*Sin[c + d*x]]*(a*b*(32*a^4 - 59*a^2*b^2 + 27*b^4) - (128*a^6
 - 272*a^4*b^2 + 165*a^2*b^4 - 21*b^6)*Sin[c + d*x]))/(280*(a^2 - b^2)^2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} - \frac{1}{7}\int \sec^6(c+$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+dx}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+dx}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+dx}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+dx}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+dx}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} - \frac{(128a^4-144}{}$$

Mathematica [A] time = 4.44, size = 338, normalized size = 0.77

$$\frac{\frac{\sec(c+dx)(a+b\sin(c+dx))\left(128a^4\sin(c+dx)-32a^3b-144a^2b^2\sin(c+dx)+40\left(a^2-b^2\right)\sec^6(c+dx)\left(\left(a^2+b^2\right)\sin(c+dx)+2ab\right)-4\left(a^2-b^2\right)\sec^4(c+dx)\left(3\left(b^2\right.\right.}{a^2-b^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((128*a^4 - 144*a^2*b^2 + 21*b^4)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/
(a + b)] - 16*a*(8*a^3 - 8*a^2*b - 3*a*b^2 + 3*b^3)*EllipticF[(-2*c + Pi -
2*d*x)/4, (2*b)/(a + b)])*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a - b) + (Se
c[c + d*x]*(a + b*Sin[c + d*x])*(-32*a^3*b + 27*a*b^3 + 128*a^4*Sin[c + d*x
] - 144*a^2*b^2*Sin[c + d*x] + 21*b^4*Sin[c + d*x] + 2*(a^2 - b^2)*Sec[c +
d*x]^2*(-4*a*b + (32*a^2 - 7*b^2)*Sin[c + d*x]) - 4*(a^2 - b^2)*Sec[c + d*x
]^4*(a*b + 3*(-4*a^2 + b^2)*Sin[c + d*x]) + 40*(a^2 - b^2)*Sec[c + d*x]^6*(
2*a*b + (a^2 + b^2)*Sin[c + d*x])))/(a^2 - b^2))/(280*d*Sqrt[a + b*Sin[c +
d*x]])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\,ab\sec(dx+c)^8\sin(dx+c)-\left(b^2\cos(dx+c)^2-a^2-b^2\right)\sec(dx+c)^8\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*sec(d*x + c)^8*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^8)*sqrt(b*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 8.63, size = 1888, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/280*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)/cos(d*x+c)^9/(a+b*sin(d*x+c))
^(3/2)/b/(a^2-b^2)*(2*cos(d*x+c)^4*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*
a)^(1/2)*b^2*(4*a^4-5*a^2*b^2+b^4)+40*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)
^2*a)^(1/2)*b^2*(3*a^4-2*a^2*b^2-b^4)+4*cos(d*x+c)^2*(cos(d*x+c)^2*sin(d*x+
c)*b+cos(d*x+c)^2*a)^(1/2)*b^2*(a^4-14*a^2*b^2+13*b^4)-cos(d*x+c)^8*(cos(d*
x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*b^2*(128*a^4-144*a^2*b^2+21*b^4)+
16*cos(d*x+c)^2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b*(3*a^4
-4*a^2*b^2+b^4)*sin(d*x+c)+40*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1
/2)*a*b*(a^4+2*a^2*b^2-3*b^4)*sin(d*x+c)+16*cos(d*x+c)^6*(cos(d*x+c)^2*sin(
d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b*(8*a^4-11*a^2*b^2+3*b^4)*sin(d*x+c)+2*co
s(d*x+c)^4*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b*(32*a^4-43*
a^2*b^2+11*b^4)*sin(d*x+c)-cos(d*x+c)^6*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+
c)^2*a)^(1/2)*(128*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a
+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b
))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^5*b-96*EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b
))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2)*a^4*b^2-176*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a
+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b
))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b^3+117*EllipticF((b/(a-b
)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(
a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2)*a^2*b^4+48*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/
(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a
-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b^5-21*EllipticF((b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*
a)^(1/2)*b^6-128*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))
^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^6+272*(-b/(a+b)*sin(d*x+c)+b/
(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2)*a^4*b^2-165*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^
(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^4+21*(-b/(a+b)*sin(d*x+c)+
b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))
^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2)*b^6-32*a^4*b^2+39*a^2*b^4-7*b^6))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^8, x)
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^8,x)
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.506 $\int \frac{\cos^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=152

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{2\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{2(a+b\,\mathrm{s}}{9}$$

```
[Out] -8/3*a*(a^2-b^2)*(a+b*sin(d*x+c))^(3/2)/b^5/d+4/5*(3*a^2-b^2)*(a+b*sin(d*x+
c))^(5/2)/b^5/d-8/7*a*(a+b*sin(d*x+c))^(7/2)/b^5/d+2/9*(a+b*sin(d*x+c))^(9/
2)/b^5/d+2*(a^2-b^2)^2*(a+b*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A] time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2668, 697}

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{2\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{2(a+b\,\mathrm{s}}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a^2 - b^2)^2*Sqrt[a + b*Sin[c + d*x]])/(b^5*d) - (8*a*(a^2 - b^2)*(a +
b*Sin[c + d*x])^(3/2))/(3*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^(5
/2))/(5*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) + (2*(a + b*Sin
[c + d*x])^(9/2))/(9*b^5*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{\sqrt{a+x}}\,dx, x, b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{\sqrt{a+x}}-4\left(a^3-ab^2\right)\sqrt{a+x}+2\left(3a^2-b^2\right)(a+x)^{3/2}-4a(a+x)^{5/2}+(a\right.\right.}{b^5d}\\ &= \frac{2\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}{b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{4\left(3a^2-b^2\right)(a}{}\end{aligned}$$

Mathematica [A] time = 0.31, size = 118, normalized size = 0.78

$$\frac{\sqrt{a+b\sin(c+dx)}\left(1024a^4-512a^3b\sin(c+dx)-2496a^2b^2-4\left(48a^2b^2-91b^4\right)\cos(2(c+dx))+1104ab^3\sin(c\right.}{1260b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(1024*a^4 - 2496*a^2*b^2 + 2121*b^4 - 4*(48*a^2*b
^2 - 91*b^4)*Cos[2*(c + d*x)] + 35*b^4*Cos[4*(c + d*x)] - 512*a^3*b*Sin[c +
 d*x] + 1104*a*b^3*Sin[c + d*x] + 80*a*b^3*Sin[3*(c + d*x)]))/(1260*b^5*d)
```

fricas [A] time = 0.78, size = 111, normalized size = 0.73

$$\frac{2\left(35\,b^4\cos\left(dx+c\right)^4+128\,a^4-288\,a^2b^2+224\,b^4-8\left(6\,a^2b^2-7\,b^4\right)\cos\left(dx+c\right)^2+8\left(5\,ab^3\cos\left(dx+c\right)^2-8\right.}{315\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*cos(d*x + c)^4 + 128*a^4 - 288*a^2*b^2 + 224*b^4 - 8*(6*a^2*b
^2 - 7*b^4)*cos(d*x + c)^2 + 8*(5*a*b^3*cos(d*x + c)^2 - 8*a^3*b + 16*a*b^3
)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^5*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^5}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/sqrt(b*sin(d*x + c) + a), x)
```

maple [A] time = 0.33, size = 126, normalized size = 0.83

$$\frac{2\sqrt{a+b\sin\left(dx+c\right)}\,\left(35b^4\left(\cos^4\left(dx+c\right)\right)+40a\,b^3\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)-48a^2b^2\left(\cos^2\left(dx+c\right)\right)+56b\right.}{315b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/315/b^5*(a+b*sin(d*x+c))^(1/2)*(35*b^4*cos(d*x+c)^4+40*a*b^3*cos(d*x+c)^2
*sin(d*x+c)-48*a^2*b^2*cos(d*x+c)^2+56*b^4*cos(d*x+c)^2-64*a^3*b*sin(d*x+c)
+128*a*b^3*sin(d*x+c)+128*a^4-288*a^2*b^2+224*b^4)/d
```

maxima [A] time = 0.34, size = 160, normalized size = 1.05

$$\frac{2\left(315\,\sqrt{b\sin\left(dx+c\right)+a}-\frac{42\left(3\left(b\sin(dx+c)+a\right)^{\frac{5}{2}}-10\left(b\sin(dx+c)+a\right)^{\frac{3}{2}}a+15\,\sqrt{b\sin(dx+c)+a}\,a^2\right)}{b^2}+\frac{35\left(b\sin(dx+c)+a\right)^{\frac{9}{2}}-180\left(b\sin(\right.}{}\right.}{315\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(b*sin(d*x + c) + a) - 42*(3*(b*sin(d*x + c) + a)^(5/2) - 10
*(b*sin(d*x + c) + a)^(3/2)*a + 15*sqrt(b*sin(d*x + c) + a)*a^2)/b^2 + (35*
(b*sin(d*x + c) + a)^(9/2) - 180*(b*sin(d*x + c) + a)^(7/2)*a + 378*(b*sin(
d*x + c) + a)^(5/2)*a^2 - 420*(b*sin(d*x + c) + a)^(3/2)*a^3 + 315*sqrt(b*s
in(d*x + c) + a)*a^4)/b^4)/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d\,x)^5}{\sqrt{a+b\,\sin (c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.507 $\int \frac{\cos^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=81

$$-\frac{2\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^3d}-\frac{2(a+b\sin(c+dx))^{5/2}}{5b^3d}+\frac{4a(a+b\sin(c+dx))^{3/2}}{3b^3d}$$

```
[Out] 4/3*a*(a+b*sin(d*x+c))^(3/2)/b^3/d-2/5*(a+b*sin(d*x+c))^(5/2)/b^3/d-2*(a^2-
b^2)*(a+b*sin(d*x+c))^(1/2)/b^3/d
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$-\frac{2\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^3d}-\frac{2(a+b\sin(c+dx))^{5/2}}{5b^3d}+\frac{4a(a+b\sin(c+dx))^{3/2}}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*(a^2 - b^2)*Sqrt[a + b*Sin[c + d*x]])/(b^3*d) + (4*a*(a + b*Sin[c + d*x
])^(3/2))/(3*b^3*d) - (2*(a + b*Sin[c + d*x])^(5/2))/(5*b^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx&=\frac{\text{Subst}\left(\int \frac{b^2-x^2}{\sqrt{a+x}}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\&=\frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{\sqrt{a+x}}+2a\sqrt{a+x}-(a+x)^{3/2}\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\&=-\frac{2\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^3d}+\frac{4a(a+b\sin(c+dx))^{3/2}}{3b^3d}-\frac{2(a+b\sin(c+dx))^{5/2}}{5b^3d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 58, normalized size = 0.72

$$\frac{2\sqrt{a+b\sin(c+dx)}\left(-8a^2+4ab\sin(c+dx)-3b^2\sin^2(c+dx)+15b^2\right)}{15b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sin[c + d*x]]*(-8*a^2 + 15*b^2 + 4*a*b*Sin[c + d*x] - 3*b^2*S
in[c + d*x]^2))/(15*b^3*d)
```

fricas [A] time = 0.89, size = 54, normalized size = 0.67

$$\frac{2\left(3\,b^2\cos\left(dx+c\right)^2+4\,ab\sin\left(dx+c\right)-8\,a^2+12\,b^2\right)\sqrt{b\sin\left(dx+c\right)+a}}{15\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*cos(d*x + c)^2 + 4*a*b*sin(d*x + c) - 8*a^2 + 12*b^2)*sqrt(b*si
n(d*x + c) + a)/(b^3*d)
```

giac [A] time = 1.88, size = 75, normalized size = 0.93

$$\frac{2\left(15\,\sqrt{b\sin\left(dx+c\right)+a}-\frac{3\,(b\sin(dx+c)+a)^{\frac{5}{2}}-10\,(b\sin(dx+c)+a)^{\frac{3}{2}}a+15\,\sqrt{b\sin(dx+c)+a}\,a^2}{b^2}\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*sin(d*x + c) + a) - (3*(b*sin(d*x + c) + a)^(5/2) - 10*(b*s
in(d*x + c) + a)^(3/2)*a + 15*sqrt(b*sin(d*x + c) + a)*a^2)/b^2)/(b*d)
```

maple [A] time = 0.30, size = 55, normalized size = 0.68

$$-\frac{2\sqrt{a+b\sin\left(dx+c\right)}\,\left(-3b^2\left(\cos^2\left(dx+c\right)\right)-4ab\sin\left(dx+c\right)+8a^2-12b^2\right)}{15b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/15/b^3*(a+b*sin(d*x+c))^(1/2)*(-3*b^2*cos(d*x+c)^2-4*a*b*sin(d*x+c)+8*a^
2-12*b^2)/d
```

maxima [A] time = 0.32, size = 75, normalized size = 0.93

$$\frac{2\left(15\,\sqrt{b\sin\left(dx+c\right)+a}-\frac{3\,(b\sin(dx+c)+a)^{\frac{5}{2}}-10\,(b\sin(dx+c)+a)^{\frac{3}{2}}a+15\,\sqrt{b\sin(dx+c)+a}\,a^2}{b^2}\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(b*sin(d*x + c) + a) - (3*(b*sin(d*x + c) + a)^(5/2) - 10*(b*s
in(d*x + c) + a)^(3/2)*a + 15*sqrt(b*sin(d*x + c) + a)*a^2)/b^2)/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^(1/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.508 $\int \frac{\cos(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=22

$$\frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

```
[Out] 2*(a+b*sin(d*x+c))^(1/2)/b/d
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 32}

$$\frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sin[c + d*x]])/(b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+x}}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{2\sqrt{a+b\sin(c+dx)}}{bd}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sin[c + d*x]])/(b*d)
```

fricas [A] time = 1.32, size = 20, normalized size = 0.91

$$\frac{2\sqrt{b\sin(dx+c)+a}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*sin(d*x + c) + a)/(b*d)
```

giac [A] time = 1.79, size = 20, normalized size = 0.91

$$\frac{2\sqrt{b\sin(dx+c)+a}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*sin(d*x + c) + a)/(b*d)
```

maple [A] time = 0.02, size = 21, normalized size = 0.95

$$\frac{2\sqrt{a+b\sin(dx+c)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*(a+b*sin(d*x+c))^(1/2)/b/d
```

maxima [A] time = 0.32, size = 20, normalized size = 0.91

$$\frac{2\sqrt{b\sin(dx+c)+a}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*sin(d*x + c) + a)/(b*d)
```

mupad [B] time = 6.22, size = 20, normalized size = 0.91

$$\frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] (2*(a + b*sin(c + d*x))^(1/2))/(b*d)
```

sympy [A] time = 1.14, size = 54, normalized size = 2.45

$$\begin{cases} \frac{x\cos(c)}{\sqrt{a}} & \text{for } b=0 \wedge d=0 \\ \frac{\sin(c+dx)}{\sqrt{a}d} & \text{for } b=0 \\ \frac{x\cos(c)}{\sqrt{a+b\sin(c)}} & \text{for } d=0 \\ \frac{2\sqrt{a+b\sin(c+dx)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((x*cos(c)/sqrt(a), Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(sqrt(a)*d
), Eq(b, 0)), (x*cos(c)/sqrt(a + b*sin(c)), Eq(d, 0)), (2*sqrt(a + b*sin(c
+ d*x))/(b*d), True))
```

3.509 $\int \frac{\sec(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

```
[Out] -arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+arctanh((a+b*sin
(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2668, 708, 1093, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh[
Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\,(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{(2b)\,\text{Subst}\left(\int \frac{1}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh[
Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)}{\sqrt{b\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

maple [A] time = 0.47, size = 62, normalized size = 0.84

$$\frac{\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)}{d\sqrt{-a+b}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/d/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+arctanh((a+b*s
in(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\sqrt{a+b\sin\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a + b*sin(c + d*x)), x)
```

3.510 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=144

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d\left(a^2-b^2\right)}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{4d(a+b)^{3/2}}$$

```
[Out] -1/4*(2*a-3*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d+1/
4*(2*a+3*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d-1/2*s
ec(d*x+c)^2*(b-a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d/(a^2-b^2)
```

Rubi [A] time = 0.31, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2668, 741, 827, 1166, 206}

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d\left(a^2-b^2\right)}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{4d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/
2)*d) + ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a +
 b)^(3/2)*d) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]
])/(2*(a^2 - b^2)*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2\left(a^2-b^2\right)d} + \frac{b\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(2a^2-3b^2\right)+\frac{ax}{2}}{\sqrt{a+x}\left(b^2-x^2\right)}\,dx, x,\right.}{2\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2\left(a^2-b^2\right)d} + \frac{b\,\text{Subst}\left(\int \frac{-\frac{a^2}{2}+\frac{1}{2}\left(2a^2-3b^2\right)+\frac{ax^2}{2}}{-a^2+b^2+2ax^2-x^4}\right.}{\left(a^2-b^2\right.}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2\left(a^2-b^2\right)d} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x\right.}{4(a-b)d}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} + \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{3/2}d} - \frac{\sec^2(c+dx)}{}$$

Mathematica [A] time = 0.51, size = 176, normalized size = 1.22

$$\frac{\sqrt{a+b}\left(2a^2-ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)-\sqrt{a-b}\left(\left(2a^2+ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2\sqrt{a+b}\,\text{se}\right.}{4d\sqrt{a-b}\sqrt{a+b}\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a + b]*(2*a^2 - a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a
- b]] - Sqrt[a - b]*((2*a^2 + a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]
/Sqrt[a + b]] + 2*Sqrt[a + b]*Sec[c + d*x]^2*(-b + a*Sin[c + d*x])*Sqrt[a +
 b*Sin[c + d*x]]))/(4*Sqrt[a - b]*Sqrt[a + b]*(-a^2 + b^2)*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

maple [A] time = 0.82, size = 218, normalized size = 1.51

$$-\frac{b\sqrt{a+b\sin\left(dx+c\right)}}{4d\left(a-b\right)\left(b\sin\left(dx+c\right)+b\right)}+\frac{\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)a}{2d\left(a-b\right)\sqrt{-a+b}}-\frac{3\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)b}{4d\left(a-b\right)\sqrt{-a+b}}-\frac{b\sqrt{a+b\sin\left(dx+c\right)}}{4d\left(a+b\right)\left(b\sin\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d*b/(a-b)*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c)+b)+1/2/d/(a-b)/(-a+b)^(
1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a-3/4/d/(a-b)/(-a+b)^(1/2)
*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*b-1/4/d*b/(a+b)*(a+b*sin(d*x+c
))^(1/2)/(b*sin(d*x+c)-b)+1/2/d/(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/
(a+b)^(1/2))*a+3/4/d/(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2)
)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^3\,\sqrt{a+b\,\sin\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}{\left(c + d x \right)}}{\sqrt{a + b \sin{\left(c + d x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a + b*sin(c + d*x)), x)
```

3.511 $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=230

$$-\frac{3\left(4a^2-10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{5/2}}+\frac{3\left(4a^2+10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{5/2}}-\frac{\sec^4(c+dx)(b-a\,\mathrm{s}}{4}$$

```
[Out] -3/32*(4*a^2-10*a*b+7*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b
)^(5/2)/d+3/32*(4*a^2+10*a*b+7*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1
/2))/(a+b)^(5/2)/d-1/4*sec(d*x+c)^4*(b-a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)
/d/(a^2-b^2)-1/16*sec(d*x+c)^2*(b*(a^2-7*b^2)-6*a*(a^2-2*b^2)*sin(d*x+c))*(
a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d
```

Rubi [A] time = 0.39, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2668, 741, 823, 827, 1166, 206}

$$-\frac{3\left(4a^2-10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{5/2}}+\frac{3\left(4a^2+10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{5/2}}-\frac{\sec^4(c+dx)(b-a\,\mathrm{s}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-3*(4*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])
/(32*(a - b)^(5/2)*d) + (3*(4*a^2 + 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[
c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(5/2)*d) - (Sec[c + d*x]^4*(b - a*Sin[c
 + d*x])*Sqrt[a + b*Sin[c + d*x]])/(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*Sqrt
[a + b*Sin[c + d*x]]*(b*(a^2 - 7*b^2) - 6*a*(a^2 - 2*b^2)*Sin[c + d*x]))/(1
6*(a^2 - b^2)^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4\left(a^2-b^2\right)d} + \frac{b^3\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(6a^2-7b^2\right)+\frac{5ax}{2}}{\sqrt{a+x}\left(b^2-x^2\right)^2}\, \right.}{4\left(a^2-b^2\right.}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\sqrt{a+b\sin(c+}}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\sqrt{a+b\sin(c+}}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\sqrt{a+b\sin(c+}}{}$$

$$= -\frac{3\left(4a^2-10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{5/2}d} + \frac{3\left(4a^2+10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-}}{}\right.}{32(a+b)^{5/2}d}$$

Mathematica [A] time = 1.90, size = 244, normalized size = 1.06

$$\frac{\sqrt{a-b}\left(3(a-b)^2\left(4a^2+10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+\sqrt{a+b}\,\sec^4(c+dx)\sqrt{a+b\sin(c+dx)}\left(3\left(a^3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-3*(a + b)^(5/2)*(4*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]
/Sqrt[a - b]] + Sqrt[a - b]*(3*(a - b)^2*(4*a^2 + 10*a*b + 7*b^2)*ArcTanh[S
qrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + Sqrt[a + b]*Sec[c + d*x]^4*Sqrt[a +
b*Sin[c + d*x]]*(-9*a^2*b + 15*b^3 + (-(a^2*b) + 7*b^3)*Cos[2*(c + d*x)] +
a*(11*a^2 - 14*b^2)*Sin[c + d*x] + 3*(a^3 - 2*a*b^2)*Sin[3*(c + d*x)])))/(3
2*Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)^2*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx+c)^5}{\sqrt{b \sin (dx+c)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 1.11, size = 618, normalized size = 2.69

$$-\frac{3b (a+b\sin (dx+c))^{\frac{3}{2}} a}{16d (b\sin (dx+c)+b)^2 \left(a^2-2ab+b^2\right)}+\frac{9b^2 (a+b\sin (dx+c))^{\frac{3}{2}}}{32d (b\sin (dx+c)+b)^2 \left(a^2-2ab+b^2\right)}+\frac{3b\sqrt{a+b\sin (dx+c)}\, a}{16d (b\sin (dx+c)+b)^2 (a-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -3/16/d/(b*sin(d*x+c)+b)^2*b/(a^2-2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)*a+9/32/
d/(b*sin(d*x+c)+b)^2*b^2/(a^2-2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)+3/16/d/(b*s
in(d*x+c)+b)^2*b/(a-b)*(a+b*sin(d*x+c))^(1/2)*a-11/32/d/(b*sin(d*x+c)+b)^2*
b^2/(a-b)*(a+b*sin(d*x+c))^(1/2)+3/8/d/(a^2-2*a*b+b^2)/(-a+b)^(1/2)*arctan(
(a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-15/16/d/(a^2-2*a*b+b^2)/(-a+b)^(1/
2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*b+21/32/d/(a^2-2*a*b+b^2)/
(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*b^2-3/16/d/(b*sin(
d*x+c)-b)^2*b/(a^2+2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)*a-9/32/d/(b*sin(d*x+c)
-b)^2*b^2/(a^2+2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)+3/16/d/(b*sin(d*x+c)-b)^2*
b/(a+b)*(a+b*sin(d*x+c))^(1/2)*a+11/32/d/(b*sin(d*x+c)-b)^2*b^2/(a+b)*(a+b*
sin(d*x+c))^(1/2)+3/8/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c)
)^(1/2)/(a+b)^(1/2))*a^2+15/16/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctanh((a+b*s
in(d*x+c))^(1/2)/(a+b)^(1/2))*a*b+21/32/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arcta
nh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}{\left(c+dx\right)}}{\sqrt{a+b\sin{\left(c+dx\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/sqrt(a + b*sin(c + d*x)), x)
```

3.512 $\int \frac{\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=247

$$-\frac{32a\left(a^2-2b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{35b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a^2-3ab\sin(c+d\right.}{35b^3d}$$

```
[Out] 2/7*cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/b/d-4/35*cos(d*x+c)*(4*a^2-5*b^2-3*
a*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^3/d+32/35*a*(a^2-2*b^2)*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*si
n(d*x+c))/(a+b))^(1/2)-8/35*(4*a^4-9*a^2*b^2+5*b^4)*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*sin(d*x+
c))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2695, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a^2-3ab\sin(c+dx)-5b^2\right)}{35b^3d}+\frac{8\left(-9a^2b^2+4a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+\right.\right.}{35b^4d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(7*b*d) - (32*a*(a^2 - 2*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(35*b
^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^4 - 9*a^2*b^2 + 5*b^4)*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(35*b^4*d*Sqrt[a + b*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[c
 + d*x]]*(4*a^2 - 5*b^2 - 3*a*b*Sin[c + d*x]))/(35*b^3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} + \frac{6\int \frac{\cos^2(c+dx)(b+a\sin(c+dx))}{\sqrt{a+b\sin(c+dx)}}\,dx}{7b}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a^2-5b^2-\right.}{35b^3d}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a^2-5b^2-\right.}{35b^3d}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a^2-5b^2-\right.}{35b^3d}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{32a\left(a^2-2b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b}}{35b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.07, size = 219, normalized size = 0.89

$$\frac{-16\left(4a^4-9a^2b^2+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+b\cos(c+dx)\left(-32a^3+\left(45b^3-8a^2b\right)\sin(c\right.}{70b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (64*a*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 16*(4*a^4 - 9*a^2*b^2 + 5*b
^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)] + b*Cos[c + d*x]*(-32*a^3 + 62*a*b^2 - 2*a*b^2*Cos[2*(c + d*x)]
 + (-8*a^2*b + 45*b^3)*Sin[c + d*x] + 5*b^3*Sin[3*(c + d*x)]))/(70*b^4*d*Sq
rt[a + b*Sin[c + d*x]])
```

fricas [F] time = 1.37, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{4}}{\sqrt{b\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{4}}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 0.85, size = 942, normalized size = 3.81

$$-\frac{2\left(-5b^{5}\left(\sin^{5}\left(dx+c\right)\right)+16\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\;\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^{4}b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/35*(-5*b^5*sin(d*x+c)^5+16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-12*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-36*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*
b^3+12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a*b^4+20*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^5-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+48*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-32*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a*b^4+a*b^4*sin(d*x+c)^4-2*a^2*b^3*sin(d*x+c)^3+20*b^5*sin(d*x+c)^3-8*a^3*
```

```
b^2*sin(d*x+c)^2+14*a*b^4*sin(d*x+c)^2+2*a^2*b^3*sin(d*x+c)-15*b^5*sin(d*x+
c)+8*a^3*b^2-15*a*b^4)/b^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4}{\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

3.513 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=175

$$-\frac{4\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\cos(c+dx)\sqrt{}}{3}$$

```
[Out] 2/3*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b/d-4/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1
/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/d/((a+b*sin(d*x+c))/(a+b))^
(1/2)+4/3*(a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/
2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*s
in(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2695, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\cos(c+dx)\sqrt{}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b*d) + (4*a*EllipticE[(c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*d*Sqrt[(a + b*S
in[c + d*x])/(a + b)]) - (4*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d
*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx &= \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd}+\frac{2\int \frac{b+a\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{3b}\\ &= \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd}+\frac{1}{3}\left(2\left(1-\frac{a^2}{b^2}\right)\right)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx+\frac{(2a}{\quad}\\ &= \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd}+\frac{\left(2a\sqrt{a+b\sin(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}\,d}{3b^2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}\\ &= \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd}+\frac{4aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}\end{aligned}$$

Mathematica [A] time = 0.82, size = 145, normalized size = 0.83

$$\frac{4\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+2b\cos(c+dx)(a+b\sin(c+dx))-4a(a+b)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{3b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*b*Cos[c + d*x]*(a + b*Sin[c + d*x]) - 4*a*(a + b)*EllipticE[(-2*c + Pi -
 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 4*(a^2 - b^2
)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^2}{\sqrt{b\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 0.74, size = 462, normalized size = 2.64

$$\frac{\frac{4\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^2b}{3}-\frac{4\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\mathrm{Elli}}{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^2*b-2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*b^3-2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3+2*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^2-b^3*sin(d*x+c)
^3-a*b^2*sin(d*x+c)^2+b^3*sin(d*x+c)+a*b^2)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a + b*sin(c + d*x)), x)
```

3.514 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=183

$$-\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d\left(a^2-b^2\right)}-\frac{a\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{\sqrt{\frac{a+b\sin(c+dx}{a+b}}}{d\sqrt{a}}$$

```
[Out] -sec(d*x+c)*(b-a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d/(a^2-b^2)+a*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d/(
(a+b*sin(d*x+c))/(a+b))^(1/2)-(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2
))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2696, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d\left(a^2-b^2\right)}-\frac{a\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{\sqrt{\frac{a+b\sin(c+dx}{a+b}}}{d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((Sec[c + d*x]*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/((a^2 - b^2)
*d)) - (a*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d
*x]])/((a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (EllipticF[(c -
Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a
 + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{\left(a^2-b^2\right)d}+\frac{\int \frac{\frac{b^2}{2}+\frac{1}{2}ab\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{-a^2+b^2}$$

$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{\left(a^2-b^2\right)d}+\frac{1}{2}\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx - \cdots$$

$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{\left(a^2-b^2\right)d}-\frac{\left(a\sqrt{a+b\sin(c+dx)}\right)\int \sqrt{\frac{a}{a+b} \cdots}}{2\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c \cdots}{a+b}}}$$

$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{\left(a^2-b^2\right)d}-\frac{aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b \cdots}}{\left(a^2-b^2\right)d\sqrt{\frac{a+b\sin(c+ \cdots}{a+b}}}$$

Mathematica [A] time = 0.63, size = 177, normalized size = 0.97

$$\frac{-\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+a^2\tan(c+dx)-ab\sec(c+dx)+ab\sin(c+dx)\tan(c+dx) \cdots}{d(a-b)(a+b)\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-(a*b*Sec[c + d*x]) + a*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - (a^2 - b^2)*EllipticF[(-2*c + Pi
 - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + a^2*Tan[c
+ d*x] - b^2*Tan[c + d*x] + a*b*Sin[c + d*x]*Tan[c + d*x])/((a - b)*(a + b)
*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 0.80, size = 640, normalized size = 3.50

$$\frac{\sqrt{\left(\cos^2(dx+c)\right)\sin(dx+c)\,b+\left(\cos^2(dx+c)\right)a}\,\left(\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\,\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\,\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x)
[Out] 1/b*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*((b/(a-b)*sin(d*x+c)+1
/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/
(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*a^3-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b)
)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1
/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2
)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b
+(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(
-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*b^3-a*b^2*cos(d*x+c)^2+a^2*b*sin(d*x+c)-b^3*sin
(d*x+c))/(a+b)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(a-b
)/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^2\,\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a + b*sin(c + d*x)), x)
```

3.515 $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=291

$$-\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3d\left(a^2-b^2\right)}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(b\left(a^2-5b^2\right)-4a\left(a^2-2b\right.\right.}{6d\left(a^2-b^2\right)^2}$$

```
[Out] -1/3*sec(d*x+c)^3*(b-a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d/(a^2-b^2)-1/6*s
ec(d*x+c)*(b*(a^2-5*b^2)-4*a*(a^2-2*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)
/(a^2-b^2)^2/d+2/3*a*(a^2-2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1
/6*(4*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(
d*x+c))/(a+b))^(1/2)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2696, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3d\left(a^2-b^2\right)}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(b\left(a^2-5b^2\right)-4a\left(a^2-2b\right.\right.}{6d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -(Sec[c + d*x]^3*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(3*(a^2 - b
^2)*d) - (2*a*(a^2 - 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sq
rt[a + b*Sin[c + d*x]])/(3*(a^2 - b^2)^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b
)]) + ((4*a^2 - 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)])/(6*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - (
Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(b*(a^2 - 5*b^2) - 4*a*(a^2 - 2*b^2)*
Sin[c + d*x]))/(6*(a^2 - b^2)^2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^2(c+dx)\left(-2a^2+\frac{5b^2}{2}-\frac{3}{2}ab\sin(c+dx\right.}{\sqrt{a+b\sin(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}}{6}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}}{6}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}}{6}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{2a\left(a^2-2b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right.\right.}{3\left(a^2-b^2\right)^2 d\sqrt{}}$$

Mathematica [A] time = 4.14, size = 306, normalized size = 1.05

$$-4\left(4a^4-9a^2b^2+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+16a\left(a^3+a^2b-2ab^2-2b^3\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}E\Big($$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (16*a*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 4*(4*a^4 - 9*a^2*b^2 + 5*b^
4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)] + Sec[c + d*x]^3*(-4*a^3*b + 10*a*b^3 + (-6*a^3*b + 14*a*b^3)*Co
s[2*(c + d*x)] + (-2*a^3*b + 4*a*b^3)*Cos[4*(c + d*x)] + 12*a^4*Sin[c + d*x
] - 25*a^2*b^2*Sin[c + d*x] + 13*b^4*Sin[c + d*x] + 4*a^4*Sin[3*(c + d*x)]
- 9*a^2*b^2*Sin[3*(c + d*x)] + 5*b^4*Sin[3*(c + d*x)]))/(24*(a - b)^2*(a +
b)^2*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{4}}{\sqrt{b\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)^{4}}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

maple [B] time = 2.26, size = 1314, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/6*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)/cos(d*x+c)^5/(a+b*sin(d*x+c))^(
3/2)/b/(a^4-2*a^2*b^2+b^4)*(-4*cos(d*x+c)^4*(cos(d*x+c)^2*sin(d*x+c)*b+cos(
d*x+c)^2*a)^(1/2)*a*b^2*(a^2-2*b^2)+cos(d*x+c)^2*(cos(d*x+c)^2*sin(d*x+c)*b
+cos(d*x+c)^2*a)^(1/2)*b*(4*a^4-9*a^2*b^2+5*b^4)*sin(d*x+c)+2*(cos(d*x+c)^2
*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*b*(a^4-2*a^2*b^2+b^4)*sin(d*x+c)-cos(d*
x+c)^2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*(4*(-b/(a-b)*sin(d*
x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b
)*a)^(1/2)*a^4*b-3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)
+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b)
)^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b^2-9*(-b/(a-b)*sin(d*x+c
)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a
```

```
)^(1/2)*a^2*b^3+3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+
b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))
^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b^4+5*(-b/(a-b)*sin(d*x+c)-b
/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2)*b^5-4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b)
)^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5+12*(-b/(a-b)*sin(d*x+c)-b/(a-b))^
(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3
*b^2-8*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1
/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a
-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-a^3*b^2+a*b^4))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^4}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^4\,\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

3.516 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^5d}-\frac{8a\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^5d}-\frac{2\left(a^2-b^2\right)^2}{b^5d\sqrt{a+b\sin(c+dx)}}+\frac{2(a+b\sin(c+d}{7b^5d}$$

```
[Out] 4/3*(3*a^2-b^2)*(a+b*sin(d*x+c))^(3/2)/b^5/d-8/5*a*(a+b*sin(d*x+c))^(5/2)/b
^5/d+2/7*(a+b*sin(d*x+c))^(7/2)/b^5/d-2*(a^2-b^2)^2/b^5/d/(a+b*sin(d*x+c))^
(1/2)-8*a*(a^2-b^2)*(a+b*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2668, 697}

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^5d}-\frac{8a\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^5d}-\frac{2\left(a^2-b^2\right)^2}{b^5d\sqrt{a+b\sin(c+dx)}}+\frac{2(a+b\sin(c+d}{7b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)^2)/(b^5*d*Sqrt[a + b*Sin[c + d*x]]) - (8*a*(a^2 - b^2)*Sqrt
[a + b*Sin[c + d*x]])/(b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^(3/2)
)/(3*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) + (2*(a + b*Sin[c
+ d*x])^(7/2))/(7*b^5*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{(a+x)^{3/2}}\,dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{(a+x)^{3/2}}-\frac{4\left(a^3-ab^2\right)}{\sqrt{a+x}}+2\left(3a^2-b^2\right)\sqrt{a+x}-4a(a+x)^{3/2}+(a+x)^{5/2}\right)}{b^5d}\\ &= -\frac{2\left(a^2-b^2\right)^2}{b^5d\sqrt{a+b\sin(c+dx)}}-\frac{8a\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{4\left(3a^2-b^2\right)(a+b}{3b^5}\end{aligned}$$

Mathematica [A] time = 0.26, size = 116, normalized size = 0.77

$$\frac{30b^4\cos^4(c+dx)-16\left(48a^4+ab\left(24a^2-35b^2\right)\sin(c+dx)-70a^2b^2+\left(5b^4-6a^2b^2\right)\sin^2(c+dx)+3ab^3\sin}{105b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (30*b^4*Cos[c + d*x]^4 - 16*(48*a^4 - 70*a^2*b^2 + 15*b^4 + a*b*(24*a^2 - 3
5*b^2)*Sin[c + d*x] + (-6*a^2*b^2 + 5*b^4)*Sin[c + d*x]^2 + 3*a*b^3*Sin[c +
 d*x]^3))/(105*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [A] time = 0.54, size = 125, normalized size = 0.83

$$\frac{2\left(15\,b^4\cos\left(dx+c\right)^4-384\,a^4+608\,a^2b^2-160\,b^4-8\left(6\,a^2b^2-5\,b^4\right)\cos\left(dx+c\right)^2+8\left(3\,ab^3\cos\left(dx+c\right)^2-24\,a\right.\right.}{105\left(b^6d\sin\left(dx+c\right)+ab^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^4*cos(d*x + c)^4 - 384*a^4 + 608*a^2*b^2 - 160*b^4 - 8*(6*a^2*b
^2 - 5*b^4)*cos(d*x + c)^2 + 8*(3*a*b^3*cos(d*x + c)^2 - 24*a^3*b + 32*a*b^
3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^6*d*sin(d*x + c) + a*b^5*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^5}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.35, size = 116, normalized size = 0.77

$$\frac{\frac{16a\,b^3\left(\cos^2(dx+c)\right)\sin(dx+c)}{35}+\frac{2\left(-192a^3b+256a\,b^3\right)\sin(dx+c)}{105}+\frac{2b^4\left(\cos^4(dx+c)\right)}{7}+\frac{2\left(-48a^2b^2+40b^4\right)\left(\cos^2(dx+c)\right)}{105}-\frac{256a^4}{35}+\frac{1216a^2b^2}{105}-6}{b^5\sqrt{a+b\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/105/b^5*(24*a*b^3*cos(d*x+c)^2*sin(d*x+c)+(-192*a^3*b+256*a*b^3)*sin(d*x+
c)+15*b^4*cos(d*x+c)^4+(-48*a^2*b^2+40*b^4)*cos(d*x+c)^2-384*a^4+608*a^2*b^
2-160*b^4)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [A] time = 0.32, size = 124, normalized size = 0.83

$$\frac{2\left(\frac{15\,(b\sin(dx+c)+a)^{\frac{7}{2}}-84\,(b\sin(dx+c)+a)^{\frac{5}{2}}a+70\left(3\,a^2-b^2\right)(b\sin(dx+c)+a)^{\frac{3}{2}}-420\left(a^3-ab^2\right)\sqrt{b\sin(dx+c)+a}}{b^4}-\frac{105\left(a^4-2\,a^2b^2+b^4\right)}{\sqrt{b\sin(dx+c)+a}\,b^4}\right)}{105\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(b*sin(d*x + c) + a)^(7/2) - 84*(b*sin(d*x + c) + a)^(5/2)*a + 7
0*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(3/2) - 420*(a^3 - a*b^2)*sqrt(b*sin(d
*x + c) + a))/b^4 - 105*(a^4 - 2*a^2*b^2 + b^4)/(sqrt(b*sin(d*x + c) + a)*b
^4))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^5}{(a+b\,\sin(c+dx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.517 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2\left(a^2-b^2\right)}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2(a+b\sin(c+dx))^{3/2}}{3b^3d}+\frac{4a\sqrt{a+b\sin(c+dx)}}{b^3d}$$

```
[Out] -2/3*(a+b*sin(d*x+c))^(3/2)/b^3/d+2*(a^2-b^2)/b^3/d/(a+b*sin(d*x+c))^(1/2)+
4*a*(a+b*sin(d*x+c))^(1/2)/b^3/d
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{2\left(a^2-b^2\right)}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2(a+b\sin(c+dx))^{3/2}}{3b^3d}+\frac{4a\sqrt{a+b\sin(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - b^2))/(b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (4*a*Sqrt[a + b*Sin[c +
d*x]])/(b^3*d) - (2*(a + b*Sin[c + d*x])^(3/2))/(3*b^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^{3/2}}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(a+x)^{3/2}}+\frac{2a}{\sqrt{a+x}}-\sqrt{a+x}\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{2\left(a^2-b^2\right)}{b^3d\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}}{b^3d}-\frac{2(a+b\sin(c+dx))^{3/2}}{3b^3d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.72

$$\frac{16a^2+8ab\sin(c+dx)+b^2\cos(2(c+dx))-7b^2}{3b^3d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a^2 - 7*b^2 + b^2*Cos[2*(c + d*x)] + 8*a*b*Sin[c + d*x])/(3*b^3*d*Sqrt[
a + b*Sin[c + d*x]])
```

fricas [A] time = 0.80, size = 67, normalized size = 0.85

$$\frac{2\left(b^{2}\cos\left(dx+c\right)^{2}+4\,ab\sin\left(dx+c\right)+8\,a^{2}-4\,b^{2}\right)\sqrt{b\sin\left(dx+c\right)+a}}{3\left(b^{4}d\sin\left(dx+c\right)+ab^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*cos(d*x + c)^2 + 4*a*b*sin(d*x + c) + 8*a^2 - 4*b^2)*sqrt(b*sin(d*
x + c) + a)/(b^4*d*sin(d*x + c) + a*b^3*d)
```

giac [A] time = 0.56, size = 72, normalized size = 0.91

$$\frac{2\left(\frac{3\left(a^{2}-b^{2}\right)}{\sqrt{b\sin(dx+c)+a}\,b^{3}}-\frac{(b\sin(dx+c)+a)^{\frac{3}{2}}b^{6}-6\,\sqrt{b\sin(dx+c)+a}\,ab^{6}}{b^{9}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(a^2 - b^2)/(sqrt(b*sin(d*x + c) + a)*b^3) - ((b*sin(d*x + c) + a)^(
3/2)*b^6 - 6*sqrt(b*sin(d*x + c) + a)*a*b^6)/b^9)/d
```

maple [A] time = 0.33, size = 54, normalized size = 0.68

$$\frac{\frac{2b^{2}\left(\cos^{2}(dx+c)\right)}{3}+\frac{8ab\sin(dx+c)}{3}+\frac{16a^{2}}{3}-\frac{8b^{2}}{3}}{b^{3}\sqrt{a+b\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/3/b^3/(a+b*sin(d*x+c))^(1/2)*(b^2*cos(d*x+c)^2+4*a*b*sin(d*x+c)+8*a^2-4*b
^2)/d
```

maxima [A] time = 0.33, size = 67, normalized size = 0.85

$$-\frac{2\left(\frac{(b\sin(dx+c)+a)^{\frac{3}{2}}-6\,\sqrt{b\sin(dx+c)+a}\,a}{b^{2}}-\frac{3\left(a^{2}-b^{2}\right)}{\sqrt{b\sin(dx+c)+a}\,b^{2}}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*(((b*sin(d*x + c) + a)^(3/2) - 6*sqrt(b*sin(d*x + c) + a)*a)/b^2 - 3*(
a^2 - b^2)/(sqrt(b*sin(d*x + c) + a)*b^2))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.518 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2}{bd\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -2/b/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2668, 32}

$$-\frac{2}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
[Out] -2/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{2}{bd\sqrt{a+b\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$-\frac{2}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
[Out] -2/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [A] time = 0.64, size = 32, normalized size = 1.45

$$-\frac{2\sqrt{b\sin(dx+c)+a}}{b^2d\sin(dx+c)+abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(b*sin(d*x + c) + a)/(b^2*d*sin(d*x + c) + a*b*d)
```

giac [**A**] time = 1.90, size = 20, normalized size = 0.91

$$-\frac{2}{\sqrt{b\sin(dx+c)+a}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(b*sin(d*x + c) + a)*b*d)
```

maple [**A**] time = 0.02, size = 21, normalized size = 0.95

$$-\frac{2}{bd\sqrt{a+b\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x)
[Out] -2/b/d/(a+b*sin(d*x+c))^(1/2)
```

maxima [**A**] time = 0.32, size = 20, normalized size = 0.91

$$-\frac{2}{\sqrt{b\sin(dx+c)+a}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*sin(d*x + c) + a)*b*d)
```

mupad [**B**] time = 6.14, size = 51, normalized size = 2.32

$$-\frac{4\,(a+b\,\sin(c+d\,x))^{3/2}}{b\,d\,\left(2\,a^2+4\,a\,b\,\sin(c+d\,x)+2\,b^2\,\sin(c+d\,x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^(3/2),x)
[Out] -(4*(a + b*sin(c + d*x))^(3/2))/(b*d*(2*a^2 + 2*b^2*sin(c + d*x)^2 + 4*a*b*
sin(c + d*x)))
```

sympy [**A**] time = 3.01, size = 56, normalized size = 2.55

$$\begin{cases} \frac{x\cos(c)}{a^{\frac{3}{2}}} & \text{for}\ b=0\wedge d=0 \\ \frac{\sin(c+dx)}{a^{\frac{3}{2}}d} & \text{for}\ b=0 \\ \frac{x\cos(c)}{(a+b\sin(c))^{\frac{3}{2}}} & \text{for}\ d=0 \\ -\frac{2}{bd\sqrt{a+b\sin(c+dx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((x*cos(c)/a**(3/2), Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**(3/2)
*d), Eq(b, 0)), (x*cos(c)/(a + b*sin(c))**(3/2), Eq(d, 0)), (-2/(b*d*sqrt(a
 + b*sin(c + d*x))), True))
```

3.519 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$\frac{2b}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

```
[Out] -arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d+arctanh((a+b*sin
(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d+2*b/(a^2-b^2)/d/(a+b*sin(d*x+c))^
(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2668, 710, 827, 1166, 206}

$$\frac{2b}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d)) + ArcTan
h[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b)/((a^2 - b
^2)*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{2b}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{b\,\text{Subst}\left(\int \frac{a-x}{\sqrt{a+x}\,(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{(a^2-b^2)\,d}$$
$$= \frac{2b}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{(2b)\,\text{Subst}\left(\int \frac{2a-x^2}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sin(c}\right.}{(a^2-b^2)\,d}$$
$$= \frac{2b}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{(a-b)d} + -\frac{\text{S}}{}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{2b}{(a^2-b^2)\,d\sqrt{a+b\sin(c+}}$$

Mathematica [C] time = 0.09, size = 91, normalized size = 0.87

$$\frac{(a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(b-a)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)}{d(a-b)(a+b)\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)] + (-
a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a + b)])/((a -
 b)*(a + b)*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\sec(dx+c)}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.56, size = 99, normalized size = 0.94

$$\frac{2b}{d\left(a-b\right)\left(a+b\right)\sqrt{a+b\sin\left(dx+c\right)}}+\frac{\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)}{d\left(a-b\right)\sqrt{-a+b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)}{\left(a+b\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/d*b/(a-b)/(a+b)/(a+b*sin(d*x+c))^(1/2)+1/d/(a-b)/(-a+b)^(1/2)*arctan((a+b
*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2)
)/(a+b)^(3/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**(3/2), x)
```

3.520 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{b\left(a^2+5b^2\right)}{2d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{5/2}}+\frac{(2a+$$

```
[Out] -1/4*(2*a-5*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+1/
4*(2*a+5*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d-1/2*b
*(a^2+5*b^2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)-1/2*sec(d*x+c)^2*(b-a*sin
(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2668, 741, 829, 827, 1166, 206}

$$-\frac{b\left(a^2+5b^2\right)}{2d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{5/2}}+\frac{(2a+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2*a - 5*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/
2)*d) + ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a +
 b)^(5/2)*d) - (b*(a^2 + 5*b^2))/(2*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]
]) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*Sqrt[a + b*Sin[
c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
```

```
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{b^3\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^{3/2}\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{b\,\mathrm{Subst}\left(\int \frac{\frac{1}{2}\left(2a^2-5b^2\right)+\frac{3ax}{2}}{(a+x)^{3/2}\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= -\frac{b\left(a^2+5b^2\right)}{2\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{b\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{b\left(a^2+5b^2\right)}{2\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{b\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{b\left(a^2+5b^2\right)}{2\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{(2a-5b)\,\mathrm{S}}{}$$

$$= -\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d} + \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{5/2}d} - \frac{}{2\left(a^2-b^2\right.}$$

Mathematica [C] time = 1.20, size = 221, normalized size = 1.19

$$\frac{\frac{\left(a^2+5b^2\right)\left((a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(b-a)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)\right)}{(a-b)(a+b)\sqrt{a+b\sin(c+dx)}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{3a\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \frac{2\sec^2(c}{\sqrt{}}}{4d\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((3*a*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] - (3*a*Arc
Tanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] + ((a^2 + 5*b^2)*((
a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)] + (-a
```

```
+ b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a + b)]))/((a -
b)*(a + b)*Sqrt[a + b*Sin[c + d*x]]) + (2*Sec[c + d*x]^2*(b - a*Sin[c + d*x
]))/Sqrt[a + b*Sin[c + d*x]])/(4*(-a^2 + b^2)*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.89, size = 250, normalized size = 1.34

$$-\frac{2b^3}{d(a+b)^2(a-b)^2\sqrt{a+b\sin(dx+c)}}-\frac{b\sqrt{a+b\sin(dx+c)}}{4d(a-b)^2(b\sin(dx+c)+b)}+\frac{\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)a}{2d(a-b)^2\sqrt{-a+b}}-\frac{5b\arctan\left(\right.}{4d(a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/d*b^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))^(1/2)-1/4/d*b/(a-b)^2*(a+b*sin(d*
x+c))^(1/2)/(b*sin(d*x+c)+b)+1/2/d/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*sin(d*x
+c))^(1/2)/(-a+b)^(1/2))*a-5/4/d*b/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*sin(d*x
+c))^(1/2)/(-a+b)^(1/2))-1/4/d*b/(a+b)^2*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+
c)-b)+1/2/d/(a+b)^(5/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+5/4/d
*b/(a+b)^(5/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^3\,(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c + dx)}{(a + b\sin(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**(3/2), x)
```

3.521 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{3\left(4a^2-14ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{7/2}}+\frac{3\left(4a^2+14ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{7/2}}-\frac{\sec^4(c+dx)(b}{4d\left(a^2-b^2\right)\sqrt{}}$$

```
[Out] -3/32*(4*a^2-14*a*b+15*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-
b)^(7/2)/d+3/32*(4*a^2+14*a*b+15*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^
(1/2))/(a+b)^(7/2)/d-3/16*b*(2*a^4-7*a^2*b^2-15*b^4)/(a^2-b^2)^3/d/(a+b*sin
(d*x+c))^(1/2)-1/4*sec(d*x+c)^4*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c
))^(1/2)+1/16*sec(d*x+c)^2*(b*(a^2+9*b^2)+2*a*(3*a^2-8*b^2)*sin(d*x+c))/(a^
2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2668, 741, 823, 829, 827, 1166, 206}

$$-\frac{3b\left(-7a^2b^2+2a^4-15b^4\right)}{16d\left(a^2-b^2\right)^3\sqrt{a+b\sin(c+dx)}}-\frac{3\left(4a^2-14ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{7/2}}+\frac{3\left(4a^2+14ab+15b^2\right)\tan}{32d(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*(4*a^2 - 14*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]
)/(32*(a - b)^(7/2)*d) + (3*(4*a^2 + 14*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Si
n[c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(7/2)*d) - (3*b*(2*a^4 - 7*a^2*b^2 -
15*b^4))/(16*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]^4*(b
 - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d
*x]^2*(b*(a^2 + 9*b^2) + 2*a*(3*a^2 - 8*b^2)*Sin[c + d*x]))/(16*(a^2 - b^2)
^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{b^3\,\text{Subst}\left(\int \frac{\frac{3}{2}\left(2a^2-3b^2\right)+\frac{7ax}{2}}{(a+x)^{3/2}\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+d\right.}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{\sec^2(c+dx)\left(b\left(a^2+9b^2\right)+2a\left(3a^2-8b^2\right)\right.}{16\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx}}$$

$$= -\frac{3b\left(2a^4-7a^2b^2-15b^4\right)}{16\left(a^2-b^2\right)^3 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{\sec^2(c}{}$$

$$= -\frac{3b\left(2a^4-7a^2b^2-15b^4\right)}{16\left(a^2-b^2\right)^3 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{\sec^2(c}{}$$

$$= -\frac{3b\left(2a^4-7a^2b^2-15b^4\right)}{16\left(a^2-b^2\right)^3 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{\sec^2(c}{}$$

$$= -\frac{3\left(4a^2-14ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{7/2}d} + \frac{3\left(4a^2+14ab+15b^2\right)\tanh^{-1}}{32(a+b)^{7/2}d}$$

Mathematica [C] time = 2.24, size = 324, normalized size = 1.14

$$\frac{3a\sqrt{a-b}\sqrt{a+b}\left(3a^2-8b^2\right)\sqrt{a+b\sin(c+dx)}\left(\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)-\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+}}{\sqrt{a+b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((3*(2*a^4 - 7*a^2*b^2 - 15*b^4)*((a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (
a + b*Sin[c + d*x])/(a - b)] + (-a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a
+ b*Sin[c + d*x])/(a + b)]))/2 - 4*(a - b)^2*(a + b)^2*Sec[c + d*x]^4*(-b +
 a*Sin[c + d*x]) + 3*a*Sqrt[a - b]*Sqrt[a + b]*(3*a^2 - 8*b^2)*(Sqrt[a + b]
*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]] - Sqrt[a - b]*ArcTanh[Sqrt[a
 + b*Sin[c + d*x]]/Sqrt[a + b]])*Sqrt[a + b*Sin[c + d*x]] - (a - b)*(a + b)
*Sec[c + d*x]^2*(b*(a^2 + 9*b^2) + 2*a*(3*a^2 - 8*b^2)*Sin[c + d*x]))/(16*(
a^2 - b^2)^2*(-a^2 + b^2)*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\sec(dx+c)^5}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^5}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.10, size = 649, normalized size = 2.29

$$\frac{2b^5}{d(a-b)^3(a+b)^3\sqrt{a+b\sin(dx+c)}}-\frac{3b(a+b\sin(dx+c))^{\frac{3}{2}}a}{16d(a-b)^3(b\sin(dx+c)+b)^2}+\frac{13b^2(a+b\sin(dx+c))^{\frac{3}{2}}}{32d(a-b)^3(b\sin(dx+c)+b)^2}+\frac{3}{16d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/d*b^5/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c))^(1/2)-3/16/d*b/(a-b)^3/(b*sin(d*x+
c)+b)^2*(a+b*sin(d*x+c))^(3/2)*a+13/32/d*b^2/(a-b)^3/(b*sin(d*x+c)+b)^2*(a+
b*sin(d*x+c))^(3/2)+3/16/d*b/(a-b)^3/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1
/2)*a^2-21/32/d*b^2/(a-b)^3/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)*a+15/
32/d*b^3/(a-b)^3/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)+3/8/d/(a-b)^3/(-
a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-21/16/d*b/(a-b)^
3/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a+45/32/d*b^2/(a
-b)^3/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))-3/16/d*b/(a+
b)^3/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(3/2)*a-13/32/d*b^2/(a+b)^3/(b*sin
(d*x+c)-b)^2*(a+b*sin(d*x+c))^(3/2)+3/16/d*b/(a+b)^3/(b*sin(d*x+c)-b)^2*(a+
b*sin(d*x+c))^(1/2)*a^2+21/32/d*b^2/(a+b)^3/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x
+c))^(1/2)*a+15/32/d*b^3/(a+b)^3/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1/2)+
3/8/d/(a+b)^(7/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+21/16/d*b
/(a+b)^(7/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+45/32/d*b^2/(a+b
)^(7/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^5\,{(a+b\,\sin(c+d\,x))}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x))**(3/2), x)
```

3.522 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=313

$$-\frac{8\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(a\left(32a^2-33b^2\right)-3b\left(8a^2-7b^2\right)\sin(c+dx)\right)}{63b^5d}+\frac{16a\left(32a^4-65a^2b^2+33b^4\right)\sqrt{\ }}{63b^6d\sqrt{a+}}$$

```
[Out] -2*cos(d*x+c)^5/b/d/(a+b*sin(d*x+c))^(1/2)+20/63*cos(d*x+c)^3*(8*a-7*b*sin(
d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^3/d-8/63*cos(d*x+c)*(a*(32*a^2-33*b^2)-3*b
*(8*a^2-7*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^5/d+16/63*(32*a^4-57*a^
2*b^2+21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+
c))^(1/2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-16/63*a*(32*a^4-65*a^2*b^2+3
3*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a
+b))^(1/2)/b^6/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2693, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{8\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(a\left(32a^2-33b^2\right)-3b\left(8a^2-7b^2\right)\sin(c+dx)\right)}{63b^5d}+\frac{16a\left(-65a^2b^2+32a^4+33b^4\right)\ }{63b^6d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5)/(b*d*Sqrt[a + b*Sin[c + d*x]]) + (20*Cos[c + d*x]^3*(8*
a - 7*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(63*b^3*d) - (16*(32*a^4 -
57*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
b*Sin[c + d*x]])/(63*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (16*a*(32*
a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/(63*b^6*d*Sqrt[a + b*Sin[c + d*x]]) - (8*C
os[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*(32*a^2 - 33*b^2) - 3*b*(8*a^2 - 7*
b^2)*Sin[c + d*x]))/(63*b^5*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{10\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}dx}{b}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d}$$

Mathematica [A] time = 1.51, size = 273, normalized size = 0.87

$$-64a\left(32a^4-65a^2b^2+33b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+b\cos(c+dx)\left(-1024a^4-256a^3b\sin(c+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (64*(32*a^5 + 32*a^4*b - 57*a^3*b^2 - 57*a^2*b^3 + 21*a*b^4 + 21*b^5)*Ellip
ticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b
)] - 64*a*(32*a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*Cos[c + d*x]*(-1024*a^
4 + 1760*a^2*b^2 - 595*b^4 + (-64*a^2*b^2 + 84*b^4)*Cos[2*(c + d*x)] + 7*b^
4*Cos[4*(c + d*x)] - 256*a^3*b*Sin[c + d*x] + 404*a*b^3*Sin[c + d*x] + 20*a
*b^3*Sin[3*(c + d*x)]))/(252*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\cos(dx+c)^6}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^6/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^6}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 0.82, size = 1195, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/63*(7*b^6*sin(d*x+c)^6-10*a*b^5*sin(d*x+c)^5+256*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-192*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^4*b^2-520*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^3+360*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+264*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-168*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
b^6-256*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^6+712*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-624*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+168*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6+16*
a^2*b^4*sin(d*x+c)^4-35*b^6*sin(d*x+c)^4-32*a^3*b^3*sin(d*x+c)^3+68*a*b^5*s
in(d*x+c)^3-128*a^4*b^2*sin(d*x+c)^2+196*a^2*b^4*sin(d*x+c)^2-35*b^6*sin(d*
x+c)^2+32*a^3*b^3*sin(d*x+c)-58*a*b^5*sin(d*x+c)+128*a^4*b^2-212*a^2*b^4+63
*b^6)/b^7/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^6}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(b*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^6}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.523 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{32a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{a+b\sin(c+dx)}}+\frac{8\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -2*cos(d*x+c)^3/b/d/(a+b*sin(d*x+c))^(1/2)+4/5*cos(d*x+c)*(4*a-3*b*sin(d*x+
c))*(a+b*sin(d*x+c))^(1/2)/b^3/d-8/5*(4*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2
^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*sin(d*x+c))/(a+b
))^(1/2)+32/5*a*(a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(
(a+b*sin(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2693, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{32a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{a+b\sin(c+dx)}}+\frac{8\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3)/(b*d*Sqrt[a + b*Sin[c + d*x]]) + (4*Cos[c + d*x]*(4*a -
 3*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(5*b^3*d) + (8*(4*a^2 - 3*b^2)
*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(5*
b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (32*a*(a^2 - b^2)*EllipticF[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(5*b^4*
d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{6\int \frac{\cos^2(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5b^3d} -$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5b^3d} +$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5b^3d} +$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5b^3d} +$$

Mathematica [A] time = 1.12, size = 187, normalized size = 0.82

$$\frac{b\cos(c+dx)\left(16a^2+4ab\sin(c+dx)+b^2\cos(2(c+dx))-11b^2\right)+32a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\right.}{5b^4d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*(4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 32*a*(a^2 - b^2)*Elliptic
F[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
+ b*Cos[c + d*x]*(16*a^2 - 11*b^2 + b^2*Cos[2*(c + d*x)] + 4*a*b*Sin[c + d*
x]))/(5*b^4*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{4}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 0.70, size = 797, normalized size = 3.48

$$\frac{32\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^{3}b}{5}-\frac{24a^{2}\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c)}{a-b}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5*(16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^3*b-12*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2-16*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^3+12*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4-16*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^4+28*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^2*b^2-12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4+b^4*sin(d*x+c)^4-2*a*b^3*sin(d*x+c)
^3-8*a^2*b^2*sin(d*x+c)^2+4*b^4*sin(d*x+c)^2+2*a*b^3*sin(d*x+c)+8*a^2*b^2-5
*b^4)/b^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^4}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^4}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)**4/(a + b*sin(c + d*x))**(3/2), x)
```

3.524 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{4a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\sin(c+dx)}}-\frac{4\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -2*cos(d*x+c)/b/d/(a+b*sin(d*x+c))^(1/2)+4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)
-4*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b
))^(1/2)/b^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2693, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\sin(c+dx)}}-\frac{4\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x])/(b*d*Sqrt[a + b*Sin[c + d*x]]) - (4*EllipticE[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(b^2*d*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)]) + (4*a*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[
(a + b*Sin[c + d*x])/(a + b)])/(b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{2\int \sqrt{a+b\sin(c+dx)}\,dx}{b^2} + \frac{(2a)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx}{b^2}$$

$$= -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{\left(2\sqrt{a+b\sin(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}\,dx}{b^2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{\left(2a\sqrt{\frac{a+}{}}\right.}{}$$

$$= -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{4aF\left(\frac{1}{2}\left(c\right.\right.}{b}$$

Mathematica [A] time = 2.77, size = 125, normalized size = 0.78

$$\frac{4(a+b)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-2\left(2a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+b\cos(c+dx)\right)}{b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (4*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[
c + d*x])/(a + b)] - 2*(b*Cos[c + d*x] + 2*a*EllipticF[(-2*c + Pi - 2*d*x)/
4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]))/(b^2*d*Sqrt[a + b*Si
n[c + d*x]])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\cos(dx+c)^2}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 0.75, size = 434, normalized size = 2.71

$$\frac{4\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\mathrm{EllipticE}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^{2}-4\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2*(2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^2-2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*b^2-2*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b+2*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2+b^2*sin(d*x+c)^2-b^2
)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^2}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sin(c + d*x))**(3/2), x)
```

3.525 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab-\left(a^2+3b^2\right)\sin(c+dx)\right)}{d\left(a^2-b^2\right)^2}+\frac{2b\sec(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F}{d\left(a^2-b^2\right)\sqrt{}}$$

```
[Out] 2*b*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)-sec(d*x+c)*(4*a*b-(a^2+3*
b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d+(a^2+3*b^2)*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d
/((a+b*sin(d*x+c))/(a+b))^(1/2)-a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^
(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2694, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab-\left(a^2+3b^2\right)\sin(c+dx)\right)}{d\left(a^2-b^2\right)^2}+\frac{2b\sec(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F}{d\left(a^2-b^2\right)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sec[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - ((a^2 + 3*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/((
a^2 - b^2)^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (a*EllipticF[(c - Pi/2
 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/((a^2 - b^2)*
d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(4*a*b
 - (a^2 + 3*b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2b\sec(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{\sec^2(c+dx)\left(-\frac{a}{2}+\frac{3}{2}b\sin(c+dx)\right)}{\sqrt{a+b\sin(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2b\sec(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab-\left(a^2+3b\right.\right.}{\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab-\left(a^2+3b\right.\right.}{\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab-\left(a^2+3b\right.\right.}{\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\left(a^2+3b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c}}{\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.75, size = 205, normalized size = 0.82

$$\frac{-a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-\frac{1}{2}\sec(c+dx)\left(-2a\left(a^2-b^2\right)\sin(c+dx)+b\left(a^2+3b^2\right)c\right.}{d(a-b)^2(a+b)^2\sqrt{a+b\,\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a^3 + a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - a*(a^2 - b^2)*EllipticF[(-2*c +
Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - (Sec[c +
 d*x]*(3*a^2*b + b^3 + b*(a^2 + 3*b^2)*Cos[2*(c + d*x)] - 2*a*(a^2 - b^2)*S
in[c + d*x]))/2)/((a - b)^2*(a + b)^2*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{2}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.12, size = 1062, normalized size = 4.23

$$\frac{\sqrt{\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)b+\left(\cos^{2}\left(dx+c\right)\right)a}\ \left(\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\ \sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\ \mathrm{EllipticE}\left(\sqrt{\frac{b\sin(dx+c)}{a-b}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/b*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*((-b/(a-b)*sin(d*x+c)-
b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^
(1/2)*a^4+2*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2
))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^2*b^2-3*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)
*b^4-(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/
(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^3*b-3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)
*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^2*b^2+
(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*El
lipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*a*b^3+3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(
a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b^4-a^2*b^2*c
```

```
os(d*x+c)^2-3*b^4*cos(d*x+c)^2+a^3*b*sin(d*x+c)-a*b^3*sin(d*x+c)-a^2*b^2+b^
4)/(a^2-b^2)/(a-b)/(a+b)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^
(1/2)/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**(3/2), x)
```

3.526 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=359

$$-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab-\left(a^2+7b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{2b\sec^3(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{2a\left(a^2-3b^2\right)\sqrt{}}{3d\left(a^2\right.}$$

```
[Out] 2*b*sec(d*x+c)^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)-1/3*sec(d*x+c)^3*(8*a*b
-(a^2+7*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d-1/6*sec(d*x+c
)*(a*b*(a^2-33*b^2)-(4*a^4-15*a^2*b^2-21*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^
(1/2)/(a^2-b^2)^3/d+1/6*(4*a^4-15*a^2*b^2-21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d/((a+b*sin(d*x+c
))/(a+b))^(1/2)-2/3*a*(a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^
(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2694, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab-\left(a^2+7b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{2b\sec^3(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{\sec(c+dx)\sqrt{a}}{3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sec[c + d*x]^3)/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - ((4*a^4 - 1
5*a^2*b^2 - 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b
*Sin[c + d*x]])/(6*(a^2 - b^2)^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2
*a*(a^2 - 3*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[
c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a*b - (a^2 + 7*b^2)*Sin[c + d*x]))/(
3*(a^2 - b^2)^2*d) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*b*(a^2 - 33*
b^2) - (4*a^4 - 15*a^2*b^2 - 21*b^4)*Sin[c + d*x]))/(6*(a^2 - b^2)^3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{\sec^4(c+dx)\left(-\frac{a}{2}+\frac{7}{2}b\sin(c+dx)\right)}{\sqrt{a+b\sin(c+dx)}}dx}{a^2-b^2}$$

$$= \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(8ab-\left(a^2+7b^2\right)\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(8ab-\left(a^2+7b^2\right)\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(8ab-\left(a^2+7b^2\right)\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(8ab-\left(a^2+7b^2\right)\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\left(4a^4-15a^2b^2-21b^4\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a}}{6\left(a^2-b^2\right)^3 d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 3.11, size = 348, normalized size = 0.97

$$\frac{-4a\left(a^4-4a^2b^2+3b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(4a^5+4a^4b-15a^3b^2-15a^2b^3-21ab^4-21b^5\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((4*a^5 + 4*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 - 21*a*b^4 - 21*b^5)*EllipticE[
(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] -
4*a*(a^4 - 4*a^2*b^2 + 3*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b
)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + (Sec[c + d*x]^3*(-24*a^4*b + 101*a^
2*b^3 + 19*b^5 + (-12*a^4*b + 84*a^2*b^3 + 56*b^5)*Cos[2*(c + d*x)] + (-4*a
^4*b + 15*a^2*b^3 + 21*b^5)*Cos[4*(c + d*x)] + 24*a^5*Sin[c + d*x] - 64*a^3
*b^2*Sin[c + d*x] + 40*a*b^4*Sin[c + d*x] + 8*a^5*Sin[3*(c + d*x)] - 32*a^3
*b^2*Sin[3*(c + d*x)] + 24*a*b^4*Sin[3*(c + d*x)]))/8)/(6*(a - b)^3*(a + b)
^3*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^4}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx+c)^4}{(b\sin (dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [B] time = 4.73, size = 1646, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/6*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)/cos(d*x+c)^5/(a+b*sin(d*x+c))^(
3/2)/b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*(-2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x
+c)^2*a)^(1/2)*b^2*(a^4-2*a^2*b^2+b^4)-cos(d*x+c)^4*(cos(d*x+c)^2*sin(d*x+c
)*b+cos(d*x+c)^2*a)^(1/2)*b^2*(4*a^4-15*a^2*b^2-21*b^4)+4*cos(d*x+c)^2*(cos
(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b*(a^4-4*a^2*b^2+3*b^4)*sin(
d*x+c)+2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*a*b*(a^4-2*a^2*b^
2+b^4)*sin(d*x+c)+cos(d*x+c)^2*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(
1/2)*(4*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1
/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^6-19*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/
2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/
(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^4*b^
2-6*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-
b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^4+21*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/
2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/
(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^6-4*
EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+
b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*a^5*b+3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*s
in(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^4*b^2+16*El
lipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b^3+18*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^4-12*E
llipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b
)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2)*a*b^5-21*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*s
in(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^6+a^4*b^2+6
*a^2*b^4-7*b^6))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}{\left(c + dx \right)}}{\left(a + b \sin{\left(c + dx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x))**(3/2), x)
```

3.527 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=150

$$\frac{4\left(3a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{8a\left(a^2-b^2\right)}{b^5d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(a^2-b^2\right)^2}{3b^5d(a+b\sin(c+dx))^{3/2}}+\frac{2(a+b\sin(c+dx))^{5/2}}{5b^5d}-$$

