Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.1.4-d-tan-^n-a+b-sec-^m

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [365]. This is test number [120].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.73 (364)	% 0.27 (1)
Mathematica	% 94.25 (344)	% 5.75 (21)
Maple	% 90.68 (331)	% 9.32 (34)
Maxima	% 58.36 (213)	% 41.64 (152)
Fricas	% 71.23 (260)	% 28.77 (105)
Sympy	% 10.96 (40)	% 89.04 (325)
Giac	% 68.77 (251)	% 31.23 (114)
Mupad	% 49.59 (181)	% 50.41 (184)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

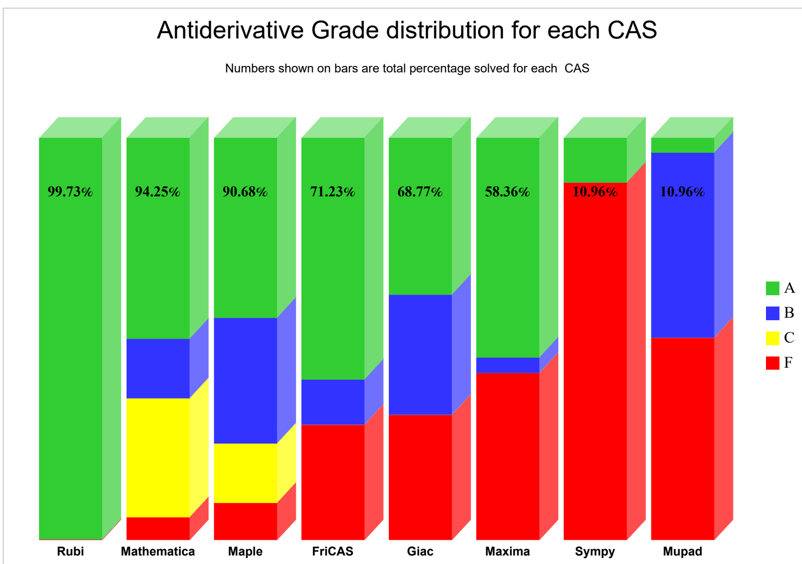
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

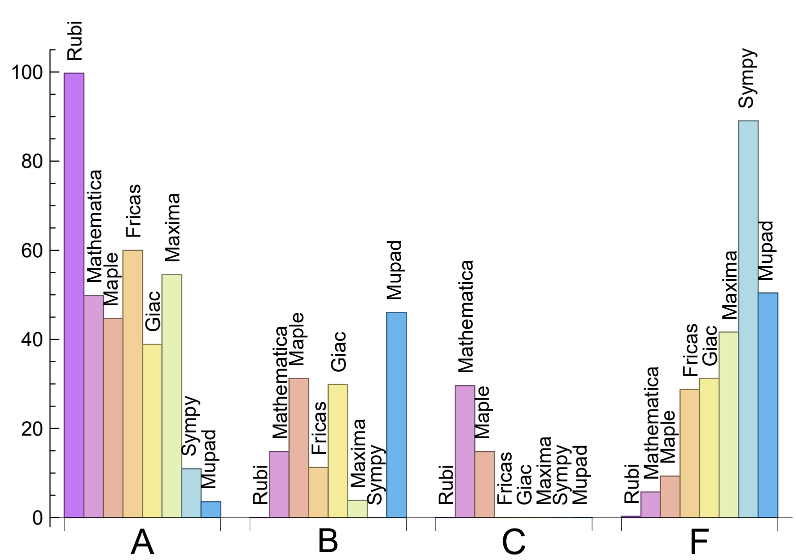
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.73	0.00	0.00	0.27
Mathematica	49.86	14.79	29.59	5.75
Maple	44.66	31.23	14.79	9.32
Maxima	54.52	3.84	0.00	41.64
Fricas	60.00	11.23	0.00	28.77
Sympy	10.96	0.00	0.00	89.04
Giac	38.90	29.86	0.00	31.23
Mupad	3.56	46.03	0.00	50.41

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	21	95.24 %	4.76 %	0.00 %
Maple	34	100.00 %	0.00 %	0.00 %
Maxima	152	64.47 %	28.95 %	6.58 %
Fricas	105	39.05 %	60.95 %	0.00 %
Sympy	325	82.46 %	17.54 %	0.00 %
Giac	114	92.98 %	0.88 %	6.14 %
Mupad	184	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

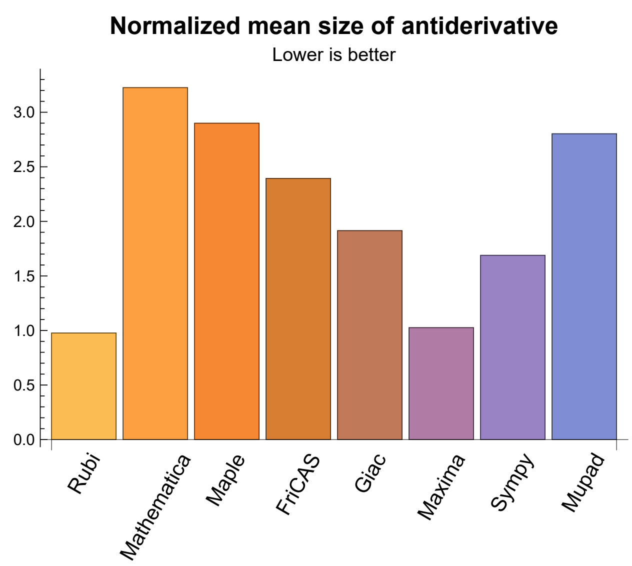
1.3 Performance

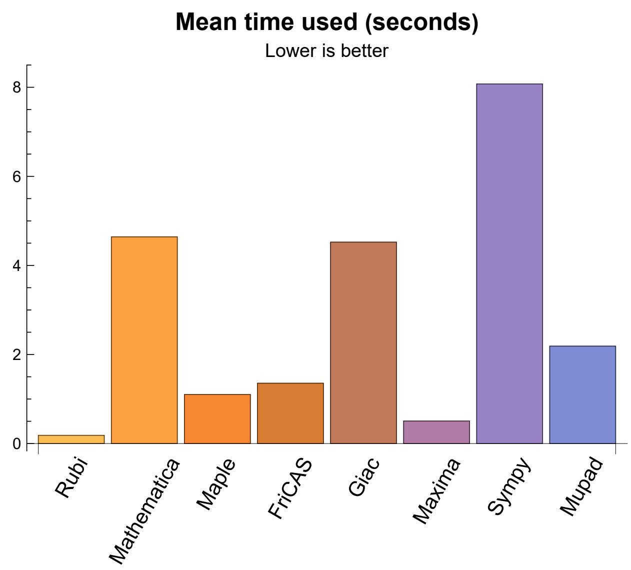
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	169.12	0.98	129.50	1.00
Mathematica	4.64	569.06	3.23	138.00	1.04
Maple	1.10	612.34	2.90	226.00	1.81
Maxima	0.51	116.34	1.03	105.00	0.95
Fricas	1.35	335.08	2.39	178.00	1.58
Sympy	8.08	131.63	1.69	97.00	1.36
Giac	4.52	223.54	1.91	173.00	1.63
Mupad	2.19	416.45	2.80	118.00	1.45

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {65, 107, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 133, 134, 143, 144, 145, 146, 156, 157, 158, 169, 170, 177, 178, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 201, 202, 203, 204, 205, 206, 207, 209, 213, 226, 227, 229, 230, 231, 232, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 286, 299, 312, 316, 317, 322, 330, 331, 334, 338, 339, 340, 341, 342, 347, 357}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
           leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

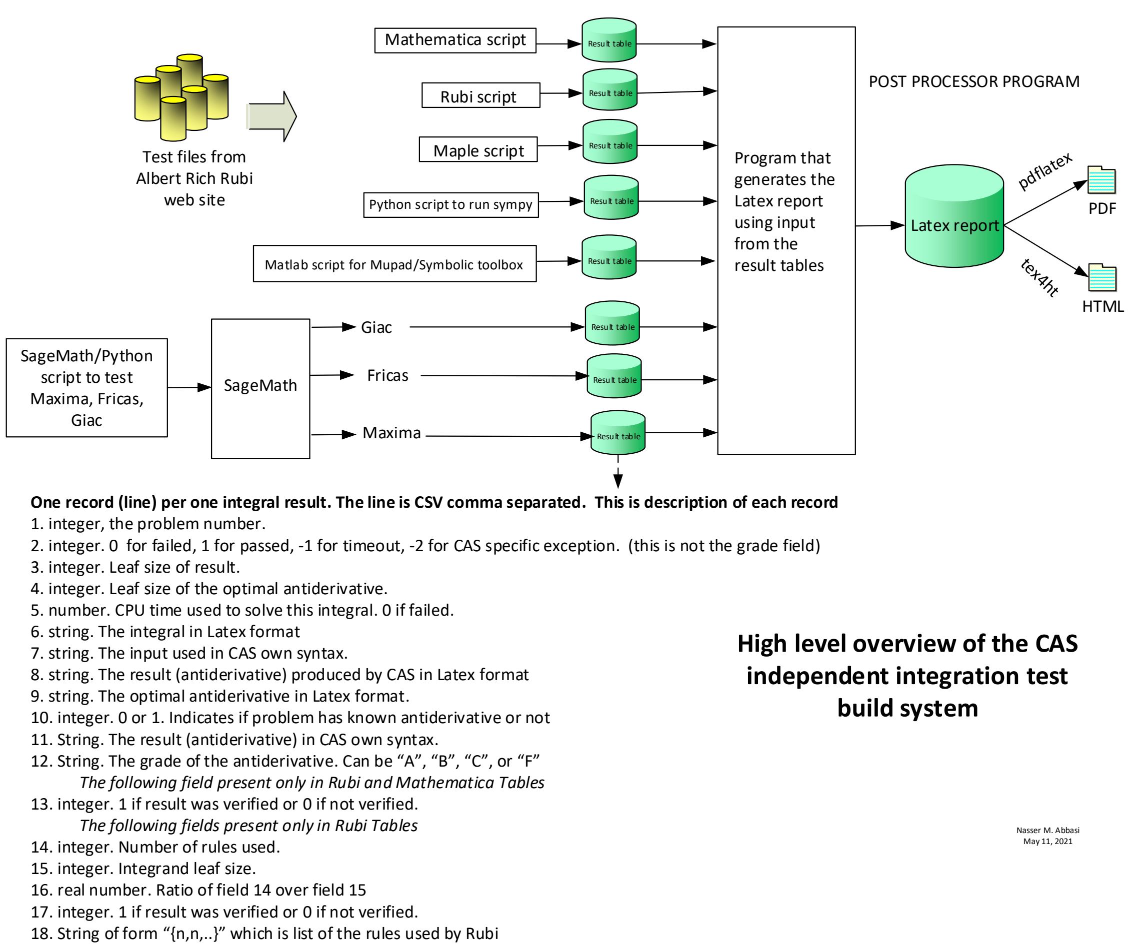
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { }

C grade: { }

F grade: { 347 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 101, 103, 135, 136, 137, 138, 141, 142, 147, 148, 149, 150, 151, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 179, 183, 184, 190, 195, 196, 211, 219, 220, 221, 222, 223, 224, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 271, 272, 273, 274, 275, 276, 277, 278, 279, 282, 283, 284, 285, 288, 289, 290, 291, 292, 296, 297, 301, 302, 303, 304, 309, 310, 311, 318, 319, 324, 325, 326, 333, 335, 336, 340, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 29, 30, 31, 35, 36, 49, 50, 54, 64, 65, 66, 67, 69, 70, 79, 80, 81, 84, 85, 97, 98, 99, 100, 102, 201, 226, 227, 229, 230, 231, 232, 280, 281, 286, 287, 294, 295, 298, 299, 300, 307, 308, 320, 321, 322, 327, 328, 329, 330, 331, 337, 338, 339, 347 }

C grade: { 14, 15, 16, 17, 18, 32, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 133, 134, 139, 140, 143, 144, 145, 146, 152, 157, 158, 169, 170, 174, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 188, 189, 191, 192, 193, 194, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 209, 210, 213, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 267, 268, 269, 270, 293, 305, 306, 312, 313, 314, 315, 316, 317, 323, 334, 341, 342, 343 }

F grade: { 127, 128, 129, 130, 132, 208, 212, 214, 215, 216, 217, 218, 225, 228, 250, 251, 252, 253, 254, 255, 332 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 137, 138, 149, 150, 156, 161, 162, 173, 184, 185, 190, 197, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 297, 298, 300, 301, 302, 303, 304, 305, 306, 309, 310, 311, 320, 324, 328, 333, 337, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 20, 22, 38, 52, 53, 54, 55, 64, 65, 66, 67, 79, 80, 81, 96, 97, 98, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 152, 153, 154, 155, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 256, 257, 258, 294, 295, 296, 299, 307, 308, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 341, 342, 343 }

C grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

F grade: { 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 135, 136, 137, 147, 148, 149, 159, 160, 161, 171, 172, 173, 183, 184, 185, 197, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 299, 300, 301, 302, 303, 304, 305, 318, 319, 320, 326, 327, 328, 335, 336, 337, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 64, 65, 66, 67, 79, 80, 81, 96, 97, 98, 99, 195, 196, 306 }

C grade: { }

F grade: { 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 294, 295, 296, 297, 298, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 153, 154, 155, 157, 158, 159, 160, 161, 162, 165, 166, 167, 169, 170, 171, 172, 176, 177, 178, 179, 180, 181, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 308, 310, 318, 319, 320, 326, 327, 335, 336, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 9, 13, 17, 18, 28, 46, 150, 151, 152, 156, 163, 164, 168, 173, 174, 175, 185, 186, 187, 188, 196, 197, 198, 199, 200, 266, 293, 298, 305, 306, 307, 309, 311, 321, 322, 328, 329, 330, 337, 338, 339 }

C grade: { }

F grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 323, 324, 325, 331, 332, 333, 334, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 19, 20, 21, 22, 23, 37, 38, 39, 40, 41, 60, 75, 90, 91, 256, 257, 258, 259, 271, 272, 273, 274, 275, 290, 348, 349, 350, 351, 352, 358, 359, 360, 363, 364, 365 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357, 361, 362 }

2.1.7 Giac

A grade: { 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 171, 174, 175, 176, 180, 181, 182, 183, 186, 187, 188, 191, 192, 193, 194, 197, 198, 200, 202, 203, 204, 205, 206, 260, 267, 276, 279, 282, 309, 310, 311, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 13, 20, 21, 22, 23, 25, 38, 39, 40, 41, 42, 43, 44, 57, 58, 59, 66, 67, 71, 72, 73, 81, 87, 88, 89, 143, 144, 145, 156, 157, 158, 168, 169, 170, 172, 173, 177, 178, 179, 184, 185, 189, 190, 195, 196, 199, 201, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 301, 302, 303, 304, 305, 306, 307, 308, 318, 319, 320, 322, 326, 327, 328, 336 }

C grade: { }

F grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 300, 312, 313, 314, 315, 316, 317, 321, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.8 Mupad

A grade: { 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 137, 149, 161, 173, 185, 197, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 320, 328, 337 }

C grade: { }

F grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	134	273	116	123	184	293	259
normalized size	1	1.00	0.89	1.81	0.77	0.81	1.22	1.94	1.72
time (sec)	N/A	0.072	0.479	0.852	0.803	0.709	21.438	24.072	5.133

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	106	216	94	101	148	247	204
normalized size	1	1.00	0.90	1.83	0.80	0.86	1.25	2.09	1.73
time (sec)	N/A	0.062	0.467	0.859	0.350	0.797	8.462	7.977	5.693

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	161	72	79	112	201	151
normalized size	1	1.00	0.94	1.85	0.83	0.91	1.29	2.31	1.74
time (sec)	N/A	0.049	0.366	0.737	0.609	0.564	3.063	3.088	5.734

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	104	50	57	76	155	96
normalized size	1	1.00	0.96	1.82	0.88	1.00	1.33	2.72	1.68
time (sec)	N/A	0.039	0.115	0.729	0.483	0.710	0.916	0.955	1.993

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	26	34	37	106	40
normalized size	1	1.00	1.00	1.00	1.04	1.36	1.48	4.24	1.60
time (sec)	N/A	0.019	0.020	0.282	0.789	0.720	0.260	0.499	1.172

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	29	29	14	16	0	58	34
normalized size	1	1.00	1.81	1.81	0.88	1.00	0.00	3.62	2.12
time (sec)	N/A	0.020	0.025	0.442	0.515	0.671	0.000	0.201	1.239

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	114	60	42	69	0	103	46
normalized size	1	1.00	2.00	1.05	0.74	1.21	0.00	1.81	0.81
time (sec)	N/A	0.044	0.783	0.658	0.718	0.612	0.000	0.246	1.275

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	127	93	86	150	0	149	88
normalized size	1	1.00	1.34	0.98	0.91	1.58	0.00	1.57	0.93
time (sec)	N/A	0.064	0.544	0.546	0.588	0.615	0.000	0.301	1.178

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	165	124	126	241	0	197	118
normalized size	1	1.00	1.24	0.93	0.95	1.81	0.00	1.48	0.89
time (sec)	N/A	0.082	0.415	0.612	0.569	0.616	0.000	0.422	1.227

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	115	227	164	156	0	174	242
normalized size	1	1.00	0.89	1.76	1.27	1.21	0.00	1.35	1.88
time (sec)	N/A	0.129	1.894	0.543	0.527	0.758	0.000	12.224	2.469

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	95	178	134	134	0	146	188
normalized size	1	1.00	0.93	1.75	1.31	1.31	0.00	1.43	1.84
time (sec)	N/A	0.094	1.247	0.510	0.817	0.695	0.000	8.792	2.383

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	127	102	112	0	118	134
normalized size	1	1.00	1.03	1.74	1.40	1.53	0.00	1.62	1.84
time (sec)	N/A	0.062	0.440	0.481	0.894	0.805	0.000	1.543	1.852

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	60	78	65	87	0	88	80
normalized size	1	1.00	1.33	1.73	1.44	1.93	0.00	1.96	1.78
time (sec)	N/A	0.034	0.030	0.464	0.687	0.828	0.000	0.902	1.139

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	43	35	31	33	0	26	19
normalized size	1	1.00	1.65	1.35	1.19	1.27	0.00	1.00	0.73
time (sec)	N/A	0.024	0.027	0.476	1.222	0.941	0.000	0.231	1.073

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	62	86	59	72	0	56	53
normalized size	1	1.00	1.13	1.56	1.07	1.31	0.00	1.02	0.96
time (sec)	N/A	0.052	0.038	0.788	1.159	0.635	0.000	0.516	1.200

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	129	79	139	0	83	156
normalized size	1	1.00	0.94	1.54	0.94	1.65	0.00	0.99	1.86
time (sec)	N/A	0.081	0.056	0.812	0.620	0.509	0.000	0.305	1.449

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	92	162	100	210	0	113	204
normalized size	1	1.00	0.83	1.46	0.90	1.89	0.00	1.02	1.84
time (sec)	N/A	0.114	0.055	0.951	1.029	0.738	0.000	0.345	1.983

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	111	205	119	279	0	140	252
normalized size	1	1.00	0.79	1.46	0.85	1.99	0.00	1.00	1.80
time (sec)	N/A	0.145	0.082	0.973	0.823	0.893	0.000	0.507	3.175

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	140	327	149	156	314	342	308
normalized size	1	1.00	0.73	1.70	0.78	0.81	1.64	1.78	1.60
time (sec)	N/A	0.099	0.509	0.855	0.320	0.873	33.746	25.568	5.142

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	264	110	117	252	292	249
normalized size	1	1.00	0.83	2.00	0.83	0.89	1.91	2.21	1.89
time (sec)	N/A	0.079	0.306	0.782	0.427	0.727	14.177	10.996	4.819

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	125	203	97	104	189	242	192
normalized size	1	1.00	1.04	1.69	0.81	0.87	1.58	2.02	1.60
time (sec)	N/A	0.073	0.417	0.736	0.697	0.748	5.335	3.347	5.052

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	83	140	58	65	126	192	133
normalized size	1	1.00	1.28	2.15	0.89	1.00	1.94	2.95	2.05
time (sec)	N/A	0.056	0.210	0.719	0.355	0.632	1.815	1.427	3.870

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	46	43	52	60	142	76
normalized size	1	1.00	1.06	0.96	0.90	1.08	1.25	2.96	1.58
time (sec)	N/A	0.036	0.146	0.297	0.318	0.622	0.480	0.394	1.214

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	29	34	31	35	0	64	36
normalized size	1	1.00	0.83	0.97	0.89	1.00	0.00	1.83	1.03
time (sec)	N/A	0.040	0.038	0.532	0.392	0.774	0.000	0.257	1.255

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	51	34	48	0	111	50
normalized size	1	1.00	1.40	1.28	0.85	1.20	0.00	2.78	1.25
time (sec)	N/A	0.050	0.071	0.680	0.529	0.669	0.000	0.444	1.229

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	86	87	72	122	0	138	62
normalized size	1	1.00	1.01	1.02	0.85	1.44	0.00	1.62	0.73
time (sec)	N/A	0.067	0.268	0.582	0.644	0.583	0.000	0.560	1.260

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	114	122	109	191	0	186	113
normalized size	1	1.00	0.90	0.96	0.86	1.50	0.00	1.46	0.89
time (sec)	N/A	0.087	0.250	0.628	0.560	0.702	0.000	0.424	1.269

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	146	159	165	322	0	238	149
normalized size	1	1.00	0.86	0.94	0.98	1.91	0.00	1.41	0.88
time (sec)	N/A	0.111	0.345	0.705	0.454	0.604	0.000	1.377	1.589

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	337	226	151	165	0	180	234
normalized size	1	1.00	2.09	1.40	0.94	1.02	0.00	1.12	1.45
time (sec)	N/A	0.179	1.580	0.540	0.666	0.669	0.000	6.010	2.648

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	558	169	119	139	0	148	174
normalized size	1	1.00	4.69	1.42	1.00	1.17	0.00	1.24	1.46
time (sec)	N/A	0.139	5.632	0.519	0.637	0.609	0.000	1.816	1.953

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	773	112	83	111	0	99	101
normalized size	1	1.00	10.74	1.56	1.15	1.54	0.00	1.38	1.40
time (sec)	N/A	0.109	6.330	0.493	0.820	0.726	0.000	2.329	1.208

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	50	48	42	0	31	24
normalized size	1	1.00	1.31	1.43	1.37	1.20	0.00	0.89	0.69
time (sec)	N/A	0.072	0.037	0.725	0.576	0.682	0.000	0.248	1.078

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	112	112	77	81	0	50	39
normalized size	1	1.00	1.62	1.62	1.12	1.17	0.00	0.72	0.57
time (sec)	N/A	0.111	0.271	0.890	0.717	0.511	0.000	0.434	1.096

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	194	155	97	118	0	80	78
normalized size	1	1.00	1.81	1.45	0.91	1.10	0.00	0.75	0.73
time (sec)	N/A	0.128	0.812	1.038	0.543	0.716	0.000	0.348	1.259

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	312	188	117	173	0	112	182
normalized size	1	1.00	2.24	1.35	0.84	1.24	0.00	0.81	1.31
time (sec)	N/A	0.141	1.152	1.103	0.834	0.590	0.000	0.392	1.755

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	428	231	137	274	0	145	230
normalized size	1	1.00	2.39	1.29	0.77	1.53	0.00	0.81	1.28
time (sec)	N/A	0.155	1.987	1.029	0.757	0.613	0.000	0.434	2.778

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	214	351	162	169	439	367	337
normalized size	1	1.00	1.02	1.67	0.77	0.80	2.09	1.75	1.60
time (sec)	N/A	0.103	0.883	0.868	0.655	0.873	51.795	19.684	5.263

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	110	288	110	117	350	317	278
normalized size	1	1.00	0.80	2.10	0.80	0.85	2.55	2.31	2.03
time (sec)	N/A	0.077	0.438	0.806	0.363	0.832	22.571	28.275	5.169

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	140	227	110	117	255	267	221
normalized size	1	1.00	1.01	1.64	0.80	0.85	1.85	1.93	1.60
time (sec)	N/A	0.077	0.424	0.792	0.619	0.485	9.008	4.000	5.483

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	92	164	84	91	165	217	162
normalized size	1	1.00	0.93	1.66	0.85	0.92	1.67	2.19	1.64
time (sec)	N/A	0.067	0.301	0.784	0.466	0.487	3.239	1.521	5.546

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	62	58	65	76	167	105
normalized size	1	1.00	0.97	0.94	0.88	0.98	1.15	2.53	1.59
time (sec)	N/A	0.039	0.162	0.304	0.676	0.659	0.923	0.505	1.940

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	36	47	43	61	0	145	86
normalized size	1	1.00	0.75	0.98	0.90	1.27	0.00	3.02	1.79
time (sec)	N/A	0.046	0.091	0.514	0.598	0.786	0.000	0.318	1.201

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	46	51	34	50	0	109	48
normalized size	1	1.00	1.15	1.28	0.85	1.25	0.00	2.72	1.20
time (sec)	N/A	0.050	0.133	0.728	0.612	0.731	0.000	0.366	1.234

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	72	68	59	82	0	138	78
normalized size	1	1.00	1.18	1.11	0.97	1.34	0.00	2.26	1.28
time (sec)	N/A	0.059	0.169	0.617	0.409	0.857	0.000	1.012	1.220

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	102	104	96	178	0	165	94
normalized size	1	1.00	0.95	0.97	0.90	1.66	0.00	1.54	0.88
time (sec)	N/A	0.076	0.679	0.625	0.524	1.172	0.000	1.483	1.291

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	130	141	142	272	0	213	130
normalized size	1	1.00	0.87	0.95	0.95	1.83	0.00	1.43	0.87
time (sec)	N/A	0.098	0.399	0.750	0.373	0.759	0.000	0.507	1.204

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	363	250	262	178	0	196	263
normalized size	1	1.00	1.53	1.05	1.11	0.75	0.00	0.83	1.11
time (sec)	N/A	0.299	2.178	0.585	0.822	0.824	0.000	5.201	2.429

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	303	193	210	152	0	164	203
normalized size	1	1.00	1.79	1.14	1.24	0.90	0.00	0.97	1.20
time (sec)	N/A	0.224	1.292	0.568	0.455	0.822	0.000	2.588	2.439

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	230	137	147	113	0	132	146
normalized size	1	1.00	2.35	1.40	1.50	1.15	0.00	1.35	1.49
time (sec)	N/A	0.158	0.849	0.544	0.452	0.559	0.000	1.045	1.909

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	109	68	85	84	0	66	35
normalized size	1	1.00	2.22	1.39	1.73	1.71	0.00	1.35	0.71
time (sec)	N/A	0.097	0.229	0.833	0.434	0.732	0.000	0.333	1.200

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	112	125	90	82	0	50	39
normalized size	1	1.00	1.62	1.81	1.30	1.19	0.00	0.72	0.57
time (sec)	N/A	0.132	0.241	0.854	0.431	0.750	0.000	0.319	1.198

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	112	232	122	118	0	66	62
normalized size	1	1.00	1.05	2.17	1.14	1.10	0.00	0.62	0.58
time (sec)	N/A	0.164	0.674	1.066	0.437	0.749	0.000	0.364	1.446

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	252	293	152	160	0	96	91
normalized size	1	1.00	1.79	2.08	1.08	1.13	0.00	0.68	0.65
time (sec)	N/A	0.178	0.983	1.183	0.450	0.486	0.000	0.499	1.700

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	370	364	182	235	0	128	206
normalized size	1	1.00	2.07	2.03	1.02	1.31	0.00	0.72	1.15
time (sec)	N/A	0.199	1.415	1.204	0.454	0.549	0.000	0.534	1.944

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	268	425	212	314	0	161	254
normalized size	1	1.00	1.26	2.00	1.00	1.47	0.00	0.76	1.19
time (sec)	N/A	0.221	6.047	1.273	0.442	0.523	0.000	0.652	3.032

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	137	125	90	95	0	245	208
normalized size	1	1.00	1.01	0.93	0.67	0.70	0.00	1.81	1.54
time (sec)	N/A	0.078	0.562	0.626	0.319	0.503	0.000	17.820	5.142

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	103	93	70	75	0	201	153
normalized size	1	1.00	1.06	0.96	0.72	0.77	0.00	2.07	1.58
time (sec)	N/A	0.068	0.273	0.558	0.326	0.499	0.000	6.786	6.021

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	62	50	55	0	157	99
normalized size	1	1.00	0.98	0.94	0.76	0.83	0.00	2.38	1.50
time (sec)	N/A	0.057	0.194	0.536	0.320	0.491	0.000	4.200	2.193

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	30	28	33	0	111	44
normalized size	1	1.00	0.75	1.07	1.00	1.18	0.00	3.96	1.57
time (sec)	N/A	0.048	0.072	0.313	0.317	0.496	0.000	1.121	1.224

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	33	17	19	41	31	21
normalized size	1	1.00	1.12	1.94	1.00	1.12	2.41	1.82	1.24
time (sec)	N/A	0.026	0.018	0.115	0.321	0.764	3.805	0.282	1.211

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	67	54	47	60	0	86	49
normalized size	1	1.00	1.10	0.89	0.77	0.98	0.00	1.41	0.80
time (sec)	N/A	0.056	0.120	0.613	0.324	0.735	0.000	1.238	1.252

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	90	91	139	0	157	76
normalized size	1	1.00	1.04	0.87	0.88	1.35	0.00	1.52	0.74
time (sec)	N/A	0.076	0.621	0.805	0.689	0.707	0.000	0.253	1.343

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	135	126	130	217	0	211	132
normalized size	1	1.00	0.93	0.87	0.90	1.50	0.00	1.46	0.91
time (sec)	N/A	0.098	0.538	0.690	0.328	0.894	0.000	0.335	1.313

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	301	312	329	127	0	149	193
normalized size	1	1.00	2.87	2.97	3.13	1.21	0.00	1.42	1.84
time (sec)	N/A	0.144	0.899	0.538	0.787	0.575	0.000	9.294	2.524

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	893	228	247	107	0	123	139
normalized size	1	1.00	11.45	2.92	3.17	1.37	0.00	1.58	1.78
time (sec)	N/A	0.109	6.450	0.530	0.541	0.807	0.000	4.102	1.997

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	241	144	163	86	0	96	83
normalized size	1	1.00	4.92	2.94	3.33	1.76	0.00	1.96	1.69
time (sec)	N/A	0.077	0.878	0.499	0.780	0.615	0.000	2.267	1.287

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	60	59	78	35	0	50	25
normalized size	1	1.00	2.86	2.81	3.71	1.67	0.00	2.38	1.19
time (sec)	N/A	0.050	0.094	0.324	0.641	0.828	0.000	0.565	1.108

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	100	74	93	64	0	66	65
normalized size	1	1.00	1.64	1.21	1.52	1.05	0.00	1.08	1.07
time (sec)	N/A	0.097	0.799	0.616	0.603	0.490	0.000	0.240	1.293

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	254	113	137	134	0	98	158
normalized size	1	1.00	2.89	1.28	1.56	1.52	0.00	1.11	1.80
time (sec)	N/A	0.127	0.901	0.659	0.965	0.458	0.000	0.669	1.434

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	359	150	177	198	0	127	206
normalized size	1	1.00	3.07	1.28	1.51	1.69	0.00	1.09	1.76
time (sec)	N/A	0.162	1.097	0.771	0.590	0.466	0.000	0.394	2.036

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	125	110	80	85	0	223	193
normalized size	1	1.00	1.04	0.92	0.67	0.71	0.00	1.86	1.61
time (sec)	N/A	0.074	0.526	0.627	0.333	0.500	0.000	26.201	5.181

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	83	63	50	55	0	180	135
normalized size	1	1.00	1.28	0.97	0.77	0.85	0.00	2.77	2.08
time (sec)	N/A	0.058	0.200	0.605	0.439	0.486	0.000	8.972	3.869

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	46	40	45	0	136	77
normalized size	1	1.00	1.06	0.96	0.83	0.94	0.00	2.83	1.60
time (sec)	N/A	0.050	0.123	0.545	0.873	0.475	0.000	8.112	1.350

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	34	31	31	0	33	22
normalized size	1	1.00	0.91	1.03	0.94	0.94	0.00	1.00	0.67
time (sec)	N/A	0.045	0.068	0.515	0.326	0.465	0.000	0.982	1.388

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	56	50	35	43	177	57	35
normalized size	1	1.00	1.56	1.39	0.97	1.19	4.92	1.58	0.97
time (sec)	N/A	0.035	0.133	0.189	0.410	0.483	19.471	0.467	1.130

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	83	72	74	106	0	117	62
normalized size	1	1.00	1.02	0.89	0.91	1.31	0.00	1.44	0.77
time (sec)	N/A	0.061	0.191	0.699	0.322	0.492	0.000	0.257	1.263

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	108	110	162	0	186	89
normalized size	1	1.00	0.98	0.88	0.89	1.32	0.00	1.51	0.72
time (sec)	N/A	0.087	0.393	0.936	0.326	0.538	0.000	0.353	1.366

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	154	144	167	283	0	236	151
normalized size	1	1.00	0.93	0.87	1.01	1.72	0.00	1.43	0.92
time (sec)	N/A	0.111	0.820	1.016	0.454	0.605	0.000	0.413	1.245

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	495	269	301	117	0	136	179
normalized size	1	1.00	4.16	2.26	2.53	0.98	0.00	1.14	1.50
time (sec)	N/A	0.190	5.905	0.661	0.607	0.760	0.000	13.722	2.258

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	767	185	196	97	0	99	111
normalized size	1	1.00	10.65	2.57	2.72	1.35	0.00	1.38	1.54
time (sec)	N/A	0.149	6.306	0.602	0.821	0.763	0.000	4.132	1.389

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	177	102	123	66	0	79	61
normalized size	1	1.00	5.21	3.00	3.62	1.94	0.00	2.32	1.79
time (sec)	N/A	0.065	0.523	0.404	0.513	0.571	0.000	2.656	1.192

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	35	42	37	49	42	0	29	22
normalized size	1	1.06	1.27	1.12	1.48	1.27	0.00	0.88	0.67
time (sec)	N/A	0.112	0.020	0.476	0.916	0.774	0.000	2.820	1.096

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	149	94	113	106	0	84	78
normalized size	1	1.00	1.39	0.88	1.06	0.99	0.00	0.79	0.73
time (sec)	N/A	0.174	1.348	0.703	0.591	0.666	0.000	0.355	1.395

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	314	132	157	154	0	114	182
normalized size	1	1.00	2.26	0.95	1.13	1.11	0.00	0.82	1.31
time (sec)	N/A	0.191	1.067	0.825	0.604	0.771	0.000	0.313	1.645

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	802	170	197	250	0	144	230
normalized size	1	1.00	4.48	0.95	1.10	1.40	0.00	0.80	1.28
time (sec)	N/A	0.208	6.571	0.852	0.885	0.698	0.000	0.453	2.507

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	140	127	90	95	0	246	225
normalized size	1	1.00	1.02	0.93	0.66	0.69	0.00	1.80	1.64
time (sec)	N/A	0.077	0.345	0.799	0.417	0.658	0.000	93.139	5.287

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	93	70	75	0	202	167
normalized size	1	1.00	0.94	0.94	0.71	0.76	0.00	2.04	1.69
time (sec)	N/A	0.067	0.366	0.648	0.327	0.711	0.000	20.747	5.916

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	64	63	50	55	0	158	109
normalized size	1	1.00	0.98	0.97	0.77	0.85	0.00	2.43	1.68
time (sec)	N/A	0.057	0.181	0.558	0.314	0.999	0.000	9.285	2.033

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	46	45	53	0	112	72
normalized size	1	1.00	0.78	1.00	0.98	1.15	0.00	2.43	1.57
time (sec)	N/A	0.054	0.116	0.521	0.332	0.658	0.000	5.494	1.269

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	51	36	42	457	56	36
normalized size	1	1.00	0.94	1.46	1.03	1.20	13.06	1.60	1.03
time (sec)	N/A	0.051	0.059	0.630	0.616	0.821	23.815	1.901	1.171

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	79	68	60	76	411	87	48
normalized size	1	1.00	1.41	1.21	1.07	1.36	7.34	1.55	0.86
time (sec)	N/A	0.041	0.131	0.303	0.498	0.697	23.677	0.514	1.154

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	90	98	151	0	143	75
normalized size	1	1.00	0.96	0.89	0.97	1.50	0.00	1.42	0.74
time (sec)	N/A	0.069	0.326	0.815	0.372	0.661	0.000	0.335	1.245

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	140	126	146	240	0	212	102
normalized size	1	1.00	0.98	0.88	1.02	1.68	0.00	1.48	0.71
time (sec)	N/A	0.097	0.639	0.925	0.439	0.669	0.000	0.394	1.399

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	169	162	188	317	0	261	170
normalized size	1	1.00	0.91	0.88	1.02	1.71	0.00	1.41	0.92
time (sec)	N/A	0.124	1.242	0.864	1.396	0.814	0.000	1.910	1.328

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	362	396	429	147	0	175	265
normalized size	1	1.00	1.53	1.67	1.81	0.62	0.00	0.74	1.12
time (sec)	N/A	0.363	1.339	0.848	0.624	0.601	0.000	116.124	2.564

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	303	312	343	127	0	149	208
normalized size	1	1.00	1.79	1.85	2.03	0.75	0.00	0.88	1.23
time (sec)	N/A	0.268	0.919	0.820	0.702	0.538	0.000	64.838	2.355

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	230	228	257	97	0	123	148
normalized size	1	1.00	2.32	2.30	2.60	0.98	0.00	1.24	1.49
time (sec)	N/A	0.204	0.757	0.621	0.521	0.783	0.000	11.182	1.934

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	241	144	171	87	0	97	92
normalized size	1	1.00	3.65	2.18	2.59	1.32	0.00	1.47	1.39
time (sec)	N/A	0.091	0.962	0.600	0.447	0.619	0.000	7.541	1.309

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	117	76	98	83	0	63	37
normalized size	1	1.04	2.54	1.65	2.13	1.80	0.00	1.37	0.80
time (sec)	N/A	0.141	0.268	0.600	0.514	0.785	0.000	3.669	1.161

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	71	125	56	72	80	0	50	35
normalized size	1	1.18	2.08	0.93	1.20	1.33	0.00	0.83	0.58
time (sec)	N/A	0.173	0.403	0.584	0.448	0.598	0.000	0.785	1.161

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	252	113	133	142	0	99	91
normalized size	1	1.00	1.76	0.79	0.93	0.99	0.00	0.69	0.64
time (sec)	N/A	0.236	1.233	0.805	0.434	0.752	0.000	0.486	1.703

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	366	151	177	216	0	131	205
normalized size	1	1.00	2.07	0.85	1.00	1.22	0.00	0.74	1.16
time (sec)	N/A	0.253	1.184	0.882	0.430	1.208	0.000	0.469	2.075

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	394	189	218	282	0	160	254
normalized size	1	1.00	1.83	0.88	1.01	1.31	0.00	0.74	1.18
time (sec)	N/A	0.278	3.725	1.017	0.559	1.158	0.000	1.468	3.237

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	186	1495	177	0	0	0	-1
normalized size	1	1.00	0.60	4.82	0.57	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	2.387	1.900	0.463	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	214	688	171	0	0	0	-1
normalized size	1	1.00	0.76	2.44	0.61	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.274	2.107	1.799	0.497	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	182	1409	154	0	0	0	-1
normalized size	1	1.00	0.67	5.18	0.57	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	1.571	1.968	0.464	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	220	284	156	0	0	0	-1
normalized size	1	1.00	0.90	1.16	0.64	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	1.764	1.803	0.450	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	196	1390	168	0	0	0	-1
normalized size	1	1.00	0.64	4.56	0.55	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	2.663	1.776	0.437	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	200	648	169	0	0	0	-1
normalized size	1	1.00	0.71	2.30	0.60	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	1.526	1.697	0.432	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	254	1427	196	0	0	0	-1
normalized size	1	1.00	0.73	4.12	0.57	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	2.558	1.764	0.431	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	117	1518	200	0	0	0	-1
normalized size	1	1.00	0.32	4.15	0.55	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.433	6.212	1.959	0.608	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	257	721	194	0	0	0	-1
normalized size	1	1.00	0.77	2.15	0.58	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.385	11.887	1.946	0.468	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	106	1480	177	0	0	0	-1
normalized size	1	1.00	0.34	4.79	0.57	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	1.269	2.038	0.455	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	220	653	178	0	0	0	-1
normalized size	1	1.00	0.79	2.35	0.64	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	2.467	1.995	0.457	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	238	1392	190	0	0	0	-1
normalized size	1	1.00	0.77	4.49	0.61	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	7.591	1.816	0.483	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	224	650	191	0	0	0	-1
normalized size	1	1.00	0.71	2.06	0.60	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.386	5.160	1.808	0.440	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	2820	1429	218	0	0	0	-1
normalized size	1	1.00	7.62	3.86	0.59	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	14.276	1.899	0.437	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	332	734	0	0	0	0	-1
normalized size	1	1.00	1.01	2.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.419	21.943	1.733	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	129	1505	0	0	0	0	-1
normalized size	1	1.00	0.40	4.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.394	13.012	1.689	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	271	698	0	0	0	0	-1
normalized size	1	1.00	0.92	2.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	54.601	1.699	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	105	1419	0	0	0	0	-1
normalized size	1	1.00	0.37	4.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	5.508	2.087	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	1211	319	0	0	0	0	-1
normalized size	1	1.00	4.71	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	13.206	1.829	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	2715	352	0	0	0	0	-1
normalized size	1	1.00	8.62	1.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.375	8.089	1.766	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	1253	1269	0	0	0	0	-1
normalized size	1	1.00	4.32	4.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	9.068	1.731	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	180	2113	0	0	0	0	-1
normalized size	1	1.00	0.50	5.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.452	13.966	1.618	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	1299	1896	0	0	0	0	-1
normalized size	1	1.00	3.96	5.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	9.574	1.682	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	0	1518	0	0	0	0	-1
normalized size	1	1.00	0.00	4.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.492	13.807	1.760	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	0	721	0	0	0	0	-1
normalized size	1	1.00	0.00	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.458	74.213	1.692	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	0	1480	0	0	0	0	-1
normalized size	1	1.00	0.00	4.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.411	4.601	1.734	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	0	653	0	0	0	0	-1
normalized size	1	1.00	0.00	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	4.116	1.778	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	812	360	0	0	0	0	-1
normalized size	1	1.00	2.62	1.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	6.710	1.715	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	0	1267	0	0	0	0	-1
normalized size	1	1.00	0.00	4.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	16.519	1.842	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	2792	2117	0	0	0	0	-1
normalized size	1	1.00	7.69	5.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.525	8.450	1.875	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	1281	1896	0	0	0	0	-1
normalized size	1	1.00	3.51	5.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	8.914	1.901	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	102	359	145	299	0	193	-1
normalized size	1	1.00	0.69	2.44	0.99	2.03	0.00	1.31	-0.01
time (sec)	N/A	0.115	0.648	1.377	0.625	1.022	0.000	4.820	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	80	221	107	259	0	152	-1
normalized size	1	1.00	0.81	2.23	1.08	2.62	0.00	1.54	-0.01
time (sec)	N/A	0.084	0.168	1.262	0.566	0.980	0.000	2.507	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	42	67	184	0	75	47
normalized size	1	1.00	1.18	0.82	1.31	3.61	0.00	1.47	0.92
time (sec)	N/A	0.045	0.048	0.239	0.582	1.084	0.000	0.733	1.481

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	72	98	0	242	0	88	-1
normalized size	1	1.00	0.99	1.34	0.00	3.32	0.00	1.21	-0.01
time (sec)	N/A	0.071	0.056	1.142	0.000	0.833	0.000	2.114	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	87	267	0	426	0	140	-1
normalized size	1	1.00	0.66	2.04	0.00	3.25	0.00	1.07	-0.01
time (sec)	N/A	0.118	0.297	1.376	0.000	1.126	0.000	0.787	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	102	407	0	529	0	201	-1
normalized size	1	1.00	0.53	2.11	0.00	2.74	0.00	1.04	-0.01
time (sec)	N/A	0.158	0.328	1.469	0.000	0.968	0.000	0.878	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	134	566	0	371	0	284	-1
normalized size	1	1.00	0.60	2.55	0.00	1.67	0.00	1.28	-0.00
time (sec)	N/A	0.106	7.282	1.436	0.000	0.776	0.000	10.327	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	110	317	0	331	0	246	-1
normalized size	1	1.00	0.69	1.98	0.00	2.07	0.00	1.54	-0.01
time (sec)	N/A	0.095	5.913	1.360	0.000	0.692	0.000	5.441	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	226	210	0	283	0	208	-1
normalized size	1	1.00	2.35	2.19	0.00	2.95	0.00	2.17	-0.01
time (sec)	N/A	0.077	4.154	1.065	0.000	0.798	0.000	4.776	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	5502	188	0	422	0	236	-1
normalized size	1	1.00	50.48	1.72	0.00	3.87	0.00	2.17	-0.01
time (sec)	N/A	0.099	24.189	1.236	0.000	0.971	0.000	1.179	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	5552	381	0	547	0	365	-1
normalized size	1	1.00	28.33	1.94	0.00	2.79	0.00	1.86	-0.01
time (sec)	N/A	0.203	24.008	1.250	0.000	1.340	0.000	4.655	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	5594	573	0	646	0	476	-1
normalized size	1	1.00	19.98	2.05	0.00	2.31	0.00	1.70	-0.00
time (sec)	N/A	0.271	24.370	1.584	0.000	2.964	0.000	3.907	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	112	429	162	334	0	218	-1
normalized size	1	1.00	0.66	2.54	0.96	1.98	0.00	1.29	-0.01
time (sec)	N/A	0.136	0.553	1.207	0.462	1.693	0.000	4.765	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	92	291	124	290	0	173	-1
normalized size	1	1.00	0.76	2.40	1.02	2.40	0.00	1.43	-0.01
time (sec)	N/A	0.104	0.270	1.138	0.425	1.333	0.000	2.126	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	57	86	238	0	122	67
normalized size	1	1.00	0.96	0.78	1.18	3.26	0.00	1.67	0.92
time (sec)	N/A	0.055	0.150	0.174	0.454	0.695	0.000	1.134	1.460

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	72	101	0	243	0	89	-1
normalized size	1	1.00	0.99	1.38	0.00	3.33	0.00	1.22	-0.01
time (sec)	N/A	0.079	0.064	1.019	0.000	0.748	0.000	1.215	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	99	258	0	378	0	138	-1
normalized size	1	1.00	0.91	2.37	0.00	3.47	0.00	1.27	-0.01
time (sec)	N/A	0.107	0.362	1.181	0.000	0.680	0.000	0.908	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	104	502	0	589	0	211	-1
normalized size	1	1.00	0.61	2.94	0.00	3.44	0.00	1.23	-0.01
time (sec)	N/A	0.143	0.319	1.270	0.000	0.847	0.000	1.161	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	147	656	0	415	0	369	-1
normalized size	1	1.00	0.57	2.54	0.00	1.61	0.00	1.43	-0.00
time (sec)	N/A	0.122	8.622	1.356	0.000	0.720	0.000	6.468	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	123	407	0	371	0	310	-1
normalized size	1	1.00	0.63	2.10	0.00	1.91	0.00	1.60	-0.01
time (sec)	N/A	0.110	6.502	1.279	0.000	0.766	0.000	3.384	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	97	300	0	321	0	224	-1
normalized size	1	1.00	0.76	2.34	0.00	2.51	0.00	1.75	-0.01
time (sec)	N/A	0.093	5.592	1.024	0.000	0.577	0.000	5.496	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	102	113	0	264	0	197	-1
normalized size	1	1.00	1.59	1.77	0.00	4.12	0.00	3.08	-0.02
time (sec)	N/A	0.071	0.379	1.044	0.000	0.666	0.000	1.437	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	5542	372	0	531	0	369	-1
normalized size	1	1.00	38.49	2.58	0.00	3.69	0.00	2.56	-0.01
time (sec)	N/A	0.148	24.005	1.107	0.000	0.778	0.000	2.461	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	5582	720	0	708	0	519	-1
normalized size	1	1.00	24.70	3.19	0.00	3.13	0.00	2.30	-0.00
time (sec)	N/A	0.231	24.119	1.611	0.000	0.724	0.000	6.014	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	156	500	181	386	0	244	-1
normalized size	1	1.00	0.81	2.59	0.94	2.00	0.00	1.26	-0.01
time (sec)	N/A	0.146	0.778	1.319	0.599	0.730	0.000	9.541	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	102	362	143	334	0	198	-1
normalized size	1	1.00	0.70	2.50	0.99	2.30	0.00	1.37	-0.01
time (sec)	N/A	0.118	0.609	1.244	0.605	0.787	0.000	3.923	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	82	74	105	282	0	148	92
normalized size	1	1.00	0.85	0.76	1.08	2.91	0.00	1.53	0.95
time (sec)	N/A	0.065	0.236	0.194	0.638	1.257	0.000	1.729	1.823

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	124	0	300	0	112	-1
normalized size	1	1.00	0.87	1.31	0.00	3.16	0.00	1.18	-0.01
time (sec)	N/A	0.089	0.131	1.008	0.000	0.600	0.000	1.031	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	115	248	0	398	0	140	-1
normalized size	1	1.00	1.08	2.34	0.00	3.75	0.00	1.32	-0.01
time (sec)	N/A	0.103	0.288	1.133	0.000	0.763	0.000	6.337	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	138	376	0	503	0	177	-1
normalized size	1	1.00	0.94	2.56	0.00	3.42	0.00	1.20	-0.01
time (sec)	N/A	0.127	1.388	1.327	0.000	0.587	0.000	1.763	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	173	747	0	477	0	397	-1
normalized size	1	1.00	0.60	2.58	0.00	1.64	0.00	1.37	-0.00
time (sec)	N/A	0.128	10.335	1.668	0.000	0.800	0.000	10.859	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	149	498	0	425	0	339	-1
normalized size	1	1.00	0.67	2.22	0.00	1.90	0.00	1.51	-0.00
time (sec)	N/A	0.111	7.499	1.425	0.000	0.496	0.000	4.629	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	125	391	0	374	0	281	-1
normalized size	1	1.00	0.78	2.44	0.00	2.34	0.00	1.76	-0.01
time (sec)	N/A	0.099	5.918	1.061	0.000	0.651	0.000	12.613	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	124	192	0	270	0	192	-1
normalized size	1	1.00	1.88	2.91	0.00	4.09	0.00	2.91	-0.02
time (sec)	N/A	0.071	0.826	1.007	0.000	0.707	0.000	2.439	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	81	214	0	355	0	311	-1
normalized size	1	1.00	0.84	2.23	0.00	3.70	0.00	3.24	-0.01
time (sec)	N/A	0.076	0.293	1.184	0.000	0.636	0.000	5.969	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	5562	542	0	650	0	481	-1
normalized size	1	1.00	31.60	3.08	0.00	3.69	0.00	2.73	-0.01
time (sec)	N/A	0.184	24.179	1.409	0.000	0.865	0.000	12.537	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	88	293	129	285	0	190	-1
normalized size	1	1.00	0.70	2.33	1.02	2.26	0.00	1.51	-0.01
time (sec)	N/A	0.101	0.184	1.329	0.538	0.792	0.000	3.768	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	155	91	241	0	149	-1
normalized size	1	1.00	0.85	1.99	1.17	3.09	0.00	1.91	-0.01
time (sec)	N/A	0.074	0.091	1.327	0.993	0.528	0.000	1.748	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	44	26	49	137	0	55	27
normalized size	1	1.00	1.42	0.84	1.58	4.42	0.00	1.77	0.87
time (sec)	N/A	0.039	0.040	0.172	0.607	0.468	0.000	4.377	1.415

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	57	259	0	384	0	150	-1
normalized size	1	1.00	0.62	2.82	0.00	4.17	0.00	1.63	-0.01
time (sec)	N/A	0.086	0.058	1.320	0.000	0.730	0.000	1.233	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	90	504	0	546	0	229	-1
normalized size	1	1.00	0.59	3.32	0.00	3.59	0.00	1.51	-0.01
time (sec)	N/A	0.135	0.215	1.519	0.000	0.593	0.000	7.086	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	102	746	0	705	0	296	-1
normalized size	1	1.00	0.48	3.49	0.00	3.29	0.00	1.38	-0.00
time (sec)	N/A	0.178	0.282	1.854	0.000	0.678	0.000	8.554	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	467	480	0	355	0	353	-1
normalized size	1	1.00	2.47	2.54	0.00	1.88	0.00	1.87	-0.01
time (sec)	N/A	0.101	19.224	1.403	0.000	0.746	0.000	9.449	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	238	231	0	311	0	283	-1
normalized size	1	1.00	1.90	1.85	0.00	2.49	0.00	2.26	-0.01
time (sec)	N/A	0.084	3.112	1.269	0.000	0.840	0.000	6.154	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	119	116	0	235	0	191	-1
normalized size	1	1.00	1.89	1.84	0.00	3.73	0.00	3.03	-0.02
time (sec)	N/A	0.063	0.771	0.934	0.000	0.816	0.000	1.907	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	5534	374	0	503	0	123	-1
normalized size	1	1.00	33.54	2.27	0.00	3.05	0.00	0.75	-0.01
time (sec)	N/A	0.141	24.297	1.302	0.000	0.576	0.000	1.425	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	5574	722	0	666	0	256	-1
normalized size	1	1.00	22.21	2.88	0.00	2.65	0.00	1.02	-0.00
time (sec)	N/A	0.226	24.181	1.508	0.000	2.153	0.000	1.556	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	5618	1068	0	823	0	387	-1
normalized size	1	1.00	16.77	3.19	0.00	2.46	0.00	1.16	-0.00
time (sec)	N/A	0.322	24.150	1.500	0.000	0.915	0.000	1.838	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	79	224	110	261	0	168	-1
normalized size	1	1.00	0.79	2.24	1.10	2.61	0.00	1.68	-0.01
time (sec)	N/A	0.097	0.178	1.126	0.547	0.592	0.000	4.901	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	81	71	191	0	99	-1
normalized size	1	1.00	1.04	1.50	1.31	3.54	0.00	1.83	-0.02
time (sec)	N/A	0.075	0.083	1.046	0.810	0.531	0.000	3.200	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	45	70	244	0	102	50
normalized size	1	1.00	0.70	0.83	1.30	4.52	0.00	1.89	0.93
time (sec)	N/A	0.049	0.038	0.135	0.591	0.774	0.000	2.723	1.586

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	60	376	0	485	0	185	-1
normalized size	1	1.00	0.50	3.13	0.00	4.04	0.00	1.54	-0.01
time (sec)	N/A	0.107	0.060	1.293	0.000	0.706	0.000	2.424	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	90	514	0	592	0	281	-1
normalized size	1	1.00	0.51	2.92	0.00	3.36	0.00	1.60	-0.01
time (sec)	N/A	0.162	0.176	1.362	0.000	0.641	0.000	1.626	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	99	866	0	837	0	350	-1
normalized size	1	1.00	0.42	3.64	0.00	3.52	0.00	1.47	-0.00
time (sec)	N/A	0.205	0.319	1.378	0.000	0.639	0.000	1.896	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	248	391	0	343	0	338	-1
normalized size	1	1.00	1.58	2.49	0.00	2.18	0.00	2.15	-0.01
time (sec)	N/A	0.097	2.880	1.372	0.000	0.585	0.000	7.384	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	162	142	0	295	0	258	-1
normalized size	1	1.00	1.71	1.49	0.00	3.11	0.00	2.72	-0.01
time (sec)	N/A	0.080	4.827	1.234	0.000	0.484	0.000	7.995	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	4739	142	0	295	0	73	-1
normalized size	1	1.00	55.75	1.67	0.00	3.47	0.00	0.86	-0.01
time (sec)	N/A	0.086	23.844	0.903	0.000	0.632	0.000	2.968	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	5578	542	0	603	0	164	-1
normalized size	1	1.00	25.94	2.52	0.00	2.80	0.00	0.76	-0.00
time (sec)	N/A	0.195	24.648	1.330	0.000	0.665	0.000	1.489	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	5620	732	0	712	0	289	-1
normalized size	1	1.00	18.55	2.42	0.00	2.35	0.00	0.95	-0.00
time (sec)	N/A	0.282	24.008	1.472	0.000	0.902	0.000	1.953	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	5662	1240	0	955	0	414	-1
normalized size	1	1.00	14.63	3.20	0.00	2.47	0.00	1.07	-0.00
time (sec)	N/A	0.368	24.209	1.545	0.000	0.670	0.000	2.515	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	155	163	241	0	140	-1
normalized size	1	1.00	0.88	1.99	2.09	3.09	0.00	1.79	-0.01
time (sec)	N/A	0.090	0.123	1.190	0.457	0.476	0.000	4.513	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	154	125	245	0	104	-1
normalized size	1	1.00	0.93	2.85	2.31	4.54	0.00	1.93	-0.02
time (sec)	N/A	0.072	0.057	1.136	0.453	0.668	0.000	4.922	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	40	62	87	321	0	130	69
normalized size	1	1.00	0.51	0.79	1.12	4.12	0.00	1.67	0.88
time (sec)	N/A	0.058	0.059	0.128	0.441	0.584	0.000	1.496	1.907

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	60	496	0	573	0	227	-1
normalized size	1	1.00	0.42	3.44	0.00	3.98	0.00	1.58	-0.01
time (sec)	N/A	0.125	0.074	1.329	0.000	0.636	0.000	4.291	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	90	744	0	748	0	323	-1
normalized size	1	1.00	0.45	3.72	0.00	3.74	0.00	1.62	-0.00
time (sec)	N/A	0.175	0.212	1.572	0.000	0.608	0.000	2.021	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	99	986	0	905	0	392	-1
normalized size	1	1.00	0.38	3.76	0.00	3.45	0.00	1.50	-0.00
time (sec)	N/A	0.227	0.317	1.490	0.000	0.691	0.000	2.666	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	447	302	0	323	0	292	-1
normalized size	1	1.00	3.52	2.38	0.00	2.54	0.00	2.30	-0.01
time (sec)	N/A	0.087	6.079	1.216	0.000	0.459	0.000	6.434	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	5491	326	0	414	0	73	-1
normalized size	1	1.00	48.59	2.88	0.00	3.66	0.00	0.65	-0.01
time (sec)	N/A	0.107	23.737	1.209	0.000	0.523	0.000	3.050	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	5521	370	0	492	0	54	-1
normalized size	1	1.00	43.47	2.91	0.00	3.87	0.00	0.43	-0.01
time (sec)	N/A	0.106	23.811	0.925	0.000	0.719	0.000	10.292	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	5604	714	0	691	0	205	-1
normalized size	1	1.00	21.15	2.69	0.00	2.61	0.00	0.77	-0.00
time (sec)	N/A	0.233	23.931	1.399	0.000	0.683	0.000	5.577	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	5646	1066	0	868	0	331	-1
normalized size	1	1.00	15.90	3.00	0.00	2.45	0.00	0.93	-0.00
time (sec)	N/A	0.329	24.157	1.559	0.000	0.710	0.000	2.690	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	5688	1412	0	1023	0	454	-1
normalized size	1	1.00	12.96	3.22	0.00	2.33	0.00	1.03	-0.00
time (sec)	N/A	0.420	24.274	1.624	0.000	0.812	0.000	3.674	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	227	5584	724	0	674	0	0	-1
normalized size	1	1.28	31.55	4.09	0.00	3.81	0.00	0.00	-0.01
time (sec)	N/A	0.180	24.142	1.167	0.000	0.814	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	1.289	2.550	0.000	0.565	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	391	0	0	0	0	0	-1
normalized size	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	6.227	1.901	0.000	0.499	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	358	0	0	0	0	0	-1
normalized size	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	3.197	2.372	0.000	0.526	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	105	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.734	2.452	0.000	0.602	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.533	2.451	0.000	0.530	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	329	0	0	0	0	0	-1
normalized size	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.272	6.918	1.722	0.000	0.474	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	11.602	1.848	0.000	0.610	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	3.757	1.610	0.000	0.836	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	7.999	1.614	0.000	0.683	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	2.350	1.668	0.000	0.565	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	10.465	1.548	0.000	0.504	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	87	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.456	1.392	0.000	0.538	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	72	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.170	1.186	0.000	0.533	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	49	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.053	1.160	0.000	0.513	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.045	1.148	0.000	0.484	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.037	1.438	0.000	0.622	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	96	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.257	1.063	0.000	0.479	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	1.347	1.035	0.000	0.515	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	910	0	0	0	0	0	-1
normalized size	1	1.00	8.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	11.327	0.761	0.000	0.734	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	893	0	0	0	0	0	-1
normalized size	1	1.00	8.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	4.183	1.029	0.000	0.634	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	1.705	1.134	0.000	0.496	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	2072	0	0	0	0	0	-1
normalized size	1	1.00	18.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	19.020	1.756	0.000	0.503	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	238	0	0	0	0	0	-1
normalized size	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	2.229	1.733	0.000	0.518	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	229	0	0	0	0	0	-1
normalized size	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	1.612	1.536	0.000	0.503	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	2164	0	0	0	0	0	-1
normalized size	1	1.00	19.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	15.865	1.443	0.000	0.551	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	185	648	186	0	0	0	-1
normalized size	1	1.00	0.58	2.02	0.58	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.264	3.797	2.483	0.471	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	191	1390	180	0	0	0	-1
normalized size	1	1.00	0.55	4.02	0.52	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	1.196	2.348	0.497	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	169	284	166	0	0	0	-1
normalized size	1	1.00	0.62	1.04	0.61	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.201	1.830	2.365	0.467	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	189	1409	166	0	0	0	-1
normalized size	1	1.00	0.63	4.71	0.56	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.233	1.602	2.362	0.497	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	224	688	188	0	0	0	-1
normalized size	1	1.00	0.70	2.15	0.59	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	2.601	2.077	0.452	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	93	650	214	0	0	0	-1
normalized size	1	1.00	0.26	1.82	0.60	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	2.333	2.681	0.496	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	220	1392	208	0	0	0	-1
normalized size	1	1.00	0.64	4.06	0.61	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	5.247	2.542	0.465	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	118	655	193	0	0	0	-1
normalized size	1	1.00	0.38	2.11	0.62	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	1.845	2.724	0.468	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	221	1480	194	0	0	0	-1
normalized size	1	1.00	0.65	4.37	0.57	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.318	11.463	2.354	0.514	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	127	721	216	0	0	0	-1
normalized size	1	1.00	0.34	1.92	0.58	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.328	6.274	2.278	0.489	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	316	2113	0	0	0	0	-1
normalized size	1	1.00	0.78	5.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	4.048	2.002	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	135	1269	0	0	0	0	-1
normalized size	1	1.00	0.42	3.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.325	1.712	2.119	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	249	352	0	0	0	0	-1
normalized size	1	1.00	0.72	1.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.357	3.706	2.069	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	112	319	0	0	0	0	-1
normalized size	1	1.00	0.39	1.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	8.863	1.971	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	194	1419	0	0	0	0	-1
normalized size	1	1.00	0.60	4.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	62.674	2.396	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	130	698	0	0	0	0	-1
normalized size	1	1.00	0.39	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	15.929	2.385	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	261	1505	0	0	0	0	-1
normalized size	1	1.00	0.70	4.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	19.743	2.086	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	0	2117	0	0	0	0	-1
normalized size	1	1.00	0.00	5.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	13.846	2.317	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	0	1267	0	0	0	0	-1
normalized size	1	1.00	0.00	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.389	10.121	2.044	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	0	360	0	0	0	0	-1
normalized size	1	1.00	0.00	1.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.418	5.222	2.138	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	0	653	0	0	0	0	-1
normalized size	1	1.00	0.00	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	5.099	2.233	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	0	1480	0	0	0	0	-1
normalized size	1	1.00	0.00	4.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.394	55.435	2.086	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	0	721	0	0	0	0	-1
normalized size	1	1.00	0.00	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.480	13.534	2.109	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	106	216	94	101	148	317	221
normalized size	1	1.00	0.95	1.95	0.85	0.91	1.33	2.86	1.99
time (sec)	N/A	0.156	0.471	0.625	0.323	1.292	8.484	11.584	5.240

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	161	72	79	112	248	162
normalized size	1	1.00	0.98	1.92	0.86	0.94	1.33	2.95	1.93
time (sec)	N/A	0.094	0.201	0.641	0.504	1.134	3.128	3.013	5.846

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	104	50	57	76	179	102
normalized size	1	1.00	1.00	1.89	0.91	1.04	1.38	3.25	1.85
time (sec)	N/A	0.066	0.136	0.643	0.506	0.564	0.915	0.951	2.228

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	26	34	37	107	40
normalized size	1	1.00	1.00	1.00	1.04	1.36	1.48	4.28	1.60
time (sec)	N/A	0.037	0.015	0.176	0.563	1.021	0.262	1.034	1.302

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	60	35	34	38	0	61	51
normalized size	1	1.00	1.40	0.81	0.79	0.88	0.00	1.42	1.19
time (sec)	N/A	0.081	0.038	0.459	0.522	0.816	0.000	0.257	1.306

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	114	85	62	99	0	170	86
normalized size	1	1.00	1.58	1.18	0.86	1.38	0.00	2.36	1.19
time (sec)	N/A	0.108	1.350	0.648	0.353	0.491	0.000	2.501	1.356

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	166	134	99	168	0	266	128
normalized size	1	1.00	1.63	1.31	0.97	1.65	0.00	2.61	1.25
time (sec)	N/A	0.131	0.396	0.536	0.501	0.478	0.000	0.968	1.344

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	216	185	133	237	0	358	170
normalized size	1	1.00	1.66	1.42	1.02	1.82	0.00	2.75	1.31
time (sec)	N/A	0.176	0.618	0.619	0.369	0.529	0.000	0.567	1.503

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	103	178	134	134	0	228	331
normalized size	1	1.00	1.01	1.75	1.31	1.31	0.00	2.24	3.25
time (sec)	N/A	0.095	1.069	0.428	0.867	0.559	0.000	4.537	2.510

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	79	127	102	112	0	172	267
normalized size	1	1.00	1.08	1.74	1.40	1.53	0.00	2.36	3.66
time (sec)	N/A	0.065	0.596	0.430	0.620	0.522	0.000	1.576	2.234

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	60	78	65	87	0	115	96
normalized size	1	1.00	1.33	1.73	1.44	1.93	0.00	2.56	2.13
time (sec)	N/A	0.035	0.026	0.412	0.743	0.565	0.000	1.524	1.391

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	43	35	31	33	0	52	48
normalized size	1	1.00	1.65	1.35	1.19	1.27	0.00	2.00	1.85
time (sec)	N/A	0.026	0.022	0.454	0.543	0.635	0.000	0.208	1.305

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	62	86	59	87	0	112	90
normalized size	1	1.00	1.13	1.56	1.07	1.58	0.00	2.04	1.64
time (sec)	N/A	0.052	0.030	0.812	0.574	0.508	0.000	0.261	1.561

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	129	79	130	0	170	132
normalized size	1	1.00	0.94	1.54	0.94	1.55	0.00	2.02	1.57
time (sec)	N/A	0.081	0.043	0.815	0.694	0.602	0.000	0.372	1.386

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	92	162	100	179	0	225	174
normalized size	1	1.00	0.83	1.46	0.90	1.61	0.00	2.03	1.57
time (sec)	N/A	0.111	0.053	0.911	0.662	0.512	0.000	0.350	1.617

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	217	173	317	174	181	314	489	344
normalized size	1	1.17	0.94	1.71	0.94	0.98	1.70	2.64	1.86
time (sec)	N/A	0.131	0.425	0.769	0.705	0.607	34.471	19.614	5.071

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	169	138	256	139	146	252	415	280
normalized size	1	1.13	0.93	1.72	0.93	0.98	1.69	2.79	1.88
time (sec)	N/A	0.111	0.352	0.644	0.441	0.568	14.240	8.885	5.173

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	131	105	197	108	115	189	341	215
normalized size	1	1.14	0.91	1.71	0.94	1.00	1.64	2.97	1.87
time (sec)	N/A	0.091	0.266	0.661	0.331	0.559	5.436	5.390	4.968

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	74	136	75	82	126	267	151
normalized size	1	1.00	0.85	1.56	0.86	0.94	1.45	3.07	1.74
time (sec)	N/A	0.069	0.434	0.665	0.500	0.526	1.857	3.750	3.733

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	45	42	51	60	191	81
normalized size	1	1.00	0.89	0.96	0.89	1.09	1.28	4.06	1.72
time (sec)	N/A	0.034	0.064	0.192	0.378	0.559	0.493	0.873	1.533

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	53	62	68	0	101	96
normalized size	1	1.00	0.87	0.87	1.02	1.11	0.00	1.66	1.57
time (sec)	N/A	0.100	0.108	0.547	0.419	0.900	0.000	0.268	1.429

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	108	72	113	0	209	98
normalized size	1	1.00	0.89	1.17	0.78	1.23	0.00	2.27	1.07
time (sec)	N/A	0.129	0.470	0.693	0.502	0.509	0.000	1.566	1.364

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	148	169	122	203	0	360	164
normalized size	1	1.00	1.17	1.34	0.97	1.61	0.00	2.86	1.30
time (sec)	N/A	0.159	3.191	0.582	0.434	0.515	0.000	0.363	1.445

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	293	219	150	184	0	282	403
normalized size	1	1.00	1.87	1.39	0.96	1.17	0.00	1.80	2.57
time (sec)	N/A	0.198	1.351	0.409	0.694	0.508	0.000	9.201	2.645

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	355	164	118	151	0	220	332
normalized size	1	1.00	3.06	1.41	1.02	1.30	0.00	1.90	2.86
time (sec)	N/A	0.152	0.965	0.391	0.597	0.554	0.000	2.389	2.563

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	201	109	82	115	0	158	227
normalized size	1	1.00	2.87	1.56	1.17	1.64	0.00	2.26	3.24
time (sec)	N/A	0.114	1.204	0.393	0.658	0.634	0.000	1.669	1.669

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	39	49	47	44	0	80	58
normalized size	1	1.00	0.81	1.02	0.98	0.92	0.00	1.67	1.21
time (sec)	N/A	0.075	0.381	0.684	0.556	0.564	0.000	1.034	1.431

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	122	111	76	102	0	176	118
normalized size	1	1.00	1.44	1.31	0.89	1.20	0.00	2.07	1.39
time (sec)	N/A	0.115	0.477	0.818	0.572	0.499	0.000	0.321	1.460

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	198	154	96	152	0	273	191
normalized size	1	1.00	1.62	1.26	0.79	1.25	0.00	2.24	1.57
time (sec)	N/A	0.135	0.590	0.806	0.659	0.481	0.000	1.213	1.497

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	257	187	116	206	0	366	258
normalized size	1	1.00	1.68	1.22	0.76	1.35	0.00	2.39	1.69
time (sec)	N/A	0.148	0.838	0.952	0.526	0.500	0.000	0.434	1.497

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	520	460	268	293	0	1768	631
normalized size	1	1.00	2.08	1.84	1.07	1.17	0.00	7.07	2.52
time (sec)	N/A	0.197	6.235	0.551	0.623	0.647	0.000	17.339	2.969

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	371	292	183	205	0	1052	395
normalized size	1	1.00	2.18	1.72	1.08	1.21	0.00	6.19	2.32
time (sec)	N/A	0.139	6.191	0.480	0.601	0.584	0.000	11.517	2.403

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	163	110	129	0	560	227
normalized size	1	1.00	1.00	1.51	1.02	1.19	0.00	5.19	2.10
time (sec)	N/A	0.096	0.400	0.437	0.403	0.546	0.000	3.135	1.881

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	70	57	69	0	289	115
normalized size	1	1.00	0.88	1.19	0.97	1.17	0.00	4.90	1.95
time (sec)	N/A	0.071	0.130	0.388	0.422	0.509	0.000	2.071	1.538

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	19	35	19	19	82	114	71
normalized size	1	1.00	0.54	1.00	0.54	0.54	2.34	3.26	2.03
time (sec)	N/A	0.032	0.038	0.128	0.437	0.484	5.725	0.350	1.468

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	70	80	68	75	0	257	93
normalized size	1	1.00	0.74	0.85	0.72	0.80	0.00	2.73	0.99
time (sec)	N/A	0.102	0.107	0.709	0.504	0.518	0.000	0.310	1.749

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	141	167	144	263	0	403	174
normalized size	1	1.00	0.90	1.06	0.92	1.68	0.00	2.57	1.11
time (sec)	N/A	0.181	1.053	0.876	0.342	0.603	0.000	0.859	1.845

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	625	308	289	576	0	649	290
normalized size	1	1.00	2.67	1.32	1.24	2.46	0.00	2.77	1.24
time (sec)	N/A	0.295	6.251	0.684	0.516	0.768	0.000	1.138	2.266

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	271	907	785	0	603	0	746	9148
normalized size	1	1.37	4.58	3.96	0.00	3.05	0.00	3.77	46.20
time (sec)	N/A	0.373	6.198	0.460	0.000	1.053	0.000	4.210	4.129

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	287	374	0	444	0	476	6062
normalized size	1	1.00	2.28	2.97	0.00	3.52	0.00	3.78	48.11
time (sec)	N/A	0.343	2.320	0.442	0.000	0.926	0.000	1.483	3.292

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	115	153	0	253	0	140	121
normalized size	1	1.00	1.51	2.01	0.00	3.33	0.00	1.84	1.59
time (sec)	N/A	0.185	0.149	0.422	0.000	0.547	0.000	1.469	1.625

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	135	147	123	0	362	0	582	1002
normalized size	1	1.27	1.39	1.16	0.00	3.42	0.00	5.49	9.45
time (sec)	N/A	0.243	0.424	0.625	0.000	0.506	0.000	0.447	3.940

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	256	416	238	0	742	0	1073	3859
normalized size	1	1.45	2.35	1.34	0.00	4.19	0.00	6.06	21.80
time (sec)	N/A	0.387	6.200	0.698	0.000	0.530	0.000	0.608	11.109

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	528	498	321	423	0	1696	760
normalized size	1	1.00	2.07	1.95	1.26	1.66	0.00	6.65	2.98
time (sec)	N/A	0.205	6.314	0.560	0.429	0.640	0.000	21.611	4.769

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	383	324	227	312	0	0	505
normalized size	1	1.00	2.14	1.81	1.27	1.74	0.00	0.00	2.82
time (sec)	N/A	0.146	6.221	0.492	0.512	0.584	0.000	0.000	3.090

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	187	192	149	219	0	568	286
normalized size	1	1.00	1.55	1.59	1.23	1.81	0.00	4.69	2.36
time (sec)	N/A	0.104	0.601	0.477	0.340	0.546	0.000	6.544	2.127

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	62	93	74	102	0	313	124
normalized size	1	1.00	0.84	1.26	1.00	1.38	0.00	4.23	1.68
time (sec)	N/A	0.084	0.276	0.528	0.387	0.509	0.000	1.027	1.651

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	54	41	46	0	238	257
normalized size	1	1.00	1.00	1.00	0.76	0.85	0.00	4.41	4.76
time (sec)	N/A	0.043	0.040	0.130	0.439	0.506	0.000	1.412	1.484

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	189	141	142	234	0	303	160
normalized size	1	1.00	1.37	1.02	1.03	1.70	0.00	2.20	1.16
time (sec)	N/A	0.142	0.358	0.606	0.319	0.603	0.000	0.283	1.978

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	351	226	303	693	0	656	313
normalized size	1	1.00	1.78	1.15	1.54	3.52	0.00	3.33	1.59
time (sec)	N/A	0.229	2.272	0.719	0.544	0.770	0.000	1.930	2.484

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	473	367	558	1378	0	795	471
normalized size	1	1.00	1.70	1.32	2.01	4.96	0.00	2.86	1.69
time (sec)	N/A	0.371	3.137	0.743	0.375	1.102	0.000	1.192	2.967

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	283	865	723	0	843	0	411	9452
normalized size	1	1.42	4.32	3.62	0.00	4.22	0.00	2.06	47.26
time (sec)	N/A	0.431	6.247	0.506	0.000	0.920	0.000	4.336	4.816

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	327	353	0	584	0	294	7044
normalized size	1	1.00	2.18	2.35	0.00	3.89	0.00	1.96	46.96
time (sec)	N/A	0.326	1.585	0.398	0.000	0.654	0.000	1.552	3.468

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	120	0	383	0	144	551
normalized size	1	1.00	0.94	1.41	0.00	4.51	0.00	1.69	6.48
time (sec)	N/A	0.146	0.262	0.432	0.000	0.532	0.000	0.663	2.145

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	209	255	0	705	0	332	6093
normalized size	1	1.00	0.92	1.12	0.00	3.11	0.00	1.46	26.84
time (sec)	N/A	0.411	1.725	0.623	0.000	0.567	0.000	0.309	6.458

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	303	416	0	1481	0	487	8348
normalized size	1	1.00	0.84	1.16	0.00	4.11	0.00	1.35	23.19
time (sec)	N/A	0.567	2.442	0.694	0.000	0.663	0.000	0.363	6.972

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	761	761	1846	3747	0	0	0	0	-1
normalized size	1	1.00	2.43	4.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.175	26.439	1.765	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	740	740	202	1801	0	0	0	0	-1
normalized size	1	1.00	0.27	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.015	6.910	1.637	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	224	859	0	0	0	0	-1
normalized size	1	1.00	0.54	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.705	5.480	1.748	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	246	2313	0	0	0	0	-1
normalized size	1	1.00	0.58	5.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.564	12.804	1.727	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	863	863	1571	6426	0	0	0	0	-1
normalized size	1	1.00	1.82	7.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.250	27.535	1.709	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	836	836	2169	16178	0	0	0	0	-1
normalized size	1	1.00	2.59	19.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.063	23.902	1.934	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	254	3268	191	425	0	966	-1
normalized size	1	1.00	1.50	19.34	1.13	2.51	0.00	5.72	-0.01
time (sec)	N/A	0.170	6.307	2.049	0.428	1.638	0.000	3.761	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	194	2342	108	311	0	539	-1
normalized size	1	1.00	1.94	23.42	1.08	3.11	0.00	5.39	-0.01
time (sec)	N/A	0.113	6.224	1.720	0.422	1.447	0.000	2.168	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	137	42	67	192	0	185	47
normalized size	1	1.00	2.69	0.82	1.31	3.76	0.00	3.63	0.92
time (sec)	N/A	0.048	0.284	0.243	0.424	0.710	0.000	0.412	1.861

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	333	575	0	2132	0	0	-1
normalized size	1	1.00	3.14	5.42	0.00	20.11	0.00	0.00	-0.01
time (sec)	N/A	0.159	4.380	1.517	0.000	1.082	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	937	2844	0	3523	0	514	-1
normalized size	1	1.00	4.36	13.23	0.00	16.39	0.00	2.39	-0.00
time (sec)	N/A	0.295	18.948	1.479	0.000	3.139	0.000	1.887	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	692	1106	0	0	0	0	-1
normalized size	1	1.00	2.01	3.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.385	17.881	1.404	0.000	22.659	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	151	215	0	0	0	0	-1
normalized size	1	1.00	1.21	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.260	1.143	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	154	628	0	0	0	0	-1
normalized size	1	1.00	0.63	2.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	3.581	1.320	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	248	4997	175	381	0	722	-1
normalized size	1	1.00	1.68	33.76	1.18	2.57	0.00	4.88	-0.01
time (sec)	N/A	0.143	6.321	1.752	0.427	2.010	0.000	6.082	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	194	3003	92	273	0	288	-1
normalized size	1	1.00	2.46	38.01	1.16	3.46	0.00	3.65	-0.01
time (sec)	N/A	0.098	1.225	1.683	0.425	1.194	0.000	1.438	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	108	26	49	145	0	109	27
normalized size	1	1.00	3.48	0.84	1.58	4.68	0.00	3.52	0.87
time (sec)	N/A	0.043	0.203	0.145	0.414	1.607	0.000	1.611	1.634

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	218	690	0	2420	0	0	-1
normalized size	1	1.00	2.06	6.51	0.00	22.83	0.00	0.00	-0.01
time (sec)	N/A	0.134	5.979	1.539	0.000	4.410	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	1022	4203	0	4336	0	0	-1
normalized size	1	1.00	3.93	16.17	0.00	16.68	0.00	0.00	-0.00
time (sec)	N/A	0.263	6.892	1.733	0.000	42.439	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	404	610	835	1780	0	0	0	0	-1
normalized size	1	1.51	2.07	4.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.763	16.941	1.804	0.000	0.834	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	0	823	0	0	0	0	-1
normalized size	1	1.00	0.00	2.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	0.000	1.394	0.000	22.980	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	138	180	0	0	0	0	-1
normalized size	1	1.00	1.30	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.230	1.247	0.000	24.123	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	1198	1409	0	0	0	0	-1
normalized size	1	1.00	3.32	3.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	19.025	1.709	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	263	6612	194	467	0	0	-1
normalized size	1	1.00	1.78	44.68	1.31	3.16	0.00	0.00	-0.01
time (sec)	N/A	0.172	6.386	2.393	0.433	1.417	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	167	2830	110	317	0	307	-1
normalized size	1	1.00	1.90	32.16	1.25	3.60	0.00	3.49	-0.01
time (sec)	N/A	0.124	1.033	1.662	0.425	1.174	0.000	2.059	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	128	45	70	260	0	0	50
normalized size	1	1.00	2.37	0.83	1.30	4.81	0.00	0.00	0.93
time (sec)	N/A	0.053	0.392	0.139	0.420	1.038	0.000	0.000	1.955

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	1020	2766	0	3924	0	0	-1
normalized size	1	1.00	7.18	19.48	0.00	27.63	0.00	0.00	-0.01
time (sec)	N/A	0.197	6.943	1.315	0.000	38.669	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	316	1114	10977	0	8098	0	0	-1
normalized size	1	1.34	4.72	46.51	0.00	34.31	0.00	0.00	-0.00
time (sec)	N/A	0.385	7.526	1.962	0.000	76.759	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	530	907	859	1544	0	0	0	0	-1
normalized size	1	1.71	1.62	2.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.298	17.186	1.579	0.000	23.514	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	5162	633	0	0	0	0	-1
normalized size	1	1.00	15.01	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.404	23.591	1.188	0.000	0.774	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	1249	1209	0	0	0	0	-1
normalized size	1	1.00	3.60	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	6.161	1.253	0.000	23.569	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	664	663	2238	0	0	0	0	-1
normalized size	1	1.48	1.48	4.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.953	13.640	1.324	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	238	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	3.515	1.803	0.000	0.823	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	178	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	1.296	2.770	0.000	0.689	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	106	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.675	2.539	0.000	1.418	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	266	0	786	0	0	0	0	0	-1
normalized size	1	0.00	2.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.050	4.735	2.485	0.000	0.934	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	8.061	1.531	0.000	0.607	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	0.709	1.648	0.000	0.607	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.062	3.001	1.780	0.000	0.633	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.070	3.894	1.566	0.000	0.903	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	3.490	2.120	0.000	0.804	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	298	0	0	0	0	0	-1
normalized size	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	3.313	1.244	0.000	0.523	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	118	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	1.346	1.142	0.000	0.504	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.448	1.001	0.000	0.434	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	163	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	1.694	1.357	0.000	0.450	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	256	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	7.010	1.134	0.000	0.737	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	6.654	1.103	0.000	0.516	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.330	3.550	0.839	0.000	0.469	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	3.723	1.048	0.000	0.561	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	6.239	1.199	0.000	0.945	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	4.408	1.894	0.000	0.632	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	5.596	1.653	0.000	0.610	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	5.087	1.581	0.000	0.609	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	6.184	1.448	0.000	0.744	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [316] had the largest ratio of [.9200]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	19	0.105
2	A	3	2	1.00	19	0.105
3	A	3	2	1.00	19	0.105
4	A	3	2	1.00	19	0.105
5	A	3	2	1.00	17	0.118
6	A	2	2	1.00	17	0.118
7	A	3	2	1.00	19	0.105
8	A	3	2	1.00	19	0.105
9	A	3	2	1.00	19	0.105
10	A	6	2	1.00	19	0.105
11	A	5	2	1.00	19	0.105
12	A	4	2	1.00	19	0.105
13	A	3	2	1.00	19	0.105
14	A	2	2	1.00	19	0.105
15	A	3	2	1.00	19	0.105
16	A	4	2	1.00	19	0.105
17	A	5	2	1.00	19	0.105
18	A	6	2	1.00	19	0.105
19	A	3	2	1.00	21	0.095
20	A	3	2	1.00	21	0.095
21	A	3	2	1.00	21	0.095
22	A	3	2	1.00	21	0.095
23	A	3	2	1.00	19	0.105
24	A	3	2	1.00	19	0.105
25	A	3	2	1.00	21	0.095
26	A	3	2	1.00	21	0.095
27	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	3	2	1.00	21	0.095
29	A	12	7	1.00	21	0.333
30	A	10	7	1.00	21	0.333
31	A	8	7	1.00	21	0.333
32	A	8	5	1.00	21	0.238
33	A	9	6	1.00	21	0.286
34	A	11	7	1.00	21	0.333
35	A	12	7	1.00	21	0.333
36	A	13	7	1.00	21	0.333
37	A	3	2	1.00	21	0.095
38	A	3	2	1.00	21	0.095
39	A	3	2	1.00	21	0.095
40	A	3	2	1.00	21	0.095
41	A	3	2	1.00	19	0.105
42	A	3	2	1.00	19	0.105
43	A	3	2	1.00	21	0.095
44	A	3	2	1.00	21	0.095
45	A	3	2	1.00	21	0.095
46	A	3	2	1.00	21	0.095
47	A	17	8	1.00	21	0.381
48	A	14	8	1.00	21	0.381
49	A	11	8	1.00	21	0.381
50	A	11	8	1.00	21	0.381
51	A	11	6	1.00	21	0.286
52	A	14	8	1.00	21	0.381
53	A	15	8	1.00	21	0.381
54	A	16	8	1.00	21	0.381
55	A	17	8	1.00	21	0.381
56	A	3	2	1.00	21	0.095
57	A	3	2	1.00	21	0.095
58	A	3	2	1.00	21	0.095
59	A	3	2	1.00	21	0.095
60	A	2	2	1.00	19	0.105
61	A	3	2	1.00	19	0.105
62	A	3	2	1.00	21	0.095
63	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
64	A	6	3	1.00	21	0.143
65	A	5	3	1.00	21	0.143
66	A	4	3	1.00	21	0.143
67	A	3	2	1.00	21	0.095
68	A	4	3	1.00	21	0.143
69	A	5	3	1.00	21	0.143
70	A	6	3	1.00	21	0.143
71	A	3	2	1.00	21	0.095
72	A	3	2	1.00	21	0.095
73	A	3	2	1.00	21	0.095
74	A	3	2	1.00	21	0.095
75	A	3	2	1.00	19	0.105
76	A	3	2	1.00	19	0.105
77	A	3	2	1.00	21	0.095
78	A	3	2	1.00	21	0.095
79	A	11	8	1.00	21	0.381
80	A	9	8	1.00	21	0.381
81	A	5	5	1.00	21	0.238
82	A	9	6	1.06	21	0.286
83	A	12	8	1.00	21	0.381
84	A	13	8	1.00	21	0.381
85	A	14	8	1.00	21	0.381
86	A	3	2	1.00	21	0.095
87	A	3	2	1.00	21	0.095
88	A	3	2	1.00	21	0.095
89	A	3	2	1.00	21	0.095
90	A	3	2	1.00	21	0.095
91	A	3	2	1.00	19	0.105
92	A	3	2	1.00	19	0.105
93	A	3	2	1.00	21	0.095
94	A	3	2	1.00	21	0.095
95	A	18	9	1.00	21	0.429
96	A	15	9	1.00	21	0.429
97	A	12	9	1.00	21	0.429
98	A	6	6	1.00	21	0.286
99	A	12	9	1.04	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	12	7	1.18	21	0.333
101	A	16	9	1.00	21	0.429
102	A	17	9	1.00	21	0.429
103	A	18	9	1.00	21	0.429
104	A	17	14	1.00	23	0.609
105	A	16	13	1.00	23	0.565
106	A	16	13	1.00	23	0.565
107	A	15	12	1.00	23	0.522
108	A	17	14	1.00	23	0.609
109	A	16	13	1.00	23	0.565
110	A	18	14	1.00	23	0.609
111	A	21	17	1.00	25	0.680
112	A	20	16	1.00	25	0.640
113	A	19	15	1.00	25	0.600
114	A	18	14	1.00	25	0.560
115	A	20	16	1.00	25	0.640
116	A	20	16	1.00	25	0.640
117	A	22	17	1.00	25	0.680
118	A	18	14	1.00	25	0.560
119	A	18	15	1.00	25	0.600
120	A	17	14	1.00	25	0.560
121	A	17	14	1.00	25	0.560
122	A	16	13	1.00	25	0.520
123	A	18	15	1.00	25	0.600
124	A	17	14	1.00	25	0.560
125	A	19	15	1.00	25	0.600
126	A	18	14	1.00	25	0.560
127	A	22	18	1.00	25	0.720
128	A	21	17	1.00	25	0.680
129	A	20	16	1.00	25	0.640
130	A	19	15	1.00	25	0.600
131	A	21	17	1.00	25	0.680
132	A	21	17	1.00	25	0.680
133	A	23	18	1.00	25	0.720
134	A	23	17	1.00	25	0.680
135	A	8	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	6	5	1.00	23	0.217
137	A	4	4	1.00	21	0.190
138	A	6	4	1.00	21	0.190
139	A	8	6	1.00	23	0.261
140	A	10	7	1.00	23	0.304
141	A	4	3	1.00	23	0.130
142	A	4	3	1.00	23	0.130
143	A	4	4	1.00	23	0.174
144	A	5	4	1.00	23	0.174
145	A	7	5	1.00	23	0.217
146	A	9	6	1.00	23	0.261
147	A	9	5	1.00	23	0.217
148	A	7	5	1.00	23	0.217
149	A	5	4	1.00	21	0.190
150	A	6	4	1.00	21	0.190
151	A	7	5	1.00	23	0.217
152	A	9	7	1.00	23	0.304
153	A	4	3	1.00	23	0.130
154	A	4	3	1.00	23	0.130
155	A	4	3	1.00	23	0.130
156	A	3	3	1.00	23	0.130
157	A	6	5	1.00	23	0.217
158	A	8	5	1.00	23	0.217
159	A	10	5	1.00	23	0.217
160	A	8	5	1.00	23	0.217
161	A	6	4	1.00	21	0.190
162	A	7	5	1.00	21	0.238
163	A	7	5	1.00	23	0.217
164	A	8	6	1.00	23	0.261
165	A	4	3	1.00	23	0.130
166	A	4	3	1.00	23	0.130
167	A	4	3	1.00	23	0.130
168	A	3	3	1.00	23	0.130
169	A	4	3	1.00	23	0.130
170	A	7	5	1.00	23	0.217
171	A	7	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	5	5	1.00	23	0.217
173	A	3	3	1.00	21	0.143
174	A	7	5	1.00	21	0.238
175	A	9	6	1.00	23	0.261
176	A	11	7	1.00	23	0.304
177	A	4	3	1.00	23	0.130
178	A	5	4	1.00	23	0.174
179	A	3	3	1.00	23	0.130
180	A	6	5	1.00	23	0.217
181	A	8	6	1.00	23	0.261
182	A	10	6	1.00	23	0.261
183	A	6	5	1.00	23	0.217
184	A	4	4	1.00	23	0.174
185	A	4	4	1.00	21	0.190
186	A	8	6	1.00	21	0.286
187	A	10	6	1.00	23	0.261
188	A	12	7	1.00	23	0.304
189	A	5	4	1.00	23	0.174
190	A	4	3	1.00	23	0.130
191	A	4	3	1.00	23	0.130
192	A	7	6	1.00	23	0.261
193	A	9	6	1.00	23	0.261
194	A	11	6	1.00	23	0.261
195	A	5	4	1.00	23	0.174
196	A	4	4	1.00	23	0.174
197	A	5	4	1.00	21	0.190
198	A	9	6	1.00	21	0.286
199	A	11	6	1.00	23	0.261
200	A	13	7	1.00	23	0.304
201	A	4	3	1.00	23	0.130
202	A	5	4	1.00	23	0.174
203	A	5	4	1.00	23	0.174
204	A	8	6	1.00	23	0.261
205	A	10	6	1.00	23	0.261
206	A	12	6	1.00	23	0.261
207	A	7	5	1.28	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	1	1	1.00	23	0.043
209	A	8	6	1.00	23	0.261
210	A	7	6	1.00	23	0.261
211	A	4	4	1.00	21	0.190
212	A	5	5	1.00	23	0.217
213	A	8	7	1.00	23	0.304
214	A	9	7	1.00	23	0.304
215	A	1	1	1.00	25	0.040
216	A	1	1	1.00	25	0.040
217	A	1	1	1.00	25	0.040
218	A	1	1	1.00	25	0.040
219	A	4	3	1.00	21	0.143
220	A	4	3	1.00	21	0.143
221	A	3	3	1.00	21	0.143
222	A	2	2	1.00	19	0.105
223	A	4	4	1.00	19	0.210
224	A	5	5	1.00	21	0.238
225	A	1	1	1.00	21	0.048
226	A	1	1	1.00	21	0.048
227	A	1	1	1.00	21	0.048
228	A	1	1	1.00	21	0.048
229	A	1	1	1.00	23	0.043
230	A	1	1	1.00	23	0.043
231	A	1	1	1.00	23	0.043
232	A	1	1	1.00	23	0.043
233	A	17	14	1.00	23	0.609
234	A	18	15	1.00	23	0.652
235	A	16	13	1.00	23	0.565
236	A	17	14	1.00	23	0.609
237	A	17	14	1.00	23	0.609
238	A	21	17	1.00	25	0.680
239	A	21	17	1.00	25	0.680
240	A	19	15	1.00	25	0.600
241	A	20	16	1.00	25	0.640
242	A	21	17	1.00	25	0.680
243	A	20	16	1.00	25	0.640

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
244	A	18	15	1.00	25	0.600
245	A	19	16	1.00	25	0.640
246	A	17	14	1.00	25	0.560
247	A	18	15	1.00	25	0.600
248	A	18	15	1.00	25	0.600
249	A	19	16	1.00	25	0.640
250	A	24	19	1.00	25	0.760
251	A	22	18	1.00	25	0.720
252	A	22	18	1.00	25	0.720
253	A	20	16	1.00	25	0.640
254	A	21	17	1.00	25	0.680
255	A	22	18	1.00	25	0.720
256	A	7	5	1.00	19	0.263
257	A	6	5	1.00	19	0.263
258	A	5	5	1.00	19	0.263
259	A	4	4	1.00	17	0.235
260	A	5	4	1.00	17	0.235
261	A	6	5	1.00	19	0.263
262	A	7	5	1.00	19	0.263
263	A	8	5	1.00	19	0.263
264	A	5	2	1.00	19	0.105
265	A	4	2	1.00	19	0.105
266	A	3	2	1.00	19	0.105
267	A	2	2	1.00	19	0.105
268	A	3	2	1.00	19	0.105
269	A	4	2	1.00	19	0.105
270	A	5	2	1.00	19	0.105
271	A	3	2	1.17	21	0.095
272	A	3	2	1.13	21	0.095
273	A	3	2	1.14	21	0.095
274	A	3	2	1.00	21	0.095
275	A	3	2	1.00	19	0.105
276	A	3	2	1.00	19	0.105
277	A	4	3	1.00	21	0.143
278	A	5	4	1.00	21	0.190
279	A	12	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	10	7	1.00	21	0.333
281	A	8	7	1.00	21	0.333
282	A	8	5	1.00	21	0.238
283	A	9	6	1.00	21	0.286
284	A	11	7	1.00	21	0.333
285	A	12	7	1.00	21	0.333
286	A	3	2	1.00	21	0.095
287	A	3	2	1.00	21	0.095
288	A	3	2	1.00	21	0.095
289	A	3	2	1.00	21	0.095
290	A	4	4	1.00	19	0.210
291	A	3	2	1.00	19	0.105
292	A	3	2	1.00	21	0.095
293	A	3	2	1.00	21	0.095
294	A	15	8	1.37	21	0.381
295	A	6	6	1.00	21	0.286
296	A	7	7	1.00	21	0.333
297	A	9	8	1.27	21	0.381
298	A	15	8	1.45	21	0.381
299	A	3	2	1.00	21	0.095
300	A	3	2	1.00	21	0.095
301	A	3	2	1.00	21	0.095
302	A	3	2	1.00	21	0.095
303	A	3	2	1.00	19	0.105
304	A	3	2	1.00	19	0.105
305	A	3	2	1.00	21	0.095
306	A	3	2	1.00	21	0.095
307	A	16	10	1.42	21	0.476
308	A	6	6	1.00	21	0.286
309	A	6	6	1.00	21	0.286
310	A	11	7	1.00	21	0.333
311	A	15	8	1.00	21	0.381
312	A	38	22	1.00	25	0.880
313	A	35	19	1.00	25	0.760
314	A	21	16	1.00	25	0.640
315	A	19	14	1.00	25	0.560

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	39	23	1.00	25	0.920
317	A	36	20	1.00	25	0.800
318	A	5	4	1.00	23	0.174
319	A	5	4	1.00	23	0.174
320	A	4	4	1.00	21	0.190
321	A	7	5	1.00	21	0.238
322	A	13	9	1.00	23	0.391
323	A	7	7	1.00	23	0.304
324	A	1	1	1.00	14	0.071
325	A	5	4	1.00	23	0.174
326	A	5	4	1.00	23	0.174
327	A	5	4	1.00	23	0.174
328	A	3	3	1.00	21	0.143
329	A	7	5	1.00	21	0.238
330	A	11	6	1.00	23	0.261
331	A	11	8	1.51	23	0.348
332	A	6	6	1.00	23	0.261
333	A	1	1	1.00	14	0.071
334	A	9	8	1.00	23	0.348
335	A	5	4	1.00	23	0.174
336	A	5	4	1.00	23	0.174
337	A	4	4	1.00	21	0.190
338	A	7	4	1.00	21	0.190
339	A	11	5	1.34	23	0.217
340	A	17	11	1.71	23	0.478
341	A	7	7	1.00	23	0.304
342	A	6	6	1.00	14	0.429
343	A	14	11	1.48	23	0.478
344	A	8	6	1.00	23	0.261
345	A	7	6	1.00	23	0.261
346	A	4	4	1.00	21	0.190
347	F	0	0	N/A	0	N/A
348	A	0	0	0.00	0	0.000
349	A	0	0	0.00	0	0.000
350	A	0	0	0.00	0	0.000
351	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	0	0	0.00	0	0.000
353	A	5	4	1.00	21	0.190
354	A	4	4	1.00	21	0.190
355	A	2	2	1.00	19	0.105
356	A	8	5	1.00	19	0.263
357	A	10	5	1.00	21	0.238
358	A	0	0	0.00	0	0.000
359	A	0	0	0.00	0	0.000
360	A	0	0	0.00	0	0.000
361	A	0	0	0.00	0	0.000
362	A	0	0	0.00	0	0.000
363	A	0	0	0.00	0	0.000
364	A	0	0	0.00	0	0.000
365	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int (a + a\sec(c + dx))\tan^9(c + dx)\, dx$

Optimal. Leaf size=151

$$\frac{a\sec^9(c+dx)}{9d}+\frac{a\sec^8(c+dx)}{8d}-\frac{4a\sec^7(c+dx)}{7d}-\frac{2a\sec^6(c+dx)}{3d}+\frac{6a\sec^5(c+dx)}{5d}+\frac{3a\sec^4(c+dx)}{2d}-\frac{4a\sec^3(c}{3d}$$

```
[Out] -a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d-2*a*sec(d*x+c)^2/d-4/3*a*sec(d*x+c)^3/d+
3/2*a*sec(d*x+c)^4/d+6/5*a*sec(d*x+c)^5/d-2/3*a*sec(d*x+c)^6/d-4/7*a*sec(d*
x+c)^7/d+1/8*a*sec(d*x+c)^8/d+1/9*a*sec(d*x+c)^9/d
```

Rubi [A] time = 0.07, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{a\sec^9(c+dx)}{9d}+\frac{a\sec^8(c+dx)}{8d}-\frac{4a\sec^7(c+dx)}{7d}-\frac{2a\sec^6(c+dx)}{3d}+\frac{6a\sec^5(c+dx)}{5d}+\frac{3a\sec^4(c+dx)}{2d}-\frac{4a\sec^3(c}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^9,x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d - (2*a*Sec[c + d*x]^2)/d -
(4*a*Sec[c + d*x]^3)/(3*d) + (3*a*Sec[c + d*x]^4)/(2*d) + (6*a*Sec[c + d*x]
^5)/(5*d) - (2*a*Sec[c + d*x]^6)/(3*d) - (4*a*Sec[c + d*x]^7)/(7*d) + (a*Se
c[c + d*x]^8)/(8*d) + (a*Sec[c + d*x]^9)/(9*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a + a\sec(c+dx))\tan^9(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^5}{x^{10}}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{10}} + \frac{a^9}{x^9} - \frac{4a^9}{x^8} - \frac{4a^9}{x^7} + \frac{6a^9}{x^6} + \frac{6a^9}{x^5} - \frac{4a^9}{x^4} - \frac{4a^9}{x^3} + \frac{a^9}{x^2} + \frac{a^9}{x}\right) dx, x,$$
$$a^8 d$$
$$= -\frac{a\log(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} - \frac{2a\sec^2(c+dx)}{d} - \frac{4a\sec^3(c+dx)}{3d}$$

Mathematica [A] time = 0.48, size = 134, normalized size = 0.89

$$\frac{a\sec^9(c+dx)}{9d} - \frac{4a\sec^7(c+dx)}{7d} + \frac{6a\sec^5(c+dx)}{5d} - \frac{4a\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)}{d} - \frac{a\left(-3\tan^8(c+dx) + 4\tan^6(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^9,x]
```

```
[Out] (a*Sec[c + d*x])/d - (4*a*Sec[c + d*x]^3)/(3*d) + (6*a*Sec[c + d*x]^5)/(5*d
) - (4*a*Sec[c + d*x]^7)/(7*d) + (a*Sec[c + d*x]^9)/(9*d) - (a*(24*Log[Cos[
c + d*x]] + 12*Tan[c + d*x]^2 - 6*Tan[c + d*x]^4 + 4*Tan[c + d*x]^6 - 3*Tan
[c + d*x]^8))/(24*d)
```

fricas [A] time = 0.71, size = 123, normalized size = 0.81

$$-\frac{2520\,a\cos(dx+c)^9\log(-\cos(dx+c)) - 2520\,a\cos(dx+c)^8 + 5040\,a\cos(dx+c)^7 + 3360\,a\cos(dx+c)^6 - 3}{2520\,d\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a*cos(d*x + c)^9*log(-cos(d*x + c)) - 2520*a*cos(d*x + c)^8 +
 5040*a*cos(d*x + c)^7 + 3360*a*cos(d*x + c)^6 - 3780*a*cos(d*x + c)^5 - 30
24*a*cos(d*x + c)^4 + 1680*a*cos(d*x + c)^3 + 1440*a*cos(d*x + c)^2 - 315*a
*cos(d*x + c) - 280*a)/(d*cos(d*x + c)^9)
```

giac [B] time = 24.07, size = 293, normalized size = 1.94

$$\frac{2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{9177\,a + \frac{87633\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{375732\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{953988\,a(\cos(}{(\cos(dx+c}}{}}{2520\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2520*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (9177*a + 87633*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 375732*a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 953988*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1594
782*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1336734*a*(cos(d*x + c) -
 1)^5/(cos(d*x + c) + 1)^5 + 781956*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 + 302004*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 69201*a*(cos(d*
x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 7129*a*(cos(d*x + c) - 1)^9/(cos(d*x +
 c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^9)/d
```

maple [A] time = 0.85, size = 273, normalized size = 1.81

$$\frac{a\left(\tan^{8}(dx+c)\right)}{8d}-\frac{a\left(\tan^{6}(dx+c)\right)}{6d}+\frac{a\left(\tan^{4}(dx+c)\right)}{4d}-\frac{a\left(\tan^{2}(dx+c)\right)}{2d}-\frac{a\ln(\cos(dx+c))}{d}+\frac{a\left(\sin^{10}(dx+c\right.}{9d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^9,x)
```

```
[Out] 1/8/d*a*tan(d*x+c)^8-1/6/d*a*tan(d*x+c)^6+1/4*a*tan(d*x+c)^4/d-1/2*a*tan(d*
x+c)^2/d-a*ln(cos(d*x+c))/d+1/9/d*a*sin(d*x+c)^10/cos(d*x+c)^9-1/63/d*a*sin
(d*x+c)^10/cos(d*x+c)^7+1/105/d*a*sin(d*x+c)^10/cos(d*x+c)^5-1/63/d*a*sin(d
*x+c)^10/cos(d*x+c)^3+1/9/d*a*sin(d*x+c)^10/cos(d*x+c)+128/315*a*cos(d*x+c)
/d+1/9/d*cos(d*x+c)*sin(d*x+c)^8*a+8/63/d*a*cos(d*x+c)*sin(d*x+c)^6+16/105/
d*a*cos(d*x+c)*sin(d*x+c)^4+64/315/d*a*cos(d*x+c)*sin(d*x+c)^2
```

maxima [A] time = 0.80, size = 116, normalized size = 0.77

$$-\frac{2520\,a\log(\cos(dx+c))-\frac{2520\,a\cos(dx+c)^{8}-5040\,a\cos(dx+c)^{7}-3360\,a\cos(dx+c)^{6}+3780\,a\cos(dx+c)^{5}+3024\,a\cos(dx+c)^{4}-1680\,a\cos}{\cos(dx+c)^{9}}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a*log(cos(d*x + c)) - (2520*a*cos(d*x + c)^8 - 5040*a*cos(d*x
 + c)^7 - 3360*a*cos(d*x + c)^6 + 3780*a*cos(d*x + c)^5 + 3024*a*cos(d*x +
c)^4 - 1680*a*cos(d*x + c)^3 - 1440*a*cos(d*x + c)^2 + 315*a*cos(d*x + c) +
 280*a)/cos(d*x + c)^9)/d
```

mupad [B] time = 5.13, size = 259, normalized size = 1.72

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-18\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+\frac{218\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}-174\,a\tan}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-84\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((256*a)/315 - (326*a*tan(c/2 + (d*x)
/2)^2)/35 + (1654*a*tan(c/2 + (d*x)/2)^4)/35 - (2114*a*tan(c/2 + (d*x)/2)^6
)/15 + (1382*a*tan(c/2 + (d*x)/2)^8)/5 - 174*a*tan(c/2 + (d*x)/2)^10 + (218
*a*tan(c/2 + (d*x)/2)^12)/3 - 18*a*tan(c/2 + (d*x)/2)^14 + 2*a*tan(c/2 + (d
*x)/2)^16)/(d*(9*tan(c/2 + (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/
2 + (d*x)/2)^6 - 126*tan(c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 - 84*
tan(c/2 + (d*x)/2)^12 + 36*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)^16
+ tan(c/2 + (d*x)/2)^18 - 1))
```

sympy [A] time = 21.44, size = 184, normalized size = 1.22

$$\begin{cases}\frac{a\log\left(\tan^{2}(c+dx)+1\right)}{2d}+\frac{a\tan^{8}(c+dx)\sec(c+dx)}{9d}+\frac{a\tan^{8}(c+dx)}{8d}-\frac{8a\tan^{6}(c+dx)\sec(c+dx)}{63d}-\frac{a\tan^{6}(c+dx)}{6d}+\frac{16a\tan^{4}(c+dx)\sec(c+dx}{105d} \\ x\,(a\sec(c)+a)\tan^{9}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**8*sec(c + d*x
)/(9*d) + a*tan(c + d*x)**8/(8*d) - 8*a*tan(c + d*x)**6*sec(c + d*x)/(63*d)
 - a*tan(c + d*x)**6/(6*d) + 16*a*tan(c + d*x)**4*sec(c + d*x)/(105*d) + a*
tan(c + d*x)**4/(4*d) - 64*a*tan(c + d*x)**2*sec(c + d*x)/(315*d) - a*tan(c
 + d*x)**2/(2*d) + 128*a*sec(c + d*x)/(315*d), Ne(d, 0)), (x*(a*sec(c) + a)
*tan(c)**9, True))
```

3.2 $\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx$

Optimal. Leaf size=118

$$\frac{a\sec^7(c+dx)}{7d}+\frac{a\sec^6(c+dx)}{6d}-\frac{3a\sec^5(c+dx)}{5d}-\frac{3a\sec^4(c+dx)}{4d}+\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}-\frac{a\sec(c+dx)}{d}$$