```
[Out] -2/3*(a^2-b^2)^2/b^5/d/(a+b*sin(d*x+c))^(3/2)-8/3*a*(a+b*sin(d*x+c))^(3/2)/
b^5/d+2/5*(a+b*sin(d*x+c))^(5/2)/b^5/d+8*a*(a^2-b^2)/b^5/d/(a+b*sin(d*x+c))
^(1/2)+4*(3*a^2-b^2)*(a+b*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{4\left(3a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{8a\left(a^2-b^2\right)}{b^5d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(a^2-b^2\right)^2}{3b^5d(a+b\sin(c+dx))^{3/2}}+\frac{2(a+b\sin(c+dx))^{5/2}}{5b^5d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)^2)/(3*b^5*d*(a + b*Sin[c + d*x])^(3/2)) + (8*a*(a^2 - b^2))
/(b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (4*(3*a^2 - b^2)*Sqrt[a + b*Sin[c + d*x
]])/(b^5*d) - (8*a*(a + b*Sin[c + d*x])^(3/2))/(3*b^5*d) + (2*(a + b*Sin[c
+ d*x])^(5/2))/(5*b^5*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{(a+x)^{5/2}}\,dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{(a+x)^{5/2}}-\frac{4\left(a^3-ab^2\right)}{(a+x)^{3/2}}+\frac{2\left(3a^2-b^2\right)}{\sqrt{a+x}}-4a\sqrt{a+x}+(a+x)^{3/2}\right)dx,x,b\sin(c\right.}{b^5d}\\ &= -\frac{2\left(a^2-b^2\right)^2}{3b^5d(a+b\sin(c+dx))^{3/2}}+\frac{8a\left(a^2-b^2\right)}{b^5d\sqrt{a+b\sin(c+dx)}}+\frac{4\left(3a^2-b^2\right)\sqrt{a+b\sin(}{b^5d}\end{aligned}$$

Mathematica [A] time = 0.30, size = 117, normalized size = 0.78

$$\frac{16\left(16a^4+3ab\left(8a^2-5b^2\right)\sin(c+dx)-10a^2b^2+\left(6a^2b^2-3b^4\right)\sin^2(c+dx)-ab^3\sin^3(c+dx)-b^4\right)+6b^4\text{co}}{15b^5d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (6*b^4*Cos[c + d*x]^4 + 16*(16*a^4 - 10*a^2*b^2 - b^4 + 3*a*b*(8*a^2 - 5*b^
2)*Sin[c + d*x] + (6*a^2*b^2 - 3*b^4)*Sin[c + d*x]^2 - a*b^3*Sin[c + d*x]^3
))/(15*b^5*d*(a + b*Sin[c + d*x])^(3/2))
```

fricas [A] time = 0.81, size = 147, normalized size = 0.98

$$-\frac{2\left(3\,b^4\cos\left(dx+c\right)^4+128\,a^4-32\,a^2b^2-32\,b^4-24\left(2\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2+8\left(ab^3\cos\left(dx+c\right)^2+24\,a^3b-\right.}{15\left(b^7d\cos\left(dx+c\right)^2-2\,ab^6d\sin\left(dx+c\right)-\left(a^2b^5+b^7\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*b^4*cos(d*x + c)^4 + 128*a^4 - 32*a^2*b^2 - 32*b^4 - 24*(2*a^2*b^2
 - b^4)*cos(d*x + c)^2 + 8*(a*b^3*cos(d*x + c)^2 + 24*a^3*b - 16*a*b^3)*sin
(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^7*d*cos(d*x + c)^2 - 2*a*b^6*d*sin(d
*x + c) - (a^2*b^5 + b^7)*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^5}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [A] time = 0.43, size = 116, normalized size = 0.77

$$\frac{\frac{16a\,b^3\left(\cos^2(dx+c)\right)\sin(dx+c)}{15}+\frac{2\left(192a^3b-128a\,b^3\right)\sin(dx+c)}{15}+\frac{2b^4\left(\cos^4(dx+c)\right)}{5}+\frac{2\left(-48a^2b^2+24b^4\right)\left(\cos^2(dx+c)\right)}{15}+\frac{256a^4}{15}-\frac{64a^2b^2}{15}-\frac{64b^4}{15}}{b^5\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15/b^5*(8*a*b^3*cos(d*x+c)^2*sin(d*x+c)+(192*a^3*b-128*a*b^3)*sin(d*x+c)+
3*b^4*cos(d*x+c)^4+(-48*a^2*b^2+24*b^4)*cos(d*x+c)^2+128*a^4-32*a^2*b^2-32*
b^4)/(a+b*sin(d*x+c))^(3/2)/d
```

maxima [A] time = 0.75, size = 122, normalized size = 0.81

$$\frac{2\left(\frac{3\,(b\sin(dx+c)+a)^{\frac{5}{2}}-20\,(b\sin(dx+c)+a)^{\frac{3}{2}}a+30\left(3\,a^2-b^2\right)\sqrt{b\sin(dx+c)+a}}{b^4}-\frac{5\left(a^4-2\,a^2b^2+b^4-12\left(a^3-ab^2\right)(b\sin(dx+c)+a)\right)}{(b\sin(dx+c)+a)^{\frac{3}{2}}b^4}\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(b*sin(d*x + c) + a)^(5/2) - 20*(b*sin(d*x + c) + a)^(3/2)*a + 30*
(3*a^2 - b^2)*sqrt(b*sin(d*x + c) + a))/b^4 - 5*(a^4 - 2*a^2*b^2 + b^4 - 12
*(a^3 - a*b^2)*(b*sin(d*x + c) + a))/((b*sin(d*x + c) + a)^(3/2)*b^4))/(b*d
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + d\,x)^5}{(a + b\,\sin(c + d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.528 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2(a^2-b^2)}{3b^3d(a+b\sin(c+dx))^{3/2}}-\frac{4a}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\sqrt{a+b\sin(c+dx)}}{b^3d}$$

```
[Out] 2/3*(a^2-b^2)/b^3/d/(a+b*sin(d*x+c))^(3/2)-4*a/b^3/d/(a+b*sin(d*x+c))^(1/2)
-2*(a+b*sin(d*x+c))^(1/2)/b^3/d
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{2668, 697\}$

$$\frac{2(a^2-b^2)}{3b^3d(a+b\sin(c+dx))^{3/2}}-\frac{4a}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\sqrt{a+b\sin(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^2 - b^2))/(3*b^3*d*(a + b*Sin[c + d*x])^(3/2)) - (4*a)/(b^3*d*Sqrt[a
+ b*Sin[c + d*x]]) - (2*Sqrt[a + b*Sin[c + d*x]])/(b^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^{5/2}}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(a+x)^{5/2}}+\frac{2a}{(a+x)^{3/2}}-\frac{1}{\sqrt{a+x}}\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= \frac{2(a^2-b^2)}{3b^3d(a+b\sin(c+dx))^{3/2}}-\frac{4a}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\sqrt{a+b\sin(c+dx)}}{b^3d}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 56, normalized size = 0.71

$$-\frac{2\left(8a^2+12ab\sin(c+dx)+3b^2\sin^2(c+dx)+b^2\right)}{3b^3d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^2 + b^2 + 12*a*b*Sin[c + d*x] + 3*b^2*Sin[c + d*x]^2))/(3*b^3*d*(a
 + b*Sin[c + d*x])^(3/2))
```

fricas [A] time = 0.90, size = 91, normalized size = 1.15

$$-\frac{2\left(3\,b^2\cos\left(dx+c\right)^2-12\,ab\sin\left(dx+c\right)-8\,a^2-4\,b^2\right)\sqrt{b\sin\left(dx+c\right)+a}}{3\left(b^5d\cos\left(dx+c\right)^2-2\,ab^4d\sin\left(dx+c\right)-\left(a^2b^3+b^5\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b^2*cos(d*x + c)^2 - 12*a*b*sin(d*x + c) - 8*a^2 - 4*b^2)*sqrt(b*si
n(d*x + c) + a)/(b^5*d*cos(d*x + c)^2 - 2*a*b^4*d*sin(d*x + c) - (a^2*b^3 +
 b^5)*d)
```

giac [A] time = 0.86, size = 61, normalized size = 0.77

$$-\frac{2\left(\frac{3\,\sqrt{b\sin\left(dx+c\right)+a}}{b^3}+\frac{6\left(b\sin\left(dx+c\right)+a\right)a-a^2+b^2}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}b^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(b*sin(d*x + c) + a)/b^3 + (6*(b*sin(d*x + c) + a)*a - a^2 + b^
2)/((b*sin(d*x + c) + a)^(3/2)*b^3))/d
```

maple [A] time = 0.39, size = 55, normalized size = 0.70

$$-\frac{2\left(-3b^2\left(\cos^2\left(dx+c\right)\right)+12ab\sin\left(dx+c\right)+8a^2+4b^2\right)}{3b^3\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3/b^3*(-3*b^2*cos(d*x+c)^2+12*a*b*sin(d*x+c)+8*a^2+4*b^2)/(a+b*sin(d*x+c
))^(3/2)/d
```

maxima [A] time = 0.73, size = 64, normalized size = 0.81

$$-\frac{2\left(\frac{3\,\sqrt{b\sin\left(dx+c\right)+a}}{b^2}+\frac{6\left(b\sin\left(dx+c\right)+a\right)a-a^2+b^2}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}b^2}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(b*sin(d*x + c) + a)/b^2 + (6*(b*sin(d*x + c) + a)*a - a^2 + b^
2)/((b*sin(d*x + c) + a)^(3/2)*b^2))/(b*d)
```

mupad [B] time = 11.89, size = 1402, normalized size = 17.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] (16*a^2*b^2*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i)*(a + (b*(
cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*1i)/2 - (b*(cos(d*x) + sin(d*x
)*1i)*(cos(c) + sin(c)*1i)*1i)/2)^(1/2))/(3*(a^2*b^5*d - b^7*d + 2*b^7*d*(c
os(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i) - b^7*d*(cos(4*d*x) + s
in(4*d*x)*1i)*(cos(4*c) + sin(4*c)*1i) - a*b^6*d*(cos(3*d*x) + sin(3*d*x)*1
i)*(cos(3*c) + sin(3*c)*1i)*4i + a*b^6*d*(cos(d*x) + sin(d*x)*1i)*(cos(c) +
 sin(c)*1i)*4i + 2*a^2*b^5*d*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2
*c)*1i) - 4*a^4*b^3*d*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i)
 + a^3*b^4*d*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*4i + a^2
*b^5*d*(cos(4*d*x) + sin(4*d*x)*1i)*(cos(4*c) + sin(4*c)*1i) - a^3*b^4*d*(c
os(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*4i)) - (8*a^4*(cos(2*d*x) + sin
(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i)*(a + (b*(cos(d*x) - sin(d*x)*1i)*(cos(
c) - sin(c)*1i)*1i)/2 - (b*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1i
)/2)^(1/2))/(3*(a^2*b^5*d - b^7*d + 2*b^7*d*(cos(2*d*x) + sin(2*d*x)*1i)*(c
os(2*c) + sin(2*c)*1i) - b^7*d*(cos(4*d*x) + sin(4*d*x)*1i)*(cos(4*c) + sin
(4*c)*1i) - a*b^6*d*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*4
i + a*b^6*d*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*4i + 2*a^2*b^5*d*
(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i) - 4*a^4*b^3*d*(cos(2*
d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i) + a^3*b^4*d*(cos(3*d*x) + si
n(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*4i + a^2*b^5*d*(cos(4*d*x) + sin(4*d*
x)*1i)*(cos(4*c) + sin(4*c)*1i) - a^3*b^4*d*(cos(d*x) + sin(d*x)*1i)*(cos(c
) + sin(c)*1i)*4i)) - (8*b^4*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2
*c)*1i)*(a + (b*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*1i)/2 - (b*(c
os(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1i)/2)^(1/2))/(3*(a^2*b^5*d - b
^7*d + 2*b^7*d*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i) - b^7*
d*(cos(4*d*x) + sin(4*d*x)*1i)*(cos(4*c) + sin(4*c)*1i) - a*b^6*d*(cos(3*d*
x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*4i + a*b^6*d*(cos(d*x) + sin(d
*x)*1i)*(cos(c) + sin(c)*1i)*4i + 2*a^2*b^5*d*(cos(2*d*x) + sin(2*d*x)*1i)*
(cos(2*c) + sin(2*c)*1i) - 4*a^4*b^3*d*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*
c) + sin(2*c)*1i) + a^3*b^4*d*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(
3*c)*1i)*4i + a^2*b^5*d*(cos(4*d*x) + sin(4*d*x)*1i)*(cos(4*c) + sin(4*c)*1
i) - a^3*b^4*d*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*4i)) - (a*(cos
(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*(a + (b*(cos(d*x) - sin(d*x)*1i)*
(cos(c) - sin(c)*1i)*1i)/2 - (b*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1
i)*1i)/2)^(1/2)*8i)/(b^4*d*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c
)*1i) - b^4*d + a*b^3*d*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*2i) -
 (2*(a + (b*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*1i)/2 - (b*(cos(d
*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1i)/2)^(1/2))/(b^3*d)
```

sympy [A] time = 23.93, size = 304, normalized size = 3.85

$$\begin{cases} \frac{x\cos^3(c)}{a^{\frac{5}{2}}} \\ \frac{\frac{2\sin^3(c+dx)}{3d}+\frac{\sin(c+dx)\cos^2(c+dx)}{d}}{a^{\frac{5}{2}}} \\ \frac{x\cos^3(c)}{(a+b\sin(c))^{\frac{5}{2}}} \\ -\frac{16a^2}{3ab^3d\sqrt{a+b\sin(c+dx)}+3b^4d\sqrt{a+b\sin(c+dx)}\sin(c+dx)} - \frac{24ab\sin(c+dx)}{3ab^3d\sqrt{a+b\sin(c+dx)}+3b^4d\sqrt{a+b\sin(c+dx)}\sin(c+dx)} - \frac{8b^2}{3ab^3d\sqrt{a+b\sin(c+dx)}+3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((x*cos(c)**3/a**(5/2), Eq(b, 0) & Eq(d, 0)), ((2*sin(c + d*x)**3/
(3*d) + sin(c + d*x)*cos(c + d*x)**2/d)/a**(5/2), Eq(b, 0)), (x*cos(c)**3/(
a + b*sin(c))**(5/2), Eq(d, 0)), (-16*a**2/(3*a*b**3*d*sqrt(a + b*sin(c + d
```

```
*x)) + 3*b**4*d*sqrt(a + b*sin(c + d*x))*sin(c + d*x)) - 24*a*b*sin(c + d*x
)/(3*a*b**3*d*sqrt(a + b*sin(c + d*x)) + 3*b**4*d*sqrt(a + b*sin(c + d*x))*
sin(c + d*x)) - 8*b**2*sin(c + d*x)**2/(3*a*b**3*d*sqrt(a + b*sin(c + d*x))
 + 3*b**4*d*sqrt(a + b*sin(c + d*x))*sin(c + d*x)) - 2*b**2*cos(c + d*x)**2
/(3*a*b**3*d*sqrt(a + b*sin(c + d*x)) + 3*b**4*d*sqrt(a + b*sin(c + d*x))*s
in(c + d*x)), True))
```

3.529 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=24

$$-\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}$$

```
[Out] -2/3/b/d/(a+b*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 32}

$$-\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$-\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

fricas [B] time = 0.57, size = 55, normalized size = 2.29

$$\frac{2\sqrt{b\sin(dx+c)+a}}{3\left(b^3d\cos(dx+c)^2-2ab^2d\sin(dx+c)-\left(a^2b+b^3\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*sin(d*x + c) + a)/(b^3*d*cos(d*x + c)^2 - 2*a*b^2*d*sin(d*x + c)
 - (a^2*b + b^3)*d)
```

giac [A] time = 0.89, size = 20, normalized size = 0.83

$$-\frac{2}{3\,(b\sin\left(dx+c\right)+a)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((b*sin(d*x + c) + a)^(3/2)*b*d)
```

maple [A] time = 0.02, size = 21, normalized size = 0.88

$$-\frac{2}{3bd\,(a+b\sin\left(dx+c\right))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3/b/d/(a+b*sin(d*x+c))^(3/2)
```

maxima [A] time = 0.47, size = 20, normalized size = 0.83

$$-\frac{2}{3\,(b\sin\left(dx+c\right)+a)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((b*sin(d*x + c) + a)^(3/2)*b*d)
```

mupad [B] time = 7.25, size = 157, normalized size = 6.54

$$-\frac{8\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,\left(2\,a^2+b^2-b^2\,\cos\left(2\,c+2\,d\,x\right)+4\,a\,b\right.}{3\,b\,d\,\left(8\,a^4+3\,b^4+24\,a^2\,b^2-4\,b^4\,\cos\left(2\,c+2\,d\,x\right)+b^4\,\cos\left(4\,c+4\,d\,x\right)-8\,a\,b^3\,\sin\left(3\,c+3\,d\,x\right)-24\,a^2\,b^2\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] -(8*(a + b*sin(c + d*x))^(1/2)*(2*a^2 + b^2 - b^2*cos(2*c + 2*d*x) + 4*a*b*
sin(c + d*x)))/(3*b*d*(8*a^4 + 3*b^4 + 24*a^2*b^2 - 4*b^4*cos(2*c + 2*d*x)
+ b^4*cos(4*c + 4*d*x) - 8*a*b^3*sin(3*c + 3*d*x) - 24*a^2*b^2*cos(2*c + 2*
d*x) + 24*a*b^3*sin(c + d*x) + 32*a^3*b*sin(c + d*x)))
```

sympy [A] time = 22.74, size = 87, normalized size = 3.62

$$\begin{cases} \frac{x\cos\left(c\right)}{a^{\frac{5}{2}}} & \text{for}\: b=0\wedge d=0 \\ \frac{\sin\left(c+dx\right)}{a^{\frac{5}{2}}d} & \text{for}\: b=0 \\ \frac{x\cos\left(c\right)}{(a+b\sin\left(c\right))^{\frac{5}{2}}} & \text{for}\: d=0 \\ -\frac{2}{3abd\sqrt{a+b\sin\left(c+dx\right)}+3b^2d\sqrt{a+b\sin\left(c+dx\right)}\sin\left(c+dx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((x*cos(c)/a**(5/2), Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**(5/2)
*d), Eq(b, 0)), (x*cos(c)/(a + b*sin(c))**(5/2), Eq(d, 0)), (-2/(3*a*b*d*sq
rt(a + b*sin(c + d*x)) + 3*b**2*d*sqrt(a + b*sin(c + d*x))*sin(c + d*x)), T
rue))
```

3.530 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=139

$$\frac{4ab}{d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+\frac{2b}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}$$

```
[Out] -arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+arctanh((a+b*sin
(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d+2/3*b/(a^2-b^2)/d/(a+b*sin(d*x+c)
)^(3/2)+4*a*b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2668, 710, 829, 827, 1166, 206}

$$\frac{4ab}{d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+\frac{2b}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/((a - b)^(5/2)*d)) + ArcTan
h[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/((a + b)^(5/2)*d) + (2*b)/(3*(a^2 -
 b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (4*a*b)/((a^2 - b^2)^2*d*Sqrt[a + b*S
in[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{2b}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{b\,\text{Subst}\left(\int \frac{a-x}{(a+x)^{3/2}\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d}$$

$$= \frac{2b}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{4ab}{\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{b\,\text{Subst}\left(\int \frac{}{}\right.}{}$$

$$= \frac{2b}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{4ab}{\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{(2b)\,\text{Subst}\left(\right.}{}$$

$$= \frac{2b}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{4ab}{\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\text{Subst}\left(\int \frac{}{a-b}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} + \frac{2b}{3\left(a^2-b^2\right)d(a+b\sin(c+dx)}$$

Mathematica [C] time = 0.08, size = 94, normalized size = 0.68

$$\frac{(a+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(b-a)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)}{3d(a-b)(a+b)(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[c + d*x])/(a - b)] + (
-a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[c + d*x])/(a + b)])/(3*
(a - b)*(a + b)*d*(a + b*Sin[c + d*x])^(3/2))
```

fricas [B] time = 1.65, size = 3225, normalized size = 23.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2
 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*sin(d*x + c))*sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4 +
 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 +
9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2
 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^
3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a
^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
+ 8)) + 3*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b^2
 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2
*b^3 + a*b^4)*sin(d*x + c))*sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4 -
 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 +
9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2
 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^
3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a
^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
+ 8)) + 16*(7*a^4*b - 8*a^2*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c))*s
qrt(b*sin(d*x + c) + a))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x
 + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^8 -
 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d), 1/24*(6*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^
2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^
2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x + c))*sqrt(-a - b)*ar
ctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*s
in(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b
^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x
 + c))) - 3*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b
^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a
^2*b^3 + a*b^4)*sin(d*x + c))*sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4
 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3
+ 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b
^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*
b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112
*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x +
c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c
) + 8)) - 16*(7*a^4*b - 8*a^2*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c))
*sqrt(b*sin(d*x + c) + a))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d
*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^8
 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d), 1/24*(6*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4*
a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c
)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*sin(d*x + c))*sqrt(-a + b)*
arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*
sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*
b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*
x + c))) - 3*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*
b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*
a^2*b^3 - a*b^4)*sin(d*x + c))*sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^
4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3
 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*
b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8
*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 11
2*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x +
c) + 8)) - 16*(7*a^4*b - 8*a^2*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c)
)*sqrt(b*sin(d*x + c) + a))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(
d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^
8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d), 1/12*(3*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4
```

```
*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x +
c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*sin(d*x + c))*sqrt(-a + b)
*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)
*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a
*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d
*x + c))) + 3*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3
*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3
*a^2*b^3 - a*b^4)*sin(d*x + c))*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^
2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x
+ c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos
(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c))) - 8*(7*a^4*b - 8*a^2
*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/((
a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*
b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^
8)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [A] time = 0.75, size = 130, normalized size = 0.94

$$\frac{2b}{3d(a-b)(a+b)(a+b\sin(dx+c))^{\frac{3}{2}}}+\frac{4ba}{d(a-b)^2(a+b)^2\sqrt{a+b\sin(dx+c)}}+\frac{\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)}{d(a-b)^2\sqrt{-a+b}}+\frac{\operatorname{arctanh}\left(\right.}{(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*b/(a-b)/(a+b)/(a+b*sin(d*x+c))^(3/2)+4/d*b*a/(a-b)^2/(a+b)^2/(a+b*sin
(d*x+c))^(1/2)+1/d/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b
)^(1/2))+arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,{\left(a+b\,\sin(c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**(5/2), x)
```

3.531 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=231

$$-\frac{ab\left(a^2+19b^2\right)}{2d\left(a^2-b^2\right)^3\sqrt{a+b\sin(c+dx)}}-\frac{b\left(3a^2+7b^2\right)}{6d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{(2a-}{}$$

```
[Out] -1/4*(2*a-7*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(7/2)/d+1/
4*(2*a+7*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/d-1/6*b
*(3*a^2+7*b^2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(3/2)-1/2*sec(d*x+c)^2*(b-a*s
in(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))^(3/2)-1/2*a*b*(a^2+19*b^2)/(a^2-b^2
)^3/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2668, 741, 829, 827, 1166, 206}

$$-\frac{ab\left(a^2+19b^2\right)}{2d\left(a^2-b^2\right)^3\sqrt{a+b\sin(c+dx)}}-\frac{b\left(3a^2+7b^2\right)}{6d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{(2a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2*a - 7*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(7/
2)*d) + ((2*a + 7*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a +
 b)^(7/2)*d) - (b*(3*a^2 + 7*b^2))/(6*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^
(3/2)) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[
c + d*x])^(3/2)) - (a*b*(a^2 + 19*b^2))/(2*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c
 + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
```

```
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{b\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(2a^2-7b^2\right)+\frac{5ax}{2}}{(a+x)^{5/2}\left(b^2-x^2\right)}\,dx, x, b\sin(c+d$$

$$= -\frac{b\left(3a^2+7b^2\right)}{6\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{b\,\text{Su}}{}$$

$$= -\frac{b\left(3a^2+7b^2\right)}{6\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{}{2\left(a^2\right.}$$

$$= -\frac{b\left(3a^2+7b^2\right)}{6\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{}{2\left(a^2\right.}$$

$$= -\frac{b\left(3a^2+7b^2\right)}{6\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{}{2\left(a^2\right.}$$

$$= -\frac{(2a-7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{7/2}d} + \frac{(2a+7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{7/2}d} - \frac{}{6\left(a^2-\right.}$$

Mathematica [C] time = 0.88, size = 245, normalized size = 1.06

$$-\frac{\left(\left(3a^3+3a^2b+7ab^2+7b^3\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)\right)+\left(3a^3-3a^2b+7ab^2-7b^3\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(}{a+l}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-((3*a^3 + 3*a^2*b + 7*a*b^2 + 7*b^3)*Hypergeometric2F1[-3/2, 1, -1/2, (a
+ b*Sin[c + d*x])/(a - b)]) + (3*a^3 - 3*a^2*b + 7*a*b^2 - 7*b^3)*Hypergeom
etric2F1[-3/2, 1, -1/2, (a + b*Sin[c + d*x])/(a + b)] + 15*a*(a + b)*Hyperg
eometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)]*(a + b*Sin[c + d*x]
) - 3*(a - b)*(-2*(a + b)*Sec[c + d*x]^2*(-b + a*Sin[c + d*x]) + 5*a*Hyperg
eometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x]
)))/(12*(a - b)^2*(a + b)^2*d*(a + b*Sin[c + d*x])^(3/2))
```

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{3}}{3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^3/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{3}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [A] time = 0.93, size = 283, normalized size = 1.23

$$-\frac{2b^{3}}{3d\left(a+b\right)^{2}\left(a-b\right)^{2}\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}}-\frac{8b^{3}a}{d\left(a+b\right)^{3}\left(a-b\right)^{3}\sqrt{a+b\sin\left(dx+c\right)}}-\frac{b\sqrt{a+b\sin\left(dx+c\right)}}{4d\left(a-b\right)^{3}\left(b\sin\left(dx+c\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*b^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))^(3/2)-8/d*b^3*a/(a+b)^3/(a-b)^3
/(a+b*sin(d*x+c))^(1/2)-1/4/d*b/(a-b)^3*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c
)+b)+1/2/d/(a-b)^3/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))
*a-7/4/d*b/(a-b)^3/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))
-1/4/d*b/(a+b)^3*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c)-b)+1/2/d/(a+b)^(7/2)*
arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+7/4/d*b/(a+b)^(7/2)*arctanh((
a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos (c+d\,x)^3\,(a+b\,\sin (c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3 (c+dx)}{(a+b\sin (c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**(5/2), x)
```

3.532 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=339

$$-\frac{\left(12a^2-54ab+77b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{9/2}}+\frac{\left(12a^2+54ab+77b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{9/2}}-\frac{\sec^4(c+dx)(b-}{4d\left(a^2-b^2\right)(a+b}$$

```
[Out] -1/32*(12*a^2-54*a*b+77*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a
-b)^(9/2)/d+1/32*(12*a^2+54*a*b+77*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b
)^(1/2))/(a+b)^(9/2)/d-1/48*b*(18*a^4-81*a^2*b^2-77*b^4)/(a^2-b^2)^3/d/(a+b
*sin(d*x+c))^(3/2)-1/4*sec(d*x+c)^4*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d
*x+c))^(3/2)+1/16*sec(d*x+c)^2*(b*(3*a^2+11*b^2)+2*a*(3*a^2-10*b^2)*sin(d*x
+c))/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(3/2)-1/8*a*b*(3*a^4-16*a^2*b^2-127*b^4
)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2668, 741, 823, 829, 827, 1166, 206}

$$-\frac{ab\left(-16a^2b^2+3a^4-127b^4\right)}{8d\left(a^2-b^2\right)^4\sqrt{a+b\sin(c+dx)}}-\frac{b\left(-81a^2b^2+18a^4-77b^4\right)}{48d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^{3/2}}-\frac{\left(12a^2-54ab+77b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c}}{\sqrt{a-b}}\right)}{32d(a-b)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((12*a^2 - 54*a*b + 77*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])
/(32*(a - b)^(9/2)*d) + ((12*a^2 + 54*a*b + 77*b^2)*ArcTanh[Sqrt[a + b*Sin[
c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(9/2)*d) - (b*(18*a^4 - 81*a^2*b^2 - 77
*b^4))/(48*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^(3/2)) - (Sec[c + d*x]^4*(b
 - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) - (a*b*(3*
a^4 - 16*a^2*b^2 - 127*b^4))/(8*(a^2 - b^2)^4*d*Sqrt[a + b*Sin[c + d*x]]) +
 (Sec[c + d*x]^2*(b*(3*a^2 + 11*b^2) + 2*a*(3*a^2 - 10*b^2)*Sin[c + d*x]))/
(16*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{b^3\,\text{Subst}\left(\int \frac{\frac{1}{2}(6a^2-11b^2)+\frac{9ax}{2}}{(a+x)^{5/2}(b^2-x^2)^2}\,dx, x, b\sin(c+dx\right.}{4(a^2-b^2)\,d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{\sec^2(c+dx)\left(b\left(3a^2+11b^2\right)+2a\left(3a^2-10b^2\right.\right.}{16\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))}$$

$$= -\frac{b\left(18a^4-81a^2b^2-77b^4\right)}{48\left(a^2-b^2\right)^3 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{\sec^2(c}{}$$

$$= -\frac{b\left(18a^4-81a^2b^2-77b^4\right)}{48\left(a^2-b^2\right)^3 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{ab}{8\left(a^2-\right.}$$

$$= -\frac{b\left(18a^4-81a^2b^2-77b^4\right)}{48\left(a^2-b^2\right)^3 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{ab}{8\left(a^2-\right.}$$

$$= -\frac{b\left(18a^4-81a^2b^2-77b^4\right)}{48\left(a^2-b^2\right)^3 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{ab}{8\left(a^2-\right.}$$

$$= -\frac{\left(12a^2-54ab+77b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{9/2}d} + \frac{\left(12a^2+54ab+77b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{32(a+b)^{9/2}d}$$

Mathematica [C] time = 3.43, size = 296, normalized size = 0.87

$$\frac{-15a\left(3a^2-10b^2\right)(a+b\sin(c+dx))\left((a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(b-a)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)\right)+\frac{1}{2}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((18*a^4 - 81*a^2*b^2 - 77*b^4)*((a + b)*Hypergeometric2F1[-3/2, 1, -1/2,
(a + b*Sin[c + d*x])/(a - b)] + (-a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (
a + b*Sin[c + d*x])/(a + b)]))/2 - 12*(a - b)^2*(a + b)^2*Sec[c + d*x]^4*(-
b + a*Sin[c + d*x]) - 15*a*(3*a^2 - 10*b^2)*((a + b)*Hypergeometric2F1[-1/2
, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)] + (-a + b)*Hypergeometric2F1[-1/2,
1, 1/2, (a + b*Sin[c + d*x])/(a + b)])*(a + b*Sin[c + d*x]) - 3*(a - b)*(a
+ b)*Sec[c + d*x]^2*(3*a^2*b + 11*b^3 + (6*a^3 - 20*a*b^2)*Sin[c + d*x]))/(
48*(a^2 - b^2)^2*(-a^2 + b^2)*d*(a + b*Sin[c + d*x])^(3/2))
```

fricas [F] time = 2.36, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\sec(dx+c)^5}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^5}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.05, size = 682, normalized size = 2.01

$$\frac{2b^5}{3d(a-b)^3(a+b)^3(a+b\sin(dx+c))^{\frac{3}{2}}}+\frac{12b^5a}{d(a-b)^4(a+b)^4\sqrt{a+b\sin(dx+c)}}-\frac{3b(a+b\sin(dx+c))^{\frac{3}{2}}a}{16d(a-b)^4(b\sin(dx+c)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*b^5/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c))^(3/2)+12/d*b^5*a/(a-b)^4/(a+b)^4
/(a+b*sin(d*x+c))^(1/2)-3/16/d*b/(a-b)^4/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c)
)^(3/2)*a+17/32/d*b^2/(a-b)^4/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(3/2)+3/1
6/d*b/(a-b)^4/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)*a^2-25/32/d*b^2/(a-
b)^4/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)*a+19/32/d*b^3/(a-b)^4/(b*sin
(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)+3/8/d/(a-b)^4/(-a+b)^(1/2)*arctan((a+b*
sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-27/16/d*b/(a-b)^4/(-a+b)^(1/2)*arctan((
a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a+77/32/d*b^2/(a-b)^4/(-a+b)^(1/2)*arct
an((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))-3/16/d*b/(a+b)^4/(b*sin(d*x+c)-b)^2
*(a+b*sin(d*x+c))^(3/2)*a-17/32/d*b^2/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(d
*x+c))^(3/2)+3/16/d*b/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1/2)*a^2
+25/32/d*b^2/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1/2)*a+19/32/d*b^
3/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1/2)+3/8/d/(a+b)^(9/2)*arcta
nh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+27/16/d*b/(a+b)^(9/2)*arctanh((a
+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+77/32/d*b^2/(a+b)^(9/2)*arctanh((a+b*si
n(d*x+c))^(1/2)/(a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x))**(5/2), x)
```

3.533 $\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=384

$$-\frac{128a\left(8a^2-9b^2\right)\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{99b^8d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{40\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{99b^5}$$

```
[Out] -2/3*cos(d*x+c)^7/b/d/(a+b*sin(d*x+c))^(3/2)-28/33*cos(d*x+c)^5*(12*a+b*sin
(d*x+c))/b^3/d/(a+b*sin(d*x+c))^(1/2)+40/99*cos(d*x+c)^3*(32*a^2-3*b^2-28*a
*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^5/d-16/99*cos(d*x+c)*(128*a^4-144*a
^2*b^2+15*b^4-3*a*b*(32*a^2-31*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^7/
d+128/99*a*(8*a^2-9*b^2)*(4*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a
+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^8/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-32/
99*(128*a^6-272*a^4*b^2+159*a^2*b^4-15*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*
(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^8/d/(a+b*sin(d*x+c))^(1/2
)
```

Rubi [A] time = 0.76, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2693, 2863, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{40\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-28ab\sin(c+dx)-3b^2\right)}{99b^5d}-\frac{16\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(-3ab\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) - (128*a*(8*a^2 - 9*
b^2)*(4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
b*Sin[c + d*x]])/(99*b^8*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*(128*a
^6 - 272*a^4*b^2 + 159*a^2*b^4 - 15*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(99*b^8*d*Sqrt[a + b*Sin[c +
 d*x]]) - (28*Cos[c + d*x]^5*(12*a + b*Sin[c + d*x]))/(33*b^3*d*Sqrt[a + b*
Sin[c + d*x]]) + (40*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 3*b^
2 - 28*a*b*Sin[c + d*x]))/(99*b^5*d) - (16*Cos[c + d*x]*Sqrt[a + b*Sin[c +
d*x]]*(128*a^4 - 144*a^2*b^2 + 15*b^4 - 3*a*b*(32*a^2 - 31*b^2)*Sin[c + d*x
]))/(99*b^7*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{14\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{280\int \frac{\cos^4(}{}}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c+}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c+}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c+}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c+}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{128a\left(8a^2-9b^2\right)\left(4a^2-3b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)}{99b^8d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.79, size = 356, normalized size = 0.93

$$\frac{256(a+b)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}\left(4\left(32a^5-60a^3b^2+27ab^4\right)\left((a+b)E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{4}(-2c-2dx+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (256*(a + b)*(b*(32*a^4*b - 51*a^2*b^3 + 15*b^5)*EllipticF[(-2*c + Pi - 2*d
*x)/4, (2*b)/(a + b)] + 4*(32*a^5 - 60*a^3*b^2 + 27*a*b^4)*((a + b)*Ellipti
cE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/
4, (2*b)/(a + b)]))*((a + b*Sin[c + d*x])/(a + b))^(3/2) + (b*Cos[c + d*x]*
(-32768*a^6 + 55296*a^4*b^2 - 18144*a^2*b^4 - 2574*b^6 + (2048*a^4*b^2 - 36
48*a^2*b^4 + 1383*b^6)*Cos[2*(c + d*x)] + (-96*a^2*b^4 + 126*b^6)*Cos[4*(c
+ d*x)] + 9*b^6*Cos[6*(c + d*x)] - 40960*a^5*b*Sin[c + d*x] + 74112*a^3*b^3
*Sin[c + d*x] - 30920*a*b^5*Sin[c + d*x] - 384*a^3*b^3*Sin[3*(c + d*x)] + 5
96*a*b^5*Sin[3*(c + d*x)] + 28*a*b^5*Sin[5*(c + d*x)]))/2)/(792*b^8*d*(a +
b*Sin[c + d*x])^(3/2))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\cos(dx+c)^8}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^8/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^8}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^8/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.89, size = 2253, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/99*(-14*a*b^7*sin(d*x+c)*cos(d*x+c)^6+(48*a^3*b^5-64*a*b^7)*cos(d*x+c)^4
*sin(d*x+c)+(1280*a^5*b^3-2328*a^3*b^5+984*a*b^7)*cos(d*x+c)^2*sin(d*x+c)-1
6*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b*(128*EllipticE((b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^7-368*EllipticE((b/(a-b)*sin(d*x+c)+1
/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2+348*EllipticE((b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4-108*EllipticE((b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^6-128*EllipticF((b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^6*b+96*EllipticF((b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2+272*EllipticF((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3-189*Ellipt
icF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4-159*E
llipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5+
93*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^
6+15*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^
7)*sin(d*x+c)-9*b^8*cos(d*x+c)^8+(24*a^2*b^6-18*b^8)*cos(d*x+c)^6+(-128*a^4
*b^4+204*a^2*b^6-60*b^8)*cos(d*x+c)^4+(1024*a^6*b^2-1664*a^4*b^4+456*a^2*b^
6+120*b^8)*cos(d*x+c)^2+2048*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-1536*(b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((
a-b)/(a+b))^(1/2))*a^6*b^2-4352*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF
((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^3+3024*(b/
(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(
a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2),((a-b)/(a+b))^(1/2))*a^4*b^4+2544*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(
-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Elli
pticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^5-148
8*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*
(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6-240*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*
EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^7-2
048*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2),((a-b)/(a+b))^(1/2))*a^8+5888*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2
)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*E
llipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-
5568*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/
```

```
2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b
)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+1728*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1
/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2
*b^6)/(a+b*sin(d*x+c))^(3/2)/b^9/cos(d*x+c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^8}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^8/(b*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^8}{(a+b\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^8/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.534 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=293

$$\frac{16a\left(32a^2-29b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{21b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{8\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(32a^2-24ab\sin(\right.}{21b^5d}$$

```
[Out] -2/3*cos(d*x+c)^5/b/d/(a+b*sin(d*x+c))^(3/2)-20/21*cos(d*x+c)^3*(8*a+b*sin(
d*x+c))/b^3/d/(a+b*sin(d*x+c))^(1/2)+8/21*cos(d*x+c)*(32*a^2-5*b^2-24*a*b*s
in(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^5/d-16/21*a*(32*a^2-29*b^2)*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^6/d/((a+b*sin
(d*x+c))/(a+b))^(1/2)+16/21*(32*a^4-37*a^2*b^2+5*b^4)*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x
),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^6/d/(a+b*sin(d*
x+c))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2693, 2863, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{8\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(32a^2-24ab\sin(c+dx)-5b^2\right)}{21b^5d}-\frac{16\left(-37a^2b^2+32a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\right.}{21b^6d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) + (16*a*(32*a^2 - 29
*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]]
)/(21*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (16*(32*a^4 - 37*a^2*b^2
+ 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(21*b^6*d*Sqrt[a + b*Sin[c + d*x]]) - (20*Cos[c + d*x]^3*(8*
a + b*Sin[c + d*x]))/(21*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (8*Cos[c + d*x]*
Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 5*b^2 - 24*a*b*Sin[c + d*x]))/(21*b^5*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{10\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\int \frac{\cos^2(c+dx)(\cdots}{\sqrt{a+b\cdots}}}{7\cdots}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{\cdots}}{\cdots}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{\cdots}}{\cdots}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{\cdots}}{\cdots}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{16a\left(32a^2-29b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(\cdots}}{21b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.22, size = 244, normalized size = 0.83

$$\frac{\frac{1}{2}b\cos(c+dx)\left(1024a^4+1280a^3b\sin(c+dx)-736a^2b^2+\left(52b^4-64a^2b^2\right)\cos(2(c+dx))-1076ab^3\sin(c+dx)\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32*(a + b)*(a*(32*a^3 + 32*a^2*b - 29*a*b^2 - 29*b^3)*EllipticE[(-2*c + P
i - 2*d*x)/4, (2*b)/(a + b)] + (-32*a^4 + 37*a^2*b^2 - 5*b^4)*EllipticF[(-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*x])/(a + b))^(3/2) +
(b*Cos[c + d*x]*(1024*a^4 - 736*a^2*b^2 - 111*b^4 + (-64*a^2*b^2 + 52*b^4)*
Cos[2*(c + d*x)] + 3*b^4*Cos[4*(c + d*x)] + 1280*a^3*b*Sin[c + d*x] - 1076*
a*b^3*Sin[c + d*x] + 12*a*b^3*Sin[3*(c + d*x)]))/2)/(42*b^6*d*(a + b*Sin[c
+ d*x])^(3/2))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\cos(dx+c)^6}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^6/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^6}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.78, size = 1642, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/21*(6*a*b^5*sin(d*x+c)*cos(d*x+c)^4+(160*a^3*b^3-136*a*b^5)*cos(d*x+c)^2*
sin(d*x+c)+8*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+
b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b*(32*EllipticF((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-24*EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-37*EllipticF((b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+24*EllipticF((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+5*EllipticF((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^5-32*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5+61*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-29*Ellipti
cE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)*sin(d*x
+c)+3*b^6*cos(d*x+c)^6+(-16*a^2*b^4+10*b^6)*cos(d*x+c)^4+(128*a^4*b^2-84*a^
2*b^4-20*b^6)*cos(d*x+c)^2-256*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*Elliptic
E((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d
*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^6+488*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(
(a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)
+1/(a-b)*a)^(1/2)*a^4*b^2-232*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE
((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*
x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^4+256*Ellipt
icF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin
(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2)*a^5*b-192*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d
*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^4*b^2-296*Ellip
ticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*si
n(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2)*a^3*b^3+192*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*si
n(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^4+40*Ell
ipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*
sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2)*a*b^5)/(a+b*sin(d*x+c))^(3/2)/b^7/cos(d*x+c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{6}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(b*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^6}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.535 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=221

$$\frac{8\left(4a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^4d\sqrt{a+b\sin(c+dx)}}-\frac{32a\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\cos(c+d}{3b^3d\sqrt{}}$$

```
[Out] -2/3*cos(d*x+c)^3/b/d/(a+b*sin(d*x+c))^(3/2)-4/3*cos(d*x+c)*(4*a+b*sin(d*x+
c))/b^3/d/(a+b*sin(d*x+c))^(1/2)+32/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(
a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/
3*(4*a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x
+c))/(a+b))^(1/2)/b^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2693, 2863, 2752, 2663, 2661, 2655, 2653}

$$\frac{8\left(4a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^4d\sqrt{a+b\sin(c+dx)}}-\frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}}-\frac{32a\sqrt{a+b\sin(c+dx)}\,E}{3b^4d\sqrt{\frac{a+b}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) - (32*a*EllipticE[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^4*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^4*d*Sqrt[a + b*S
in[c + d*x]]) - (4*Cos[c + d*x]*(4*a + b*Sin[c + d*x]))/(3*b^3*d*Sqrt[a + b
*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx &= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{b} \\
&= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\int \frac{-\frac{b}{2}-2a\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}}{3b^3} \\
&= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}} - \frac{(16a)\int \sqrt{a+b\sin}}{3b^4} \\
&= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}} - \frac{\left(16a\sqrt{a+b\sin(c\right.}}{3b} \\
&= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{32aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{8(4}{}
\end{aligned}$$

Mathematica [A] time = 1.03, size = 174, normalized size = 0.79

$$\frac{b\cos(c+dx)\left(-16a^2-20ab\sin(c+dx)+b^2\cos(2(c+dx))-3b^2\right)-8\left(4a^2-b^2\right)(a+b)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}F\left(\frac{1}{4}(-2\right.}{3b^4d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (32*a*(a + b)^2*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin
[c + d*x])/(a + b))^(3/2) - 8*(a + b)*(4*a^2 - b^2)*EllipticF[(-2*c + Pi -
2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) + b*Cos[c + d
*x]*(-16*a^2 - 3*b^2 + b^2*Cos[2*(c + d*x)] - 20*a*b*Sin[c + d*x]))/(3*b^4*
d*(a + b*Sin[c + d*x])^(3/2))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{4}}{3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.78, size = 1047, normalized size = 4.74

$$-\frac{2\left(10a\,b^{3}\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)+4\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\,\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\,\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\,b\left(4\,\text{Ellipti}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*(10*a*b^3*cos(d*x+c)^2*sin(d*x+c)+4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2
)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*
b*(4*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^
2*b-3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a
*b^2-EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^
3-4*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3
+4*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^
2)*sin(d*x+c)-b^4*cos(d*x+c)^4+(8*a^2*b^2+2*b^4)*cos(d*x+c)^2+16*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c
)+b/(a+b))^(1/2)*a^3*b-12*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-
b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^2*b^2-4*(-b/(a-b)*si
n(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(
a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+
b/(a+b))^(1/2)*a*b^3-16*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)
/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^4+16*(-b/(a-b)*sin(d*x
+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+
b))^(1/2)*a^2*b^2)/(a+b*sin(d*x+c))^(3/2)/b^5/cos(d*x+c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4}{(a+b\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.536 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{4a\cos(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\right.\right.}{3b^2d\sqrt{a+b\sin(c+d}}$$

```
[Out] -2/3*cos(d*x+c)/b/d/(a+b*sin(d*x+c))^(3/2)+4/3*a*cos(d*x+c)/b/(a^2-b^2)/d/(
a+b*sin(d*x+c))^(1/2)-4/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))
*(a+b*sin(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+4/3*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1
/2)/b^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2693, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a\cos(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\right.\right.}{3b^2d\sqrt{a+b\sin(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) + (4*a*Cos[c + d*x])/(
3*b*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + (4*a*EllipticE[(c - Pi/2 + d*
x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*(a^2 - b^2)*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) - (4*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \frac{\frac{b}{2}+\frac{1}{2}a\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{1}{\sqrt{a+b\sin(c+dx)}}}{3b^2}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{\left(2a\sqrt{a+b\sin(c+\right.}{3b^2\left(a^2\right.}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{4aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{3b^2\left(a^2-\right.}$$

Mathematica [A] time = 1.03, size = 167, normalized size = 0.76

$$\frac{2b\cos(c+dx)\left(a^2+2ab\sin(c+dx)+b^2\right)+4(a-b)(a+b)^2\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-4a(a+b}{3b^2d(a-b)(a+b)(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*a*(a + b)^2*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin
[c + d*x])/(a + b))^(3/2) + 4*(a - b)*(a + b)^2*EllipticF[(-2*c + Pi - 2*d*
x)/4, (2*b)/(a + b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) + 2*b*Cos[c + d*x
]*(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(3*(a - b)*b^2*(a + b)*d*(a + b*Sin[c +
 d*x])^(3/2))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2}}{3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.82, size = 864, normalized size = 3.95

$$\frac{\frac{4a\,b^{3}\left(\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+\frac{4\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\,\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\,\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\,b\left(\text{EllipticF}\left(\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^{2}b-\text{EllipticF}\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/3*(2*a*b^3*cos(d*x+c)^2*sin(d*x+c)+2*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*
(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b*
(EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b-
EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^3-Ell
ipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3+Ellipt
icE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2)*sin(d*
x+c)+(a^2*b^2+b^4)*cos(d*x+c)^2+2*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^3*b-2*(-b/(a
-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*Elliptic
F((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d
*x+c)+b/(a+b))^(1/2)*a*b^3-2*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(
(a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^4+2*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b
/(a+b))^(1/2)*a^2*b^2)/(a^2-b^2)/(a+b*sin(d*x+c))^(3/2)/b^3/cos(d*x+c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sin(c + d*x))**(5/2), x)
```

3.537 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=325

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(b\left(27a^2+5b^2\right)-a\left(3a^2+29b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^3}+\frac{16ab\sec(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+d}}$$

```
[Out] 2/3*b*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(3/2)+16/3*a*b*sec(d*x+c)/(a^
2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)-1/3*sec(d*x+c)*(b*(27*a^2+5*b^2)-a*(3*a^2
+29*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d+1/3*a*(3*a^2+29*b
^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/
(a^2-b^2)^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/3*(3*a^2+5*b^2)*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*P
i+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/(a^2-b^2
)^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2694, 2864, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,\left(b\left(27a^2+5b^2\right)-a\left(3a^2+29b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^3}+\frac{16ab\sec(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*b*Sec[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (16*a*b*S
ec[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (a*(3*a^2 + 29*
b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])
/(3*(a^2 - b^2)^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((3*a^2 + 5*b^2)*
EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
 b)])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]]*(b*(27*a^2 + 5*b^2) - a*(3*a^2 + 29*b^2)*Sin[c + d*x]))/(3
*(a^2 - b^2)^3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\sec^2(c+dx)\left(-\frac{3a}{2}+\frac{5}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \frac{\sec^2}{\cdots}}{\cdots}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+\cdots}{\cdots}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+\cdots}{\cdots}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+\cdots}{\cdots}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{a\left(3a^2+\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.89, size = 241, normalized size = 0.74

$$\frac{\frac{\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}\left(\left(3a^3+29ab^2\right)E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(-3a^3+3a^2b-5ab^2+5b^3\right)F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)\right)}{(a-b)^3(a+b)} - \frac{2b^3\left(a^2-b^2\right)\cos(c+dx)+3\sec(c+dx)(a\cdots}{\cdots}}{3d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((3*a^3 + 29*a*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + (-3
*a^3 + 3*a^2*b - 5*a*b^2 + 5*b^3)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
 + b)])*((a + b*Sin[c + d*x])/(a + b))^(3/2))/((a - b)^3*(a + b)) - (2*b^3*
(a^2 - b^2)*Cos[c + d*x] + 20*a*b^3*Cos[c + d*x]*(a + b*Sin[c + d*x]) + 3*S
ec[c + d*x]*(a + b*Sin[c + d*x])^2*(3*a^2*b + b^3 - a*(a^2 + 3*b^2)*Sin[c +
 d*x]))/(a^2 - b^2)^3)/(3*d*(a + b*Sin[c + d*x])^(3/2))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\sec(dx+c)^2}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 3.85, size = 1653, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] (-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*(1/2/(a+b)^3/b/cos(d*x+c)^2/(a+b*si
n(d*x+c))*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*(EllipticE((b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b
/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(
a-b))^(1/2)*a^2-EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b)
)^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^2-b^2*cos(d*x+c)^2+a*b*sin(d*x
+c)+b^2*sin(d*x+c)+a*b+b^2)-2*a*b^2/(a+b)^2/(a-b)^2*(2*b*cos(d*x+c)^2/(a^2-
b^2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)+2*a/(a^2-b^2)*(a/b-1)*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a
-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+2*b/(a^2-b^2)*(a/b-1)*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1
/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))))-b^2/(a+b)/(a-b)*(2/3/b/(a^2-b^2)*(-(-a-b*sin(
d*x+c))*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^2+8/3*b*cos(d*x+c)^2/(a^2-b^2)
^2*a/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)+2*(3*a^2+b^2)/(3*a^4-6*a^2*b^2
+3*b^4)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/
2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+8/3*a*b/(a^2-
b^2)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2
)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*(
(-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))+1/2/(a-b)^2*(-(
-sin(d*x+c)^2*b-a*sin(d*x+c)+b*sin(d*x+c)+a)/(a-b)/((-a-b*sin(d*x+c))*(sin(
d*x+c)-1)*(1+sin(d*x+c)))^(1/2)-2*b/(2*a-2*b)*(a/b-1)*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-
(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))-b/(a-b)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*
(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x
+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2)))))/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx + c\right)^{2}}{{\left(b \sin\left(dx + c\right) + a\right)}^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**(5/2), x)
```

3.538 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=425

$$-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(29a^2+3b^2\right)-a\left(a^2+31b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^3}+\frac{8ab\sec^3(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+$$

```
[Out] 2/3*b*sec(d*x+c)^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^(3/2)+8*a*b*sec(d*x+c)^3/(a
^2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)-1/3*sec(d*x+c)^3*(b*(29*a^2+3*b^2)-a*(a^
2+31*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d-1/6*sec(d*x+c)*(
b*(a^4-114*a^2*b^2-15*b^4)-4*a*(a^4-6*a^2*b^2-27*b^4)*sin(d*x+c))*(a+b*sin(
d*x+c))^(1/2)/(a^2-b^2)^4/d+2/3*a*(a^4-6*a^2*b^2-27*b^4)*(sin(1/2*c+1/4*Pi+
1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*
d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^4/d/((a+b*si
n(d*x+c))/(a+b))^(1/2)-1/6*(4*a^4-21*a^2*b^2-15*b^4)*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/(a^2-b^2)^3/d/(a+b
*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2694, 2864, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(29a^2+3b^2\right)-a\left(a^2+31b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^3}+\frac{8ab\sec^3(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (8*a*b*
Sec[c + d*x]^3)/((a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (2*a*(a^4 - 6*
a^2*b^2 - 27*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(3*(a^2 - b^2)^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((4*
a^4 - 21*a^2*b^2 - 15*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/(6*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c + d*x]
]) - (Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(b*(29*a^2 + 3*b^2) - a*(a^2
+ 31*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^3*d) - (Sec[c + d*x]*Sqrt[a + b*Sin
[c + d*x]]*(b*(a^4 - 114*a^2*b^2 - 15*b^4) - 4*a*(a^4 - 6*a^2*b^2 - 27*b^4)
*Sin[c + d*x]))/(6*(a^2 - b^2)^4*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\sec^4(c+dx)\left(-\frac{3a}{2}+\frac{9}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}}dx}{3(a^2-b^2)}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2 d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \frac{\sec^4(c+dx)}{\cdots}}{\cdots}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)}{\cdots}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)}{\cdots}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)}{\cdots}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)}{\cdots}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{2a(a^4-6a^2\cdots}{\cdots}$$

Mathematica [A] time = 2.50, size = 341, normalized size = 0.80

$$\frac{\frac{2(a^2-b^2)\sec^3(c+dx)(a+b\sin(c+dx))^2\left(a(a^2+3b^2)\sin(c+dx)-b(3a^2+b^2)\right)+4b^5(a^2-b^2)\cos(c+dx)+\sec(c+dx)(a+b\sin(c+dx))^2\left(-a^4b+54a^2b^3+4a(a^4-6a^2\cdots\right.}{(a^2-b^2)^4}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((4*(a^5 - 6*a^3*b^2 - 27*a*b^4)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
 + b)] + (-4*a^5 + 4*a^4*b + 21*a^3*b^2 - 21*a^2*b^3 + 15*a*b^4 - 15*b^5)*E
llipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*x])/(a +
b))^(3/2))/((a - b)^4*(a + b)^2) + (4*b^5*(a^2 - b^2)*Cos[c + d*x] + 64*a*b
^5*Cos[c + d*x]*(a + b*Sin[c + d*x]) + 2*(a^2 - b^2)*Sec[c + d*x]^3*(a + b*
Sin[c + d*x])^2*(-(b*(3*a^2 + b^2)) + a*(a^2 + 3*b^2)*Sin[c + d*x]) + Sec[c
 + d*x]*(a + b*Sin[c + d*x])^2*(-(a^4*b) + 54*a^2*b^3 + 11*b^5 + 4*a*(a^4 -
 6*a^2*b^2 - 11*b^4)*Sin[c + d*x]))/(a^2 - b^2)^4)/(6*d*(a + b*Sin[c + d*x]
)^(3/2))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,\sec(dx+c)^4}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^4}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [B] time = 5.84, size = 2585, normalized size = 6.08

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] (-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*(-1/4*(-a-3*b)/(a+b)^4/b/cos(d*x+c)
^2/(a+b*sin(d*x+c))*(cos(d*x+c)^2*sin(d*x+c)*b+cos(d*x+c)^2*a)^(1/2)*(Ellip
ticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*si
n(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(
d*x+c)-b/(a-b))^(1/2)*a^2-EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((
a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^2-b^2*cos(d*x+c)^2+a
*b*sin(d*x+c)+b^2*sin(d*x+c)+a*b+b^2)+4*a*b^4/(a+b)^3/(a-b)^3*(2*b*cos(d*x+
c)^2/(a^2-b^2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)+2*a/(a^2-b^2)*(a/b-1
)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*
x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+2*b/(a^2-b^2)*(a/b-1)*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b
/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))+1/4/(a+b)^2*(1/3/(a+b)*(-(-a-b*sin(
d*x+c))*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)-1)^2-1/3*(-sin(d*x+c)^2*b-a*sin(d*x
+c)-b*sin(d*x+c)-a)/(a+b)^2*(a+3*b)/((-a-b*sin(d*x+c))*(sin(d*x+c)-1)*(1+si
n(d*x+c)))^(1/2)+2*b^2/(3*a^2+6*a*b+3*b^2)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a
-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))-1/3*b*(a+3*b)/(a+b)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-
a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))))+b^4/(a+b)^2/(a-b)^2*(2/3/b/(a^2-b^2)*(-(-a-b*sin(d*x+
c))*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^2+8/3*b*cos(d*x+c)^2/(a^2-b^2)^2*a
/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)+2*(3*a^2+b^2)/(3*a^4-6*a^2*b^2+3*b
^4)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*(
(-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+8/3*a*b/(a^2-b^2)
^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((
-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/
b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))+1/4*(a-3*b)/(a-b)^3
*(-(-sin(d*x+c)^2*b-a*sin(d*x+c)+b*sin(d*x+c)+a)/(a-b)/((-a-b*sin(d*x+c))*(
sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)-2*b/(2*a-2*b)*(a/b-1)*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2
)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
```

```
)^(1/2),((a-b)/(a+b))^(1/2))-b/(a-b)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin
(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))))+1/4/(a-b)^2*(-1/3/(a-b)*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1
/2)/(1+sin(d*x+c))^2-1/3*(-sin(d*x+c)^2*b-a*sin(d*x+c)+b*sin(d*x+c)+a)/(a-b
)^2*(a-3*b)/((-a-b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)+2*b^2/(
3*a^2-6*a*b+3*b^2)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))
/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c
)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))-1/
3*b*(a-3*b)/(a-b)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c)
)/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+
c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)))))/c
os(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}{\left(c + dx \right)}}{\left(a + b \sin{\left(c + dx \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x))**(5/2), x)
```

3.539 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=124

$$\frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{10ae^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{5/2}}{7d}-\frac{2b(e$$

```
[Out] -2/9*b*(e*cos(d*x+c))^(9/2)/d/e+2/7*a*e*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/d+1
0/21*a*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+10/21*a*e^3*s
in(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2669, 2635, 2642, 2641}

$$\frac{10ae^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}+\frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{5/2}}{7d}-\frac{2b(e$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(e*Cos[c + d*x])^(9/2))/(9*d*e) + (10*a*e^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (10*a*e^3*Sqrt[e*Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))\,dx = -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + a\int (e\cos(c+dx))^{7/2}\,dx$$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{2ae(e\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}\left(5ae^2\right)$$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2ae(e\cos}{}$$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2ae(e\cos}{}$$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}} + \frac{10ae^3}{}$$

Mathematica [A] time = 0.90, size = 104, normalized size = 0.84

$$\frac{e^3\sqrt{e\cos(c+dx)}\left(\sqrt{\cos(c+dx)}\,(138a\sin(c+dx)+18a\sin(3(c+dx))-28b\cos(2(c+dx))-7b\cos(4(c+dx))\right.}{252d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (e^3*Sqrt[e*Cos[c + d*x]]*(120*a*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d
*x]]*(-21*b - 28*b*Cos[2*(c + d*x)] - 7*b*Cos[4*(c + d*x)] + 138*a*Sin[c +
d*x] + 18*a*Sin[3*(c + d*x)])))/(252*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(be^3\cos(dx+c)^3\sin(dx+c) + ae^3\cos(dx+c)^3\right)\sqrt{e\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*e^3*cos(d*x + c)^3*sin(d*x + c) + a*e^3*cos(d*x + c)^3)*sqrt(e*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a), x)
```

maple [A] time = 1.40, size = 259, normalized size = 2.09

$$-\frac{2e^4\left(-224b\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+144a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+560b\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-216a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] -2/63/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-224*b*si
n(1/2*d*x+1/2*c)^11+144*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+560*b*sin
(1/2*d*x+1/2*c)^9-216*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-560*b*sin(1
/2*d*x+1/2*c)^7+168*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+280*b*sin(1/2
*d*x+1/2*c)^5+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-48*a*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2-70*b*sin(1/2*d*x+1/2*c)^3+7*b*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{7/2}\,(a+b\,\sin(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.540 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{6ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{3/2}}{5d}-\frac{2b(e\cos(c+dx))^{7/2}}{7de}$$

```
[Out] -2/7*b*(e*cos(d*x+c))^(7/2)/d/e+2/5*a*e*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/d+6
/5*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2635, 2640, 2639}

$$\frac{6ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{3/2}}{5d}-\frac{2b(e\cos(c+dx))^{7/2}}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(e*Cos[c + d*x])^(7/2))/(7*d*e) + (6*a*e^2*Sqrt[e*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*e*(e*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))\,dx = -\frac{2b(e\cos(c+dx))^{7/2}}{7de} + a\int (e\cos(c+dx))^{5/2}\,dx$$
$$= -\frac{2b(e\cos(c+dx))^{7/2}}{7de} + \frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\Big(3ae$$
$$= -\frac{2b(e\cos(c+dx))^{7/2}}{7de} + \frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\big(3ae^2\sqrt{}}{}$$
$$= -\frac{2b(e\cos(c+dx))^{7/2}}{7de} + \frac{6ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2a}{}$$

Mathematica [A] time = 0.49, size = 79, normalized size = 0.83

$$\frac{(e\cos(c+dx))^{5/2}\left(\cos^{\frac{3}{2}}(c+dx)(14a\sin(c+dx)-5b\cos(2(c+dx))-5b)+42aE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(42*a*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^(3/2
)*(-5*b - 5*b*Cos[2*(c + d*x)] + 14*a*Sin[c + d*x])))/(35*d*Cos[c + d*x]^(5
/2))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(be^2\cos(dx+c)^2\sin(dx+c)+ae^2\cos(dx+c)^2\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*e^2*cos(d*x + c)^2*sin(d*x + c) + a*e^2*cos(d*x + c)^2)*sqrt(e*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a), x)
```

maple [B] time = 1.41, size = 222, normalized size = 2.34

$$\frac{2e^3\left(-80b\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+56a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+160b\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-56a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] 2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(-80*b*sin(
1/2*d*x+1/2*c)^9+56*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+160*b*sin(1/2
*d*x+1/2*c)^7-56*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-120*b*sin(1/2*d*
x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+14*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2+40*b*sin(1/2*d*x+1/2*c)^3-5*b*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{5/2}\,\left(a+b\,\sin(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.541 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}+\frac{2ae\sin(c+dx)\sqrt{e\cos(c+dx)}}{3d}-\frac{2b(e\cos(c+dx))^{5/2}}{5de}$$

```
[Out] -2/5*b*(e*cos(d*x+c))^(5/2)/d/e+2/3*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*
cos(d*x+c))^(1/2)+2/3*a*e*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2635, 2642, 2641}

$$\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}+\frac{2ae\sin(c+dx)\sqrt{e\cos(c+dx)}}{3d}-\frac{2b(e\cos(c+dx))^{5/2}}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(e*Cos[c + d*x])^(5/2))/(5*d*e) + (2*a*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) + (2*a*e*Sqrt[e*Cos[c + d*x]
]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))\,dx = -\frac{2b(e\cos(c+dx))^{5/2}}{5de} + a\int (e\cos(c+dx))^{3/2}\,dx$$
$$= -\frac{2b(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\left(ae^2\right)\int$$
$$= -\frac{2b(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{\left(ae^2\sqrt{\cos(c}\right.}{3}$$
$$= -\frac{2b(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}} + 2ae\sqrt{e}$$

Mathematica [A] time = 0.48, size = 79, normalized size = 0.83

$$\frac{(e\cos(c+dx))^{3/2}\left(\sqrt{\cos(c+dx)}\,(10a\sin(c+dx)-3b\cos(2(c+dx))-3b)+10aF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(10*a*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(-3*b - 3*b*Cos[2*(c + d*x)] + 10*a*Sin[c + d*x])))/(15*d*Cos[c + d*x]^(3
/2))
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left((be\cos(dx+c)\sin(dx+c)+ae\cos(dx+c))\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*e*cos(d*x + c)*sin(d*x + c) + a*e*cos(d*x + c))*sqrt(e*cos(d*x
+ c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a), x)
```

maple [A] time = 1.29, size = 185, normalized size = 1.95

$$-\frac{2e^2\left(-24b\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+36b\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}\right.}{15\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] -2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(-24*b*sin
(1/2*d*x+1/2*c)^7+20*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*b*sin(1/2
*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-10*a*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2-18*b*sin(1/2*d*x+1/2*c)^3+3*b*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.542 $\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=63

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}}{3de}$$

```
[Out] -2/3*b*(e*cos(d*x+c))^(3/2)/d/e+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2669, 2640, 2639}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(e*Cos[c + d*x])^(3/2))/(3*d*e) + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))\,dx &= -\frac{2b(e\cos(c+dx))^{3/2}}{3de}+a\int \sqrt{e\cos(c+dx)}\,dx\\ &= -\frac{2b(e\cos(c+dx))^{3/2}}{3de}+\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{2b(e\cos(c+dx))^{3/2}}{3de}+\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 56, normalized size = 0.89

$$-\frac{2\sqrt{e\cos(c+dx)}\left(b\cos^{\frac{3}{2}}(c+dx)-3aE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*(b*Cos[c + d*x]^(3/2) - 3*a*EllipticE[(c + d*x)/2,
 2]))/(3*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a), x)
```

maple [A] time = 1.12, size = 123, normalized size = 1.95

$$\frac{2e\left(-4b\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a+4b\left(\sin^{3}\right.\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(-4*b*sin(1/2*
d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+4*b*sin(1/2*d*x+1/2*c)^3-b*sin(1/
2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+b\,\sin\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(e*cos(c + d*x))*(a + b*sin(c + d*x)), x)
```

3.543 $\int \frac{a+b\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}}{de}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-2*b*(e*cos(d*x+c))^(
1/2)/d/e
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2669, 2642, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[e*Cos[c + d*x]])/(d*e) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx &= -\frac{2b\sqrt{e\cos(c+dx)}}{de}+a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx\\ &= -\frac{2b\sqrt{e\cos(c+dx)}}{de}+\frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}}\\ &= -\frac{2b\sqrt{e\cos(c+dx)}}{de}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.20, size = 50, normalized size = 0.82

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-2b\cos(c+dx)}{d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*b*Cos[c + d*x] + 2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*S
qrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)}{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)/(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(dx+c\right)+a}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

maple [A] time = 0.84, size = 106, normalized size = 1.74

$$-\frac{2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-2b\left(\sin^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*((sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a-2*b*sin(1/2*d*x+1/2*c)^3+b*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(dx+c\right)+a}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

mupad [B] time = 6.49, size = 47, normalized size = 0.77

$$-\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\sqrt{\cos\left(c+d\,x\right)}-a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d\,\sqrt{e\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*cos(c + d*x)^(1/2)*(b*cos(c + d*x)^(1/2) - a*ellipticF(c/2 + (d*x)/2, 2
)))/(d*(e*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(c+dx\right)}{\sqrt{e\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x))/sqrt(e*cos(c + d*x)), x)
```

3.544 $\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}+\frac{2b}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*b/d/e/(e*cos(d*x+c))^(1/2)+2*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(1/2)-2*a*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2640, 2639}