```
[Out] a*ln(cos(d*x+c))/d-a*sec(d*x+c)/d+3/2*a*sec(d*x+c)^2/d+a*sec(d*x+c)^3/d-3/4
*a*sec(d*x+c)^4/d-3/5*a*sec(d*x+c)^5/d+1/6*a*sec(d*x+c)^6/d+1/7*a*sec(d*x+c
)^7/d
```

Rubi [A] time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3879, 88}

$$\frac{a\sec^7(c+dx)}{7d}+\frac{a\sec^6(c+dx)}{6d}-\frac{3a\sec^5(c+dx)}{5d}-\frac{3a\sec^4(c+dx)}{4d}+\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}-\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (a*Sec[c + d*x])/d + (3*a*Sec[c + d*x]^2)/(2*d) +
 (a*Sec[c + d*x]^3)/d - (3*a*Sec[c + d*x]^4)/(4*d) - (3*a*Sec[c + d*x]^5)/(
5*d) + (a*Sec[c + d*x]^6)/(6*d) + (a*Sec[c + d*x]^7)/(7*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx&=-\frac{\text{Subst}\left(\int\frac{(a-ax)^3(a+ax)^4}{x^8}\,dx,x,\cos(c+dx)\right)}{a^6d}\\&=-\frac{\text{Subst}\left(\int\left(\frac{a^7}{x^8}+\frac{a^7}{x^7}-\frac{3a^7}{x^6}-\frac{3a^7}{x^5}+\frac{3a^7}{x^4}+\frac{3a^7}{x^3}-\frac{a^7}{x^2}-\frac{a^7}{x}\right)dx,x,\cos(c+dx)\right)}{a^6d}\\&=\frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.47, size = 106, normalized size = 0.90

$$\frac{a\sec^7(c+dx)}{7d}-\frac{3a\sec^5(c+dx)}{5d}+\frac{a\sec^3(c+dx)}{d}-\frac{a\sec(c+dx)}{d}+\frac{a\left(2\tan^6(c+dx)-3\tan^4(c+dx)+6\tan^2(c+dx)\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/d - (3*a*Sec[c + d*x]^5)/(5*d) +
 (a*Sec[c + d*x]^7)/(7*d) + (a*(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3
*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

fricas [A] time = 0.80, size = 101, normalized size = 0.86

$$\frac{420\,a\cos\left(dx+c\right)^{7}\log\left(-\cos\left(dx+c\right)\right)-420\,a\cos\left(dx+c\right)^{6}+630\,a\cos\left(dx+c\right)^{5}+420\,a\cos\left(dx+c\right)^{4}-315\,a\cos}{420\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(420*a*cos(d*x + c)^7*log(-cos(d*x + c)) - 420*a*cos(d*x + c)^6 + 630
*a*cos(d*x + c)^5 + 420*a*cos(d*x + c)^4 - 315*a*cos(d*x + c)^3 - 252*a*cos
(d*x + c)^2 + 70*a*cos(d*x + c) + 60*a)/(d*cos(d*x + c)^7)
```

giac [B] time = 7.98, size = 247, normalized size = 2.09

$$-\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{1473\,a+\frac{11151\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{36813\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{69475\,a(\cos(dx+}{(\cos(dx+c)+1}}{}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1473*a + 11151*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 36813*a*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 + 69475*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a*(cos(d*x + c) - 1)^5/(c
os(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 108
9*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 1)^7)/d
```

maple [A] time = 0.86, size = 216, normalized size = 1.83

$$\frac{a\left(\tan^{6}\left(dx+c\right)\right)}{6d}-\frac{a\left(\tan^{4}\left(dx+c\right)\right)}{4d}+\frac{a\left(\tan^{2}\left(dx+c\right)\right)}{2d}+\frac{a\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{a\left(\sin^{8}\left(dx+c\right)\right)}{7d\cos\left(dx+c\right)^{7}}-\frac{a\left(\sin^{8}\left(dx+c\right)\right)}{35d\cos\left(dx+c\right)^{5}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^7,x)
```

```
[Out] 1/6/d*a*tan(d*x+c)^6-1/4*a*tan(d*x+c)^4/d+1/2*a*tan(d*x+c)^2/d+a*ln(cos(d*x
+c))/d+1/7/d*a*sin(d*x+c)^8/cos(d*x+c)^7-1/35/d*a*sin(d*x+c)^8/cos(d*x+c)^5
+1/35/d*a*sin(d*x+c)^8/cos(d*x+c)^3-1/7/d*a*sin(d*x+c)^8/cos(d*x+c)-16/35*a
*cos(d*x+c)/d-1/7/d*a*cos(d*x+c)*sin(d*x+c)^6-6/35/d*a*cos(d*x+c)*sin(d*x+c
)^4-8/35/d*a*cos(d*x+c)*sin(d*x+c)^2
```

maxima [A] time = 0.35, size = 94, normalized size = 0.80

$$\frac{420\,a\log\left(\cos\left(dx+c\right)\right)-\frac{420\,a\cos(dx+c)^{6}-630\,a\cos(dx+c)^{5}-420\,a\cos(dx+c)^{4}+315\,a\cos(dx+c)^{3}+252\,a\cos(dx+c)^{2}-70\,a\cos(dx+c)-60\,a}{\cos(dx+c)^{7}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(420*a*log(cos(d*x + c)) - (420*a*cos(d*x + c)^6 - 630*a*cos(d*x + c)
^5 - 420*a*cos(d*x + c)^4 + 315*a*cos(d*x + c)^3 + 252*a*cos(d*x + c)^2 - 7
0*a*cos(d*x + c) - 60*a)/cos(d*x + c)^7)/d
```

mupad [B] time = 5.69, size = 204, normalized size = 1.73

$$\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}-\frac{224\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}+\frac{166\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5}-\frac{42\,a}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-21\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x)),x)
```

```
[Out] ((32*a)/35 - (42*a*tan(c/2 + (d*x)/2)^2)/5 + (166*a*tan(c/2 + (d*x)/2)^4)/5
 - (224*a*tan(c/2 + (d*x)/2)^6)/3 + (128*a*tan(c/2 + (d*x)/2)^8)/3 - 14*a*t
an(c/2 + (d*x)/2)^10 + 2*a*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^
2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/
2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)
/2)^14 - 1)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 8.46, size = 148, normalized size = 1.25

$$\begin{cases} -\frac{a\log\left(\tan^{2}(c+dx)+1\right)}{2d}+\frac{a\tan^{6}(c+dx)\sec(c+dx)}{7d}+\frac{a\tan^{6}(c+dx)}{6d}-\frac{6a\tan^{4}(c+dx)\sec(c+dx)}{35d}-\frac{a\tan^{4}(c+dx)}{4d}+\frac{8a\tan^{2}(c+dx)\sec(c+d}{35d} \\ x\left(a\sec(c)+a\right)\tan^{7}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**6*sec(c + d*
x)/(7*d) + a*tan(c + d*x)**6/(6*d) - 6*a*tan(c + d*x)**4*sec(c + d*x)/(35*d
) - a*tan(c + d*x)**4/(4*d) + 8*a*tan(c + d*x)**2*sec(c + d*x)/(35*d) + a*t
an(c + d*x)**2/(2*d) - 16*a*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) +
a)*tan(c)**7, True))
```

3.3 $\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=87

$$\frac{a\sec^5(c+dx)}{5d}+\frac{a\sec^4(c+dx)}{4d}-\frac{2a\sec^3(c+dx)}{3d}-\frac{a\sec^2(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d-a*sec(d*x+c)^2/d-2/3*a*sec(d*x+c)^3/d+1/
4*a*sec(d*x+c)^4/d+1/5*a*sec(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{a\sec^5(c+dx)}{5d}+\frac{a\sec^4(c+dx)}{4d}-\frac{2a\sec^3(c+dx)}{3d}-\frac{a\sec^2(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d - (a*Sec[c + d*x]^2)/d - (2
*a*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^5)/(5
*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^3}{x^6}\,dx,x,\cos(c+dx)\right)}{a^4d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{a^5}{x^5}-\frac{2a^5}{x^4}-\frac{2a^5}{x^3}+\frac{a^5}{x^2}+\frac{a^5}{x}\right)dx,x,\cos(c+dx)\right)}{a^4d}\\
&= -\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\sec^2(c+dx)}{d}-\frac{2a\sec^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 82, normalized size = 0.94

$$\frac{a\sec^5(c+dx)}{5d}-\frac{2a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (a*Sec[c + d*x])/d - (2*a*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]^5)/(5*d)
- (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

fricas [A] time = 0.56, size = 79, normalized size = 0.91

$$-\frac{60\,a\cos\left(dx+c\right)^{5}\log\left(-\cos\left(dx+c\right)\right)-60\,a\cos\left(dx+c\right)^{4}+60\,a\cos\left(dx+c\right)^{3}+40\,a\cos\left(dx+c\right)^{2}-15\,a\cos}{60\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^5*log(-cos(d*x + c)) - 60*a*cos(d*x + c)^4 + 60*a*
cos(d*x + c)^3 + 40*a*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 12*a)/(d*cos(d*x
 + c)^5)
```

giac [B] time = 3.09, size = 201, normalized size = 2.31

$$\frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{201\,a+\frac{1125\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2610\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1970\,a(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (201*a + 1125*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2610*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a*(cos(d*x + c) - 1)^5/(cos(d*x + c)
+ 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5)/d
```

maple [A] time = 0.74, size = 161, normalized size = 1.85

$$\frac{a\left(\tan^{4}\left(dx+c\right)\right)}{4d}-\frac{a\left(\tan^{2}\left(dx+c\right)\right)}{2d}-\frac{a\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{a\left(\sin^{6}\left(dx+c\right)\right)}{5d\cos\left(dx+c\right)^{5}}-\frac{a\left(\sin^{6}\left(dx+c\right)\right)}{15d\cos\left(dx+c\right)^{3}}+\frac{a\left(\sin^{6}\left(dx+c\right)\right.}{5d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^5,x)
```

```
[Out] 1/4*a*tan(d*x+c)^4/d-1/2*a*tan(d*x+c)^2/d-a*ln(cos(d*x+c))/d+1/5/d*a*sin(d*
x+c)^6/cos(d*x+c)^5-1/15/d*a*sin(d*x+c)^6/cos(d*x+c)^3+1/5/d*a*sin(d*x+c)^6
/cos(d*x+c)+8/15*a*cos(d*x+c)/d+1/5/d*a*cos(d*x+c)*sin(d*x+c)^4+4/15/d*a*co
s(d*x+c)*sin(d*x+c)^2
```

maxima [A] time = 0.61, size = 72, normalized size = 0.83

$$-\frac{60\,a\log\left(\cos\left(dx+c\right)\right)-\frac{60\,a\cos(dx+c)^{4}-60\,a\cos(dx+c)^{3}-40\,a\cos(dx+c)^{2}+15\,a\cos(dx+c)+12\,a}{\cos(dx+c)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(cos(d*x + c)) - (60*a*cos(d*x + c)^4 - 60*a*cos(d*x + c)^3
- 40*a*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 12*a)/cos(d*x + c)^5)/d
```

mupad [B] time = 5.73, size = 151, normalized size = 1.74

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{62\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{1}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((16*a)/15 - (22*a*tan(c/2 + (d*x)/2)
^2)/3 + (62*a*tan(c/2 + (d*x)/2)^4)/3 - 10*a*tan(c/2 + (d*x)/2)^6 + 2*a*tan
(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 1
0*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1
))
```

sympy [A] time = 3.06, size = 112, normalized size = 1.29

$$\begin{cases} \frac{a\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a\tan^4(c+dx)\sec(c+dx)}{5d}+\frac{a\tan^4(c+dx)}{4d}-\frac{4a\tan^2(c+dx)\sec(c+dx)}{15d}-\frac{a\tan^2(c+dx)}{2d}+\frac{8a\sec(c+dx)}{15d} & \text{for } d\neq 0 \\ x\left(a\sec(c)+a\right)\tan^5(c) & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4*sec(c + d*x
)/(5*d) + a*tan(c + d*x)**4/(4*d) - 4*a*tan(c + d*x)**2*sec(c + d*x)/(15*d)
 - a*tan(c + d*x)**2/(2*d) + 8*a*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a*sec(
c) + a)*tan(c)**5, True))
```

3.4 $\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=57

$$\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec^2(c+dx)}{2d}-\frac{a\sec(c+dx)}{d}+\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] a*ln(cos(d*x+c))/d-a*sec(d*x+c)/d+1/2*a*sec(d*x+c)^2/d+1/3*a*sec(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 75}

$$\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec^2(c+dx)}{2d}-\frac{a\sec(c+dx)}{d}+\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^2)/(2*d) + (
a*Sec[c + d*x]^3)/(3*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)(a+ax)^2}{x^4}\,dx,x,\cos(c+dx)\right)}{a^2d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{a^3}{x^3}-\frac{a^3}{x^2}-\frac{a^3}{x}\right)dx,x,\cos(c+dx)\right)}{a^2d}\\ &= \frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 0.96

$$\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}+\frac{a\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*(2*Log[Cos[c + d*x]]
+ Tan[c + d*x]^2))/(2*d)
```

fricas [A] time = 0.71, size = 57, normalized size = 1.00

$$\frac{6\,a\cos\left(dx+c\right)^{3}\log\left(-\cos\left(dx+c\right)\right)-6\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+2\,a}{6\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*cos(d*x + c)^3*log(-cos(d*x + c)) - 6*a*cos(d*x + c)^2 + 3*a*cos(d
*x + c) + 2*a)/(d*cos(d*x + c)^3)
```

giac [B] time = 0.96, size = 155, normalized size = 2.72

$$-\frac{6\,a\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)-6\,a\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}-1\right|\right)+\frac{19\,a+\frac{69\,a\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+\frac{45\,a\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{11\,a\left(\cos\left(dx+c\right)-1\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}}{\left(\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (19*a + 69*a*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 11*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1)^3)/d
```

maple [A] time = 0.73, size = 104, normalized size = 1.82

$$\frac{a\left(\tan^{2}\left(dx+c\right)\right)}{2d}+\frac{a\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{a\left(\sin^{4}\left(dx+c\right)\right)}{3d\cos\left(dx+c\right)^{3}}-\frac{a\left(\sin^{4}\left(dx+c\right)\right)}{3d\cos\left(dx+c\right)}-\frac{a\cos\left(dx+c\right)\left(\sin^{2}\left(dx+c\right)\right)}{3d}-\frac{2a\cos\left(\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^3,x)
```

```
[Out] 1/2*a*tan(d*x+c)^2/d+a*ln(cos(d*x+c))/d+1/3/d*a*sin(d*x+c)^4/cos(d*x+c)^3-1
/3/d*a*sin(d*x+c)^4/cos(d*x+c)-1/3/d*a*cos(d*x+c)*sin(d*x+c)^2-2/3*a*cos(d*
x+c)/d
```

maxima [A] time = 0.48, size = 50, normalized size = 0.88

$$\frac{6\,a\log\left(\cos\left(dx+c\right)\right)-\frac{6\,a\cos\left(dx+c\right)^{2}-3\,a\cos\left(dx+c\right)-2\,a}{\cos\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a*log(cos(d*x + c)) - (6*a*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 2*a)/
cos(d*x + c)^3)/d
```

mupad [B] time = 1.99, size = 96, normalized size = 1.68

$$\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{4\,a}{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{2\,a\,\mathrm{atanh}\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] ((4*a)/3 - 6*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c
/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*
a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 0.92, size = 76, normalized size = 1.33

$$\begin{cases} -\frac{a \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a \tan^2(c+dx)\sec(c+dx)}{3d} + \frac{a\tan^2(c+dx)}{2d} - \frac{2a\sec(c+dx)}{3d} & \text{for } d \neq 0 \\ x\,(a\sec(c)+a)\tan^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2*sec(c + d*
x)/(3*d) + a*tan(c + d*x)**2/(2*d) - 2*a*sec(c + d*x)/(3*d), Ne(d, 0)), (x*
(a*sec(c) + a)*tan(c)**3, True))
```

3.5 $\int (a + a\sec(c + dx))\tan(c + dx)\,dx$

Optimal. Leaf size=25

$$\frac{a\sec(c+dx)}{d} - \frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3879, 43}

$$\frac{a\sec(c+dx)}{d} - \frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\tan(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{a+ax}{x^2}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{a}{x}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a\log(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{a\sec(c+dx)}{d} - \frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d
```

fricas [A] time = 0.72, size = 34, normalized size = 1.36

$$-\frac{a\cos(dx+c)\log(-\cos(dx+c)) - a}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - a)/(d*cos(d*x + c))
```

giac [B] time = 0.50, size = 106, normalized size = 4.24

$$\frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{3a+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3*a + a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

maple [A] time = 0.28, size = 25, normalized size = 1.00

$$\frac{a\sec(dx+c)}{d}+\frac{a\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c),x)
```

```
[Out] a*sec(d*x+c)/d+a/d*ln(sec(d*x+c))
```

maxima [A] time = 0.79, size = 26, normalized size = 1.04

$$-\frac{a\log(\cos(dx+c))-\frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*log(cos(d*x + c)) - a/cos(d*x + c))/d
```

mupad [B] time = 1.17, size = 40, normalized size = 1.60

$$\frac{2a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2a}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [A] time = 0.26, size = 37, normalized size = 1.48

$$\begin{cases}\frac{a\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a\sec(c+dx)}{d} & \text{for } d\neq 0\\ x\left(a\sec(c)+a\right)\tan(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*sec(c + d*x)/d, Ne(d, 0)),
(x*(a*sec(c) + a)*tan(c), True))
```

3.6 $\int \cot(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=16

$$\frac{a\log(1-\cos(c+dx))}{d}$$

```
[Out] a*ln(1-cos(d*x+c))/d
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3879, 31}

$$\frac{a\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x]),x]
[Out] (a*Log[1 - Cos[c + d*x]])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{a^2\,\text{Subst}\left(\int \frac{1}{a-ax}\,dx, x, \cos(c+dx)\right)}{d}\\ &= \frac{a\log(1-\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.81

$$\frac{2a\left(\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x]),x]
[Out] (2*a*(Log[Cos[(c + d*x)/2]] + Log[Tan[(c + d*x)/2]]))/d
```

fricas [A] time = 0.67, size = 16, normalized size = 1.00

$$\frac{a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] a*log(-1/2*cos(d*x + c) + 1/2)/d
```

giac [B] time = 0.20, size = 58, normalized size = 3.62

$$\frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a*log(abs(-(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

maple [A] time = 0.44, size = 29, normalized size = 1.81

$$-\frac{a\ln\left(\sec\left(dx+c\right)\right)}{d}+\frac{a\ln\left(-1+\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] -a/d*ln(sec(d*x+c))+a/d*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.52, size = 14, normalized size = 0.88

$$\frac{a\log\left(\cos\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] a*log(cos(d*x + c) - 1)/d
```

mupad [B] time = 1.24, size = 34, normalized size = 2.12

$$\frac{a\left(2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\cot\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)*sec(c + d*x), x) + Integral(cot(c + d*x), x))
```

3.7 $\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=57

$$-\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx)+1)}{4d}$$

```
[Out] -1/2*a/d/(1-cos(d*x+c))-3/4*a*ln(1-cos(d*x+c))/d-1/4*a*ln(1+cos(d*x+c))/d
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$-\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx)+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a/(2*d*(1 - Cos[c + d*x])) - (3*a*Log[1 - Cos[c + d*x]])/(4*d) - (a*Log[1
+ Cos[c + d*x]])/(4*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{a^4\,\text{Subst}\left(\int \frac{x^2}{(a-ax)^2(a+ax)}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{2a^3(-1+x)^2}+\frac{3}{4a^3(-1+x)}+\frac{1}{4a^3(1+x)}\right)dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(1+\cos(c+dx))}{4d}\end{aligned}$$

Mathematica [A] time = 0.78, size = 114, normalized size = 2.00

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\left(\cot^2(c+dx)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[T
an[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

fricas [A] time = 0.61, size = 69, normalized size = 1.21

$$-\frac{(a\cos(dx+c)-a)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3(a\cos(dx+c)-a)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2a}{4(d\cos(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x + c)
- a)*log(-1/2*cos(d*x + c) + 1/2) - 2*a)/(d*cos(d*x + c) - d)
```

giac [B] time = 0.25, size = 103, normalized size = 1.81

$$-\frac{3a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-4a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+\frac{3a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(3*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + 3*a*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

maple [A] time = 0.66, size = 60, normalized size = 1.05

$$\frac{a\ln(\sec(dx+c))}{d}-\frac{a}{2d(-1+\sec(dx+c))}-\frac{3a\ln(-1+\sec(dx+c))}{4d}-\frac{a\ln(1+\sec(dx+c))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c)),x)
```

```
[Out] a/d*ln(sec(d*x+c))-1/2*a/d/(-1+sec(d*x+c))-3/4*a/d*ln(-1+sec(d*x+c))-1/4*a/
d*ln(1+sec(d*x+c))
```

maxima [A] time = 0.72, size = 42, normalized size = 0.74

$$-\frac{a\log(\cos(dx+c)+1)+3a\log(\cos(dx+c)-1)-\frac{2a}{\cos(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(a*log(cos(d*x + c) + 1) + 3*a*log(cos(d*x + c) - 1) - 2*a/(cos(d*x +
c) - 1))/d
```

mupad [B] time = 1.28, size = 46, normalized size = 0.81

$$-\frac{a\left(\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2+6\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(6*log(tan(c/2 + (d*x)/2)) - 4*log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot^{3}(c+dx)\sec(c+dx)\,dx+\int\cot^{3}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**3*sec(c + d*x), x) + Integral(cot(c + d*x)**3, x)
)
```

3.8 $\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{3a}{4d(1-\cos(c+dx))}+\frac{a}{8d(\cos(c+dx)+1)}-\frac{a}{8d(1-\cos(c+dx))^2}+\frac{11a\log(1-\cos(c+dx))}{16d}+\frac{5a\log(\cos(c+dx)+}{16d}$$

```
[Out] -1/8*a/d/(1-cos(d*x+c))^2+3/4*a/d/(1-cos(d*x+c))+1/8*a/d/(1+cos(d*x+c))+11/
16*a*ln(1-cos(d*x+c))/d+5/16*a*ln(1+cos(d*x+c))/d
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{3a}{4d(1-\cos(c+dx))}+\frac{a}{8d(\cos(c+dx)+1)}-\frac{a}{8d(1-\cos(c+dx))^2}+\frac{11a\log(1-\cos(c+dx))}{16d}+\frac{5a\log(\cos(c+dx)+}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a/(8*d*(1 - Cos[c + d*x])^2) + (3*a)/(4*d*(1 - Cos[c + d*x])) + a/(8*d*(1
+ Cos[c + d*x])) + (11*a*Log[1 - Cos[c + d*x]])/(16*d) + (5*a*Log[1 + Cos[c
 + d*x]])/(16*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx=-\frac{a^6\,\text{Subst}\left(\int \frac{x^4}{(a-ax)^3(a+ax)^2}\,dx,x,\cos(c+dx)\right)}{d}$$

$$=-\frac{a^6\,\text{Subst}\left(\int\left(-\frac{1}{4a^5(-1+x)^3}-\frac{3}{4a^5(-1+x)^2}-\frac{11}{16a^5(-1+x)}+\frac{1}{8a^5(1+x)^2}-\frac{5}{16a^5(1+x)}\right)\right.}{d}$$

$$=-\frac{a}{8d(1-\cos(c+dx))^2}+\frac{3a}{4d(1-\cos(c+dx))}+\frac{a}{8d(1+\cos(c+dx))}+\ldots$$

Mathematica [A] time = 0.54, size = 127, normalized size = 1.34

$$\frac{a\left(-16\cot^4(c+dx)+32\cot^2(c+dx)-\csc^4\left(\frac{1}{2}(c+dx)\right)+10\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)-10\sec^2\left(\frac{1}{2}(c+\right.\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(32*Cot[c + d*x]^2 - 16*Cot[c + d*x]^4 + 10*Csc[(c + d*x)/2]^2 - Csc[(c
+ d*x)/2]^4 - 24*Log[Cos[(c + d*x)/2]] + 64*Log[Cos[c + d*x]] + 24*Log[Sin[
(c + d*x)/2]] + 64*Log[Tan[c + d*x]] - 10*Sec[(c + d*x)/2]^2 + Sec[(c + d*x
)/2]^4))/(64*d)
```

fricas [A] time = 0.61, size = 150, normalized size = 1.58

$$-\frac{10\,a\cos\left(dx+c\right)^{2}+6\,a\cos\left(dx+c\right)-5\,\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+a\right)\log\left(\frac{1}{2}\,\cos\left(dx\right.\right.}{16\,\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*a*cos(d*x + c)^2 + 6*a*cos(d*x + c) - 5*(a*cos(d*x + c)^3 - a*cos
(d*x + c)^2 - a*cos(d*x + c) + a)*log(1/2*cos(d*x + c) + 1/2) - 11*(a*cos(d
*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-1/2*cos(d*x + c) +
1/2) - 12*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)
```

giac [A] time = 0.30, size = 149, normalized size = 1.57

$$\frac{22\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-32\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)^{2}}{(\cos(dx+c)-1)^{2}}-\frac{2\,a(\cos(dx+c}{\cos(dx+c)}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(22*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + 10*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 33*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/d
```

maple [A] time = 0.55, size = 93, normalized size = 0.98

$$-\frac{a\ln\left(\sec\left(dx+c\right)\right)}{d}-\frac{a}{8d\left(-1+\sec\left(dx+c\right)\right)^{2}}+\frac{a}{2d\left(-1+\sec\left(dx+c\right)\right)}+\frac{11a\ln\left(-1+\sec\left(dx+c\right)\right)}{16d}-\frac{a}{8d\left(1+\sec\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c)),x)
```

```
[Out] -a/d*ln(sec(d*x+c))-1/8*a/d/(-1+sec(d*x+c))^2+1/2*a/d/(-1+sec(d*x+c))+11/16
*a/d*ln(-1+sec(d*x+c))-1/8*a/d/(1+sec(d*x+c))+5/16*a/d*ln(1+sec(d*x+c))
```

maxima [A] time = 0.59, size = 86, normalized size = 0.91

$$\frac{5\,a\log\left(\cos\left(dx+c\right)+1\right)+11\,a\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(5\,a\cos(dx+c)^{2}+3\,a\cos(dx+c)-6\,a\right)}{\cos(dx+c)^{3}-\cos(dx+c)^{2}-\cos(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(5*a*log(cos(d*x + c) + 1) + 11*a*log(cos(d*x + c) - 1) - 2*(5*a*cos(d
*x + c)^2 + 3*a*cos(d*x + c) - 6*a)/(cos(d*x + c)^3 - cos(d*x + c)^2 - cos(
d*x + c) + 1))/d
```

mupad [B] time = 1.18, size = 88, normalized size = 0.93

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a}{4}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}\right)}{8\,d}+\frac{11\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(16*d) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (co
t(c/2 + (d*x)/2)^4*(a/4 - (5*a*tan(c/2 + (d*x)/2)^2)/2))/(8*d) + (11*a*log(
tan(c/2 + (d*x)/2)))/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot^5(c+dx)\sec(c+dx)\,dx+\int\cot^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**5*sec(c + d*x), x) + Integral(cot(c + d*x)**5, x)
)
```

3.9 $\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=133

$$-\frac{15a}{16d(1-\cos(c+dx))}-\frac{a}{4d(\cos(c+dx)+1)}+\frac{9a}{32d(1-\cos(c+dx))^2}+\frac{a}{32d(\cos(c+dx)+1)^2}-\frac{a}{24d(1-\cos(c+$$

```
[Out] -1/24*a/d/(1-cos(d*x+c))^3+9/32*a/d/(1-cos(d*x+c))^2-15/16*a/d/(1-cos(d*x+c
))+1/32*a/d/(1+cos(d*x+c))^2-1/4*a/d/(1+cos(d*x+c))-21/32*a*ln(1-cos(d*x+c)
)/d-11/32*a*ln(1+cos(d*x+c))/d
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$-\frac{15a}{16d(1-\cos(c+dx))}-\frac{a}{4d(\cos(c+dx)+1)}+\frac{9a}{32d(1-\cos(c+dx))^2}+\frac{a}{32d(\cos(c+dx)+1)^2}-\frac{a}{24d(1-\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a/(24*d*(1 - Cos[c + d*x])^3) + (9*a)/(32*d*(1 - Cos[c + d*x])^2) - (15*a)
/(16*d*(1 - Cos[c + d*x])) + a/(32*d*(1 + Cos[c + d*x])^2) - a/(4*d*(1 + Co
s[c + d*x])) - (21*a*Log[1 - Cos[c + d*x]])/(32*d) - (11*a*Log[1 + Cos[c +
d*x]])/(32*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{a^8\,\text{Subst}\left(\int \frac{x^6}{(a-ax)^4(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^8\,\text{Subst}\left(\int \left(\frac{1}{8a^7(-1+x)^4}+\frac{9}{16a^7(-1+x)^3}+\frac{15}{16a^7(-1+x)^2}+\frac{21}{32a^7(-1+x)}+\frac{1}{16a^7(1}\right.\right.}{d}\\ &= -\frac{a}{24d(1-\cos(c+dx))^3}+\frac{9a}{32d(1-\cos(c+dx))^2}-\frac{15a}{16d(1-\cos(c+dx}\end{aligned}$$

Mathematica [A] time = 0.41, size = 165, normalized size = 1.24

$$-\frac{a\left(64\cot^6(c+dx)-96\cot^4(c+dx)+192\cot^2(c+dx)+\csc^6\left(\frac{1}{2}(c+dx)\right)-12\csc^4\left(\frac{1}{2}(c+dx)\right)+66\csc^2\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/384*(a*(192*Cot[c + d*x]^2 - 96*Cot[c + d*x]^4 + 64*Cot[c + d*x]^6 + 66*
Csc[(c + d*x)/2]^2 - 12*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 - 120*Log[C
os[(c + d*x)/2]] + 384*Log[Cos[c + d*x]] + 120*Log[Sin[(c + d*x)/2]] + 384*
Log[Tan[c + d*x]] - 66*Sec[(c + d*x)/2]^2 + 12*Sec[(c + d*x)/2]^4 - Sec[(c
+ d*x)/2]^6))/d
```

fricas [B] time = 0.62, size = 241, normalized size = 1.81

$$\frac{66\,a\cos\left(dx+c\right)^{4}+78\,a\cos\left(dx+c\right)^{3}-158\,a\cos\left(dx+c\right)^{2}-58\,a\cos\left(dx+c\right)-33\,\left(a\cos\left(dx+c\right)^{5}-a\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*a*cos(d*x + c)^4 + 78*a*cos(d*x + c)^3 - 158*a*cos(d*x + c)^2 - 58
*a*cos(d*x + c) - 33*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c
)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2)
- 63*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*
x + c)^2 + a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/2) + 88*a)/(d*cos(
d*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d
*cos(d*x + c) - d)
```

giac [A] time = 0.42, size = 197, normalized size = 1.48

$$-\frac{252\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(2\,a+\frac{21\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{462\,a(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}\right)(\cos(dx+}{(\cos(dx+c)-1)^{3}}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/384*(252*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (2*a + 21*a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 132*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 462*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(
cos(d*x + c) - 1)^3 - 42*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

maple [A] time = 0.61, size = 124, normalized size = 0.93

$$\frac{a\ln\left(\sec\left(dx+c\right)\right)}{d}-\frac{a}{24d\left(-1+\sec\left(dx+c\right)\right)^{3}}+\frac{5a}{32d\left(-1+\sec\left(dx+c\right)\right)^{2}}-\frac{a}{2d\left(-1+\sec\left(dx+c\right)\right)}-\frac{21a\ln\left(-1+\sec\left(\right.\right.}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c)),x)
```

```
[Out] a/d*ln(sec(d*x+c))-1/24*a/d/(-1+sec(d*x+c))^3+5/32*a/d/(-1+sec(d*x+c))^2-1/
2*a/d/(-1+sec(d*x+c))-21/32*a/d*ln(-1+sec(d*x+c))+1/32*a/d/(1+sec(d*x+c))^2
+3/16*a/d/(1+sec(d*x+c))-11/32*a/d*ln(1+sec(d*x+c))
```

maxima [A] time = 0.57, size = 126, normalized size = 0.95

$$-\frac{33\,a\log\left(\cos\left(dx+c\right)+1\right)+63\,a\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(33\,a\cos(dx+c)^{4}+39\,a\cos(dx+c)^{3}-79\,a\cos(dx+c)^{2}-29\,a\cos(dx+c)+44\right.}{\cos(dx+c)^{5}-\cos(dx+c)^{4}-2\,\cos(dx+c)^{3}+2\,\cos(dx+c)^{2}+\cos(dx+c)-1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(33*a*log(cos(d*x + c) + 1) + 63*a*log(cos(d*x + c) - 1) - 2*(33*a*co
s(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 79*a*cos(d*x + c)^2 - 29*a*cos(d*x + c
) + 44*a)/(cos(d*x + c)^5 - cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 2*cos(d*x +
 c)^2 + cos(d*x + c) - 1))/d
```

mupad [B] time = 1.23, size = 118, normalized size = 0.89

$$\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(11\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\frac{7\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4}+\frac{a}{6}\right)}{32\,d}-\frac{7\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64\,d}+\frac{a\tan\left(\frac{c}{2}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2)^6*(a/6 - (7*a*tan
(c/2 + (d*x)/2)^2)/4 + 11*a*tan(c/2 + (d*x)/2)^4))/(32*d) - (7*a*tan(c/2 +
(d*x)/2)^2)/(64*d) + (a*tan(c/2 + (d*x)/2)^4)/(128*d) - (21*a*log(tan(c/2 +
 (d*x)/2)))/(16*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot^7(c+dx)\sec(c+dx)\,dx+\int\cot^7(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**7*sec(c + d*x), x) + Integral(cot(c + d*x)**7, x)
)
```

3.10 $\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx$

Optimal. Leaf size=129

$$\frac{35a\tanh^{-1}(\sin(c+dx))}{128d}+\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}-\frac{\tan^5(c+dx)(35a\sec(c+dx)+48a)}{240d}+\frac{\tan^3(c+dx)}{}$$

```
[Out] a*x+35/128*a*arctanh(sin(d*x+c))/d-1/128*(128*a+35*a*sec(d*x+c))*tan(d*x+c)
/d+1/192*(64*a+35*a*sec(d*x+c))*tan(d*x+c)^3/d-1/240*(48*a+35*a*sec(d*x+c))
*tan(d*x+c)^5/d+1/56*(8*a+7*a*sec(d*x+c))*tan(d*x+c)^7/d
```

Rubi [A] time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3881, 3770}

$$\frac{35a\tanh^{-1}(\sin(c+dx))}{128d}+\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}-\frac{\tan^5(c+dx)(35a\sec(c+dx)+48a)}{240d}+\frac{\tan^3(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^8,x]
```

```
[Out] a*x + (35*a*ArcTanh[Sin[c + d*x]])/(128*d) - ((128*a + 35*a*Sec[c + d*x])*T
an[c + d*x])/(128*d) + ((64*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^3)/(192*d)
 - ((48*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^5)/(240*d) + ((8*a + 7*a*Sec[c +
 d*x])*Tan[c + d*x]^7)/(56*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx &= \frac{(8a+7a\sec(c+dx))\tan^7(c+dx)}{56d}-\frac{1}{8}\int (8a+7a\sec(c+dx))\tan^6(c+ \\
&= -\frac{(48a+35a\sec(c+dx))\tan^5(c+dx)}{240d}+\frac{(8a+7a\sec(c+dx))\tan^7(c+}{56d} \\
&= \frac{(64a+35a\sec(c+dx))\tan^3(c+dx)}{192d}-\frac{(48a+35a\sec(c+dx))\tan^5(c+}{240d} \\
&= -\frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d}+\frac{(64a+35a\sec(c+dx))\tan^3(c}{192d} \\
&= ax-\frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d}+\frac{(64a+35a\sec(c+dx))\tan}{192d} \\
&= ax+\frac{35a\tanh^{-1}(\sin(c+dx))}{128d}-\frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d}+
\end{aligned}$$

Mathematica [A] time = 1.89, size = 115, normalized size = 0.89

$$a\Big(13440\tan^{-1}(\tan(c+dx))+3675\tanh^{-1}(\sin(c+dx))-\frac{1}{32}(223232\cos(c+dx)+75915\cos(2(c+dx))+1479$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^8,x]
```

```
[Out] (a*(13440*ArcTan[Tan[c + d*x]] + 3675*ArcTanh[Sin[c + d*x]] - ((18970 + 223
232*Cos[c + d*x] + 75915*Cos[2*(c + d*x)] + 147968*Cos[3*(c + d*x)] + 12950
*Cos[4*(c + d*x)] + 47616*Cos[5*(c + d*x)] + 9765*Cos[6*(c + d*x)] + 11264*
Cos[7*(c + d*x)])*Sec[c + d*x]^7*Tan[c + d*x])/32))/(13440*d)
```

fricas [A] time = 0.76, size = 156, normalized size = 1.21

$$\frac{26880\,adx\cos\left(dx+c\right)^{8}+3675\,a\cos\left(dx+c\right)^{8}\log\left(\sin\left(dx+c\right)+1\right)-3675\,a\cos\left(dx+c\right)^{8}\log\left(-\sin\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/26880*(26880*a*d*x*cos(d*x + c)^8 + 3675*a*cos(d*x + c)^8*log(sin(d*x + c
) + 1) - 3675*a*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(22528*a*cos(d*x
+ c)^7 + 9765*a*cos(d*x + c)^6 - 15616*a*cos(d*x + c)^5 - 11410*a*cos(d*x +
 c)^4 + 8448*a*cos(d*x + c)^3 + 7000*a*cos(d*x + c)^2 - 1920*a*cos(d*x + c)
 - 1680*a)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

giac [A] time = 12.22, size = 174, normalized size = 1.35

$$\frac{13440\left(dx+c\right)a+3675\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3675\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9765\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)*a + 3675*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 36
75*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9765*a*tan(1/2*d*x + 1/2*c)^15
 - 83825*a*tan(1/2*d*x + 1/2*c)^13 + 321013*a*tan(1/2*d*x + 1/2*c)^11 - 724
649*a*tan(1/2*d*x + 1/2*c)^9 + 1078359*a*tan(1/2*d*x + 1/2*c)^7 - 508683*a*
tan(1/2*d*x + 1/2*c)^5 + 140175*a*tan(1/2*d*x + 1/2*c)^3 - 17115*a*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

maple [A] time = 0.54, size = 227, normalized size = 1.76

$$\frac{a\left(\tan^{7}\left(dx+c\right)\right)}{7d}-\frac{a\left(\tan^{5}\left(dx+c\right)\right)}{5d}+\frac{a\left(\tan^{3}\left(dx+c\right)\right)}{3d}-\frac{a\tan\left(dx+c\right)}{d}+ax+\frac{ca}{d}+\frac{a\left(\sin^{9}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{8}}-\frac{a\left(\sin^{9}\left(dx\right.\right.}{48d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^8,x)
```

```
[Out] 1/7*a*tan(d*x+c)^7/d-1/5*a*tan(d*x+c)^5/d+1/3*a*tan(d*x+c)^3/d-a*tan(d*x+c)
/d+a*x+1/d*c*a+1/8/d*a*sin(d*x+c)^9/cos(d*x+c)^8-1/48/d*a*sin(d*x+c)^9/cos(
d*x+c)^6+1/64/d*a*sin(d*x+c)^9/cos(d*x+c)^4-5/128/d*a*sin(d*x+c)^9/cos(d*x+
c)^2-5/128*a*sin(d*x+c)^7/d-7/128*a*sin(d*x+c)^5/d-35/384*a*sin(d*x+c)^3/d-
35/128*a*sin(d*x+c)/d+35/128/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.53, size = 164, normalized size = 1.27

$$\frac{256\left(15\,\tan\left(dx+c\right)^{7}-21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+105\,dx+105\,c-105\,\tan\left(dx+c\right)\right)a+35\,a\left(\frac{2\left(279\right.}{}\right.}{26880}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/26880*(256*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 1
05*d*x + 105*c - 105*tan(d*x + c))*a + 35*a*(2*(279*sin(d*x + c)^7 - 511*si
n(d*x + c)^5 + 385*sin(d*x + c)^3 - 105*sin(d*x + c))/(sin(d*x + c)^8 - 4*s
in(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) + 105*log(sin(d*x
+ c) + 1) - 105*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.47, size = 242, normalized size = 1.88

$$a\,x-\frac{-\frac{93\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{2395\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}-\frac{45859\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{960}+\frac{724649\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{6720}-\frac{359453\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2240}+\frac{24223\,a}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+28\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-56\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+70\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-56\,\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x - ((163*a*tan(c/2 + (d*x)/2))/64 - (1335*a*tan(c/2 + (d*x)/2)^3)/64 + (
24223*a*tan(c/2 + (d*x)/2)^5)/320 - (359453*a*tan(c/2 + (d*x)/2)^7)/2240 +
(724649*a*tan(c/2 + (d*x)/2)^9)/6720 - (45859*a*tan(c/2 + (d*x)/2)^11)/960
+ (2395*a*tan(c/2 + (d*x)/2)^13)/192 - (93*a*tan(c/2 + (d*x)/2)^15)/64)/(d*
(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6
 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/
2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1)) + (35*a*atanh
(tan(c/2 + (d*x)/2)))/(64*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \tan^{8}{\left(c+dx\right)}\sec{\left(c+dx\right)}\,dx+\int \tan^{8}{\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**8,x)
```

```
[Out] a*(Integral(tan(c + d*x)**8*sec(c + d*x), x) + Integral(tan(c + d*x)**8, x)
)
```

3.11 $\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx$

Optimal. Leaf size=102

$$-\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}-\frac{\tan^3(c+dx)(5a\sec(c+dx)+8a)}{24d}+\frac{\tan(c+dx)(\ldots}{\ldots}$$

```
[Out] -a*x-5/16*a*arctanh(sin(d*x+c))/d+1/16*(16*a+5*a*sec(d*x+c))*tan(d*x+c)/d-1
/24*(8*a+5*a*sec(d*x+c))*tan(d*x+c)^3/d+1/30*(6*a+5*a*sec(d*x+c))*tan(d*x+c
)^5/d
```

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3881, 3770}

$$-\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}-\frac{\tan^3(c+dx)(5a\sec(c+dx)+8a)}{24d}+\frac{\tan(c+dx)(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*x) - (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((16*a + 5*a*Sec[c + d*x])*Ta
n[c + d*x])/(16*d) - ((8*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^3)/(24*d) + ((6
*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\tan^6(c+dx)\,dx &= \frac{(6a+5a\sec(c+dx))\tan^5(c+dx)}{30d}-\frac{1}{6}\int (6a+5a\sec(c+dx))\tan^4(c \ldots \\
&= -\frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d}+\frac{(6a+5a\sec(c+dx))\tan^5(c+\ldots}{30d} \\
&= \frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5a\sec(c+dx))\tan^3(c+d\ldots}{24d} \\
&= -ax+\frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5a\sec(c+dx))\tan^3\ldots}{24d} \\
&= -ax-\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d}-\ldots
\end{aligned}$$

Mathematica [A] time = 1.25, size = 95, normalized size = 0.93

$$-\frac{a\left(240\tan^{-1}(\tan(c+dx))+75\tanh^{-1}(\sin(c+dx))-\frac{1}{8}(1168\cos(c+dx)+140\cos(2(c+dx))+568\cos(3(c\ldots\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -1/240*(a*(240*ArcTan[Tan[c + d*x]] + 75*ArcTanh[Sin[c + d*x]] - ((295 + 11
68*Cos[c + d*x] + 140*Cos[2*(c + d*x)] + 568*Cos[3*(c + d*x)] + 165*Cos[4*(
c + d*x)] + 184*Cos[5*(c + d*x)])*Sec[c + d*x]^5*Tan[c + d*x])/8))/d
```

fricas [A] time = 0.69, size = 134, normalized size = 1.31

$$-\frac{480\,adx\cos\left(dx+c\right)^{6}+75\,a\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-75\,a\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)-2\,(}{480}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 + 75*a*cos(d*x + c)^6*log(sin(d*x + c) + 1
) - 75*a*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*a*cos(d*x + c)^5 +
165*a*cos(d*x + c)^4 - 176*a*cos(d*x + c)^3 - 130*a*cos(d*x + c)^2 + 48*a*c
os(d*x + c) + 40*a)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

giac [A] time = 8.79, size = 146, normalized size = 1.43

$$-\frac{240\,(dx+c)a+75\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-75\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1095\,a\,\mathrm{t}}{}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)*a + 75*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 75*a*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*a*tan(1/2*d*x + 1/2*c)^11 - 1095*
a*tan(1/2*d*x + 1/2*c)^9 + 3138*a*tan(1/2*d*x + 1/2*c)^7 - 5118*a*tan(1/2*d
*x + 1/2*c)^5 + 1945*a*tan(1/2*d*x + 1/2*c)^3 - 315*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 0.51, size = 178, normalized size = 1.75

$$\frac{a\left(\tan^{5}\left(dx+c\right)\right)}{5d}-\frac{a\left(\tan^{3}\left(dx+c\right)\right)}{3d}+\frac{a\tan\left(dx+c\right)}{d}-ax-\frac{ca}{d}+\frac{a\left(\sin^{7}\left(dx+c\right)\right)}{6d\cos\left(dx+c\right)^{6}}-\frac{a\left(\sin^{7}\left(dx+c\right)\right)}{24d\cos\left(dx+c\right)^{4}}+\frac{a\left(\sin^{7}\left(dx+c\right.\right.}{16d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^6,x)
```

```
[Out] 1/5*a*tan(d*x+c)^5/d-1/3*a*tan(d*x+c)^3/d+a*tan(d*x+c)/d-a*x-1/d*c*a+1/6/d*
a*sin(d*x+c)^7/cos(d*x+c)^6-1/24/d*a*sin(d*x+c)^7/cos(d*x+c)^4+1/16/d*a*sin
(d*x+c)^7/cos(d*x+c)^2+1/16*a*sin(d*x+c)^5/d+5/48*a*sin(d*x+c)^3/d+5/16*a*s
in(d*x+c)/d-5/16/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.82, size = 134, normalized size = 1.31

$$\frac{32\left(3\,\tan\left(dx+c\right)^{5}-5\,\tan\left(dx+c\right)^{3}-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a-5\,a\left(\frac{2\left(33\,\sin(dx+c)^{5}-40\,\sin(dx+c)^{3}+15\,\sin(dx+c)\right.}{\sin(dx+c)^{6}-3\,\sin(dx+c)^{4}+3\,\sin(dx+c)^{2}-1}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x
 + c))*a - 5*a*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d*
x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.38, size = 188, normalized size = 1.84

$$\frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{73\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8}-\frac{523\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{853\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}-\frac{389\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{21\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-6\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

```
[Out] ((21*a*tan(c/2 + (d*x)/2))/8 - (389*a*tan(c/2 + (d*x)/2)^3)/24 + (853*a*tan
(c/2 + (d*x)/2)^5)/20 - (523*a*tan(c/2 + (d*x)/2)^7)/20 + (73*a*tan(c/2 + (
d*x)/2)^9)/8 - (11*a*tan(c/2 + (d*x)/2)^11)/8)/(d*(15*tan(c/2 + (d*x)/2)^4
- 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^
8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - a*x - (5*a*atan
h(tan(c/2 + (d*x)/2)))/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\tan^{6}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\tan^{6}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**6,x)
```

```
[Out] a*(Integral(tan(c + d*x)**6*sec(c + d*x), x) + Integral(tan(c + d*x)**6, x)
)
```

3.12 $\int (a+a\sec(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan^3(c+dx)(3a\sec(c+dx)+4a)}{12d}-\frac{\tan(c+dx)(3a\sec(c+dx)+8a)}{8d}+ax$$

```
[Out] a*x+3/8*a*arctanh(sin(d*x+c))/d-1/8*(8*a+3*a*sec(d*x+c))*tan(d*x+c)/d+1/12*
(4*a+3*a*sec(d*x+c))*tan(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3770}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan^3(c+dx)(3a\sec(c+dx)+4a)}{12d}-\frac{\tan(c+dx)(3a\sec(c+dx)+8a)}{8d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x + (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - ((8*a + 3*a*Sec[c + d*x])*Tan[c +
 d*x])/(8*d) + ((4*a + 3*a*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\tan^4(c+dx)\,dx &= \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d}-\frac{1}{4}\int (4a+3a\sec(c+dx))\tan^2(c+ \\ &= -\frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d} \\ &= ax-\frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+3a\sec(c+dx))\tan^3(c+ \\ &= ax+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d}+(4a+\end{aligned}$$

Mathematica [A] time = 0.44, size = 75, normalized size = 1.03

$$\frac{a\left(24\tan^{-1}(\tan(c+dx))+9\tanh^{-1}(\sin(c+dx))-\frac{1}{2}(32\cos(c+dx)+15\cos(2(c+dx))+16\cos(3(c+dx))+3)\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*(24*ArcTan[Tan[c + d*x]] + 9*ArcTanh[Sin[c + d*x]] - ((3 + 32*Cos[c + d*
x] + 15*Cos[2*(c + d*x)] + 16*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Tan[c + d*x]
)/2))/(24*d)
```

fricas [A] time = 0.80, size = 112, normalized size = 1.53

$$\frac{48\,adx\cos\left(dx+c\right)^4+9\,a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-9\,a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-2\,(32}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 + 9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
9*a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*a*cos(d*x + c)^3 + 15*a*c
os(d*x + c)^2 - 8*a*cos(d*x + c) - 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 1.54, size = 118, normalized size = 1.62

$$\frac{24\,(dx+c)a+9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-71\,a\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a + 9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 71*a*tan(1/2
*d*x + 1/2*c)^5 + 137*a*tan(1/2*d*x + 1/2*c)^3 - 33*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.48, size = 127, normalized size = 1.74

$$\frac{a\left(\tan^3\left(dx+c\right)\right)}{3d}-\frac{a\tan\left(dx+c\right)}{d}+ax+\frac{ca}{d}+\frac{a\left(\sin^5\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}-\frac{a\left(\sin^5\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^2}-\frac{a\left(\sin^3\left(dx+c\right)\right)}{8d}-\frac{3a\sin\left(dx+\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^4,x)
```

```
[Out] 1/3*a*tan(d*x+c)^3/d-a*tan(d*x+c)/d+a*x+1/d*c*a+1/4/d*a*sin(d*x+c)^5/cos(d*
x+c)^4-1/8/d*a*sin(d*x+c)^5/cos(d*x+c)^2-1/8*a*sin(d*x+c)^3/d-3/8*a*sin(d*x
+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.89, size = 102, normalized size = 1.40

$$\frac{16\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a+3\,a\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\mathrm{lo}\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a + 3*a*(2*(5*sin(
d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*lo
g(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 1.85, size = 134, normalized size = 1.84

$$a\,x-\frac{-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{71\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}-\frac{137\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x - ((11*a*tan(c/2 + (d*x)/2))/4 - (137*a*tan(c/2 + (d*x)/2)^3)/12 + (71*
a*tan(c/2 + (d*x)/2)^5)/12 - (5*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \tan^{4}{\left(c + dx \right)}\sec{\left(c + dx \right)}\, dx + \int \tan^{4}{\left(c + dx \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] a*(Integral(tan(c + d*x)**4*sec(c + d*x), x) + Integral(tan(c + d*x)**4, x)
)
```

3.13 $\int (a+a\sec(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=45

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)(a\sec(c+dx)+2a)}{2d}-ax$$

```
[Out] -a*x-1/2*a*arctanh(sin(d*x+c))/d+1/2*(2*a+a*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3770}

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)(a\sec(c+dx)+2a)}{2d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) - (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a + a*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\tan^2(c+dx)\,dx &= \frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d}-\frac{1}{2}\int (2a+a\sec(c+dx))\,dx\\ &= -ax+\frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d}-\frac{1}{2}a\int \sec(c+dx)\,dx\\ &= -ax-\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.33

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan(c+dx)}{d}-\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) - (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c
+ d*x])/d + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

fricas [B] time = 0.83, size = 87, normalized size = 1.93

$$-\frac{4\,adx\cos(dx+c)^2+a\cos(dx+c)^2\log(\sin(dx+c)+1)-a\cos(dx+c)^2\log(-\sin(dx+c)+1)-2\,(2\,a\cos(}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 + a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*c
os(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*cos(d*x + c) + a)*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.90, size = 88, normalized size = 1.96

$$-\frac{2\,(dx+c)a+a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.46, size = 78, normalized size = 1.73

$$-ax+\frac{a\tan(dx+c)}{d}-\frac{ca}{d}+\frac{a\left(\sin^3(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{a\sin(dx+c)}{2d}-\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^2,x)
```

```
[Out] -a*x+a*tan(d*x+c)/d-1/d*c*a+1/2/d*a*sin(d*x+c)^3/cos(d*x+c)^2+1/2*a*sin(d*x
+c)/d-1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.69, size = 65, normalized size = 1.44

$$-\frac{4\,(dx+c-\tan(dx+c))a+a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c - tan(d*x + c))*a + a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 1.14, size = 80, normalized size = 1.78

$$\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-a\,x-\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
- 2*tan(c/2 + (d*x)/2)^2 + 1)) - a*x - (a*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \tan^2(c+dx)\sec(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] a*(Integral(tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)**2, x)
)
```

3.14 $\int \cot^2(c + dx)(a + a\sec(c + dx))\,dx$

Optimal. Leaf size=26

$$-\frac{\cot(c + dx)(a\sec(c + dx) + a)}{d} - ax$$

```
[Out] -a*x-cot(d*x+c)*(a+a*sec(d*x+c))/d
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot(c + dx)(a\sec(c + dx) + a)}{d} - ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x]),x]
[Out] -(a*x) - (Cot[c + d*x]*(a + a*Sec[c + d*x]))/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c + dx)(a + a\sec(c + dx))\,dx &= -\frac{\cot(c + dx)(a + a\sec(c + dx))}{d} - \int a\,dx\\ &= -ax - \frac{\cot(c + dx)(a + a\sec(c + dx))}{d}\end{aligned}$$

Mathematica [C] time = 0.03, size = 43, normalized size = 1.65

$$-\frac{a\cot(c + dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c + dx)\right)}{d} - \frac{a\csc(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x]),x]
[Out] -((a*Csc[c + d*x])/d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[c + d*x]^2])/d
```

fricas [A] time = 0.94, size = 33, normalized size = 1.27

$$-\frac{adx\sin(dx + c) + a\cos(dx + c) + a}{d\sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -(a*d*x*sin(d*x + c) + a*cos(d*x + c) + a)/(d*sin(d*x + c))
```

giac [A] time = 0.23, size = 26, normalized size = 1.00

$$-\frac{(dx+c)a+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -((d*x + c)*a + a/tan(1/2*d*x + 1/2*c))/d
```

maple [A] time = 0.48, size = 35, normalized size = 1.35

$$\frac{a\left(-\cot\left(dx+c\right)-dx-c\right)-\frac{a}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c)),x)
[Out] 1/d*(a*(-cot(d*x+c)-d*x-c)-a/sin(d*x+c))
```

maxima [A] time = 1.22, size = 31, normalized size = 1.19

$$-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a+\frac{a}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -((d*x + c + 1/tan(d*x + c))*a + a/sin(d*x + c))/d
```

mupad [B] time = 1.07, size = 19, normalized size = 0.73

$$-\frac{a\left(\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x)),x)
[Out] -(a*(cot(c/2 + (d*x)/2) + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cot^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(cot(c + d*x)**2*sec(c + d*x), x) + Integral(cot(c + d*x)**2, x)
)
```

3.15 $\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=55

$$-\frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}+\frac{\cot(c+dx)(2a\sec(c+dx)+3a)}{3d}+ax$$

```
[Out] a*x-1/3*cot(d*x+c)^3*(a+a*sec(d*x+c))/d+1/3*cot(d*x+c)*(3*a+2*a*sec(d*x+c))
/d
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}+\frac{\cot(c+dx)(2a\sec(c+dx)+3a)}{3d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^3*(a + a*Sec[c + d*x]))/(3*d) + (Cot[c + d*x]*(3*a + 2*
a*Sec[c + d*x]))/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{1}{3}\int \cot^2(c+dx)(-3a-2a\sec(c+dx\\ &= -\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2a\sec(c+dx))}{3d}+\frac{1}{3}\\ &= ax-\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2a\sec(c+dx))}{3d}\end{aligned}$$

Mathematica [C] time = 0.04, size = 62, normalized size = 1.13

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]^3*Hypergeom
etric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d)
```

fricas [A] time = 0.63, size = 72, normalized size = 1.31

$$\frac{4\,a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+3\,\left(adx\cos\left(dx+c\right)-adx\right)\sin\left(dx+c\right)-2\,a}{3\,\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(4*a*cos(d*x + c)^2 - a*cos(d*x + c) + 3*(a*d*x*cos(d*x + c) - a*d*x)*s
in(d*x + c) - 2*a)/((d*cos(d*x + c) - d)*sin(d*x + c))
```

giac [A] time = 0.52, size = 56, normalized size = 1.02

$$\frac{12\,(dx+c)a-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(d*x + c)*a - 3*a*tan(1/2*d*x + 1/2*c) + (12*a*tan(1/2*d*x + 1/2*c
)^2 - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.79, size = 86, normalized size = 1.56

$$\frac{a\left(-\frac{\left(\cot^{3}(dx+c)\right)}{3}+\cot\left(dx+c\right)+dx+c\right)+a\left(-\frac{\cos^{4}(dx+c)}{3\sin(dx+c)^{3}}+\frac{\cos^{4}(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+a*(-1/3/sin(d*x+c)^3*cos(d*x+c)
^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 1.16, size = 59, normalized size = 1.07

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^{2}-1}{\tan(dx+c)^{3}}\right)a+\frac{\left(3\,\sin(dx+c)^{2}-1\right)a}{\sin(dx+c)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + (3*sin(d*x +
 c)^2 - 1)*a/sin(d*x + c)^3)/d
```

mupad [B] time = 1.20, size = 53, normalized size = 0.96

$$a\,x-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}-\frac{\frac{a}{12}-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x - (a*tan(c/2 + (d*x)/2))/(4*d) - (a/12 - a*tan(c/2 + (d*x)/2)^2)/(d*tan
(c/2 + (d*x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cot^4(c+dx)\sec(c+dx)\,dx + \int \cot^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**4*sec(c + d*x), x) + Integral(cot(c + d*x)**4, x)
)
```

3.16 $\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=84

$$-\frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}+\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{15d}-\frac{\cot(c+dx)(8a\sec(c+dx)+15a)}{15d}-ax$$

```
[Out] -a*x-1/5*cot(d*x+c)^5*(a+a*sec(d*x+c))/d+1/15*cot(d*x+c)^3*(5*a+4*a*sec(d*x
+c))/d-1/15*cot(d*x+c)*(15*a+8*a*sec(d*x+c))/d
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}+\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{15d}-\frac{\cot(c+dx)(8a\sec(c+dx)+15a)}{15d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]^5*(a + a*Sec[c + d*x]))/(5*d) + (Cot[c + d*x]^3*(5*a
 + 4*a*Sec[c + d*x]))/(15*d) - (Cot[c + d*x]*(15*a + 8*a*Sec[c + d*x]))/(15
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d}+\frac{1}{5}\int \cot^4(c+dx)(-5a-4a\sec(c+\\ &= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d}\\ &= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d}\\ &= -ax-\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4a\sec(c+\end{aligned}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.94

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d}-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

[Out] -((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*d)

fricas [A] time = 0.51, size = 139, normalized size = 1.65

$$-\frac{23\,a\cos\left(dx+c\right)^{4}-8\,a\cos\left(dx+c\right)^{3}-27\,a\cos\left(dx+c\right)^{2}+7\,a\cos\left(dx+c\right)+15\left(adx\cos\left(dx+c\right)^{3}-adx\cos\left(dx\right.\right.}{15\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)+d\right)\sin\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/15*(23*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^3 - 27*a*cos(d*x + c)^2 + 7*a*cos(d*x + c) + 15*(a*d*x*cos(d*x + c)^3 - a*d*x*cos(d*x + c)^2 - a*d*x*cos(d*x + c) + a*d*x)*sin(d*x + c) + 8*a)/((d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))

giac [A] time = 0.31, size = 83, normalized size = 0.99

$$-\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+240\,(dx+c)a-90\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] -1/240*(5*a*tan(1/2*d*x + 1/2*c)^3 + 240*(d*x + c)*a - 90*a*tan(1/2*d*x + 1/2*c) + 3*(80*a*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1/2*d*x + 1/2*c)^2 + a)/tan(1/2*d*x + 1/2*c)^5)/d

maple [A] time = 0.81, size = 129, normalized size = 1.54

$$\frac{a\left(-\frac{\left(\cot^{5}(dx+c)\right)}{5}+\frac{\left(\cot^{3}(dx+c)\right)}{3}-\cot\left(dx+c\right)-dx-c\right)+a\left(-\frac{\cos^{6}(dx+c)}{5\sin(dx+c)^{5}}+\frac{\cos^{6}(dx+c)}{15\sin(dx+c)^{3}}-\frac{\cos^{6}(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4(\cos\right.}{}}{5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c)),x)

[Out] 1/d*(a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))

maxima [A] time = 0.62, size = 79, normalized size = 0.94

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^{4}-5\,\tan(dx+c)^{2}+3}{\tan(dx+c)^{5}}\right)a+\frac{\left(15\,\sin(dx+c)^{4}-10\,\sin(dx+c)^{2}+3\right)a}{\sin(dx+c)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*a + (15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a/sin(d*x + c)^5)/d

mupad [B] time = 1.45, size = 156, normalized size = 1.86

$$-\frac{a\left(3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-90\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+240\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-3\right.}{240\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(3*cos(c/2 + (d*x)/2)^8 + 5*sin(c/2 + (d*x)/2)^8 - 90*cos(c/2 + (d*x)/2
)^2*sin(c/2 + (d*x)/2)^6 + 240*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 -
30*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 + 240*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2)^5*(c + d*x)))/(240*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2
)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot^6(c+dx)\sec(c+dx)\,dx+\int\cot^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**6*sec(c + d*x), x) + Integral(cot(c + d*x)**6, x)
)
```

3.17 $\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}+\frac{\cot^5(c+dx)(6a\sec(c+dx)+7a)}{35d}-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{105d}+\frac{\cot(c$$

```
[Out] a*x-1/7*cot(d*x+c)^7*(a+a*sec(d*x+c))/d+1/35*cot(d*x+c)^5*(7*a+6*a*sec(d*x+
c))/d+1/35*cot(d*x+c)*(35*a+16*a*sec(d*x+c))/d-1/105*cot(d*x+c)^3*(35*a+24*
a*sec(d*x+c))/d
```

Rubi [A] time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3882, 8}

$$-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}+\frac{\cot^5(c+dx)(6a\sec(c+dx)+7a)}{35d}-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{105d}+\frac{\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^7*(a + a*Sec[c + d*x]))/(7*d) + (Cot[c + d*x]^5*(7*a +
6*a*Sec[c + d*x]))/(35*d) + (Cot[c + d*x]*(35*a + 16*a*Sec[c + d*x]))/(35*d
) - (Cot[c + d*x]^3*(35*a + 24*a*Sec[c + d*x]))/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{1}{7}\int \cot^6(c+dx)(-7a-6a\sec(c+dx\\ &= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d}+\\ &= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d}-\\ &= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d}+\\ &= ax-\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d}\end{aligned}$$

Mathematica [C] time = 0.06, size = 92, normalized size = 0.83

$$-\frac{a\cot^7(c+dx)\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\tan^2(c+dx)\right)}{7d}-\frac{a\csc^7(c+dx)}{7d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{d}+\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*a*Csc[c + d*x]^5)/(5*d) - (a
*Csc[c + d*x]^7)/(7*d) - (a*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2,
 -Tan[c + d*x]^2])/(7*d)
```

fricas [B] time = 0.74, size = 210, normalized size = 1.89

$$\frac{176\,a\cos\left(dx+c\right)^{6}-71\,a\cos\left(dx+c\right)^{5}-335\,a\cos\left(dx+c\right)^{4}+125\,a\cos\left(dx+c\right)^{3}+225\,a\cos\left(dx+c\right)^{2}-57\,a}{105\left(d\cos\left(dx+c\right)^{5}-d\cos\left(dx+c\right)^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(176*a*cos(d*x + c)^6 - 71*a*cos(d*x + c)^5 - 335*a*cos(d*x + c)^4 +
125*a*cos(d*x + c)^3 + 225*a*cos(d*x + c)^2 - 57*a*cos(d*x + c) + 105*(a*d*
x*cos(d*x + c)^5 - a*d*x*cos(d*x + c)^4 - 2*a*d*x*cos(d*x + c)^3 + 2*a*d*x*
cos(d*x + c)^2 + a*d*x*cos(d*x + c) - a*d*x)*sin(d*x + c) - 48*a)/((d*cos(d
*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*
cos(d*x + c) - d)*sin(d*x + c))
```

giac [A] time = 0.35, size = 113, normalized size = 1.02

$$-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6720\,(dx+c)a+3045\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{6720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(21*a*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^3 - 6720*
(d*x + c)*a + 3045*a*tan(1/2*d*x + 1/2*c) - (6720*a*tan(1/2*d*x + 1/2*c)^6
- 1015*a*tan(1/2*d*x + 1/2*c)^4 + 168*a*tan(1/2*d*x + 1/2*c)^2 - 15*a)/tan(
1/2*d*x + 1/2*c)^7)/d
```

maple [A] time = 0.95, size = 162, normalized size = 1.46

$$\frac{a\left(-\frac{\left(\cot^{7}(dx+c)\right)}{7}+\frac{\left(\cot^{5}(dx+c)\right)}{5}-\frac{\left(\cot^{3}(dx+c)\right)}{3}+\cot\left(dx+c\right)+dx+c\right)+a\left(-\frac{\cos^{8}(dx+c)}{7\sin(dx+c)^{7}}+\frac{\cos^{8}(dx+c)}{35\sin(dx+c)^{5}}-\frac{\cos^{8}(dx+c)}{35\sin(dx+c)^{3}}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+
c)+a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/si
n(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^6+
6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 1.03, size = 100, normalized size = 0.90

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^{6}-35\,\tan(dx+c)^{4}+21\,\tan(dx+c)^{2}-15}{\tan(dx+c)^{7}}\right)a+\frac{3\left(35\,\sin(dx+c)^{6}-35\,\sin(dx+c)^{4}+21\,\sin(dx+c)^{2}-5\right)a}{\sin(dx+c)^{7}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a + 3*(35*sin(d*x + c)^6 - 35*sin(d*x + c)
^4 + 21*sin(d*x + c)^2 - 5)*a/sin(d*x + c)^7)/d
```

mupad [B] time = 1.98, size = 204, normalized size = 1.84

$$-\frac{a\left(15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-280\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+3045\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(15*cos(c/2 + (d*x)/2)^12 + 21*sin(c/2 + (d*x)/2)^12 - 280*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 3045*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 1015*cos(c/2 + (d*x
)/2)^8*sin(c/2 + (d*x)/2)^4 - 168*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^
2 - 6720*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7*(c + d*x)))/(6720*d*cos(
c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.18 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=140

$$-\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}+\frac{\cot^7(c+dx)(8a\sec(c+dx)+9a)}{63d}-\frac{\cot^5(c+dx)(16a\sec(c+dx)+21a)}{105d}+\frac{\mathrm{co}}{}$$

```
[Out] -a*x-1/9*cot(d*x+c)^9*(a+a*sec(d*x+c))/d+1/63*cot(d*x+c)^7*(9*a+8*a*sec(d*x
+c))/d-1/105*cot(d*x+c)^5*(21*a+16*a*sec(d*x+c))/d+1/315*cot(d*x+c)^3*(105*
a+64*a*sec(d*x+c))/d-1/315*cot(d*x+c)*(315*a+128*a*sec(d*x+c))/d
```

Rubi [A] time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3882, 8}

$$-\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}+\frac{\cot^7(c+dx)(8a\sec(c+dx)+9a)}{63d}-\frac{\cot^5(c+dx)(16a\sec(c+dx)+21a)}{105d}+\frac{\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]^9*(a + a*Sec[c + d*x]))/(9*d) + (Cot[c + d*x]^7*(9*a
 + 8*a*Sec[c + d*x]))/(63*d) - (Cot[c + d*x]^5*(21*a + 16*a*Sec[c + d*x]))/
(105*d) + (Cot[c + d*x]^3*(105*a + 64*a*Sec[c + d*x]))/(315*d) - (Cot[c + d
*x]*(315*a + 128*a*Sec[c + d*x]))/(315*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d}+\frac{1}{9}\int \cot^8(c+dx)(-9a-8a\sec(c\\
&= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d}+\frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d}\\
&= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d}+\frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d}\\
&= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d}+\frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d}\\
&= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d}+\frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d}\\
&= -ax-\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d}+\frac{\cot^7(c+dx)(9a+8a\sec(c+}{63d}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 111, normalized size = 0.79

$$-\frac{a\cot^9(c+dx)\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};-\tan^2(c+dx)\right)}{9d}-\frac{a\csc^9(c+dx)}{9d}+\frac{4a\csc^7(c+dx)}{7d}-\frac{6a\csc^5(c+dx)}{5d}+\frac{4a\csc^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (4*a*Csc[c + d*x]^3)/(3*d) - (6*a*Csc[c + d*x]^5)/(
5*d) + (4*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d) - (a*Cot[c + d
*x]^9*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[c + d*x]^2])/(9*d)
```

fricas [B] time = 0.89, size = 279, normalized size = 1.99

$$-\frac{563\,a\cos\left(dx+c\right)^{8}-248\,a\cos\left(dx+c\right)^{7}-1498\,a\cos\left(dx+c\right)^{6}+658\,a\cos\left(dx+c\right)^{5}+1610\,a\cos\left(dx+c\right)^{4}-60}{315\,\left(d\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/315*(563*a*cos(d*x + c)^8 - 248*a*cos(d*x + c)^7 - 1498*a*cos(d*x + c)^6
 + 658*a*cos(d*x + c)^5 + 1610*a*cos(d*x + c)^4 - 602*a*cos(d*x + c)^3 - 76
3*a*cos(d*x + c)^2 + 187*a*cos(d*x + c) + 315*(a*d*x*cos(d*x + c)^7 - a*d*x
*cos(d*x + c)^6 - 3*a*d*x*cos(d*x + c)^5 + 3*a*d*x*cos(d*x + c)^4 + 3*a*d*x
*cos(d*x + c)^3 - 3*a*d*x*cos(d*x + c)^2 - a*d*x*cos(d*x + c) + a*d*x)*sin(
d*x + c) + 128*a)/((d*cos(d*x + c)^7 - d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^
5 + 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 - 3*d*cos(d*x + c)^2 - d*cos(d*
x + c) + d)*sin(d*x + c))
```

giac [A] time = 0.51, size = 140, normalized size = 1.00

$$-\frac{45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4830\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+80640\,(dx+c)a-40950\,a\tan\left(\frac{1}{2}\,d\right.}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(45*a*tan(1/2*d*x + 1/2*c)^7 - 630*a*tan(1/2*d*x + 1/2*c)^5 + 4830
*a*tan(1/2*d*x + 1/2*c)^3 + 80640*(d*x + c)*a - 40950*a*tan(1/2*d*x + 1/2*c
) + (80640*a*tan(1/2*d*x + 1/2*c)^8 - 13650*a*tan(1/2*d*x + 1/2*c)^6 + 2898
*a*tan(1/2*d*x + 1/2*c)^4 - 450*a*tan(1/2*d*x + 1/2*c)^2 + 35*a)/tan(1/2*d*
x + 1/2*c)^9)/d
```

maple [A] time = 0.97, size = 205, normalized size = 1.46

$$\frac{a\left(-\frac{\left(\cot^{9}(dx+c)\right)}{9}+\frac{\left(\cot^{7}(dx+c)\right)}{7}-\frac{\left(\cot^{5}(dx+c)\right)}{5}+\frac{\left(\cot^{3}(dx+c)\right)}{3}-\cot\left(dx+c\right)-dx-c\right)+a\left(-\frac{\cos^{10}(dx+c)}{9\sin(dx+c)^{9}}+\frac{\cos^{10}(dx+c)}{63\sin(dx+c)^{7}}-\frac{\mathrm{co}}{105}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^
3-cot(d*x+c)-d*x-c)+a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^10+1/63/sin(d*x+c)^7*co
s(d*x+c)^10-1/105/sin(d*x+c)^5*cos(d*x+c)^10+1/63/sin(d*x+c)^3*cos(d*x+c)^1
0-1/9/sin(d*x+c)*cos(d*x+c)^10-1/9*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48
/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.82, size = 119, normalized size = 0.85

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan(dx+c)^{8}-105\,\tan(dx+c)^{6}+63\,\tan(dx+c)^{4}-45\,\tan(dx+c)^{2}+35}{\tan(dx+c)^{9}}\right)a+\frac{\left(315\,\sin(dx+c)^{8}-420\,\sin(dx+c)^{6}+378\,\sin(dx+\right.}{\sin(dx+c)^{9}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a + (315*sin(d*x + c
)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35)*a/
sin(d*x + c)^9)/d
```

mupad [B] time = 3.18, size = 252, normalized size = 1.80

$$-\frac{a\left(35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+45\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+4830\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^10*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(35*cos(c/2 + (d*x)/2)^16 + 45*sin(c/2 + (d*x)/2)^16 - 630*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 4830*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^12 - 40950*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 80640*cos(c/2 +
(d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 13650*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*
x)/2)^6 + 2898*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 450*cos(c/2 + (
d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 80640*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x
)/2)^9*(c + d*x)))/(80640*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.19 $\int (a + a\sec(c + dx))^2 \tan^9(c + dx)\, dx$

Optimal. Leaf size=192

$$\frac{a^2\sec^{10}(c+dx)}{10d}+\frac{2a^2\sec^9(c+dx)}{9d}-\frac{3a^2\sec^8(c+dx)}{8d}-\frac{8a^2\sec^7(c+dx)}{7d}+\frac{a^2\sec^6(c+dx)}{3d}+\frac{12a^2\sec^5(c+dx)}{5d}+\frac{a^2 s}{}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d-3/2*a^2*sec(d*x+c)^2/d-8/3*a^2*sec
(d*x+c)^3/d+1/2*a^2*sec(d*x+c)^4/d+12/5*a^2*sec(d*x+c)^5/d+1/3*a^2*sec(d*x+
c)^6/d-8/7*a^2*sec(d*x+c)^7/d-3/8*a^2*sec(d*x+c)^8/d+2/9*a^2*sec(d*x+c)^9/d
+1/10*a^2*sec(d*x+c)^10/d
```

Rubi [A] time = 0.10, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3879, 88}

$$\frac{a^2\sec^{10}(c+dx)}{10d}+\frac{2a^2\sec^9(c+dx)}{9d}-\frac{3a^2\sec^8(c+dx)}{8d}-\frac{8a^2\sec^7(c+dx)}{7d}+\frac{a^2\sec^6(c+dx)}{3d}+\frac{12a^2\sec^5(c+dx)}{5d}+\frac{a^2 s}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d - (3*a^2*Sec[c + d*x]
^2)/(2*d) - (8*a^2*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]^4)/(2*d) + (12
*a^2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(3*d) - (8*a^2*Sec[c + d*
x]^7)/(7*d) - (3*a^2*Sec[c + d*x]^8)/(8*d) + (2*a^2*Sec[c + d*x]^9)/(9*d) +
 (a^2*Sec[c + d*x]^10)/(10*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^6}{x^{11}}\,dx, x, \cos(c+dx)\right)}{a^8 d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^{10}}{x^{11}}+\frac{2a^{10}}{x^{10}}-\frac{3a^{10}}{x^9}-\frac{8a^{10}}{x^8}+\frac{2a^{10}}{x^7}+\frac{12a^{10}}{x^6}+\frac{2a^{10}}{x^5}-\frac{8a^{10}}{x^4}-\frac{3a^{10}}{x^3}+\right.\right.}{a^8 d}\\ &= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{3a^2\sec^2(c+dx)}{2d}-\frac{8a^2\sec^3(c}{3d}\end{aligned}$$

Mathematica [A] time = 0.51, size = 140, normalized size = 0.73

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-252\sec^{10}(c+dx)-560\sec^9(c+dx)+945\sec^8(c+dx)+2880\sec^7(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -1/10080*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(2520*Log[Cos[c + d*x
]] - 5040*Sec[c + d*x] + 3780*Sec[c + d*x]^2 + 6720*Sec[c + d*x]^3 - 1260*S
ec[c + d*x]^4 - 6048*Sec[c + d*x]^5 - 840*Sec[c + d*x]^6 + 2880*Sec[c + d*x
]^7 + 945*Sec[c + d*x]^8 - 560*Sec[c + d*x]^9 - 252*Sec[c + d*x]^10))/d
```

fricas [A] time = 0.87, size = 156, normalized size = 0.81

$$-\frac{2520\,a^2\cos\left(dx+c\right)^{10}\log\left(-\cos\left(dx+c\right)\right)-5040\,a^2\cos\left(dx+c\right)^9+3780\,a^2\cos\left(dx+c\right)^8+6720\,a^2\cos\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a^2*cos(d*x + c)^10*log(-cos(d*x + c)) - 5040*a^2*cos(d*x + c
)^9 + 3780*a^2*cos(d*x + c)^8 + 6720*a^2*cos(d*x + c)^7 - 1260*a^2*cos(d*x
+ c)^6 - 6048*a^2*cos(d*x + c)^5 - 840*a^2*cos(d*x + c)^4 + 2880*a^2*cos(d*
x + c)^3 + 945*a^2*cos(d*x + c)^2 - 560*a^2*cos(d*x + c) - 252*a^2)/(d*cos(
d*x + c)^10)
```

giac [A] time = 25.57, size = 342, normalized size = 1.78

$$\frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{11477\,a^2+\frac{119810\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{566865\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11477*a^2 + 1
19810*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 566865*a^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 1605720*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 3031770*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 2995020*
a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 2171610*a^2*(cos(d*x + c) -
 1)^6/(cos(d*x + c) + 1)^6 + 1114200*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c)
 + 1)^7 + 382545*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 78850*a^2*
(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9 + 7381*a^2*(cos(d*x + c) - 1)^10/
(cos(d*x + c) + 1)^10)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^10)/d
```

maple [A] time = 0.86, size = 327, normalized size = 1.70

$$\frac{a^2\left(\tan^8\left(dx+c\right)\right)}{8d}-\frac{a^2\left(\tan^6\left(dx+c\right)\right)}{6d}+\frac{a^2\left(\tan^4\left(dx+c\right)\right)}{4d}-\frac{a^2\left(\tan^2\left(dx+c\right)\right)}{2d}-\frac{a^2\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{2a^2\left(\sin^{10}\right.}{9d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x)
```

```
[Out] 1/8/d*a^2*tan(d*x+c)^8-1/6/d*a^2*tan(d*x+c)^6+1/4*a^2*tan(d*x+c)^4/d-1/2*a^
2*tan(d*x+c)^2/d-a^2*ln(cos(d*x+c))/d+2/9/d*a^2*sin(d*x+c)^10/cos(d*x+c)^9-
2/63/d*a^2*sin(d*x+c)^10/cos(d*x+c)^7+2/105/d*a^2*sin(d*x+c)^10/cos(d*x+c)^
5-2/63/d*a^2*sin(d*x+c)^10/cos(d*x+c)^3+2/9/d*a^2*sin(d*x+c)^10/cos(d*x+c)+
256/315*a^2*cos(d*x+c)/d+2/9/d*a^2*cos(d*x+c)*sin(d*x+c)^8+16/63/d*a^2*cos(
d*x+c)*sin(d*x+c)^6+32/105/d*a^2*cos(d*x+c)*sin(d*x+c)^4+128/315/d*a^2*cos(
d*x+c)*sin(d*x+c)^2+1/10/d*a^2*sin(d*x+c)^10/cos(d*x+c)^10
```

maxima [A] time = 0.32, size = 149, normalized size = 0.78

$$-\frac{2520\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{5040\,a^2\cos(dx+c)^9-3780\,a^2\cos(dx+c)^8-6720\,a^2\cos(dx+c)^7+1260\,a^2\cos(dx+c)^6+6048\,a^2\cos(dx+c)^5+840\,a^2 \cdots}{\cos(dx+c)^{10}}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a^2*log(cos(d*x + c)) - (5040*a^2*cos(d*x + c)^9 - 3780*a^2*c
os(d*x + c)^8 - 6720*a^2*cos(d*x + c)^7 + 1260*a^2*cos(d*x + c)^6 + 6048*a^
2*cos(d*x + c)^5 + 840*a^2*cos(d*x + c)^4 - 2880*a^2*cos(d*x + c)^3 - 945*a
^2*cos(d*x + c)^2 + 560*a^2*cos(d*x + c) + 252*a^2)/cos(d*x + c)^10)/d
```

mupad [B] time = 5.14, size = 308, normalized size = 1.60

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-20\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+\frac{272\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{3}-\frac{740\,a^2\tan\left(\cdots\right.}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+45\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-120\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+2 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - ((1150*a^2*tan(c/2 + (d*x)/2)^2)/63
 - (652*a^2*tan(c/2 + (d*x)/2)^4)/7 + (2000*a^2*tan(c/2 + (d*x)/2)^6)/7 - 5
88*a^2*tan(c/2 + (d*x)/2)^8 + (2252*a^2*tan(c/2 + (d*x)/2)^10)/5 - (740*a^2
*tan(c/2 + (d*x)/2)^12)/3 + (272*a^2*tan(c/2 + (d*x)/2)^14)/3 - 20*a^2*tan(
c/2 + (d*x)/2)^16 + 2*a^2*tan(c/2 + (d*x)/2)^18 - (512*a^2)/315)/(d*(45*tan
(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 - 120*tan(c/2 + (d*x)/2)^6 + 21
0*tan(c/2 + (d*x)/2)^8 - 252*tan(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)
^12 - 120*tan(c/2 + (d*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^16 - 10*tan(c/2 + (
d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1))
```

sympy [A] time = 33.75, size = 314, normalized size = 1.64

$$\begin{cases} \frac{a^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^2\tan^8(c+dx)\sec^2(c+dx)}{10d}+\frac{2a^2\tan^8(c+dx)\sec(c+dx)}{9d}+\frac{a^2\tan^8(c+dx)}{8d}-\frac{a^2\tan^6(c+dx)\sec^2(c+dx)}{10d}-\frac{16a^2\tan^6(\cdots}{} \\ x\left(a\sec(c)+a\right)^2\tan^9(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**8*sec(c
 + d*x)**2/(10*d) + 2*a**2*tan(c + d*x)**8*sec(c + d*x)/(9*d) + a**2*tan(c
+ d*x)**8/(8*d) - a**2*tan(c + d*x)**6*sec(c + d*x)**2/(10*d) - 16*a**2*tan
(c + d*x)**6*sec(c + d*x)/(63*d) - a**2*tan(c + d*x)**6/(6*d) + a**2*tan(c
+ d*x)**4*sec(c + d*x)**2/(10*d) + 32*a**2*tan(c + d*x)**4*sec(c + d*x)/(10
5*d) + a**2*tan(c + d*x)**4/(4*d) - a**2*tan(c + d*x)**2*sec(c + d*x)**2/(1
0*d) - 128*a**2*tan(c + d*x)**2*sec(c + d*x)/(315*d) - a**2*tan(c + d*x)**2
/(2*d) + a**2*sec(c + d*x)**2/(10*d) + 256*a**2*sec(c + d*x)/(315*d), Ne(d,
 0)), (x*(a*sec(c) + a)**2*tan(c)**9, True))
```

3.20 $\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx$

Optimal. Leaf size=132

$$\frac{a^2\sec^8(c+dx)}{8d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{a^2\sec^6(c+dx)}{3d}-\frac{6a^2\sec^5(c+dx)}{5d}+\frac{2a^2\sec^3(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}-\frac{2a^2}{}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a^2*sec(d*x+c)/d+a^2*sec(d*x+c)^2/d+2*a^2*sec(d*x+c)
^3/d-6/5*a^2*sec(d*x+c)^5/d-1/3*a^2*sec(d*x+c)^6/d+2/7*a^2*sec(d*x+c)^7/d+1
/8*a^2*sec(d*x+c)^8/d
```

Rubi [A] time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{a^2\sec^8(c+dx)}{8d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{a^2\sec^6(c+dx)}{3d}-\frac{6a^2\sec^5(c+dx)}{5d}+\frac{2a^2\sec^3(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}-\frac{2a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (a^2*Sec[c + d*x]^2)/d
 + (2*a^2*Sec[c + d*x]^3)/d - (6*a^2*Sec[c + d*x]^5)/(5*d) - (a^2*Sec[c + d
*x]^6)/(3*d) + (2*a^2*Sec[c + d*x]^7)/(7*d) + (a^2*Sec[c + d*x]^8)/(8*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^5}{x^9}\,dx,x,\cos(c+dx)\right)}{a^6d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^8}{x^9}+\frac{2a^8}{x^8}-\frac{2a^8}{x^7}-\frac{6a^8}{x^6}+\frac{6a^8}{x^4}+\frac{2a^8}{x^3}-\frac{2a^8}{x^2}-\frac{a^8}{x}\right)dx,x,\cos(c+\right.}{a^6d}\\
&= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}+\frac{2a^2\sec^3(c}{d}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 110, normalized size = 0.83

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(105\sec^8(c+dx)+240\sec^7(c+dx)-280\sec^6(c+dx)-1008\sec^5(c+\right.}{3360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(840*Log[Cos[c + d*x]] - 1680*
Sec[c + d*x] + 840*Sec[c + d*x]^2 + 1680*Sec[c + d*x]^3 - 1008*Sec[c + d*x]
^5 - 280*Sec[c + d*x]^6 + 240*Sec[c + d*x]^7 + 105*Sec[c + d*x]^8))/(3360*d
)
```

fricas [A] time = 0.73, size = 117, normalized size = 0.89

$$\frac{840\,a^2\cos(dx+c)^8\log(-\cos(dx+c)) - 1680\,a^2\cos(dx+c)^7 + 840\,a^2\cos(dx+c)^6 + 1680\,a^2\cos(dx+c)^5 - 1}{840\,d\cos(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(840*a^2*cos(d*x + c)^8*log(-cos(d*x + c)) - 1680*a^2*cos(d*x + c)^7
+ 840*a^2*cos(d*x + c)^6 + 1680*a^2*cos(d*x + c)^5 - 1008*a^2*cos(d*x + c)^
3 - 280*a^2*cos(d*x + c)^2 + 240*a^2*cos(d*x + c) + 105*a^2)/(d*cos(d*x + c
)^8)
```

giac [B] time = 11.00, size = 292, normalized size = 2.21

$$-\frac{840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{3819\,a^2 + \frac{32232\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{120372\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{261464\,a}{(\cos}}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/840*(840*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 840*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3819*a^2 + 3223
2*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 120372*a^2*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 261464*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 + 258370*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 175448*a^2*(co
s(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 77364*a^2*(cos(d*x + c) - 1)^6/(co
s(d*x + c) + 1)^6 + 19944*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 2
283*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8)/((cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 1)^8)/d
```

maple [B] time = 0.78, size = 264, normalized size = 2.00

$$\frac{a^2\left(\tan^6(dx+c)\right)}{6d} - \frac{a^2\left(\tan^4(dx+c)\right)}{4d} + \frac{a^2\left(\tan^2(dx+c)\right)}{2d} + \frac{a^2\ln(\cos(dx+c))}{d} + \frac{2a^2\left(\sin^8(dx+c)\right)}{7d\cos(dx+c)^7} - \frac{2a^2\left(\sin^8(dx\right.}{35d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x)
```

```
[Out] 1/6/d*a^2*tan(d*x+c)^6-1/4*a^2*tan(d*x+c)^4/d+1/2*a^2*tan(d*x+c)^2/d+a^2*ln
(cos(d*x+c))/d+2/7/d*a^2*sin(d*x+c)^8/cos(d*x+c)^7-2/35/d*a^2*sin(d*x+c)^8/
cos(d*x+c)^5+2/35/d*a^2*sin(d*x+c)^8/cos(d*x+c)^3-2/7/d*a^2*sin(d*x+c)^8/co
s(d*x+c)-32/35*a^2*cos(d*x+c)/d-2/7/d*a^2*cos(d*x+c)*sin(d*x+c)^6-12/35/d*a
^2*cos(d*x+c)*sin(d*x+c)^4-16/35/d*a^2*cos(d*x+c)*sin(d*x+c)^2+1/8/d*a^2*si
n(d*x+c)^8/cos(d*x+c)^8
```

maxima [A] time = 0.43, size = 110, normalized size = 0.83

$$\frac{840\,a^2\log(\cos(dx+c)) - \frac{1680\,a^2\cos(dx+c)^7 - 840\,a^2\cos(dx+c)^6 - 1680\,a^2\cos(dx+c)^5 + 1008\,a^2\cos(dx+c)^3 + 280\,a^2\cos(dx+c)^2 - 240\,a^2\cos(dx}{\cos(dx+c)^8}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/840*(840*a^2*log(cos(d*x + c)) - (1680*a^2*cos(d*x + c)^7 - 840*a^2*cos(d
*x + c)^6 - 1680*a^2*cos(d*x + c)^5 + 1008*a^2*cos(d*x + c)^3 + 280*a^2*cos
(d*x + c)^2 - 240*a^2*cos(d*x + c) - 105*a^2)/cos(d*x + c)^8)/d
```

mupad [B] time = 4.82, size = 249, normalized size = 1.89

$$\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-16\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\frac{170\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{3}-\frac{352\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}+\frac{2386\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{15}-}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-56\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+70\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-56\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((582*a^2*tan(c/2 + (d*x)/2)^2)/35 - (336*a^2*tan(c/2 + (d*x)/2)^4)/5 + (23
86*a^2*tan(c/2 + (d*x)/2)^6)/15 - (352*a^2*tan(c/2 + (d*x)/2)^8)/3 + (170*a
^2*tan(c/2 + (d*x)/2)^10)/3 - 16*a^2*tan(c/2 + (d*x)/2)^12 + 2*a^2*tan(c/2
+ (d*x)/2)^14 - (64*a^2)/35)/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x
)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (
d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 +
 (d*x)/2)^16 + 1)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 14.18, size = 252, normalized size = 1.91

$$\begin{cases} -\frac{a^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^2\tan^6(c+dx)\sec^2(c+dx)}{8d}+\frac{2a^2\tan^6(c+dx)\sec(c+dx)}{7d}+\frac{a^2\tan^6(c+dx)}{6d}-\frac{a^2\tan^4(c+dx)\sec^2(c+dx)}{8d}-\frac{12a^2}{} \\ x\left(a\sec(c)+a\right)^2\tan^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**6*sec(
c + d*x)**2/(8*d) + 2*a**2*tan(c + d*x)**6*sec(c + d*x)/(7*d) + a**2*tan(c
+ d*x)**6/(6*d) - a**2*tan(c + d*x)**4*sec(c + d*x)**2/(8*d) - 12*a**2*tan(
c + d*x)**4*sec(c + d*x)/(35*d) - a**2*tan(c + d*x)**4/(4*d) + a**2*tan(c +
 d*x)**2*sec(c + d*x)**2/(8*d) + 16*a**2*tan(c + d*x)**2*sec(c + d*x)/(35*d
) + a**2*tan(c + d*x)**2/(2*d) - a**2*sec(c + d*x)**2/(8*d) - 32*a**2*sec(c
 + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**7, True))
```

3.21 $\int (a + a\sec(c + dx))^2 \tan^5(c + dx)\,dx$

Optimal. Leaf size=120

$$\frac{a^2\sec^6(c+dx)}{6d}+\frac{2a^2\sec^5(c+dx)}{5d}-\frac{a^2\sec^4(c+dx)}{4d}-\frac{4a^2\sec^3(c+dx)}{3d}-\frac{a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(c$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d-1/2*a^2*sec(d*x+c)^2/d-4/3*a^2*sec
(d*x+c)^3/d-1/4*a^2*sec(d*x+c)^4/d+2/5*a^2*sec(d*x+c)^5/d+1/6*a^2*sec(d*x+c
)^6/d
```

Rubi [A] time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3879, 88}

$$\frac{a^2\sec^6(c+dx)}{6d}+\frac{2a^2\sec^5(c+dx)}{5d}-\frac{a^2\sec^4(c+dx)}{4d}-\frac{4a^2\sec^3(c+dx)}{3d}-\frac{a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d - (a^2*Sec[c + d*x]^2
)/(2*d) - (4*a^2*Sec[c + d*x]^3)/(3*d) - (a^2*Sec[c + d*x]^4)/(4*d) + (2*a^
2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(6*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^4}{x^7}\,dx,x,\cos(c+dx)\right)}{a^4d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^6}{x^7}+\frac{2a^6}{x^6}-\frac{a^6}{x^5}-\frac{4a^6}{x^4}-\frac{a^6}{x^3}+\frac{2a^6}{x^2}+\frac{a^6}{x}\right)dx,x,\cos(c+dx)\right)}{a^4d}\\ &= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\sec^2(c+dx)}{2d}-\frac{4a^2\sec^3(c\end{aligned}$$

Mathematica [A] time = 0.42, size = 125, normalized size = 1.04

$$\frac{a^2\sec^6(c+dx)(312\cos(c+dx)-5(-28\cos(3(c+dx))+6\cos(4(c+dx))-12\cos(5(c+dx))+18\cos(4(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (a^2*(312*Cos[c + d*x] - 5*(14 - 28*Cos[3*(c + d*x)] + 6*Cos[4*(c + d*x)] -
 12*Cos[5*(c + d*x)] + 30*Log[Cos[c + d*x]] + 18*Cos[4*(c + d*x)]*Log[Cos[c
 + d*x]] + 3*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 9*Cos[2*(c + d*x)]*(4 + 5
*Log[Cos[c + d*x]])))*Sec[c + d*x]^6)/(480*d)
```

fricas [A] time = 0.75, size = 104, normalized size = 0.87

$$-\frac{60\,a^2\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)-120\,a^2\cos\left(dx+c\right)^5+30\,a^2\cos\left(dx+c\right)^4+80\,a^2\cos\left(dx+c\right)^3+15}{60\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a^2*cos(d*x + c)^6*log(-cos(d*x + c)) - 120*a^2*cos(d*x + c)^5 +
30*a^2*cos(d*x + c)^4 + 80*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c)^2 - 24*
a^2*cos(d*x + c) - 10*a^2)/(d*cos(d*x + c)^6)
```

giac [B] time = 3.35, size = 242, normalized size = 2.02

$$\frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{275\,a^2+\frac{1770\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4845\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4780\,a^2(\cos(dx}{(\cos(dx+c)}}{\left(\frac{\cos(}{\cos(}\right.}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (275*a^2 + 1770*a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4845*a^2*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 4780*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925
*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^2*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 + 147*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^6)/d
```

maple [A] time = 0.74, size = 203, normalized size = 1.69

$$\frac{a^2\left(\tan^4\left(dx+c\right)\right)}{4d}-\frac{a^2\left(\tan^2\left(dx+c\right)\right)}{2d}-\frac{a^2\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{2a^2\left(\sin^6\left(dx+c\right)\right)}{5d\cos\left(dx+c\right)^5}-\frac{2a^2\left(\sin^6\left(dx+c\right)\right)}{15d\cos\left(dx+c\right)^3}+\frac{2a^2\left(\sin^6\right.}{5d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x)
```

```
[Out] 1/4*a^2*tan(d*x+c)^4/d-1/2*a^2*tan(d*x+c)^2/d-a^2*ln(cos(d*x+c))/d+2/5/d*a^
2*sin(d*x+c)^6/cos(d*x+c)^5-2/15/d*a^2*sin(d*x+c)^6/cos(d*x+c)^3+2/5/d*a^2*
sin(d*x+c)^6/cos(d*x+c)+16/15*a^2*cos(d*x+c)/d+2/5/d*a^2*cos(d*x+c)*sin(d*x
+c)^4+8/15/d*a^2*cos(d*x+c)*sin(d*x+c)^2+1/6/d*a^2*sin(d*x+c)^6/cos(d*x+c)^
6
```

maxima [A] time = 0.70, size = 97, normalized size = 0.81

$$-\frac{60\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{120\,a^2\cos(dx+c)^5-30\,a^2\cos(dx+c)^4-80\,a^2\cos(dx+c)^3-15\,a^2\cos(dx+c)^2+24\,a^2\cos(dx+c)+10\,a^2}{\cos(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a^2*log(cos(d*x + c)) - (120*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x
+ c)^4 - 80*a^2*cos(d*x + c)^3 - 15*a^2*cos(d*x + c)^2 + 24*a^2*cos(d*x + c
) + 10*a^2)/cos(d*x + c)^6)/d
```

mupad [B] time = 5.05, size = 192, normalized size = 1.60

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{92\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}-44\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+15\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - ((74*a^2*tan(c/2 + (d*x)/2)^2)/5 -
44*a^2*tan(c/2 + (d*x)/2)^4 + (92*a^2*tan(c/2 + (d*x)/2)^6)/3 - 12*a^2*tan(
c/2 + (d*x)/2)^8 + 2*a^2*tan(c/2 + (d*x)/2)^10 - (32*a^2)/15)/(d*(15*tan(c/
2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(
c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [A] time = 5.33, size = 189, normalized size = 1.58

$$\begin{cases} \frac{a^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^2\tan^4(c+dx)\sec^2(c+dx)}{6d}+\frac{2a^2\tan^4(c+dx)\sec(c+dx)}{5d}+\frac{a^2\tan^4(c+dx)}{4d}-\frac{a^2\tan^2(c+dx)\sec^2(c+dx)}{6d}-\frac{8a^2\tan^2(c+\cdots}{} \\ x\,(a\sec(c)+a)^2\tan^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**4*sec(c
 + d*x)**2/(6*d) + 2*a**2*tan(c + d*x)**4*sec(c + d*x)/(5*d) + a**2*tan(c +
 d*x)**4/(4*d) - a**2*tan(c + d*x)**2*sec(c + d*x)**2/(6*d) - 8*a**2*tan(c
+ d*x)**2*sec(c + d*x)/(15*d) - a**2*tan(c + d*x)**2/(2*d) + a**2*sec(c + d
*x)**2/(6*d) + 16*a**2*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a*sec(c) + a)**2
*tan(c)**5, True))
```

3.22 $\int (a + a\sec(c + dx))^2 \tan^3(c + dx)\,dx$

Optimal. Leaf size=65

$$\frac{a^2\sec^4(c+dx)}{4d}+\frac{2a^2\sec^3(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\log(\cos(c+dx))}{d}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a^2*sec(d*x+c)/d+2/3*a^2*sec(d*x+c)^3/d+1/4*a^2*sec(
d*x+c)^4/d
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3879, 75}

$$\frac{a^2\sec^4(c+dx)}{4d}+\frac{2a^2\sec^3(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (2*a^2*Sec[c + d*x]^3)
/(3*d) + (a^2*Sec[c + d*x]^4)/(4*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx&=-\frac{\text{Subst}\left(\int \frac{(a-ax)(a+ax)^3}{x^5}\,dx,x,\cos(c+dx)\right)}{a^2d}\\&=-\frac{\text{Subst}\left(\int \left(\frac{a^4}{x^5}+\frac{2a^4}{x^4}-\frac{2a^4}{x^2}-\frac{a^4}{x}\right)dx,x,\cos(c+dx)\right)}{a^2d}\\&=\frac{a^2\log(\cos(c+dx))}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\sec^3(c+dx)}{3d}+\frac{a^2\sec^4(c}{4d}\end{aligned}$$

Mathematica [A] time = 0.21, size = 83, normalized size = 1.28

$$\frac{a^2\sec^4(c+dx)(3(-4\cos(3(c+dx))+4\cos(2(c+dx))\log(\cos(c+dx))+\cos(4(c+dx))\log(\cos(c+dx))+3}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(-20*Cos[c + d*x] + 3*(2 - 4*Cos[3*(c + d*x)] + 3*Log[Cos[c + d*x]] +
4*Cos[2*(c + d*x)]*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c + d*x]]))
*Sec[c + d*x]^4)/(24*d)
```

fricas [A] time = 0.63, size = 65, normalized size = 1.00

$$\frac{12\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)-24\,a^2\cos\left(dx+c\right)^3+8\,a^2\cos\left(dx+c\right)+3\,a^2}{12\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) - 24*a^2*cos(d*x + c)^3 + 8*
a^2*cos(d*x + c) + 3*a^2)/(d*cos(d*x + c)^4)
```

giac [B] time = 1.43, size = 192, normalized size = 2.95

$$-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{57\,a^2+\frac{252\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{246\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{124\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (57*a^2 + 252*a^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 246*a^2*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 25*a^2*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1)^4)/d
```

maple [B] time = 0.72, size = 140, normalized size = 2.15

$$\frac{a^2\left(\tan^2\left(dx+c\right)\right)}{2d}+\frac{a^2\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{2a^2\left(\sin^4\left(dx+c\right)\right)}{3d\cos\left(dx+c\right)^3}-\frac{2a^2\left(\sin^4\left(dx+c\right)\right)}{3d\cos\left(dx+c\right)}-\frac{2a^2\cos\left(dx+c\right)\left(\sin^2\left(dx+c\right)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x)
```

```
[Out] 1/2*a^2*tan(d*x+c)^2/d+a^2*ln(cos(d*x+c))/d+2/3/d*a^2*sin(d*x+c)^4/cos(d*x+
c)^3-2/3/d*a^2*sin(d*x+c)^4/cos(d*x+c)-2/3/d*a^2*cos(d*x+c)*sin(d*x+c)^2-4/
3*a^2*cos(d*x+c)/d+1/4/d*a^2*sin(d*x+c)^4/cos(d*x+c)^4
```

maxima [A] time = 0.36, size = 58, normalized size = 0.89

$$\frac{12\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{24\,a^2\cos(dx+c)^3-8\,a^2\cos(dx+c)-3\,a^2}{\cos(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*a^2*log(cos(d*x + c)) - (24*a^2*cos(d*x + c)^3 - 8*a^2*cos(d*x + c
) - 3*a^2)/cos(d*x + c)^4)/d
```

mupad [B] time = 3.87, size = 133, normalized size = 2.05

$$\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{38\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{8\,a^2}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((38*a^2*tan(c/2 + (d*x)/2)^2)/3 - 8*a^2*tan(c/2 + (d*x)/2)^4 + 2*a^2*tan(c
/2 + (d*x)/2)^6 - (8*a^2)/3)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)
/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (2*a^2*atanh(
tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 1.82, size = 126, normalized size = 1.94

$$\begin{cases} -\frac{a^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^2\tan^2(c+dx)\sec^2(c+dx)}{4d}+\frac{2a^2\tan^2(c+dx)\sec(c+dx)}{3d}+\frac{a^2\tan^2(c+dx)}{2d}-\frac{a^2\sec^2(c+dx)}{4d}-\frac{4a^2\sec(c+dx)}{3d} \\ x\left(a\sec(c)+a\right)^2\tan^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2*sec(
c + d*x)**2/(4*d) + 2*a**2*tan(c + d*x)**2*sec(c + d*x)/(3*d) + a**2*tan(c
+ d*x)**2/(2*d) - a**2*sec(c + d*x)**2/(4*d) - 4*a**2*sec(c + d*x)/(3*d), N
e(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**3, True))
```

3.23 $\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d+1/2*a^2*sec(d*x+c)^2/d
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 43}

$$\frac{a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d + (a^2*Sec[c + d*x]^2
)/(2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a+ax)^2}{x^3}\,dx,x,\cos(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2a^2}{x^2}+\frac{a^2}{x}\right)dx,x,\cos(c+dx)\right)}{d}\\
&= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 51, normalized size = 1.06

$$-\frac{a^2\sec^2(c+dx)(-4\cos(c+dx)+\cos(2(c+dx))\log(\cos(c+dx))+\log(\cos(c+dx))-1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -1/2*(a^2*(-1 - 4*Cos[c + d*x] + Log[Cos[c + d*x]] + Cos[2*(c + d*x)]*Log[C
os[c + d*x]])*Sec[c + d*x]^2)/d
```

fricas [A] time = 0.62, size = 52, normalized size = 1.08

$$-\frac{2\,a^2\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-4\,a^2\cos\left(dx+c\right)-a^2}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*cos(d*x + c)^2*log(-cos(d*x + c)) - 4*a^2*cos(d*x + c) - a^2)/(
d*cos(d*x + c)^2)
```

giac [B] time = 0.39, size = 142, normalized size = 2.96

$$\frac{2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{11\,a^2+\frac{10\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11*a^2 + 10*a^2*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

maple [A] time = 0.30, size = 46, normalized size = 0.96

$$\frac{a^2\left(\sec^2\left(dx+c\right)\right)}{2d}+\frac{2a^2\sec\left(dx+c\right)}{d}+\frac{a^2\ln\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c),x)
```

```
[Out] 1/2*a^2*sec(d*x+c)^2/d+2*a^2*sec(d*x+c)/d+a^2/d*ln(sec(d*x+c))
```

maxima [A] time = 0.32, size = 43, normalized size = 0.90

$$-\frac{2\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{4\,a^2\cos(dx+c)+a^2}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*log(cos(d*x + c)) - (4*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^2)/
d
```

mupad [B] time = 1.21, size = 76, normalized size = 1.58

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a^2*tan(c/2 + (d*x)/2)^2 - 4*a^2
)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [A] time = 0.48, size = 60, normalized size = 1.25

$$\begin{cases} \frac{a^2 \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^2 \sec^2(c+dx)}{2d} + \frac{2a^2 \sec(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a\sec(c)+a\right)^2 \tan(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*sec(c + d*x)**2/(2*d)
 + 2*a**2*sec(c + d*x)/d, Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c), True))
```

3.24 $\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=35

$$\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

```
[Out] 2*a^2*ln(1-cos(d*x+c))/d-a^2*ln(cos(d*x+c))/d
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 72}

$$\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^2,x]
[Out] (2*a^2*Log[1 - Cos[c + d*x]])/d - (a^2*Log[Cos[c + d*x]])/d
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx &= -\frac{a^2\,\text{Subst}\left(\int \frac{a+ax}{x(a-ax)}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^2\,\text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= \frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 29, normalized size = 0.83

$$-\frac{a^2\left(\log(\cos(c+dx))-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^2,x]
[Out] -((a^2*(Log[Cos[c + d*x]] - 4*Log[Sin[(c + d*x)/2]]))/d)
```

fricas [A] time = 0.77, size = 35, normalized size = 1.00

$$-\frac{a^2\log\left(-\cos\left(dx+c\right)\right)-2\,a^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(a^2*log(-cos(d*x + c)) - 2*a^2*log(-1/2*cos(d*x + c) + 1/2))/d
```

giac [A] time = 0.26, size = 64, normalized size = 1.83

$$\frac{2\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^2\log\left(\left|\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] (2*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^2*log(abs((cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)))/d
```

maple [A] time = 0.53, size = 34, normalized size = 0.97

$$-\frac{a^2\ln\left(\sec\left(dx+c\right)\right)}{d}+\frac{2a^2\ln\left(-1+\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^2,x)
[Out] -a^2/d*ln(sec(d*x+c))+2*a^2/d*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.39, size = 31, normalized size = 0.89

$$\frac{2\,a^2\log\left(\cos\left(dx+c\right)-1\right)-a^2\log\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] (2*a^2*log(cos(d*x + c) - 1) - a^2*log(cos(d*x + c)))/d
```

mupad [B] time = 1.25, size = 36, normalized size = 1.03

$$\frac{a^2\,\left(4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^2,x)
[Out] (a^2*(4*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^4 - 1)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cot\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cot\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**2,x)
[Out] a**2*(Integral(2*cot(c + d*x)*sec(c + d*x), x) + Integral(cot(c + d*x)*sec(
c + d*x)**2, x) + Integral(cot(c + d*x), x))
```

3.25 $\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=40

$$-\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}$$

```
[Out] -a^2/d/(1-cos(d*x+c))-a^2*ln(1-cos(d*x+c))/d
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$-\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2/(d*(1 - Cos[c + d*x]))) - (a^2*Log[1 - Cos[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx &= -\frac{a^4\,\text{Subst}\left(\int \frac{x}{(a-ax)^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{a^2(-1+x)^2}+\frac{1}{a^2(-1+x)}\right)dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 1.40

$$\frac{a^2\csc^2\left(\frac{1}{2}(c+dx)\right)\left(-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\cos(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[(c + d*x)/2]^2*(-1 - 2*Log[Sin[(c + d*x)/2]] + 2*Cos[c + d*x]*Log[
Sin[(c + d*x)/2]]))/(2*d)
```

fricas [A] time = 0.67, size = 48, normalized size = 1.20

$$\frac{a^2 - \left(a^2 \cos\left(dx + c\right) - a^2\right) \log\left(-\frac{1}{2} \cos\left(dx + c\right) + \frac{1}{2}\right)}{d \cos\left(dx + c\right) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2 - (a^2*cos(d*x + c) - a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x +
c) - d)
```

giac [B] time = 0.44, size = 111, normalized size = 2.78

$$-\frac{2\,a^2 \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 2\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - \frac{\left(a^2 + \frac{2\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a^2*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^2 + 2*a^2*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

maple [A] time = 0.68, size = 51, normalized size = 1.28

$$\frac{a^2 \ln\left(\sec\left(dx + c\right)\right)}{d} - \frac{a^2}{d\left(-1 + \sec\left(dx + c\right)\right)} - \frac{a^2 \ln\left(-1 + \sec\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2/d*ln(sec(d*x+c))-a^2/d/(-1+sec(d*x+c))-a^2/d*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.53, size = 34, normalized size = 0.85

$$-\frac{a^2 \log\left(\cos\left(dx + c\right) - 1\right) - \frac{a^2}{\cos(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a^2*log(cos(d*x + c) - 1) - a^2/(cos(d*x + c) - 1))/d
```

mupad [B] time = 1.23, size = 50, normalized size = 1.25

$$-\frac{a^2 \left(\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2} + 2\,\ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right) - \ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(a^2*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot^3(c+dx)\sec(c+dx)\,dx+\int \cot^3(c+dx)\sec^2(c+dx)\,dx+\int \cot^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**3*sec(c + d*x), x) + Integral(cot(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**3, x))
```

3.26 $\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=85

$$\frac{5a^2}{4d(1-\cos(c+dx))}-\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\log(\cos(c+dx)+1)}{8d}$$

```
[Out] -1/4*a^2/d/(1-cos(d*x+c))^2+5/4*a^2/d/(1-cos(d*x+c))+7/8*a^2*ln(1-cos(d*x+c
))/d+1/8*a^2*ln(1+cos(d*x+c))/d
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{5a^2}{4d(1-\cos(c+dx))}-\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\log(\cos(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -a^2/(4*d*(1 - Cos[c + d*x])^2) + (5*a^2)/(4*d*(1 - Cos[c + d*x])) + (7*a^2
*Log[1 - Cos[c + d*x]])/(8*d) + (a^2*Log[1 + Cos[c + d*x]])/(8*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx &= -\frac{a^6\,\text{Subst}\left(\int \frac{x^3}{(a-ax)^3(a+ax)}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{2a^4(-1+x)^3}-\frac{5}{4a^4(-1+x)^2}-\frac{7}{8a^4(-1+x)}-\frac{1}{8a^4(1+x)}\right)dx, x, \cos(c\right.}{d}\\ &= -\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{5a^2}{4d(1-\cos(c+dx))}+\frac{7a^2\log(1-\cos(c+dx))}{8d}\end{aligned}$$

Mathematica [A] time = 0.27, size = 86, normalized size = 1.01

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^4\left(\frac{1}{2}(c+dx)\right)-10\csc^2\left(\frac{1}{2}(c+dx)\right)-4\left(7\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\right.\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/64*(a^2*(1 + Cos[c + d*x])^2*(-10*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^
4 - 4*(Log[Cos[(c + d*x)/2]] + 7*Log[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4
)/d
```

fricas [A] time = 0.58, size = 122, normalized size = 1.44

$$-\frac{10\,a^2\cos\left(dx+c\right)-8\,a^2-\left(a^2\cos\left(dx+c\right)^2-2\,a^2\cos\left(dx+c\right)+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-7\left(a^2\cos\left(dx\right.\right.}{8\left(d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(10*a^2*cos(d*x + c) - 8*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c
) + a^2)*log(1/2*cos(d*x + c) + 1/2) - 7*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*
x + c) + a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - 2*d*cos(d*x
 + c) + d)
```

giac [A] time = 0.56, size = 138, normalized size = 1.62

$$\frac{14\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-16\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^2+\frac{8\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{21\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(14*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 16*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^2 + 8*a^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 21*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

maple [A] time = 0.58, size = 87, normalized size = 1.02

$$-\frac{a^2\ln\left(\sec\left(dx+c\right)\right)}{d}-\frac{a^2}{4d\left(-1+\sec\left(dx+c\right)\right)^2}+\frac{3a^2}{4d\left(-1+\sec\left(dx+c\right)\right)}+\frac{7a^2\ln\left(-1+\sec\left(dx+c\right)\right)}{8d}+\frac{a^2\ln\left(1+\mathrm{se}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -a^2/d*ln(sec(d*x+c))-1/4*a^2/d/(-1+sec(d*x+c))^2+3/4*a^2/d/(-1+sec(d*x+c))
+7/8*a^2/d*ln(-1+sec(d*x+c))+1/8*a^2/d*ln(1+sec(d*x+c))
```

maxima [A] time = 0.64, size = 72, normalized size = 0.85

$$\frac{a^2\log\left(\cos\left(dx+c\right)+1\right)+7\,a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(5\,a^2\cos(dx+c)-4\,a^2\right)}{\cos(dx+c)^2-2\,\cos(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*log(cos(d*x + c) + 1) + 7*a^2*log(cos(d*x + c) - 1) - 2*(5*a^2*cos
(d*x + c) - 4*a^2)/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1))/d
```

mupad [B] time = 1.26, size = 62, normalized size = 0.73

$$\frac{a^2\left(-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{7\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*((7*log(tan(c/2 + (d*x)/2)))/4 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c
/2 + (d*x)/2)^2/2 - cot(c/2 + (d*x)/2)^4/16))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot^5(c+dx)\sec(c+dx)\,dx+\int\cot^5(c+dx)\sec^2(c+dx)\,dx+\int\cot^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**5*sec(c + d*x), x) + Integral(cot(c + d*x)**
5*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**5, x))
```

3.27 $\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=127

$$-\frac{23a^2}{16d(1-\cos(c+dx))}-\frac{a^2}{16d(\cos(c+dx)+1)}+\frac{a^2}{2d(1-\cos(c+dx))^2}-\frac{a^2}{12d(1-\cos(c+dx))^3}-\frac{13a^2\log(1-\cos(c+dx))}{16d}$$

```
[Out] -1/12*a^2/d/(1-cos(d*x+c))^3+1/2*a^2/d/(1-cos(d*x+c))^2-23/16*a^2/d/(1-cos(
d*x+c))-1/16*a^2/d/(1+cos(d*x+c))-13/16*a^2*ln(1-cos(d*x+c))/d-3/16*a^2*ln(
1+cos(d*x+c))/d
```

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$-\frac{23a^2}{16d(1-\cos(c+dx))}-\frac{a^2}{16d(\cos(c+dx)+1)}+\frac{a^2}{2d(1-\cos(c+dx))^2}-\frac{a^2}{12d(1-\cos(c+dx))^3}-\frac{13a^2\log(1-\cos(c+dx))}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -a^2/(12*d*(1 - Cos[c + d*x])^3) + a^2/(2*d*(1 - Cos[c + d*x])^2) - (23*a^2
)/(16*d*(1 - Cos[c + d*x])) - a^2/(16*d*(1 + Cos[c + d*x])) - (13*a^2*Log[1
 - Cos[c + d*x]])/(16*d) - (3*a^2*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx &= -\frac{a^8\,\text{Subst}\left(\int \frac{x^5}{(a-ax)^4(a+ax)^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^8\,\text{Subst}\left(\int \left(\frac{1}{4a^6(-1+x)^4}+\frac{1}{a^6(-1+x)^3}+\frac{23}{16a^6(-1+x)^2}+\frac{13}{16a^6(-1+x)}-\frac{1}{16a^6(1+\cdots}\right.\right.}{d}\\ &= -\frac{a^2}{12d(1-\cos(c+dx))^3}+\frac{a^2}{2d(1-\cos(c+dx))^2}-\frac{23a^2}{16d(1-\cos(c+dx)}\cdots\end{aligned}$$

Mathematica [A] time = 0.25, size = 114, normalized size = 0.90

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^6\left(\frac{1}{2}(c+dx)\right)-12\csc^4\left(\frac{1}{2}(c+dx)\right)+69\csc^2\left(\frac{1}{2}(c+dx)\right)+3\left(\sec^2\cdots\right.\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(69*Csc[(c + d*x)/2]^2
- 12*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 + 3*(12*Log[Cos[(c + d*x)/2]]
+ 52*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2)))/d
```

fricas [A] time = 0.70, size = 191, normalized size = 1.50

$$\frac{66\,a^2\cos(dx+c)^3 - 36\,a^2\cos(dx+c)^2 - 74\,a^2\cos(dx+c) + 52\,a^2 - 9\left(a^2\cos(dx+c)^4 - 2\,a^2\cos(dx+c)^3 + 2\right.}{48\left(d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(66*a^2*cos(d*x + c)^3 - 36*a^2*cos(d*x + c)^2 - 74*a^2*cos(d*x + c) +
 52*a^2 - 9*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)
 - a^2)*log(1/2*cos(d*x + c) + 1/2) - 39*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*
x + c)^3 + 2*a^2*cos(d*x + c) - a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d
*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c) - d)
```

giac [A] time = 0.42, size = 186, normalized size = 1.46

$$-\frac{78\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 96\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{3\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{\left(a^2+\frac{9\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{143\,a^2(}{(\cos}\right.}{(\cos(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*(78*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 96*a^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 3*a^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - (a^2 + 9*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 48*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 143*a^2*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)^3)/d
```

maple [A] time = 0.63, size = 122, normalized size = 0.96

$$\frac{a^2\ln(\sec(dx+c))}{d} - \frac{a^2}{12d(-1+\sec(dx+c))^3} + \frac{a^2}{4d(-1+\sec(dx+c))^2} - \frac{11a^2}{16d(-1+\sec(dx+c))} - \frac{13a^2\ln(-1+\text{se}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2/d*ln(sec(d*x+c))-1/12*a^2/d/(-1+sec(d*x+c))^3+1/4*a^2/d/(-1+sec(d*x+c))
^2-11/16*a^2/d/(-1+sec(d*x+c))-13/16*a^2/d*ln(-1+sec(d*x+c))+1/16*a^2/d/(1+
sec(d*x+c))-3/16*a^2/d*ln(1+sec(d*x+c))
```

maxima [A] time = 0.56, size = 109, normalized size = 0.86

$$-\frac{9\,a^2\log(\cos(dx+c)+1) + 39\,a^2\log(\cos(dx+c)-1) - \frac{2\left(33\,a^2\cos(dx+c)^3 - 18\,a^2\cos(dx+c)^2 - 37\,a^2\cos(dx+c) + 26\,a^2\right)}{\cos(dx+c)^4 - 2\cos(dx+c)^3 + 2\cos(dx+c) - 1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/48*(9*a^2*log(cos(d*x + c) + 1) + 39*a^2*log(cos(d*x + c) - 1) - 2*(33*a
^2*cos(d*x + c)^3 - 18*a^2*cos(d*x + c)^2 - 37*a^2*cos(d*x + c) + 26*a^2)/(
cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 2*cos(d*x + c) - 1))/d
```

mupad [B] time = 1.27, size = 113, normalized size = 0.89

$$\frac{a^2 \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{13\,a^2 \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 \left(8\,a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{3\,a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{a^2}{6}\right)}{16\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (13*a^2*log(tan(c/2 + (d*x)/2)))/(8
*d) - (cot(c/2 + (d*x)/2)^6*(8*a^2*tan(c/2 + (d*x)/2)^4 - (3*a^2*tan(c/2 +
(d*x)/2)^2)/2 + a^2/6))/(16*d) - (a^2*tan(c/2 + (d*x)/2)^2)/(32*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int 2\cot^7(c+dx)\sec(c+dx)\,dx+\int \cot^7(c+dx)\sec^2(c+dx)\,dx+\int \cot^7(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**7*sec(c + d*x), x) + Integral(cot(c + d*x)**
7*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**7, x))
```

3.28 $\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=169

$$\frac{51a^2}{32d(1-\cos(c+dx))}+\frac{9a^2}{64d(\cos(c+dx)+1)}-\frac{3a^2}{4d(1-\cos(c+dx))^2}-\frac{a^2}{64d(\cos(c+dx)+1)^2}+\frac{11a^2}{48d(1-\cos(c+dx))}$$

```
[Out] -1/32*a^2/d/(1-cos(d*x+c))^4+11/48*a^2/d/(1-cos(d*x+c))^3-3/4*a^2/d/(1-cos(
d*x+c))^2+51/32*a^2/d/(1-cos(d*x+c))-1/64*a^2/d/(1+cos(d*x+c))^2+9/64*a^2/d
/(1+cos(d*x+c))+99/128*a^2*ln(1-cos(d*x+c))/d+29/128*a^2*ln(1+cos(d*x+c))/d
```

Rubi [A] time = 0.11, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{3879, 88\}$

$$\frac{51a^2}{32d(1-\cos(c+dx))}+\frac{9a^2}{64d(\cos(c+dx)+1)}-\frac{3a^2}{4d(1-\cos(c+dx))^2}-\frac{a^2}{64d(\cos(c+dx)+1)^2}+\frac{11a^2}{48d(1-\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -a^2/(32*d*(1 - Cos[c + d*x])^4) + (11*a^2)/(48*d*(1 - Cos[c + d*x])^3) - (
3*a^2)/(4*d*(1 - Cos[c + d*x])^2) + (51*a^2)/(32*d*(1 - Cos[c + d*x])) - a^
2/(64*d*(1 + Cos[c + d*x])^2) + (9*a^2)/(64*d*(1 + Cos[c + d*x])) + (99*a^2
*Log[1 - Cos[c + d*x]])/(128*d) + (29*a^2*Log[1 + Cos[c + d*x]])/(128*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx &= -\frac{a^{10}\,\text{Subst}\left(\int \frac{x^7}{(a-ax)^5(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^{10}\,\text{Subst}\left(\int \left(-\frac{1}{8a^8(-1+x)^5}-\frac{11}{16a^8(-1+x)^4}-\frac{3}{2a^8(-1+x)^3}-\frac{51}{32a^8(-1+x)^2}-\frac{9}{128a^8(}\right.\right.}{d}\\ &= -\frac{a^2}{32d(1-\cos(c+dx))^4}+\frac{11a^2}{48d(1-\cos(c+dx))^3}-\frac{3a^2}{4d(1-\cos(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.34, size = 146, normalized size = 0.86

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(3\csc^8\left(\frac{1}{2}(c+dx)\right)-44\csc^6\left(\frac{1}{2}(c+dx)\right)+288\csc^4\left(\frac{1}{2}(c+dx)\right)-1224\,\text{cs}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6144*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-1224*Csc[(c + d*x)/2
]^2 + 288*Csc[(c + d*x)/2]^4 - 44*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8
 - 6*(116*Log[Cos[(c + d*x)/2]] + 396*Log[Sin[(c + d*x)/2]] + 18*Sec[(c + d
*x)/2]^2 - Sec[(c + d*x)/2]^4)))/d
```

fricas [B] time = 0.60, size = 322, normalized size = 1.91

$$-\frac{558\,a^2\cos\left(dx+c\right)^5-156\,a^2\cos\left(dx+c\right)^4-1268\,a^2\cos\left(dx+c\right)^3+676\,a^2\cos\left(dx+c\right)^2+686\,a^2\cos\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(558*a^2*cos(d*x + c)^5 - 156*a^2*cos(d*x + c)^4 - 1268*a^2*cos(d*x
+ c)^3 + 676*a^2*cos(d*x + c)^2 + 686*a^2*cos(d*x + c) - 448*a^2 - 87*(a^2*
cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^5 - a^2*cos(d*x + c)^4 + 4*a^2*cos(d*x
+ c)^3 - a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*log(1/2*cos(d*x + c
) + 1/2) - 297*(a^2*cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^5 - a^2*cos(d*x + c
)^4 + 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)
*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^5 - d*c
os(d*x + c)^4 + 4*d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - 2*d*cos(d*x + c) +
d)
```

giac [A] time = 1.38, size = 238, normalized size = 1.41

$$\frac{1188\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-1536\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{96\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{\left(3\,a^2+\frac{32\,a^2(\cos(}{\cos(dx}\right.}{}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(1188*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 1536*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 96*a^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 6*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 - (3*a^2 + 32*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 174*a^2*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 768*a^2*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 + 2475*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)*(cos(d*x
 + c) + 1)^4/(cos(d*x + c) - 1)^4)/d
```

maple [A] time = 0.70, size = 159, normalized size = 0.94

$$-\frac{a^2\ln\left(\sec\left(dx+c\right)\right)}{d}-\frac{a^2}{32d\left(-1+\sec\left(dx+c\right)\right)^4}+\frac{5a^2}{48d\left(-1+\sec\left(dx+c\right)\right)^3}-\frac{a^2}{4d\left(-1+\sec\left(dx+c\right)\right)^2}+\frac{2}{32d\,(-1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -a^2/d*ln(sec(d*x+c))-1/32*a^2/d/(-1+sec(d*x+c))^4+5/48*a^2/d/(-1+sec(d*x+c
))^3-1/4*a^2/d/(-1+sec(d*x+c))^2+21/32*a^2/d/(-1+sec(d*x+c))+99/128*a^2/d*l
n(-1+sec(d*x+c))-1/64*a^2/d/(1+sec(d*x+c))^2-7/64*a^2/d/(1+sec(d*x+c))+29/1
28*a^2/d*ln(1+sec(d*x+c))
```

maxima [A] time = 0.45, size = 165, normalized size = 0.98

$$\frac{87\,a^2\log\left(\cos\left(dx+c\right)+1\right)+297\,a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(279\,a^2\cos(dx+c)^5-78\,a^2\cos(dx+c)^4-634\,a^2\cos(dx+c)^3+338\,a^2\cos}{\cos(dx+c)^6-2\cos(dx+c)^5-\cos(dx+c)^4+4\cos(dx+c)^3-\cos}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/384*(87*a^2*log(cos(d*x + c) + 1) + 297*a^2*log(cos(d*x + c) - 1) - 2*(27
9*a^2*cos(d*x + c)^5 - 78*a^2*cos(d*x + c)^4 - 634*a^2*cos(d*x + c)^3 + 338
*a^2*cos(d*x + c)^2 + 343*a^2*cos(d*x + c) - 224*a^2)/(cos(d*x + c)^6 - 2*c
os(d*x + c)^5 - cos(d*x + c)^4 + 4*cos(d*x + c)^3 - cos(d*x + c)^2 - 2*cos(
d*x + c) + 1))/d
```

mupad [B] time = 1.59, size = 149, normalized size = 0.88

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{99\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{29\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{4}\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(16*d) - (a^2*tan(c/2 + (d*x)/2)^4)/(256*d) + (9
9*a^2*log(tan(c/2 + (d*x)/2)))/(64*d) + (cot(c/2 + (d*x)/2)^8*((4*a^2*tan(c
/2 + (d*x)/2)^2)/3 - (29*a^2*tan(c/2 + (d*x)/2)^4)/4 + 32*a^2*tan(c/2 + (d*
x)/2)^6 - a^2/8))/(64*d) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.29 $\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{a^2\tan^7(c+dx)}{7d}+\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan^5(c+dx)\text{s}}{3d}$$

```
[Out] -a^2*x-5/8*a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+5/8*a^2*sec(d*x+c)*ta
n(d*x+c)/d-1/3*a^2*tan(d*x+c)^3/d-5/12*a^2*sec(d*x+c)*tan(d*x+c)^3/d+1/5*a^
2*tan(d*x+c)^5/d+1/3*a^2*sec(d*x+c)*tan(d*x+c)^5/d+1/7*a^2*tan(d*x+c)^7/d
```

Rubi [A] time = 0.18, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan^7(c+dx)}{7d}+\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan^5(c+dx)\text{s}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -(a^2*x) - (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5*
a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a^2*
Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a^2*Sec
[c + d*x]*Tan[c + d*x]^5)/(3*d) + (a^2*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a + a\sec(c+dx))^2 \tan^6(c+dx)\,dx = \int \left(a^2\tan^6(c+dx) + 2a^2\sec(c+dx)\tan^6(c+dx) + a^2\sec^2(c+dx)\tan^6\right.$$

$$= a^2\int \tan^6(c+dx)\,dx + a^2\int \sec^2(c+dx)\tan^6(c+dx)\,dx + \left(2a^2\right)\int \sec$$

$$= \frac{a^2\tan^5(c+dx)}{5d} + \frac{a^2\sec(c+dx)\tan^5(c+dx)}{3d} - a^2\int \tan^4(c+dx)\,dx -$$

$$= -\frac{a^2\tan^3(c+dx)}{3d} - \frac{5a^2\sec(c+dx)\tan^3(c+dx)}{12d} + \frac{a^2\tan^5(c+dx)}{5d} + \frac{a^2}{}$$

$$= \frac{a^2\tan(c+dx)}{d} + \frac{5a^2\sec(c+dx)\tan(c+dx)}{8d} - \frac{a^2\tan^3(c+dx)}{3d} - \frac{5a^2\text{se}}{}$$

$$= -a^2x - \frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2\tan(c+dx)}{d} + \frac{5a^2\sec(c+dx)\tan(}{8d}$$

Mathematica [B] time = 1.58, size = 337, normalized size = 2.09

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\left(33600\cos^7(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^7*(33600*Cos[c +
d*x]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]) + Sec[c]*(-14700*d*x*Cos[d*x] - 14700*d*x*Cos[2*c + d*x]
- 8820*d*x*Cos[2*c + 3*d*x] - 8820*d*x*Cos[4*c + 3*d*x] - 2940*d*x*Cos[4*c
+ 5*d*x] - 2940*d*x*Cos[6*c + 5*d*x] - 420*d*x*Cos[6*c + 7*d*x] - 420*d*x*C
os[8*c + 7*d*x] + 24640*Sin[d*x] - 16240*Sin[2*c + d*x] + 2975*Sin[c + 2*d*
x] + 2975*Sin[3*c + 2*d*x] + 14448*Sin[2*c + 3*d*x] - 10080*Sin[4*c + 3*d*x
] + 980*Sin[3*c + 4*d*x] + 980*Sin[5*c + 4*d*x] + 6496*Sin[4*c + 5*d*x] - 1
680*Sin[6*c + 5*d*x] + 1155*Sin[5*c + 6*d*x] + 1155*Sin[7*c + 6*d*x] + 1168
*Sin[6*c + 7*d*x])))/(215040*d)
```

fricas [A] time = 0.67, size = 165, normalized size = 1.02

$$-\frac{1680\,a^2dx\cos\left(dx+c\right)^7 + 525\,a^2\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right) - 525\,a^2\cos\left(dx+c\right)^7\log\left(-\sin\left(dx+c\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/1680*(1680*a^2*d*x*cos(d*x + c)^7 + 525*a^2*cos(d*x + c)^7*log(sin(d*x +
 c) + 1) - 525*a^2*cos(d*x + c)^7*log(-sin(d*x + c) + 1) - 2*(1168*a^2*cos(
d*x + c)^6 + 1155*a^2*cos(d*x + c)^5 - 256*a^2*cos(d*x + c)^4 - 910*a^2*cos
(d*x + c)^3 - 192*a^2*cos(d*x + c)^2 + 280*a^2*cos(d*x + c) + 120*a^2)*sin(
d*x + c))/(d*cos(d*x + c)^7)
```

giac [A] time = 6.01, size = 180, normalized size = 1.12

$$-\frac{840\,(dx+c)a^2 + 525\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 525\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)*a^2 + 525*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 52
5*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(315*a^2*tan(1/2*d*x + 1/2*c)^
13 - 2660*a^2*tan(1/2*d*x + 1/2*c)^11 + 9863*a^2*tan(1/2*d*x + 1/2*c)^9 - 2
1216*a^2*tan(1/2*d*x + 1/2*c)^7 + 29673*a^2*tan(1/2*d*x + 1/2*c)^5 - 9660*a
^2*tan(1/2*d*x + 1/2*c)^3 + 1365*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 - 1)^7)/d
```

maple [A] time = 0.54, size = 226, normalized size = 1.40

$$\frac{a^2\left(\tan^5(dx+c)\right)}{5d} - \frac{a^2\left(\tan^3(dx+c)\right)}{3d} + \frac{a^2\tan(dx+c)}{d} - a^2x - \frac{a^2c}{d} + \frac{a^2\left(\sin^7(dx+c)\right)}{3d\cos(dx+c)^6} - \frac{a^2\left(\sin^7(dx+c)\right)}{12d\cos(dx+c)^4} + \frac{a^2\left(\mathrm{s}\right.}{8d\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x)
```

```
[Out] 1/5*a^2*tan(d*x+c)^5/d-1/3*a^2*tan(d*x+c)^3/d+a^2*tan(d*x+c)/d-a^2*x-1/d*a^
2*c+1/3/d*a^2*sin(d*x+c)^7/cos(d*x+c)^6-1/12/d*a^2*sin(d*x+c)^7/cos(d*x+c)^
4+1/8/d*a^2*sin(d*x+c)^7/cos(d*x+c)^2+1/8*a^2*sin(d*x+c)^5/d+5/24*a^2*sin(d
*x+c)^3/d+5/8*a^2*sin(d*x+c)/d-5/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/7/d*a^
2*sin(d*x+c)^7/cos(d*x+c)^7
```

maxima [A] time = 0.67, size = 151, normalized size = 0.94

$$\frac{240\,a^2\tan(dx+c)^7 + 112\left(3\,\tan(dx+c)^5 - 5\,\tan(dx+c)^3 - 15\,dx - 15\,c + 15\,\tan(dx+c)\right)a^2 - 35\,a^2\left(\frac{2\left(33\,\mathrm{s}\right.}{\mathrm{sin}(}\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/1680*(240*a^2*tan(d*x + c)^7 + 112*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 -
 15*d*x - 15*c + 15*tan(d*x + c))*a^2 - 35*a^2*(2*(33*sin(d*x + c)^5 - 40*s
in(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin
(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.65, size = 234, normalized size = 1.45

$$-a^2\,x - \frac{5\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4} - \frac{19\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3} + \frac{1409\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{60} - \frac{1768\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} - 7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + 21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

```
[Out] - a^2*x - (5*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((1413*a^2*tan(c/2 + (d
*x)/2)^5)/20 - 23*a^2*tan(c/2 + (d*x)/2)^3 - (1768*a^2*tan(c/2 + (d*x)/2)^7
```

```
)/35 + (1409*a^2*tan(c/2 + (d*x)/2)^9)/60 - (19*a^2*tan(c/2 + (d*x)/2)^11)/
3 + (3*a^2*tan(c/2 + (d*x)/2)^13)/4 + (13*a^2*tan(c/2 + (d*x)/2))/4)/(d*(7*
tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 -
35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^1
2 + tan(c/2 + (d*x)/2)^14 - 1))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\tan^6(c+dx)\sec(c+dx)\,dx+\int \tan^6(c+dx)\sec^2(c+dx)\,dx+\int \tan^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] a**2*(Integral(2*tan(c + d*x)**6*sec(c + d*x), x) + Integral(tan(c + d*x)**
6*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**6, x))
```

3.30 $\int (a+a\sec (c+dx))^2 \tan ^4(c+dx) \, dx$

Optimal. Leaf size=119

$$\frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3a^2\,\mathrm{t}}{}$$

```
[Out] a^2*x+3/4*a^2*arctanh(sin(d*x+c))/d-a^2*tan(d*x+c)/d-3/4*a^2*sec(d*x+c)*tan
(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+1/2*a^2*sec(d*x+c)*tan(d*x+c)^3/d+1/5*a^2*
tan(d*x+c)^5/d
```

Rubi [A] time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3a^2\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] a^2*x + (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a^2
*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a^2*Sec[c
 + d*x]*Tan[c + d*x]^3)/(2*d) + (a^2*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^4(c+dx)\,dx = \int \left(a^2\tan^4(c+dx)+2a^2\sec(c+dx)\tan^4(c+dx)+a^2\sec^2(c+dx)\tan^4\right.$$

$$= a^2\int \tan^4(c+dx)\,dx + a^2\int \sec^2(c+dx)\tan^4(c+dx)\,dx + \left(2a^2\right)\int \sec$$

$$= \frac{a^2\tan^3(c+dx)}{3d} + \frac{a^2\sec(c+dx)\tan^3(c+dx)}{2d} - a^2\int \tan^2(c+dx)\,dx -$$

$$= -\frac{a^2\tan(c+dx)}{d} - \frac{3a^2\sec(c+dx)\tan(c+dx)}{4d} + \frac{a^2\tan^3(c+dx)}{3d} + \frac{a^2\text{se}}{}$$

$$= a^2x + \frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d} - \frac{a^2\tan(c+dx)}{d} - \frac{3a^2\sec(c+dx)\tan(c}{4d}$$

Mathematica [B] time = 5.63, size = 558, normalized size = 4.69

$$\frac{1}{960}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\cos\left(\frac{c}{2}\right)\left(\frac{151}{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{36}{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx\right.\right.}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(240*x - (180*Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]])/d + (180*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
)/d - ((293*Cos[(d*x)/2] + 333*Cos[2*c + (3*d*x)/2] + 287*Cos[2*c + (5*d*x)
/2] + 67*Cos[4*c + (7*d*x)/2] + 68*Cos[4*c + (9*d*x)/2])*Sec[c]*Sec[c + d*x
]^5*Sin[(d*x)/2])/(2*d) - (24*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^4) + (149*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - (24*Sin[c/2])/(d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + (149*Sin[c/2])/(d*(Cos[c/2]
 + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Cos[c/2]*(36/((Cos
[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4) - 151/((Cos[c/2]
 - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 36/((Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + 151/((Cos[c/2] + Sin[c/2]
)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)))/d))/960
```

fricas [A] time = 0.61, size = 139, normalized size = 1.17

$$\frac{120\,a^2dx\cos\left(dx+c\right)^5+45\,a^2\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-45\,a^2\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)-2}{120\,d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*d*x*cos(d*x + c)^5 + 45*a^2*cos(d*x + c)^5*log(sin(d*x + c)
+ 1) - 45*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(68*a^2*cos(d*x + c
```

```
)^4 + 75*a^2*cos(d*x + c)^3 + 4*a^2*cos(d*x + c)^2 - 30*a^2*cos(d*x + c) -
12*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [A] time = 1.82, size = 148, normalized size = 1.24

$$\frac{60\,(dx+c)a^2 + 45\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 45\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 110\,a\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*a^2 + 45*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a^2
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 - 11
0*a^2*tan(1/2*d*x + 1/2*c)^7 + 328*a^2*tan(1/2*d*x + 1/2*c)^5 - 530*a^2*tan
(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^5)/d
```

maple [A] time = 0.52, size = 169, normalized size = 1.42

$$\frac{a^2\left(\tan^3(dx+c)\right)}{3d} - \frac{a^2\tan(dx+c)}{d} + a^2x + \frac{a^2c}{d} + \frac{a^2\left(\sin^5(dx+c)\right)}{2d\cos(dx+c)^4} - \frac{a^2\left(\sin^5(dx+c)\right)}{4d\cos(dx+c)^2} - \frac{a^2\left(\sin^3(dx+c)\right)}{4d} - \frac{3a^2\,\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x)
```

```
[Out] 1/3*a^2*tan(d*x+c)^3/d-a^2*tan(d*x+c)/d+a^2*x+1/d*a^2*c+1/2/d*a^2*sin(d*x+c
)^5/cos(d*x+c)^4-1/4/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2-1/4*a^2*sin(d*x+c)^3/d
-3/4*a^2*sin(d*x+c)/d+3/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/5/d*a^2*sin(d*x
+c)^5/cos(d*x+c)^5
```

maxima [A] time = 0.64, size = 119, normalized size = 1.00

$$\frac{24\,a^2\tan(dx+c)^5 + 40\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^2 + 15\,a^2\left(\frac{2\left(5\,\sin(dx+c)^3 - 3\,\sin(dx+c)\right)}{\sin(dx+c)^4 - 2\,\sin(dx+c)^2 + 1} + 3\,\log\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/120*(24*a^2*tan(d*x + c)^5 + 40*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^2 + 15*a^2*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4
- 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)
))/d
```

mupad [B] time = 1.95, size = 174, normalized size = 1.46

$$a^2\,x + \frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d} + \frac{\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2} - \frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3} + \frac{164\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15} - \frac{53\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} + }{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (3*a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d) + ((164*a^2*tan(c/2 + (d*x)
/2)^5)/15 - (53*a^2*tan(c/2 + (d*x)/2)^3)/3 - (11*a^2*tan(c/2 + (d*x)/2)^7)
```

```
/3 + (a^2*tan(c/2 + (d*x)/2)^9)/2 + (7*a^2*tan(c/2 + (d*x)/2))/2)/(d*(5*tan
(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*t
an(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\tan^4(c+dx)\sec(c+dx)\,dx+\int\tan^4(c+dx)\sec^2(c+dx)\,dx+\int\tan^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] a**2*(Integral(2*tan(c + d*x)**4*sec(c + d*x), x) + Integral(tan(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**4, x))
```

3.31 $\int (a + a\sec(c + dx))^2 \tan^2(c + dx)\,dx$

Optimal. Leaf size=72

$$\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}-a^2x$$

```
[Out] -a^2*x-a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)
/d+1/3*a^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a^2*Sec[
c + d*x]*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a^2\tan^2(c+dx)+2a^2\sec(c+dx)\tan^2(c+dx)+a^2\sec^2(c+dx)\tan^2\right.$$
$$= a^2\int \tan^2(c+dx)\,dx + a^2\int \sec^2(c+dx)\tan^2(c+dx)\,dx + \left(2a^2\right)\int \sec$$
$$= \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} - a^2\int 1\,dx - a^2\int \sec(c+$$
$$= -a^2x - \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c-}{d}$$

Mathematica [B] time = 6.33, size = 773, normalized size = 10.74

$$-\frac{1}{4}x\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2+\frac{\sin\left(\frac{dx}{2}\right)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2}{6d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\sin\left(\right.}{6d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/4*(x*Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2) + (Cos[
c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^
4*(a + a*Sec[c + d*x])^2)/(4*d) - (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] +
Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(4*d) + (C
os[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(24
*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Co
s[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(7*Cos[c/2] - 5*Si
n[c/2]))/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2])^2) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d
*x)/2])/(6*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
)) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/
2])/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3
) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(-7*Cos[c/2
] - 5*Sin[c/2]))/(48*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2])^2) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^
2*Sin[(d*x)/2])/(6*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 +
(d*x)/2]))
```

fricas [A] time = 0.73, size = 111, normalized size = 1.54

$$-\frac{6\,a^2dx\cos(dx+c)^3+3\,a^2\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,a^2\cos(dx+c)^3\log(-\sin(dx+c)+1)-2\left(2\right.}{6\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*a^2*cos(d*x + c)^2 +
3*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 2.33, size = 99, normalized size = 1.38

$$-\frac{3\,(dx+c)a^2+3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*(a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.49, size = 112, normalized size = 1.56

$$-a^2x+\frac{a^2\tan(dx+c)}{d}-\frac{a^2c}{d}+\frac{a^2\left(\sin^3(dx+c)\right)}{d\cos(dx+c)^2}+\frac{a^2\sin(dx+c)}{d}-\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\left(\sin^3(d\right.}{3d\cos(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x)
```

```
[Out] -a^2*x+a^2*tan(d*x+c)/d-1/d*a^2*c+1/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+a^2*sin
(d*x+c)/d-1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*a^2*sin(d*x+c)^3/cos(d*x+
c)^3
```

maxima [A] time = 0.82, size = 83, normalized size = 1.15

$$\frac{2\,a^2\tan(dx+c)^3-6\,(dx+c-\tan(dx+c))a^2-3\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a^2*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

mupad [B] time = 1.21, size = 101, normalized size = 1.40

$$\frac{\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((4*a^2*tan(c/2 + (d*x)/2)^3)/3 - 4*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 +
 (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*a^2*
atanh(tan(c/2 + (d*x)/2)))/d - a^2*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\tan^2(c+dx)\sec(c+dx)\,dx+\int\tan^2(c+dx)\sec^2(c+dx)\,dx+\int\tan^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x))
```

3.32 $\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=35

$$-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+a^2(-x)$$

[Out] -a^2*x-2*a^2*cot(d*x+c)/d-2*a^2*csc(d*x+c)/d

Rubi [A] time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3886, 3473, 8, 2606, 3767}

$$-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+a^2(-x)$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]

[Out] -(a^2*x) - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = \int \left(a^2\cot^2(c+dx)+2a^2\cot(c+dx)\csc(c+dx)+a^2\csc^2(c+dx)\right)dx$$
$$= a^2\int \cot^2(c+dx)\,dx + a^2\int \csc^2(c+dx)\,dx + \left(2a^2\right)\int \cot(c+dx)\csc(c$$
$$= -\frac{a^2\cot(c+dx)}{d} - a^2\int 1\,dx - \frac{a^2\,\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{d} - \frac{\left(2a^2\right)\text{S}}{}$$
$$= -a^2x - \frac{2a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d}$$

Mathematica [C] time = 0.04, size = 46, normalized size = 1.31

$$-\frac{2a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Cot[c/2 + (d*x)/2]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c/2 + (d*x)
/2]^2])/d
```

fricas [A] time = 0.68, size = 42, normalized size = 1.20

$$-\frac{a^2dx\sin(dx+c)+2\,a^2\cos(dx+c)+2\,a^2}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*sin(d*x + c) + 2*a^2*cos(d*x + c) + 2*a^2)/(d*sin(d*x + c))
```

giac [A] time = 0.25, size = 31, normalized size = 0.89

$$-\frac{(dx+c)a^2+\frac{2\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^2 + 2*a^2/tan(1/2*d*x + 1/2*c))/d
```

maple [A] time = 0.72, size = 50, normalized size = 1.43

$$\frac{a^2(-\cot(dx+c)-dx-c)-\frac{2a^2}{\sin(dx+c)}-a^2\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-cot(d*x+c)-d*x-c)-2*a^2/sin(d*x+c)-a^2*cot(d*x+c))
```

maxima [A] time = 0.58, size = 48, normalized size = 1.37

$$-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2+\frac{2\,a^2}{\sin(dx+c)}+\frac{a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c + 1/tan(d*x + c))*a^2 + 2*a^2/sin(d*x + c) + a^2/tan(d*x + c))/d
```

mupad [B] time = 1.08, size = 24, normalized size = 0.69

$$-a^2\,x-\frac{2\,a^2\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

```
[Out] - a^2*x - (2*a^2*cot(c/2 + (d*x)/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot^2(c+dx)\sec(c+dx)\,dx+\int\cot^2(c+dx)\sec^2(c+dx)\,dx+\int\cot^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**2*sec(c + d*x), x) + Integral(cot(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))
```

3.33 $\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=69

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}+a^2x$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-2/3*a^2*cot(d*x+c)^3/d+2*a^2*csc(d*x+c)/d-2/3*a^2*cs
c(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3886, 3473, 8, 2606, 2607, 30}

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3*d) + (2*a^2*Csc[c
+ d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)+2a^2\cot^3(c+dx)\csc(c+dx)+a^2\cot^2(c+dx)\text{cs}\right.$$
$$= a^2\int \cot^4(c+dx)\,dx + a^2\int \cot^2(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right)\int \text{c}$$
$$= -\frac{a^2\cot^3(c+dx)}{3d} - a^2\int \cot^2(c+dx)\,dx + \frac{a^2\,\text{Subst}\left(\int x^2\,dx, x, -\cot\right.}{d}$$
$$= \frac{a^2\cot(c+dx)}{d} - \frac{2a^2\cot^3(c+dx)}{3d} + \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+d}{3d}$$
$$= a^2x + \frac{a^2\cot(c+dx)}{d} - \frac{2a^2\cot^3(c+dx)}{3d} + \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3}{3}$$

Mathematica [A] time = 0.27, size = 112, normalized size = 1.62

$$\frac{a^2\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(-12\sin\left(c+\frac{dx}{2}\right)+10\sin\left(c+\frac{3dx}{2}\right)-9dx\cos\left(c+\frac{dx}{2}\right)-3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\,\text{c}\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c/2]*Csc[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] - 9*d*x*Cos[c + (d*x)/
2] - 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 18*Sin[(d*x)/2
] - 12*Sin[c + (d*x)/2] + 10*Sin[c + (3*d*x)/2]))/(24*d)
```

fricas [A] time = 0.51, size = 81, normalized size = 1.17

$$\frac{5\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)-4\,a^2+3\left(a^2dx\cos\left(dx+c\right)-a^2dx\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(5*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - 4*a^2 + 3*(a^2*d*x*cos(d*x +
 c) - a^2*d*x)*sin(d*x + c))/((d*cos(d*x + c) - d)*sin(d*x + c))
```

giac [A] time = 0.43, size = 50, normalized size = 0.72

$$\frac{6\left(dx+c\right)a^2+\frac{9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*a^2 + (9*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2)/tan(1/2*d*x + 1
/2*c)^3)/d
```

maple [A] time = 0.89, size = 112, normalized size = 1.62

$$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot\left(dx+c\right)+dx+c\right)+2a^2\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}\right)-\frac{a^2\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a^2*(-1/3/sin(d*x+c)^3*cos(
d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c))-1/3*a
^2/sin(d*x+c)^3*cos(d*x+c)^3)
```

maxima [A] time = 0.72, size = 77, normalized size = 1.12

$$\frac{\left(3\,dx + 3\,c + \frac{3\,\tan\left(dx+c\right)^{2}-1}{\tan\left(dx+c\right)^{3}}\right)a^{2} + \frac{2\,\left(3\,\sin\left(dx+c\right)^{2}-1\right)a^{2}}{\sin\left(dx+c\right)^{3}} - \frac{a^{2}}{\tan\left(dx+c\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 + 2*(3*sin(d
*x + c)^2 - 1)*a^2/sin(d*x + c)^3 - a^2/tan(d*x + c)^3)/d
```

mupad [B] time = 1.10, size = 39, normalized size = 0.57

$$a^{2}\,x + \frac{a^{2}\,\left(9\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right) - \cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (a^2*(9*cot(c/2 + (d*x)/2) - cot(c/2 + (d*x)/2)^3))/(6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\cot^{4}\left(c + dx\right)\sec\left(c + dx\right)dx + \int \cot^{4}\left(c + dx\right)\sec^{2}\left(c + dx\right)dx + \int \cot^{4}\left(c + dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**4*sec(c + d*x), x) + Integral(cot(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**4, x))
```

3.34 $\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=107

$$-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{4a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}-a^2x$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-2/5*a^2*cot(d*x+c)^5/d-2*a^2
*csc(d*x+c)/d+4/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{4a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot[c
 + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d + (4*a^2*Csc[c + d*x]^3)/(3*d) -
(2*a^2*Csc[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)+2a^2\cot^5(c+dx)\csc(c+dx)+a^2\cot^4(c+dx)\csc^2(\right.$$
$$= a^2\int \cot^6(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right)\int \cot^5$$
$$= -\frac{a^2\cot^5(c+dx)}{5d} - a^2\int \cot^4(c+dx)\,dx + \frac{a^2\,\text{Subst}\left(\int x^4\,dx, x, -\cot(c\right.}{d}$$
$$= \frac{a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} + a^2\int \cot^2(c+dx)\,dx - \frac{\left(2a^2\right)\text{Subst}}{}$$
$$= -\frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc(c+dx)}{d}$$
$$= -a^2x - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc(c}{d}$$

Mathematica [A] time = 0.81, size = 194, normalized size = 1.81

$$\frac{a^2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^5\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(445\sin(c+dx)-356\sin(2(c+dx))+89\sin(3(c+dx))+24}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c/2]*Csc[(c + d*x)/2]^5*Sec[c/2]*Sec[(c + d*x)/2]*(-150*d*x*Cos[d*
x] + 150*d*x*Cos[2*c + d*x] + 120*d*x*Cos[c + 2*d*x] - 120*d*x*Cos[3*c + 2*
d*x] - 30*d*x*Cos[2*c + 3*d*x] + 30*d*x*Cos[4*c + 3*d*x] - 80*Sin[c] + 280*
Sin[d*x] + 445*Sin[c + d*x] - 356*Sin[2*(c + d*x)] + 89*Sin[3*(c + d*x)] +
240*Sin[2*c + d*x] - 296*Sin[c + 2*d*x] - 120*Sin[3*c + 2*d*x] + 104*Sin[2*
c + 3*d*x]))/(3840*d)
```

fricas [A] time = 0.72, size = 118, normalized size = 1.10

$$-\frac{26\,a^2\cos(dx+c)^3 - 22\,a^2\cos(dx+c)^2 - 17\,a^2\cos(dx+c) + 16\,a^2 + 15\left(a^2dx\cos(dx+c)^2 - 2\,a^2dx\cos(dx+\right.}{15\left(d\cos(dx+c)^2 - 2\,d\cos(dx+c) + d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(26*a^2*cos(d*x + c)^3 - 22*a^2*cos(d*x + c)^2 - 17*a^2*cos(d*x + c)
+ 16*a^2 + 15*(a^2*d*x*cos(d*x + c)^2 - 2*a^2*d*x*cos(d*x + c) + a^2*d*x)*s
in(d*x + c))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

giac [A] time = 0.35, size = 80, normalized size = 0.75

$$-\frac{120\,(dx+c)a^2 - 15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{165\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/120*(120*(d*x + c)*a^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + (165*a^2*tan(1/2*
d*x + 1/2*c)^4 - 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2)/tan(1/2*d*x + 1/2*c
)^5)/d
```

maple [A] time = 1.04, size = 155, normalized size = 1.45

$$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2a^2\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a^2*(-1/5/
sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos
(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/5*a^2/sin(d
*x+c)^5*cos(d*x+c)^5)
```

maxima [A] time = 0.54, size = 97, normalized size = 0.91

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2+\frac{2\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)a^2}{\sin(dx+c)^5}+\frac{3\,a^2}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^2 + 2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a^2/sin(d*x + c
)^5 + 3*a^2/tan(d*x + c)^5)/d
```

mupad [B] time = 1.26, size = 78, normalized size = 0.73

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{8}-\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{24}+\frac{a^2}{40}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}-a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2))/(8*d) - ((11*a^2*tan(c/2 + (d*x)/2)^4)/8 - (5*a^2*
tan(c/2 + (d*x)/2)^2)/24 + a^2/40)/(d*tan(c/2 + (d*x)/2)^5) - a^2*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot^6(c+dx)\sec(c+dx)\,dx+\int\cot^6(c+dx)\sec^2(c+dx)\,dx+\int\cot^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**6*sec(c + d*x), x) + Integral(cot(c + d*x)**
6*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**6, x))
```

3.35 $\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=139

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}+\frac{6a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc}{}$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d+1/5*a^2*cot(d*x+c)^5/d-2/7*a^
2*cot(d*x+c)^7/d+2*a^2*csc(d*x+c)/d-2*a^2*csc(d*x+c)^3/d+6/5*a^2*csc(d*x+c)
^5/d-2/7*a^2*csc(d*x+c)^7/d
```

Rubi [A] time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}+\frac{6a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*
x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) + (2*a^2*Csc[c + d*x])/d - (2*a^
2*Csc[c + d*x]^3)/d + (6*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)
/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx = \int \left(a^2\cot^8(c+dx)+2a^2\cot^7(c+dx)\csc(c+dx)+a^2\cot^6(c+dx)\,\mathrm{cs}\right.$$

$$= a^2\int \cot^8(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right)\int \mathrm{c}$$

$$= -\frac{a^2\cot^7(c+dx)}{7d} - a^2\int \cot^6(c+dx)\,dx + \frac{a^2\,\mathrm{Subst}\left(\int x^6\,dx, x, -\cot\right.}{d}$$

$$= \frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} + a^2\int \cot^4(c+dx)\,dx - \frac{\left(2a^2\right)\mathrm{Su}}{}$$

$$= -\frac{a^2\cot^3(c+dx)}{3d} + \frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} + \frac{2a^2\csc(c+}{d}$$

$$= \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d}$$

$$= a^2x + \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c}{7d}$$

Mathematica [B] time = 1.15, size = 312, normalized size = 2.24

$$\frac{a^2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)(-16002\sin(c+dx)+9144\sin(2(c+dx))+3429\sin(3(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c/2]*Csc[(c + d*x)/2]^7*Sec[c/2]*Sec[(c + d*x)/2]^3*(5880*d*x*Cos[
d*x] - 5880*d*x*Cos[2*c + d*x] - 3360*d*x*Cos[c + 2*d*x] + 3360*d*x*Cos[3*c
 + 2*d*x] - 1260*d*x*Cos[2*c + 3*d*x] + 1260*d*x*Cos[4*c + 3*d*x] + 1680*d*
x*Cos[3*c + 4*d*x] - 1680*d*x*Cos[5*c + 4*d*x] - 420*d*x*Cos[4*c + 5*d*x] +
 420*d*x*Cos[6*c + 5*d*x] + 4032*Sin[c] - 9632*Sin[d*x] - 16002*Sin[c + d*x
] + 9144*Sin[2*(c + d*x)] + 3429*Sin[3*(c + d*x)] - 4572*Sin[4*(c + d*x)] +
 1143*Sin[5*(c + d*x)] - 11760*Sin[2*c + d*x] + 8864*Sin[c + 2*d*x] + 3360*
Sin[3*c + 2*d*x] + 2064*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] - 4432*Sin
[3*c + 4*d*x] - 1680*Sin[5*c + 4*d*x] + 1528*Sin[4*c + 5*d*x]))/(860160*d)
```

fricas [A] time = 0.59, size = 173, normalized size = 1.24

$$\frac{191\,a^2\cos(dx+c)^5 - 172\,a^2\cos(dx+c)^4 - 253\,a^2\cos(dx+c)^3 + 258\,a^2\cos(dx+c)^2 + 87\,a^2\cos(dx+c) - 9}{105\left(d\cos(dx+c)^4 - 2\,d\cos(dx+c)^3 + 2\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(191*a^2*cos(d*x + c)^5 - 172*a^2*cos(d*x + c)^4 - 253*a^2*cos(d*x +
c)^3 + 258*a^2*cos(d*x + c)^2 + 87*a^2*cos(d*x + c) - 96*a^2 + 105*(a^2*d*x
*cos(d*x + c)^4 - 2*a^2*d*x*cos(d*x + c)^3 + 2*a^2*d*x*cos(d*x + c) - a^2*d
*x)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c
) - d)*sin(d*x + c))
```

giac [A] time = 0.39, size = 112, normalized size = 0.81

$$\frac{35\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3360\,(dx+c)a^{2}-735\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4410\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-770\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+147\,a^{2}\,\mathrm{ta}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(35*a^2*tan(1/2*d*x + 1/2*c)^3 + 3360*(d*x + c)*a^2 - 735*a^2*tan(1/
2*d*x + 1/2*c) + (4410*a^2*tan(1/2*d*x + 1/2*c)^6 - 770*a^2*tan(1/2*d*x + 1
/2*c)^4 + 147*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/
d
```

maple [A] time = 1.10, size = 188, normalized size = 1.35

$$\frac{a^{2}\left(-\frac{\left(\cot^{7}(dx+c)\right)}{7}+\frac{\left(\cot^{5}(dx+c)\right)}{5}-\frac{\left(\cot^{3}(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2a^{2}\left(-\frac{\cos^{8}(dx+c)}{7\sin(dx+c)^{7}}+\frac{\cos^{8}(dx+c)}{35\sin(dx+c)^{5}}-\frac{\cos^{8}(dx+c)}{35\sin(dx+c)^{3}}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*
x+c)+2*a^2*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1
/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/7*a^2/sin(d*x+c)^7*c
os(d*x+c)^7)
```

maxima [A] time = 0.83, size = 117, normalized size = 0.84

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^{6}-35\,\tan(dx+c)^{4}+21\,\tan(dx+c)^{2}-15}{\tan(dx+c)^{7}}\right)a^{2}+\frac{6\left(35\,\sin(dx+c)^{6}-35\,\sin(dx+c)^{4}+21\,\sin(dx+c)^{2}-5\right)a^{2}}{\sin(dx+c)^{7}}-\frac{15\,a^{2}}{\tan(dx+c)^{7}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^2 + 6*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a^2/sin(d*x + c)^7 - 15*a^2/tan(d*x + c)^7)/d
```

mupad [B] time = 1.75, size = 182, normalized size = 1.31

$$\frac{a^{2}\left(35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-735\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+4410\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\right.}{3360\,d\cos\left(\frac{c}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*(35*sin(c/2 + (d*x)/2)^10 - 15*cos(c/2 + (d*x)/2)^10 - 735*cos(c/2 + (
d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 4410*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^6 - 770*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 147*cos(c/2 + (d*x)/
```

```
2)^8*sin(c/2 + (d*x)/2)^2 + 3360*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7*
(c + d*x)))/(3360*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.36 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=179

$$-\frac{2a^2\cot^9(c+dx)}{9d}+\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^9(c+dx)}{9d}+\frac{8a^2\csc^7}{7}$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-1/5*a^2*cot(d*x+c)^5/d+1/7*a
^2*cot(d*x+c)^7/d-2/9*a^2*cot(d*x+c)^9/d-2*a^2*csc(d*x+c)/d+8/3*a^2*csc(d*x
+c)^3/d-12/5*a^2*csc(d*x+c)^5/d+8/7*a^2*csc(d*x+c)^7/d-2/9*a^2*csc(d*x+c)^9
/d
```

Rubi [A] time = 0.15, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2a^2\cot^9(c+dx)}{9d}+\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^9(c+dx)}{9d}+\frac{8a^2\csc^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c +
 d*x]^5)/(5*d) + (a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d)
- (2*a^2*Csc[c + d*x])/d + (8*a^2*Csc[c + d*x]^3)/(3*d) - (12*a^2*Csc[c + d
*x]^5)/(5*d) + (8*a^2*Csc[c + d*x]^7)/(7*d) - (2*a^2*Csc[c + d*x]^9)/(9*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \int \left(a^2\cot^{10}(c+dx)+2a^2\cot^9(c+dx)\csc(c+dx)+a^2\cot^8(c+dx)\right.$$
$$= a^2\int \cot^{10}(c+dx)\,dx + a^2\int \cot^8(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right)\int$$
$$= -\frac{a^2\cot^9(c+dx)}{9d} - a^2\int \cot^8(c+dx)\,dx + \frac{a^2\,\mathrm{Subst}\left(\int x^8\,dx, x, -\mathrm{co}\right.}{d}$$
$$= \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} + a^2\int \cot^6(c+dx)\,dx - \frac{\left(2a^2\right)\mathrm{S}}{}$$
$$= -\frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{2a^2\csc(c+}{d}$$
$$= \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+d}{9d}$$
$$= -\frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+dx}{7d}$$
$$= -a^2x - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(}{7}$$

Mathematica [B] time = 1.99, size = 428, normalized size = 2.39

$$-\frac{a^2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^9\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(-1152405\sin(c+dx)+512180\sin(2(c+dx))+486571}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/330301440*(a^2*Csc[c/2]*Csc[(c + d*x)/2]^9*Sec[c/2]*Sec[(c + d*x)/2]^5*(
453600*d*x*Cos[d*x] - 453600*d*x*Cos[2*c + d*x] - 201600*d*x*Cos[c + 2*d*x]
 + 201600*d*x*Cos[3*c + 2*d*x] - 191520*d*x*Cos[2*c + 3*d*x] + 191520*d*x*C
os[4*c + 3*d*x] + 161280*d*x*Cos[3*c + 4*d*x] - 161280*d*x*Cos[5*c + 4*d*x]
 + 10080*d*x*Cos[4*c + 5*d*x] - 10080*d*x*Cos[6*c + 5*d*x] - 40320*d*x*Cos[
5*c + 6*d*x] + 40320*d*x*Cos[7*c + 6*d*x] + 10080*d*x*Cos[6*c + 7*d*x] - 10
080*d*x*Cos[8*c + 7*d*x] + 259584*Sin[c] - 897024*Sin[d*x] - 1152405*Sin[c
+ d*x] + 512180*Sin[2*(c + d*x)] + 486571*Sin[3*(c + d*x)] - 409744*Sin[4*(
c + d*x)] - 25609*Sin[5*(c + d*x)] + 102436*Sin[6*(c + d*x)] - 25609*Sin[7*
(c + d*x)] - 825216*Sin[2*c + d*x] + 622976*Sin[c + 2*d*x] + 142464*Sin[3*c
 + 2*d*x] + 297088*Sin[2*c + 3*d*x] + 430080*Sin[4*c + 3*d*x] - 424192*Sin[
3*c + 4*d*x] - 188160*Sin[5*c + 4*d*x] + 2048*Sin[4*c + 5*d*x] - 40320*Sin[
6*c + 5*d*x] + 112768*Sin[5*c + 6*d*x] + 40320*Sin[7*c + 6*d*x] - 38272*Sin
[6*c + 7*d*x]))/d
```

fricas [A] time = 0.61, size = 274, normalized size = 1.53

$$-\frac{598\,a^2\cos(dx+c)^7 - 566\,a^2\cos(dx+c)^6 - 1212\,a^2\cos(dx+c)^5 + 1310\,a^2\cos(dx+c)^4 + 860\,a^2\cos(dx+}{315\left(d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(598*a^2*cos(d*x + c)^7 - 566*a^2*cos(d*x + c)^6 - 1212*a^2*cos(d*x
+ c)^5 + 1310*a^2*cos(d*x + c)^4 + 860*a^2*cos(d*x + c)^3 - 1014*a^2*cos(d*
x + c)^2 - 197*a^2*cos(d*x + c) + 256*a^2 + 315*(a^2*d*x*cos(d*x + c)^6 - 2
*a^2*d*x*cos(d*x + c)^5 - a^2*d*x*cos(d*x + c)^4 + 4*a^2*d*x*cos(d*x + c)^3
 - a^2*d*x*cos(d*x + c)^2 - 2*a^2*d*x*cos(d*x + c) + a^2*d*x)*sin(d*x + c))
/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^5 - d*cos(d*x + c)^4 + 4*d*cos(d*x +
 c)^3 - d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

giac [A] time = 0.43, size = 145, normalized size = 0.81

$$\frac{63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-945\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40320\,(dx+c)a^2+11655\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{51345\,a^2\tan\left(\frac{1}{2}\,dx\right.}{}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40320*(63*a^2*tan(1/2*d*x + 1/2*c)^5 - 945*a^2*tan(1/2*d*x + 1/2*c)^3 - 4
0320*(d*x + c)*a^2 + 11655*a^2*tan(1/2*d*x + 1/2*c) - (51345*a^2*tan(1/2*d*
x + 1/2*c)^8 - 9765*a^2*tan(1/2*d*x + 1/2*c)^6 + 2331*a^2*tan(1/2*d*x + 1/2
*c)^4 - 405*a^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

maple [A] time = 1.03, size = 231, normalized size = 1.29

$$\frac{a^2\left(-\frac{\left(\cot^9(dx+c)\right)}{9}+\frac{\left(\cot^7(dx+c)\right)}{7}-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2a^2\left(-\frac{\cos^{10}(dx+c)}{9\sin(dx+c)^9}+\frac{\cos^{10}(dx+c)}{63\sin(dx+c)^7}-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7-1/5*cot(d*x+c)^5+1/3*cot(d*x+c
)^3-cot(d*x+c)-d*x-c)+2*a^2*(-1/9/sin(d*x+c)^9*cos(d*x+c)^10+1/63/sin(d*x+c
)^7*cos(d*x+c)^10-1/105/sin(d*x+c)^5*cos(d*x+c)^10+1/63/sin(d*x+c)^3*cos(d*
x+c)^10-1/9/sin(d*x+c)*cos(d*x+c)^10-1/9*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c
)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-1/9*a^2/sin(d*x+c)^9
*cos(d*x+c)^9)
```

maxima [A] time = 0.76, size = 137, normalized size = 0.77

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan(dx+c)^8-105\,\tan(dx+c)^6+63\,\tan(dx+c)^4-45\,\tan(dx+c)^2+35}{\tan(dx+c)^9}\right)a^2+\frac{2\left(315\,\sin(dx+c)^8-420\,\sin(dx+c)^6+378\,\sin(d\right.}{\sin(dx+c)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a^2 + 2*(315*sin(d*x
 + c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35
)*a^2/sin(d*x + c)^9 + 35*a^2/tan(d*x + c)^9)/d
```

mupad [B] time = 2.78, size = 230, normalized size = 1.28

$$-\frac{a^2\left(35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-63\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+945\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-11655\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^10*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(a^2*(35*cos(c/2 + (d*x)/2)^14 - 63*sin(c/2 + (d*x)/2)^14 + 945*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 11655*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*
x)/2)^10 + 51345*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 9765*cos(c/2 +
 (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 2331*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*
x)/2)^4 - 405*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 40320*cos(c/2 +
(d*x)/2)^5*sin(c/2 + (d*x)/2)^9*(c + d*x)))/(40320*d*cos(c/2 + (d*x)/2)^5*s
in(c/2 + (d*x)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.37 $\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx$

Optimal. Leaf size=210

$$\frac{a^3\sec^{11}(c+dx)}{11d}+\frac{3a^3\sec^{10}(c+dx)}{10d}-\frac{a^3\sec^9(c+dx)}{9d}-\frac{11a^3\sec^8(c+dx)}{8d}-\frac{6a^3\sec^7(c+dx)}{7d}+\frac{7a^3\sec^6(c+dx)}{3d}+\frac{14}{}$$

```
[Out] -a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d-1/2*a^3*sec(d*x+c)^2/d-11/3*a^3*se
c(d*x+c)^3/d-3/2*a^3*sec(d*x+c)^4/d+14/5*a^3*sec(d*x+c)^5/d+7/3*a^3*sec(d*x
+c)^6/d-6/7*a^3*sec(d*x+c)^7/d-11/8*a^3*sec(d*x+c)^8/d-1/9*a^3*sec(d*x+c)^9
/d+3/10*a^3*sec(d*x+c)^10/d+1/11*a^3*sec(d*x+c)^11/d
```

Rubi [A] time = 0.10, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{a^3\sec^{11}(c+dx)}{11d}+\frac{3a^3\sec^{10}(c+dx)}{10d}-\frac{a^3\sec^9(c+dx)}{9d}-\frac{11a^3\sec^8(c+dx)}{8d}-\frac{6a^3\sec^7(c+dx)}{7d}+\frac{7a^3\sec^6(c+dx)}{3d}+\frac{14}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^9,x]
```

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d - (a^3*Sec[c + d*x]^2
)/(2*d) - (11*a^3*Sec[c + d*x]^3)/(3*d) - (3*a^3*Sec[c + d*x]^4)/(2*d) + (1
4*a^3*Sec[c + d*x]^5)/(5*d) + (7*a^3*Sec[c + d*x]^6)/(3*d) - (6*a^3*Sec[c +
 d*x]^7)/(7*d) - (11*a^3*Sec[c + d*x]^8)/(8*d) - (a^3*Sec[c + d*x]^9)/(9*d)
 + (3*a^3*Sec[c + d*x]^10)/(10*d) + (a^3*Sec[c + d*x]^11)/(11*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^7}{x^{12}}\,dx,x,\cos(c+dx)\right)}{a^8d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^{11}}{x^{12}}+\frac{3a^{11}}{x^{11}}-\frac{a^{11}}{x^{10}}-\frac{11a^{11}}{x^9}-\frac{6a^{11}}{x^8}+\frac{14a^{11}}{x^7}+\frac{14a^{11}}{x^6}-\frac{6a^{11}}{x^5}-\frac{11a^{11}}{x^4}-\right.\right.}{a^8d}\\ &= -\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\sec^2(c+dx)}{2d}-\frac{11a^3\sec^3(c}{3d}\end{aligned}$$

Mathematica [A] time = 0.88, size = 214, normalized size = 1.02

$$-\frac{a^3\sec^{11}(c+dx)(-1613260\cos(2(c+dx))+960960\cos(3(c+dx))-1131504\cos(4(c+dx))+314160\cos(5(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^9,x]
```

```
[Out] -1/3548160*(a^3*(-1151740 - 1613260*Cos[2*(c + d*x)] + 960960*Cos[3*(c + d*
x)] - 1131504*Cos[4*(c + d*x)] + 314160*Cos[5*(c + d*x)] - 432894*Cos[6*(c
+ d*x)] + 145530*Cos[7*(c + d*x)] - 106260*Cos[8*(c + d*x)] + 6930*Cos[9*(c
 + d*x)] - 20790*Cos[10*(c + d*x)] + 1143450*Cos[3*(c + d*x)]*Log[Cos[c + d
*x]] + 571725*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 190575*Cos[7*(c + d*x)]*
Log[Cos[c + d*x]] + 38115*Cos[9*(c + d*x)]*Log[Cos[c + d*x]] + 3465*Cos[11*
(c + d*x)]*Log[Cos[c + d*x]] + 462*Cos[c + d*x]*(2606 + 3465*Log[Cos[c + d*
x]]))*Sec[c + d*x]^11)/d
```

fricas [A] time = 0.87, size = 169, normalized size = 0.80

$$-\frac{27720\,a^3\cos\left(dx+c\right)^{11}\log\left(-\cos\left(dx+c\right)\right)-83160\,a^3\cos\left(dx+c\right)^{10}+13860\,a^3\cos\left(dx+c\right)^{9}+101640\,a^3\cos\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/27720*(27720*a^3*cos(d*x + c)^11*log(-cos(d*x + c)) - 83160*a^3*cos(d*x
+ c)^10 + 13860*a^3*cos(d*x + c)^9 + 101640*a^3*cos(d*x + c)^8 + 41580*a^3*
cos(d*x + c)^7 - 77616*a^3*cos(d*x + c)^6 - 64680*a^3*cos(d*x + c)^5 + 2376
0*a^3*cos(d*x + c)^4 + 38115*a^3*cos(d*x + c)^3 + 3080*a^3*cos(d*x + c)^2 -
 8316*a^3*cos(d*x + c) - 2520*a^3)/(d*cos(d*x + c)^11)
```

giac [A] time = 19.68, size = 367, normalized size = 1.75

$$\frac{27720\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-27720\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{153343\,a^3+\frac{1742213\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9043705\,a^3(\cos(dx+c\ldots}{(\cos(dx+c)+1)^2}\ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/27720*(27720*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2
7720*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (153343*a^3
 + 1742213*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9043705*a^3*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 + 28369275*a^3*(cos(d*x + c) - 1)^3/(cos(
d*x + c) + 1)^3 + 59954070*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
67458930*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 57997170*a^3*(cos(
d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 36975510*a^3*(cos(d*x + c) - 1)^7/(c
os(d*x + c) + 1)^7 + 16879995*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8
 + 5213945*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9 + 976261*a^3*(cos(
d*x + c) - 1)^10/(cos(d*x + c) + 1)^10 + 83711*a^3*(cos(d*x + c) - 1)^11/(c
os(d*x + c) + 1)^11)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^11)/d
```

maple [A] time = 0.87, size = 351, normalized size = 1.67

$$\frac{4352a^3\cos\left(dx+c\right)}{3465d}+\frac{34a^3\left(\sin^{10}\left(dx+c\right)\right)}{99d\cos\left(dx+c\right)^9}-\frac{34a^3\left(\sin^{10}\left(dx+c\right)\right)}{693d\cos\left(dx+c\right)^7}+\frac{34a^3\left(\sin^{10}\left(dx+c\right)\right)}{1155d\cos\left(dx+c\right)^5}-\frac{34a^3\left(\sin^{10}\left(dx+c\right)\right)}{693d\cos\left(dx+c\right)^3}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x)
```

```
[Out] 4352/3465*a^3*cos(d*x+c)/d+34/99/d*a^3*sin(d*x+c)^10/cos(d*x+c)^9-34/693/d*
a^3*sin(d*x+c)^10/cos(d*x+c)^7+34/1155/d*a^3*sin(d*x+c)^10/cos(d*x+c)^5-34/
```

```
693/d*a^3*sin(d*x+c)^10/cos(d*x+c)^3+34/99/d*a^3*sin(d*x+c)^10/cos(d*x+c)+1
/11/d*a^3*sin(d*x+c)^10/cos(d*x+c)^11+3/10/d*a^3*sin(d*x+c)^10/cos(d*x+c)^1
0+34/99/d*a^3*cos(d*x+c)*sin(d*x+c)^8+272/693/d*a^3*cos(d*x+c)*sin(d*x+c)^6
+544/1155/d*a^3*cos(d*x+c)*sin(d*x+c)^4+2176/3465/d*a^3*cos(d*x+c)*sin(d*x+
c)^2+1/8/d*a^3*tan(d*x+c)^8-1/6/d*a^3*tan(d*x+c)^6+1/4*a^3*tan(d*x+c)^4/d-1
/2*a^3*tan(d*x+c)^2/d-a^3*ln(cos(d*x+c))/d
```

maxima [A] time = 0.65, size = 162, normalized size = 0.77

$$-\frac{27720\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{83160\,a^3\cos(dx+c)^{10}-13860\,a^3\cos(dx+c)^9-101640\,a^3\cos(dx+c)^8-41580\,a^3\cos(dx+c)^7+77616\,a^3\cos(dx+c)}{\text{co}}}{27720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/27720*(27720*a^3*log(cos(d*x + c)) - (83160*a^3*cos(d*x + c)^10 - 13860*
a^3*cos(d*x + c)^9 - 101640*a^3*cos(d*x + c)^8 - 41580*a^3*cos(d*x + c)^7 +
 77616*a^3*cos(d*x + c)^6 + 64680*a^3*cos(d*x + c)^5 - 23760*a^3*cos(d*x +
c)^4 - 38115*a^3*cos(d*x + c)^3 - 3080*a^3*cos(d*x + c)^2 + 8316*a^3*cos(d*
x + c) + 2520*a^3)/cos(d*x + c)^11)/d
```

mupad [B] time = 5.26, size = 337, normalized size = 1.60

$$\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}-22\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+\frac{332\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{3}-\frac{1012\,a^3\,\mathrm{ta}}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{22}-11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}+55\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-165\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - ((10090*a^3*tan(c/2 + (d*x)/2)^4)/6
3 - (9334*a^3*tan(c/2 + (d*x)/2)^2)/315 - (3676*a^3*tan(c/2 + (d*x)/2)^6)/7
 + (8164*a^3*tan(c/2 + (d*x)/2)^8)/7 - (5192*a^3*tan(c/2 + (d*x)/2)^10)/5 +
 (10456*a^3*tan(c/2 + (d*x)/2)^12)/15 - (1012*a^3*tan(c/2 + (d*x)/2)^14)/3
+ (332*a^3*tan(c/2 + (d*x)/2)^16)/3 - 22*a^3*tan(c/2 + (d*x)/2)^18 + 2*a^3*
tan(c/2 + (d*x)/2)^20 + (8704*a^3)/3465)/(d*(11*tan(c/2 + (d*x)/2)^2 - 55*t
an(c/2 + (d*x)/2)^4 + 165*tan(c/2 + (d*x)/2)^6 - 330*tan(c/2 + (d*x)/2)^8 +
 462*tan(c/2 + (d*x)/2)^10 - 462*tan(c/2 + (d*x)/2)^12 + 330*tan(c/2 + (d*x
)/2)^14 - 165*tan(c/2 + (d*x)/2)^16 + 55*tan(c/2 + (d*x)/2)^18 - 11*tan(c/2
 + (d*x)/2)^20 + tan(c/2 + (d*x)/2)^22 - 1))
```

sympy [A] time = 51.80, size = 439, normalized size = 2.09

$$\begin{cases}\frac{a^3\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^3\tan^8(c+dx)\sec^3(c+dx)}{11d}+\frac{3a^3\tan^8(c+dx)\sec^2(c+dx)}{10d}+\frac{a^3\tan^8(c+dx)\sec(c+dx)}{3d}+\frac{a^3\tan^8(c+dx)}{8d}-\frac{8a^3\tan^6(c\cdot}{} \\ x\left(a\sec(c)+a\right)^3\tan^9(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**8*sec(c
 + d*x)**3/(11*d) + 3*a**3*tan(c + d*x)**8*sec(c + d*x)**2/(10*d) + a**3*ta
n(c + d*x)**8*sec(c + d*x)/(3*d) + a**3*tan(c + d*x)**8/(8*d) - 8*a**3*tan(
c + d*x)**6*sec(c + d*x)**3/(99*d) - 3*a**3*tan(c + d*x)**6*sec(c + d*x)**2
/(10*d) - 8*a**3*tan(c + d*x)**6*sec(c + d*x)/(21*d) - a**3*tan(c + d*x)**6
```

```
/(6*d) + 16*a**3*tan(c + d*x)**4*sec(c + d*x)**3/(231*d) + 3*a**3*tan(c + d
*x)**4*sec(c + d*x)**2/(10*d) + 16*a**3*tan(c + d*x)**4*sec(c + d*x)/(35*d)
 + a**3*tan(c + d*x)**4/(4*d) - 64*a**3*tan(c + d*x)**2*sec(c + d*x)**3/(11
55*d) - 3*a**3*tan(c + d*x)**2*sec(c + d*x)**2/(10*d) - 64*a**3*tan(c + d*x
)**2*sec(c + d*x)/(105*d) - a**3*tan(c + d*x)**2/(2*d) + 128*a**3*sec(c + d
*x)**3/(3465*d) + 3*a**3*sec(c + d*x)**2/(10*d) + 128*a**3*sec(c + d*x)/(10
5*d), Ne(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**9, True))
```

3.38 $\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx$

Optimal. Leaf size=137

$$\frac{a^3\sec^9(c+dx)}{9d}+\frac{3a^3\sec^8(c+dx)}{8d}-\frac{4a^3\sec^6(c+dx)}{3d}-\frac{6a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{2d}+\frac{8a^3\sec^3(c+dx)}{3d}-\frac{3a^3 s}{}$$

```
[Out] a^3*ln(cos(d*x+c))/d-3*a^3*sec(d*x+c)/d+8/3*a^3*sec(d*x+c)^3/d+3/2*a^3*sec(
d*x+c)^4/d-6/5*a^3*sec(d*x+c)^5/d-4/3*a^3*sec(d*x+c)^6/d+3/8*a^3*sec(d*x+c)
^8/d+1/9*a^3*sec(d*x+c)^9/d
```

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{3879, 88\}$

$$\frac{a^3\sec^9(c+dx)}{9d}+\frac{3a^3\sec^8(c+dx)}{8d}-\frac{4a^3\sec^6(c+dx)}{3d}-\frac{6a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{2d}+\frac{8a^3\sec^3(c+dx)}{3d}-\frac{3a^3 s}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^3*Sec[c + d*x])/d + (8*a^3*Sec[c + d*x]^3)
/(3*d) + (3*a^3*Sec[c + d*x]^4)/(2*d) - (6*a^3*Sec[c + d*x]^5)/(5*d) - (4*a
^3*Sec[c + d*x]^6)/(3*d) + (3*a^3*Sec[c + d*x]^8)/(8*d) + (a^3*Sec[c + d*x]
^9)/(9*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^6}{x^{10}}\,dx,x,\cos(c+dx)\right)}{a^6d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{10}}+\frac{3a^9}{x^9}-\frac{8a^9}{x^7}-\frac{6a^9}{x^6}+\frac{6a^9}{x^5}+\frac{8a^9}{x^4}-\frac{3a^9}{x^2}-\frac{a^9}{x}\right)dx,x,\cos(c+d\right.}{a^6d}\\ &= \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{8a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^4(c-}{2d}\end{aligned}$$

Mathematica [A] time = 0.44, size = 110, normalized size = 0.80

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(40\sec^9(c+dx)+135\sec^8(c+dx)-480\sec^6(c+dx)-432\sec^5(c+dx)+\right.}{2880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(360*Log[Cos[c + d*x]] - 1080*
Sec[c + d*x] + 960*Sec[c + d*x]^3 + 540*Sec[c + d*x]^4 - 432*Sec[c + d*x]^5
 - 480*Sec[c + d*x]^6 + 135*Sec[c + d*x]^8 + 40*Sec[c + d*x]^9))/(2880*d)
```

fricas [A] time = 0.83, size = 117, normalized size = 0.85

$$\frac{360\,a^3\cos(dx+c)^9\log(-\cos(dx+c)) - 1080\,a^3\cos(dx+c)^8 + 960\,a^3\cos(dx+c)^6 + 540\,a^3\cos(dx+c)^5 - \cdots}{360\,d\cos(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/360*(360*a^3*cos(d*x + c)^9*log(-cos(d*x + c)) - 1080*a^3*cos(d*x + c)^8
+ 960*a^3*cos(d*x + c)^6 + 540*a^3*cos(d*x + c)^5 - 432*a^3*cos(d*x + c)^4
- 480*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x + c) + 40*a^3)/(d*cos(d*x + c)^9
)
```

giac [B] time = 28.27, size = 317, normalized size = 2.31

$$-\frac{2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{14297\,a^3 + \frac{133713\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{560052\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/2520*(2520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 25
20*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (14297*a^3 +
133713*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 560052*a^3*(cos(d*x + c)
 - 1)^2/(cos(d*x + c) + 1)^2 + 1384068*a^3*(cos(d*x + c) - 1)^3/(cos(d*x +
c) + 1)^3 + 1594782*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1336734
*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 781956*a^3*(cos(d*x + c) -
 1)^6/(cos(d*x + c) + 1)^6 + 302004*a^3*(cos(d*x + c) - 1)^7/(cos(d*x + c)
+ 1)^7 + 69201*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 7129*a^3*(co
s(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 1)^9)/d
```

maple [B] time = 0.81, size = 288, normalized size = 2.10

$$\frac{a^3\left(\tan^6(dx+c)\right)}{6d} - \frac{a^3\left(\tan^4(dx+c)\right)}{4d} + \frac{a^3\left(\tan^2(dx+c)\right)}{2d} + \frac{a^3\ln(\cos(dx+c))}{d} + \frac{4a^3\left(\sin^8(dx+c)\right)}{9d\cos(dx+c)^7} - \frac{4a^3\left(\sin^8 \cdots\right.}{45d\cos \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x)
```

```
[Out] 1/6/d*a^3*tan(d*x+c)^6-1/4*a^3*tan(d*x+c)^4/d+1/2*a^3*tan(d*x+c)^2/d+a^3*ln
(cos(d*x+c))/d+4/9/d*a^3*sin(d*x+c)^8/cos(d*x+c)^7-4/45/d*a^3*sin(d*x+c)^8/
cos(d*x+c)^5+4/45/d*a^3*sin(d*x+c)^8/cos(d*x+c)^3-4/9/d*a^3*sin(d*x+c)^8/co
s(d*x+c)-64/45*a^3*cos(d*x+c)/d-4/9/d*a^3*cos(d*x+c)*sin(d*x+c)^6-8/15/d*a^
3*cos(d*x+c)*sin(d*x+c)^4-32/45/d*a^3*cos(d*x+c)*sin(d*x+c)^2+3/8/d*a^3*sin
(d*x+c)^8/cos(d*x+c)^8+1/9/d*a^3*sin(d*x+c)^8/cos(d*x+c)^9
```

maxima [A] time = 0.36, size = 110, normalized size = 0.80

$$\frac{360\,a^3\log(\cos(dx+c)) - \frac{1080\,a^3\cos(dx+c)^8 - 960\,a^3\cos(dx+c)^6 - 540\,a^3\cos(dx+c)^5 + 432\,a^3\cos(dx+c)^4 + 480\,a^3\cos(dx+c)^3 - 135\,a^3\cos(\cdots}{\cos(dx+c)^9}}{360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/360*(360*a^3*log(cos(d*x + c)) - (1080*a^3*cos(d*x + c)^8 - 960*a^3*cos(d
*x + c)^6 - 540*a^3*cos(d*x + c)^5 + 432*a^3*cos(d*x + c)^4 + 480*a^3*cos(d
*x + c)^3 - 135*a^3*cos(d*x + c) - 40*a^3)/cos(d*x + c)^9)/d
```

mupad [B] time = 5.17, size = 278, normalized size = 2.03

$$\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-18\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+\frac{218\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}-174\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{1382\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-84\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+126\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-126\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((602*a^3*tan(c/2 + (d*x)/2)^4)/5 - (138*a^3*tan(c/2 + (d*x)/2)^2)/5 - (155
8*a^3*tan(c/2 + (d*x)/2)^6)/5 + (1382*a^3*tan(c/2 + (d*x)/2)^8)/5 - 174*a^3
*tan(c/2 + (d*x)/2)^10 + (218*a^3*tan(c/2 + (d*x)/2)^12)/3 - 18*a^3*tan(c/2
 + (d*x)/2)^14 + 2*a^3*tan(c/2 + (d*x)/2)^16 + (128*a^3)/45)/(d*(9*tan(c/2
+ (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 - 126*tan(
c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 - 84*tan(c/2 + (d*x)/2)^12 + 3
6*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/2)^18 -
 1)) - (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 22.57, size = 350, normalized size = 2.55

$$\begin{cases} -\frac{a^3\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^3\tan^6(c+dx)\sec^3(c+dx)}{9d}+\frac{3a^3\tan^6(c+dx)\sec^2(c+dx)}{8d}+\frac{3a^3\tan^6(c+dx)\sec(c+dx)}{7d}+\frac{a^3\tan^6(c+dx)}{6d}-\frac{2a^3\tan^4}{} \\ x\,(a\sec(c)+a)^3\tan^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**6*sec(
c + d*x)**3/(9*d) + 3*a**3*tan(c + d*x)**6*sec(c + d*x)**2/(8*d) + 3*a**3*t
an(c + d*x)**6*sec(c + d*x)/(7*d) + a**3*tan(c + d*x)**6/(6*d) - 2*a**3*tan
(c + d*x)**4*sec(c + d*x)**3/(21*d) - 3*a**3*tan(c + d*x)**4*sec(c + d*x)**
2/(8*d) - 18*a**3*tan(c + d*x)**4*sec(c + d*x)/(35*d) - a**3*tan(c + d*x)**
4/(4*d) + 8*a**3*tan(c + d*x)**2*sec(c + d*x)**3/(105*d) + 3*a**3*tan(c + d
*x)**2*sec(c + d*x)**2/(8*d) + 24*a**3*tan(c + d*x)**2*sec(c + d*x)/(35*d)
+ a**3*tan(c + d*x)**2/(2*d) - 16*a**3*sec(c + d*x)**3/(315*d) - 3*a**3*sec
(c + d*x)**2/(8*d) - 48*a**3*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) +
 a)**3*tan(c)**7, True))
```

3.39 $\int (a+a\sec(c+dx))^3\tan^5(c+dx)\,dx$

Optimal. Leaf size=138

$$\frac{a^3\sec^7(c+dx)}{7d}+\frac{a^3\sec^6(c+dx)}{2d}+\frac{a^3\sec^5(c+dx)}{5d}-\frac{5a^3\sec^4(c+dx)}{4d}-\frac{5a^3\sec^3(c+dx)}{3d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3 s}{}$$

```
[Out] -a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d-5/3*a^3*sec
(d*x+c)^3/d-5/4*a^3*sec(d*x+c)^4/d+1/5*a^3*sec(d*x+c)^5/d+1/2*a^3*sec(d*x+c
)^6/d+1/7*a^3*sec(d*x+c)^7/d
```

Rubi [A] time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{a^3\sec^7(c+dx)}{7d}+\frac{a^3\sec^6(c+dx)}{2d}+\frac{a^3\sec^5(c+dx)}{5d}-\frac{5a^3\sec^4(c+dx)}{4d}-\frac{5a^3\sec^3(c+dx)}{3d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3 s}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2
)/(2*d) - (5*a^3*Sec[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]^4)/(4*d) + (a^
3*Sec[c + d*x]^5)/(5*d) + (a^3*Sec[c + d*x]^6)/(2*d) + (a^3*Sec[c + d*x]^7)
/(7*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\tan^5(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^5}{x^8}\,dx,x,\cos(c+dx)\right)}{a^4d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}+\frac{3a^7}{x^7}+\frac{a^7}{x^6}-\frac{5a^7}{x^5}-\frac{5a^7}{x^4}+\frac{a^7}{x^3}+\frac{3a^7}{x^2}+\frac{a^7}{x}\right)dx,x,\cos(c+\right.}{a^4d}\\ &= -\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}-\frac{5a^3\sec^3}{3}\end{aligned}$$

Mathematica [A] time = 0.42, size = 140, normalized size = 1.01

$$-\frac{a^3\sec^7(c+dx)(-4522\cos(2(c+dx))+1050\cos(3(c+dx))-2380\cos(4(c+dx))-210\cos(5(c+dx))-63}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -1/6720*(a^3*(-3732 - 4522*Cos[2*(c + d*x)] + 1050*Cos[3*(c + d*x)] - 2380*
Cos[4*(c + d*x)] - 210*Cos[5*(c + d*x)] - 630*Cos[6*(c + d*x)] + 2205*Cos[3
*(c + d*x)]*Log[Cos[c + d*x]] + 735*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 10
5*Cos[7*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[c + d*x]*(8 + 35*Log[Cos[c +
 d*x]]))*Sec[c + d*x]^7)/d
```

fricas [A] time = 0.49, size = 117, normalized size = 0.85

$$-\frac{420\,a^3\cos\left(dx+c\right)^7\log\left(-\cos\left(dx+c\right)\right)-1260\,a^3\cos\left(dx+c\right)^6-210\,a^3\cos\left(dx+c\right)^5+700\,a^3\cos\left(dx+c\right)^4+5}{420\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/420*(420*a^3*cos(d*x + c)^7*log(-cos(d*x + c)) - 1260*a^3*cos(d*x + c)^6
 - 210*a^3*cos(d*x + c)^5 + 700*a^3*cos(d*x + c)^4 + 525*a^3*cos(d*x + c)^3
 - 84*a^3*cos(d*x + c)^2 - 210*a^3*cos(d*x + c) - 60*a^3)/(d*cos(d*x + c)^7
)
```

giac [B] time = 4.00, size = 267, normalized size = 1.93

$$\frac{420\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2497\,a^3+\frac{18319\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{58317\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{69475\,a^3(\cos}{(\cos(dx}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/420*(420*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a
^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2497*a^3 + 18319
*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 58317*a^3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 69475*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 + 56035*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^3*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8463*a^3*(cos(d*x + c) - 1)^6/(cos(d*x
+ c) + 1)^6 + 1089*a^3*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 1)^7)/d
```

maple [A] time = 0.79, size = 227, normalized size = 1.64

$$\frac{a^3\left(\tan^4\left(dx+c\right)\right)}{4d}-\frac{a^3\left(\tan^2\left(dx+c\right)\right)}{2d}-\frac{a^3\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{22a^3\left(\sin^6\left(dx+c\right)\right)}{35d\cos\left(dx+c\right)^5}-\frac{22a^3\left(\sin^6\left(dx+c\right)\right)}{105d\cos\left(dx+c\right)^3}+\frac{22a^3\left(\sin\right.}{35d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x)
```

```
[Out] 1/4*a^3*tan(d*x+c)^4/d-1/2*a^3*tan(d*x+c)^2/d-a^3*ln(cos(d*x+c))/d+22/35/d*
a^3*sin(d*x+c)^6/cos(d*x+c)^5-22/105/d*a^3*sin(d*x+c)^6/cos(d*x+c)^3+22/35/
d*a^3*sin(d*x+c)^6/cos(d*x+c)+176/105*a^3*cos(d*x+c)/d+22/35/d*a^3*cos(d*x+
c)*sin(d*x+c)^4+88/105/d*a^3*cos(d*x+c)*sin(d*x+c)^2+1/2/d*a^3*sin(d*x+c)^6
/cos(d*x+c)^6+1/7/d*a^3*sin(d*x+c)^6/cos(d*x+c)^7
```

maxima [A] time = 0.62, size = 110, normalized size = 0.80

$$-\frac{420\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{1260\,a^3\cos(dx+c)^6+210\,a^3\cos(dx+c)^5-700\,a^3\cos(dx+c)^4-525\,a^3\cos(dx+c)^3+84\,a^3\cos(dx+c)^2+210\,a^3\cos(dx+}{\cos(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/420*(420*a^3*log(cos(d*x + c)) - (1260*a^3*cos(d*x + c)^6 + 210*a^3*cos(
d*x + c)^5 - 700*a^3*cos(d*x + c)^4 - 525*a^3*cos(d*x + c)^3 + 84*a^3*cos(d
*x + c)^2 + 210*a^3*cos(d*x + c) + 60*a^3)/cos(d*x + c)^7)/d
```

mupad [B] time = 5.48, size = 221, normalized size = 1.60

$$\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-14\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}-\frac{224\,a^3\tan\left(\frac{c}{2}\right.}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - ((422*a^3*tan(c/2 + (d*x)/2)^4)/5 -
 (382*a^3*tan(c/2 + (d*x)/2)^2)/15 - (224*a^3*tan(c/2 + (d*x)/2)^6)/3 + (12
8*a^3*tan(c/2 + (d*x)/2)^8)/3 - 14*a^3*tan(c/2 + (d*x)/2)^10 + 2*a^3*tan(c/
2 + (d*x)/2)^12 + (352*a^3)/105)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 +
(d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2
 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

sympy [A] time = 9.01, size = 255, normalized size = 1.85

$$\begin{cases} \frac{a^3\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^3\tan^4(c+dx)\sec^3(c+dx)}{7d}+\frac{a^3\tan^4(c+dx)\sec^2(c+dx)}{2d}+\frac{3a^3\tan^4(c+dx)\sec(c+dx)}{5d}+\frac{a^3\tan^4(c+dx)}{4d}-\frac{4a^3\tan}{} \\ x\left(a\sec(c)+a\right)^3\tan^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**4*sec(c
 + d*x)**3/(7*d) + a**3*tan(c + d*x)**4*sec(c + d*x)**2/(2*d) + 3*a**3*tan(
c + d*x)**4*sec(c + d*x)/(5*d) + a**3*tan(c + d*x)**4/(4*d) - 4*a**3*tan(c
+ d*x)**2*sec(c + d*x)**3/(35*d) - a**3*tan(c + d*x)**2*sec(c + d*x)**2/(2*
d) - 4*a**3*tan(c + d*x)**2*sec(c + d*x)/(5*d) - a**3*tan(c + d*x)**2/(2*d)
 + 8*a**3*sec(c + d*x)**3/(105*d) + a**3*sec(c + d*x)**2/(2*d) + 8*a**3*sec
(c + d*x)/(5*d), Ne(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**5, True))
```

3.40 $\int (a + a\sec(c + dx))^3 \tan^3(c + dx)\, dx$

Optimal. Leaf size=99

$$\frac{a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{4d}+\frac{2a^3\sec^3(c+dx)}{3d}-\frac{a^3\sec^2(c+dx)}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}$$

```
[Out] a^3*ln(cos(d*x+c))/d-3*a^3*sec(d*x+c)/d-a^3*sec(d*x+c)^2/d+2/3*a^3*sec(d*x+
c)^3/d+3/4*a^3*sec(d*x+c)^4/d+1/5*a^3*sec(d*x+c)^5/d
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 75}

$$\frac{a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{4d}+\frac{2a^3\sec^3(c+dx)}{3d}-\frac{a^3\sec^2(c+dx)}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^3*Sec[c + d*x])/d - (a^3*Sec[c + d*x]^2)/d
 + (2*a^3*Sec[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]^4)/(4*d) + (a^3*Sec[c
 + d*x]^5)/(5*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a-ax)(a+ax)^4}{x^6}\,dx, x, \cos(c+dx)\right)}{a^2d}\\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{3a^5}{x^5}+\frac{2a^5}{x^4}-\frac{2a^5}{x^3}-\frac{3a^5}{x^2}-\frac{a^5}{x}\right)dx, x, \cos(c+dx)\right)}{a^2d}\\
&= \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\sec^2(c+dx)}{d}+\frac{2a^3\sec^3(c+
\end{aligned}$$

Mathematica [A] time = 0.30, size = 92, normalized size = 0.93

$$-\frac{a^3\sec^5(c+dx)(280\cos(2(c+dx))+90\cos(4(c+dx))+\cos(3(c+dx))(60-75\log(\cos(c+dx)))-150\cos(c-}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] -1/240*(a^3*(142 + 280*Cos[2*(c + d*x)] + 90*Cos[4*(c + d*x)] + Cos[3*(c +
d*x)]*(60 - 75*Log[Cos[c + d*x]]) - 150*Cos[c + d*x]*Log[Cos[c + d*x]] - 15
*Cos[5*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^5)/d
```

fricas [A] time = 0.49, size = 91, normalized size = 0.92

$$\frac{60\,a^3\cos(dx+c)^5\log(-\cos(dx+c)) - 180\,a^3\cos(dx+c)^4 - 60\,a^3\cos(dx+c)^3 + 40\,a^3\cos(dx+c)^2 + 45\,a}{60\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a^3*cos(d*x + c)^5*log(-cos(d*x + c)) - 180*a^3*cos(d*x + c)^4 - 6
0*a^3*cos(d*x + c)^3 + 40*a^3*cos(d*x + c)^2 + 45*a^3*cos(d*x + c) + 12*a^3
)/(d*cos(d*x + c)^5)
```

giac [B] time = 1.52, size = 217, normalized size = 2.19

$$-\frac{60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{393\,a^3 + \frac{2085\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{2610\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{1970\,a^3(\cos(}{(\cos(dx+}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\right.}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^3
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (393*a^3 + 2085*a^3
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2610*a^3*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + 1970*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805
*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*x + c) - 1)
^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5)/d
```

maple [A] time = 0.78, size = 164, normalized size = 1.66

$$\frac{a^3\left(\tan^2(dx+c)\right)}{2d} + \frac{a^3\ln(\cos(dx+c))}{d} + \frac{16a^3\left(\sin^4(dx+c)\right)}{15d\cos(dx+c)^3} - \frac{16a^3\left(\sin^4(dx+c)\right)}{15d\cos(dx+c)} - \frac{16a^3\cos(dx+c)\left(\sin^2(dx\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x)
```

```
[Out] 1/2*a^3*tan(d*x+c)^2/d+a^3*ln(cos(d*x+c))/d+16/15/d*a^3*sin(d*x+c)^4/cos(d*
x+c)^3-16/15/d*a^3*sin(d*x+c)^4/cos(d*x+c)-16/15/d*a^3*cos(d*x+c)*sin(d*x+c
)^2-32/15*a^3*cos(d*x+c)/d+3/4/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4+1/5/d*a^3*si
n(d*x+c)^4/cos(d*x+c)^5
```

maxima [A] time = 0.47, size = 84, normalized size = 0.85

$$\frac{60\,a^3\log(\cos(dx+c)) - \frac{180\,a^3\cos(dx+c)^4+60\,a^3\cos(dx+c)^3-40\,a^3\cos(dx+c)^2-45\,a^3\cos(dx+c)-12\,a^3}{\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/60*(60*a^3*log(cos(d*x + c)) - (180*a^3*cos(d*x + c)^4 + 60*a^3*cos(d*x +
 c)^3 - 40*a^3*cos(d*x + c)^2 - 45*a^3*cos(d*x + c) - 12*a^3)/cos(d*x + c)^
5)/d
```

mupad [B] time = 5.55, size = 162, normalized size = 1.64

$$\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{62\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{70\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{64\,a^3}{15}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,a^3\,\mathrm{atanh}\left(\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((62*a^3*tan(c/2 + (d*x)/2)^4)/3 - (70*a^3*tan(c/2 + (d*x)/2)^2)/3 - 10*a^3
*tan(c/2 + (d*x)/2)^6 + 2*a^3*tan(c/2 + (d*x)/2)^8 + (64*a^3)/15)/(d*(5*tan
(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*t
an(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (2*a^3*atanh(tan(c/2 +
(d*x)/2)^2))/d
```

sympy [A] time = 3.24, size = 165, normalized size = 1.67

$$\begin{cases} -\frac{a^3\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^3\tan^2(c+dx)\sec^3(c+dx)}{5d}+\frac{3a^3\tan^2(c+dx)\sec^2(c+dx)}{4d}+\frac{a^3\tan^2(c+dx)\sec(c+dx)}{d}+\frac{a^3\tan^2(c+dx)}{2d}-\frac{2a^3\sec^3(}{15a} \\ x\left(a\sec(c)+a\right)^3\tan^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**2*sec(
c + d*x)**3/(5*d) + 3*a**3*tan(c + d*x)**2*sec(c + d*x)**2/(4*d) + a**3*tan
(c + d*x)**2*sec(c + d*x)/d + a**3*tan(c + d*x)**2/(2*d) - 2*a**3*sec(c + d
*x)**3/(15*d) - 3*a**3*sec(c + d*x)**2/(4*d) - 2*a**3*sec(c + d*x)/d, Ne(d,
 0)), (x*(a*sec(c) + a)**3*tan(c)**3, True))
```

3.41 $\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=66

$$\frac{a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\log(\cos(c+dx))}{d}$$

```
[Out] -a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+3/2*a^3*sec(d*x+c)^2/d+1/3*a^3*sec
(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 43}

$$\frac{a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d + (3*a^3*Sec[c + d*x]
^2)/(2*d) + (a^3*Sec[c + d*x]^3)/(3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx&=-\frac{\text{Subst}\left(\int\frac{(a+ax)^3}{x^4}\,dx,x,\cos(c+dx)\right)}{d}\\&=-\frac{\text{Subst}\left(\int\left(\frac{a^3}{x^4}+\frac{3a^3}{x^3}+\frac{3a^3}{x^2}+\frac{a^3}{x}\right)dx,x,\cos(c+dx)\right)}{d}\\&=-\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{3a^3\sec^2(c+dx)}{2d}+\frac{a^3\sec^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.16, size = 64, normalized size = 0.97

$$-\frac{a^3\sec^3(c+dx)(-18\cos(2(c+dx))+9\cos(c+dx)(\log(\cos(c+dx))-2)+3\cos(3(c+dx))\log(\cos(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -1/12*(a^3*(-22 - 18*Cos[2*(c + d*x)] + 9*Cos[c + d*x]*(-2 + Log[Cos[c + d*
x]]) + 3*Cos[3*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^3)/d
```

fricas [A] time = 0.66, size = 65, normalized size = 0.98

$$-\frac{6\,a^3\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)-18\,a^3\cos\left(dx+c\right)^2-9\,a^3\cos\left(dx+c\right)-2\,a^3}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^3*cos(d*x + c)^3*log(-cos(d*x + c)) - 18*a^3*cos(d*x + c)^2 - 9*a
^3*cos(d*x + c) - 2*a^3)/(d*cos(d*x + c)^3)
```

giac [B] time = 0.51, size = 167, normalized size = 2.53

$$\frac{6\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-6\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{51\,a^3+\frac{69\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a^3*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (51*a^3 + 69*a^3*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 45*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 11*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 1)^3)/d
```

maple [A] time = 0.30, size = 62, normalized size = 0.94

$$\frac{a^3\left(\sec^3\left(dx+c\right)\right)}{3d}+\frac{3a^3\left(\sec^2\left(dx+c\right)\right)}{2d}+\frac{3a^3\sec\left(dx+c\right)}{d}+\frac{a^3\ln\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c),x)
```

```
[Out] 1/3*a^3*sec(d*x+c)^3/d+3/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d+a^3/d*ln(s
ec(d*x+c))
```

maxima [A] time = 0.68, size = 58, normalized size = 0.88

$$-\frac{6\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{18\,a^3\cos(dx+c)^2+9\,a^3\cos(dx+c)+2\,a^3}{\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/6*(6*a^3*log(cos(d*x + c)) - (18*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c)
 + 2*a^3)/cos(d*x + c)^3)/d
```

mupad [B] time = 1.94, size = 105, normalized size = 1.59

$$\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{20\,a^3}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a^3*tan(c/2 + (d*x)/2)^4 - 6*a^3
*tan(c/2 + (d*x)/2)^2 + (20*a^3)/3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2
+ (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [**A**] time = 0.92, size = 76, normalized size = 1.15

$$\begin{cases} \frac{a^3\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^3\sec^3(c+dx)}{3d} + \frac{3a^3\sec^2(c+dx)}{2d} + \frac{3a^3\sec(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a\sec(c)+a\right)^3\tan(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*sec(c + d*x)**3/(3*d)
 + 3*a**3*sec(c + d*x)**2/(2*d) + 3*a**3*sec(c + d*x)/d, Ne(d, 0)), (x*(a*s
ec(c) + a)**3*tan(c), True))
```

3.42 $\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=48

$$\frac{a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

```
[Out] 4*a^3*ln(1-cos(d*x+c))/d-3*a^3*ln(cos(d*x+c))/d+a^3*sec(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (4*a^3*Log[1 - Cos[c + d*x]])/d - (3*a^3*Log[Cos[c + d*x]])/d + (a^3*Sec[c + d*x])/d
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m - 1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx &= -\frac{a^2\,\text{Subst}\left(\int \frac{(a+ax)^2}{x^2(a-ax)}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^2\,\text{Subst}\left(\int \left(-\frac{4a}{-1+x}+\frac{a}{x^2}+\frac{3a}{x}\right)dx,x,\cos(c+dx)\right)}{d}\\ &= \frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}+\frac{a^3\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 36, normalized size = 0.75

$$\frac{a^3\left(\sec(c+dx)+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(-3*Log[Cos[c + d*x]] + 8*Log[Sin[(c + d*x)/2]] + Sec[c + d*x]))/d
```

fricas [A] time = 0.79, size = 61, normalized size = 1.27

$$-\frac{3\,a^{3}\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-4\,a^{3}\cos\left(dx+c\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-a^{3}}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*a^3*cos(d*x + c)*log(-cos(d*x + c)) - 4*a^3*cos(d*x + c)*log(-1/2*cos(d
*x + c) + 1/2) - a^3)/(d*cos(d*x + c))
```

giac [B] time = 0.32, size = 145, normalized size = 3.02

$$\frac{4\,a^{3}\log\left(\frac{\left|-\cos\left(dx+c\right)+1\right|}{\left|\cos\left(dx+c\right)+1\right|}\right)-a^{3}\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)-3\,a^{3}\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}-1\right|\right)+\frac{5\,a^{3}+\frac{3\,a^{3}\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}}{\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (4*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^3*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 3*a^3*log(abs(-(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 1)) + (5*a^3 + 3*a^3*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

maple [A] time = 0.51, size = 47, normalized size = 0.98

$$\frac{a^{3}\sec\left(dx+c\right)}{d}-\frac{a^{3}\ln\left(\sec\left(dx+c\right)\right)}{d}+\frac{4a^{3}\ln\left(-1+\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] a^3*sec(d*x+c)/d-a^3/d*ln(sec(d*x+c))+4*a^3/d*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.60, size = 43, normalized size = 0.90

$$\frac{4\,a^{3}\log\left(\cos\left(dx+c\right)-1\right)-3\,a^{3}\log\left(\cos\left(dx+c\right)\right)+\frac{a^{3}}{\cos\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (4*a^3*log(cos(d*x + c) - 1) - 3*a^3*log(cos(d*x + c)) + a^3/cos(d*x + c))/
d
```

mupad [B] time = 1.20, size = 86, normalized size = 1.79

$$\frac{8\,a^{3}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a^{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}-1\right)}-\frac{3\,a^{3}\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}-1\right)}{d}-\frac{a^{3}\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (8*a^3*log(tan(c/2 + (d*x)/2)))/d - (2*a^3)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
- (3*a^3*log(tan(c/2 + (d*x)/2)^2 - 1))/d - (a^3*log(tan(c/2 + (d*x)/2)^2 +
 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cot(c+dx)\sec(c+dx)\,dx+\int 3\cot(c+dx)\sec^2(c+dx)\,dx+\int \cot(c+dx)\sec^3(c+dx)\,dx+\int \cot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)*sec(c + d*x), x) + Integral(3*cot(c + d*x)*se
c(c + d*x)**2, x) + Integral(cot(c + d*x)*sec(c + d*x)**3, x) + Integral(co
t(c + d*x), x))
```

3.43 $\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=40

$$-\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3\log(1-\cos(c+dx))}{d}$$

```
[Out] -2*a^3/d/(1-cos(d*x+c))-a^3*ln(1-cos(d*x+c))/d
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$-\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^3)/(d*(1 - Cos[c + d*x])) - (a^3*Log[1 - Cos[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx &= -\frac{a^4\,\text{Subst}\left(\int \frac{a+ax}{(a-ax)^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^4\,\text{Subst}\left(\int \left(\frac{2}{a(-1+x)^2}+\frac{1}{a(-1+x)}\right)dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3\log(1-\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.13, size = 46, normalized size = 1.15

$$-\frac{a^3\left(\cot^2\left(\frac{1}{2}(c+dx)\right)+2\left(\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((a^3*(Cot[(c + d*x)/2]^2 + 2*(Log[Cos[(c + d*x)/2]] + Log[Tan[(c + d*x)/2
]]))))/d)
```

fricas [A] time = 0.73, size = 50, normalized size = 1.25

$$\frac{2 a^3 - \left(a^3 \cos(dx+c) - a^3\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right)}{d \cos(dx+c) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (2*a^3 - (a^3*cos(d*x + c) - a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x
+ c) - d)
```

giac [B] time = 0.37, size = 109, normalized size = 2.72

$$-\frac{a^3 \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - \frac{\left(a^3 + \frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^3*log(abs(-(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^3 + a^3*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

maple [A] time = 0.73, size = 51, normalized size = 1.28

$$\frac{a^3 \ln(\sec(dx+c))}{d} - \frac{2a^3}{d(-1+\sec(dx+c))} - \frac{a^3 \ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x)
```

```
[Out] a^3/d*ln(sec(d*x+c))-2*a^3/d/(-1+sec(d*x+c))-a^3/d*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.61, size = 34, normalized size = 0.85

$$-\frac{a^3 \log(\cos(dx+c) - 1) - \frac{2a^3}{\cos(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(a^3*log(cos(d*x + c) - 1) - 2*a^3/(cos(d*x + c) - 1))/d
```

mupad [B] time = 1.23, size = 48, normalized size = 1.20

$$-\frac{a^3 \left(\cot\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 2 \ln\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \ln\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cot^3(c+dx)\sec(c+dx)\,dx+\int 3\cot^3(c+dx)\sec^2(c+dx)\,dx+\int \cot^3(c+dx)\sec^3(c+dx)\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**3*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**3, x))
```

3.44 $\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=61

$$\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{a^3\log(1-\cos(c+dx))}{d}$$

```
[Out] -1/2*a^3/d/(1-cos(d*x+c))^2+2*a^3/d/(1-cos(d*x+c))+a^3*ln(1-cos(d*x+c))/d
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{a^3\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -a^3/(2*d*(1 - Cos[c + d*x])^2) + (2*a^3)/(d*(1 - Cos[c + d*x])) + (a^3*Log
[1 - Cos[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx &= -\frac{a^6\,\text{Subst}\left(\int \frac{x^2}{(a-ax)^3}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{a^3(-1+x)^3}-\frac{2}{a^3(-1+x)^2}-\frac{1}{a^3(-1+x)}\right)dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{2a^3}{d(1-\cos(c+dx))}+\frac{a^3\log(1-\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.17, size = 72, normalized size = 1.18

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\csc^4\left(\frac{1}{2}(c+dx)\right)-8\csc^2\left(\frac{1}{2}(c+dx)\right)-16\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/64*(a^3*(1 + Cos[c + d*x])^3*(-8*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^4
 - 16*Log[Sin[(c + d*x)/2]])*Sec[(c + d*x)/2]^6)/d
```

fricas [A] time = 0.86, size = 82, normalized size = 1.34

$$-\frac{4\,a^3\cos\left(dx+c\right)-3\,a^3-2\,\left(a^3\cos\left(dx+c\right)^2-2\,a^3\cos\left(dx+c\right)+a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,\left(d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*cos(d*x + c) - 3*a^3 - 2*(a^3*cos(d*x + c)^2 - 2*a^3*cos(d*x +
c) + a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c
) + d)
```

giac [B] time = 1.01, size = 138, normalized size = 2.26

$$\frac{8\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-8\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^3+\frac{6\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*a^3*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^3 + 6*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 12*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

maple [A] time = 0.62, size = 68, normalized size = 1.11

$$-\frac{a^3\ln\left(\sec\left(dx+c\right)\right)}{d}+\frac{a^3}{d\left(-1+\sec\left(dx+c\right)\right)}-\frac{a^3}{2d\left(-1+\sec\left(dx+c\right)\right)^2}+\frac{a^3\ln\left(-1+\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -a^3/d*ln(sec(d*x+c))+a^3/d/(-1+sec(d*x+c))-1/2*a^3/d/(-1+sec(d*x+c))^2+a^3
/d*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.41, size = 59, normalized size = 0.97

$$\frac{2\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{4\,a^3\cos(dx+c)-3\,a^3}{\cos(dx+c)^2-2\,\cos(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3*log(cos(d*x + c) - 1) - (4*a^3*cos(d*x + c) - 3*a^3)/(cos(d*x +
c)^2 - 2*cos(d*x + c) + 1))/d
```

mupad [B] time = 1.22, size = 78, normalized size = 1.28

$$\frac{2\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-\frac{a^3}{8}}{d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}-\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*log(tan(c/2 + (d*x)/2)))/d + ((3*a^3*tan(c/2 + (d*x)/2)^2)/4 - a^3/8
)/(d*tan(c/2 + (d*x)/2)^4) - (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\cot^{5}(c+dx)\sec(c+dx)\,dx+\int 3\cot^{5}(c+dx)\sec^{2}(c+dx)\,dx+\int\cot^{5}(c+dx)\sec^{3}(c+dx)\,dx+\int c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**5*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**5*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**5*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**5, x))
```

3.45 $\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=107

$$-\frac{17a^3}{8d(1-\cos(c+dx))}+\frac{7a^3}{8d(1-\cos(c+dx))^2}-\frac{a^3}{6d(1-\cos(c+dx))^3}-\frac{15a^3\log(1-\cos(c+dx))}{16d}-\frac{a^3\log(\cos(c+\ldots}{16d}$$

```
[Out] -1/6*a^3/d/(1-cos(d*x+c))^3+7/8*a^3/d/(1-cos(d*x+c))^2-17/8*a^3/d/(1-cos(d*
x+c))-15/16*a^3*ln(1-cos(d*x+c))/d-1/16*a^3*ln(1+cos(d*x+c))/d
```