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}+\frac{2b}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b)/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(d*e*Sqrt[e*Co
s[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{2b}{de\sqrt{e\cos(c+dx)}}+a\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2b}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{a\int \sqrt{e\cos(c+dx)}\,dx}{e^2}\\ &= \frac{2b}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{e^2\sqrt{\cos(c+dx)}}\\ &= \frac{2b}{de\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 54, normalized size = 0.59

$$\frac{2\left(a\sin(c+dx)-a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
[Out] (2*(b - a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + a*Sin[c + d*x]))/(
d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)}{e^2\cos\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)/(e^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

maple [A] time = 1.62, size = 119, normalized size = 1.31

$$-\frac{2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{e\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a-2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-b*sin(1/2*d*x+1/2*
c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin(c+d\,x)}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(3/2),x)
[Out] Integral((a + b*sin(c + d*x))/(e*cos(c + d*x))**(3/2), x)
```

3.545 $\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}+\frac{2b}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*b/d/e/(e*cos(d*x+c))^(3/2)+2/3*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(3/2)+2/
3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2642, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}+\frac{2b}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*b)/(3*d*e*(e*Cos[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*d*e*(
e*Cos[c + d*x])^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2b}{3de(e\cos(c+dx))^{3/2}} + a\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$
$$= \frac{2b}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}} + \frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2\sqrt{e\cos(c+dx)}}$$
$$= \frac{2b}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.14, size = 55, normalized size = 0.57

$$\frac{2\left(a\sin(c+dx)+a\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+b\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + a*Sin[c + d*x]))/(
3*d*e*(e*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,(b\sin(dx+c)+a)}{e^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)/(e^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

maple [A] time = 2.08, size = 193, normalized size = 1.99

$$-\frac{2\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\cdots}\right.}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*sin(1/2*d*x+1/2*c)^2-(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*a+2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+b*sin(1/2*d*
x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sin(dx+c) + a}{(e \cos(dx+c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\, \sin(c + d\, x)}{(e\, \cos(c + d\, x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.546 $\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{2b}{5de(e\cos(c+dx))^{5/2}}$$

```
[Out] 2/5*b/d/e/(e*cos(d*x+c))^(5/2)+2/5*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(5/2)+6/
5*a*sin(d*x+c)/d/e^3/(e*cos(d*x+c))^(1/2)-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(
1/2)/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2640, 2639}

$$\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{2b}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*b)/(5*d*e*(e*Cos[c + d*x])^(5/2)) - (6*a*Sqrt[e*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(5*d*e*(
e*Cos[c + d*x])^(5/2)) + (6*a*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2b}{5de(e\cos(c+dx))^{5/2}} + a\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b}{5de(e\cos(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} + \frac{(3a)\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2}$$
$$= \frac{2b}{5de(e\cos(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} + \frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} - \frac{(3a)\int \sqrt{e\cos}}{5e}$$
$$= \frac{2b}{5de(e\cos(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} + \frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} - \frac{\left(3a\sqrt{e\cos(c\right.}}{5e}$$
$$= \frac{2b}{5de(e\cos(c+dx))^{5/2}} - \frac{6a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} +$$

Mathematica [A] time = 0.34, size = 70, normalized size = 0.56

$$\frac{7a\sin(c+dx)+3a\sin(3(c+dx))-12a\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+4b}{10de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
[Out] (4*b - 12*a*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 7*a*Sin[c + d*x]
 + 3*a*Sin[3*(c + d*x)])/(10*d*e*(e*Cos[c + d*x])^(5/2))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,(b\sin(dx+c)+a)}{e^4\cos(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)/(e^4*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

maple [B] time = 3.24, size = 310, normalized size = 2.46

$$-\frac{2\left(12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24a\cos\left(\frac{dx}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*sin(1
/2*d*x+1/2*c)^4-24*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*sin(1/2*d*x+1/2*c)^2+24*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-8*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2-b*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sin(dx + c) + a}{(e \cos(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\, \sin(c + d\, x)}{(e\, \cos(c + d\, x))^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.547 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=188

$$\frac{10e^4(11a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{10e^3(11a^2+2b^2)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{2e\left(11a^2+2b^2\right.}{}$$

```
[Out] -26/99*a*b*(e*cos(d*x+c))^(9/2)/d/e+2/77*(11*a^2+2*b^2)*e*(e*cos(d*x+c))^(5
/2)*sin(d*x+c)/d-2/11*b*(e*cos(d*x+c))^(9/2)*(a+b*sin(d*x+c))/d/e+10/231*(1
1*a^2+2*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+10/231*
(11*a^2+2*b^2)*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2692, 2669, 2635, 2642, 2641}

$$\frac{10e^3(11a^2+2b^2)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{10e^4(11a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{2e\left(11a^2+2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-26*a*b*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (10*(11*a^2 + 2*b^2)*e^4*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (10*
(11*a^2 + 2*b^2)*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(11*a^
2 + 2*b^2)*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(77*d) - (2*b*(e*Cos[c +
d*x])^(9/2)*(a + b*Sin[c + d*x]))/(11*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2\,dx = -\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{11de}+\frac{2}{11}\int (e\cos(c+dx))^{7/2}\Big(1$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}-\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{11de}+$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{2\left(11a^2+2b^2\right)e(e\cos(c+dx))^{5/2}\sin(c+}{77d}$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{10\left(11a^2+2b^2\right)e^3\sqrt{e\cos(c+dx)}\,\sin(c+}{231d}$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{10\left(11a^2+2b^2\right)e^3\sqrt{e\cos(c+dx)}\,\sin(c+}{231d}$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{10\left(11a^2+2b^2\right)e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{231d\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 1.98, size = 160, normalized size = 0.85

$$\frac{(e\cos(c+dx))^{7/2}\left(40\left(11a^2+2b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{1}{6}\sqrt{\cos(c+dx)}\,\left(6\left(572a^2+41b^2\right)\sin(c+dx)+8\cos(2(c+\right.\right.}{924d\cos^{\frac{7}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*(-154*a*b*Sqrt[Cos[c + d*x]] + 40*(11*a^2 + 2*b^2)*
EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(6*(572*a^2 + 41*b^2)*Sin[c
 + d*x] - 14*b*Cos[4*(c + d*x)]*(22*a + 9*b*Sin[c + d*x]) + 8*Cos[2*(c + d*
x)]*(-154*a*b + 9*(11*a^2 - 5*b^2)*Sin[c + d*x])))/6))/(924*d*Cos[c + d*x]^
(7/2))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2e^3\cos(dx+c)^5-2\,abe^3\cos(dx+c)^3\sin(dx+c)-\left(a^2+b^2\right)e^3\cos(dx+c)^3\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*e^3*cos(d*x + c)^5 - 2*a*b*e^3*cos(d*x + c)^3*sin(d*x + c) -
 (a^2 + b^2)*e^3*cos(d*x + c)^3)*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^2, x)
```

maple [B] time = 1.75, size = 473, normalized size = 2.52

$$-\frac{2e^4\left(-4032b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4928ab\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10080b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2/693/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-4032*b^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-4928*a*b*sin(1/2*d*x+1/2*c)^11+1
0080*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+1584*a^2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^8+12320*a*b*sin(1/2*d*x+1/2*c)^9-9792*b^2*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^8-2376*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^6-12320*a*b*sin(1/2*d*x+1/2*c)^7+4608*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^6+1848*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6160*a*b*sin(1/2*d
*x+1/2*c)^5-924*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+165*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2+30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-528*a^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-1540*a*b*sin(1/2*d*x+1/2*c)^3+30*b^2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2+154*a*b*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{7/2}\,(a+b\,\sin(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.548 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=149

$$\frac{2e^2\left(9a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2e\left(9a^2+2b^2\right)\sin(c+dx)(e\cos(c+dx))^{3/2}}{45d}-\frac{22ab(e\cos(c+dx)}{63de}$$

```
[Out] -22/63*a*b*(e*cos(d*x+c))^(7/2)/d/e+2/45*(9*a^2+2*b^2)*e*(e*cos(d*x+c))^(3/
2)*sin(d*x+c)/d-2/9*b*(e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))/d/e+2/15*(9*a^2
+2*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2692, 2669, 2635, 2640, 2639}

$$\frac{2e^2\left(9a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2e\left(9a^2+2b^2\right)\sin(c+dx)(e\cos(c+dx))^{3/2}}{45d}-\frac{22ab(e\cos(c+dx)}{63de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-22*a*b*(e*Cos[c + d*x])^(7/2))/(63*d*e) + (2*(9*a^2 + 2*b^2)*e^2*Sqrt[e*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(9*a
^2 + 2*b^2)*e*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) - (2*b*(e*Cos[c +
 d*x])^(7/2)*(a + b*Sin[c + d*x]))/(9*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
```

```
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2\,dx = -\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{9de}+\frac{2}{9}\int (e\cos(c+dx))^{5/2}($$
$$= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}-\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{9de}$$
$$= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}+\frac{2\left(9a^2+2b^2\right)e(e\cos(c+dx))^{3/2}\sin(c}{45d}$$
$$= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}+\frac{2\left(9a^2+2b^2\right)e(e\cos(c+dx))^{3/2}\sin(c}{45d}$$
$$= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}+\frac{2\left(9a^2+2b^2\right)e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{15d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.92, size = 113, normalized size = 0.76

$$\frac{(e\cos(c+dx))^{5/2}\left(84\left(9a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^{\frac{3}{2}}(c+dx)\left(21\left(12a^2+b^2\right)\sin(c+dx)-5b(36a+7b\sin\right.\right.}{630d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(84*(9*a^2 + 2*b^2)*EllipticE[(c + d*x)/2, 2] + Cos
[c + d*x]^(3/2)*(-180*a*b*Cos[2*(c + d*x)] + 21*(12*a^2 + b^2)*Sin[c + d*x]
 - 5*b*(36*a + 7*b*Sin[3*(c + d*x)]))))/(630*d*Cos[c + d*x]^(5/2))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2e^2\cos(dx+c)^4-2\,abe^2\cos(dx+c)^2\sin(dx+c)-\left(a^2+b^2\right)e^2\cos(dx+c)^2\right)\sqrt{e\cos(dx+c)},x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*e^2*cos(d*x + c)^4 - 2*a*b*e^2*cos(d*x + c)^2*sin(d*x + c) -
 (a^2 + b^2)*e^2*cos(d*x + c)^2)*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^2, x)
```

maple [B] time = 1.51, size = 408, normalized size = 2.74

$$2e^3\left(-1120b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1440ab\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2240b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(-1120*b^2
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-1440*a*b*sin(1/2*d*x+1/2*c)^9+224
0*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+504*a^2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^6+2880*a*b*sin(1/2*d*x+1/2*c)^7-1568*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^6-504*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2160
*a*b*sin(1/2*d*x+1/2*c)^5+448*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+1
89*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+126*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+720*a*b*sin(1/2*d*x+1/2*c)^3-42*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-90*a*b*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{5/2}\,(a+b\,\sin(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.549 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=149

$$\frac{2e^2\left(7a^2+2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2e\left(7a^2+2b^2\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{18ab(e\cos(c+dx))}{35de}$$

```
[Out] -18/35*a*b*(e*cos(d*x+c))^(5/2)/d/e-2/7*b*(e*cos(d*x+c))^(5/2)*(a+b*sin(d*x
+c))/d/e+2/21*(7*a^2+2*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c)
)^(1/2)+2/21*(7*a^2+2*b^2)*e*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2692, 2669, 2635, 2642, 2641}

$$\frac{2e^2\left(7a^2+2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2e\left(7a^2+2b^2\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{18ab(e\cos(c+dx))}{35de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-18*a*b*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (2*(7*a^2 + 2*b^2)*e^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (2*(7*a
^2 + 2*b^2)*e*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(21*d) - (2*b*(e*Cos[c + d
*x])^(5/2)*(a + b*Sin[c + d*x]))/(7*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
```

```
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2\,dx = -\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{7de} + \frac{2}{7}\int (e\cos(c+dx))^{3/2}\Big(\frac{7a}{2}$$
$$= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de} - \frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{7de} + \frac{1}{7}$$
$$= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de} + \frac{2\left(7a^2+2b^2\right)e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d}$$
$$= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de} + \frac{2\left(7a^2+2b^2\right)e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d}$$
$$= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de} + \frac{2\left(7a^2+2b^2\right)e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{21d\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 1.18, size = 115, normalized size = 0.77

$$\frac{(e\cos(c+dx))^{3/2}\left(20\left(7a^2+2b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\left(5\left(28a^2+5b^2\right)\sin(c+dx)-3b(28a+5b\sin(\right.\right.}{210d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(20*(7*a^2 + 2*b^2)*EllipticF[(c + d*x)/2, 2] + Sqr
t[Cos[c + d*x]]*(-84*a*b*Cos[2*(c + d*x)] + 5*(28*a^2 + 5*b^2)*Sin[c + d*x]
 - 3*b*(28*a + 5*b*Sin[3*(c + d*x)]))))/(210*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2 e\cos(dx+c)^3 - 2\,abe\cos(dx+c)\sin(dx+c) - \left(a^2+b^2\right)e\cos(dx+c)\right)\sqrt{e\cos(dx+c)}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*e*cos(d*x + c)^3 - 2*a*b*e*cos(d*x + c)*sin(d*x + c) - (a^2
+ b^2)*e*cos(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^2, x)
```

maple [B] time = 1.72, size = 343, normalized size = 2.30

$$-\frac{2e^2\left(-240b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-336ab\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+360b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+140a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2/105/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(-240*b^2
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-336*a*b*sin(1/2*d*x+1/2*c)^7+360*b
^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+140*a^2*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^4+504*a*b*sin(1/2*d*x+1/2*c)^5-140*b^2*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^4+35*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+10*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*b^2-70*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-252*a*b*sin(1/2*d*x+1
/2*c)^3+10*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+42*a*b*sin(1/2*d*x+1
/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{3/2}\,(a+b\,\sin(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.550 $\int \sqrt{e \cos(c+dx)}\,(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=109

$$\frac{2\left(5a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{14ab(e\cos(c+dx))^{3/2}}{15de}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de}$$

```
[Out] -14/15*a*b*(e*cos(d*x+c))^(3/2)/d/e-2/5*b*(e*cos(d*x+c))^(3/2)*(a+b*sin(d*x
+c))/d/e+2/5*(5*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/
2)
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2692, 2669, 2640, 2639}

$$\frac{2\left(5a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{14ab(e\cos(c+dx))^{3/2}}{15de}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-14*a*b*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (2*(5*a^2 + 2*b^2)*Sqrt[e*Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*b*(e*Cos[
c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(5*d*e)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2\,dx = -\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de} + \frac{2}{5}\int \sqrt{e\cos(c+dx)}\,\Big(\frac{5a}{2}$$

$$= -\frac{14ab(e\cos(c+dx))^{3/2}}{15de} - \frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de} +$$

$$= -\frac{14ab(e\cos(c+dx))^{3/2}}{15de} - \frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de} +$$

$$= -\frac{14ab(e\cos(c+dx))^{3/2}}{15de} + \frac{2\left(5a^2+2b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.33, size = 80, normalized size = 0.73

$$\frac{\sqrt{e\cos(c+dx)}\left(6\left(5a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)-2b\cos^{\frac{3}{2}}(c+dx)(10a+3b\sin(c+dx))\right)}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(6*(5*a^2 + 2*b^2)*EllipticE[(c + d*x)/2, 2] - 2*b*Co
s[c + d*x]^(3/2)*(10*a + 3*b*Sin[c + d*x])))/(15*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(e*cos(
d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^2, x)
```

maple [B] time = 1.73, size = 251, normalized size = 2.30

$$\frac{2e\left(-24b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40ab\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\sqrt{\frac{1}{2}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(-24*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-40*a*b*sin(1/2*d*x+1/2*c)^5+24*b^2*cos(
```

```
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+6*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*b^2+40*a*b*sin(1/2*d*x+1/2*c)^3-6*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-10*a*b*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos(c+d\,x)}\,\left(a+b\,\sin(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*(e*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.551 $\int \frac{(a+b\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=109

$$\frac{2\left(3a^2+2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{10ab\sqrt{e\cos(c+dx)}}{3de}-\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de}$$

```
[Out] 2/3*(3*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-10/3*a
*b*(e*cos(d*x+c))^(1/2)/d/e-2/3*b*(a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2692, 2669, 2642, 2641}

$$\frac{2\left(3a^2+2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{10ab\sqrt{e\cos(c+dx)}}{3de}-\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-10*a*b*Sqrt[e*Cos[c + d*x]])/(3*d*e) + (2*(3*a^2 + 2*b^2)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) - (2*b*Sqrt[e*Cos
[c + d*x]]*(a + b*Sin[c + d*x]))/(3*d*e)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}dx = -\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de} + \frac{2}{3}\int \frac{\frac{3a^2}{2}+b^2+\frac{5}{2}ab\sin(c+dx)}{\sqrt{e\cos(c+dx)}}dx$$
$$= -\frac{10ab\sqrt{e\cos(c+dx)}}{3de} - \frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de} + \frac{1}{3}\left(3a^2+2b^2\right)\int \frac{}{\sqrt{e}}$$
$$= -\frac{10ab\sqrt{e\cos(c+dx)}}{3de} - \frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de} + \frac{\left(\left(3a^2+2b^2\right)\sqrt{\cos(c}\right.}{3\sqrt{e\cos}}$$
$$= -\frac{10ab\sqrt{e\cos(c+dx)}}{3de} + \frac{2\left(3a^2+2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}} - \frac{2b\sqrt{e\cos(c+}}{}$$

Mathematica [A] time = 0.43, size = 75, normalized size = 0.69

$$\frac{2\left(3a^2+2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right) - 2b\cos(c+dx)(6a+b\sin(c+dx))}{3d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (2*(3*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - 2*b*Cos[c
 + d*x]*(6*a + b*Sin[c + d*x]))/(3*d*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2\cos(dx+c)^2 - 2\,ab\sin(dx+c) - a^2 - b^2\right)\sqrt{e\cos(dx+c)}}{e\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(e*cos(
d*x + c))/(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^2}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/sqrt(e*cos(d*x + c)), x)
```

maple [A] time = 1.07, size = 210, normalized size = 1.93

$$-\frac{2\left(-4b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-4*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b^2-12*a*b*sin(1/2*d*x+1/2*c)^3+2*b^2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2+6*a*b*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^2}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^2}{\sqrt{e\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.552 $\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{2\left(a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}}{de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*a*b*(e*cos(d*x+c))^(3/2)/d/e^3+2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d/e/(e
*cos(d*x+c))^(1/2)-2*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(
d*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2691, 2669, 2640, 2639}

$$-\frac{2\left(a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}}{de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a*b*(e*Cos[c + d*x])^(3/2))/(d*e^3) - (2*(a^2 + 2*b^2)*Sqrt[e*Cos[c + d*
x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*(b + a*Sin[c
 + d*x])*(a + b*Sin[c + d*x]))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}} - \frac{2\int \sqrt{e\cos(c+dx)}\left(\frac{a^2}{2}+b^2+\frac{3}{2}ab\sin(c$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}} - \frac{\left(a^2+2b^2\right)\int \sqrt{e}}{e^2}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}} - \frac{\left(\left(a^2+2b^2\right)\sqrt{e\,c}\right.}{}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}}{de^3} - \frac{2\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}} + \frac{2(b+a\sin}{}$$

Mathematica [A] time = 0.24, size = 71, normalized size = 0.63

$$\frac{2\left(a^2+b^2\right)\sin(c+dx)-2\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+4ab}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*a*b - 2*(a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(
a^2 + b^2)*Sin[c + d*x])/(d*e*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2\right)\sqrt{e\cos\left(dx+c\right)}}{e^2\cos\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(e*cos(
d*x + c))/(e^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^2}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(3/2), x)
```

maple [A] time = 1.80, size = 197, normalized size = 1.74

$$-\frac{2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^2+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{e\sqrt{-2\left(\mathrm{si}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-2*a^2*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^2-2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*a*b*sin(1/
2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sin(dx + c) + a)^2}{(e \cos(dx + c))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \sin(c + d x))^2}{(e \cos(c + d x))^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.553 $\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=119

$$\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d/e/(e*cos(d*x+c))^(3/2)+2/3*(a^2-2*b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)+2/3*a*b*(e*cos(d
*x+c))^(1/2)/d/e^3
```

Rubi [A] time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2691, 2669, 2642, 2641}

$$\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a*b*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*(b + a*Sin
[c + d*x])*(a + b*Sin[c + d*x]))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}+b^2+\frac{1}{2}ab\sin(c+dx)}{\sqrt{e\cos(c+dx)}}dx}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}} + \frac{(a^2-2b^2)\int \frac{1}{\sqrt{e\cos(c+}}}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}} + \frac{((a^2-2b^2)\sqrt{\cos(c+}}{3e^2\sqrt{e\cos}}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+d}{3de(e\cos}$$

Mathematica [A] time = 0.26, size = 72, normalized size = 0.61

$$\frac{2\left(\left(a^2+b^2\right)\sin(c+dx)+\left(a^2-2b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2ab\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(2*a*b + (a^2 - 2*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + (a
^2 + b^2)*Sin[c + d*x]))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2\right)\sqrt{e\cos(dx+c)}}{e^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(e*cos(
d*x + c))/(e^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(5/2), x)
```

maple [B] time = 2.18, size = 333, normalized size = 2.80

$$-\frac{2\left(2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,a^2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\,\text{EllipticF}\left(\cos\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2-4*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*b^2+2*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*a*b*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^2}{(e\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.554 $\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=160

$$-\frac{2\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2\left(3a^2-2b^2\right)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2ab}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2(a\sin(c+dx)}{5de(e\text{c}}$$

```
[Out] 2/5*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d/e/(e*cos(d*x+c))^(5/2)+2/5*a*b/d/e^
3/(e*cos(d*x+c))^(1/2)+2/5*(3*a^2-2*b^2)*sin(d*x+c)/d/e^3/(e*cos(d*x+c))^(1
/2)-2/5*(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^4/cos(d*x+c)^(1/2
)
```

Rubi [A] time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2691, 2669, 2636, 2640, 2639}

$$\frac{2\left(3a^2-2b^2\right)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{2\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2ab}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2(a\sin(c+dx)+}{5de(e\text{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*a*b)/(5*d*e^3*Sqrt[e*Cos[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Sqrt[e*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*(3*a^2 -
 2*b^2)*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]]) + (2*(b + a*Sin[c + d*
x])*(a + b*Sin[c + d*x]))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
```

```
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{5de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+b^2-\frac{1}{2}ab\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2}$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{5de(e\cos(c+dx))^{5/2}} + \frac{\left(3a^2-2b^2\right)\int \frac{}{(e\,c}}{5e^2}$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2\left(3a^2-2b^2\right)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a+b\sin(}{5de(e\cos(c+dx))^{5/2}}$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2\left(3a^2-2b^2\right)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a+b\sin(}{5de(e\cos(c+dx))^{5/2}}$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} - \frac{2\left(3a^2-2b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{2\left(3a^2-2\right.}{5de^3\sqrt{e}}$$

Mathematica [A] time = 0.56, size = 105, normalized size = 0.66

$$\frac{\left(7a^2+2b^2\right)\sin(c+dx)-4\left(3a^2-2b^2\right)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+3a^2\sin(3(c+dx))+8ab-2b^2\sin(3(c}{10de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (8*a*b - 4*(3*a^2 - 2*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + (
7*a^2 + 2*b^2)*Sin[c + d*x] + 3*a^2*Sin[3*(c + d*x)] - 2*b^2*Sin[3*(c + d*x
)])/(10*d*e*(e*Cos[c + d*x])^(5/2))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2\right)\sqrt{e\cos\left(dx+c\right)}}{e^4\cos\left(dx+c\right)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(e*cos(
d*x + c))/(e^4*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^2}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(7/2), x)
```

maple [B] time = 4.00, size = 564, normalized size = 3.52

$$-\frac{2\left(12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ a^2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*sin
(1/2*d*x+1/2*c)^4-8*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4-24*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+16*b^2*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2+8*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^2+24*a^2*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^4-16*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-8*a^2*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*a*b*si
n(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.555 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=237

$$\frac{10ae^4\left(11a^2+6b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{10ae^3\left(11a^2+6b^2\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{2b\left(177a^2+\right.}{}$$

```
[Out] -2/1287*b*(177*a^2+44*b^2)*(e*cos(d*x+c))^(9/2)/d/e+2/77*a*(11*a^2+6*b^2)*e
*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/d-34/143*a*b*(e*cos(d*x+c))^(9/2)*(a+b*sin
(d*x+c))/d/e-2/13*b*(e*cos(d*x+c))^(9/2)*(a+b*sin(d*x+c))^2/d/e+10/231*a*(1
1*a^2+6*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+10/231*
a*(11*a^2+6*b^2)*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.31, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2692, 2862, 2669, 2635, 2642, 2641}

$$\frac{10ae^3\left(11a^2+6b^2\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{10ae^4\left(11a^2+6b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{2b\left(177a^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(177*a^2 + 44*b^2)*(e*Cos[c + d*x])^(9/2))/(1287*d*e) + (10*a*(11*a^2
 + 6*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*C
os[c + d*x]]) + (10*a*(11*a^2 + 6*b^2)*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x
])/(231*d) + (2*a*(11*a^2 + 6*b^2)*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
77*d) - (34*a*b*(e*Cos[c + d*x])^(9/2)*(a + b*Sin[c + d*x]))/(143*d*e) - (2
*b*(e*Cos[c + d*x])^(9/2)*(a + b*Sin[c + d*x])^2)/(13*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3\,dx = -\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^2}{13de} + \frac{2}{13}\int (e\cos(c+dx))^{7/2}(a$$

$$= -\frac{34ab(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{143de} - \frac{2b(e\cos(c+dx))^{9/2}(a+}{13de}$$

$$= -\frac{2b\left(177a^2+44b^2\right)(e\cos(c+dx))^{9/2}}{1287de} - \frac{34ab(e\cos(c+dx))^{9/2}(a+b}{143de}$$

$$= -\frac{2b\left(177a^2+44b^2\right)(e\cos(c+dx))^{9/2}}{1287de} + \frac{2a\left(11a^2+6b^2\right)e(e\cos(c+}{77d}$$

$$= -\frac{2b\left(177a^2+44b^2\right)(e\cos(c+dx))^{9/2}}{1287de} + \frac{10a\left(11a^2+6b^2\right)e^3\sqrt{e\cos(c}}{231d}$$

$$= -\frac{2b\left(177a^2+44b^2\right)(e\cos(c+dx))^{9/2}}{1287de} + \frac{10a\left(11a^2+6b^2\right)e^3\sqrt{e\cos(c}}{231d}$$

$$= -\frac{2b\left(177a^2+44b^2\right)(e\cos(c+dx))^{9/2}}{1287de} + \frac{10a\left(11a^2+6b^2\right)e^4\sqrt{\cos(c-}}{231d\sqrt{e\cos(c}}$$

Mathematica [A] time = 2.09, size = 205, normalized size = 0.86

$$(e\cos(c+dx))^{7/2}\left(2080\left(11a^3+6ab^2\right)F\left(\tfrac{1}{2}(c+dx)\middle|2\right)-154b\left(78a^2+11b^2\right)\sqrt{\cos(c+dx)}+\tfrac{1}{3}\sqrt{\cos(c+dx)}\,(156$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*(-154*b*(78*a^2 + 11*b^2)*Sqrt[Cos[c + d*x]] + 2080
*(11*a^3 + 6*a*b^2)*EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(-77*b*
(624*a^2 + 73*b^2)*Cos[2*(c + d*x)] + 154*b*(-78*a^2 + b^2)*Cos[4*(c + d*x)
] + 693*b^3*Cos[6*(c + d*x)] + 156*a*(506*a^2 + 213*b^2)*Sin[c + d*x] + 234
*a*(44*a^2 - 39*b^2)*Sin[3*(c + d*x)] - 4914*a*b^2*Sin[5*(c + d*x)]))/3))/(
48048*d*Cos[c + d*x]^(7/2))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,ab^2e^3\cos\left(dx+c\right)^5-\left(a^3+3\,ab^2\right)e^3\cos\left(dx+c\right)^3+\left(b^3e^3\cos\left(dx+c\right)^5-\left(3\,a^2b+b^3\right)e^3\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*e^3*cos(d*x + c)^5 - (a^3 + 3*a*b^2)*e^3*cos(d*x + c)^3
+ (b^3*e^3*cos(d*x + c)^5 - (3*a^2*b + b^3)*e^3*cos(d*x + c)^3)*sin(d*x + c
))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e \cos \left(dx+c\right)\right)^{\frac{7}{2}} \left(b \sin \left(dx+c\right)+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^3, x)
```

maple [B] time = 3.47, size = 618, normalized size = 2.61

$$-\frac{2e^4\left(3003a^2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)-308b^3\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+433664b^3\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-310464b^3\left(\sin^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -2/9009/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(3003*a^
2*b*sin(1/2*d*x+1/2*c)+308*b^3*sin(1/2*d*x+1/2*c)+88704*b^3*sin(1/2*d*x+1/2
*c)^15-308000*b^3*sin(1/2*d*x+1/2*c)^9+113960*b^3*sin(1/2*d*x+1/2*c)^7-1817
2*b^3*sin(1/2*d*x+1/2*c)^5-308*b^3*sin(1/2*d*x+1/2*c)^3+433664*b^3*sin(1/2*
d*x+1/2*c)^11-310464*b^3*sin(1/2*d*x+1/2*c)^13+20592*a^3*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^8-30888*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+240
24*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-6864*a^3*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2-96096*a^2*b*sin(1/2*d*x+1/2*c)^11+240240*a^2*b*sin(1/2
*d*x+1/2*c)^9-240240*a^2*b*sin(1/2*d*x+1/2*c)^7+120120*a^2*b*sin(1/2*d*x+1/
2*c)^5-30030*a^2*b*sin(1/2*d*x+1/2*c)^3+1170*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2
-381888*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+179712*a*b^2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36036*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+1170*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2145*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a^3-157248*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+39
3120*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e \cos \left(dx+c\right)\right)^{\frac{7}{2}} \left(b \sin \left(dx+c\right)+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

3.556 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=197

$$\frac{2ae^2\left(3a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de}+\frac{2ae\left(3a^2+2b^2\right)\sin(c+\ldots}{15\ldots}$$

```
[Out] -2/231*b*(43*a^2+12*b^2)*(e*cos(d*x+c))^(7/2)/d/e+2/15*a*(3*a^2+2*b^2)*e*(e
*cos(d*x+c))^(3/2)*sin(d*x+c)/d-10/33*a*b*(e*cos(d*x+c))^(7/2)*(a+b*sin(d*x
+c))/d/e-2/11*b*(e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2/d/e+2/5*a*(3*a^2+2*
b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2692, 2862, 2669, 2635, 2640, 2639}

$$\frac{2ae^2\left(3a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de}+\frac{2ae\left(3a^2+2b^2\right)\sin(c+\ldots}{15\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(43*a^2 + 12*b^2)*(e*Cos[c + d*x])^(7/2))/(231*d*e) + (2*a*(3*a^2 + 2
*b^2)*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
 d*x]]) + (2*a*(3*a^2 + 2*b^2)*e*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*d
) - (10*a*b*(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x]))/(33*d*e) - (2*b*(e
*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2)/(11*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3\,dx = -\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}{11de}+\frac{2}{11}\int (e\cos(c+dx))^{5/2}(a$$
$$= -\frac{10ab(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{33de}-\frac{2b(e\cos(c+dx))^{7/2}(a+}{11de}$$
$$= -\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de}-\frac{10ab(e\cos(c+dx))^{7/2}(a+b\,\mathrm{s}}{33de}$$
$$= -\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de}+\frac{2a\left(3a^2+2b^2\right)e(e\cos(c+dx}{15d}$$
$$= -\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de}+\frac{2a\left(3a^2+2b^2\right)e(e\cos(c+dx}{15d}$$
$$= -\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de}+\frac{2a\left(3a^2+2b^2\right)e^2\sqrt{e\cos(c+}}{5d\sqrt{\cos(c+}}$$

Mathematica [A] time = 1.41, size = 150, normalized size = 0.76

$$\frac{(e\cos(c+dx))^{5/2}\left(1848\left(3a^3+2ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^{\frac{3}{2}}(c+dx)\left(1848a^3\sin(c+dx)-60\left(33a^2b+4b^3\right)\cos(\right.\right.}{4620d\cos^{\frac{5}{2}}(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(1848*(3*a^3 + 2*a*b^2)*EllipticE[(c + d*x)/2, 2] +
 Cos[c + d*x]^(3/2)*(-1980*a^2*b - 345*b^3 - 60*(33*a^2*b + 4*b^3)*Cos[2*(c
 + d*x)] + 105*b^3*Cos[4*(c + d*x)] + 1848*a^3*Sin[c + d*x] + 462*a*b^2*Sin
[c + d*x] - 770*a*b^2*Sin[3*(c + d*x)])))/(4620*d*Cos[c + d*x]^(5/2))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(3\,ab^2e^2\cos\left(dx+c\right)^4-\left(a^3+3\,ab^2\right)e^2\cos\left(dx+c\right)^2+\left(b^3e^2\cos\left(dx+c\right)^4-\left(3\,a^2b+b^3\right)e^2\cos\left(dx+c\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*e^2*cos(d*x + c)^4 - (a^3 + 3*a*b^2)*e^2*cos(d*x + c)^2
+ (b^3*e^2*cos(d*x + c)^4 - (3*a^2*b + b^3)*e^2*cos(d*x + c)^2)*sin(d*x + c
))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^3, x)
```

maple [B] time = 3.09, size = 534, normalized size = 2.71

$$2e^3\left(6720b^3\left(\sin^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12320a\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20160b^3\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7920a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x)
[Out] 2/1155/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(6720*b^3
*sin(1/2*d*x+1/2*c)^13-12320*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10
-20160*b^3*sin(1/2*d*x+1/2*c)^11-7920*a^2*b*sin(1/2*d*x+1/2*c)^9+24640*a*b^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+22560*b^3*sin(1/2*d*x+1/2*c)^9+18
48*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+15840*a^2*b*sin(1/2*d*x+1/2*
c)^7-17248*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-11520*b^3*sin(1/2*
d*x+1/2*c)^7-1848*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-11880*a^2*b*s
in(1/2*d*x+1/2*c)^5+4928*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+2340
*b^3*sin(1/2*d*x+1/2*c)^5+693*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+462*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b^2+462*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3960*a^2
*b*sin(1/2*d*x+1/2*c)^3-462*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6
0*b^3*sin(1/2*d*x+1/2*c)^3-495*a^2*b*sin(1/2*d*x+1/2*c)-60*b^3*sin(1/2*d*x+
1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+d\,x))^{5/2}\,(a+b\sin(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.557 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=197

$$\frac{2ae^2\left(7a^2+6b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de}+\frac{2ae\left(7a^2+6b^2\right)\sin(c+a}{21d}$$

```
[Out] -2/315*b*(89*a^2+28*b^2)*(e*cos(d*x+c))^(5/2)/d/e-26/63*a*b*(e*cos(d*x+c))^
(5/2)*(a+b*sin(d*x+c))/d/e-2/9*b*(e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2/d/
e+2/21*a*(7*a^2+6*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/
2)+2/21*a*(7*a^2+6*b^2)*e*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2692, 2862, 2669, 2635, 2642, 2641}

$$\frac{2ae^2\left(7a^2+6b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de}+\frac{2ae\left(7a^2+6b^2\right)\sin(c+a}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(89*a^2 + 28*b^2)*(e*Cos[c + d*x])^(5/2))/(315*d*e) + (2*a*(7*a^2 + 6
*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c
+ d*x]]) + (2*a*(7*a^2 + 6*b^2)*e*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(21*d)
 - (26*a*b*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x]))/(63*d*e) - (2*b*(e*
Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2)/(9*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3\,dx = -\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}{9de}+\frac{2}{9}\int (e\cos(c+dx))^{3/2}(a$$

$$= -\frac{26ab(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{63de}-\frac{2b(e\cos(c+dx))^{5/2}(a+}{9de}$$

$$= -\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de}-\frac{26ab(e\cos(c+dx))^{5/2}(a+b\,s}{63de}$$

$$= -\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de}+\frac{2a\left(7a^2+6b^2\right)e\sqrt{e\cos(c+d}}{21d}$$

$$= -\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de}+\frac{2a\left(7a^2+6b^2\right)e\sqrt{e\cos(c+d}}{21d}$$

$$= -\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de}+\frac{2a\left(7a^2+6b^2\right)e^2\sqrt{\cos(c+d}}{21d\sqrt{e\cos(c+}}$$

Mathematica [A] time = 1.45, size = 153, normalized size = 0.78

$$\frac{(e\cos(c+dx))^{3/2}\left(80\left(7a^3+6ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{2}{3}\sqrt{\cos(c+dx)}\left(840a^3\sin(c+dx)-28\left(27a^2b+4b^3\right)\cos(2\right.\right.}{840d\cos^{\frac{3}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(80*(7*a^3 + 6*a*b^2)*EllipticF[(c + d*x)/2, 2] + (
2*Sqrt[Cos[c + d*x]]*(-756*a^2*b - 147*b^3 - 28*(27*a^2*b + 4*b^3)*Cos[2*(c
 + d*x)] + 35*b^3*Cos[4*(c + d*x)] + 840*a^3*Sin[c + d*x] + 450*a*b^2*Sin[c
 + d*x] - 270*a*b^2*Sin[3*(c + d*x)]))/3))/(840*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,ab^2e\cos\left(dx+c\right)^3-\left(a^3+3\,ab^2\right)e\cos\left(dx+c\right)+\left(b^3e\cos\left(dx+c\right)^3-\left(3\,a^2b+b^3\right)e\cos\left(dx+c\right)\right)\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*e*cos(d*x + c)^3 - (a^3 + 3*a*b^2)*e*cos(d*x + c) + (b^3
*e*cos(d*x + c)^3 - (3*a^2*b + b^3)*e*cos(d*x + c))*sin(d*x + c))*sqrt(e*co
s(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}} (b\sin(dx+c)+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^3, x)
```

maple [B] time = 2.62, size = 450, normalized size = 2.28

$$-\frac{2e^2\left(1120b^3\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2160a\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2800b^3\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1512a^2b\left(\text{si}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(1120*b^3
*sin(1/2*d*x+1/2*c)^11-2160*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2
800*b^3*sin(1/2*d*x+1/2*c)^9-1512*a^2*b*sin(1/2*d*x+1/2*c)^7+3240*a*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2296*b^3*sin(1/2*d*x+1/2*c)^7+420*a^3*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+2268*a^2*b*sin(1/2*d*x+1/2*c)^5-126
0*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-644*b^3*sin(1/2*d*x+1/2*c)^
5+105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+90*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-210*a^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-1134*a^2*b*sin(1/2*d*x+1/2*c)^3+9
0*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-28*b^3*sin(1/2*d*x+1/2*c)^3
+189*a^2*b*sin(1/2*d*x+1/2*c)+28*b^3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}} (b\sin(dx+c)+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{3/2}\,(a+b\,\sin(c+d\,x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.558 $\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=156

$$-\frac{2b\left(57a^2+20b^2\right)(e\cos(c+dx))^{3/2}}{105de}+\frac{2a\left(5a^2+6b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}(a+}{7de}$$

```
[Out] -2/105*b*(57*a^2+20*b^2)*(e*cos(d*x+c))^(3/2)/d/e-22/35*a*b*(e*cos(d*x+c))^
(3/2)*(a+b*sin(d*x+c))/d/e-2/7*b*(e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2/d/
e+2/5*a*(5*a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2692, 2862, 2669, 2640, 2639}

$$-\frac{2b\left(57a^2+20b^2\right)(e\cos(c+dx))^{3/2}}{105de}+\frac{2a\left(5a^2+6b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}(a+}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(57*a^2 + 20*b^2)*(e*Cos[c + d*x])^(3/2))/(105*d*e) + (2*a*(5*a^2 + 6
*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) - (22*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(35*d*e) - (2*b*
(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2)/(7*d*e)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3\,dx = -\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{7de} + \frac{2}{7}\int \sqrt{e\cos(c+dx)}\,(a+b$$

$$= -\frac{22ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{35de} - \frac{2b(e\cos(c+dx))^{3/2}(a+}{7de}$$

$$= -\frac{2b\left(57a^2+20b^2\right)(e\cos(c+dx))^{3/2}}{105de} - \frac{22ab(e\cos(c+dx))^{3/2}(a+b\,\mathrm{si}}{35de}$$

$$= -\frac{2b\left(57a^2+20b^2\right)(e\cos(c+dx))^{3/2}}{105de} - \frac{22ab(e\cos(c+dx))^{3/2}(a+b\,\mathrm{si}}{35de}$$

$$= -\frac{2b\left(57a^2+20b^2\right)(e\cos(c+dx))^{3/2}}{105de} + \frac{2a\left(5a^2+6b^2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx}}$$

Mathematica [A] time = 0.63, size = 101, normalized size = 0.65

$$\frac{\sqrt{e\cos(c+dx)}\left(42\left(5a^3+6ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\cos^{\frac{3}{2}}(c+dx)\left(-210a^2-126ab\sin(c+dx)+15b^2\cos(2(c+\right.\right.}{105d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(42*(5*a^3 + 6*a*b^2)*EllipticE[(c + d*x)/2, 2] + b*C
os[c + d*x]^(3/2)*(-210*a^2 - 55*b^2 + 15*b^2*Cos[2*(c + d*x)] - 126*a*b*Si
n[c + d*x])))/(105*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(3\,ab^2\cos\left(dx+c\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^3, x)
```

maple [B] time = 2.12, size = 339, normalized size = 2.17

$$\frac{2e\left(240b^{3}\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-504a\,b^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480b^{3}\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-420a^{2}b\left(\sin^{5}\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/105/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(240*b^3*sin
(1/2*d*x+1/2*c)^9-504*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-480*b^3
*sin(1/2*d*x+1/2*c)^7-420*a^2*b*sin(1/2*d*x+1/2*c)^5+504*a*b^2*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+220*b^3*sin(1/2*d*x+1/2*c)^5+105*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a^3+126*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+420*a^2*b*sin(1/2*d*x+1/2
*c)^3-126*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+20*b^3*sin(1/2*d*x+
1/2*c)^3-105*a^2*b*sin(1/2*d*x+1/2*c)-20*b^3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+b\,\sin\left(c+d\,x\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.559 $\int \frac{(a+b\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{2b\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{5de}+\frac{2a\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+a}{5de}$$

```
[Out] 2*a*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-2/5*b*(1
1*a^2+4*b^2)*(e*cos(d*x+c))^(1/2)/d/e-6/5*a*b*(a+b*sin(d*x+c))*(e*cos(d*x+c
))^(1/2)/d/e-2/5*b*(a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A] time = 0.24, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2692, 2862, 2669, 2642, 2641}

$$-\frac{2b\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{5de}+\frac{2a\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+a}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*b*(11*a^2 + 4*b^2)*Sqrt[e*Cos[c + d*x]])/(5*d*e) + (2*a*(a^2 + 2*b^2)*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]]) - (6*
a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))/(5*d*e) - (2*b*Sqrt[e*Cos[c
+ d*x]]*(a + b*Sin[c + d*x])^2)/(5*d*e)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}{5de} + \frac{2}{5}\int \frac{(a+b\sin(c+dx))\left(\frac{5a^2}{2}+2b^2+\frac{9}{2}\right.}{\sqrt{e\cos(c+dx)}}$$

$$= -\frac{6ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{5de} - \frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}{5de} +$$

$$= -\frac{2b\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{5de} - \frac{6ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{5de} - 2b\sqrt{e}$$

$$= -\frac{2b\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{5de} - \frac{6ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{5de} - 2b\sqrt{e}$$

$$= -\frac{2b\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{5de} + \frac{2a\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}} -$$

Mathematica [A] time = 0.79, size = 94, normalized size = 0.62

$$\frac{b\cos(c+dx)\left(-30a^2-10ab\sin(c+dx)+b^2\cos(2(c+dx))-9b^2\right)+10a\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{5d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (10*a*(a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Cos[c
+ d*x]*(-30*a^2 - 9*b^2 + b^2*Cos[2*(c + d*x)] - 10*a*b*Sin[c + d*x]))/(5*d
*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,ab^2\cos\left(dx+c\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^3}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/sqrt(e*cos(d*x + c)), x)
```

maple [A] time = 1.76, size = 279, normalized size = 1.84

$$-\frac{2\left(8b^3\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20a\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12b^3\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(8*b^3*sin(1/2*
d*x+1/2*c)^7-20*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-12*b^3*sin(1/
2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+10*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
*b^2-30*a^2*b*sin(1/2*d*x+1/2*c)^3+10*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-4*b^3*sin(1/2*d*x+1/2*c)^3+15*a^2*b*sin(1/2*d*x+1/2*c)+4*b^3*sin(1
/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^3}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^3}{\sqrt{e\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.560 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{2b\left(3a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{3de^3}-\frac{2a\left(a^2+6b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin}{de^3}$$

```
[Out] 2/3*b*(3*a^2+4*b^2)*(e*cos(d*x+c))^(3/2)/d/e^3+2*a*b*(e*cos(d*x+c))^(3/2)*(
a+b*sin(d*x+c))/d/e^3+2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^2/d/e/(e*cos(d*x+
c))^(1/2)-2*a*(a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^
(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2691, 2862, 2669, 2640, 2639}

$$\frac{2b\left(3a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{3de^3}-\frac{2a\left(a^2+6b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin}{de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b*(3*a^2 + 4*b^2)*(e*Cos[c + d*x])^(3/2))/(3*d*e^3) - (2*a*(a^2 + 6*b^2)
*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]])
 + (2*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(d*e^3) + (2*(b + a*
Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{de\sqrt{e\cos(c+dx)}} - \frac{2\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))\left(\frac{a^2}{2}\right.}{e^2}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{de\sqrt{e\cos(c+dx)}} -$$

$$= \frac{2b\left(3a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{3de^3} + \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3} + \frac{2(b+a}{}$$

$$= \frac{2b\left(3a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{3de^3} + \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3} + \frac{2(b+a}{}$$

$$= \frac{2b\left(3a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{3de^3} - \frac{2a\left(a^2+6b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}} + \frac{2}{}$$

Mathematica [A] time = 0.41, size = 98, normalized size = 0.61

$$\frac{-6a\left(a^2+6b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+6\left(a\left(a^2+3b^2\right)\sin(c+dx)+3a^2b+b^3\right)+2b^3\cos^2(c+dx)}{3de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b^3*Cos[c + d*x]^2 - 6*a*(a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 6*(3*a^2*b + b^3 + a*(a^2 + 3*b^2)*Sin[c + d*x]))/(3*d*e*Sqrt
[e*Cos[c + d*x]])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,ab^2\cos\left(dx+c\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^2\cos\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^3}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(3/2), x)
```

maple [A] time = 2.69, size = 248, normalized size = 1.55

$$-\frac{2\left(-4b^3\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^3+18\sqrt{\frac{1}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-4*b^3*sin(1
/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+18*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a*b^2-6*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*a*b^2*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+4*b^3*sin(1/2*d*x+1/2*c)^3-9*a^2*b*sin(1/2*d*x+1/
2*c)-4*b^3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^3}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.561 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$\frac{2b\left(a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2a\left(a^2-6b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx)}{3de^3}$$

```
[Out] 2/3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^2/d/e/(e*cos(d*x+c))^(3/2)+2/3*a*(a^2
-6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)+2/3*b*(a^2+4
*b^2)*(e*cos(d*x+c))^(1/2)/d/e^3+2/3*a*b*(a+b*sin(d*x+c))*(e*cos(d*x+c))^(1
/2)/d/e^3
```

Rubi [A] time = 0.25, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2691, 2862, 2669, 2642, 2641}

$$\frac{2b\left(a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2a\left(a^2-6b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx)}{3de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*b*(a^2 + 4*b^2)*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*a*(a^2 - 6*b^2)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) +
 (2*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))/(3*d*e^3) + (2*(b + a*Si
n[c + d*x])*(a + b*Sin[c + d*x])^2)/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{(a+b\sin(c+dx))\left(-\frac{a^2}{2}+2b^2+\frac{3}{2}ab\sin(c+dx)\right)}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3de(e\cos(c+dx))^{3/2}} -$$

$$= \frac{2b\left(a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3} + \frac{2(b+a\sin}{}$$

$$= \frac{2b\left(a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3} + \frac{2(b+a\sin}{}$$

$$= \frac{2b\left(a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2a\left(a^2-6b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{2a}{}$$

Mathematica [A] time = 0.71, size = 103, normalized size = 0.63

$$\frac{2a^3\sin(c+dx)+2a\left(a^2-6b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+6a^2b+6ab^2\sin(c+dx)+3b^3\cos(2(c+dx))+}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*a^2*b + 5*b^3 + 3*b^3*Cos[2*(c + d*x)] + 2*a*(a^2 - 6*b^2)*Cos[c + d*x]^
(3/2)*EllipticF[(c + d*x)/2, 2] + 2*a^3*Sin[c + d*x] + 6*a*b^2*Sin[c + d*x]
)/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,ab^2\cos\left(dx+c\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^3\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^3}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(5/2), x)
```

maple [B] time = 2.60, size = 384, normalized size = 2.34

$$-\frac{2\left(2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,a^3\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-12*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+12*b^3*sin(1/2*d*x+1/2*c)^
5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*a^3+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+2*a^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2-12*b^3*sin(1/2*d*x+1/2*c)^3+3*a^2*b*sin(1/2*d*x+1/2*c)+4*b^3*sin(1/
2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^3}{(e\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.562 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2b\left(3a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{5de^5}-\frac{6a\left(a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}-\frac{2\left(ab-\left(3a^2-4b^2\right)\sin(c+d\right.}{5de^3\sqrt{e\cos(c}}$$

```
[Out] 2/5*b*(3*a^2-4*b^2)*(e*cos(d*x+c))^(3/2)/d/e^5+2/5*(b+a*sin(d*x+c))*(a+b*si
n(d*x+c))^2/d/e/(e*cos(d*x+c))^(5/2)-2/5*(a+b*sin(d*x+c))*(a*b-(3*a^2-4*b^2
)*sin(d*x+c))/d/e^3/(e*cos(d*x+c))^(1/2)-6/5*a*(a^2-2*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*co
s(d*x+c))^(1/2)/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2691, 2861, 2669, 2640, 2639}

$$\frac{2b\left(3a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{5de^5}-\frac{2\left(ab-\left(3a^2-4b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx))}{5de^3\sqrt{e\cos(c+dx)}}-\frac{6a\left(a^2-2b^2\right)E\left(\frac{1}{2}(c+\right.}{5de^4\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*b*(3*a^2 - 4*b^2)*(e*Cos[c + d*x])^(3/2))/(5*d*e^5) - (6*a*(a^2 - 2*b^2)
*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]
]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(5*d*e*(e*Cos[c + d*x]
)^(5/2)) - (2*(a + b*Sin[c + d*x])*(a*b - (3*a^2 - 4*b^2)*Sin[c + d*x]))/(5
*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] && !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{(a+b\sin(c+dx))\left(-\frac{3a^2}{2}+2b^2+\frac{1}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2(a+b\sin(c+dx))\left(ab-\left(3a^2-4b^2\right)\sin(\right.}{5de^3\sqrt{e\cos(c+dx)}}$$

$$= \frac{2b\left(3a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2(a+b}{}$$

$$= \frac{2b\left(3a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2(a+b}{}$$

$$= \frac{2b\left(3a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} - \frac{6a\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{2}{}$$

Mathematica [A] time = 0.74, size = 126, normalized size = 0.67

$$\frac{2\left(3a^3\sin(c+dx)+b\left(3a^2+b^2\right)\sec^2(c+dx)-3a\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(a^2+3b^2\right)\tan(c\right.}{5de^3\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-5*b^3 - 3*a*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
 + b*(3*a^2 + b^2)*Sec[c + d*x]^2 + 3*a^3*Sin[c + d*x] - 6*a*b^2*Sin[c + d*
x] + a*(a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x]))/(5*d*e^3*Sqrt[e*Cos[c + d*
x]])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,ab^2\cos\left(dx+c\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^4\cos\left(dx+c\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^4*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^3}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(7/2), x)
```

maple [B] time = 5.15, size = 618, normalized size = 3.30

$$-\frac{2\left(12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,a^{3}\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\,\mathrm{EllipticE}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin
(1/2*d*x+1/2*c)^4-24*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^4-24
*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+48*a*b^2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2+24*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+24*a^3*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4-48*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20
*b^3*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-6*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a*b^2-8*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*a*b^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-20*b^3*sin(1/2*d*x+1/2*c)^3-3*a^2*b*sin(1
/2*d*x+1/2*c)+4*b^3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^{3}}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^{3}}{(e\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.563 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=188

$$\frac{2b(5a^2-4b^2)\sqrt{e\cos(c+dx)}}{21de^5}+\frac{2a(5a^2-6b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}+\frac{2\left((5a^2-4b^2)\sin(c+dx)+ab\right.}{21de^3(e\cos(c+d}$$

```
[Out] 2/7*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^2/d/e/(e*cos(d*x+c))^(7/2)+2/21*(a+b*
sin(d*x+c))*(a*b+(5*a^2-4*b^2)*sin(d*x+c))/d/e^3/(e*cos(d*x+c))^(3/2)+2/21*
a*(5*a^2-6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^4/(e*cos(d*x+c))^(1/2)+2/21
*b*(5*a^2-4*b^2)*(e*cos(d*x+c))^(1/2)/d/e^5
```

Rubi [A] time = 0.27, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2691, 2861, 2669, 2642, 2641}

$$\frac{2b(5a^2-4b^2)\sqrt{e\cos(c+dx)}}{21de^5}+\frac{2\left((5a^2-4b^2)\sin(c+dx)+ab\right)(a+b\sin(c+dx))}{21de^3(e\cos(c+dx))^{3/2}}+\frac{2a(5a^2-6b^2)\sqrt{\cos(c+}}{21de^4\sqrt{e\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*b*(5*a^2 - 4*b^2)*Sqrt[e*Cos[c + d*x]])/(21*d*e^5) + (2*a*(5*a^2 - 6*b^2
)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[c + d*
x]]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(7*d*e*(e*Cos[c + d*
x])^(7/2)) + (2*(a + b*Sin[c + d*x])*(a*b + (5*a^2 - 4*b^2)*Sin[c + d*x]))/
(21*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} - \frac{2\int \frac{(a+b\sin(c+dx))\left(-\frac{5a^2}{2}+2b^2-\frac{1}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{5/2}}\,dx}{7e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} + \frac{2(a+b\sin(c+dx))\left(ab+\left(5a^2-4b^2\right)\sin\right.}{21de^3(e\cos(c+dx))^{3/2}}$$

$$= \frac{2b\left(5a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} + \frac{2(a+b\,\mathrm{s}}{}$$

$$= \frac{2b\left(5a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} + \frac{2(a+b\,\mathrm{s}}{}$$

$$= \frac{2b\left(5a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2a\left(5a^2-6b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}} + \frac{2(b}{}$$

Mathematica [A] time = 0.65, size = 140, normalized size = 0.74

$$\frac{\sec^4(c+dx)\sqrt{e\cos(c+dx)}\left(17a^3\sin(c+dx)+5a^3\sin(3(c+dx))+4a\left(5a^2-6b^2\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{42de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*(36*a^2*b - 2*b^3 - 14*b^3*Cos[2*(c +
d*x)] + 4*a*(5*a^2 - 6*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] +
17*a^3*Sin[c + d*x] + 30*a*b^2*Sin[c + d*x] + 5*a^3*Sin[3*(c + d*x)] - 6*a*
b^2*Sin[3*(c + d*x)]))/(42*d*e^5)
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(3\,ab^2\cos\left(dx+c\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^5\cos\left(dx+c\right)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^5*cos(d*x + c)^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(9/2), x)
```

maple [B] time = 5.73, size = 750, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(40*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^6-48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^
2*sin(1/2*d*x+1/2*c)^6-60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^4
+72*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^4+40*a^3*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^6-48*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
6+30*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-36*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2-40*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4+48*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-28*b^3*sin(1/2*d*x+
1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+16
*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*a*b^2*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2+28*b^3*sin(1/2*d*x+1/2*c)^3+9*a^2*b*sin(1/2*d*x+1/2*c)-4*
b^3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^3}{(e\,\cos(c+d\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

3.564 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=305

$$-\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de}-\frac{2b\left(93a^2+26b^2\right)(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{715de}+\frac{2e^4\left(55a^4+60\right.}{}$$

```
[Out] -34/6435*a*b*(53*a^2+38*b^2)*(e*cos(d*x+c))^(9/2)/d/e+2/385*(55*a^4+60*a^2*
b^2+4*b^4)*e*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/d-2/715*b*(93*a^2+26*b^2)*(e*c
os(d*x+c))^(9/2)*(a+b*sin(d*x+c))/d/e-14/65*a*b*(e*cos(d*x+c))^(9/2)*(a+b*s
in(d*x+c))^2/d/e-2/15*b*(e*cos(d*x+c))^(9/2)*(a+b*sin(d*x+c))^3/d/e+2/231*(
55*a^4+60*a^2*b^2+4*b^4)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(
1/2)+2/231*(55*a^4+60*a^2*b^2+4*b^4)*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.55, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2692, 2862, 2669, 2635, 2642, 2641}

$$\frac{2e^3\left(60a^2b^2+55a^4+4b^4\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{2e^4\left(60a^2b^2+55a^4+4b^4\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|2}{231d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-34*a*b*(53*a^2 + 38*b^2)*(e*Cos[c + d*x])^(9/2))/(6435*d*e) + (2*(55*a^4
+ 60*a^2*b^2 + 4*b^4)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(23
1*d*Sqrt[e*Cos[c + d*x]]) + (2*(55*a^4 + 60*a^2*b^2 + 4*b^4)*e^3*Sqrt[e*Cos
[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(55*a^4 + 60*a^2*b^2 + 4*b^4)*e*(e*Co
s[c + d*x])^(5/2)*Sin[c + d*x])/(385*d) - (2*b*(93*a^2 + 26*b^2)*(e*Cos[c +
 d*x])^(9/2)*(a + b*Sin[c + d*x]))/(715*d*e) - (14*a*b*(e*Cos[c + d*x])^(9/
2)*(a + b*Sin[c + d*x])^2)/(65*d*e) - (2*b*(e*Cos[c + d*x])^(9/2)*(a + b*Si
n[c + d*x])^3)/(15*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^3}{15de}+\frac{2}{15}\int (e\cos(c+dx))^{7/2}(a$$

$$= -\frac{14ab(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^2}{65de}-\frac{2b(e\cos(c+dx))^{9/2}(a}{15de}$$

$$= -\frac{2b\left(93a^2+26b^2\right)(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{715de}-\frac{14ab(e\cos}{}$$

$$= -\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de}-\frac{2b\left(93a^2+26b^2\right)(e\cos(c+}{715}$$

$$= -\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de}+\frac{2\left(55a^4+60a^2b^2+4b^4\right)e(}{3}$$

$$= -\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de}+\frac{2\left(55a^4+60a^2b^2+4b^4\right)e^3}{23}$$

$$= -\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de}+\frac{2\left(55a^4+60a^2b^2+4b^4\right)e^3}{23}$$

$$= -\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de}+\frac{2\left(55a^4+60a^2b^2+4b^4\right)e^4}{231d\sqrt{e}}$$

Mathematica [A] time = 4.81, size = 251, normalized size = 0.82

$$\frac{(e\cos(c+dx))^{7/2}\left(-154ab\left(26a^2+11b^2\right)\sqrt{\cos(c+dx)}+104\left(55a^4+60a^2b^2+4b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{1}{120}\sqrt{\cos(}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*(-154*a*b*(26*a^2 + 11*b^2)*Sqrt[Cos[c + d*x]] + 10
4*(55*a^4 + 60*a^2*b^2 + 4*b^4)*EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d
*x]]*(156*(5720*a^4 + 2460*a^2*b^2 + 87*b^4)*Sin[c + d*x] + 462*b^3*Cos[6*(
c + d*x)]*(60*a + 13*b*Sin[c + d*x]) - 28*b*Cos[4*(c + d*x)]*(220*a*(26*a^2
 - b^2) + 39*b*(180*a^2 + b^2)*Sin[c + d*x]) + Cos[2*(c + d*x)]*(-3080*(208
*a^3*b + 73*a*b^3) + 78*(2640*a^4 - 7200*a^2*b^2 - 557*b^4)*Sin[c + d*x]))
/120))/(12012*d*Cos[c + d*x]^(7/2))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

integral $\left(\left(b^4 e^3 \cos(dx+c)^7 - 2\left(3a^2b^2+b^4\right)e^3\cos(dx+c)^5 + \left(a^4+6a^2b^2+b^4\right)e^3\cos(dx+c)^3 - 4\left(ab^3e^3 \cos\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*e^3*cos(d*x + c)^7 - 2*(3*a^2*b^2 + b^4)*e^3*cos(d*x + c)^5 +
 (a^4 + 6*a^2*b^2 + b^4)*e^3*cos(d*x + c)^3 - 4*(a*b^3*e^3*cos(d*x + c)^5 -
 (a^3*b + a*b^3)*e^3*cos(d*x + c)^3)*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}} (b\sin(dx+c)+a)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^4, x)
```

maple [B] time = 3.41, size = 863, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x)
```

```
[Out] -2/45045/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(373900
8*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-2620800*b^4*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^10+102960*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8
+946608*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-154440*a^4*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^6-144456*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6+120120*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-34320*a^4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2+780*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2-2690688*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+768768*b^4*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^16+8673280*a*b^3*sin(1/2*d*x+1/2*c)^11-620928
0*a*b^3*sin(1/2*d*x+1/2*c)^13+1774080*a*b^3*sin(1/2*d*x+1/2*c)^15-640640*a^
3*b*sin(1/2*d*x+1/2*c)^11+1601600*a^3*b*sin(1/2*d*x+1/2*c)^9-6160000*a*b^3*
sin(1/2*d*x+1/2*c)^9-1601600*a^3*b*sin(1/2*d*x+1/2*c)^7+2279200*a*b^3*sin(1
/2*d*x+1/2*c)^7+800800*a^3*b*sin(1/2*d*x+1/2*c)^5-363440*a*b^3*sin(1/2*d*x+
1/2*c)^5-200200*a^3*b*sin(1/2*d*x+1/2*c)^3-6160*a*b^3*sin(1/2*d*x+1/2*c)^3+
20020*a^3*b*sin(1/2*d*x+1/2*c)+6160*a*b^3*sin(1/2*d*x+1/2*c)+11700*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*a^2*b^2-1572480*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^12+3931200*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-3818880*a^2
*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+1797120*a^2*b^2*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^6-360360*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4+11700*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+10725*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*a^4+780*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos(c+d\,x))^{7/2}\,(a+b\,\sin(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.565 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=258

$$-\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de}-\frac{2b\left(73a^2+22b^2\right)(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{429de}+\frac{2e^2\left(39a^4+5\right.}{}$$

```
[Out] -10/3003*a*b*(115*a^2+94*b^2)*(e*cos(d*x+c))^(7/2)/d/e+2/195*(39*a^4+52*a^2
*b^2+4*b^4)*e*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/d-2/429*b*(73*a^2+22*b^2)*(e*
cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))/d/e-38/143*a*b*(e*cos(d*x+c))^(7/2)*(a+b
*sin(d*x+c))^2/d/e-2/13*b*(e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3/d/e+2/65*
(39*a^4+52*a^2*b^2+4*b^4)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^
(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2692, 2862, 2669, 2635, 2640, 2639}

$$\frac{2e^2\left(52a^2b^2+39a^4+4b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{65d\sqrt{\cos(c+dx)}}-\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de}-\frac{2b\left(73a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-10*a*b*(115*a^2 + 94*b^2)*(e*Cos[c + d*x])^(7/2))/(3003*d*e) + (2*(39*a^4
 + 52*a^2*b^2 + 4*b^4)*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/
(65*d*Sqrt[Cos[c + d*x]]) + (2*(39*a^4 + 52*a^2*b^2 + 4*b^4)*e*(e*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(195*d) - (2*b*(73*a^2 + 22*b^2)*(e*Cos[c + d*x])^
(7/2)*(a + b*Sin[c + d*x]))/(429*d*e) - (38*a*b*(e*Cos[c + d*x])^(7/2)*(a +
 b*Sin[c + d*x])^2)/(143*d*e) - (2*b*(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c +
d*x])^3)/(13*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3}{13de} + \frac{2}{13}\int (e\cos(c+dx))^{5/2}(a$$

$$= -\frac{38ab(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}{143de} - \frac{2b(e\cos(c+dx))^{7/2}(a}{13de}$$

$$= -\frac{2b\left(73a^2+22b^2\right)(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{429de} - \frac{38ab(e\cos}{}$$

$$= -\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de} - \frac{2b\left(73a^2+22b^2\right)(e\cos(c}{42}$$

$$= -\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de} + \frac{2\left(39a^4+52a^2b^2+4b^4\right)e}{1}$$

$$= -\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de} + \frac{2\left(39a^4+52a^2b^2+4b^4\right)e}{1}$$

$$= -\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de} + \frac{2\left(39a^4+52a^2b^2+4b^4\right)e}{65d\sqrt{}}$$

Mathematica [A] time = 2.15, size = 209, normalized size = 0.81

$$(e\cos(c+dx))^{5/2}\left(2\left(39a^4+52a^2b^2+4b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+65\sqrt{\cos(c+dx)}\left(-\frac{1}{78}b^2\left(13a^2+b^2\right)\sin(4(c+dx))\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(2*(39*a^4 + 52*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x
)/2, 2] + 65*Sqrt[Cos[c + d*x]]*(-1/77*(a*b*(66*a^2 + 31*b^2)*Cos[c + d*x])
 - (a*b*(44*a^2 + 9*b^2)*Cos[3*(c + d*x)])/154 + (a*b^3*Cos[5*(c + d*x)])/2
2 + ((624*a^4 - 208*a^2*b^2 - 61*b^4)*Sin[2*(c + d*x)])/3120 - (b^2*(13*a^2
 + b^2)*Sin[4*(c + d*x)])/78 + (b^4*Sin[6*(c + d*x)])/208)))/(65*d*Cos[c +
d*x]^(5/2))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4e^2\cos\left(dx+c\right)^6-2\left(3\,a^2b^2+b^4\right)e^2\cos\left(dx+c\right)^4+\left(a^4+6\,a^2b^2+b^4\right)e^2\cos\left(dx+c\right)^2-4\left(ab^3e^2\text{co}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*e^2*cos(d*x + c)^6 - 2*(3*a^2*b^2 + b^4)*e^2*cos(d*x + c)^4 +
 (a^4 + 6*a^2*b^2 + b^4)*e^2*cos(d*x + c)^2 - 4*(a*b^3*e^2*cos(d*x + c)^4 -
 (a^3*b + a*b^3)*e^2*cos(d*x + c)^2)*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^4, x)
```

maple [B] time = 3.00, size = 776, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x)
```

```
[Out] 2/15015/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(-443520
*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+12012*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^2*b^2+9009*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+924*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^
4+492800*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-246400*b^4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^8+24024*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6+48664*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-24024*a^4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6006*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2-924*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+147840*b^4*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^14-1048320*a*b^3*sin(1/2*d*x+1/2*c)^11+349440*a*b
^3*sin(1/2*d*x+1/2*c)^13-137280*a^3*b*sin(1/2*d*x+1/2*c)^9+1173120*a*b^3*si
n(1/2*d*x+1/2*c)^9+274560*a^3*b*sin(1/2*d*x+1/2*c)^7-599040*a*b^3*sin(1/2*d
*x+1/2*c)^7-205920*a^3*b*sin(1/2*d*x+1/2*c)^5+121680*a*b^3*sin(1/2*d*x+1/2*
c)^5+68640*a^3*b*sin(1/2*d*x+1/2*c)^3+3120*a*b^3*sin(1/2*d*x+1/2*c)^3-8580*
a^3*b*sin(1/2*d*x+1/2*c)-3120*a*b^3*sin(1/2*d*x+1/2*c)+616*b^4*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4-320320*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^10+640640*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-448448*a^2*
b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+128128*a^2*b^2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4-12012*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cos(c+d\,x))^{5/2}\,(a+b\sin(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**4,x)
[Out] Timed out
```

3.566 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=258

$$-\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de}-\frac{2b\left(167a^2+54b^2\right)(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{693de}+\frac{2e^2\left(77a^4+1\right.}{}$$