Rubi [A] time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$-\frac{17a^3}{8d(1-\cos(c+dx))}+\frac{7a^3}{8d(1-\cos(c+dx))^2}-\frac{a^3}{6d(1-\cos(c+dx))^3}-\frac{15a^3\log(1-\cos(c+dx))}{16d}-\frac{a^3\log(\cos(c+\ldots}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -a^3/(6*d*(1 - Cos[c + d*x])^3) + (7*a^3)/(8*d*(1 - Cos[c + d*x])^2) - (17*
a^3)/(8*d*(1 - Cos[c + d*x])) - (15*a^3*Log[1 - Cos[c + d*x]])/(16*d) - (a^
3*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx &= -\frac{a^8\,\text{Subst}\left(\int \frac{x^4}{(a-ax)^4(a+ax)}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^8\,\text{Subst}\left(\int \left(\frac{1}{2a^5(-1+x)^4}+\frac{7}{4a^5(-1+x)^3}+\frac{17}{8a^5(-1+x)^2}+\frac{15}{16a^5(-1+x)}+\frac{1}{16a^5(1\ldots}\right.\right.}{d}\\ &= -\frac{a^3}{6d(1-\cos(c+dx))^3}+\frac{7a^3}{8d(1-\cos(c+dx))^2}-\frac{17a^3}{8d(1-\cos(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.68, size = 102, normalized size = 0.95

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2\csc^6\left(\frac{1}{2}(c+dx)\right)-21\csc^4\left(\frac{1}{2}(c+dx)\right)+102\csc^2\left(\frac{1}{2}(c+dx)\right)+12\left(1\ldots\right.\right.}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/768*(a^3*(1 + Cos[c + d*x])^3*(102*Csc[(c + d*x)/2]^2 - 21*Csc[(c + d*x)
/2]^4 + 2*Csc[(c + d*x)/2]^6 + 12*(Log[Cos[(c + d*x)/2]] + 15*Log[Sin[(c +
d*x)/2]]))*Sec[(c + d*x)/2]^6)/d
```

fricas [A] time = 1.17, size = 178, normalized size = 1.66

$$\frac{102\,a^3\cos\left(dx+c\right)^2-162\,a^3\cos\left(dx+c\right)+68\,a^3-3\left(a^3\cos\left(dx+c\right)^3-3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)-a^3\right.}{48\left(d\cos\left(dx+c\right)^3-3\,d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(102*a^3*cos(d*x + c)^2 - 162*a^3*cos(d*x + c) + 68*a^3 - 3*(a^3*cos(d
*x + c)^3 - 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) - a^3)*log(1/2*cos(d*
x + c) + 1/2) - 45*(a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d
*x + c) - a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - 3*d*cos(d*
x + c)^2 + 3*d*cos(d*x + c) - d)
```

giac [A] time = 1.48, size = 165, normalized size = 1.54

$$-\frac{90\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-96\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(2\,a^3+\frac{15\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{66\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{165\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(d}{(\cos(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(90*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 96*a^3*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (2*a^3 + 15*a^3*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 66*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 165*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c)
+ 1)^3/(cos(d*x + c) - 1)^3)/d
```

maple [A] time = 0.62, size = 104, normalized size = 0.97

$$\frac{a^3\ln\left(\sec\left(dx+c\right)\right)}{d}-\frac{a^3}{6d\left(-1+\sec\left(dx+c\right)\right)^3}+\frac{3a^3}{8d\left(-1+\sec\left(dx+c\right)\right)^2}-\frac{7a^3}{8d\left(-1+\sec\left(dx+c\right)\right)}-\frac{15a^3\ln\left(-1+\sec\left(\right.\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x)
```

```
[Out] a^3/d*ln(sec(d*x+c))-1/6*a^3/d/(-1+sec(d*x+c))^3+3/8*a^3/d/(-1+sec(d*x+c))^
2-7/8*a^3/d/(-1+sec(d*x+c))-15/16*a^3/d*ln(-1+sec(d*x+c))-1/16*a^3/d*ln(1+s
ec(d*x+c))
```

maxima [A] time = 0.52, size = 96, normalized size = 0.90

$$-\frac{3\,a^3\log\left(\cos\left(dx+c\right)+1\right)+45\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(51\,a^3\cos(dx+c)^2-81\,a^3\cos(dx+c)+34\,a^3\right)}{\cos(dx+c)^3-3\,\cos(dx+c)^2+3\,\cos(dx+c)-1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^3*log(cos(d*x + c) + 1) + 45*a^3*log(cos(d*x + c) - 1) - 2*(51*a
^3*cos(d*x + c)^2 - 81*a^3*cos(d*x + c) + 34*a^3)/(cos(d*x + c)^3 - 3*cos(d
*x + c)^2 + 3*cos(d*x + c) - 1))/d
```

mupad [B] time = 1.29, size = 94, normalized size = 0.88

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\frac{11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-\frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{a^3}{6}}{8\,d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}-\frac{15\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - ((11*a^3*tan(c/2 + (d*x)/2)^4)/2 -
(5*a^3*tan(c/2 + (d*x)/2)^2)/4 + a^3/6)/(8*d*tan(c/2 + (d*x)/2)^6) - (15*a^
3*log(tan(c/2 + (d*x)/2)))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.46 $\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=149

$$\frac{9a^3}{4d(1-\cos(c+dx))}+\frac{a^3}{32d(\cos(c+dx)+1)}-\frac{39a^3}{32d(1-\cos(c+dx))^2}+\frac{5a^3}{12d(1-\cos(c+dx))^3}-\frac{a^3}{16d(1-\cos(c+dx))^4}$$

```
[Out] -1/16*a^3/d/(1-cos(d*x+c))^4+5/12*a^3/d/(1-cos(d*x+c))^3-39/32*a^3/d/(1-cos
(d*x+c))^2+9/4*a^3/d/(1-cos(d*x+c))+1/32*a^3/d/(1+cos(d*x+c))+57/64*a^3*ln(
1-cos(d*x+c))/d+7/64*a^3*ln(1+cos(d*x+c))/d
```

Rubi [A] time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{9a^3}{4d(1-\cos(c+dx))}+\frac{a^3}{32d(\cos(c+dx)+1)}-\frac{39a^3}{32d(1-\cos(c+dx))^2}+\frac{5a^3}{12d(1-\cos(c+dx))^3}-\frac{a^3}{16d(1-\cos(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -a^3/(16*d*(1 - Cos[c + d*x])^4) + (5*a^3)/(12*d*(1 - Cos[c + d*x])^3) - (3
9*a^3)/(32*d*(1 - Cos[c + d*x])^2) + (9*a^3)/(4*d*(1 - Cos[c + d*x])) + a^3
/(32*d*(1 + Cos[c + d*x])) + (57*a^3*Log[1 - Cos[c + d*x]])/(64*d) + (7*a^3
*Log[1 + Cos[c + d*x]])/(64*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx &= -\frac{a^{10}\,\text{Subst}\left(\int \frac{x^6}{(a-ax)^5(a+ax)^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^{10}\,\text{Subst}\left(\int \left(-\frac{1}{4a^7(-1+x)^5}-\frac{5}{4a^7(-1+x)^4}-\frac{39}{16a^7(-1+x)^3}-\frac{9}{4a^7(-1+x)^2}-\frac{57}{64a^7(-1}\right.\right.}{d}\\ &= -\frac{a^3}{16d(1-\cos(c+dx))^4}+\frac{5a^3}{12d(1-\cos(c+dx))^3}-\frac{39a^3}{32d(1-\cos(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.40, size = 130, normalized size = 0.87

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-3\csc^8\left(\frac{1}{2}(c+dx)\right)+40\csc^6\left(\frac{1}{2}(c+dx)\right)-234\csc^4\left(\frac{1}{2}(c+dx)\right)+864\csc\right.}{6144d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(864*Csc[(c + d*x)/2]^2 - 234*
Csc[(c + d*x)/2]^4 + 40*Csc[(c + d*x)/2]^6 - 3*Csc[(c + d*x)/2]^8 + 12*(14*
Log[Cos[(c + d*x)/2]] + 114*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2)))/(
6144*d)
```

fricas [B] time = 0.76, size = 272, normalized size = 1.83

$$-\frac{426\,a^3\cos(dx+c)^4-606\,a^3\cos(dx+c)^3-190\,a^3\cos(dx+c)^2+666\,a^3\cos(dx+c)-272\,a^3-21\left(a^3\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(426*a^3*cos(d*x + c)^4 - 606*a^3*cos(d*x + c)^3 - 190*a^3*cos(d*x +
 c)^2 + 666*a^3*cos(d*x + c) - 272*a^3 - 21*(a^3*cos(d*x + c)^5 - 3*a^3*cos
(d*x + c)^4 + 2*a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x +
 c) + a^3)*log(1/2*cos(d*x + c) + 1/2) - 171*(a^3*cos(d*x + c)^5 - 3*a^3*co
s(d*x + c)^4 + 2*a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x
+ c) + a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^5 - 3*d*cos(d*x +
 c)^4 + 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - 3*d*cos(d*x + c) + d)
```

giac [A] time = 0.51, size = 213, normalized size = 1.43

$$\frac{684\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-768\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{\left(3\,a^3+\frac{28\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a^3(\cos(dx+c)-1}{(\cos(dx+c)+1)^2}\right.}{}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(684*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 768*a^3*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - (3*a^3 + 28*a^3*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 132*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 504*a^3*(cos(d*
x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1425*a^3*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4)*(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4)/d
```

maple [A] time = 0.75, size = 141, normalized size = 0.95

$$-\frac{a^3\ln(\sec(dx+c))}{d}-\frac{a^3}{16d(-1+\sec(dx+c))^4}+\frac{a^3}{6d(-1+\sec(dx+c))^3}-\frac{11a^3}{32d(-1+\sec(dx+c))^2}+\frac{13}{16d(-1+\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -a^3/d*ln(sec(d*x+c))-1/16*a^3/d/(-1+sec(d*x+c))^4+1/6*a^3/d/(-1+sec(d*x+c)
)^3-11/32*a^3/d/(-1+sec(d*x+c))^2+13/16*a^3/d/(-1+sec(d*x+c))+57/64*a^3/d*l
n(-1+sec(d*x+c))-1/32*a^3/d/(1+sec(d*x+c))+7/64*a^3/d*ln(1+sec(d*x+c))
```

maxima [A] time = 0.37, size = 142, normalized size = 0.95

$$\frac{21\,a^3\log(\cos(dx+c)+1)+171\,a^3\log(\cos(dx+c)-1)-\frac{2\left(213\,a^3\cos(dx+c)^4-303\,a^3\cos(dx+c)^3-95\,a^3\cos(dx+c)^2+333\,a^3\right.}{\cos(dx+c)^5-3\cos(dx+c)^4+2\cos(dx+c)^3+2\cos(dx+c)^2-3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/192*(21*a^3*log(cos(d*x + c) + 1) + 171*a^3*log(cos(d*x + c) - 1) - 2*(21
3*a^3*cos(d*x + c)^4 - 303*a^3*cos(d*x + c)^3 - 95*a^3*cos(d*x + c)^2 + 333
*a^3*cos(d*x + c) - 136*a^3)/(cos(d*x + c)^5 - 3*cos(d*x + c)^4 + 2*cos(d*x
 + c)^3 + 2*cos(d*x + c)^2 - 3*cos(d*x + c) + 1))/d
```

mupad [B] time = 1.20, size = 130, normalized size = 0.87

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}+\frac{57\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32\,d}+\frac{21\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}-\frac{a^3}{8}}{32\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}-\frac{a^3\,\ln\left(\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(64*d) + (57*a^3*log(tan(c/2 + (d*x)/2)))/(32*d)
 + ((7*a^3*tan(c/2 + (d*x)/2)^2)/6 - (11*a^3*tan(c/2 + (d*x)/2)^4)/2 + 21*a
^3*tan(c/2 + (d*x)/2)^6 - a^3/8)/(32*d*tan(c/2 + (d*x)/2)^8) - (a^3*log(tan
(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.47 $\int (a + a\sec(c + dx))^3 \tan^6(c + dx) \, dx$

Optimal. Leaf size=237

$$\frac{3a^3\tan^7(c+dx)}{7d}+\frac{a^3\tan^5(c+dx)}{5d}-\frac{a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}-\frac{125a^3\tanh^{-1}(\sin(c+dx))}{128d}+\frac{a^3\tan^5(c+$$

```
[Out] -a^3*x-125/128*a^3*arctanh(sin(d*x+c))/d+a^3*tan(d*x+c)/d+115/128*a^3*sec(d
*x+c)*tan(d*x+c)/d+5/64*a^3*sec(d*x+c)^3*tan(d*x+c)/d-1/3*a^3*tan(d*x+c)^3/
d-5/8*a^3*sec(d*x+c)*tan(d*x+c)^3/d-5/48*a^3*sec(d*x+c)^3*tan(d*x+c)^3/d+1/
5*a^3*tan(d*x+c)^5/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)^5/d+1/8*a^3*sec(d*x+c)^3
*tan(d*x+c)^5/d+3/7*a^3*tan(d*x+c)^7/d
```

Rubi [A] time = 0.30, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$\frac{3a^3\tan^7(c+dx)}{7d}+\frac{a^3\tan^5(c+dx)}{5d}-\frac{a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}-\frac{125a^3\tanh^{-1}(\sin(c+dx))}{128d}+\frac{a^3\tan^5(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] -(a^3*x) - (125*a^3*ArcTanh[Sin[c + d*x]])/(128*d) + (a^3*Tan[c + d*x])/d +
 (115*a^3*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[c
+ d*x])/(64*d) - (a^3*Tan[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]*Tan[c + d
*x]^3)/(8*d) - (5*a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*d) + (a^3*Tan[c +
d*x]^5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x]^5)/(2*d) + (a^3*Sec[c + d*x]
^3*Tan[c + d*x]^5)/(8*d) + (3*a^3*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\tan^6(c+dx)\,dx &= \int \left(a^3\tan^6(c+dx)+3a^3\sec(c+dx)\tan^6(c+dx)+3a^3\sec^2(c+dx)\tan\right.\\ &= a^3\int \tan^6(c+dx)\,dx+a^3\int \sec^3(c+dx)\tan^6(c+dx)\,dx+\left(3a^3\right)\int \sec\\ &= \frac{a^3\tan^5(c+dx)}{5d}+\frac{a^3\sec(c+dx)\tan^5(c+dx)}{2d}+\frac{a^3\sec^3(c+dx)\tan^5(c+}{8d}\\ &= -\frac{a^3\tan^3(c+dx)}{3d}-\frac{5a^3\sec(c+dx)\tan^3(c+dx)}{8d}-\frac{5a^3\sec^3(c+dx)\tan^3}{48d}\\ &= \frac{a^3\tan(c+dx)}{d}+\frac{15a^3\sec(c+dx)\tan(c+dx)}{16d}+\frac{5a^3\sec^3(c+dx)\tan(c}{64d}\\ &= -a^3x-\frac{15a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan(c+dx)}{d}+\frac{115a^3\sec(c+dx)\,t}{128d}\\ &= -a^3x-\frac{125a^3\tanh^{-1}(\sin(c+dx))}{128d}+\frac{a^3\tan(c+dx)}{d}+\frac{115a^3\sec(c+dx)}{128d}\end{aligned}$$

Mathematica [A] time = 2.18, size = 363, normalized size = 1.53

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^8(c+dx)\left(1680000\cos^8(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^8*(1680000*Cos[c
+ d*x]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]) - Sec[c]*(470400*d*x*Cos[c] + 376320*d*x*Cos[c + 2*d*x]
 + 376320*d*x*Cos[3*c + 2*d*x] + 188160*d*x*Cos[3*c + 4*d*x] + 188160*d*x*C
os[5*c + 4*d*x] + 53760*d*x*Cos[5*c + 6*d*x] + 53760*d*x*Cos[7*c + 6*d*x] +
 6720*d*x*Cos[7*c + 8*d*x] + 6720*d*x*Cos[9*c + 8*d*x] + 519680*Sin[c] - 13
3175*Sin[d*x] - 133175*Sin[2*c + d*x] - 544768*Sin[c + 2*d*x] + 286720*Sin[
3*c + 2*d*x] - 63595*Sin[2*c + 3*d*x] - 63595*Sin[4*c + 3*d*x] - 254464*Sin
[3*c + 4*d*x] + 161280*Sin[5*c + 4*d*x] - 65135*Sin[4*c + 5*d*x] - 65135*Si
n[6*c + 5*d*x] - 118784*Sin[5*c + 6*d*x] - 27195*Sin[6*c + 7*d*x] - 27195*S
in[8*c + 7*d*x] - 14848*Sin[7*c + 8*d*x])))/(13762560*d)
```

fricas [A] time = 0.82, size = 178, normalized size = 0.75

$$-\frac{26880\,a^{3}dx\cos\left(dx+c\right)^{8}+13125\,a^{3}\cos\left(dx+c\right)^{8}\log\left(\sin\left(dx+c\right)+1\right)-13125\,a^{3}\cos\left(dx+c\right)^{8}\log\left(-\sin\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/26880*(26880*a^3*d*x*cos(d*x + c)^8 + 13125*a^3*cos(d*x + c)^8*log(sin(d
*x + c) + 1) - 13125*a^3*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(14848*a
^3*cos(d*x + c)^7 + 27195*a^3*cos(d*x + c)^6 + 7424*a^3*cos(d*x + c)^5 - 17
710*a^3*cos(d*x + c)^4 - 14592*a^3*cos(d*x + c)^3 + 1960*a^3*cos(d*x + c)^2
 + 5760*a^3*cos(d*x + c) + 1680*a^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

giac [A] time = 5.20, size = 196, normalized size = 0.83

$$-\frac{13440\,(dx+c)a^{3}+13125\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-13125\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(315\,a^{3}\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/13440*(13440*(d*x + c)*a^3 + 13125*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 13125*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(315*a^3*tan(1/2*d*x +
 1/2*c)^15 - 11375*a^3*tan(1/2*d*x + 1/2*c)^13 + 79723*a^3*tan(1/2*d*x + 1/
2*c)^11 - 269879*a^3*tan(1/2*d*x + 1/2*c)^9 + 550089*a^3*tan(1/2*d*x + 1/2*
c)^7 - 749973*a^3*tan(1/2*d*x + 1/2*c)^5 + 212625*a^3*tan(1/2*d*x + 1/2*c)^
3 - 26565*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

maple [A] time = 0.58, size = 250, normalized size = 1.05

$$\frac{a^{3}\left(\tan^{5}\left(dx+c\right)\right)}{5d}-\frac{a^{3}\left(\tan^{3}\left(dx+c\right)\right)}{3d}+\frac{a^{3}\tan\left(dx+c\right)}{d}-a^{3}x-\frac{a^{3}c}{d}+\frac{25a^{3}\left(\sin^{7}\left(dx+c\right)\right)}{48d\cos\left(dx+c\right)^{6}}-\frac{25a^{3}\left(\sin^{7}\left(dx+c\right)\right)}{192d\cos\left(dx+c\right)^{4}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x)
```

```
[Out] 1/5*a^3*tan(d*x+c)^5/d-1/3*a^3*tan(d*x+c)^3/d+a^3*tan(d*x+c)/d-a^3*x-1/d*a^
3*c+25/48/d*a^3*sin(d*x+c)^7/cos(d*x+c)^6-25/192/d*a^3*sin(d*x+c)^7/cos(d*x
+c)^4+25/128/d*a^3*sin(d*x+c)^7/cos(d*x+c)^2+25/128*a^3*sin(d*x+c)^5/d+125/
384*a^3*sin(d*x+c)^3/d+125/128*a^3*sin(d*x+c)/d-125/128/d*a^3*ln(sec(d*x+c)
+tan(d*x+c))+3/7/d*a^3*sin(d*x+c)^7/cos(d*x+c)^7+1/8/d*a^3*sin(d*x+c)^7/cos
(d*x+c)^8
```

maxima [A] time = 0.82, size = 262, normalized size = 1.11

$$\frac{11520\,a^{3}\tan\left(dx+c\right)^{7}+1792\left(3\,\tan\left(dx+c\right)^{5}-5\,\tan\left(dx+c\right)^{3}-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a^{3}+35\,a^{3}\left(\frac{2\left(\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/26880*(11520*a^3*tan(d*x + c)^7 + 1792*(3*tan(d*x + c)^5 - 5*tan(d*x + c)
^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^3 + 35*a^3*(2*(15*sin(d*x + c)^7 +
```

```
73*sin(d*x + c)^5 - 55*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^8 -
4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*
x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 840*a^3*(2*(33*sin(d*x + c)^5 - 4
0*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*
sin(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))
/d
```

mupad [B] time = 2.43, size = 263, normalized size = 1.11

$$-a^3\,x-\frac{125\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,d}-\frac{\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}-\frac{325\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}+\frac{11389\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{960}-\frac{269879\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6720}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+28\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-56\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] - a^3*x - (125*a^3*atanh(tan(c/2 + (d*x)/2)))/(64*d) - ((2025*a^3*tan(c/2 +
 (d*x)/2)^3)/64 - (35713*a^3*tan(c/2 + (d*x)/2)^5)/320 + (183363*a^3*tan(c/
2 + (d*x)/2)^7)/2240 - (269879*a^3*tan(c/2 + (d*x)/2)^9)/6720 + (11389*a^3*
tan(c/2 + (d*x)/2)^11)/960 - (325*a^3*tan(c/2 + (d*x)/2)^13)/192 + (3*a^3*t
an(c/2 + (d*x)/2)^15)/64 - (253*a^3*tan(c/2 + (d*x)/2))/64)/(d*(28*tan(c/2
+ (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/
2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*ta
n(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\tan^6\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\tan^6\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\tan^6\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**6,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**6*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**6*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**6*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**6, x))
```

3.48 $\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{3a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+\frac{19a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan^3(c+dx)\sec^3(c+dx)}{6d}-\frac{a}{}$$

```
[Out] a^3*x+19/16*a^3*arctanh(sin(d*x+c))/d-a^3*tan(d*x+c)/d-17/16*a^3*sec(d*x+c)
*tan(d*x+c)/d-1/8*a^3*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)^3/d+3/4*
a^3*sec(d*x+c)*tan(d*x+c)^3/d+1/6*a^3*sec(d*x+c)^3*tan(d*x+c)^3/d+3/5*a^3*t
an(d*x+c)^5/d
```

Rubi [A] time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.381$, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$\frac{3a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+\frac{19a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan^3(c+dx)\sec^3(c+dx)}{6d}-\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] a^3*x + (19*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (a^3*Tan[c + d*x])/d - (17*
a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(
8*d) + (a^3*Tan[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x]^3)/(4*
d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d) + (3*a^3*Tan[c + d*x]^5)/(5*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx &= \int \left(a^3\tan^4(c+dx)+3a^3\sec(c+dx)\tan^4(c+dx)+3a^3\sec^2(c+dx)\tan\right.\\ &= a^3\int \tan^4(c+dx)\,dx+a^3\int \sec^3(c+dx)\tan^4(c+dx)\,dx+\left(3a^3\right)\int \sec\\ &= \frac{a^3\tan^3(c+dx)}{3d}+\frac{3a^3\sec(c+dx)\tan^3(c+dx)}{4d}+\frac{a^3\sec^3(c+dx)\tan^3(c}{6d}\\ &= -\frac{a^3\tan(c+dx)}{d}-\frac{9a^3\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^3\sec^3(c+dx)\tan(c+}{8d}\\ &= a^3x+\frac{9a^3\tanh^{-1}(\sin(c+dx))}{8d}-\frac{a^3\tan(c+dx)}{d}-\frac{17a^3\sec(c+dx)\tan(}{16d}\\ &= a^3x+\frac{19a^3\tanh^{-1}(\sin(c+dx))}{16d}-\frac{a^3\tan(c+dx)}{d}-\frac{17a^3\sec(c+dx)\tan}{16d}\end{aligned}$$

Mathematica [A] time = 1.29, size = 303, normalized size = 1.79

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(\sec(c)(210\sin(2c+dx)-1440\sin(c+2dx)+1200\sin(3c+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^6*(-9120*Cos[c +
d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]) + Sec[c]*(2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 180
0*d*x*Cos[3*c + 2*d*x] + 720*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x
] + 120*d*x*Cos[5*c + 6*d*x] + 120*d*x*Cos[7*c + 6*d*x] + 1760*Sin[c] + 210
*Sin[d*x] + 210*Sin[2*c + d*x] - 1440*Sin[c + 2*d*x] + 1200*Sin[3*c + 2*d*x
] - 865*Sin[2*c + 3*d*x] - 865*Sin[4*c + 3*d*x] - 1296*Sin[3*c + 4*d*x] - 2
40*Sin[5*c + 4*d*x] - 435*Sin[4*c + 5*d*x] - 435*Sin[6*c + 5*d*x] - 176*Sin
[5*c + 6*d*x])))/(61440*d)
```

fricas [A] time = 0.82, size = 152, normalized size = 0.90

$$\frac{480\,a^3dx\cos(dx+c)^6+285\,a^3\cos(dx+c)^6\log(\sin(dx+c)+1)-285\,a^3\cos(dx+c)^6\log(-\sin(dx+c)+1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/480*(480*a^3*d*x*cos(d*x + c)^6 + 285*a^3*cos(d*x + c)^6*log(sin(d*x + c)
 + 1) - 285*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(176*a^3*cos(d*x
+ c)^5 + 435*a^3*cos(d*x + c)^4 + 208*a^3*cos(d*x + c)^3 - 110*a^3*cos(d*x
+ c)^2 - 144*a^3*cos(d*x + c) - 40*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

giac [A] time = 2.59, size = 164, normalized size = 0.97

$$\frac{240\,(dx+c)a^3 + 285\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 285\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}\right.}{}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/240*(240*(d*x + c)*a^3 + 285*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 285
*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*a^3*tan(1/2*d*x + 1/2*c)^11
 - 95*a^3*tan(1/2*d*x + 1/2*c)^9 - 366*a^3*tan(1/2*d*x + 1/2*c)^7 + 1746*a^
3*tan(1/2*d*x + 1/2*c)^5 - 3135*a^3*tan(1/2*d*x + 1/2*c)^3 + 525*a^3*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 0.57, size = 193, normalized size = 1.14

$$\frac{a^3\left(\tan^3(dx+c)\right)}{3d}-\frac{a^3\tan(dx+c)}{d}+a^3x+\frac{a^3c}{d}+\frac{19a^3\left(\sin^5(dx+c)\right)}{24d\cos(dx+c)^4}-\frac{19a^3\left(\sin^5(dx+c)\right)}{48d\cos(dx+c)^2}-\frac{19a^3\left(\sin^3(dx+c)\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x)
```

```
[Out] 1/3*a^3*tan(d*x+c)^3/d-a^3*tan(d*x+c)/d+a^3*x+1/d*a^3*c+19/24/d*a^3*sin(d*x
+c)^5/cos(d*x+c)^4-19/48/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2-19/48*a^3*sin(d*x+
c)^3/d-19/16*a^3*sin(d*x+c)/d+19/16/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/5/d*a
^3*sin(d*x+c)^5/cos(d*x+c)^5+1/6/d*a^3*sin(d*x+c)^5/cos(d*x+c)^6
```

maxima [A] time = 0.46, size = 210, normalized size = 1.24

$$288\,a^3\tan(dx+c)^5 + 160\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^3 - 5\,a^3\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(d\right.}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/480*(288*a^3*tan(d*x + c)^5 + 160*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d
*x + c))*a^3 - 5*a^3*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x +
c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 90*a^3*(2*(5*sin(d*x + c)^3 - 3*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c)
+ 1) - 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.44, size = 203, normalized size = 1.20

$$a^3\,x+\frac{19\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}+\frac{-\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{19\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{61\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}-\frac{291\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{20}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

```
[Out] a^3*x + (19*a^3*atanh(tan(c/2 + (d*x)/2)))/(8*d) + ((209*a^3*tan(c/2 + (d*x
)/2)^3)/8 - (291*a^3*tan(c/2 + (d*x)/2)^5)/20 + (61*a^3*tan(c/2 + (d*x)/2)^
7)/20 + (19*a^3*tan(c/2 + (d*x)/2)^9)/24 - (3*a^3*tan(c/2 + (d*x)/2)^11)/8
- (35*a^3*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 +
(d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2
+ (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int 3 \tan^4 (c + dx) \sec (c + dx)\, dx + \int 3 \tan^4 (c + dx) \sec^2 (c + dx)\, dx + \int \tan^4 (c + dx) \sec^3 (c + dx)\, dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**4,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**4*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**4, x))
```

3.49 $\int (a + a\sec(c + dx))^3 \tan^2(c + dx)\,dx$

Optimal. Leaf size=98

$$\frac{a^3\tan^3(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}-\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{11a^3\tan(c+dx)\sec}{8d}$$

```
[Out] -a^3*x-13/8*a^3*arctanh(sin(d*x+c))/d+a^3*tan(d*x+c)/d+11/8*a^3*sec(d*x+c)*
tan(d*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+a^3*tan(d*x+c)^3/d
```

Rubi [A] time = 0.16, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$\frac{a^3\tan^3(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}-\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{11a^3\tan(c+dx)\sec}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -(a^3*x) - (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*Tan[c + d*x])/d + (1
1*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/
(4*d) + (a^3*Tan[c + d*x]^3)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a + a\sec(c + dx))^3 \tan^2(c + dx)\,dx = \int \left(a^3\tan^2(c + dx) + 3a^3\sec(c + dx)\tan^2(c + dx) + 3a^3\sec^2(c + dx)\tan\right.$$
$$= a^3\int \tan^2(c + dx)\,dx + a^3\int \sec^3(c + dx)\tan^2(c + dx)\,dx + \left(3a^3\right)\int \sec$$
$$= \frac{a^3\tan(c + dx)}{d} + \frac{3a^3\sec(c + dx)\tan(c + dx)}{2d} + \frac{a^3\sec^3(c + dx)\tan(c +}{4d}$$
$$= -a^3x - \frac{3a^3\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a^3\tan(c + dx)}{d} + \frac{11a^3\sec(c + dx)\tan}{8d}$$
$$= -a^3x - \frac{13a^3\tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^3\tan(c + dx)}{d} + \frac{11a^3\sec(c + dx)\tan}{8d}$$

Mathematica [B] time = 0.85, size = 230, normalized size = 2.35

$$-\frac{a^3\sec^4(c + dx)\left(-38\sin(c + dx) - 32\sin(2(c + dx)) - 22\sin(3(c + dx)) - 39\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c +\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -1/64*(a^3*Sec[c + d*x]^4*(24*d*x - 39*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + 39*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*Cos[2*(c + d*x)]*(8*
d*x - 13*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 13*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) + Cos[4*(c + d*x)]*(8*d*x - 13*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] + 13*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 38*Sin[
c + d*x] - 32*Sin[2*(c + d*x)] - 22*Sin[3*(c + d*x)]))/d
```

fricas [A] time = 0.56, size = 113, normalized size = 1.15

$$-\frac{16\,a^3dx\cos(dx + c)^4 + 13\,a^3\cos(dx + c)^4\log(\sin(dx + c) + 1) - 13\,a^3\cos(dx + c)^4\log(-\sin(dx + c) + 1) - 2}{16\,d\cos(dx + c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^3*d*x*cos(d*x + c)^4 + 13*a^3*cos(d*x + c)^4*log(sin(d*x + c) +
 1) - 13*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(11*a^3*cos(d*x + c)
^2 + 8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 1.04, size = 132, normalized size = 1.35

$$-\frac{8(dx+c)a^3 + 13a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 13a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(5a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-13a^3\,\mathrm{t}\right.}{}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*(8*(d*x + c)*a^3 + 13*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^3*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 13*a
^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*a^3*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.54, size = 137, normalized size = 1.40

$$-a^3x+\frac{a^3\tan(dx+c)}{d}-\frac{a^3c}{d}+\frac{13a^3\left(\sin^3(dx+c)\right)}{8d\cos(dx+c)^2}+\frac{13a^3\sin(dx+c)}{8d}-\frac{13a^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{a^3(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x)
```

```
[Out] -a^3*x+a^3*tan(d*x+c)/d-1/d*a^3*c+13/8/d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+13/8
*a^3*sin(d*x+c)/d-13/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3*sin(d*x+c)^3
/cos(d*x+c)^3+1/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4
```

maxima [A] time = 0.45, size = 147, normalized size = 1.50

$$\frac{16a^3\tan(dx+c)^3 - 16(dx+c-\tan(dx+c))a^3 + a^3\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - \log(\sin(dx+c)+1) + \log(\mathrm{sin}\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^3*tan(d*x + c)^3 - 16*(d*x + c - tan(d*x + c))*a^3 + a^3*(2*(sin
(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 1.91, size = 146, normalized size = 1.49

$$\frac{\frac{5a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4}-\frac{13a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}+\frac{3a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4}+\frac{21a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-a^3x-\frac{13a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((3*a^3*tan(c/2 + (d*x)/2)^3)/4 - (13*a^3*tan(c/2 + (d*x)/2)^5)/4 + (5*a^3*
tan(c/2 + (d*x)/2)^7)/4 + (21*a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d
*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x
)/2)^8 + 1)) - a^3*x - (13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\tan^2(c+dx)\sec(c+dx)\,dx+\int 3\tan^2(c+dx)\sec^2(c+dx)\,dx+\int\tan^2(c+dx)\sec^3(c+dx)\,dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**2*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**2, x))
```

3.50 $\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=49

$$-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+a^3(-x)$$

```
[Out] -a^3*x+a^3*arctanh(sin(d*x+c))/d-4*a^3*cot(d*x+c)/d-4*a^3*csc(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2606, 3767, 2621, 321, 207}

$$-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+a^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (a^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*Cot[c + d*x])/d - (4*a^3*
Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^2(c+dx)+3a^3\cot(c+dx)\csc(c+dx)+3a^3\csc^2(c+dx)+a^3\right.$$
$$= a^3\int \cot^2(c+dx)\,dx + a^3\int \csc^2(c+dx)\sec(c+dx)\,dx + \left(3a^3\right)\int \cot(c$$
$$= -\frac{a^3\cot(c+dx)}{d} - a^3\int 1\,dx - \frac{a^3\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{(3}{}$$
$$= -a^3x - \frac{4a^3\cot(c+dx)}{d} - \frac{4a^3\csc(c+dx)}{d} - \frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(\right.}{d}$$
$$= -a^3x + \frac{a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^3\cot(c+dx)}{d} - \frac{4a^3\csc(c+dx)}{d}$$

Mathematica [B] time = 0.23, size = 109, normalized size = 2.22

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-4\csc\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/8*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(d*x + Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 4*Csc[c
/2]*Csc[(c + d*x)/2]*Sin[(d*x)/2]))/d
```

fricas [A] time = 0.73, size = 84, normalized size = 1.71

$$-\frac{2\,a^3dx\sin(dx+c) - a^3\log(\sin(dx+c)+1)\sin(dx+c) + a^3\log(-\sin(dx+c)+1)\sin(dx+c) + 8\,a^3\cos(dx}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*d*x*sin(d*x + c) - a^3*log(sin(d*x + c) + 1)*sin(d*x + c) + a^3
*log(-sin(d*x + c) + 1)*sin(d*x + c) + 8*a^3*cos(d*x + c) + 8*a^3)/(d*sin(d
*x + c))
```

giac [A] time = 0.33, size = 66, normalized size = 1.35

$$-\frac{(dx+c)a^3 - a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{4\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + a^3*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 4*a^3/tan(1/2*d*x + 1/2*c))/d
```

maple [A] time = 0.83, size = 68, normalized size = 1.39

$$-a^{3}x-\frac{4a^{3}\cot \left(dx+c\right)}{d}-\frac{a^{3}c}{d}-\frac{4a^{3}}{d\sin \left(dx+c\right)}+\frac{a^{3}\ln \left(\sec \left(dx+c\right)+\tan \left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -a^3*x-4*a^3*cot(d*x+c)/d-1/d*a^3*c-4/d*a^3/sin(d*x+c)+1/d*a^3*ln(sec(d*x+c
)+tan(d*x+c))
```

maxima [A] time = 0.43, size = 85, normalized size = 1.73

$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^{3}+a^{3}\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{6\,a^{3}}{\sin(dx+c)}+\frac{6\,a^{3}}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a^3 + a^3*(2/sin(d*x + c) - log(sin(d*x
+ c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3/sin(d*x + c) + 6*a^3/tan(d*x + c
))/d
```

mupad [B] time = 1.20, size = 35, normalized size = 0.71

$$-\frac{a^{3}\left(4\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(4*cot(c/2 + (d*x)/2) - 2*atanh(tan(c/2 + (d*x)/2)) + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\cot^{2}\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\cot^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int \cot^{2}\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**2*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**2, x))
```

3.51 $\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=69

$$-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

```
[Out] a^3*x+a^3*cot(d*x+c)/d-4/3*a^3*cot(d*x+c)^3/d+3*a^3*csc(d*x+c)/d-4/3*a^3*cs
c(d*x+c)^3/d
```

Rubi [A] time = 0.13, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3886, 3473, 8, 2606, 2607, 30}

$$-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (4*a^3*Cot[c + d*x]^3)/(3*d) + (3*a^3*Csc[c
+ d*x])/d - (4*a^3*Csc[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)+3a^3\cot^3(c+dx)\csc(c+dx)+3a^3\cot^2(c+dx)\,c\right.$$
$$= a^3\int \cot^4(c+dx)\,dx + a^3\int \cot(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int \text{co}$$
$$= -\frac{a^3\cot^3(c+dx)}{3d} - a^3\int \cot^2(c+dx)\,dx - \frac{a^3\,\text{Subst}\left(\int x^2\,dx, x, \csc(c\right.}{d}$$
$$= \frac{a^3\cot(c+dx)}{d} - \frac{4a^3\cot^3(c+dx)}{3d} + \frac{3a^3\csc(c+dx)}{d} - \frac{4a^3\csc^3(c+d}{3d}$$
$$= a^3x + \frac{a^3\cot(c+dx)}{d} - \frac{4a^3\cot^3(c+dx)}{3d} + \frac{3a^3\csc(c+dx)}{d} - \frac{4a^3\csc^3}{3}$$

Mathematica [A] time = 0.24, size = 112, normalized size = 1.62

$$\frac{a^3\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(-18\sin\left(c+\frac{dx}{2}\right)+14\sin\left(c+\frac{3dx}{2}\right)-9dx\cos\left(c+\frac{dx}{2}\right)-3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\,\text{c}\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] - 9*d*x*Cos[c + (d*x)/
2] - 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 24*Sin[(d*x)/2
] - 18*Sin[c + (d*x)/2] + 14*Sin[c + (3*d*x)/2]))/(24*d)
```

fricas [A] time = 0.75, size = 82, normalized size = 1.19

$$\frac{7\,a^3\cos\left(dx+c\right)^2+2\,a^3\cos\left(dx+c\right)-5\,a^3+3\,\left(a^3dx\cos\left(dx+c\right)-a^3dx\right)\sin\left(dx+c\right)}{3\,\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(7*a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) - 5*a^3 + 3*(a^3*d*x*cos(d*x
 + c) - a^3*d*x)*sin(d*x + c))/((d*cos(d*x + c) - d)*sin(d*x + c))
```

giac [A] time = 0.32, size = 50, normalized size = 0.72

$$\frac{3\,(dx+c)a^3+\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^3 + (6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3)/tan(1/2*d*x + 1
/2*c)^3)/d
```

maple [A] time = 0.85, size = 125, normalized size = 1.81

$$\frac{a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot\left(dx+c\right)+dx+c\right)+3a^3\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}\right)-\frac{a^3\left(\cos^3(dx+c)\right)}{\sin(dx+c)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*a^3*(-1/3/sin(d*x+c)^3*cos(
d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c))-a^3/s
in(d*x+c)^3*cos(d*x+c)^3-1/3*a^3/sin(d*x+c)^3)
```

maxima [A] time = 0.43, size = 90, normalized size = 1.30

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^{2}-1}{\tan\left(dx+c\right)^{3}}\right)a^{3}+\frac{3\left(3\,\sin\left(dx+c\right)^{2}-1\right)a^{3}}{\sin\left(dx+c\right)^{3}}-\frac{a^{3}}{\sin\left(dx+c\right)^{3}}-\frac{3\,a^{3}}{\tan\left(dx+c\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + 3*(3*sin(d
*x + c)^2 - 1)*a^3/sin(d*x + c)^3 - a^3/sin(d*x + c)^3 - 3*a^3/tan(d*x + c)
^3)/d
```

mupad [B] time = 1.20, size = 39, normalized size = 0.57

$$a^{3}\,x+\frac{a^{3}\left(6\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

```
[Out] a^3*x + (a^3*(6*cot(c/2 + (d*x)/2) - cot(c/2 + (d*x)/2)^3))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\cot^{4}\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\cot^{4}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int \cot^{4}\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\int c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**4*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**4, x))
```

3.52 $\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=107

$$-\frac{4a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^5(c+dx)}{5d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}-a^3x$$

```
[Out] -a^3*x-a^3*cot(d*x+c)/d+1/3*a^3*cot(d*x+c)^3/d-4/5*a^3*cot(d*x+c)^5/d-3*a^3
*csc(d*x+c)/d+7/3*a^3*csc(d*x+c)^3/d-4/5*a^3*csc(d*x+c)^5/d
```

Rubi [A] time = 0.16, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30, 14}

$$-\frac{4a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^5(c+dx)}{5d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}-a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (4*a^3*Cot[c
 + d*x]^5)/(5*d) - (3*a^3*Csc[c + d*x])/d + (7*a^3*Csc[c + d*x]^3)/(3*d) -
(4*a^3*Csc[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)+3a^3\cot^5(c+dx)\csc(c+dx)+3a^3\cot^4(c+dx)\csc^2\right.$$

$$= a^3\int \cot^6(c+dx)\,dx + a^3\int \cot^3(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int \cot^5$$

$$= -\frac{a^3\cot^5(c+dx)}{5d} - a^3\int \cot^4(c+dx)\,dx - \frac{a^3\,\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)dx,\right.}{d}$$

$$= \frac{a^3\cot^3(c+dx)}{3d} - \frac{4a^3\cot^5(c+dx)}{5d} + a^3\int \cot^2(c+dx)\,dx - \frac{a^3\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\csc(c+dx)}{d}$$

$$= -a^3x - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\csc(c}{d}$$

Mathematica [A] time = 0.67, size = 112, normalized size = 1.05

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{c}{2}\right)(13\cos(c+dx)-10)\csc^4\left(\frac{1}{2}(c+dx)\right)+\csc\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)(51\cos(c\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/480*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(60*d*x + (-10 + 13*Cos
[c + d*x])*Cot[c/2]*Csc[(c + d*x)/2]^4 + (-38 + 51*Cos[c + d*x] - 16*Cos[2*
(c + d*x)])*Csc[c/2]*Csc[(c + d*x)/2]^5*Sin[(d*x)/2]))/d
```

fricas [A] time = 0.75, size = 118, normalized size = 1.10

$$-\frac{32\,a^3\cos(dx+c)^3-19\,a^3\cos(dx+c)^2-29\,a^3\cos(dx+c)+22\,a^3+15\left(a^3dx\cos(dx+c)^2-2\,a^3dx\cos(dx+\right.}{15\left(d\cos(dx+c)^2-2\,d\cos(dx+c)+d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(32*a^3*cos(d*x + c)^3 - 19*a^3*cos(d*x + c)^2 - 29*a^3*cos(d*x + c)
+ 22*a^3 + 15*(a^3*d*x*cos(d*x + c)^2 - 2*a^3*d*x*cos(d*x + c) + a^3*d*x)*s
in(d*x + c))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

giac [A] time = 0.36, size = 66, normalized size = 0.62

$$-\frac{60\,(dx+c)a^3+\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)*a^3 + (105*a^3*tan(1/2*d*x + 1/2*c)^4 - 20*a^3*tan(1/2*
d*x + 1/2*c)^2 + 3*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [B] time = 1.07, size = 232, normalized size = 2.17

$$\frac{a^3\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+3a^3\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+3*a^3*(-1/5/
sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos
(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-3/5*a^3/sin(d
*x+c)^5*cos(d*x+c)^5+a^3*(-1/5/sin(d*x+c)^5*cos(d*x+c)^4-1/15/sin(d*x+c)^3*
cos(d*x+c)^4+1/15/sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))
)
```

maxima [A] time = 0.44, size = 122, normalized size = 1.14

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3+\frac{3\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)a^3}{\sin(dx+c)^5}-\frac{\left(5\,\sin(dx+c)^2-3\right)a^3}{\sin(dx+c)^5}+\frac{9\,a^3}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^3 + 3*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a^3/sin(d*x + c
)^5 - (5*sin(d*x + c)^2 - 3)*a^3/sin(d*x + c)^5 + 9*a^3/tan(d*x + c)^5)/d
```

mupad [B] time = 1.45, size = 62, normalized size = 0.58

$$\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,d}-a^3\,x-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20\,d}-\frac{7\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^3)/(3*d) - a^3*x - (a^3*cot(c/2 + (d*x)/2)^5)/(20*d
) - (7*a^3*cot(c/2 + (d*x)/2))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.53 $\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=141

$$-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^7(c+dx)}{7d}+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{10a^3 c}{}$$

```
[Out] a^3*x+a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d+1/5*a^3*cot(d*x+c)^5/d-4/7*a^
3*cot(d*x+c)^7/d+3*a^3*csc(d*x+c)/d-10/3*a^3*csc(d*x+c)^3/d+11/5*a^3*csc(d*
x+c)^5/d-4/7*a^3*csc(d*x+c)^7/d
```

Rubi [A] time = 0.18, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^7(c+dx)}{7d}+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{10a^3 c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*
x]^5)/(5*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) + (3*a^3*Csc[c + d*x])/d - (10*a
^3*Csc[c + d*x]^3)/(3*d) + (11*a^3*Csc[c + d*x]^5)/(5*d) - (4*a^3*Csc[c + d
*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^8(c+dx)+3a^3\cot^7(c+dx)\csc(c+dx)+3a^3\cot^6(c+dx)\,c\right.$$
$$= a^3\int \cot^8(c+dx)\,dx + a^3\int \cot^5(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int c$$
$$= -\frac{a^3\cot^7(c+dx)}{7d} - a^3\int \cot^6(c+dx)\,dx - \frac{a^3\,\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^2\right.}{d}$$
$$= \frac{a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7(c+dx)}{7d} + a^3\int \cot^4(c+dx)\,dx - \frac{a^3\,\mathrm{Subst}}{}$$
$$= -\frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7(c+dx)}{7d} + \frac{3a^3\csc(c+}{d}$$
$$= \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7(c+dx)}{7d}$$
$$= a^3x + \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7(c}{7d}$$

Mathematica [A] time = 0.98, size = 252, normalized size = 1.79

$$\frac{a^3\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^7\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(-23282\sin(c+dx)+23282\sin(2(c+dx))-9978\sin(3(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[(c + d*x)/2]^7*Sec[c/2]*Sec[(c + d*x)/2]*(5880*d*x*Cos[d*
x] - 5880*d*x*Cos[2*c + d*x] - 5880*d*x*Cos[c + 2*d*x] + 5880*d*x*Cos[3*c +
 2*d*x] + 2520*d*x*Cos[2*c + 3*d*x] - 2520*d*x*Cos[4*c + 3*d*x] - 420*d*x*C
os[3*c + 4*d*x] + 420*d*x*Cos[5*c + 4*d*x] + 4200*Sin[c] - 11032*Sin[d*x] -
 23282*Sin[c + d*x] + 23282*Sin[2*(c + d*x)] - 9978*Sin[3*(c + d*x)] + 1663
*Sin[4*(c + d*x)] - 13720*Sin[2*c + d*x] + 15512*Sin[c + 2*d*x] + 9240*Sin[
3*c + 2*d*x] - 8088*Sin[2*c + 3*d*x] - 2520*Sin[4*c + 3*d*x] + 1768*Sin[3*c
 + 4*d*x]))/(215040*d)
```

fricas [A] time = 0.49, size = 160, normalized size = 1.13

$$\frac{221\,a^3\cos(dx+c)^4-348\,a^3\cos(dx+c)^3-25\,a^3\cos(dx+c)^2+303\,a^3\cos(dx+c)-136\,a^3+105\left(a^3dx\cos\right.}{105\left(d\cos(dx+c)^3-3\,d\cos(dx+c)^2+3\,d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(221*a^3*cos(d*x + c)^4 - 348*a^3*cos(d*x + c)^3 - 25*a^3*cos(d*x + c
)^2 + 303*a^3*cos(d*x + c) - 136*a^3 + 105*(a^3*d*x*cos(d*x + c)^3 - 3*a^3*
d*x*cos(d*x + c)^2 + 3*a^3*d*x*cos(d*x + c) - a^3*d*x)*sin(d*x + c))/((d*co
s(d*x + c)^3 - 3*d*cos(d*x + c)^2 + 3*d*cos(d*x + c) - d)*sin(d*x + c))
```

giac [A] time = 0.50, size = 96, normalized size = 0.68

$$\frac{1680\,(dx+c)a^3 - 105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{2730\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 126\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 15\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1680*(1680*(d*x + c)*a^3 - 105*a^3*tan(1/2*d*x + 1/2*c) + (2730*a^3*tan(1
/2*d*x + 1/2*c)^6 - 560*a^3*tan(1/2*d*x + 1/2*c)^4 + 126*a^3*tan(1/2*d*x +
1/2*c)^2 - 15*a^3)/tan(1/2*d*x + 1/2*c)^7)/d
```

maple [B] time = 1.18, size = 293, normalized size = 2.08

$$a^3\left(-\frac{\left(\cot^7(dx+c)\right)}{7}+\frac{\left(\cot^5(dx+c)\right)}{5}-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^3\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*
x+c)+3*a^3*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1
/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-3/7*a^3/sin(d*x+c)^7*c
os(d*x+c)^7+a^3*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c
)^6+1/105/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.45, size = 152, normalized size = 1.08

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a^3+\frac{9\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)a^3}{\sin(dx+c)^7}-\frac{(35\,\sin}{}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^3 + 9*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a^3/sin(d*x + c)^7 - (35*sin(d*x + c)^4 - 42*
sin(d*x + c)^2 + 15)*a^3/sin(d*x + c)^7 - 45*a^3/tan(d*x + c)^7)/d
```

mupad [B] time = 1.70, size = 91, normalized size = 0.65

$$\frac{a^3\left(126\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-560\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2730\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-105\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+1680\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7(c+\right.}{1680\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*(126*tan(c/2 + (d*x)/2)^2 - 560*tan(c/2 + (d*x)/2)^4 + 2730*tan(c/2 +
(d*x)/2)^6 - 105*tan(c/2 + (d*x)/2)^8 + 1680*tan(c/2 + (d*x)/2)^7*(c + d*x)
 - 15))/(1680*d*tan(c/2 + (d*x)/2)^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.54 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=179

$$-\frac{4a^3\cot^9(c+dx)}{9d}+\frac{a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^9(c+dx)}{9d}+\frac{15a^3\csc}{7}$$

```
[Out] -a^3*x-a^3*cot(d*x+c)/d+1/3*a^3*cot(d*x+c)^3/d-1/5*a^3*cot(d*x+c)^5/d+1/7*a
^3*cot(d*x+c)^7/d-4/9*a^3*cot(d*x+c)^9/d-3*a^3*csc(d*x+c)/d+13/3*a^3*csc(d*
x+c)^3/d-21/5*a^3*csc(d*x+c)^5/d+15/7*a^3*csc(d*x+c)^7/d-4/9*a^3*csc(d*x+c)
^9/d
```

Rubi [A] time = 0.20, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$-\frac{4a^3\cot^9(c+dx)}{9d}+\frac{a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^9(c+dx)}{9d}+\frac{15a^3\csc}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot[c +
 d*x]^5)/(5*d) + (a^3*Cot[c + d*x]^7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d)
- (3*a^3*Csc[c + d*x])/d + (13*a^3*Csc[c + d*x]^3)/(3*d) - (21*a^3*Csc[c +
d*x]^5)/(5*d) + (15*a^3*Csc[c + d*x]^7)/(7*d) - (4*a^3*Csc[c + d*x]^9)/(9*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^{10}(c+dx)+3a^3\cot^9(c+dx)\csc(c+dx)+3a^3\cot^8(c+dx\right.$$
$$= a^3\int \cot^{10}(c+dx)\,dx + a^3\int \cot^7(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int$$
$$= -\frac{a^3\cot^9(c+dx)}{9d} - a^3\int \cot^8(c+dx)\,dx - \frac{a^3\,\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^3\right.}{d}$$
$$= \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} + a^3\int \cot^6(c+dx)\,dx - \frac{a^3\,\mathrm{Subs}}{}$$
$$= -\frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} - \frac{3a^3\csc(c+}{d}$$
$$= \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+d}{9d}$$
$$= -\frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx}{7d}$$
$$= -a^3x - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(}{7}$$

Mathematica [B] time = 1.42, size = 370, normalized size = 2.07

$$\frac{a^3\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^9\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)(675036\sin(c+dx)-506277\sin(2(c+dx))-37502\sin(3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[(c + d*x)/2]^9*Sec[c/2]*Sec[(c + d*x)/2]^3*(-181440*d*x*C
os[d*x] + 181440*d*x*Cos[2*c + d*x] + 136080*d*x*Cos[c + 2*d*x] - 136080*d*
x*Cos[3*c + 2*d*x] + 10080*d*x*Cos[2*c + 3*d*x] - 10080*d*x*Cos[4*c + 3*d*x
] - 60480*d*x*Cos[3*c + 4*d*x] + 60480*d*x*Cos[5*c + 4*d*x] + 30240*d*x*Cos
[4*c + 5*d*x] - 30240*d*x*Cos[6*c + 5*d*x] - 5040*d*x*Cos[5*c + 6*d*x] + 50
40*d*x*Cos[7*c + 6*d*x] - 169344*Sin[c] + 338112*Sin[d*x] + 675036*Sin[c +
d*x] - 506277*Sin[2*(c + d*x)] - 37502*Sin[3*(c + d*x)] + 225012*Sin[4*(c +
 d*x)] - 112506*Sin[5*(c + d*x)] + 18751*Sin[6*(c + d*x)] + 431424*Sin[2*c
+ d*x] - 375552*Sin[c + 2*d*x] - 201600*Sin[3*c + 2*d*x] + 41248*Sin[2*c +
3*d*x] - 84000*Sin[4*c + 3*d*x] + 155712*Sin[3*c + 4*d*x] + 100800*Sin[5*c
+ 4*d*x] - 98016*Sin[4*c + 5*d*x] - 30240*Sin[6*c + 5*d*x] + 21376*Sin[5*c
+ 6*d*x]))/(41287680*d)
```

fricas [A] time = 0.55, size = 235, normalized size = 1.31

$$-\frac{668\,a^3\cos(dx+c)^6-1059\,a^3\cos(dx+c)^5-573\,a^3\cos(dx+c)^4+1813\,a^3\cos(dx+c)^3-393\,a^3\cos(dx+c)^2}{315\left(d\cos(dx+c)^5-3\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/315*(668*a^3*cos(d*x + c)^6 - 1059*a^3*cos(d*x + c)^5 - 573*a^3*cos(d*x
+ c)^4 + 1813*a^3*cos(d*x + c)^3 - 393*a^3*cos(d*x + c)^2 - 789*a^3*cos(d*x
 + c) + 368*a^3 + 315*(a^3*d*x*cos(d*x + c)^5 - 3*a^3*d*x*cos(d*x + c)^4 +
2*a^3*d*x*cos(d*x + c)^3 + 2*a^3*d*x*cos(d*x + c)^2 - 3*a^3*d*x*cos(d*x + c
) + a^3*d*x)*sin(d*x + c))/((d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 2*d*co
s(d*x + c)^3 + 2*d*cos(d*x + c)^2 - 3*d*cos(d*x + c) + d)*sin(d*x + c))
```

giac [A] time = 0.53, size = 128, normalized size = 0.72

$$-\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20160\,(dx+c)a^3-2520\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{31185\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-6720\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+}{\tan\left(\frac{1}{2}\right.}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20160*(105*a^3*tan(1/2*d*x + 1/2*c)^3 + 20160*(d*x + c)*a^3 - 2520*a^3*t
an(1/2*d*x + 1/2*c) + (31185*a^3*tan(1/2*d*x + 1/2*c)^8 - 6720*a^3*tan(1/2*
d*x + 1/2*c)^6 + 1827*a^3*tan(1/2*d*x + 1/2*c)^4 - 360*a^3*tan(1/2*d*x + 1/
2*c)^2 + 35*a^3)/tan(1/2*d*x + 1/2*c)^9)/d
```

maple [B] time = 1.20, size = 364, normalized size = 2.03

$$a^3\left(-\frac{\left(\cot^9(dx+c)\right)}{9}+\frac{\left(\cot^7(dx+c)\right)}{7}-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+3a^3\left(-\frac{\cos^{10}(dx+c)}{9\sin(dx+c)^9}+\frac{\cos^{10}(dx+c)}{63\sin(dx+c)^7}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7-1/5*cot(d*x+c)^5+1/3*cot(d*x+c
)^3-cot(d*x+c)-d*x-c)+3*a^3*(-1/9/sin(d*x+c)^9*cos(d*x+c)^10+1/63/sin(d*x+c
)^7*cos(d*x+c)^10-1/105/sin(d*x+c)^5*cos(d*x+c)^10+1/63/sin(d*x+c)^3*cos(d*
x+c)^10-1/9/sin(d*x+c)*cos(d*x+c)^10-1/9*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c
)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-1/3*a^3/sin(d*x+c)^9
*cos(d*x+c)^9+a^3*(-1/9/sin(d*x+c)^9*cos(d*x+c)^8-1/63/sin(d*x+c)^7*cos(d*x
+c)^8+1/315/sin(d*x+c)^5*cos(d*x+c)^8-1/315/sin(d*x+c)^3*cos(d*x+c)^8+1/63/
sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*
x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.45, size = 182, normalized size = 1.02

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan(dx+c)^8-105\,\tan(dx+c)^6+63\,\tan(dx+c)^4-45\,\tan(dx+c)^2+35}{\tan(dx+c)^9}\right)a^3+\frac{3\left(315\,\sin(dx+c)^8-420\,\sin(dx+c)^6+378\,\sin(d\right.}{\sin(dx+c)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a^3 + 3*(315*sin(d*x
 + c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35
)*a^3/sin(d*x + c)^9 - (105*sin(d*x + c)^6 - 189*sin(d*x + c)^4 + 135*sin(d
*x + c)^2 - 35)*a^3/sin(d*x + c)^9 + 105*a^3/tan(d*x + c)^9)/d
```

mupad [B] time = 1.94, size = 206, normalized size = 1.15

$$-\frac{a^3\left(35\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+105\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-2520\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+31185\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^10*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(35*cos(c/2 + (d*x)/2)^12 + 105*sin(c/2 + (d*x)/2)^12 - 2520*cos(c/2
+ (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 31185*cos(c/2 + (d*x)/2)^4*sin(c/2 + (
d*x)/2)^8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 1827*cos(c/2 +
 (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 360*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x
)/2)^2 + 20160*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9*(c + d*x)))/(20160
*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.55 $\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=213

$$-\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{a^3\cot^9(c+dx)}{9d}-\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^{11}}{11}$$

```
[Out] a^3*x+a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d+1/5*a^3*cot(d*x+c)^5/d-1/7*a^
3*cot(d*x+c)^7/d+1/9*a^3*cot(d*x+c)^9/d-4/11*a^3*cot(d*x+c)^11/d+3*a^3*csc(
d*x+c)/d-16/3*a^3*csc(d*x+c)^3/d+34/5*a^3*csc(d*x+c)^5/d-36/7*a^3*csc(d*x+c
)^7/d+19/9*a^3*csc(d*x+c)^9/d-4/11*a^3*csc(d*x+c)^11/d
```

Rubi [A] time = 0.22, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$-\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{a^3\cot^9(c+dx)}{9d}-\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^12*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*
x]^5)/(5*d) - (a^3*Cot[c + d*x]^7)/(7*d) + (a^3*Cot[c + d*x]^9)/(9*d) - (4*
a^3*Cot[c + d*x]^11)/(11*d) + (3*a^3*Csc[c + d*x])/d - (16*a^3*Csc[c + d*x]
^3)/(3*d) + (34*a^3*Csc[c + d*x]^5)/(5*d) - (36*a^3*Csc[c + d*x]^7)/(7*d) +
 (19*a^3*Csc[c + d*x]^9)/(9*d) - (4*a^3*Csc[c + d*x]^11)/(11*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^{12}(c+dx)+3a^3\cot^{11}(c+dx)\csc(c+dx)+3a^3\cot^{10}(c+
$$

$$= a^3\int \cot^{12}(c+dx)\,dx + a^3\int \cot^9(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int
$$

$$= -\frac{a^3\cot^{11}(c+dx)}{11d} - a^3\int \cot^{10}(c+dx)\,dx - \frac{a^3\,\mathrm{Subst}\left(\int x^2\left(-1+x^2\right.\right.}{d}
$$

$$= \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a^3\cot^{11}(c+dx)}{11d} + a^3\int \cot^8(c+dx)\,dx - \frac{a^3\,\mathrm{Sub}}{}
$$

$$= -\frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a^3\cot^{11}(c+dx)}{11d} + \frac{3a^3\csc(c}{d}
$$

$$= \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a^3\cot^{11}(c+}{11d}
$$

$$= -\frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\cot^9(c+}{9d}
$$

$$= \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d}
$$

$$= a^3x + \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c}{7d}
$$

Mathematica [A] time = 6.05, size = 268, normalized size = 1.26

$$-\frac{a^3\tan\left(\frac{c}{2}\right)(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(20\cot^2\left(\frac{c}{2}\right)(-4528480\cos(c+dx)+2388316\cos(2(c+dx))\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^12*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/3633315840*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(20*(2786111 - 4
528480*Cos[c + d*x] + 2388316*Cos[2*(c + d*x)] - 750112*Cos[3*(c + d*x)] +
112229*Cos[4*(c + d*x)])*Cot[c/2]^2*Csc[(c + d*x)/2]^10 - 5*Cot[c/2]*(90832
896*d*x + (-32611198 + 54812150*Cos[c + d*x] - 32118776*Cos[2*(c + d*x)] +
12626567*Cos[3*(c + d*x)] - 3023754*Cos[4*(c + d*x)] + 347267*Cos[5*(c + d*
x)])*Csc[c/2]*Csc[(c + d*x)/2]^11*Sin[(d*x)/2]) + 7392*Csc[c/2]*Sec[(c + d*
x)/2]^5*(4370*Sin[(d*x)/2] - 3060*Sin[c + (d*x)/2] + 2860*Sin[c + (3*d*x)/2
] - 855*Sin[2*c + (3*d*x)/2] + 743*Sin[2*c + (5*d*x)/2]))*Tan[c/2])/d
```

fricas [A] time = 0.52, size = 314, normalized size = 1.47

$$7453\,a^3\cos\left(dx+c\right)^8 - 11964\,a^3\cos\left(dx+c\right)^7 - 11866\,a^3\cos\left(dx+c\right)^6 + 30542\,a^3\cos\left(dx+c\right)^5 + 90\,a^3\cos\left(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(7453*a^3*cos(d*x + c)^8 - 11964*a^3*cos(d*x + c)^7 - 11866*a^3*cos(
d*x + c)^6 + 30542*a^3*cos(d*x + c)^5 + 90*a^3*cos(d*x + c)^4 - 26438*a^3*c
os(d*x + c)^3 + 8539*a^3*cos(d*x + c)^2 + 7671*a^3*cos(d*x + c) - 3712*a^3
+ 3465*(a^3*d*x*cos(d*x + c)^7 - 3*a^3*d*x*cos(d*x + c)^6 + a^3*d*x*cos(d*x
 + c)^5 + 5*a^3*d*x*cos(d*x + c)^4 - 5*a^3*d*x*cos(d*x + c)^3 - a^3*d*x*cos
(d*x + c)^2 + 3*a^3*d*x*cos(d*x + c) - a^3*d*x)*sin(d*x + c))/((d*cos(d*x +
 c)^7 - 3*d*cos(d*x + c)^6 + d*cos(d*x + c)^5 + 5*d*cos(d*x + c)^4 - 5*d*co
s(d*x + c)^3 - d*cos(d*x + c)^2 + 3*d*cos(d*x + c) - d)*sin(d*x + c))
```

giac [A] time = 0.65, size = 161, normalized size = 0.76

$$-\frac{693\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 11550\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 887040\,(dx+c)a^3 + 159390\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{5\left(26472\right.}{}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/887040*(693*a^3*tan(1/2*d*x + 1/2*c)^5 - 11550*a^3*tan(1/2*d*x + 1/2*c)^
3 - 887040*(d*x + c)*a^3 + 159390*a^3*tan(1/2*d*x + 1/2*c) - 5*(264726*a^3*
tan(1/2*d*x + 1/2*c)^10 - 59136*a^3*tan(1/2*d*x + 1/2*c)^8 + 18018*a^3*tan(
1/2*d*x + 1/2*c)^6 - 4554*a^3*tan(1/2*d*x + 1/2*c)^4 + 770*a^3*tan(1/2*d*x
+ 1/2*c)^2 - 63*a^3)/tan(1/2*d*x + 1/2*c)^11)/d
```

maple [B] time = 1.27, size = 425, normalized size = 2.00

$$a^3\left(-\frac{\left(\cot^{11}(dx+c)\right)}{11}+\frac{\left(\cot^{9}(dx+c)\right)}{9}-\frac{\left(\cot^{7}(dx+c)\right)}{7}+\frac{\left(\cot^{5}(dx+c)\right)}{5}-\frac{\left(\cot^{3}(dx+c)\right)}{3}+\cot\left(dx+c\right)+dx+c\right)+3a^3\left(-\frac{\cos^{12}(dx+c)}{11\sin(dx+c)^{11}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/11*cot(d*x+c)^11+1/9*cot(d*x+c)^9-1/7*cot(d*x+c)^7+1/5*cot(d*x
+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*a^3*(-1/11/sin(d*x+c)^11*cos(d*x
+c)^12+1/99/sin(d*x+c)^9*cos(d*x+c)^12-1/231/sin(d*x+c)^7*cos(d*x+c)^12+1/2
31/sin(d*x+c)^5*cos(d*x+c)^12-1/99/sin(d*x+c)^3*cos(d*x+c)^12+1/11/sin(d*x+
c)*cos(d*x+c)^12+1/11*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x
+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))-3/11*a^3/sin(d*x+
c)^11*cos(d*x+c)^11+a^3*(-1/11/sin(d*x+c)^11*cos(d*x+c)^10-1/99/sin(d*x+c)^
9*cos(d*x+c)^10+1/693/sin(d*x+c)^7*cos(d*x+c)^10-1/1155/sin(d*x+c)^5*cos(d*
x+c)^10+1/693/sin(d*x+c)^3*cos(d*x+c)^10-1/99/sin(d*x+c)*cos(d*x+c)^10-1/99
*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^
2)*sin(d*x+c)))
```

maxima [A] time = 0.44, size = 212, normalized size = 1.00

$$\left(3465\,dx+3465\,c+\frac{3465\,\tan(dx+c)^{10}-1155\,\tan(dx+c)^{8}+693\,\tan(dx+c)^{6}-495\,\tan(dx+c)^{4}+385\,\tan(dx+c)^{2}-315}{\tan(dx+c)^{11}}\right)a^3+\frac{15\left(693\,\sin(dx+c)^{10}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3465*((3465*d*x + 3465*c + (3465*tan(d*x + c)^10 - 1155*tan(d*x + c)^8 +
693*tan(d*x + c)^6 - 495*tan(d*x + c)^4 + 385*tan(d*x + c)^2 - 315)/tan(d*x
 + c)^11)*a^3 + 15*(693*sin(d*x + c)^10 - 1155*sin(d*x + c)^8 + 1386*sin(d*
x + c)^6 - 990*sin(d*x + c)^4 + 385*sin(d*x + c)^2 - 63)*a^3/sin(d*x + c)^1
1 - (1155*sin(d*x + c)^8 - 2772*sin(d*x + c)^6 + 2970*sin(d*x + c)^4 - 1540
*sin(d*x + c)^2 + 315)*a^3/sin(d*x + c)^11 - 945*a^3/tan(d*x + c)^11)/d
```

mupad [B] time = 3.03, size = 254, normalized size = 1.19

$$-\frac{a^3\left(315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+693\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-11550\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+159390\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^12*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(315*cos(c/2 + (d*x)/2)^16 + 693*sin(c/2 + (d*x)/2)^16 - 11550*cos(c/
2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 159390*cos(c/2 + (d*x)/2)^4*sin(c/2
+ (d*x)/2)^12 - 1323630*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 295680
*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 90090*cos(c/2 + (d*x)/2)^10*si
n(c/2 + (d*x)/2)^6 + 22770*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 385
0*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 - 887040*cos(c/2 + (d*x)/2)^5*
sin(c/2 + (d*x)/2)^11*(c + d*x)))/(887040*d*cos(c/2 + (d*x)/2)^5*sin(c/2 +
(d*x)/2)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**12*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.56 $\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{\sec^7(c+dx)}{7ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\sec^5(c+dx)}{5ad}+\frac{3\sec^4(c+dx)}{4ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos}{a$$

```
[Out] -ln(cos(d*x+c))/a/d-sec(d*x+c)/a/d-3/2*sec(d*x+c)^2/a/d+sec(d*x+c)^3/a/d+3/
4*sec(d*x+c)^4/a/d-3/5*sec(d*x+c)^5/a/d-1/6*sec(d*x+c)^6/a/d+1/7*sec(d*x+c)
^7/a/d
```

Rubi [A] time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{\sec^7(c+dx)}{7ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\sec^5(c+dx)}{5ad}+\frac{3\sec^4(c+dx)}{4ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos}{a$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Sec[c + d*x]/(a*d) - (3*Sec[c + d*x]^2)/(2*a*d
) + Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^4)/(4*a*d) - (3*Sec[c + d*x]^5)/
(5*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x]^7/(7*a*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^3}{x^8}\,dx,x,\cos(c+dx)\right)}{a^8d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}-\frac{a^7}{x^7}-\frac{3a^7}{x^6}+\frac{3a^7}{x^5}+\frac{3a^7}{x^4}-\frac{3a^7}{x^3}-\frac{a^7}{x^2}+\frac{a^7}{x}\right)dx,x,\cos(c+dx)\right)}{a^8d}\\ &= -\frac{\log(\cos(c+dx))}{ad}-\frac{\sec(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}+\frac{\sec^3(c+dx)}{ad}+\frac{3\sec^4(c+dx)}{4ad}-\frac{3}{}\end{aligned}$$

Mathematica [A] time = 0.56, size = 137, normalized size = 1.01

$$-\frac{\sec^7(c+dx)(35\cos(c+dx)(105\log(\cos(c+dx))+104)+3(602\cos(2(c+dx))+140\cos(4(c+dx))+210\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/6720*((35*Cos[c + d*x]*(104 + 105*Log[Cos[c + d*x]]) + 3*(212 + 602*Cos[
2*(c + d*x)] + 140*Cos[4*(c + d*x)] + 210*Cos[5*(c + d*x)] + 70*Cos[6*(c +
d*x)] + 245*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 35*Cos[7*(c + d*x)]*Log[Co
s[c + d*x]] + 105*Cos[3*(c + d*x)]*(6 + 7*Log[Cos[c + d*x]])))*Sec[c + d*x]
^7)/(a*d)
```

fricas [A] time = 0.50, size = 95, normalized size = 0.70

$$-\frac{420\cos(dx+c)^7\log(-\cos(dx+c))+420\cos(dx+c)^6+630\cos(dx+c)^5-420\cos(dx+c)^4-315\cos}{420\,ad\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/420*(420*cos(d*x + c)^7*log(-cos(d*x + c)) + 420*cos(d*x + c)^6 + 630*co
s(d*x + c)^5 - 420*cos(d*x + c)^4 - 315*cos(d*x + c)^3 + 252*cos(d*x + c)^2
 + 70*cos(d*x + c) - 60)/(a*d*cos(d*x + c)^7)
```

giac [A] time = 17.82, size = 245, normalized size = 1.81

$$\frac{\frac{420\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{420\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a}+\frac{\frac{5775(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{20685(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{42595(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{56035(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{2874}{(}}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*(420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 420*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (5775*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 20685*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
42595*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*(cos(d*x + c) - 1)^
4/(cos(d*x + c) + 1)^4 + 28749*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 +
8463*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*(cos(d*x + c) - 1)^7/
(cos(d*x + c) + 1)^7 + 705)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^
7))/d
```

maple [A] time = 0.63, size = 125, normalized size = 0.93

$$\frac{\sec^7(dx+c)}{7da}-\frac{\sec^6(dx+c)}{6da}-\frac{3\left(\sec^5(dx+c)\right)}{5da}+\frac{3\left(\sec^4(dx+c)\right)}{4da}+\frac{\sec^3(dx+c)}{da}-\frac{3\left(\sec^2(dx+c)\right)}{2da}-\frac{\sec(dx+}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/7*sec(d*x+c)^7/d/a-1/6*sec(d*x+c)^6/d/a-3/5*sec(d*x+c)^5/d/a+3/4*sec(d*x+
c)^4/d/a+sec(d*x+c)^3/d/a-3/2*sec(d*x+c)^2/d/a-sec(d*x+c)/d/a+1/a/d*ln(sec(
d*x+c))
```

maxima [A] time = 0.32, size = 90, normalized size = 0.67

$$-\frac{\frac{420\log(\cos(dx+c))}{a}+\frac{420\cos(dx+c)^6+630\cos(dx+c)^5-420\cos(dx+c)^4-315\cos(dx+c)^3+252\cos(dx+c)^2+70\cos(dx+c)-60}{a\cos(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(420*log(cos(d*x + c))/a + (420*cos(d*x + c)^6 + 630*cos(d*x + c)^5
- 420*cos(d*x + c)^4 - 315*cos(d*x + c)^3 + 252*cos(d*x + c)^2 + 70*cos(d*x
 + c) - 60)/(a*cos(d*x + c)^7))/d
```

mupad [B] time = 5.14, size = 208, normalized size = 1.54

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a\,d}-\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+\frac{32\tan(}{}}{d\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-21\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+35\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9/(a + a/cos(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d) - ((26*tan(c/2 + (d*x)/2)^4)/5 - (22*
tan(c/2 + (d*x)/2)^2)/5 + (32*tan(c/2 + (d*x)/2)^6)/3 - (128*tan(c/2 + (d*x
)/2)^8)/3 + 14*tan(c/2 + (d*x)/2)^10 - 2*tan(c/2 + (d*x)/2)^12 + 32/35)/(d*
(a - 7*a*tan(c/2 + (d*x)/2)^2 + 21*a*tan(c/2 + (d*x)/2)^4 - 35*a*tan(c/2 +
(d*x)/2)^6 + 35*a*tan(c/2 + (d*x)/2)^8 - 21*a*tan(c/2 + (d*x)/2)^10 + 7*a*t
an(c/2 + (d*x)/2)^12 - a*tan(c/2 + (d*x)/2)^14))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^9(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**9/(sec(c + d*x) + 1), x)/a
```

3.57 $\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)} \, dx$

Optimal. Leaf size=97

$$\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^4(c+dx)}{4ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec^2(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

```
[Out] ln(cos(d*x+c))/a/d+sec(d*x+c)/a/d+sec(d*x+c)^2/a/d-2/3*sec(d*x+c)^3/a/d-1/4
*sec(d*x+c)^4/a/d+1/5*sec(d*x+c)^5/a/d
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^4(c+dx)}{4ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec^2(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^2/(a*d) - (2*Se
c[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^4/(4*a*d) + Sec[c + d*x]^5/(5*a*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)} \, dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^2}{x^6} \, dx, x, \cos(c+dx)\right)}{a^6 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}-\frac{a^5}{x^5}-\frac{2a^5}{x^4}+\frac{2a^5}{x^3}+\frac{a^5}{x^2}-\frac{a^5}{x}\right) dx, x, \cos(c+dx)\right)}{a^6 d}\\
&= \frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^2(c+dx)}{ad}-\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec^4(c+dx)}{4ad}+\frac{s
\end{aligned}$$

Mathematica [A] time = 0.27, size = 103, normalized size = 1.06

$$\frac{\sec^5(c+dx)(40\cos(2(c+dx))+60\cos(3(c+dx))+30\cos(4(c+dx))+75\cos(3(c+dx))\log(\cos(c+dx))+}{240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((58 + 40*Cos[2*(c + d*x)] + 60*Cos[3*(c + d*x)] + 30*Cos[4*(c + d*x)] + 75
*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[5*(c + d*x)]*Log[Cos[c + d*x]]
 + 30*Cos[c + d*x]*(4 + 5*Log[Cos[c + d*x]]))*Sec[c + d*x]^5)/(240*a*d)
```

fricas [A] time = 0.50, size = 75, normalized size = 0.77

$$\frac{60\,\cos\left(dx+c\right)^{5}\log\left(-\cos\left(dx+c\right)\right)+60\,\cos\left(dx+c\right)^{4}+60\,\cos\left(dx+c\right)^{3}-40\,\cos\left(dx+c\right)^{2}-15\,\cos\left(dx+c\right)+}{60\,ad\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(60*cos(d*x + c)^5*log(-cos(d*x + c)) + 60*cos(d*x + c)^4 + 60*cos(d*x
 + c)^3 - 40*cos(d*x + c)^2 - 15*cos(d*x + c) + 12)/(a*d*cos(d*x + c)^5)
```

giac [B] time = 6.79, size = 201, normalized size = 2.07

$$-\frac{\frac{60\,\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)}{a}-\frac{60\,\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}-1\right|\right)}{a}+\frac{\frac{485\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1330\,(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1970\,(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{805\,(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{137\,(\cos(dx+c)-}{(\cos(dx+c)+1}}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 60*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (485*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1330*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 + 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 73)/(a*((
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

maple [A] time = 0.56, size = 93, normalized size = 0.96

$$\frac{\sec^{5}\left(dx+c\right)}{5da}-\frac{\sec^{4}\left(dx+c\right)}{4da}-\frac{2\left(\sec^{3}\left(dx+c\right)\right)}{3da}+\frac{\sec^{2}\left(dx+c\right)}{da}+\frac{\sec\left(dx+c\right)}{da}-\frac{\ln\left(\sec\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/5*sec(d*x+c)^5/d/a-1/4*sec(d*x+c)^4/d/a-2/3*sec(d*x+c)^3/d/a+sec(d*x+c)^2
/d/a+sec(d*x+c)/d/a-1/a/d*ln(sec(d*x+c))
```

maxima [A] time = 0.33, size = 70, normalized size = 0.72

$$\frac{\frac{60\,\log\left(\cos\left(dx+c\right)\right)}{a}+\frac{60\,\cos\left(dx+c\right)^{4}+60\,\cos\left(dx+c\right)^{3}-40\,\cos\left(dx+c\right)^{2}-15\,\cos\left(dx+c\right)+12}{a\cos\left(dx+c\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*log(cos(d*x + c))/a + (60*cos(d*x + c)^4 + 60*cos(d*x + c)^3 - 40*
cos(d*x + c)^2 - 15*cos(d*x + c) + 12)/(a*cos(d*x + c)^5))/d
```

mupad [B] time = 6.02, size = 153, normalized size = 1.58

$$\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{3}-\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3}+\frac{16}{15}}{d\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+a\right)}-\frac{2\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + a/cos(c + d*x)),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^4)/3 - (10*tan(c/2 + (d*x)/2)^2)/3 + 10*tan(c/2 + (d
*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^8 + 16/15)/(d*(a - 5*a*tan(c/2 + (d*x)/2)^2
 + 10*a*tan(c/2 + (d*x)/2)^4 - 10*a*tan(c/2 + (d*x)/2)^6 + 5*a*tan(c/2 + (d
*x)/2)^8 - a*tan(c/2 + (d*x)/2)^10)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^7(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x) + 1), x)/a
```

3.58 $\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

```
[Out] -ln(cos(d*x+c))/a/d-sec(d*x+c)/a/d-1/2*sec(d*x+c)^2/a/d+1/3*sec(d*x+c)^3/a/
d
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 75}

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Sec[c + d*x]/(a*d) - Sec[c + d*x]^2/(2*a*d) +
Sec[c + d*x]^3/(3*a*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)}{x^4}\,dx,x,\cos(c+dx)\right)}{a^4d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^3}{x^3}-\frac{a^3}{x^2}+\frac{a^3}{x}\right)dx,x,\cos(c+dx)\right)}{a^4d}\\ &= -\frac{\log(\cos(c+dx))}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^2(c+dx)}{2ad}+\frac{\sec^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.19, size = 65, normalized size = 0.98

$$-\frac{\sec^3(c+dx)(6\cos(2(c+dx))+3\cos(3(c+dx))\log(\cos(c+dx))+\cos(c+dx)(9\log(\cos(c+dx))+6)+2)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/12*((2 + 6*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + Cos
[c + d*x]*(6 + 9*Log[Cos[c + d*x]]))*Sec[c + d*x]^3)/(a*d)
```

fricas [A] time = 0.49, size = 55, normalized size = 0.83

$$-\frac{6 \cos\left(dx+c\right)^{3} \log\left(-\cos\left(dx+c\right)\right) + 6 \cos\left(dx+c\right)^{2} + 3 \cos\left(dx+c\right) - 2}{6 \, ad \cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(6*cos(d*x + c)^3*log(-cos(d*x + c)) + 6*cos(d*x + c)^2 + 3*cos(d*x +
c) - 2)/(a*d*cos(d*x + c)^3)
```

giac [B] time = 4.20, size = 157, normalized size = 2.38

$$\frac{\frac{6 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{6 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a} + \frac{\frac{21 \, (\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{45 \, (\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{11 \, (\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}} + 3}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{3}}}{6 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 6*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (21*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 45*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3)/(a*((cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 1)^3))/d
```

maple [A] time = 0.54, size = 62, normalized size = 0.94

$$\frac{\sec^{3}\left(dx+c\right)}{3da} - \frac{\sec^{2}\left(dx+c\right)}{2da} - \frac{\sec\left(dx+c\right)}{da} + \frac{\ln\left(\sec\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/3*sec(d*x+c)^3/d/a-1/2*sec(d*x+c)^2/d/a-sec(d*x+c)/d/a+1/a/d*ln(sec(d*x+c
))
```

maxima [A] time = 0.32, size = 50, normalized size = 0.76

$$-\frac{\frac{6 \log\left(\cos\left(dx+c\right)\right)}{a} + \frac{6 \cos\left(dx+c\right)^{2} + 3 \cos\left(dx+c\right) - 2}{a \cos\left(dx+c\right)^{3}}}{6 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*log(cos(d*x + c))/a + (6*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)/(a*co
s(d*x + c)^3))/d
```

mupad [B] time = 2.19, size = 99, normalized size = 1.50

$$\frac{2 \, \mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a\,d} + \frac{2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - \frac{4}{3}}{d\left(-a \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 3\,a \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 3\,a \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d) + (2*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2
 + (d*x)/2)^4 - 4/3)/(d*(a - 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)
/2)^4 - a*tan(c/2 + (d*x)/2)^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

3.59 $\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

```
[Out] ln(cos(d*x+c))/a/d+sec(d*x+c)/a/d
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3879, 43}

$$\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m - 1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{a-ax}{x^2}\,dx, x, \cos(c+dx)\right)}{a^2 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{a}{x}\right)dx, x, \cos(c+dx)\right)}{a^2 d}\\
&= \frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 21, normalized size = 0.75

$$\frac{\sec(c+dx)+\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Log[Cos[c + d*x]] + Sec[c + d*x])/(a*d)
```

fricas [A] time = 0.50, size = 33, normalized size = 1.18

$$\frac{\cos(dx+c)\log(-\cos(dx+c))+1}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (cos(d*x + c)*log(-cos(d*x + c)) + 1)/(a*d*cos(d*x + c))
```

giac [B] time = 1.12, size = 111, normalized size = 3.96

$$-\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a}+\frac{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - log(abs(-(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + ((cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 1)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

maple [A] time = 0.31, size = 30, normalized size = 1.07

$$\frac{\sec(dx+c)}{da}-\frac{\ln(\sec(dx+c))}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] sec(d*x+c)/d/a-1/a/d*ln(sec(d*x+c))
```

maxima [A] time = 0.32, size = 28, normalized size = 1.00

$$\frac{\frac{\log(\cos(dx+c))}{a}+\frac{1}{a\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(cos(d*x + c))/a + 1/(a*cos(d*x + c)))/d
```

mupad [B] time = 1.22, size = 44, normalized size = 1.57

$$\frac{2}{d\left(a-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

```
[Out] 2/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

3.60 $\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=17

$$-\frac{\log(\cos(c+dx)+1)}{ad}$$

```
[Out] -ln(1+cos(d*x+c))/a/d
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3879, 31}

$$-\frac{\log(\cos(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[1 + Cos[c + d*x]]/(a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m - 1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{\log(1+\cos(c+dx))}{ad}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.12

$$-\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*Log[Cos[(c + d*x)/2]])/(a*d)
```

fricas [A] time = 0.76, size = 19, normalized size = 1.12

$$-\frac{\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -log(1/2*cos(d*x + c) + 1/2)/(a*d)
```

giac [A] time = 0.28, size = 31, normalized size = 1.82

$$\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/(a*d)
```

maple [A] time = 0.12, size = 33, normalized size = 1.94

$$\frac{\ln(\sec(dx+c))}{ad}-\frac{\ln(1+\sec(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c)),x)
[Out] 1/a/d*ln(sec(d*x+c))-1/d/a*ln(1+sec(d*x+c))
```

maxima [A] time = 0.32, size = 17, normalized size = 1.00

$$-\frac{\log(\cos(dx+c)+1)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -log(cos(d*x + c) + 1)/(a*d)
```

mupad [B] time = 1.21, size = 21, normalized size = 1.24

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x)),x)
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

sympy [A] time = 3.80, size = 41, normalized size = 2.41

$$\begin{cases}\frac{\log\left(\tan^2(c+dx)+1\right)}{2ad}-\frac{\log(\sec(c+dx)+1)}{ad} & \text{for } d\neq 0\\ \frac{x\tan(c)}{a\sec(c)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x)
[Out] Piecewise((log(tan(c + d*x)**2 + 1)/(2*a*d) - log(sec(c + d*x) + 1)/(a*d),
Ne(d, 0)), (x*tan(c)/(a*sec(c) + a), True))
```

3.61 $\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{1}{2ad(\cos(c+dx)+1)}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(\cos(c+dx)+1)}{4ad}$$

```
[Out] 1/2/a/d/(1+cos(d*x+c))+1/4*ln(1-cos(d*x+c))/a/d+3/4*ln(1+cos(d*x+c))/a/d
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{1}{2ad(\cos(c+dx)+1)}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(\cos(c+dx)+1)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] 1/(2*a*d*(1 + Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(4*a*d) + (3*Log[1 + C
os[c + d*x]])/(4*a*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{a^2\,\text{Subst}\left(\int \frac{x^2}{(a-ax)(a+ax)^2}\,dx,x,\cos(c+dx)\right)}{d}\\
&= -\frac{a^2\,\text{Subst}\left(\int \left(-\frac{1}{4a^3(-1+x)}+\frac{1}{2a^3(1+x)^2}-\frac{3}{4a^3(1+x)}\right)dx,x,\cos(c+dx)\right)}{d}\\
&= \frac{1}{2ad(1+\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(1+\cos(c+dx))}{4ad}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 67, normalized size = 1.10

$$\frac{\sec(c+dx)\left(2\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+1\right)}{2ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((1 + 2*Cos[(c + d*x)/2]^2*(3*Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]
))*Sec[c + d*x])/(2*a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 0.74, size = 60, normalized size = 0.98

$$\frac{3\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2}{4\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c) + 1)*
log(-1/2*cos(d*x + c) + 1/2) + 2)/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.24, size = 86, normalized size = 1.41

$$\frac{\frac{\log\left(\frac{\left|-\cos\left(dx+c\right)+1\right|}{\left|\cos\left(dx+c\right)+1\right|}\right)}{a}-\frac{4\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)}{a}-\frac{\cos\left(dx+c\right)-1}{a\left(\cos\left(dx+c\right)+1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a - 4*log(abs(-(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - (cos(d*x + c) - 1)/(a*(cos(d*x +
 c) + 1)))/d
```

maple [A] time = 0.61, size = 54, normalized size = 0.89

$$\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{4da}+\frac{1}{2ad\left(1+\cos\left(dx+c\right)\right)}+\frac{3\ln\left(1+\cos\left(dx+c\right)\right)}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/4/d/a*ln(-1+cos(d*x+c))+1/2/a/d/(1+cos(d*x+c))+3/4*ln(1+cos(d*x+c))/d/a
```

maxima [A] time = 0.32, size = 47, normalized size = 0.77

$$\frac{\frac{3\log\left(\cos\left(dx+c\right)+1\right)}{a}+\frac{\log\left(\cos\left(dx+c\right)-1\right)}{a}+\frac{2}{a\cos\left(dx+c\right)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(3*log(cos(d*x + c) + 1)/a + log(cos(d*x + c) - 1)/a + 2/(a*cos(d*x + c
) + a))/d
```

mupad [B] time = 1.25, size = 49, normalized size = 0.80

$$\frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2}-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{4}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))/2 - log(tan(c/2 + (d*x)/2)^2 + 1) + tan(c/2 + (d*x
)/2)^2/4)/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x) + 1), x)/a
```

3.62 $\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{8ad(1-\cos(c+dx))}-\frac{3}{4ad(\cos(c+dx)+1)}+\frac{1}{8ad(\cos(c+dx)+1)^2}-\frac{5\log(1-\cos(c+dx))}{16ad}-\frac{11\log(\cos(c+}{16ad}$$

```
[Out] -1/8/a/d/(1-cos(d*x+c))+1/8/a/d/(1+cos(d*x+c))^2-3/4/a/d/(1+cos(d*x+c))-5/1
6*ln(1-cos(d*x+c))/a/d-11/16*ln(1+cos(d*x+c))/a/d
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3879, 88}

$$-\frac{1}{8ad(1-\cos(c+dx))}-\frac{3}{4ad(\cos(c+dx)+1)}+\frac{1}{8ad(\cos(c+dx)+1)^2}-\frac{5\log(1-\cos(c+dx))}{16ad}-\frac{11\log(\cos(c+}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/(8*a*d*(1 - Cos[c + d*x])) + 1/(8*a*d*(1 + Cos[c + d*x])^2) - 3/(4*a*d*(
1 + Cos[c + d*x])) - (5*Log[1 - Cos[c + d*x]])/(16*a*d) - (11*Log[1 + Cos[c
 + d*x]])/(16*a*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{a^4\,\text{Subst}\left(\int \frac{x^4}{(a-ax)^2(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{8a^5(-1+x)^2}+\frac{5}{16a^5(-1+x)}+\frac{1}{4a^5(1+x)^3}-\frac{3}{4a^5(1+x)^2}+\frac{11}{16a^5(1+x)}\right)dx, x, \cos(c\right.}{d}\\
&= -\frac{1}{8ad(1-\cos(c+dx))}+\frac{1}{8ad(1+\cos(c+dx))^2}-\frac{3}{4ad(1+\cos(c+dx))}-\frac{5\log(1-}{1}
\end{aligned}$$

Mathematica [A] time = 0.62, size = 107, normalized size = 1.04

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^4\left(\frac{1}{2}(c+dx)\right)+12\sec^2\left(\frac{1}{2}(c+dx)\right)+20\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.}{16ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/16*(Cos[(c + d*x)/2]^2*(2*Csc[(c + d*x)/2]^2 + 44*Log[Cos[(c + d*x)/2]]
+ 20*Log[Sin[(c + d*x)/2]] + 12*Sec[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^4)*Se
c[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 0.71, size = 139, normalized size = 1.35

$$-\frac{10\cos\left(dx+c\right)^2+11\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2-\cos\left(dx+c\right)-1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+5\left(\cos\left(dx+c\right.\right.}{16\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right)^2-ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*cos(d*x + c)^2 + 11*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x +
c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 5*(cos(d*x + c)^3 + cos(d*x + c)^2 -
cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c) - 12)/(a*d*
cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)
```

giac [A] time = 0.25, size = 157, normalized size = 1.52

$$\frac{\frac{2\left(\frac{5\left(\cos(dx+c)-1\right)}{\cos(dx+c)+1}+1\right)\left(\cos(dx+c)+1\right)}{a\left(\cos(dx+c)-1\right)}-\frac{10\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right)}{a}+\frac{32\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}+\frac{\frac{10\,a\left(\cos(dx+c)-1\right)}{\cos(dx+c)+1}+\frac{a\left(\cos(dx+c)-1\right)^2}{\left(\cos(dx+c)+1\right)^2}}{a^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(2*(5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
*(cos(d*x + c) - 1)) - 10*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))
/a + 32*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a + (10*a*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2)/a^2)/d
```

maple [A] time = 0.80, size = 90, normalized size = 0.87

$$\frac{1}{8ad\left(-1+\cos\left(dx+c\right)\right)}-\frac{5\ln\left(-1+\cos\left(dx+c\right)\right)}{16da}+\frac{1}{8ad\left(1+\cos\left(dx+c\right)\right)^2}-\frac{3}{4ad\left(1+\cos\left(dx+c\right)\right)}-\frac{11\ln\left(1+\cos\right.}{16da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/8/a/d/(-1+cos(d*x+c))-5/16/d/a*ln(-1+cos(d*x+c))+1/8/a/d/(1+cos(d*x+c))^2
-3/4/a/d/(1+cos(d*x+c))-11/16*ln(1+cos(d*x+c))/d/a
```

maxima [A] time = 0.69, size = 91, normalized size = 0.88

$$-\frac{\frac{2\left(5\cos(dx+c)^2-3\cos(dx+c)-6\right)}{a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a}+\frac{11\log(\cos(dx+c)+1)}{a}+\frac{5\log(\cos(dx+c)-1)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(5*cos(d*x + c)^2 - 3*cos(d*x + c) - 6)/(a*cos(d*x + c)^3 + a*cos(
d*x + c)^2 - a*cos(d*x + c) - a) + 11*log(cos(d*x + c) + 1)/a + 5*log(cos(d
*x + c) - 1)/a)/d
```

mupad [B] time = 1.34, size = 76, normalized size = 0.74

$$-\frac{\frac{5\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

```
[Out] -((5*log(tan(c/2 + (d*x)/2)))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2 +
 (d*x)/2)^2/16 + (5*tan(c/2 + (d*x)/2)^2)/16 - tan(c/2 + (d*x)/2)^4/32)/(a*
d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

3.63 $\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=145

$$\frac{1}{4ad(1-\cos(c+dx))}+\frac{15}{16ad(\cos(c+dx)+1)}-\frac{1}{32ad(1-\cos(c+dx))^2}-\frac{9}{32ad(\cos(c+dx)+1)^2}+\frac{1}{24ad(\cos(c+dx$$

```
[Out] -1/32/a/d/(1-cos(d*x+c))^2+1/4/a/d/(1-cos(d*x+c))+1/24/a/d/(1+cos(d*x+c))^3
-9/32/a/d/(1+cos(d*x+c))^2+15/16/a/d/(1+cos(d*x+c))+11/32*ln(1-cos(d*x+c))/
a/d+21/32*ln(1+cos(d*x+c))/a/d
```

Rubi [A] time = 0.10, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3879, 88}

$$\frac{1}{4ad(1-\cos(c+dx))}+\frac{15}{16ad(\cos(c+dx)+1)}-\frac{1}{32ad(1-\cos(c+dx))^2}-\frac{9}{32ad(\cos(c+dx)+1)^2}+\frac{1}{24ad(\cos(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/(32*a*d*(1 - Cos[c + d*x])^2) + 1/(4*a*d*(1 - Cos[c + d*x])) + 1/(24*a*d
*(1 + Cos[c + d*x])^3) - 9/(32*a*d*(1 + Cos[c + d*x])^2) + 15/(16*a*d*(1 +
Cos[c + d*x])) + (11*Log[1 - Cos[c + d*x]])/(32*a*d) + (21*Log[1 + Cos[c +
d*x]])/(32*a*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{a^6\,\text{Subst}\left(\int \frac{x^6}{(a-ax)^3(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{16a^7(-1+x)^3}-\frac{1}{4a^7(-1+x)^2}-\frac{11}{32a^7(-1+x)}+\frac{1}{8a^7(1+x)^4}-\frac{9}{16a^7(1+x)^3}+\frac{15}{16a^7(1+x)^2}-\right.\right.}{d}\\ &= -\frac{1}{32ad(1-\cos(c+dx))^2}+\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{24ad(1+\cos(c+dx))^3}-\frac{}{32ad(1+}\end{aligned}$$

Mathematica [A] time = 0.54, size = 135, normalized size = 0.93

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(3\csc^4\left(\frac{1}{2}(c+dx)\right)-48\csc^2\left(\frac{1}{2}(c+dx)\right)-2\sec^6\left(\frac{1}{2}(c+dx)\right)+27\sec^4\left(\frac{1}{2}(c+dx)\right)\right.}{192ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/192*(Cos[(c + d*x)/2]^2*(-48*Csc[(c + d*x)/2]^2 + 3*Csc[(c + d*x)/2]^4 -
 504*Log[Cos[(c + d*x)/2]] - 264*Log[Sin[(c + d*x)/2]] - 180*Sec[(c + d*x)/
2]^2 + 27*Sec[(c + d*x)/2]^4 - 2*Sec[(c + d*x)/2]^6)*Sec[c + d*x])/(a*d*(1
+ Sec[c + d*x]))
```

fricas [A] time = 0.89, size = 217, normalized size = 1.50

$$\frac{66 \cos(dx+c)^4 - 78 \cos(dx+c)^3 - 158 \cos(dx+c)^2 + 63 \left(\cos(dx+c)^5 + \cos(dx+c)^4 - 2 \cos(dx+c)^3 - \right.}{96 \left(ad \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*cos(d*x + c)^4 - 78*cos(d*x + c)^3 - 158*cos(d*x + c)^2 + 63*(cos(
d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x
 + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 33*(cos(d*x + c)^5 + cos(d*x + c)^
4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*log(-1/2*cos(d*
x + c) + 1/2) + 58*cos(d*x + c) + 88)/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c
)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.34, size = 211, normalized size = 1.46

$$-\frac{\frac{3\left(\frac{14(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{66(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2} - \frac{132 \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} + \frac{384 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} + \frac{\frac{132 a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{21 a^2(\cdots}{(\cos\cdots}}{\cdots}}{384 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/384*(3*(14*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 66*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2)
- 132*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + 384*log(abs(-(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a + (132*a^2*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 21*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2*a^
2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^3)/d
```

maple [A] time = 0.69, size = 126, normalized size = 0.87

$$-\frac{1}{32ad(-1+\cos(dx+c))^2} - \frac{1}{4ad(-1+\cos(dx+c))} + \frac{11 \ln(-1+\cos(dx+c))}{32da} + \frac{1}{24ad(1+\cos(dx+c))^3} - \frac{\cdots}{32a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/32/a/d/(-1+cos(d*x+c))^2-1/4/a/d/(-1+cos(d*x+c))+11/32/d/a*ln(-1+cos(d*x
+c))+1/24/a/d/(1+cos(d*x+c))^3-9/32/a/d/(1+cos(d*x+c))^2+15/16/a/d/(1+cos(d
*x+c))+21/32*ln(1+cos(d*x+c))/d/a
```

maxima [A] time = 0.33, size = 130, normalized size = 0.90

$$\frac{\frac{2\left(33 \cos(dx+c)^4 - 39 \cos(dx+c)^3 - 79 \cos(dx+c)^2 + 29 \cos(dx+c) + 44\right)}{a \cos(dx+c)^5 + a \cos(dx+c)^4 - 2 a \cos(dx+c)^3 - 2 a \cos(dx+c)^2 + a \cos(dx+c) + a} + \frac{63 \log(\cos(dx+c)+1)}{a} + \frac{33 \log(\cos(dx+c)-1)}{a}}{96 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(33*cos(d*x + c)^4 - 39*cos(d*x + c)^3 - 79*cos(d*x + c)^2 + 29*cos
(d*x + c) + 44)/(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 -
 2*a*cos(d*x + c)^2 + a*cos(d*x + c) + a) + 63*log(cos(d*x + c) + 1)/a + 33
*log(cos(d*x + c) - 1)/a)/d
```

mupad [B] time = 1.31, size = 132, normalized size = 0.91

$$\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{32\,a\,d}-\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{128\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{192\,a\,d}+\frac{11\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{16\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

```
[Out] (11*tan(c/2 + (d*x)/2)^2)/(32*a*d) - (7*tan(c/2 + (d*x)/2)^4)/(128*a*d) + t
an(c/2 + (d*x)/2)^6/(192*a*d) + (11*log(tan(c/2 + (d*x)/2)))/(16*a*d) - log
(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) + (cot(c/2 + (d*x)/2)^4*((7*tan(c/2 + (d*x
)/2)^2)/2 - 1/4))/(32*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

3.64 $\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)(6-5\sec(c+dx))}{30ad}+\frac{\tan^3(c+dx)(8-5\sec(c+dx))}{24ad}-\frac{\tan(c+dx)(16-5}{16ad}$$

```
[Out] x/a-5/16*arctanh(sin(d*x+c))/a/d-1/16*(16-5*sec(d*x+c))*tan(d*x+c)/a/d+1/24
*(8-5*sec(d*x+c))*tan(d*x+c)^3/a/d-1/30*(6-5*sec(d*x+c))*tan(d*x+c)^5/a/d
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3881, 3770}

$$-\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)(6-5\sec(c+dx))}{30ad}+\frac{\tan^3(c+dx)(8-5\sec(c+dx))}{24ad}-\frac{\tan(c+dx)(16-5}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a - (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - ((16 - 5*Sec[c + d*x])*Tan[c + d
*x])/(16*a*d) + ((8 - 5*Sec[c + d*x])*Tan[c + d*x]^3)/(24*a*d) - ((6 - 5*Se
c[c + d*x])*Tan[c + d*x]^5)/(30*a*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int(-a+a\sec(c+dx))\tan^6(c+dx)\,dx}{a^2}$$
$$= -\frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad} - \frac{\int(-6a+5a\sec(c+dx))\tan^4(c+dx)\,dx}{6a^2}$$
$$= \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad} + \frac{\int(-24a+15a\,\mathrm{se}$$
$$= -\frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\sec(c+d}{30}$$
$$= \frac{x}{a} - \frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\sec(c}{}$$
$$= \frac{x}{a} - \frac{5\tanh^{-1}(\sin(c+dx))}{16ad} - \frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan}{24ad}$$

Mathematica [B] time = 0.90, size = 301, normalized size = 2.87

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(2400\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(2400*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x]^6*(
2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 1800*d*x*Cos[3*c + 2*d*x] + 720
*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x] + 120*d*x*Cos[5*c + 6*d*x]
 + 120*d*x*Cos[7*c + 6*d*x] + 3680*Sin[c] + 450*Sin[d*x] + 450*Sin[2*c + d*
x] - 3360*Sin[c + 2*d*x] + 2160*Sin[3*c + 2*d*x] - 25*Sin[2*c + 3*d*x] - 25
*Sin[4*c + 3*d*x] - 1488*Sin[3*c + 4*d*x] + 720*Sin[5*c + 4*d*x] + 165*Sin[
4*c + 5*d*x] + 165*Sin[6*c + 5*d*x] - 368*Sin[5*c + 6*d*x])))/(3840*a*d*(1
+ Sec[c + d*x]))
```

fricas [A] time = 0.57, size = 127, normalized size = 1.21

$$\frac{480\,dx\cos(dx+c)^6 - 75\cos(dx+c)^6\log(\sin(dx+c)+1) + 75\cos(dx+c)^6\log(-\sin(dx+c)+1) - 2\left(368\right.}{480\,ad\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(480*d*x*cos(d*x + c)^6 - 75*cos(d*x + c)^6*log(sin(d*x + c) + 1) + 7
5*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*cos(d*x + c)^5 - 165*cos(d
*x + c)^4 - 176*cos(d*x + c)^3 + 130*cos(d*x + c)^2 + 48*cos(d*x + c) - 40)
*sin(d*x + c))/(a*d*cos(d*x + c)^6)
```

giac [A] time = 9.29, size = 149, normalized size = 1.42

$$\frac{\frac{240\,(dx+c)}{a} - \frac{75\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a} + \frac{75\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a} + \frac{2\left(315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11} - 1945\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 + 5118\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 -\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(240*(d*x + c)/a - 75*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + 75*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a + 2*(315*tan(1/2*d*x + 1/2*c)^11 - 1945*tan
(1/2*d*x + 1/2*c)^9 + 5118*tan(1/2*d*x + 1/2*c)^7 - 3138*tan(1/2*d*x + 1/2*
c)^5 + 1095*tan(1/2*d*x + 1/2*c)^3 - 165*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*
x + 1/2*c)^2 - 1)^6*a))/d
```

maple [B] time = 0.54, size = 312, normalized size = 2.97

$$\frac{1}{6ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{6}}+\frac{7}{10ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{3}{4ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{5}{12ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{}{16ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/6/a/d/(tan(1/2*d*x+1/2*c)-1)^6+7/10/a/d/(tan(1/2*d*x+1/2*c)-1)^5+3/4/a/d/
(tan(1/2*d*x+1/2*c)-1)^4-5/12/a/d/(tan(1/2*d*x+1/2*c)-1)^3-9/16/a/d/(tan(1/
2*d*x+1/2*c)-1)^2+21/16/a/d/(tan(1/2*d*x+1/2*c)-1)+5/16/a/d*ln(tan(1/2*d*x+
1/2*c)-1)-1/6/a/d/(tan(1/2*d*x+1/2*c)+1)^6+7/10/a/d/(tan(1/2*d*x+1/2*c)+1)^
5-3/4/a/d/(tan(1/2*d*x+1/2*c)+1)^4-5/12/a/d/(tan(1/2*d*x+1/2*c)+1)^3+9/16/a
/d/(tan(1/2*d*x+1/2*c)+1)^2+21/16/a/d/(tan(1/2*d*x+1/2*c)+1)-5/16/a/d*ln(ta
n(1/2*d*x+1/2*c)+1)+2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.79, size = 329, normalized size = 3.13

$$-\frac{\frac{2\left(\frac{165\sin(dx+c)}{\cos(dx+c)+1}-\frac{1095\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3138\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{5118\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{1945\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a-\frac{6a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{15a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{20a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{75\log\left(\frac{\sin(dx}{\cos(dx+}\right.}{a}}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(2*(165*sin(d*x + c)/(cos(d*x + c) + 1) - 1095*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3138*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5118*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 1945*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 315*
sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a - 6*a*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*sin(d*
x + c)^10/(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)
- 480*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 75*log(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)/a - 75*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

mupad [B] time = 2.52, size = 193, normalized size = 1.84

$$\frac{x}{a}-\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}-\frac{-\frac{21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{389\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-\frac{853\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{523\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8/(a + a/cos(c + d*x)),x)
```

```
[Out] x/a - (5*atanh(tan(c/2 + (d*x)/2)))/(8*a*d) - ((11*tan(c/2 + (d*x)/2))/8 -
(73*tan(c/2 + (d*x)/2)^3)/8 + (523*tan(c/2 + (d*x)/2)^5)/20 - (853*tan(c/2
+ (d*x)/2)^7)/20 + (389*tan(c/2 + (d*x)/2)^9)/24 - (21*tan(c/2 + (d*x)/2)^1
```

```
1)/8)/(d*(a - 6*a*tan(c/2 + (d*x)/2)^2 + 15*a*tan(c/2 + (d*x)/2)^4 - 20*a*t
an(c/2 + (d*x)/2)^6 + 15*a*tan(c/2 + (d*x)/2)^8 - 6*a*tan(c/2 + (d*x)/2)^10
 + a*tan(c/2 + (d*x)/2)^12))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^8(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x) + 1), x)/a
```

3.65 $\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)(4-3\sec(c+dx))}{12ad}+\frac{\tan(c+dx)(8-3\sec(c+dx))}{8ad}-\frac{x}{a}$$

```
[Out] -x/a+3/8*arctanh(sin(d*x+c))/a/d+1/8*(8-3*sec(d*x+c))*tan(d*x+c)/a/d-1/12*(
4-3*sec(d*x+c))*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3881, 3770}

$$\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)(4-3\sec(c+dx))}{12ad}+\frac{\tan(c+dx)(8-3\sec(c+dx))}{8ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + ((8 - 3*Sec[c + d*x])*Tan[c +
d*x])/(8*a*d) - ((4 - 3*Sec[c + d*x])*Tan[c + d*x]^3)/(12*a*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int(-a+a\sec(c+dx))\tan^4(c+dx)\,dx}{a^2}\\
&= -\frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad}-\frac{\int(-4a+3a\sec(c+dx))\tan^2(c+dx)\,dx}{4a^2}\\
&= \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad}-\frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad}+\frac{\int(-8a+3a\sec}{8a^2}\\
&= -\frac{x}{a}+\frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad}-\frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad}+\frac{3\int\sec(c}{8}\\
&= -\frac{x}{a}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad}-\frac{(4-3\sec(c+dx))}{12ad}
\end{aligned}$$

Mathematica [B] time = 6.45, size = 893, normalized size = 11.45

$$-\frac{2x\sec(c+dx)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\sec(c+dx)a+a}-\frac{3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec(c+dx)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d(\sec(c+dx)a+a)}+\frac{3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*x*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x])/(a + a*Sec[c + d*x]) - (3*Cos[c/2
+ (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x])/(4*
d*(a + a*Sec[c + d*x])) + (3*Cos[c/2 + (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] +
Sin[c/2 + (d*x)/2]]*Sec[c + d*x])/(4*d*(a + a*Sec[c + d*x])) + (Cos[c/2 + (
d*x)/2]^2*Sec[c + d*x])/(8*d*(a + a*Sec[c + d*x])*(Cos[c/2 + (d*x)/2] - Sin
[c/2 + (d*x)/2])^4) - (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d
*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
 (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*(-19*Cos[c/2] + 11*Sin[c
/2]))/(24*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2]
- Sin[c/2 + (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2
])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Si
n[c/2 + (d*x)/2])) - (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x])/(8*d*(a + a*Sec[c
+ d*x])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) - (Cos[c/2 + (d*x)/2]^
2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] + Sin[c/2]
)*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^2*Sec[
c + d*x]*(19*Cos[c/2] + 11*Sin[c/2]))/(24*d*(a + a*Sec[c + d*x])*(Cos[c/2]
+ Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (8*Cos[c/2 + (d*
x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] + Si
n[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 0.81, size = 107, normalized size = 1.37

$$-\frac{48\,dx\cos\left(dx+c\right)^4-9\,\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)+9\,\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-2\left(32\cos\right.}{48\,ad\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(48*d*x*cos(d*x + c)^4 - 9*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 9*c
os(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 - 15*cos(d*x +
c)^2 - 8*cos(d*x + c) + 6)*sin(d*x + c))/(a*d*cos(d*x + c)^4)
```

giac [A] time = 4.10, size = 123, normalized size = 1.58

$$-\frac{\frac{24\,(dx+c)}{a}-\frac{9\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{9\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-137\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+71\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(24*(d*x + c)/a - 9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + 9*log(abs(
tan(1/2*d*x + 1/2*c) - 1))/a + 2*(33*tan(1/2*d*x + 1/2*c)^7 - 137*tan(1/2*d
*x + 1/2*c)^5 + 71*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 - 1)^4*a))/d
```

maple [B] time = 0.53, size = 228, normalized size = 2.92

$$\frac{1}{4ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{5}{6ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{3}{8ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{11}{8ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/4/a/d/(tan(1/2*d*x+1/2*c)-1)^4+5/6/a/d/(tan(1/2*d*x+1/2*c)-1)^3+3/8/a/d/(
tan(1/2*d*x+1/2*c)-1)^2-11/8/a/d/(tan(1/2*d*x+1/2*c)-1)-3/8/a/d*ln(tan(1/2*
d*x+1/2*c)-1)-1/4/a/d/(tan(1/2*d*x+1/2*c)+1)^4+5/6/a/d/(tan(1/2*d*x+1/2*c)+
1)^3-3/8/a/d/(tan(1/2*d*x+1/2*c)+1)^2-11/8/a/d/(tan(1/2*d*x+1/2*c)+1)+3/8/a
/d*ln(tan(1/2*d*x+1/2*c)+1)-2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.54, size = 247, normalized size = 3.17

$$\frac{\frac{2\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{71\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{137\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{33\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{a-\frac{4\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{4\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}-\frac{48\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/24*(2*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 71*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 137*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 33*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 48*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - 9*log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a)/d
```

mupad [B] time = 2.00, size = 139, normalized size = 1.78

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d}-\frac{x}{a}+\frac{-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{137\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{12}-\frac{71\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{12}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) - x/a + ((5*tan(c/2 + (d*x)/2))/4 - (
71*tan(c/2 + (d*x)/2)^3)/12 + (137*tan(c/2 + (d*x)/2)^5)/12 - (11*tan(c/2 +
 (d*x)/2)^7)/4)/(d*(a - 4*a*tan(c/2 + (d*x)/2)^2 + 6*a*tan(c/2 + (d*x)/2)^4
 - 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2)^8))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{6}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

3.66 $\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)(2-\sec(c+dx))}{2ad}+\frac{x}{a}$$

```
[Out] x/a-1/2*arctanh(sin(d*x+c))/a/d-1/2*(2-sec(d*x+c))*tan(d*x+c)/a/d
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3881, 3770}

$$-\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)(2-\sec(c+dx))}{2ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a - ArcTanh[Sin[c + d*x]]/(2*a*d) - ((2 - Sec[c + d*x])*Tan[c + d*x])/(2*
a*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int (-a+a\sec(c+dx))\tan^2(c+dx)\,dx}{a^2}\\ &= -\frac{(2-\sec(c+dx))\tan(c+dx)}{2ad}-\frac{\int (-2a+a\sec(c+dx))\,dx}{2a^2}\\ &= \frac{x}{a}-\frac{(2-\sec(c+dx))\tan(c+dx)}{2ad}-\frac{\int \sec(c+dx)\,dx}{2a}\\ &= \frac{x}{a}-\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(2-\sec(c+dx))\tan(c+dx)}{2ad}\end{aligned}$$

Mathematica [B] time = 0.88, size = 241, normalized size = 4.92

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\frac{4\sin(dx)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+ -\frac{}{d(}\right.}{2a(\mathrm{se}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(4*x + (2*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]])/d - (2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + 1/(d*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2) - (4*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(2
*a*(1 + Sec[c + d*x]))
```

fricas [A] time = 0.61, size = 86, normalized size = 1.76

$$\frac{4\,dx\cos(dx+c)^2 - \cos(dx+c)^2\log(\sin(dx+c)+1) + \cos(dx+c)^2\log(-\sin(dx+c)+1) - 2\,(2\,\cos(dx+}{4\,ad\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*x*cos(d*x + c)^2 - cos(d*x + c)^2*log(sin(d*x + c) + 1) + cos(d*x
+ c)^2*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) - 1)*sin(d*x + c))/(a*d*c
os(d*x + c)^2)
```

giac [B] time = 2.27, size = 96, normalized size = 1.96

$$\frac{\frac{2\,(dx+c)}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} + \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2 a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

maple [B] time = 0.50, size = 144, normalized size = 2.94

$$\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}-\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{2ad\left(\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)+1/2/a/d*ln(
tan(1/2*d*x+1/2*c)-1)-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2+3/2/a/d/(tan(1/2*d*x
+1/2*c)+1)-1/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)+2/a/d*arctan(tan(1/2*d*x+1/2*c)
)
```

maxima [B] time = 0.78, size = 163, normalized size = 3.33

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4) - 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a)/d
```

mupad [B] time = 1.29, size = 83, normalized size = 1.69

$$\frac{x}{a}-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] x/a - atanh(tan(c/2 + (d*x)/2))/(a*d) - (tan(c/2 + (d*x)/2) - 3*tan(c/2 + (
d*x)/2)^3)/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.67 $\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{x}{a}$$

```
[Out] -x/a+arctanh(sin(d*x+c))/a/d
```

Rubi [A] time = 0.05, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3888, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + ArcTanh[Sin[c + d*x]]/(a*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int (-a+a\sec(c+dx))\,dx}{a^2}\\ &= -\frac{x}{a}+\frac{\int \sec(c+dx)\,dx}{a}\\ &= -\frac{x}{a}+\frac{\tanh^{-1}(\sin(c+dx))}{ad}\end{aligned}$$

Mathematica [B] time = 0.09, size = 60, normalized size = 2.86

$$-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+dx}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((d*x + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]])/(a*d))
```

fricas [A] time = 0.83, size = 35, normalized size = 1.67

$$-\frac{2\,dx-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(-\sin\left(dx+c\right)+1\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*x - log(sin(d*x + c) + 1) + log(-sin(d*x + c) + 1))/(a*d)
```

giac [B] time = 0.57, size = 50, normalized size = 2.38

$$-\frac{\frac{dx+c}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2*d*x + 1/2*c) - 1))/a)/d
```

maple [B] time = 0.32, size = 59, normalized size = 2.81

$$-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad} - \frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)-2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.64, size = 78, normalized size = 3.71

$$-\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

mupad [B] time = 1.11, size = 25, normalized size = 1.19

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} - \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a*d) - x/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{2}(c+dx)}{\sec(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.68 $\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}-\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}-\frac{x}{a}$$

```
[Out] -x/a-1/3*cot(d*x+c)*(3-2*sec(d*x+c))/a/d+1/3*cot(d*x+c)^3*(1-sec(d*x+c))/a/
d
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3882, 8}

$$\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}-\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) - (Cot[c + d*x]*(3 - 2*Sec[c + d*x]))/(3*a*d) + (Cot[c + d*x]^3*(1 -
 Sec[c + d*x]))/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int \cot^4(c+dx)(-a+a\sec(c+dx))\,dx}{a^2}\\
&= \frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}+\frac{\int \cot^2(c+dx)(3a-2a\sec(c+dx))\,dx}{3a^2}\\
&= -\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}+\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}+\frac{\int -3a\,dx}{3a^2}\\
&= -\frac{x}{a}-\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}+\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}
\end{aligned}$$

Mathematica [A] time = 0.80, size = 100, normalized size = 1.64

$$\frac{\sec(c+dx)\left(-12dx\cos^2\left(\frac{1}{2}(c+dx)\right)-\tan\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{dx}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(3\csc\left(\frac{c}{2}\right)\cot\left(\frac{1}{2}(c+dx)\right)+\right.\right.}{6ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(-12*d*x*Cos[(c + d*x)/2]^2 + Cos[(c + d*x)/2]*(3*Cot[(c + d*
x)/2]*Csc[c/2] + 13*Sec[c/2])*Sin[(d*x)/2] - Tan[(c + d*x)/2]))/(6*a*d*(1 +
 Sec[c + d*x]))
```

fricas [A] time = 0.49, size = 64, normalized size = 1.05

$$-\frac{4\cos\left(dx+c\right)^{2}+3\left(dx\cos\left(dx+c\right)+dx\right)\sin\left(dx+c\right)+\cos\left(dx+c\right)-2}{3\left(ad\cos\left(dx+c\right)+ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(4*cos(d*x + c)^2 + 3*(d*x*cos(d*x + c) + d*x)*sin(d*x + c) + cos(d*x
+ c) - 2)/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

giac [A] time = 0.24, size = 66, normalized size = 1.08

$$-\frac{\frac{12\left(dx+c\right)}{a}+\frac{a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-12a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{3}}+\frac{3}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*(d*x + c)/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*tan(1/2*d*x +
1/2*c))/a^3 + 3/(a*tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.62, size = 74, normalized size = 1.21

$$-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{4ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/12/a/d*tan(1/2*d*x+1/2*c)^3+1/a/d*tan(1/2*d*x+1/2*c)-1/4/a/d/tan(1/2*d*x
+1/2*c)-2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.60, size = 93, normalized size = 1.52

$$\frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{3\left(\cos(dx+c)+1\right)}{a\sin(dx+c)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3*(cos(d*x + c) +
1)/(a*sin(d*x + c)))/d
```

mupad [B] time = 1.29, size = 65, normalized size = 1.07

$$-\frac{x}{a}-\frac{\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{7\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}+\frac{1}{12}}{a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] - x/a - ((4*cos(c/2 + (d*x)/2)^4)/3 - (7*cos(c/2 + (d*x)/2)^2)/6 + 1/12)/(a
*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.69 $\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}+\frac{x}{a}$$

```
[Out] x/a+1/15*cot(d*x+c)*(15-8*sec(d*x+c))/a/d-1/15*cot(d*x+c)^3*(5-4*sec(d*x+c)
)/a/d+1/5*cot(d*x+c)^5*(1-sec(d*x+c))/a/d
```

Rubi [A] time = 0.13, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3882, 8}

$$\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a + (Cot[c + d*x]*(15 - 8*Sec[c + d*x]))/(15*a*d) - (Cot[c + d*x]^3*(5 -
4*Sec[c + d*x]))/(15*a*d) + (Cot[c + d*x]^5*(1 - Sec[c + d*x]))/(5*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int \cot^6(c+dx)(-a+a\sec(c+dx))\,dx}{a^2}\\
&= \frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}+\frac{\int \cot^4(c+dx)(5a-4a\sec(c+dx))\,dx}{5a^2}\\
&= -\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}+\frac{\int \cot^2(c+dx)(-15}{1}\\
&= \frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot^5(c+dx)(1-\text{se}}{5ad}\\
&= \frac{x}{a}+\frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot^5(c+dx)(1}{5}
\end{aligned}$$

Mathematica [B] time = 0.90, size = 254, normalized size = 2.89

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^3(c+dx)\sec(c+dx)(534\sin(c+dx)+178\sin(2(c+dx))-178\sin(3(c+dx))-89\sin(4(c$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*Sec[c + d*x]*(360*d*x*Cos[d*x] - 360*d*x*
Cos[2*c + d*x] + 120*d*x*Cos[c + 2*d*x] - 120*d*x*Cos[3*c + 2*d*x] - 120*d*
x*Cos[2*c + 3*d*x] + 120*d*x*Cos[4*c + 3*d*x] - 60*d*x*Cos[3*c + 4*d*x] + 6
0*d*x*Cos[5*c + 4*d*x] - 200*Sin[c] - 584*Sin[d*x] + 534*Sin[c + d*x] + 178
*Sin[2*(c + d*x)] - 178*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)] - 520*Sin[2*
c + d*x] - 248*Sin[c + 2*d*x] - 120*Sin[3*c + 2*d*x] + 248*Sin[2*c + 3*d*x]
 + 120*Sin[4*c + 3*d*x] + 184*Sin[3*c + 4*d*x]))/(1920*a*d*(1 + Sec[c + d*x
]))
```

fricas [A] time = 0.46, size = 134, normalized size = 1.52

$$\frac{23\cos(dx+c)^4+8\cos(dx+c)^3-27\cos(dx+c)^2+15\left(dx\cos(dx+c)^3+dx\cos(dx+c)^2-dx\cos(dx+}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 27*cos(d*x + c)^2 + 15*(d*x*co
s(d*x + c)^3 + d*x*cos(d*x + c)^2 - d*x*cos(d*x + c) - d*x)*sin(d*x + c) -
7*cos(d*x + c) + 8)/((a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x
 + c) - a*d)*sin(d*x + c))
```

giac [A] time = 0.67, size = 98, normalized size = 1.11

$$\frac{\frac{240\,(dx+c)}{a}+\frac{5\left(18\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{3\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(240*(d*x + c)/a + 5*(18*tan(1/2*d*x + 1/2*c)^2 - 1)/(a*tan(1/2*d*x +
 1/2*c)^3) - 3*(a^4*tan(1/2*d*x + 1/2*c)^5 - 10*a^4*tan(1/2*d*x + 1/2*c)^3
+ 80*a^4*tan(1/2*d*x + 1/2*c))/a^5)/d
```

maple [A] time = 0.66, size = 113, normalized size = 1.28

$$-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{80ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{48ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/80/a/d*tan(1/2*d*x+1/2*c)^5+1/8/a/d*tan(1/2*d*x+1/2*c)^3-1/a/d*tan(1/2*d
*x+1/2*c)-1/48/a/d/tan(1/2*d*x+1/2*c)^3+3/8/a/d/tan(1/2*d*x+1/2*c)+2/a/d*ar
ctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.97, size = 137, normalized size = 1.56

$$-\frac{\frac{3\left(\frac{80\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a}-\frac{480\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{5\left(\frac{18\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(3*(80*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 480*arctan(sin(d*x +
 c)/(cos(d*x + c) + 1))/a - 5*(18*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*
(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

mupad [B] time = 1.43, size = 158, normalized size = 1.80

$$-\frac{5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-30\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+240\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-90\cos}{240\,a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^8 + 3*sin(c/2 + (d*x)/2)^8 - 30*cos(c/2 + (d*x)/2)^2
*sin(c/2 + (d*x)/2)^6 + 240*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 - 90*
cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 - 240*cos(c/2 + (d*x)/2)^5*sin(c/
2 + (d*x)/2)^3*(c + d*x))/(240*a*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^
3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.70 $\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}-\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}+\frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}-\frac{\cot(c+dx}{}$$

```
[Out] -x/a+1/105*cot(d*x+c)^3*(35-24*sec(d*x+c))/a/d-1/35*cot(d*x+c)*(35-16*sec(d
*x+c))/a/d-1/35*cot(d*x+c)^5*(7-6*sec(d*x+c))/a/d+1/7*cot(d*x+c)^7*(1-sec(d
*x+c))/a/d
```

Rubi [A] time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3888, 3882, 8}

$$\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}-\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}+\frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}-\frac{\cot(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (Cot[c + d*x]^3*(35 - 24*Sec[c + d*x]))/(105*a*d) - (Cot[c + d*x]*
(35 - 16*Sec[c + d*x]))/(35*a*d) - (Cot[c + d*x]^5*(7 - 6*Sec[c + d*x]))/(3
5*a*d) + (Cot[c + d*x]^7*(1 - Sec[c + d*x]))/(7*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))\,dx}{a^2}$$
$$= \frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad} + \frac{\int \cot^6(c+dx)(7a-6a\sec(c+dx))\,dx}{7a^2}$$
$$= -\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad} + \frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad} + \frac{\int \cot^4(c+dx)(-35$$
$$= \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad} - \frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad} + \frac{\cot^7(c+dx)(1-}{7ad}$$
$$= \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad} - \frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad} - \frac{\cot^5(c+dx)(7}{35}$$
$$= -\frac{x}{a} + \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad} - \frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad} - \cot^5(c+$$

Mathematica [B] time = 1.10, size = 359, normalized size = 3.07

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^5(c+dx)\sec(c+dx)(-22860\sin(c+dx)-5715\sin(2(c+dx))+11430\sin(3(c+dx))+4572}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^5*Sec[c/2]*Sec[c + d*x]*(-16800*d*x*Cos[d*x] + 16800
*d*x*Cos[2*c + d*x] - 4200*d*x*Cos[c + 2*d*x] + 4200*d*x*Cos[3*c + 2*d*x] +
 8400*d*x*Cos[2*c + 3*d*x] - 8400*d*x*Cos[4*c + 3*d*x] + 3360*d*x*Cos[3*c +
 4*d*x] - 3360*d*x*Cos[5*c + 4*d*x] - 1680*d*x*Cos[4*c + 5*d*x] + 1680*d*x*
Cos[6*c + 5*d*x] - 840*d*x*Cos[5*c + 6*d*x] + 840*d*x*Cos[7*c + 6*d*x] + 31
36*Sin[c] + 30112*Sin[d*x] - 22860*Sin[c + d*x] - 5715*Sin[2*(c + d*x)] + 1
1430*Sin[3*(c + d*x)] + 4572*Sin[4*(c + d*x)] - 2286*Sin[5*(c + d*x)] - 114
3*Sin[6*(c + d*x)] + 26208*Sin[2*c + d*x] + 14080*Sin[c + 2*d*x] - 16400*Si
n[2*c + 3*d*x] - 11760*Sin[4*c + 3*d*x] - 7904*Sin[3*c + 4*d*x] - 3360*Sin[
5*c + 4*d*x] + 3952*Sin[4*c + 5*d*x] + 1680*Sin[6*c + 5*d*x] + 2816*Sin[5*c
 + 6*d*x]))/(107520*a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 0.47, size = 198, normalized size = 1.69

$$-\frac{176\cos(dx+c)^6 + 71\cos(dx+c)^5 - 335\cos(dx+c)^4 - 125\cos(dx+c)^3 + 225\cos(dx+c)^2 + 105\left(dx\cos\right.}{105\left(ad\cos(dx+c)^5 + ad\cos(dx+c)^4 - 2\,ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/105*(176*cos(d*x + c)^6 + 71*cos(d*x + c)^5 - 335*cos(d*x + c)^4 - 125*c
os(d*x + c)^3 + 225*cos(d*x + c)^2 + 105*(d*x*cos(d*x + c)^5 + d*x*cos(d*x
+ c)^4 - 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c)^2 + d*x*cos(d*x + c) + d
*x)*sin(d*x + c) + 57*cos(d*x + c) - 48)/((a*d*cos(d*x + c)^5 + a*d*cos(d*x
 + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) +
a*d)*sin(d*x + c))
```

giac [A] time = 0.39, size = 127, normalized size = 1.09

$$-\frac{\frac{6720\,(dx+c)}{a} + \frac{7\left(435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 3\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5} + \frac{15a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 168a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 1015a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 6720a^6}{a^7}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(6720*(d*x + c)/a + 7*(435*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x
+ 1/2*c)^2 + 3)/(a*tan(1/2*d*x + 1/2*c)^5) + (15*a^6*tan(1/2*d*x + 1/2*c)^7
 - 168*a^6*tan(1/2*d*x + 1/2*c)^5 + 1015*a^6*tan(1/2*d*x + 1/2*c)^3 - 6720*
a^6*tan(1/2*d*x + 1/2*c))/a^7)/d
```

maple [A] time = 0.77, size = 150, normalized size = 1.28

$$-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{448ad}+\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{40ad}-\frac{29\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{320ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}+\frac{1}{24ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/448/a/d*tan(1/2*d*x+1/2*c)^7+1/40/a/d*tan(1/2*d*x+1/2*c)^5-29/192/a/d*ta
n(1/2*d*x+1/2*c)^3+1/a/d*tan(1/2*d*x+1/2*c)-1/320/a/d/tan(1/2*d*x+1/2*c)^5+
1/24/a/d/tan(1/2*d*x+1/2*c)^3-29/64/a/d/tan(1/2*d*x+1/2*c)-2/a/d*arctan(tan
(1/2*d*x+1/2*c))
```

maxima [A] time = 0.59, size = 177, normalized size = 1.51

$$\frac{\frac{\frac{6720\sin(dx+c)}{\cos(dx+c)+1}-\frac{1015\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{168\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a}-\frac{13440\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{7\left(\frac{40\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{435\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-3\right)(\cos(dx+c)+1)^{5}}{a\sin(dx+c)^{5}}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6720*((6720*sin(d*x + c)/(cos(d*x + c) + 1) - 1015*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 168*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a - 13440*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 7*(40*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 435*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a*sin(d*x + c)^5))/d
```

mupad [B] time = 2.04, size = 206, normalized size = 1.76

$$-\frac{21\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-168\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+1015\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

```
[Out] -(21*cos(c/2 + (d*x)/2)^12 + 15*sin(c/2 + (d*x)/2)^12 - 168*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^10 + 1015*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^
8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 3045*cos(c/2 + (d*x)/2
)^8*sin(c/2 + (d*x)/2)^4 - 280*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2 +
 6720*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5*(c + d*x))/(6720*a*d*cos(c/
2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^{6}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

3.71 $\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{\sec^6(c+dx)}{6a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{\sec^4(c+dx)}{4a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec^2(c+dx)}{2a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-2*sec(d*x+c)/a^2/d-1/2*sec(d*x+c)^2/a^2/d+4/3*sec(d*x
+c)^3/a^2/d-1/4*sec(d*x+c)^4/a^2/d-2/5*sec(d*x+c)^5/a^2/d+1/6*sec(d*x+c)^6/
a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{\sec^6(c+dx)}{6a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{\sec^4(c+dx)}{4a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec^2(c+dx)}{2a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) - Sec[c + d*x]^2/(2
*a^2*d) + (4*Sec[c + d*x]^3)/(3*a^2*d) - Sec[c + d*x]^4/(4*a^2*d) - (2*Sec[
c + d*x]^5)/(5*a^2*d) + Sec[c + d*x]^6/(6*a^2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^2}{x^7}\,dx,x,\cos(c+dx)\right)}{a^8d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^6}{x^7}-\frac{2a^6}{x^6}-\frac{a^6}{x^5}+\frac{4a^6}{x^4}-\frac{a^6}{x^3}-\frac{2a^6}{x^2}+\frac{a^6}{x}\right)dx,x,\cos(c+dx)\right)}{a^8d}\\ &= -\frac{\log(\cos(c+dx))}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\sec^2(c+dx)}{2a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec^4(c+d}{4a^2d}\end{aligned}$$

Mathematica [A] time = 0.53, size = 125, normalized size = 1.04

$$-\frac{\sec^6(c+dx)(312\cos(c+dx)+5(28\cos(3(c+dx))+6\cos(4(c+dx))+12\cos(5(c+dx))+18\cos(4(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/480*((312*Cos[c + d*x] + 5*(14 + 28*Cos[3*(c + d*x)] + 6*Cos[4*(c + d*x)
] + 12*Cos[5*(c + d*x)] + 30*Log[Cos[c + d*x]] + 18*Cos[4*(c + d*x)]*Log[Co
s[c + d*x]] + 3*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 9*Cos[2*(c + d*x)]*(4
+ 5*Log[Cos[c + d*x]])))*Sec[c + d*x]^6)/(a^2*d)
```

fricas [A] time = 0.50, size = 85, normalized size = 0.71

$$-\frac{60\cos(dx+c)^6\log(-\cos(dx+c))+120\cos(dx+c)^5+30\cos(dx+c)^4-80\cos(dx+c)^3+15\cos(dx+c}{60\,a^2d\cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^6*log(-cos(d*x + c)) + 120*cos(d*x + c)^5 + 30*cos(d
*x + c)^4 - 80*cos(d*x + c)^3 + 15*cos(d*x + c)^2 + 24*cos(d*x + c) - 10)/(
a^2*d*cos(d*x + c)^6)
```

giac [B] time = 26.20, size = 223, normalized size = 1.86

$$\frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2}+\frac{\frac{234(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1005(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2220(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2925(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{1002(\cos(dx+c)-}{(\cos(dx+c)+1)}}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 60*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 + (234*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 1005*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2
220*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 + 1002*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 147*
(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 19)/(a^2*((cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 1)^6))/d
```

maple [A] time = 0.63, size = 110, normalized size = 0.92

$$\frac{\sec^6(dx+c)}{6a^2d}-\frac{2\left(\sec^5(dx+c)\right)}{5a^2d}-\frac{\sec^4(dx+c)}{4a^2d}+\frac{4\left(\sec^3(dx+c)\right)}{3a^2d}-\frac{\sec^2(dx+c)}{2a^2d}-\frac{2\sec(dx+c)}{a^2d}+\frac{\ln(\sec(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*sec(d*x+c)^6/a^2/d-2/5*sec(d*x+c)^5/a^2/d-1/4*sec(d*x+c)^4/a^2/d+4/3*se
c(d*x+c)^3/a^2/d-1/2*sec(d*x+c)^2/a^2/d-2*sec(d*x+c)/a^2/d+1/a^2/d*ln(sec(d
*x+c))
```

maxima [A] time = 0.33, size = 80, normalized size = 0.67

$$-\frac{\frac{60\log(\cos(dx+c))}{a^2}+\frac{120\cos(dx+c)^5+30\cos(dx+c)^4-80\cos(dx+c)^3+15\cos(dx+c)^2+24\cos(dx+c)-10}{a^2\cos(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(cos(d*x + c))/a^2 + (120*cos(d*x + c)^5 + 30*cos(d*x + c)^4 -
 80*cos(d*x + c)^3 + 15*cos(d*x + c)^2 + 24*cos(d*x + c) - 10)/(a^2*cos(d*x
 + c)^6))/d
```

mupad [B] time = 5.18, size = 193, normalized size = 1.61

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^2\,d}+\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-20\,\mathrm{t}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,a^2\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^2*d) + ((54*tan(c/2 + (d*x)/2)^2)/5 - 20
*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 12*tan(c/2 + (d*x)/2)^8 -
 2*tan(c/2 + (d*x)/2)^10 - 32/15)/(d*(15*a^2*tan(c/2 + (d*x)/2)^4 - 6*a^2*t
an(c/2 + (d*x)/2)^2 - 20*a^2*tan(c/2 + (d*x)/2)^6 + 15*a^2*tan(c/2 + (d*x)/
2)^8 - 6*a^2*tan(c/2 + (d*x)/2)^10 + a^2*tan(c/2 + (d*x)/2)^12 + a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{9}{\left(c+dx \right)}}{\sec^{2}{\left(c+dx \right)}+2\sec{\left(c+dx \right)}+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**9/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.72 $\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sec^4(c+dx)}{4a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}+\frac{2\sec(c+dx)}{a^2d}+\frac{\log(\cos(c+dx))}{a^2d}$$

```
[Out] ln(cos(d*x+c))/a^2/d+2*sec(d*x+c)/a^2/d-2/3*sec(d*x+c)^3/a^2/d+1/4*sec(d*x+
c)^4/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 75}

$$\frac{\sec^4(c+dx)}{4a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}+\frac{2\sec(c+dx)}{a^2d}+\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + (2*Sec[c + d*x])/(a^2*d) - (2*Sec[c + d*x]^3)/(
3*a^2*d) + Sec[c + d*x]^4/(4*a^2*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)}{x^5}\,dx, x, \cos(c+dx)\right)}{a^6d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^4}{x^5}-\frac{2a^4}{x^4}+\frac{2a^4}{x^2}-\frac{a^4}{x}\right)dx, x, \cos(c+dx)\right)}{a^6d}\\ &= \frac{\log(\cos(c+dx))}{a^2d}+\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}+\frac{\sec^4(c+dx)}{4a^2d}\end{aligned}$$

Mathematica [A] time = 0.20, size = 83, normalized size = 1.28

$$\frac{\sec^4(c+dx)(20\cos(c+dx)+3(4\cos(3(c+dx))+4\cos(2(c+dx))\log(\cos(c+dx))+\cos(4(c+dx))\log(\cos(c}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((20*Cos[c + d*x] + 3*(2 + 4*Cos[3*(c + d*x)] + 3*Log[Cos[c + d*x]] + 4*Cos
[2*(c + d*x)]*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c + d*x]]))*Sec[
c + d*x]^4)/(24*a^2*d)
```

fricas [A] time = 0.49, size = 55, normalized size = 0.85

$$\frac{12\cos\left(dx+c\right)^{4}\log\left(-\cos\left(dx+c\right)\right)+24\cos\left(dx+c\right)^{3}-8\cos\left(dx+c\right)+3}{12\,a^{2}d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(12*cos(d*x + c)^4*log(-cos(d*x + c)) + 24*cos(d*x + c)^3 - 8*cos(d*x
+ c) + 3)/(a^2*d*cos(d*x + c)^4)
```

giac [B] time = 8.97, size = 180, normalized size = 2.77

$$-\frac{\frac{12\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{2}}-\frac{12\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^{2}}-\frac{\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{54(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{124(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{25(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}+7}{a^{2}\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(12*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 12*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 - (4*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 54*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 124*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 25*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 + 7)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^4))/d
```

maple [A] time = 0.60, size = 63, normalized size = 0.97

$$\frac{\sec^{4}\left(dx+c\right)}{4a^{2}d}-\frac{2\left(\sec^{3}\left(dx+c\right)\right)}{3a^{2}d}+\frac{2\sec\left(dx+c\right)}{a^{2}d}-\frac{\ln\left(\sec\left(dx+c\right)\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/4*sec(d*x+c)^4/a^2/d-2/3*sec(d*x+c)^3/a^2/d+2*sec(d*x+c)/a^2/d-1/a^2/d*ln
(sec(d*x+c))
```

maxima [A] time = 0.44, size = 50, normalized size = 0.77

$$\frac{\frac{12\log(\cos(dx+c))}{a^{2}}+\frac{24\cos(dx+c)^{3}-8\cos(dx+c)+3}{a^{2}\cos(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*log(cos(d*x + c))/a^2 + (24*cos(d*x + c)^3 - 8*cos(d*x + c) + 3)/(
a^2*cos(d*x + c)^4))/d
```

mupad [B] time = 3.87, size = 135, normalized size = 2.08

$$\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-\frac{26\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{3}+\frac{8}{3}}{d\left(a^{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-4\,a^{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+6\,a^{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-4\,a^{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+a^{2}\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\right.\right.}{a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (8*tan(c/2 + (d*x)/2)^4 - (26*tan(c/2 + (d*x)/2)^2)/3 + 2*tan(c/2 + (d*x)/2
)^6 + 8/3)/(d*(6*a^2*tan(c/2 + (d*x)/2)^4 - 4*a^2*tan(c/2 + (d*x)/2)^2 - 4*
a^2*tan(c/2 + (d*x)/2)^6 + a^2*tan(c/2 + (d*x)/2)^8 + a^2)) - (2*atanh(tan(
c/2 + (d*x)/2)^2))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^7(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.73 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=48

$$\frac{\sec^2(c+dx)}{2a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-2*sec(d*x+c)/a^2/d+1/2*sec(d*x+c)^2/a^2/d
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$\frac{\sec^2(c+dx)}{2a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) + Sec[c + d*x]^2/(2
*a^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2}{x^3}\,dx,x,\cos(c+dx)\right)}{a^4d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}-\frac{2a^2}{x^2}+\frac{a^2}{x}\right)dx,x,\cos(c+dx)\right)}{a^4d}\\
&= -\frac{\log(\cos(c+dx))}{a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{\sec^2(c+dx)}{2a^2d}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 51, normalized size = 1.06

$$-\frac{\sec^2(c+dx)(4\cos(c+dx)+\cos(2(c+dx))\log(\cos(c+dx))+\log(\cos(c+dx))-1)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*((-1 + 4*Cos[c + d*x] + Log[Cos[c + d*x]] + Cos[2*(c + d*x)]*Log[Cos[c
 + d*x]])*Sec[c + d*x]^2)/(a^2*d)
```

fricas [A] time = 0.48, size = 45, normalized size = 0.94

$$-\frac{2\cos\left(dx+c\right)^{2}\log\left(-\cos\left(dx+c\right)\right)+4\cos\left(dx+c\right)-1}{2\,a^{2}d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(d*x + c)^2*log(-cos(d*x + c)) + 4*cos(d*x + c) - 1)/(a^2*d*cos(
d*x + c)^2)
```

giac [B] time = 8.11, size = 136, normalized size = 2.83

$$\frac{\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{2}}-\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^{2}}-\frac{\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+5}{a^{2}\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 2*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 - (6*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5)/(a^2*(
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2))/d
```

maple [A] time = 0.54, size = 46, normalized size = 0.96

$$\frac{\sec^{2}\left(dx+c\right)}{2a^{2}d}-\frac{2\sec\left(dx+c\right)}{a^{2}d}+\frac{\ln\left(\sec\left(dx+c\right)\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2*sec(d*x+c)^2/a^2/d-2*sec(d*x+c)/a^2/d+1/a^2/d*ln(sec(d*x+c))
```

maxima [A] time = 0.87, size = 40, normalized size = 0.83

$$-\frac{\frac{2\log(\cos(dx+c))}{a^{2}}+\frac{4\cos(dx+c)-1}{a^{2}\cos(dx+c)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(cos(d*x + c))/a^2 + (4*cos(d*x + c) - 1)/(a^2*cos(d*x + c)^2))/
d
```

mupad [B] time = 1.35, size = 77, normalized size = 1.60

$$\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-4}{d\,\left(a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-2\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+a^{2}\right)}+\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\right)}{a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (6*tan(c/2 + (d*x)/2)^2 - 4)/(d*(a^2*tan(c/2 + (d*x)/2)^4 - 2*a^2*tan(c/2 +
 (d*x)/2)^2 + a^2)) + (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.74 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=33

$$\frac{2\log(\cos(c+dx)+1)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d+2*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 72}

$$\frac{2\log(\cos(c+dx)+1)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + (2*Log[1 + Cos[c + d*x]])/(a^2*d)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{a-ax}{x(a+ax)}\,dx,x,\cos(c+dx)\right)}{a^2d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{1+x}\right)dx,x,\cos(c+dx)\right)}{a^2d}\\ &= -\frac{\log(\cos(c+dx))}{a^2d}+\frac{2\log(1+\cos(c+dx))}{a^2d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 30, normalized size = 0.91

$$\frac{4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Log[Cos[(c + d*x)/2]] - Log[Cos[c + d*x]])/(a^2*d)
```

fricas [A] time = 0.46, size = 31, normalized size = 0.94

$$-\frac{\log\left(-\cos\left(dx+c\right)\right)-2\,\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(log(-cos(d*x + c)) - 2*log(1/2*cos(d*x + c) + 1/2))/(a^2*d)
```

giac [A] time = 0.98, size = 33, normalized size = 1.00

$$-\frac{\log\left(\left|\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right|\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -log(abs((cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1))/(a^2*d)
```

maple [A] time = 0.52, size = 34, normalized size = 1.03

$$-\frac{\ln\left(\sec\left(dx+c\right)\right)}{a^{2}d}+\frac{2\ln\left(1+\sec\left(dx+c\right)\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
[Out] -1/a^2/d*ln(sec(d*x+c))+2/a^2/d*ln(1+sec(d*x+c))
```

maxima [A] time = 0.33, size = 31, normalized size = 0.94

$$\frac{\frac{2\,\log(\cos(dx+c)+1)}{a^2}-\frac{\log(\cos(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] (2*log(cos(d*x + c) + 1)/a^2 - log(cos(d*x + c))/a^2)/d
```

mupad [B] time = 1.39, size = 22, normalized size = 0.67

$$-\frac{\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-1\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
[Out] -log(tan(c/2 + (d*x)/2)^4 - 1)/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^{3}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.75 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=36

$$-\frac{1}{a^2d(\cos(c+dx)+1)}-\frac{\log(\cos(c+dx)+1)}{a^2d}$$

```
[Out] -1/a^2/d/(1+cos(d*x+c))-ln(1+cos(d*x+c))/a^2/d
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3879, 43}

$$-\frac{1}{a^2d(\cos(c+dx)+1)}-\frac{\log(\cos(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(1/(a^2*d*(1 + Cos[c + d*x]))) - Log[1 + Cos[c + d*x]]/(a^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{x}{(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2(1+x)^2}+\frac{1}{a^2(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{1}{a^2d(1+\cos(c+dx))}-\frac{\log(1+\cos(c+dx))}{a^2d}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 56, normalized size = 1.56

$$-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+1\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*((1 + 2*Log[Cos[(c + d*x)/2]] + 2*Cos[c + d*x]*Log[Cos[(c + d*x)/2]])*
Sec[(c + d*x)/2]^2)/(a^2*d)
```

fricas [A] time = 0.48, size = 43, normalized size = 1.19

$$-\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+1}{a^2d\cos(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 1)/(a^2*d*cos(d*x + c) +
 a^2*d)
```

giac [A] time = 0.47, size = 57, normalized size = 1.58

$$\frac{\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}+\frac{\cos(dx+c)-1}{a^2(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (cos(d*x
+ c) - 1)/(a^2*(cos(d*x + c) + 1)))/d
```

maple [A] time = 0.19, size = 50, normalized size = 1.39

$$\frac{\ln(\sec(dx+c))}{a^2d}+\frac{1}{a^2d(1+\sec(dx+c))}-\frac{\ln(1+\sec(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/a^2/d*ln(sec(d*x+c))+1/a^2/d/(1+sec(d*x+c))-1/a^2/d*ln(1+sec(d*x+c))
```

maxima [A] time = 0.41, size = 35, normalized size = 0.97

$$-\frac{\frac{1}{a^2\cos(dx+c)+a^2}+\frac{\log(\cos(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(1/(a^2*cos(d*x + c) + a^2) + log(cos(d*x + c) + 1)/a^2)/d
```

mupad [B] time = 1.13, size = 35, normalized size = 0.97

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1) - tan(c/2 + (d*x)/2)^2/2)/(a^2*d)
```

sympy [A] time = 19.47, size = 177, normalized size = 4.92

$$\begin{cases} \frac{\log\left(\tan^{2}(c+dx)+1\right)\sec(c+dx)}{2a^{2}d\sec(c+dx)+2a^{2}d}+\frac{\log\left(\tan^{2}(c+dx)+1\right)}{2a^{2}d\sec(c+dx)+2a^{2}d}-\frac{2\log(\sec(c+dx)+1)\sec(c+dx)}{2a^{2}d\sec(c+dx)+2a^{2}d}-\frac{2\log(\sec(c+dx)+1)}{2a^{2}d\sec(c+dx)+2a^{2}d}+\frac{2}{2a^{2}d\sec(c+dx)+2a^{2}d} \\ \frac{x\tan(c)}{(a\sec(c)+a)^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)*sec(c + d*x)/(2*a**2*d*sec(c + d*x) + 2
*a**2*d) + log(tan(c + d*x)**2 + 1)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) - 2*
log(sec(c + d*x) + 1)*sec(c + d*x)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) - 2*l
og(sec(c + d*x) + 1)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) + 2/(2*a**2*d*sec(c
 + d*x) + 2*a**2*d), Ne(d, 0)), (x*tan(c)/(a*sec(c) + a)**2, True))
```

3.76 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=81

$$\frac{5}{4a^2d(\cos(c+dx)+1)}-\frac{1}{4a^2d(\cos(c+dx)+1)^2}+\frac{\log(1-\cos(c+dx))}{8a^2d}+\frac{7\log(\cos(c+dx)+1)}{8a^2d}$$

```
[Out] -1/4/a^2/d/(1+cos(d*x+c))^2+5/4/a^2/d/(1+cos(d*x+c))+1/8*ln(1-cos(d*x+c))/a
^2/d+7/8*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{5}{4a^2d(\cos(c+dx)+1)}-\frac{1}{4a^2d(\cos(c+dx)+1)^2}+\frac{\log(1-\cos(c+dx))}{8a^2d}+\frac{7\log(\cos(c+dx)+1)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/(4*a^2*d*(1 + Cos[c + d*x])^2) + 5/(4*a^2*d*(1 + Cos[c + d*x])) + Log[1
- Cos[c + d*x]]/(8*a^2*d) + (7*Log[1 + Cos[c + d*x]])/(8*a^2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{a^2\operatorname{Subst}\left(\int \frac{x^3}{(a-ax)(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^2\operatorname{Subst}\left(\int \left(-\frac{1}{8a^4(-1+x)}-\frac{1}{2a^4(1+x)^3}+\frac{5}{4a^4(1+x)^2}-\frac{7}{8a^4(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{1}{4a^2d(1+\cos(c+dx))^2}+\frac{5}{4a^2d(1+\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{8a^2d}+7\log\end{aligned}$$

Mathematica [A] time = 0.19, size = 83, normalized size = 1.02

$$\frac{\sec^2(c+dx)\left(10\cos^2\left(\frac{1}{2}(c+dx)\right)+4\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+7\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)-1\right)}{4a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((-1 + 10*Cos[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^4*(7*Log[Cos[(c + d*x)/2]
] + Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^2)/(4*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.49, size = 106, normalized size = 1.31

$$\frac{7\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\,\cos\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(7*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) +
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 10*cos
(d*x + c) + 8)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.26, size = 117, normalized size = 1.44

$$\frac{\frac{2\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^{2}}-\frac{16\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{2}}-\frac{\frac{8\,a^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^{2}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}}{a^{4}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 - 16*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - (8*a^2*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/a^4)/
d
```

maple [A] time = 0.70, size = 72, normalized size = 0.89

$$\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{8a^{2}d}-\frac{1}{4a^{2}d\left(1+\cos\left(dx+c\right)\right)^{2}}+\frac{5}{4a^{2}d\left(1+\cos\left(dx+c\right)\right)}+\frac{7\ln\left(1+\cos\left(dx+c\right)\right)}{8a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/8/a^2/d*ln(-1+cos(d*x+c))-1/4/a^2/d/(1+cos(d*x+c))^2+5/4/a^2/d/(1+cos(d*x
+c))+7/8*ln(1+cos(d*x+c))/a^2/d
```

maxima [A] time = 0.32, size = 74, normalized size = 0.91

$$\frac{\frac{2\,(5\,\cos(dx+c)+4)}{a^{2}\cos(dx+c)^{2}+2\,a^{2}\cos(dx+c)+a^{2}}+\frac{7\,\log(\cos(dx+c)+1)}{a^{2}}+\frac{\log(\cos(dx+c)-1)}{a^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(5*cos(d*x + c) + 4)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
 + 7*log(cos(d*x + c) + 1)/a^2 + log(cos(d*x + c) - 1)/a^2)/d
```

mupad [B] time = 1.26, size = 62, normalized size = 0.77

$$\frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4}-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{2}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}}{16}}{a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))/4 - log(tan(c/2 + (d*x)/2)^2 + 1) + tan(c/2 + (d*x
)/2)^2/2 - tan(c/2 + (d*x)/2)^4/16)/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.77 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=123

$$-\frac{1}{16a^2d(1-\cos(c+dx))}-\frac{23}{16a^2d(\cos(c+dx)+1)}+\frac{1}{2a^2d(\cos(c+dx)+1)^2}-\frac{1}{12a^2d(\cos(c+dx)+1)^3}-\frac{3\log(1-c}{16}$$

```
[Out] -1/16/a^2/d/(1-cos(d*x+c))-1/12/a^2/d/(1+cos(d*x+c))^3+1/2/a^2/d/(1+cos(d*x
+c))^2-23/16/a^2/d/(1+cos(d*x+c))-3/16*ln(1-cos(d*x+c))/a^2/d-13/16*ln(1+co
s(d*x+c))/a^2/d
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$-\frac{1}{16a^2d(1-\cos(c+dx))}-\frac{23}{16a^2d(\cos(c+dx)+1)}+\frac{1}{2a^2d(\cos(c+dx)+1)^2}-\frac{1}{12a^2d(\cos(c+dx)+1)^3}-\frac{3\log(1-c}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/(16*a^2*d*(1 - Cos[c + d*x])) - 1/(12*a^2*d*(1 + Cos[c + d*x])^3) + 1/(2
*a^2*d*(1 + Cos[c + d*x])^2) - 23/(16*a^2*d*(1 + Cos[c + d*x])) - (3*Log[1
- Cos[c + d*x]])/(16*a^2*d) - (13*Log[1 + Cos[c + d*x]])/(16*a^2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{a^4\,\text{Subst}\left(\int \frac{x^5}{(a-ax)^2(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{16a^6(-1+x)^2}+\frac{3}{16a^6(-1+x)}-\frac{1}{4a^6(1+x)^4}+\frac{1}{a^6(1+x)^3}-\frac{23}{16a^6(1+x)^2}+\frac{13}{16a^6(1+x)}\right) d}{d}\\ &= -\frac{1}{16a^2d(1-\cos(c+dx))}-\frac{1}{12a^2d(1+\cos(c+dx))^3}+\frac{1}{2a^2d(1+\cos(c+dx))^2}-\frac{}{16a}\end{aligned}$$

Mathematica [A] time = 0.39, size = 121, normalized size = 0.98

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(3\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^6\left(\frac{1}{2}(c+dx)\right)-12\sec^4\left(\frac{1}{2}(c+dx)\right)+69\sec^2\left(\frac{1}{2}(c+dx)\right)\right.}{24a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/24*(Cos[(c + d*x)/2]^4*(3*Csc[(c + d*x)/2]^2 + 156*Log[Cos[(c + d*x)/2]]
 + 36*Log[Sin[(c + d*x)/2]] + 69*Sec[(c + d*x)/2]^2 - 12*Sec[(c + d*x)/2]^4
 + Sec[(c + d*x)/2]^6)*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.54, size = 162, normalized size = 1.32

$$-\frac{66\,\cos\left(dx+c\right)^{3}+36\,\cos\left(dx+c\right)^{2}+39\left(\cos\left(dx+c\right)^{4}+2\,\cos\left(dx+c\right)^{3}-2\,\cos\left(dx+c\right)-1\right)\log\left(\frac{1}{2}\,\cos\left(dx\right.\right.}{48\left(a^{2}d\cos\left(dx+c\right)^{4}+2\,a^{2}d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(66*cos(d*x + c)^3 + 36*cos(d*x + c)^2 + 39*(cos(d*x + c)^4 + 2*cos(d
*x + c)^3 - 2*cos(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 9*(cos(d*x +
c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2)
- 74*cos(d*x + c) - 52)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*
a^2*d*cos(d*x + c) - a^2*d)
```

giac [A] time = 0.35, size = 186, normalized size = 1.51

$$\frac{\frac{3\left(\frac{6\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^{2}(\cos(dx+c)-1)}-\frac{18\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^{2}}+\frac{96\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{2}}+\frac{\frac{48\,a^{4}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,a^{4}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{4}(\cos(dx+c)-}{(\cos(dx+c)+1)}}{a^{6}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
^2*(cos(d*x + c) - 1)) - 18*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1
))/a^2 + 96*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (48*
a^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^4*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^6)/d
```

maple [A] time = 0.94, size = 108, normalized size = 0.88

$$\frac{1}{16a^{2}d\left(-1+\cos\left(dx+c\right)\right)}-\frac{3\ln\left(-1+\cos\left(dx+c\right)\right)}{16a^{2}d}-\frac{1}{12a^{2}d\left(1+\cos\left(dx+c\right)\right)^{3}}+\frac{1}{2a^{2}d\left(1+\cos\left(dx+c\right)\right)^{2}}-\frac{}{16a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/16/a^2/d/(-1+cos(d*x+c))-3/16/a^2/d*ln(-1+cos(d*x+c))-1/12/a^2/d/(1+cos(d
*x+c))^3+1/2/a^2/d/(1+cos(d*x+c))^2-23/16/a^2/d/(1+cos(d*x+c))-13/16*ln(1+c
os(d*x+c))/a^2/d
```

maxima [A] time = 0.33, size = 110, normalized size = 0.89

$$-\frac{\frac{2\left(33\,\cos(dx+c)^{3}+18\,\cos(dx+c)^{2}-37\,\cos(dx+c)-26\right)}{a^{2}\cos(dx+c)^{4}+2\,a^{2}\cos(dx+c)^{3}-2\,a^{2}\cos(dx+c)-a^{2}}+\frac{39\,\log(\cos(dx+c)+1)}{a^{2}}+\frac{9\,\log(\cos(dx+c)-1)}{a^{2}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(33*cos(d*x + c)^3 + 18*cos(d*x + c)^2 - 37*cos(d*x + c) - 26)/(a^
2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c) - a^2) + 39*lo
g(cos(d*x + c) + 1)/a^2 + 9*log(cos(d*x + c) - 1)/a^2)/d
```

mupad [B] time = 1.37, size = 89, normalized size = 0.72

$$-\frac{\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{32}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{96}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -((3*log(tan(c/2 + (d*x)/2)))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2 +
 (d*x)/2)^2/32 + tan(c/2 + (d*x)/2)^2/2 - (3*tan(c/2 + (d*x)/2)^4)/32 + tan
(c/2 + (d*x)/2)^6/96)/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.78 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=165

$$\frac{9}{64a^2d(1-\cos(c+dx))}+\frac{51}{32a^2d(\cos(c+dx)+1)}-\frac{1}{64a^2d(1-\cos(c+dx))^2}-\frac{3}{4a^2d(\cos(c+dx)+1)^2}+\frac{}{48a^2d(\text{cc}}$$

```
[Out] -1/64/a^2/d/(1-cos(d*x+c))^2+9/64/a^2/d/(1-cos(d*x+c))-1/32/a^2/d/(1+cos(d*
x+c))^4+11/48/a^2/d/(1+cos(d*x+c))^3-3/4/a^2/d/(1+cos(d*x+c))^2+51/32/a^2/d
/(1+cos(d*x+c))+29/128*ln(1-cos(d*x+c))/a^2/d+99/128*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A] time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{9}{64a^2d(1-\cos(c+dx))}+\frac{51}{32a^2d(\cos(c+dx)+1)}-\frac{1}{64a^2d(1-\cos(c+dx))^2}-\frac{3}{4a^2d(\cos(c+dx)+1)^2}+\frac{}{48a^2d(\text{cc}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/(64*a^2*d*(1 - Cos[c + d*x])^2) + 9/(64*a^2*d*(1 - Cos[c + d*x])) - 1/(3
2*a^2*d*(1 + Cos[c + d*x])^4) + 11/(48*a^2*d*(1 + Cos[c + d*x])^3) - 3/(4*a
^2*d*(1 + Cos[c + d*x])^2) + 51/(32*a^2*d*(1 + Cos[c + d*x])) + (29*Log[1 -
 Cos[c + d*x]])/(128*a^2*d) + (99*Log[1 + Cos[c + d*x]])/(128*a^2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{a^6\,\text{Subst}\left(\int \frac{x^7}{(a-ax)^3(a+ax)^5}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{32a^8(-1+x)^3}-\frac{9}{64a^8(-1+x)^2}-\frac{29}{128a^8(-1+x)}-\frac{1}{8a^8(1+x)^5}+\frac{11}{16a^8(1+x)^4}-\frac{}{2a^8(}\right.\right.}{d}\\ &= -\frac{1}{64a^2d(1-\cos(c+dx))^2}+\frac{9}{64a^2d(1-\cos(c+dx))}-\frac{1}{32a^2d(1+\cos(c+dx))^4}+\end{aligned}$$

Mathematica [A] time = 0.82, size = 154, normalized size = 0.93

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(6\csc^4\left(\frac{1}{2}(c+dx)\right)-108\csc^2\left(\frac{1}{2}(c+dx)\right)+3\sec^8\left(\frac{1}{2}(c+dx)\right)-44\sec^6\left(\frac{1}{2}(c+\right.\right.}{384a^2d(\text{se}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(Cos[(c + d*x)/2]^4*(-108*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4
- 24*(99*Log[Cos[(c + d*x)/2]] + 29*Log[Sin[(c + d*x)/2]]) - 1224*Sec[(c +
d*x)/2]^2 + 288*Sec[(c + d*x)/2]^4 - 44*Sec[(c + d*x)/2]^6 + 3*Sec[(c + d*x
)/2]^8)*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.60, size = 283, normalized size = 1.72

$$\frac{558 \cos(dx+c)^5 + 156 \cos(dx+c)^4 - 1268 \cos(dx+c)^3 - 676 \cos(dx+c)^2 + 297 \left(\cos(dx+c)^6 + 2 \cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(558*cos(d*x + c)^5 + 156*cos(d*x + c)^4 - 1268*cos(d*x + c)^3 - 676*
cos(d*x + c)^2 + 297*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 -
4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c
) + 1/2) + 87*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d
*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/
2) + 686*cos(d*x + c) + 448)/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5
 - a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2
*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.41, size = 236, normalized size = 1.43

$$-\frac{\frac{6\left(\frac{16(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{87(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a^2(\cos(dx+c)-1)^2} - \frac{348\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{1536\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} + \frac{\frac{768a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{174a^6(\cos}{(\cos(dx}}{}}{1536d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/1536*(6*(16*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 87*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a^2*(cos(d*x + c) - 1)^
2) - 348*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + 1536*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (768*a^6*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 174*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 32*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^6*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4)/a^8)/d
```

maple [A] time = 1.02, size = 144, normalized size = 0.87

$$-\frac{1}{64a^2d(-1+\cos(dx+c))^2}-\frac{9}{64a^2d(-1+\cos(dx+c))}+\frac{29\ln(-1+\cos(dx+c))}{128a^2d}-\frac{1}{32a^2d(1+\cos(dx+c))^4}+\frac{}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/64/a^2/d/(-1+cos(d*x+c))^2-9/64/a^2/d/(-1+cos(d*x+c))+29/128/a^2/d*ln(-1
+cos(d*x+c))-1/32/a^2/d/(1+cos(d*x+c))^4+11/48/a^2/d/(1+cos(d*x+c))^3-3/4/a
^2/d/(1+cos(d*x+c))^2+51/32/a^2/d/(1+cos(d*x+c))+99/128*ln(1+cos(d*x+c))/a^
2/d
```

maxima [A] time = 0.45, size = 167, normalized size = 1.01

$$\frac{\frac{2\left(279\cos(dx+c)^5+78\cos(dx+c)^4-634\cos(dx+c)^3-338\cos(dx+c)^2+343\cos(dx+c)+224\right)}{a^2\cos(dx+c)^6+2a^2\cos(dx+c)^5-a^2\cos(dx+c)^4-4a^2\cos(dx+c)^3-a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2}+\frac{297\log(\cos(dx+c)+1)}{a^2}+\frac{87\log(\cos(dx+c)-1}{a^2}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/384*(2*(279*cos(d*x + c)^5 + 78*cos(d*x + c)^4 - 634*cos(d*x + c)^3 - 338
*cos(d*x + c)^2 + 343*cos(d*x + c) + 224)/(a^2*cos(d*x + c)^6 + 2*a^2*cos(d
*x + c)^5 - a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2
+ 2*a^2*cos(d*x + c) + a^2) + 297*log(cos(d*x + c) + 1)/a^2 + 87*log(cos(d*
x + c) - 1)/a^2)/d
```

mupad [B] time = 1.24, size = 151, normalized size = 0.92

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,a^2\,d}-\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{48\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{512\,a^2\,d}+\frac{29\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\right.}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(2*a^2*d) - (29*tan(c/2 + (d*x)/2)^4)/(256*a^2*d) + ta
n(c/2 + (d*x)/2)^6/(48*a^2*d) - tan(c/2 + (d*x)/2)^8/(512*a^2*d) + (29*log(
tan(c/2 + (d*x)/2)))/(64*a^2*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) + (
cot(c/2 + (d*x)/2)^4*(4*tan(c/2 + (d*x)/2)^2 - 1/4))/(64*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.79 $\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{\tan^3(c+dx)\sec(c+dx)}{2a^2d}+\frac{3\tan(c+dx)\sec(c-}{4a^2d}$$

```
[Out] x/a^2-3/4*arctanh(sin(d*x+c))/a^2/d-tan(d*x+c)/a^2/d+3/4*sec(d*x+c)*tan(d*x
+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d-1/2*sec(d*x+c)*tan(d*x+c)^3/a^2/d+1/5*tan(
d*x+c)^5/a^2/d
```

Rubi [A] time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3888, 3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{\tan^3(c+dx)\sec(c+dx)}{2a^2d}+\frac{3\tan(c+dx)\sec(c-}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (3*ArcTanh[Sin[c + d*x]])/(4*a^2*d) - Tan[c + d*x]/(a^2*d) + (3*Sec
[c + d*x]*Tan[c + d*x])/(4*a^2*d) + Tan[c + d*x]^3/(3*a^2*d) - (Sec[c + d*x
]*Tan[c + d*x]^3)/(2*a^2*d) + Tan[c + d*x]^5/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int (-a+a\sec(c+dx))^2\tan^4(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\tan^4(c+dx) - 2a^2\sec(c+dx)\tan^4(c+dx) + a^2\sec^2(c+dx)\tan^4(c+dx)\right)}{a^4}$$
$$= \frac{\int \tan^4(c+dx)\,dx}{a^2} + \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a^2} - \frac{2\int \sec(c+dx)\tan^4(c+d}{a^2}$$
$$= \frac{\tan^3(c+dx)}{3a^2d} - \frac{\sec(c+dx)\tan^3(c+dx)}{2a^2d} - \frac{\int \tan^2(c+dx)\,dx}{a^2} + \frac{3\int \sec(c+dx)\,t}{2a^2}$$
$$= -\frac{\tan(c+dx)}{a^2d} + \frac{3\sec(c+dx)\tan(c+dx)}{4a^2d} + \frac{\tan^3(c+dx)}{3a^2d} - \frac{\sec(c+dx)\tan^3(c+}{2a^2d}$$
$$= \frac{x}{a^2} - \frac{3\tanh^{-1}(\sin(c+dx))}{4a^2d} - \frac{\tan(c+dx)}{a^2d} + \frac{3\sec(c+dx)\tan(c+dx)}{4a^2d} + \frac{\tan^3(c}{3a^2}$$

Mathematica [B] time = 5.91, size = 495, normalized size = 4.16

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-\frac{151\sin\left(\frac{c}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{151\sin\left(\frac{c}{2}\right)}{d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(240*x + (180*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/d - (180*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d - ((29
3*Cos[(d*x)/2] + 333*Cos[2*c + (3*d*x)/2] + 287*Cos[2*c + (5*d*x)/2] + 67*C
os[4*c + (7*d*x)/2] + 68*Cos[4*c + (9*d*x)/2])*Sec[c]*Sec[c + d*x]^5*Sin[(d
*x)/2])/(2*d) + (36*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^4) - (151*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^2) + (36*Sin[c/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^4) - (151*Sin[c/2])/(d*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Cos[c/2]*Sec[c]*Sec[c + d*x]
^4*(308*Sin[c/2] - 43*Sin[c/2 + d*x] - 43*Sin[(3*c)/2 + d*x] - 346*Sin[(3*c
)/2 + 2*d*x] + 346*Sin[(5*c)/2 + 2*d*x] + 149*Sin[(5*c)/2 + 3*d*x] + 149*Si
n[(7*c)/2 + 3*d*x]))/(4*d)))/(60*a^2*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.76, size = 117, normalized size = 0.98

$$\frac{120\,dx\cos(dx+c)^5 - 45\cos(dx+c)^5\log(\sin(dx+c)+1) + 45\cos(dx+c)^5\log(-\sin(dx+c)+1) - 2\,(6}{120\,a^2d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/120*(120*d*x*cos(d*x + c)^5 - 45*cos(d*x + c)^5*log(sin(d*x + c) + 1) + 4
5*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(68*cos(d*x + c)^4 - 75*cos(d*x
 + c)^3 + 4*cos(d*x + c)^2 + 30*cos(d*x + c) - 12)*sin(d*x + c))/(a^2*d*cos
(d*x + c)^5)
```

giac [A] time = 13.72, size = 136, normalized size = 1.14

$$\frac{\frac{60\,(dx+c)}{a^2} - \frac{45\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} + \frac{45\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} + \frac{2\left(105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 530\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 328\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 110 \cdots}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5 a^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)/a^2 - 45*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 45*log
(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^9 - 530*t
an(1/2*d*x + 1/2*c)^7 + 328*tan(1/2*d*x + 1/2*c)^5 - 110*tan(1/2*d*x + 1/2*
c)^3 + 15*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^2))/d
```

maple [B] time = 0.66, size = 269, normalized size = 2.26

$$-\frac{1}{5a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{19}{12a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{\cdots}{4a^2d \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/5/a^2/d/(tan(1/2*d*x+1/2*c)-1)^5-1/a^2/d/(tan(1/2*d*x+1/2*c)-1)^4-19/12/
a^2/d/(tan(1/2*d*x+1/2*c)-1)^3-1/8/a^2/d/(tan(1/2*d*x+1/2*c)-1)^2+7/4/a^2/d
/(tan(1/2*d*x+1/2*c)-1)+3/4/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)-1/5/a^2/d/(tan(1
/2*d*x+1/2*c)+1)^5+1/a^2/d/(tan(1/2*d*x+1/2*c)+1)^4-19/12/a^2/d/(tan(1/2*d*
x+1/2*c)+1)^3+1/8/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2+7/4/a^2/d/(tan(1/2*d*x+1/2
*c)+1)-3/4/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)+2/a^2/d*arctan(tan(1/2*d*x+1/2*c)
)
```

maxima [B] time = 0.61, size = 301, normalized size = 2.53

$$-\frac{\frac{2\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{110\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{328\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{530\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{105\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)}{a^2-\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{45\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}-\frac{45\log \cdots}{\cdots}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(2*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 110*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 328*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 530*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7 + 105*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^2 - 5*a^
2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 10*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 120*ar
```

```
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 45*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a^2 - 45*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

mupad [B] time = 2.26, size = 179, normalized size = 1.50

$$\frac{x}{a^2} - \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d} + \frac{\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2} - \frac{53\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3} + \frac{164\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15} - \frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} -}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 10\,a^2\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8/(a + a/cos(c + d*x))^2,x)
```

```
[Out] x/a^2 - (3*atanh(tan(c/2 + (d*x)/2)))/(2*a^2*d) + (tan(c/2 + (d*x)/2)/2 - (
11*tan(c/2 + (d*x)/2)^3)/3 + (164*tan(c/2 + (d*x)/2)^5)/15 - (53*tan(c/2 +
(d*x)/2)^7)/3 + (7*tan(c/2 + (d*x)/2)^9)/2)/(d*(5*a^2*tan(c/2 + (d*x)/2)^2
- 10*a^2*tan(c/2 + (d*x)/2)^4 + 10*a^2*tan(c/2 + (d*x)/2)^6 - 5*a^2*tan(c/2
 + (d*x)/2)^8 + a^2*tan(c/2 + (d*x)/2)^10 - a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^8(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.80 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=72

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\tan(c+dx)\sec(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[Out] -x/a^2+arctanh(sin(d*x+c))/a^2/d+tan(d*x+c)/a^2/d-sec(d*x+c)*tan(d*x+c)/a^2
/d+1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.15, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3888, 3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\tan(c+dx)\sec(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) + ArcTanh[Sin[c + d*x]]/(a^2*d) + Tan[c + d*x]/(a^2*d) - (Sec[c +
d*x]*Tan[c + d*x])/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int (-a+a\sec(c+dx))^2\tan^2(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\tan^2(c+dx) - 2a^2\sec(c+dx)\tan^2(c+dx) + a^2\sec^2(c+dx)\tan^2(c+dx)\right)}{a^4}$$
$$= \frac{\int \tan^2(c+dx)\,dx}{a^2} + \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2} - \frac{2\int \sec(c+dx)\tan^2(c+d}{a^2}$$
$$= \frac{\tan(c+dx)}{a^2d} - \frac{\sec(c+dx)\tan(c+dx)}{a^2d} - \frac{\int 1\,dx}{a^2} + \frac{\int \sec(c+dx)\,dx}{a^2} + \frac{\text{Subst}\left(\int x\right.}{}$$
$$= -\frac{x}{a^2} + \frac{\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{\tan(c+dx)}{a^2d} - \frac{\sec(c+dx)\tan(c+dx)}{a^2d} + \frac{\tan^3(c+}{3a^2d}$$

Mathematica [B] time = 6.31, size = 767, normalized size = 10.65

$$-\frac{4x\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)}{(a\sec(c+dx)+a)^2} + \frac{8\sin\left(\frac{dx}{2}\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)}{3d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(a\sec(c+dx)+a)^2\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} + \frac{}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*x*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2 - (4*Cos[
c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^
2)/(d*(a + a*Sec[c + d*x])^2) + (4*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^2)/(d*(a + a*Sec[c + d*x])^2) + (2*C
os[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])^
2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos
[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(-5*Cos[c/2] + 7*Sin[c/2]))/(3*d*(a + a*Se
c[c + d*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2
])^2) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Se
c[c + d*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2
])) + (2*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Sec[
c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])
^3) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(5*Cos[c/2] + 7*Sin[c/2]))/(3*d*
(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*
(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2]))
```

fricas [A] time = 0.76, size = 97, normalized size = 1.35

$$-\frac{6\,dx\cos\left(dx+c\right)^3 - 3\,\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right) + 3\,\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right) - 2\left(2\,\cos\right.}{6\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*d*x*cos(d*x + c)^3 - 3*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*cos
(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c)^2 - 3*cos(d*x + c) +
 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3)
```

giac [A] time = 4.13, size = 99, normalized size = 1.38

$$-\frac{\frac{3(dx+c)}{a^2}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}+\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{4\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 - 3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 3*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 4*(3*tan(1/2*d*x + 1/2*c)^5 - tan(1/2*d*
x + 1/2*c)^3)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2))/d
```

maple [B] time = 0.60, size = 185, normalized size = 2.57

$$-\frac{1}{3a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3}{2a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}-\frac{}{3a^2d\left(\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/3/a^2/d/(tan(1/2*d*x+1/2*c)-1)^3-3/2/a^2/d/(tan(1/2*d*x+1/2*c)-1)^2-2/a^
2/d/(tan(1/2*d*x+1/2*c)-1)-1/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)-1/3/a^2/d/(tan(
1/2*d*x+1/2*c)+1)^3+3/2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2-2/a^2/d/(tan(1/2*d*x
+1/2*c)+1)+1/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)-2/a^2/d*arctan(tan(1/2*d*x+1/2*
c))
```

maxima [B] time = 0.82, size = 196, normalized size = 2.72

$$-\frac{\frac{4\left(\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{6\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(4*(sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5)/(a^2 - 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 6
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 3*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

mupad [B] time = 1.39, size = 111, normalized size = 1.54

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{x}{a^2}+\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - x/a^2 + ((4*tan(c/2 + (d*x)/2)^3)/3
 - 4*tan(c/2 + (d*x)/2)^5)/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*tan(c/2 +
 (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 - a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.81 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=34

$$\frac{\tan(c+dx)}{a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{x}{a^2}$$

```
[Out] x/a^2-2*arctanh(sin(d*x+c))/a^2/d+tan(d*x+c)/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3888, 3773, 3770, 3767, 8}

$$\frac{\tan(c+dx)}{a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*ArcTanh[Sin[c + d*x]])/(a^2*d) + Tan[c + d*x]/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\int (-a+a\sec(c+dx))^2\,dx}{a^4}\\ &= \frac{x}{a^2}+\frac{\int \sec^2(c+dx)\,dx}{a^2}-\frac{2\int \sec(c+dx)\,dx}{a^2}\\ &= \frac{x}{a^2}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\text{Subst}(\int 1\,dx,x,-\tan(c+dx))}{a^2d}\\ &= \frac{x}{a^2}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{a^2d}\end{aligned}$$

Mathematica [B] time = 0.52, size = 177, normalized size = 5.21

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+2\right.}{a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c
/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.57, size = 66, normalized size = 1.94

$$\frac{dx\cos(dx+c)-\cos(dx+c)\log(\sin(dx+c)+1)+\cos(dx+c)\log(-\sin(dx+c)+1)+\sin(dx+c)}{a^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (d*x*cos(d*x + c) - cos(d*x + c)*log(sin(d*x + c) + 1) + cos(d*x + c)*log(-
sin(d*x + c) + 1) + sin(d*x + c))/(a^2*d*cos(d*x + c))
```

giac [B] time = 2.66, size = 79, normalized size = 2.32

$$\frac{\frac{dx+c}{a^2}-\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}+\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^2 - 2*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 2*log(abs(tan(1
/2*d*x + 1/2*c) - 1))/a^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*a^2))/d
```

maple [B] time = 0.40, size = 102, normalized size = 3.00

$$-\frac{1}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}-\frac{1}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}+\frac{2\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/a^2/d/(tan(1/2*d*x+1/2*c)-1)+2/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)-1/a^2/d/(t
an(1/2*d*x+1/2*c)+1)-2/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)+2/a^2/d*arctan(tan(1/
2*d*x+1/2*c))
```

maxima [B] time = 0.51, size = 123, normalized size = 3.62

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^2 + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + sin(d*x
 + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))
/d
```

mupad [B] time = 1.19, size = 61, normalized size = 1.79

$$\frac{x}{a^2}-\frac{4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

```
[Out] x/a^2 - (4*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (2*tan(c/2 + (d*x)/2))/(d*(
a^2*tan(c/2 + (d*x)/2)^2 - a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.82 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=33

$$\frac{2\tan(c+dx)}{ad(a\sec(c+dx)+a)}-\frac{x}{a^2}$$

```
[Out] -x/a^2+2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3888, 3886, 3473, 8, 2606, 3767}

$$-\frac{2\cot(c+dx)}{a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Cot[c + d*x])/(a^2*d) + (2*Csc[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \cot^2(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^2(c+dx) - 2a^2\cot(c+dx)\csc(c+dx) + a^2\csc^2(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \cot^2(c+dx)\,dx}{a^2} + \frac{\int \csc^2(c+dx)\,dx}{a^2} - \frac{2\int \cot(c+dx)\csc(c+dx)\,dx}{a^2}$$
$$= -\frac{\cot(c+dx)}{a^2d} - \frac{\int 1\,dx}{a^2} - \frac{\text{Subst}(\int 1\,dx, x, \cot(c+dx))}{a^2d} + \frac{2\,\text{Subst}(\int 1\,dx, x, \csc(c+a}{a^2d}$$
$$= -\frac{x}{a^2} - \frac{2\cot(c+dx)}{a^2d} + \frac{2\csc(c+dx)}{a^2d}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.27

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{2\tan^{-1}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
[Out] ((-2*ArcTan[Tan[c/2 + (d*x)/2]])/d + (2*Tan[c/2 + (d*x)/2])/d)/a^2
```

fricas [A] time = 0.77, size = 42, normalized size = 1.27

$$-\frac{dx\cos(dx+c) + dx - 2\sin(dx+c)}{a^2d\cos(dx+c) + a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(d*x*cos(d*x + c) + d*x - 2*sin(d*x + c))/(a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 2.82, size = 29, normalized size = 0.88

$$-\frac{\frac{dx+c}{a^2} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -((d*x + c)/a^2 - 2*tan(1/2*d*x + 1/2*c)/a^2)/d
```

maple [A] time = 0.48, size = 37, normalized size = 1.12

$$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^2d} - \frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
[Out] 2/a^2/d*tan(1/2*d*x+1/2*c)-2/a^2/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.92, size = 49, normalized size = 1.48

$$-\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d*
x + c) + 1)))/d
```

mupad [B] time = 1.10, size = 22, normalized size = 0.67

$$\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{d\,x}{2}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*(tan(c/2 + (d*x)/2) - (d*x)/2))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.83 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=107

$$-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[Out] -x/a^2-cot(d*x+c)/a^2/d+1/3*cot(d*x+c)^3/a^2/d-2/5*cot(d*x+c)^5/a^2/d+2*csc
(d*x+c)/a^2/d-4/3*csc(d*x+c)^3/a^2/d+2/5*csc(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.17, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - Cot[c + d*x]/(a^2*d) + Cot[c + d*x]^3/(3*a^2*d) - (2*Cot[c + d*x
]^5)/(5*a^2*d) + (2*Csc[c + d*x])/(a^2*d) - (4*Csc[c + d*x]^3)/(3*a^2*d) +
(2*Csc[c + d*x]^5)/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \cot^6(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^6(c+dx)-2a^2\cot^5(c+dx)\csc(c+dx)+a^2\cot^4(c+dx)\csc^2(c+dx)\right)\,}{a^4}$$
$$= \frac{\int \cot^6(c+dx)\,dx}{a^2}+\frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot^5(c+dx)\csc(c+dx}{a^2}$$
$$= -\frac{\cot^5(c+dx)}{5a^2d}-\frac{\int \cot^4(c+dx)\,dx}{a^2}+\frac{\mathrm{Subst}\left(\int x^4\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{2\,\mathrm{Subst}}{}$$
$$= \frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\int \cot^2(c+dx)\,dx}{a^2}+\frac{2\,\mathrm{Subst}\left(\int\left(1-2x^2+x^4\right)\,d\right.}{a^2d}$$
$$= -\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+$$
$$= -\frac{x}{a^2}-\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{4\csc^3(c+}{3a^2d}$$

Mathematica [A] time = 1.35, size = 149, normalized size = 1.39

$$\frac{\sec^2(c+dx)\left(-120dx\cos^4\left(\frac{1}{2}(c+dx)\right)+3\tan\left(\frac{1}{2}(c+dx)\right)-31\tan\left(\frac{c}{2}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)-31\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\right.}{30a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(-120*d*x*Cos[(c + d*x)/2]^4 - 31*Cos[(c + d*x)/2]*Sec[c/2]
*Sin[(d*x)/2] + Cos[(c + d*x)/2]^3*(15*Cot[(c + d*x)/2]*Csc[c/2] + 193*Sec[
c/2])*Sin[(d*x)/2] - 31*Cos[(c + d*x)/2]^2*Tan[c/2] + 3*Tan[(c + d*x)/2]))/
(30*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.67, size = 106, normalized size = 0.99

$$-\frac{26\cos(dx+c)^3+22\cos(dx+c)^2+15\left(dx\cos(dx+c)^2+2\,dx\cos(dx+c)+dx\right)\sin(dx+c)-17\cos(dx}{15\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(26*cos(d*x + c)^3 + 22*cos(d*x + c)^2 + 15*(d*x*cos(d*x + c)^2 + 2*d
*x*cos(d*x + c) + d*x)*sin(d*x + c) - 17*cos(d*x + c) - 16)/((a^2*d*cos(d*x
 + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

giac [A] time = 0.36, size = 84, normalized size = 0.79

$$-\frac{\frac{120\,(dx+c)}{a^2}+\frac{15}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{3\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-25\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+165\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/120*(120*(d*x + c)/a^2 + 15/(a^2*tan(1/2*d*x + 1/2*c)) - (3*a^8*tan(1/2*
d*x + 1/2*c)^5 - 25*a^8*tan(1/2*d*x + 1/2*c)^3 + 165*a^8*tan(1/2*d*x + 1/2*
c))/a^10)/d
```

maple [A] time = 0.70, size = 94, normalized size = 0.88

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{40a^2d}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24a^2d}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d}-\frac{1}{8a^2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/40/a^2/d*tan(1/2*d*x+1/2*c)^5-5/24/a^2/d*tan(1/2*d*x+1/2*c)^3+11/8/a^2/d*
tan(1/2*d*x+1/2*c)-1/8/a^2/d/tan(1/2*d*x+1/2*c)-2/a^2/d*arctan(tan(1/2*d*x+
1/2*c))
```

maxima [A] time = 0.59, size = 113, normalized size = 1.06

$$\frac{\frac{\frac{165\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{240\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{15\,(\cos(dx+c)+1)}{a^2\sin(dx+c)}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 240*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2 - 15*(cos(d*x + c) + 1)/(a^2*sin(d*x + c)))/d
```

mupad [B] time = 1.40, size = 78, normalized size = 0.73

$$-\frac{x}{a^2}-\frac{\frac{26\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}-\frac{28\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}+\frac{17\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{60}-\frac{1}{40}}{a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

```
[Out] - x/a^2 - ((17*cos(c/2 + (d*x)/2)^2)/60 - (28*cos(c/2 + (d*x)/2)^4)/15 + (2
6*cos(c/2 + (d*x)/2)^6)/15 - 1/40)/(a^2*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d
*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.84 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=139

$$-\frac{2\cot^7(c+dx)}{7a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{6\csc^5(c+dx)}{5a^2d}+\frac{2\csc^3(c+dx)}{a^2d}-\frac{2\csc\text{(}}{a}$$

```
[Out] x/a^2+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+1/5*cot(d*x+c)^5/a^2/d-2/7*co
t(d*x+c)^7/a^2/d-2*csc(d*x+c)/a^2/d+2*csc(d*x+c)^3/a^2/d-6/5*csc(d*x+c)^5/a
^2/d+2/7*csc(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2\cot^7(c+dx)}{7a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{6\csc^5(c+dx)}{5a^2d}+\frac{2\csc^3(c+dx)}{a^2d}-\frac{2\csc\text{(}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 + Cot[c + d*x]/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d) + Cot[c + d*x]^5/(5
*a^2*d) - (2*Cot[c + d*x]^7)/(7*a^2*d) - (2*Csc[c + d*x])/(a^2*d) + (2*Csc[
c + d*x]^3)/(a^2*d) - (6*Csc[c + d*x]^5)/(5*a^2*d) + (2*Csc[c + d*x]^7)/(7*
a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^8(c+dx) - 2a^2\cot^7(c+dx)\csc(c+dx) + a^2\cot^6(c+dx)\csc^2(c+dx)\right)}{a^4}$$

$$= \frac{\int \cot^8(c+dx)\,dx}{a^2} + \frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a^2} - \frac{2\int \cot^7(c+dx)\csc(c+dx}{a^2}$$

$$= -\frac{\cot^7(c+dx)}{7a^2d} - \frac{\int \cot^6(c+dx)\,dx}{a^2} + \frac{\text{Subst}\left(\int x^6\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\,\text{Subst}}{}$$

$$= \frac{\cot^5(c+dx)}{5a^2d} - \frac{2\cot^7(c+dx)}{7a^2d} + \frac{\int \cot^4(c+dx)\,dx}{a^2} + \frac{2\,\text{Subst}\left(\int \left(-1+3x^2-3x^4\right.\right.}{a^2d}$$

$$= -\frac{\cot^3(c+dx)}{3a^2d} + \frac{\cot^5(c+dx)}{5a^2d} - \frac{2\cot^7(c+dx)}{7a^2d} - \frac{2\csc(c+dx)}{a^2d} + \frac{2\csc^3(c+dx)}{a^2d}$$

$$= \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{\cot^5(c+dx)}{5a^2d} - \frac{2\cot^7(c+dx)}{7a^2d} - \frac{2\csc(c+dx)}{a^2d} + \frac{2\,c}{}$$

$$= \frac{x}{a^2} + \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{\cot^5(c+dx)}{5a^2d} - \frac{2\cot^7(c+dx)}{7a^2d} - \frac{2\csc(c+dx)}{a^2d}$$

Mathematica [B] time = 1.07, size = 314, normalized size = 2.26

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^3(c+dx)\sec^2(c+dx)(16002\sin(c+dx)+9144\sin(2(c+dx))-3429\sin(3(c+dx))-457}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*Sec[c + d*x]^2*(5880*d*x*Cos[d*x] - 5880*
d*x*Cos[2*c + d*x] + 3360*d*x*Cos[c + 2*d*x] - 3360*d*x*Cos[3*c + 2*d*x] -
1260*d*x*Cos[2*c + 3*d*x] + 1260*d*x*Cos[4*c + 3*d*x] - 1680*d*x*Cos[3*c +
4*d*x] + 1680*d*x*Cos[5*c + 4*d*x] - 420*d*x*Cos[4*c + 5*d*x] + 420*d*x*Cos
[6*c + 5*d*x] - 4032*Sin[c] - 9632*Sin[d*x] + 16002*Sin[c + d*x] + 9144*Sin
[2*(c + d*x)] - 3429*Sin[3*(c + d*x)] - 4572*Sin[4*(c + d*x)] - 1143*Sin[5*
(c + d*x)] - 11760*Sin[2*c + d*x] - 8864*Sin[c + 2*d*x] - 3360*Sin[3*c + 2*
d*x] + 2064*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] + 4432*Sin[3*c + 4*d*x
] + 1680*Sin[5*c + 4*d*x] + 1528*Sin[4*c + 5*d*x]))/(26880*a^2*d*(1 + Sec[c
 + d*x])^2)
```

fricas [A] time = 0.77, size = 154, normalized size = 1.11

$$\frac{191\,\cos\left(dx+c\right)^{5}+172\,\cos\left(dx+c\right)^{4}-253\,\cos\left(dx+c\right)^{3}-258\,\cos\left(dx+c\right)^{2}+105\left(dx\cos\left(dx+c\right)^{4}+2\,dx\cos\right.}{105\left(a^{2}d\cos\left(dx+c\right)^{4}+2\,a^{2}d\cos\left(dx+c\right)^{3}-2\,a^{2}d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(191*cos(d*x + c)^5 + 172*cos(d*x + c)^4 - 253*cos(d*x + c)^3 - 258*c
os(d*x + c)^2 + 105*(d*x*cos(d*x + c)^4 + 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(
d*x + c) - d*x)*sin(d*x + c) + 87*cos(d*x + c) + 96)/((a^2*d*cos(d*x + c)^4
 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

giac [A] time = 0.31, size = 114, normalized size = 0.82

$$\frac{\frac{3360\,(dx+c)}{a^{2}}+\frac{35\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}+\frac{15\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-147\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+770\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4410\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{14}}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(3360*(d*x + c)/a^2 + 35*(21*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1/
2*d*x + 1/2*c)^3) + (15*a^12*tan(1/2*d*x + 1/2*c)^7 - 147*a^12*tan(1/2*d*x
+ 1/2*c)^5 + 770*a^12*tan(1/2*d*x + 1/2*c)^3 - 4410*a^12*tan(1/2*d*x + 1/2*
c))/a^14)/d
```

maple [A] time = 0.82, size = 132, normalized size = 0.95

$$\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{224a^{2}d}-\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160a^{2}d}+\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48a^{2}d}-\frac{21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^{2}d}-\frac{1}{96a^{2}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{7}{32a^{2}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/224/a^2/d*tan(1/2*d*x+1/2*c)^7-7/160/a^2/d*tan(1/2*d*x+1/2*c)^5+11/48/a^2
/d*tan(1/2*d*x+1/2*c)^3-21/16/a^2/d*tan(1/2*d*x+1/2*c)-1/96/a^2/d/tan(1/2*d
*x+1/2*c)^3+7/32/a^2/d/tan(1/2*d*x+1/2*c)+2/a^2/d*arctan(tan(1/2*d*x+1/2*c)
)
```

maxima [A] time = 0.60, size = 157, normalized size = 1.13

$$-\frac{\frac{\frac{4410\sin(dx+c)}{\cos(dx+c)+1}-\frac{770\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{147\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{2}}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}-\frac{35\left(\frac{21\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1\right)(\cos(dx+c)+1)^{3}}{a^{2}\sin(dx+c)^{3}}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*((4410*sin(d*x + c)/(cos(d*x + c) + 1) - 770*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^2 - 6720*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^2 - 35*(21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(
a^2*sin(d*x + c)^3))/d
```

mupad [B] time = 1.65, size = 182, normalized size = 1.31

$$\frac{15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-147\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+770\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-}{3360\,a^2\,d\cos\left(\frac{c}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (15*sin(c/2 + (d*x)/2)^10 - 35*cos(c/2 + (d*x)/2)^10 - 147*cos(c/2 + (d*x)/
2)^2*sin(c/2 + (d*x)/2)^8 + 770*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 -
 4410*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 735*cos(c/2 + (d*x)/2)^8*
sin(c/2 + (d*x)/2)^2 + 3360*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3*(c +
d*x))/(3360*a^2*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.85 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{12\csc^5}{5a}$$

```
[Out] -x/a^2-cot(d*x+c)/a^2/d+1/3*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+c)^5/a^2/d+1/7*c
ot(d*x+c)^7/a^2/d-2/9*cot(d*x+c)^9/a^2/d+2*csc(d*x+c)/a^2/d-8/3*csc(d*x+c)^
3/a^2/d+12/5*csc(d*x+c)^5/a^2/d-8/7*csc(d*x+c)^7/a^2/d+2/9*csc(d*x+c)^9/a^2
/d
```

Rubi [A] time = 0.21, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{12\csc^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - Cot[c + d*x]/(a^2*d) + Cot[c + d*x]^3/(3*a^2*d) - Cot[c + d*x]^5
/(5*a^2*d) + Cot[c + d*x]^7/(7*a^2*d) - (2*Cot[c + d*x]^9)/(9*a^2*d) + (2*C
sc[c + d*x])/(a^2*d) - (8*Csc[c + d*x]^3)/(3*a^2*d) + (12*Csc[c + d*x]^5)/(
5*a^2*d) - (8*Csc[c + d*x]^7)/(7*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \cot^{10}(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^{10}(c+dx) - 2a^2\cot^9(c+dx)\csc(c+dx) + a^2\cot^8(c+dx)\csc^2(c+dx)\right)}{a^4}$$

$$= \frac{\int \cot^{10}(c+dx)\,dx}{a^2} + \frac{\int \cot^8(c+dx)\csc^2(c+dx)\,dx}{a^2} - \frac{2\int \cot^9(c+dx)\csc(c+d}{a^2}$$

$$= -\frac{\cot^9(c+dx)}{9a^2d} - \frac{\int \cot^8(c+dx)\,dx}{a^2} + \frac{\mathrm{Subst}\left(\int x^8\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\,\mathrm{Subst}}{}$$

$$= \frac{\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+dx)}{9a^2d} + \frac{\int \cot^6(c+dx)\,dx}{a^2} + \frac{2\,\mathrm{Subst}\left(\int \left(1-4x^2+6x^4-\right.\right.}{a}$$

$$= -\frac{\cot^5(c+dx)}{5a^2d} + \frac{\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+dx)}{9a^2d} + \frac{2\csc(c+dx)}{a^2d} - \frac{8\csc^3(c+dx)}{3a^2d}$$

$$= \frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5a^2d} + \frac{\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+dx)}{9a^2d} + \frac{2\csc(c+dx)}{a^2d} - 8$$

$$= -\frac{\cot(c+dx)}{a^2d} + \frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5a^2d} + \frac{\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+dx)}{9a^2d} + 2$$

$$= -\frac{x}{a^2} - \frac{\cot(c+dx)}{a^2d} + \frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5a^2d} + \frac{\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+dx}{9a^2d}$$

Mathematica [B] time = 6.57, size = 802, normalized size = 4.48

$$\frac{\sec\left(\frac{c}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{288d(\sec(c+dx)a+a)^2} + \frac{\sec^2(c+dx)\tan\left(\frac{c}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{288d(\sec(c+dx)a+a)^2} - \frac{109\sec\left(\frac{c}{2}\right)\sec^2(c+dx)\sin\left(\right.}{2016d(\sec(c+dx)a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*x*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2 + (17*Cos
[c/2 + (d*x)/2]^2*Cot[c/2]*Cot[c/2 + (d*x)/2]^2*Sec[c + d*x]^2)/(160*d*(a +
 a*Sec[c + d*x])^2) - (Cot[c/2]*Cot[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(160*d
*(a + a*Sec[c + d*x])^2) + (201*Cos[c/2 + (d*x)/2]^3*Cot[c/2 + (d*x)/2]*Csc
[c/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/(160*d*(a + a*Sec[c + d*x])^2) - (17*Cos
[c/2 + (d*x)/2]*Cot[c/2 + (d*x)/2]^3*Csc[c/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/
(160*d*(a + a*Sec[c + d*x])^2) + (Cot[c/2 + (d*x)/2]^4*Csc[c/2]*Csc[c/2 + (
d*x)/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/(160*d*(a + a*Sec[c + d*x])^2) - (7891
```

```
*Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c + d*x]^2*Sin[(d*x)/2]])/(5040*d*(a + a*Se
c[c + d*x])^2) + (63881*Cos[c/2 + (d*x)/2]^3*Sec[c/2]*Sec[c + d*x]^2*Sin[(d
*x)/2]])/(10080*d*(a + a*Sec[c + d*x])^2) + (313*Sec[c/2]*Sec[c/2 + (d*x)/2]
*Sec[c + d*x]^2*Sin[(d*x)/2]])/(840*d*(a + a*Sec[c + d*x])^2) - (109*Sec[c/2
]*Sec[c/2 + (d*x)/2]^3*Sec[c + d*x]^2*Sin[(d*x)/2]])/(2016*d*(a + a*Sec[c +
d*x])^2) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^5*Sec[c + d*x]^2*Sin[(d*x)/2]])/(288
*d*(a + a*Sec[c + d*x])^2) + (313*Sec[c + d*x]^2*Tan[c/2]])/(840*d*(a + a*Se
c[c + d*x])^2) - (7891*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]^2*Tan[c/2]])/(5040*
d*(a + a*Sec[c + d*x])^2) - (109*Sec[c/2 + (d*x)/2]^2*Sec[c + d*x]^2*Tan[c/
2]])/(2016*d*(a + a*Sec[c + d*x])^2) + (Sec[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*
Tan[c/2]])/(288*d*(a + a*Sec[c + d*x])^2)
```

fricas [A] time = 0.70, size = 250, normalized size = 1.40

$$-\frac{598\cos(dx+c)^7+566\cos(dx+c)^6-1212\cos(dx+c)^5-1310\cos(dx+c)^4+860\cos(dx+c)^3+1014\cos}{315\left(a^2d\cos(dx+c)^6+2a^2d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(598*cos(d*x + c)^7 + 566*cos(d*x + c)^6 - 1212*cos(d*x + c)^5 - 131
0*cos(d*x + c)^4 + 860*cos(d*x + c)^3 + 1014*cos(d*x + c)^2 + 315*(d*x*cos(
d*x + c)^6 + 2*d*x*cos(d*x + c)^5 - d*x*cos(d*x + c)^4 - 4*d*x*cos(d*x + c)
^3 - d*x*cos(d*x + c)^2 + 2*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 197*cos(
d*x + c) - 256)/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos
(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)*sin(d*x + c))
```

giac [A] time = 0.45, size = 144, normalized size = 0.80

$$-\frac{\frac{40320\,(dx+c)}{a^2}+\frac{63\left(185\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}-\frac{35a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-405a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+2331a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-97}{a^{18}}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40320*(40320*(d*x + c)/a^2 + 63*(185*tan(1/2*d*x + 1/2*c)^4 - 15*tan(1/2
*d*x + 1/2*c)^2 + 1)/(a^2*tan(1/2*d*x + 1/2*c)^5) - (35*a^16*tan(1/2*d*x +
1/2*c)^9 - 405*a^16*tan(1/2*d*x + 1/2*c)^7 + 2331*a^16*tan(1/2*d*x + 1/2*c)
^5 - 9765*a^16*tan(1/2*d*x + 1/2*c)^3 + 51345*a^16*tan(1/2*d*x + 1/2*c))/a^
18)/d
```

maple [A] time = 0.85, size = 170, normalized size = 0.95

$$\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{1152a^2d}-\frac{9\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{896a^2d}+\frac{37\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640a^2d}-\frac{31\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128a^2d}+\frac{163\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128a^2d}-\frac{1}{640a^2d\tan\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/1152/a^2/d*tan(1/2*d*x+1/2*c)^9-9/896/a^2/d*tan(1/2*d*x+1/2*c)^7+37/640/a
^2/d*tan(1/2*d*x+1/2*c)^5-31/128/a^2/d*tan(1/2*d*x+1/2*c)^3+163/128/a^2/d*t
an(1/2*d*x+1/2*c)-1/640/a^2/d/tan(1/2*d*x+1/2*c)^5+3/128/a^2/d/tan(1/2*d*x+
1/2*c)^3-37/128/a^2/d/tan(1/2*d*x+1/2*c)-2/a^2/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.89, size = 197, normalized size = 1.10

$$\frac{\frac{\frac{51345\sin(dx+c)}{\cos(dx+c)+1}-\frac{9765\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2331\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{405\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2}-\frac{80640\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{63\left(\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{185\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\right.}{a^2\sin(dx+c)^5}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/40320*((51345*sin(d*x + c)/(cos(d*x + c) + 1) - 9765*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 2331*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 405*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 - 8
0640*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 63*(15*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 185*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)*(cos(d*x
+ c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

mupad [B] time = 2.51, size = 230, normalized size = 1.28

$$-\frac{63\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+405\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-2331\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(63*cos(c/2 + (d*x)/2)^14 - 35*sin(c/2 + (d*x)/2)^14 + 405*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^12 - 2331*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^
10 + 9765*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 51345*cos(c/2 + (d*x)
/2)^8*sin(c/2 + (d*x)/2)^6 + 11655*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)
^4 - 945*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 40320*cos(c/2 + (d*x)
/2)^9*sin(c/2 + (d*x)/2)^5*(c + d*x))/(40320*a^2*d*cos(c/2 + (d*x)/2)^9*sin
(c/2 + (d*x)/2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.86 $\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=137

$$\frac{\sec^7(c+dx)}{7a^3d}-\frac{\sec^6(c+dx)}{2a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{5\sec^4(c+dx)}{4a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}-\frac{\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos}{a}$$

```
[Out] ln(cos(d*x+c))/a^3/d+3*sec(d*x+c)/a^3/d-1/2*sec(d*x+c)^2/a^3/d-5/3*sec(d*x+
c)^3/a^3/d+5/4*sec(d*x+c)^4/a^3/d+1/5*sec(d*x+c)^5/a^3/d-1/2*sec(d*x+c)^6/a
^3/d+1/7*sec(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{\sec^7(c+dx)}{7a^3d}-\frac{\sec^6(c+dx)}{2a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{5\sec^4(c+dx)}{4a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}-\frac{\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^3*d) + (3*Sec[c + d*x])/(a^3*d) - Sec[c + d*x]^2/(2*a^
3*d) - (5*Sec[c + d*x]^3)/(3*a^3*d) + (5*Sec[c + d*x]^4)/(4*a^3*d) + Sec[c
+ d*x]^5/(5*a^3*d) - Sec[c + d*x]^6/(2*a^3*d) + Sec[c + d*x]^7/(7*a^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a-ax)^5(a+ax)^2}{x^8}\,dx, x, \cos(c+dx)\right)}{a^{10}d}\\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{a^7}{x^8}-\frac{3a^7}{x^7}+\frac{a^7}{x^6}+\frac{5a^7}{x^5}-\frac{5a^7}{x^4}-\frac{a^7}{x^3}+\frac{3a^7}{x^2}-\frac{a^7}{x}\right)dx, x, \cos(c+dx)\right)}{a^{10}d}\\
&= \frac{\log(\cos(c+dx))}{a^3d}+\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^2(c+dx)}{2a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}+\frac{5\sec^4(c+dx)}{4a^3d}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 140, normalized size = 1.02

$$\frac{\sec^7(c+dx)(4522\cos(2(c+dx))+1050\cos(3(c+dx))+2380\cos(4(c+dx))-210\cos(5(c+dx))+630\cos(6(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((3732 + 4522*Cos[2*(c + d*x)] + 1050*Cos[3*(c + d*x)] + 2380*Cos[4*(c + d*
x)] - 210*Cos[5*(c + d*x)] + 630*Cos[6*(c + d*x)] + 2205*Cos[3*(c + d*x)]*L
og[Cos[c + d*x]] + 735*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[7*(c +
d*x)]*Log[Cos[c + d*x]] + 105*Cos[c + d*x]*(8 + 35*Log[Cos[c + d*x]]))*Sec[
c + d*x]^7)/(6720*a^3*d)
```

fricas [A] time = 0.66, size = 95, normalized size = 0.69

$$\frac{420 \cos(dx+c)^7 \log(-\cos(dx+c)) + 1260 \cos(dx+c)^6 - 210 \cos(dx+c)^5 - 700 \cos(dx+c)^4 + 525 \cos}{420 a^3 d \cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/420*(420*cos(d*x + c)^7*log(-cos(d*x + c)) + 1260*cos(d*x + c)^6 - 210*co
s(d*x + c)^5 - 700*cos(d*x + c)^4 + 525*cos(d*x + c)^3 + 84*cos(d*x + c)^2
- 210*cos(d*x + c) + 60)/(a^3*d*cos(d*x + c)^7)
```

giac [A] time = 93.14, size = 246, normalized size = 1.80

$$-\frac{\frac{420 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{420 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{1393 (\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{819 (\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{6755 (\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{20195 (\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - \frac{28749}{(\cos}}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^7}}{420 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/420*(420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 420*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (1393*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 819*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 6755*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 20195*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 28749*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
- 8463*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*(cos(d*x + c) - 1)^
7/(cos(d*x + c) + 1)^7 + 319)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)^7))/d
```

maple [A] time = 0.80, size = 127, normalized size = 0.93

$$\frac{\sec^7(dx+c)}{7a^3d} - \frac{\sec^6(dx+c)}{2a^3d} + \frac{\sec^5(dx+c)}{5a^3d} + \frac{5\left(\sec^4(dx+c)\right)}{4a^3d} - \frac{5\left(\sec^3(dx+c)\right)}{3a^3d} - \frac{\sec^2(dx+c)}{2a^3d} + \frac{3\sec(dx+c)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/7*sec(d*x+c)^7/a^3/d-1/2*sec(d*x+c)^6/a^3/d+1/5*sec(d*x+c)^5/a^3/d+5/4*se
c(d*x+c)^4/a^3/d-5/3*sec(d*x+c)^3/a^3/d-1/2*sec(d*x+c)^2/a^3/d+3*sec(d*x+c)
/a^3/d-1/d/a^3*ln(sec(d*x+c))
```

maxima [A] time = 0.42, size = 90, normalized size = 0.66

$$\frac{\frac{420 \log(\cos(dx+c))}{a^3} + \frac{1260 \cos(dx+c)^6 - 210 \cos(dx+c)^5 - 700 \cos(dx+c)^4 + 525 \cos(dx+c)^3 + 84 \cos(dx+c)^2 - 210 \cos(dx+c) + 60}{a^3 \cos(dx+c)^7}}{420 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/420*(420*log(cos(d*x + c))/a^3 + (1260*cos(d*x + c)^6 - 210*cos(d*x + c)^
5 - 700*cos(d*x + c)^4 + 525*cos(d*x + c)^3 + 84*cos(d*x + c)^2 - 210*cos(d
*x + c) + 60)/(a^3*cos(d*x + c)^7))/d
```

mupad [B] time = 5.29, size = 225, normalized size = 1.64

$$-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^3\,d}-\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}-\frac{224}{}}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^11/(a + a/cos(c + d*x))^3,x)
```

```
[Out] - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d) - ((282*tan(c/2 + (d*x)/2)^4)/5 -
 (322*tan(c/2 + (d*x)/2)^2)/15 - (224*tan(c/2 + (d*x)/2)^6)/3 + (128*tan(c/
2 + (d*x)/2)^8)/3 + 14*tan(c/2 + (d*x)/2)^10 - 2*tan(c/2 + (d*x)/2)^12 + 35
2/105)/(d*(7*a^3*tan(c/2 + (d*x)/2)^2 - 21*a^3*tan(c/2 + (d*x)/2)^4 + 35*a^
3*tan(c/2 + (d*x)/2)^6 - 35*a^3*tan(c/2 + (d*x)/2)^8 + 21*a^3*tan(c/2 + (d*
x)/2)^10 - 7*a^3*tan(c/2 + (d*x)/2)^12 + a^3*tan(c/2 + (d*x)/2)^14 - a^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{11}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**11/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**11/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.87 $\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$\frac{\sec^5(c+dx)}{5a^3d}-\frac{3\sec^4(c+dx)}{4a^3d}+\frac{2\sec^3(c+dx)}{3a^3d}+\frac{\sec^2(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{\log(\cos(c+dx))}{a^3d}$$

```
[Out] -ln(cos(d*x+c))/a^3/d-3*sec(d*x+c)/a^3/d+sec(d*x+c)^2/a^3/d+2/3*sec(d*x+c)^
3/a^3/d-3/4*sec(d*x+c)^4/a^3/d+1/5*sec(d*x+c)^5/a^3/d
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 75}

$$\frac{\sec^5(c+dx)}{5a^3d}-\frac{3\sec^4(c+dx)}{4a^3d}+\frac{2\sec^3(c+dx)}{3a^3d}+\frac{\sec^2(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{\log(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^3*d)) - (3*Sec[c + d*x])/(a^3*d) + Sec[c + d*x]^2/(a
^3*d) + (2*Sec[c + d*x]^3)/(3*a^3*d) - (3*Sec[c + d*x]^4)/(4*a^3*d) + Sec[c
 + d*x]^5/(5*a^3*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)}{x^6}\,dx, x, \cos(c+dx)\right)}{a^8d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}-\frac{3a^5}{x^5}+\frac{2a^5}{x^4}+\frac{2a^5}{x^3}-\frac{3a^5}{x^2}+\frac{a^5}{x}\right)dx, x, \cos(c+dx)\right)}{a^8d}\\ &= -\frac{\log(\cos(c+dx))}{a^3d}-\frac{3\sec(c+dx)}{a^3d}+\frac{\sec^2(c+dx)}{a^3d}+\frac{2\sec^3(c+dx)}{3a^3d}-\frac{3\sec^4(c+}{4a^3d}\end{aligned}$$

Mathematica [A] time = 0.37, size = 93, normalized size = 0.94

$$-\frac{\sec^5(c+dx)(280\cos(2(c+dx))+90\cos(4(c+dx))+150\cos(c+dx)\log(\cos(c+dx))+15\cos(5(c+dx))1}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/240*((142 + 280*Cos[2*(c + d*x)] + 90*Cos[4*(c + d*x)] + 150*Cos[c + d*x
]*Log[Cos[c + d*x]] + 15*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[3*(c +
 d*x)]*(-4 + 5*Log[Cos[c + d*x]]))*Sec[c + d*x]^5)/(a^3*d)
```

fricas [A] time = 0.71, size = 75, normalized size = 0.76

$$-\frac{60\cos(dx+c)^5\log(-\cos(dx+c))+180\cos(dx+c)^4-60\cos(dx+c)^3-40\cos(dx+c)^2+45\cos(dx+c}{60\,a^3d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^5*log(-cos(d*x + c)) + 180*cos(d*x + c)^4 - 60*cos(d
*x + c)^3 - 40*cos(d*x + c)^2 + 45*cos(d*x + c) - 12)/(a^3*d*cos(d*x + c)^5
)
```

giac [B] time = 20.75, size = 202, normalized size = 2.04

$$\frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3}-\frac{\frac{475(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{590(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{50(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{805(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{137(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 60*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (475*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 590*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 50
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 - 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 119)/(a^3
*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

maple [A] time = 0.65, size = 93, normalized size = 0.94

$$\frac{\sec^5(dx+c)}{5a^3d}-\frac{3\left(\sec^4(dx+c)\right)}{4a^3d}+\frac{2\left(\sec^3(dx+c)\right)}{3a^3d}+\frac{\sec^2(dx+c)}{a^3d}-\frac{3\sec(dx+c)}{a^3d}+\frac{\ln(\sec(dx+c))}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/5*sec(d*x+c)^5/a^3/d-3/4*sec(d*x+c)^4/a^3/d+2/3*sec(d*x+c)^3/a^3/d+sec(d*
x+c)^2/a^3/d-3*sec(d*x+c)/a^3/d+1/d/a^3*ln(sec(d*x+c))
```

maxima [A] time = 0.33, size = 70, normalized size = 0.71

$$-\frac{\frac{60\log(\cos(dx+c))}{a^3}+\frac{180\cos(dx+c)^4-60\cos(dx+c)^3-40\cos(dx+c)^2+45\cos(dx+c)-12}{a^3\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(cos(d*x + c))/a^3 + (180*cos(d*x + c)^4 - 60*cos(d*x + c)^3 -
 40*cos(d*x + c)^2 + 45*cos(d*x + c) - 12)/(a^3*cos(d*x + c)^5))/d
```

<u>**mupad**</u> **[B]** time = 5.92, size = 167, normalized size = 1.69

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^3\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{98\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{58\tan\left(\frac{c}{2}+\frac{d}{}\right.}{3}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d) - ((58*tan(c/2 + (d*x)/2)^2)/3 - (9
8*tan(c/2 + (d*x)/2)^4)/3 + 22*tan(c/2 + (d*x)/2)^6 + 2*tan(c/2 + (d*x)/2)^
8 - 64/15)/(d*(5*a^3*tan(c/2 + (d*x)/2)^2 - 10*a^3*tan(c/2 + (d*x)/2)^4 + 1
0*a^3*tan(c/2 + (d*x)/2)^6 - 5*a^3*tan(c/2 + (d*x)/2)^8 + a^3*tan(c/2 + (d*
x)/2)^10 - a^3))
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^9(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**9/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.88 $\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=65

$$\frac{\sec^3(c+dx)}{3a^3d}-\frac{3\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos(c+dx))}{a^3d}$$

```
[Out] ln(cos(d*x+c))/a^3/d+3*sec(d*x+c)/a^3/d-3/2*sec(d*x+c)^2/a^3/d+1/3*sec(d*x+
c)^3/a^3/d
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 43}

$$\frac{\sec^3(c+dx)}{3a^3d}-\frac{3\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^3*d) + (3*Sec[c + d*x])/(a^3*d) - (3*Sec[c + d*x]^2)/(
2*a^3*d) + Sec[c + d*x]^3/(3*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{a^6d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{3a^3}{x^3}+\frac{3a^3}{x^2}-\frac{a^3}{x}\right)dx, x, \cos(c+dx)\right)}{a^6d}\\
&= \frac{\log(\cos(c+dx))}{a^3d}+\frac{3\sec(c+dx)}{a^3d}-\frac{3\sec^2(c+dx)}{2a^3d}+\frac{\sec^3(c+dx)}{3a^3d}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 64, normalized size = 0.98

$$\frac{\sec^3(c+dx)(18\cos(2(c+dx))+9\cos(c+dx)(\log(\cos(c+dx))-2)+3\cos(3(c+dx))\log(\cos(c+dx))+22)}{12a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((22 + 18*Cos[2*(c + d*x)] + 9*Cos[c + d*x]*(-2 + Log[Cos[c + d*x]]) + 3*Co
s[3*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^3)/(12*a^3*d)
```

fricas [A] time = 1.00, size = 55, normalized size = 0.85

$$\frac{6\cos(dx+c)^3\log(-\cos(dx+c))+18\cos(dx+c)^2-9\cos(dx+c)+2}{6a^3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*cos(d*x + c)^3*log(-cos(d*x + c)) + 18*cos(d*x + c)^2 - 9*cos(d*x +
c) + 2)/(a^3*d*cos(d*x + c)^3)
```

giac [B] time = 9.29, size = 158, normalized size = 2.43

$$-\frac{\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3}-\frac{\frac{75(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{51(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{11(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+29}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 6*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (75*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 51*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 11*(co
s(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 29)/(a^3*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 1)^3))/d
```

maple [A] time = 0.56, size = 63, normalized size = 0.97

$$\frac{\sec^3(dx+c)}{3a^3d}-\frac{3\left(\sec^2(dx+c)\right)}{2a^3d}+\frac{3\sec(dx+c)}{a^3d}-\frac{\ln(\sec(dx+c))}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/3*sec(d*x+c)^3/a^3/d-3/2*sec(d*x+c)^2/a^3/d+3*sec(d*x+c)/a^3/d-1/d/a^3*ln
(sec(d*x+c))
```

maxima [A] time = 0.31, size = 50, normalized size = 0.77

$$\frac{\frac{6\log(\cos(dx+c))}{a^3}+\frac{18\cos(dx+c)^2-9\cos(dx+c)+2}{a^3\cos(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(cos(d*x + c))/a^3 + (18*cos(d*x + c)^2 - 9*cos(d*x + c) + 2)/(a^
3*cos(d*x + c)^3))/d
```

mupad [B] time = 2.03, size = 109, normalized size = 1.68

$$-\frac{14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{20}{3}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + a/cos(c + d*x))^3,x)
```