```
[Out] -26/3465*a*b*(79*a^2+74*b^2)*(e*cos(d*x+c))^(5/2)/d/e-2/693*b*(167*a^2+54*b
^2)*(e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))/d/e-34/99*a*b*(e*cos(d*x+c))^(5/2
)*(a+b*sin(d*x+c))^2/d/e-2/11*b*(e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3/d/e
+2/231*(77*a^4+132*a^2*b^2+12*b^4)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos
(d*x+c))^(1/2)+2/231*(77*a^4+132*a^2*b^2+12*b^4)*e*sin(d*x+c)*(e*cos(d*x+c)
)^(1/2)/d
```

Rubi [A] time = 0.51, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2692, 2862, 2669, 2635, 2642, 2641}

$$\frac{2e^2\left(132a^2b^2+77a^4+12b^4\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de}-\frac{2b\left(167a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-26*a*b*(79*a^2 + 74*b^2)*(e*Cos[c + d*x])^(5/2))/(3465*d*e) + (2*(77*a^4
+ 132*a^2*b^2 + 12*b^4)*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(
231*d*Sqrt[e*Cos[c + d*x]]) + (2*(77*a^4 + 132*a^2*b^2 + 12*b^4)*e*Sqrt[e*C
os[c + d*x]]*Sin[c + d*x])/(231*d) - (2*b*(167*a^2 + 54*b^2)*(e*Cos[c + d*x
])^(5/2)*(a + b*Sin[c + d*x]))/(693*d*e) - (34*a*b*(e*Cos[c + d*x])^(5/2)*(
a + b*Sin[c + d*x])^2)/(99*d*e) - (2*b*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c
+ d*x])^3)/(11*d*e)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3}{11de}+\frac{2}{11}\int (e\cos(c+dx))^{3/2}(a$$

$$= -\frac{34ab(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}{99de}-\frac{2b(e\cos(c+dx))^{5/2}(a}{11de}$$

$$= -\frac{2b\left(167a^2+54b^2\right)(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{693de}-\frac{34ab(e\,\mathrm{co}}{}$$

$$= -\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de}-\frac{2b\left(167a^2+54b^2\right)(e\cos(c}{69}$$

$$= -\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de}+\frac{2\left(77a^4+132a^2b^2+12b^4\right)}{2}$$

$$= -\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de}+\frac{2\left(77a^4+132a^2b^2+12b^4\right)}{2}$$

$$= -\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de}+\frac{2\left(77a^4+132a^2b^2+12b^4\right)}{231d\sqrt{}}$$

Mathematica [A] time = 2.81, size = 189, normalized size = 0.73

$$\frac{(e\cos(c+dx))^{3/2}\left(240\left(77a^4+132a^2b^2+12b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\left(-2464\left(9a^3b+4ab^3\right)\cos(2(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(240*(77*a^4 + 132*a^2*b^2 + 12*b^4)*EllipticF[(c +
 d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(-1848*b*(12*a^3 + 7*a*b^2) - 2464*(9*a^3*
b + 4*a*b^3)*Cos[2*(c + d*x)] + 3080*a*b^3*Cos[4*(c + d*x)] + 30*(616*a^4 +
 660*a^2*b^2 + 39*b^4)*Sin[c + d*x] - 45*b*(264*a^2*b + 31*b^3)*Sin[3*(c +
d*x)] + 315*b^4*Sin[5*(c + d*x)])))/(27720*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4 e\cos\left(dx+c\right)^5 - 2\left(3\,a^2b^2+b^4\right)e\cos\left(dx+c\right)^3 + \left(a^4+6\,a^2b^2+b^4\right)e\cos\left(dx+c\right) - 4\left(ab^3 e\cos\left(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*e*cos(d*x + c)^5 - 2*(3*a^2*b^2 + b^4)*e*cos(d*x + c)^3 + (a^
4 + 6*a^2*b^2 + b^4)*e*cos(d*x + c) - 4*(a*b^3*e*cos(d*x + c)^3 - (a^3*b +
a*b^3)*e*cos(d*x + c))*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^4, x)
```

maple [B] time = 2.76, size = 639, normalized size = 2.48

$$-\frac{2e^2\left(20160b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+49280a\,b^3\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-50400b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x)
```

```
[Out] -2/3465/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(20160*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+49280*a*b^3*sin(1/2*d*x+1/2*c)^
11-50400*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-47520*a^2*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-123200*a*b^3*sin(1/2*d*x+1/2*c)^9+41040*b^
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-22176*a^3*b*sin(1/2*d*x+1/2*c)^7+
71280*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+101024*a*b^3*sin(1/2*
d*x+1/2*c)^7-11160*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4620*a^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+33264*a^3*b*sin(1/2*d*x+1/2*c)^5-27720
*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-28336*a*b^3*sin(1/2*d*x+1/
2*c)^5+1155*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+1980*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*
b^2+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-2310*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2-16632*a^3*b*sin(1/2*d*x+1/2*c)^3+1980*a^2*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2-1232*a*b^3*sin(1/2*d*x+1/2*c)^3+180*b^4*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2+2772*a^3*b*sin(1/2*d*x+1/2*c)+1232*a*b^3*sin(1/2
*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**4,x)
[Out] Timed out
```

3.567 $\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=210

$$-\frac{22ab\left(17a^2+18b^2\right)(e\cos(c+dx))^{3/2}}{315de}-\frac{2b\left(41a^2+14b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{105de}+\frac{2\left(15a^4+36a^2\right.}{}$$

```
[Out] -22/315*a*b*(17*a^2+18*b^2)*(e*cos(d*x+c))^(3/2)/d/e-2/105*b*(41*a^2+14*b^2
)*(e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))/d/e-10/21*a*b*(e*cos(d*x+c))^(3/2)*
(a+b*sin(d*x+c))^2/d/e-2/9*b*(e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3/d/e+2/
15*(15*a^4+36*a^2*b^2+4*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(
1/2)
```

Rubi [A] time = 0.44, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2692, 2862, 2669, 2640, 2639}

$$-\frac{22ab\left(17a^2+18b^2\right)(e\cos(c+dx))^{3/2}}{315de}-\frac{2b\left(41a^2+14b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{105de}+\frac{2\left(36a^2b^2+15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-22*a*b*(17*a^2 + 18*b^2)*(e*Cos[c + d*x])^(3/2))/(315*d*e) + (2*(15*a^4 +
 36*a^2*b^2 + 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*
Sqrt[Cos[c + d*x]]) - (2*b*(41*a^2 + 14*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*
Sin[c + d*x]))/(105*d*e) - (10*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*
x])^2)/(21*d*e) - (2*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3)/(9*d*
e)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3}{9de}+\frac{2}{9}\int \sqrt{e\cos(c+dx)}\,(a+b$$

$$=-\frac{10ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{21de}-\frac{2b(e\cos(c+dx))^{3/2}(a+}{9de}$$

$$=-\frac{2b\left(41a^2+14b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{105de}-\frac{10ab(e\cos(c}{}$$

$$=-\frac{22ab\left(17a^2+18b^2\right)(e\cos(c+dx))^{3/2}}{315de}-\frac{2b\left(41a^2+14b^2\right)(e\cos(c+}{105d}$$

$$=-\frac{22ab\left(17a^2+18b^2\right)(e\cos(c+dx))^{3/2}}{315de}-\frac{2b\left(41a^2+14b^2\right)(e\cos(c+}{105d}$$

$$=-\frac{22ab\left(17a^2+18b^2\right)(e\cos(c+dx))^{3/2}}{315de}+\frac{2\left(15a^4+36a^2b^2+4b^4\right)\sqrt{e}}{15d\sqrt{\cos}}$$

Mathematica [A] time = 1.10, size = 137, normalized size = 0.65

$$\frac{\sqrt{e\cos(c+dx)}\left(84\left(15a^4+36a^2b^2+4b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)-b\cos^{\frac{3}{2}}(c+dx)\left(5\left(336a^3+264ab^2-7b^3\sin(3(c+\right.\right.\right.}{630d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(84*(15*a^4 + 36*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x)
/2, 2] - b*Cos[c + d*x]^(3/2)*(-360*a*b^2*Cos[2*(c + d*x)] + 21*b*(72*a^2 +
 13*b^2)*Sin[c + d*x] + 5*(336*a^3 + 264*a*b^2 - 7*b^3*Sin[3*(c + d*x)]))))
/(630*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4\cos\left(dx+c\right)^4+a^4+6\,a^2b^2+b^4-2\left(3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2-4\left(ab^3\cos\left(dx+c\right)^2-a^3b-ab^3\right)\text{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^4, x)
```

maple [B] time = 2.59, size = 525, normalized size = 2.50

$$2e\left(1120b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2880a\,b^3\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2240b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(1120*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+2880*a*b^3*sin(1/2*d*x+1/2*c)^9-2240
*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-3024*a^2*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^6-5760*a*b^3*sin(1/2*d*x+1/2*c)^7+1064*b^4*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^6-1680*a^3*b*sin(1/2*d*x+1/2*c)^5+3024*a^2*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+2640*a*b^3*sin(1/2*d*x+1/2*c)^5+56*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+315*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4
+756*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+84*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+1680*
a^3*b*sin(1/2*d*x+1/2*c)^3-756*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2+240*a*b^3*sin(1/2*d*x+1/2*c)^3-84*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2-420*a^3*b*sin(1/2*d*x+1/2*c)-240*a*b^3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.568 $\int \frac{(a+b\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=210

$$-\frac{6ab\left(31a^2+34b^2\right)\sqrt{e\cos(c+dx)}}{35de}-\frac{2b\left(29a^2+10b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{35de}+\frac{2\left(7a^4+28a^2b^2+4b^4\right)}{7d\sqrt{e}}$$

```
[Out] 2/7*(7*a^4+28*a^2*b^2+4*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(
1/2)-6/35*a*b*(31*a^2+34*b^2)*(e*cos(d*x+c))^(1/2)/d/e-2/35*b*(29*a^2+10*b^
2)*(a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/d/e-26/35*a*b*(a+b*sin(d*x+c))^2*(
e*cos(d*x+c))^(1/2)/d/e-2/7*b*(a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A] time = 0.45, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2692, 2862, 2669, 2642, 2641}

$$-\frac{6ab\left(31a^2+34b^2\right)\sqrt{e\cos(c+dx)}}{35de}-\frac{2b\left(29a^2+10b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{35de}+\frac{2\left(28a^2b^2+7a^4+4b^4\right)}{7d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-6*a*b*(31*a^2 + 34*b^2)*Sqrt[e*Cos[c + d*x]])/(35*d*e) + (2*(7*a^4 + 28*a
^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*d*Sqrt[e*C
os[c + d*x]]) - (2*b*(29*a^2 + 10*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c +
d*x]))/(35*d*e) - (26*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2)/(35*
d*e) - (2*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3)/(7*d*e)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}{7de} + \frac{2}{7}\int \frac{(a+b\sin(c+dx))^2\left(\frac{7a^2}{2}+3b^2+\frac{1}{2}\right.}{\sqrt{e\cos(c+dx)}}$$

$$= -\frac{26ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}{35de} - \frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}{7de}$$

$$= -\frac{2b\left(29a^2+10b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{35de} - \frac{26ab\sqrt{e\cos(c+dx)}\,(a+b}{35de}$$

$$= -\frac{6ab\left(31a^2+34b^2\right)\sqrt{e\cos(c+dx)}}{35de} - \frac{2b\left(29a^2+10b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(}{35de}$$

$$= -\frac{6ab\left(31a^2+34b^2\right)\sqrt{e\cos(c+dx)}}{35de} - \frac{2b\left(29a^2+10b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(}{35de}$$

$$= -\frac{6ab\left(31a^2+34b^2\right)\sqrt{e\cos(c+dx)}}{35de} + \frac{2\left(7a^4+28a^2b^2+4b^4\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{7d\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 1.13, size = 130, normalized size = 0.62

$$\frac{20\left(7a^4+28a^2b^2+4b^4\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right) - b\cos(c+dx)\left(560a^3+5b\left(56a^2+11b^2\right)\sin(c+dx)\right. -}{70d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (20*(7*a^4 + 28*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2] - b*Cos[c + d*x]*(560*a^3 + 504*a*b^2 - 56*a*b^2*Cos[2*(c + d*x)] + 5*b*
(56*a^2 + 11*b^2)*Sin[c + d*x] - 5*b^3*Sin[3*(c + d*x)]))/(70*d*Sqrt[e*Cos[
c + d*x]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\cos\left(dx+c\right)^4+a^4+6\,a^2b^2+b^4-2\left(3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2-4\left(ab^3\cos\left(dx+c\right)^2-a^3b-ab^3\right.\right.}{e\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/sqrt(e*cos(d*x + c)), x)
```

maple [A] time = 2.02, size = 412, normalized size = 1.96

$$-\frac{2\left(80b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+224a\,b^3\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-280a^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(80*b^4*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+224*a*b^3*sin(1/2*d*x+1/2*c)^7-120*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-280*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^4-336*a*b^3*sin(1/2*d*x+1/2*c)^5+35*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
^4+140*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-280
*a^3*b*sin(1/2*d*x+1/2*c)^3+140*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2-112*a*b^3*sin(1/2*d*x+1/2*c)^3+20*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2+140*a^3*b*sin(1/2*d*x+1/2*c)+112*a*b^3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^4}{\sqrt{e\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.569 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=218

$$\frac{2ab\left(15a^2+62b^2\right)(e\cos(c+dx))^{3/2}}{15de^3}+\frac{2b\left(5a^2+6b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^3}-\frac{2\left(5a^4+60a^2b^2+1\right.}{5}$$

```
[Out] 2/15*a*b*(15*a^2+62*b^2)*(e*cos(d*x+c))^(3/2)/d/e^3+2/5*b*(5*a^2+6*b^2)*(e*
cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))/d/e^3+2*a*b*(e*cos(d*x+c))^(3/2)*(a+b*si
n(d*x+c))^2/d/e^3+2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^3/d/e/(e*cos(d*x+c))^
(1/2)-2/5*(5*a^4+60*a^2*b^2+12*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/c
os(d*x+c)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2691, 2862, 2669, 2640, 2639}

$$\frac{2ab\left(15a^2+62b^2\right)(e\cos(c+dx))^{3/2}}{15de^3}+\frac{2b\left(5a^2+6b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^3}-\frac{2\left(60a^2b^2+5a^4+1\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a*b*(15*a^2 + 62*b^2)*(e*Cos[c + d*x])^(3/2))/(15*d*e^3) - (2*(5*a^4 + 6
0*a^2*b^2 + 12*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^
2*Sqrt[Cos[c + d*x]]) + (2*b*(5*a^2 + 6*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*
Sin[c + d*x]))/(5*d*e^3) + (2*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x
])^2)/(d*e^3) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(d*e*Sqrt[e
*Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
```

```
|| IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{de\sqrt{e\cos(c+dx)}} - \frac{2\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2\left(\frac{a^2}{2}\right.}{e^2}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{de\sqrt{e\cos(c+dx)}} -$$

$$= \frac{2b\left(5a^2+6b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^3} + \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin}{de^3}$$

$$= \frac{2ab\left(15a^2+62b^2\right)(e\cos(c+dx))^{3/2}}{15de^3} + \frac{2b\left(5a^2+6b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+}{5de^3}$$

$$= \frac{2ab\left(15a^2+62b^2\right)(e\cos(c+dx))^{3/2}}{15de^3} + \frac{2b\left(5a^2+6b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+}{5de^3}$$

$$= \frac{2ab\left(15a^2+62b^2\right)(e\cos(c+dx))^{3/2}}{15de^3} - \frac{2\left(5a^4+60a^2b^2+12b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(\right.}{5de^2\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.60, size = 135, normalized size = 0.62

$$\frac{\frac{1}{2}\left(240a^3b+\left(60a^4+360a^2b^2+63b^4\right)\sin(c+dx)+40ab^3\cos(2(c+dx))+280ab^3+3b^4\sin(3(c+dx))\right)-6\left(5a^4\right.}{15de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-6*(5*a^4 + 60*a^2*b^2 + 12*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2] + (240*a^3*b + 280*a*b^3 + 40*a*b^3*Cos[2*(c + d*x)] + (60*a^4 + 360*a^
2*b^2 + 63*b^4)*Sin[c + d*x] + 3*b^4*Sin[3*(c + d*x)])/2)/(15*d*e*Sqrt[e*Co
s[c + d*x]])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^4\cos(dx+c)^4+a^4+6a^2b^2+b^4-2\left(3a^2b^2+b^4\right)\cos(dx+c)^2-4\left(ab^3\cos(dx+c)^2-a^3b-ab^3\right)\mathrm{si}\right.}{e^2\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(3/2), x)
```

maple [A] time = 3.02, size = 378, normalized size = 1.73

$$-\frac{2\left(-24b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-80a\,b^3\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\sqrt{\frac{1}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/15/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-24*b^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-80*a*b^3*sin(1/2*d*x+1/2*c)^5+24*b^4*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+180*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+36*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-30*a^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-180*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2+80*a*b^3*sin(1/2*d*x+1/2*c)^3-36*b^4*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2-60*a^3*b*sin(1/2*d*x+1/2*c)-80*a*b^3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.570 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=216

$$\frac{2ab\left(a^2+14b^2\right)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2b\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3}+\frac{2\left(a^4-12a^2b^2-4b^4\right)\sqrt{\cos}}{3de^2\sqrt{e\cos}}$$

```
[Out] 2/3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^3/d/e/(e*cos(d*x+c))^(3/2)+2/3*(a^4-1
2*a^2*b^2-4*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)+2/3
*a*b*(a^2+14*b^2)*(e*cos(d*x+c))^(1/2)/d/e^3+2/3*b*(a^2+2*b^2)*(a+b*sin(d*x
+c))*(e*cos(d*x+c))^(1/2)/d/e^3+2/3*a*b*(a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(
1/2)/d/e^3
```

Rubi [A] time = 0.45, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2691, 2862, 2669, 2642, 2641}

$$\frac{2ab\left(a^2+14b^2\right)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2b\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3}+\frac{2\left(-12a^2b^2+a^4-4b^4\right)\sqrt{c}}{3de^2\sqrt{e\,co}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a*b*(a^2 + 14*b^2)*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*(a^4 - 12*a^2*b^
2 - 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Co
s[c + d*x]]) + (2*b*(a^2 + 2*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])
)/(3*d*e^3) + (2*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2)/(3*d*e^3)
 + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(3*d*e*(e*Cos[c + d*x])^
(3/2))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
```

```
|| IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{3de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{a^2}{2}+3b^2+\frac{5}{2}ab\sin(c+dx)\right)}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{3de(e\cos(c+dx))^{3/2}} - $$

$$= \frac{2b\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3} + \frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+ }{3de^3}$$

$$= \frac{2ab\left(a^2+14b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2b\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3} + $$

$$= \frac{2ab\left(a^2+14b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2b\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3} + $$

$$= \frac{2ab\left(a^2+14b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2\left(a^4-12a^2b^2-4b^4\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|2}{3de^2\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 1.19, size = 137, normalized size = 0.63

$$\frac{4a^4\sin(c+dx)+16a^3b+24a^2b^2\sin(c+dx)+4\left(a^4-12a^2b^2-4b^4\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+24ab^3\cos(2}{6de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (16*a^3*b + 40*a*b^3 + 24*a*b^3*Cos[2*(c + d*x)] + 4*(a^4 - 12*a^2*b^2 - 4*
b^4)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 4*a^4*Sin[c + d*x] + 24
*a^2*b^2*Sin[c + d*x] + 5*b^4*Sin[c + d*x] + b^4*Sin[3*(c + d*x)])/(6*d*e*(
e*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\cos(dx+c)^4+a^4+6a^2b^2+b^4-2\left(3a^2b^2+b^4\right)\cos(dx+c)^2-4\left(ab^3\cos(dx+c)^2-a^3b-ab^3\right)\text{si}}{e^3\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(5/2), x)
```

maple [B] time = 2.66, size = 575, normalized size = 2.66

$$-\frac{2\left(8b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(8*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2-24*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2
*sin(1/2*d*x+1/2*c)^2-8*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+4
8*a*b^3*sin(1/2*d*x+1/2*c)^5-8*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*a^4+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+4*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*b^4+2*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+12*a^2*b
^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-48*a*b^3*sin(1/2*d*x+1/2*c)^3+4*
b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+4*a^3*b*sin(1/2*d*x+1/2*c)+16*a
*b^3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.571 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2ab\left(3a^2-10b^2\right)(e\cos(c+dx))^{3/2}}{5de^5}+\frac{6b\left(a^2-2b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^5}-\frac{6\left(ab-\left(a^2-2b^2\right)\sin(\right.}{5de^3\sqrt{e}}$$

```
[Out] 2/5*a*b*(3*a^2-10*b^2)*(e*cos(d*x+c))^(3/2)/d/e^5+6/5*b*(a^2-2*b^2)*(e*cos(
d*x+c))^(3/2)*(a+b*sin(d*x+c))/d/e^5+2/5*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^
3/d/e/(e*cos(d*x+c))^(5/2)-6/5*(a+b*sin(d*x+c))^2*(a*b-(a^2-2*b^2)*sin(d*x+
c))/d/e^3/(e*cos(d*x+c))^(1/2)-6/5*(a^4-4*a^2*b^2-4*b^4)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(
d*x+c))^(1/2)/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2691, 2861, 2862, 2669, 2640, 2639}

$$\frac{2ab\left(3a^2-10b^2\right)(e\cos(c+dx))^{3/2}}{5de^5}-\frac{6\left(ab-\left(a^2-2b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx))^2}{5de^3\sqrt{e\cos(c+dx)}}+\frac{6b\left(a^2-2b^2\right)(e\cos(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*a*b*(3*a^2 - 10*b^2)*(e*Cos[c + d*x])^(3/2))/(5*d*e^5) - (6*(a^4 - 4*a^2
*b^2 - 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt
[Cos[c + d*x]]) + (6*b*(a^2 - 2*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c +
d*x]))/(5*d*e^5) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(5*d*e*(
e*Cos[c + d*x])^(5/2)) - (6*(a + b*Sin[c + d*x])^2*(a*b - (a^2 - 2*b^2)*Sin
[c + d*x]))/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
```

```
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{5de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{3a^2}{2}+3b^2+\frac{3}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{5de(e\cos(c+dx))^{5/2}} - \frac{6(a+b\sin(c+dx))^2\left(ab-\left(a^2-2b^2\right)\sin(\right.}{5de^3\sqrt{e\cos(c+dx)}}$$

$$= \frac{6b\left(a^2-2b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c}{5de(e\cos(c+dx))^{5/2}}$$

$$= \frac{2ab\left(3a^2-10b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} + \frac{6b\left(a^2-2b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx}{5de^5}$$

$$= \frac{2ab\left(3a^2-10b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} + \frac{6b\left(a^2-2b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx}{5de^5}$$

$$= \frac{2ab\left(3a^2-10b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} - \frac{6\left(a^4-4a^2b^2-4b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{5de^4\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.63, size = 152, normalized size = 0.64

$$\frac{2\left(3a^4\sin(c+dx)-12a^2b^2\sin(c+dx)+4ab\left(a^2+b^2\right)\sec^2(c+dx)-3\left(a^4-4a^2b^2-4b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.\right.}{5de^3\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-20*a*b^3 - 3*(a^4 - 4*a^2*b^2 - 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 4*a*b*(a^2 + b^2)*Sec[c + d*x]^2 + 3*a^4*Sin[c + d*x] - 12*
```

```
a^2*b^2*Sin[c + d*x] - 7*b^4*Sin[c + d*x] + (a^4 + 6*a^2*b^2 + b^4)*Sec[c +
 d*x]*Tan[c + d*x]))/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\cos(dx+c)^4+a^4+6a^2b^2+b^4-2\left(3a^2b^2+b^4\right)\cos(dx+c)^2-4\left(ab^3\cos(dx+c)^2-a^3b-ab^3\right.\right.}{e^4\cos(dx+c)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e^4*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(7/2), x)
```

maple [B] time = 5.96, size = 874, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^4*sin
(1/2*d*x+1/2*c)^4-48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^4-
48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^4-24*a^4*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^6+96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6
+56*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*a^4*sin(1/2*d*x+1/2*c)^2+48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1
/2*c)^2+48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+24*a^4*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+80*a*b^3*sin(1/2*d*x+1/2*c)^5-56*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b
^2-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-8*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2+12*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-80*a*b^3*sin(1/2*d
*x+1/2*c)^3+12*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-4*a^3*b*sin(1/2*
d*x+1/2*c)+16*a*b^3*sin(1/2*d*x+1/2*c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(7/2),x)
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.572 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=241

$$\frac{10ab\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}}{21de^5}+\frac{2b\left(5a^2-6b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{21de^5}-\frac{2\left(ab-\left(5a^2-6b^2\right)\sin(c+\right.}{21de^3(e\cos(}$$

```
[Out] 2/7*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^3/d/e/(e*cos(d*x+c))^(7/2)-2/21*(a+b*
sin(d*x+c))^2*(a*b-(5*a^2-6*b^2)*sin(d*x+c))/d/e^3/(e*cos(d*x+c))^(3/2)+2/2
1*(5*a^4-12*a^2*b^2+12*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^4/(e*cos(d*x+c)
)^(1/2)+10/21*a*b*(a^2-2*b^2)*(e*cos(d*x+c))^(1/2)/d/e^5+2/21*b*(5*a^2-6*b^
2)*(a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/d/e^5
```

Rubi [A] time = 0.46, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2691, 2861, 2862, 2669, 2642, 2641}

$$\frac{10ab\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}}{21de^5}-\frac{2\left(ab-\left(5a^2-6b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx))^2}{21de^3(e\cos(c+dx))^{3/2}}+\frac{2b\left(5a^2-6b^2\right)\sqrt{e\cos(c}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (10*a*b*(a^2 - 2*b^2)*Sqrt[e*Cos[c + d*x]])/(21*d*e^5) + (2*(5*a^4 - 12*a^2
*b^2 + 12*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt
[e*Cos[c + d*x]]) + (2*b*(5*a^2 - 6*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c
+ d*x]))/(21*d*e^5) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(7*d*
e*(e*Cos[c + d*x])^(7/2)) - (2*(a + b*Sin[c + d*x])^2*(a*b - (5*a^2 - 6*b^2
)*Sin[c + d*x]))/(21*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
```

```
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{7de(e\cos(c+dx))^{7/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{5a^2}{2}+3b^2+\frac{1}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{5/2}}\,dx}{7e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{7de(e\cos(c+dx))^{7/2}} - \frac{2(a+b\sin(c+dx))^2\left(ab-\left(5a^2-6b^2\right)\sin\right.}{21de^3(e\cos(c+dx))^{3/2}}$$

$$= \frac{2b\left(5a^2-6b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{21de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c\right.}{7de(e\cos(c+dx))^{7/2}}$$

$$= \frac{10ab\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2b\left(5a^2-6b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{21de^5}$$

$$= \frac{10ab\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2b\left(5a^2-6b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{21de^5}$$

$$= \frac{10ab\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2\left(5a^4-12a^2b^2+12b^4\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{21de^4\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 0.93, size = 177, normalized size = 0.73

$$\frac{\sec^4(c+dx)\sqrt{e\cos(c+dx)}\left(17a^4\sin(c+dx)+5a^4\sin(3(c+dx))+48a^3b+60a^2b^2\sin(c+dx)-12a^2b^2\sin(3(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*(48*a^3*b - 8*a*b^3 - 56*a*b^3*Cos[2*(
c + d*x)] + 4*(5*a^4 - 12*a^2*b^2 + 12*b^4)*Cos[c + d*x]^(7/2)*EllipticF[(c
 + d*x)/2, 2] + 17*a^4*Sin[c + d*x] + 60*a^2*b^2*Sin[c + d*x] + 3*b^4*Sin[c
 + d*x] + 5*a^4*Sin[3*(c + d*x)] - 12*a^2*b^2*Sin[3*(c + d*x)] - 9*b^4*Sin[
3*(c + d*x)]))/(42*d*e^5)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\cos\left(dx+c\right)^4+a^4+6\,a^2b^2+b^4-2\left(3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2-4\left(ab^3\cos\left(dx+c\right)^2-a^3b-ab^3\right.\right.}{e^5\cos\left(dx+c\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e^5*cos(d*x + c)^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^4}{\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(9/2), x)
```

maple [B] time = 7.29, size = 1067, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(30*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2+72*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*
sin(1/2*d*x+1/2*c)^2+144*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c
)^4-96*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^6+40*a^4*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-72*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6-40*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+16*a^4*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-12*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-112*
a*b^3*sin(1/2*d*x+1/2*c)^5+112*a*b^3*sin(1/2*d*x+1/2*c)^3+12*a^3*b*sin(1/2*
d*x+1/2*c)-16*a*b^3*sin(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+
72*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-96*a^2*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^6+96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+1
2*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^4-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-72*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*s
in(1/2*d*x+1/2*c)^2+40*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
```

```
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^6+96
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^6-60*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^4*sin(1/2*d*x+1/2*c)^4-144*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1
/2*c)^4)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(9/2),x)
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(9/2),x)
[Out] Timed out
```

3.573 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=264

$$\frac{2ab\left(21a^2-22b^2\right)(e\cos(c+dx))^{3/2}}{45de^7}-\frac{2\left(b\left(7a^2-6b^2\right)-a\left(21a^2-22b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx))}{45de^5\sqrt{e\cos(c+dx)}}+\frac{2((7}{}$$

```
[Out] 2/45*a*b*(21*a^2-22*b^2)*(e*cos(d*x+c))^(3/2)/d/e^7+2/9*(b+a*sin(d*x+c))*(a
+b*sin(d*x+c))^3/d/e/(e*cos(d*x+c))^(9/2)+2/45*(a+b*sin(d*x+c))^2*(a*b+(7*a
^2-6*b^2)*sin(d*x+c))/d/e^3/(e*cos(d*x+c))^(5/2)-2/45*(a+b*sin(d*x+c))*(b*(
7*a^2-6*b^2)-a*(21*a^2-22*b^2)*sin(d*x+c))/d/e^5/(e*cos(d*x+c))^(1/2)-2/15*
(7*a^4-12*a^2*b^2+4*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^6/cos(d*x+c)^(
1/2)
```

Rubi [A] time = 0.48, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2691, 2861, 2669, 2640, 2639}

$$\frac{2ab\left(21a^2-22b^2\right)(e\cos(c+dx))^{3/2}}{45de^7}+\frac{2\left(\left(7a^2-6b^2\right)\sin(c+dx)+ab\right)(a+b\sin(c+dx))^2}{45de^3(e\cos(c+dx))^{5/2}}-\frac{2\left(b\left(7a^2-6b^2\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*a*b*(21*a^2 - 22*b^2)*(e*Cos[c + d*x])^(3/2))/(45*d*e^7) - (2*(7*a^4 - 1
2*a^2*b^2 + 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^
6*Sqrt[Cos[c + d*x]]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(9*
d*e*(e*Cos[c + d*x])^(9/2)) - (2*(a + b*Sin[c + d*x])*(b*(7*a^2 - 6*b^2) -
a*(21*a^2 - 22*b^2)*Sin[c + d*x]))/(45*d*e^5*Sqrt[e*Cos[c + d*x]]) + (2*(a
+ b*Sin[c + d*x])^2*(a*b + (7*a^2 - 6*b^2)*Sin[c + d*x]))/(45*d*e^3*(e*Cos[
c + d*x])^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
```

```
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{7a^2}{2}+3b^2-\frac{1}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{7/2}}\,dx}{9e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} + \frac{2(a+b\sin(c+dx))^2\left(ab+\left(7a^2-6b^2\right)\sin\right.}{45de^3(e\cos(c+dx))^{5/2}}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} - \frac{2(a+b\sin(c+dx))\left(b\left(7a^2-6b^2\right)-a\left(21\right.\right.}{45de^5\sqrt{e\cos(c+dx}}$$

$$= \frac{2ab\left(21a^2-22b^2\right)(e\cos(c+dx))^{3/2}}{45de^7} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} - \frac{2(a}{}$$

$$= \frac{2ab\left(21a^2-22b^2\right)(e\cos(c+dx))^{3/2}}{45de^7} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} - \frac{2(a}{}$$

$$= \frac{2ab\left(21a^2-22b^2\right)(e\cos(c+dx))^{3/2}}{45de^7} - \frac{2\left(7a^4-12a^2b^2+4b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{15de^6\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 1.62, size = 219, normalized size = 0.83

$$\frac{\sec^5(c+dx)\sqrt{e\cos(c+dx)}\left(150a^4\sin(c+dx)+91a^4\sin(3(c+dx))+21a^4\sin(5(c+dx))+320a^3b+360a^2b^2\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^5*(320*a^3*b + 32*a*b^3 - 288*a*b^3*Cos[
2*(c + d*x)] - 48*(7*a^4 - 12*a^2*b^2 + 4*b^4)*Cos[c + d*x]^(9/2)*EllipticE
[(c + d*x)/2, 2] + 150*a^4*Sin[c + d*x] + 360*a^2*b^2*Sin[c + d*x] + 60*b^4
*Sin[c + d*x] + 91*a^4*Sin[3*(c + d*x)] - 156*a^2*b^2*Sin[3*(c + d*x)] - 8*
b^4*Sin[3*(c + d*x)] + 21*a^4*Sin[5*(c + d*x)] - 36*a^2*b^2*Sin[5*(c + d*x)
] + 12*b^4*Sin[5*(c + d*x)]))/(360*d*e^6)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\cos(dx+c)^4+a^4+6a^2b^2+b^4-2\left(3a^2b^2+b^4\right)\cos(dx+c)^2-4\left(ab^3\cos(dx+c)^2-a^3b-ab^3\right)\sin\right.}{e^6\cos(dx+c)^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e^6*cos(d*x + c)^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(11/2), x)
```

maple [B] time = 9.88, size = 1416, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e
+e)^(1/2)/e^5*(-36*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+21*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*a^4+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-384*b^4*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^10+1344*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+768*b^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-1064*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^6-488*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+392*a^4*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-66*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2-12*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+144*a*b^3*sin(1/2*d*x+1/
2*c)^5-144*a*b^3*sin(1/2*d*x+1/2*c)^3-20*a^3*b*sin(1/2*d*x+1/2*c)+16*a*b^3*
sin(1/2*d*x+1/2*c)-672*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+104*b^4
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+1152*a^2*b^2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^10-2304*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+18
24*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-672*a^2*b^2*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+36*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2-576*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2*sin(1/2*d*x+1/2*c)^8+1152*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a^2*b^2*sin(1/2*d*x+1/2*c)^6-384*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
b^4*sin(1/2*d*x+1/2*c)^6-864*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1
/2*c)^4+288*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2+504*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^4+288*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b
^4*sin(1/2*d*x+1/2*c)^4-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^4*sin(1/2*d*x+1/2*c)
^2-96*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+336*(2*sin(1/2*d*x+
```

```
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^8+192*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1/2
*d*x+1/2*c)^8-672*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^6)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\sin(c+d\,x))^4}{(e\cos(c+d\,x))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(11/2),x)
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(11/2),x)
[Out] Timed out
```

3.574 $\int \frac{(e\cos(c+dx))^{11/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=531

$$-\frac{e^{11/2}\left(b^2-a^2\right)^{9/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}d}-\frac{e^{11/2}\left(b^2-a^2\right)^{9/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}d}-\frac{ae^6\left(a^2-b^2\right)^3\sqrt{\cos(c+dx)}}{b^6d\left(a^2-b\left(b-\sqrt{b^2}\right.\right.}$$

```
[Out] -(-a^2+b^2)^(9/4)*e^(11/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(11/2)/d-(-a^2+b^2)^(9/4)*e^(11/2)*arctanh(b^(1/2)*(e*cos(d
*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d+2/9*e*(e*cos(d*x+c))^(9/2
)/b/d-2/35*e^3*(e*cos(d*x+c))^(5/2)*(7*a^2-7*b^2-5*a*b*sin(d*x+c))/b^3/d+2/
21*a*(21*a^4-49*a^2*b^2+33*b^4)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(e*co
s(d*x+c))^(1/2)-a*(a^2-b^2)^3*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(
d*x+c)^(1/2)/b^6/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-a*(a^2
-b^2)^3*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(
1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(a^
2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+2/21*e^5*(21*(a^2-b^2)^2-a*b
*(7*a^2-12*b^2)*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^5/d
```

Rubi [A] time = 1.91, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2695, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{2e^5\sqrt{e\cos(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\sin(c+dx)\right)}{21b^5d}-\frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c\right.}{35b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*d)) - ((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTa
nh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*
d) + (2*e*(e*Cos[c + d*x])^(9/2))/(9*b*d) + (2*a*(21*a^4 - 49*a^2*b^2 + 33*
b^4)*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^6*d*Sqrt[e*Cos
[c + d*x]]) - (a*(a^2 - b^2)^3*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b -
 Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*
d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)^3*e^6*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^6*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (2*e^3*(e*Cos[c + d*x])^(5/2)*(7*(a^2
 - b^2) - 5*a*b*Sin[c + d*x]))/(35*b^3*d) + (2*e^5*Sqrt[e*Cos[c + d*x]]*(21
*(a^2 - b^2)^2 - a*b*(7*a^2 - 12*b^2)*Sin[c + d*x]))/(21*b^5*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{9/2}}{9bd} + \frac{e^2\int \frac{(e\cos(c+dx))^{7/2}(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{\left(2e^4\right)}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{2e^5\sqrt{e}}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{2e^5\sqrt{e}}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{2e^5\sqrt{e}}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} + \frac{2a\left(21a^4-49a^2b^2+33b^4\right)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^6d\sqrt{e\cos(c+dx)}} -$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} + \frac{2a\left(21a^4-49a^2b^2+33b^4\right)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^6d\sqrt{e\cos(c+dx)}} -$$

$$= -\frac{\left(-a^2+b^2\right)^{9/4}e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2}d} - \frac{\left(-a^2+b^2\right)^{9/4}e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+}}{\sqrt[4]{-a^2+b^2}\sqrt{}}\right.}{b^{11/2}d}$$

Mathematica [C] time = 28.30, size = 2035, normalized size = 3.83

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(11/2)*((-2*(280*a^4 - 636*a^2*b^2 + 721*b^4)*(a + b*Sqrt
[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Co
s[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4,
3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*
x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 +
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c +
d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 -
 Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + ((840*a^4 - 1764*a^2*b^2 + 959*b^4
)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 +
 b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/
(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^
(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c +
 d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b
^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(
5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^
2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^
2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x
]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^
2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[
c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*
x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*(-392*a^3*b + 722*
a*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/
2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c +
d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4
, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/
4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2
- b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcT
an[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1
 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 -
 b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*
x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c +
d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*
x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(1680*b^4*d*Cos[c + d*x
]^(11/2)) + ((e*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*(((-9*a^2 + 14*b^2)*Cos
[2*(c + d*x)])/(45*b^3) + Cos[4*(c + d*x)]/(36*b) - (a*(28*a^2 - 51*b^2)*Si
n[c + d*x])/(42*b^4) + (a*Sin[3*(c + d*x)])/(14*b^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 5.14, size = 3711, normalized size = 6.99

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a^5/sin(
1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^6/(-2*sin(1/2*d*x+1/2
*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+14/3/d*(e*(2
*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a^3/sin(1/2*d*x+1/
2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^4/(-2*sin(1/2*d*x+1/2*c)^4*e+si
n(1/2*d*x+1/2*c)^2*e)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-22/7/d*(e*(2*cos(1/2*d
*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a/sin(1/2*d*x+1/2*c)/(e*(2*c
os(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/
2*c)^2*e)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6/d*e^5/b^3*(e*(2*cos(1/2*d*x+1/2*
c)^2-1))^(1/2)*a^2-2/d*e^7/b^5*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_
R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/
2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(
16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^6+6/d*e^7/b^3*sum((_
R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3
)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)
-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3
*_Z^2+b^2*e^4))*a^4-6/d*e^7/b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R
^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2
)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(1
6*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+32/9/d*e^5/b*cos(1/
2*d*x+1/2*c)^8*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-64/9/d*e^5/b*cos(1/2*d*x+
1/2*c)^6*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+104/15/d*e^5/b*cos(1/2*d*x+1/2*
c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-152/45/d*e^5/b*cos(1/2*d*x+1/2*c)^2
*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+8/5/d*e^5/b^3*(2*cos(1/2*d*x+1/2*c)^2*e
-e)^(1/2)*a^2+2/d*e^5/b^5*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*a^4+1/8/d*(e
*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a^7/sin(1/2*d*x
+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^8*sum(1/_alpha/(2*_alpha^2-1
)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha
^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+
b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/
(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*
(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4
*_Z^2*b^2+a^2))-3/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*e^6*a^5/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^6*su
m(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b
^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^
2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1
))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_a
lpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+3/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)
```

```
*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a^3/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1
/2*c)^2-1))^(1/2)/b^4*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b
^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(
1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2
^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(
2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(
_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/8/d*(e*(2
*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a/sin(1/2*d*x+1/2*
c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2*sum(1/_alpha/(2*_alpha^2-1)*(2^
(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)
/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_
alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(
2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alp
ha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-si
n(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2
*b^2+a^2))-152/45/d*e^5/b*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+6/d*e^5/b*(e*(
2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)+2/d*e^7*b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R
^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*
b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))+8/5/d*e^5/
b^3*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2-8/5/d*e^5/b
^3*cos(1/2*d*x+1/2*c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2+48/7/d*(e*(2
*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a*sin(1/2*d*x+1/2*
c)^5/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*e+si
n(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)-32/7/d*(e*(2*cos(1/2*d*x+1/2
*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a*sin(1/2*d*x+1/2*c)^7/(e*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)
^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+8/3/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*e^6*a^3*sin(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2
-1))^(1/2)/b^4/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos
(1/2*d*x+1/2*c)-8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*e^6*a*sin(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*
sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)-4/3
/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a^3*sin(1/
2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^4/(-2*sin(1/2*d*x+1/2*c
)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+20/7/d*(e*(2*cos(1/2
*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a*sin(1/2*d*x+1/2*c)/(e*(2
*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+
1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(11/2)/(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{11/2}}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.575 $\int \frac{(e\cos(c+dx))^{9/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=446

$$\frac{e^{9/2}\left(b^2-a^2\right)^{7/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d}-\frac{e^{9/2}\left(b^2-a^2\right)^{7/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d}+\frac{ae^5\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\Pi\left(\right.}{b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e}}$$

```
[Out] (-a^2+b^2)^(7/4)*e^(9/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(9/2)/d-(-a^2+b^2)^(7/4)*e^(9/2)*arctanh(b^(1/2)*(e*cos(d*x+c
))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d+2/7*e*(e*cos(d*x+c))^(7/2)/b/d
-2/15*e^3*(e*cos(d*x+c))^(3/2)*(5*a^2-5*b^2-3*a*b*sin(d*x+c))/b^3/d+a*(a^2-
b^2)^2*e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/d/(b-(
-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+a*(a^2-b^2)^2*e^5*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2
)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c)
)^(1/2)-2/5*a*(5*a^2-8*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^4/d/cos(d
*x+c)^(1/2)
```

Rubi [A] time = 1.29, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2695, 2865, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{2e^3(e\cos(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{15b^3d}+\frac{e^{9/2}\left(b^2-a^2\right)^{7/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d}-\frac{e^{9/2}\left(b^2-a^2\right)^{7/4}\tan}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d) - ((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d) + (2
*e*(e*Cos[c + d*x])^(7/2))/(7*b*d) - (2*a*(5*a^2 - 8*b^2)*e^4*Sqrt[e*Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c + d*x]]) + (a*(a^2 -
 b^2)^2*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c
+ d*x)/2, 2])/(b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^2
 - b^2)^2*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (
c + d*x)/2, 2])/(b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (2*e^
3*(e*Cos[c + d*x])^(3/2)*(5*(a^2 - b^2) - 3*a*b*Sin[c + d*x]))/(15*b^3*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
```

```
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{7/2}}{7bd} + \frac{e^2\int \frac{(e\cos(c+dx))^{5/2}(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2e^3(e\cos(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{15b^3d} + \frac{\left(2e^4\right)\int -\sqrt{}}{}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2e^3(e\cos(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{15b^3d} - \frac{\left(a\left(5a^2-\right.\right.}{}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2e^3(e\cos(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{15b^3d} - \frac{\left(a\left(a^2-b\right.\right.}{}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2a\left(5a^2-8b^2\right)e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} - \frac{2e^3(e\cos(c+}{}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2a\left(5a^2-8b^2\right)e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{a\left(a^2-b^2\right)^2 e}{b^5(}$$

$$= \frac{\left(-a^2+b^2\right)^{7/4}e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d} - \frac{\left(-a^2+b^2\right)^{7/4}e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d} + \cdots$$

Mathematica [C] time = 27.17, size = 834, normalized size = 1.87

$$\frac{(e\cos(c+dx))^{9/2}\sec^4(c+dx)\left(\frac{\left(37b^2-28a^2\right)\cos(c+dx)}{42b^3}+\frac{\cos(3(c+dx))}{14b}+\frac{a\sin(2(c+dx))}{5b^2}\right)}{d} - \frac{(e\cos(c+dx))^{9/2}\left(-\frac{\left(5a^3-8ab^2\right)\left(a+b\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*((e*Cos[c + d*x])^(9/2)*((-2*(2*a^2*b - 5*b^3)*(a + b*Sqrt[1 - Cos[c +
 d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^
2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcT
an[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[
1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^
2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[
c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[
Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d
*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((5*a^3 - 8*a*b^2)*(
a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[
c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[
2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(
a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2
- b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(
a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12
*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(b^3*d*C
os[c + d*x]^(9/2)) + ((e*Cos[c + d*x])^(9/2)*Sec[c + d*x]^4*(((-28*a^2 + 37
*b^2)*Cos[c + d*x])/(42*b^3) + Cos[3*(c + d*x)]/(14*b) + (a*Sin[2*(c + d*x)
])/(5*b^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 3.57, size = 2126, normalized size = 4.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 16/7/d*e^4/b*cos(1/2*d*x+1/2*c)^6*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-24/7/d
*e^4/b*cos(1/2*d*x+1/2*c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+64/21/d*e^4/
b*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+64/21/d*e^4/b*(2*
cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-4/3/d*e^4/b^3*cos(1/2*d*x+1/2*c)^2*(2*cos(1
/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2-4/3/d*e^4/b^3*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(
1/2)*a^2+2/d*e^4/b^3*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*a^2-4/d*e^4/b*(e*
(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)+1/2/d*e^5/b^3*sum((_R^6-_R^4*e-_R^2*e^2+e
^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2
*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=
RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^
2*e^4))*a^4-1/d*e^5/b*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e
+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)
^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z
```

```
^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+1/2/d*e^5*b*su
m((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b
^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d
*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^
2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-16/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*e^5*a/b^2/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(
1/2*d*x+1/2*c)^7+32/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*e^5*a/b^2/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/s
in(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(1/2*d*x+1/2*c)^5
-4/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^5*a/b^2/(-
e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(1/2*d*x+1/2*c)^3-2/d*(e*(2*cos(1/2*
d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^5*a^3/b^4/(-e*(2*sin(1/2*d*x+
1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+
1/2*c)^2-1))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+16/5/d*(e*(2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^5*a/b^2/(-e*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1)
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+4/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*e^5*a/b^2/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(
1/2*d*x+1/2*c)-1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*e^5/a/b^6/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*sum((
a^4-2*a^2*b^2+b^4)/_alpha*(8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1
/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos
(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*
2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^
4-sin(1/2*d*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2
*c)^2-1))^(1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*d*x+1/2
*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b
^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(9/2)/(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{9/2}}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.576 $\int \frac{(e\cos(c+dx))^{7/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=461

$$-\frac{e^{7/2}\left(b^2-a^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d}-\frac{e^{7/2}\left(b^2-a^2\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d}-\frac{2ae^4\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}}{3b^4d\sqrt{e\cos(c+d}}$$

```
[Out] -(-a^2+b^2)^(5/4)*e^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(7/2)/d-(-a^2+b^2)^(5/4)*e^(7/2)*arctanh(b^(1/2)*(e*cos(d*x+
c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d+2/5*e*(e*cos(d*x+c))^(5/2)/b/
d-2/3*a*(3*a^2-4*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(e*cos(d*x+c))^
(1/2)+a*(a^2-b^2)^2*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/
2)/b^4/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+a*(a^2-b^2)^2*e^
4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(a^2-b*(b+(-a
^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-2/3*e^3*(3*a^2-3*b^2-a*b*sin(d*x+c))*(
e*cos(d*x+c))^(1/2)/b^3/d
```

Rubi [A] time = 1.32, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2695, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{2e^3\sqrt{e\cos(c+dx)}\left(3\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{3b^3d}-\frac{e^{7/2}\left(b^2-a^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d}-\frac{e^{7/2}\left(b^2-a^2\right)^{5/4}\tanh^{-1}}{b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d)) - ((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d) +
 (2*e*(e*Cos[c + d*x])^(5/2))/(5*b*d) - (2*a*(3*a^2 - 4*b^2)*e^4*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^4*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^
2 - b^2)^2*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x
]]) + (a*(a^2 - b^2)^2*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a
^2 + b^2]), (c + d*x)/2, 2])/(b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e
*Cos[c + d*x]]) - (2*e^3*Sqrt[e*Cos[c + d*x]]*(3*(a^2 - b^2) - a*b*Sin[c +
d*x]))/(3*b^3*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{5/2}}{5bd} + \frac{e^2\int \frac{(e\cos(c+dx))^{3/2}(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2e^3\sqrt{e\cos(c+dx)}\left(3\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{3b^3d} + \frac{\left(2e^4\right)\int \frac{-\frac{1}{2}b(2}{\cdots}}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2e^3\sqrt{e\cos(c+dx)}\left(3\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{3b^3d} - \frac{\left(a\left(3a^2-4b^2\right.\right.}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2e^3\sqrt{e\cos(c+dx)}\left(3\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{3b^3d} - \frac{\left(a\left(-a^2+b^2\right)\right.}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2a\left(3a^2-4b^2\right)e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\sqrt{e\cos(c+dx)}} - \frac{2e^3\sqrt{e\cos(c+}}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2a\left(3a^2-4b^2\right)e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\sqrt{e\cos(c+dx)}} + \frac{a\left(-a^2+b^2\right)^{3/2}}{b^4\left(\cdots\right.}$$

$$= -\frac{\left(-a^2+b^2\right)^{5/4}e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d} - \frac{\left(-a^2+b^2\right)^{5/4}e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d} + \cdots$$

Mathematica [C] time = 29.04, size = 1955, normalized size = 4.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*(Cos[2*(c + d*x)]/(5*b) + (2*a*Sin[c
 + d*x])/(3*b^2)))/d - ((e*Cos[c + d*x])^(7/2)*((-2*(10*a^2 - 27*b^2)*(a +
b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[
```

```
1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^
2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4,
 Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1
[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[
c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*Ar
cTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTa
n[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-
a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Co
s[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqr
t[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sq
rt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + ((30*a^2 - 33*b^2)*(a + b*Sq
rt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTa
n[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-
a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]
*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4
*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b
^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*
a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^
2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^
2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*
x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^
2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos
[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)
*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2
)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] +
(1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/
(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 +
 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) + (28*a*b*(a + b*Sqrt[1 - Cos[c +
d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])
/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/
4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2
 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Co
s[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c
+ d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a
+ b*Sin[c + d*x]))))/(60*b^2*d*Cos[c + d*x]^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(b*sin(d*x + c) + a), x)
```

maple [C] time = 4.47, size = 2329, normalized size = 5.05

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 8/5/d*e^3/b*cos(1/2*d*x+1/2*c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-8/5/d*e
^3/b*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-8/5/d*e^3/b*(2
*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-2/d*e^3/b^3*(e*(2*cos(1/2*d*x+1/2*c)^2-1))
^(1/2)*a^2+4/d*e^3/b*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)+2/d*e^5/b^3*sum((
_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^
3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2
)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^
3*_Z^2+b^2*e^4))*a^4-4/d*e^5/b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_
R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/
2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(
16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+2/d*e^5*b*sum((_R^
4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*
ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_
R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_
Z^2+b^2*e^4))-8/3/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*e^4*a*sin(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*
sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+4/3
/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a*sin(1/2*
d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^
4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+2/d*(e*(2*cos(1/2*d*x+
1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a^3/sin(1/2*d*x+1/2*c)/(e*(2*co
s(1/2*d*x+1/2*c)^2-1))^(1/2)/b^4/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2
*c)^2*e)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8/3/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2
*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*e^4*a^5/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(
1/2)/b^6*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)
/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c
)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*
_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x
+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2
^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/4/d*(e*(2*cos(1/2*d*x+
1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a^3/sin(1/2*d*x+1/2*c)/(e*(2*co
s(1/2*d*x+1/2*c)^2-1))^(1/2)/b^4*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2
*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b
^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a
^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1
/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-
1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a/sin(1
/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2*sum(1/_alpha/(2*_alp
ha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*
_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2
*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^
```

```
(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_
alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4
*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(7/2)/(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{7/2}}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.577 $\int \frac{(e\cos(c+dx))^{5/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=384

$$\frac{e^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{e^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{ae^3\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b-}\right.}{b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e}}$$

```
[Out] (-a^2+b^2)^(3/4)*e^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(5/2)/d-(-a^2+b^2)^(3/4)*e^(5/2)*arctanh(b^(1/2)*(e*cos(d*x+c
))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/d+2/3*e*(e*cos(d*x+c))^(3/2)/b/d
-a*(a^2-b^2)*e^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi
(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/
d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-a*(a^2-b^2)*e^3*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2
+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*
x+c))^(1/2)+2*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2695, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{e^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{e^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{ae^3\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b-}\right.}{b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d) - ((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d) + (2
*e*(e*Cos[c + d*x])^(3/2))/(3*b*d) + (2*a*e^2*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) - (a*(a^2 - b^2)*e^3*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^3*(
b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^3*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^3*(
b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
```

```
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}\,(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{\left(ae^2\right)\int \sqrt{e\cos(c+dx)}\,dx}{b^2} + \frac{\left(\left(-a^2+b^2\right)e^2\right)\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{\left(a\left(a^2-b^2\right)e^3\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2b^3} - \frac{\left(a\left(a^2-b^2\right.\right.}{}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{2ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} - \frac{\left(2\left(a^2-b^2\right)e^3\right)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{2ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} - \frac{a\left(a^2-b^2\right)e^3\sqrt{\cos(c+dx}}{b^3\left(b-\sqrt{-a^2+}\right.}$$

$$= \frac{\left(-a^2+b^2\right)^{3/4}e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d} - \frac{\left(-a^2+b^2\right)^{3/4}e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d} +$$

Mathematica [C] time = 21.81, size = 709, normalized size = 1.85

$$\frac{(e\cos(c+dx))^{5/2}\left(-\frac{6b\sin(c+dx)\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(\frac{a\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)}{3\left(a^2-b^2\right)}+\left(\frac{1}{8}+\frac{i}{8}\right)\left(-\log\left(-(1+i)\sqrt{b}\sqrt[4]{b^2-a^2}\sqrt{\cos(c+dx)}+\sqrt{\right.\right.\right.}{\sqrt{\sin^2(c+}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(2*Cos[c + d*x]^(3/2) - (a*(8*b^(5/2)*AppellF1[3/4,
 -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d
*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sq
rt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[C
os[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a
^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2]
+ Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*
(a + b*Sqrt[Sin[c + d*x]^2]))/(4*b^(3/2)*(-a^2 + b^2)*(a + b*Sin[c + d*x]))
 - (6*b*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)
/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan
[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1
+ ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2
+ b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c
+ d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Co
s[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x
```

```
]*(a + b*Sqrt[Sin[c + d*x]^2]))/(Sqrt[Sin[c + d*x]^2]*(a + b*Sin[c + d*x]))
))/(3*b*d*Cos[c + d*x]^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(b*sin(d*x + c) + a), x)
```

maple [C] time = 3.67, size = 1131, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 4/3/d*e^2/b*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+4/3/d*e
^2/b*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-2/d*e^2/b*(e*(2*cos(1/2*d*x+1/2*c)^
2-1))^(1/2)-1/2/d*e^3/b*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2
*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+
e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*
_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+1/2/d*e^3*b*
sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3
*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2
*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*
e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))+2/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*e^3*a/b^2/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*e^3/a/b^4/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)
*sum((a^2-b^2)/_alpha*(8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))
*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2
*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1
/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^
2-1))^(1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^
2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a
^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.578 $\int \frac{(e\cos(c+dx))^{3/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=397

$$-\frac{ae^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{ae^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}$$

```
[Out] -(-a^2+b^2)^(1/4)*e^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(3/2)/d-(-a^2+b^2)^(1/4)*e^(3/2)*arctanh(b^(1/2)*(e*cos(d*x+
c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/d+2*a*e^2*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/b^2/d/(e*cos(d*x+c))^(1/2)-a*(a^2-b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2
)),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+
c))^(1/2)-a*(a^2-b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(
1/2)/b^2/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+2*e*(e*cos(d*x
+c))^(1/2)/b/d
```

Rubi [A] time = 0.88, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2695, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{e^{3/2}\sqrt[4]{b^2-a^2}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d}-\frac{e^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d}-\frac{ae^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b-}\right.}{b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d)) - ((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d) +
 (2*e*Sqrt[e*Cos[c + d*x]])/(b*d) + (2*a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2])/(b^2*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^2*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^2*(a
^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^2
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2
])/(b^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
```

```
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{a+b\sin(c+dx)}\,dx = \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{e^2\int \frac{b+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx}{b}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{(ae^2)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{b^2} + \frac{((-a^2+b^2)e^2)\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+d}}{b^2}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} - \frac{\left(a\sqrt{-a^2+b^2}\,e^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2b^2} - \frac{\left(a\sqrt{-a^2\right.}{}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}} - \frac{\left(2\left(a^2-b^2\right)e^3\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}} + \frac{a\sqrt{-a^2+b^2}\,e^2\sqrt{\cos(c+d}}{b^2\left(b-\sqrt{-a^2+\right.}}$$

$$= -\frac{\sqrt[4]{-a^2+b^2}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{3/2}d} - \frac{\sqrt[4]{-a^2+b^2}\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{3/2}d} + \frac{2}{}$$

Mathematica [C] time = 4.67, size = 219, normalized size = 0.55

$$-\frac{e\csc^2(c+dx)\sqrt{e\cos(c+dx)}\left(a^2+b^2\cos^2(c+dx)-b^2\right)\left(2b\tan(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\left(1-\frac{a^2}{b^2}\right)\sec^2(c+dx)\right)\right.}{b^2d(a\csc(c+dx)+b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((e*Sqrt[e*Cos[c + d*x]]*(a^2 - b^2 + b^2*Cos[c + d*x]^2)*Csc[c + d*x]^2*(
2*b*Hypergeometric2F1[-1/4, 1, 3/4, (1 - a^2/b^2)*Sec[c + d*x]^2]*Tan[c + d
*x] + a*(EllipticPi[-(Sqrt[-a^2 + b^2]/b), ArcSin[(Sec[c + d*x]^2)^(1/4)],
-1] + EllipticPi[Sqrt[-a^2 + b^2]/b, ArcSin[(Sec[c + d*x]^2)^(1/4)], -1])*(
Sec[c + d*x]^2)^(1/4)*Sqrt[-Tan[c + d*x]^2]))/(b^2*d*(b + a*Csc[c + d*x])*(
-(a*Sqrt[Sec[c + d*x]^2]) + b*Tan[c + d*x])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a), x)
```

maple [C] time = 3.23, size = 1266, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 2/d*e/b*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)-2/d*e^3/b*sum((_R^4+_R^2*e)/(_
R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1
/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf
(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4)
)*a^2+2/d*e^3*b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_
R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(
1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b
^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-2/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a*e^2/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^
2-1))^(1/2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^3*e^2/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)/b^4*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^
2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2
*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_al
pha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/
2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1
/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/8/d*(e*(2*cos(1/2*d*x+1/2
*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*e^2/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2
*d*x+1/2*c)^2-1))^(1/2)/b^2*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alp
ha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(
4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*
b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/
2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)
^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2
/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{3/2}}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.579 $\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=292

$$\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,d\sqrt[4]{b^2-a^2}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,d\sqrt[4]{b^2-a^2}}+\frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{ae\sqrt{\cos(c+}}{bd\left(\sqrt{b^2}\right.}$$

```
[Out] arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2
+b^2)^(1/4)/d/b^(1/2)-arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)
/e^(1/2))*e^(1/2)/(-a^2+b^2)^(1/4)/d/b^(1/2)+a*e*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)
),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+a
*e*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b+(-a^2+b^2)^
(1/2))/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,d\sqrt[4]{b^2-a^2}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,d\sqrt[4]{b^2-a^2}}+\frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{ae\sqrt{\cos(c+}}{bd\left(\sqrt{b^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d) - (Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d) + (a
*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
 2])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*e*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b*(b + Sqr
t[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx = -\frac{(ae)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}dx}{2b} + \frac{(ae)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}+b\cos(c+dx)\right)}dx}{2b}$$

$$= \frac{(2be)\,\mathrm{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}dx, x, \sqrt{e\cos(c+dx)}\right)}{d} - \frac{\left(ae\sqrt{\cos(c+dx)}\right)\int \frac{}{\sqrt{\cos(c+dx)}}}{2b\sqrt{e\cos(c+}}$$

$$= \frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}} + \frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{-a^2+b^2}};\frac{1}{2}(c+\right.}{b\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+d}}$$

$$= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}\,d} - \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}\,d} + \frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2}{b-\sqrt{-}}\right.}{b\left(b-\sqrt{-a^2+b^2}\right)d}$$

Mathematica [C] time = 16.03, size = 361, normalized size = 1.24

$$-\frac{2\sin(c+dx)\sqrt{e\cos(c+dx)}\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(\frac{a\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)}{3(a^2-b^2)} + \frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(-\log\left(-(1+\right.\right.}{}\right.}{d\sqrt{\sin^2(c+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8
 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1
/4)))*Sin[c + d*x]*(a + b*Sqrt[Sin[c + d*x]^2]))/(d*Sqrt[Cos[c + d*x]]*Sqrt
[Sin[c + d*x]^2]*(a + b*Sin[c + d*x]))
```

fricas [F] time = 167.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}}{b\sin\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a), x)
```

maple [C] time = 2.84, size = 682, normalized size = 2.34

$$\frac{eb\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^8-4b^2e\_Z^6+\left(16a^2e^2-10b^2e^2\right)\_Z^4-4b^2e^3\_Z^2+b^2e^4\right)}\frac{\left(\_R^6-\_R^4e-\_R^2e^2+e^3\right)\ln\left(\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}-\sqrt{e}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}-\_R\right)}{\_R^7b^2-3\_R^5b^2e+8\_R^3a^2e^2-5\_R^3b^2e^2-\_R\,b^2e^3}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d*e*b*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*
e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/
2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e
^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-1/8/d*(e*(2*cos(1/2*d*x+1/2*c)
^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e/a/b^2*sum(1/_alpha*(8*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*
_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2)
)*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_al
pha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)
^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/
2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+
1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)
```

```
/b^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alp
ha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+
1/2*c)^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(dx+c)}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\cos(c+d\,x)}}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.580 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\,\left(b^2-a^2\right)^{3/4}}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\,\left(b^2-a^2\right)^{3/4}}+\frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a\sqrt{\cos(c+}{d\left(a^2-b\left(\right.\right.}$$

```
[Out] -arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^
2+b^2)^(3/4)/d/e^(1/2)-arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4
)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(3/4)/d/e^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2))
,2^(1/2))*cos(d*x+c)^(1/2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1
/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*
x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/d/(a^2-b*(b+(-a
^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\,\left(b^2-a^2\right)^{3/4}}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\,\left(b^2-a^2\right)^{3/4}}+\frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a\sqrt{\cos(c+}{d\left(a^2-b\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e])) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e]) +
 (a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2
, 2])/((a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (a*Sqrt[C
os[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^
2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx = -\frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2\sqrt{-a^2+b^2}} - \frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}+b\cos(c+dx)\right)}\,dx}{2\sqrt{-a^2+b^2}}$$

$$= \frac{(2be)\,\text{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\cos(c+dx)}\right)}{d} - \frac{\left(a\sqrt{\cos(c+d}\right.}{2}$$

$$= \frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\cos(c+dx)}} + \frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b+}\right.}{\left(a^2-b\left(b+\sqrt{-a^2+\right.\right.}$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{3/4}d\sqrt{e}} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{3/4}d\sqrt{e}} + \frac{a\sqrt{\cos}}{\left(a^2-\right.}$$

Mathematica [C] time = 16.13, size = 558, normalized size = 1.87

$$-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(\frac{5a\left(a^2-b^2\right)\sqrt{}}{\sqrt{\sin^2(c+dx)}\left(a^2+b^2\cos^2(c+dx)-b^2\right)\left(5\left(a^2-b^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\cos^2(c+dx),\frac{b^2\cos}{b^2}\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((
1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2]
- (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
 - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Co
s[c + d*x]])/((a^2 - b^2 + b^2*Cos[c + d*x]^2)*(5*(a^2 - b^2)*AppellF1[1/4,
 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2
*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*Sqrt[Sin[c + d*x]^2]))*(a + b*Sqrt[S
in[c + d*x]^2]))/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2]*(a + b*Sin[c
+ d*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)), x)
```

maple [C] time = 2.77, size = 678, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*b*e*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*
e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+
1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*
_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/a/b^2*sum(1/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
4*b^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_a
lpha^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*
(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_alph
a^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2
+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+1/
2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alpha
=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/
2*c)^2-1))^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos(dx+c)}\,(b\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,\cos(c+d\,x)}\,\left(a+b\,\sin(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))),x)
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.581 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=411

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{2(b-a\sin(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}-\frac{ab\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{de\left(a^2-b^2\right)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\cdots$$

```
[Out] b^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2
+b^2)^(5/4)/d/e^(3/2)-b^(3/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/(-a^2+b^2)^(5/4)/d/e^(3/2)-2*(b-a*sin(d*x+c))/(a^2-b^2)/d
/e/(e*cos(d*x+c))^(1/2)-a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)
^(1/2)/(a^2-b^2)/d/e/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-a*b*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(
b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)/d/e/(b+(-a^2+b^2)^(
1/2))/(e*cos(d*x+c))^(1/2)-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/(a^2-b^2)/d/
e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2696, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{2(b-a\sin(c+\cdots}{de\left(a^2-b^2\right)\sqrt{e\cos(\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(5/4)*d*e^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(5/4)*d*e^(3/2)) - (2
*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/((a^2 - b^2)*d*e^2*Sqrt[
Cos[c + d*x]]) - (a*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[
c + d*x]]) - (a*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]
), (c + d*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c +
d*x]]) - (2*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
```

```
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
```

```
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}\,dx = -\frac{2(b-a\sin(c+dx))}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}} - \frac{2\int \frac{\sqrt{e\cos(c+dx)}\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{1}{2}ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{(a^2-b^2)\,e^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}} - \frac{a\int\sqrt{e\cos(c+dx)}\,dx}{(a^2-b^2)\,e^2} - \frac{b^2\int \frac{\sqrt{e\cos(c}}{a+b\sin(}}{(a^2-b^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}} + \frac{(ab)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}}{2(a^2-b^2)\,e}$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,de^2\sqrt{\cos(c+dx)}} - \frac{2(b-a\sin(c+dx))}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}} -$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,de^2\sqrt{\cos(c+dx)}} - \frac{ab\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}}\right.}{(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)de}$$

$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{5/4}de^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{5/4}de^{3/2}} - \frac{2a\sqrt{e\cos}}{(a^2-}$$

Mathematica [C] time = 22.85, size = 791, normalized size = 1.92

$$\frac{2\cos(c+dx)(a\sin(c+dx)-b)}{d\left(a^2-b^2\right)(e\cos(c+dx))^{3/2}} - \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{a\sin^2(c+dx)\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos}{b^2}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] (2*Cos[c + d*x]*(-b + a*Sin[c + d*x]))/((a^2 - b^2)*d*(e*Cos[c + d*x])^(3/2
)) - (Cos[c + d*x]^(3/2)*((-2*(a^2 + b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*
((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
 b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1
 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 +
I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] -
 (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*
x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt
[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (a*(a + b*Sqrt[1 - Cos[c + d*x
]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d
*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*
```

```
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcT
an[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
+ d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 -
 Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/((a - b)*(a + b)*d*(e*Cos[c + d*x]
)^(3/2))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)), x)
```

maple [**C**] time = 4.30, size = 1103, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x)
[Out] 1/2/d/e^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))*(-2*sin(1/2*
d*x+1/2*c)^2*e+e)^(1/2)-1/2/d/e*b^3/(a-b)/(a+b)*sum((_R^6-_R^4*e-_R^2*e^2+e
^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2
*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=
RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^
2*e^4))-1/2/d/e^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))*(-2*
sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-4/d/e*a/(a+b)/(a-b)/sin(1/2*d*x+1/2*c)/(e*(
2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(1/2*d*x+1/2*c)^3-2/d/e*a/(a+b)/(a-b)/(
-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/
(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+4/d/e*a/(a+b)/(a-b)/sin(1/2*d
*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(1/2*d*x+1/2*c)+1/8/d/e/a
/(a+b)/(a-b)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*sum(1/
_alpha*(8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2
*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b
^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+a^2*2
^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a
^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alph
a^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c
)^2))^(1/2))*(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2))/(e
*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*
```

```
d*x+1/2*c)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(e*(2*cos(
1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))),x)
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.582 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=434

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}-\frac{ab^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{de^2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{ab^2\sqrt{\cos(c+dx)}}{de^2\left(a^2-b^2\right)\left(a^2-b\left(\right.\right.}$$

```
[Out] -b^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^
2+b^2)^(7/4)/d/e^(5/2)-b^(5/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(7/4)/d/e^(5/2)-2/3*(b-a*sin(d*x+c))/(a^2-b^2
)/d/e/(e*cos(d*x+c))^(3/2)+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)/d/e^
2/(e*cos(d*x+c))^(1/2)-a*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c
)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-a
*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/
(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.00, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2696, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}-\frac{ab^2\sqrt{\cos(c+}}{de^2\left(a^2-b^2\right)\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(7/4)*d*e^(5/2))) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(7/4)*d*e^(5/2)) +
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*(a^2 - b^2)*d*e^2*Sq
rt[e*Cos[c + d*x]]) - (a*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[
-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2])
)*d*e^2*Sqrt[e*Cos[c + d*x]]) - (a*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/
(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (2*(b - a*Sin[c + d*x]))/(3*(a^2
- b^2)*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
```

```
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}\,dx = -\frac{2(b-a\sin(c+dx))}{3\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}+\frac{3b^2}{2}-\frac{1}{2}ab\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx}{3\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{3\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2} - \frac{b^2\int \frac{}{\sqrt{e\,\mathrm{co}}}}{}$$

$$= -\frac{2(b-a\sin(c+dx))}{3\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}} - \frac{\left(ab^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\,\mathrm{c}\right.}}{2\left(-a^2+b^2\right)^{3/2}e^2}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3\left(a^2-b^2\right)de^2\sqrt{e\cos(c+dx)}} - \frac{2(b-a\sin(c+dx))}{3\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3\left(a^2-b^2\right)de^2\sqrt{e\cos(c+dx)}} + \frac{ab^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2}}\right.}{\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2+b^2}\right.}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{7/4}de^{5/2}} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{7/4}de^{5/2}} + \frac{2a\sqrt{}}{3(}$$

Mathematica [C] time = 24.60, size = 1192, normalized size = 2.75

$$\left(-\frac{2ab\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\frac{5b\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{1-\cos^2(c+dx)}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)b^2+\left(a^2-b^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)\right)\cos^2(c+dx)-5\left(a^2-b^2\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\right.}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] (2*Cos[c + d*x]*(-b + a*Sin[c + d*x]))/(3*(a^2 - b^2)*d*(e*Cos[c + d*x])^(5
/2)) + (Cos[c + d*x]^(5/2)*((-2*(a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^
2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(
-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b
]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*S
qrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqr
```

```
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-
a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Co
s[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(
a + b*Sin[c + d*x])) - (2*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 -
 b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
 + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2
)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b
^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c +
d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b
^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^
(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c
 + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/
(3*(a - b)*(a + b)*d*(e*Cos[c + d*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)), x)
```

maple [C] time = 4.96, size = 1083, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x)
[Out] -2/d/e*b^3/(a-b)/(a+b)*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*
e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/
2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e
^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-1/12/d/e^3*b/(a^2-b^2)/(cos(1/
2*d*x+1/2*c)+1/2*2^(1/2))^2*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/12/d/e^3*
b*2^(1/2)/(a^2-b^2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)
^2*e+e)^(1/2)-1/12/d/e^3*b/(a^2-b^2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))^2*(-2
*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+1/12/d/e^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*d*
x+1/2*c)-1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+1/8/d*(e*(2*cos(1
/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/e^2/sin(1/2*d*x+1/2*c)/(e*
(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a-b)/(a+b)*sum(1/_alpha/(2*_alpha^2-1)*(
2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-
3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2
*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_a
```

```
lpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z
^2*b^2+a^2))+1/3/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a/e^3/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)*c
os(1/2*d*x+1/2*c)*(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2-2/3/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a/e^2/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(
1/2)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(b\sin\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))),x)
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.583 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=486

$$-\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}-\frac{2(b-a\sin(c+dx))}{5de\left(a^2-b^2\right)(e\cos(c+dx))^{5/2}}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\left(b^2-a^2\right)^{9/4}}-\frac{b^{7/}}{}$$

```
[Out] b^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2
+b^2)^(9/4)/d/e^(7/2)-b^(7/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(7/2)-2/5*(b-a*sin(d*x+c))/(a^2-b^2)
/d/e/(e*cos(d*x+c))^(5/2)+2/5*(5*b^3+a*(3*a^2-8*b^2)*sin(d*x+c))/(a^2-b^2)^
2/d/e^3/(e*cos(d*x+c))^(1/2)+a*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos
(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+a
*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^
3/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-2/5*a*(3*a^2-8*b^2)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
(e*cos(d*x+c))^(1/2)/(a^2-b^2)^2/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.33, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2696, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{2\left(a\left(3a^2-8b^2\right)\sin(c+dx)+5b^3\right)}{5de^3\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\left(b^2-a^2\right)^{9/4}}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\left(b^2-a^2\right)^{9/4}}-\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\right.}{5de^4\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] (b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(9/4)*d*e^(7/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(9/4)*d*e^(7/2)) - (2
*a*(3*a^2 - 8*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*(a^2
- b^2)^2*d*e^4*Sqrt[Cos[c + d*x]]) + (a*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)^2*(b - Sqrt[-a^2
 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (a*b^3*Sqrt[Cos[c + d*x]]*EllipticPi
[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)^2*(b + Sqrt[-a
^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) - (2*(b - a*Sin[c + d*x]))/(5*(a^2 -
 b^2)*d*e*(e*Cos[c + d*x])^(5/2)) + (2*(5*b^3 + a*(3*a^2 - 8*b^2)*Sin[c + d
*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}\,dx = -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+\frac{5b^2}{2}-\frac{3}{2}ab\sin(c+dx)}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}\,dx}{5\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} + \frac{2\left(5b^3+a\left(3a^2-8b^2\right)\sin(c+dx\right.}{5\left(a^2-b^2\right)^2 de^3\sqrt{e\cos(c+dx)}}$$

$$= -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} + \frac{2\left(5b^3+a\left(3a^2-8b^2\right)\sin(c+dx\right.}{5\left(a^2-b^2\right)^2 de^3\sqrt{e\cos(c+dx)}}$$

$$= -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} + \frac{2\left(5b^3+a\left(3a^2-8b^2\right)\sin(c+dx\right.}{5\left(a^2-b^2\right)^2 de^3\sqrt{e\cos(c+dx)}}$$

$$= -\frac{2a\left(3a^2-8b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5\left(a^2-b^2\right)^2 de^4\sqrt{\cos(c+dx)}} - \frac{2(b-a\sin(}{5\left(a^2-b^2\right)de(e\cos}$$

$$= -\frac{2a\left(3a^2-8b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5\left(a^2-b^2\right)^2 de^4\sqrt{\cos(c+dx)}} + \frac{ab^3\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)^2\left(b-\sqrt{\right.}}$$

$$= \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{9/4}de^{7/2}} - \frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{9/4}de^{7/2}} - \frac{2a\left(3a^2\right.}{}$$

Mathematica [C] time = 6.76, size = 881, normalized size = 1.81

$$\frac{\cos^4(c+dx)\left(\frac{2(a\sin(c+dx)-b)\sec^3(c+dx)}{5\left(a^2-b^2\right)}+\frac{2\left(3\sin(c+dx)a^3-8b^2\sin(c+dx)a+5b^3\right)\sec(c+dx)}{5\left(a^2-b^2\right)^2}\right)}{d(e\cos(c+dx))^{7/2}} - \frac{\cos^{\frac{7}{2}}(c+dx)\left(-\frac{\left(3a^3b-8ab^3\right)\left(a+b\sqrt{1-\cos\right.}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] -1/5*(Cos[c + d*x]^(7/2)*((-2*(3*a^4 - 8*a^2*b^2 - 5*b^4)*(a + b*Sqrt[1 - C
os[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(
2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*A
rcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sq
rt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*
b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin
[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((3*a^3*b - 8*
a*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7
/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) +
 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c +
d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]
])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x
]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/
((a - b)^2*(a + b)^2*d*(e*Cos[c + d*x])^(7/2)) + (Cos[c + d*x]^4*((2*Sec[c
+ d*x]^3*(-b + a*Sin[c + d*x]))/(5*(a^2 - b^2)) + (2*Sec[c + d*x]*(5*b^3 +
3*a^3*Sin[c + d*x] - 8*a*b^2*Sin[c + d*x]))/(5*(a^2 - b^2)^2)))/(d*(e*Cos[c
 + d*x])^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)), x)
```

maple [C] time = 7.17, size = 2399, normalized size = 4.94

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/80/d/e^4*b/(a^2-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))^3*(-2*sin(1
/2*d*x+1/2*c)^2*e+e)^(1/2)+3/80/d/e^4*b/(a^2-b^2)/(cos(1/2*d*x+1/2*c)+1/2*2
^(1/2))^2*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+3/80/d/e^4*b/(a^2-b^2)*2^(1/2
)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/2/
d/e^4*b^3/(a^2-b^2)^2*2^(1/2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))*(-2*sin(1/2*
d*x+1/2*c)^2*e+e)^(1/2)+1/2/d/e^3*b^5/(a-b)^2/(a+b)^2*sum((_R^6-_R^4*e-_R^2
*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*
```

```
ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_
R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_
Z^2+b^2*e^4))+3/80/d/e^4*b/(a^2-b^2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))^2*(-2
*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-3/80/d/e^4*b*2^(1/2)/(a^2-b^2)/(cos(1/2*d*
x+1/2*c)-1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/80/d/e^4*b/(a^2
-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))^3*(-2*sin(1/2*d*x+1/2*c)^2*e
+e)^(1/2)+1/2/d/e^4*b^3/(a^2-b^2)^2*2^(1/2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2)
)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+4/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin
(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^
2-1))^(1/2)/(a^2-b^2)^2*b^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*
c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)-2/d*(e*(2*cos(1/2*d
*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a/sin(1/2*d*x+1/2*c)^3/(e*(2
*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)^2*b^2/(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1
/2*c)^2*e)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/8/d*(e*(2*cos(1/2*
d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^3*a/sin(1/2*d*x+1/2*c)/(e*(2*
cos(1/2*d*x+1/2*c)^2-1))^(1/2)*b^2/(a-b)^2/(a+b)^2*sum(1/_alpha*(2^(1/2)/(e
*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-
3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-
3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*
x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2
))-48/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a*s
in(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1
/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+24/5/d*(
e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a*sin(1/2*d*x+
1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)
^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)
^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)+48/5/d*(e*(2*co
s(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a*sin(1/2*d*x+1/2*c)/
(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)^6-12*s
in(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+si
n(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)-24/5/d*(e*(2*cos(1/2*d*x+1/2
*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2
*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/
2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2
*c)^2*e)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-16/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1
)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/
2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+
6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e
)^(1/2)*cos(1/2*d*x+1/2*c)+6/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+
1/2*c)^2)^(1/2)/e^4*a/sin(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(
1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e \cos\left(dx + c\right)\right)^{\frac{7}{2}} \left(b \sin\left(dx + c\right) + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{7/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))),x)
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.584 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=543

$$-\frac{9ae^{11/2}\left(b^2-a^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}-\frac{9ae^{11/2}\left(b^2-a^2\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}+\frac{9a^2e^6\left(a^2-b^2\right)^2\sqrt{\cos(c}}{2b^6d\left(a^2-b\left(b-\right.\right.}$$

```
[Out] -9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+
b^2)^(1/4)/e^(1/2))/b^(11/2)/d-9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctanh(b^(1
/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d+9/35*e^3*(e*c
os(d*x+c))^(5/2)*(7*a-5*b*sin(d*x+c))/b^3/d-e*(e*cos(d*x+c))^(9/2)/b/d/(a+b
*sin(d*x+c))-3/7*(21*a^4-28*a^2*b^2+5*b^4)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/
b^6/d/(e*cos(d*x+c))^(1/2)+9/2*a^2*(a^2-b^2)^2*e^6*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/
2)),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x
+c))^(1/2)+9/2*a^2*(a^2-b^2)^2*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos
(d*x+c)^(1/2)/b^6/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-3/7*e
^5*(21*a*(a^2-b^2)-b*(7*a^2-5*b^2)*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^5/d
```

Rubi [A] time = 1.52, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2693, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{3e^5\sqrt{e\cos(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\sin(c+dx)\right)}{7b^5d}-\frac{9ae^{11/2}\left(b^2-a^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}-\frac{9ae^{11}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-9*a*(-a^2 + b^2)^(5/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(11/2)*d) - (9*a*(-a^2 + b^2)^(5/4)*e^(11/
2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2
*b^(11/2)*d) - (3*(21*a^4 - 28*a^2*b^2 + 5*b^4)*e^6*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2])/(7*b^6*d*Sqrt[e*Cos[c + d*x]]) + (9*a^2*(a^2 - b^2)^
2*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)
/2, 2])/(2*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (
9*a^2*(a^2 - b^2)^2*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
+ b^2]), (c + d*x)/2, 2])/(2*b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*
Cos[c + d*x]]) + (9*e^3*(e*Cos[c + d*x])^(5/2)*(7*a - 5*b*Sin[c + d*x]))/(3
5*b^3*d) - (e*(e*Cos[c + d*x])^(9/2))/(b*d*(a + b*Sin[c + d*x])) - (3*e^5*S
qrt[e*Cos[c + d*x]]*(21*a*(a^2 - b^2) - b*(7*a^2 - 5*b^2)*Sin[c + d*x]))/(7
*b^5*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{\left(9e^2\right)\int \frac{(e\cos(c+dx))^{7/2}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{\left(9e^4\right)\int \frac{(e\cos(c+d\ldots}{\ldots}}{\ldots}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{3e^5\sqrt{e\cos(c+d\ldots}}{\ldots}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{3e^5\sqrt{e\cos(c+d\ldots}}{\ldots}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{3e^5\sqrt{e\cos(c+d\ldots}}{\ldots}$$

$$= -\frac{3\left(21a^4-28a^2b^2+5b^4\right)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7b^6d\sqrt{e\cos(c+dx)}} + \frac{9e^3(e\cos(c+dx))^{5/2}(7a\ldots}{35b^3d\ldots}$$

$$= -\frac{3\left(21a^4-28a^2b^2+5b^4\right)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7b^6d\sqrt{e\cos(c+dx)}} + \frac{9a^2\left(-a^2+b^2\right)^{3/2}e^6\sqrt{\mathrm{co}\ldots}}{2b^6\left(b-\sqrt{-\ldots}\right.}$$

$$= -\frac{9a\left(-a^2+b^2\right)^{5/4}e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d} - \frac{9a\left(-a^2+b^2\right)^{5/4}e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\,\mathrm{co}\ldots}}{\sqrt[4]{-a^2\ldots}}\right.}{2b^{11/2}d}$$

Mathematica [C] time = 27.80, size = 2030, normalized size = 3.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/70*((e*Cos[c + d*x])^(11/2)*((-2*(70*a^3*b - 93*a*b^3)*(a + b*Sqrt[1 - C
os[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c +
d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[
c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x
]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1
, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*
(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1
 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 +
I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] -
 (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
- Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*
x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c
 + d*x]^2]*(a + b*Sin[c + d*x])) + ((140*a^3*b - 147*a*b^3)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 +
b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[
Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a
^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
- 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d
*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^
2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/
2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos
[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*(35*a^4 - 126*a^2*b^2 + 75*b^4)*(a
+ b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4,
 Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt
[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2
, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*App
ellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])
*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sq
rt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[
Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*
Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1
- Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(b^5*d*Cos[c + d*x]^(11/2)) + ((e
*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*((2*a*Cos[2*(c + d*x)])/(5*b^3) - ((-2
8*a^2 + 17*b^2)*Sin[c + d*x])/(14*b^4) - (-a^2 + b^2)^2/(b^5*(a + b*Sin[c +
 d*x])) - Sin[3*(c + d*x)]/(14*b^2)))/d
```

<u>fricas</u> [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 13.80, size = 19829, normalized size = 36.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(11/2)/(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{11/2}}{(a+b\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c))**2,x)
[Out] Timed out
```

3.585 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=459

$$\frac{7ae^{9/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}-\frac{7ae^{9/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}-\frac{7a^2e^5\left(a^2-b^2\right)\sqrt{\cos(c+}}{2b^5d\left(b-\sqrt{b^2}\right.}$$

```
[Out] 7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/b^(9/2)/d-7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctanh(b^(1/2)*
(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d+7/15*e^3*(e*cos(d*
x+c))^(3/2)*(5*a-3*b*sin(d*x+c))/b^3/d-e*(e*cos(d*x+c))^(7/2)/b/d/(a+b*sin(
d*x+c))-7/2*a^2*(a^2-b^2)*e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+
c)^(1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-7/2*a^2*(a^2-b^2)*
e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/d/(b+(-a^2+b^
2)^(1/2))/(e*cos(d*x+c))^(1/2)+7/5*(5*a^2-3*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+
c))^(1/2)/b^4/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.12, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2693, 2865, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{7ae^{9/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}-\frac{7ae^{9/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}+\frac{7e^4\left(5a^2-3b^2\right)E\left(\frac{1}{2}(c+\right.}{5b^4d\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(9/2)*d) - (7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*A
rcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(
9/2)*d) + (7*(5*a^2 - 3*b^2)*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2])/(5*b^4*d*Sqrt[Cos[c + d*x]]) - (7*a^2*(a^2 - b^2)*e^5*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^5*(b - S
qrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (7*a^2*(a^2 - b^2)*e^5*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^5*
(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (7*e^3*(e*Cos[c + d*x])^(3
/2)*(5*a - 3*b*Sin[c + d*x]))/(15*b^3*d) - (e*(e*Cos[c + d*x])^(7/2))/(b*d*
(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} - \frac{\left(7e^2\right)\int \frac{(e\cos(c+dx))^{5/2}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d} - \frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} - \frac{\left(7e^4\right)\int \frac{\sqrt{e\cos}}{}}{}$$

$$= \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d} - \frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} + \frac{\left(7\left(5a^2-3b^2\right.\right.}{}$$

$$= \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d} - \frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} + \frac{\left(7a^2\left(a^2-b^2\right.\right.}{}$$

$$= \frac{7\left(5a^2-3b^2\right)e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\,\mathrm{si}}{15b^3d}$$

$$= \frac{7\left(5a^2-3b^2\right)e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} - \frac{7a^2\left(a^2-b^2\right)e^5\sqrt{\cos(c+dx)}\,\Pi}{2b^5\left(b-\sqrt{-a^2+b^2}\right)}$$

$$= \frac{7a\left(-a^2+b^2\right)^{3/4}e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d} - \frac{7a\left(-a^2+b^2\right)^{3/4}e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\,\mathrm{co}}}{\sqrt[4]{-a^2+}}\right.}{2b^{9/2}d}$$

Mathematica [C] time = 26.82, size = 835, normalized size = 1.82

$$7\left(-\frac{\left(5a^2-3b^2\right)\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)\cos^{\frac{3}{2}}(c+dx)b^{5/2}+3\sqrt{2}\,a\left(a^2-b^2\right)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\right.\right.}{12b^{3/2}\left(b^2-a^2\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (7*(e*Cos[c + d*x])^(9/2)*((-4*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*
Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I
*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Co
s[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((5*a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[
c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3
/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b
*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^
2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(10*b^3*d*Cos[c + d*x]^(9/2
)) + ((e*Cos[c + d*x])^(9/2)*Sec[c + d*x]^4*((4*a*Cos[c + d*x])/(3*b^3) + (
a^2*Cos[c + d*x] - b^2*Cos[c + d*x])/(b^3*(a + b*Sin[c + d*x])) - Sin[2*(c
+ d*x)]/(5*b^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 10.70, size = 20346, normalized size = 44.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c))**2,x)
[Out] Timed out
```

3.586 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=473

$$-\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}-\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}+\frac{5e^4\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}F\left(\right.}{3b^4d\sqrt{e\cos(c+dx}}$$

```
[Out] -5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/b^(7/2)/d-5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctanh(b^(1/2)
*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d-e*(e*cos(d*x+c))^
(5/2)/b/d/(a+b*sin(d*x+c))+5/3*(3*a^2-b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/
b^4/d/(e*cos(d*x+c))^(1/2)-5/2*a^2*(a^2-b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)
),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c
))^(1/2)-5/2*a^2*(a^2-b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x
+c)^(1/2)/b^4/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+5/3*e^3*(
3*a-b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^3/d
```

Rubi [A] time = 1.12, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2693, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}-\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}+\frac{5e^4\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}F\left(\right.}{3b^4d\sqrt{e\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a
^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(7/2)*d) - (5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*
ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^
(7/2)*d) + (5*(3*a^2 - b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
])/(3*b^4*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - b^2)*e^4*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^4*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - b^2)*e^4*
Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2]
)/(2*b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (5*e^3*
Sqrt[e*Cos[c + d*x]]*(3*a - b*Sin[c + d*x]))/(3*b^3*d) - (e*(e*Cos[c + d*x]
)^(5/2))/(b*d*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a-b\sin(c+dx))}{3b^3d} - \frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{\left(5e^4\right)\int \frac{-ab-\frac{1}{2}\left(3a^2-\right.}{\sqrt{e\cos(c+dx)}\,(}}{3b^3}$$

$$= \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a-b\sin(c+dx))}{3b^3d} - \frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{\left(5a\left(a^2-b^2\right)e^4\right)\int}{}$$

$$= \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a-b\sin(c+dx))}{3b^3d} - \frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{\left(5a^2\sqrt{-a^2+b^2}\,e^4\right)}{}$$

$$= \frac{5\left(3a^2-b^2\right)e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\sqrt{e\cos(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a-b\sin(c+dx}{3b^3d}$$

$$= \frac{5\left(3a^2-b^2\right)e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\sqrt{e\cos(c+dx)}} + \frac{5a^2\sqrt{-a^2+b^2}\,e^4\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b-}\right.}{2b^4\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\,c}}$$

$$= -\frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{7/2}d} - \frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{7/2}d}$$

Mathematica [C] time = 27.04, size = 1956, normalized size = 4.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*((-2*Sin[c + d*x])/(3*b^2) + (a^2 -
b^2)/(b^3*(a + b*Sin[c + d*x]))))/d + ((e*Cos[c + d*x])^(7/2)*((-8*a*b*(a +
 b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, C
```

```
os[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt
[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]
^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4
, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF
1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos
[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*A
rcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcT
an[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[
-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*C
os[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(S
qrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + (6*a*b*(a + b*Sqrt[1 - Cos[
c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^
(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c
 + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c
 + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^
2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1
[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(
2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2
))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-
a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b
^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqr
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-
a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c +
d*x]^2)*(a + b*Sin[c + d*x])) - (2*(3*a^2 - 5*b^2)*(a + b*Sqrt[1 - Cos[c +
d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])
/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/
4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2
 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Co
s[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c
+ d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a
+ b*Sin[c + d*x]))))/(6*b^3*d*Cos[c + d*x]^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 10.76, size = 14392, normalized size = 30.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e \cos\left(dx + c\right)\right)^{\frac{7}{2}}}{\left(b \sin\left(dx + c\right) + a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(7/2)/(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**2,x)
[Out] Timed out
```

3.587 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=390

$$\frac{3ae^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}-\frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}+\frac{3a^2e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+$$

```
[Out] 3/2*a*e^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
/b^(5/2)/(-a^2+b^2)^(1/4)/d-3/2*a*e^(5/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1
/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(1/4)/d-e*(e*cos(d*x+c))^(
3/2)/b/d/(a+b*sin(d*x+c))+3/2*a^2*e^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*
cos(d*x+c)^(1/2)/b^3/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+3/2*a^2*e^
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/d/(b+(-a^2+b^2)
^(1/2))/(e*cos(d*x+c))^(1/2)-3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^2/d/co
s(d*x+c)^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2693, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{3ae^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}-\frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}+\frac{3a^2e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*b^(5/2)*(-a^2 + b^2)^(1/4)*d) - (3*a*e^(5/2)*ArcTanh[(Sqrt[b]*Sqr
t[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(5/2)*(-a^2 + b^2)^(
1/4)*d) - (3*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqr
t[Cos[c + d*x]]) + (3*a^2*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[
c + d*x]]) + (3*a^2*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
+ b^2]), (c + d*x)/2, 2])/(2*b^3*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*
x]]) - (e*(e*Cos[c + d*x])^(3/2))/(b*d*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
```

```
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} - \frac{\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} - \frac{\left(3e^2\right)\int \sqrt{e\cos(c+dx)}\,dx}{2b^2} + \frac{\left(3ae^2\right)\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx}{2b^2}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} - \frac{\left(3a^2e^3\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{4b^3} + \frac{\left(3a^2e^3\right)\int}{}$$

$$= -\frac{3e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} + \frac{\left(3ae^3\right)\mathrm{Subst}\left(\int \right.}{}$$

$$= -\frac{3e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{3a^2e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+d\right.}{2b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}}$$

$$= \frac{3ae^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}\,d} - \frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}\,d} - \frac{3e^2\sqrt{e\cos(c+dx}}{b^2d\sqrt{\cos}}$$

Mathematica [C] time = 37.95, size = 371, normalized size = 0.95

$$\frac{(e\cos(c+dx))^{5/2}\left(-\frac{\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)+3\sqrt{2}\,a\left(a^2-b^2\right)^{3/4}\left(-\log\left(-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\right.\right.\right.}{}\right.}{8b^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(-8*b^(3/2)*Cos[c + d*x]^(3/2) - ((8*b^(5/2)*Appell
F1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Co
s[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d
*x]]))*(a + b*Sqrt[Sin[c + d*x]^2]))/(a^2 - b^2)))/(8*b^(5/2)*d*Cos[c + d*x
]^(5/2)*(a + b*Sin[c + d*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] Timed out
```

maple [**C**] time = 8.14, size = 13221, normalized size = 33.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x)
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cos(dx+c))^{\frac{5}{2}}}{(b \sin(dx+c)+a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(5/2)/(b*sin(d*x + c) + a)^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{(a+b\,\sin(c+d\,x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**2,x)
[Out] Timed out
```

3.588 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=404

$$\frac{a^2e^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a^2e^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^2d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\ldots}}{\sqrt{\ldots}}\right)}{2b^{3/2}d\left(b^2-\ldots\right.}$$

```
[Out] -1/2*a*e^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/b^(3/2)/(-a^2+b^2)^(3/4)/d-1/2*a*e^(3/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(
1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(3/4)/d-e^2*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)/b^2/d/(e*cos(d*x+c))^(1/2)+1/2*a^2*e^2*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^
(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(
d*x+c))^(1/2)+1/2*a^2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(
1/2)/b^2/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-e*(e*cos(d*x+c
))^(1/2)/b/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.89, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2693, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}}-\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}}+\frac{a^2e^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a^2\ldots}{2b\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(2*b^(3/2)*(-a^2 + b^2)^(3/4)*d) - (a*e^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e
*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(3/2)*(-a^2 + b^2)^(3/4
)*d) - (e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[e*Cos
[c + d*x]]) + (a^2*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
 b^2]), (c + d*x)/2, 2])/(2*b^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*C
os[c + d*x]]) + (a^2*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(2*b^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e
*Cos[c + d*x]]) - (e*Sqrt[e*Cos[c + d*x]])/(b*d*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
```

```
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} - \frac{e^2\int \frac{\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx}{2b}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{2b^2} + \frac{(ae^2)\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx}{2b^2}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} - \frac{(a^2e^2)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{4b^2\sqrt{-a^2+b^2}} - \frac{(a^2e^2)\int \cdots}{\cdots}$$

$$= -\frac{e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}} - \frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} + \frac{(ae^3)\,\mathrm{Subst}\left(\int \frac{\cdots}{(a^2-b^2\cdots}\right.}{\cdots}$$

$$= -\frac{e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}} + \frac{a^2e^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right.}{2b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\cos(c+dx\cdots}}$$

$$= -\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{e^2\sqrt{\cos(c+dx)}\,F\left(\cdots\right.}{b^2d\sqrt{e\cos(\cdots}}$$

Mathematica [C] time = 12.53, size = 614, normalized size = 1.52

$$\frac{\sin^2(c+dx)(e\cos(c+dx))^{3/2}\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\frac{5b\left(a^2-b^2\right)\sqrt{\cos(c+\cdots}}{\left(a^2+b^2\left(\cos^2(c+dx)-1\right)\right)\left(2\cos^2(c+dx)\left(2b^2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(c+dx),\frac{b^2}{\cdots}\right.\right.\right.}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((e*Cos[c + d*x])^(3/2)*Sec[c + d*x])/(b*d*(a + b*Sin[c + d*x]))) + ((e*C
os[c + d*x])^(3/2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*Appel
lF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*S
qrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4,
-1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2
*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) +
 (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)]
+ 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*S
qrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4))
)*Sin[c + d*x]^2)/(b*d*Cos[c + d*x]^(3/2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c
 + d*x]))
```

fricas [F] time = 123.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos(dx+c)}\,e\cos(dx+c)}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e*cos(d*x + c)/(b^2*cos(d*x + c)^2 - 2*a*b*s
in(d*x + c) - a^2 - b^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 8.24, size = 9301, normalized size = 23.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{3/2}}{(a+b\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.589 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=422

$$\frac{b(e\cos(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}\,d\left(b^2-a^2\right)^{5/4}}+\frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}\,d\left(b^2-a^2\right)^{5/4}}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\ }}{d\left(a^2-b^2\right)\sqrt{co}}$$

```
[Out] b*(e*cos(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))-1/2*a*arctan(b^(1/2)*
(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(5/4)/d/b
^(1/2)+1/2*a*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
*e^(1/2)/(-a^2+b^2)^(5/4)/d/b^(1/2)+1/2*a^2*e*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2
^(1/2))*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^
(1/2)+1/2*a^2*e*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/(a^
2-b^2)/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c)
)^(1/2)/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2694, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{b(e\cos(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}\,d\left(b^2-a^2\right)^{5/4}}+\frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}\,d\left(b^2-a^2\right)^{5/4}}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\ }}{d\left(a^2-b^2\right)\sqrt{co}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(2*Sqrt[b]*(-a^2 + b^2)^(5/4)*d) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e
*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*Sqrt[b]*(-a^2 + b^2)^(5/4
)*d) + (Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/((a^2 - b^2)*d*Sqrt
[Cos[c + d*x]]) + (a^2*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(2*b*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e
*Cos[c + d*x]]) + (a^2*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(2*b*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e
*Cos[c + d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Sin[c
+ d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
```

```
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^2}\,dx = \frac{b(e\cos(c+dx))^{3/2}}{\left(a^2-b^2\right)de(a+b\sin(c+dx))} + \frac{\int \frac{\sqrt{e\cos(c+dx)}\left(-a-\frac{1}{2}b\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{\left(a^2-b^2\right)de(a+b\sin(c+dx))} + \frac{\int \sqrt{e\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)} + \frac{a\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{\left(a^2-b^2\right)de(a+b\sin(c+dx))} - \frac{\left(a^2e\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{4b\left(a^2-b^2\right)} + \frac{\left(a^2e\right.\cdots}{\cdots}$$

$$= \frac{\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b(e\cos(c+dx))^{3/2}}{\left(a^2-b^2\right)de(a+b\sin(c+dx))} + \frac{(abe)\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{a^2e\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}} \cdots$$

$$= -\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\left(-a^2+b^2\right)^{5/4}d} + \frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\left(-a^2+b^2\right)^{5/4}d} + \frac{\sqrt{e\cos(c+dx)}\,E\left(\cdots\right.}{\left(a^2-b^2\right)d\sqrt{\cos\cdots}}$$

Mathematica [C] time = 16.38, size = 787, normalized size = 1.86

$$-\frac{b\cos(c+dx)\sqrt{e\cos(c+dx)}}{d\left(b^2-a^2\right)(a+b\sin(c+dx))} + \frac{\sqrt{e\cos(c+dx)}\left(-\frac{\sin^2(c+dx)\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b*Cos[c + d*x]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)*d*(a + b*Sin[c + d*x]
))) + (Sqrt[e*Cos[c + d*x]]*((-4*a*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*
Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I
*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Co
s[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*
b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-
a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1
- (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (S
```

```
qrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
 - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] +
 Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 - Cos[c +
d*x]^2)*(a + b*Sin[c + d*x]))))/(2*(a - b)*(a + b)*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 113.33, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{\left(b\sin\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^2, x)
```

maple [C] time = 9.15, size = 7033, normalized size = 16.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{\left(b\sin\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.590 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=429

$$\frac{b\sqrt{e\cos(c+dx)}}{de\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\left(b^2-a^2\right)^{7/4}}+\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\left(b^2-a^2\right)^{7/4}}-\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{d\left(a^2-b^2\right)\sqrt{e\cos}}$$

```
[Out] 3/2*a*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)
/(-a^2+b^2)^(7/4)/d/e^(1/2)+3/2*a*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^
2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(7/4)/d/e^(1/2)-(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)/(a^2-b^2)/d/(e*cos(d*x+c))^(1/2)+3/2*a^2*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/
2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*c
os(d*x+c))^(1/2)+3/2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1
/2)/(a^2-b^2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+b*(e*cos(
d*x+c))^(1/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.90, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2694, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{b\sqrt{e\cos(c+dx)}}{de\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\left(b^2-a^2\right)^{7/4}}+\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\left(b^2-a^2\right)^{7/4}}-\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{d\left(a^2-b^2\right)\sqrt{e\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (3*a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(7/4)*d*Sqrt[e]) + (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqr
t[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(7/4)*d*S
qrt[e]) - (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/((a^2 - b^2)*d*Sqr
t[e*Cos[c + d*x]]) + (3*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*d*Sqrt[e*Cos[c + d*x]]) + (3*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b
+ Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^
2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c + d*x]])/((a^2 - b^2)*
d*e*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
```

```
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}\,dx = \frac{b\sqrt{e\cos(c+dx)}}{\left(a^2-b^2\right)de(a+b\sin(c+dx))} + \frac{\int \frac{-a+\frac{1}{2}b\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx}{-a^2+b^2}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{\left(a^2-b^2\right)de(a+b\sin(c+dx))} - \frac{\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{2\left(a^2-b^2\right)} + \frac{(3a)\int \frac{}{\sqrt{e\cos(c+dx}}}{2\left(a^2\right.}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{\left(a^2-b^2\right)de(a+b\sin(c+dx))} + \frac{\left(3a^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+d\right.}}{4\left(-a^2+b^2\right)^{3/2}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}} + \frac{b\sqrt{e\cos(c+dx)}}{\left(a^2-b^2\right)de(a+b\sin(c+dx))} + \cdots$$

$$= -\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}} - \frac{3a^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\right.}{2\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)d}$$

$$= \frac{3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}} + \frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}} - \frac{\sqrt{\cos}}{\left(a^2\right.}$$

Mathematica [C] time = 23.76, size = 1181, normalized size = 2.75

$$\frac{b\cos(c+dx)}{\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))} + \frac{\left(2b\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\frac{5b\left(a^2-b^2\right)\sqrt{\cos(c}}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)b^2+\left(a^2-b^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\right.\right.\right.}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (b*Cos[c + d*x])/((a^2 - b^2)*d*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))
+ (Sqrt[Cos[c + d*x]]*((-4*a*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 -
b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
+ b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*Appell
F1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2
*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a
^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*C
os[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]
^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*
x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]]
)/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 +
```

```
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a
^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x
])) + (2*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4,
-1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[
c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1,
5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF
1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (
a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcT
an[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
+ d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c
+ d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(2*(a - b)*(a + b)*
d*Sqrt[e*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e \cos(dx + c)}\,(b \sin(dx + c) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^2), x)
```

maple [C] time = 8.90, size = 4457, normalized size = 10.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x)
[Out] -512/d*a*b*e^(7/2)/(1024*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2
)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*e^(7/2)*2^(1/2)*(-2*sin(
1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256
*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/
2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2
*c)^8-192*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d
*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^
4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b
^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*2^(1/2)*cos(1/2*d*x+1/2*
c)^5+160/d*a*b*e^3/(1024*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2
)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*e^(7/2)*2^(1/2)*(-2*sin(
1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256
*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/
2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2
```

```
*c)^8-192*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d
*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^
4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b
^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*(e*(2*cos(1/2*d*x+1/2*c)
^2-1))^(1/2)*cos(1/2*d*x+1/2*c)^4+384/d*a*b*e^(7/2)/(1024*e^(7/2)*2^(1/2)*(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6-1792*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*
(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+
1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^
6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^
2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(
a^2-b^2)*2^(1/2)*cos(1/2*d*x+1/2*c)^3+160/d*a*b*e^2/(1024*e^(7/2)*2^(1/2)*(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6-1792*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*
(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+
1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^
6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^
2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(
a^2-b^2)*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(3/2)*cos(1/2*d*x+1/2*c)^2-64/d*a*b
*e^(7/2)/(1024*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/
2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*e^(7/2)*2
^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*
e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)-
5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2
*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin
(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*2^(1/2)*cos(1/2*d*x+1/2*c)+8/d*a*b
*e/(1024*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*e^(7/2)*2^(1/2)
*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2+768*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*e^(7/2
)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b
^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2*e^4*s
in(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d
*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(5/2)-48/
d*a*b*e^3/(1024*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1
/2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*e^(7/2)*
2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192
*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)
-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^
2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*si
n(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1
/2)*cos(1/2*d*x+1/2*c)^2-8/d*a*b*e^2/(1024*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+
1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*e^(7/2
)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+256*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*
```

```
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x
+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e
^4*sin(1/2*d*x+1/2*c)^8-192*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(
1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*s
in(1/2*d*x+1/2*c)^4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d
*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*(e*(2*
cos(1/2*d*x+1/2*c)^2-1))^(3/2)+3/d*a*b*e/(a^2-b^2)*sum((_R^4+_R^2*e)/(_R^7*
b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d
*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2
*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))+1/
8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2*sum(1/_alpha/(2*_alpha^2-1)*
(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2
-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^
2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-
e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_
alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_
Z^2*b^2+a^2))-2/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*b^2/(a^2-b^2)/e*co
s(1/2*d*x+1/2*c)*(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(
4*b^2*cos(1/2*d*x+1/2*c)^4-4*b^2*cos(1/2*d*x+1/2*c)^2+a^2)+1/d*(e*(2*cos(1/
2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(
1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/16/d*(e*(2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2
*c)^2-1))^(1/2)/b^2*sum((-5*a^2+2*b^2)/(a-b)/(a+b)/(2*_alpha^2-1)/_alpha*(2
^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3
)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*
_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*
(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_al
pha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-s
in(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^
2*b^2+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^2),x)
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.591 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=492

$$-\frac{\left(2a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}-\frac{5ab-\left(2a^2+3b^2\right)\sin(c+dx)}{de\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}+\frac{b}{de\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}\,(a+$$

```
[Out] -5/2*a*b^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/(-a^2+b^2)^(9/4)/d/e^(3/2)+5/2*a*b^(3/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(
1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(3/2)+b/(a^2-b^2)/d/e/(
a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2)+(-5*a*b+(2*a^2+3*b^2)*sin(d*x+c))/(a^2
-b^2)^2/d/e/(e*cos(d*x+c))^(1/2)-5/2*a^2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1
/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/e/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(
1/2)-5/2*a^2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(s
in(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b
^2)^2/d/e/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-(2*a^2+3*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*(e*cos(d*x+c))^(1/2)/(a^2-b^2)^2/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.22, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2694, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}}+\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}}-\frac{\left(2a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (-5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(9/4)*d*e^(3/2)) + (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(9/4)*d*
e^(3/2)) - ((2*a^2 + 3*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])
/((a^2 - b^2)^2*d*e^2*Sqrt[Cos[c + d*x]]) - (5*a^2*b*Sqrt[Cos[c + d*x]]*Ell
ipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^2*(b
- Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) - (5*a^2*b*Sqrt[Cos[c + d*x]]
*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^2
*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/((a^2 - b^2)*d*e*Sqrt
[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])) - (5*a*b - (2*a^2 + 3*b^2)*Sin[c + d
*x])/((a^2 - b^2)^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}\,dx = \frac{b}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))} + \frac{\int \frac{-a+\frac{3}{2}b\sin}{(e\cos(c+dx))^{3/2}(a}}{-a^2+}$$

$$= \frac{b}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))} - \frac{5ab-\left(2a^2+3b\right.}{\left(a^2-b^2\right)^2 de\sqrt{}}$$

$$= \frac{b}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))} - \frac{5ab-\left(2a^2+3b\right.}{\left(a^2-b^2\right)^2 de\sqrt{}}$$

$$= \frac{b}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))} - \frac{5ab-\left(2a^2+3b\right.}{\left(a^2-b^2\right)^2 de\sqrt{}}$$

$$= -\frac{\left(2a^2+3b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)^2 de^2\sqrt{\cos(c+dx)}} + \frac{}{\left(a^2-b^2\right)de\sqrt{e\,c}}$$

$$= -\frac{\left(2a^2+3b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)^2 de^2\sqrt{\cos(c+dx)}} - \frac{5a^2b\sqrt{\cos(c+d}}{2\left(a^2-b^2\right)^2\left(b-\right.}$$

$$= -\frac{5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} + \frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}}$$

Mathematica [C] time = 6.41, size = 777, normalized size = 1.58

$$\cos^{\frac{3}{2}}(c+dx)\left(\frac{12\left(-\left(2a^2b+3b^3\right)\cos(2(c+dx))+4a\left(a^2-b^2\right)\sin(c+dx)-6a^2b+b^3\right)}{\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(-\frac{\left(2a^2+3b^2\right)\csc(c+dx)\left(8b^{5/2}\cos^{\frac{3}{2}}(c+d\right.}{\cdots}\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*((12*(-6*a^2*b + b^3 - (2*a^2*b + 3*b^3)*Cos[2*(c + d*x
)] + 4*a*(a^2 - b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*Sqrt[Cos[c + d*x]]) - (S
in[c + d*x]*(-(((2*a^2 + 3*b^2)*Csc[c + d*x]*(8*b^(5/2)*AppellF1[3/4, -1/2,
 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3
/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos
[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c +
 d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]])))/(Sqrt
[b]*(-a^2 + b^2))) - (48*a*(a^2 + 4*b^2)*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos
[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2
 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/
(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a
^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*S
qrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]
*(-a^2 + b^2)^(1/4))))/Sqrt[Sin[c + d*x]^2])*(a + b*Sqrt[Sin[c + d*x]^2]))/
((a - b)^2*(a + b)^2)))/(24*d*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^2), x)
```

maple [C] time = 13.27, size = 8216, normalized size = 16.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\,(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.592 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=514

$$\frac{\left(2a^2+5b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}-\frac{7a^2b^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2de^2\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{7a^2b^2\sqrt{\cdots}}{2de^2\left(a^2-b^{\cdots}\right.}$$

```
[Out] 7/2*a*b^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
/(-a^2+b^2)^(11/4)/d/e^(5/2)+7/2*a*b^(5/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(
1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(11/4)/d/e^(5/2)+b/(a^2-b^2)/d/e/
(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))+1/3*(-7*a*b+(2*a^2+5*b^2)*sin(d*x+c))
/(a^2-b^2)^2/d/e/(e*cos(d*x+c))^(3/2)+1/3*(2*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)/(a^2-b^2)^2/d/e^2/(e*cos(d*x+c))^(1/2)-7/2*a^2*b^2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2
+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-b*(b-(-a^2+b^
2)^(1/2)))/(e*cos(d*x+c))^(1/2)-7/2*a^2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(
1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*co
s(d*x+c))^(1/2)
```

Rubi [A] time = 1.31, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2694, 2866, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}+\frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}+\frac{\left(2a^2+5b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}-\frac{7a\cdots}{2de^2\left(a\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (7*a*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(11/4)*d*e^(5/2)) + (7*a*b^(5/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(11/4)*d
*e^(5/2)) + ((2*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(3*(a^2 - b^2)^2*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a^2*b^2*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^
2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a^2*b^2
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2
])/(2*(a^2 - b^2)^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d
*x]]) + b/((a^2 - b^2)*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])) - (
7*a*b - (2*a^2 + 5*b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])
^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}\,dx = \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} + \frac{\int \frac{-a+\frac{5}{2}b\sin(\cdots}{(e\cos(c+dx))^{5/2}(a+\cdots}}{-a^2+\cdots}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} - \frac{7ab-\left(2a^2+5b\cdots\right.}{3\left(a^2-b^2\right)^2 de(e\cdots}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} - \frac{7ab-\left(2a^2+5b\cdots\right.}{3\left(a^2-b^2\right)^2 de(e\cdots}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} - \frac{7ab-\left(2a^2+5b\cdots\right.}{3\left(a^2-b^2\right)^2 de(e\cdots}$$

$$= \frac{\left(2a^2+5b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3\left(a^2-b^2\right)^2 de^2\sqrt{e\cos(c+dx)}} + \frac{\cdots}{\left(a^2-b^2\right)de(e\cos(c+\cdots}$$

$$= \frac{\left(2a^2+5b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3\left(a^2-b^2\right)^2 de^2\sqrt{e\cos(c+dx)}} - \frac{7a^2b^2\sqrt{\cos(c+dx)}\,\mathrm{I}\cdots}{2\left(-a^2+b^2\right)^{5/2}\left(b-\sqrt{\cdots}\right.}$$

$$= \frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}} + \frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}} + \frac{(2\cdots}{\cdots}$$

Mathematica [C] time = 24.34, size = 1258, normalized size = 2.45

$$\frac{\left(\frac{2\sec^2(c+dx)\left(\sin(c+dx)a^2-2ba+b^2\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2} - \frac{b^3}{\left(a^2-b^2\right)^2(a+b\sin(c+dx))}\right)\cos^3(c+dx)}{d(e\cos(c+dx))^{5/2}} + \frac{\left(-\frac{2\left(5b^3+2a^2b\right)\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\cdots\right.}{\left(2\left({}_2F_1\left(\frac{5}{4};-\frac{1}{2}\cdots\right.\right.\right.}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*((-2*(2*a^3 - 16*a*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2]
)*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^
2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-
a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[
c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c +
 d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1
 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*
Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqr
t[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^
2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[
c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a
+ b*Sin[c + d*x])) - (2*(2*a^2*b + 5*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*
((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(
-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c
+ d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-
1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x
]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/
(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4
)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c
 + d*x]))))/(6*(a - b)^2*(a + b)^2*d*(e*Cos[c + d*x])^(5/2)) + (Cos[c + d*x
]^3*(-(b^3/((a^2 - b^2)^2*(a + b*Sin[c + d*x]))) + (2*Sec[c + d*x]^2*(-2*a*
b + a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(a^2 - b^2)^2)))/(d*(e*Cos[c +
 d*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^2), x)
```

maple [C] time = 17.26, size = 6022, normalized size = 11.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(e\,\cos(c+d\,x))}^{5/2}\,{(a+b\,\sin(c+d\,x))}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.593 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=574

$$\frac{b}{de\left(a^2-b^2\right)(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}-\frac{9ab-\left(2a^2+7b^2\right)\sin(c+dx)}{5de\left(a^2-b^2\right)^2(e\cos(c+dx))^{5/2}}-\frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}$$

```
[Out] -9/2*a*b^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/(-a^2+b^2)^(13/4)/d/e^(7/2)+9/2*a*b^(7/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^
(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(7/2)+b/(a^2-b^2)/d/e
/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))+1/5*(-9*a*b+(2*a^2+7*b^2)*sin(d*x+c)
)/(a^2-b^2)^2/d/e/(e*cos(d*x+c))^(5/2)+3/5*(15*a*b^3+(2*a^4-10*a^2*b^2-7*b^
4)*sin(d*x+c))/(a^2-b^2)^3/d/e^3/(e*cos(d*x+c))^(1/2)+9/2*a^2*b^3*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b
-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^3/(b-(-a^2+b^2
)^(1/2))/(e*cos(d*x+c))^(1/2)+9/2*a^2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/
2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^
(1/2)-3/5*(2*a^4-10*a^2*b^2-7*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/(a^2-b^2
)^3/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.62, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2694, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{3\left(\left(-10a^2b^2+2a^4-7b^4\right)\sin(c+dx)+15ab^3\right)}{5de^3\left(a^2-b^2\right)^3\sqrt{e\cos(c+dx)}}-\frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}+\frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (-9*a*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(13/4)*d*e^(7/2)) + (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*S
qrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(13/4)*
d*e^(7/2)) - (3*(2*a^4 - 10*a^2*b^2 - 7*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(5*(a^2 - b^2)^3*d*e^4*Sqrt[Cos[c + d*x]]) + (9*a^2*b^3*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(2*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (9*a
^2*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x
)/2, 2])/(2*(a^2 - b^2)^3*(b + Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]
) + b/((a^2 - b^2)*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])) - (9*a*
b - (2*a^2 + 7*b^2)*Sin[c + d*x])/(5*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(5/
2)) + (3*(15*a*b^3 + (2*a^4 - 10*a^2*b^2 - 7*b^4)*Sin[c + d*x]))/(5*(a^2 -
b^2)^3*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}\,dx = \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} + \frac{\int \frac{-a+\frac{7}{2}b\cdots}{(e\cos(c+dx))^{7/\cdots}}}{-a^2\cdots}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-\left(2a^2+\cdots\right.}{5\left(a^2-b^2\right)^2 de\cdots}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-\left(2a^2+\cdots\right.}{5\left(a^2-b^2\right)^2 de\cdots}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-\left(2a^2+\cdots\right.}{5\left(a^2-b^2\right)^2 de\cdots}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-\left(2a^2+\cdots\right.}{5\left(a^2-b^2\right)^2 de\cdots}$$

$$= -\frac{3\left(2a^4-10a^2b^2-7b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5\left(a^2-b^2\right)^3 de^4\sqrt{\cos(c+dx)}} + \frac{\cdots}{\left(a^2-\cdots\right.}$$

$$= -\frac{3\left(2a^4-10a^2b^2-7b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5\left(a^2-b^2\right)^3 de^4\sqrt{\cos(c+dx)}} + \frac{9a^2b\cdots}{2\left(a^2\cdots\right.}$$

$$= -\frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4} de^{7/2}} + \frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4} de^{7/2}} \cdots$$

Mathematica [C] time = 6.77, size = 949, normalized size = 1.65

$$\frac{\cos^4(c+dx)\left(\frac{\cos(c+dx)b^5}{(a^2-b^2)^3(a+b\sin(c+dx))}+\frac{2\sec^3(c+dx)\left(\sin(c+dx)a^2-2ba+b^2\sin(c+dx)\right)}{5(a^2-b^2)^2}+\frac{2\sec(c+dx)\left(3\sin(c+dx)a^4-15b^2\sin(c+dx)a^2+20b^3a\right.}{5(a^2-b^2)^3}\right.}{d(e\cos(c+dx))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(7/2)*((-2*(2*a^5 - 10*a^3*b^2 - 22*a*b^4)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)
*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[
Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*S
in[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((2*a^4*b -
10*a^2*b^3 - 7*b^5)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/
4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c +
 d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*
Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt
[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*
(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2
] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]])
)*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[
c + d*x]))))/(10*(a - b)^3*(a + b)^3*d*(e*Cos[c + d*x])^(7/2)) + (Cos[c + d
*x]^4*((b^5*Cos[c + d*x])/((a^2 - b^2)^3*(a + b*Sin[c + d*x])) + (2*Sec[c +
 d*x]^3*(-2*a*b + a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(5*(a^2 - b^2)^2) +
 (2*Sec[c + d*x]*(20*a*b^3 + 3*a^4*Sin[c + d*x] - 15*a^2*b^2*Sin[c + d*x] -
 8*b^4*Sin[c + d*x]))/(5*(a^2 - b^2)^3)))/(d*(e*Cos[c + d*x])^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(b\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^2), x)
```

maple [C] time = 25.88, size = 10743, normalized size = 18.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{7/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.594 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=575

$$\frac{11ae^6\left(45a^2-37b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{20b^6d\sqrt{\cos(c+dx)}}+\frac{11e^5(e\cos(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\sin(c+dx)\right)}{60b^5d}-\cdots$$

```
[Out] -11/8*(9*a^4-11*a^2*b^2+2*b^4)*e^(13/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)
/(-a^2+b^2)^(1/4)/e^(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d+11/8*(9*a^4-11*a^2*b
^2+2*b^4)*e^(13/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^
(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-1/2*e*(e*cos(d*x+c))^(11/2)/b/d/(a+b*sin
(d*x+c))^2-11/28*e^3*(e*cos(d*x+c))^(7/2)*(9*a+2*b*sin(d*x+c))/b^3/d/(a+b*s
in(d*x+c))+11/60*e^5*(e*cos(d*x+c))^(3/2)*(45*a^2-10*b^2-27*a*b*sin(d*x+c))
/b^5/d-11/8*a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1
/2))*cos(d*x+c)^(1/2)/b^7/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-11/8*
a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x
+c)^(1/2)/b^7/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+11/20*a*(45*a^2-3
7*b^2)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^6/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.41, antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2693, 2863, 2865, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{11e^5(e\cos(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\sin(c+dx)\right)}{60b^5d}-\frac{11e^{13/2}\left(-11a^2b^2+9a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{13/2}d\sqrt[4]{b^2-a^2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-11*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d
*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) + (1
1*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x
]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) + (11*
a*(45*a^2 - 37*b^2)*e^6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(20
*b^6*d*Sqrt[Cos[c + d*x]]) - (11*a*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^7*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*b^
7*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (11*a*(9*a^4 - 11*a^2*b^
2 + 2*b^4)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(8*b^7*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e
*(e*Cos[c + d*x])^(11/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (11*e^3*(e*Cos[c
 + d*x])^(7/2)*(9*a + 2*b*Sin[c + d*x]))/(28*b^3*d*(a + b*Sin[c + d*x])) +
(11*e^5*(e*Cos[c + d*x])^(3/2)*(5*(9*a^2 - 2*b^2) - 27*a*b*Sin[c + d*x]))/(
60*b^5*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{\left(11e^2\right)\int \frac{(e\cos(c+dx))^{9/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{\left(11e^4\right)\int}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{11e^5(e\,c}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{11e^5(e\,c}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{11e^5(e\,c}{}$$

$$= \frac{11a\left(45a^2-37b^2\right)e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{20b^6d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} -$$

$$= \frac{11a\left(45a^2-37b^2\right)e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{20b^6d\sqrt{\cos(c+dx)}} - \frac{11a\left(9a^4-11a^2b^2+2b^4\right)e}{8b^7\left(b-\sqrt{}\right.}$$

$$= -\frac{11\left(9a^4-11a^2b^2+2b^4\right)e^{13/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}\,d} + \frac{11\left(9a^4-11a^2b^2+2b^4\right)e^{1}}{8b^{13/2}\sqrt[4]{-}}$$

Mathematica [C] time = 27.05, size = 932, normalized size = 1.62

$$11\left(-\frac{\left(45a^3-37ab^2\right)\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)\cos^{\frac{3}{2}}(c+dx)b^{5/2}+3\sqrt{2}\,a\left(a^2-b^2\right)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(c+d}}{\sqrt[4]{a^2-b^2}}\right.\right.\right.}{12b^{3/2}\left(b^2-\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (11*(e*Cos[c + d*x])^(13/2)*((-2*(18*a^2*b - 10*b^3)*(a + b*Sqrt[1 - Cos[c
+ d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]
^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*Arc
Tan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan
[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a
^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos
[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt
[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c +
d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((45*a^3 - 37*a*b^2
)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, C
os[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sq
rt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]]
)/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a
```

```
^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*
Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/
(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(40*b
^5*d*Cos[c + d*x]^(13/2)) + ((e*Cos[c + d*x])^(13/2)*Sec[c + d*x]^6*(-1/42*
((-168*a^2 + 65*b^2)*Cos[c + d*x])/b^5 - Cos[3*(c + d*x)]/(14*b^3) + (-(a^4
*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c + d*x])/(2*b^5*(a + b*S
in[c + d*x])^2) + (19*(a^3*Cos[c + d*x] - a*b^2*Cos[c + d*x]))/(4*b^5*(a +
b*Sin[c + d*x])) - (3*a*Sin[2*(c + d*x)])/(5*b^4)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 36.49, size = 111631, normalized size = 194.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{13}{2}}}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(13/2)/(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{13/2}}{(a+b\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(13/2)/(a + b*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(13/2)/(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.595 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=589

$$\frac{3ae^6\left(21a^2-13b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^6d\sqrt{e\cos(c+dx)}}+\frac{3e^5\sqrt{e\cos(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\sin(c+dx)\right)}{4b^5d}+\frac{9e^{11/2}\left(7\right.}{}$$

```
[Out] 9/8*(7*a^4-9*a^2*b^2+2*b^4)*e^(11/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-
a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/(-a^2+b^2)^(3/4)/d+9/8*(7*a^4-9*a^2*b^2+2*
b^4)*e^(11/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/b^(11/2)/(-a^2+b^2)^(3/4)/d-1/2*e*(e*cos(d*x+c))^(9/2)/b/d/(a+b*sin(d*x+c
))^2-9/20*e^3*(e*cos(d*x+c))^(5/2)*(7*a+2*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+
c))+3/4*a*(21*a^2-13*b^2)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(e*cos(d*x+
c))^(1/2)-9/8*a*(7*a^4-9*a^2*b^2+2*b^4)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(
1/2))*cos(d*x+c)^(1/2)/b^6/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1
/2)-9/8*a*(7*a^4-9*a^2*b^2+2*b^4)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*
cos(d*x+c)^(1/2)/b^6/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+3/
4*e^5*(21*a^2-6*b^2-7*a*b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^5/d
```

Rubi [A] time = 1.49, antiderivative size = 589, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2693, 2863, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{3e^5\sqrt{e\cos(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\sin(c+dx)\right)}{4b^5d}+\frac{9e^{11/2}\left(-9a^2b^2+7a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{11/2}d\left(b^2-a^2\right)^{3/4}}+\frac{9e^{11/2}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (9*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]
])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) + (9*(7
*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/(
(-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) + (3*a*(21*
a^2 - 13*b^2)*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(4*b^6*d*Sq
rt[e*Cos[c + d*x]]) - (9*a*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^6*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*b^6*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (9*a*(7*a^4 - 9*a^2*b^2
 + 2*b^4)*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (
c + d*x)/2, 2])/(8*b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*
x]]) - (e*(e*Cos[c + d*x])^(9/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (9*e^3*(
e*Cos[c + d*x])^(5/2)*(7*a + 2*b*Sin[c + d*x]))/(20*b^3*d*(a + b*Sin[c + d*
x])) + (3*e^5*Sqrt[e*Cos[c + d*x]]*(3*(7*a^2 - 2*b^2) - 7*a*b*Sin[c + d*x])
)/(4*b^5*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{(9e^2)\int \frac{(e\cos(c+dx))^{7/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{(9e^4)\int -}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{3e^5\sqrt{e\cos}}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{3e^5\sqrt{e\cos}}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{3e^5\sqrt{e\cos}}{}$$

$$= \frac{3a(21a^2-13b^2)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^6d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - 9e^3$$

$$= \frac{3a(21a^2-13b^2)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^6d\sqrt{e\cos(c+dx)}} - \frac{9a(7a^4-9a^2b^2+2b^4)e^6\sqrt{\cos}}{8b^6\left(a^2-b\left(b-\sqrt{-}\right.\right.}$$

$$= \frac{9(7a^4-9a^2b^2+2b^4)e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{11/2}(-a^2+b^2)^{3/4}d} + \frac{9(7a^4-9a^2b^2+2b^4)e^{11/2}\tan}{8b^{11/2}(-a^2+b}$$

Mathematica [C] time = 27.62, size = 2024, normalized size = 3.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*(-1/5*Cos[2*(c + d*x)]/b^3 - (2*a*S
in[c + d*x])/b^4 - (-a^2 + b^2)^2/(2*b^5*(a + b*Sin[c + d*x])^2) + (17*a*(a
^2 - b^2))/(4*b^5*(a + b*Sin[c + d*x]))))/d + (3*(e*Cos[c + d*x])^(11/2)*((
-2*(30*a^2*b - 16*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)
]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4
, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^
2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2)))
- ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(
-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^
2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b
^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) +
((40*a^2*b - 14*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1
/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-
a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^
2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^
```

```
(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1
/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]
^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1
- Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
 (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, C
os[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5
/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c
+ d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*
Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2
+ b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c
 + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(
Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2
*(25*a^3 - 37*a*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4
, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b
^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
 b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c
+ d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2)))
 + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)
] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] -
Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]]
+ b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4
)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(40*b^5*d
*Cos[c + d*x]^(11/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 39.14, size = 85607, normalized size = 145.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.596 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=483

$$\frac{7e^{9/2}\left(5a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}-\frac{7e^{9/2}\left(5a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}+\frac{7ae^5\left(5a^2-2b^2\right)\sqrt{\cos(c+dx}}{8b^5d\left(b-\sqrt{b^2-a^2}\right.}$$

```
[Out] 7/8*(5*a^2-2*b^2)*e^(9/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(1/4)/d-7/8*(5*a^2-2*b^2)*e^(9/2)*arctanh(b
^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(1
/4)/d-1/2*e*(e*cos(d*x+c))^(7/2)/b/d/(a+b*sin(d*x+c))^2-7/12*e^3*(e*cos(d*x
+c))^(3/2)*(5*a+2*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))+7/8*a*(5*a^2-2*b^2)*
e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/d/(b-(-a^2+b^
2)^(1/2))/(e*cos(d*x+c))^(1/2)+7/8*a*(5*a^2-2*b^2)*e^5*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)
^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))
^(1/2)-35/4*a*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^4/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2693, 2863, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{7e^{9/2}\left(5a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}-\frac{7e^{9/2}\left(5a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}+\frac{7ae^5\left(5a^2-2b^2\right)\sqrt{\cos(c+dx}}{8b^5d\left(b-\sqrt{b^2-a^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (7*(5*a^2 - 2*b^2)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) - (7*(5*a^2 - 2*b^2)*
e^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) - (35*a*e^4*Sqrt[e*Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2])/(4*b^4*d*Sqrt[Cos[c + d*x]]) + (7*a*(5*a^2 - 2*b^2)*e^
5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
2])/(8*b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (7*a*(5*a^2 - 2
*b^2)*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c +
d*x)/2, 2])/(8*b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*C
os[c + d*x])^(7/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (7*e^3*(e*Cos[c + d*x]
)^(3/2)*(5*a + 2*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{\left(7e^2\right)\int \frac{(e\cos(c+dx))^{5/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+2b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))} + \frac{\left(7e^4\right)\int \frac{\sqrt{e\cos\ldots}}{\ldots}}{\ldots}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+2b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))} - \frac{\left(35ae^4\right)\int \sqrt{\ldots}}{\ldots}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+2b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))} - \frac{\left(7a\left(5a^2-2\ldots\right.\right.}{\ldots}$$

$$= -\frac{35ae^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx\ldots}{12b^3d(a\ldots}$$

$$= -\frac{35ae^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{\cos(c+dx)}} + \frac{7a\left(5a^2-2b^2\right)e^5\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b\ldots}}\right.}{8b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c\ldots}}$$

$$= \frac{7\left(5a^2-2b^2\right)e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}\,d} - \frac{7\left(5a^2-2b^2\right)e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}\,d}$$

Mathematica [C] time = 25.99, size = 777, normalized size = 1.61

$$(e\cos(c+dx))^{9/2}\left(\frac{28\sin(c+dx)\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(\frac{a\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)}{3\left(a^2-b^2\right)}+\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(-\log\left(-(1+i)\sqrt{b}\sqrt[4]{b^2-a^2}\sqrt{\cos(c+dx)}+\sqrt{b^2\ldots}\right.\right.}{\ldots}\right.}{b^2\sqrt{\sin^2(c+d\ldots}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(9/2)*((-16*Cos[c + d*x]^(3/2))/(3*b^3) + (4*(a^2 - b^2)*
Cos[c + d*x]^(3/2))/(b^3*(a + b*Sin[c + d*x])^2) - (22*a*Cos[c + d*x]^(3/2)
)/(b^3*(a + b*Sin[c + d*x])) + (35*a*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4,
 Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*
Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x
]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/
(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4
)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*(a + b*Sqrt[Si
n[c + d*x]^2]))/(12*b^(9/2)*(-a^2 + b^2)*(a + b*Sin[c + d*x])) + (28*((a*Ap
pellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*
Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 +
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[
Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x]*(a + b*Sqrt[
Sin[c + d*x]^2]))/(b^2*Sqrt[Sin[c + d*x]^2]*(a + b*Sin[c + d*x]))))/(8*d*Co
s[c + d*x]^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 29.46, size = 85489, normalized size = 177.00

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cos (dx + c))^{\frac{9}{2}}}{(b \sin (dx + c) + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

3.597 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=497

$$-\frac{5e^{7/2}\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}-\frac{5e^{7/2}\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}+\frac{5ae^4\left(3a^2-2b^2\right)\sqrt{\cos(c\cdots}}{8b^4d\left(a^2-b\left(b-\sqrt{\cdots}\right.\right.}$$

```
[Out] -5/8*(3*a^2-2*b^2)*e^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(3/4)/d-5/8*(3*a^2-2*b^2)*e^(7/2)*arctanh(
b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(
3/4)/d-1/2*e*(e*cos(d*x+c))^(5/2)/b/d/(a+b*sin(d*x+c))^2-15/4*a*e^4*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)/b^4/d/(e*cos(d*x+c))^(1/2)+5/8*a*(3*a^2-2*b^2)*e^4*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(a^2-b*(b-(-a^2+b^
2)^(1/2)))/(e*cos(d*x+c))^(1/2)+5/8*a*(3*a^2-2*b^2)*e^4*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2
)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*co
s(d*x+c))^(1/2)-5/4*e^3*(3*a+2*b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^3/d/(a+
b*sin(d*x+c))
```

Rubi [A] time = 1.08, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2693, 2863, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{5e^{7/2}\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}-\frac{5e^{7/2}\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}+\frac{5ae^4\left(3a^2-2b^2\right)\sqrt{\cos(c\cdots}}{8b^4d\left(a^2-b\left(b-\sqrt{\cdots}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-5*(3*a^2 - 2*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) - (5*(3*a^2 - 2*b^2)
*e^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) - (15*a*e^4*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(4*b^4*d*Sqrt[e*Cos[c + d*x]]) + (5*a*(3*a^2 - 2*b^2)*e
^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
 2])/(8*b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (5*a
*(3*a^2 - 2*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (c + d*x)/2, 2])/(8*b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos
[c + d*x]]) - (e*(e*Cos[c + d*x])^(5/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (
5*e^3*Sqrt[e*Cos[c + d*x]]*(3*a + 2*b*Sin[c + d*x]))/(4*b^3*d*(a + b*Sin[c
+ d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a+2b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))} + \frac{\left(5e^4\right)\int \frac{}{\sqrt{e}\cdots}}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a+2b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))} - \frac{\left(15ae^4\right)\int \cdots}{8\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a+2b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))} - \frac{\left(5a\left(3a^2-\cdots\right.\right.}{\cdots}$$

$$= -\frac{15ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+\cdots}}{4b^3d(a\cdots}$$

$$= -\frac{15ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{e\cos(c+dx)}} + \frac{5a\left(3a^2-2b^2\right)e^4\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2\cdots}}\right.}{8b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\cdots}}$$

$$= -\frac{5\left(3a^2-2b^2\right)e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{5\left(3a^2-2b^2\right)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+\cdots}}{\sqrt[4]{-a^2+b^2}\sqrt{\cdots}}\right.}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d}$$

Mathematica [C] time = 26.37, size = 1954, normalized size = 3.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*((a^2 - b^2)/(2*b^3*(a + b*Sin[c + d
*x])^2) - (9*a)/(4*b^3*(a + b*Sin[c + d*x]))))/d - ((e*Cos[c + d*x])^(7/2)*
((-12*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/
```

```
2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c +
d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4,
 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 +
b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
+ b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*
Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
 + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[
c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + (4*b*(a + b*Sqr
t[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan
[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a
^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*
Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a
*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2
)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2
)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x
]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[
c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)
*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(
b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 +
2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (14*a*(a + b*Sqrt[1 - Cos[c + d*x
]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((
-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x
]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (
b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 +
b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c
 + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d
*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b
*Sin[c + d*x]))))/(8*b^3*d*Cos[c + d*x]^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C]   time = 29.36, size = 65216, normalized size = 131.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x)
[Out] result too large to display
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e \cos \left(dx + c\right)\right)^{\frac{7}{2}}}{\left(b \sin \left(dx + c\right) + a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(7/2)/(b*sin(d*x + c) + a)^3, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

3.598 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=505

$$\frac{3ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{4b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{3ae(e\cos(c+dx))^{3/2}}{4bd\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3e^{5/2}\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\left(b^2-a^2\right)^{5/4}}-3$$