```
[Out] - (14*tan(c/2 + (d*x)/2)^4 - 18*tan(c/2 + (d*x)/2)^2 + 20/3)/(d*(3*a^3*tan(
c/2 + (d*x)/2)^2 - 3*a^3*tan(c/2 + (d*x)/2)^4 + a^3*tan(c/2 + (d*x)/2)^6 -
a^3)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^7(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.89 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=46

$$\frac{\sec(c+dx)}{a^3d}+\frac{3\log(\cos(c+dx))}{a^3d}-\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

```
[Out] 3*ln(cos(d*x+c))/a^3/d-4*ln(1+cos(d*x+c))/a^3/d+sec(d*x+c)/a^3/d
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3879, 88}

$$\frac{\sec(c+dx)}{a^3d}+\frac{3\log(\cos(c+dx))}{a^3d}-\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Log[Cos[c + d*x]])/(a^3*d) - (4*Log[1 + Cos[c + d*x]])/(a^3*d) + Sec[c + d*x]/(a^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m - 1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2}{x^2(a+ax)}\,dx,x,\cos(c+dx)\right)}{a^4d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{3a}{x}+\frac{4a}{1+x}\right)dx,x,\cos(c+dx)\right)}{a^4d}\\ &= \frac{3\log(\cos(c+dx))}{a^3d}-\frac{4\log(1+\cos(c+dx))}{a^3d}+\frac{\sec(c+dx)}{a^3d}\end{aligned}$$

Mathematica [A] time = 0.12, size = 36, normalized size = 0.78

$$\frac{\sec(c+dx)-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3\log(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-8*Log[Cos[(c + d*x)/2]] + 3*Log[Cos[c + d*x]] + Sec[c + d*x])/(a^3*d)
```

fricas [A] time = 0.66, size = 53, normalized size = 1.15

$$\frac{3\cos(dx+c)\log(-\cos(dx+c)) - 4\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+1}{a^3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (3*cos(d*x + c)*log(-cos(d*x + c)) - 4*cos(d*x + c)*log(1/2*cos(d*x + c) +
1/2) + 1)/(a^3*d*cos(d*x + c))
```

giac [B] time = 5.49, size = 112, normalized size = 2.43

$$\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3}-\frac{\frac{3(\cos(dx+c)-1)}{\cos(dx+c)+1}+1}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + 3*log(abs(-(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (3*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 1)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

maple [A] time = 0.52, size = 46, normalized size = 1.00

$$\frac{\sec(dx+c)}{a^3d}+\frac{\ln(\sec(dx+c))}{d\,a^3}-\frac{4\ln(1+\sec(dx+c))}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

```
[Out] sec(d*x+c)/a^3/d+1/d/a^3*ln(sec(d*x+c))-4/d/a^3*ln(1+sec(d*x+c))
```

maxima [A] time = 0.33, size = 45, normalized size = 0.98

$$-\frac{\frac{4\log(\cos(dx+c)+1)}{a^3}-\frac{3\log(\cos(dx+c))}{a^3}-\frac{1}{a^3\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(4*log(cos(d*x + c) + 1)/a^3 - 3*log(cos(d*x + c))/a^3 - 1/(a^3*cos(d*x +
c)))/d
```

mupad [B] time = 1.27, size = 72, normalized size = 1.57

$$\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{a^3\,d}-\frac{2}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (3*log(tan(c/2 + (d*x)/2)^2 - 1))/(a^3*d) - 2/(d*(a^3*tan(c/2 + (d*x)/2)^2
- a^3)) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.90 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=35

$$\frac{2}{a^3d(\cos(c+dx)+1)}+\frac{\log(\cos(c+dx)+1)}{a^3d}$$

```
[Out] 2/a^3/d/(1+cos(d*x+c))+ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3879, 43}

$$\frac{2}{a^3d(\cos(c+dx)+1)}+\frac{\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 2/(a^3*d*(1 + Cos[c + d*x])) + Log[1 + Cos[c + d*x]]/(a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{a-ax}{(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{a^2d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{2}{a(1+x)^2}-\frac{1}{a(1+x)}\right)dx, x, \cos(c+dx)\right)}{a^2d}\\
&= \frac{2}{a^3d(1+\cos(c+dx))}+\frac{\log(1+\cos(c+dx))}{a^3d}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 33, normalized size = 0.94

$$\frac{\tan^2\left(\frac{1}{2}(c+dx)\right)+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Log[Cos[(c + d*x)/2]] + Tan[(c + d*x)/2]^2)/(a^3*d)
```

fricas [A]   time = 0.82, size = 42, normalized size = 1.20

$$\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2}{a^3d\cos(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 2)/(a^3*d*cos(d*x + c) +
a^3*d)
```

giac [A]   time = 1.90, size = 56, normalized size = 1.60

$$-\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\cos(dx+c)-1}{a^3(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (cos(d*x + c)
- 1)/(a^3*(cos(d*x + c) + 1)))/d
```

maple [A]   time = 0.63, size = 51, normalized size = 1.46

$$-\frac{\ln(\sec(dx+c))}{d\,a^3}-\frac{2}{d\,a^3\,(1+\sec(dx+c))}+\frac{\ln(1+\sec(dx+c))}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*ln(sec(d*x+c))-2/d/a^3/(1+sec(d*x+c))+1/d/a^3*ln(1+sec(d*x+c))
```

maxima [A]   time = 0.62, size = 36, normalized size = 1.03

$$\frac{\frac{2}{a^3\cos(dx+c)+a^3}+\frac{\log(\cos(dx+c)+1)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2/(a^3*cos(d*x + c) + a^3) + log(cos(d*x + c) + 1)/a^3)/d
```

mupad [B]   time = 1.17, size = 36, normalized size = 1.03

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(log(tan(c/2 + (d*x)/2)^2 + 1) - tan(c/2 + (d*x)/2)^2)/(a^3*d)
```

sympy [A] time = 23.81, size = 457, normalized size = 13.06

$$\begin{cases} -\frac{\log\left(\tan^2(c+dx)+1\right)\sec^2(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{2\log\left(\tan^2(c+dx)+1\right)\sec(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{\log\left(\tan^2(c+dx)+1\right)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \frac{2\log(\sec}{2a^3d\sec^2(c+} \\ \frac{x\tan^3(c)}{(a\sec(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Piecewise((-log(tan(c + d*x)**2 + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)
**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 2*log(tan(c + d*x)**2 + 1)*sec(c
+ d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - log(
tan(c + d*x)**2 + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*
a**3*d) + 2*log(sec(c + d*x) + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2
 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 4*log(sec(c + d*x) + 1)*sec(c + d*x)
/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 2*log(sec(
c + d*x) + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
 + tan(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a*
*3*d) - 2*sec(c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) +
2*a**3*d) - 2/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
, Ne(d, 0)), (x*tan(c)**3/(a*sec(c) + a)**3, True))
```

3.91 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=56

$$-\frac{2}{a^3d(\cos(c+dx)+1)}+\frac{1}{2a^3d(\cos(c+dx)+1)^2}-\frac{\log(\cos(c+dx)+1)}{a^3d}$$

```
[Out] 1/2/a^3/d/(1+cos(d*x+c))^2-2/a^3/d/(1+cos(d*x+c))-ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 43}

$$-\frac{2}{a^3d(\cos(c+dx)+1)}+\frac{1}{2a^3d(\cos(c+dx)+1)^2}-\frac{\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 1/(2*a^3*d*(1 + Cos[c + d*x])^2) - 2/(a^3*d*(1 + Cos[c + d*x])) - Log[1 + C
os[c + d*x]]/(a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^3(1+x)^3}-\frac{2}{a^3(1+x)^2}+\frac{1}{a^3(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= \frac{1}{2a^3d(1+\cos(c+dx))^2}-\frac{2}{a^3d(1+\cos(c+dx))}-\frac{\log(1+\cos(c+dx))}{a^3d}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 79, normalized size = 1.41

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(8\cos^2\left(\frac{1}{2}(c+dx)\right)+16\cos^4\left(\frac{1}{2}(c+dx)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-1\right)}{a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((Cos[(c + d*x)/2]^2*(-1 + 8*Cos[(c + d*x)/2]^2 + 16*Cos[(c + d*x)/2]^4*Lo
g[Cos[(c + d*x)/2]])*Sec[c + d*x]^3)/(a^3*d*(1 + Sec[c + d*x])^3))
```

fricas [A] time = 0.70, size = 76, normalized size = 1.36

$$-\frac{2\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+4\,\cos\left(dx+c\right)+3}{2\left(a^{3}d\cos\left(dx+c\right)^{2}+2\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) +
 4*cos(d*x + c) + 3)/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.51, size = 87, normalized size = 1.55

$$\frac{\frac{8\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{3}}+\frac{\frac{6\,a^{3}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^{3}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}}{a^{6}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (6*a^3*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2)/a^6)/d
```

maple [A] time = 0.30, size = 68, normalized size = 1.21

$$\frac{\ln\left(\sec\left(dx+c\right)\right)}{d\,a^{3}}+\frac{1}{2a^{3}d\left(1+\sec\left(dx+c\right)\right)^{2}}+\frac{1}{d\,a^{3}\left(1+\sec\left(dx+c\right)\right)}-\frac{\ln\left(1+\sec\left(dx+c\right)\right)}{d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*ln(sec(d*x+c))+1/2/a^3/d/(1+sec(d*x+c))^2+1/d/a^3/(1+sec(d*x+c))-1/
d/a^3*ln(1+sec(d*x+c))
```

maxima [A] time = 0.50, size = 60, normalized size = 1.07

$$-\frac{\frac{4\,\cos(dx+c)+3}{a^{3}\,\cos(dx+c)^{2}+2\,a^{3}\,\cos(dx+c)+a^{3}}+\frac{2\,\log(\cos(dx+c)+1)}{a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*cos(d*x + c) + 3)/(a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) + a^3)
+ 2*log(cos(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 1.15, size = 48, normalized size = 0.86

$$\frac{\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)-\frac{3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{4}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{8}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1) - (3*tan(c/2 + (d*x)/2)^2)/4 + tan(c/2 + (d*
x)/2)^4/8)/(a^3*d)
```

sympy [A] time = 23.68, size = 411, normalized size = 7.34

$$\begin{cases} \frac{\log\left(\tan^2(c+dx)+1\right)\sec^2(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \frac{2\log\left(\tan^2(c+dx)+1\right)\sec(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \frac{\log\left(\tan^2(c+dx)+1\right)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{2\log(\mathrm{se}}{2a^3d\sec^2(c} \\ \frac{x\tan(c)}{(a\sec(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)*
*2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 2*log(tan(c + d*x)**2 + 1)*sec(c +
 d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + log(t
an(c + d*x)**2 + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a
**3*d) - 2*log(sec(c + d*x) + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2
+ 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 4*log(sec(c + d*x) + 1)*sec(c + d*x)/
(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 2*log(sec(c
 + d*x) + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
+ 2*sec(c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3
*d) + 3/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d), Ne(d
, 0)), (x*tan(c)/(a*sec(c) + a)**3, True))
```

3.92 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=101

$$\frac{17}{8a^3d(\cos(c+dx)+1)}-\frac{7}{8a^3d(\cos(c+dx)+1)^2}+\frac{1}{6a^3d(\cos(c+dx)+1)^3}+\frac{\log(1-\cos(c+dx))}{16a^3d}+\frac{15\log(\cos(c+d}{16a^3d}$$

```
[Out] 1/6/a^3/d/(1+cos(d*x+c))^3-7/8/a^3/d/(1+cos(d*x+c))^2+17/8/a^3/d/(1+cos(d*x
+c))+1/16*ln(1-cos(d*x+c))/a^3/d+15/16*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3879, 88}

$$\frac{17}{8a^3d(\cos(c+dx)+1)}-\frac{7}{8a^3d(\cos(c+dx)+1)^2}+\frac{1}{6a^3d(\cos(c+dx)+1)^3}+\frac{\log(1-\cos(c+dx))}{16a^3d}+\frac{15\log(\cos(c+d}{16a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 1/(6*a^3*d*(1 + Cos[c + d*x])^3) - 7/(8*a^3*d*(1 + Cos[c + d*x])^2) + 17/(8
*a^3*d*(1 + Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(16*a^3*d) + (15*Log[1 +
 Cos[c + d*x]])/(16*a^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{a^2\,\text{Subst}\left(\int \frac{x^4}{(a-ax)(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^2\,\text{Subst}\left(\int \left(-\frac{1}{16a^5(-1+x)}+\frac{1}{2a^5(1+x)^4}-\frac{7}{4a^5(1+x)^3}+\frac{17}{8a^5(1+x)^2}-\frac{15}{16a^5(1+x)}\right)dx, x, \cos(c-\right.}{d}\\ &= \frac{1}{6a^3d(1+\cos(c+dx))^3}-\frac{7}{8a^3d(1+\cos(c+dx))^2}+\frac{17}{8a^3d(1+\cos(c+dx))}+\frac{\log(1-}{}\end{aligned}$$

Mathematica [A] time = 0.33, size = 97, normalized size = 0.96

$$\frac{\sec^3(c+dx)\left(102\cos^4\left(\frac{1}{2}(c+dx)\right)-21\cos^2\left(\frac{1}{2}(c+dx)\right)+12\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+15\log\left(c\right.\right.\right.}{12a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((2 - 21*Cos[(c + d*x)/2]^2 + 102*Cos[(c + d*x)/2]^4 + 12*Cos[(c + d*x)/2]^
6*(15*Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^3)/(12*a
^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.66, size = 151, normalized size = 1.50

$$\frac{102 \cos(dx+c)^2 + 45\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + 3\left(\cos\right.}{48\left(a^3 d\cos(dx+c)^3 + 3a^3 d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(102*cos(d*x + c)^2 + 45*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^3 + 3*cos(d*x + c
)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 162*cos(d*x + c) +
 68)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
+ a^3*d)
```

giac [A] time = 0.34, size = 143, normalized size = 1.42

$$\frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} - \frac{96\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{\frac{66a^6(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{15a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{2a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^9}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 - 96*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - (66*a^6*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 15*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
2*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^9)/d
```

maple [A] time = 0.82, size = 90, normalized size = 0.89

$$\frac{\ln(-1+\cos(dx+c))}{16d\,a^3} + \frac{1}{6d\,a^3(1+\cos(dx+c))^3} - \frac{7}{8d\,a^3(1+\cos(dx+c))^2} + \frac{17}{8d\,a^3(1+\cos(dx+c))} + \frac{15\ln(1\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/16/d/a^3*ln(-1+cos(d*x+c))+1/6/d/a^3/(1+cos(d*x+c))^3-7/8/d/a^3/(1+cos(d*
x+c))^2+17/8/d/a^3/(1+cos(d*x+c))+15/16*ln(1+cos(d*x+c))/a^3/d
```

maxima [A] time = 0.37, size = 98, normalized size = 0.97

$$\frac{\frac{2\left(51\cos(dx+c)^2+81\cos(dx+c)+34\right)}{a^3\cos(dx+c)^3+3a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3} + \frac{45\log(\cos(dx+c)+1)}{a^3} + \frac{3\log(\cos(dx+c)-1)}{a^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(2*(51*cos(d*x + c)^2 + 81*cos(d*x + c) + 34)/(a^3*cos(d*x + c)^3 + 3*
a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 45*log(cos(d*x + c) + 1)/a
^3 + 3*log(cos(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 1.24, size = 75, normalized size = 0.74

$$\frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{48}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + (11*tan(c/2 +
(d*x)/2)^2)/16 - (5*tan(c/2 + (d*x)/2)^4)/32 + tan(c/2 + (d*x)/2)^6/48)/(a^
3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

3.93 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=143

$$-\frac{1}{32a^3d(1-\cos(c+dx))}-\frac{9}{4a^3d(\cos(c+dx)+1)}+\frac{39}{32a^3d(\cos(c+dx)+1)^2}-\frac{5}{12a^3d(\cos(c+dx)+1)^3}+\frac{1}{16a^3d(c$$

```
[Out] -1/32/a^3/d/(1-cos(d*x+c))+1/16/a^3/d/(1+cos(d*x+c))^4-5/12/a^3/d/(1+cos(d*
x+c))^3+39/32/a^3/d/(1+cos(d*x+c))^2-9/4/a^3/d/(1+cos(d*x+c))-7/64*ln(1-cos
(d*x+c))/a^3/d-57/64*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3879, 88}

$$-\frac{1}{32a^3d(1-\cos(c+dx))}-\frac{9}{4a^3d(\cos(c+dx)+1)}+\frac{39}{32a^3d(\cos(c+dx)+1)^2}-\frac{5}{12a^3d(\cos(c+dx)+1)^3}+\frac{1}{16a^3d(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/(32*a^3*d*(1 - Cos[c + d*x])) + 1/(16*a^3*d*(1 + Cos[c + d*x])^4) - 5/(1
2*a^3*d*(1 + Cos[c + d*x])^3) + 39/(32*a^3*d*(1 + Cos[c + d*x])^2) - 9/(4*a
^3*d*(1 + Cos[c + d*x])) - (7*Log[1 - Cos[c + d*x]])/(64*a^3*d) - (57*Log[1
 + Cos[c + d*x]])/(64*a^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{a^4\,\text{Subst}\left(\int \frac{x^6}{(a-ax)^2(a+ax)^5}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{32a^7(-1+x)^2}+\frac{7}{64a^7(-1+x)}+\frac{1}{4a^7(1+x)^5}-\frac{5}{4a^7(1+x)^4}+\frac{39}{16a^7(1+x)^3}-\frac{9}{4a^7(1+x)}\right.\right.}{d}\\ &= -\frac{1}{32a^3d(1-\cos(c+dx))}+\frac{1}{16a^3d(1+\cos(c+dx))^4}-\frac{5}{12a^3d(1+\cos(c+dx))^3}+\end{aligned}$$

Mathematica [A] time = 0.64, size = 140, normalized size = 0.98

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)-3\sec^8\left(\frac{1}{2}(c+dx)\right)+40\sec^6\left(\frac{1}{2}(c+dx)\right)-234\sec^4\left(\frac{1}{2}(c\right.\right.}{96a^3d(\sec(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/96*(Cos[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 + 24*(57*Log[Cos[(c + d*x)
/2]] + 7*Log[Sin[(c + d*x)/2]]) + 864*Sec[(c + d*x)/2]^2 - 234*Sec[(c + d*x
)/2]^4 + 40*Sec[(c + d*x)/2]^6 - 3*Sec[(c + d*x)/2]^8)*Sec[c + d*x]^3)/(a^3
*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.67, size = 240, normalized size = 1.68

$$-\frac{426 \cos(dx+c)^4 + 606 \cos(dx+c)^3 - 190 \cos(dx+c)^2 + 171 \left(\cos(dx+c)^5 + 3 \cos(dx+c)^4 + 2 \cos(dx+\right.}{192 \left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(426*cos(d*x + c)^4 + 606*cos(d*x + c)^3 - 190*cos(d*x + c)^2 + 171*
(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 -
3*cos(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 21*(cos(d*x + c)^5 + 3*co
s(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*lo
g(-1/2*cos(d*x + c) + 1/2) - 666*cos(d*x + c) - 272)/(a^3*d*cos(d*x + c)^5
+ 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2
- 3*a^3*d*cos(d*x + c) - a^3*d)
```

giac [A] time = 0.39, size = 212, normalized size = 1.48

$$\frac{\frac{12\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^3(\cos(dx+c)-1)} - \frac{84\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} + \frac{768\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{\frac{504a^9(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132a^9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{28a^9(\cos(dx+}{(\cos(dx+c)}}{a^{12}}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(12*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/
(a^3*(cos(d*x + c) - 1)) - 84*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/a^3 + 768*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (
504*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 132*a^9*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 28*a^9*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
 3*a^9*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^12)/d
```

maple [A] time = 0.92, size = 126, normalized size = 0.88

$$\frac{1}{32a^3d(-1+\cos(dx+c))} - \frac{7\ln(-1+\cos(dx+c))}{64d\,a^3} + \frac{1}{16a^3d(1+\cos(dx+c))^4} - \frac{5}{12d\,a^3(1+\cos(dx+c))^3} + \frac{}{32d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/32/a^3/d/(-1+cos(d*x+c))-7/64/d/a^3*ln(-1+cos(d*x+c))+1/16/a^3/d/(1+cos(d
*x+c))^4-5/12/d/a^3/(1+cos(d*x+c))^3+39/32/d/a^3/(1+cos(d*x+c))^2-9/4/d/a^3
/(1+cos(d*x+c))-57/64*ln(1+cos(d*x+c))/a^3/d
```

maxima [A] time = 0.44, size = 146, normalized size = 1.02

$$-\frac{\frac{2\left(213\cos(dx+c)^4+303\cos(dx+c)^3-95\cos(dx+c)^2-333\cos(dx+c)-136\right)}{a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+2a^3\cos(dx+c)^3-2a^3\cos(dx+c)^2-3a^3\cos(dx+c)-a^3} + \frac{171\log(\cos(dx+c)+1)}{a^3} + \frac{21\log(\cos(dx+c)-1)}{a^3}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(213*cos(d*x + c)^4 + 303*cos(d*x + c)^3 - 95*cos(d*x + c)^2 - 33
3*cos(d*x + c) - 136)/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 2*a^3*co
s(d*x + c)^3 - 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x + c) - a^3) + 171*log(c
os(d*x + c) + 1)/a^3 + 21*log(cos(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 1.40, size = 102, normalized size = 0.71

$$-\frac{\frac{7\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32}-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)+\frac{{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{64}+\frac{21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{32}-\frac{11\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64}+\frac{7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{192}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{256}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -((7*log(tan(c/2 + (d*x)/2)))/32 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2/64 + (21*tan(c/2 + (d*x)/2)^2)/32 - (11*tan(c/2 + (d*x)/2)^4)/
64 + (7*tan(c/2 + (d*x)/2)^6)/192 - tan(c/2 + (d*x)/2)^8/256)/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{3}{\left(c+dx\right)}}{\sec^{3}{\left(c+dx\right)}+3\sec^{2}{\left(c+dx\right)}+3\sec{\left(c+dx\right)}+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.94 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=185

$$\frac{5}{64a^3d(1-\cos(c+dx))}+\frac{303}{128a^3d(\cos(c+dx)+1)}-\frac{1}{128a^3d(1-\cos(c+dx))^2}-\frac{99}{64a^3d(\cos(c+dx)+1)^2}+\frac{}{48a^3d(\mathrm{co}}$$

```
[Out] -1/128/a^3/d/(1-cos(d*x+c))^2+5/64/a^3/d/(1-cos(d*x+c))+1/40/a^3/d/(1+cos(d
*x+c))^5-13/64/a^3/d/(1+cos(d*x+c))^4+35/48/a^3/d/(1+cos(d*x+c))^3-99/64/a^
3/d/(1+cos(d*x+c))^2+303/128/a^3/d/(1+cos(d*x+c))+37/256*ln(1-cos(d*x+c))/a
^3/d+219/256*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] time = 0.12, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3879, 88}

$$\frac{5}{64a^3d(1-\cos(c+dx))}+\frac{303}{128a^3d(\cos(c+dx)+1)}-\frac{1}{128a^3d(1-\cos(c+dx))^2}-\frac{99}{64a^3d(\cos(c+dx)+1)^2}+\frac{}{48a^3d(\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/(128*a^3*d*(1 - Cos[c + d*x])^2) + 5/(64*a^3*d*(1 - Cos[c + d*x])) + 1/(
40*a^3*d*(1 + Cos[c + d*x])^5) - 13/(64*a^3*d*(1 + Cos[c + d*x])^4) + 35/(4
8*a^3*d*(1 + Cos[c + d*x])^3) - 99/(64*a^3*d*(1 + Cos[c + d*x])^2) + 303/(1
28*a^3*d*(1 + Cos[c + d*x])) + (37*Log[1 - Cos[c + d*x]])/(256*a^3*d) + (21
9*Log[1 + Cos[c + d*x]])/(256*a^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{a^6\,\mathrm{Subst}\left(\int \frac{x^8}{(a-ax)^3(a+ax)^6}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^6\,\mathrm{Subst}\left(\int \left(-\frac{1}{64a^9(-1+x)^3}-\frac{5}{64a^9(-1+x)^2}-\frac{37}{256a^9(-1+x)}+\frac{1}{8a^9(1+x)^6}-\frac{13}{16a^9(1+x)^5}+\frac{35}{16a^9(1+}\right.\right.}{d}\\ &= -\frac{1}{128a^3d(1-\cos(c+dx))^2}+\frac{5}{64a^3d(1-\cos(c+dx))}+\frac{1}{40a^3d(1+\cos(c+dx))^5}-\end{aligned}$$

Mathematica [A] time = 1.24, size = 169, normalized size = 0.91

$$\sec^4\left(\tfrac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(1400\cos^4\left(\tfrac{1}{2}(c+dx)\right)-195\cos^2\left(\tfrac{1}{2}(c+dx)\right)+60\cos^8\left(\tfrac{1}{2}(c+dx)\right)\left(10\cot^2\left(\tfrac{1}{2}(c+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((12 - 195*Cos[(c + d*x)/2]^2 + 1400*Cos[(c + d*x)/2]^4 + 60*Cos[(c + d*x)/
2]^8*(303 + 10*Cot[(c + d*x)/2]^2) - 30*Cos[(c + d*x)/2]^6*(198 + Cot[(c +
d*x)/2]^4) + 120*Cos[(c + d*x)/2]^10*(219*Log[Cos[(c + d*x)/2]] + 37*Log[Si
n[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3)/(1920*a^3*d*(1 + Sec[c
 + d*x])^3)
```

fricas [A] time = 0.81, size = 317, normalized size = 1.71

$$\frac{8790\,\cos\left(dx+c\right)^{6}+11010\,\cos\left(dx+c\right)^{5}-13880\,\cos\left(dx+c\right)^{4}-25560\,\cos\left(dx+c\right)^{3}-734\,\cos\left(dx+c\right)^{2}+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3840*(8790*cos(d*x + c)^6 + 11010*cos(d*x + c)^5 - 13880*cos(d*x + c)^4 -
 25560*cos(d*x + c)^3 - 734*cos(d*x + c)^2 + 3285*(cos(d*x + c)^7 + 3*cos(d
*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x
+ c)^2 + 3*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 555*(cos(d*x + c
)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)
^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 13
878*cos(d*x + c) + 5536)/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a
^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.91, size = 261, normalized size = 1.41

$$-\frac{\frac{30\left(\frac{18\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{111\,(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)(\cos(dx+c)+1)^{2}}{a^{3}(\cos(dx+c)-1)^{2}}-\frac{2220\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^{3}}+\frac{15360\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{3}}+\frac{\frac{9780\,a^{12}(\cos(dx+c)-1)}{\cos(dx+c)+1}}{}}{15360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/15360*(30*(18*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 111*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c) -
1)^2) - 2220*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + 15360*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (9780*a^12*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 2790*a^12*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + 740*a^12*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 135*a^12
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 12*a^12*(cos(d*x + c) - 1)^5/(
cos(d*x + c) + 1)^5)/a^15)/d
```

maple [A] time = 0.86, size = 162, normalized size = 0.88

$$-\frac{1}{128a^{3}d\left(-1+\cos\left(dx+c\right)\right)^{2}}-\frac{5}{64a^{3}d\left(-1+\cos\left(dx+c\right)\right)}+\frac{37\ln\left(-1+\cos\left(dx+c\right)\right)}{256d\,a^{3}}+\frac{1}{40a^{3}d\left(1+\cos\left(dx+c\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/128/a^3/d/(-1+cos(d*x+c))^2-5/64/a^3/d/(-1+cos(d*x+c))+37/256/d/a^3*ln(-
1+cos(d*x+c))+1/40/a^3/d/(1+cos(d*x+c))^5-13/64/a^3/d/(1+cos(d*x+c))^4+35/4
8/d/a^3/(1+cos(d*x+c))^3-99/64/d/a^3/(1+cos(d*x+c))^2+303/128/d/a^3/(1+cos(
d*x+c))+219/256*ln(1+cos(d*x+c))/a^3/d
```

maxima [A] time = 1.40, size = 188, normalized size = 1.02

$$\frac{\frac{2\left(4395\cos(dx+c)^6+5505\cos(dx+c)^5-6940\cos(dx+c)^4-12780\cos(dx+c)^3-367\cos(dx+c)^2+6939\cos(dx+c)+2768\right)}{a^3\cos(dx+c)^7+3a^3\cos(dx+c)^6+a^3\cos(dx+c)^5-5a^3\cos(dx+c)^4-5a^3\cos(dx+c)^3+a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}+\frac{3285\log(\cos(dx+c)+1)}{a^3}+}{3840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(4395*cos(d*x + c)^6 + 5505*cos(d*x + c)^5 - 6940*cos(d*x + c)^4
- 12780*cos(d*x + c)^3 - 367*cos(d*x + c)^2 + 6939*cos(d*x + c) + 2768)/(a^
3*cos(d*x + c)^7 + 3*a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 5*a^3*cos(d*
x + c)^4 - 5*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
 a^3) + 3285*log(cos(d*x + c) + 1)/a^3 + 555*log(cos(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 1.33, size = 170, normalized size = 0.92

$$\frac{163\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{256a^3d}-\frac{93\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{512a^3d}+\frac{37\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{768a^3d}-\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{1024a^3d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{1280a^3d}+\frac{37\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (163*tan(c/2 + (d*x)/2)^2)/(256*a^3*d) - (93*tan(c/2 + (d*x)/2)^4)/(512*a^3
*d) + (37*tan(c/2 + (d*x)/2)^6)/(768*a^3*d) - (9*tan(c/2 + (d*x)/2)^8)/(102
4*a^3*d) + tan(c/2 + (d*x)/2)^10/(1280*a^3*d) + (37*log(tan(c/2 + (d*x)/2))
)/(128*a^3*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d) + (cot(c/2 + (d*x)/2)
^4*((9*tan(c/2 + (d*x)/2)^2)/2 - 1/4))/(128*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.95 $\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=237

$$-\frac{3\tan^7(c+dx)}{7a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{\tan(c+dx)}{a^3d}-\frac{125\tanh^{-1}(\sin(c+dx))}{128a^3d}+\frac{\tan^5(c+dx)\sec^3(c+d}{8a^3d}$$

```
[Out] x/a^3-125/128*arctanh(sin(d*x+c))/a^3/d-tan(d*x+c)/a^3/d+115/128*sec(d*x+c)
*tan(d*x+c)/a^3/d+5/64*sec(d*x+c)^3*tan(d*x+c)/a^3/d+1/3*tan(d*x+c)^3/a^3/d
-5/8*sec(d*x+c)*tan(d*x+c)^3/a^3/d-5/48*sec(d*x+c)^3*tan(d*x+c)^3/a^3/d-1/5
*tan(d*x+c)^5/a^3/d+1/2*sec(d*x+c)*tan(d*x+c)^5/a^3/d+1/8*sec(d*x+c)^3*tan(
d*x+c)^5/a^3/d-3/7*tan(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.36, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{3\tan^7(c+dx)}{7a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{\tan(c+dx)}{a^3d}-\frac{125\tanh^{-1}(\sin(c+dx))}{128a^3d}+\frac{\tan^5(c+dx)\sec^3(c+d}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^12/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 - (125*ArcTanh[Sin[c + d*x]])/(128*a^3*d) - Tan[c + d*x]/(a^3*d) + (1
15*Sec[c + d*x]*Tan[c + d*x])/(128*a^3*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])
/(64*a^3*d) + Tan[c + d*x]^3/(3*a^3*d) - (5*Sec[c + d*x]*Tan[c + d*x]^3)/(8
*a^3*d) - (5*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*a^3*d) - Tan[c + d*x]^5/(5*
a^3*d) + (Sec[c + d*x]*Tan[c + d*x]^5)/(2*a^3*d) + (Sec[c + d*x]^3*Tan[c +
d*x]^5)/(8*a^3*d) - (3*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\int (-a+a\sec(c+dx))^3\tan^6(c+dx)\,dx}{a^6}\\
&= \frac{\int \left(-a^3\tan^6(c+dx)+3a^3\sec(c+dx)\tan^6(c+dx)-3a^3\sec^2(c+dx)\tan^6(c+dx)+\right.}{a^6}\\
&= -\frac{\int \tan^6(c+dx)\,dx}{a^3}+\frac{\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{a^3}+\frac{3\int \sec(c+dx)\tan^6(c+dx)}{a^3}\\
&= -\frac{\tan^5(c+dx)}{5a^3d}+\frac{\sec(c+dx)\tan^5(c+dx)}{2a^3d}+\frac{\sec^3(c+dx)\tan^5(c+dx)}{8a^3d}-\frac{5\int \sec^3(c}{}\\
&= \frac{\tan^3(c+dx)}{3a^3d}-\frac{5\sec(c+dx)\tan^3(c+dx)}{8a^3d}-\frac{5\sec^3(c+dx)\tan^3(c+dx)}{48a^3d}-\frac{\tan^5(c+}{5a^3d}\\
&= -\frac{\tan(c+dx)}{a^3d}+\frac{15\sec(c+dx)\tan(c+dx)}{16a^3d}+\frac{5\sec^3(c+dx)\tan(c+dx)}{64a^3d}+\frac{\tan^3(c+}{3a^3d}\\
&= \frac{x}{a^3}-\frac{15\tanh^{-1}(\sin(c+dx))}{16a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{115\sec(c+dx)\tan(c+dx)}{128a^3d}+\frac{5\sec^3(c}{}\\
&= \frac{x}{a^3}-\frac{125\tanh^{-1}(\sin(c+dx))}{128a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{115\sec(c+dx)\tan(c+dx)}{128a^3d}+\frac{5\sec^3}{}
\end{aligned}$$

Mathematica [A] time = 1.34, size = 362, normalized size = 1.53

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(1680000\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^12/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(1680000*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x
]^8*(470400*d*x*Cos[c] + 376320*d*x*Cos[c + 2*d*x] + 376320*d*x*Cos[3*c + 2
*d*x] + 188160*d*x*Cos[3*c + 4*d*x] + 188160*d*x*Cos[5*c + 4*d*x] + 53760*d
*x*Cos[5*c + 6*d*x] + 53760*d*x*Cos[7*c + 6*d*x] + 6720*d*x*Cos[7*c + 8*d*x
] + 6720*d*x*Cos[9*c + 8*d*x] + 519680*Sin[c] + 133175*Sin[d*x] + 133175*Si
n[2*c + d*x] - 544768*Sin[c + 2*d*x] + 286720*Sin[3*c + 2*d*x] + 63595*Sin[
2*c + 3*d*x] + 63595*Sin[4*c + 3*d*x] - 254464*Sin[3*c + 4*d*x] + 161280*Si
n[5*c + 4*d*x] + 65135*Sin[4*c + 5*d*x] + 65135*Sin[6*c + 5*d*x] - 118784*S
in[5*c + 6*d*x] + 27195*Sin[6*c + 7*d*x] + 27195*Sin[8*c + 7*d*x] - 14848*S
in[7*c + 8*d*x])))/(215040*a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.60, size = 147, normalized size = 0.62

$$\frac{26880\,dx\cos(dx+c)^8 - 13125\,\cos(dx+c)^8\log(\sin(dx+c)+1) + 13125\,\cos(dx+c)^8\log(-\sin(dx+c)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/26880*(26880*d*x*cos(d*x + c)^8 - 13125*cos(d*x + c)^8*log(sin(d*x + c) +
 1) + 13125*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(14848*cos(d*x + c)^7
 - 27195*cos(d*x + c)^6 + 7424*cos(d*x + c)^5 + 17710*cos(d*x + c)^4 - 1459
2*cos(d*x + c)^3 - 1960*cos(d*x + c)^2 + 5760*cos(d*x + c) - 1680)*sin(d*x
+ c))/(a^3*d*cos(d*x + c)^8)
```

giac [A] time = 116.12, size = 175, normalized size = 0.74

$$\frac{\frac{13440\,(dx+c)}{a^3} - \frac{13125\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} + \frac{13125\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} + \frac{2\left(26565\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} - 212625\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} + 749\cdots}{\cdots}}{1344\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)/a^3 - 13125*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3
 + 13125*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(26565*tan(1/2*d*x + 1/
2*c)^15 - 212625*tan(1/2*d*x + 1/2*c)^13 + 749973*tan(1/2*d*x + 1/2*c)^11 -
 550089*tan(1/2*d*x + 1/2*c)^9 + 269879*tan(1/2*d*x + 1/2*c)^7 - 79723*tan(
1/2*d*x + 1/2*c)^5 + 11375*tan(1/2*d*x + 1/2*c)^3 - 315*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^8*a^3))/d
```

maple [A] time = 0.85, size = 396, normalized size = 1.67

$$\frac{1}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^8}+\frac{13}{14a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^7}+\frac{65}{24a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{143}{40a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/8/a^3/d/(tan(1/2*d*x+1/2*c)-1)^8+13/14/a^3/d/(tan(1/2*d*x+1/2*c)-1)^7+65/
24/a^3/d/(tan(1/2*d*x+1/2*c)-1)^6+143/40/a^3/d/(tan(1/2*d*x+1/2*c)-1)^5+79/
64/a^3/d/(tan(1/2*d*x+1/2*c)-1)^4-49/32/a^3/d/(tan(1/2*d*x+1/2*c)-1)^3-29/1
28/a^3/d/(tan(1/2*d*x+1/2*c)-1)^2+253/128/a^3/d/(tan(1/2*d*x+1/2*c)-1)+125/
```

```
128/a^3/d*ln(tan(1/2*d*x+1/2*c)-1)-1/8/a^3/d/(tan(1/2*d*x+1/2*c)+1)^8+13/14
/a^3/d/(tan(1/2*d*x+1/2*c)+1)^7-65/24/a^3/d/(tan(1/2*d*x+1/2*c)+1)^6+143/40
/a^3/d/(tan(1/2*d*x+1/2*c)+1)^5-79/64/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4-49/32/
a^3/d/(tan(1/2*d*x+1/2*c)+1)^3+29/128/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2+253/12
8/a^3/d/(tan(1/2*d*x+1/2*c)+1)-125/128/a^3/d*ln(tan(1/2*d*x+1/2*c)+1)+2/d/a
^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.62, size = 429, normalized size = 1.81

$$-\frac{\frac{2\left(\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{11375\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{79723\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{269879\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{550089\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{749973\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{212625\,\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{26565\,\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}\right)}{a^{3}-\frac{8\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{28\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{56\,a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{70\,a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{56\,a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28\,a^{3}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{8\,a^{3}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^{3}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-2\cdots}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/13440*(2*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 11375*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 79723*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 269879*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 550089*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 749973*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 212625*sin(d*x + c)^13/(
cos(d*x + c) + 1)^13 - 26565*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^3 -
8*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 - 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 28*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^3*sin(d*x + c)^14/(cos(
d*x + c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 26880*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 13125*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a^3 - 13125*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

mupad [B] time = 2.56, size = 265, normalized size = 1.12

$$\frac{x}{a^{3}}-\frac{125\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a^{3}\,d}-\frac{-\frac{253\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{2025\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}-\frac{35713\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{320}+\frac{1833\cdots}{\cdots}}{d\left(a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-8\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+28\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-56\,a^{3}\tan\left(\frac{c}{2}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^12/(a + a/cos(c + d*x))^3,x)
```

```
[Out] x/a^3 - (125*atanh(tan(c/2 + (d*x)/2)))/(64*a^3*d) - ((3*tan(c/2 + (d*x)/2)
)/64 - (325*tan(c/2 + (d*x)/2)^3)/192 + (11389*tan(c/2 + (d*x)/2)^5)/960 -
(269879*tan(c/2 + (d*x)/2)^7)/6720 + (183363*tan(c/2 + (d*x)/2)^9)/2240 - (
35713*tan(c/2 + (d*x)/2)^11)/320 + (2025*tan(c/2 + (d*x)/2)^13)/64 - (253*t
an(c/2 + (d*x)/2)^15)/64)/(d*(28*a^3*tan(c/2 + (d*x)/2)^4 - 8*a^3*tan(c/2 +
 (d*x)/2)^2 - 56*a^3*tan(c/2 + (d*x)/2)^6 + 70*a^3*tan(c/2 + (d*x)/2)^8 - 5
6*a^3*tan(c/2 + (d*x)/2)^10 + 28*a^3*tan(c/2 + (d*x)/2)^12 - 8*a^3*tan(c/2
+ (d*x)/2)^14 + a^3*tan(c/2 + (d*x)/2)^16 + a^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{12}{(c+dx)}}{\sec^{3}{(c+dx)}+3\sec^{2}{(c+dx)}+3\sec{(c+dx)}+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**12/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**12/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.96 $\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=169

$$-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan(c+dx)}{a^3d}+\frac{19\tanh^{-1}(\sin(c+dx))}{16a^3d}+\frac{\tan^3(c+dx)\sec^3(c+dx)}{6a^3d}-\frac{\tan(c+dx)}{8a}$$

```
[Out] -x/a^3+19/16*arctanh(sin(d*x+c))/a^3/d+tan(d*x+c)/a^3/d-17/16*sec(d*x+c)*ta
n(d*x+c)/a^3/d-1/8*sec(d*x+c)^3*tan(d*x+c)/a^3/d-1/3*tan(d*x+c)^3/a^3/d+3/4
*sec(d*x+c)*tan(d*x+c)^3/a^3/d+1/6*sec(d*x+c)^3*tan(d*x+c)^3/a^3/d-3/5*tan(
d*x+c)^5/a^3/d
```

Rubi [A] time = 0.27, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan(c+dx)}{a^3d}+\frac{19\tanh^{-1}(\sin(c+dx))}{16a^3d}+\frac{\tan^3(c+dx)\sec^3(c+dx)}{6a^3d}-\frac{\tan(c+dx)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^10/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) + (19*ArcTanh[Sin[c + d*x]])/(16*a^3*d) + Tan[c + d*x]/(a^3*d) - (
17*Sec[c + d*x]*Tan[c + d*x])/(16*a^3*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(8
*a^3*d) - Tan[c + d*x]^3/(3*a^3*d) + (3*Sec[c + d*x]*Tan[c + d*x]^3)/(4*a^3
*d) + (Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int(-a+a\sec(c+dx))^3\tan^4(c+dx)\,dx}{a^6}$$
$$= \frac{\int\left(-a^3\tan^4(c+dx)+3a^3\sec(c+dx)\tan^4(c+dx)-3a^3\sec^2(c+dx)\tan^4(c+dx)+\right.}{a^6}$$
$$= -\frac{\int\tan^4(c+dx)\,dx}{a^3}+\frac{\int\sec^3(c+dx)\tan^4(c+dx)\,dx}{a^3}+\frac{3\int\sec(c+dx)\tan^4(c+dx)}{a^3}$$
$$= -\frac{\tan^3(c+dx)}{3a^3d}+\frac{3\sec(c+dx)\tan^3(c+dx)}{4a^3d}+\frac{\sec^3(c+dx)\tan^3(c+dx)}{6a^3d}-\frac{\int\sec^3(c}{}$$
$$= \frac{\tan(c+dx)}{a^3d}-\frac{9\sec(c+dx)\tan(c+dx)}{8a^3d}-\frac{\sec^3(c+dx)\tan(c+dx)}{8a^3d}-\frac{\tan^3(c+dx)}{3a^3d}$$
$$= -\frac{x}{a^3}+\frac{9\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{\tan(c+dx)}{a^3d}-\frac{17\sec(c+dx)\tan(c+dx)}{16a^3d}-\frac{\sec^3(c+}{}$$
$$= -\frac{x}{a^3}+\frac{19\tanh^{-1}(\sin(c+dx))}{16a^3d}+\frac{\tan(c+dx)}{a^3d}-\frac{17\sec(c+dx)\tan(c+dx)}{16a^3d}-\frac{\sec^3(c}{}$$

Mathematica [A] time = 0.92, size = 303, normalized size = 1.79

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(9120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^10/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/960*(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(9120*(Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c +
 d*x]^6*(2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 1800*d*x*Cos[3*c + 2*d
*x] + 720*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x] + 120*d*x*Cos[5*c
 + 6*d*x] + 120*d*x*Cos[7*c + 6*d*x] + 1760*Sin[c] - 210*Sin[d*x] - 210*Sin
```

```
[2*c + d*x] - 1440*Sin[c + 2*d*x] + 1200*Sin[3*c + 2*d*x] + 865*Sin[2*c + 3
*d*x] + 865*Sin[4*c + 3*d*x] - 1296*Sin[3*c + 4*d*x] - 240*Sin[5*c + 4*d*x]
 + 435*Sin[4*c + 5*d*x] + 435*Sin[6*c + 5*d*x] - 176*Sin[5*c + 6*d*x])))/(a
^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.54, size = 127, normalized size = 0.75

$$-\frac{480\,dx\cos\left(dx+c\right)^{6}-285\,\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)+285\,\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)-2}{480\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/480*(480*d*x*cos(d*x + c)^6 - 285*cos(d*x + c)^6*log(sin(d*x + c) + 1) +
 285*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(176*cos(d*x + c)^5 - 435*co
s(d*x + c)^4 + 208*cos(d*x + c)^3 + 110*cos(d*x + c)^2 - 144*cos(d*x + c) +
 40)*sin(d*x + c))/(a^3*d*cos(d*x + c)^6)
```

giac [A] time = 64.84, size = 149, normalized size = 0.88

$$-\frac{\frac{240\,(dx+c)}{a^{3}}-\frac{285\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}+\frac{285\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{3}}+\frac{2\left(525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-3135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1746\,\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\left(\frac{1}{2}\right.\right.}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)/a^3 - 285*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 28
5*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(525*tan(1/2*d*x + 1/2*c)^11 -
 3135*tan(1/2*d*x + 1/2*c)^9 + 1746*tan(1/2*d*x + 1/2*c)^7 - 366*tan(1/2*d*
x + 1/2*c)^5 - 95*tan(1/2*d*x + 1/2*c)^3 + 45*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 - 1)^6*a^3))/d
```

maple [B] time = 0.82, size = 312, normalized size = 1.85

$$\frac{1}{6a^{3}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{6}}+\frac{11}{10a^{3}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{11}{4a^{3}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{11}{4a^{3}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/6/a^3/d/(tan(1/2*d*x+1/2*c)-1)^6+11/10/a^3/d/(tan(1/2*d*x+1/2*c)-1)^5+11/
4/a^3/d/(tan(1/2*d*x+1/2*c)-1)^4+11/4/a^3/d/(tan(1/2*d*x+1/2*c)-1)^3-5/16/a
^3/d/(tan(1/2*d*x+1/2*c)-1)^2-35/16/a^3/d/(tan(1/2*d*x+1/2*c)-1)-19/16/a^3/
d*ln(tan(1/2*d*x+1/2*c)-1)-1/6/a^3/d/(tan(1/2*d*x+1/2*c)+1)^6+11/10/a^3/d/(
tan(1/2*d*x+1/2*c)+1)^5-11/4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4+11/4/a^3/d/(tan
(1/2*d*x+1/2*c)+1)^3+5/16/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2-35/16/a^3/d/(tan(1
/2*d*x+1/2*c)+1)+19/16/a^3/d*ln(tan(1/2*d*x+1/2*c)+1)-2/d/a^3*arctan(tan(1/
2*d*x+1/2*c))
```

maxima [B] time = 0.70, size = 343, normalized size = 2.03

$$-\frac{\frac{2\left(\frac{45\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{95\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{366\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{1746\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{3135\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{525\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^{3}-\frac{6\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{15\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{20\,a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15\,a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{6\,a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^{3}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{480\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}-\frac{285\,\log\left(\frac{\sin}{\cos}\right.}{a^{3}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(2*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 95*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 366*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1746*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - 3135*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 525*sin(
d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^3 - 6*a^3*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^
3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x + c)
 + 1)^12) + 480*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 285*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 285*log(sin(d*x + c)/(cos(d*x + c) +
1) - 1)/a^3)/d
```

mupad [B] time = 2.35, size = 208, normalized size = 1.23

$$\frac{19\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^3\,d}-\frac{x}{a^3}-\frac{\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{209\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8}+\frac{291\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}-\frac{61\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{20}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,a^3\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^10/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (19*atanh(tan(c/2 + (d*x)/2)))/(8*a^3*d) - x/a^3 - ((3*tan(c/2 + (d*x)/2))/
8 - (19*tan(c/2 + (d*x)/2)^3)/24 - (61*tan(c/2 + (d*x)/2)^5)/20 + (291*tan(
c/2 + (d*x)/2)^7)/20 - (209*tan(c/2 + (d*x)/2)^9)/8 + (35*tan(c/2 + (d*x)/2
)^11)/8)/(d*(15*a^3*tan(c/2 + (d*x)/2)^4 - 6*a^3*tan(c/2 + (d*x)/2)^2 - 20*
a^3*tan(c/2 + (d*x)/2)^6 + 15*a^3*tan(c/2 + (d*x)/2)^8 - 6*a^3*tan(c/2 + (d
*x)/2)^10 + a^3*tan(c/2 + (d*x)/2)^12 + a^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{10}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**10/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**10/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.97 $\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$-\frac{\tan^3(c+dx)}{a^3d}-\frac{\tan(c+dx)}{a^3d}-\frac{13\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{\tan(c+dx)\sec^3(c+dx)}{4a^3d}+\frac{11\tan(c+dx)\sec(c+dx)}{8a^3d}+$$

```
[Out] x/a^3-13/8*arctanh(sin(d*x+c))/a^3/d-tan(d*x+c)/a^3/d+11/8*sec(d*x+c)*tan(d
*x+c)/a^3/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a^3/d-tan(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.20, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{\tan^3(c+dx)}{a^3d}-\frac{\tan(c+dx)}{a^3d}-\frac{13\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{\tan(c+dx)\sec^3(c+dx)}{4a^3d}+\frac{11\tan(c+dx)\sec(c+dx)}{8a^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 - (13*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - Tan[c + d*x]/(a^3*d) + (11*S
ec[c + d*x]*Tan[c + d*x])/(8*a^3*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a^3*
d) - Tan[c + d*x]^3/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int (-a+a\sec(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\tan^2(c+dx)+3a^3\sec(c+dx)\tan^2(c+dx)-3a^3\sec^2(c+dx)\tan^2(c+dx)+\right.}{a^6}$$
$$= -\frac{\int \tan^2(c+dx)\,dx}{a^3}+\frac{\int \sec^3(c+dx)\tan^2(c+dx)\,dx}{a^3}+\frac{3\int \sec(c+dx)\tan^2(c+dx)}{a^3}$$
$$= -\frac{\tan(c+dx)}{a^3d}+\frac{3\sec(c+dx)\tan(c+dx)}{2a^3d}+\frac{\sec^3(c+dx)\tan(c+dx)}{4a^3d}-\frac{\int \sec^3(c+}{4a^3}$$
$$= \frac{x}{a^3}-\frac{3\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{11\sec(c+dx)\tan(c+dx)}{8a^3d}+\frac{\sec^3(c+}{}$$
$$= \frac{x}{a^3}-\frac{13\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{11\sec(c+dx)\tan(c+dx)}{8a^3d}+\frac{\sec^3(c+}{}$$

Mathematica [B] time = 0.76, size = 230, normalized size = 2.32

$$\frac{\sec^4(c+dx)\left(38\sin(c+dx)-32\sin(2(c+dx))+22\sin(3(c+dx))+39\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^4*(24*d*x + 39*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*C
os[2*(c + d*x)]*(8*d*x + 13*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 13*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[4*(c + d*x)]*(8*d*x + 13*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 13*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) - 39*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 38*Sin[c + d*x] -
 32*Sin[2*(c + d*x)] + 22*Sin[3*(c + d*x)]))/(64*a^3*d)
```

fricas [A] time = 0.78, size = 97, normalized size = 0.98

$$\frac{16\,dx\cos(dx+c)^4-13\,\cos(dx+c)^4\log(\sin(dx+c)+1)+13\,\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(11\,\cos\right.}{16\,a^3d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(16*d*x*cos(d*x + c)^4 - 13*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 13*
cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(11*cos(d*x + c)^2 - 8*cos(d*x +
c) + 2)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4)
```

giac [A] time = 11.18, size = 123, normalized size = 1.24

$$\frac{\frac{8\,(dx+c)}{a^3} - \frac{13\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} + \frac{13\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} + \frac{2\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-13\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4 a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/a^3 - 13*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 13*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(21*tan(1/2*d*x + 1/2*c)^7 + 3*tan(1/
2*d*x + 1/2*c)^5 - 13*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*c))/((ta
n(1/2*d*x + 1/2*c)^2 - 1)^4*a^3))/d
```

maple [B] time = 0.62, size = 228, normalized size = 2.30

$$\frac{1}{4a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{3}{2a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{27}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{21}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{13\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/a^3/d/(tan(1/2*d*x+1/2*c)-1)^4+3/2/a^3/d/(tan(1/2*d*x+1/2*c)-1)^3+27/8/
a^3/d/(tan(1/2*d*x+1/2*c)-1)^2+21/8/a^3/d/(tan(1/2*d*x+1/2*c)-1)+13/8/a^3/d
*ln(tan(1/2*d*x+1/2*c)-1)-1/4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4+3/2/a^3/d/(tan
(1/2*d*x+1/2*c)+1)^3-27/8/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2+21/8/a^3/d/(tan(1/
2*d*x+1/2*c)+1)-13/8/a^3/d*ln(tan(1/2*d*x+1/2*c)+1)+2/d/a^3*arctan(tan(1/2*
d*x+1/2*c))
```

maxima [B] time = 0.52, size = 257, normalized size = 2.60

$$\frac{\frac{2\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{13\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{21\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^3-\frac{4\,a^3\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a^3\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\,a^3\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{16\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{13\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{13\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 13*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^3 - 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 16*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^3 - 13*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 13*l
og(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

mupad [B] time = 1.93, size = 148, normalized size = 1.49

$$\frac{x}{a^3}-\frac{13\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d}+\frac{\frac{21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8/(a + a/cos(c + d*x))^3,x)
```

```
[Out] x/a^3 - (13*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((5*tan(c/2 + (d*x)/2))/
4 - (13*tan(c/2 + (d*x)/2)^3)/4 + (3*tan(c/2 + (d*x)/2)^5)/4 + (21*tan(c/2
+ (d*x)/2)^7)/4)/(d*(6*a^3*tan(c/2 + (d*x)/2)^4 - 4*a^3*tan(c/2 + (d*x)/2)^
2 - 4*a^3*tan(c/2 + (d*x)/2)^6 + a^3*tan(c/2 + (d*x)/2)^8 + a^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^8(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.98 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=66

$$-\frac{5\tan(c+dx)}{2a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{\tan(c+dx)(1-\sec(c+dx))}{2a^3d}-\frac{x}{a^3}$$

```
[Out] -x/a^3+7/2*arctanh(sin(d*x+c))/a^3/d-5/2*tan(d*x+c)/a^3/d-1/2*(1-sec(d*x+c)
)*tan(d*x+c)/a^3/d
```

Rubi [A] time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3888, 3775, 3914, 3767, 8, 3770}

$$-\frac{5\tan(c+dx)}{2a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{\tan(c+dx)(1-\sec(c+dx))}{2a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) + (7*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (5*Tan[c + d*x])/(2*a^3*d)
 - ((1 - Sec[c + d*x])*Tan[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\int (-a+a\sec(c+dx))^3\,dx}{a^6}\\ &= -\frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d}-\frac{\int (-a+a\sec(c+dx))(-2a+5a\sec(c+dx))\,dx}{2a^5}\\ &= -\frac{x}{a^3}-\frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d}-\frac{5\int \sec^2(c+dx)\,dx}{2a^3}+\frac{7\int \sec(c+dx)\,dx}{2a^3}\\ &= -\frac{x}{a^3}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d}+\frac{5\,\text{Subst}(\int 1\,dx,x,-}{2a^3d}\\ &= -\frac{x}{a^3}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{5\tan(c+dx)}{2a^3d}-\frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d}\end{aligned}$$

Mathematica [B] time = 0.96, size = 241, normalized size = 3.65

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-\frac{12\sin(dx)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{}{d(}\right.}{a^3(\sec}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-4*x - (14*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/d + (14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + 1/(d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2) - (12*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]
)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]))))/(a^3*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.62, size = 87, normalized size = 1.32

$$-\frac{4\,dx\cos(dx+c)^2-7\,\cos(dx+c)^2\log(\sin(dx+c)+1)+7\,\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(6\,\cos(d}{4\,a^3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*x*cos(d*x + c)^2 - 7*cos(d*x + c)^2*log(sin(d*x + c) + 1) + 7*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*cos(d*x + c) - 1)*sin(d*x + c))/(
a^3*d*cos(d*x + c)^2)
```

giac [A] time = 7.54, size = 97, normalized size = 1.47

$$-\frac{\frac{2\,(dx+c)}{a^3}-\frac{7\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}+\frac{7\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{2\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)/a^3 - 7*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 7*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 2*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

maple [B] time = 0.60, size = 144, normalized size = 2.18

$$\frac{1}{2a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{2a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^3d}-\frac{1}{2a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{2a^3d\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2/a^3/d/(tan(1/2*d*x+1/2*c)-1)^2+7/2/a^3/d/(tan(1/2*d*x+1/2*c)-1)-7/2/a^3
/d*ln(tan(1/2*d*x+1/2*c)-1)-1/2/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2+7/2/a^3/d/(t
an(1/2*d*x+1/2*c)+1)+7/2/a^3/d*ln(tan(1/2*d*x+1/2*c)+1)-2/d/a^3*arctan(tan(
1/2*d*x+1/2*c))
```

maxima [B] time = 0.45, size = 171, normalized size = 2.59

$$-\frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 -
 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 7*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3)/d
```

mupad [B] time = 1.31, size = 92, normalized size = 1.39

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{x}{a^3}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - x/a^3 - (5*tan(c/2 + (d*x)/2) - 7*t
an(c/2 + (d*x)/2)^3)/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2
)^2 + a^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.99 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{x}{a^3}-\frac{4\tan(c+dx)}{a^2d(a\sec(c+dx)+a)}$$

```
[Out] x/a^3+arctanh(sin(d*x+c))/a^3/d-4*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2606, 3767, 2621, 321, 207}

$$\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}+\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 + ArcTanh[Sin[c + d*x]]/(a^3*d) + (4*Cot[c + d*x])/(a^3*d) - (4*Csc[c
 + d*x])/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\int \cot^2(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^2(c+dx)+3a^3\cot(c+dx)\csc(c+dx)-3a^3\csc^2(c+dx)+a^3\csc^2(c+\right.}{a^6} \\
&= -\frac{\int \cot^2(c+dx)\,dx}{a^3}+\frac{\int \csc^2(c+dx)\sec(c+dx)\,dx}{a^3}+\frac{3\int \cot(c+dx)\csc(c+dx}{a^3} \\
&= \frac{\cot(c+dx)}{a^3d}+\frac{\int 1\,dx}{a^3}-\frac{\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{a^3d}+\frac{3\,\mathrm{Subst}(\int 1\,dx,x,c}{a^3d} \\
&= \frac{x}{a^3}+\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&= \frac{x}{a^3}+\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}
\end{aligned}
$$

Mathematica [B] time = 0.27, size = 117, normalized size = 2.54

$$\frac{8\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (8*Cos[(c + d*x)/2]^5*Sec[c + d*x]^3*(Cos[(c + d*x)/2]*(d*x - Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 4
*Sec[c/2]*Sin[(d*x)/2]))/(a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.78, size = 83, normalized size = 1.80

$$\frac{2\,dx\cos(dx+c)+2\,dx+(\cos(dx+c)+1)\log(\sin(dx+c)+1)-(\cos(dx+c)+1)\log(-\sin(dx+c)+1)}{2\left(a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x*cos(d*x + c) + 2*d*x + (cos(d*x + c) + 1)*log(sin(d*x + c) + 1)
- (cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 8*sin(d*x + c))/(a^3*d*cos(d*
x + c) + a^3*d)
```

giac [A] time = 3.67, size = 63, normalized size = 1.37

$$\frac{\frac{dx+c}{a^3} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} - \frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^3 + log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^3 - 4*tan(1/2*d*x + 1/2*c)/a^3)/d
```

maple [A] time = 0.60, size = 76, normalized size = 1.65

$$-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3 d} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3 d} + \frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/d/a^3*tan(1/2*d*x+1/2*c)-1/a^3/d*ln(tan(1/2*d*x+1/2*c)-1)+1/a^3/d*ln(tan
(1/2*d*x+1/2*c)+1)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.51, size = 98, normalized size = 2.13

$$\frac{\frac{2\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3} - \frac{4\,\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^3 - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3 - 4*sin(d
*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 1.16, size = 37, normalized size = 0.80

$$\frac{x}{a^3} + \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right) - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

```
[Out] x/a^3 + (2*atanh(tan(c/2 + (d*x)/2)) - 4*tan(c/2 + (d*x)/2))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.100 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$-\frac{x}{a^3}+\frac{2\tan(c+dx)}{a^2d(a\sec(c+dx)+a)}-\frac{\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^3}$$

```
[Out] -x/a^3+2*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))-1/3*tan(d*x+c)^3/d/(a+a*sec(d*x+
c))^3
```

Rubi [A] time = 0.17, antiderivative size = 71, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3888, 3886, 3473, 8, 2606, 2607, 30}

$$\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (3*Csc[c +
 d*x])/(a^3*d) - (4*Csc[c + d*x]^3)/(3*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\int \cot^4(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(-a^3\cot^4(c+dx)+3a^3\cot^3(c+dx)\csc(c+dx)-3a^3\cot^2(c+dx)\csc^2(c+dx\right.}{a^6}\\
&= -\frac{\int \cot^4(c+dx)\,dx}{a^3}+\frac{\int \cot(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^3(c+dx)\csc(c+dx}{a^3}\\
&= \frac{\cot^3(c+dx)}{3a^3d}+\frac{\int \cot^2(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int x^2\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\,\text{Subst}\left(\int\right.}{}\\
&= -\frac{\cot(c+dx)}{a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}-\frac{\int 1\,dx}{a^3}\\
&= -\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}
\end{aligned}$$

Mathematica [B] time = 0.40, size = 125, normalized size = 2.08

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(351\sin\left(c+\frac{dx}{2}\right)-277\sin\left(c+\frac{3dx}{2}\right)-3\sin\left(2c+\frac{3dx}{2}\right)+180dx\cos\left(c+\frac{dx}{2}\right)+60dx\right.}{480a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/480*(Sec[c/2]*Sec[(c + d*x)/2]^3*(180*d*x*Cos[(d*x)/2] + 180*d*x*Cos[c +
 (d*x)/2] + 60*d*x*Cos[c + (3*d*x)/2] + 60*d*x*Cos[2*c + (3*d*x)/2] - 471*S
in[(d*x)/2] + 351*Sin[c + (d*x)/2] - 277*Sin[c + (3*d*x)/2] - 3*Sin[2*c + (
3*d*x)/2]))/(a^3*d)
```

fricas [A] time = 0.60, size = 80, normalized size = 1.33

$$-\frac{3\,dx\cos\left(dx+c\right)^2+6\,dx\cos\left(dx+c\right)+3\,dx-\left(7\,\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)}{3\left(a^3d\cos\left(dx+c\right)^2+2\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (7*cos(d*x + c) +
 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.79, size = 50, normalized size = 0.83

$$-\frac{\frac{3\,(dx+c)}{a^3}+\frac{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^9}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^3 + (a^6*tan(1/2*d*x + 1/2*c)^3 - 6*a^6*tan(1/2*d*x + 1
/2*c))/a^9)/d
```

maple [A] time = 0.58, size = 56, normalized size = 0.93

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d\,a^3}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/3/d/a^3*tan(1/2*d*x+1/2*c)^3+2/d/a^3*tan(1/2*d*x+1/2*c)-2/d/a^3*arctan(t
an(1/2*d*x+1/2*c))
```

maxima [A] time = 0.45, size = 72, normalized size = 1.20

$$\frac{\frac{\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{6\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*((6*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/a^3 - 6*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 1.16, size = 35, normalized size = 0.58

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,d\,x}{3\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^3 - 6*tan(c/2 + (d*x)/2) + 3*d*x)/(3*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.101 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=143

$$\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{11\csc^5(c+dx)}{5a^3d}-\frac{10\csc^3(c+dx)}{3a^3d}+3$$

```
[Out] -x/a^3-cot(d*x+c)/a^3/d+1/3*cot(d*x+c)^3/a^3/d-1/5*cot(d*x+c)^5/a^3/d+4/7*c
ot(d*x+c)^7/a^3/d+3*csc(d*x+c)/a^3/d-10/3*csc(d*x+c)^3/a^3/d+11/5*csc(d*x+c
)^5/a^3/d-4/7*csc(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.24, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{11\csc^5(c+dx)}{5a^3d}-\frac{10\csc^3(c+dx)}{3a^3d}+3$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + Cot[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]^5
/(5*a^3*d) + (4*Cot[c + d*x]^7)/(7*a^3*d) + (3*Csc[c + d*x])/(a^3*d) - (10*
Csc[c + d*x]^3)/(3*a^3*d) + (11*Csc[c + d*x]^5)/(5*a^3*d) - (4*Csc[c + d*x]
^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(-a^3\cot^8(c+dx)+3a^3\cot^7(c+dx)\csc(c+dx)-3a^3\cot^6(c+dx)\csc^2(c+dx)+\right.}{a^6}\\
&= -\frac{\int \cot^8(c+dx)\,dx}{a^3}+\frac{\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^7(c+dx)\csc(c+dx)}{a^3}\\
&= \frac{\cot^7(c+dx)}{7a^3d}+\frac{\int \cot^6(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\,\text{S}}{}\\
&= -\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\int \cot^4(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,\right.}{a^3d}\\
&= \frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{10\csc^3(c+dx)}{3a^3d}+\frac{1}{}\\
&= -\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{10\,\text{c}}{}\\
&= -\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d}-
\end{aligned}$$

Mathematica [A] time = 1.23, size = 252, normalized size = 1.76

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)\sec^7\left(\frac{1}{2}(c+dx)\right)(-23282\sin(c+dx)-23282\sin(2(c+dx))-9978\sin(3(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*Csc[(c + d*x)/2]*Sec[c/2]*Sec[(c + d*x)/2]^7*(-5880*d*x*Cos[d*x]
+ 5880*d*x*Cos[2*c + d*x] - 5880*d*x*Cos[c + 2*d*x] + 5880*d*x*Cos[3*c + 2*
d*x] - 2520*d*x*Cos[2*c + 3*d*x] + 2520*d*x*Cos[4*c + 3*d*x] - 420*d*x*Cos[
3*c + 4*d*x] + 420*d*x*Cos[5*c + 4*d*x] + 4200*Sin[c] + 11032*Sin[d*x] - 23
```

```
282*Sin[c + d*x] - 23282*Sin[2*(c + d*x)] - 9978*Sin[3*(c + d*x)] - 1663*Si
n[4*(c + d*x)] + 13720*Sin[2*c + d*x] + 15512*Sin[c + 2*d*x] + 9240*Sin[3*c
 + 2*d*x] + 8088*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] + 1768*Sin[3*c +
4*d*x]))/(215040*a^3*d)
```

fricas [A] time = 0.75, size = 142, normalized size = 0.99

$$-\frac{221\cos(dx+c)^4+348\cos(dx+c)^3-25\cos(dx+c)^2+105\left(dx\cos(dx+c)^3+3\,dx\cos(dx+c)^2+3\,dx\cos\right.}{105\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/105*(221*cos(d*x + c)^4 + 348*cos(d*x + c)^3 - 25*cos(d*x + c)^2 + 105*(
d*x*cos(d*x + c)^3 + 3*d*x*cos(d*x + c)^2 + 3*d*x*cos(d*x + c) + d*x)*sin(d
*x + c) - 303*cos(d*x + c) - 136)/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))
```

giac [A] time = 0.49, size = 99, normalized size = 0.69

$$-\frac{\frac{1680\,(dx+c)}{a^3}+\frac{105}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{15\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-126\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+560\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2730\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{21}}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/1680*(1680*(d*x + c)/a^3 + 105/(a^3*tan(1/2*d*x + 1/2*c)) + (15*a^18*tan
(1/2*d*x + 1/2*c)^7 - 126*a^18*tan(1/2*d*x + 1/2*c)^5 + 560*a^18*tan(1/2*d*
x + 1/2*c)^3 - 2730*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

maple [A] time = 0.80, size = 113, normalized size = 0.79

$$-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{112a^3d}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^3}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d\,a^3}+\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}-\frac{1}{16a^3d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\right.\right.}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/112/a^3/d*tan(1/2*d*x+1/2*c)^7+3/40/d/a^3*tan(1/2*d*x+1/2*c)^5-1/3/d/a^3
*tan(1/2*d*x+1/2*c)^3+13/8/d/a^3*tan(1/2*d*x+1/2*c)-1/16/a^3/d/tan(1/2*d*x+
1/2*c)-2/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.43, size = 133, normalized size = 0.93

$$\frac{\frac{\frac{2730\sin(dx+c)}{\cos(dx+c)+1}-\frac{560\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{126\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}-\frac{3360\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{105\,(\cos(dx+c)+1)}{a^3\sin(dx+c)}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1680*((2730*sin(d*x + c)/(cos(d*x + c) + 1) - 560*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 126*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/a^3 - 3360*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 105*(cos(d*x + c) + 1)/(a^3*sin(d*x + c)))/d
```

mupad [B] time = 1.70, size = 91, normalized size = 0.64

$$-\frac{x}{a^3}-\frac{\frac{221\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{105}-\frac{268\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{105}+\frac{257\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{420}-\frac{31\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{280}+\frac{1}{112}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] - x/a^3 - ((257*cos(c/2 + (d*x)/2)^4)/420 - (31*cos(c/2 + (d*x)/2)^2)/280 -
 (268*cos(c/2 + (d*x)/2)^6)/105 + (221*cos(c/2 + (d*x)/2)^8)/105 + 1/112)/(
a^3*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.102 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=177

$$\frac{4\cot^9(c+dx)}{9a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{15\csc^7(c+dx)}{7a^3d}-\frac{21\,\mathrm{cs}}{}$$

```
[Out] x/a^3+cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+1/5*cot(d*x+c)^5/a^3/d-1/7*co
t(d*x+c)^7/a^3/d+4/9*cot(d*x+c)^9/a^3/d-3*csc(d*x+c)/a^3/d+13/3*csc(d*x+c)^
3/a^3/d-21/5*csc(d*x+c)^5/a^3/d+15/7*csc(d*x+c)^7/a^3/d-4/9*csc(d*x+c)^9/a^
3/d
```

Rubi [A] time = 0.25, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$\frac{4\cot^9(c+dx)}{9a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{15\csc^7(c+dx)}{7a^3d}-\frac{21\,\mathrm{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 + Cot[c + d*x]/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + Cot[c + d*x]^5/(5
*a^3*d) - Cot[c + d*x]^7/(7*a^3*d) + (4*Cot[c + d*x]^9)/(9*a^3*d) - (3*Csc[
c + d*x])/(a^3*d) + (13*Csc[c + d*x]^3)/(3*a^3*d) - (21*Csc[c + d*x]^5)/(5*
a^3*d) + (15*Csc[c + d*x]^7)/(7*a^3*d) - (4*Csc[c + d*x]^9)/(9*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \cot^{10}(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^{10}(c+dx)+3a^3\cot^9(c+dx)\csc(c+dx)-3a^3\cot^8(c+dx)\csc^2(c+dx)+\right.}{a^6}$$
$$= -\frac{\int \cot^{10}(c+dx)\,dx}{a^3}+\frac{\int \cot^7(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^9(c+dx)\csc(c+dx)}{a^3}$$
$$= \frac{\cot^9(c+dx)}{9a^3d}+\frac{\int \cot^8(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\,\text{S}}{}$$
$$= -\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{\int \cot^6(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int\left(-x^2+3x^4-3x^6+\right.\right.}{a^3d}$$
$$= \frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(c+dx)}{a^3d}+\frac{13\csc^3(c+dx)}{3a^3d}-\frac{2}{}$$
$$= -\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(c+dx)}{a^3d}+\frac{13}{}$$
$$= \frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(}{a}$$
$$= \frac{x}{a^3}+\frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3}{}$$

Mathematica [B] time = 1.18, size = 366, normalized size = 2.07

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^3(2(c+dx))(675036\sin(c+dx)+506277\sin(2(c+dx))-37502\sin(3(c+dx))-225012\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*Csc[2*(c + d*x)]^3*Sec[c/2]*(181440*d*x*Cos[d*x] - 181440*d*x*Cos
[2*c + d*x] + 136080*d*x*Cos[c + 2*d*x] - 136080*d*x*Cos[3*c + 2*d*x] - 100
```

```
80*d*x*Cos[2*c + 3*d*x] + 10080*d*x*Cos[4*c + 3*d*x] - 60480*d*x*Cos[3*c +
4*d*x] + 60480*d*x*Cos[5*c + 4*d*x] - 30240*d*x*Cos[4*c + 5*d*x] + 30240*d*
x*Cos[6*c + 5*d*x] - 5040*d*x*Cos[5*c + 6*d*x] + 5040*d*x*Cos[7*c + 6*d*x]
- 169344*Sin[c] - 338112*Sin[d*x] + 675036*Sin[c + d*x] + 506277*Sin[2*(c +
 d*x)] - 37502*Sin[3*(c + d*x)] - 225012*Sin[4*(c + d*x)] - 112506*Sin[5*(c
 + d*x)] - 18751*Sin[6*(c + d*x)] - 431424*Sin[2*c + d*x] - 375552*Sin[c +
2*d*x] - 201600*Sin[3*c + 2*d*x] - 41248*Sin[2*c + 3*d*x] + 84000*Sin[4*c +
 3*d*x] + 155712*Sin[3*c + 4*d*x] + 100800*Sin[5*c + 4*d*x] + 98016*Sin[4*c
 + 5*d*x] + 30240*Sin[6*c + 5*d*x] + 21376*Sin[5*c + 6*d*x]))/(80640*a^3*d*
(1 + Sec[c + d*x])^3)
```

fricas [A] time = 1.21, size = 216, normalized size = 1.22

$$\frac{668\,\cos\left(dx+c\right)^{6}+1059\,\cos\left(dx+c\right)^{5}-573\,\cos\left(dx+c\right)^{4}-1813\,\cos\left(dx+c\right)^{3}-393\,\cos\left(dx+c\right)^{2}+315\,\left(dx\right.}{315\,\left(a^{3}d\cos\left(dx+c\right)^{5}+3\,a^{3}d\cos\left(dx+c\right)^{4}+2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/315*(668*cos(d*x + c)^6 + 1059*cos(d*x + c)^5 - 573*cos(d*x + c)^4 - 1813
*cos(d*x + c)^3 - 393*cos(d*x + c)^2 + 315*(d*x*cos(d*x + c)^5 + 3*d*x*cos(
d*x + c)^4 + 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c)^2 - 3*d*x*cos(d*x +
c) - d*x)*sin(d*x + c) + 789*cos(d*x + c) + 368)/((a^3*d*cos(d*x + c)^5 + 3
*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3
*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

giac [A] time = 0.47, size = 131, normalized size = 0.74

$$\frac{\frac{20160\,(dx+c)}{a^{3}}+\frac{105\left(24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}-\frac{35\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-360\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+1827\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6720\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{a^{27}}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/20160*(20160*(d*x + c)/a^3 + 105*(24*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*tan
(1/2*d*x + 1/2*c)^3) - (35*a^24*tan(1/2*d*x + 1/2*c)^9 - 360*a^24*tan(1/2*d
*x + 1/2*c)^7 + 1827*a^24*tan(1/2*d*x + 1/2*c)^5 - 6720*a^24*tan(1/2*d*x +
1/2*c)^3 + 31185*a^24*tan(1/2*d*x + 1/2*c))/a^27)/d
```

maple [A] time = 0.88, size = 151, normalized size = 0.85

$$-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{576a^{3}d}+\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56a^{3}d}-\frac{29\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320d\,a^{3}}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d\,a^{3}}-\frac{99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d\,a^{3}}-\frac{1}{192a^{3}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/576/a^3/d*tan(1/2*d*x+1/2*c)^9+1/56/a^3/d*tan(1/2*d*x+1/2*c)^7-29/320/d/
a^3*tan(1/2*d*x+1/2*c)^5+1/3/d/a^3*tan(1/2*d*x+1/2*c)^3-99/64/d/a^3*tan(1/2
*d*x+1/2*c)-1/192/a^3/d/tan(1/2*d*x+1/2*c)^3+1/8/a^3/d/tan(1/2*d*x+1/2*c)+2
/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.43, size = 177, normalized size = 1.00

$$-\frac{\frac{\frac{31185\sin(dx+c)}{\cos(dx+c)+1}-\frac{6720\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1827\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{360\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{a^{3}}-\frac{40320\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}-\frac{105\left(\frac{24\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1\right)(\cos(dx+\right.}{a^{3}\sin(dx+c)^{3}}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/20160*((31185*sin(d*x + c)/(cos(d*x + c) + 1) - 6720*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 1827*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 360*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^3 -
40320*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 105*(24*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^3*sin(d*x + c)^3))/d
```

mupad [B] time = 2.08, size = 205, normalized size = 1.16

$$\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(c+d\,x\right)-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(c+d\,x\right)}{a^3\,d\,\left({\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9-{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\right)}-\frac{\frac{668\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}}{315}-\frac{983\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{210}+\frac{346\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}}{105}-\frac{2291\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{2520}}{a^3\,d\,\left({\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\cos\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (cos(c/2 + (d*x)/2)^9*(c + d*x) - cos(c/2 + (d*x)/2)^11*(c + d*x))/(a^3*d*(
cos(c/2 + (d*x)/2)^9 - cos(c/2 + (d*x)/2)^11)) - ((173*cos(c/2 + (d*x)/2)^4
)/840 - (19*cos(c/2 + (d*x)/2)^2)/672 - (2291*cos(c/2 + (d*x)/2)^6)/2520 +
(346*cos(c/2 + (d*x)/2)^8)/105 - (983*cos(c/2 + (d*x)/2)^10)/210 + (668*cos
(c/2 + (d*x)/2)^12)/315 + 1/576)/(a^3*d*(cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*
x)/2) - cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.103 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=215

$$\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{19\csc\ldots}{9\ldots}$$

```
[Out] -x/a^3-cot(d*x+c)/a^3/d+1/3*cot(d*x+c)^3/a^3/d-1/5*cot(d*x+c)^5/a^3/d+1/7*c
ot(d*x+c)^7/a^3/d-1/9*cot(d*x+c)^9/a^3/d+4/11*cot(d*x+c)^11/a^3/d+3*csc(d*x
+c)/a^3/d-16/3*csc(d*x+c)^3/a^3/d+34/5*csc(d*x+c)^5/a^3/d-36/7*csc(d*x+c)^7
/a^3/d+19/9*csc(d*x+c)^9/a^3/d-4/11*csc(d*x+c)^11/a^3/d
```

Rubi [A] time = 0.28, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3888, 3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{19\csc\ldots}{9\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + Cot[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]^5
/(5*a^3*d) + Cot[c + d*x]^7/(7*a^3*d) - Cot[c + d*x]^9/(9*a^3*d) + (4*Cot[c
 + d*x]^11)/(11*a^3*d) + (3*Csc[c + d*x])/(a^3*d) - (16*Csc[c + d*x]^3)/(3*
a^3*d) + (34*Csc[c + d*x]^5)/(5*a^3*d) - (36*Csc[c + d*x]^7)/(7*a^3*d) + (1
9*Csc[c + d*x]^9)/(9*a^3*d) - (4*Csc[c + d*x]^11)/(11*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \cot^{12}(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-a^3\cot^{12}(c+dx)+3a^3\cot^{11}(c+dx)\csc(c+dx)-3a^3\cot^{10}(c+dx)\csc^2(c+dx\right.}{a^6}$$

$$= -\frac{\int \cot^{12}(c+dx)\,dx}{a^3}+\frac{\int \cot^9(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^{11}(c+dx)\csc(c+dx}{a^3}$$

$$= \frac{\cot^{11}(c+dx)}{11a^3d}+\frac{\int \cot^{10}(c+dx)\,dx}{a^3}-\frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^4\,dx,x,\csc(c+dx)\right)}{a^3d}-$$

$$= -\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{\int \cot^8(c+dx)\,dx}{a^3}-\frac{\mathrm{Subst}\left(\int \left(x^2-4x^4+6x^6-\right.\right.}{a}$$

$$= \frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{16\csc^3(c+dx)}{3a^3d}+$$

$$= -\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{3\csc(c+dx)}{a^3d}-$$

$$= \frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+$$

$$= -\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+$$

$$= -\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+$$

Mathematica [A] time = 3.73, size = 394, normalized size = 1.83

$$-\frac{\tan\left(\frac{c}{2}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(315\sec^{10}\left(\frac{1}{2}(c+dx)\right)-5425\sec^8\left(\frac{1}{2}(c+dx)\right)+41320\sec^6\left(\frac{1}{2}(c+dx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/110880*(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(231*(-25 + 28*Cos[c + d*x])*C
ot[c/2]^2*Csc[(c + d*x)/2]^4 + 561145*Sec[(c + d*x)/2]^2 - 184650*Sec[(c +
d*x)/2]^4 + 41320*Sec[(c + d*x)/2]^6 - 5425*Sec[(c + d*x)/2]^8 + 315*Sec[(c
 + d*x)/2]^10 - 1736335*Csc[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 561145*Csc
[c/2]*Sec[(c + d*x)/2]^3*Sin[(d*x)/2] - 184650*Csc[c/2]*Sec[(c + d*x)/2]^5*
Sin[(d*x)/2] + 41320*Csc[c/2]*Sec[(c + d*x)/2]^7*Sin[(d*x)/2] - 5425*Csc[c/
2]*Sec[(c + d*x)/2]^9*Sin[(d*x)/2] + 315*Csc[c/2]*Sec[(c + d*x)/2]^11*Sin[(
d*x)/2] + 6468*Csc[c/2]^3*Csc[(c + d*x)/2]^3*Sin[c]*Sin[(d*x)/2] + 231*Cot[
c/2]*(3840*d*x - Csc[c/2]*Csc[(c + d*x)/2]*(743 + 3*Csc[(c + d*x)/2]^4)*Sin
[(d*x)/2]))*Tan[c/2])/(a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [**A**] time = 1.16, size = 282, normalized size = 1.31

$$-\frac{7453\cos(dx+c)^8+11964\cos(dx+c)^7-11866\cos(dx+c)^6-30542\cos(dx+c)^5+90\cos(dx+c)^4+\cdots}{3465\left(a^3d\cos(dx+c)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3465*(7453*cos(d*x + c)^8 + 11964*cos(d*x + c)^7 - 11866*cos(d*x + c)^6
- 30542*cos(d*x + c)^5 + 90*cos(d*x + c)^4 + 26438*cos(d*x + c)^3 + 8539*co
s(d*x + c)^2 + 3465*(d*x*cos(d*x + c)^7 + 3*d*x*cos(d*x + c)^6 + d*x*cos(d*
x + c)^5 - 5*d*x*cos(d*x + c)^4 - 5*d*x*cos(d*x + c)^3 + d*x*cos(d*x + c)^2
 + 3*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 7671*cos(d*x + c) - 3712)/((a^3
*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(d*x + c)^5 - 5*a^3*d
*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)*sin(d*x + c))
```

giac [**A**] time = 1.47, size = 160, normalized size = 0.74

$$-\frac{\frac{887040\,(dx+c)}{a^3}+\frac{231\left(690\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-50\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}+\frac{5\left(63\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-770\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+4554\,a^{30}\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}{\cdots}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/887040*(887040*(d*x + c)/a^3 + 231*(690*tan(1/2*d*x + 1/2*c)^4 - 50*tan(
1/2*d*x + 1/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) + 5*(63*a^30*tan(1/2*d
*x + 1/2*c)^11 - 770*a^30*tan(1/2*d*x + 1/2*c)^9 + 4554*a^30*tan(1/2*d*x +
1/2*c)^7 - 18018*a^30*tan(1/2*d*x + 1/2*c)^5 + 59136*a^30*tan(1/2*d*x + 1/2
*c)^3 - 264726*a^30*tan(1/2*d*x + 1/2*c))/a^33)/d
```

maple [**A**] time = 1.02, size = 189, normalized size = 0.88

$$-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2816a^3d}+\frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1152a^3d}-\frac{23\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{896a^3d}+\frac{13\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d\,a^3}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d\,a^3}+\frac{191\tan\left(\frac{dx}{2}+\cdots\right)}{128d\,a^3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/2816/a^3/d*tan(1/2*d*x+1/2*c)^11+5/1152/a^3/d*tan(1/2*d*x+1/2*c)^9-23/89
6/a^3/d*tan(1/2*d*x+1/2*c)^7+13/128/d/a^3*tan(1/2*d*x+1/2*c)^5-1/3/d/a^3*ta
n(1/2*d*x+1/2*c)^3+191/128/d/a^3*tan(1/2*d*x+1/2*c)-1/1280/a^3/d/tan(1/2*d*
```

```
x+1/2*c)^5+5/384/a^3/d/tan(1/2*d*x+1/2*c)^3-23/128/a^3/d/tan(1/2*d*x+1/2*c)
-2/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.56, size = 218, normalized size = 1.01

$$\frac{\frac{5\left(\frac{264726 \sin(dx+c)}{\cos(dx+c)+1} - \frac{59136 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{18018 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{4554 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{770 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{63 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^3} - \frac{1774080 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{231\left(\frac{50 \sin}{(\cos(dx}\right.}{}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/887040*(5*(264726*sin(d*x + c)/(cos(d*x + c) + 1) - 59136*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 + 18018*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4554*sin
(d*x + c)^7/(cos(d*x + c) + 1)^7 + 770*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
- 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^3 - 1774080*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3 + 231*(50*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
690*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^3*sin(
d*x + c)^5))/d
```

mupad [B] time = 3.24, size = 254, normalized size = 1.18

$$-\frac{693 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16} + 315 \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16} - 3850 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} + 22770 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} \sin\left(\frac{c}{2}+\frac{d}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(693*cos(c/2 + (d*x)/2)^16 + 315*sin(c/2 + (d*x)/2)^16 - 3850*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 22770*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)
/2)^12 - 90090*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 295680*cos(c/2
+ (d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 1323630*cos(c/2 + (d*x)/2)^10*sin(c/2 +
 (d*x)/2)^6 + 159390*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 11550*cos
(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 887040*cos(c/2 + (d*x)/2)^11*sin(
c/2 + (d*x)/2)^5*(c + d*x))/(887040*a^3*d*cos(c/2 + (d*x)/2)^11*sin(c/2 + (
d*x)/2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.104 $\int (a + a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=310

$$\frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{ae^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+d}\right.}{2\sqrt{2}d}$$

```
[Out] 1/2*a*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/2*
a*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/4*a*e^
(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)
+1/4*a*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d*2^(1/2)-6/5*a*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x
)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d*x+2*c
)^(1/2)-6/5*a*e*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d+2/15*e*(5*a+3*a*sec(d*x+c
))*(e*tan(d*x+c))^(3/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3881, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{ae^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+d}\right.}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d)
- (a*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
d*x]]])/(2*Sqrt[2]*d) + (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (6*a*e^2*Cos[c + d*x]*EllipticE
[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*c + 2*d*x]]) - (6
*a*e*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*d) + (2*e*(5*a + 3*a*Sec[c + d
*x])*(e*Tan[c + d*x])^(3/2))/(15*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a + a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx = \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} - \frac{1}{5}\left(2e^2\right)\int\left(\frac{5a}{2}+\frac{3}{2}a\,\mathrm{se}\right.$$

$$= \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} - \frac{1}{5}\left(3ae^2\right)\int \sec(c+dx$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan}{15d}$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan}{15d}$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan}{15d}$$

$$= \frac{6ae^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} - \frac{6ae\cos(c+d}{}$$

$$= -\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d} + \frac{ae^{5/2}\log}{}$$

$$= \frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} - \frac{ae^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} -$$

Mathematica [C] time = 2.39, size = 186, normalized size = 0.60

$$a(\cos(c+dx)+1)\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(e\tan(c+dx))^{5/2}\left(\frac{24\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(c+dx)\right)}{\sqrt{\sec^2(c+dx)}} - 36\cos^2(c+dx)+20\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*(12 + 20*Cos[c + d*x]
 - 36*Cos[c + d*x]^2 + (24*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2
])/Sqrt[Sec[c + d*x]^2] + 15*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Cot[c + d*
x]^2*Sqrt[Sin[2*(c + d*x)]] + 15*Cot[c + d*x]^2*Log[Cos[c + d*x] + Sin[c +
d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]])*(e*Tan[c + d*x])^(5/
2))/(60*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx + c) + a) (e \tan (dx + c))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2), x)

maple [C] time = 1.90, size = 1495, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x)

[Out] 1/30*a/d*(-1+cos(d*x+c))^2*(15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+8*2^(1/2)*cos(d*x+c)^3-24*cos(d*x+c)^2*2^(1/2)+10*cos(d*x+c)*2^(1/2)+6*2^(1/2))*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(5/2)/sin(d*x+c)^7*2^(1/2)

maxima [A] time = 0.46, size = 177, normalized size = 0.57

$$-\frac{\left(3\,e^4\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{e}+2\,\sqrt{e\tan(dx+c)}\right)}{2\,\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{e}-2\,\sqrt{e\tan(dx+c)}\right)}{2\,\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\,\log\left(e\tan(dx+c)+\sqrt{2}\,\sqrt{e\tan(dx+c)}\,\sqrt{e}+e\right)}{\sqrt{e}}\right.\right.}{12\,de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*e^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(
d*x + c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e
) - 2*sqrt(e*tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(e*tan(d*x + c) +
 sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e) + sqrt(2)*log(e*tan(d*x
+ c) - sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e)) - 8*(e*tan(d*x +
c))^(3/2)*e^2)*a/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\tan(c+d\,x))^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.105 $\int (a + a\sec(c + dx))(e\tan(c + dx))^{3/2}\,dx$

Optimal. Leaf size=282

$$\frac{ae^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right.}{2\sqrt{2}d}$$

```
[Out] 1/2*a*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/2*
a*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/4*a*e^
(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)
-1/4*a*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d*2^(1/2)+1/3*a*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF
(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(d*x+c)
)^(1/2)+2/3*e*(3*a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3881, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{ae^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right.}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d)
- (a*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) + (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
d*x]]])/(2*Sqrt[2]*d) - (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a*e^2*EllipticF[c - Pi/4 + d*x
, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan[c + d*x]]) + (2*e
*(3*a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
```

```
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx = \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} - \frac{1}{3}\left(2e^2\right)\int \frac{\frac{3a}{2}+\frac{1}{2}a\sec(c+d}{\sqrt{e\tan(c+dx)}}$$
$$= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} - \frac{1}{3}\left(ae^2\right)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,d$$
$$= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} - \frac{\left(ae^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx,\right.}{d}$$
$$= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} - \frac{\left(2ae^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\right.}{d}$$
$$= -\frac{ae^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2e(3a+a\sec(c}{}$$
$$= -\frac{ae^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2e(3a+a\sec(c}{}$$
$$= \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d} - \frac{ae^{3/2}\log\left(\sqrt{\right.}{}$$
$$= \frac{ae^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} - \frac{ae^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} + \frac{a}{}$$

Mathematica [C] time = 2.11, size = 214, normalized size = 0.76

$$-\frac{ae\cos(2(c+dx))\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec^2(c+dx)}\sqrt{e\tan(c+dx)}\left(\sqrt{\sec^2(c+dx)}\left(12\sin(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/12*(a*e*Cos[2*(c + d*x)]*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c +
d*x]^2]*Sqrt[e*Tan[c + d*x]]*(4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*S
qrt[Tan[c + d*x]]], -1]*Sqrt[Tan[c + d*x]] + Sqrt[Sec[c + d*x]^2]*(12*Sin[c
 + d*x] + 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] - 3*
Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d
*x)]] + 4*Tan[c + d*x])))/(d*(-1 + Tan[c + d*x]^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2), x)
```

maple [C] time = 1.80, size = 688, normalized size = 2.44

$$\frac{a(-1+\cos(dx+c))\left(-3i\cos(dx+c)\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/6*a/d*(-1+cos(d*x+c))*(-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-4*cos(d*x+c)*sin(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*
2^(1/2)-2*2^(1/2))*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(3/2)/sin(d*x
+c)^5*2^(1/2)
```

maxima [A] time = 0.50, size = 171, normalized size = 0.61

$$-\frac{\left(2\sqrt{2}e^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)+2\sqrt{2}e^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)+\sqrt{2}e^{\frac{5}{2}}\log\left(e\tan(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*e^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(
d*x + c)))/sqrt(e)) + 2*sqrt(2)*e^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e
) - 2*sqrt(e*tan(d*x + c)))/sqrt(e)) + sqrt(2)*e^(5/2)*log(e*tan(d*x + c) +
 sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e) - sqrt(2)*e^(5/2)*log(e*tan(d*x
+ c) - sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e) - 8*sqrt(e*tan(d*x + c))*e
^2)*a/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\tan(c+d\,x))^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (e\tan{(c+dx)})^{\frac{3}{2}}\,dx+\int (e\tan{(c+dx)})^{\frac{3}{2}}\sec{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**(3/2), x) + Integral((e*tan(c + d*x))**(3/2)*
sec(c + d*x), x))
```

3.106 $\int (a + a\sec(c + dx))\sqrt{e\tan(c + dx)}\,dx$

Optimal. Leaf size=272

$$-\frac{a\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+}\right.}{2\sqrt{2}d}$$

```
[Out] -1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/4*a*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/d*2^(1/2
)-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)
/d*2^(1/2)+2*a*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ell
ipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d*x+2*c)^(1/
2)+2*a*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d/e
```

Rubi [A] time = 0.24, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$-\frac{a\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+}\right.}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
)) + (a*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*d) + (a*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*d) - (a*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a*Cos[c + d*x]*EllipticE[
c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*a*
Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*e)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
```

```
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx = a\int \sqrt{e\tan(c+dx)}\,dx + a\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx$$

$$= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - (2a)\int \cos(c+dx)\sqrt{e\tan(c+dx)}$$

$$= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} + \frac{(2ae)\,\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan}\right.}{d}$$

$$= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{(ae)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(}\right.}{d}$$

$$= -\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} + \frac{2a\cos(c+dx)(}{d}$$

$$= \frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a\sqrt{e}\log\left(\right.}{}$$

$$= -\frac{a\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} +$$

Mathematica [C] time = 1.57, size = 182, normalized size = 0.67

$$-\frac{a(\cos(c+dx)+1)\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\left(8\tan^2(c+dx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(c+dx)\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -1/12*(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[e*Tan[c +
d*x]]*(3*Sqrt[Sec[c + d*x]^2]*(-4*Sin[c + d*x]^2 + ArcSin[Cos[c + d*x] - Si
n[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt
[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) + 8*Hypergeometric2F1[3/4, 3/2,
 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^2))/(d*Sqrt[Sec[c + d*x]^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)\sqrt{e\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c)), x)
```

maple [C] time = 1.97, size = 1409, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/2*a/d*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*
(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-I*Elli
pticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*
cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+I*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)-2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2
))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+4*Elliptic
E(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)-2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+4*EllipticE(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)-2*cos(d*x+c)*2^(1/2)+2*2^(1/2))/sin(d*x+c)^5*2^(1/2
)
```

maxima [A] time = 0.46, size = 154, normalized size = 0.57

$$\frac{ae\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log\left(e\tan(dx+c)+\sqrt{2}\sqrt{e\tan(dx+c)}\sqrt{e}+e\right)}{\sqrt{e}}+\frac{\sqrt{2}\,\mathrm{l}}{}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(d*x +
 c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) - 2
```

```
*sqrt(e*tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(e*tan(d*x + c) + sqrt
(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e) + sqrt(2)*log(e*tan(d*x + c)
- sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\tan(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sqrt{e\tan(c+dx)}\,dx+\int \sqrt{e\tan(c+dx)}\,\sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*tan(c + d*x)), x) + Integral(sqrt(e*tan(c + d*x))*sec(c
+ d*x), x))
```

3.107 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=244

$$-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}}+\cdots$$

```
[Out] -1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)-1/4*a*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)/e^(1/2
)+1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/
2)/e^(1/2)-a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+
1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(d*x+c))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[e]
)) + (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt
[e]) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]
])/(2*Sqrt[2]*d*Sqrt[e]) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a*EllipticF[c - Pi/4 + d*x,
 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx + a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$$
$$= \frac{(ae)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{d} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,d}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$
$$= \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} + \frac{\left(a\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+}}}{\sqrt{e\tan(c+dx)}}$$
$$= \frac{aF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d}$$
$$= \frac{aF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c}\right.}{2d}$$
$$= -\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\right.}{2\sqrt{2}d\sqrt{e}}$$
$$= -\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+\right.}{2\sqrt{2}}$$

Mathematica [C] time = 1.76, size = 220, normalized size = 0.90

$$\frac{20a\sin(c+dx)\cos^2\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)F_1\left(\frac{1}{4}\right.}{d\sqrt{e\tan(c+dx)}\left(2(\cos(c+dx)-1)\left(2F_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-F_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\tan^2\left(\frac{1}{2}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (20*a*AppellF1[1/4, 1/2, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[(c + d*x)/2]^2*(1 + Sec[c + d*x])*Sin[c + d*x])/(d*(2*(2*AppellF1[5/4, 1
/2, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[5/4, 3/2, 1
, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 5*Ap
pellF1[1/4, 1/2, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[
c + d*x]))*Sqrt[e*Tan[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*tan(d*x + c)), x)

maple [C] time = 1.80, size = 284, normalized size = 1.16

$$-\frac{a\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\mathrm{Elliptic}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x)

[Out] -1/2*a/d*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2 *I,1/2*2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1 /2+1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/ 2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)) ^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)) ^(1/2)*(-1+cos(d*x+c))/sin(d*x+c)^2/cos(d*x+c)*(1+cos(d*x+c))^2/(e*sin(d*x+ c)/cos(d*x+c))^(1/2)*2^(1/2)

maxima [A] time = 0.45, size = 156, normalized size = 0.64

$$\frac{\left(2\sqrt{2}\sqrt{e}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)+2\sqrt{2}\sqrt{e}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)+\sqrt{2}\sqrt{e}\log\left(e\tan(d\right.\right.}{4\,de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/4*(2*sqrt(2)*sqrt(e)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(d *x + c)))/sqrt(e)) + 2*sqrt(2)*sqrt(e)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) - 2*sqrt(e*tan(d*x + c)))/sqrt(e)) + sqrt(2)*sqrt(e)*log(e*tan(d*x + c) + sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e) - sqrt(2)*sqrt(e)*log(e*tan(d*x + c) - sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e))*a/(d*e)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{e\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(1/2),x)

[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\sqrt{e\tan\left(c+dx\right)}}\,dx+\int \frac{\sec\left(c+dx\right)}{\sqrt{e\tan\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)

[Out] a*(Integral(1/sqrt(e*tan(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*tan(c + d*x)), x))

3.108 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=305

$$\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}}+\frac{a\,l}{}$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/2*
a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/4*a*ln
(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2)*2^(1/2)
+1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2
)*2^(1/2)-2*(a+a*sec(d*x+c))/d/e/(e*tan(d*x+c))^(1/2)+2*a*cos(d*x+c)*(sin(c
+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2)
)*(e*tan(d*x+c))^(1/2)/d/e^2/sin(2*d*x+2*c)^(1/2)+2*a*cos(d*x+c)*(e*tan(d*x
+c))^(3/2)/d/e^3
```

Rubi [A] time = 0.30, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}}+\frac{a\,l}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2))
- (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)
) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/
(2*Sqrt[2]*d*e^(3/2)) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (2*(a + a*Sec[c + d*x]))/(d*e*S
qrt[e*Tan[c + d*x]]) - (2*a*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[
e*Tan[c + d*x]])/(d*e^2*Sqrt[Sin[2*c + 2*d*x]]) + (2*a*Cos[c + d*x]*(e*Tan[
c + d*x])^(3/2))/(d*e^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}+\frac{2\int\left(-\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}-\frac{a\int\sqrt{e\tan(c+dx)}\,dx}{e^2}+\frac{a\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3}-\frac{(2a)\int\cos(c+dx)\sqrt{e\tan(c}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3}-\frac{(2a)\,\mathrm{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx,x,\sqrt{}\right.}{de}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3}+\frac{a\,\mathrm{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\,\mathrm{ta}}\right.}{de}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}-\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}}+\frac{2a\cos(c+}{}$$

$$= -\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{3/2}}+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\right.}{2\sqrt{2}\,de^{3/2}}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}-\frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}\,d}$$

Mathematica [C] time = 2.66, size = 196, normalized size = 0.64

$$-\frac{a(\cos(c+dx)+1)\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\left(8\tan^2(c+dx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(c+dx)\right)+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/12*(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[e*Tan[c +
d*x]]*(3*Sqrt[Sec[c + d*x]^2]*(2 + 4*Cos[c + d*x] + 2*Cos[2*(c + d*x)] - Ar
cSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] - Log[Cos[c + d*x]
 + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) + 8*Hyper
geometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^2))/(d*e^2*Sqrt[S
ec[c + d*x]^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sec(dx + c) + a}{(e \tan(dx + c))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(3/2), x)
```

maple [C] time = 1.78, size = 1390, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x)
[Out] -1/2*a/d*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-Elli
pticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-4*EllipticE(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-I
*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1
/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)-4*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2*EllipticF((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/s
```

```
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)+4*cos(d*x+c)*2^(1/2))*sin(d*x+c)/cos(d*x+c
)^2/(e*sin(d*x+c)/cos(d*x+c))^(3/2)*2^(1/2)
```

maxima [A] time = 0.44, size = 168, normalized size = 0.55

$$-\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log\left(e\tan(dx+c)+\sqrt{2}\sqrt{e\tan(dx+c)}\sqrt{e}+e\right)}{\sqrt{e}}+\frac{\sqrt{2}\,\mathrm{l}}{}\right.}{4\,de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(d*x +
c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) - 2*
sqrt(e*tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(e*tan(d*x + c) + sqrt(
2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e) + sqrt(2)*log(e*tan(d*x + c) -
 sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e) + 8/sqrt(e*tan(d*x + c))
)/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{(e\tan(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*tan(c +
 d*x))**(3/2), x))
```

3.109 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=282

$$\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{5/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}-$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/2*
a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/4*a*ln
(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2)*2^(1/2)
-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2
)*2^(1/2)+1/3*a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos
(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/e^2/(e*tan(d*x+c)
)^(1/2)-2/3*(a+a*sec(d*x+c))/d/e/(e*tan(d*x+c))^(3/2)
```

Rubi [A] time = 0.28, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{5/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2))
- (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)
) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/
(2*Sqrt[2]*d*e^(5/2)) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (2*(a + a*Sec[c + d*x]))/(3*d*e
*(e*Tan[c + d*x])^(3/2)) - (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqr
t[Sin[2*c + 2*d*x]])/(3*d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx = -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3e^2} - \frac{a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{de} - \frac{\left(a\sqrt{\sin(c+dx\right.}}{3e^2\sqrt{\cos(\right.}}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de} - \frac{\left(a\sec(c+dx\right.}{}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{aF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}} - \frac{a\,\text{Subst}\left(\right.}{}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{aF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}} + \frac{a\,\text{Subst}\left(\right.}{}$$

$$= \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{5/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\right.}{2\sqrt{2}\,de^{5/2}}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}} - \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c\right.}{2}$$

Mathematica [C] time = 1.53, size = 200, normalized size = 0.71

$$-\frac{a\csc(c+dx)\sqrt{e\tan(c+dx)}\left(\sqrt{\sec^2(c+dx)}\left(2\cot\left(\frac{1}{2}(c+dx)\right)-3\sqrt{\sin(2(c+dx))}\sin^{-1}(\cos(c+dx)-\sin(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/6*(a*Csc[c + d*x]*(Sqrt[Sec[c + d*x]^2]*(2*Cot[(c + d*x)/2] + 2*Cos[(3*(
c + d*x))/2]*Csc[(c + d*x)/2] - 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[
Sin[2*(c + d*x)]] + 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x
)]]]*Sqrt[Sin[2*(c + d*x)]]) - 4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*
Sqrt[Tan[c + d*x]]], -1]*Sqrt[Tan[c + d*x]])*Sqrt[e*Tan[c + d*x]])/(d*e^3*S
qrt[Sec[c + d*x]^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(5/2), x)
```

maple [C] time = 1.70, size = 648, normalized size = 2.30

$$a(1+\cos(dx+c))^2(-1+\cos(dx+c))\left(3i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(}{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/6*a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(3*I*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*sin(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*sin(d*x+c)-4*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+
2*cos(d*x+c)*2^(1/2))/cos(d*x+c)^3/(e*sin(d*x+c)/cos(d*x+c))^(5/2)/sin(d*x+
c)*2^(1/2)
```

maxima [A] time = 0.43, size = 169, normalized size = 0.60

$$-\frac{a\left(\frac{6\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{6\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{3\sqrt{2}\log\left(e\tan(dx+c)+\sqrt{2}\sqrt{e\tan(dx+c)}\sqrt{e}+e\right)}{e^{\frac{3}{2}}}-3\sqrt{}\right.}{12de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*a*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(d*x +
 c)))/sqrt(e))/e^(3/2) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) - 2
*sqrt(e*tan(d*x + c)))/sqrt(e))/e^(3/2) + 3*sqrt(2)*log(e*tan(d*x + c) + sq
rt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/e^(3/2) - 3*sqrt(2)*log(e*tan(d*x +
 c) - sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/e^(3/2) + 8/(e*tan(d*x + c)
)^(3/2))/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(e\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int \frac{\sec\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**(-5/2), x) + Integral(sec(c + d*x)/(e*tan(c +
 d*x))**(5/2), x))
```

3.110 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{7/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{7/2}}-$$

```
[Out] -1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/4*a*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/2)*2^(1/2
)-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/
2)*2^(1/2)+2/5*(5*a+3*a*sec(d*x+c))/d/e^3/(e*tan(d*x+c))^(1/2)-6/5*a*cos(d*
x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d
*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/e^4/sin(2*d*x+2*c)^(1/2)-2/5*(a+a*sec(d
*x+c))/d/e/(e*tan(d*x+c))^(5/2)-6/5*a*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d/e^5
```

Rubi [A] time = 0.37, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{7/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] -((a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/2)
)) + (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7
/2)) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]
])/(2*Sqrt[2]*d*e^(7/2)) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (2*(a + a*Sec[c + d*x]))/(5*
d*e*(e*Tan[c + d*x])^(5/2)) + (2*(5*a + 3*a*Sec[c + d*x]))/(5*d*e^3*Sqrt[e*
Tan[c + d*x]]) + (6*a*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[
c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]]) - (6*a*Cos[c + d*x]*(e*Tan[c +
d*x])^(3/2))/(5*d*e^5)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx = -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2\int \frac{-\frac{5a}{2}-\frac{3}{2}a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{5e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} + \frac{4\int \left(\frac{5a}{4}-\frac{3}{4}a\sec(c+dx)\right)\sqrt{e\tan(c}}{5e^4}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} - \frac{(3a)\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5e^4}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} - \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5} + $$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} - \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5} + $$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} - \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5} - $$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} + \frac{6a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e}}{5de^4\sqrt{\sin(2c+2dx)}}$$

$$= \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{7/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\right.}{2\sqrt{2}\,de^{7/2}}$$

$$= -\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+\right.}{2\sqrt{2}}$$

Mathematica [C] time = 2.56, size = 254, normalized size = 0.73

$$-\frac{a\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)\left(-8\sin^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec^2(c+dx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] -1/20*(a*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(1 + Sec[c + d*x])*(2*Cot[(c + d
*x)/2] - 19*Sin[c + d*x] + 12*Sin[c + d*x]^2*Tan[(c + d*x)/2] - 8*Hypergeom
```

```
etric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Sqrt[Sec[c + d*x]^2]*Sin[c + d*x]^
2*Tan[(c + d*x)/2] + 5*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c +
d*x)]]*Tan[(c + d*x)/2] + 5*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c
 + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]*Tan[(c + d*x)/2] + 5*Sin[c + d*x]*Tan[(c
+ d*x)/2]^2))/(d*e^3*Sqrt[e*Tan[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sec (dx + c) + a}{(e \tan (dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(7/2), x)
```

maple [C] time = 1.76, size = 1427, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x)
[Out] 1/10*a/d*(5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))-5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Elli
pticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-5*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+6*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-5*I*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)+5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(
1/2))+12*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+5*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))
```

```
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+5*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)-6*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+18*cos(d*x+c
)^2*2^(1/2)-16*cos(d*x+c)*2^(1/2))*sin(d*x+c)^3/(-1+cos(d*x+c))/cos(d*x+c)^
4/(e*sin(d*x+c)/cos(d*x+c))^(7/2)*2^(1/2)
```

maxima [A] time = 0.43, size = 196, normalized size = 0.57

$$\frac{a\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log\left(e\tan(dx+c)+\sqrt{2}\sqrt{e\tan(dx+c)}\sqrt{e}+e\right)}{\sqrt{e}}+\frac{\sqrt{2}\log\left(e\tan(dx+c)-\sqrt{2}\sqrt{\cdots}\right.}{\sqrt{e}}\cdots}{e^2}\cdots\right)}{20de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*a*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(d*x
 + c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) -
 2*sqrt(e*tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(e*tan(d*x + c) + sq
rt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e) + sqrt(2)*log(e*tan(d*x + c
) - sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e))/e^2 + 8*(5*e^2*tan(d
*x + c)^2 - e^2)/((e*tan(d*x + c))^(5/2)*e^2))/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(e\,\tan\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(7/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.111 $\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=366

$$\frac{a^2 e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2 e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{a^2 e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+\right.}{2\sqrt{2}d}$$