```
[Out] 3/8*(a^2-2*b^2)*e^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4
)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/d-3/8*(a^2-2*b^2)*e^(5/2)*arctanh(b^(1/
2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/
d-1/2*e*(e*cos(d*x+c))^(3/2)/b/d/(a+b*sin(d*x+c))^2+3/4*a*e*(e*cos(d*x+c))^
(3/2)/b/(a^2-b^2)/d/(a+b*sin(d*x+c))-3/8*a*(a^2-2*b^2)*e^3*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+
b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/(a^2-b^2)/d/(b-(-a^2+b^2)^(1/2))/
(e*cos(d*x+c))^(1/2)-3/8*a*(a^2-2*b^2)*e^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1
/2))*cos(d*x+c)^(1/2)/b^3/(a^2-b^2)/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(
1/2)+3/4*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/cos(d*x+c)^(
1/2)
```

Rubi [A] time = 1.10, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2693, 2864, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{3e^{5/2}\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\left(b^2-a^2\right)^{5/4}}-\frac{3e^{5/2}\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\left(b^2-a^2\right)^{5/4}}+\frac{3ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{4b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*(a^2 - 2*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2
)^(1/4)*Sqrt[e])])/(8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) - (3*(a^2 - 2*b^2)*e^(5
/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(
8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) + (3*a*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) - (3*a*(a^2 - 2*b^
2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x
)/2, 2])/(8*b^3*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
- (3*a*(a^2 - 2*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(8*b^3*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(3/2))/(2*b*d*(a + b*Sin[c + d*x])^
2) + (3*a*e*(e*Cos[c + d*x])^(3/2))/(4*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} - \frac{\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}\left(b+\right.}{a+b\sin(}}{4b\left(a^2-\right.}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\left(3ae^2\right)\int\sqrt{e\cos(c+}}{8b^2\left(a^2-b^2\right)}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\left(3a\left(a^2-2b^2\right)e^3\right)\int}{16}$$

$$= \frac{3ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+}{4b\left(a^2-b^2\right)d(a+b}$$

$$= \frac{3ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{3a\left(a^2-2b^2\right)e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\right.}{8b^3\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c}}$$

$$= \frac{3\left(a^2-2b^2\right)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d} - \frac{3\left(a^2-2b^2\right)e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d} + $$

Mathematica [C] time = 24.06, size = 831, normalized size = 1.65

$$\frac{\sec^2(c+dx)\left(-\frac{3a\cos(c+dx)}{4b\left(b^2-a^2\right)(a+b\sin(c+dx))} - \frac{\cos(c+dx)}{2b(a+b\sin(c+dx))^2}\right)(e\cos(c+dx))^{5/2}}{d} + \frac{3\left(-\frac{a\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*(-1/2*Cos[c + d*x]/(b*(a + b*Sin[c +
 d*x])^2) - (3*a*Cos[c + d*x])/(4*b*(-a^2 + b^2)*(a + b*Sin[c + d*x]))))/d
+ (3*(e*Cos[c + d*x])^(5/2)*((-4*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*
Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I
*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Co
s[c + d*x]^2]*(a + b*Sin[c + d*x])) - (a*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(
8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[
1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 +
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^
2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]
 + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x
]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c
+ d*x]^2)*(a + b*Sin[c + d*x]))))/(8*(a - b)*b*(a + b)*d*Cos[c + d*x]^(5/2)
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 31.06, size = 63272, normalized size = 125.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

3.599 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=519

$$-\frac{ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}+\frac{ae^2\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^2d\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{ae^2\left(a^2+2b^2\right)\sqrt{\cdots}}{8b^2d\left(a^2-b^2\right)\left(\cdots\right.}$$

```
[Out] 1/8*(a^2+2*b^2)*e^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4
)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)/d+1/8*(a^2+2*b^2)*e^(3/2)*arctanh(b^(1/
2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)/
d-1/4*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/(e*cos(d*x+c))^(1/2
)+1/8*a*(a^2+2*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/
2)/b^2/(a^2-b^2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+1/8*a*
(a^2+2*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^2/(
a^2-b^2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-1/2*e*(e*cos(d
*x+c))^(1/2)/b/d/(a+b*sin(d*x+c))^2+1/4*a*e*(e*cos(d*x+c))^(1/2)/b/(a^2-b^2
)/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 1.14, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2693, 2864, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{e^{3/2}\left(a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\left(b^2-a^2\right)^{7/4}}+\frac{e^{3/2}\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\left(b^2-a^2\right)^{7/4}}-\frac{ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{4b^2d\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^
(1/4)*Sqrt[e])])/(8*b^(3/2)*(-a^2 + b^2)^(7/4)*d) + ((a^2 + 2*b^2)*e^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^
(3/2)*(-a^2 + b^2)^(7/4)*d) - (a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2])/(4*b^2*(a^2 - b^2)*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^2 + 2*b^2)*e^2*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(8*b^2*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]
) + (a*(a^2 + 2*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(8*b^2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2
]))*d*Sqrt[e*Cos[c + d*x]]) - (e*Sqrt[e*Cos[c + d*x]])/(2*b*d*(a + b*Sin[c
+ d*x])^2) + (a*e*Sqrt[e*Cos[c + d*x]])/(4*b*(a^2 - b^2)*d*(a + b*Sin[c + d
*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} - \frac{e^2\int \frac{\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(c+dx)}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{e^2\int \frac{b-\frac{1}{2}a\sin(c}{\sqrt{e\cos(c+dx)}\,(a+}}{4b\left(a^2-\right.}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(c+dx)}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\left(ae^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}}{8b^2\left(a^2-b^2\right)}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(c+dx)}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\left(a\left(a^2+2b^2\right)e^2\right)\int}{1}$$

$$= -\frac{ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}} - \frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(}}{4b\left(a^2-b^2\right)d(a+}$$

$$= -\frac{ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}} - \frac{a\left(a^2+2b^2\right)e^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\right.}{8b^2\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos}}$$

$$= \frac{\left(a^2+2b^2\right)e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}\left(-a^2+b^2\right)^{7/4}d} + \frac{\left(a^2+2b^2\right)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}\left(-a^2+b^2\right)^{7/4}d} -$$

Mathematica [C] time = 23.75, size = 1211, normalized size = 2.33

$$\frac{(e\cos(c+dx))^{3/2}\sec(c+dx)\left(-\frac{a}{4b\left(b^2-a^2\right)(a+b\sin(c+dx))} - \frac{1}{2b(a+b\sin(c+dx))^2}\right)}{d} - \frac{(e\cos(c+dx))^{3/2}\left(\frac{4b\left(a+b\sqrt{1-\cos^2(c+dx)}\right)}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*Sec[c + d*x]*(-1/2*1/(b*(a + b*Sin[c + d*x])^2) - a
/(4*b*(-a^2 + b^2)*(a + b*Sin[c + d*x]))))/d - ((e*Cos[c + d*x])^(3/2)*((4*
b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])
/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c
+ d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2,
2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*A
ppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)
])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b
]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] -
2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
 I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*
x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (2*a*(a + b*Sqrt[1 -
Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]
^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c +
d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*
Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c
+ d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/
2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]
^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqr
t[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]
*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^
2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]
))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x
]^2)*(a + b*Sin[c + d*x]))))/(8*(a - b)*b*(a + b)*d*Cos[c + d*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 30.14, size = 45147, normalized size = 86.99

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\ \cos(c+d\,x))^{3/2}}{(a+b\ \sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

3.600 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=514

$$\frac{5ab(e\cos(c+dx))^{3/2}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b(e\cos(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\sin(c+dx))^2}+\frac{\sqrt{e}\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8\sqrt{b}\,d\left(b^2-a^2\right)^{9/4}}-\frac{\sqrt{e}}{}$$

```
[Out] 1/2*b*(e*cos(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^2+5/4*a*b*(e*cos(
d*x+c))^(3/2)/(a^2-b^2)^2/d/e/(a+b*sin(d*x+c))+1/8*(3*a^2+2*b^2)*arctan(b^(
1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(9/4
)/d/b^(1/2)-1/8*(3*a^2+2*b^2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(9/4)/d/b^(1/2)+1/8*a*(3*a^2+2*b^2)*e*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*
c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(b-(-
a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+1/8*a*(3*a^2+2*b^2)*e*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+
b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(b+(-a^2+b^2)^(1/2))/
(e*cos(d*x+c))^(1/2)+5/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/(a^2-b^2)^2/d/co
s(d*x+c)^(1/2)
```

Rubi [A] time = 1.19, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2694, 2864, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{5ab(e\cos(c+dx))^{3/2}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b(e\cos(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\sin(c+dx))^2}+\frac{\sqrt{e}\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8\sqrt{b}\,d\left(b^2-a^2\right)^{9/4}}-\frac{\sqrt{e}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((3*a^2 + 2*b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2
)^(1/4)*Sqrt[e])])/(8*Sqrt[b]*(-a^2 + b^2)^(9/4)*d) - ((3*a^2 + 2*b^2)*Sqrt
[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(
8*Sqrt[b]*(-a^2 + b^2)^(9/4)*d) + (5*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2])/(4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(3*a^2 + 2*b^2)*e*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(8*b*(a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*(3*
a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(8*b*(a^2 - b^2)^2*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c +
 d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/(2*(a^2 - b^2)*d*e*(a + b*Sin[c + d*x]
)^2) + (5*a*b*(e*Cos[c + d*x])^(3/2))/(4*(a^2 - b^2)^2*d*e*(a + b*Sin[c + d
*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^3}\,dx = \frac{b(e\cos(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} - \frac{\int \frac{\sqrt{e\cos(c+dx)}\left(-2a+\frac{1}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{5ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))} + \frac{\int \frac{\sqrt{e\cos(c+dx}}{}}{}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{5ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))} + \frac{(5a)\int \sqrt{e\cos}}{8\left(a^2\right.}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{5ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))} - \frac{\left(a\left(3a^2+2b\right.\right.}{}$$

$$= \frac{5a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{b(e\cos(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{5ab(}{4\left(a^2-b^2\right)}$$

$$= \frac{5a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{a\left(3a^2+2b^2\right)e\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c\right.}{8b\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+}}$$

$$= \frac{\left(3a^2+2b^2\right)\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\left(-a^2+b^2\right)^{9/4}d} - \frac{\left(3a^2+2b^2\right)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\left(-a^2+b^2\right)^{9/4}d} + \frac{5}{}$$

Mathematica [C] time = 24.33, size = 837, normalized size = 1.63

$$\frac{\sqrt{e\cos(c+dx)}\left(\frac{5ab\cos(c+dx)}{4\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\cos(c+dx)}{2\left(a^2-b^2\right)(a+b\sin(c+dx))^2}\right)}{d} + \frac{\sqrt{e\cos(c+dx)}\left(-\frac{5a\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*((b*Cos[c + d*x])/(2*(a^2 - b^2)*(a + b*Sin[c + d*x])
^2) + (5*a*b*Cos[c + d*x])/(4*(a^2 - b^2)^2*(a + b*Sin[c + d*x]))))/d + (Sq
rt[e*Cos[c + d*x]]*((-2*(8*a^2 + 2*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((
a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] +
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1
 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (5*a*(a + b*Sqrt[1 - Cos[c + d*x
]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d
*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcT
an[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
+ d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 -
 Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(8*(a - b)^2*(a + b)^2*d*Sqrt[Cos[
c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cos\left(dx + c\right)}}{\left(b \sin\left(dx + c\right) + a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^3, x)
```

maple [C] time = 30.27, size = 36688, normalized size = 71.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cos\left(dx + c\right)}}{\left(b \sin\left(dx + c\right) + a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.601 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=520

$$\frac{7ab\sqrt{e\cos(c+dx)}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sqrt{e\cos(c+dx)}}{2de\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{3\sqrt{b}\left(5a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8d\sqrt{e}\left(b^2-a^2\right)^{11/4}}-$$

```
[Out] -3/8*(5*a^2+2*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(
1/2))*b^(1/2)/(-a^2+b^2)^(11/4)/d/e^(1/2)-3/8*(5*a^2+2*b^2)*arctanh(b^(1/2)
*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(11/4)/d
/e^(1/2)-7/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/(e*cos(d*x+c))^(1/
2)+3/8*a*(5*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2
)/(a^2-b^2)^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+3/8*a*(5*
a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^
2/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+1/2*b*(e*cos(d*x+c))^
(1/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^2+7/4*a*b*(e*cos(d*x+c))^(1/2)/(a^2-b^
2)^2/d/e/(a+b*sin(d*x+c))
```

Rubi [A] time = 1.23, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2694, 2864, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{7ab\sqrt{e\cos(c+dx)}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sqrt{e\cos(c+dx)}}{2de\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{3\sqrt{b}\left(5a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8d\sqrt{e}\left(b^2-a^2\right)^{11/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (-3*Sqrt[b]*(5*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) - (3*Sqrt[b]*(5*a^2
 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) - (7*a*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2])/(4*(a^2 - b^2)^2*d*Sqrt[e*Cos[c + d*x]]) + (3*a*(5*a^2 + 2
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)
/2, 2])/(8*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c +
d*x]]) + (3*a*(5*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^2*(a^2 - b*(b + Sqrt[-a^2 +
b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c + d*x]])/(2*(a^2 - b^2)*d*
e*(a + b*Sin[c + d*x])^2) + (7*a*b*Sqrt[e*Cos[c + d*x]])/(4*(a^2 - b^2)^2*d
*e*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}\,dx = \frac{b\sqrt{e\cos(c+dx)}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} - \frac{\int \frac{-2a+\frac{3}{2}b\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{7ab\sqrt{e\cos(c+dx)}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+d \ldots}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{7ab\sqrt{e\cos(c+dx)}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+d \ldots}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{7ab\sqrt{e\cos(c+dx)}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+d \ldots}$$

$$= -\frac{7a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{e\cos(c+dx)}} + \frac{b\sqrt{e\cos(c+dx)}}{2\left(a^2-b^2\right)de(a+b\sin(c+d \ldots}$$

$$= -\frac{7a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{e\cos(c+dx)}} + \frac{3a\left(5a^2+2b^2\right)\sqrt{\cos(c+dx)}}{8\left(-a^2+b^2\right)^{5/2}\left(b-\sqrt{-a^2 \ldots}\right.}$$

$$= -\frac{3\sqrt{b}\left(5a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{11/4} d\sqrt{e}} - \frac{3\sqrt{b}\left(5a^2+2b^2\right)\tanh \ldots}{8\left(-a^2+b^2\right) \ldots}$$

Mathematica [C] time = 24.55, size = 1226, normalized size = 2.36

$$\frac{\cos(c+dx)\left(\frac{7ab}{4\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b}{2\left(a^2-b^2\right)(a+b\sin(c+dx))^2}\right)}{d\sqrt{e\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\left(\frac{14ab\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\ldots\right.}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos \ldots\right.\right.\right.}\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (Cos[c + d*x]*(b/(2*(a^2 - b^2)*(a + b*Sin[c + d*x])^2) + (7*a*b)/(4*(a^2 -
 b^2)^2*(a + b*Sin[c + d*x]))))/(d*Sqrt[e*Cos[c + d*x]]) + (Sqrt[Cos[c + d*
x]]*((-2*(8*a^2 + 6*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*
b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
 b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))
) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])
/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-
a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 +
 b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x]))
+ (14*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4,
-1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c
 + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1,
5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1
[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a
^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(
-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*A
rcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTa
n[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^
2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c +
 d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c
 + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c +
 d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(8*(a - b)^2*(a + b)
^2*d*Sqrt[e*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^3), x)
```

maple [C] time = 28.64, size = 25322, normalized size = 48.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos(dx+c)}\,(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,\cos(c+d\,x)}\,\left(a+b\,\sin(c+d\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.602 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=596

$$-\frac{a\left(8a^2+37b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{4de^2\left(a^2-b^2\right)^3\sqrt{\cos(c+dx)}}+\frac{9ab}{4de\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}+\frac{}{2de\left(a^2-b^2\right)}$$

```
[Out] 5/8*b^(3/2)*(7*a^2+2*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(3/2)-5/8*b^(3/2)*(7*a^2+2*b^2)*arctanh(
b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/
e^(3/2)+1/2*b/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2)+9/4*a*b
/(a^2-b^2)^2/d/e/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2)+1/4*(-5*b*(7*a^2+2*b
^2)+a*(8*a^2+37*b^2)*sin(d*x+c))/(a^2-b^2)^3/d/e/(e*cos(d*x+c))^(1/2)-5/8*a
*b*(7*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi
(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2
-b^2)^3/d/e/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-5/8*a*b*(7*a^2+2*b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/d/e/(b+(
-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-1/4*a*(8*a^2+37*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*co
s(d*x+c))^(1/2)/(a^2-b^2)^3/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.59, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2694, 2864, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{5b^{3/2}\left(7a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}-\frac{5b^{3/2}\left(7a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}-\frac{a\left(8a^2+37b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4de^2\left(a^2-b^2\right)^3\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (5*b^(3/2)*(7*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) - (5*b^(3/2)*(7*a^2
+ 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) - (a*(8*a^2 + 37*b^2)*Sqrt[e*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2])/(4*(a^2 - b^2)^3*d*e^2*Sqrt[Cos[c + d*x]])
 - (5*a*b*(7*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^
2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e*Sqr
t[e*Cos[c + d*x]]) - (5*a*b*(7*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(b + Sqrt[-a
^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/(2*(a^2 - b^2)*d*e*Sqrt[e*Cos[c +
d*x]]*(a + b*Sin[c + d*x])^2) + (9*a*b)/(4*(a^2 - b^2)^2*d*e*Sqrt[e*Cos[c +
 d*x]]*(a + b*Sin[c + d*x])) - (5*b*(7*a^2 + 2*b^2) - a*(8*a^2 + 37*b^2)*Si
n[c + d*x])/(4*(a^2 - b^2)^3*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3}\,dx = \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2} - \frac{\int \frac{-2a+\frac{5}{2}}{(e\cos(c+dx))^3}}{2\,(}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2 c}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2 c}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2 c}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2 c}$$

$$= -\frac{a\left(8a^2+37b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^3 de^2\sqrt{\cos(c+dx)}} + \frac{}{2\left(a^2-b^2\right)de}$$

$$= -\frac{a\left(8a^2+37b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^3 de^2\sqrt{\cos(c+dx)}} - \frac{5ab\left(7a^2+2b\right.}{8\left(a^2-b\right.}$$

$$= \frac{5b^{3/2}\left(7a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}} - \frac{5b^{3/2}\left(7a^2+2b^2\right)\tanh}{8\left(-a^2+b^2\right.}$$

Mathematica [C] time = 6.73, size = 922, normalized size = 1.55

$$\frac{\cos^2(c+dx)\left(-\frac{13a\cos(c+dx)b^3}{4\left(a^2-b^2\right)^3(a+b\sin(c+dx))} - \frac{\cos(c+dx)b^3}{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2} + \frac{2\sec(c+dx)\left(\sin(c+dx)a^3-3ba^2+3b^2\sin(c+dx)a-b^3\right)}{\left(a^2-b^2\right)^3}\right)}{d(e\cos(c+dx))^{3/2}} - \frac{\cos^{\frac{3}{2}}(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] -1/8*(Cos[c + d*x]^(3/2)*((-2*(8*a^4 + 72*a^2*b^2 + 10*b^4)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)
*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[
Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*S
in[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((8*a^3*b +
37*a*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1
```

```
, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2
) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c
 + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d
*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c +
d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))
))/((a - b)^3*(a + b)^3*d*(e*Cos[c + d*x])^(3/2)) + (Cos[c + d*x]^2*(-1/2*(
b^3*Cos[c + d*x])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) - (13*a*b^3*Cos[c
+ d*x])/(4*(a^2 - b^2)^3*(a + b*Sin[c + d*x])) + (2*Sec[c + d*x]*(-3*a^2*b
- b^3 + a^3*Sin[c + d*x] + 3*a*b^2*Sin[c + d*x]))/(a^2 - b^2)^3))/(d*(e*Cos
[c + d*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^3), x)
```

maple [C] time = 57.54, size = 46134, normalized size = 77.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^3),x)
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.603 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=614

$$\frac{a\left(8a^2+69b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12de^2\left(a^2-b^2\right)^3\sqrt{e\cos(c+dx)}}-\frac{7ab^2\left(9a^2+2b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8de^2\left(a^2-b^2\right)^3\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{7ab^2\left(9\right.}{8de^2\left(\right.}$$

```
[Out] -7/8*b^(5/2)*(9*a^2+2*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/(-a^2+b^2)^(15/4)/d/e^(5/2)-7/8*b^(5/2)*(9*a^2+2*b^2)*arctanh
(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(15/4)/d
/e^(5/2)+1/2*b/(a^2-b^2)/d/e/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2+11/4*a
*b/(a^2-b^2)^2/d/e/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))+1/12*(-7*b*(9*a^2+
2*b^2)+a*(8*a^2+69*b^2)*sin(d*x+c))/(a^2-b^2)^3/d/e/(e*cos(d*x+c))^(3/2)+1/
12*a*(8*a^2+69*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^2/(e*cos(d*
x+c))^(1/2)-7/8*a*b^2*(9*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*co
s(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c)
)^(1/2)-7/8*a*b^2*(9*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*
x+c)^(1/2)/(a^2-b^2)^3/d/e^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1
/2)
```

Rubi [A] time = 1.72, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2694, 2864, 2866, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{7b^{5/2}\left(9a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}-\frac{7b^{5/2}\left(9a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}+\frac{a\left(8a^2+69b^2\right)\sqrt{\cos(c+dx)}}{12de^2\left(a^2-b^2\right)^3\sqrt{e\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (-7*b^(5/2)*(9*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) - (7*b^(5/2)*(9*a^2
 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) + (a*(8*a^2 + 69*b^2)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2])/(12*(a^2 - b^2)^3*d*e^2*Sqrt[e*Cos[c + d*x]
]) - (7*a*b^2*(9*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(a^2 - b*(b - Sqrt[-a^2 +
b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a*b^2*(9*a^2 + 2*b^2)*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b
^2)^3*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) + b/(2*(
a^2 - b^2)*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2) + (11*a*b)/(4
*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])) - (7*b*(9*a
^2 + 2*b^2) - a*(8*a^2 + 69*b^2)*Sin[c + d*x])/(12*(a^2 - b^2)^3*d*e*(e*Cos
[c + d*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3}\,dx = \frac{b}{2(a^2-b^2)\,de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} - \frac{\int \frac{-2a\cdots}{(e\cos(c+dx\cdots}}{2\cdots}$$

$$= \frac{b}{2(a^2-b^2)\,de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{\cdots}{4(a^2-b^2)^{\cdots}}$$

$$= \frac{b}{2(a^2-b^2)\,de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{\cdots}{4(a^2-b^2)^{\cdots}}$$

$$= \frac{b}{2(a^2-b^2)\,de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{\cdots}{4(a^2-b^2)^{\cdots}}$$

$$= \frac{b}{2(a^2-b^2)\,de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{\cdots}{4(a^2-b^2)^{\cdots}}$$

$$= \frac{a(8a^2+69b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12(a^2-b^2)^3\,de^2\sqrt{e\cos(c+dx)}} + \frac{\cdots}{2(a^2-b^2)\,de(e\cdots}$$

$$= \frac{a(8a^2+69b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12(a^2-b^2)^3\,de^2\sqrt{e\cos(c+dx)}} + \frac{7ab^2(9a^2+2b^2\cdots}{8(-a^2+b^2)\cdots}$$

$$= -\frac{7b^{5/2}(9a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8(-a^2+b^2)^{15/4}\,de^{5/2}} - \frac{7b^{5/2}(9a^2+2b^2)\tan\cdots}{8(-a^2+b\cdots}$$

Mathematica [C] time = 23.49, size = 1308, normalized size = 2.13

$$\frac{\left(-\frac{15ab^3}{4(a^2-b^2)^3(a+b\sin(c+dx))} - \frac{b^3}{2(a^2-b^2)^2(a+b\sin(c+dx))^2} + \frac{2\sec^2(c+dx)\left(\sin(c+dx)a^3-3ba^2+3b^2\sin(c+dx)a-b^3\right)}{3(a^2-b^2)^3}\right)\cos^3(c+dx)}{d(e\cos(c+dx))^{5/2}} + \cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*((-2*(8*a^4 - 120*a^2*b^2 - 42*b^4)*(a + b*Sqrt[1 - Cos
[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*
x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c
+ d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^
2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1,
9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a
^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] -
```

```
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c +
 d*x]^2]*(a + b*Sin[c + d*x])) - (2*(8*a^3*b + 69*a*b^3)*(a + b*Sqrt[1 - Co
s[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*
x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2
)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*S
qrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(
a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2]
 + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))
/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^
2)*(a + b*Sin[c + d*x]))))/(24*(a - b)^3*(a + b)^3*d*(e*Cos[c + d*x])^(5/2)
) + (Cos[c + d*x]^3*(-1/2*b^3/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) - (15*
a*b^3)/(4*(a^2 - b^2)^3*(a + b*Sin[c + d*x])) + (2*Sec[c + d*x]^2*(-3*a^2*b
 - b^3 + a^3*Sin[c + d*x] + 3*a*b^2*Sin[c + d*x]))/(3*(a^2 - b^2)^3)))/(d*(
e*Cos[c + d*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^3), x)
```

maple [C] time = 78.67, size = 32645, normalized size = 53.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,\cos(c+d\,x))^{5/2}\,(a+b\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^3),x)
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

3.604 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=685

$$\frac{13ab}{4de\left(a^2-b^2\right)^2(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}+\frac{b}{2de\left(a^2-b^2\right)(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}-\frac{9b\left(11a^2\right.}{20}$$

```
[Out] 9/8*b^(7/2)*(11*a^2+2*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/d/e^(7/2)-9/8*b^(7/2)*(11*a^2+2*b^2)*arctan
h(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/
d/e^(7/2)+1/2*b/(a^2-b^2)/d/e/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2+13/4*
a*b/(a^2-b^2)^2/d/e/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))+1/20*(-9*b*(11*a^
2+2*b^2)+a*(8*a^2+109*b^2)*sin(d*x+c))/(a^2-b^2)^3/d/e/(e*cos(d*x+c))^(5/2)
+3/20*(15*b^3*(11*a^2+2*b^2)+a*(8*a^4-64*a^2*b^2-139*b^4)*sin(d*x+c))/(a^2-
b^2)^4/d/e^3/(e*cos(d*x+c))^(1/2)+9/8*a*b^3*(11*a^2+2*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+
b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^4/d/e^3/(b-(-a^2+b^2)^(1/2)
)/(e*cos(d*x+c))^(1/2)+9/8*a*b^3*(11*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2))
,2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^4/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*cos(d*
x+c))^(1/2)-3/20*a*(8*a^4-64*a^2*b^2-139*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/
2)/(a^2-b^2)^4/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A] time = 2.03, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2694, 2864, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{3\left(a\left(-64a^2b^2+8a^4-139b^4\right)\sin(c+dx)+15b^3\left(11a^2+2b^2\right)\right)}{20de^3\left(a^2-b^2\right)^4\sqrt{e\cos(c+dx)}}+\frac{9b^{7/2}\left(11a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{7/2}\left(b^2-a^2\right)^{17/4}}-\frac{9b^{7/2}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (9*b^(7/2)*(11*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) - (9*b^(7/2)*(11*a^
2 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) - (3*a*(8*a^4 - 64*a^2*b^2 - 139*b^
4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(20*(a^2 - b^2)^4*d*e^4*
Sqrt[Cos[c + d*x]]) + (9*a*b^3*(11*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^4*(b - Sqr
t[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (9*a*b^3*(11*a^2 + 2*b^2)*Sqrt
[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8
*(a^2 - b^2)^4*(b + Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + b/(2*(a
^2 - b^2)*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2) + (13*a*b)/(4*
(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])) - (9*b*(11*a
^2 + 2*b^2) - a*(8*a^2 + 109*b^2)*Sin[c + d*x])/(20*(a^2 - b^2)^3*d*e*(e*Co
s[c + d*x])^(5/2)) + (3*(15*b^3*(11*a^2 + 2*b^2) + a*(8*a^4 - 64*a^2*b^2 -
139*b^4)*Sin[c + d*x]))/(20*(a^2 - b^2)^4*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3}dx = \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} - \frac{\int \frac{-2a-}{(e\cos(c+dx}}{2}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2}$$

$$= -\frac{3a\left(8a^4-64a^2b^2-139b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{20\left(a^2-b^2\right)^4 de^4\sqrt{\cos(c+dx)}} + \frac{}{2}$$

$$= -\frac{3a\left(8a^4-64a^2b^2-139b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{20\left(a^2-b^2\right)^4 de^4\sqrt{\cos(c+dx)}} + \frac{9}{}$$

$$= \frac{9b^{7/2}\left(11a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}} - \frac{9b^{7/2}\left(11a^2+2b^2\right)\tan}{8\left(-a^2+b\right.}$$

Mathematica [C] time = 6.90, size = 1014, normalized size = 1.48

$$\frac{\cos^4(c+dx)\left(\frac{21a\cos(c+dx)b^5}{4\left(a^2-b^2\right)^4(a+b\sin(c+dx))} + \frac{\cos(c+dx)b^5}{2\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2} + \frac{2\sec^3(c+dx)\left(\sin(c+dx)a^3-3ba^2+3b^2\sin(c+dx)a-b^3\right)}{5\left(a^2-b^2\right)^3} + \frac{2\sec(c+}{}\right.}{d(e\cos(c+dx))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (-3*Cos[c + d*x]^(7/2)*((-2*(8*a^6 - 64*a^4*b^2 - 304*a^2*b^4 - 30*b^6)*(a
+ b*Sqrt[1 - Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) +
((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2
)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(
1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^
```

```
2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^
2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) -
((8*a^5*b - 64*a^3*b^3 - 139*a*b^5)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(
5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (
Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt
[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Lo
g[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x
]^2)*(a + b*Sin[c + d*x]))))/(40*(a - b)^4*(a + b)^4*d*(e*Cos[c + d*x])^(7/
2)) + (Cos[c + d*x]^4*((b^5*Cos[c + d*x])/(2*(a^2 - b^2)^3*(a + b*Sin[c + d
*x])^2) + (21*a*b^5*Cos[c + d*x])/(4*(a^2 - b^2)^4*(a + b*Sin[c + d*x])) +
(2*Sec[c + d*x]^3*(-3*a^2*b - b^3 + a^3*Sin[c + d*x] + 3*a*b^2*Sin[c + d*x]
))/(5*(a^2 - b^2)^3) + (2*Sec[c + d*x]*(50*a^2*b^3 + 10*b^5 + 3*a^5*Sin[c +
 d*x] - 24*a^3*b^2*Sin[c + d*x] - 39*a*b^4*Sin[c + d*x]))/(5*(a^2 - b^2)^4)
))/(d*(e*Cos[c + d*x])^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(b\sin\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^3), x)
```

maple [C] time = 111.40, size = 49016, normalized size = 71.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.605 $\int \frac{(e\cos(c+dx))^{15/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=671

$$\frac{13e^7\sqrt{e\cos(c+dx)}\left(21a\left(11a^2-6b^2\right)-b\left(77a^2-20b^2\right)\sin(c+dx)\right)}{56b^7d}-\frac{39e^5(e\cos(c+dx))^{5/2}\left(77a^2+22ab\sin(c+\right.}{280b^5d(a+b\sin(c+dx))}$$

```
[Out] 39/16*a*(11*a^4-17*a^2*b^2+6*b^4)*e^(15/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1
/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(15/2)/(-a^2+b^2)^(3/4)/d+39/16*a*(11*a^4-1
7*a^2*b^2+6*b^4)*e^(15/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(15/2)/(-a^2+b^2)^(3/4)/d-1/3*e*(e*cos(d*x+c))^(13/2)/b/d/(
a+b*sin(d*x+c))^3-13/84*e^3*(e*cos(d*x+c))^(9/2)*(11*a+4*b*sin(d*x+c))/b^3/
d/(a+b*sin(d*x+c))^2-39/280*e^5*(e*cos(d*x+c))^(5/2)*(77*a^2-20*b^2+22*a*b*
sin(d*x+c))/b^5/d/(a+b*sin(d*x+c))+13/56*(231*a^4-203*a^2*b^2+20*b^4)*e^8*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/b^8/d/(e*cos(d*x+c))^(1/2)-39/16*a^2*(11*a^4-17*
a^2*b^2+6*b^4)*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^
8/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-39/16*a^2*(11*a^4-17*
a^2*b^2+6*b^4)*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^
8/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+13/56*e^7*(21*a*(11*a
^2-6*b^2)-b*(77*a^2-20*b^2)*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^7/d
```

Rubi [A] time = 1.83, antiderivative size = 671, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2693, 2863, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{13e^7\sqrt{e\cos(c+dx)}\left(21a\left(11a^2-6b^2\right)-b\left(77a^2-20b^2\right)\sin(c+dx)\right)}{56b^7d}-\frac{39e^5(e\cos(c+dx))^{5/2}\left(77a^2+22ab\sin(c+\right.}{280b^5d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(15/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (39*a*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^(15/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c +
 d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(15/2)*(-a^2 + b^2)^(3/4)*d) +
 (39*a*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^(15/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
 + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(15/2)*(-a^2 + b^2)^(3/4)*d)
 + (13*(231*a^4 - 203*a^2*b^2 + 20*b^4)*e^8*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(56*b^8*d*Sqrt[e*Cos[c + d*x]]) - (39*a^2*(11*a^4 - 17*a^2*b
^2 + 6*b^4)*e^8*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (c + d*x)/2, 2])/(16*b^8*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c +
 d*x]]) - (39*a^2*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^8*Sqrt[Cos[c + d*x]]*Elli
pticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^8*(a^2 - b*(b +
 Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(13/2))/(
3*b*d*(a + b*Sin[c + d*x])^3) - (13*e^3*(e*Cos[c + d*x])^(9/2)*(11*a + 4*b*
Sin[c + d*x]))/(84*b^3*d*(a + b*Sin[c + d*x])^2) - (39*e^5*(e*Cos[c + d*x])
^(5/2)*(77*a^2 - 20*b^2 + 22*a*b*Sin[c + d*x]))/(280*b^5*d*(a + b*Sin[c + d
*x])) + (13*e^7*Sqrt[e*Cos[c + d*x]]*(21*a*(11*a^2 - 6*b^2) - b*(77*a^2 - 2
0*b^2)*Sin[c + d*x]))/(56*b^7*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{15/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{(13e^2)\int \frac{(e\cos(c+dx))^{11/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} + \frac{(39e^4)}{}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39e^5(e}{}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39e^5(e}{}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39e^5(e}{}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39e^5(e}{}$$

$$= \frac{13(231a^4-203a^2b^2+20b^4)e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{56b^8d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx)}{3bd(a+b\sin(c+}$$

$$= \frac{13(231a^4-203a^2b^2+20b^4)e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{56b^8d\sqrt{e\cos(c+dx)}} - \frac{39a^2(11a^4-17a}{16b^8\sqrt{}}$$

$$= \frac{39a(11a^4-17a^2b^2+6b^4)e^{15/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{15/2}(-a^2+b^2)^{3/4}d} + \frac{39a(11a^4-17a^2b^2+6b}{16b^{15/2}(}$$

Mathematica [C] time = 27.81, size = 2102, normalized size = 3.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(15/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(15/2)*((-2*(4410*a^3*b - 3418*a*b^3)*(a + b*Sqrt[1 - Cos
[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*
x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c
+ d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^
2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1,
9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a
^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] -
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c +
 d*x]^2]*(a + b*Sin[c + d*x])) + ((5600*a^3*b - 3472*a*b^3)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 +
b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
```

```
Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[
Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a
^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
- 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d
*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^
2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/
2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos
[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*(3815*a^4 - 6251*a^2*b^2 + 1300*b^4
)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1,
 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]
*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos
[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1
/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2
)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1
- (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (S
qrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
 - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] +
 Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)
/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(560*b^7*d*Cos[c + d*x]^(15/
2)) + ((e*Cos[c + d*x])^(15/2)*Sec[c + d*x]^7*((-4*a*Cos[2*(c + d*x)])/(5*b
^5) + ((-280*a^2 + 79*b^2)*Sin[c + d*x])/(42*b^6) - (-a^2 + b^2)^3/(3*b^7*(
a + b*Sin[c + d*x])^3) - (37*a*(a^2 - b^2)^2)/(12*b^7*(a + b*Sin[c + d*x])^
2) + ((-a^2 + b^2)*(-393*a^2 + 76*b^2))/(24*b^7*(a + b*Sin[c + d*x])) + Sin
[3*(c + d*x)]/(14*b^4)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 139.89, size = 300244, normalized size = 447.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x)
[Out] result too large to display
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,\cos\left(c+d\,x\right)\right)^{15/2}}{\left(a+b\,\sin\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(15/2)/(a + b*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(15/2)/(a + b*sin(c + d*x))^4, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(15/2)/(a+b*sin(d*x+c))**4,x)
[Out] Timed out
```

3.606 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=557

$$\frac{77ae^{13/2}\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{13/2}d\sqrt[4]{b^2-a^2}}-\frac{77ae^{13/2}\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{13/2}d\sqrt[4]{b^2-a^2}}+\frac{77a^2e^7\left(3a^2-2b^2\right)\sqrt{\mathrm{co}}}{16b^7d\left(b-\right.}$$

```
[Out] 77/16*a*(3*a^2-2*b^2)*e^(13/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-77/16*a*(3*a^2-2*b^2)*e^(13/2
)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(13/2)/(
-a^2+b^2)^(1/4)/d-1/3*e*(e*cos(d*x+c))^(11/2)/b/d/(a+b*sin(d*x+c))^3-11/60*
e^3*(e*cos(d*x+c))^(7/2)*(9*a+4*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))^2-77/1
20*e^5*(e*cos(d*x+c))^(3/2)*(15*a^2-4*b^2+6*a*b*sin(d*x+c))/b^5/d/(a+b*sin(
d*x+c))+77/16*a^2*(3*a^2-2*b^2)*e^7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*co
s(d*x+c)^(1/2)/b^7/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+77/16*a^2*(3
*a^2-2*b^2)*e^7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^7/d
/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-77/40*(15*a^2-4*b^2)*e^6*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*(e*cos(d*x+c))^(1/2)/b^6/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.36, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2693, 2863, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{77e^5(e\cos(c+dx))^{3/2}\left(15a^2+6ab\sin(c+dx)-4b^2\right)}{120b^5d(a+b\sin(c+dx))}+\frac{77ae^{13/2}\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{13/2}d\sqrt[4]{b^2-a^2}}-\frac{77ae^{13/2}\left(3a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (77*a*(3*a^2 - 2*b^2)*e^(13/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(16*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (77*a*(3*a^2
- 2*b^2)*e^(13/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4
)*Sqrt[e])])/(16*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (77*(15*a^2 - 4*b^2)*e^6*
Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(40*b^6*d*Sqrt[Cos[c + d*x]
]) + (77*a^2*(3*a^2 - 2*b^2)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - S
qrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^7*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*
Cos[c + d*x]]) + (77*a^2*(3*a^2 - 2*b^2)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[
(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^7*(b + Sqrt[-a^2 + b^2
])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(11/2))/(3*b*d*(a + b*Sin[
c + d*x])^3) - (11*e^3*(e*Cos[c + d*x])^(7/2)*(9*a + 4*b*Sin[c + d*x]))/(60
*b^3*d*(a + b*Sin[c + d*x])^2) - (77*e^5*(e*Cos[c + d*x])^(3/2)*(15*a^2 - 4
*b^2 + 6*a*b*Sin[c + d*x]))/(120*b^5*d*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{\left(11e^2\right)\int \frac{(e\cos(c+dx))^{9/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} + \frac{\left(77e^4\right)\int -\frac{(e}{}}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} - \frac{77e^5(e\cos(}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} - \frac{77e^5(e\cos(}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} - \frac{77e^5(e\cos(}{}$$

$$= -\frac{77\left(15a^2-4b^2\right)e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{40b^6d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3}{}$$

$$= -\frac{77\left(15a^2-4b^2\right)e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{40b^6d\sqrt{\cos(c+dx)}} + \frac{77a^2\left(3a^2-2b^2\right)e^7\sqrt{\cos(c+a}}{16b^7\left(b-\sqrt{-a^2+b}\right.}$$

$$= \frac{77a\left(3a^2-2b^2\right)e^{13/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{13/2}\sqrt[4]{-a^2+b^2}\,d} - \frac{77a\left(3a^2-2b^2\right)e^{13/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(}}{\sqrt[4]{-a^2+b^2}}\right.}{16b^{13/2}\sqrt[4]{-a^2+b^2}\,d}$$

Mathematica [C] time = 26.85, size = 937, normalized size = 1.68

$$\frac{(e\cos(c+dx))^{13/2}\sec^6(c+dx)\left(-\frac{8a\cos(c+dx)}{3b^5}+\frac{\sin(2(c+dx))}{5b^4}+\frac{20b^2\cos(c+dx)-71a^2\cos(c+dx)}{8b^5(a+b\sin(c+dx))}+\frac{9\left(a^3\cos(c+dx)-ab^2\cos(c+dx)\right)}{4b^5(a+b\sin(c+dx))^2}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-77*(e*Cos[c + d*x])^(13/2)*((-12*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a
*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 +
I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*
Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1
 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + L
og[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1
- Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((15*a^2 - 4*b^2)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b
^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^
2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/
4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)]
- Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2
 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(80*b^5*d*Cos[c + d*x]
^(13/2)) + ((e*Cos[c + d*x])^(13/2)*Sec[c + d*x]^6*((-8*a*Cos[c + d*x])/(3*
b^5) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c + d*x])/(3
*b^5*(a + b*Sin[c + d*x])^3) + (9*(a^3*Cos[c + d*x] - a*b^2*Cos[c + d*x]))/
(4*b^5*(a + b*Sin[c + d*x])^2) + (-71*a^2*Cos[c + d*x] + 20*b^2*Cos[c + d*x
])/(8*b^5*(a + b*Sin[c + d*x])) + Sin[2*(c + d*x)]/(5*b^4)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 94.46, size = 180834, normalized size = 324.66

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+d\,x))^{13/2}}{(a+b\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(13/2)/(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(13/2)/(a + b*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.607 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=571

$$-\frac{15ae^{11/2}\left(7a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{11/2}d\left(b^2-a^2\right)^{3/4}}-\frac{15ae^{11/2}\left(7a^2-6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{11/2}d\left(b^2-a^2\right)^{3/4}}-\frac{5e^6\left(21a^2-4b^2\right)\sqrt{\cdot}}{8b^6d\cdot}$$

```
[Out] -15/16*a*(7*a^2-6*b^2)*e^(11/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/b^(11/2)/(-a^2+b^2)^(3/4)/d-15/16*a*(7*a^2-6*b^2)*e^(11/
2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/
(-a^2+b^2)^(3/4)/d-1/3*e*(e*cos(d*x+c))^(9/2)/b/d/(a+b*sin(d*x+c))^3-1/4*e^
3*(e*cos(d*x+c))^(5/2)*(7*a+4*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))^2-5/8*(2
1*a^2-4*b^2)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(e*cos(d*x+c))^(1/2)+15/
16*a^2*(7*a^2-6*b^2)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1
/2)/b^6/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+15/16*a^2*(7*a^
2-6*b^2)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin
(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(a
^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-5/8*e^5*(21*a^2-4*b^2+14*a*
b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^5/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 1.38, antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2693, 2863, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{5e^5\sqrt{e\cos(c+dx)}\left(21a^2+14ab\sin(c+dx)-4b^2\right)}{8b^5d(a+b\sin(c+dx))}-\frac{15ae^{11/2}\left(7a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{11/2}d\left(b^2-a^2\right)^{3/4}}-\frac{15ae^{11/2}\left(7a\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-15*a*(7*a^2 - 6*b^2)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(11/2)*(-a^2 + b^2)^(3/4)*d) - (15*a*(7*a^2
 - 6*b^2)*e^(11/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[e])])/(16*b^(11/2)*(-a^2 + b^2)^(3/4)*d) - (5*(21*a^2 - 4*b^2)*e^6*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(8*b^6*d*Sqrt[e*Cos[c + d*x]]
) + (15*a^2*(7*a^2 - 6*b^2)*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sq
rt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*
d*Sqrt[e*Cos[c + d*x]]) + (15*a^2*(7*a^2 - 6*b^2)*e^6*Sqrt[Cos[c + d*x]]*El
lipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^6*(a^2 - b*(b
 + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(9/2))/
(3*b*d*(a + b*Sin[c + d*x])^3) - (e^3*(e*Cos[c + d*x])^(5/2)*(7*a + 4*b*Sin
[c + d*x]))/(4*b^3*d*(a + b*Sin[c + d*x])^2) - (5*e^5*Sqrt[e*Cos[c + d*x]]*
(21*a^2 - 4*b^2 + 14*a*b*Sin[c + d*x]))/(8*b^5*d*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{\left(3e^2\right)\int \frac{(e\cos(c+dx))^{7/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{2b}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} + \frac{\left(5e^4\right)\int \cdots}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} - \frac{5e^5\sqrt{e\cos \cdots}}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} - \frac{5e^5\sqrt{e\cos \cdots}}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} - \frac{5e^5\sqrt{e\cos \cdots}}{\cdots}$$

$$= -\frac{5\left(21a^2-4b^2\right)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^6d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e \cdots}{\cdots}$$

$$= -\frac{5\left(21a^2-4b^2\right)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^6d\sqrt{e\cos(c+dx)}} + \frac{15a^2\left(7a^2-6b^2\right)e^6\sqrt{\cos(c+a \cdots}}{16b^6\left(a^2-b\left(b-\sqrt{-a^2 \cdots}\right.\right.}$$

$$= -\frac{15a\left(7a^2-6b^2\right)e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{11/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{15a\left(7a^2-6b^2\right)e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e \cdots}}{\sqrt[4]{- \cdots}}\right)}{16b^{11/2}\left(-a^2+b^2\right)^{3/4}d}$$

Mathematica [C] time = 26.50, size = 2020, normalized size = 3.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*((2*Sin[c + d*x])/(3*b^4) - (-a^2 +
 b^2)^2/(3*b^5*(a + b*Sin[c + d*x])^3) + (25*a*(a^2 - b^2))/(12*b^5*(a + b*
Sin[c + d*x])^2) + (-165*a^2 + 52*b^2)/(24*b^5*(a + b*Sin[c + d*x]))))/d -
((e*Cos[c + d*x])^(11/2)*((-76*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2
)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^
2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c
+ d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[
c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c +
 d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
+ b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2]
+ (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c
 + d*x])) + (32*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/
2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a
^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2
)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(
3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/
2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^
(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 -
 Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/
4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c +
 d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*L
og[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 +
 b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c
+ d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(S
qrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*
(41*a^2 - 20*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*Appell
F1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sq
rt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*
AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d
*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) +
(a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] +
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b
*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))
*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(16*b^5*d*Co
s[c + d*x]^(11/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 111.92, size = 144252, normalized size = 252.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x))^4,x)
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c))**4,x)
[Out] Timed out
```

3.608 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=591

$$\frac{7ae^{9/2}\left(5a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{9/2}d\left(b^2-a^2\right)^{5/4}}-\frac{7ae^{9/2}\left(5a^2-6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{9/2}d\left(b^2-a^2\right)^{5/4}}-\frac{7a^2e^5\left(5a^2-6b^2\right)\sqrt{\cos(c+}}{16b^5d\left(a^2-b^2\right)\left(b-\right.}$$

```
[Out] 7/16*a*(5*a^2-6*b^2)*e^(9/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)
^(1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(5/4)/d-7/16*a*(5*a^2-6*b^2)*e^(9/2)*arc
tanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/(-a^2+b
^2)^(5/4)/d-1/3*e*(e*cos(d*x+c))^(7/2)/b/d/(a+b*sin(d*x+c))^3+7/8*(5*a^2-4*
b^2)*e^3*(e*cos(d*x+c))^(3/2)/b^3/(a^2-b^2)/d/(a+b*sin(d*x+c))-7/12*e^3*(e*
cos(d*x+c))^(3/2)*(5*a+4*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))^2-7/16*a^2*(5
*a^2-6*b^2)*e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/(
a^2-b^2)/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-7/16*a^2*(5*a^2-6*b^2)
*e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/(a^2-b^2)/d/
(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+7/8*(5*a^2-4*b^2)*e^4*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
(e*cos(d*x+c))^(1/2)/b^4/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.43, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2693, 2863, 2864, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{7e^3\left(5a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{8b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{7ae^{9/2}\left(5a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{9/2}d\left(b^2-a^2\right)^{5/4}}-\frac{7ae^{9/2}\left(5a^2-6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(}}{\sqrt{e}\sqrt[4]{b^2-}}\right.}{16b^{9/2}d\left(b^2-a^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (7*a*(5*a^2 - 6*b^2)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(16*b^(9/2)*(-a^2 + b^2)^(5/4)*d) - (7*a*(5*a^2 - 6*
b^2)*e^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(16*b^(9/2)*(-a^2 + b^2)^(5/4)*d) + (7*(5*a^2 - 4*b^2)*e^4*Sqrt[e*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(8*b^4*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]]) - (7*a^2*(5*a^2 - 6*b^2)*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b -
 Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^5*(a^2 - b^2)*(b - Sqrt[-a^2 + b
^2])*d*Sqrt[e*Cos[c + d*x]]) - (7*a^2*(5*a^2 - 6*b^2)*e^5*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^5*(a^2 -
b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(
7/2))/(3*b*d*(a + b*Sin[c + d*x])^3) + (7*(5*a^2 - 4*b^2)*e^3*(e*Cos[c + d*
x])^(3/2))/(8*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (7*e^3*(e*Cos[c + d
*x])^(3/2)*(5*a + 4*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*x])^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} - \frac{\left(7e^2\right)\int \frac{(e\cos(c+dx))^{5/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+4b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))^2} + \frac{\left(7e^4\right)\int \cdots}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7\left(5a^2-4b^2\right)e^3(e\cos(c+dx))^{3/2}}{8b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{7e^3(e\cos(c+dx)\cdots}{12b^3d(a\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7\left(5a^2-4b^2\right)e^3(e\cos(c+dx))^{3/2}}{8b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{7e^3(e\cos(c+dx)\cdots}{12b^3d(a\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7\left(5a^2-4b^2\right)e^3(e\cos(c+dx))^{3/2}}{8b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{7e^3(e\cos(c+dx)\cdots}{12b^3d(a\cdots}$$

$$= \frac{7\left(5a^2-4b^2\right)e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^4\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7\left(5\cdots\right.}{8b^3\left(\cdots\right.}$$

$$= \frac{7\left(5a^2-4b^2\right)e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^4\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{7a^2\left(5a^2-6b^2\right)e^5\sqrt{\cos(c+dx)}\cdots}{16b^5\left(a^2-b^2\right)\left(b-\sqrt{-a^2\cdots}\right.}$$

$$= \frac{7a\left(5a^2-6b^2\right)e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{9/2}\left(-a^2+b^2\right)^{5/4}d} - \frac{7a\left(5a^2-6b^2\right)e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+\cdots}}{\sqrt[4]{-a^2+b^2}\cdots}\right.}{16b^{9/2}\left(-a^2+b^2\right)^{5/4}d}$$

Mathematica [C] time = 26.83, size = 900, normalized size = 1.52

$$\frac{\sec^4(c+dx)\left(-\frac{5a\cos(c+dx)}{4b^3(a+b\sin(c+dx))^2} + \frac{12b^2\cos(c+dx)-19a^2\cos(c+dx)}{8b^3\left(b^2-a^2\right)(a+b\sin(c+dx))} + \frac{a^2\cos(c+dx)-b^2\cos(c+dx)}{3b^3(a+b\sin(c+dx))^3}\right)(e\cos(c+dx))^{9/2}}{d} + \frac{7\left(-\frac{(5a^2\cdots}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(9/2)*Sec[c + d*x]^4*((a^2*Cos[c + d*x] - b^2*Cos[c + d*x
])/(3*b^3*(a + b*Sin[c + d*x])^3) - (5*a*Cos[c + d*x])/(4*b^3*(a + b*Sin[c
+ d*x])^2) + (-19*a^2*Cos[c + d*x] + 12*b^2*Cos[c + d*x])/(8*b^3*(-a^2 + b^
2)*(a + b*Sin[c + d*x]))))/d + (7*(e*Cos[c + d*x])^(9/2)*((-4*a*b*(a + b*Sq
rt[1 - Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8
+ I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4
)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
- Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*
x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^
2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/
4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((5*a^
```

```
2 - 4*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2,
1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/
2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[
c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c +
d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^
2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c +
 d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x])
)))/(16*(a - b)*b^3*(a + b)*d*Cos[c + d*x]^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 100.68, size = 237416, normalized size = 401.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{9/2}}{(a+b\,\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.609 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=597

$$-\frac{5ae^{7/2}\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{7/2}d\left(b^2-a^2\right)^{7/4}}-\frac{5ae^{7/2}\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{7/2}d\left(b^2-a^2\right)^{7/4}}+\frac{5e^4\left(3a^2-4b^2\right)\sqrt{\cos(c+dx}}{24b^4d\left(a^2-b^2\right)\sqrt{e\cos}}$$

```
[Out] -5/16*a*(a^2-2*b^2)*e^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(7/4)/d-5/16*a*(a^2-2*b^2)*e^(7/2)*arctan
h(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)
^(7/4)/d-1/3*e*(e*cos(d*x+c))^(5/2)/b/d/(a+b*sin(d*x+c))^3+5/24*(3*a^2-4*b^
2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^4/(a^2-b^2)/d/(e*cos(d*x+c))^(1/2)-5/1
6*a^2*(a^2-2*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)
/b^4/(a^2-b^2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-5/16*a^2
*(a^2-2*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi
(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^4/
(a^2-b^2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-5/24*(3*a^2-4
*b^2)*e^3*(e*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d/(a+b*sin(d*x+c))+5/12*e^3*(3
*a+4*b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^3/d/(a+b*sin(d*x+c))^2
```

Rubi [A] time = 1.52, antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2693, 2863, 2864, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{5e^3\left(3a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{24b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{5ae^{7/2}\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{7/2}d\left(b^2-a^2\right)^{7/4}}-\frac{5ae^{7/2}\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(}}{\sqrt{e}\sqrt[4]{b^2-}}\right)}{16b^{7/2}d\left(b^2-a^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-5*a*(a^2 - 2*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(16*b^(7/2)*(-a^2 + b^2)^(7/4)*d) - (5*a*(a^2 - 2*b^2
)*e^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(16*b^(7/2)*(-a^2 + b^2)^(7/4)*d) + (5*(3*a^2 - 4*b^2)*e^4*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2])/(24*b^4*(a^2 - b^2)*d*Sqrt[e*Cos[c + d*x
]]) - (5*a^2*(a^2 - 2*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqr
t[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^4*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^
2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - 2*b^2)*e^4*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^4*(a^
2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*C
os[c + d*x])^(5/2))/(3*b*d*(a + b*Sin[c + d*x])^3) - (5*(3*a^2 - 4*b^2)*e^3
*Sqrt[e*Cos[c + d*x]])/(24*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (5*e^3
*Sqrt[e*Cos[c + d*x]]*(3*a + 4*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*
x])^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} + \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a+4b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))^2} - \frac{\left(5e^4\right)\int \frac{}{\sqrt{e}}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5\left(3a^2-4b^2\right)e^3\sqrt{e\cos(c+dx)}}{24b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{5e^3\sqrt{e\cos(c+d}}{12b^3d(a}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5\left(3a^2-4b^2\right)e^3\sqrt{e\cos(c+dx)}}{24b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{5e^3\sqrt{e\cos(c+d}}{12b^3d(a}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5\left(3a^2-4b^2\right)e^3\sqrt{e\cos(c+dx)}}{24b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{5e^3\sqrt{e\cos(c+d}}{12b^3d(a}$$

$$= \frac{5\left(3a^2-4b^2\right)e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24b^4\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5\left(3a\right.}{24b^3\left(\right.}$$

$$= \frac{5\left(3a^2-4b^2\right)e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24b^4\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}} + \frac{5a^2\left(a^2-2b^2\right)e^4\sqrt{\cos(c+dx)}\,\Pi}{16b^4\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2}\right.}$$

$$= -\frac{5a\left(a^2-2b^2\right)e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{7/2}\left(-a^2+b^2\right)^{7/4}d} - \frac{5a\left(a^2-2b^2\right)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+}}{\sqrt[4]{-a^2+b^2}\sqrt{}}\right.}{16b^{7/2}\left(-a^2+b^2\right)^{7/4}d}$$

<u>**Mathematica**</u> [C] time = 24.06, size = 1263, normalized size = 2.12

$$\frac{\sec^3(c+dx)\left(-\frac{13a}{12b^3(a+b\sin(c+dx))^2}+\frac{28b^2-33a^2}{24b^3\left(b^2-a^2\right)(a+b\sin(c+dx))}+\frac{a^2-b^2}{3b^3(a+b\sin(c+dx))^3}\right)(e\cos(c+dx))^{7/2}}{d} + \frac{5\left(-\frac{2\left(3a^2-4b^2\right)\left(\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*((a^2 - b^2)/(3*b^3*(a + b*Sin[c + d
*x])^3) - (13*a)/(12*b^3*(a + b*Sin[c + d*x])^2) + (-33*a^2 + 28*b^2)/(24*b
^3*(-a^2 + b^2)*(a + b*Sin[c + d*x]))))/d + (5*(e*Cos[c + d*x])^(7/2)*((-4*
a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1
, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]
])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2
, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)
*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt
[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
- 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + L
```

```
og[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c +
d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (2*(3*a^2 - 4*b^2)*
(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*S
qrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2
, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*
AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2
)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 -
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + L
og[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
 b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/(
(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(48*(a - b)*b^3*(a + b)*d*Cos[
c + d*x]^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 114.16, size = 192036, normalized size = 321.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+d\,x))^{7/2}}{(a+b\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.610 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=574

$$\frac{e^2\left(a^2+4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{8b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}+\frac{e\left(a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{8bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{ae(e\cos(c+dx))^{3/2}}{4bd\left(a^2-b^2\right)(a+b\sin(c+dx)}$$

```
[Out] -1/16*a*(a^2-6*b^2)*e^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(9/4)/d+1/16*a*(a^2-6*b^2)*e^(5/2)*arctan
h(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)
^(9/4)/d-1/3*e*(e*cos(d*x+c))^(3/2)/b/d/(a+b*sin(d*x+c))^3+1/4*a*e*(e*cos(d
*x+c))^(3/2)/b/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+1/8*(a^2+4*b^2)*e*(e*cos(d*x+
c))^(3/2)/b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-1/16*a^2*(a^2-6*b^2)*e^3*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/
(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d/(b-(-a^2+b
^2)^(1/2))/(e*cos(d*x+c))^(1/2)-1/16*a^2*(a^2-6*b^2)*e^3*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^
2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d/(b+(-a^2+b^2)^(1/2))/
(e*cos(d*x+c))^(1/2)+1/8*(a^2+4*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^
2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.45, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2693, 2864, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{ae^{5/2}\left(a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{5/2}d\left(b^2-a^2\right)^{9/4}}+\frac{ae^{5/2}\left(a^2-6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{5/2}d\left(b^2-a^2\right)^{9/4}}+\frac{e^2\left(a^2+4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] -(a*(a^2 - 6*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^
2)^(1/4)*Sqrt[e])])/(16*b^(5/2)*(-a^2 + b^2)^(9/4)*d) + (a*(a^2 - 6*b^2)*e^
(5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])
/(16*b^(5/2)*(-a^2 + b^2)^(9/4)*d) + ((a^2 + 4*b^2)*e^2*Sqrt[e*Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2])/(8*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) - (
a^2*(a^2 - 6*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
b^2]), (c + d*x)/2, 2])/(16*b^3*(a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*Sqrt
[e*Cos[c + d*x]]) - (a^2*(a^2 - 6*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^3*(a^2 - b^2)^2*(b + Sqr
t[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(3/2))/(3*b*d*
(a + b*Sin[c + d*x])^3) + (a*e*(e*Cos[c + d*x])^(3/2))/(4*b*(a^2 - b^2)*d*(
a + b*Sin[c + d*x])^2) + ((a^2 + 4*b^2)*e*(e*Cos[c + d*x])^(3/2))/(8*b*(a^2
 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{2b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}\left(2b-\frac{1}{2}\right.}{(a+b\sin(c+}}{4b\left(a^2-b\right.}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(a^2+4b^2\right)e(e\cos}{8b\left(a^2-b^2\right)^2d(a+b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(a^2+4b^2\right)e(e\cos}{8b\left(a^2-b^2\right)^2d(a+b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(a^2+4b^2\right)e(e\cos}{8b\left(a^2-b^2\right)^2d(a+b}$$

$$= \frac{\left(a^2+4b^2\right)e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(}{4b\left(a^2-b^2\right.}$$

$$= \frac{\left(a^2+4b^2\right)e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} - \frac{a^2\left(a^2-6b^2\right)e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b-\sqrt{-}}\right.}{16b^3\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)}$$

$$= -\frac{a\left(a^2-6b^2\right)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{5/2}\left(-a^2+b^2\right)^{9/4}d} + \frac{a\left(a^2-6b^2\right)e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{5/2}\left(-a^2+b^2\right)^{9/4}d} +$$

Mathematica [C] time = 26.72, size = 892, normalized size = 1.55

$$\frac{\sec^2(c+dx)\left(-\frac{a\cos(c+dx)}{4b(b^2-a^2)(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)}{3b(a+b\sin(c+dx))^3}+\frac{\cos(c+dx)a^2+4b^2\cos(c+dx)}{8b(b^2-a^2)^2(a+b\sin(c+dx))}\right)(e\cos(c+dx))^{5/2}}{d}+\frac{\left(-\frac{(a^2+4b^2)(a}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*(-1/3*Cos[c + d*x]/(b*(a + b*Sin[c +
 d*x])^3) - (a*Cos[c + d*x])/(4*b*(-a^2 + b^2)*(a + b*Sin[c + d*x])^2) + (a
^2*Cos[c + d*x] + 4*b^2*Cos[c + d*x])/(8*b*(-a^2 + b^2)^2*(a + b*Sin[c + d*
x]))))/d + ((e*Cos[c + d*x])^(5/2)*((-20*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2
])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1
 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x
]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c +
 d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(S
qrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((a^2 + 4*b^2)*(a + b*Sqrt[
1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2,
 (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 -
 b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^
(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4
)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^
(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-
a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(16*(a - b)^2*b*(a
+ b)^2*d*Cos[c + d*x]^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 115.35, size = 179434, normalized size = 312.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{(a+b\,\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.611 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=592

$$-\frac{e^2\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24b^2d\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}+\frac{a^2e^2\left(a^2+6b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16b^2d\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a^2e}{16b}\cdots$$

```
[Out] -1/16*a*(a^2+6*b^2)*e^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(11/4)/d-1/16*a*(a^2+6*b^2)*e^(3/2)*arcta
nh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2
)^(11/4)/d-1/24*(3*a^2+4*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)
^2/d/(e*cos(d*x+c))^(1/2)+1/16*a^2*(a^2+6*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/
2)),2^(1/2))*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2))
)/(e*cos(d*x+c))^(1/2)+1/16*a^2*(a^2+6*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2))
,2^(1/2))*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(
e*cos(d*x+c))^(1/2)-1/3*e*(e*cos(d*x+c))^(1/2)/b/d/(a+b*sin(d*x+c))^3+1/12*
a*e*(e*cos(d*x+c))^(1/2)/b/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+1/24*(3*a^2+4*b^2
)*e*(e*cos(d*x+c))^(1/2)/b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 1.47, antiderivative size = 592, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2693, 2864, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{ae^{3/2}\left(a^2+6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{3/2}d\left(b^2-a^2\right)^{11/4}}-\frac{ae^{3/2}\left(a^2+6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{3/2}d\left(b^2-a^2\right)^{11/4}}-\frac{e^2\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\cdots}{24b^2d\left(a^2-b^2\right)^2\sqrt{e\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] -(a*(a^2 + 6*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^
2)^(1/4)*Sqrt[e])])/(16*b^(3/2)*(-a^2 + b^2)^(11/4)*d) - (a*(a^2 + 6*b^2)*e
^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])]
)/(16*b^(3/2)*(-a^2 + b^2)^(11/4)*d) - ((3*a^2 + 4*b^2)*e^2*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2])/(24*b^2*(a^2 - b^2)^2*d*Sqrt[e*Cos[c + d*x]]
) + (a^2*(a^2 + 6*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a
^2 + b^2]), (c + d*x)/2, 2])/(16*b^2*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2
+ b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (a^2*(a^2 + 6*b^2)*e^2*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^2*(a^2 -
b^2)^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*Sqrt[e
*Cos[c + d*x]])/(3*b*d*(a + b*Sin[c + d*x])^3) + (a*e*Sqrt[e*Cos[c + d*x]])
/(12*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 + 4*b^2)*e*Sqrt[e*Co
s[c + d*x]])/(24*b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} - \frac{e^2\int \frac{\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{e^2\int \frac{2b-\frac{3}{2}a\,\mathrm{s}}{\sqrt{e\cos(c+dx)}\,(}}{12b\left(a^2\right.}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(3a^2+4b^2\right)e}{24b\left(a^2-b^2\right)^2 d}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(3a^2+4b^2\right)e}{24b\left(a^2-b^2\right)^2 d}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(3a^2+4b^2\right)e}{24b\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(3a^2+4b^2\right)e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24b^2\left(a^2-b^2\right)^2 d\sqrt{e\cos(c+dx)}} - \frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{}{12b\left(a\right.}$$

$$= -\frac{\left(3a^2+4b^2\right)e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24b^2\left(a^2-b^2\right)^2 d\sqrt{e\cos(c+dx)}} + \frac{a^2\left(a^2+6b^2\right)e^2\sqrt{\cos(c+dx)}\,\Pi\left(\right.}{16b^2\left(-a^2+b^2\right)^{5/2}\left(b-\sqrt{-a^2}\right.}$$

$$= -\frac{a\left(a^2+6b^2\right)e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{3/2}\left(-a^2+b^2\right)^{11/4} d} - \frac{a\left(a^2+6b^2\right)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right.}{16b^{3/2}\left(-a^2+b^2\right)^{11/4} d}$$

Mathematica [C] time = 23.85, size = 1263, normalized size = 2.13

$$\frac{(e\cos(c+dx))^{3/2}\sec(c+dx)\left(-\frac{a}{12b(b^2-a^2)(a+b\sin(c+dx))^2}+\frac{3a^2+4b^2}{24b(b^2-a^2)^2(a+b\sin(c+dx))}-\frac{1}{3b(a+b\sin(c+dx))^3}\right)}{d}-\frac{(e\cos(c+dx))}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*Sec[c + d*x]*(-1/3*1/(b*(a + b*Sin[c + d*x])^3) - a
/(12*b*(-a^2 + b^2)*(a + b*Sin[c + d*x])^2) + (3*a^2 + 4*b^2)/(24*b*(-a^2 +
 b^2)^2*(a + b*Sin[c + d*x]))))/d - ((e*Cos[c + d*x])^(3/2)*((28*a*b*(a + b
*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos
[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1
 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[
5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c
 + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*Arc
Tan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan
[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a
^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos
[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt
[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqr
t[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (2*(3*a^2 + 4*b^2)*(a + b*Sqr
t[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c
+ d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Co
s[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2,
 (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5
/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c
+ d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*S
qrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[
b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^
2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c +
 d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c
 + d*x]^2)*(a + b*Sin[c + d*x]))))/(48*(a - b)^2*b*(a + b)^2*d*Cos[c + d*x]
^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 116.41, size = 138380, normalized size = 233.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(b\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{3/2}}{(a+b\,\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**4,x)
[Out] Timed out
```

3.612 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=579

$$\frac{3ab(e\cos(c+dx))^{3/2}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{b\left(11a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{8de\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b(e\cos(c+dx))^{3/2}}{3de\left(a^2-b^2\right)(a+b\sin(c+dx))^3}-\frac{5a\sqrt{e}\,(}{}$$

```
[Out] 1/3*b*(e*cos(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^3+3/4*a*b*(e*cos(
d*x+c))^(3/2)/(a^2-b^2)^2/d/e/(a+b*sin(d*x+c))^2+1/8*b*(11*a^2+4*b^2)*(e*co
s(d*x+c))^(3/2)/(a^2-b^2)^3/d/e/(a+b*sin(d*x+c))-5/16*a*(a^2+2*b^2)*arctan(
b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(
13/4)/d/b^(1/2)+5/16*a*(a^2+2*b^2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a
^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(13/4)/d/b^(1/2)+5/16*a^2*(a^2+2*
b^2)*e*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d
*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/(a^2-b^2)^3/
d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+5/16*a^2*(a^2+2*b^2)*e*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(
b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/(a^2-b^2)^3/d/(b+(-a^2+b^2)
^(1/2))/(e*cos(d*x+c))^(1/2)+1/8*(11*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(
1/2)/(a^2-b^2)^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.53, antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2694, 2864, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{3ab(e\cos(c+dx))^{3/2}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{b\left(11a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{8de\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b(e\cos(c+dx))^{3/2}}{3de\left(a^2-b^2\right)(a+b\sin(c+dx))^3}-\frac{5a\sqrt{e}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-5*a*(a^2 + 2*b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(16*Sqrt[b]*(-a^2 + b^2)^(13/4)*d) + (5*a*(a^2 + 2*b^
2)*Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(16*Sqrt[b]*(-a^2 + b^2)^(13/4)*d) + ((11*a^2 + 4*b^2)*Sqrt[e*Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2])/(8*(a^2 - b^2)^3*d*Sqrt[Cos[c + d*x]]) +
(5*a^2*(a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
 b^2]), (c + d*x)/2, 2])/(16*b*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[
e*Cos[c + d*x]]) + (5*a^2*(a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*
b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b*(a^2 - b^2)^3*(b + Sqrt[-
a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/(3*(a^2 -
b^2)*d*e*(a + b*Sin[c + d*x])^3) + (3*a*b*(e*Cos[c + d*x])^(3/2))/(4*(a^2 -
 b^2)^2*d*e*(a + b*Sin[c + d*x])^2) + (b*(11*a^2 + 4*b^2)*(e*Cos[c + d*x])^
(3/2))/(8*(a^2 - b^2)^3*d*e*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^4}\,dx = \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} - \frac{\int \frac{\sqrt{e\cos(c+dx)}\left(-3a+\frac{3}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))^2} + \frac{\int \frac{\sqrt{e\cos(c+d}\cdots}{\cdots}}{\cdots}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))^2} + \frac{b\left(11a^2+4\cdots\right.}{8\left(a^2-b^2\right)\cdots}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))^2} + \frac{b\left(11a^2+4\cdots\right.}{8\left(a^2-b^2\right)\cdots}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))^2} + \frac{b\left(11a^2+4\cdots\right.}{8\left(a^2-b^2\right)\cdots}$$

$$= \frac{\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8\left(a^2-b^2\right)^3 d\sqrt{\cos(c+dx)}} + \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \cdots$$

$$= \frac{\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8\left(a^2-b^2\right)^3 d\sqrt{\cos(c+dx)}} + \frac{5a^2\left(a^2+2b^2\right)e\sqrt{\cos(c+dx)}\,\Pi\left(\frac{\cdots}{b-\sqrt{\cdots}}\cdots\right.}{16b\left(a^2-b^2\right)^3\left(b-\sqrt{-a^2+b^2}\right)\cdots}$$

$$= -\frac{5a\left(a^2+2b^2\right)\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\sqrt{b}\left(-a^2+b^2\right)^{13/4}d} + \frac{5a\left(a^2+2b^2\right)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\sqrt{b}\left(-a^2+b^2\right)^{13/4}d}\cdots$$