```
[Out] 1/2*a^2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/
2*a^2*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/4*
a^2*e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2
^(1/2)+1/4*a^2*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/d*2^(1/2)-12/5*a^2*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c
+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/si
n(2*d*x+2*c)^(1/2)+2/3*a^2*e*(e*tan(d*x+c))^(3/2)/d-12/5*a^2*e*cos(d*x+c)*(
e*tan(d*x+c))^(3/2)/d+4/5*a^2*e*sec(d*x+c)*(e*tan(d*x+c))^(3/2)/d+2/7*a^2*(
e*tan(d*x+c))^(7/2)/d/e
```

Rubi [A] time = 0.43, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3886, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2611, 2613, 2615, 2572, 2639, 2607, 32}

$$\frac{a^2 e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2 e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{a^2 e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+\right.}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a^2*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) - (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]])/(2*Sqrt[2]*d) + (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (12*a^2*e^2*Cos[c + d*x
]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*c + 2*
d*x]]) + (2*a^2*e*(e*Tan[c + d*x])^(3/2))/(3*d) - (12*a^2*e*Cos[c + d*x]*(e
*Tan[c + d*x])^(3/2))/(5*d) + (4*a^2*e*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))
/(5*d) + (2*a^2*(e*Tan[c + d*x])^(7/2))/(7*d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
```

```
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx = \int \left(a^2(e\tan(c+dx))^{5/2}+2a^2\sec(c+dx)(e\tan(c+dx))^{5/2}+a^2\sec^2\right.$$
$$= a^2\int (e\tan(c+dx))^{5/2}\,dx + a^2\int \sec^2(c+dx)(e\tan(c+dx))^{5/2}\,dx +$$
$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} + \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{a^2\,\mathrm{Sub}}{}$$
$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{4a^2e\,\mathrm{s}}{}$$
$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{4a^2e\,\mathrm{s}}{}$$
$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{4a^2e\,\mathrm{s}}{}$$
$$= \frac{12a^2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} + \frac{2a^2e(e\tan(c}{3d}$$
$$= -\frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d} + \frac{a^2e^{5/2}\,\mathrm{lo}}{}$$
$$= \frac{a^2e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} - \frac{a^2e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} -$$

Mathematica [C] time = 6.21, size = 117, normalized size = 0.32

$$\frac{2a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)(e\tan(c+dx))^{3/2}\sec^4\left(\frac{1}{2}\tan^{-1}(\tan(c+dx))\right)\left(-42\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)-35\,{}_2F_1\left(\frac{3}{4},\right.\right.}{105d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*e*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(e*Tan[c + d*x])^
(3/2)*(35 - 42*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 35*Hyper
geometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2] + 42*Sqrt[Sec[c + d*x]^2] + 15*T
an[c + d*x]^2))/(105*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\,(e\tan(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^(5/2), x)
```

maple [C] time = 1.96, size = 1518, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/210*a^2/d*(-1+cos(d*x+c))^2*(-105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*cos(d*x+c)^4+105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*cos(d*x+c)^4+105*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)^4+105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*cos(d*x+c)^3+105*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^4-105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*cos(d*x+c)^3+252*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d
*x+c)^4-504*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4+105*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+105*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+252*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticF(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-504*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticE(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+212*cos(d*x+c)^4*2^(1/2)-336*2^(1/2)*cos
(d*x+c)^3+10*cos(d*x+c)^2*2^(1/2)+84*cos(d*x+c)*2^(1/2)+30*2^(1/2))*(1+cos(
d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(5/2)/sin(d*x+c)^7/cos(d*x+c)*2^(1/2)
```

maxima [A] time = 0.61, size = 200, normalized size = 0.55

$$\frac{\frac{24\,(e\tan(dx+c))^{\frac{7}{2}}a^2}{e} - \frac{7\left(3e^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2}\log\left(e\tan(dx+c)+\sqrt{2}\sqrt{e\tan(dx+c)}\cdots\right.}{\sqrt{e}}\cdots\right.}{e}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/84*(24*(e*tan(d*x + c))^(7/2)*a^2/e - 7*(3*e^4*(2*sqrt(2)*arctan(1/2*sqrt
(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(d*x + c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) - 2*sqrt(e*tan(d*x + c)))/sqrt(e))/sq
rt(e) - sqrt(2)*log(e*tan(d*x + c) + sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) +
 e)/sqrt(e) + sqrt(2)*log(e*tan(d*x + c) - sqrt(2)*sqrt(e*tan(d*x + c))*sqr
t(e) + e)/sqrt(e)) - 8*(e*tan(d*x + c))^(3/2)*e^2)*a^2/e)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\tan (c+d\,x))^{5/2}\left(a+\frac{a}{\cos (c+d\,x)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.112 $\int (a+a\sec(c+dx))^2 (e\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=335

$$\frac{a^2e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{a^2e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+\right.}{2\sqrt{2}d}$$

```
[Out] 1/2*a^2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/
2*a^2*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/4*
a^2*e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2
^(1/2)-1/4*a^2*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/d*2^(1/2)+2/3*a^2*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*
tan(d*x+c))^(1/2)+2*a^2*e*(e*tan(d*x+c))^(1/2)/d+4/3*a^2*e*sec(d*x+c)*(e*ta
n(d*x+c))^(1/2)/d+2/5*a^2*(e*tan(d*x+c))^(5/2)/d/e
```

Rubi [A] time = 0.39, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3886, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2611, 2614, 2573, 2641, 2607, 32}

$$\frac{a^2e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{a^2e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+\right.}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a^2*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) + (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a^2*e^2*EllipticF[c
- Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan[c + d
*x]]) + (2*a^2*e*Sqrt[e*Tan[c + d*x]])/d + (4*a^2*e*Sec[c + d*x]*Sqrt[e*Tan
[c + d*x]])/(3*d) + (2*a^2*(e*Tan[c + d*x])^(5/2))/(5*d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx = \int \left(a^2(e\tan(c+dx))^{3/2}+2a^2\sec(c+dx)(e\tan(c+dx))^{3/2}+a^2\text{se}\right.$$

$$= a^2\int (e\tan(c+dx))^{3/2}\,dx + a^2\int \sec^2(c+dx)(e\tan(c+dx))^{3/2}\,dx$$

$$= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{a^2\,\text{Subs}}{}$$

$$= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{2a^2(e\,\text{ta}}{}$$

$$= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{2a^2(e\,\text{ta}}{}$$

$$= -\frac{2a^2e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2e\sqrt{e\,\text{ta}}}{d}$$

$$= -\frac{2a^2e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2e\sqrt{e\,\text{ta}}}{d}$$

$$= \frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a^2e^{3/2}\,\text{l}}{}$$

$$= \frac{a^2e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{a^2e^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$

Mathematica [C] time = 11.89, size = 257, normalized size = 0.77

$$\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)(e\tan(c+dx))^{3/2}\sec^4\left(\frac{1}{2}\tan^{-1}(\tan(c+dx))\right)\left(-80\sqrt{\tan(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(e*Tan[c + d*x])^(3/2
)*(30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 30*Sqrt[2]*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]] + 15*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] - 15*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]] + 120*Sqrt[Tan[c + d*x]] - 80*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan
[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 80*Sqrt[Sec[c + d*x]^2]*Sqrt[Tan[c + d*x]
] + 24*Tan[c + d*x]^(5/2)))/(60*d*Tan[c + d*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx + c) + a)^2 (e \tan (dx + c))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^(3/2), x)
```

maple [C] time = 1.95, size = 721, normalized size = 2.15

$$\frac{a^2 (-1 + \cos (dx + c)) \left(-15i \sin (dx + c) \operatorname{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}, \frac{1}{2} + \frac{i}{2}, \frac{\sqrt{2}}{2}\right) \sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}} \sqrt{\frac{-1+\cos(dx+c}{\sin(d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/30*a^2/d*(-1+cos(d*x+c))*(-15*I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*I*sin(d*x+c)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-10*sin(d*x+c)*
EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+24*2^(1/2)*cos(d*x
+c)^3-4*cos(d*x+c)^2*2^(1/2)-14*cos(d*x+c)*2^(1/2)-6*2^(1/2))*(1+cos(d*x+c)
)^2*(e*sin(d*x+c)/cos(d*x+c))^(3/2)/sin(d*x+c)^5/cos(d*x+c)*2^(1/2)
```

maxima [A] time = 0.47, size = 194, normalized size = 0.58

$$\frac{\frac{8 (e \tan(dx+c))^{\frac{5}{2}} a^2}{e} - \frac{5\left(2 \sqrt{2} e^{\frac{5}{2}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right) + 2\sqrt{2} e^{\frac{5}{2}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right) + \sqrt{2} e^{\frac{5}{2}} \log\left(e\tan(dx+c) + \sqrt{2}\sqrt{e\tan}\right.\right.}{e}}{20\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(8*(e*tan(d*x + c))^(5/2)*a^2/e - 5*(2*sqrt(2)*e^(5/2)*arctan(1/2*sqrt
(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(d*x + c)))/sqrt(e)) + 2*sqrt(2)*e^(5/2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) - 2*sqrt(e*tan(d*x + c)))/sqrt(e)) +
sqrt(2)*e^(5/2)*log(e*tan(d*x + c) + sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) +
 e) - sqrt(2)*e^(5/2)*log(e*tan(d*x + c) - sqrt(2)*sqrt(e*tan(d*x + c))*sqr
t(e) + e) - 8*sqrt(e*tan(d*x + c))*e^2)*a^2/e)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\tan (c+d\,x))^{3/2}\,\left(a+\frac{a}{\cos (c+d\,x)}\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int (e\tan (c+dx))^{\frac{3}{2}}\, dx+\int 2\,(e\tan (c+dx))^{\frac{3}{2}}\sec (c+dx)\, dx+\int (e\tan (c+dx))^{\frac{3}{2}}\sec^{2}(c+dx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*tan(c + d*x))**(3/2), x) + Integral(2*(e*tan(c + d*x))**(
3/2)*sec(c + d*x), x) + Integral((e*tan(c + d*x))**(3/2)*sec(c + d*x)**2, x
))
```

3.113 $\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx$

Optimal. Leaf size=309

$$-\frac{a^2\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{2a^2(e\tan(c+dx))^{3/2}}{3de}+\frac{a^2\sqrt{e}\log\left(\sqrt{e}\tan(c+\right.}{}$$

```
[Out] -1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/4
*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/d*
2^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
*e^(1/2)/d*2^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*P
i+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d*
x+2*c)^(1/2)+2/3*a^2*(e*tan(d*x+c))^(3/2)/d/e+4*a^2*cos(d*x+c)*(e*tan(d*x+c
))^(3/2)/d/e
```

Rubi [A] time = 0.34, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3886, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 2607, 32}

$$-\frac{a^2\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{2a^2(e\tan(c+dx))^{3/2}}{3de}+\frac{a^2\sqrt{e}\log\left(\sqrt{e}\tan(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a^2*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*d)) + (a^2*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d) + (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e
*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c +
 d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (4*a^2*Cos[c + d*x]*
EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]
]) + (2*a^2*(e*Tan[c + d*x])^(3/2))/(3*d*e) + (4*a^2*Cos[c + d*x]*(e*Tan[c
+ d*x])^(3/2))/(d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx = \int \left(a^2\sqrt{e\tan(c+dx)}+2a^2\sec(c+dx)\sqrt{e\tan(c+dx)}+a^2\sec^2(c+a$$

$$= a^2\int \sqrt{e\tan(c+dx)}\,dx + a^2\int \sec^2(c+dx)\sqrt{e\tan(c+dx)}\,dx + \left(2a^2\right.$$

$$= \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \left(4a^2\right)\int \cos(c+dx)\sqrt{e\tan(c+dx}$$

$$= \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} + \frac{\left(2a^2e\right)\mathrm{Sul}}{}$$

$$= \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{\left(a^2e\right)\mathrm{Subs}}{}$$

$$= -\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} + \frac{2a^2(e\tan(c+dx}{3de}$$

$$= \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a^2\sqrt{e}\log\left(\sqrt{\right.}}{}$$

$$= -\frac{a^2\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} +$$

Mathematica [C] time = 1.27, size = 106, normalized size = 0.34

$$\frac{4a^2\sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\tan(c+dx)}\left(2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\mathrm{ta}\right.\right.}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (4*a^2*Cos[(c + d*x)/2]^5*(1 + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c +
d*x]^2] + Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Sec[c + d*x]*Sec
[ArcTan[Tan[c + d*x]]/2]^4*Sin[(c + d*x)/2]*Sqrt[e*Tan[c + d*x]])/(3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx + c) + a)^2 \sqrt{e \tan (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c)), x)
```

maple [C] time = 2.04, size = 1480, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/6*a^2/d*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^
2*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/
2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-
3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2
^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+24*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-12*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-3*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)-3*I*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)+24*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)-12*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))
*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-3*EllipticPi
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*
x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-3*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-14*cos(d*x+c)^2*2^(1/2)+12
*cos(d*x+c)*2^(1/2)+2*2^(1/2))/cos(d*x+c)/sin(d*x+c)^5*2^(1/2)
```

maxima [A] time = 0.46, size = 177, normalized size = 0.57

$$\frac{3\,a^2e\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{e}+2\,\sqrt{e\tan(dx+c)}\right)}{2\,\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{e}-2\,\sqrt{e\tan(dx+c)}\right)}{2\,\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log\left(e\tan(dx+c)+\sqrt{2}\,\sqrt{e\tan(dx+c)}\,\sqrt{e}+e\right)}{\sqrt{e}}+\cdots}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^2*e*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan
(d*x + c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
e) - 2*sqrt(e*tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(e*tan(d*x + c)
+ sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e) + sqrt(2)*log(e*tan(d*x
 + c) - sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e)) + 8*(e*tan(d*x +
 c))^(3/2)*a^2/e)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\tan\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \sqrt{e\tan(c+dx)}\,dx+\int 2\sqrt{e\tan(c+dx)}\sec(c+dx)\,dx+\int \sqrt{e\tan(c+dx)}\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*tan(c + d*x)), x) + Integral(2*sqrt(e*tan(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*tan(c + d*x))*sec(c + d*x)**2, x))
```

3.114 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=278

$$-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}}+\frac{2a^2\sqrt{e\tan(c+dx)}}{de}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{}\right.}{2\sqrt{2}d}$$

```
[Out] -1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)-1/4
*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)/
e^(1/2)+1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/d*2^(1/2)/e^(1/2)-2*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(d
*x+c))^(1/2)+2*a^2*(e*tan(d*x+c))^(1/2)/d/e
```

Rubi [A] time = 0.30, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3886, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 2607, 32}

$$-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}}+\frac{2a^2\sqrt{e\tan(c+dx)}}{de}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{}\right.}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[
e])) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
Sqrt[e]) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (2*a^2*EllipticF[c -
 Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]
]) + (2*a^2*Sqrt[e*Tan[c + d*x]])/(d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx = \int\left(\frac{a^2}{\sqrt{e\tan(c+dx)}}+\frac{2a^2\sec(c+dx)}{\sqrt{e\tan(c+dx)}}+\frac{a^2\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\right)dx$$

$$= a^2\int\frac{1}{\sqrt{e\tan(c+dx)}}\,dx + a^2\int\frac{\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx + \left(2a^2\right)\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$$

$$= \frac{a^2\,\text{Subst}\left(\int\frac{1}{\sqrt{ex}}\,dx,x,\tan(c+dx)\right)}{d} + \frac{\left(a^2e\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx,x,e\tan(c+dx\right.}{d}$$

$$= \frac{2a^2\sqrt{e\tan(c+dx)}}{de} + \frac{\left(2a^2e\right)\text{Subst}\left(\int\frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{d} + \frac{\left(2a^2\sec(c+\right.}{}$$

$$= \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{2a^2\sqrt{e\tan(c+dx)}}{de} + \frac{a^2\,\text{Subs}}{}$$

$$= \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{2a^2\sqrt{e\tan(c+dx)}}{de} + \frac{a^2\,\text{Subs}}{}$$

$$= -\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d\sqrt{e}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+\right.}{2\sqrt{2}}$$

$$= -\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}} + \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

Mathematica [C] time = 2.47, size = 220, normalized size = 0.79

$$\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\tan(c+dx)}\sec^4\left(\frac{1}{2}\tan^{-1}(\tan(c+dx))\right)\left(16\sqrt{\tan(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(-2*Sqrt[2]*ArcTan[1
- Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d
*x]]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Sqrt[2
]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 8*Sqrt[Tan[c + d*x]]
 + 16*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]])
*Sqrt[Tan[c + d*x]])/(4*d*Sqrt[e*Tan[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*tan(d*x + c)), x)
```

maple [C] time = 2.00, size = 653, normalized size = 2.35

$$-\frac{a^2(1+\cos(dx+c))^2(-1+\cos(dx+c))\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(I*EllipticPi(((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*sin(d*x+c)+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*sin(d*x+c)+2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2
^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-2*cos
(d*x+c)*2^(1/2)+2*2^(1/2))/cos(d*x+c)/sin(d*x+c)^3/(e*sin(d*x+c)/cos(d*x+c)
)^(1/2)*2^(1/2)
```

maxima [A] time = 0.46, size = 178, normalized size = 0.64

$$\frac{\frac{\left(2\sqrt{2}\sqrt{e}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)+2\sqrt{2}\sqrt{e}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)+\sqrt{2}\sqrt{e}\log\left(e\tan(dx+c)+\sqrt{2}\sqrt{e\tan(dx+c)}\sqrt{e}+e\right)-\sqrt{2}\sqrt{e}}{e}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*sqrt(2)*sqrt(e)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(
d*x + c)))/sqrt(e)) + 2*sqrt(2)*sqrt(e)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e
) - 2*sqrt(e*tan(d*x + c)))/sqrt(e)) + sqrt(2)*sqrt(e)*log(e*tan(d*x + c) +
 sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e) - sqrt(2)*sqrt(e)*log(e*tan(d*x
+ c) - sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e))*a^2/e + 8*sqrt(e*tan(d*x
+ c))*a^2/e)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{e\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\sqrt{e\tan\left(c+dx\right)}}\,dx + \int \frac{2\sec\left(c+dx\right)}{\sqrt{e\tan\left(c+dx\right)}}\,dx + \int \frac{\sec^2\left(c+dx\right)}{\sqrt{e\tan\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*tan(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
tan(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*tan(c + d*x)), x))
```

3.115 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=310

$$\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}}+$$

```
[Out] 1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/
2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/4*
a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2)*2
^(1/2)+1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d/e^(3/2)*2^(1/2)-4*a^2/d/e/(e*tan(d*x+c))^(1/2)-4*a^2*cos(d*x+c)/d/e/(e*ta
n(d*x+c))^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d
*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/e^2/sin(2*d
*x+2*c)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2608, 2615, 2572, 2639, 2607, 32}

$$\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)
) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(
3/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*d*e^(3/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Tan
[c + d*x]]) - (4*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Tan[c + d*x]]) - (4*a^2*Cos[
c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*e^2*Sqrt[Sin
[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
```

```
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \int \left(\frac{a^2}{(e\tan(c+dx))^{3/2}} + \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{3/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\right) dx$$

$$= a^2\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{3/2}}$$

$$= -\frac{2a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{(ex)^{3/2}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2}{}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{a^2\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{de} - \frac{(}{}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right.}{de}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c}}{de^2\sqrt{\sin(2c+2dx)}}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c}}{de^2\sqrt{\sin(2c+2dx)}}$$

$$= -\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{3/2}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx\right.}{2\sqrt{2}\,de}$$

$$= \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}} - \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

Mathematica [C] time = 7.59, size = 238, normalized size = 0.77

$$\frac{a^2\left(8e^{3i(c+dx)}\sqrt{1-e^{4i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{4i(c+dx)}\right)-24\left(e^{i(c+dx)}+e^{2i(c+dx)}+e^{3i(c+dx)}+1\right)-3\sqrt{-1+e^{4i(c+dx)}}\tan^{-}\right.}{6de\left(1+e^{2i(c+dx)}\right)\sqrt{e\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-24*(1 + E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)))
 - 3*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
+ 6*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sq
rt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]] + 8*E^((3*I)*(c +
 d*x))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4
*I)*(c + d*x))]))/(6*d*e*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Tan[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(3/2), x)
```

maple [C] time = 1.82, size = 1392, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/2*a^2/d*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x
+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+4*EllipticF(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-8*EllipticE(((1-cos(d*x+c)+sin(d*x+c))
```

```
/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)+4*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-8*EllipticE
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+8*cos(d*x+c)*2^(1/2))*sin(d*x+c)/cos(d*x
+c)^2/(e*sin(d*x+c)/cos(d*x+c))^(3/2)*2^(1/2)
```

maxima [A] time = 0.48, size = 190, normalized size = 0.61

$$-\frac{\frac{a^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log\left(e\tan(dx+c)+\sqrt{2}\sqrt{e\tan(dx+c)}\sqrt{e}+e\right)}{\sqrt{e}}+\frac{\sqrt{2}\log\left(e\tan(dx+c)-\sqrt{2}\sqrt{e}\cdots\right.}{\sqrt{e}}\cdots\right.}{e}\cdots}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(d*x
 + c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) -
 2*sqrt(e*tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(e*tan(d*x + c) + sq
rt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e) + sqrt(2)*log(e*tan(d*x + c
) - sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e) + 8/sqrt(e*tan(d*x +
c)))/e + 8*a^2/(sqrt(e*tan(d*x + c))*e))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(e\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\left(e\tan\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{2\sec\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{\sec^2\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*tan(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*ta
n(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*tan(c + d*x))**(3/2),
x))
```

3.116 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=316

$$\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{5/2}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}-$$

```
[Out] 1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/
2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/4*
a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2)*2
^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d/e^(5/2)*2^(1/2)+2/3*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ell
ipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/e^2/(e*
tan(d*x+c))^(1/2)-4/3*a^2/d/e/(e*tan(d*x+c))^(3/2)-4/3*a^2*sec(d*x+c)/d/e/(
e*tan(d*x+c))^(3/2)
```

Rubi [A] time = 0.39, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3886, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2609, 2614, 2573, 2641, 2607, 32}

$$\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{5/2}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)
) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(
5/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*d*e^(5/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Tan[c
 + d*x])^(3/2)) - (4*a^2*Sec[c + d*x])/(3*d*e*(e*Tan[c + d*x])^(3/2)) - (2*
a^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*
e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = \int \left(\frac{a^2}{(e\tan(c+dx))^{5/2}} + \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{5/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\right)dx$$

$$= a^2\int \frac{1}{(e\tan(c+dx))^{5/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{5/2}}$$

$$= -\frac{2a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{(ex)^{5/2}}\,dx, x, \tan(c+dx\right.}{d}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx, x, e\tan(c\right.}{de}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c\right.}{de}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{s}}{3de^2\sqrt{e\tan(c+dx)}}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{s}}{3de^2\sqrt{e\tan(c+dx)}}$$

$$= \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{5/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\,\tan(c+dx)\right.}{2\sqrt{2}\,de^{5/}}$$

$$= \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}} - \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\,\tan\right.}{}$$

Mathematica [C] time = 5.16, size = 224, normalized size = 0.71

$$-\frac{a^2\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\tan^{-1}(\tan(c+dx))\right)\left(16\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\tan^2(c+dx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/24*(a^2*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Cot[(c + d*x)/2]*Sec[ArcTan[Tan[
c + d*x]]/2]^4*(16*Hypergeometric2F1[-3/4, 1/2, 1/4, -Tan[c + d*x]^2] + 16*
Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*(2*ArcTan[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Lo
g[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])*Tan[c + d*x]^(3/2)))/(d*e^2*Sqrt[e*Tan[c + d*x]
])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(5/2), x)
```

maple [C] time = 1.81, size = 650, normalized size = 2.06

$$\frac{a^2(1+\cos(dx+c))^2(-1+\cos(dx+c))\left(3i\,\text{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/6*a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(3*I*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*sin(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
```

```
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*sin(d*x+c)-2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c
)+4*cos(d*x+c)*2^(1/2))/cos(d*x+c)^3/(e*sin(d*x+c)/cos(d*x+c))^(5/2)/sin(d*
x+c)*2^(1/2)
```

maxima [A] time = 0.44, size = 191, normalized size = 0.60

$$-\frac{a^2\left(\frac{6\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{6\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{3\sqrt{2}\log\left(e\tan(dx+c)+\sqrt{2}\sqrt{e\tan(dx+c)}\sqrt{e}+e\right)}{e^{\frac{3}{2}}}-\frac{3\sqrt{2}\log\left(e\tan(dx+c)-\sqrt{2}\right.}{e^{\frac{3}{2}}}}{e}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(a^2*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(d*
x + c)))/sqrt(e))/e^(3/2) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e)
- 2*sqrt(e*tan(d*x + c)))/sqrt(e))/e^(3/2) + 3*sqrt(2)*log(e*tan(d*x + c) +
 sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/e^(3/2) - 3*sqrt(2)*log(e*tan(d*
x + c) - sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/e^(3/2) + 8/(e*tan(d*x +
 c))^(3/2))/e + 8*a^2/((e*tan(d*x + c))^(3/2)*e))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2}{{\left(e\tan\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int \frac{2\sec\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int \frac{\sec^2\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(5/2),x)
```

```
[Out] a**2*(Integral((e*tan(c + d*x))**(-5/2), x) + Integral(2*sec(c + d*x)/(e*ta
n(c + d*x))**(5/2), x) + Integral(sec(c + d*x)**2/(e*tan(c + d*x))**(5/2),
x))
```

3.117 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}de^{7/2}}$$

```
[Out] -1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/4
*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/2)*
2^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/d/e^(7/2)*2^(1/2)+2*a^2/d/e^3/(e*tan(d*x+c))^(1/2)+12/5*a^2*cos(d*x+c)/d/e
^3/(e*tan(d*x+c))^(1/2)-12/5*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin
(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/
e^4/sin(2*d*x+2*c)^(1/2)-4/5*a^2/d/e/(e*tan(d*x+c))^(5/2)-4/5*a^2*sec(d*x+c
)/d/e/(e*tan(d*x+c))^(5/2)
```

Rubi [A] time = 0.46, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2609, 2608, 2615, 2572, 2639, 2607, 32}

$$-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}}+\frac{2a^2}{de^3\sqrt{e\tan(c+dx)}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\right.}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] -((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/
2))) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
e^(7/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (4*a^2)/(5*d*e*(e*Ta
n[c + d*x])^(5/2)) - (4*a^2*Sec[c + d*x])/(5*d*e*(e*Tan[c + d*x])^(5/2)) +
(2*a^2)/(d*e^3*Sqrt[e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x])/(5*d*e^3*Sqrt[
e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e
*Tan[c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
```

```
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \int \left(\frac{a^2}{(e\tan(c+dx))^{7/2}} + \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{7/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\right) dx$$

$$= a^2 \int \frac{1}{(e\tan(c+dx))^{7/2}}\,dx + a^2 \int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx + \left(2a^2\right) \int \frac{\sec(c+dx)}{(e\tan(c+dx))^{7/2}}$$

$$= -\frac{2a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{(ex)^{7/2}}\,dx, x, \tan(c+dx\right)}{d}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c}{5de^3\sqrt{e\tan(c}}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c}{5de^3\sqrt{e\tan(c}}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c}{5de^3\sqrt{e\tan(c}}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c}{5de^3\sqrt{e\tan(c}}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c}{5de^3\sqrt{e\tan(c}}$$

$$= \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{7/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)\right.}{2\sqrt{2}\,de^{7/}}$$

$$= -\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}} + \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}} + a^2\log\left(\sqrt{e}+\sqrt{e}\,\mathrm{ta}\right.$$

Mathematica [C] time = 14.28, size = 2820, normalized size = 7.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] (Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((7*Cot[c/2])/(10*d) - (Cot[c/
2]*Csc[c/2 + (d*x)/2]^2)/(20*d) - (3*(4*Cos[c/2] - Cos[(3*c)/2] + Cos[(5*c)
/2])*Cos[d*x]*Sec[2*c]*Sin[c/2])/(10*d*(-1 + 2*Cos[c])) - (7*Csc[c/2]*Csc[c
/2 + (d*x)/2]*Sin[(d*x)/2])/(10*d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]^3*Sin[(d*
x)/2])/(20*d) - (3*(2 - 5*Cos[c] + 6*Cos[2*c] + Cos[3*c])*Sec[2*c]*Sin[d*x]
)/(20*d*(-1 + 2*Cos[c])))*Sin[c + d*x]^2*Tan[c + d*x]^2)/(e*Tan[c + d*x])^(
7/2) + ((E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*
I)*(c + d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d
*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*
Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c +
 d*x]^(7/2))/(16*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((
2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x]
)^(7/2)) + ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)
*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x
)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
Tan[c + d*x]^(7/2))/(16*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x)))
)/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*T
```

```
an[c + d*x])^(7/2)) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan
[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c
 + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*
(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*C
os[c])*(e*Tan[c + d*x])^(7/2)) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sq
rt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))
]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1
+ E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a +
 a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2
*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 +
 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTa
n[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d
*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))
/(1 + E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*
(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((2*I)*c)*Sqrt[((-I)*(-1
 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)
))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) + ((Sqrt[-1 + E^((4*I)*(c + d*x)
)]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((6*I)*c)*Sqrt[-1 + E^((2*I
)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c
+ d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((3*I)*c)*Sqrt[(
(-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(
c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*(3 - 3
*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4
*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[
2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(20*d*
E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
 d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2))
 - (Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(1 + E
^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
Tan[c + d*x]^(7/2))/(20*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/
(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan
[c + d*x])^(7/2)) + (E^(I*(c - d*x))*Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*
x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hyperge
ometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]
^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(8*d*Sqrt[((-I)*(-1 + E^((2*I
)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2
*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*x
)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hy
pergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(40*d*E^(I*(3*c + d*x))*
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((
2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*
(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*d*x)*(1 + E^((6*I)*c)
)*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(
c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*
x]^(7/2))/(10*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2
*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])
^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\tan(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(7/2), x)
```

maple [C] time = 1.90, size = 1429, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x)
```

```
[Out] -1/10*a^2/d*(5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos
(d*x+c)^2-5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*
x+c)^2+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)+24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellipt
icE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-12*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+5*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))+5*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)-5*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-24*Ellip
ticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+12*EllipticF(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)-5*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-26*cos(
d*x+c)^2*2^(1/2)+22*cos(d*x+c)*2^(1/2))*sin(d*x+c)^3/(-1+cos(d*x+c))/cos(d*
x+c)^4/(e*sin(d*x+c)/cos(d*x+c))^(7/2)*2^(1/2)
```

maxima [A] time = 0.44, size = 218, normalized size = 0.59

$$\frac{\frac{a^2\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{e\tan(dx+c)}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log\left(e\tan(dx+c)+\sqrt{2}\sqrt{e\tan(dx+c)}\sqrt{e}+e\right)}{\sqrt{e}}+\frac{\sqrt{2}\log\left(e\tan(dx+c)-\sqrt{\cdots}\right.}{\sqrt{\cdots}}\cdots\right)}{e^2}\cdots\right.}{e}\cdots}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*(a^2*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e*tan(
d*x + c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e
) - 2*sqrt(e*tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(e*tan(d*x + c) +
 sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e) + sqrt(2)*log(e*tan(d*x
+ c) - sqrt(2)*sqrt(e*tan(d*x + c))*sqrt(e) + e)/sqrt(e))/e^2 + 8*(5*e^2*ta
n(d*x + c)^2 - e^2)/((e*tan(d*x + c))^(5/2)*e^2))/e - 8*a^2/((e*tan(d*x + c
))^(5/2)*e))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(e\tan\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(7/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.118 $\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=330

$$\frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}-\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{e^{11/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}ad}$$

```
[Out] 1/2*e^(11/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2
*e^(11/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e^
(11/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1
/2)-1/4*e^(11/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c)
)/a/d*2^(1/2)-5/21*e^6*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellipt
icF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d
*x+c))^(1/2)+2/21*e^5*(21-5*sec(d*x+c))*(e*tan(d*x+c))^(1/2)/a/d-2/35*e^3*(
7-5*sec(d*x+c))*(e*tan(d*x+c))^(5/2)/a/d
```

Rubi [A] time = 0.42, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3888, 3881, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}-\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{e^{11/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^(11/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
 - (e^(11/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
*d) + (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + S
qrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (5*e^6*EllipticF[c - Pi/4 +
 d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(21*a*d*Sqrt[e*Tan[c + d*x]])
 + (2*e^5*(21 - 5*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(21*a*d) - (2*e^3*(7
- 5*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2))/(35*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{7/2}\,dx}{a^2}$$

$$= -\frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{\left(2e^4\right)\int\left(-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)\right)(e\tan(c+$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} + \frac{(4}{}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} + \frac{(5}{}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{e^7}{}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{(2}{}$$

$$= \frac{5e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+}}{21ad}$$

$$= \frac{5e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+}}{21ad}$$

$$= \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}a}$$

$$= \frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{11/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

Mathematica [C] time = 21.94, size = 332, normalized size = 1.01

$$\frac{e^5\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\tan(c+dx)}\left(-320\sqrt{\tan(c+dx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\tan^2(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^5*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sqrt[e*Tan[
c + d*x]]*(70*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 70*Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 35*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]] - 35*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]] + 280*Sqrt[Tan[c + d*x]] - 320*Hypergeometric2F1[-1/2, 1/4,
 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 280*Hypergeometric2F1[1/4, 1/2,
 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 40*Sqrt[Sec[c + d*x]^2]*Sqrt[Ta
n[c + d*x]] - 56*Tan[c + d*x]^(5/2) + 40*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x]^
(5/2)))/(70*a*d*(1 + Sec[c + d*x])^2*Sqrt[Tan[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \tan(dx+c))^{\frac{11}{2}}}{a \sec(dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a), x)
```

maple [C] time = 1.73, size = 734, normalized size = 2.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/210/a/d*(-1+cos(d*x+c))*(-105*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+105*I*sin(d*x+c)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+105*sin(d*x+c)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+105*sin(d*x+c)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-260*sin(d*x
+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*Ellipti
cF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+252*cos(d*x+c)
^4*2^(1/2)-332*2^(1/2)*cos(d*x+c)^3+38*cos(d*x+c)^2*2^(1/2)+72*cos(d*x+c)*2
^(1/2)-30*2^(1/2))*cos(d*x+c)^2*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^
(11/2)/sin(d*x+c)^9*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(e\tan(c+d\,x)\right)^{11/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(11/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(11/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(11/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.119 $\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=326

$$-\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right.}{2\sqrt{2}ad}$$

```
[Out] -1/2*e^(9/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/2
*e^(9/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e^(
9/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2
)-1/4*e^(9/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a
/d*2^(1/2)-6/5*e^4*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x+2*
c)^(1/2)-6/5*e^3*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d-2/15*e^3*(5-3*sec(d*x+
c))*(e*tan(d*x+c))^(3/2)/a/d
```

Rubi [A] time = 0.39, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3888, 3881, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$-\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right.}{2\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (e^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (6*e^4*Cos[c + d*x]*Elliptic
E[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a*d*Sqrt[Sin[2*c + 2*d*x]]) -
 (6*e^3*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*a*d) - (2*e^3*(5 - 3*Sec[c
+ d*x])*(e*Tan[c + d*x])^(3/2))/(15*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx}{a^2}$$

$$= -\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} - \frac{\left(2e^4\right)\int\left(-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)\right)\sqrt{e\tan(c}}{5a^2}$$

$$= -\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} - \frac{\left(3e^4\right)\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5a} +$$

$$= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} + \frac{\left(6e^4\right.}{}$$

$$= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} + \frac{\left(2e^5\right.}{}$$

$$= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} - \frac{e^5\,\mathrm{Su}}{}$$

$$= \frac{6e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ad\sqrt{\sin(2c+2dx)}} - \frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad}$$

$$= \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad} - \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}\,a}$$

$$= -\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} + \frac{e^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} + \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{\right.}}{}$$

Mathematica [C] time = 13.01, size = 129, normalized size = 0.40

$$\frac{4e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)(e\tan(c+dx))^{3/2}\left({}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)-{}_2F_1\left(\right.\right.}{3ad(\sec(c+dx)+1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*e^3*Cos[(c + d*x)/2]^2*(-1 + Hypergeometric2F1[-1/2, 3/4, 7/4, -Tan[c +
d*x]^2] - Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + Hypergeometri
c2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])
*(e*Tan[c + d*x])^(3/2))/(3*a*d*(1 + Sec[c + d*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \tan (dx+c))^{\frac{9}{2}}}{a \sec (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a), x)
```

maple [C] time = 1.69, size = 1505, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/30/a/d*(-1+cos(d*x+c))^2*(-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))+15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))-15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))-15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))-36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(
d*x+c)^3*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)+18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*Ellipt
icF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-15*I*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*I*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)
```

```
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-36*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2*2^(1/2))+28*2^(1/2)*cos(d*x+c)^3-24*cos(d*x+c)^2*2^(1/2)-10*cos(
d*x+c)*2^(1/2)+6*2^(1/2))*cos(d*x+c)^2*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d
*x+c))^(9/2)/sin(d*x+c)^9*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(e\tan(c+d\,x)\right)^{9/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(9/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(9/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(9/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

3.120 $\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=295

$$-\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}-\frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}ad}$$

```
[Out] -1/2*e^(7/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/2
*e^(7/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/4*e^(
7/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2
)+1/4*e^(7/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a
/d*2^(1/2)+1/3*e^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(
cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x+c
))^(1/2)-2/3*e^3*(3-sec(d*x+c))*(e*tan(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.35, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3888, 3881, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}-\frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) - (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) + (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) - (e^4*EllipticF[c - Pi/4 + d*
x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*a*d*Sqrt[e*Tan[c + d*x]]) - (
2*e^3*(3 - Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx}{a^2}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} - \frac{\left(2e^4\right)\int\frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} - \frac{e^4\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a} + \frac{e^4\int\frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} + \frac{e^5\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad} - \frac{(e}{}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} + \frac{\left(2e^5\right)\mathrm{Subst}\left(\int\frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{(}{}$$

$$= -\frac{e^4F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} - \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad}$$

$$= -\frac{e^4F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} - \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad}$$

$$= -\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad} + \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx\right.}{2\sqrt{2}\,ac}$$

$$= -\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} + \frac{e^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} - \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\,t\right.}{}$$

Mathematica [C] time = 54.60, size = 271, normalized size = 0.92

$$\frac{e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\tan(c+dx)}\left(8\sqrt{\tan(c+dx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\tan^2(c+dx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*(-2*Sqrt[2]
*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 8*Sqrt[Tan
[c + d*x]] + 8*Hypergeometric2F1[-1/2, 1/4, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[
```

```
c + d*x]] - 8*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c
+ d*x]])*Sqrt[e*Tan[c + d*x]])/(2*a*d*(1 + Sec[c + d*x])^2*Sqrt[Tan[c + d*x
]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

maple [C] time = 1.70, size = 698, normalized size = 2.37

$$-\frac{(-1+\cos(dx+c))\left(3i\cos(dx+c)\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\ \mathrm{E}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/6/a/d*(-1+cos(d*x+c))*(3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*cos(d*x+c)*sin(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*cos(d*x+c)^2*2^(1/2)-8*cos(d*x+c)*
2^(1/2)+2*2^(1/2))*cos(d*x+c)^2*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^
(7/2)/sin(d*x+c)^7*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e\,\tan\left(c+d\,x\right)\right)}^{7/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(7/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.121 $\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=285

$$\frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad}-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}\,ad}$$

```
[Out] 1/2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2*
e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/4*e^(5
/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)
+1/4*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/
d*2^(1/2)+2*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*El
lipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x+2*c)^
(1/2)+2*e*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d
```

Rubi [A] time = 0.33, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3888, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad}-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}\,ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
- (e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
) - (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*a*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) - (2*e^2*Cos[c + d*x]*EllipticE[c
 - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[2*c + 2*d*x]]) + (2*e
*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx}{a^2}$$

$$= -\frac{e^2\int\sqrt{e\tan(c+dx)}\,dx}{a} + \frac{e^2\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} - \frac{\left(2e^2\right)\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a} - \frac{e^3\,\mathrm{Subst}}{}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} - \frac{\left(2e^3\right)\mathrm{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad} - \frac{(2e}{}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} + \frac{e^3\,\mathrm{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad} - \frac{e^3\,\mathrm{Sub}}{}$$

$$= -\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} + \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad}$$

$$= -\frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad} + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+\right.}{2\sqrt{}}$$

$$= \frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} - \frac{e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} - \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

Mathematica [C] time = 5.51, size = 105, normalized size = 0.37

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)(e\tan(c+dx))^{5/2}\left({}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)-{}_2F_1\left(\frac{3}{4},1;\right.\right.}{3ad(\sec(c+dx)+1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Csc[c + d*x]*(Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c
 + d*x]^2] - Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*(1 + Sqrt[Sec
[c + d*x]^2])*(e*Tan[c + d*x])^(5/2))/(3*a*d*(1 + Sec[c + d*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

maple [C] time = 2.09, size = 1419, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2/a/d*(-1+cos(d*x+c))^2*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/
2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-I*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)-4*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)-4*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2*cos(d*x+c)*2^(1/2)-2*2
^(1/2))*cos(d*x+c)^2*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(5/2)/sin(d
*x+c)^7*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\tan\left(dx+c\right)\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,\left(e\tan\left(c+d\,x\right)\right)^{5/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(5/2)/(sec(c + d*x) + 1), x)/a
```

3.122 $\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=257

$$\frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{}\right.}{2\sqrt{2}ad}$$

```
[Out] 1/2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2*
e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e^(3
/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)
-1/4*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/
d*2^(1/2)-e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c
+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x+c))^(1
/2)
```

Rubi [A] time = 0.29, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3888, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{}\right.}{2\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
- (e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
) + (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*a*d) - (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (e^2*EllipticF[c - Pi/4 + d*x,
2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(a*d*Sqrt[e*Tan[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
```

```
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2}$$

$$= -\frac{e^2\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} + \frac{e^2\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= -\frac{e^3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad} + \frac{\left(e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{\sin(c+dx)}}\,d}{a\sqrt{\cos(c+dx)}\,\sqrt{e\tan(c+dx)}}$$

$$= -\frac{\left(2e^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{\left(e^2\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{}{\sqrt{\sin(}}}{a\sqrt{e\tan(c+dx)}}$$

$$= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}} - \frac{e^2\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+d}\right.}{ad}$$

$$= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}} + \frac{e^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\,t}\right.}{2\sqrt{2}\,ad}$$

$$= \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)\right.}{2\sqrt{2}\,ad}$$

$$= \frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} - \frac{e^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} + \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

Mathematica [C] time = 13.21, size = 1211, normalized size = 4.71

$$-\frac{4\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\middle|-1\right)(e\tan(c+dx))^{3/2}\sec^4(c+dx)}{d(\sec(c+dx)a+a)\tan^{\frac{3}{2}}(c+dx)\left(\tan^2(c+dx)+1\right)^{3/2}} - \frac{2e^{-i(c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c}\right.}{1+e^{2i(c+}}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^2*Csc[c + d*x]*((8*Cos[c]*Cos[d*x]*Sec[2*c]*Sin[c/2]^2)
/d - (16*Cos[c/2]*Sec[2*c]*Sin[c/2]^3*Sin[d*x])/d)*(e*Tan[c + d*x])^(3/2))/
(a + a*Sec[c + d*x]) - (2*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2
*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Se
c[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*
Tan[c + d*x]^(3/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)
*(c + d*x)))]*(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)
))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(2
*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]
^(3/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))
]*(Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] +
2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/
2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(2*d*E^((2*I)*
```

```
c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/2)) + (S
qrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 +
(d*x)/2]^2*(3*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4
*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(3*d*E^(I*(2
*c + d*x))*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/
2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Co
s[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 +
 E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4
, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(3*d*
E^(I*d*x)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/2
)) - (4*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt
[Tan[c + d*x]]], -1]*Sec[c + d*x]^4*(e*Tan[c + d*x])^(3/2))/(d*(a + a*Sec[c
 + d*x])*Tan[c + d*x]^(3/2)*(1 + Tan[c + d*x]^2)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

maple [C] time = 1.83, size = 319, normalized size = 1.24

$$\frac{(1+\cos(dx+c))^2\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
[Out] 1/2/a/d*(1+cos(d*x+c))^2*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticF(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(e*sin(d*x+c)/cos(d*x+c))^(3/2)*cos(d*x+c
)/sin(d*x+c)^4*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)\,(e\tan(c+dx))^{3/2}}{a\,(\cos(c+dx)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.123 $\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=315

$$-\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a/d*2^(1/2
)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a
/d*2^(1/2)+2*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(1/2)+2*cos(d*x+c)*(sin(c+
1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))
*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x+2*c)^(1/2)+2*cos(d*x+c)*(e*tan(d*x+c))^
(3/2)/a/d/e
```

Rubi [A] time = 0.38, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3888, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$-\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (2*e*(1 - Sec[c + d*x]))/(a*
d*Sqrt[e*Tan[c + d*x]]) - (2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt
[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c +
 d*x])^(3/2))/(a*d*e)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\int \left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{\int \sqrt{e\tan(c+dx)}\,dx}{a} + \frac{\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} - \frac{2\int \cos(c+dx)\sqrt{e\tan(c+d}}{a}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} + \frac{(2e)\,\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e}\right.}{ad}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} - \frac{e\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan}\right.}{ad}$$

$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} + \frac{2\cos(c+dx}{}$$

$$= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{\right.}}{}$$

Mathematica [C] time = 8.09, size = 2715, normalized size = 8.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*((-2*Cos[c/2]*Cos[d*x]*Sec[2*c]*(4*Sin[c
/2] + Sin[(3*c)/2] + Sin[(5*c)/2]))/(d*(1 + 2*Cos[c])) - (4*Sec[c/2]*Sec[c/
2 + (d*x)/2]*Sin[(d*x)/2])/d - ((-2 - 5*Cos[c] - 6*Cos[2*c] + Cos[3*c])*Sec
[2*c]*Sin[d*x])/(d*(1 + 2*Cos[c])) - (4*Tan[c/2])/d)*Sqrt[e*Tan[c + d*x]])/
(a + a*Sec[c + d*x]) + ((E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[
Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)
*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d
*x]])/(2*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c
+ d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqr
t[Tan[c + d*x]]) - ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sq
rt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 +
E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*
(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*
x]])/(2*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*
(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*
Sqrt[Tan[c + d*x]]) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan
[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c +
 d*x]])/(2*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*
I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x
])*Sqrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]] - 2*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c +
 d*x]])/(2*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((
4*I)*(c + d*x))]] - 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x
]])/(2*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((
4*I)*(c + d*x))]] - 2*E^((6*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x
]])/(2*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^(
(4*I)*(c + d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeome
tric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*T
an[c + d*x]])/(3*d*E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))
/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a
*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*
(c + d*x)) + E^((2*I)*(c + 2*d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c +
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec
[c + d*x]*Sqrt[e*Tan[c + d*x]])/(3*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c
 + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[
c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (5*E^(I*(c - d*x))*Cos[c/2 +
 (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sq
rt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c +
d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]])/(6*d*Sqrt[((-I)*(-1 + E
^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(
1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) - (Cos[c/2 + (d*x)/2
]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt
[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*
```

```
x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]])/(6*d*E^(I*(3*c + d*x))*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (2
*Cos[c/2 + (d*x)/2]^2*(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)
*d*x)*(1 + E^((6*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]
])/(3*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
 d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt
[Tan[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\tan(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a), x)
```

maple [C] time = 1.77, size = 352, normalized size = 1.12

$$-\frac{\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{e\sin(dx+c)}{\cos(dx+c)}}\,(1+\cos(dx+c))^2\,(-1+\cos(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2/a/d*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(e*sin(d*x+c)/c
os(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(I*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*EllipticE(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*EllipticF(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/sin(d*x+c)^3*
2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\tan(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)\sqrt{e\tan(c+dx)}}{a(\cos(c+dx)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
[Out] Integral(sqrt(e*tan(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.124 $\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=290

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad\sqrt{e}}-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad\sqrt{e}}+\frac{\text{lo}}{}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)-1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)/e^(1/2
)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/
2)/e^(1/2)+1/3*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(
c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x+c))^(
1/2)+2/3*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(3/2)
```

Rubi [A] time = 0.35, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3888, 3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad\sqrt{e}}-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad\sqrt{e}}+\frac{\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e]))
 + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e])
 - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sq
rt[2]*a*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]]/(2*Sqrt[2]*a*d*Sqrt[e]) + (2*e*(1 - Sec[c + d*x]))/(3*a*d*(e*T
an[c + d*x])^(3/2)) - (EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2
*c + 2*d*x]])/(3*a*d*Sqrt[e*Tan[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a} + \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx, x, e\tan(c+dx)\right)}{ad} -$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad\sqrt{e}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}} - \frac{\log\left(\sqrt{e}\right.}{}$$

Mathematica [C] time = 9.07, size = 1253, normalized size = 4.32

$$\frac{4\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\middle|-1\right)\sqrt{\tan(c+dx)}\sec^4(c+dx)}{3d(\sec(c+dx)a+a)\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)+1\right)^{3/2}} + \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{2\sec^2\left(\frac{c}{2}\right.}{3d}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (2*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E
^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c +
 d*x]])/(3*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (
```

```
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^((4*I)
*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] +
 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqr
t[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2
]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(2*d*E^((2*I)*c)*(-1 + E^((2*
I)*(c + d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Sqrt[((-I)*(-1
 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((4*I)*(c
+ d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqrt[-1 + E
^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*
I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*S
ec[c + d*x]*Sqrt[Tan[c + d*x]])/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))
*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) - (Sqrt[((-I)*(-1 + E^((2*I)*(c
 + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3*(-1 + E^((4*I
)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c
 + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*S
ec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d*E^(I*(2*c + d*x))*(-1 + E^((2*I)*(c +
d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Sqrt[((-I)*(-1 + E^((2
*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3 - 3*E^(
(4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*
I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2
*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d*E^(I*d*x)*(-1 + E^((2*I)*(c + d*x
)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*Sec[
c + d*x]*(-4/(3*d) + (2*(3 - 2*Cos[c] + 3*Cos[2*c])*Cos[d*x]*Sec[2*c])/(3*d
) + (2*Sec[c/2 + (d*x)/2]^2)/(3*d) - (2*Sec[2*c]*(-2*Sin[c] + 3*Sin[2*c])*S
in[d*x])/(3*d))*Tan[c + d*x])/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) +
 (4*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan
[c + d*x]]], -1]*Sec[c + d*x]^4*Sqrt[Tan[c + d*x]])/(3*d*(a + a*Sec[c + d*x
])*Sqrt[e*Tan[c + d*x]]*(1 + Tan[c + d*x]^2)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec(dx + c) + a)\sqrt{e \tan(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

maple [C] time = 1.73, size = 1269, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/6/a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(3*I*cos(d*x+c)*sin(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*sin(d*x+c)
```

```
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*cos(d*x+c)*sin(d*x+
c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*sin(d*x+c)-8*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*sin(d*x+c)+2*cos(d*x+c)^2*2^(1/2)-2*cos(d*x+c)*2^(1/2))/sin(d*x+c)^5/co
s(d*x+c)/(e*sin(d*x+c)/cos(d*x+c))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{a\,\sqrt{e\tan(c+d\,x)}\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*tan(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}\sec(c+dx)+\sqrt{e\tan(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*tan(c + d*x))*sec(c + d*x) + sqrt(e*tan(c + d*x))), x)/a
```

3.125 $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=359

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ade^{3/2}}-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{3/2}}+\frac{\log\left(\sqrt{\right.}}{}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)*2^(1/2)-1/2*
arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)*2^(1/2)-1/4*ln(e
^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(3/2)*2^(1/2)
+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(3/2
)*2^(1/2)-2/5*(5-3*sec(d*x+c))/a/d/e/(e*tan(d*x+c))^(1/2)-6/5*cos(d*x+c)*(s
in(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(
1/2))*(e*tan(d*x+c))^(1/2)/a/d/e^2/sin(2*d*x+2*c)^(1/2)+2/5*e*(1-sec(d*x+c)
)/a/d/(e*tan(d*x+c))^(5/2)-6/5*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d/e^3
```

Rubi [A] time = 0.45, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3888, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ade^{3/2}}-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{3/2}}+\frac{\log\left(\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(3/2)) -
ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(3/2)) -
Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sqrt[
2]*a*d*e^(3/2)) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]]/(2*Sqrt[2]*a*d*e^(3/2)) + (2*e*(1 - Sec[c + d*x]))/(5*a*d*(e*Tan[
c + d*x])^(5/2)) - (2*(5 - 3*Sec[c + d*x]))/(5*a*d*e*Sqrt[e*Tan[c + d*x]])
+ (6*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a*d
*e^2*Sqrt[Sin[2*c + 2*d*x]]) - (6*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*a
*d*e^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} + \frac{2\int \frac{\frac{5a}{2}-\frac{3}{2}a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{5a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} + \frac{4\int\left(-\frac{5a}{4}-\frac{3}{4}a\sec(c+\right.}{5}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{3\int\sec(c+dx)\sqrt{e\tan}}{5ae^2}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{6\cos(c+dx)(e\tan(c}{5ade^3}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{6\cos(c+dx)(e\tan(c}{5ade^3}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{6\cos(c+dx)(e\tan(c}{5ade^3}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} + \frac{6\cos(c+dx)E\left(c-\right.}{5ade^2\sqrt{s}}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ade^{3/2}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{3/2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{3/2}} - \frac{\log\left(\sqrt{e}+\right.}{}$$

Mathematica [C] time = 13.97, size = 180, normalized size = 0.50

$$-\frac{4\sin^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\tan(c+dx)}\left(-5\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+5\,{}_2F_1\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (-4*Csc[c + d*x]*(15*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 3*Cot[c + d*x]^4*H
ypergeometric2F1[-5/4, -1/2, -1/4, -Tan[c + d*x]^2] - 15*Cot[c + d*x]^2*Hyp
ergeometric2F1[-1/2, -1/4, 3/4, -Tan[c + d*x]^2] - 5*Hypergeometric2F1[1/2,
 3/4, 7/4, -Tan[c + d*x]^2] + 5*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x
]^2])*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2*Sqrt[e*Tan[c + d*x]])/(
15*a*d*e^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

maple [C] time = 1.62, size = 2113, normalized size = 5.89

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/10/a/d*(-1+cos(d*x+c))*(-10*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))*cos(d*x+c)+5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))+5*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)-5*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+
```

```
sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+12*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-6*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))-5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))+10*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)-10*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)-10*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)+24*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*c
os(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-12*EllipticF((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-5*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)-5*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)+12*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/
2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-6*EllipticF(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)+6*cos(d*x+c)^2*2^(1/2)+4*cos(d*x+c)*2^(1/2))/cos(
d*x+c)^2/(e*sin(d*x+c)/cos(d*x+c))^(3/2)/sin(d*x+c)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{a\,(e\tan(c+d\,x))^{3/2}\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*tan(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\tan(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*tan(c + d*x))**(3/2)*sec(c + d*x) + (e*tan(c + d*x))**(3/2))
, x)/a
```

3.126 $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=328

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ade^{5/2}}+\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{5/2}}-\frac{\log\left(\sqrt{\right.}}{}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)*2^(1/2)-1/2*
arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)*2^(1/2)+1/4*ln(e
^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(5/2)*2^(1/2)
-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(5/2
)*2^(1/2)-5/21*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(
c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/e^2/(e*tan(d*x+c
))^(1/2)+2/7*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(7/2)-2/21*(7-5*sec(d*x+c)
)/a/d/e/(e*tan(d*x+c))^(3/2)
```

Rubi [A] time = 0.42, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3888, 3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ade^{5/2}}+\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{5/2}}-\frac{\log\left(\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(5/2)) -
ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(5/2)) +
Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sqrt[
2]*a*d*e^(5/2)) - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]]/(2*Sqrt[2]*a*d*e^(5/2)) + (2*e*(1 - Sec[c + d*x]))/(7*a*d*(e*Tan[
c + d*x])^(7/2)) - (2*(7 - 5*Sec[c + d*x]))/(21*a*d*e*(e*Tan[c + d*x])^(3/2
)) + (5*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(
21*a*d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{9/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} + \frac{2\int \frac{\frac{7a}{2}-\frac{5}{2}a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{7a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{4\int \frac{-\frac{21a}{4}+\frac{5}{4}a\sec(c+dx}{\sqrt{e\tan(c+dx)}}}{21a^2e^2}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{21ae^2}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right.}\right.}{}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,d\right.}{}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5F\left(c-\frac{\pi}{4}+dx\middle|2\right)}{21ade}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5F\left(c-\frac{\pi}{4}+dx\middle|2\right)}{21ade}$$

$$= \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ade^{5/2}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\,\text{ta}\right.}{}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{5/2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{5/2}} + \frac{\log\left(\sqrt{e}+\right.}{}$$

Mathematica [C] time = 9.57, size = 1299, normalized size = 3.96

$$-\frac{20\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\middle|-1\right)\tan^{\frac{5}{2}}(c+dx)\sec^4(c+dx)}{21d(\sec(c+dx)a+a)(e\tan(c+dx))^{5/2}\left(\tan^2(c+dx)+1\right)^{3/2}} + \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right.}{14d}\right.}{}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]

[Out] (-10*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(21*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) + (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(3*d*E^(I*(2*c + d*x))*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(3*d*E^(I*d*x)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) + (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*(40/(21*d) - Csc[c/2 + (d*x)/2]^2/(6*d) - (2*(21 - 10*Cos[c] + 21*Cos[2*c])*Cos[d*x]*Sec[2*c])/(21*d) - (13*Sec[c/2 + (d*x)/2]^2)/(14*d) + Sec[c/2 + (d*x)/2]^4/(14*d) + (2*Sec[2*c]*(-10*Sin[c] + 21*Sin[2*c])*Sin[d*x])/(21*d))*Tan[c + d*x]^3)/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) - (20*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]], -1]*Sec[c + d*x]^4*Tan[c + d*x]^(5/2))/(21*d*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)*(1 + Tan[c + d*x]^2)^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)

maple [C] time = 1.68, size = 1896, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x)

[Out] -1/42/a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^3*(-21*I*sin(d*x+c)*cos(d*x+c)^2 *((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+ c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+42*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((1-c os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+21*I*((-1+co s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d *x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-21*I*((-1+cos(d* x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-c os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-21*sin(d*x+c)*Ellipt icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1 /2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-21*sin(d*x+c) *EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1 /2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x +c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+52*sin (d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)) *((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)) ^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-42*I*((-1+ cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2) *((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*Ellipti cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+21* I*sin(d*x+c)*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1 +cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-42*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)) ^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/ 2*2^(1/2))-42*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1 +cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d* x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2 *I,1/2*2^(1/2))+104*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/ sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/ 2*2^(1/2))-21*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1 /2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d* x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d* x+c)-21*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1 /2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/ sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+5 2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+ cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2) *((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-20*2^(1/2)*cos(d*x +c)^3-4*cos(d*x+c)^2*2^(1/2)+10*cos(d*x+c)*2^(1/2))/sin(d*x+c)^5/cos(d*x+c) ^3/(e*sin(d*x+c)/cos(d*x+c))^(5/2)*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\left(e\tan\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)}{a\,(e\tan(c+dx))^{5/2}\,(\cos(c+dx)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*tan(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\tan(c+dx))^{\frac{5}{2}}\sec(c+dx)+(e\tan(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((e*tan(c + d*x))**(5/2)*sec(c + d*x) + (e*tan(c + d*x))**(5/2))
, x)/a
```

3.127 $\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=372

$$\frac{e^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{13/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}-\frac{e^{13/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}a^2d}$$

```
[Out] 1/2*e^(13/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1
/2*e^(13/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/
4*e^(13/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/
d*2^(1/2)+1/4*e^(13/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/a^2/d*2^(1/2)+12/5*e^6*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c
+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/
d/sin(2*d*x+2*c)^(1/2)+2/3*e^5*(e*tan(d*x+c))^(3/2)/a^2/d+12/5*e^5*cos(d*x+
c)*(e*tan(d*x+c))^(3/2)/a^2/d-4/5*e^5*sec(d*x+c)*(e*tan(d*x+c))^(3/2)/a^2/d
+2/7*e^3*(e*tan(d*x+c))^(7/2)/a^2/d
```

Rubi [A] time = 0.49, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3888, 3886, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2611, 2613, 2615, 2572, 2639, 2607, 32}

$$\frac{e^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{13/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}-\frac{e^{13/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(13/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*
d) - (e^(13/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*a^2*d) - (e^(13/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(13/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (12*e^6*Cos[c + d*x
]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a^2*d*Sqrt[Sin[2*c
+ 2*d*x]]) + (2*e^5*(e*Tan[c + d*x])^(3/2))/(3*a^2*d) + (12*e^5*Cos[c + d*x
]*(e*Tan[c + d*x])^(3/2))/(5*a^2*d) - (4*e^5*Sec[c + d*x]*(e*Tan[c + d*x])^
(3/2))/(5*a^2*d) + (2*e^3*(e*Tan[c + d*x])^(7/2))/(7*a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
```

```
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx}{a^4}$$
$$= \frac{e^4\int\left(a^2(e\tan(c+dx))^{5/2} - 2a^2\sec(c+dx)(e\tan(c+dx))^{5/2} + a^2\sec^2(c+dx)(e\,\text{ta}\right.}{a^4}$$
$$= \frac{e^4\int(e\tan(c+dx))^{5/2}\,dx}{a^2} + \frac{e^4\int\sec^2(c+dx)(e\tan(c+dx))^{5/2}\,dx}{a^2} - \frac{(2e^4)\int\sec(c}{}$$
$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d} - \frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d} + \frac{e^4\,\text{Subst}\left(\int(ex)^{5/2}\,dx,\right.}{a^2d}$$
$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d} + \frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d} - \frac{4e^5\sec(c+dx)(e\tan}{5a^2d}$$
$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d} + \frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d} - \frac{4e^5\sec(c+dx)(e\tan}{5a^2d}$$
$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d} + \frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d} - \frac{4e^5\sec(c+dx)(e\tan}{5a^2d}$$
$$= -\frac{12e^6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}} + \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d} + \frac{12e}{}$$
$$= -\frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d} + \frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$
$$= \frac{e^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d} - \frac{e^{13/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d} - \frac{e^{13/2}\log\left(\sqrt{e}\right.}{}$$

Mathematica [F] time = 13.81, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2,x]
[Out] Integrate[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 1.76, size = 1518, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/210/a^2/d*(-1+cos(d*x+c))^2*(105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*cos(d*x+c)^4-105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*cos(d*x+c)^4+105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*cos(d*x+c)^3-105*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)^3+105*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*cos(d*x+c)^4+105*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^4+504*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*
x+c)^4-252*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/
2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4+105*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+105*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+504*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticE(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-252*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticF(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))-292*cos(d*x+c)^4*2^(1/2)+336*2^(1/2)*cos(
d*x+c)^3+10*cos(d*x+c)^2*2^(1/2)-84*cos(d*x+c)*2^(1/2)+30*2^(1/2))*cos(d*x+
c)^3*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(13/2)/sin(d*x+c)^11*2^(1/2
)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan\left(c+d\,x\right)\right)}^{13/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(13/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(13/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(13/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.128 $\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=339

$$\frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{e^{11/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}a^2d}$$

```
[Out] 1/2*e^(11/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1
/2*e^(11/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/
4*e^(11/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/
d*2^(1/2)-1/4*e^(11/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/a^2/d*2^(1/2)-2/3*e^6*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d
/(e*tan(d*x+c))^(1/2)+2*e^5*(e*tan(d*x+c))^(1/2)/a^2/d-4/3*e^5*sec(d*x+c)*(
e*tan(d*x+c))^(1/2)/a^2/d+2/5*e^3*(e*tan(d*x+c))^(5/2)/a^2/d
```

Rubi [A] time = 0.46, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3888, 3886, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2611, 2614, 2573, 2641, 2607, 32}

$$\frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}+\frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}+\frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}+\frac{e^{11/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(11/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*
d) - (e^(11/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*a^2*d) + (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (2*e^6*EllipticF[c
- Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*Sqrt[e*Tan[c
 + d*x]]) + (2*e^5*Sqrt[e*Tan[c + d*x]])/(a^2*d) - (4*e^5*Sec[c + d*x]*Sqrt
[e*Tan[c + d*x]])/(3*a^2*d) + (2*e^3*(e*Tan[c + d*x])^(5/2))/(5*a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx}{a^4}$$
$$= \frac{e^4\int\left(a^2(e\tan(c+dx))^{3/2}-2a^2\sec(c+dx)(e\tan(c+dx))^{3/2}+a^2\sec^2(c+dx)(e\tan\right.}{a^4}$$
$$= \frac{e^4\int(e\tan(c+dx))^{3/2}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)(e\tan(c+dx))^{3/2}\,dx}{a^2}-\frac{\left(2e^4\right)\int\sec(c}{}$$
$$= \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d}+\frac{e^4\,\text{Subst}\left(\int(ex)^{3/2}\,dx,x,\text{ta}\right.}{a^2d}$$
$$= \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d}+\frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}-\frac{e^7}{}$$
$$= \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d}+\frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}-\frac{(2e}{}$$
$$= \frac{2e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(}{}$$
$$= \frac{2e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(}{}$$
$$= \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d}-\frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c\right.}{2}$$
$$= \frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{11/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{11/2}\log\left(\sqrt{e}\right.}{}$$

<u>**Mathematica**</u> **[F]** time = 74.21, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2,x]
[Out] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2, x]
```

<u>**fricas**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

<u>**giac**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{11}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a)^2, x)

maple [C] time = 1.69, size = 721, normalized size = 2.13

$$\frac{(-1+\cos(dx+c))\left(-15i\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)+s}{\sin(dx+c}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x)

[Out] 1/30/a^2/d*(-1+cos(d*x+c))*(-15*I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+15*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-50*sin(d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+24*2^(1/2)*cos(d*x+c)^3-44*cos(d*x+c)^2*2^(1/2)+26*cos(d*x+c)*2^(1/2)-6*2^(1/2))*(e*sin(d*x+c)/cos(d*x+c))^(11/2)*cos(d*x+c)^3*(1+cos(d*x+c))^2/sin(d*x+c)^9*2^(1/2)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan\left(c+d\,x\right)\right)}^{11/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*tan(c + d*x))^(11/2)/(a + a/cos(c + d*x))^2,x)

[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(11/2))/(a^2*(cos(c + d*x) + 1)^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*tan(d*x+c))**(11/2)/(a+a*sec(d*x+c))**2,x)

[Out] Timed out

3.129 $\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=312

$$-\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right.}{2\sqrt{2}\,a^2d}$$

```
[Out] -1/2*e^(9/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1
/2*e^(9/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/4
*e^(9/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*
2^(1/2)-1/4*e^(9/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x
+c))/a^2/d*2^(1/2)-4*e^4*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*P
i+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/d/sin(
2*d*x+2*c)^(1/2)+2/3*e^3*(e*tan(d*x+c))^(3/2)/a^2/d-4*e^3*cos(d*x+c)*(e*tan
(d*x+c))^(3/2)/a^2/d
```

Rubi [A] time = 0.41, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3888, 3886, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 2607, 32}

$$-\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (e^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) + (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (4*e^4*Cos[c + d*x]*
EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a^2*d*Sqrt[Sin[2*c + 2*
d*x]]) + (2*e^3*(e*Tan[c + d*x])^(3/2))/(3*a^2*d) - (4*e^3*Cos[c + d*x]*(e*
Tan[c + d*x])^(3/2))/(a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
```

```
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx}{a^4}$$

$$= \frac{e^4\int\left(a^2\sqrt{e\tan(c+dx)}-2a^2\sec(c+dx)\sqrt{e\tan(c+dx)}+a^2\sec^2(c+dx)\sqrt{e\tan\right.}{a^4}$$

$$= \frac{e^4\int\sqrt{e\tan(c+dx)}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a^2}-\frac{\left(2e^4\right)\int\sec(c+}{}$$

$$= -\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}+\frac{\left(4e^4\right)\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a^2}+\frac{e^4 S}{}$$

$$= \frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}+\frac{\left(2e^5\right)\mathrm{Subst}\left(\int\frac{x^2}{e^2+x^4}\,d\right.}{a^2}$$

$$= \frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}-\frac{e^5\,\mathrm{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x\right.}{a^2d}$$

$$= \frac{4e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos}{}$$

$$= \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}-\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+\right.}{2\sqrt{}}$$

$$= -\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{e^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{e^{9/2}\log\left(\sqrt{e}+\right.}{}$$

Mathematica [F] time = 4.60, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 1.73, size = 1480, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2/d*(-1+cos(d*x+c))^2*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*cos(d*x+c)^2-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^2+3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)-3*I*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c
)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))+12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellip
ticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-24*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
```

```
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-3*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-3*EllipticPi(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+12*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)-24*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+
10*cos(d*x+c)^2*2^(1/2)-12*cos(d*x+c)*2^(1/2)+2*2^(1/2))*(e*sin(d*x+c)/cos(
d*x+c))^(9/2)*cos(d*x+c)^3*(1+cos(d*x+c))^2/sin(d*x+c)^9*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan\left(c+d\,x\right)\right)}^{9/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(9/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(9/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.130 $\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=281

$$-\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}-\frac{e^{7/2}\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\,+\right.}{2\sqrt{2}\,a^2d}$$

```
[Out] -1/2*e^(7/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1
/2*e^(7/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/4
*e^(7/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*
2^(1/2)+1/4*e^(7/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x
+c))/a^2/d*2^(1/2)+2*e^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*t
an(d*x+c))^(1/2)+2*e^3*(e*tan(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.38, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3888, 3886, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 2607, 32}

$$-\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}+\frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d}-\frac{e^{7/2}\log\left(\sqrt{e}\,\tan(c+dx)-\right.}{2\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) - (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (2*e^4*EllipticF[c -
 Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(a^2*d*Sqrt[e*Tan[c +
d*x]]) + (2*e^3*Sqrt[e*Tan[c + d*x]])/(a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2}{\sqrt{e\tan(c+dx)}} - \frac{2a^2\sec(c+dx)}{\sqrt{e\tan(c+dx)}} + \frac{a^2\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2} - \frac{\left(2e^4\right)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2}$$

$$= \frac{e^4\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{ex}}\,dx, x, \tan(c+dx)\right)}{a^2d} + \frac{e^5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx, x, e\tan(c+dx)\right)}{a^2d} - \cdots$$

$$= \frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d} + \frac{\left(2e^5\right)\mathrm{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d} - \frac{\left(2e^4\sec(c+dx)\cdots\right.}{\cdots}$$

$$= -\frac{2e^4F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2d\sqrt{e\tan(c+dx)}} + \frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d} + \frac{e^4\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{2e^4F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2d\sqrt{e\tan(c+dx)}} + \frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d} - \frac{e^{7/2}\,\mathrm{Subst}\cdots}{\cdots}$$

$$= -\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d} + \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+\cdots\right.}{2\sqrt{2}\cdots}$$

$$= -\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d} + \frac{e^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d} - \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{\cdots}\right.}{\cdots}$$

Mathematica [F] time = 4.12, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
[Out] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{(a\sec(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 1.78, size = 653, normalized size = 2.32

$$-\frac{\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x)
[Out] -1/2/a^2/d*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-6*EllipticF(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-2*cos(d*x+c)*2^(1/2)+2*2^(1/2))*(-1+co
s(d*x+c))*cos(d*x+c)^3*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(7/2)/sin
(d*x+c)^7*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,(e\tan(c+d\,x))^{7/2}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(7/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.131 $\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=310

$$\frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}\,a^2d}$$

```
[Out] 1/2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/
2*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/4*
e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*2
^(1/2)+1/4*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+
c))/a^2/d*2^(1/2)-4*e^3/a^2/d/(e*tan(d*x+c))^(1/2)+4*e^3*cos(d*x+c)/a^2/d/(
e*tan(d*x+c))^(1/2)-4*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*
Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/d/sin
(2*d*x+2*c)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3888, 3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2608, 2615, 2572, 2639, 2607, 32}

$$\frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}-\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}}-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*d
) - (e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
^2*d) - (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
+ d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(a^2*d*Sqrt[e*T
an[c + d*x]]) + (4*e^3*Cos[c + d*x])/(a^2*d*Sqrt[e*Tan[c + d*x]]) + (4*e^2*
Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a^2*d*Sqrt
[Sin[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
```

```
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2}{(e\tan(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{3/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} - \frac{\left(2e^4\right)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2}$$

$$= -\frac{2e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} - \frac{e^2\int \sqrt{e\tan(c+dx)}\,dx}{a^2} + \frac{\left(4e^2\right)\int \cos(c}{}$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} - \frac{e^3\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{a^2d}$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} - \frac{\left(2e^3\right)\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+a}\right.}{a^2d}$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan}}{a^2d\sqrt{\sin(2c+2dx)}}$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan}}{a^2d\sqrt{\sin(2c+2dx)}}$$

$$= -\frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d} + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+\right.}{2\sqrt{2}}$$

$$= \frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d} - \frac{e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d} - \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

Mathematica [C] time = 6.71, size = 812, normalized size = 2.62

$$\frac{\csc^2(c+dx)\left(\frac{32\cos\left(\frac{c}{2}\right)\cos(dx)\sec(2c)\sin\left(\frac{c}{2}\right)}{d} + \frac{16\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{d} - \frac{16\cos(c)\sec(2c)\sin(dx)}{d} + \frac{16\tan\left(\frac{c}{2}\right)}{d}\right)(e\tan(c+dx))^5}{(\sec(c+dx)a+a)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^4*Csc[c + d*x]^2*((32*Cos[c/2]*Cos[d*x]*Sec[2*c]*Sin[c/
2])/d + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (16*Cos[c]*Sec[2*
c]*Sin[d*x])/d + (16*Tan[c/2])/d)*(e*Tan[c + d*x])^(5/2))/(a + a*Sec[c + d*
x])^2 + ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^(
(4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))
]])*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(d
*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)
))]*(1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2)) -
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(d*E^((2*I)*c
```

```
)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^
((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2)) - (8*E^(I*(c
- d*x))*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1
+ E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/
4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(3
*d*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E
^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [**C**] time = 1.72, size = 360, normalized size = 1.16

$$\frac{(1+\cos(dx+c))^2\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2/a^2/d*(1+cos(d*x+c))^2*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticF(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+8*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-1+cos(d*x+c))*(e*sin(d*x+c)/cos(d*x+c))^(5/2)*cos(d*x+c)^2
/sin(d*x+c)^5*2^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan\left(c+d\,x\right)\right)}^{5/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.132 $\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=316

$$\frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}+\frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\right.}{2\sqrt{2}\,a^2d}$$

```
[Out] 1/2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/
2*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/4*
e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*2
^(1/2)-1/4*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+
c))/a^2/d*2^(1/2)-2/3*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ell
ipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*
tan(d*x+c))^(1/2)-4/3*e^3/a^2/d/(e*tan(d*x+c))^(3/2)+4/3*e^3*sec(d*x+c)/a^2
/d/(e*tan(d*x+c))^(3/2)
```

Rubi [A] time = 0.46, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3888, 3886, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2609, 2614, 2573, 2641, 2607, 32}

$$\frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}-\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*d
) - (e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
^2*d) + (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
+ d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(3*a^2*d*(e*Tan
[c + d*x])^(3/2)) + (4*e^3*Sec[c + d*x])/(3*a^2*d*(e*Tan[c + d*x])^(3/2)) +
 (2*e^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(
3*a^2*d*Sqrt[e*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx}{a^4}$$

$$= \frac{e^4\int\left(\frac{a^2}{(e\tan(c+dx))^{5/2}}-\frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{5/2}}+\frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{5/2}}\,dx}{a^2}+\frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2}-\frac{\left(2e^4\right)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2}$$

$$= -\frac{2e^3}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{\left(2e^2\right)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2}-\frac{e^2\int \frac{}{\sqrt{e}}}{}$$

$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}}-\frac{e^3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx,x,e\tan\right.}{a^2d}$$

$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}}-\frac{\left(2e^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan}\right.}{a^2d}$$

$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{2e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)}{3a^2d\sqrt{e\tan(c+dx}}$$

$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{2e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)}{3a^2d\sqrt{e\tan(c+dx}}$$

$$= \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d}-\frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}\,a}$$

$$= \frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

Mathematica [F] time = 16.52, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
[Out] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 1.84, size = 1267, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*(-1+cos(d*x+c))^2*(3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*I*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*I*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-10*cos(d*x+c)*sin(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*sin(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*sin(d*x+c)-10*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+
c)+4*cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2^(1/2))*cos(d*x+c)*(1+cos(d*x+c))^2
*(e*sin(d*x+c)/cos(d*x+c))^(3/2)/sin(d*x+c)^7*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan\left(c+d\,x\right)\right)}^{3/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*tan(c + d*x))**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
/a**2
```

3.133 $\int \frac{\sqrt{e \tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=363

$$-\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}}+\frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}}-\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1
/2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1/4
*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a^2/d*
2^(1/2)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(
1/2)/a^2/d*2^(1/2)+2*e/a^2/d/(e*tan(d*x+c))^(1/2)-12/5*e*cos(d*x+c)/a^2/d/(
e*tan(d*x+c))^(1/2)+12/5*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*P
i+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/d/sin(
2*d*x+2*c)^(1/2)-4/5*e^3/a^2/d/(e*tan(d*x+c))^(5/2)+4/5*e^3*sec(d*x+c)/a^2/
d/(e*tan(d*x+c))^(5/2)
```

Rubi [A] time = 0.52, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3888, 3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2609, 2608, 2615, 2572, 2639, 2607, 32}

$$-\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}}+\frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}}-\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(5*a^2*d*(e*
Tan[c + d*x])^(5/2)) + (4*e^3*Sec[c + d*x])/(5*a^2*d*(e*Tan[c + d*x])^(5/2)
) + (2*e)/(a^2*d*Sqrt[e*Tan[c + d*x]]) - (12*e*Cos[c + d*x])/(5*a^2*d*Sqrt[
e*Tan[c + d*x]]) - (12*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan
[c + d*x]])/(5*a^2*d*Sqrt[Sin[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
```

```
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx}{a^4}$$

$$= \frac{e^4\int\left(\frac{a^2}{(e\tan(c+dx))^{7/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{7/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int\frac{1}{(e\tan(c+dx))^{7/2}}\,dx}{a^2} + \frac{e^4\int\frac{\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2} - \frac{\left(2e^4\right)\int\frac{\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2}$$

$$= -\frac{2e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} - \frac{e^2\int\frac{1}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} + \frac{\left(6e^2\right)\int \frac{}{(e}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(}{5a^2d\sqrt{e\tan}}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(}{5a^2d\sqrt{e\tan}}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(}{5a^2d\sqrt{e\tan}}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(}{5a^2d\sqrt{e\tan}}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(}{5a^2d\sqrt{e\tan}}$$

$$= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}a}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{\right.}{}$$

Mathematica [C] time = 8.45, size = 2792, normalized size = 7.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*((-24*Cos[c/2]*Cos[d*x]*Sec[2*c]*(4*Si
n[c/2] + Sin[(3*c)/2] + Sin[(5*c)/2]))/(5*d*(1 + 2*Cos[c])) - (56*Sec[c/2]*
Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*S
in[(d*x)/2])/(5*d) - (12*(-2 - 5*Cos[c] - 6*Cos[2*c] + Cos[3*c])*Sec[2*c]*S
in[d*x])/(5*d*(1 + 2*Cos[c])) - (56*Tan[c/2])/(5*d) + (4*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(5*d))*Sqrt[e*Tan[c + d*x]])/(a + a*Sec[c + d*x])^2 + ((E^((2*
I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
 - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[S
qrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)
/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^(I*c)*Sqrt[((-I)*(
-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*
x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - ((-(E^((4*
I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[
Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x
```

```
)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((2*I)*c)*Sqrt[((
-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c
 + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - ((-(E
^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*
x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 +
 (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((3*I)*c)*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) +
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^(I*c)*Sqrt
[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)
*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + ((
Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E
^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 +
 (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((2*I)*c)*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) +
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((6*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((3*I)*c)*
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((
2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]])
+ (4*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*
(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4,
 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(
5*d*E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*
(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^
2*Sqrt[Tan[c + d*x]]) + (4*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x))
+ E^((2*I)*(c + 2*d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hyp
ergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2
*Sqrt[e*Tan[c + d*x]])/(5*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x)))
)/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a +
a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + (2*E^(I*(c - d*x))*Cos[c/2 + (d*x)/
2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sqrt[1 -
E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])
*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[((-I)*(-1 + E^((2*I)
*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*C
os[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - (2*Cos[c/2 + (d*x)/2]^4
*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt[1
- E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))
])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(5*d*E^(I*(3*c + d*x))*Sqr
t[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I
)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + (
8*Cos[c/2 + (d*x)/2]^4*(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I
)*d*x)*(1 + E^((6*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/
2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d
*x]])/(5*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2
*Sqrt[Tan[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \tan(dx+c)}}{(a \sec(dx+c)+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 1.88, size = 2117, normalized size = 5.83

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/10/a^2/d*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c)
)^3*(-10*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c
)+5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+5
*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-5*I*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))+24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*cos(d*x+c)^2*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2*2^(1/2))-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(
d*x+c)^2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)-5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+1
0*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-10*El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-10*Elliptic
Pi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(
d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+48*EllipticE(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-24*EllipticF(((1-cos(d*x+c)+sin(d*
```

```
x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)-5*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)-5*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+24*EllipticE
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-12*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)+2*cos(d*x+c)^2*2^(1/2)-2*cos(d*x+c)*2^(1/2))/sin(d*x+c)^7*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \tan(dx+c)}}{(a \sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,\sqrt{e\tan(c+d\,x)}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e \tan(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*tan(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

3.134 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=365

$$-\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}}+\frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d\sqrt{e}}+\frac{}{3a^2d(e\,\mathrm{ta}}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)+1
/2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)-1/4
*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*2^(1/2)/
e^(1/2)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2
/d*2^(1/2)/e^(1/2)+10/21*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*t
an(d*x+c))^(1/2)-4/7*e^3/a^2/d/(e*tan(d*x+c))^(7/2)+4/7*e^3*sec(d*x+c)/a^2/
d/(e*tan(d*x+c))^(7/2)+2/3*e/a^2/d/(e*tan(d*x+c))^(3/2)-20/21*e*sec(d*x+c)/
a^2/d/(e*tan(d*x+c))^(3/2)
```

Rubi [A] time = 0.53, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3888, 3886, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2609, 2614, 2573, 2641, 2607, 32}

$$-\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}}+\frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d\sqrt{e}}+\frac{}{3a^2d(e\,\mathrm{ta}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a^2*d*Sqrt[e]
)) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a^2*d*Sqrt
[e]) - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(
2*Sqrt[2]*a^2*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]]/(2*Sqrt[2]*a^2*d*Sqrt[e]) - (4*e^3)/(7*a^2*d*(e*Tan[c +
d*x])^(7/2)) + (4*e^3*Sec[c + d*x])/(7*a^2*d*(e*Tan[c + d*x])^(7/2)) + (2*e
)/(3*a^2*d*(e*Tan[c + d*x])^(3/2)) - (20*e*Sec[c + d*x])/(21*a^2*d*(e*Tan[c
 + d*x])^(3/2)) - (10*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*
c + 2*d*x]])/(21*a^2*d*Sqrt[e*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{9/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2}{(e\tan(c+dx))^{9/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{9/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{9/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{9/2}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{9/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{9/2}}}{a^2}$$

$$= -\frac{2e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} - \frac{e^2\int \frac{1}{(e\tan(c+dx))}}{a^2}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d\sqrt{e}} + \frac{\log\left(\sqrt{e}+\right.}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d\sqrt{e}} - \frac{\log\left(\right.}{}$$

Mathematica [C] time = 8.91, size = 1281, normalized size = 3.51

$$\frac{80\sqrt[4]{-1}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\middle|-1\right)\sqrt{\tan(c+dx)}\sec^5(c+dx)}{21d(\sec(c+dx)a+a)^2\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)+1\right)^{3/2}} + \frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{2\sec^4}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (40*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 +
E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[
c + d*x]])/(21*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]
]) + (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^
((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x
))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTa
nh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (
d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(d*E^((2*I)*c)*(-1 +
E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (Sqrt[(
(-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((
```

```
4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqr
t[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1
+ E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^4*Se
c[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(d*E^((2*I)*c)*(-1 + E^((2*I)*(c
+ d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) - (2*Sqrt[((-I)*(-1 +
 E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^4*(3*(
-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 -
 E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))]
)*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(3*d*E^(I*(2*c + d*x))*(-1 +
E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt
[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*
x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c
))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*
(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(3*d*E^(I*d*x)*(-1
 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (Cos
[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(-104/(21*d) + (4*(21 - 20*Cos[c] + 21*Cos
[2*c])*Cos[d*x]*Sec[2*c])/(21*d) + (64*Sec[c/2 + (d*x)/2]^2)/(21*d) - (2*Se
c[c/2 + (d*x)/2]^4)/(7*d) - (4*Sec[2*c]*(-20*Sin[c] + 21*Sin[2*c])*Sin[d*x]
)/(21*d))*Tan[c + d*x])/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (80
*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^4*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c
+ d*x]]], -1]*Sec[c + d*x]^5*Sqrt[Tan[c + d*x]])/(21*d*(a + a*Sec[c + d*x])
^2*Sqrt[e*Tan[c + d*x]]*(1 + Tan[c + d*x]^2)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c))), x)
```

maple [C] time = 1.90, size = 1896, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x)
[Out] 1/42/a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^3*(-21*I*sin(d*x+c)*cos(d*x+c)^
2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+42*I*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+21*I*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-21*I*((-1+cos(d
```

```
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-21*sin(d*x+c)*Ellip
ticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-21*sin(d*x+c
)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+62*si
n(d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-42*I*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*Ellipt
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+21
*I*sin(d*x+c)*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))-42*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))-42*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/
2*I,1/2*2^(1/2))+124*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2*2^(1/2))-21*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d
*x+c)-21*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+
62*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-26*2^(1/2)*cos(d*
x+c)^3+6*cos(d*x+c)^2*2^(1/2)+20*cos(d*x+c)*2^(1/2))/sin(d*x+c)^7/cos(d*x+c
)/(e*sin(d*x+c)/cos(d*x+c))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{a^2\,\sqrt{e\tan(c+d\,x)}\,\left(\cos(c+d\,x)+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*tan(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}\,\sec^2(c+dx)+2\sqrt{e\tan(c+dx)}\,\sec(c+dx)+\sqrt{e\tan(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*tan(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*tan(c + d*x))*s
ec(c + d*x) + sqrt(e*tan(c + d*x))), x)/a**2
```

3.135 $\int \sqrt{a + a\sec(c + dx)}\,\tan^5(c + dx)\,dx$

Optimal. Leaf size=147

$$\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^4d}-\frac{6(a\sec(c+dx)+a)^{7/2}}{7a^3d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}+\frac{2\sqrt{a\sec(c+}}{d}$$

```
[Out] 2/3*(a+a*sec(d*x+c))^(3/2)/a/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^2/d-6/7*(a+a*se
c(d*x+c))^(7/2)/a^3/d+2/9*(a+a*sec(d*x+c))^(9/2)/a^4/d-2*arctanh((a+a*sec(d
*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 88, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^4d}-\frac{6(a\sec(c+dx)+a)^{7/2}}{7a^3d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}+\frac{2\sqrt{a\sec(c+}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Se
c[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a*d) + (2*(a + a*Sec[c +
 d*x])^(5/2))/(5*a^2*d) - (6*(a + a*Sec[c + d*x])^(7/2))/(7*a^3*d) + (2*(a
+ a*Sec[c + d*x])^(9/2))/(9*a^4*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{5/2} + \frac{a^2(a+ax)^{5/2}}{x} + a(a+ax)^{7/2}\right) dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= -\frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d} + \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,d\right.}{a^2 c}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d} + \frac{2(a+a\sec(c+dx))^{9}}{9a^4d}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{6(a+a\sec(c+dx))^{7}}{7a^3d}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+}{3ad}$$

Mathematica [A] time = 0.65, size = 102, normalized size = 0.69

$$\frac{2\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)+1}\left(35\sec^4(c+dx)+5\sec^3(c+dx)-132\sec^2(c+dx)-34\sec(c+dx)+\right.\right.}{315d\sqrt{\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*(-315*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[
1 + Sec[c + d*x]]*(383 - 34*Sec[c + d*x] - 132*Sec[c + d*x]^2 + 5*Sec[c + d
*x]^3 + 35*Sec[c + d*x]^4)))/(315*d*Sqrt[1 + Sec[c + d*x]])
```

fricas [A] time = 1.02, size = 299, normalized size = 2.03

$$\left[\frac{315\sqrt{a}\cos(dx+c)^4\log\left(-8a\cos(dx+c)^2 + 4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8a\cos(dx\right.}{630d\cos(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*sqrt(a)*cos(d*x + c)^4*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) + 4*(383*cos(d*x + c)^4 - 34*cos(d*x + c)^3 - 132*cos(d*
x + c)^2 + 5*cos(d*x + c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d
*cos(d*x + c)^4), 1/315*(315*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^4 +
2*(383*cos(d*x + c)^4 - 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 + 5*cos(d*x
+ c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4)]
```

giac [A] time = 4.82, size = 193, normalized size = 1.31

$$\frac{\sqrt{2}\left(\frac{315\,\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(315\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{4}a-210\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}a^{2}+252\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}a^{3}\cdots}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{4}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\cdots\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(315*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(315*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*a - 2
10*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^2 + 252*(a*tan(1/2*d*x + 1/2*c)^2 - a
)^2*a^3 + 1080*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^4 + 560*a^5)/((a*tan(1/2*d*
x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(cos(d*x + c))
/d
```

maple [B] time = 1.38, size = 359, normalized size = 2.44

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(315\left(\cos^{4}(dx+c)\right)\sqrt{2}\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{9}{2}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)+1260\left(\cos^{3}(dx+c)\right)\sqrt{2}\,\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x)
```

```
[Out] 1/5040/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(315*cos(d*x+c)^4*2^(1/2)*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*2^(1/2))+1260*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/
2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+1890*cos(d*x+c)
^2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctan(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2))+1260*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(9/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+31
5*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(9/2)+12256*cos(d*x+c)^4-1088*cos(d*x+c)^3-4224*cos(
d*x+c)^2+160*cos(d*x+c)+1120)/cos(d*x+c)^4
```

maxima [A] time = 0.63, size = 145, normalized size = 0.99

$$\frac{315\,\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+630\,\sqrt{a+\frac{a}{\cos(dx+c)}}+\frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^{4}}-\frac{270\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^{3}}+\frac{126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^{2}}+\frac{210\cdots}{\cdots}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/315*(315*sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos
(d*x + c)) + sqrt(a))) + 630*sqrt(a + a/cos(d*x + c)) + 70*(a + a/cos(d*x +
 c))^(9/2)/a^4 - 270*(a + a/cos(d*x + c))^(7/2)/a^3 + 126*(a + a/cos(d*x +
c))^(5/2)/a^2 + 210*(a + a/cos(d*x + c))^(3/2)/a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan (c + d\,x)^5 \sqrt{a + \frac{a}{\cos (c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a (\sec (c + dx) + 1)}\, \tan^5 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**5, x)
```

3.136 $\int \sqrt{a+a\sec(c+dx)}\,\tan^3(c+dx)\,dx$

Optimal. Leaf size=99

$$\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}-\frac{2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

```
[Out] -2/3*(a+a*sec(d*x+c))^(3/2)/a/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^2/d+2*arctanh(
(a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3880, 80, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}-\frac{2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + a*Sec
[c + d*x]])/d - (2*(a + a*Sec[c + d*x])^(3/2))/(3*a*d) + (2*(a + a*Sec[c +
d*x])^(5/2))/(5*a^2*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
```

```
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{3/2}}{x}\,dx,x,\sec(c+dx)\right)}{a^2d}$$
$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx,x,\sec(c+dx)\right)}{ad}$$
$$= -\frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx,\right.}{d}$$
$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/}}{5a^2d}$$
$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/}}{5a^2d}$$
$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+d}{3ad}$$

Mathematica [A] time = 0.17, size = 80, normalized size = 0.81

$$\frac{2\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)+1}\left(3\sec^2(c+dx)+\sec(c+dx)-17\right)+15\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)\right)}{15d\sqrt{\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*(15*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[1
+ Sec[c + d*x]]*(-17 + Sec[c + d*x] + 3*Sec[c + d*x]^2)))/(15*d*Sqrt[1 + Se
c[c + d*x]])
```

fricas [A] time = 0.98, size = 259, normalized size = 2.62

$$\left[\frac{15\sqrt{a}\cos(dx+c)^2\log\left(-8a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\cos(dx+\right.}{30d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) - 4*(17*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2), -1/15*(15*sqrt(-a)*arctan(2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
 c) + a))*cos(d*x + c)^2 + 2*(17*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2)]
```

giac [A] time = 2.51, size = 152, normalized size = 1.54

$$-\frac{\sqrt{2}\left(\frac{15\,\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2a^2-10\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a^3-12\,a^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)\operatorname{sgn}(\cos(dx+c}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/15*sqrt(2)*(15*sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a^2 -
 10*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^3 - 12*a^4)/((a*tan(1/2*d*x + 1/2*c)^2
 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(cos(d*x + c))/(a*d)
```

maple [B] time = 1.26, size = 221, normalized size = 2.23

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(15\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{2}\,\left(\cos^2(dx+c)\right)+30\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x)
```

```
[Out] -1/60/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*arctan(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*
cos(d*x+c)^2+30*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*cos(d*x+c)+15*2^(1/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)+136*cos(d*x+c)^2-8*cos(d*x+c)-24)/cos(d*x+c)^2
```

maxima [A] time = 0.57, size = 107, normalized size = 1.08

$$-\frac{15\,\sqrt{a}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+30\,\sqrt{a+\frac{a}{\cos(dx+c)}}-\frac{6\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^2}+\frac{10\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(
d*x + c)) + sqrt(a))) + 30*sqrt(a + a/cos(d*x + c)) - 6*(a + a/cos(d*x + c)
)^(5/2)/a^2 + 10*(a + a/cos(d*x + c))^(3/2)/a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^3\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\tan^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**3,x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**3, x)
```

3.137 $\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3880, 50, 63, 207}

$$\frac{2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x],x]

[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Sec[c + d*x]])/d

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}$$
$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 1.18

$$\frac{\sqrt{a(\sec(c+dx)+1)}\left(2\sqrt{\sec(c+dx)+1} - 2\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)\right)}{d\sqrt{\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] (Sqrt[a*(1 + Sec[c + d*x])]*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 2*Sqrt[1
+ Sec[c + d*x]]))/(d*Sqrt[1 + Sec[c + d*x]])
```

fricas [A] time = 1.08, size = 184, normalized size = 3.61

$$\left[\frac{\sqrt{a}\log\left(-8\,a\cos(dx+c)^2 + 4\left(2\cos(dx+c)^2 + \cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8\,a\cos(dx+c) - a\right) + 4\sqrt{\frac{a\,\cdots}{\cdots}}}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) +
4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d, (sqrt(-a)*arctan(2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))
 + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d]
```

giac [A] time = 0.73, size = 75, normalized size = 1.47

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\,a}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)\mathrm{sgn}\left(\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/s
qrt(-a))/sqrt(-a) + 2*a/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))*sgn(cos(d*x +
c))/d
```

maple [A] time = 0.24, size = 42, normalized size = 0.82

$$\frac{2\sqrt{a+a\sec\left(dx+c\right)}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x)
```

```
[Out] 1/d*(2*(a+a*sec(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1
/2)))
```

maxima [A] time = 0.58, size = 67, normalized size = 1.31

$$\frac{\sqrt{a}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+2\sqrt{a+\frac{a}{\cos(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] (sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
 + sqrt(a))) + 2*sqrt(a + a/cos(d*x + c)))/d
```

mupad [B] time = 1.48, size = 47, normalized size = 0.92

$$\frac{2\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{d}-\frac{2\sqrt{a}\ \operatorname{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(a + a/cos(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + a/cos(c + d*x))^(1
/2)/a^(1/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\ \tan\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x), x)
```

3.138 $\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh(1/2*(a+a*sec(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3880, 86, 63, 207}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (Sqrt[2]*Sqrt[a]*
ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx,\right.}{d}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-2a+x^2}\,d\right.}{}$$
$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Mathematica [A] time = 0.06, size = 72, normalized size = 0.99

$$\frac{\sqrt{a(\sec(c+dx)+1)}\left(2\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sec(c+dx)+1}}{\sqrt{2}}\right)\right)}{d\sqrt{\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]
]/Sqrt[2]])*Sqrt[a*(1 + Sec[c + d*x])])/(d*Sqrt[1 + Sec[c + d*x]])
```

fricas [A] time = 0.83, size = 242, normalized size = 3.32

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+2\sqrt{a}\log\left(-2a\cos(dx+c)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\text{c}\right.}{2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 2*sqr
t(a)*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c) - a))/d, (sqrt(2)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 2*sqr
t(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(a*cos(d*x + c) + a)))/d]
```

giac [A] time = 2.11, size = 88, normalized size = 1.21

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)\text{sgn}\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/
sqrt(-a))/sqrt(-a) - a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))
/sqrt(-a))*sgn(cos(d*x + c))/d
```

maple [A] time = 1.14, size = 98, normalized size = 1.34

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2^(1/2)*arctan(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(dx+c\right)+a}\,\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cot\left(c+d\,x\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\cot\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x), x)
```

3.139 $\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{a}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a}{2d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{7\sqrt{a}\tanh^{-1}\left(\sqrt{\cdots}\right.}{4\sqrt{2}\cdots}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+7/8*arctanh(1/2*(a+a*s
ec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d+1/4*a/d/(a+a*sec(d*x+c)
)^(1/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3880, 103, 152, 156, 63, 207}

$$\frac{a}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a}{2d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{7\sqrt{a}\tanh^{-1}\left(\sqrt{\cdots}\right.}{4\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (7*Sqrt[a]*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(4*Sqrt[2]*d) + a/(4*d*Sqrt
[a + a*Sec[c + d*x]]) + a/(2*d*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} - \frac{a\,\text{Subst}\left(\int \frac{2a^2+\frac{3a^2x}{2}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x\right.}{2d}$$

$$= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}}{}$$

$$= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} + \frac{a\,\text{Sub}}{}$$

$$= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Sub}}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{7\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{}{4d\sqrt{a+}}$$

Mathematica [C] time = 0.30, size = 87, normalized size = 0.66

$$\frac{\cot^2(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-7(\sec(c+dx)-1)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(\sec(c+dx)+1)\right)+8(\sec(c+dx)-1)\,{}_2F_1\left(\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]^2*(-2 - 7*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/
2]*(-1 + Sec[c + d*x]) + 8*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]
]*(-1 + Sec[c + d*x]))*Sqrt[a*(1 + Sec[c + d*x])])/(4*d)
```

fricas [A] time = 1.13, size = 426, normalized size = 3.25

$$\left[\frac{8\left(\cos\left(dx+c\right)^{2}-1\right)\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*(cos(d*x + c)^2 - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*
x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) -
8*a*cos(d*x + c) - a) + 7*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(a)*log((2
*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 3*a
*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2 - cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - d), -1/8*(7*
(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 8*(co
s(d*x + c)^2 - 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(3*cos(d*x + c)^2 - cos(
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - d)]
```

giac [A] time = 0.79, size = 140, normalized size = 1.07

$$\frac{\sqrt{2}\left(\frac{8\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{7\,a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\cdots\right.}}{\tan\left(\frac{1}{2}\,dx+\cdots\right.}\cdots\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(8*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/sqrt(-a) - 7*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sq
rt(-a))/sqrt(-a) + 2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)/tan(1/2*d*x + 1/2*c)^2)*sgn(cos(d*x + c))/d
```

maple [B] time = 1.38, size = 267, normalized size = 2.04

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(8\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sqrt{2}\left(\cos^{2}\left(dx+c\right)\right)+7\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\cdots}{\sqrt{-\cdots}}\right.\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(8*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos
(d*x+c)^2+7*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-8*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ar
ctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-7*(-2*cos(d*
```

```
x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+6
*cos(d*x+c)^2-2*cos(d*x+c))/sin(d*x+c)^4*(cos(d*x+c)^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec\left(dx + c\right) + a} \cot\left(dx + c\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(c+d\,x\right)^3\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \left(\sec{\left(c + d x \right)} + 1\right)} \cot^{3}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**3, x)
```

3.140 $\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=193

$$\frac{43a^2}{96d(a\sec(c+dx)+a)^{3/2}}-\frac{15a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^3}$$

```
[Out] 43/96*a^2/d/(a+a*sec(d*x+c))^(3/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+
c))^(3/2)-15/16*a^2/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)+2*arctanh((a+a*
sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-107/128*arctanh(1/2*(a+a*sec(d*x+c))^(
1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-21/64*a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3880, 103, 151, 152, 156, 63, 207}

$$\frac{43a^2}{96d(a\sec(c+dx)+a)^{3/2}}-\frac{15a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (107*Sqrt[a]*ArcT
anh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2]*d) + (43*a^2)/
(96*d*(a + a*Sec[c + d*x])^(3/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a + a*Se
c[c + d*x])^(3/2)) - (15*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])
^(3/2)) - (21*a)/(64*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a^6\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{a^3\,\text{Subst}\left(\int \frac{4a^2+\frac{7a^2x}{2}}{x(-a+ax)^2(a+}\right.}{4}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{15a^2}{16d(1-\sec(c+dx))(a}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{107\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d} + \frac{}{96d(}$$

Mathematica [C] time = 0.33, size = 102, normalized size = 0.53

$$\frac{\cot^4(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(107(\sec(c+dx)-1)^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(\sec(c+dx)+1)\right)-2\left(32(\sec(c+dx\right.\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]^4*(-2*(57 + 32*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d
*x]]*(-1 + Sec[c + d*x])^2 - 45*Sec[c + d*x]) + 107*Hypergeometric2F1[-3/2,
 1, -1/2, (1 + Sec[c + d*x])/2]*(-1 + Sec[c + d*x])^2)*Sqrt[a*(1 + Sec[c +
d*x])])/(96*d)
```

fricas [A] time = 0.97, size = 529, normalized size = 2.74

$$\left[\frac{384\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\sqrt{a}\log\left(-8a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(384*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(a)*log(-8*a*cos(d*
x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 321*(sqrt(2)*cos(d*x + c)^4 -
 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos
(d*x + c) - 1)) - 4*(205*cos(d*x + c)^4 - 71*cos(d*x + c)^3 - 149*cos(d*x +
 c)^2 + 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d), 1/384*(321*(sqrt(2)*cos(d*x + c)^4 - 2*
sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 384*(c
os(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(205
*cos(d*x + c)^4 - 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 + 63*cos(d*x + c))
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^2 + d)]
```

giac [A] time = 0.88, size = 201, normalized size = 1.04

$$-\frac{\sqrt{2}\left(\frac{384\,\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{321\,a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{8\left(\left(-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}a^{2}+15\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}\cdots}{a^{3}}\cdots\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/384*sqrt(2)*(384*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/sqrt(-a) - 321*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/sqrt(-a) + 8*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^2 + 15
*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^3)/a^3 + 3*(21*(-a*tan(1/2*d*x + 1/2
*c)^2 + a)^(3/2)*a - 19*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2)/(a^2*tan(1
/2*d*x + 1/2*c)^4))*sgn(cos(d*x + c))/d
```

maple [B] time = 1.47, size = 407, normalized size = 2.11

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(-1+\cos\left(dx+c\right)\right)^{2}\left(1+\cos\left(dx+c\right)\right)^{2}\left(384\left(\cos^{4}\left(dx+c\right)\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(}{1+\cos}}}{2}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c
))^2*(384*cos(d*x+c)^4*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+321*cos(d*x+c)^4*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))
-768*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2+410*cos(d*x+c)^4-642*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*
cos(d*x+c)^2-142*cos(d*x+c)^3+384*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arct
an(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-298*cos(d*x+c)
^2+321*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2))+126*cos(d*x+c))/sin(d*x+c)^8
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(c+d\,x\right)^5\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**5, x)
```

3.141 $\int \sqrt{a + a\sec(c + dx)}\,\tan^6(c + dx)\,dx$

Optimal. Leaf size=222

$$\frac{2a^6\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{10a^5\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{2a^4\tan^7(c+dx)}{d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2a^2\text{ta}}{3d(a\sec(}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+2*a*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2
/5*a^3*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2*a^4*tan(d*x+c)^7/d/(a+a*sec(
d*x+c))^(7/2)+10/9*a^5*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+2/11*a^6*tan(d
*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)
```

Rubi [A] time = 0.11, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3887, 461, 203}

$$\frac{2a^6\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{10a^5\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{2a^4\tan^7(c+dx)}{d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2a^2\text{ta}}{3d(a\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^6,x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^2*Tan[c + d*x]^3)/(3*d
*(a + a*Sec[c + d*x])^(3/2)) + (2*a^3*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d
*x])^(5/2)) + (2*a^4*Tan[c + d*x]^7)/(d*(a + a*Sec[c + d*x])^(7/2)) + (10*a
^5*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (2*a^6*Tan[c + d*x]^1
1)/(11*d*(a + a*Sec[c + d*x])^(11/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,\tan^6(c+dx)\,dx = -\frac{(2a^4)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^4)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+7x^6+5ax^8+a^2x^{10}-\frac{1}{a^3(1+ax^2)}\right)dx,\right.}{d}$$

$$= \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^3\tan^5(c+d}{5d(a+a\sec(c+d}$$

$$= -\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^2\tan^3(c}{3d(a+a\sec(c}$$

Mathematica [A] time = 7.28, size = 134, normalized size = 0.60

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(792\sin\left(\frac{1}{2}(c+dx)\right)-1386\sin\left(\frac{3}{2}(c+dx)\right)+495\sin\left(\frac{5}{2}(c\right.\right.}{3960}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^6,x]
```

```
[Out] -1/3960*(Sec[(c + d*x)/2]*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*(3960*S
qrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(11/2) + 792*Sin[(c +
d*x)/2] - 1386*Sin[(3*(c + d*x))/2] + 495*Sin[(5*(c + d*x))/2] - 616*Sin[(7
*(c + d*x))/2] - 247*Sin[(11*(c + d*x))/2]))/d
```

fricas [A] time = 0.78, size = 371, normalized size = 1.67

$$\left[\frac{495\left(\cos(dx+c)^6+\cos(dx+c)^5\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2+2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,(4}{495\,(d\,co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/495*(495*(cos(d*x + c)^6 + cos(d*x + c)^5)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(494*cos(d*x + c)^5 +
 247*cos(d*x + c)^4 - 186*cos(d*x + c)^3 - 155*cos(d*x + c)^2 + 50*cos(d*x
+ c) + 45)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^6 + d*cos(d*x + c)^5), 2/495*(495*(cos(d*x + c)^6 + cos(d*x + c)^5)*s
qrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c))) + (494*cos(d*x + c)^5 + 247*cos(d*x + c)^4 - 186*cos(d*x +
c)^3 - 155*cos(d*x + c)^2 + 50*cos(d*x + c) + 45)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

giac [A] time = 10.33, size = 284, normalized size = 1.28

$$\frac{\sqrt{2}\left(\frac{495\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{4\left(495a^6-\left(2805a^6-\left(6666a^6-\left(4158a^6+\left(221a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.\right.\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}\right.\right.}\right.}{990d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/990*sqrt(2)*(495*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(
2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
 4*sqrt(2)*abs(a) - 6*a))/abs(a) - 4*(495*a^6 - (2805*a^6 - (6666*a^6 - (41
58*a^6 + (221*a^6*tan(1/2*d*x + 1/2*c)^2 - 1463*a^6)*tan(1/2*d*x + 1/2*c)^2
)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*t
an(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))*sgn(cos(d*x + c))/d
```

maple [B] time = 1.44, size = 566, normalized size = 2.55

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(495\sqrt{2}\,\sin(dx+c)\left(\cos^5(dx+c)\right)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{11}{2}}+2475\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x)
```

```
[Out] -1/15840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(495*2^(1/2)*sin(d*x+c)*cos(
d*x+c)^5*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*
x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+2475*2^(1/2)*sin(d*x+c)
*cos(d*x+c)^4*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/c
os(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+4950*2^(1/2)*sin(d
*x+c)*cos(d*x+c)^3*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+4950*2^(1/2)*
sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+2475*2^(
1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+495*2
^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+
c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*sin(d*x+c)+31616*cos(d*x+
c)^6-15808*cos(d*x+c)^5-27712*cos(d*x+c)^4+1984*cos(d*x+c)^3+13120*cos(d*x+
c)^2-320*cos(d*x+c)-2880)/sin(d*x+c)/cos(d*x+c)^5
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^6\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec(c+dx)+1\right)}\,\tan^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**6,x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**6, x)
```