Mathematica [C] time = 6.63, size = 900, normalized size = 1.55

$$\frac{\sqrt{e\cos(c+dx)}\left(\frac{3ab\cos(c+dx)}{4(a^2-b^2)^2(a+b\sin(c+dx))^2}+\frac{b\cos(c+dx)}{3(a^2-b^2)(a+b\sin(c+dx))^3}-\frac{-4\cos(c+dx)b^3-11a^2\cos(c+dx)b}{8(a^2-b^2)^3(a+b\sin(c+dx))}\right)}{d}+\frac{\sqrt{e\cos(c+dx)}\left(-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*((b*Cos[c + d*x])/(3*(a^2 - b^2)*(a + b*Sin[c + d*x])
^3) + (3*a*b*Cos[c + d*x])/(4*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) - (-11*
a^2*b*Cos[c + d*x] - 4*b^3*Cos[c + d*x])/(8*(a^2 - b^2)^3*(a + b*Sin[c + d*
x]))))/d + (Sqrt[e*Cos[c + d*x]]*((-2*(16*a^3 + 14*a*b^2)*(a + b*Sqrt[1 - C
os[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(
2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*A
rcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sq
rt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*
b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin
[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((11*a^2*b + 4
*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/
4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) +
3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d
*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]]
)/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sq
rt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]
^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(
16*(a - b)^3*(a + b)^3*d*Sqrt[Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos(dx+c)}}{(b\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^4, x)
```

maple [C] time = 110.04, size = 112960, normalized size = 195.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cos(dx+c)}}{(b \sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\cos(c+d\,x)}}{(a+b\,\sin(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.613 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=593

$$\frac{11ab\sqrt{e\cos(c+dx)}}{12de\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{b\left(57a^2+20b^2\right)\sqrt{e\cos(c+dx)}}{24de\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b\sqrt{e\cos(c+dx)}}{3de\left(a^2-b^2\right)(a+b\sin(c+dx))^3}+\frac{7a}{}$$

```
[Out] 7/16*a*(5*a^2+6*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e
^(1/2))*b^(1/2)/(-a^2+b^2)^(15/4)/d/e^(1/2)+7/16*a*(5*a^2+6*b^2)*arctanh(b^
(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(15
/4)/d/e^(1/2)-1/24*(57*a^2+20*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/
d/(e*cos(d*x+c))^(1/2)+7/16*a^2*(5*a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2
^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(
d*x+c))^(1/2)+7/16*a^2*(5*a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*c
os(d*x+c)^(1/2)/(a^2-b^2)^3/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(
1/2)+1/3*b*(e*cos(d*x+c))^(1/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^3+11/12*a*b*
(e*cos(d*x+c))^(1/2)/(a^2-b^2)^2/d/e/(a+b*sin(d*x+c))^2+1/24*b*(57*a^2+20*b
^2)*(e*cos(d*x+c))^(1/2)/(a^2-b^2)^3/d/e/(a+b*sin(d*x+c))
```

Rubi [A] time = 1.57, antiderivative size = 593, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2694, 2864, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{11ab\sqrt{e\cos(c+dx)}}{12de\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{b\left(57a^2+20b^2\right)\sqrt{e\cos(c+dx)}}{24de\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b\sqrt{e\cos(c+dx)}}{3de\left(a^2-b^2\right)(a+b\sin(c+dx))^3}+\frac{7a}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] (7*a*Sqrt[b]*(5*a^2 + 6*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(16*(-a^2 + b^2)^(15/4)*d*Sqrt[e]) + (7*a*Sqrt[b]*(5
*a^2 + 6*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[e])])/(16*(-a^2 + b^2)^(15/4)*d*Sqrt[e]) - ((57*a^2 + 20*b^2)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2])/(24*(a^2 - b^2)^3*d*Sqrt[e*Cos[c + d*x]
]) + (7*a^2*(5*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^3*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*d*Sqrt[e*Cos[c + d*x]]) + (7*a^2*(5*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^3*
(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c
+ d*x]])/(3*(a^2 - b^2)*d*e*(a + b*Sin[c + d*x])^3) + (11*a*b*Sqrt[e*Cos[c
+ d*x]])/(12*(a^2 - b^2)^2*d*e*(a + b*Sin[c + d*x])^2) + (b*(57*a^2 + 20*b^
2)*Sqrt[e*Cos[c + d*x]])/(24*(a^2 - b^2)^3*d*e*(a + b*Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^4}\,dx = \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} - \frac{\int \frac{-3a+\frac{5}{2}b\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12\left(a^2-b^2\right)^2 de(a+b\sin(c+}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12\left(a^2-b^2\right)^2 de(a+b\sin(c+}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12\left(a^2-b^2\right)^2 de(a+b\sin(c+}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12\left(a^2-b^2\right)^2 de(a+b\sin(c+}$$

$$= -\frac{\left(57a^2+20b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24\left(a^2-b^2\right)^3 d\sqrt{e\cos(c+dx)}} + \frac{b\sqrt{e\cos(}}{3\left(a^2-b^2\right)de(a+}$$

$$= -\frac{\left(57a^2+20b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24\left(a^2-b^2\right)^3 d\sqrt{e\cos(c+dx)}} - \frac{7a^2\left(5a^2+6b^2\right)}{16\left(-a^2+b^2\right)}$$

$$= \frac{7a\sqrt{b}\left(5a^2+6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\left(-a^2+b^2\right)^{15/4} d\sqrt{e}} + \frac{7a\sqrt{b}\left(5a^2+6b^2\right)\tan}{16\left(-a^2+b^2\right.}$$

Mathematica [C] time = 24.46, size = 1276, normalized size = 2.15

$$\frac{\cos(c+dx)\left(\frac{(57a^2+20b^2)b}{24(a^2-b^2)^3(a+b\sin(c+dx))}+\frac{11ab}{12(a^2-b^2)^2(a+b\sin(c+dx))^2}+\frac{b}{3(a^2-b^2)(a+b\sin(c+dx))^3}\right)}{d\sqrt{e\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\left(-\frac{2(-20b^3-57a^2b\ldots}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] (Cos[c + d*x]*(b/(3*(a^2 - b^2)*(a + b*Sin[c + d*x])^3) + (11*a*b)/(12*(a^2
 - b^2)^2*(a + b*Sin[c + d*x])^2) + (b*(57*a^2 + 20*b^2))/(24*(a^2 - b^2)^3
*(a + b*Sin[c + d*x]))))/(d*Sqrt[e*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*((-
2*(48*a^3 + 106*a*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)
]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4
, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^
2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2)))
- ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(
-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^
2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b
^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) -
(2*(-57*a^2*b - 20*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(
2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2
))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1
/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x
]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(48*(a
 - b)^3*(a + b)^3*d*Sqrt[e*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cos(dx+c)}\,(b\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^4), x)
```

maple [C] time = 104.03, size = 85165, normalized size = 143.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^4),x)
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.614 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=674

$$\frac{13ab}{12de\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}+\frac{b\left(89a^2+28b^2\right)}{24de\left(a^2-b^2\right)^3\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}+\frac{}{3de\left(a^2-\right.}$$

```
[Out] -15/16*a*b^(3/2)*(7*a^2+6*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/d/e^(3/2)+15/16*a*b^(3/2)*(7*a^2+6*b^2)
*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^
(17/4)/d/e^(3/2)+1/3*b/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2
)+13/12*a*b/(a^2-b^2)^2/d/e/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2)+1/24*b*
(89*a^2+28*b^2)/(a^2-b^2)^3/d/e/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2)+1/8*(
-15*a*b*(7*a^2+6*b^2)+(16*a^4+151*a^2*b^2+28*b^4)*sin(d*x+c))/(a^2-b^2)^4/d
/e/(e*cos(d*x+c))^(1/2)-15/16*a^2*b*(7*a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2
)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^4/d/e/(b-(-a^2+b^2)^(1/2))/(e*cos(d*
x+c))^(1/2)-15/16*a^2*b*(7*a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*
cos(d*x+c)^(1/2)/(a^2-b^2)^4/d/e/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-
1/8*(16*a^4+151*a^2*b^2+28*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/(a^2-b^2)^4
/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.95, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2694, 2864, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{15ab^{3/2}\left(7a^2+6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16de^{3/2}\left(b^2-a^2\right)^{17/4}}+\frac{15ab^{3/2}\left(7a^2+6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16de^{3/2}\left(b^2-a^2\right)^{17/4}}-\frac{\left(151a^2b^2+16a^4+28b\right.}{8de^2\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] (-15*a*b^(3/2)*(7*a^2 + 6*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(16*(-a^2 + b^2)^(17/4)*d*e^(3/2)) + (15*a*b^(3/2)
*(7*a^2 + 6*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)
*Sqrt[e])])/(16*(-a^2 + b^2)^(17/4)*d*e^(3/2)) - ((16*a^4 + 151*a^2*b^2 + 2
8*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(8*(a^2 - b^2)^4*d*e
^2*Sqrt[Cos[c + d*x]]) - (15*a^2*b*(7*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^4*(b -
 Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) - (15*a^2*b*(7*a^2 + 6*b^2)*Sq
rt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/
(16*(a^2 - b^2)^4*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/(3*(
a^2 - b^2)*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3) + (13*a*b)/(12*
(a^2 - b^2)^2*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2) + (b*(89*a^2
 + 28*b^2))/(24*(a^2 - b^2)^3*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])
) - (15*a*b*(7*a^2 + 6*b^2) - (16*a^4 + 151*a^2*b^2 + 28*b^4)*Sin[c + d*x])
/(8*(a^2 - b^2)^4*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4}\,dx = \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} - \frac{\int \frac{-3a+\frac{7}{2}\cdots}{(e\cos(c+dx))^{3}\cdots}}{3\left(\cdots\right.}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} + \frac{\cdots}{12\left(a^2-b^2\right)^2\cdots}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} + \frac{\cdots}{12\left(a^2-b^2\right)^2\cdots}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} + \frac{\cdots}{12\left(a^2-b^2\right)^2\cdots}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} + \frac{\cdots}{12\left(a^2-b^2\right)^2\cdots}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} + \frac{\cdots}{12\left(a^2-b^2\right)^2\cdots}$$

$$= -\frac{\left(16a^4+151a^2b^2+28b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8\left(a^2-b^2\right)^4 de^2\sqrt{\cos(c+dx)}} + \frac{\cdots}{3\left(\cdots\right.}$$

$$= -\frac{\left(16a^4+151a^2b^2+28b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8\left(a^2-b^2\right)^4 de^2\sqrt{\cos(c+dx)}} - \frac{15a\cdots}{\cdots}$$

$$= -\frac{15ab^{3/2}\left(7a^2+6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\left(-a^2+b^2\right)^{17/4}de^{3/2}} + \frac{15ab^{3/2}\left(7a^2+6b^2\cdots\right.}{16\left(-a^2\cdots\right.}$$

Mathematica [C] time = 6.82, size = 996, normalized size = 1.48

$$\frac{\cos^2(c+dx)\left(-\frac{7a\cos(c+dx)b^3}{4\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2} - \frac{\cos(c+dx)b^3}{3\left(a^2-b^2\right)^2(a+b\sin(c+dx))^3} + \frac{2\sec(c+dx)\left(\sin(c+dx)a^4-4ba^3+6b^2\sin(c+dx)a^2-4b^3a+b^4\sin(\cdots\right.}{\left(a^2-b^2\right)^4}\cdots\right.}{d(e\cos(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] -1/16*(Cos[c + d*x]^(3/2)*((-2*(16*a^5 + 256*a^3*b^2 + 118*a*b^4)*(a + b*Sq
rt[1 - Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8
+ I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4
)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
- Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*
x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^
```

```
2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/
4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((16*a
^4*b + 151*a^2*b^3 + 28*b^5)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*Ap
pellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)
]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]
*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[
a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
 + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a
 + b*Sin[c + d*x]))))/((a - b)^4*(a + b)^4*d*(e*Cos[c + d*x])^(3/2)) + (Cos
[c + d*x]^2*(-1/3*(b^3*Cos[c + d*x])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^3)
 - (7*a*b^3*Cos[c + d*x])/(4*(a^2 - b^2)^3*(a + b*Sin[c + d*x])^2) + (-55*a
^2*b^3*Cos[c + d*x] - 12*b^5*Cos[c + d*x])/(8*(a^2 - b^2)^4*(a + b*Sin[c +
d*x])) + (2*Sec[c + d*x]*(-4*a^3*b - 4*a*b^3 + a^4*Sin[c + d*x] + 6*a^2*b^2
*Sin[c + d*x] + b^4*Sin[c + d*x]))/(a^2 - b^2)^4))/(d*(e*Cos[c + d*x])^(3/2
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^4), x)
```

maple [C] time = 184.06, size = 150599, normalized size = 223.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^4),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.615 $\int \frac{1}{\sqrt{c\cos(e+fx)}\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=183

$$\frac{2\sqrt{2}\sqrt[4]{b-a}\sqrt{c\cos(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(1-\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{a+b}\sqrt{\frac{\cos(e+fx)+\sin(e+fx)+1}{\cos(e+fx)-\sin(e+fx)+1}}}{\sqrt[4]{b-a}}\right)\middle|-1\right)}{cf\sqrt[4]{a+b}\sqrt{\frac{\sin(e+fx)+\cos(e+fx)+1}{-\sin(e+fx)+\cos(e+fx)+1}}\sqrt{a+b\sin(e+fx)}}$$

```
[Out] 2*(-a+b)^(1/4)*EllipticF((a+b)^(1/4)*((1+cos(f*x+e)+sin(f*x+e))/(1+cos(f*x+
e)-sin(f*x+e)))^(1/2)/(-a+b)^(1/4),I)*2^(1/2)*(c*cos(f*x+e))^(1/2)*((a+b*si
n(f*x+e))/(a-b)/(1-sin(f*x+e)))^(1/2)/(a+b)^(1/4)/c/f/((1+cos(f*x+e)+sin(f*
x+e))/(1+cos(f*x+e)-sin(f*x+e)))^(1/2)/(a+b*sin(f*x+e))^(1/2)
```

Rubi [B] time = 0.43, antiderivative size = 374, normalized size of antiderivative = 2.04, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2697, 220}

$$\frac{\sqrt{2}\sqrt[4]{a-b}\sqrt{c\cos(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(1-\sin(e+fx))}}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(\sin(e)(-\cos(fx))-\cos(e)\sin(fx)+1)\left(\frac{\sqrt{a+b}(\sin(e+fx)+\cos(e+fx)+1)}{\sqrt{a-b}(-\sin(e+fx)+\cos(e+fx)+1)}+1\right)^2}}\left(\frac{\sqrt{a+b}}{\sqrt{a-b}(\ldots}\right.}{cf\sqrt[4]{a+b}\sqrt{\frac{\sin(e+fx)+\cos(e+fx)+1}{-\sin(e+fx)+\cos(e+fx)+1}}\sqrt{a+b\sin(e+fx)}\sqrt{\frac{a+\ldots}{(a-b)(\sin(e)(-\ldots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[c*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[2]*(a - b)^(1/4)*Sqrt[c*Cos[e + f*x]]*EllipticF[2*ArcTan[((a + b)^(1/
4)*Sqrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e + f*x])
])/(a - b)^(1/4)], 1/2]*Sqrt[(a + b*Sin[e + f*x])/((a - b)*(1 - Sin[e + f*x
]))]*Sqrt[(a + b*Sin[e + f*x])/((a - b)*(1 - Cos[f*x]*Sin[e] - Cos[e]*Sin[f
*x])*(1 + (Sqrt[a + b]*(1 + Cos[e + f*x] + Sin[e + f*x]))/(Sqrt[a - b]*(1 +
 Cos[e + f*x] - Sin[e + f*x])))^2)]*(1 + (Sqrt[a + b]*(1 + Cos[e + f*x] + S
in[e + f*x]))/(Sqrt[a - b]*(1 + Cos[e + f*x] - Sin[e + f*x]))))/((a + b)^(1
/4)*c*f*Sqrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e +
f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/((a - b)*(1 - Cos
[f*x]*Sin[e] - Cos[e]*Sin[f*x]))])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 2697

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[(2*Sqrt[2]*Sqrt[g*Cos[e + f*x]]*Sqrt[(a + b*
Sin[e + f*x])/((a - b)*(1 - Sin[e + f*x]))])/(f*g*Sqrt[a + b*Sin[e + f*x]]*
Sqrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e + f*x])]),
 Subst[Int[1/Sqrt[1 + ((a + b)*x^4)/(a - b)], x], x, Sqrt[(1 + Cos[e + f*x]
 + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e + f*x])]], x] /; FreeQ[{a, b, e,
 f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{c\cos(e+fx)}\sqrt{a+b\sin(e+fx)}}dx = \frac{\left(2\sqrt{2}\sqrt{c\cos(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(1-\sin(e+fx))}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(a+b)x^4}{a-b}}}dx, x\right.}{cf\sqrt{\frac{1+\cos(e+fx)+\sin(e+fx)}{1+\cos(e+fx)-\sin(e+fx)}}\sqrt{a+b\sin(e+fx}}$$

$$= \frac{\sqrt{2}\sqrt[4]{a-b}\sqrt{c\cos(e+fx)}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\sqrt{\frac{1+\cos(e+fx)+\sin(e+fx)}{1+\cos(e+fx)-\sin(e+fx)}}}{\sqrt[4]{a-b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a+b}\,cf\sqrt{\frac{1+\cos(e+f}{1+\cos(e+f}}}$$

Mathematica [C] time = 0.32, size = 117, normalized size = 0.64

$$-\frac{2c(\sin(e+fx)-1)\left(\frac{(a+b)(\sin(e+fx)+1)}{(a-b)(\sin(e+fx)-1)}\right)^{3/4}\sqrt{a+b\sin(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{2(a+b\sin(e+fx))}{(a-b)(\sin(e+fx)-1)}\right)}{f(a+b)(c\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]),x]
```

```
[Out] (-2*c*Hypergeometric2F1[1/2, 3/4, 3/2, (-2*(a + b*Sin[e + f*x]))/((a - b)*(
-1 + Sin[e + f*x]))]*(-1 + Sin[e + f*x])*(((a + b)*(1 + Sin[e + f*x]))/((a
- b)*(-1 + Sin[e + f*x])))^(3/4)*Sqrt[a + b*Sin[e + f*x]])/((a + b)*f*(c*Co
s[e + f*x])^(3/2))
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c\cos\left(fx+e\right)}\sqrt{b\sin\left(fx+e\right)+a}}{bc\cos\left(fx+e\right)\sin\left(fx+e\right)+ac\cos\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(c*cos(f*x + e))*sqrt(b*sin(f*x + e) + a)/(b*c*cos(f*x + e)*si
n(f*x + e) + a*c*cos(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\cos\left(fx+e\right)}\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*cos(f*x + e))*sqrt(b*sin(f*x + e) + a)), x)
```

maple [B] time = 0.61, size = 442, normalized size = 2.42

$$\frac{4\,\text{EllipticF}\left(\sqrt{\frac{\left(\sin\left(fx+e\right)-1\right)\left(b+\sqrt{-a^2+b^2}-a\right)}{\cos\left(fx+e\right)\left(b+\sqrt{-a^2+b^2}+a\right)}}, \sqrt{\frac{\left(a-b+\sqrt{-a^2+b^2}\right)\left(b+\sqrt{-a^2+b^2}+a\right)}{\left(-b+\sqrt{-a^2+b^2}-a\right)\left(b+\sqrt{-a^2+b^2}-a\right)}}\right)\sqrt{\frac{\cos\left(fx+e\right)\sqrt{-a^2+b^2}+a\sin\left(fx+e\right)+b\cos\left(fx+\right.}{\left(1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\left(b+\sqrt{-a^2\right.}}}}{f\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] 4/f*EllipticF(((sin(f*x+e)-1)/cos(f*x+e)*(b+(-a^2+b^2)^(1/2)-a)/(b+(-a^2+b^
2)^(1/2)+a))^(1/2),((a-b+(-a^2+b^2)^(1/2))*(b+(-a^2+b^2)^(1/2)+a)/(-b+(-a^2
+b^2)^(1/2)-a)/(b+(-a^2+b^2)^(1/2)-a))^(1/2))*((cos(f*x+e)*(-a^2+b^2)^(1/2)
+a*sin(f*x+e)+b*cos(f*x+e)+(-a^2+b^2)^(1/2)+b)/(1+cos(f*x+e)+sin(f*x+e))/(b
+(-a^2+b^2)^(1/2)+a))^(1/2)*((sin(f*x+e)-1)/cos(f*x+e)*(b+(-a^2+b^2)^(1/2)-
a)/(b+(-a^2+b^2)^(1/2)+a))^(1/2)*(-(a*sin(f*x+e)-cos(f*x+e)*(-a^2+b^2)^(1/2
)+b*cos(f*x+e)-(-a^2+b^2)^(1/2)+b)/(1+cos(f*x+e)+sin(f*x+e))/(-b+(-a^2+b^2)
^(1/2)-a))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(b+(-a^2+b^2)^(1/2)+a)*
(1+sin(f*x+e))/(a+b*sin(f*x+e))^(1/2)/sin(f*x+e)^4/(c*cos(f*x+e))^(1/2)/(b+
(-a^2+b^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\cos\left(fx+e\right)}\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*cos(f*x + e))*sqrt(b*sin(f*x + e) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,\cos\left(e+f\,x\right)}\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((c*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\cos\left(e+fx\right)}\sqrt{a+b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*cos(e + f*x))*sqrt(a + b*sin(e + f*x))), x)
```

3.616 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=229

$$-\frac{a\left(a^2(p+2)+3b^2\right)\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)(p+2)\sqrt{\sin^2(c+dx)}}-\frac{b\left(a^2\left(p^2+6p+11\right)+2b^2(p\right.}{de(p+1)(p+}$$

```
[Out] -b*(2*b^2*(2+p)+a^2*(p^2+6*p+11))*(e*cos(d*x+c))^(1+p)/d/e/(3+p)/(p^2+3*p+2
)-a*b*(5+p)*(e*cos(d*x+c))^(1+p)*(a+b*sin(d*x+c))/d/e/(2+p)/(3+p)-b*(e*cos(
d*x+c))^(1+p)*(a+b*sin(d*x+c))^2/d/e/(3+p)-a*(3*b^2+a^2*(2+p))*(e*cos(d*x+c
))^(1+p)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],cos(d*x+c)^2)*sin(d*x+c)/d/
e/(1+p)/(2+p)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2692, 2862, 2669, 2643}

$$-\frac{a\left(a^2(p+2)+3b^2\right)\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)(p+2)\sqrt{\sin^2(c+dx)}}-\frac{b\left(a^2\left(p^2+6p+11\right)+2b^2(p\right.}{de(p+1)(p+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((b*(2*b^2*(2 + p) + a^2*(11 + 6*p + p^2))*(e*Cos[c + d*x])^(1 + p))/(d*e*
(1 + p)*(2 + p)*(3 + p))) - (a*(3*b^2 + a^2*(2 + p))*(e*Cos[c + d*x])^(1 +
p)*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[c + d*x
])/(d*e*(1 + p)*(2 + p)*Sqrt[Sin[c + d*x]^2]) - (a*b*(5 + p)*(e*Cos[c + d*x
])^(1 + p)*(a + b*Sin[c + d*x]))/(d*e*(2 + p)*(3 + p)) - (b*(e*Cos[c + d*x]
)^(1 + p)*(a + b*Sin[c + d*x])^2)/(d*e*(3 + p))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
```

```
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^3\,dx = -\frac{b(e\cos(c+dx))^{1+p}(a+b\sin(c+dx))^2}{de(3+p)} + \frac{\int (e\cos(c+dx))^p(a+b\sin$$

$$= -\frac{ab(5+p)(e\cos(c+dx))^{1+p}(a+b\sin(c+dx))}{de(2+p)(3+p)} - \frac{b(e\cos(c+dx))^{1+p}(}{de(3}$$

$$= -\frac{b\left(2b^2(2+p)+a^2\left(11+6p+p^2\right)\right)(e\cos(c+dx))^{1+p}}{de(1+p)\left(6+5p+p^2\right)} - \frac{ab(5+p)(e\,c}{}$$

$$= -\frac{b\left(2b^2(2+p)+a^2\left(11+6p+p^2\right)\right)(e\cos(c+dx))^{1+p}}{de(1+p)\left(6+5p+p^2\right)} - \frac{a\left(a^2+\frac{3b^2}{2+p}\right)}{}$$

Mathematica [A] time = 54.88, size = 290, normalized size = 1.27

$$\frac{8\sec^2(c+dx)^{p/2}(a+b\sin(c+dx))^3(e\cos(c+dx))^p\left(a^3\tan(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{p+4}{2};\frac{3}{2};-\tan^2(c+dx)\right)+\frac{1}{3}a\left(a^2+3b^2\right.\right.}{d\left(8a^3+2b\left(6a^2+b^2\right)\sin(2(c+dx))\sqrt{\sec^2(c+dx)}-12ab^2\cos\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (8*(e*Cos[c + d*x])^p*(Sec[c + d*x]^2)^(p/2)*(a + b*Sin[c + d*x])^3*((-3*a^
2*b*(Sec[c + d*x]^2)^(-3/2 - p/2))/(3 + p) + a^3*Hypergeometric2F1[1/2, (4
+ p)/2, 3/2, -Tan[c + d*x]^2]*Tan[c + d*x] + (a*(a^2 + 3*b^2)*Hypergeometri
c2F1[3/2, (4 + p)/2, 5/2, -Tan[c + d*x]^2]*Tan[c + d*x]^3)/3 - (b*(3*a^2 +
b^2)*(Sec[c + d*x]^2)^(-3/2 - p/2)*(2 + (3 + p)*Tan[c + d*x]^2))/((1 + p)*(
3 + p))))/(d*(8*a^3 + 12*a*b^2 - 12*a*b^2*Cos[2*(c + d*x)] + 2*b*(6*a^2 + b
^2)*Sqrt[Sec[c + d*x]^2]*Sin[2*(c + d*x)] - b^3*Sqrt[Sec[c + d*x]^2]*Sin[4*
(c + d*x)]))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,ab^2\cos\left(dx+c\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\left(e\cos\left(dx+c\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*(e*cos(d*x + c))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^3(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^3*(e*cos(d*x + c))^p, x)
```

maple [F] time = 6.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p (a+b\sin(dx+c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x)
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^3 (e\cos(dx+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^3*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cos(c+dx))^p (a+b\sin(c+dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

3.617 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=157

$$-\frac{(a^2(p+2)+b^2)\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)(p+2)\sqrt{\sin^2(c+dx)}}-\frac{ab(p+3)(e\cos(c+dx))^{p+1}}{de(p+1)(p+2)}-\frac{b(a$$

```
[Out] -a*b*(3+p)*(e*cos(d*x+c))^(1+p)/d/e/(1+p)/(2+p)-b*(e*cos(d*x+c))^(1+p)*(a+b
*sin(d*x+c))/d/e/(2+p)-(b^2+a^2*(2+p))*(e*cos(d*x+c))^(1+p)*hypergeom([1/2,
 1/2+1/2*p],[3/2+1/2*p],cos(d*x+c)^2)*sin(d*x+c)/d/e/(1+p)/(2+p)/(sin(d*x+c
)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2692, 2669, 2643}

$$-\frac{(a^2(p+2)+b^2)\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)(p+2)\sqrt{\sin^2(c+dx)}}-\frac{ab(p+3)(e\cos(c+dx))^{p+1}}{de(p+1)(p+2)}-\frac{b(a$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a*b*(3 + p)*(e*Cos[c + d*x])^(1 + p))/(d*e*(1 + p)*(2 + p))) - ((b^2 + a
^2*(2 + p))*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[1/2, (1 + p)/2, (3 +
 p)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*e*(1 + p)*(2 + p)*Sqrt[Sin[c + d*x]
^2]) - (b*(e*Cos[c + d*x])^(1 + p)*(a + b*Sin[c + d*x]))/(d*e*(2 + p))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^2\,dx = -\frac{b(e\cos(c+dx))^{1+p}(a+b\sin(c+dx))}{de(2+p)} + \frac{\int (e\cos(c+dx))^p\left(b^2+a\right.}{}$$

$$= -\frac{ab(3+p)(e\cos(c+dx))^{1+p}}{de(1+p)(2+p)} - \frac{b(e\cos(c+dx))^{1+p}(a+b\sin(c+d}{de(2+p)}$$

$$= -\frac{ab(3+p)(e\cos(c+dx))^{1+p}}{de(1+p)(2+p)} - \frac{\left(b^2+a^2(2+p)\right)(e\cos(c+dx))^{1+p}}{de(1+p)(2}$$

Mathematica [C] time = 1.05, size = 285, normalized size = 1.82

$$-\frac{(e\cos(c+dx))^p\left(-\frac{1}{2}a^2(p-1)\sin(2(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)+ab2^{-p}\left(1+e^{2i(c+dx)}\right)\left(e^{-i(c+dx)}+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((e*Cos[c + d*x])^p*((a*b*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^p*(1 + E
^((2*I)*(c + d*x)))*(-(((-1 + p)*Hypergeometric2F1[1, (1 + p)/2, (1 - p)/2,
 -E^((2*I)*(c + d*x))])/E^(I*(c + d*x))) + E^(I*(c + d*x))*(1 + p)*Hypergeo
metric2F1[1, (3 + p)/2, (3 - p)/2, -E^((2*I)*(c + d*x))])*Sqrt[Sin[c + d*x]
^2])/(2^p*Cos[c + d*x]^p) - (b^2*(-1 + p)*Hypergeometric2F1[-1/2, (1 + p)/2
, (3 + p)/2, Cos[c + d*x]^2]*Sin[2*(c + d*x)])/2 - (a^2*(-1 + p)*Hypergeome
tric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[2*(c + d*x)])/2))/((
d - d*p^2)*Sqrt[Sin[c + d*x]^2]))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2\right)(e\cos(dx+c))^p\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*(e*cos(d*x
+ c))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^2(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*(e*cos(d*x + c))^p, x)
```

maple [F] time = 4.43, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p(a+b\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^2(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^2*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+d\,x))^p\,(a+b\sin(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^p\,(a+b\sin(c+dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**2,x)
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x))**2, x)
```

3.618 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)\sqrt{\sin^2(c+dx)}}-\frac{b(e\cos(c+dx))^{p+1}}{de(p+1)}$$

```
[Out] -b*(e*cos(d*x+c))^(1+p)/d/e/(1+p)-a*(e*cos(d*x+c))^(1+p)*hypergeom([1/2, 1/
2+1/2*p],[3/2+1/2*p],cos(d*x+c)^2)*sin(d*x+c)/d/e/(1+p)/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2669, 2643}

$$-\frac{a\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)\sqrt{\sin^2(c+dx)}}-\frac{b(e\cos(c+dx))^{p+1}}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*(e*Cos[c + d*x])^(1 + p))/(d*e*(1 + p))) - (a*(e*Cos[c + d*x])^(1 + p)
*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(d*e*(1 + p)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))\,dx=-\frac{b(e\cos(c+dx))^{1+p}}{de(1+p)}+a\int (e\cos(c+dx))^p\,dx$$

$$=-\frac{b(e\cos(c+dx))^{1+p}}{de(1+p)}-\frac{a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\cos^2(c+\right.}{de(1+p)\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 0.99, size = 240, normalized size = 2.47

$$-\frac{(e\cos(c+dx))^p\left(-\frac{1}{2}a(p-1)\sin(2(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)+b2^{-p-1}\left(1+e^{2i(c+dx)}\right)\left(e^{-i(c+dx)}+\right.\right.}{\left(d-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((e*Cos[c + d*x])^p*((2^(-1 - p)*b*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))
^p*(1 + E^((2*I)*(c + d*x)))*(-(((-1 + p)*Hypergeometric2F1[1, (1 + p)/2, (
1 - p)/2, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x))) + E^(I*(c + d*x))*(1 + p)
*Hypergeometric2F1[1, (3 + p)/2, (3 - p)/2, -E^((2*I)*(c + d*x))])*Sqrt[Sin
[c + d*x]^2])/Cos[c + d*x]^p - (a*(-1 + p)*Hypergeometric2F1[1/2, (1 + p)/2
, (3 + p)/2, Cos[c + d*x]^2]*Sin[2*(c + d*x)])/2))/((d - d*p^2)*Sqrt[Sin[c
+ d*x]^2]))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(dx+c\right)+a\right)\left(e\cos\left(dx+c\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)\left(e\cos\left(dx+c\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

maple [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{p}\left(a+b\sin\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)\left(e\cos\left(dx+c\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,\left(a+b\,\sin\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(c+dx\right)\right)^{p}\left(a+b\sin\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x)), x)
```

3.619 $\int \frac{(e\cos(c+dx))^p}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(1-p)}$$

```
[Out] -e*AppellF1(1-p,1/2-1/2*p,1/2-1/2*p,2-p,(a-b)/(a+b*sin(d*x+c)),(a+b)/(a+b*s
in(d*x+c)))*(e*cos(d*x+c))^(-1+p)*(-b*(1-sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2
-1/2*p)*(b*(1+sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2-1/2*p)/b/d/(1-p)
```

Rubi [A] time = 0.08, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2703}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((e*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(1 - p)))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{a+b\sin(c+dx)}\,dx=-\frac{eF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)}{bd(1-p)}$$

Mathematica [B] time = 20.27, size = 3815, normalized size = 24.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^p*Tan[c + d*x]*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*
(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d
*x]^2]*Tan[c + d*x]) - (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (
```

(-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)))/(2*a^2*d*Sqrt[Sec[c + d*x]^2]*(a + b*Sin[c + d*x])*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])*((Sqrt[Sec[c + d*x]^2]*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x]) - (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2))))/(2*a^2*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])) - (Tan[c + d*x]^2*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x]) - (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2))))/(2*a^2*Sqrt[Sec[c + d*x]^2]*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])) + (Tan[c + d*x]*(b*Sec[c + d*x]^2 + a*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x]) - (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2))))/(2*a^2*Sqrt[Sec[c + d*x]^2]*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])) - (Tan[c + d*x]*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*(b*Sqrt[Sec[c + d*x]^2] - (b*Tan[c + d*x]^2)/Sqrt[Sec[c + d*x]^2])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x]) - (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2))))/(2*a^2*Sqrt[Sec[c + d*x]^2]*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])^2) + (Tan[c + d*x]*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2) - b*Tan[c + d*x]*((-1 + b^2/a^2)*AppellF1[2, (1 + p)/2, 2, 3, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x] - ((1 + p)*AppellF1[2, 1 + (1 + p)/2, 1, 3, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/2) + (12*a^5*(a^2 - b^2)*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*(Sec[c + d*x]^2)^(1 - p/2)*Tan[c + d*x])/((a^2 + (a^2 - b^2)*Tan[c + d*x]^2)^2*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)) + (6*a^5*p*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Tan[c + d*x])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 +

```
p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x
]^2)) - (6*a^5*(-1/3*(p*AppellF1[3/2, 1 + p/2, 1, 5/2, -Tan[c + d*x]^2, ((-
a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Sec[c + d*x]^2*Tan[c + d*x]) + (2*(-a^2 + b
^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^
2)/a^2]*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2)))/((Sec[c + d*x]^2)^(p/2)*(a^2
 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c +
d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2,
2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/
2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[
c + d*x]^2)) + (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 +
b^2)*Tan[c + d*x]^2)/a^2]*(2*(2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan
[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2
, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Sec[c + d*x]^2*T
an[c + d*x] - 3*a^2*(-1/3*(p*AppellF1[3/2, 1 + p/2, 1, 5/2, -Tan[c + d*x]^2
, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x]) + (2*(-1 + b^
2/a^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d
*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/3) + Tan[c + d*x]^2*(2*(a^2 - b^2)*((-3
*p*AppellF1[5/2, 1 + p/2, 2, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d
*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5 + (12*(-1 + b^2/a^2)*AppellF1[5/2, p/
2, 3, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*T
an[c + d*x])/5) + a^2*p*((6*(-1 + b^2/a^2)*AppellF1[5/2, (2 + p)/2, 2, 7/2,
 -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x
])/5 - (3*(2 + p)*AppellF1[5/2, 1 + (2 + p)/2, 1, 7/2, -Tan[c + d*x]^2, (-1
 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/((Sec[c + d*
x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1
, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*App
ellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a
^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c
 + d*x]^2])*Tan[c + d*x]^2)^2)))/(2*a^2*Sqrt[Sec[c + d*x]^2]*(a + (b*Tan[c
+ d*x])/Sqrt[Sec[c + d*x]^2])))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\cos(dx+c))^p}{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((e*cos(d*x + c))^p/(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a), x)
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

3.620 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(2-p;\frac{1-p}{2},\frac{1-p}{2};3-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(2-p)(a+b\sin(c+dx))}$$

```
[Out] -e*AppellF1(2-p,1/2-1/2*p,1/2-1/2*p,3-p,(a-b)/(a+b*sin(d*x+c)),(a+b)/(a+b*s
in(d*x+c)))*(e*cos(d*x+c))^(-1+p)*(-b*(1-sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2
-1/2*p)*(b*(1+sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2-1/2*p)/b/d/(2-p)/(a+b*sin(
d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2703}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(2-p;\frac{1-p}{2},\frac{1-p}{2};3-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(2-p)(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((e*AppellF1[2 - p, (1 - p)/2, (1 - p)/2, 3 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(2 - p)*(a + b*Sin[c + d*x])))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^2}\,dx = -\frac{eF_1\left(2-p;\frac{1-p}{2},\frac{1-p}{2};3-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right.}{bd(2-p)(a+b\sin(c+dx))}$$

Mathematica [B] time = 25.54, size = 4727, normalized size = 27.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^p*Tan[c + d*x]*(b*(a^2 - b^2)*AppellF1[1, (-1 + p)/2, 2,
2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x] + (3*a^5*((
```

```
-2*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c
 + d*x]^2])/((-3*a^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/
a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d
*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5
/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(b^2*T
an[c + d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^2) + ((a^2 + b^2)*AppellF1[1/2, p
/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])/((-3*a^2*Appel
lF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*
(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[
c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 +
 b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(-(b^2*Tan[c + d*x]^2) + a^2*(1
+ Tan[c + d*x]^2)))))/(1 + Tan[c + d*x]^2)^(p/2)))/(a^3*(-a^2 + b^2)*d*(a +
 b*Sin[c + d*x])^2*((Sec[c + d*x]^2*(b*(a^2 - b^2)*AppellF1[1, (-1 + p)/2,
2, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x] + (3*a^5
*((-2*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Ta
n[c + d*x]^2])/((-3*a^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b
^2/a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c
+ d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2
, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(b^
2*Tan[c + d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^2) + ((a^2 + b^2)*AppellF1[1/2
, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])/((-3*a^2*Ap
pellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] +
(2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*T
an[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-
1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(-(b^2*Tan[c + d*x]^2) + a^2*
(1 + Tan[c + d*x]^2)))))/(1 + Tan[c + d*x]^2)^(p/2)))/(a^3*(-a^2 + b^2)) +
(Tan[c + d*x]*(b*(a^2 - b^2)*AppellF1[1, (-1 + p)/2, 2, 2, -Tan[c + d*x]^2,
 (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2 + b*(a^2 - b^2)*Tan[c + d*x]
*(-1/2*((-1 + p)*AppellF1[2, 1 + (-1 + p)/2, 2, 3, -Tan[c + d*x]^2, (-1 + b
^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x]) + 2*(-1 + b^2/a^2)*App
ellF1[2, (-1 + p)/2, 3, 3, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*
Sec[c + d*x]^2*Tan[c + d*x]) - 3*a^5*p*Sec[c + d*x]^2*Tan[c + d*x]*(1 + Tan
[c + d*x]^2)^(-1 - p/2)*((-2*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*
x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])/((-3*a^2*AppellF1[1/2, p/2, 2, 3/2, -
Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*AppellF1[3/
2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*App
ellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^
2])*Tan[c + d*x]^2)*(b^2*Tan[c + d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^2) + ((
a^2 + b^2)*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c
 + d*x]^2])/((-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/
a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d
*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5
/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(-(b^2
*Tan[c + d*x]^2) + a^2*(1 + Tan[c + d*x]^2)))) + (3*a^5*((4*a^2*b^2*AppellF
1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*(-2*a^2
*Sec[c + d*x]^2*Tan[c + d*x] + 2*b^2*Sec[c + d*x]^2*Tan[c + d*x]))/((-3*a^2
*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]
 + (4*(a^2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2
)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2,
 (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(b^2*Tan[c + d*x]^2 - a^2*
(1 + Tan[c + d*x]^2))^3) - (2*a^2*b^2*(-1/3*(p*AppellF1[3/2, 1 + p/2, 2, 5/
2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d
*x]) + (4*(-1 + b^2/a^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 +
b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/3))/((-3*a^2*AppellF1
[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (4*(a^
2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c +
 d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^
2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(b^2*Tan[c + d*x]^2 - a^2*(1 + Tan[
c + d*x]^2))^2) - ((a^2 + b^2)*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2,
```

```
(-1 + b^2/a^2)*Tan[c + d*x]^2]*(2*a^2*Sec[c + d*x]^2*Tan[c + d*x] - 2*b^2*S
ec[c + d*x]^2*Tan[c + d*x]))/((-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d
*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2
, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2
, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c
 + d*x]^2)*(-(b^2*Tan[c + d*x]^2) + a^2*(1 + Tan[c + d*x]^2))^2) + ((a^2 +
b^2)*(-1/3*(p*AppellF1[3/2, 1 + p/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2
)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x]) + (2*(-1 + b^2/a^2)*AppellF1
[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c +
d*x]^2*Tan[c + d*x])/3))/((-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^
2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/
2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2
 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d
*x]^2)*(-(b^2*Tan[c + d*x]^2) + a^2*(1 + Tan[c + d*x]^2))) - ((a^2 + b^2)*A
ppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*(
2*(2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)
*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2,
(-1 + b^2/a^2)*Tan[c + d*x]^2])*Sec[c + d*x]^2*Tan[c + d*x] - 3*a^2*(-1/3*(
p*AppellF1[3/2, 1 + p/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*
x]^2]*Sec[c + d*x]^2*Tan[c + d*x]) + (2*(-1 + b^2/a^2)*AppellF1[3/2, p/2, 2
, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c
 + d*x])/3) + Tan[c + d*x]^2*(2*(a^2 - b^2)*((-3*p*AppellF1[5/2, 1 + p/2, 2
, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c
 + d*x])/5 + (12*(-1 + b^2/a^2)*AppellF1[5/2, p/2, 3, 7/2, -Tan[c + d*x]^2,
 (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5) + a^2*p*((6
*(-1 + b^2/a^2)*AppellF1[5/2, (2 + p)/2, 2, 7/2, -Tan[c + d*x]^2, (-1 + b^2
/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5 - (3*(2 + p)*AppellF1[
5/2, 1 + (2 + p)/2, 1, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]
*Sec[c + d*x]^2*Tan[c + d*x])/5))))/((-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Ta
n[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2,
 p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*Appel
lF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]
)*Tan[c + d*x]^2)^2*(-(b^2*Tan[c + d*x]^2) + a^2*(1 + Tan[c + d*x]^2))) + (
2*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c
+ d*x]^2]*(2*(4*(a^2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1
 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c
 + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Sec[c + d*x]^2*Tan[c + d*x] - 3*
a^2*(-1/3*(p*AppellF1[3/2, 1 + p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)
*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x]) + (4*(-1 + b^2/a^2)*AppellF1[
3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d
*x]^2*Tan[c + d*x])/3) + Tan[c + d*x]^2*(4*(a^2 - b^2)*((-3*p*AppellF1[5/2,
 1 + p/2, 3, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d
*x]^2*Tan[c + d*x])/5 + (18*(-1 + b^2/a^2)*AppellF1[5/2, p/2, 4, 7/2, -Tan[
c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5)
+ a^2*p*((12*(-1 + b^2/a^2)*AppellF1[5/2, (2 + p)/2, 3, 7/2, -Tan[c + d*x]^
2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5 - (3*(2 +
p)*AppellF1[5/2, 1 + (2 + p)/2, 2, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan
[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/((-3*a^2*AppellF1[1/2, p/2,
 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*A
ppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] +
 a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan
[c + d*x]^2])*Tan[c + d*x]^2)^2*(b^2*Tan[c + d*x]^2 - a^2*(1 + Tan[c + d*x]
^2))^2)))/(1 + Tan[c + d*x]^2)^(p/2)))/(a^3*(-a^2 + b^2))))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(e\cos\left(dx+c\right)\right)^{p}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(e*cos(d*x + c))^p/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^2, x)
```

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+b\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.621 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=170

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(3-p;\frac{1-p}{2},\frac{1-p}{2};4-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(3-p)(a+b\sin(c+dx))^2}$$

```
[Out] -e*AppellF1(3-p,1/2-1/2*p,1/2-1/2*p,4-p,(a-b)/(a+b*sin(d*x+c)),(a+b)/(a+b*s
in(d*x+c)))*(e*cos(d*x+c))^(-1+p)*(-b*(1-sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2
-1/2*p)*(b*(1+sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2-1/2*p)/b/d/(3-p)/(a+b*sin(
d*x+c))^2
```

Rubi [A] time = 0.07, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2703}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(3-p;\frac{1-p}{2},\frac{1-p}{2};4-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(3-p)(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((e*AppellF1[3 - p, (1 - p)/2, (1 - p)/2, 4 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(3 - p)*(a + b*Sin[c + d*x])^2))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^3}\,dx = -\frac{eF_1\left(3-p;\frac{1-p}{2},\frac{1-p}{2};4-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+d}{a+b\sin(c+d}\right.}{bd(3-p)(a+b\sin(c+dx))^2}$$

Mathematica [B] time = 28.26, size = 7781, normalized size = 45.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(e\cos(dx+c))^p}{3\,ab^2\cos(dx+c)^2 - a^3 - 3\,ab^2 + \left(b^3\cos(dx+c)^2 - 3\,a^2b - b^3\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] integral(-(e*cos(d*x + c))^p/(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3
*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^3, x)
```

maple [F] time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x)
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+b\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

3.622 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=170

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(8-p;\frac{1-p}{2},\frac{1-p}{2};9-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(8-p)(a+b\sin(c+dx))^7}$$

```
[Out] -e*AppellF1(8-p,1/2-1/2*p,1/2-1/2*p,9-p,(a-b)/(a+b*sin(d*x+c)),(a+b)/(a+b*s
in(d*x+c)))*(e*cos(d*x+c))^(-1+p)*(-b*(1-sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2
-1/2*p)*(b*(1+sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2-1/2*p)/b/d/(8-p)/(a+b*sin(
d*x+c))^7
```

Rubi [A] time = 0.07, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2703}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(8-p;\frac{1-p}{2},\frac{1-p}{2};9-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(8-p)(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -((e*AppellF1[8 - p, (1 - p)/2, (1 - p)/2, 9 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(8 - p)*(a + b*Sin[c + d*x])^7))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^8}\,dx = -\frac{eF_1\left(8-p;\frac{1-p}{2},\frac{1-p}{2};9-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+d}{a+b\sin(c+d}\right.}{bd(8-p)(a+b\sin(c+dx))^7}$$

Mathematica [F] time = 67.68, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^8, x]
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{}{b^8\cos(dx+c)^8+a^8+28\,a^6b^2+70\,a^4b^4+28\,a^2b^6+b^8-4\left(7\,a^2b^6+b^8\right)\cos(dx+c)^6+2\left(35\,a^4b^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p/(b^8*cos(d*x + c)^8 + a^8 + 28*a^6*b^2 + 70*a^4
*b^4 + 28*a^2*b^6 + b^8 - 4*(7*a^2*b^6 + b^8)*cos(d*x + c)^6 + 2*(35*a^4*b^
4 + 42*a^2*b^6 + 3*b^8)*cos(d*x + c)^4 - 4*(7*a^6*b^2 + 35*a^4*b^4 + 21*a^2
*b^6 + b^8)*cos(d*x + c)^2 - 8*(a*b^7*cos(d*x + c)^6 - a^7*b - 7*a^5*b^3 -
7*a^3*b^5 - a*b^7 - (7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 + (7*a^5*b^3 + 14*
a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^8, x)
```

maple [F] time = 8.12, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+b\,\sin(c+d\,x))^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^8,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

3.623 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=156

$$\frac{2e(a+b\sin(c+dx))^{7/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a}{}\right.}{7bd}$$

```
[Out] 2/7*e*AppellF1(7/2,1/2-1/2*p,1/2-1/2*p,9/2,(a+b*sin(d*x+c))/(a-b),(a+b*sin(
d*x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(a+b*sin(d*x+c))^(7/2)*(1+(-a-b*sin(d*
x+c))/(a-b))^(1/2-1/2*p)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d
```

Rubi [A] time = 0.13, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2704, 138}

$$\frac{2e(a+b\sin(c+dx))^{7/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a}{}\right.}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*e*AppellF1[7/2, (1 - p)/2, (1 - p)/2, 9/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(7/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(7*b*d)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{5/2}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subs}}{}$$

$$= \frac{2eF_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(a+}{7bd}$$

Mathematica [A] time = 7.46, size = 187, normalized size = 1.20

$$\frac{2e(a+b\sin(c+dx))^{7/2}(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\right.}{7bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*e*AppellF1[7/2, (1 - p)/2, (1 - p)/2, 9/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*(a + b*Sin[c + d*x
])^(7/2)*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(7*b*
d)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos(dx+c)^2 - 2\,ab\sin(dx+c) - a^2 - b^2\right)\sqrt{b\sin(dx+c)+a}\,\left(e\cos(dx+c)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(b*sin(
d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\left(e\cos(dx+c)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*(e*cos(d*x + c))^p, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(e\cos(dx+c)\right)^p (a+b\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\left(e\cos(dx+c)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.624 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=156

$$\frac{2e(a+b\sin(c+dx))^{5/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-b}\right.}{5bd}$$

```
[Out] 2/5*e*AppellF1(5/2,1/2-1/2*p,1/2-1/2*p,7/2,(a+b*sin(d*x+c))/(a-b),(a+b*sin(
d*x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(a+b*sin(d*x+c))^(5/2)*(1+(-a-b*sin(d*
x+c))/(a-b))^(1/2-1/2*p)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d
```

Rubi [A] time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2704, 138}

$$\frac{2e(a+b\sin(c+dx))^{5/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-b}\right.}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*e*AppellF1[5/2, (1 - p)/2, (1 - p)/2, 7/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(5/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(5*b*d)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{3/2}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\mathrm{Su}}{}$$

$$=\frac{2eF_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(}{5bd}$$

Mathematica [A] time = 0.79, size = 187, normalized size = 1.20

$$\frac{2e(a+b\sin(c+dx))^{5/2}(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b}{}\right.}{5bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(3/2),x]
[Out] (2*e*AppellF1[5/2, (1 - p)/2, (1 - p)/2, 7/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*(a + b*Sin[c + d*x
])^(5/2)*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(5*b*
d)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p(a+b\sin(dx+c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x)
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^p (a+b\sin(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x))**(3/2), x)
```

3.625 $\int (e\cos(c+dx))^p\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{2e(a+b\sin(c+dx))^{3/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a}{}\right.}{3bd}$$

```
[Out] 2/3*e*AppellF1(3/2,1/2-1/2*p,1/2-1/2*p,5/2,(a+b*sin(d*x+c))/(a-b),(a+b*sin(
d*x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(a+b*sin(d*x+c))^(3/2)*(1+(-a-b*sin(d*
x+c))/(a-b))^(1/2-1/2*p)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d
```

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2704, 138}

$$\frac{2e(a+b\sin(c+dx))^{3/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a}{}\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*e*AppellF1[3/2, (1 - p)/2, (1 - p)/2, 5/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(3/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(3*b*d)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^p\sqrt{a+b\sin(c+dx)}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subst}}{c}$$

$$= \frac{2eF_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(a+b}{3bd}$$

Mathematica [A] time = 0.96, size = 187, normalized size = 1.20

$$\frac{2e(a+b\sin(c+dx))^{3/2}(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\right.}{3bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*e*AppellF1[3/2, (1 - p)/2, (1 - p)/2, 5/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*(a + b*Sin[c + d*x
])^(3/2)*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(3*b*
d)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin(dx+c)+a}\,(e\cos(dx+c))^p\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin(dx+c)+a}\,(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int(e\cos(dx+c))^p\sqrt{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin(dx+c)+a}\,(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(e\cos(c+dx))^p\sqrt{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^p \sqrt{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral((e*cos(c + d*x))**p*sqrt(a + b*sin(c + d*x)), x)
```

3.626 $\int \frac{(e\cos(c+dx))^p}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=154

$$\frac{2e\sqrt{a+b\sin(c+dx)}\,(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-b},\right.}{bd}$$

```
[Out] 2*e*AppellF1(1/2,1/2-1/2*p,1/2-1/2*p,3/2,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*
x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(1+(-a-b*sin(d*x+c))/(a-b))^(1/2-1/2*p)*
(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)*(a+b*sin(d*x+c))^(1/2)/b/d
```

Rubi [A] time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2704, 138}

$$\frac{2e\sqrt{a+b\sin(c+dx)}\,(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-b},\right.}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*e*AppellF1[1/2, (1 - p)/2, (1 - p)/2, 3/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*Sqrt[a + b*Sin[c +
 d*x]]*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d
*x])/(a + b))^((1 - p)/2))/(b*d)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subst}\left(\int \frac{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{\frac{1}{2}(}}{}\right.}{d}$$

$$= \frac{2eF_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\sqrt{a+b\sin(c+dx)}\,\left(\right.}{bd}$$

Mathematica [A] time = 1.08, size = 185, normalized size = 1.20

$$\frac{2e\sqrt{a+b\sin(c+dx)}\,(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+}\right.}{bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*e*AppellF1[1/2, (1 - p)/2, (1 - p)/2, 3/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*Sqrt[a + b*Sin[c +
 d*x]]*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(b*d)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\cos(dx+c))^p}{\sqrt{b\sin(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p/sqrt(b*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/sqrt(b*sin(d*x + c) + a), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{a+b\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^p/sqrt(b*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral((e*cos(c + d*x))**p/sqrt(a + b*sin(c + d*x)), x)
```

3.627 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -2*e*AppellF1(-1/2,1/2-1/2*p,1/2-1/2*p,1/2,(a+b*sin(d*x+c))/(a-b),(a+b*sin(
d*x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(1+(-a-b*sin(d*x+c))/(a-b))^(1/2-1/2*p
)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2704, 138}

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*AppellF1[-1/2, (1 - p)/2, (1 - p)/2, 1/2, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(1 - (a + b*Sin[
c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^((1 - p)/
2))/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{b}{a-b}-\right.}{}\right.}{d}$$

$$= -\frac{2eF_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+}{a-b}\right.}{bd\sqrt{a+b\sin(c+dx)}}$$

Mathematica [A] time = 2.75, size = 185, normalized size = 1.20

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)}{bd\sqrt{a+b\sin(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*AppellF1[-1/2, (1 - p)/2, (1 - p)/2, 1/2, (a + b*Sin[c + d*x])/(a - S
qrt[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*
((Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*((Sqrt[b^2] + b*
Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,(e\cos(dx+c))^p}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*(e*cos(d*x + c))^p/(b^2*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(3/2),x)
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**(3/2),x)
[Out] Integral((e*cos(c + d*x))**p/(a + b*sin(c + d*x))**(3/2), x)
```

3.628 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{3bd(a+b\sin(c+dx))^{3/2}}$$

```
[Out] -2/3*e*AppellF1(-3/2,1/2-1/2*p,1/2-1/2*p,-1/2,(a+b*sin(d*x+c))/(a-b),(a+b*s
in(d*x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(1+(-a-b*sin(d*x+c))/(a-b))^(1/2-1/
2*p)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d/(a+b*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2704, 138}

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*AppellF1[-3/2, (1 - p)/2, (1 - p)/2, -1/2, (a + b*Sin[c + d*x])/(a -
b), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(1 - (a + b*Sin
[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^((1 - p)
/2))/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subst}\left(\int \frac{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)}{\cdots}\right.}{d}$$

$$= -\frac{2eF_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+d}{a-b}\right.}{3bd(a+b\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 3.04, size = 187, normalized size = 1.20

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)}{3bd(a+b\sin(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*AppellF1[-3/2, (1 - p)/2, (1 - p)/2, -1/2, (a + b*Sin[c + d*x])/(a -
Sqrt[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)
*((Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*((Sqrt[b^2] + b
*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(3*b*d*(a + b*Sin[c + d*x])^(
3/2))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,(e\cos(dx+c))^p}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*(e*cos(d*x + c))^p/(3*a*b^2*cos(d*x + c)
^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\cos(c+d\,x))^p}{(a+b\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(5/2),x)
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**(5/2),x)
[Out] Integral((e*cos(c + d*x))**p/(a + b*sin(c + d*x))**(5/2), x)
```

3.629 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=158

$$\frac{e(e\cos(c+dx))^{p-1}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(m+1;\frac{1-p}{2},\frac{1-p}{2};m+2;\frac{a+b\sin}{}\right.}{bd(m+1)}$$

```
[Out] e*AppellF1(1+m,1/2-1/2*p,1/2-1/2*p,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+
c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c)
)/(a-b))^(1/2-1/2*p)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2704, 138}

$$\frac{e(e\cos(c+dx))^{p-1}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(m+1;\frac{1-p}{2},\frac{1-p}{2};m+2;\frac{a+b\sin}{}\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, (1 - p)/2, (1 - p)/2, 2 + m, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d
*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin
[c + d*x])/(a + b))^((1 - p)/2))/(b*d*(1 + m))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subst}}{}$$

$$= \frac{eF_1\left(1+m;\frac{1-p}{2},\frac{1-p}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1}}{bd(1+}$$

Mathematica [F] time = 2.41, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^m, x]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((e\cos(dx+c))^p(b\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((e*cos(d*x + c))^p*(b*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^p*(b*sin(d*x + c) + a)^m, x)
```

maple [F] time = 1.33, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^p*(b*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^p\,(a+b\,\sin(c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^p (a+b\sin(c+dx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x))**m, x)
```

3.630 $\int \cos^7(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=254

$$-\frac{\left(a^2-b^2\right)^3(a+b\sin(c+dx))^{m+1}}{b^7d(m+1)}+\frac{6a\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{m+2}}{b^7d(m+2)}+\frac{4a\left(5a^2-3b^2\right)(a+b\sin(c+dx))^{m+4}}{b^7d(m+4)}$$

```
[Out] -(a^2-b^2)^3*(a+b*sin(d*x+c))^(1+m)/b^7/d/(1+m)+6*a*(a^2-b^2)^2*(a+b*sin(d*
x+c))^(2+m)/b^7/d/(2+m)-3*(5*a^4-6*a^2*b^2+b^4)*(a+b*sin(d*x+c))^(3+m)/b^7/
d/(3+m)+4*a*(5*a^2-3*b^2)*(a+b*sin(d*x+c))^(4+m)/b^7/d/(4+m)-3*(5*a^2-b^2)*
(a+b*sin(d*x+c))^(5+m)/b^7/d/(5+m)+6*a*(a+b*sin(d*x+c))^(6+m)/b^7/d/(6+m)-(
a+b*sin(d*x+c))^(7+m)/b^7/d/(7+m)
```

Rubi [A] time = 0.16, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2668, 697}

$$-\frac{\left(a^2-b^2\right)^3(a+b\sin(c+dx))^{m+1}}{b^7d(m+1)}+\frac{6a\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{m+2}}{b^7d(m+2)}-\frac{3\left(-6a^2b^2+5a^4+b^4\right)(a+b\sin(c+\cdots}{b^7d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((a^2 - b^2)^3*(a + b*Sin[c + d*x])^(1 + m))/(b^7*d*(1 + m))) + (6*a*(a^2
 - b^2)^2*(a + b*Sin[c + d*x])^(2 + m))/(b^7*d*(2 + m)) - (3*(5*a^4 - 6*a^2
*b^2 + b^4)*(a + b*Sin[c + d*x])^(3 + m))/(b^7*d*(3 + m)) + (4*a*(5*a^2 - 3
*b^2)*(a + b*Sin[c + d*x])^(4 + m))/(b^7*d*(4 + m)) - (3*(5*a^2 - b^2)*(a +
 b*Sin[c + d*x])^(5 + m))/(b^7*d*(5 + m)) + (6*a*(a + b*Sin[c + d*x])^(6 +
m))/(b^7*d*(6 + m)) - (a + b*Sin[c + d*x])^(7 + m)/(b^7*d*(7 + m))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(b^2-x^2\right)^3\,dx,x,b\sin(c+dx)\right)}{b^7d}$$

$$= \frac{\text{Subst}\left(\int\left(-\left(a^2-b^2\right)^3(a+x)^m+6a\left(a^2-b^2\right)^2(a+x)^{1+m}-3\left(5a^4-\cdots\right.\right.\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)^3(a+b\sin(c+dx))^{1+m}}{b^7d(1+m)}+\frac{6a\left(a^2-b^2\right)^2(a+b\sin(c+dx)\cdots}{b^7d(2+m)}$$

Mathematica [A] time = 6.11, size = 459, normalized size = 1.81

$$\frac{6\left((b^2-a^2)\left(\frac{4\left((b^2-a^2)\left(-\frac{(a^2-b^2)(a+b\sin(c+dx))^{m+1}}{m+1}+\frac{2a(a+b\sin(c+dx))^{m+2}}{m+2}-\frac{(a+b\sin(c+dx))^{m+3}}{m+3}\right)+a\left(-\frac{(a^2-b^2)(a+b\sin(c+dx))^{m+2}}{m+2}+\frac{2a(a+b\sin(c+dx))^{m+3}}{m+3}-\frac{(a+b\sin(c+dx}{m+4}\right.\right.}{m+5}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((b^6*Cos[c + d*x]^6*(a + b*Sin[c + d*x])^(1 + m))/(7 + m) + (6*((-a^2 + b^
2)*((b^4*Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(1 + m))/(5 + m) + (4*((-a^2 +
 b^2)*(-(((a^2 - b^2)*(a + b*Sin[c + d*x])^(1 + m))/(1 + m)) + (2*a*(a + b*
Sin[c + d*x])^(2 + m))/(2 + m) - (a + b*Sin[c + d*x])^(3 + m)/(3 + m)) + a*
(-(((a^2 - b^2)*(a + b*Sin[c + d*x])^(2 + m))/(2 + m)) + (2*a*(a + b*Sin[c
+ d*x])^(3 + m))/(3 + m) - (a + b*Sin[c + d*x])^(4 + m)/(4 + m))))/(5 + m))
 + a*((b^4*Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(2 + m))/(6 + m) + (4*((-a^2
 + b^2)*(-(((a^2 - b^2)*(a + b*Sin[c + d*x])^(2 + m))/(2 + m)) + (2*a*(a +
b*Sin[c + d*x])^(3 + m))/(3 + m) - (a + b*Sin[c + d*x])^(4 + m)/(4 + m)) +
a*(-(((a^2 - b^2)*(a + b*Sin[c + d*x])^(3 + m))/(3 + m)) + (2*a*(a + b*Sin[
c + d*x])^(4 + m))/(4 + m) - (a + b*Sin[c + d*x])^(5 + m)/(5 + m))))/(6 + m
))))/(7 + m))/(b^7*d)
```

fricas [B] time = 0.92, size = 814, normalized size = 3.20

$$-\frac{\left(720\,a^7-3024\,a^5b^2+5040\,a^3b^4-5040\,ab^6-\left(ab^6m^6+15\,ab^6m^5+85\,ab^6m^4+225\,ab^6m^3+274\,ab^6m^2+120\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -(720*a^7 - 3024*a^5*b^2 + 5040*a^3*b^4 - 5040*a*b^6 - (a*b^6*m^6 + 15*a*b^
6*m^5 + 85*a*b^6*m^4 + 225*a*b^6*m^3 + 274*a*b^6*m^2 + 120*a*b^6*m)*cos(d*x
 + c)^6 - 6*(2*a*b^6*m^5 - (5*a^3*b^4 - 23*a*b^6)*m^4 - 2*(15*a^3*b^4 - 44*
a*b^6)*m^3 - (55*a^3*b^4 - 133*a*b^6)*m^2 - 6*(5*a^3*b^4 - 11*a*b^6)*m)*cos
(d*x + c)^4 - 192*(a^3*b^4 + a*b^6)*m^3 + 288*(a^5*b^2 - 2*a^3*b^4 - 7*a*b^
6)*m^2 - 24*((a^3*b^4 + 3*a*b^6)*m^4 - 6*(a^3*b^4 - 5*a*b^6)*m^3 + (15*a^5*
b^2 - 55*a^3*b^4 + 84*a*b^6)*m^2 + 3*(5*a^5*b^2 - 16*a^3*b^4 + 19*a*b^6)*m)
*cos(d*x + c)^2 - 192*(3*a^5*b^2 - 13*a^3*b^4 + 32*a*b^6)*m - (2304*b^7 + (
b^7*m^6 + 21*b^7*m^5 + 175*b^7*m^4 + 735*b^7*m^3 + 1624*b^7*m^2 + 1764*b^7*
m + 720*b^7)*cos(d*x + c)^6 + 6*(144*b^7 + (a^2*b^5 + b^7)*m^5 + 2*(5*a^2*b
^5 + 8*b^7)*m^4 + 5*(7*a^2*b^5 + 19*b^7)*m^3 + 10*(5*a^2*b^5 + 26*b^7)*m^2
+ 12*(2*a^2*b^5 + 27*b^7)*m)*cos(d*x + c)^4 + 48*(a^4*b^3 + 6*a^2*b^5 + b^7
)*m^3 - 576*(a^4*b^3 - 4*a^2*b^5 - b^7)*m^2 + 24*(48*b^7 + (3*a^2*b^5 + b^7
)*m^4 - (5*a^4*b^3 - 24*a^2*b^5 - 13*b^7)*m^3 - (15*a^4*b^3 - 51*a^2*b^5 -
56*b^7)*m^2 - 2*(5*a^4*b^3 - 15*a^2*b^5 - 46*b^7)*m)*cos(d*x + c)^2 + 48*(1
5*a^6*b - 58*a^4*b^3 + 87*a^2*b^5 + 44*b^7)*m)*sin(d*x + c))*(b*sin(d*x + c
) + a)^m/(b^7*d*m^7 + 28*b^7*d*m^6 + 322*b^7*d*m^5 + 1960*b^7*d*m^4 + 6769*
b^7*d*m^3 + 13132*b^7*d*m^2 + 13068*b^7*d*m + 5040*b^7*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \left(\cos^{7}\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x)
```

maxima [B] time = 1.00, size = 558, normalized size = 2.20

$$\frac{\frac{(b\sin(dx+c)+a)^{m+1}}{b(m+1)} - \frac{3\left(\left(m^2+3\,m+2\right)b^3\sin(dx+c)^3+\left(m^2+m\right)ab^2\sin(dx+c)^2-2\,a^2bm\sin(dx+c)+2\,a^3\right)(b\sin(dx+c)+a)^m}{\left(m^3+6\,m^2+11\,m+6\right)b^3} + \frac{3\left(\left(m^4+10\,m^3+35\,m^2\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((b*sin(d*x + c) + a)^(m + 1)/(b*(m + 1)) - 3*((m^2 + 3*m + 2)*b^3*sin(d*x
+ c)^3 + (m^2 + m)*a*b^2*sin(d*x + c)^2 - 2*a^2*b*m*sin(d*x + c) + 2*a^3)*(
b*sin(d*x + c) + a)^m/((m^3 + 6*m^2 + 11*m + 6)*b^3) + 3*((m^4 + 10*m^3 + 3
5*m^2 + 50*m + 24)*b^5*sin(d*x + c)^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a*b^4*
sin(d*x + c)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^2*b^3*sin(d*x + c)^3 + 12*(m^2 + m
)*a^3*b^2*sin(d*x + c)^2 - 24*a^4*b*m*sin(d*x + c) + 24*a^5)*(b*sin(d*x + c
) + a)^m/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*b^5) - ((m^6 + 21
*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*b^7*sin(d*x + c)^7 + (m
^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*a*b^6*sin(d*x + c)^6 - 6*
(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*a^2*b^5*sin(d*x + c)^5 + 30*(m^4 +
6*m^3 + 11*m^2 + 6*m)*a^3*b^4*sin(d*x + c)^4 - 120*(m^3 + 3*m^2 + 2*m)*a^4*
b^3*sin(d*x + c)^3 + 360*(m^2 + m)*a^5*b^2*sin(d*x + c)^2 - 720*a^6*b*m*sin
(d*x + c) + 720*a^7)*(b*sin(d*x + c) + a)^m/((m^7 + 28*m^6 + 322*m^5 + 1960
*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*b^7))/d
```

mupad [B] time = 19.09, size = 1196, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + b*sin(c + d*x))^m,x)
```

```
[Out] ((a + b*sin(c + d*x))^m*(a*b^6*645120i - a^7*92160i - a^3*b^4*645120i + a^5
*b^2*387072i - a^3*b^4*m*401856i + a^5*b^2*m*96768i + a*b^6*m^2*436336i + a
*b^6*m^3*105000i + a*b^6*m^4*14632i + a*b^6*m^5*1176i + a*b^6*m^6*40i - a^3
*b^4*m^2*26592i - a^5*b^2*m^2*13824i + a^3*b^4*m^3*6720i + a^3*b^4*m^4*96i
+ a*b^6*m*897792i))/(128*b^7*d*(m*13068i + m^2*13132i + m^3*6769i + m^4*196
0i + m^5*322i + m^6*28i + m^7*1i + 5040i)) + (sin(7*c + 7*d*x)*(a + b*sin(c
 + d*x))^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)*1i)
/(64*d*(m*13068i + m^2*13132i + m^3*6769i + m^4*1960i + m^5*322i + m^6*28i
+ m^7*1i + 5040i)) + (sin(c + d*x)*(a + b*sin(c + d*x))^m*(194868*b^7*m + 1
76400*b^7 + 78968*b^7*m^2 + 16299*b^7*m^3 + 2027*b^7*m^4 + 153*b^7*m^5 + 5*
b^7*m^6 + 279936*a^2*b^5*m - 182016*a^4*b^3*m + 169440*a^2*b^5*m^2 - 42624*
a^4*b^3*m^2 + 29328*a^2*b^5*m^3 + 1152*a^4*b^3*m^3 + 1632*a^2*b^5*m^4 + 48*
a^2*b^5*m^5 + 46080*a^6*b*m)*1i)/(64*b^7*d*(m*13068i + m^2*13132i + m^3*676
9i + m^4*1960i + m^5*322i + m^6*28i + m^7*1i + 5040i)) + (sin(3*c + 3*d*x)*
(a + b*sin(c + d*x))^m*(3*m + m^2 + 2)*(3602*b^4*m - 640*a^4*m + 5880*b^4 +
 797*b^4*m^2 + 78*b^4*m^3 + 3*b^4*m^4 + 2208*a^2*b^2*m + 552*a^2*b^2*m^2 +
24*a^2*b^2*m^3)*3i)/(64*b^4*d*(m*13068i + m^2*13132i + m^3*6769i + m^4*1960
i + m^5*322i + m^6*28i + m^7*1i + 5040i)) + (sin(5*c + 5*d*x)*(a + b*sin(c
```

```
+ d*x))^m*(24*a^2*m + 79*b^2*m + 294*b^2 + 5*b^2*m^2)*(50*m + 35*m^2 + 10*m
^3 + m^4 + 24)*1i)/(64*b^2*d*(m*13068i + m^2*13132i + m^3*6769i + m^4*1960i
 + m^5*322i + m^6*28i + m^7*1i + 5040i)) + (a*m*cos(6*c + 6*d*x)*(a + b*sin
(c + d*x))^m*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i))/(32*b
*d*(m*13068i + m^2*13132i + m^3*6769i + m^4*1960i + m^5*322i + m^6*28i + m^
7*1i + 5040i)) + (3*a*m*cos(4*c + 4*d*x)*(a + b*sin(c + d*x))^m*(b^2*m*17i
- a^2*20i + b^2*64i + b^2*m^2*1i)*(11*m + 6*m^2 + m^3 + 6))/(16*b^3*d*(m*13
068i + m^2*13132i + m^3*6769i + m^4*1960i + m^5*322i + m^6*28i + m^7*1i + 5
040i)) + (3*a*m*cos(2*c + 2*d*x)*(m + 1)*(a + b*sin(c + d*x))^m*(b^4*m*6370
i + a^4*1920i + b^4*10008i - a^2*b^2*7104i + b^4*m^2*1411i + b^4*m^3*134i +
 b^4*m^4*5i - a^2*b^2*m*1696i - a^2*b^2*m^2*32i))/(32*b^5*d*(m*13068i + m^2
*13132i + m^3*6769i + m^4*1960i + m^5*322i + m^6*28i + m^7*1i + 5040i))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.631 $\int \cos^5(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=167

$$\frac{(a^2-b^2)^2(a+b\sin(c+dx))^{m+1}}{b^5d(m+1)}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^{m+2}}{b^5d(m+2)}+\frac{2(3a^2-b^2)(a+b\sin(c+dx))^{m+3}}{b^5d(m+3)}-\frac{4a}{}$$

```
[Out] (a^2-b^2)^2*(a+b*sin(d*x+c))^(1+m)/b^5/d/(1+m)-4*a*(a^2-b^2)*(a+b*sin(d*x+c
))^(2+m)/b^5/d/(2+m)+2*(3*a^2-b^2)*(a+b*sin(d*x+c))^(3+m)/b^5/d/(3+m)-4*a*(
a+b*sin(d*x+c))^(4+m)/b^5/d/(4+m)+(a+b*sin(d*x+c))^(5+m)/b^5/d/(5+m)
```

Rubi [A] time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2668, 697}

$$\frac{(a^2-b^2)^2(a+b\sin(c+dx))^{m+1}}{b^5d(m+1)}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^{m+2}}{b^5d(m+2)}+\frac{2(3a^2-b^2)(a+b\sin(c+dx))^{m+3}}{b^5d(m+3)}-\frac{4a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^(1 + m))/(b^5*d*(1 + m)) - (4*a*(a^2 -
b^2)*(a + b*Sin[c + d*x])^(2 + m))/(b^5*d*(2 + m)) + (2*(3*a^2 - b^2)*(a +
b*Sin[c + d*x])^(3 + m))/(b^5*d*(3 + m)) - (4*a*(a + b*Sin[c + d*x])^(4 + m
))/(b^5*d*(4 + m)) + (a + b*Sin[c + d*x])^(5 + m)/(b^5*d*(5 + m))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^m-4\left(a^3-ab^2\right)(a+x)^{1+m}+2\left(3a^2-b^2\right.\right.\right.}{b^5d}\\ &= \frac{(a^2-b^2)^2(a+b\sin(c+dx))^{1+m}}{b^5d(1+m)}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^{2+}}{b^5d(2+m)}\end{aligned}$$

Mathematica [A] time = 0.90, size = 169, normalized size = 1.01

$$\frac{(a+b\sin(c+dx))^{m+1}\left(4\left(b^2-a^2\right)\left(\frac{b^2-a^2}{m+1}-\frac{(a+b\sin(c+dx))^2}{m+3}+\frac{2a(a+b\sin(c+dx))}{m+2}\right)+4a(a+b\sin(c+dx))\left(\frac{b^2-a^2}{m+2}-\frac{(a+}{}\right.\right.}{b^5d(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*(b^4*Cos[c + d*x]^4 + 4*(-a^2 + b^2)*((-a^2 +
 b^2)/(1 + m) + (2*a*(a + b*Sin[c + d*x]))/(2 + m) - (a + b*Sin[c + d*x])^2
/(3 + m)) + 4*a*(a + b*Sin[c + d*x])*((-a^2 + b^2)/(2 + m) + (2*a*(a + b*Si
n[c + d*x]))/(3 + m) - (a + b*Sin[c + d*x])^2/(4 + m))))/(b^5*d*(5 + m))
```

fricas [B] time = 0.73, size = 381, normalized size = 2.28

$$\frac{\left(24\,a^5-80\,a^3b^2+120\,ab^4+\left(ab^4m^4+6\,ab^4m^3+11\,ab^4m^2+6\,ab^4m\right)\cos\left(dx+c\right)^4+8\left(a^3b^2+3\,ab^4\right)m^2+4\left(2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (24*a^5 - 80*a^3*b^2 + 120*a*b^4 + (a*b^4*m^4 + 6*a*b^4*m^3 + 11*a*b^4*m^2
+ 6*a*b^4*m)*cos(d*x + c)^4 + 8*(a^3*b^2 + 3*a*b^4)*m^2 + 4*(2*a*b^4*m^3 -
3*(a^3*b^2 - 3*a*b^4)*m^2 - (3*a^3*b^2 - 7*a*b^4)*m)*cos(d*x + c)^2 - 24*(a
^3*b^2 - 5*a*b^4)*m + (64*b^5 + (b^5*m^4 + 10*b^5*m^3 + 35*b^5*m^2 + 50*b^5
*m + 24*b^5)*cos(d*x + c)^4 + 8*(3*a^2*b^3 + b^5)*m^2 + 4*(8*b^5 + (a^2*b^3
 + b^5)*m^3 + (3*a^2*b^3 + 7*b^5)*m^2 + 2*(a^2*b^3 + 7*b^5)*m)*cos(d*x + c)
^2 - 24*(a^4*b - 3*a^2*b^3 - 2*b^5)*m)*sin(d*x + c))*(b*sin(d*x + c) + a)^m
/(b^5*d*m^5 + 15*b^5*d*m^4 + 85*b^5*d*m^3 + 225*b^5*d*m^2 + 274*b^5*d*m + 1
20*b^5*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \left(\cos^5\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

maxima [A] time = 1.06, size = 286, normalized size = 1.71

$$\frac{\frac{(b\sin(dx+c)+a)^{m+1}}{b(m+1)}-\frac{2\left(\left(m^2+3\,m+2\right)b^3\sin(dx+c)^3+\left(m^2+m\right)ab^2\sin(dx+c)^2-2\,a^2bm\sin(dx+c)+2\,a^3\right)(b\sin(dx+c)+a)^m}{\left(m^3+6\,m^2+11\,m+6\right)b^3}+\frac{\left(\left(m^4+10\,m^3+35\,m^2+50\,m\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((b*sin(d*x + c) + a)^(m + 1)/(b*(m + 1)) - 2*((m^2 + 3*m + 2)*b^3*sin(d*x
+ c)^3 + (m^2 + m)*a*b^2*sin(d*x + c)^2 - 2*a^2*b*m*sin(d*x + c) + 2*a^3)*(
b*sin(d*x + c) + a)^m/((m^3 + 6*m^2 + 11*m + 6)*b^3) + ((m^4 + 10*m^3 + 35*
m^2 + 50*m + 24)*b^5*sin(d*x + c)^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a*b^4*si
n(d*x + c)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^2*b^3*sin(d*x + c)^3 + 12*(m^2 + m)*
```

```
a^3*b^2*sin(d*x + c)^2 - 24*a^4*b*m*sin(d*x + c) + 24*a^5)*(b*sin(d*x + c)
+ a)^m/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*b^5))/d
```

mupad [B] time = 11.62, size = 641, normalized size = 3.84

$$(a + b\sin(c + d\,x))^m\,\Big(1920\,a\,b^4 + 1200\,b^5\,\sin(c + d\,x) + 384\,a^5 - 1280\,a^3\,b^2 + 200\,b^5\,\sin(3\,c + 3\,d\,x) + 24\,b^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^m,x)
```

```
[Out] ((a + b*sin(c + d*x))^m*(1920*a*b^4 + 1200*b^5*sin(c + d*x) + 384*a^5 - 128
0*a^3*b^2 + 200*b^5*sin(3*c + 3*d*x) + 24*b^5*sin(5*c + 5*d*x) - 480*a^3*b^
2*m + 738*a*b^4*m^2 + 100*a*b^4*m^3 + 6*a*b^4*m^4 + 374*b^5*m*sin(3*c + 3*d
*x) + 50*b^5*m*sin(5*c + 5*d*x) + 310*b^5*m^2*sin(c + d*x) + 36*b^5*m^3*sin
(c + d*x) + 2*b^5*m^4*sin(c + d*x) + 32*a^3*b^2*m^2 + 217*b^5*m^2*sin(3*c +
 3*d*x) + 46*b^5*m^3*sin(3*c + 3*d*x) + 3*b^5*m^4*sin(3*c + 3*d*x) + 35*b^5
*m^2*sin(5*c + 5*d*x) + 10*b^5*m^3*sin(5*c + 5*d*x) + b^5*m^4*sin(5*c + 5*d
*x) + 2180*a*b^4*m + 1092*b^5*m*sin(c + d*x) - 96*a^3*b^2*m*cos(2*c + 2*d*x
) + 376*a*b^4*m^2*cos(2*c + 2*d*x) + 112*a*b^4*m^3*cos(2*c + 2*d*x) + 8*a*b
^4*m^4*cos(2*c + 2*d*x) + 22*a*b^4*m^2*cos(4*c + 4*d*x) + 12*a*b^4*m^3*cos(
4*c + 4*d*x) + 2*a*b^4*m^4*cos(4*c + 4*d*x) + 32*a^2*b^3*m*sin(3*c + 3*d*x)
 + 432*a^2*b^3*m^2*sin(c + d*x) + 16*a^2*b^3*m^3*sin(c + d*x) - 384*a^4*b*m
*sin(c + d*x) - 96*a^3*b^2*m^2*cos(2*c + 2*d*x) + 48*a^2*b^3*m^2*sin(3*c +
3*d*x) + 16*a^2*b^3*m^3*sin(3*c + 3*d*x) + 272*a*b^4*m*cos(2*c + 2*d*x) + 1
2*a*b^4*m*cos(4*c + 4*d*x) + 1184*a^2*b^3*m*sin(c + d*x)))/(16*b^5*d*(274*m
 + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.632 $\int \cos^3(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=92

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^{m+1}}{b^3d(m+1)}+\frac{2a(a+b\sin(c+dx))^{m+2}}{b^3d(m+2)}-\frac{(a+b\sin(c+dx))^{m+3}}{b^3d(m+3)}$$

```
[Out] -(a^2-b^2)*(a+b*sin(d*x+c))^(1+m)/b^3/d/(1+m)+2*a*(a+b*sin(d*x+c))^(2+m)/b^
3/d/(2+m)-(a+b*sin(d*x+c))^(3+m)/b^3/d/(3+m)
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^{m+1}}{b^3d(m+1)}+\frac{2a(a+b\sin(c+dx))^{m+2}}{b^3d(m+2)}-\frac{(a+b\sin(c+dx))^{m+3}}{b^3d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((a^2 - b^2)*(a + b*Sin[c + d*x])^(1 + m))/(b^3*d*(1 + m))) + (2*a*(a + b
*Sin[c + d*x])^(2 + m))/(b^3*d*(2 + m)) - (a + b*Sin[c + d*x])^(3 + m)/(b^3
*d*(3 + m))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\left(b^2-x^2\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^m+2a(a+x)^{1+m}-(a+x)^{2+m}\right)dx,x,b\sin\right.}{b^3d}\\ &= -\frac{(a^2-b^2)(a+b\sin(c+dx))^{1+m}}{b^3d(1+m)}+\frac{2a(a+b\sin(c+dx))^{2+m}}{b^3d(2+m)}-\frac{(a+b\,\mathrm{s}}{b^3}\end{aligned}$$

Mathematica [A] time = 0.18, size = 74, normalized size = 0.80

$$\frac{(a+b\sin(c+dx))^{m+1}\left(\frac{b^2-a^2}{m+1}-\frac{(a+b\sin(c+dx))^2}{m+3}+\frac{2a(a+b\sin(c+dx))}{m+2}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*((-a^2 + b^2)/(1 + m) + (2*a*(a + b*Sin[c + d
*x]))/(2 + m) - (a + b*Sin[c + d*x])^2/(3 + m)))/(b^3*d)
```

fricas [A] time = 0.77, size = 142, normalized size = 1.54

$$\frac{\left(4\,ab^2m - 2\,a^3 + 6\,ab^2 + \left(ab^2m^2 + ab^2m\right)\cos\left(dx+c\right)^2 + \left(4\,b^3 + \left(b^3m^2 + 3\,b^3m + 2\,b^3\right)\cos\left(dx+c\right)^2 + 2\left(a^2b\right.\right.}{b^3dm^3 + 6\,b^3dm^2 + 11\,b^3dm + 6\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (4*a*b^2*m - 2*a^3 + 6*a*b^2 + (a*b^2*m^2 + a*b^2*m)*cos(d*x + c)^2 + (4*b^
3 + (b^3*m^2 + 3*b^3*m + 2*b^3)*cos(d*x + c)^2 + 2*(a^2*b + b^3)*m)*sin(d*x
 + c))*(b*sin(d*x + c) + a)^m/(b^3*d*m^3 + 6*b^3*d*m^2 + 11*b^3*d*m + 6*b^3
*d)
```

giac [B] time = 0.36, size = 340, normalized size = 3.70

$$-\frac{\left(b\sin\left(dx+c\right)+a\right)^m b^3m^2\sin\left(dx+c\right)^3 + \left(b\sin\left(dx+c\right)+a\right)^m ab^2m^2\sin\left(dx+c\right)^2 + 3\left(b\sin\left(dx+c\right)+a\right)^m b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] -((b*sin(d*x + c) + a)^m*b^3*m^2*sin(d*x + c)^3 + (b*sin(d*x + c) + a)^m*a*
b^2*m^2*sin(d*x + c)^2 + 3*(b*sin(d*x + c) + a)^m*b^3*m*sin(d*x + c)^3 - (b
*sin(d*x + c) + a)^m*b^3*m^2*sin(d*x + c) + (b*sin(d*x + c) + a)^m*a*b^2*m*
sin(d*x + c)^2 + 2*(b*sin(d*x + c) + a)^m*b^3*sin(d*x + c)^3 - (b*sin(d*x +
 c) + a)^m*a*b^2*m^2 - 2*(b*sin(d*x + c) + a)^m*a^2*b*m*sin(d*x + c) - 5*(b
*sin(d*x + c) + a)^m*b^3*m*sin(d*x + c) - 5*(b*sin(d*x + c) + a)^m*a*b^2*m
- 6*(b*sin(d*x + c) + a)^m*b^3*sin(d*x + c) + 2*(b*sin(d*x + c) + a)^m*a^3
- 6*(b*sin(d*x + c) + a)^m*a*b^2)/((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)
*d)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \left(\cos^3\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
```

maxima [A] time = 0.78, size = 117, normalized size = 1.27

$$\frac{\frac{\left(b\sin\left(dx+c\right)+a\right)^{m+1}}{b(m+1)} - \frac{\left(\left(m^2+3\,m+2\right)b^3\sin\left(dx+c\right)^3+\left(m^2+m\right)ab^2\sin\left(dx+c\right)^2-2\,a^2bm\sin\left(dx+c\right)+2\,a^3\right)\left(b\sin\left(dx+c\right)+a\right)^m}{\left(m^3+6\,m^2+11\,m+6\right)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((b*sin(d*x + c) + a)^(m + 1)/(b*(m + 1)) - ((m^2 + 3*m + 2)*b^3*sin(d*x +
c)^3 + (m^2 + m)*a*b^2*sin(d*x + c)^2 - 2*a^2*b*m*sin(d*x + c) + 2*a^3)*(b*
sin(d*x + c) + a)^m/((m^3 + 6*m^2 + 11*m + 6)*b^3))/d
```

mupad [B] time = 7.51, size = 197, normalized size = 2.14

$$\frac{\left(a+b\,\sin\left(c+d\,x\right)\right)^m\,\left(24\,a\,b^2+18\,b^3\,\sin\left(c+d\,x\right)-8\,a^3+2\,b^3\,\sin\left(3\,c+3\,d\,x\right)+2\,a\,b^2\,m^2+3\,b^3\,m\,\sin\left(3\,c+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^m,x)
```

```
[Out] ((a + b*sin(c + d*x))^m*(24*a*b^2 + 18*b^3*sin(c + d*x) - 8*a^3 + 2*b^3*sin
(3*c + 3*d*x) + 2*a*b^2*m^2 + 3*b^3*m*sin(3*c + 3*d*x) + b^3*m^2*sin(c + d*
x) + b^3*m^2*sin(3*c + 3*d*x) + 18*a*b^2*m + 11*b^3*m*sin(c + d*x) + 8*a^2*
b*m*sin(c + d*x) - 2*a*b^2*m*(2*sin(c + d*x)^2 - 1) - 2*a*b^2*m^2*(2*sin(c
+ d*x)^2 - 1)))/(4*b^3*d*(11*m + 6*m^2 + m^3 + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.633 $\int \cos(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=26

$$\frac{(a+b\sin(c+dx))^{m+1}}{bd(m+1)}$$

```
[Out] (a+b*sin(d*x+c))^(1+m)/b/d/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 32}

$$\frac{(a+b\sin(c+dx))^{m+1}}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (a + b*Sin[c + d*x])^(1 + m)/(b*d*(1 + m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^{1+m}}{bd(1+m)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\frac{(a+b\sin(c+dx))^{m+1}}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (a + b*Sin[c + d*x])^(1 + m)/(b*d*(1 + m))
```

fricas [A] time = 0.72, size = 33, normalized size = 1.27

$$\frac{(b\sin(dx+c)+a)(b\sin(dx+c)+a)^m}{bdm+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] (b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m/(b*d*m + b*d)
```

giac [A] time = 1.27, size = 26, normalized size = 1.00

$$\frac{(b\sin(dx+c)+a)^{m+1}}{bd(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] (b*sin(d*x + c) + a)^(m + 1)/(b*d*(m + 1))
```

maple [A] time = 0.03, size = 27, normalized size = 1.04

$$\frac{(a+b\sin(dx+c))^{1+m}}{bd(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^m,x)
[Out] (a+b*sin(d*x+c))^(1+m)/b/d/(1+m)
```

maxima [A] time = 0.29, size = 26, normalized size = 1.00

$$\frac{(b\sin(dx+c)+a)^{m+1}}{bd(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] (b*sin(d*x + c) + a)^(m + 1)/(b*d*(m + 1))
```

mupad [B] time = 6.32, size = 26, normalized size = 1.00

$$\frac{(a+b\,\sin(c+d\,x))^{m+1}}{b\,d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^m,x)
[Out] (a + b*sin(c + d*x))^(m + 1)/(b*d*(m + 1))
```

sympy [A] time = 2.21, size = 99, normalized size = 3.81

$$\begin{cases} \frac{x\cos(c)}{a} & \text{for}\: b = 0 \wedge d = 0 \wedge m = -1 \\ \frac{a^{m}\sin(c+dx)}{d} & \text{for}\: b = 0 \\ x(a+b\sin(c))^{m}\cos(c) & \text{for}\: d = 0 \\ \frac{\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd} & \text{for}\: m = -1 \\ \frac{a(a+b\sin(c+dx))^{m}}{bdm+bd} + \frac{b(a+b\sin(c+dx))^{m}\sin(c+dx)}{bdm+bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*cos(c)/a, Eq(b, 0) & Eq(d, 0) & Eq(m, -1)), (a**m*sin(c + d*x)
/d, Eq(b, 0)), (x*(a + b*sin(c))**m*cos(c), Eq(d, 0)), (log(a/b + sin(c + d
*x))/(b*d), Eq(m, -1)), (a*(a + b*sin(c + d*x))**m/(b*d*m + b*d) + b*(a + b
*sin(c + d*x))**m*sin(c + d*x)/(b*d*m + b*d), True))
```

3.634 $\int \sec(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=115

$$\frac{(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)}{2d(m+1)(a+b)}-\frac{(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{2d(m+1)(a-b)}$$

```
[Out] -1/2*hypergeom([1, 1+m],[2+m],(a+b*sin(d*x+c))/(a-b))*(a+b*sin(d*x+c))^(1+m
)/(a-b)/d/(1+m)+1/2*hypergeom([1, 1+m],[2+m],(a+b*sin(d*x+c))/(a+b))*(a+b*s
in(d*x+c))^(1+m)/(a+b)/d/(1+m)
```

Rubi [A] time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2668, 712, 68}

$$\frac{(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)}{2d(m+1)(a+b)}-\frac{(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{2d(m+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a - b)]*(a + b*S
in[c + d*x])^(1 + m))/(2*(a - b)*d*(1 + m)) + (Hypergeometric2F1[1, 1 + m,
2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m))/(2*(a +
b)*d*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^m}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(\frac{(a+x)^m}{2b(b-x)} + \frac{(a+x)^m}{2b(b+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{b-x}\,dx, x, b\sin(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{(a+x)^m}{b+x}\,dx, x, b\sin(c+\right.}{2d}$$
$$= -\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)(a+b\sin(c+dx))^{1+m}}{2(a-b)d(1+m)} + \frac{{}_2F_1\left(1,1\right.}{}$$

Mathematica [A] time = 0.12, size = 99, normalized size = 0.86

$$-\frac{(a+b\sin(c+dx))^{m+1}\left((a+b)\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)+(b-a)\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)\right)}{2d(m+1)(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^m,x]
[Out] -1/2*(((a + b)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a -
 b)] + (-a + b)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a
+ b)])*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*(a + b)*d*(1 + m))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^m\sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \sec(dx+c)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\sin(c + d\,x))^m}{\cos(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/cos(c + d*x),x)
```

```
[Out] int((a + b*sin(c + d*x))^m/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^m \sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**m*sec(c + d*x), x)
```

3.635 $\int \sec^3(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=183

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{m+1}}{2d\left(a^2-b^2\right)}-\frac{(a-b(1-m))(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m-\right.}{4d(m+1)(a-b)^2}$$

```
[Out] -1/4*(a-b*(1-m))*hypergeom([1, 1+m],[2+m],(a+b*sin(d*x+c))/(a-b))*(a+b*sin(
d*x+c))^(1+m)/(a-b)^2/d/(1+m)+1/4*(-b*m+a+b)*hypergeom([1, 1+m],[2+m],(a+b*
sin(d*x+c))/(a+b))*(a+b*sin(d*x+c))^(1+m)/(a+b)^2/d/(1+m)-1/2*sec(d*x+c)^2*
(b-a*sin(d*x+c))*(a+b*sin(d*x+c))^(1+m)/d/(a^2-b^2)
```

Rubi [A] time = 0.23, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2668, 741, 831, 68}

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{m+1}}{2d\left(a^2-b^2\right)}-\frac{(a-b(1-m))(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m-\right.}{4d(m+1)(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -((a - b*(1 - m))*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(
a - b)]*(a + b*Sin[c + d*x])^(1 + m))/(4*(a - b)^2*d*(1 + m)) + ((a + b - b
*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b
*Sin[c + d*x])^(1 + m))/(4*(a + b)^2*d*(1 + m)) - (Sec[c + d*x]^2*(b - a*Si
n[c + d*x])*(a + b*Sin[c + d*x])^(1 + m))/(2*(a^2 - b^2)*d)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^m}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{2\left(a^2-b^2\right)d} + \frac{b\,\text{Subst}\left(\int \frac{(a+}{}\right.}{}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{2\left(a^2-b^2\right)d} + \frac{b\,\text{Subst}\left(\int \left(\frac{(b}{}\right.\right.}{}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{2\left(a^2-b^2\right)d} + \frac{((a+b)(a-b(}{}$$

$$= -\frac{(a-b(1-m))\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)(a+b\sin(c+dx))^{1+m}}{4(a-b)^2d(1+m)}$$

Mathematica [A] time = 0.62, size = 157, normalized size = 0.86

$$\frac{(a+b\sin(c+dx))^{m+1}\left(\frac{b\left((a+b)^2(a+b(m-1))\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)-(a-b)^2(a-bm+b)\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)\right)}{(m+1)(a-b)(a+b)}+2b\sec^2(c\right.}{4bd\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*((b*((a + b)^2*(a + b*(-1 + m))*Hypergeometri
c2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a - b)] - (a - b)^2*(a + b - b*
m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]))/((a -
 b)*(a + b)*(1 + m)) + 2*b*Sec[c + d*x]^2*(b - a*Sin[c + d*x])))/(4*b*(-a^2
 + b^2)*d)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^m\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)^{m}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(c+d\,x\right)\right)^{m}}{\cos\left(c+d\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/cos(c + d*x)^3,x)
[Out] int((a + b*sin(c + d*x))^m/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

3.636 $\int \sec^5(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=305

$$-\frac{\left(3a^2-3ab(2-m)+b^2\left(m^2-4m+3\right)\right)(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{16d(m+1)(a-b)^3}+\frac{\left(3a^2+3ab(2\right.}{}$$

```
[Out] -1/16*(3*a^2-3*a*b*(2-m)+b^2*(m^2-4*m+3))*hypergeom([1, 1+m],[2+m],(a+b*sin
(d*x+c))/(a-b))*(a+b*sin(d*x+c))^(1+m)/(a-b)^3/d/(1+m)+1/16*(3*a^2+3*a*b*(2
-m)+b^2*(m^2-4*m+3))*hypergeom([1, 1+m],[2+m],(a+b*sin(d*x+c))/(a+b))*(a+b*
sin(d*x+c))^(1+m)/(a+b)^3/d/(1+m)-1/4*sec(d*x+c)^4*(b-a*sin(d*x+c))*(a+b*si
n(d*x+c))^(1+m)/d/(a^2-b^2)+1/8*sec(d*x+c)^2*(a+b*sin(d*x+c))^(1+m)*(b*(b^2
*(3-m)-a^2*(1+m))+a*(3*a^2-b^2*(5-2*m))*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.42, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2668, 741, 823, 831, 68}

$$-\frac{\left(3a^2-3ab(2-m)+b^2\left(m^2-4m+3\right)\right)(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{16d(m+1)(a-b)^3}+\frac{\left(3a^2+3ab(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -((3*a^2 - 3*a*b*(2 - m) + b^2*(3 - 4*m + m^2))*Hypergeometric2F1[1, 1 + m,
 2 + m, (a + b*Sin[c + d*x])/(a - b)]*(a + b*Sin[c + d*x])^(1 + m))/(16*(a
- b)^3*d*(1 + m)) + ((3*a^2 + 3*a*b*(2 - m) + b^2*(3 - 4*m + m^2))*Hypergeo
metric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x
])^(1 + m))/(16*(a + b)^3*d*(1 + m)) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^(1 + m))/(4*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(a + b*S
in[c + d*x])^(1 + m)*(b*(b^2*(3 - m) - a^2*(1 + m)) + a*(3*a^2 - b^2*(5 - 2
*m))*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{(a+x)^m}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4\left(a^2-b^2\right)d} + \frac{b^3\,\text{Subst}\Big(\int$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx}{}$$

$$= -\frac{\left(3a^2-3ab(2-m)+b^2\left(3-4m+m^2\right)\right)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin}{a-}\right.}{16(a-b)^3d(1+m)}$$

Mathematica [A] time = 4.00, size = 260, normalized size = 0.85

$$\frac{(a+b\sin(c+dx))^{m+1}\left(\frac{(a+b)^3\left(3a^2+3ab(m-2)+b^2\left(m^2-4m+3\right)\right)\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)-(a-b)^3\left(3a^2-3ab(m-2)+b^2\left(m^2-4m+3\right)\right)\,{}_2F_1}{(m+1)(a-b)(a+b)\left(a^2-b^2\right)}\right.}{16d\,(b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*(((a + b)^3*(3*a^2 + 3*a*b*(-2 + m) + b^2*(3
- 4*m + m^2))*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a -
b)] - (a - b)^3*(3*a^2 - 3*a*b*(-2 + m) + b^2*(3 - 4*m + m^2))*Hypergeometr
```

```
ic2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)])/((a - b)*(a + b)*(a^2
 - b^2)*(1 + m)) + 4*Sec[c + d*x]^4*(b - a*Sin[c + d*x]) + (2*Sec[c + d*x]^
2*(b^3*(-3 + m) + a^2*b*(1 + m) - a*(3*a^2 + b^2*(-5 + 2*m))*Sin[c + d*x]))
/(a^2 - b^2)))/(16*(-a^2 + b^2)*d)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^m\sec(dx+c)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \left(\sec^5(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^m}{\cos(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/cos(c + d*x)^5,x)
[Out] int((a + b*sin(c + d*x))^m/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

3.637 $\int \cos^4(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=129

$$\frac{\cos^3(c+dx)(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$

```
[Out] AppellF1(1+m,-3/2,-3/2,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*c
os(d*x+c)^3*(a+b*sin(d*x+c))^(1+m)/b/d/(1+m)/(1+(-a-b*sin(d*x+c))/(a-b))^(3
/2)/(1+(-a-b*sin(d*x+c))/(a+b))^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2704, 138}

$$\frac{\cos^3(c+dx)(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, -3/2, -3/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(1 + m))/(b*d*(1 +
 m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/2)*(1 - (a + b*Sin[c + d*x])/(a +
 b))^(3/2))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\cos^3(c+dx)\,\text{Subst}\left(\int(a+bx)^m\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{3/2}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{3/2}dx,x,\sin\right.}{d\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$
$$= \frac{F_1\left(1+m;-\frac{3}{2},-\frac{3}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\cos^3(c+dx)(a+b\sin}{bd(1+m)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$

Mathematica [F] time = 4.22, size = 0, normalized size = 0.00

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^m, x]
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^m\cos(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^m,x)
[Out] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

3.638 $\int \cos^2(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=127

$$\frac{\cos(c+dx)(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] AppellF1(1+m,-1/2,-1/2,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*c
os(d*x+c)*(a+b*sin(d*x+c))^(1+m)/b/d/(1+m)/(1+(-a-b*sin(d*x+c))/(a-b))^(1/2
)/(1+(-a-b*sin(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2704, 138}

$$\frac{\cos(c+dx)(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, -1/2, -1/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + m))/(b*d*(1 + m
)*Sqrt[1 - (a + b*Sin[c + d*x])/(a - b)]*Sqrt[1 - (a + b*Sin[c + d*x])/(a +
 b)])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sin(c+dx))^m\,dx &= \frac{\cos(c+dx)\,\text{Subst}\left(\int (a+bx)^m\sqrt{-\frac{b}{a-b}-\frac{bx}{a-b}}\sqrt{\frac{b}{a+b}-\frac{bx}{a+b}}\,dx,x,\sin(c+d\right.}{d\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}\\ &= \frac{F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\cos(c+dx)(a+b\sin}{bd(1+m)\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}\end{aligned}$$

Mathematica [F] time = 5.88, size = 0, normalized size = 0.00

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^m, x]
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^m\cos(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,(a+b\,\sin(c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^m,x)
[Out] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.639 $\int \sec^2(c+dx)(a+b\sin(c+dx))^m \, dx$

Optimal. Leaf size=129

$$\frac{\sec^3(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\right.}{bd(m+1)}$$

```
[Out] AppellF1(1+m,3/2,3/2,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*sec
(d*x+c)^3*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(3/2)*(1+(-a-b
*sin(d*x+c))/(a+b))^(3/2)/b/d/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2704, 138}

$$\frac{\sec^3(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 3/2, 3/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*
Sin[c + d*x])/(a - b))^(3/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/2))/(b*d
*(1 + m))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^m \, dx = \frac{\left(\sec^3(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}\right)\text{Subst}\left(\int \frac{}{\left(-\frac{b}{a-b}\right.}\right.}{d}$$

$$= \frac{F_1\left(1+m;\frac{3}{2},\frac{3}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec^3(c+dx)(a+b\sin}{bd(1+m)}$$

Mathematica [F] time = 2.23, size = 0, normalized size = 0.00

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m, x]
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^m\sec(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^m}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/cos(c + d*x)^2,x)
[Out] int((a + b*sin(c + d*x))^m/cos(c + d*x)^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^m \sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**m*sec(c + d*x)**2, x)
```

3.640 $\int \sec^4(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=129

$$\frac{\sec^5(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{5}{2},\frac{5}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b}{}\right.}{bd(m+1)}$$

```
[Out] AppellF1(1+m,5/2,5/2,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*sec
(d*x+c)^5*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(5/2)*(1+(-a-b
*sin(d*x+c))/(a+b))^(5/2)/b/d/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2704, 138}

$$\frac{\sec^5(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{5}{2},\frac{5}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b}{}\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 5/2, 5/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*
Sin[c + d*x])/(a - b))^(5/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/2))/(b*d
*(1 + m))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\left(\sec^5(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/2}\right)\text{Subst}\left(\int \frac{}{\left(-\frac{b}{a-b}-\frac{b}{a}\right.}\right.}{d}$$

$$= \frac{F_1\left(1+m;\frac{5}{2},\frac{5}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec^5(c+dx)(a+b\sin(c}{bd(1+m)}$$

Mathematica [F] time = 4.92, size = 0, normalized size = 0.00

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^m, x]
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\sin(dx+c)+a)^m\sec(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \left(\sec^4(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\sin(c+dx))^m}{\cos(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/cos(c + d*x)^4,x)
[Out] int((a + b*sin(c + d*x))^m/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.641 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{e(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{3}{4},-\frac{3}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}}$$

```
[Out] e*AppellF1(1+m,-3/4,-3/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))
*(e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^(1+m)/b/d/(1+m)/(1+(-a-b*sin(d*x+c))
/(a-b))^(3/4)/(1+(-a-b*sin(d*x+c))/(a+b))^(3/4)
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2704, 138}

$$\frac{e(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{3}{4},-\frac{3}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, -3/4, -3/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*
Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^(1 + m))
/(b*d*(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/4)*(1 - (a + b*Sin[c +
d*x])/(a + b))^(3/4))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{3/2}\right)\text{Subst}\left(\int (a+bx)^m\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{3/4}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{3}}{d\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}}$$
$$= \frac{eF_1\left(1+m;-\frac{3}{4},-\frac{3}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{3/}}{bd(1+m)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)}$$

Mathematica [F] time = 55.45, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^m, x]
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{e\cos(dx+c)}\,(b\sin(dx+c)+a)^m e^2\cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m*e^2*cos(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

3.642 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{e\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{1}{4},-\frac{1}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] e*AppellF1(1+m,-1/4,-1/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))
*(a+b*sin(d*x+c))^(1+m)*(e*cos(d*x+c))^(1/2)/b/d/(1+m)/(1+(-a-b*sin(d*x+c))
/(a-b))^(1/4)/(1+(-a-b*sin(d*x+c))/(a+b))^(1/4)
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2704, 138}

$$\frac{e\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{1}{4},-\frac{1}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, -1/4, -1/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*
Sin[c + d*x])/(a + b)]*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^(1 + m))/(
b*d*(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(1/4)*(1 - (a + b*Sin[c + d*
x])/(a + b))^(1/4))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^m\,dx = \frac{\left(e\sqrt{e\cos(c+dx)}\right)\text{Subst}\left(\int (a+bx)^m\sqrt[4]{-\frac{b}{a-b}-\frac{bx}{a-b}}\sqrt[4]{\frac{b}{a+b}-\frac{bx}{a+b}}\,dx,\right.}{d\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

$$= \frac{eF_1\left(1+m;-\frac{1}{4},-\frac{1}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sqrt{e\cos(c+dx)}\,(}{bd(1+m)\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [F] time = 5.70, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^m, x]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^{m}e\cos\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m*e*cos(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.643 $\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{e\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}\,(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{1}{4},\frac{1}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt{e\cos(c+dx)}}$$

```
[Out] e*AppellF1(1+m,1/4,1/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*(
a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(1/4)*(1+(-a-b*sin(d*x+c)
)/(a+b))^(1/4)/b/d/(1+m)/(e*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2704, 138}

$$\frac{e\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}\,(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{1}{4},\frac{1}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, 1/4, 1/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(1/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(1/4))/(b*d*(1 + m)*Sqrt
[e*Cos[c + d*x]])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^m\,dx = \frac{\left(e\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt[4]{-\frac{b}{a-b}-\frac{bx}{a-b}}\sqrt[4]{\frac{b}{a+b}-\frac{b}{a+}}}\right.}{d\sqrt{e\cos(c+dx)}}$$

$$= \frac{eF_1\left(1+m;\frac{1}{4},\frac{1}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx)}{bd(1+m)\sqrt{e\cos(c+dx)}}$$

Mathematica [F] time = 1.93, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^m, x]
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos\left(dx+c\right)}\;\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\;\left(a+b\,\sin\left(c+d\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))*(a + b*sin(c + d*x))**m, x)
```

3.644 $\int \frac{(a+b\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=134

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{3}{4},\frac{3}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{3/2}}$$

```
[Out] e*AppellF1(1+m,3/4,3/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*(
a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(3/4)*(1+(-a-b*sin(d*x+c)
)/(a+b))^(3/4)/b/d/(1+m)/(e*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2704, 138}

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{3}{4},\frac{3}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (e*AppellF1[1 + m, 3/4, 3/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(3/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(3/2))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\left(e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{3/4}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{3/4}}\,dx,x,\text{si}\right.}{d(e\cos(c+dx))^{3/2}}$$

$$= \frac{eF_1\left(1+m;\frac{3}{4},\frac{3}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+}{a-b}\right.}{bd(1+m)(e\cos(c+dx))^{3/2}}$$

Mathematica [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
[Out] Integrate[(a + b*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]], x]
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^{m}}{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m/(e*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^{m}}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m/sqrt(e*cos(d*x + c)), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(dx+c\right)\right)^{m}}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
[Out] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^{m}}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m/sqrt(e*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sin(c + dx))^m}{\sqrt{e\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**m/sqrt(e*cos(c + d*x)), x)
```

3.645 $\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{5}{4},\frac{5}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{5/2}}$$

```
[Out] e*AppellF1(1+m,5/4,5/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*(
a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(5/4)*(1+(-a-b*sin(d*x+c)
)/(a+b))^(5/4)/b/d/(1+m)/(e*cos(d*x+c))^(5/2)
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2704, 138}

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{5}{4},\frac{5}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (e*AppellF1[1 + m, 5/4, 5/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(5/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(5/2))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{\left(e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/4}\right)\operatorname{Subst}\left(\int \frac{(a+bx)^m}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{5/4}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{5/4}}\,dx, x\right.}{d(e\cos(c+dx))^{5/2}}\\ &= \frac{eF_1\left(1+m;\frac{5}{4},\frac{5}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin}{a}\right.}{bd(1+m)(e\cos(c+dx))^{5/2}}\end{aligned}$$

Mathematica [F] time = 1.92, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2), x]
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^m}{e^2\cos\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m/(e^2*cos(d*x + c)^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^m}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(3/2), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(dx+c\right)\right)^m}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^m}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sin (c + dx))^m}{(e \cos (c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/(e*cos(d*x+c))**(3/2),x)
[Out] Integral((a + b*sin(c + d*x))**m/(e*cos(c + d*x))**(3/2), x)
```

3.646 $\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{7/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{7/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{7}{4},\frac{7}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{7/2}}$$

```
[Out] e*AppellF1(1+m,7/4,7/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*(
a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(7/4)*(1+(-a-b*sin(d*x+c)
)/(a+b))^(7/4)/b/d/(1+m)/(e*cos(d*x+c))^(7/2)
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2704, 138}

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{7/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{7/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{7}{4},\frac{7}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (e*AppellF1[1 + m, 7/4, 7/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(7/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(7/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(7/2))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx &= \frac{\left(e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{7/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{7/4}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{7/4}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{7/4}}\,dx,x,\text{si}\right.}{d(e\cos(c+dx))^{7/2}}\\ &= \frac{eF_1\left(1+m;\frac{7}{4},\frac{7}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+}{a-b}\right.}{bd(1+m)(e\cos(c+dx))^{7/2}}\end{aligned}$$

Mathematica [F] time = 2.10, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
[Out] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2), x]
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,(b\sin(dx+c)+a)^m}{e^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m/(e^3*cos(d*x + c)^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(5/2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(dx+c))^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
[Out] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/(e*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.647 $\int (e\cos(c+dx))^{-4-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=598

$$-\frac{a2^{-\frac{m}{2}-\frac{1}{2}}\left(a^2(m+2)+2ab-b^2\right)(1-\sin(c+dx))^2(e\cos(c+dx))^{-m-3}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+3}{2}}(a+b\sin(c+dx))}{de(1-m)(m+3)(a-b)(a+b)^3}$$

```
[Out] -(e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^(1+m)/(a-b)/d/e/(3+m)+2*b*(e*cos(d*
x+c))^(-1-m)*(a+b*sin(d*x+c))^(1+m)/(a-b)^2/d/e^3/(1+m)/(3+m)+a*(e*cos(d*x+
c))^(-3-m)*(1+sin(d*x+c))*(a+b*sin(d*x+c))^(1+m)/(a^2-b^2)/d/e/(3+m)+a*(3*b
+a*(2+m))*(e*cos(d*x+c))^(-3-m)*(1-sin(d*x+c))*(1+sin(d*x+c))*(a+b*sin(d*x+
c))^(1+m)/(a-b)/(a+b)^2/d/e/(1+m)/(3+m)-2^(3/2-1/2*m)*a*b*(e*cos(d*x+c))^(-
1-m)*hypergeom([-1/2-1/2*m, 1/2+1/2*m],[1/2-1/2*m],1/2*(a-b)*(1-sin(d*x+c))
/(a+b*sin(d*x+c)))*((a+b)*(1+sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2+1/2*m)*(a+b
*sin(d*x+c))^(1+m)/(a-b)^2/(a+b)/d/e^3/(m^2+4*m+3)-2^(-1/2-1/2*m)*a*(2*a*b-
b^2+a^2*(2+m))*(e*cos(d*x+c))^(-3-m)*hypergeom([1/2-1/2*m, 3/2+1/2*m],[3/2-
1/2*m],1/2*(a-b)*(1-sin(d*x+c))/(a+b*sin(d*x+c)))*(1-sin(d*x+c))^2*((a+b)*(
1+sin(d*x+c))/(a+b*sin(d*x+c)))^(3/2+1/2*m)*(a+b*sin(d*x+c))^(1+m)/(a-b)/(a
+b)^3/d/e/(1-m)/(3+m)
```

Rubi [A] time = 1.02, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2700, 2699, 2920, 132, 129, 155, 12}

$$-\frac{a2^{-\frac{m}{2}-\frac{1}{2}}\left(a^2(m+2)+2ab-b^2\right)(1-\sin(c+dx))^2(e\cos(c+dx))^{-m-3}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+3}{2}}(a+b\sin(c+dx))}{de(1-m)(m+3)(a-b)(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-4 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-3 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(3
 + m))) + (2*b*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a -
 b)^2*d*e^3*(1 + m)*(3 + m)) + (a*(e*Cos[c + d*x])^(-3 - m)*(1 + Sin[c + d*
x])*(a + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(3 + m)) + (a*(3*b + a*(
2 + m))*(e*Cos[c + d*x])^(-3 - m)*(1 - Sin[c + d*x])*(1 + Sin[c + d*x])*(a
+ b*Sin[c + d*x])^(1 + m))/((a - b)*(a + b)^2*d*e*(1 + m)*(3 + m)) - (2^(3/
2 - m/2)*a*b*(e*Cos[c + d*x])^(-1 - m)*Hypergeometric2F1[(-1 - m)/2, (1 + m
)/2, (1 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[c + d*x]))]*(((a
 + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)*(a + b*Sin[c +
d*x])^(1 + m))/((a - b)^2*(a + b)*d*e^3*(1 + m)*(3 + m)) - (2^(-1/2 - m/2)*
a*(2*a*b - b^2 + a^2*(2 + m))*(e*Cos[c + d*x])^(-3 - m)*Hypergeometric2F1[(
1 - m)/2, (3 + m)/2, (3 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[
c + d*x]))]*(1 - Sin[c + d*x])^2*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c
 + d*x]))^((3 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*(a + b)^3*d*e*
(1 - m)*(3 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 2699

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1))/(f*g*(a - b)*(p + 1)), x] + Dist[a/(g^2*(a - b)), Int[((g*Cos[e +
 f*x])^(p + 2)*(a + b*Sin[e + f*x])^m)/(1 - Sin[e + f*x]), x], x] /; FreeQ[
{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 2, 0]
```

Rule 2700

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1))/(f*g*(a - b)*(p + 1)), x] + (-Dist[(b*(m + p + 2))/(g^2*(a - b)*(
p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] + Dist
[a/(g^2*(a - b)), Int[((g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m)/(1
- Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[m + p + 2, 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-4-m}(a+b\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{a\int \frac{(e\cos(c+dx))^{-2-}}{1-\mathrm{si}}}{(a-}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+dx)}{(a-b)^2}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+dx)}{(a-b)^2}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+dx)}{(a-b)^2}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+dx)}{(a-b)^2}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+dx)}{(a-b)^2}$$

Mathematica [A] time = 6.10, size = 826, normalized size = 1.38

$$\frac{\cos(c+dx)(a+b\sin(c+dx))^{m+1}(e\cos(c+dx))^{-m-4}}{(a-b)d(-m-3)} + \frac{2b\cos^{m+4}(c+dx)\left(\frac{2^{\frac{1}{2}(-m-1)+1}a\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{m+1}{2};\frac{1}{2}(-m-1)+1;-\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-4 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (Cos[c + d*x]*(e*Cos[c + d*x])^(-4 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a -
 b)*d*(-3 - m)) + (2*b*Cos[c + d*x]^(4 + m)*(e*Cos[c + d*x])^(-4 - m)*((Cos
[c + d*x]^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*(-1 - m)) + (2^
(1 + (-1 - m)/2)*a*Cos[c + d*x]^(-1 - m)*Hypergeometric2F1[(-1 - m)/2, (1 +
 m)/2, 1 + (-1 - m)/2, -1/2*((-a + b)*(1 - Sin[c + d*x]))/(a + b*Sin[c + d*
x])]*(1 - Sin[c + d*x])^((-1 - m)/2 + (1 + m)/2)*(1 + Sin[c + d*x])^((-1 -
m)/2 + (1 + m)/2)*(-(((-a - b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x])))^(
(1 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((-a - b)*(a - b)*d*(-1 - m))))/((
a - b)*(-3 - m)) + (a*Cos[c + d*x]*(e*Cos[c + d*x])^(-4 - m)*(1 - Sin[c + d
*x])^((3 + m)/2)*(1 + Sin[c + d*x])^((3 + m)/2)*(((1 - Sin[c + d*x])^((-3 -
 m)/2)*(1 + Sin[c + d*x])^(1 + (-3 - m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((
-a - b)*(-3 - m)) - (-1/2*((3*b + a*(2 + m))*(1 - Sin[c + d*x])^(1 + (-3 -
m)/2)*(1 + Sin[c + d*x])^(1 + (-3 - m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((-
a - b)*(1 + (-3 - m)/2)) - (2^(-1 + (-3 - m)/2)*(1 + m)*(2*a*b - b^2 + a^2*
(2 + m))*Hypergeometric2F1[2 + (-3 - m)/2, (3 + m)/2, 3 + (-3 - m)/2, -1/2*
((-a + b)*(1 - Sin[c + d*x]))/(a + b*Sin[c + d*x])]*(1 - Sin[c + d*x])^(2 +
 (-3 - m)/2)*(1 + Sin[c + d*x])^((-3 - m)/2)*(-(((-a - b)*(1 + Sin[c + d*x]
))/(a + b*Sin[c + d*x])))^((3 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((-a -
b)^2*(1 + (-3 - m)/2)*(2 + (-3 - m)/2)))/((-a - b)*(-3 - m))))/((a - b)*d)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-m-4}(b\sin(dx+c)+a)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((e*cos(d*x + c))^(-m - 4)*(b*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-4}(b\sin(dx+c)+a)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(-m - 4)*(b*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-4-m}(a+b\sin(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-4}(b\sin(dx+c)+a)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(-m - 4)*(b*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^{m}}{(e\,\cos(c+d\,x))^{m+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 4),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-4-m)*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

3.648 $\int (e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=311

$$-\frac{\left(a^2(m+1)-b^2\right)(\sin(c+dx)+1)^3\sec^4(c+dx)(e\cos(c+dx))^{-m}(a+b\sin(c+dx))^{m+1}\left(\frac{(a+b)(\sin(c+dx)+1)}{(a-b)(\sin(c+dx)-1)}\right)^{\frac{m-2}{2}}}{de^3m(m+1)(a-b)^3}$$

```
[Out] sec(d*x+c)^4*(sin(d*x+c)-1)*(1+sin(d*x+c))*(a+b*sin(d*x+c))^(1+m)/(a-b)/d/e
^3/(2+m)/((e*cos(d*x+c))^m)+(-2*b+a*(2+m))*sec(d*x+c)^4*(sin(d*x+c)-1)*(1+s
in(d*x+c))^2*(a+b*sin(d*x+c))^(1+m)/(a-b)^2/d/e^3/m/(2+m)/((e*cos(d*x+c))^m
)-(-b^2+a^2*(1+m))*hypergeom([1+m, 1/2*m],[2+m],-2*(a+b*sin(d*x+c))/(a-b)/(
sin(d*x+c)-1))*sec(d*x+c)^4*(1+sin(d*x+c))^3*((a+b)*(1+sin(d*x+c))/(a-b)/(s
in(d*x+c)-1))^(-1+1/2*m)*(a+b*sin(d*x+c))^(1+m)/(a-b)^3/d/e^3/m/(1+m)/((e*c
os(d*x+c))^m)
```

Rubi [A] time = 0.51, antiderivative size = 420, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2700, 2698, 2920, 96, 132}

$$-\frac{b(1-\sin(c+dx))(e\cos(c+dx))^{-m-2}\left(-\frac{(a-b)(1-\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)^{m/2}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(m+1,\frac{m+2}{2};m+2;\frac{2}{(a-\ldots}\right.}{de(m+1)(m+2)\left(a^2-b^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(e*Cos[c + d*x])^(-3 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-2 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(2
+ m))) - (b*(e*Cos[c + d*x])^(-2 - m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2
 + m, (2*(a + b*Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))]*(1 - Sin[c + d
*x])*(-(((a - b)*(1 - Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))))^(m/2)*(
a + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(1 + m)*(2 + m)) + (a*(e*Cos[
c + d*x])^(-2 - m)*(1 + Sin[c + d*x])*(a + b*Sin[c + d*x])^(1 + m))/((a^2 -
 b^2)*d*e*(2 + m)) + (a*(a + b + a*m)*(e*Cos[c + d*x])^(-2 - m)*Hypergeomet
ric2F1[-m/2, (2 + m)/2, (2 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*S
in[c + d*x]))]*(1 - Sin[c + d*x])*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[
c + d*x]))^((2 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/(2^(m/2)*(a - b)*(a +
b)^2*d*e*m*(2 + m))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 2698

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(1 - Sin[e + f*x])
*(a + b*Sin[e + f*x])^(m + 1)*(-(((a - b)*(1 - Sin[e + f*x]))/((a + b)*(1 +
 Sin[e + f*x]))))^(m/2)*Hypergeometric2F1[m + 1, m/2 + 1, m + 2, (2*(a + b*
Sin[e + f*x]))/((a + b)*(1 + Sin[e + f*x]))])/(f*(a + b)*(m + 1)), x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rule 2700

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1))/(f*g*(a - b)*(p + 1)), x] + (-Dist[(b*(m + p + 2))/(g^2*(a - b)*(
p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] + Dist
[a/(g^2*(a - b)), Int[((g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m)/(1
- Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[m + p + 2, 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} + \frac{a\int \frac{(e\cos(c+dx))^{-1-m}(a}{1-\sin(c+}}{(a-b)}$$

$$= -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} - \frac{b(e\cos(c+dx))^{-2-}}{}$$

$$= -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} - \frac{b(e\cos(c+dx))^{-2-}}{}$$

$$= -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} - \frac{b(e\cos(c+dx))^{-2-}}{}$$

Mathematica [A] time = 5.05, size = 319, normalized size = 1.03

$$\frac{\sec^2(c+dx)(e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\left(\frac{b(\sin(c+dx)+1)(a+b\sin(c+dx))\left(\frac{(a+b)(\sin(c+dx)+1)}{(a-b)(\sin(c+dx)-1)}\right)^{m/2}{}_2F_1\left(m+1,\frac{m+2}{2};m+2;-\frac{2(a+b}{(a-b)(s}\right.}{(m+1)(a-b)}\right.}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-3 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m*(-a - b*Sin[c + d*x] + (b*Hypergeome
tric2F1[1 + m, (2 + m)/2, 2 + m, (-2*(a + b*Sin[c + d*x]))/((a - b)*(-1 + S
in[c + d*x]))]*(1 + Sin[c + d*x])*(((a + b)*(1 + Sin[c + d*x]))/((a - b)*(-
```

```
1 + Sin[c + d*x])))^(m/2)*(a + b*Sin[c + d*x]))/((a - b)*(1 + m)) + (a*(1 -
 Sin[c + d*x])*(1 + Sin[c + d*x])*(((a + b + a*m)*Hypergeometric2F1[-1/2*m,
 (2 + m)/2, 1 - m/2, -1/2*((a - b)*(-1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]
)]*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^(m/2))/2^(m/2) - (m*
(a + b*Sin[c + d*x]))/(-1 + Sin[c + d*x])))/((a + b)*m)))/((a - b)*d*e^3*(2
 + m)*(e*Cos[c + d*x])^m)
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-m-3}(b\sin(dx+c)+a)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((e*cos(d*x + c))^(-m - 3)*(b*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-3}(b\sin(dx+c)+a)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(-m - 3)*(b*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-3-m}(a+b\sin(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-3}(b\sin(dx+c)+a)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(-m - 3)*(b*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\sin(c+dx))^{m}}{(e\cos(c+dx))^{m+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 3),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-3-m)*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

3.649 $\int (e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=201

$$\frac{a2^{\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^{-m-1}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+1}{2}}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{m+1}{2};\frac{1-m}{2};\frac{(a-b)(1-\sin(c+dx))}{2(a+b\sin(c+dx))}\right)}{de(m+1)\left(a^2-b^2\right)}$$

```
[Out] -(e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^(1+m)/(a-b)/d/e/(1+m)+2^(1/2-1/2*m)
*a*(e*cos(d*x+c))^(-1-m)*hypergeom([-1/2-1/2*m, 1/2+1/2*m],[1/2-1/2*m],1/2*
(a-b)*(1-sin(d*x+c))/(a+b*sin(d*x+c)))*((a+b)*(1+sin(d*x+c))/(a+b*sin(d*x+c
)))^(1/2+1/2*m)*(a+b*sin(d*x+c))^(1+m)/(a^2-b^2)/d/e/(1+m)
```

Rubi [A] time = 0.29, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2699, 2920, 132}

$$\frac{a2^{\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^{-m-1}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+1}{2}}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{m+1}{2};\frac{1-m}{2};\frac{(a-b)(1-\sin(c+dx))}{2(a+b\sin(c+dx))}\right)}{de(m+1)\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-2 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(1
+ m))) + (2^(1/2 - m/2)*a*(e*Cos[c + d*x])^(-1 - m)*Hypergeometric2F1[(-1 -
 m)/2, (1 + m)/2, (1 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[c +
 d*x]))]*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)*(a
 + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(1 + m))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2699

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1))/(f*g*(a - b)*(p + 1)), x] + Dist[a/(g^2*(a - b)), Int[((g*Cos[e +
 f*x])^(p + 2)*(a + b*Sin[e + f*x])^m)/(1 - Sin[e + f*x]), x], x] /; FreeQ[
{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 2, 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(1+m)} + \frac{a\int \frac{(e\cos(c+dx))^{-m}}{1-\sin}}{(a-}$$

$$= -\frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(1+m)} + \left(a(e\cos(c+dx)\right.$$

$$= -\frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(1+m)} + 2^{\frac{1}{2}-\frac{m}{2}}a(e\cos(c+$$

Mathematica [A] time = 0.96, size = 168, normalized size = 0.84

$$-\frac{2^{\frac{1}{2}(-m-1)}(e\cos(c+dx))^{-m-1}(a+b\sin(c+dx))^{m+1}\left(2^{\frac{m+1}{2}}(a+b)-2a\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+1}{2}}{}_2F_1\left(\frac{1}{2}(-m-1),\frac{m}{2}\right.\right.}{de(m+1)(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-2 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -((2^((-1 - m)/2)*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m)*(2
^((1 + m)/2)*(a + b) - 2*a*Hypergeometric2F1[(-1 - m)/2, (1 + m)/2, (1 - m)
/2, -1/2*((a - b)*(-1 + Sin[c + d*x]))/(a + b*Sin[c + d*x])]*(((a + b)*(1 +
 Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)))/((a - b)*(a + b)*d*e*(1
 + m)))
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-m-2}(b\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m - 2)*(b*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-2}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 2)*(b*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2-m}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-2}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(-m - 2)*(b*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 2),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-2-m)*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

3.650 $\int (e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=132

$$\frac{e(1-\sin(c+dx))(e\cos(c+dx))^{-m-2}\left(-\frac{(a-b)(1-\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)^{m/2}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(m+1,\frac{m+2}{2};m+2;\frac{2(a\right.}{(a+b}}{d(m+1)(a+b)}$$

```
[Out] e*(e*cos(d*x+c))^(-2-m)*hypergeom([1+m, 1+1/2*m],[2+m],2*(a+b*sin(d*x+c))/(
a+b)/(1+sin(d*x+c)))*(1-sin(d*x+c))*(-(a-b)*(1-sin(d*x+c))/(a+b)/(1+sin(d*x
+c)))^(1/2*m)*(a+b*sin(d*x+c))^(1+m)/(a+b)/d/(1+m)
```

Rubi [A] time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2698}

$$\frac{e(1-\sin(c+dx))(e\cos(c+dx))^{-m-2}\left(-\frac{(a-b)(1-\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)^{m/2}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(m+1,\frac{m+2}{2};m+2;\frac{2(a\right.}{(a+b}}{d(m+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*(e*Cos[c + d*x])^(-2 - m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2 + m, (2*
(a + b*Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))]*(1 - Sin[c + d*x])*(-((
(a - b)*(1 - Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))))^(m/2)*(a + b*Sin
[c + d*x])^(1 + m))/((a + b)*d*(1 + m))
```

Rule 2698

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(1 - Sin[e + f*x])
*(a + b*Sin[e + f*x])^(m + 1)*(-(((a - b)*(1 - Sin[e + f*x]))/((a + b)*(1 +
 Sin[e + f*x]))))^(m/2)*Hypergeometric2F1[m + 1, m/2 + 1, m + 2, (2*(a + b*
Sin[e + f*x]))/((a + b)*(1 + Sin[e + f*x]))])/(f*(a + b)*(m + 1)), x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^m\,dx = \frac{e(e\cos(c+dx))^{-2-m}\,{}_2F_1\left(1+m,\frac{2+m}{2};2+m;\frac{2(a+b\sin(c+dx))}{(a+b)(1+\sin(c+dx))}\right)(}{(a+b)d(}$$

Mathematica [A] time = 0.39, size = 132, normalized size = 1.00

$$-\frac{e(\sin(c+dx)+1)(e\cos(c+dx))^{-m-2}\left(\frac{(a+b)(\sin(c+dx)+1)}{(a-b)(\sin(c+dx)-1)}\right)^{m/2}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(m+1,\frac{m+2}{2};m+2;-\frac{2}{(a-}\right.}{d(m+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -((e*(e*Cos[c + d*x])^(-2 - m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2 + m, (
-2*(a + b*Sin[c + d*x]))/((a - b)*(-1 + Sin[c + d*x]))]*(1 + Sin[c + d*x])*
(((a + b)*(1 + Sin[c + d*x]))/((a - b)*(-1 + Sin[c + d*x])))^(m/2)*(a + b*S
in[c + d*x])^(1 + m))/((a - b)*d*(1 + m)))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-m-1}(b\sin(dx+c)+a)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((e*cos(d*x + c))^(-m - 1)*(b*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-1}(b\sin(dx+c)+a)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(-m - 1)*(b*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-1-m}(a+b\sin(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m-1}(b\sin(dx+c)+a)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(-m - 1)*(b*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 1),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-1-m)*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

3.651 $\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=152

$$\frac{e(e\cos(c+dx))^{-m-1}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{m+1}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{m+1}{2}}F_1\left(m+1;\frac{m+1}{2},\frac{m+1}{2};m+\right.}{bd(m+1)}$$

```
[Out] e*AppellF1(1+m,1/2+1/2*m,1/2+1/2*m,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+
c))/(a+b))*(e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c)
)/(a-b))^(1/2+1/2*m)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2+1/2*m)/b/d/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2704, 138}

$$\frac{e(e\cos(c+dx))^{-m-1}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{m+1}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{m+1}{2}}F_1\left(m+1;\frac{m+1}{2},\frac{m+1}{2};m+\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, (1 + m)/2, (1 + m)/2, 2 + m, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d
*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 + m)/2)*(1 - (a + b*Sin
[c + d*x])/(a + b))^((1 + m)/2))/(b*d*(1 + m))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{-1-m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1+m}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1+m}{2}}\right.}{}$$

$$= \frac{eF_1\left(1+m;\frac{1+m}{2},\frac{1+m}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+}{b}$$

Mathematica [F] time = 1.67, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
[Out] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^m, x]
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int (a+b\sin(dx+c))^m\,(e\cos(dx+c))^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
[Out] int((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^m,x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/((e*cos(d*x+c))**m),x)
[Out] Integral((e*cos(c + d*x))**(-m)*(a + b*sin(c + d*x))**m, x)
```

3.652 $\int (e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=142

$$\frac{e(e\cos(c+dx))^{-m}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{m/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{m/2}F_1\left(m+1;\frac{m}{2},\frac{m}{2};m+2;\frac{a+b\sin(c+}{a-b}\right.}{bd(m+1)}$$

```
[Out] e*AppellF1(1+m,1/2*m,1/2*m,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b
))*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(1/2*m)*(1+(-a-b*sin(
d*x+c))/(a+b))^(1/2*m)/b/d/(1+m)/((e*cos(d*x+c))^m)
```

Rubi [A] time = 0.10, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2704, 138}

$$\frac{e(e\cos(c+dx))^{-m}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{m/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{m/2}F_1\left(m+1;\frac{m}{2},\frac{m}{2};m+2;\frac{a+b\sin(c+}{a-b}\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, m/2, m/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(m/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(m/2))/(b*d*(1 + m)*(e*C
os[c + d*x])^m)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^m\,dx &= \frac{\left(e(e\cos(c+dx))^{-m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{m/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{m/2}\right)\text{Subs}}{a}\\ &= \frac{eF_1\left(1+m;\frac{m}{2},\frac{m}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-m}}{bd(1+}\end{aligned}$$

Mathematica [F] time = 5.63, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c + d*x])^m, x]
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((e\cos(dx+c))^{-m+1}(b\sin(dx+c)+a)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((e*cos(d*x + c))^(-m + 1)*(b*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m+1}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(-m + 1)*(b*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{1-m}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m+1}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(-m + 1)*(b*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{1-m}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1 - m)*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(1 - m)*(a + b*sin(c + d*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1-m)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Integral((e*cos(c + d*x))**(1 - m)*(a + b*sin(c + d*x))**m, x)
```

3.653 $\int (e\cos(c+dx))^{2-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=152

$$\frac{e(e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{m-1}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{m-1}{2}}F_1\left(m+1;\frac{m-1}{2},\frac{m-1}{2};m+2\right.}{bd(m+1)}$$

```
[Out] e*AppellF1(1+m,-1/2+1/2*m,-1/2+1/2*m,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*
x+c))/(a+b))*(e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c
))/(a-b))^(-1/2+1/2*m)*(1+(-a-b*sin(d*x+c))/(a+b))^(-1/2+1/2*m)/b/d/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2704, 138}

$$\frac{e(e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{m-1}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{m-1}{2}}F_1\left(m+1;\frac{m-1}{2},\frac{m-1}{2};m+2\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, (-1 + m)/2, (-1 + m)/2, 2 + m, (a + b*Sin[c + d*x])/(a -
 b), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c +
d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((-1 + m)/2)*(1 - (a + b*S
in[c + d*x])/(a + b))^((-1 + m)/2))/(b*d*(1 + m))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{2-m}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{1-m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1}{2}(-1+m)}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1}{2}\right.}{}$$

$$= \frac{eF_1\left(1+m;\frac{1}{2}(-1+m),\frac{1}{2}(-1+m);2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+d\right.}{}$$

Mathematica [F] time = 4.73, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{2-m}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(2 - m)*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[(e*Cos[c + d*x])^(2 - m)*(a + b*Sin[c + d*x])^m, x]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left((e\cos(dx+c))^{-m+2}(b\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((e*cos(d*x + c))^(-m + 2)*(b*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m+2}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(-m + 2)*(b*sin(d*x + c) + a)^m, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{2-m}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-m+2}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(-m + 2)*(b*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{2-m}\,(a+b\,\sin(c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(2 - m)*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(2 - m)*(a + b*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2-m)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```