3.142 $\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{2a^4\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{6a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\,\mathrm{ta}}{d\sqrt{a\,\mathrm{se}}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d-2*a*tan(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+6/
5*a^3*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^4*tan(d*x+c)^7/d/(a+a*sec
(d*x+c))^(7/2)
```

Rubi [A] time = 0.09, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 461, 203}

$$\frac{2a^4\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{6a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\,\mathrm{ta}}{d\sqrt{a\,\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^4,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Tan[c + d*x]^3)/(3*d*
(a + a*Sec[c + d*x])^(3/2)) + (6*a^3*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*
x])^(5/2)) + (2*a^4*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,\tan^4(c+dx)\,dx = -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^3)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+3x^4+ax^6+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{\tan(c+d}{\sqrt{a+a\sec(c}}\right.}{d}$$

$$= -\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{6a^3\tan^5(c+}{5d(a+a\sec(c+}$$

$$= \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\tan^3(c+}{3d(a+a\sec(c-}$$

Mathematica [A] time = 5.91, size = 110, normalized size = 0.69

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(35\sin\left(\frac{1}{2}(c+dx)\right)-28\sin\left(\frac{3}{2}(c+dx)\right)-23\sin\left(\frac{7}{2}(c+dx)\right)\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^4,x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(105*Sqrt[2]*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) + 35*Sin[(c + d*x)/2] - 2
8*Sin[(3*(c + d*x))/2] - 23*Sin[(7*(c + d*x))/2]))/(105*d)
```

fricas [A] time = 0.69, size = 331, normalized size = 2.07

$$\left[\frac{105\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\,(9}{105\left(d\cos(dx+c)^4+d\cos(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(92*cos(d*x + c)^3 +
46*cos(d*x + c)^2 - 18*cos(d*x + c) - 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), -2/105*(105*(co
s(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (92*cos(d*x + c)^3 + 46*c
os(d*x + c)^2 - 18*cos(d*x + c) - 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

giac [A] time = 5.44, size = 246, normalized size = 1.54

$$-\frac{\sqrt{2}\left(\frac{105\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{4\left(105a^4-\left(385a^4+\left(43a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-203a^4\right)\tan\left(\frac{1}{2}dx+\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\right.}}\right.}{210d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/210*sqrt(2)*(105*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
+ 4*sqrt(2)*abs(a) - 6*a))/abs(a) - 4*(105*a^4 - (385*a^4 + (43*a^4*tan(1/2
*d*x + 1/2*c)^2 - 203*a^4)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*
tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a)))*sgn(cos(d*x + c))/d
```

maple [B] time = 1.36, size = 317, normalized size = 1.98

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(105\sqrt{2}\,\sin(dx+c)\left(\cos^3(dx+c)\right)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+210\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x)
```

```
[Out] -1/420/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(105*2^(1/2)*sin(d*x+c)*cos(d*
x+c)^3*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+
c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+210*2^(1/2)*sin(d*x+c)*cos
(d*x+c)^2*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+105*2^(1/2)*sin(d*x+c)*
cos(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-736*cos(d*x+c)^4+368*c
os(d*x+c)^3+512*cos(d*x+c)^2-24*cos(d*x+c)-120)/sin(d*x+c)/cos(d*x+c)^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^4\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\tan^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**4,x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**4, x)
```

3.143 $\int \sqrt{a+a\sec(c+dx)}\,\tan^2(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+2*a*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 459, 321, 203}

$$\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Tan[c + d*x]^3)/(3*d
*(a + a*Sec[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,\tan^2(c+dx)\,dx = -\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^2\left(2+ax^2\right)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{d}$$

$$= \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{1+ax}\right.}{}$$

$$= -\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c}{3d(a+a\sec(c}$$

Mathematica [C] time = 4.15, size = 226, normalized size = 2.35

$$\frac{8\sqrt{2}\,\tan^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{7/2}\sqrt{a(\sec(c+dx)+1)}\left(-\frac{4}{7}\tan^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-2\sec(c+\right.\right.}{3d\left(1-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (8*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(7/2)*Sqrt[a*(1 + Sec[c + d*x])]*(-1/2
4*(Cos[c + d*x]*(7 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^4*Sec[(c + d*x)/2]^2*
(-3*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + (-1 + 4*Cos[c + d*x])*Sq
rt[1 - Sec[c + d*x]]))/Sqrt[1 - Sec[c + d*x]] - (4*Hypergeometric2F1[2, 7/2
, 9/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]^2)
/7)*Tan[c + d*x]^3)/(3*d*(1 - Tan[(c + d*x)/2]^2)^(5/2))
```

fricas [A] time = 0.80, size = 283, normalized size = 2.95

$$\left[\frac{3\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2+2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\sqrt{\frac{a\,c}{c}}}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 +
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c
) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d
*x + c)), 2/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x + c))/(d*cos(
d*x + c)^2 + d*cos(d*x + c))]
```

giac [B] time = 4.78, size = 208, normalized size = 2.17

$$\frac{\sqrt{2}\Bigg(\frac{3\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{4\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\Bigg)\operatorname{sgn}(\cos(dx}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*s
qrt(2)*abs(a) - 6*a))/abs(a) + 4*(a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2)*tan(1
/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)))*sgn(cos(d*x + c))/d
```

maple [B] time = 1.06, size = 210, normalized size = 2.19

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\Bigg(3\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos(dx+c)\sqrt{2}\,\sin(dx+c)+3\sqrt{2}\,\mathrm{arcta}}{6d\sin(dx+c)\cos(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x)
```

```
[Out] -1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*cos(d*x+c)*2^(1/2)*sin(d*x+c)+3*2^(1/2)*arctanh(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)*sin(d*x+c)+8*cos(d*x+c)^2-4*cos(d*x+c)-4)/sin(d*x+c)/co
s(d*x+c)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^2\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\tan^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**2, x)
```

3.144 $\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+1/2*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)-cot
(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 480, 522, 203}

$$-\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (S
qrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
Sqrt[2]*d) - (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{\text{Subst}\left(\int \frac{-3a-a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\text{ta}}{\sqrt{a+}}\right.}{d}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+}}\right.}{d}$$

$$= -\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}\,d} - \frac{\cot(c+}{}$$

Mathematica [C] time = 24.19, size = 5502, normalized size = 50.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.97, size = 422, normalized size = 3.87

$$\left[\frac{\sqrt{2}\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx+c)+2\sqrt{-a}\log\left(-\frac{8ac}{}\right.}{4d\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x +
 c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 2*sqrt(-a)*l
og(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(c
os(d*x + c) + 1))*sin(d*x + c) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c))/(d*sin(d*x + c)), -1/2*(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(
d*x + c) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin
(d*x + c) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(
d*x + c))]
```

giac [B] time = 1.18, size = 236, normalized size = 2.17

$$\frac{\sqrt{2}\left(\frac{2\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\sqrt{-a}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}d\right.}\right.\right.\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*s
qrt(2)*abs(a) - 6*a))/abs(a) + sqrt(-a)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2) + 4*sqrt(-a)*a/((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a))*sgn(cos(d*x +
c))/d
```

maple [B] time = 1.24, size = 188, normalized size = 1.72

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(2\sqrt{2}\,\sin(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}+\sin(dx+c)\ln\left(-\frac{-\sqrt{-\frac{2\cos(\right.}{1+\cos(}}}{}\right.\right.}{2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*2^(1/2)*sin(d*x+c)*arctanh(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)+sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)-2*cos(d*x+c))/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(dx+c)+a}\,\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^2\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\cot^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**2, x)
```

3.145 $\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12ad}+\frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8d}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{9\sqrt{a}\tan^{-1}\left(\frac{}{\sqrt{2}\cdots}\right.}{8\sqrt{\cdots}}$$

```
[Out] 1/12*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a/d-1/4*cos(d*x+c)*cot(d*x+c)^3*se
c(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a/d+2*arctan(a^(1/2)*tan(d*x+c)/(
a+a*sec(d*x+c))^(1/2))*a^(1/2)/d-9/16*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)
/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)+7/8*cot(d*x+c)*(a+a*sec(d*x+c))^
(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = $\{3887, 472, 583, 522, 203\}$

$$\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12ad}+\frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8d}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{9\sqrt{a}\tan^{-1}\left(\frac{}{\sqrt{2}\cdots}\right.}{8\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (9*
Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
(8*Sqrt[2]*d) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8*d) + (Cot[c +
d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12*a*d) - (Cos[c + d*x]*Cot[c + d*x]^3*
Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(4*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= -\frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{4ad} - \text{Su}$$

$$= \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} - \frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c\right.}{4ad}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad}$$

$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{9\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{2}d} + \frac{7\cot(c}{}$$

Mathematica [C] time = 24.01, size = 5552, normalized size = 28.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
[Out] Result too large to show
```

fricas [A] time = 1.34, size = 547, normalized size = 2.79

$$\left[\frac{27\left(\sqrt{2}\cos(dx+c)^2-\sqrt{2}\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(27*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(-a)*log((2*sqrt(2)*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*
cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))*sin(d*x + c) + 48*(cos(d*x + c)^2 - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3
 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d
*x + c) + 4*(31*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 21*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - d)*sin(d*x + c)), 1
/48*(48*(cos(d*x + c)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*
x + c) - a))*sin(d*x + c) + 27*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(a)*a
rctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c)))*sin(d*x + c) + 2*(31*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 21
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2
- d)*sin(d*x + c))]
```

giac [B] time = 4.65, size = 365, normalized size = 1.86

$$-\frac{\sqrt{2}\left(\frac{48\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+27\sqrt{-a}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\cdots}\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(48*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/abs(a) + 27*sqrt(-a)*log((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2) + 6*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) + 8*(15*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a - 24*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^2 + 13*sqrt(
-a)*a^3)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a)^3)*sgn(cos(d*x + c))/d
```

maple [B] time = 1.25, size = 381, normalized size = 1.94

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(48\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(\cos^2(dx+c)\right)\sin(dx+c)+27\sqrt{-\frac{2\cdots}{1\cdots}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/48/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(48*2^(1/2)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)+27*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-48*2^(1/2)*sin(d*x+c)*arctanh(1/2*
```

```
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)-27*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)-62*cos(d*x+c)^3+4*cos(d*x+c)^2+42*cos(d*x+c))/sin(d*x+c)^5*(cos(
d*x+c)^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec\left(dx + c\right) + a} \cot\left(dx + c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(c+d\,x\right)^4\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \left(\sec\left(c + dx\right) + 1\right)} \cot^{4}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**4, x)
```

3.146 $\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=280

$$\frac{87\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{160a^2d}-\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{16a^2d}-\frac{17\cos(c\cdots}{}$$

```
[Out] -23/192*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a/d+87/160*cot(d*x+c)^5*(a+a*se
c(d*x+c))^(5/2)/a^2/d-17/32*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*(a
+a*sec(d*x+c))^(5/2)/a^2/d-1/16*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*d*x+1/2*c
)^4*(a+a*sec(d*x+c))^(5/2)/a^2/d-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c
))^(1/2))*a^(1/2)/d+151/256*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(
d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)-105/128*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/
d
```

Rubi [A] time = 0.27, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{87\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{160a^2d}-\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{16a^2d}-\frac{17\cos(c\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
51*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(128*Sqrt[2]*d) - (105*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(128*d) -
(23*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(192*a*d) + (87*Cot[c + d*x]
^5*(a + a*Sec[c + d*x])^(5/2))/(160*a^2*d) - (17*Cos[c + d*x]*Cot[c + d*x]^
5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(32*a^2*d) - (Cos[c + d*x]
^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(5/2))/(16*a^2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6\left(1+ax^2\right)\left(2+ax^2\right)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2 d}$$

$$= -\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2 d} - \ldots$$

$$= -\frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{32a^2 d} - \ldots$$

$$= \frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2 d} - \frac{17\cos(c+dx)\cot^5(c+dx)\sec\ldots}{3\ldots}$$

$$= -\frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad} + \frac{87\cot^5(c+dx)(a+a\sec(c+\ldots}{160a^2 d}$$

$$= -\frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d} - \frac{23\cot^3(c+dx)(a+a\sec(c+\ldots}{192ad}$$

$$= -\frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d} - \frac{23\cot^3(c+dx)(a+a\sec(c+\ldots}{192ad}$$

$$= -\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{151\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}\,d} - \frac{105\ldots}{}$$

Mathematica [C] time = 24.37, size = 5594, normalized size = 19.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6*Sqrt[a + a*Sec[c + d*x]],x]
[Out] Result too large to show
```

fricas [A] time = 2.96, size = 646, normalized size = 2.31

$$\left[\frac{2265\left(\sqrt{2}\cos(dx+c)^4-2\sqrt{2}\cos(dx+c)^2+\sqrt{2}\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/7680*(2265*(sqrt(2)*cos(d*x + c)^4 - 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2))
*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 3840*(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2
 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*(2821*cos(
d*x + c)^5 - 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 + 230*cos(d*x + c)^2
+ 1575*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x +
 c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c)), -1/3840*(3840*(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x +
 c) - a))*sin(d*x + c) + 2265*(sqrt(2)*cos(d*x + c)^4 - 2*sqrt(2)*cos(d*x +
 c)^2 + sqrt(2))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(2821*cos(d*x +
c)^5 - 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 + 230*cos(d*x + c)^2 + 1575
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^4
- 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))]
```

giac [A] time = 3.91, size = 476, normalized size = 1.70

$$-\frac{\sqrt{2}\left(30\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-25\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{3840\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.}\right)}{|a|}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] -1/7680*sqrt(2)*(30*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*tan(1/2*d*x + 1/
2*c)^2 - 25)*tan(1/2*d*x + 1/2*c) - 3840*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt
```

```
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt
(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a) - 2265*sqrt(-a)*log(
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2) -
64*(165*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^8*sqrt(-a)*a - 555*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^6*sqrt(-a)*a^2 + 785*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^3 - 505*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^4 + 134*sqrt(
-a)*a^5)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a)^5)*sgn(cos(d*x + c))/d
```

maple [B] time = 1.58, size = 573, normalized size = 2.05

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(-3840\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^4(dx+c)\right)\sin(dx+c)\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+
c))^2*(-3840*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*2
^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+
c)*2^(1/2))-2265*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+
c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(
d*x+c))+7680*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2
*sin(d*x+c)+5642*cos(d*x+c)^5+4530*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(
-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)*cos(d*x+c)^2*sin(d*x+c)-556*cos(d*x+c)^4-3840*2^(1/2)*sin(d*x+c)*arctanh(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-7928*cos(d*x+c)^3-2265*sin(d*x+c)*ln(-(-(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+460*cos(d*x+c)^2+3150*cos(d*x+c))/sin(d*
x+c)^9
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^6\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c+dx)+1)}\,\cot^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**6, x)
```

3.147 $\int (a + a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx$

Optimal. Leaf size=169

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+dx)+a)^{11/2}}{11a^4d}-\frac{2(a\sec(c+dx)+a)^{9/2}}{3a^3d}+\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d}+\frac{2(a\,\mathrm{s}}{}$$

```
[Out] -2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*(a+a*sec(d*x+c))^(
3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/a/d+2/7*(a+a*sec(d*x+c))^(7/2)/a^2/d-2/3*
(a+a*sec(d*x+c))^(9/2)/a^3/d+2/11*(a+a*sec(d*x+c))^(11/2)/a^4/d+2*a*(a+a*se
c(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3880, 88, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{11/2}}{11a^4d}-\frac{2(a\sec(c+dx)+a)^{9/2}}{3a^3d}+\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^5,x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a*Sqrt[a + a*
Sec[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec[c +
 d*x])^(5/2))/(5*a*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2*d) - (2*(a +
a*Sec[c + d*x])^(9/2))/(3*a^3*d) + (2*(a + a*Sec[c + d*x])^(11/2))/(11*a^4*
d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{7/2}+\frac{a^2(a+ax)^{7/2}}{x}+a(a+ax)^{9/2}\right)dx, x, \sec(c+d$$

$$= -\frac{2(a+a\sec(c+dx))^{9/2}}{3a^3d}+\frac{2(a+a\sec(c+dx))^{11/2}}{11a^4d}+\frac{\text{Subst}\left(\int \frac{(a+ax)^{7/2}}{x}\right.}{a}$$

$$= \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d}-\frac{2(a+a\sec(c+dx))^{9/2}}{3a^3d}+\frac{2(a+a\sec(c+dx)}{11a^4d}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5ad}+\frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d}-\frac{2(a+a\sec(c+dx)}{3a^3d}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5ad}+\frac{2(a+a\sec(c+dx)}{7a^2d}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))}{5ad}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))}{5ad}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c}{3d}$$

Mathematica [A] time = 0.55, size = 112, normalized size = 0.66

$$\frac{2(a(\sec(c+dx)+1))^{3/2}\left(\sqrt{\sec(c+dx)+1}\left(105\sec^5(c+dx)+140\sec^4(c+dx)-325\sec^3(c+dx)-534\sec^2(c\right.\right.}{1155d(\sec(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^5,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(-1155*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sq
rt[1 + Sec[c + d*x]]*(1656 + 327*Sec[c + d*x] - 534*Sec[c + d*x]^2 - 325*Se
c[c + d*x]^3 + 140*Sec[c + d*x]^4 + 105*Sec[c + d*x]^5)))/(1155*d*(1 + Sec[
c + d*x])^(3/2))
```

fricas [A] time = 1.69, size = 334, normalized size = 1.98

$$\left[\frac{1155\,a^{\frac{3}{2}}\cos(dx+c)^5\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/2310*(1155*a^(3/2)*cos(d*x + c)^5*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x
 + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8
*a*cos(d*x + c) - a) + 4*(1656*a*cos(d*x + c)^5 + 327*a*cos(d*x + c)^4 - 53
4*a*cos(d*x + c)^3 - 325*a*cos(d*x + c)^2 + 140*a*cos(d*x + c) + 105*a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^5), 1/1155*(1155*sqrt
(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^5 + 2*(1656*a*cos(d*x + c)^5 + 327*
a*cos(d*x + c)^4 - 534*a*cos(d*x + c)^3 - 325*a*cos(d*x + c)^2 + 140*a*cos(
d*x + c) + 105*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^
5)]
```

giac [A] time = 4.76, size = 218, normalized size = 1.29

$$\frac{\sqrt{2}\left(\frac{1155\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(1155\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^5a-770\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^4a^2+924\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\cdots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^5\sqrt{-a\tan\cdots}}\right)}{1155d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/1155*sqrt(2)*(1155*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(1155*(a*tan(1/2*d*x + 1/2*c)^2 - a)^5*a
- 770*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*a^2 + 924*(a*tan(1/2*d*x + 1/2*c)^2
- a)^3*a^3 - 1320*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a^4 - 6160*(a*tan(1/2*d*
x + 1/2*c)^2 - a)*a^5 - 3360*a^6)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a)))*a*sgn(cos(d*x + c))/d
```

maple [B] time = 1.21, size = 429, normalized size = 2.54

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(1155\sqrt{2}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{11}{2}}\left(\cos^5(dx+c)\right)+5775\sqrt{2}\arctan\left(\cdots\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x)
```

```
[Out] -1/36960/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1155*2^(1/2)*arctan(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
11/2)*cos(d*x+c)^5+5775*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*cos(d*x+c)^4+11550*2^(1
/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(11/2)*cos(d*x+c)^3+11550*2^(1/2)*arctan(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*cos(
d*x+c)^2+5775*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/
2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*cos(d*x+c)+1155*2^(1/2)*arctan(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(11/2)-105984*cos(d*x+c)^5-20928*cos(d*x+c)^4+34176*cos(d*x+c)^3+20800*
cos(d*x+c)^2-8960*cos(d*x+c)-6720)/cos(d*x+c)^5*a
```

maxima [A] time = 0.46, size = 162, normalized size = 0.96

$$\frac{1155\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+770\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}+\frac{210\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{11}{2}}}{a^4}-\frac{770\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^3}+\frac{330\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^2}+\frac{462}{}}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/1155*(1155*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/c
os(d*x + c)) + sqrt(a))) + 770*(a + a/cos(d*x + c))^(3/2) + 210*(a + a/cos(
d*x + c))^(11/2)/a^4 - 770*(a + a/cos(d*x + c))^(9/2)/a^3 + 330*(a + a/cos(
d*x + c))^(7/2)/a^2 + 462*(a + a/cos(d*x + c))^(5/2)/a + 2310*sqrt(a + a/co
s(d*x + c))*a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(c+d\,x\right)^5\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\tan^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**5, x)
```

3.148 $\int (a + a\sec(c + dx))^{3/2} \tan^3(c + dx)\, dx$

Optimal. Leaf size=121

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d}-\frac{2(a\sec(c+dx)+a)^{5/2}}{5ad}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}-\frac{2a\sqrt{a\,\mathrm{s}}}{}$$

[Out] 2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2/3*(a+a*sec(d*x+c))^(3/2)/d-2/5*(a+a*sec(d*x+c))^(5/2)/a/d+2/7*(a+a*sec(d*x+c))^(7/2)/a^2/d-2*a*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 80, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2(a\sec(c+dx)+a)^{5/2}}{5ad}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}-\frac{2a\sqrt{a\,\mathrm{s}}}{}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^3,x]

[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*a*Sqrt[a + a*Sec[c + d*x]])/d - (2*(a + a*Sec[c + d*x])^(3/2))/(3*d) - (2*(a + a*Sec[c + d*x])^(5/2))/(5*a*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2*d)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= -\frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\right.}{a}$$

$$= -\frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx}{7a^2d}$$

$$= -\frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx)}{5ad}$$

$$= -\frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx)}{5ad}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+}{3d}$$

Mathematica [A] time = 0.27, size = 92, normalized size = 0.76

$$\frac{2(a(\sec(c+dx)+1))^{3/2}\left(\sqrt{\sec(c+dx)+1}\left(15\sec^3(c+dx)+24\sec^2(c+dx)-32\sec(c+dx)-146\right)+105\tanh\right.}{105d(\sec(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(105*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(-146 - 32*Sec[c + d*x] + 24*Sec[c + d*x]^2 + 15*Sec[c +
 d*x]^3)))/(105*d*(1 + Sec[c + d*x])^(3/2))
```

fricas [A] time = 1.33, size = 290, normalized size = 2.40

$$\left[\frac{105\,a^{\frac{3}{2}}\cos(dx+c)^3\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+\right.}{210\,d\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/210*(105*a^(3/2)*cos(d*x + c)^3*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
```

```
*cos(d*x + c) - a) - 4*(146*a*cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 - 24*a*c
os(d*x + c) - 15*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c
)^3), -1/105*(105*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^3 + 2*(146*a*
cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 - 24*a*cos(d*x + c) - 15*a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3)]
```

giac [A] time = 2.13, size = 173, normalized size = 1.43

$$-\frac{\sqrt{2}\left(\frac{105\,\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(105\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}a^{2}-70\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}a^{3}+84\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)a\cdots}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\cdots\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/105*sqrt(2)*(105*sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(105*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^
2 - 70*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a^3 + 84*(a*tan(1/2*d*x + 1/2*c)^2
- a)*a^4 + 120*a^5)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))*sgn(cos(d*x + c))/d
```

maple [B] time = 1.14, size = 291, normalized size = 2.40

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(105\left(\cos^{3}(dx+c)\right)\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\sqrt{2}+315\left(\cos^{2}(dx+c)\right)\arctan\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x)
```

```
[Out] 1/840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(105*cos(d*x+c)^3*arctan(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(7/2)*2^(1/2)+315*cos(d*x+c)^2*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*2^(1/2)+315*cos(d*x+c)*ar
ctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(7/2)*2^(1/2)+105*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-2336*cos(d*x+c)^3-5
12*cos(d*x+c)^2+384*cos(d*x+c)+240)/cos(d*x+c)^3*a
```

maxima [A] time = 0.42, size = 124, normalized size = 1.02

$$-\frac{105\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}-\frac{30\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^{2}}+\frac{42\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a}+210\,\sqrt{a+\frac{a}{\cos(dx+c)}}\,a\cdots}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/105*(105*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/co
s(d*x + c)) + sqrt(a))) + 70*(a + a/cos(d*x + c))^(3/2) - 30*(a + a/cos(d*x
```

```
+ c))^(7/2)/a^2 + 42*(a + a/cos(d*x + c))^(5/2)/a + 210*sqrt(a + a/cos(d*x
+ c))*a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan{(c+d\,x)}^3\,{\left(a+\frac{a}{\cos{(c+d\,x)}}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}} \tan^{3}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**3, x)
```

3.149 $\int (a + a\sec(c + dx))^{3/2}\tan(c + dx)\,dx$

Optimal. Leaf size=73

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a\sec(c+dx)+a}}{d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}$$

```
[Out] -2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*(a+a*sec(d*x+c))^(
3/2)/d+2*a*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3880, 50, 63, 207}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a\sec(c+dx)+a}}{d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x],x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a*Sqrt[a + a*
Sec[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,d\right.}{d}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\right.}{}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c}{3d}$$

Mathematica [A] time = 0.15, size = 70, normalized size = 0.96

$$\frac{2(a(\sec(c+dx)+1))^{3/2}\left(\sqrt{\sec(c+dx)+1}\,(\sec(c+dx)+4)-3\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)\right)}{3d(\sec(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x],x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(-3*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[
1 + Sec[c + d*x]]*(4 + Sec[c + d*x])))/(3*d*(1 + Sec[c + d*x])^(3/2))
```

fricas [A] time = 0.70, size = 238, normalized size = 3.26

$$\left[\frac{3\,a^{\frac{3}{2}}\cos(dx+c)\log\left(-8\,a\cos(dx+c)^2+4\left(2\,\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)\right.}{6\,d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^(3/2)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
*x + c) - a) + 4*(4*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c)))/(d*cos(d*x + c)), 1/3*(3*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)
+ 2*(4*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(
d*x + c))]
```

giac [A] time = 1.13, size = 122, normalized size = 1.67

$$\frac{\sqrt{2}\left(\frac{3\,\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a-2\,a^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)a\,\text{sgn}(\cos(dx+c))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(3*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/sqrt(-a) + 2*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a - 2*a^2)/((
a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*a*sgn(c
os(d*x + c))/d
```

maple [A] time = 0.17, size = 57, normalized size = 0.78

$$\frac{\frac{2(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+a\sec\left(dx+c\right)}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x)
```

```
[Out] 1/d*(2/3*(a+a*sec(d*x+c))^(3/2)+2*a*(a+a*sec(d*x+c))^(1/2)-2*a^(3/2)*arctan
h((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

maxima [A] time = 0.45, size = 86, normalized size = 1.18

$$\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{a}{\cos(dx+c)}}\,a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x
 + c)) + sqrt(a))) + 2*(a + a/cos(d*x + c))^(3/2) + 6*sqrt(a + a/cos(d*x +
c))*a)/d
```

mupad [B] time = 1.46, size = 67, normalized size = 0.92

$$\frac{2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{3\,d}-\frac{2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{d}+\frac{2\,a\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] (2*(a + a/cos(c + d*x))^(3/2))/(3*d) - (2*a^(3/2)*atanh((a + a/cos(c + d*x)
)^(1/2)/a^(1/2)))/d + (2*a*(a + a/cos(c + d*x))^(1/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\tan\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x), x)
```

3.150 $\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

```
[Out] 2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2*a^(3/2)*arctanh(1/2*(
a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3880, 83, 63, 207}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[2]*a^(3/2
)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+}}\right.}{d}$$
$$= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} + \frac{(4a^2)\,\text{Subst}\left(\int \right.}{}$$
$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Mathematica [A] time = 0.06, size = 72, normalized size = 0.99

$$\frac{(a(\sec(c+dx)+1))^{3/2}\left(2\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right) - 2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{\sec(c+dx)+1}}{\sqrt{2}}\right)\right)}{d(\sec(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 2*Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*
x]]/Sqrt[2]])*(a*(1 + Sec[c + d*x]))^(3/2))/(d*(1 + Sec[c + d*x])^(3/2))
```

fricas [B] time = 0.75, size = 243, normalized size = 3.33

$$\left[\frac{\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3a\cos(dx+c)-a}{\cos(dx+c)-1}\right) + a^{\frac{3}{2}}\log\left(-2a\cos(dx+c) - 2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a^(3/2)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + a^(3/2)*l
og(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c) - a))/d, 2*(sqrt(2)*sqrt(-a)*a*arctan(sqrt(2)*sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - sqrt(-a
)*a*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a
*cos(d*x + c) + a)))/d]
```

giac [A] time = 1.21, size = 89, normalized size = 1.22

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\,a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)a\text{sgn}\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/
sqrt(-a))/sqrt(-a) - 2*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a
))/sqrt(-a))*a*sgn(cos(d*x + c))/d
```

maple [A] time = 1.02, size = 101, normalized size = 1.38

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)+2\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\right)a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+2*arct
an(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*cot(c + d*x), x)
```

3.151 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=109

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}+\frac{a\sqrt{a\sec(c+dx)+a}}{2d(1-\sec(c+dx))}$$

```
[Out] -2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+5/4*a^(3/2)*arctanh(1/
2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+1/2*a*(a+a*sec(d*x+c))^
(1/2)/d/(1-sec(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 103, 156, 63, 207}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}+\frac{a\sqrt{a\sec(c+dx)+a}}{2d(1-\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (5*a^(3/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*d) + (a*Sqrt[a +
 a*Sec[c + d*x]])/(2*d*(1 - Sec[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^2\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} - \frac{a\,\text{Subst}\left(\int \frac{2a^2+\frac{a^2x}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} - \frac{(5a^3)}{}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} + \frac{a\sqrt{a+}}{2d(1-}$$

Mathematica [A] time = 0.36, size = 99, normalized size = 0.91

$$\frac{(a(\sec(c+dx)+1))^{3/2}\left(-\frac{2\sqrt{\sec(c+dx)+1}}{\sec(c+dx)-1} - 8\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right) + 5\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sec(c+dx)+1}}{\sqrt{2}}\right)\right)}{4d(\sec(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(3/2)*(-8*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 5*Sqrt[
2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]] - (2*Sqrt[1 + Sec[c + d*x]])/(-1
 + Sec[c + d*x])))/(4*d*(1 + Sec[c + d*x])^(3/2))
```

fricas [B] time = 0.68, size = 378, normalized size = 3.47

$$\left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c) + 8\,(a\cos(dx+c)-a)\sqrt{a}\log\left(-2\,a\cos(dx+c) + 2\,\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c\right.}{8\,(d\cos(dx+c)-d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 8*(a*cos(d
*x + c) - a)*sqrt(a)*log(-2*a*cos(d*x + c) + 2*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c) - a) + 5*(sqrt(2)*a*cos(d*x + c) - sqrt(2)
*a)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)))/(d*cos(d*x + c) -
 d), -1/4*(5*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-a)*arctan(sqrt(2)*s
```

```
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c
) + a)) - 8*(a*cos(d*x + c) - a)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 2*a*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*cos(d*x + c) - d)]
```

giac [A] time = 0.91, size = 138, normalized size = 1.27

$$-\frac{\frac{5\sqrt{2}a^2\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-\frac{8a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(5*sqrt(2)*a^2*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))*sg
n(cos(d*x + c))/sqrt(-a) - 8*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)/sqrt(-a))*sgn(cos(d*x + c))/sqrt(-a) + sqrt(2)*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [B] time = 1.18, size = 258, normalized size = 2.37

$$-\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(4\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sqrt{2}}{2}\right)\sqrt{2}\ \left(\cos^2(dx+c)\right)+5\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\ }{\sqrt{\ }}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*co
s(d*x+c)^2+5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+2*cos(d*x+c
)^2-5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2))+2*cos(d*x+c))/sin(d*x+c)^2*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.152 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=171

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{71a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}+\frac{7a^2}{32d\sqrt{a\sec(c+dx)+a}}-\frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a}}$$

```
[Out] 2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-71/64*a^(3/2)*arctanh(1
/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+7/32*a^2/d/(a+a*sec(d*
x+c))^(1/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(1/2)-13/16*a^2/d/(
1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3880, 103, 151, 152, 156, 63, 207}

$$\frac{7a^2}{32d\sqrt{a\sec(c+dx)+a}}-\frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a\sec(c+dx)+a}}+-\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (71*a^(3/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*d) + (7*a^2)/(3
2*d*Sqrt[a + a*Sec[c + d*x]]) - a^2/(4*d*(1 - Sec[c + d*x])^2*Sqrt[a + a*Se
c[c + d*x]]) - (13*a^2)/(16*d*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^6\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{a^3\,\text{Subst}\left(\int \frac{4a^2+\frac{5a^2x}{2}}{x(-a+ax)^2(a+ax)^{3/}}\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a+}}$$

$$= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{}{16}$$

$$= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{}{16}$$

$$= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{}{16}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{71a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d} + \frac{}{32d\sqrt{a}}$$

Mathematica [C] time = 0.32, size = 104, normalized size = 0.61

$$\frac{a^2\left(71(\sec(c+dx)-1)^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(\sec(c+dx)+1)\right)-64(\sec(c+dx)-1)^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sec(c+dx)+1\right)\right.}{32d(\sec(c+dx)-1)^2\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-34 + 71*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/2]*(-1 +
Sec[c + d*x])^2 - 64*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]]*(-1
+ Sec[c + d*x])^2 + 26*Sec[c + d*x]))/(32*d*(-1 + Sec[c + d*x])^2*Sqrt[a*(1
 + Sec[c + d*x])])
```

fricas [B] time = 0.85, size = 589, normalized size = 3.44

$$\left[\frac{64\left(a\cos(dx+c)^3-a\cos(dx+c)^2-a\cos(dx+c)+a\right)\sqrt{a}\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(64*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(
a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 71*(sqrt(2)
*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*a*cos(d*x + c) + sqr
t(2)*a)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) - 4*(27*a*cos
(d*x + c)^3 - 12*a*cos(d*x + c)^2 - 7*a*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) +
 d), 1/64*(71*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2
)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 64*(a*cos(
d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(-a)*arctan(2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c)
+ a)) - 2*(27*a*cos(d*x + c)^3 - 12*a*cos(d*x + c)^2 - 7*a*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2
 - d*cos(d*x + c) + d)]
```

giac [A] time = 1.16, size = 211, normalized size = 1.23

$$\frac{\frac{71\sqrt{2}a^2\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-\frac{128a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-8\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx\right.}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(71*sqrt(2)*a^2*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))*s
gn(cos(d*x + c))/sqrt(-a) - 128*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a)/sqrt(-a))*sgn(cos(d*x + c))/sqrt(-a) - 8*sqrt(2)*sqrt(-a*ta
```

```
n(1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c)) - (17*sqrt(2)*(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)^(3/2)*a^2*sgn(cos(d*x + c)) - 15*sqrt(2)*sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)*a^3*sgn(cos(d*x + c)))/(a^2*tan(1/2*d*x + 1/2*c)^4))/d
```

maple [B] time = 1.27, size = 502, normalized size = 2.94

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\,(1+\cos(dx+c))^2\left(64\left(\cos^3(dx+c)\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/64/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))^2
*(64*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+71*cos(d*x+c)^3*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-64*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2-71*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-64*cos(d*x+c
)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))+54*cos(d*x+c)^3-71*cos(d*x+c)*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+64*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*2^(1/2))*2^(1/2)-24*cos(d*x+c)^2+71*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-14*cos(d*x+c))/sin(d*x+c
)^6*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^5\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.153 $\int (a+a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx$

Optimal. Leaf size=258

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^8\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}}+\frac{14a^7\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{34a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
+2/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+30/7*a^5*tan(d*x+c)^7/d/(a+a
*sec(d*x+c))^(7/2)+34/9*a^6*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+14/11*a^7
*tan(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)+2/13*a^8*tan(d*x+c)^13/d/(a+a*sec(
d*x+c))^(13/2)
```

Rubi [A] time = 0.12, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 461, 203}

$$\frac{2a^8\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}}+\frac{14a^7\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{34a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{30a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{}{5d(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^6,x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^3*Tan[c + d*x]^3)/(3
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (30*a^5*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(34*a^6*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (14*a^7*Tan[c +
d*x]^11)/(11*d*(a + a*Sec[c + d*x])^(11/2)) + (2*a^8*Tan[c + d*x]^13)/(13*d
*(a + a*Sec[c + d*x])^(13/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx = -\frac{(2a^5)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^4}{1+ax^2}dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^5)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+15x^6+17ax^8+7a^2x^{10}+a^3x^{12}-\frac{}{a^3(1}\right.\right.}{d}$$

$$= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx)}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^3\tan^3(c+}{3d(a+a\sec(c-}$$

Mathematica [A] time = 8.62, size = 147, normalized size = 0.57

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(164736\sin\left(\frac{1}{2}(c+dx)\right)+81081\sin\left(\frac{3}{2}(c+dx)\right)+134849\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^6,x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^6*Sqrt[a*(1 + Sec[c + d*x])]*(-1441440*Sqr
t[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(13/2) + 164736*Sin[(c +
 d*x)/2] + 81081*Sin[(3*(c + d*x))/2] + 134849*Sin[(5*(c + d*x))/2] + 98176
*Sin[(9*(c + d*x))/2] + 45045*Sin[(11*(c + d*x))/2] + 32429*Sin[(13*(c + d*
x))/2]))/(1441440*d)
```

fricas [A] time = 0.72, size = 415, normalized size = 1.61

$$\left[\frac{45045\left(a\cos(dx+c)^7+a\cos(dx+c)^6\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/45045*(45045*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(32429*a*cos(
d*x + c)^6 + 38737*a*cos(d*x + c)^5 - 4731*a*cos(d*x + c)^4 - 26465*a*cos(d
*x + c)^3 - 6265*a*cos(d*x + c)^2 + 7875*a*cos(d*x + c) + 3465*a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^7 + d*cos(d*x
+ c)^6), 2/45045*(45045*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6)*sqrt(a)*arcta
n(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c
))) + (32429*a*cos(d*x + c)^6 + 38737*a*cos(d*x + c)^5 - 4731*a*cos(d*x + c
)^4 - 26465*a*cos(d*x + c)^3 - 6265*a*cos(d*x + c)^2 + 7875*a*cos(d*x + c)
+ 3465*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c)^7 + d*cos(d*x + c)^6)]
```

giac [A] time = 6.47, size = 369, normalized size = 1.43

$$\frac{45045\sqrt{-a}\,a^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{2\left(45045\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c))-\left(300300\sqrt{2}a^8\mathrm{sgr}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/45045*(45045*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))*sgn(cos(d*x + c))/abs(a) + 2*(45045*sqrt(2)*a^8*sgn(cos(d*x
 + c)) - (300300*sqrt(2)*a^8*sgn(cos(d*x + c)) - (861861*sqrt(2)*a^8*sgn(co
s(d*x + c)) - (573144*sqrt(2)*a^8*sgn(cos(d*x + c)) - (236951*sqrt(2)*a^8*s
gn(cos(d*x + c)) + (4751*sqrt(2)*a^8*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)
^2 - 53404*sqrt(2)*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d
*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^6*sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.36, size = 656, normalized size = 2.54

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(45045\sin(dx+c)\left(\cos^6(dx+c)\right)\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{13}{2}}\sqrt{2}+27\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x)
```

```
[Out] 1/2882880/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(45045*sin(d*x+c)*cos(d*x+c
)^6*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*2^(1/2)+270270*sin(d*x+c)*co
s(d*x+c)^5*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*2^(1/2)+675675*sin(d*
x+c)*cos(d*x+c)^4*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*2^(1/2)+900900
*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*2^(1/2)
+675675*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*
2^(1/2)+270270*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
3/2)*2^(1/2)+45045*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*sin(d
*x+c)-4150912*cos(d*x+c)^7-807424*cos(d*x+c)^6+5563904*cos(d*x+c)^5+2781952
*cos(d*x+c)^4-2585600*cos(d*x+c)^3-1809920*cos(d*x+c)^2+564480*cos(d*x+c)+4
43520)/cos(d*x+c)^6/sin(d*x+c)*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^6 \left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}} \tan^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**6,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**6, x)
```

3.154 $\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx$

Optimal. Leaf size=194

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{10a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{14a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2}{3d(}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a^2*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+
14/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+10/7*a^5*tan(d*x+c)^7/d/(a+a
*sec(d*x+c))^(7/2)+2/9*a^6*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)
```

Rubi [A] time = 0.11, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 461, 203}

$$\frac{2a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{10a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{14a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^3\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^{3/2}\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^4,x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*Tan[c + d*x]^3)/(3*
d*(a + a*Sec[c + d*x])^(3/2)) + (14*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (10*a^5*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(2*a^6*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = -\frac{(2a^4)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^4)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+7x^4+5ax^6+a^2x^8+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{}}\right.}{d}$$

$$= -\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{14a^4\tan^5(c+d}{5d(a+a\sec(c+d}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+}{3d(a+a\sec(c+}$$

Mathematica [A] time = 6.50, size = 123, normalized size = 0.63

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(126\sin\left(\frac{1}{2}(c+dx)\right)-288\sin\left(\frac{5}{2}(c+dx)\right)-315\sin\left(\frac{7}{2}(c+dx\right.\right.}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^4,x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^4*Sqrt[a*(1 + Sec[c + d*x])]*(2520*Sqrt[2]
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(9/2) + 126*Sin[(c + d*x)/2]
 - 288*Sin[(5*(c + d*x))/2] - 315*Sin[(7*(c + d*x))/2] - 169*Sin[(9*(c + d*
x))/2]))/(2520*d)
```

fricas [A] time = 0.77, size = 371, normalized size = 1.91

$$\left[\frac{315\left(a\cos(dx+c)^5+a\cos(dx+c)^4\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\,(1}{315\,(d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4)*sqrt(-a)*log((2*a*cos(d*x
 + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(169*a*cos(d*x +
c)^4 + 242*a*cos(d*x + c)^3 + 24*a*cos(d*x + c)^2 - 85*a*cos(d*x + c) - 35*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
+ d*cos(d*x + c)^4), -2/315*(315*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4)*sqrt
(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*si
n(d*x + c))) + (169*a*cos(d*x + c)^4 + 242*a*cos(d*x + c)^3 + 24*a*cos(d*x
+ c)^2 - 85*a*cos(d*x + c) - 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

giac [A] time = 3.38, size = 310, normalized size = 1.60

$$-\frac{\frac{315\sqrt{-a}\,a^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{2\left(315\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c))-\left(1470\sqrt{2}a^6\mathrm{sgn}(\cos\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/315*(315*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))*sgn(cos(d*x + c))/abs(a) + 2*(315*sqrt(2)*a^6*sgn(cos(d*x + c)
) - (1470*sqrt(2)*a^6*sgn(cos(d*x + c)) - (756*sqrt(2)*a^6*sgn(cos(d*x + c)
) + (sqrt(2)*a^6*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 162*sqrt(2)*a^6
*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2
*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.28, size = 407, normalized size = 2.10

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(315\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\sqrt{2}\,\sin(dx+c)\left(\cos^4(dx+c)\right)+945\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x)
```

```
[Out] 1/2520/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(315*arctanh(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(7/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^4+945*arctanh(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(7/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3+945*2^(1/2)*sin(d*x+c)*
cos(d*x+c)^2*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+315*2^(1/2)*sin(d*x+
c)*cos(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/c
os(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+2704*cos(d*x+c)^5+1
168*cos(d*x+c)^4-3488*cos(d*x+c)^3-1744*cos(d*x+c)^2+800*cos(d*x+c)+560)/co
s(d*x+c)^4/sin(d*x+c)*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}} \tan^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**4,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**4, x)
```

3.155 $\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx$

Optimal. Leaf size=128

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^3\tan^3(c+dx)}{d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)+2*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2
/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3887, 461, 203}

$$\frac{2a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^3\tan^3(c+dx)}{d(a\sec(c+dx)+a)^{3/2}}-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*Tan[c + d*x]^3)/(d
*(a + a*Sec[c + d*x])^(3/2)) + (2*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d
*x])^(5/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx = -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^2(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^3)\,\text{Subst}\left(\int \left(\frac{1}{a}+3x^2+ax^4-\frac{1}{a(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+dx)}{d(a+a\sec(c+dx))^{3/2}} + \frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+}{d(a+a\sec(c+}$$

Mathematica [A] time = 5.59, size = 97, normalized size = 0.76

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)-10\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(\right.\right.\right.}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^2,x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(-10*Sqrt[2]*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 5*Sin[(3*(c + d*x))/2
] + Sin[(5*(c + d*x))/2]))/(10*d)
```

fricas [A] time = 0.58, size = 321, normalized size = 2.51

$$\left[\frac{5\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(a\,c\right.}{5\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/5*(5*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(a*cos(d*x + c)^2 + 3
*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 2/5*(5*(a*cos(d*x + c)^3 + a*cos(d*x
 + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c))) + (a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(
d*x + c)^2)]
```

giac [A] time = 5.50, size = 224, normalized size = 1.75

$$\frac{\frac{5\sqrt{-a}\,a^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\text{sgn}(\cos(dx+c))}{|a|} - \frac{2\left(\sqrt{2}a^4\text{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-5\sqrt{2}a^4\text{sgn}(\cos(\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*(5*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a)
- 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(sqrt(2)*a^4*sgn(cos(d*x + c))*tan(1/2
*d*x + 1/2*c)^4 - 5*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a
*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.02, size = 300, normalized size = 2.34

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(5\sqrt{2}\ \sin(dx+c)\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+10\sqrt{2}\ \mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x)
```

```
[Out] 1/20/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(5*2^(1/2)*sin(d*x+c)*cos(d*x+c)
^2*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2
^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+10*2^(1/2)*sin(d*x+c)*cos(d*x+
c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2
^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+5*2^(1/2)*arctanh(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(5/2)*sin(d*x+c)-8*cos(d*x+c)^3-16*cos(d*x+c)^2+16*cos(d*x+
c)+8)/sin(d*x+c)/cos(d*x+c)^2*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**2, x)
```

3.156 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=64

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a*cot(d*x+
c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 325, 203}

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2
*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+ax^2\right)}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= -\frac{2a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.38, size = 102, normalized size = 1.59

$$-\frac{2\cot(c+dx)\sqrt{\frac{1}{\sec(c+dx)+1}}\,(a(\sec(c+dx)+1))^{3/2}\left(\sqrt{\cos(c+dx)}\sqrt{\frac{1}{\cos(c+dx)+1}}+\tan\left(\frac{1}{2}(c+dx)\right)\sin^{-1}\left(\frac{\tan\left(\frac{1}{2}(c+\cdots}{\sqrt{\frac{1}{\cos(c+\cdots}}}\right.\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cot[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*(a*(1 + Sec[c + d*x]))^(3/2)
*(Sqrt[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)] + ArcSin[Tan[(c + d*x)/2
]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Tan[(c + d*x)/2]))/d
```

fricas [B] time = 0.67, size = 264, normalized size = 4.12

$$\left[\frac{\sqrt{-a}\,a\log\left(-\frac{8\,a\cos(dx+c)^{3}+4\left(2\,\cos(dx+c)^{2}-\cos(dx+c)\right)\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)-7\,a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)-4\,a\sqrt{\frac{a\cos(\cdots}{\cos\cdots}}}{2\,d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x +
 c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*co
s(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*a*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c)), -(a^(3/2)*arctan(2*sqrt
(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*
cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*a*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c))]
```

giac [B] time = 1.44, size = 197, normalized size = 3.08

$$\frac{\frac{\sqrt{-a}\,a^{2}\log\left(\frac{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}\,|a|-6\,a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{2\,\sqrt{2}\,\sqrt{-a}\,a^{2}\mathrm{sgn}(\cos(dx+c))}{\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a)
)*sgn(cos(d*x + c))/abs(a) + 2*sqrt(2)*sqrt(-a)*a^2*sgn(cos(d*x + c))/((sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a))/d
```

maple [A] time = 1.04, size = 113, normalized size = 1.77

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{2}\,\sin(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}-2\cos(dx+c)\right)a}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2^(1/2)*sin(d*x+c)*arctanh(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)-2*cos(d*x+c))/sin(d*x+c)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*cot(c + d*x)**2, x)
```

3.157 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{3a\cot(c+dx)\sqrt{}}{2}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-1/3*cot(d*x+c
)^3*(a+a*sec(d*x+c))^(3/2)/d-1/4*a^(3/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1
/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)+3/2*a*cot(d*x+c)*(a+a*sec(d*x+c))^(1/
2)/d
```

Rubi [A] time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3887, 480, 583, 522, 203}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{3a\cot(c+dx)\sqrt{}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (a^
(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2
*Sqrt[2]*d) + (3*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(2*d) - (Cot[c +
d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d} - \frac{\text{Subst}\left(\int \frac{-9a-3a^2x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\right.}{3d}$$

$$= \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}$$

$$= \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,d} + \frac{3a\cot(c+}{}$$

Mathematica [C] time = 24.00, size = 5542, normalized size = 38.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.78, size = 531, normalized size = 3.69

$$\left[\frac{3\left(\sqrt{2}\,a\cos(dx+c)-\sqrt{2}\,a\right)\sqrt{-a}\,\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3\,a\cos(dx+c)^2+2\,a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx+c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-a)*log((2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a
*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1))*sin(d*x + c) + 12*(a*cos(d*x + c) - a)*sqrt(-a)*log(-(8*a*cos(d*x + c)^
3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
```

```
cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(
d*x + c) + 4*(11*a*cos(d*x + c)^2 - 9*a*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c)), 1/12*(12*(a*cos(d*
x + c) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(
d*x + c) + 3*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(a)*arctan(sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*
sin(d*x + c) + 2*(11*a*cos(d*x + c)^2 - 9*a*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c))]
```

giac [B] time = 2.46, size = 369, normalized size = 2.56

$$-\frac{3\sqrt{2}\sqrt{-a}\,a\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\frac{24\sqrt{-a}\,a^2\log\left(\ldots\right)}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(3*sqrt(2)*sqrt(-a)*a*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2)*sgn(cos(d*x + c)) + 24*sqrt(-a)*a^2*log(abs(2
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(d*x + c))/abs
(a) + 8*sqrt(2)*(6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^4*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 9*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^3*sgn(cos(d*x +
c)) + 5*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3)/d
```

maple [B] time = 1.11, size = 372, normalized size = 2.58

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(12\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)+3\sqrt{-\ldots}\ldots\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(12*2^(1/2)*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)+3*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-
1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-12*2^(1/2)*sin(d*x+c)*arctanh(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)-3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)-22*cos(d*x+c)^3-4*cos(d*x+c)^2+18*cos(d*x+c))/sin(d*x+c)^3*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + d\,x)^4 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.158 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=226

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}d}+\frac{3\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{20ad}+\frac{5\cot^3(c+$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+5/24*cot(d*x
+c)^3*(a+a*sec(d*x+c))^(3/2)/d+3/20*cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)/a/d
-1/4*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(5/2)/a/
d+11/32*a^(3/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))/d*2^(1/2)-21/16*a*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3887, 472, 583, 522, 203}

$$-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}d}+\frac{3\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{20ad}+\frac{5\cot^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
1*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(16*Sqrt[2]*d) - (21*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(16*d) + (5
*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(24*d) + (3*Cot[c + d*x]^5*(a +
 a*Sec[c + d*x])^(5/2))/(20*a*d) - (Cos[c + d*x]*Cot[c + d*x]^5*Sec[(c + d*
x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(4*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$
$$= -\frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad} - \frac{\text{Sub}}{}$$
$$= \frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad} - \frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(\right.}{4ad}$$
$$= \frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d} + \frac{3\cot^5(c+dx)(a+a\sec(c+dx))}{20ad}$$
$$= -\frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} + \frac{5\cot^3(c+dx)(a+a\sec(c+dx)}{24d}$$
$$= -\frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} + \frac{5\cot^3(c+dx)(a+a\sec(c+dx)}{24d}$$
$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,d} - \frac{21a\,\text{cc}}{}$$

Mathematica [C] time = 24.12, size = 5582, normalized size = 24.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(3/2),x]
[Out] Result too large to show
```

fricas [A] time = 0.72, size = 708, normalized size = 3.13

$$\left[\frac{165\left(\sqrt{2}\,a\cos(dx+c)^3 - \sqrt{2}\,a\cos(dx+c)^2 - \sqrt{2}\,a\cos(dx+c) + \sqrt{2}\,a\right)\sqrt{-a}\,\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\cos(dx+c)^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/960*(165*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*
a*cos(d*x + c) + sqrt(2)*a)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 -
 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c)
+ 480*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(-a)*l
og(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(c
os(d*x + c) + 1))*sin(d*x + c) - 4*(449*a*cos(d*x + c)^4 - 351*a*cos(d*x +
c)^3 - 365*a*cos(d*x + c)^2 + 315*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c)))/((d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)
*sin(d*x + c)), -1/480*(480*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*
x + c) + a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(
d*x + c) + 165*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(
2)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(4
49*a*cos(d*x + c)^4 - 351*a*cos(d*x + c)^3 - 365*a*cos(d*x + c)^2 + 315*a*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^3 -
d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))]
```

giac [B] time = 6.01, size = 519, normalized size = 2.30

$$165\sqrt{2}\sqrt{-a}\,a\log\left(\left(\sqrt{-a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+30\sqrt{2}\sqrt{-a\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/960*(165*sqrt(2)*sqrt(-a)*a*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2)*sgn(cos(d*x + c)) + 30*sqrt(2)*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c) + 960*sqrt
(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn
(cos(d*x + c))/abs(a) + 32*sqrt(2)*(60*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 195*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(
-a)*a^3*sgn(cos(d*x + c)) + 275*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 175*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^5
*sgn(cos(d*x + c)) + 47*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5)/d
```

maple [B] time = 1.61, size = 720, normalized size = 3.19

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\,(1+\cos(dx+c))^{2}\left(-480\left(\cos^{3}(dx+c)\right)\sin(dx+c)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/480/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))^
2*(-480*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^
(1/2))-165*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(
-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)+480*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*
x+c)+165*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+
480*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
+898*cos(d*x+c)^4+165*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))-480*2^(1/2)*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
-702*cos(d*x+c)^3-165*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-
730*cos(d*x+c)^2+630*cos(d*x+c))/sin(d*x+c)^7*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^6\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.159 $\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx$

Optimal. Leaf size=193

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+dx)+a)^{13/2}}{13a^4d}-\frac{6(a\sec(c+dx)+a)^{11/2}}{11a^3d}+\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}+\frac{2a^2}{}$$

```
[Out] -2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*a*(a+a*sec(d*x+c))
^(3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/d+2/7*(a+a*sec(d*x+c))^(7/2)/a/d+2/9*(a
+a*sec(d*x+c))^(9/2)/a^2/d-6/11*(a+a*sec(d*x+c))^(11/2)/a^3/d+2/13*(a+a*sec
(d*x+c))^(13/2)/a^4/d+2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 88, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{13/2}}{13a^4d}-\frac{6(a\sec(c+dx)+a)^{11/2}}{11a^3d}+\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}+\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a^{5/2}\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^5,x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d + (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec
[c + d*x])^(5/2))/(5*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a*d) + (2*(a +
a*Sec[c + d*x])^(9/2))/(9*a^2*d) - (6*(a + a*Sec[c + d*x])^(11/2))/(11*a^3*
d) + (2*(a + a*Sec[c + d*x])^(13/2))/(13*a^4*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{9/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{9/2}+\frac{a^2(a+ax)^{9/2}}{x}+a(a+ax)^{11/2}\right)dx, x, \sec(c+d\right.}{a^4d}$$

$$= -\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3d}+\frac{2(a+a\sec(c+dx))^{13/2}}{13a^4d}+\frac{\text{Subst}\left(\int \frac{(a+ax)^{9}}{x}\right.}{}$$

$$= \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d}-\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3d}+\frac{2(a+a\sec(c+dx}{13a^4d}$$

$$= \frac{2(a+a\sec(c+dx))^{7/2}}{7ad}+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d}-\frac{6(a+a\sec(c+dx)}{11a^3d}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7ad}+\frac{2(a+a\sec(c+dx)}{9a^2d}$$

$$= \frac{2a(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5d}+\frac{2(a+a\sec(c+dx}{7ad}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2a(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx}{5d}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2a(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx}{5d}$$

$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2a(a+a\sec}{3}$$

Mathematica [A] time = 0.78, size = 156, normalized size = 0.81

$$\frac{(a(\sec(c+dx)+1))^{5/2}\left(\frac{2}{13}(\sec(c+dx)+1)^{13/2}-\frac{6}{11}(\sec(c+dx)+1)^{11/2}+\frac{2}{9}(\sec(c+dx)+1)^{9/2}+\frac{2}{7}(\sec(c+dx)+\right.}{d(\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^5,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(5/2)*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 2*Sqrt[
1 + Sec[c + d*x]] + (2*(1 + Sec[c + d*x])^(3/2))/3 + (2*(1 + Sec[c + d*x])^
(5/2))/5 + (2*(1 + Sec[c + d*x])^(7/2))/7 + (2*(1 + Sec[c + d*x])^(9/2))/9
- (6*(1 + Sec[c + d*x])^(11/2))/11 + (2*(1 + Sec[c + d*x])^(13/2))/13))/(d*
(1 + Sec[c + d*x])^(5/2))
```

fricas [A] time = 0.73, size = 386, normalized size = 2.00

$$\left[\frac{45045\, a^{\frac{5}{2}} \cos\left(dx+c\right)^{6} \log\left(-8\, a \cos\left(dx+c\right)^{2} + 4\left(2\, \cos\left(dx+c\right)^{2} + \cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8\, a \cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/90090*(45045*a^(5/2)*cos(d*x + c)^6*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d
*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) -
 8*a*cos(d*x + c) - a) + 4*(71689*a^2*cos(d*x + c)^6 + 31723*a^2*cos(d*x +
c)^5 - 12531*a^2*cos(d*x + c)^4 - 27095*a^2*cos(d*x + c)^3 - 4445*a^2*cos(d
*x + c)^2 + 8505*a^2*cos(d*x + c) + 3465*a^2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)))/(d*cos(d*x + c)^6), 1/45045*(45045*sqrt(-a)*a^2*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 a))*cos(d*x + c)^6 + 2*(71689*a^2*cos(d*x + c)^6 + 31723*a^2*cos(d*x + c)^
5 - 12531*a^2*cos(d*x + c)^4 - 27095*a^2*cos(d*x + c)^3 - 4445*a^2*cos(d*x
+ c)^2 + 8505*a^2*cos(d*x + c) + 3465*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)))/(d*cos(d*x + c)^6)]
```

giac [A] time = 9.54, size = 244, normalized size = 1.26

$$\frac{\sqrt{2}\left(\frac{45045\,\sqrt{2}\,a \arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(45045\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{6} a - 30030\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{5} a^{2} + 36036\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{\left(a\tan\left(\right.\right.}\right.}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/45045*sqrt(2)*(45045*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(45045*(a*tan(1/2*d*x + 1/2*c)^2 - a)^6
*a - 30030*(a*tan(1/2*d*x + 1/2*c)^2 - a)^5*a^2 + 36036*(a*tan(1/2*d*x + 1/
2*c)^2 - a)^4*a^3 - 51480*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^4 + 80080*(a*t
an(1/2*d*x + 1/2*c)^2 - a)^2*a^5 + 393120*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^
6 + 221760*a^7)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^6*sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)))*a^2*sgn(cos(d*x + c))/d
```

maple [B] time = 1.32, size = 500, normalized size = 2.59

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(45045\left(\cos^{6}\left(dx+c\right)\right)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{13}{2}} + 270270\left(\cos^{5}\left(dx+c\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x)
```

```
[Out] 1/2882880/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(45045*cos(d*x+c)^6*2^(1/2)
*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1
```

```
+cos(d*x+c)))^(13/2)+270270*cos(d*x+c)^5*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)+675675
*cos(d*x+c)^4*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/
2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)+900900*cos(d*x+c)^3*2^(1/2)*arcta
n(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(13/2)+675675*cos(d*x+c)^2*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)+270270*cos(d
*x+c)*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(13/2)+45045*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)+9176192
*cos(d*x+c)^6+4060544*cos(d*x+c)^5-1603968*cos(d*x+c)^4-3468160*cos(d*x+c)^
3-568960*cos(d*x+c)^2+1088640*cos(d*x+c)+443520)/cos(d*x+c)^6*a^2
```

maxima [A] time = 0.60, size = 181, normalized size = 0.94

$$\frac{45045\, a^{\frac{5}{2}} \log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right) + 18018\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}} + \frac{6930\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{13}{2}}}{a^4} - \frac{24570\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{11}{2}}}{a^3} + \frac{10010\left(a+\frac{a}{\cos(dx+c)}\right.}{a^2}}{45045\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/45045*(45045*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a
/cos(d*x + c)) + sqrt(a))) + 18018*(a + a/cos(d*x + c))^(5/2) + 6930*(a + a
/cos(d*x + c))^(13/2)/a^4 - 24570*(a + a/cos(d*x + c))^(11/2)/a^3 + 10010*(
a + a/cos(d*x + c))^(9/2)/a^2 + 12870*(a + a/cos(d*x + c))^(7/2)/a + 30030*
(a + a/cos(d*x + c))^(3/2)*a + 90090*sqrt(a + a/cos(d*x + c))*a^2)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^5 \left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.160 $\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}-\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2(a\sec(c+dx)+a)^{7/2}}{7ad}-\frac{2(a\sec$$

```
[Out] 2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2/3*a*(a+a*sec(d*x+c))^
(3/2)/d-2/5*(a+a*sec(d*x+c))^(5/2)/d-2/7*(a+a*sec(d*x+c))^(7/2)/a/d+2/9*(a+
a*sec(d*x+c))^(9/2)/a^2/d-2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3880, 80, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}-\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2(a\sec(c+dx)+a)^{7/2}}{7ad}-\frac{2(a\sec$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*a^2*Sqrt[a + a
*Sec[c + d*x]])/d - (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) - (2*(a + a*Sec[
c + d*x])^(5/2))/(5*d) - (2*(a + a*Sec[c + d*x])^(7/2))/(7*a*d) + (2*(a + a
*Sec[c + d*x])^(9/2))/(9*a^2*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= -\frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\ldots\right)}{\ldots}$$

$$= -\frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx\ldots}{9a^2d}$$

$$= -\frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+d\ldots}{7ad}$$

$$= -\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+\ldots}{5d}$$

$$= -\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+\ldots}{5d}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(\ldots}{3d}$$

Mathematica [A] time = 0.61, size = 102, normalized size = 0.70

$$\frac{2(a(\sec(c+dx)+1))^{5/2}\left(\sqrt{\sec(c+dx)+1}\left(35\sec^4(c+dx)+95\sec^3(c+dx)+12\sec^2(c+dx)-226\sec(c+dx\ldots\right.\right.}{315d(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(315*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(-493 - 226*Sec[c + d*x] + 12*Sec[c + d*x]^2 + 95*Sec[c
+ d*x]^3 + 35*Sec[c + d*x]^4)))/(315*d*(1 + Sec[c + d*x])^(5/2))
```

fricas [A] time = 0.79, size = 334, normalized size = 2.30

$$\left[\frac{315\,a^{\frac{5}{2}}\cos(dx+c)^4\log\left(-8\,a\cos(dx+c)^2-4\left(2\,\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+\ldots\right.}{630\,d\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/630*(315*a^(5/2)*cos(d*x + c)^4*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) - 4*(493*a^2*cos(d*x + c)^4 + 226*a^2*cos(d*x + c)^3 - 1
2*a^2*cos(d*x + c)^2 - 95*a^2*cos(d*x + c) - 35*a^2)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c)))/(d*cos(d*x + c)^4), -1/315*(315*sqrt(-a)*a^2*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + a))*cos(d*x + c)^4 + 2*(493*a^2*cos(d*x + c)^4 + 226*a^2*cos(d*x + c)^
3 - 12*a^2*cos(d*x + c)^2 - 95*a^2*cos(d*x + c) - 35*a^2)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4)]
```

giac [A] time = 3.92, size = 198, normalized size = 1.37

$$-\frac{\sqrt{2}\left(\frac{315\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(315\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^4a^2-210\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3a^3+252\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^4\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right.}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/315*sqrt(2)*(315*sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(315*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*a^
2 - 210*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^3 + 252*(a*tan(1/2*d*x + 1/2*c)^
2 - a)^2*a^4 - 360*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^5 - 560*a^6)/((a*tan(1/
2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*a*sgn(cos(d*x
 + c))/d
```

maple [B] time = 1.24, size = 362, normalized size = 2.50

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(315\sqrt{2}\left(\cos^4(dx+c)\right)\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{9}{2}}+1260\left(\cos^3(dx+c)\right)\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x)
```

```
[Out] -1/5040/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(315*2^(1/2)*cos(d*x+c)^4*arc
tan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(9/2)+1260*2^(1/2)*cos(d*x+c)^3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+1890*2^(1/2)*c
os(d*x+c)^2*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(9/2)+1260*2^(1/2)*cos(d*x+c)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+3
15*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(9/2)+15776*cos(d*x+c)^4+7232*cos(d*x+c)^3-384*cos(
d*x+c)^2-3040*cos(d*x+c)-1120)/cos(d*x+c)^4*a^2
```

maxima [A] time = 0.61, size = 143, normalized size = 0.99

$$-\frac{315a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}-\frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^2}+\frac{90\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a}+210\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/315*(315*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/co
s(d*x + c)) + sqrt(a))) + 126*(a + a/cos(d*x + c))^(5/2) - 70*(a + a/cos(d*
x + c))^(9/2)/a^2 + 90*(a + a/cos(d*x + c))^(7/2)/a + 210*(a + a/cos(d*x +
c))^(3/2)*a + 630*sqrt(a + a/cos(d*x + c))*a^2)/d
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(c+d\,x\right)^3\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.161 $\int (a + a\sec(c + dx))^{5/2} \tan(c + dx)\, dx$

Optimal. Leaf size=97

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5d}$$

```
[Out] -2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*a*(a+a*sec(d*x+c))
^(3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/d+2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3880, 50, 63, 207}

$$\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2a(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x],x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d + (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec
[c + d*x])^(5/2))/(5*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{a\,\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\right.}{d}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx)}{5d}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx)}{5d}$$

$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(}{3d}$$

Mathematica [A] time = 0.24, size = 82, normalized size = 0.85

$$\frac{2(a(\sec(c+dx)+1))^{5/2}\left(\sqrt{\sec(c+dx)+1}\left(3\sec^2(c+dx)+11\sec(c+dx)+23\right)-15\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)\right.}{15d(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x],x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(-15*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(23 + 11*Sec[c + d*x] + 3*Sec[c + d*x]^2)))/(15*d*(1 + S
ec[c + d*x])^(5/2))
```

fricas [A] time = 1.26, size = 282, normalized size = 2.91

$$\left[\frac{15\,a^{\frac{5}{2}}\cos(dx+c)^2\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+\right.}{30\,d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^(5/2)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) + 4*(23*a^2*cos(d*x + c)^2 + 11*a^2*cos(d*x + c) + 3*a^2)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2), 1/15*(15*sqrt(
-a)*a^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^2 + 2*(23*a^2*cos(d*x + c)^2 + 11*
a^2*cos(d*x + c) + 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d
*x + c)^2)]
```

giac [A] time = 1.73, size = 148, normalized size = 1.53

$$\frac{\sqrt{2}\left(\frac{15\,\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2a-10\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a^2+12\,a^3\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)a^2\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(15*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)/sqrt(-a))/sqrt(-a) + 2*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a - 10*(
a*tan(1/2*d*x + 1/2*c)^2 - a)*a^2 + 12*a^3)/((a*tan(1/2*d*x + 1/2*c)^2 - a)
^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*a^2*sgn(cos(d*x + c))/d
```

maple [A] time = 0.19, size = 74, normalized size = 0.76

$$\frac{\frac{2(a+a\sec(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2a^2\sqrt{a+a\sec\left(dx+c\right)}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x)
```

```
[Out] 1/d*(2/5*(a+a*sec(d*x+c))^(5/2)+2/3*a*(a+a*sec(d*x+c))^(3/2)+2*a^2*(a+a*sec
(d*x+c))^(1/2)-2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

maxima [A] time = 0.64, size = 105, normalized size = 1.08

$$\frac{15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+6\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}+10\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a+30\,\sqrt{a+\frac{a}{\cos(dx+c)}}\,a^2}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/15*(15*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d
*x + c)) + sqrt(a))) + 6*(a + a/cos(d*x + c))^(5/2) + 10*(a + a/cos(d*x + c
))^(3/2)*a + 30*sqrt(a + a/cos(d*x + c))*a^2)/d
```

mupad [B] time = 1.82, size = 92, normalized size = 0.95

$$\frac{2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{5\,d}+\frac{2\,a\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{3\,d}+\frac{2\,a^2\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{d}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,1\mathrm{i}}{\sqrt{a}}\right)2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] (2*(a + a/cos(c + d*x))^(5/2))/(5*d) + (a^(5/2)*atan(((a + a/cos(c + d*x))^
(1/2)*1i)/a^(1/2))*2i)/d + (2*a*(a + a/cos(c + d*x))^(3/2))/(3*d) + (2*a^2*
(a + a/cos(c + d*x))^(1/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sec (c + dx) + 1))^{\frac{5}{2}} \tan (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x), x)
```

3.162 $\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}$$

```
[Out] 2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-4*a^(5/2)*arctanh(1/2*(
a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+2*a^2*(a+a*sec(d*x+c))^(1/
2)/d
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3880, 84, 156, 63, 207}

$$\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (4*Sqrt[2]*a^(5/2
)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{a\,\text{Subst}\left(\int \frac{a^3+3a^3x}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{(4a^4$$
$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx}$$
$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{}$$

Mathematica [A] time = 0.13, size = 83, normalized size = 0.87

$$\frac{2(a(\sec(c+dx)+1))^{5/2}\left(\sqrt{\sec(c+dx)+1}+\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sec(c+dx)+1}}{\sqrt{2}}\right)\right)}{d(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 2*Sqrt[2
]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]] + Sqrt[1 + Sec[c + d*x]]))/(d*(1
+ Sec[c + d*x])^(5/2))
```

fricas [A] time = 0.60, size = 300, normalized size = 3.16

$$\left[\frac{2\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+a^{\frac{5}{2}}\log\left(-2a\cos(dx+c)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(d\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [(2*sqrt(2)*a^(5/2)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + a^(5/2)
*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c) - a) + 2*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d, 2*(2*
sqrt(2)*sqrt(-a)*a^2*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - sqrt(-a)*a^2*arctan(sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a))
 + a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d]
```

giac [A] time = 1.03, size = 112, normalized size = 1.18

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{4\,a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\,a}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\right)a^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/
sqrt(-a))/sqrt(-a) - 4*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a
))/sqrt(-a) - 2*a/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2*sgn(cos(d*x + c)
)/d
```

maple [A] time = 1.01, size = 124, normalized size = 1.31

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sqrt{2}}{2}\right)\sqrt{2}+4\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)-2\right)a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+4*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2))-2)*a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec\left(dx+c\right)+a)^{\frac{5}{2}}\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(c+d\,x\right)\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.163 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=106

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d}+\frac{a^2\sqrt{a\sec(c+dx)+a}}{d(1-\sec(c+dx))}$$

```
[Out] -2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+3/2*a^(5/2)*arctanh(1/
2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+a^2*(a+a*sec(d*x+c))^(1
/2)/d/(1-sec(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3880, 99, 156, 63, 207}

$$\frac{a^2\sqrt{a\sec(c+dx)+a}}{d(1-\sec(c+dx))}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (3*a^(5/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*d) + (a^2*Sqrt[a +
 a*Sec[c + d*x]])/(d*(1 - Sec[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^4\,\mathrm{Subst}\left(\int \frac{\sqrt{a+ax}}{x(-a+ax)^2}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))} + \frac{a^3\,\mathrm{Subst}\left(\int \frac{-a-\frac{ax}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))} + \frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} - \frac{(3}{}$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))} + \frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+}\right.}{d}$$

$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d} + \frac{a^2\sqrt{}}{d(}$$

Mathematica [A] time = 0.29, size = 115, normalized size = 1.08

$$-\frac{(a(\sec(c+dx)+1))^{5/2}\left(2\sqrt{\sec(c+dx)+1}+4(\sec(c+dx)-1)\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-3\sqrt{2}\,(\sec(c+d\right.}{2d(\sec(c+dx)-1)(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/2*((a*(1 + Sec[c + d*x]))^(5/2)*(4*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*(-1 +
 Sec[c + d*x]) - 3*Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]]*(-1 + Se
c[c + d*x]) + 2*Sqrt[1 + Sec[c + d*x]]))/(d*(-1 + Sec[c + d*x])*(1 + Sec[c
+ d*x])^(5/2))
```

fricas [B] time = 0.76, size = 398, normalized size = 3.75

$$\left[\frac{4\,a^2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)+4\left(a^2\cos(dx+c)-a^2\right)\sqrt{a}\,\log\left(-2\,a\cos(dx+c)+2\,\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos\right.}{4\,(d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 4*(a^2*c
os(d*x + c) - a^2)*sqrt(a)*log(-2*a*cos(d*x + c) + 2*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c) - a) + 3*(sqrt(2)*a^2*cos(d*x + c) -
 sqrt(2)*a^2)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)))/(d*cos(
```

```
d*x + c) - d), 1/2*(2*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c) - 3*(sqrt(2)*a^2*cos(d*x + c) - sqrt(2)*a^2)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c)
 + a)) + 4*(a^2*cos(d*x + c) - a^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)))/(d*cos(d*x +
c) - d)]
```

giac [A] time = 6.34, size = 140, normalized size = 1.32

$$-\frac{\frac{3\sqrt{2}a^3\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-\frac{4a^3\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^2\mathrm{sg}}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*sqrt(2)*a^3*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))*sg
n(cos(d*x + c))/sqrt(-a) - 4*a^3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)/sqrt(-a))*sgn(cos(d*x + c))/sqrt(-a) + sqrt(2)*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)*a^2*sgn(cos(d*x + c))/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [B] time = 1.13, size = 248, normalized size = 2.34

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(2\cos\left(dx+c\right)\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)+3\cos\left(dx+c\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{arcta}\right.}{2d\,(-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*cos(d*x+c)*2^(1/2)*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2
^(1/2))+3*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2))-2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-3*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2*cos
(d*x+c))/(-1+cos(d*x+c))*a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec\left(dx+c\right)+a)^{\frac{5}{2}}\cot\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(c+d\,x\right)^3\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.164 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=147

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{43a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}-\frac{11a^2\sqrt{a\sec(c+dx)+a}}{16d(1-\sec(c+dx))}-\frac{a^2\sqrt{a\sec(c+dx)+a}}{4d(1-\sec(c+dx))^2}$$

```
[Out] 2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-43/32*a^(5/2)*arctanh(1
/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-1/4*a^2*(a+a*sec(d*x+c
))^(1/2)/d/(1-sec(d*x+c))^2-11/16*a^2*(a+a*sec(d*x+c))^(1/2)/d/(1-sec(d*x+c
))
```

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3880, 103, 151, 156, 63, 207}

$$-\frac{11a^2\sqrt{a\sec(c+dx)+a}}{16d(1-\sec(c+dx))}-\frac{a^2\sqrt{a\sec(c+dx)+a}}{4d(1-\sec(c+dx))^2}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{43a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (43*a^(5/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*d) - (a^2*Sqrt[
a + a*Sec[c + d*x]])/(4*d*(1 - Sec[c + d*x])^2) - (11*a^2*Sqrt[a + a*Sec[c
+ d*x]])/(16*d*(1 - Sec[c + d*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^6\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^3\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{a^3\,\text{Subst}\left(\int \frac{4a^2+\frac{3a^2x}{2}}{x(-a+ax)^2\sqrt{a+ax}}\,dx, x, \sec(c+a\right.}{4d}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))} + \frac{\text{Subst}\left(\int \frac{8a^4}{x(-a+\right.}{}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))} - \frac{a^3\,\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))} - \frac{\left(2a^2\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{43a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d} - \frac{a^2\sqrt{}}{4d(}$$

Mathematica [A] time = 1.39, size = 138, normalized size = 0.94

$$\frac{(a(\sec(c+dx)+1))^{5/2}\left(\sqrt{\sec(c+dx)+1}\,(11\sec(c+dx)-15)+32(\sec(c+dx)-1)^2\tanh^{-1}\left(\sqrt{\sec(c+dx)+}\right.\right.}{16d(\sec(c+dx)-1)^2(\sec(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(5/2)*(32*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*(-1 + Sec
[c + d*x])^2 + Sqrt[1 + Sec[c + d*x]]*(-15 + 11*Sec[c + d*x]) - 86*Sqrt[2]*
ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]]*Sec[c + d*x]^2*Sin[(c + d*x)/2]^4))
/(16*d*(-1 + Sec[c + d*x])^2*(1 + Sec[c + d*x])^(5/2))
```

fricas [B] time = 0.59, size = 503, normalized size = 3.42

$$\left[\frac{64\left(a^2\cos(dx+c)^2-2a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(-2a\cos(dx+c)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-a\right)+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(64*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(a)*log(-2*a*
cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c) - a) + 43*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqr
t(2)*a^2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) - 4*(15*a^2
*cos(d*x + c)^2 - 11*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c)))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d), 1/32*(43*(sqrt(2)*a^2*cos(d
*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x
 + c) + a)) - 64*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(-a)*a
rctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(
d*x + c) + a)) - 2*(15*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)]
```

giac [A] time = 1.76, size = 177, normalized size = 1.20

$$\frac{\frac{43\sqrt{2}a^3\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-\frac{64a^3\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-\frac{\sqrt{2}\left(13\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\cdots}\cdots\right)}{\cdots}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(43*sqrt(2)*a^3*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))*s
gn(cos(d*x + c))/sqrt(-a) - 64*a^3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)/sqrt(-a))*sgn(cos(d*x + c))/sqrt(-a) - sqrt(2)*(13*(-a*tan(1
/2*d*x + 1/2*c)^2 + a)^(3/2)*a^3*sgn(cos(d*x + c)) - 11*sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)*a^4*sgn(cos(d*x + c)))/(a^2*tan(1/2*d*x + 1/2*c)^4))/d
```

maple [B] time = 1.33, size = 376, normalized size = 2.56

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(1+\cos(dx+c)\right)^2\left(32\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sqrt{2}\left(\cos^2(dx+c)\right)+43\sqrt{-\frac{2\cos\cdots}{1+\cos\cdots}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(32*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
2^(1/2))*2^(1/2)*cos(d*x+c)^2+43*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcta
```

```
n(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-64*cos(d*x+c)*2^(1/2
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))-86*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+32*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+3
0*cos(d*x+c)^2+43*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2))-22*cos(d*x+c))/sin(d*x+c)^4*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^5\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.165 $\int (a + a\sec(c + dx))^{5/2}\tan^6(c + dx)\,dx$

Optimal. Leaf size=290

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^{10}\tan^{15}(c+dx)}{15d(a\sec(c+dx)+a)^{15/2}}+\frac{18a^9\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}}+\frac{62a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^3*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
+2/5*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+62/7*a^6*tan(d*x+c)^7/d/(a+a
*sec(d*x+c))^(7/2)+98/9*a^7*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+62/11*a^8
*tan(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)+18/13*a^9*tan(d*x+c)^13/d/(a+a*sec
(d*x+c))^(13/2)+2/15*a^10*tan(d*x+c)^15/d/(a+a*sec(d*x+c))^(15/2)
```

Rubi [A] time = 0.13, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 461, 203}

$$\frac{2a^{10}\tan^{15}(c+dx)}{15d(a\sec(c+dx)+a)^{15/2}}+\frac{18a^9\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}}+\frac{62a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{98a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{6}{7d(a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^6,x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^4*Tan[c + d*x]^3)/(3
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^5*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (62*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(98*a^7*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (62*a^8*Tan[c +
d*x]^11)/(11*d*(a + a*Sec[c + d*x])^(11/2)) + (18*a^9*Tan[c + d*x]^13)/(13*
d*(a + a*Sec[c + d*x])^(13/2)) + (2*a^10*Tan[c + d*x]^15)/(15*d*(a + a*Sec[
c + d*x])^(15/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx = -\frac{(2a^6)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^5}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= -\frac{(2a^6)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+31x^6+49ax^8+31a^2x^{10}+9a^3x^{12}\right.\right.}{d}$$
$$= \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^5\tan^5(c+}{5d(a+a\sec(c+}$$
$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^4\tan^3(}{3d(a+a\sec}$$

Mathematica [A] time = 10.33, size = 173, normalized size = 0.60

$$a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(604890\sin\left(\frac{1}{2}(c+dx)\right)-87230\sin\left(\frac{3}{2}(c+dx)\right)+45045\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^6,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^7*Sqrt[a*(1 + Sec[c + d*x])]*(-2882880*S
qrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(15/2) + 604890*Sin[(c
 + d*x)/2] - 87230*Sin[(3*(c + d*x))/2] + 450450*Sin[(5*(c + d*x))/2] - 137
670*Sin[(7*(c + d*x))/2] + 210210*Sin[(9*(c + d*x))/2] + 75450*Sin[(11*(c +
 d*x))/2] + 90090*Sin[(13*(c + d*x))/2] + 16066*Sin[(15*(c + d*x))/2]))/(28
82880*d)
```

fricas [A] time = 0.80, size = 477, normalized size = 1.64

$$\left[\frac{45045\left(a^2\cos(dx+c)^8+a^2\cos(dx+c)^7\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/45045*(45045*(a^2*cos(d*x + c)^8 + a^2*cos(d*x + c)^7)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(16066*a^
2*cos(d*x + c)^7 + 53078*a^2*cos(d*x + c)^6 + 17286*a^2*cos(d*x + c)^5 - 30
640*a^2*cos(d*x + c)^4 - 26810*a^2*cos(d*x + c)^3 + 2898*a^2*cos(d*x + c)^2
 + 10164*a^2*cos(d*x + c) + 3003*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7), 2/45045*(45045*(a^2
*cos(d*x + c)^8 + a^2*cos(d*x + c)^7)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (16066*a^2*cos(d*x
 + c)^7 + 53078*a^2*cos(d*x + c)^6 + 17286*a^2*cos(d*x + c)^5 - 30640*a^2*c
os(d*x + c)^4 - 26810*a^2*cos(d*x + c)^3 + 2898*a^2*cos(d*x + c)^2 + 10164*
a^2*cos(d*x + c) + 3003*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7)]
```

giac [A] time = 10.86, size = 397, normalized size = 1.37

$$\frac{\frac{45045\sqrt{-a}\,a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}-\frac{2\left(45045\sqrt{2}\,a^{10}\mathrm{sgn}(\cos(dx+c))-\left(345345\sqrt{2}\,a^{10}\mathrm{sgn}(c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/45045*(45045*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(45045*sqrt(2)*a^10*sgn(cos(d*
x + c)) - (345345*sqrt(2)*a^10*sgn(cos(d*x + c)) - (1162161*sqrt(2)*a^10*sg
n(cos(d*x + c)) - (611325*sqrt(2)*a^10*sgn(cos(d*x + c)) - (77935*sqrt(2)*a
^10*sgn(cos(d*x + c)) + (109005*sqrt(2)*a^10*sgn(cos(d*x + c)) + (11633*sqr
t(2)*a^10*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 64725*sqrt(2)*a^10*sgn
(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
 + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^7*sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.67, size = 747, normalized size = 2.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x)
```

```
[Out] -1/5765760/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(45045*sin(d*x+c)*cos(d*x+
c)^7*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)*2^(1/2)+315315*sin(d*x+c)*c
os(d*x+c)^6*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)*2^(1/2)+945945*sin(d
*x+c)*cos(d*x+c)^5*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)*2^(1/2)+15765
75*sin(d*x+c)*cos(d*x+c)^4*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)*2^(1/
2)+1576575*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/
2)*2^(1/2)+945945*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(15/2)*2^(1/2)+315315*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(15/2)*2^(1/2)+45045*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
5/2)*sin(d*x+c)+4112896*cos(d*x+c)^8+9475072*cos(d*x+c)^7-9162752*cos(d*x+c
)^6-12269056*cos(d*x+c)^5+980480*cos(d*x+c)^4+7605248*cos(d*x+c)^3+1860096*
cos(d*x+c)^2-1833216*cos(d*x+c)-768768)/sin(d*x+c)/cos(d*x+c)^7*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^6 \left(a + \frac{a}{\cos(c + dx)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.166 $\int (a+a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx$

Optimal. Leaf size=224

$$\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{14a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{34a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{6a}{d(a\,\mathrm{se}}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a^3*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+
6*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+34/7*a^6*tan(d*x+c)^7/d/(a+a*se
c(d*x+c))^(7/2)+14/9*a^7*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+2/11*a^8*tan
(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)
```

Rubi [A] time = 0.11, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3887, 461, 203}

$$\frac{2a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{14a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{34a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{6a^5\tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^4\,\mathrm{ta}}{3d(a\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^4,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^4*Tan[c + d*x]^3)/(3*
d*(a + a*Sec[c + d*x])^(3/2)) + (6*a^5*Tan[c + d*x]^5)/(d*(a + a*Sec[c + d*
x])^(5/2)) + (34*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (14
*a^7*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (2*a^8*Tan[c + d*x]
^11)/(11*d*(a + a*Sec[c + d*x])^(11/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx = -\frac{(2a^5)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)^4}{1+ax^2}dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^5)\,\text{Subst}\left(\int\left(-\frac{1}{a^2}+\frac{x^2}{a}+15x^4+17ax^6+7a^2x^8+a^3x^{10}+\frac{1}{a^2(1+\ldots}\right.\right.}{d}$$

$$= -\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{6a^5\tan^5(c+\ldots}{d(a+a\sec(c+\ldots}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^4\tan^3(c\ldots}{3d(a+a\sec(c\ldots}$$

Mathematica [A] time = 7.50, size = 149, normalized size = 0.67

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-1386\sin\left(\frac{1}{2}(c+dx)\right)+1584\sin\left(\frac{3}{2}(c+dx)\right)-1386\sin\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^4,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*(5544*Sqrt[
2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(11/2) - 1386*Sin[(c + d*x
)/2] + 1584*Sin[(3*(c + d*x))/2] - 1386*Sin[(5*(c + d*x))/2] - 143*Sin[(7*(
c + d*x))/2] - 693*Sin[(9*(c + d*x))/2] - 26*Sin[(11*(c + d*x))/2]))/(5544*
d)
```

fricas [A] time = 0.50, size = 425, normalized size = 1.90

$$\left[\frac{693\left(a^2\cos(dx+c)^6+a^2\cos(dx+c)^5\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/693*(693*(a^2*cos(d*x + c)^6 + a^2*cos(d*x + c)^5)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(52*a^2*cos(d
*x + c)^5 + 719*a^2*cos(d*x + c)^4 + 366*a^2*cos(d*x + c)^3 - 157*a^2*cos(d
*x + c)^2 - 224*a^2*cos(d*x + c) - 63*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5), -2/693*(693*(a
^2*cos(d*x + c)^6 + a^2*cos(d*x + c)^5)*sqrt(a)*arctan(sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (52*a^2*cos(d*x
+ c)^5 + 719*a^2*cos(d*x + c)^4 + 366*a^2*cos(d*x + c)^3 - 157*a^2*cos(d*x
+ c)^2 - 224*a^2*cos(d*x + c) - 63*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

giac [A] time = 4.63, size = 339, normalized size = 1.51

$$-\frac{\frac{693\sqrt{-a}a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}-\frac{2\left(693\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c))-\left(3927\sqrt{2}a^8\mathrm{sgn}(\cos(dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/693*(693*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(693*sqrt(2)*a^8*sgn(cos(d*x + c)
) - (3927*sqrt(2)*a^8*sgn(cos(d*x + c)) - (462*sqrt(2)*a^8*sgn(cos(d*x + c)
) + (1782*sqrt(2)*a^8*sgn(cos(d*x + c)) + (305*sqrt(2)*a^8*sgn(cos(d*x + c)
)*tan(1/2*d*x + 1/2*c)^2 - 1331*sqrt(2)*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1
/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.42, size = 498, normalized size = 2.22

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(693\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{9}{2}}\left(\cos^5(dx+c)\right)\sin(dx+c)+2772\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x)
```

```
[Out] -1/11088/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(693*2^(1/2)*arctanh(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)^5*sin(d*x+c)+2772*2^(1/2)*arctanh(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)^4*sin(d*x+c)+4158*2^(1/2)*arctanh
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)^3*sin(d*x+c)+2772*arctanh(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+693*arctanh(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)*cos(d*x+c)*sin(d*x+c)-1664*cos(d*
x+c)^6-21344*cos(d*x+c)^5+11296*cos(d*x+c)^4+16736*cos(d*x+c)^3+2144*cos(d*
x+c)^2-5152*cos(d*x+c)-2016)/cos(d*x+c)^5/sin(d*x+c)*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+dx)^4 \left(a+\frac{a}{\cos(c+dx)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**4,x)
[Out] Timed out
```

3.167 $\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx$

Optimal. Leaf size=160

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^5\tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}}+\frac{14a^4\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^3 \ldots}{d\sqrt{a\sec \ldots}}$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^3*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)+14/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2
)+2*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^6*tan(d*x+c)^7/d/(a+a*s
ec(d*x+c))^(7/2)
```

Rubi [A] time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3887, 461, 203}

$$\frac{2a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^5\tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}}+\frac{14a^4\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^3\,\text{ta}\ldots}{d\sqrt{a\sec \ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (14*a^4*Tan[c + d*x]^3)/(
3*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^5*Tan[c + d*x]^5)/(d*(a + a*Sec[c +
d*x])^(5/2)) + (2*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = -\frac{(2a^4)\,\text{Subst}\left(\int \frac{x^2(2+ax^2)^3}{1+ax^2}dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^4)\,\text{Subst}\left(\int \left(\frac{1}{a}+7x^2+5ax^4+a^2x^6-\frac{1}{a(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{14a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}} + \ldots$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{14a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \ldots$$

Mathematica [**A**] time = 5.92, size = 125, normalized size = 0.78

$$-\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-35\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+\ldots\right)\right)}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^2,x]
```

```
[Out] -1/42*(a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(42*S
qrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) - 35*Sin[(c + d*
x)/2] + 7*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*
x))/2]))/d
```

fricas [**A**] time = 0.65, size = 374, normalized size = 2.34

$$\left[\frac{21\left(a^2\cos(dx+c)^4+a^2\cos(dx+c)^3\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-\ldots}{21\left(d\cos(dx+c)^4+d\cos\ldots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/21*(21*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d
*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(10*a^2*cos(d*x
 + c)^3 - 16*a^2*cos(d*x + c)^2 - 12*a^2*cos(d*x + c) - 3*a^2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c
)^3), 2/21*(21*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
 (10*a^2*cos(d*x + c)^3 - 16*a^2*cos(d*x + c)^2 - 12*a^2*cos(d*x + c) - 3*a
^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
 + d*cos(d*x + c)^3)]
```

giac [A] time = 12.61, size = 281, normalized size = 1.76

$$\frac{\frac{21\,\sqrt{-a}\,a^3\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\,\sqrt{2}\,|a|-6\,a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\,\sqrt{2}\,|a|-6\,a}\right|\right)\mathrm{sgn}(\cos(dx+c))}{|a|}-\frac{2\left(21\,\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))+\left(35\,\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))+\left(1\right.\right.\right.}{}}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/21*(21*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a
) - 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(21*sqrt(2)*a^6*sgn(cos(d*x + c)) +
(35*sqrt(2)*a^6*sgn(cos(d*x + c)) + (17*sqrt(2)*a^6*sgn(cos(d*x + c))*tan(1
/2*d*x + 1/2*c)^2 - 49*sqrt(2)*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^
2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2
- a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.06, size = 391, normalized size = 2.44

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(21\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\sqrt{2}\,\sin(dx+c)\left(\cos^3(dx+c)\right)+63\sqrt{2}\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x)
```

```
[Out] -1/168/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(21*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(7/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3+63*2^(1/2)*sin(d*x+c)*cos(d
*x+c)^2*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+63*2^(1/2)*sin(d*x+c)*cos
(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+21*2^(1/2)*arctanh(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)-160*cos(d*x+c)^4+416*cos(d*x+c)^3-64
*cos(d*x+c)^2-144*cos(d*x+c)-48)/sin(d*x+c)/cos(d*x+c)^3*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(c+d\,x\right)^2\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{5}{2}}\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x)**2, x)
```

3.168 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=66

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{4a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-4*a^2*cot(d*
x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 453, 203}

$$-\frac{4a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (4
*a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx &= -\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{2+ax^2}{x^2\left(1+ax^2\right)}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= -\frac{4a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}+\frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{4a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.83, size = 124, normalized size = 1.88

$$-\frac{\sqrt{2}\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{\sec(c+dx)+1}\right)^{3/2}(a(\sec(c+dx)+1))^{5/2}\left(2\cos(c+dx)-\frac{(\cos(c+dx)-1)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{\sqrt{1-\sec(c+dx)}}\right)}{d\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -((Sqrt[2]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*(2*Cos[c + d*x] - (ArcTanh[Sqrt[
1 - Sec[c + d*x]]]*(-1 + Cos[c + d*x]))/Sqrt[1 - Sec[c + d*x]])*((1 + Sec[c
 + d*x])^(-1))^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2))/(d*Sqrt[1 - Tan[(c + d*x
)/2]^2]))
```

fricas [B] time = 0.71, size = 270, normalized size = 4.09

$$\left[\frac{\sqrt{-a}\,a^2\log\left(-\frac{8\,a\cos(dx+c)^3+4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7\,a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)-8\,a^2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,d\sin(dx+c)}, \ldots\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a^2*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x
 + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*
cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 8*a^2*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c)), -(a^(5/2)*arctan(2*
sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(
2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 4*a^2*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c))]
```

giac [B] time = 2.44, size = 192, normalized size = 2.91

$$\frac{\sqrt{2}\sqrt{-a}\,a^4\left(\frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{a|a|}+\frac{8}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)a}\right)\mathrm{sgn}\left(\cos(dx+c)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(-a)*a^4*(sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*s
qrt(2)*abs(a) - 6*a))/(a*abs(a)) + 8/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*a))*sgn(cos(d*x + c))/d
```

maple [B] time = 1.01, size = 192, normalized size = 2.91

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\left(\cos^2(dx+c)\right)\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)-\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\operatorname{arcta}}{d\left(\cos^2(dx+c)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)/cos(d*x+c)*2^(1/2))-2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/
2))+4*cos(d*x+c)*sin(d*x+c))/(cos(d*x+c)^2-1)*a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.169 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=96

$$\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2/3*a*cot(d*x
+c)^3*(a+a*sec(d*x+c))^(3/2)/d+2*a^2*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 325, 203}

$$\frac{2a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d - (2*a*Cot[c + d*x]^3*(a + a*S
ec[c + d*x])^(3/2))/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x,\right.}{d}$$

$$= \frac{2a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a\cot^3(c+dx)(a+a\sec(c+dx))}{3d}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a}{}$$

Mathematica [C] time = 0.29, size = 81, normalized size = 0.84

$$-\frac{2\left(\frac{1}{\cos(c+dx)+1}\right)^{3/2}\cot^3(c+dx)(a(\sec(c+dx)+1))^{5/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{\frac{1}{\sec(c+dx)+1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*((1 + Cos[c + d*x])^(-1))^(3/2)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2,
-3/2, -1/2, 2*Sin[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5/2))/(3*d*Sqrt[(
1 + Sec[c + d*x])^(-1)])
```

fricas [A] time = 0.64, size = 355, normalized size = 3.70

$$\left[\frac{3\left(a^2\cos(dx+c)-a^2\right)\sqrt{-a}\,\log\left(-\frac{8a\cos(dx+c)^3-4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(d}{6\left(d\cos(dx+c)-d\right)\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*cos(d*x + c) - a^2)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*c
os(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) +
 4*(4*a^2*cos(d*x + c)^2 - 3*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c)), 1/3*(3*(a^2*cos(d*x + c)
- a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x +
 c) + 2*(4*a^2*cos(d*x + c)^2 - 3*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c))]
```

giac [B] time = 5.97, size = 311, normalized size = 3.24

$$-\frac{\frac{3\sqrt{-a}\,a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{\sqrt{2}\left(9\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right.}\right.\right.}{}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a)
 - 6*a))*sgn(cos(d*x + c))/abs(a) + sqrt(2)*(9*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^3*sgn(cos(d*x + c))
- 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 7*sqrt(-a)*a^5*sgn(cos(d*x + c)))/((sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3)/
d
```

maple [B] time = 1.18, size = 214, normalized size = 2.23

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\cos(dx+c)\sin(dx+c)\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)-3\sqrt{2}\ \sin\right.}{3d\sin(dx+c)(-1+\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*cos(d*x+c)*sin(d*x+c)*2^(1/2)
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-3*2^(1/2)*sin(d*x+c)*arctanh(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)-8*cos(d*x+c)^2+6*cos(d*x+c))/sin(d*x+c)/(-1+
cos(d*x+c))*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.170 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=176

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}d}-\frac{7a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}-\frac{\cot^5(c+dx)(}{}$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/2*a*cot(d*
x+c)^3*(a+a*sec(d*x+c))^(3/2)/d-1/5*cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)/d+1
/8*a^(5/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/d*
2^(1/2)-7/4*a^2*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3887, 480, 583, 522, 203}

$$-\frac{7a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}d}-\frac{\cot^5(c+dx)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
4*Sqrt[2]*d) - (7*a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*d) + (a*Cot
[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(2*d) - (Cot[c + d*x]^5*(a + a*Sec[
c + d*x])^(5/2))/(5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6\left(1+ax^2\right)\left(2+ax^2\right)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d} - \frac{\text{Subst}\left(\int \frac{-15a-5a^2x^2}{x^4\left(1+ax^2\right)\left(2+ax^2\right)}\,dx, x, -\right.}{5d}$$

$$= \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d}$$

$$= -\frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec(c+dx)}{2d}$$

$$= -\frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec(c+dx)}{2d}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}\,d} - \frac{7a^2\cot(}{}$$

Mathematica [C] time = 24.18, size = 5562, normalized size = 31.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.86, size = 650, normalized size = 3.69

$$\left[\frac{5\left(\sqrt{2}a^2\cos(dx+c)^2 - 2\sqrt{2}a^2\cos(dx+c) + \sqrt{2}a^2\right)\sqrt{-a}\,\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/80*(5*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)
*a^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
```

```
c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 40*(a^2*cos(d*x + c)
^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos
(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4
*(49*a^2*cos(d*x + c)^3 - 80*a^2*cos(d*x + c)^2 + 35*a^2*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c)
+ d)*sin(d*x + c)), -1/40*(40*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^
2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c)
+ 5*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)
*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(49*a^2*cos(d*x + c)^3 - 80*a^2*
cos(d*x + c)^2 + 35*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
)))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))]
```

giac [B] time = 12.54, size = 481, normalized size = 2.73

$$\frac{5\sqrt{2}\sqrt{-a}\,a^2\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\frac{80\sqrt{-a}\,a^3\log\left(\left|2\left(\sqrt{\cdots}\right.\right.\right.}{\left|2\left(\sqrt{\cdots}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/80*(5*sqrt(2)*sqrt(-a)*a^2*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2)*sgn(cos(d*x + c)) + 80*sqrt(-a)*a^3*log(abs(
2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(d*x + c))/ab
s(a) + 4*sqrt(2)*(55*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^8*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 170*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a^4*sgn(cos(d*
x + c)) + 240*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^4*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 150*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^6*sgn(cos(d*x + c))
 + 41*sqrt(-a)*a^7*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5)/d
```

maple [B] time = 1.41, size = 542, normalized size = 3.08

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(1+\cos\left(dx+c\right)\right)^2\left(40\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(\cos^2\left(dx+c\right)\right)\sin\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/40/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(40*2^(1/2)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)-80*cos(d*x+c
)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*c
```

```
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+5*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+40*2^(1/2)*sin(d*x+
c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2
^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-10*cos(d*x+c)*sin(d*x+c)*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-98*cos(d*x+c)^3+5*sin(d*x+c)*ln(-(-(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+160*cos(d*x+c)^2-70*cos(d*x+c))/sin(d*x+c)
^5*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^6\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.171 $\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^4d}-\frac{6(a\sec(c+dx)+a)^{5/2}}{5a^3d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d}+\frac{2\sqrt{a\sec(c+dx)+a}}{ad}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] 2/3*(a+a*sec(d*x+c))^(3/2)/a^2/d-6/5*(a+a*sec(d*x+c))^(5/2)/a^3/d+2/7*(a+a*
sec(d*x+c))^(7/2)/a^4/d-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
+2*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3880, 88, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^4d}-\frac{6(a\sec(c+dx)+a)^{5/2}}{5a^3d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d}+\frac{2\sqrt{a\sec(c+dx)+a}}{ad}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (2*Sqrt[a + a*
Sec[c + d*x]])/(a*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2*d) - (6*(a + a
*Sec[c + d*x])^(5/2))/(5*a^3*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^4*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{3/2} + \frac{a^2(a+ax)^{3/2}}{x} + a(a+ax)^{5/2}\right)dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$= -\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3 d} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^4 d} + \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+\right.}{a^2 d}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{6(a+a\sec(c+dx))^{5/2}}{5a^3 d} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^4 d} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{6(a+a\sec(c+dx))^{5/2}}{5a^3 d} + \frac{2(a+a\,\text{se}}{7}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{6(a+a\sec(c+dx))^{5/2}}{5a^3 d} + \frac{2(a+a\,\text{se}}{7}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{6(a+}{}$$

Mathematica [A] time = 0.18, size = 88, normalized size = 0.70

$$\frac{2\left(15\sec^4(c+dx) - 3\sec^3(c+dx) - 64\sec^2(c+dx) + 46\sec(c+dx) - 105\sqrt{\sec(c+dx)+1}\,\tanh^{-1}\left(\sqrt{\sec(c+a\right.\right.}{105d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*(92 + 46*Sec[c + d*x] - 64*Sec[c + d*x]^2 - 3*Sec[c + d*x]^3 + 15*Sec[c
+ d*x]^4 - 105*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(10
5*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.79, size = 285, normalized size = 2.26

$$\left[\frac{105\sqrt{a}\,\cos(dx+c)^3\log\left(-8a\cos(dx+c)^2 + 4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8a\cos(dx\right.}{210\,ad\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*sqrt(a)*cos(d*x + c)^3*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
```

```
*cos(d*x + c) - a) + 4*(92*cos(d*x + c)^3 - 46*cos(d*x + c)^2 - 18*cos(d*x
+ c) + 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3), 1
/105*(105*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^3 + 2*(92*cos(d*x + c)^
3 - 46*cos(d*x + c)^2 - 18*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)))/(a*d*cos(d*x + c)^3)]
```

giac [A] time = 3.77, size = 190, normalized size = 1.51

$$-\frac{\sqrt{2}\left(\frac{105\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}+\frac{2\left(105\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}-70\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}a-252\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)a^{2}-12\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/105*sqrt(2)*(105*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*(105*(a*ta
n(1/2*d*x + 1/2*c)^2 - a)^3 - 70*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a - 252*(
a*tan(1/2*d*x + 1/2*c)^2 - a)*a^2 - 120*a^3)/((a*tan(1/2*d*x + 1/2*c)^2 - a
)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.33, size = 293, normalized size = 2.33

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(105\left(\cos^{3}(dx+c)\right)\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\sqrt{2}+315\left(\cos^{2}(dx+c)\right)\mathrm{arcta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(105*cos(d*x+c)^3*arctan(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(7/2)*2^(1/2)+315*cos(d*x+c)^2*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*2^(1/2)+315*cos(d*x+c)*a
rctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(7/2)*2^(1/2)+105*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-1472*cos(d*x+c)^3+
736*cos(d*x+c)^2+288*cos(d*x+c)-240)/cos(d*x+c)^3/a
```

maxima [A] time = 0.54, size = 129, normalized size = 1.02

$$\frac{\frac{105\,\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{30\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^{4}}-\frac{126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^{3}}+\frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^{2}}+\frac{210\,\sqrt{a+\frac{a}{\cos(dx+c)}}}{a}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c
)) + sqrt(a)))/sqrt(a) + 30*(a + a/cos(d*x + c))^(7/2)/a^4 - 126*(a + a/cos
```

```
(d*x + c))^(5/2)/a^3 + 70*(a + a/cos(d*x + c))^(3/2)/a^2 + 210*sqrt(a + a/c
os(d*x + c))/a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(c+d\,x\right)^5}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**5/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.172 $\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d}-\frac{2\sqrt{a\sec(c+dx)+a}}{ad}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2/3*(a+a*sec(d*x+c))^(3/2)/a^2/d+2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/
d/a^(1/2)-2*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 80, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d}-\frac{2\sqrt{a\sec(c+dx)+a}}{ad}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*Sqrt[a + a*S
ec[c + d*x]])/(a*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
```

```
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$
$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx\right.}{d}$$
$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\,\text{se}\right.}{ad}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d}$$

Mathematica [A] time = 0.09, size = 66, normalized size = 0.85

$$\frac{2\left(\sec^2(c+dx) - \sec(c+dx) + 3\sqrt{\sec(c+dx)+1}\,\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right) - 2\right)}{3d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*(-2 - Sec[c + d*x] + Sec[c + d*x]^2 + 3*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*
Sqrt[1 + Sec[c + d*x]]))/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.53, size = 241, normalized size = 3.09

$$\left[\frac{3\sqrt{a}\cos(dx+c)\log\left(-8a\cos(dx+c)^2 - 4\left(2\cos(dx+c)^2 + \cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8a\cos(dx+c\right.}{6ad\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
*x + c) - a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) -
1))/(a*d*cos(d*x + c)), -1/3*(3*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c) +
 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) - 1))/(a*d*cos(d
*x + c))]
```

giac [B] time = 1.75, size = 149, normalized size = 1.91

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}+\frac{2\left(3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)a+2\,a^{2}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(3*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*(3*(a*tan(1/2
*d*x + 1/2*c)^2 - a)*a + 2*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/(a*d)
```

maple [B] time = 1.33, size = 155, normalized size = 1.99

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\cos\left(dx+c\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+3\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\left(\right.}{6d\cos\left(dx+c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*cos(d*x+c)*2^(1/2)*arctan(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)+3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-8*cos(d*x+c)+4)/cos(d*x+c)/a
```

maxima [A] time = 0.99, size = 91, normalized size = 1.17

$$-\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^{2}}+\frac{6\sqrt{a+\frac{a}{\cos(dx+c)}}}{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
+ sqrt(a)))/sqrt(a) - 2*(a + a/cos(d*x + c))^(3/2)/a^2 + 6*sqrt(a + a/cos(d
*x + c))/a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(c+d\,x\right)^3}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.173 $\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3880, 63, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{ad}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 1.42

$$-\frac{2\sqrt{\sec(c+dx)+1}\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[a*(1 +
Sec[c + d*x])])
```

fricas [B] time = 0.47, size = 137, normalized size = 4.42

$$\left[\frac{\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)}{2\,\sqrt{a}\,d}, \frac{\sqrt{-a}\,\text{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a)/(sqrt(a)*d)
, sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(2*a*cos(d*x + c) + a))/(a*d)]
```

giac [B] time = 4.38, size = 55, normalized size = 1.77

$$-\frac{2\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,d\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-
a)*d*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
```

maple [A] time = 0.17, size = 26, normalized size = 0.84

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

maxima [A] time = 0.61, size = 49, normalized size = 1.58

$$\frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt(a
)))/(sqrt(a)*d)
```

mupad [B] time = 1.41, size = 27, normalized size = 0.87

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*atanh((a + a/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.174 $\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=92

$$-\frac{1}{d\sqrt{a\sec(c+dx)+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-1/2*arctanh(1/2*(a+a*se
c(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-1/d/(a+a*sec(d*x+c))^(1/
2)
```

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3880, 85, 156, 63, 207}

$$-\frac{1}{d\sqrt{a\sec(c+dx)+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) - 1/(d*Sqrt[a + a*
Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2ad}$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{}{(-a+ax}\right.}{}$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} - \frac{2\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} - \frac{1}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.06, size = 57, normalized size = 0.62

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(\sec(c+dx)+1)\right) - 2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sec(c+dx)+1\right)}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2F
1[-1/2, 1, 1/2, 1 + Sec[c + d*x]])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [B] time = 0.73, size = 384, normalized size = 4.17

$$\left[\frac{2\sqrt{a}\left(\cos(dx+c)+1\right)\log\left(-8\,a\cos(dx+c)^2 - 4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8\,a\cos\right.}{4\left(ad\cos(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
```

```
*cos(d*x + c) - a) + sqrt(2)*(a*cos(d*x + c) + a)*log(-(2*sqrt(2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/sqrt(a) - 3*cos(d*x + c) - 1)/(
cos(d*x + c) - 1))/sqrt(a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c))/(a*d*cos(d*x + c) + a*d), 1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(
-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos
(d*x + c)/(cos(d*x + c) + 1)) - 2*sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c)
 + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a*d*cos(d
*x + c) + a*d)]
```

giac [A] time = 1.23, size = 150, normalized size = 1.63

$$\frac{\sqrt{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - arctan(sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)))/d
```

maple [B] time = 1.32, size = 259, normalized size = 2.82

$$\frac{\left(2\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sqrt{2}\,\left(\cos^2(dx+c)\right)+\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\left(\cos^2(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2+(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*2^(1/2)-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2*cos(d*x+c)^2-2*cos(d*x+c))*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.175 $\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{a}{12d(a\sec(c+dx)+a)^{3/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}+\frac{7}{8d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+}}{\sqrt{a}}\right.}{\sqrt{a}\,d}$$

```
[Out] -1/12*a/d/(a+a*sec(d*x+c))^(3/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(3
/2)-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)+9/16*arctanh(1/2*(a
+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)+7/8/d/(a+a*sec(d*x+
c))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3880, 103, 152, 156, 63, 207}

$$-\frac{a}{12d(a\sec(c+dx)+a)^{3/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}+\frac{7}{8d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+}}{\sqrt{a}}\right.}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (9*ArcTanh[Sqr
t[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*Sqrt[a]*d) - a/(12*d*(
a + a*Sec[c + d*x])^(3/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])
^(3/2)) + 7/(8*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{2a^2+\frac{5a^2x}{2}}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c \ldots\right.}{2d}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{\ldots}{8d\sqrt{a+\ldots}}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{\ldots}{8d\sqrt{a+\ldots}}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{\ldots}{8d\sqrt{a+\ldots}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{9\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d} - \frac{a}{12d(a+a\sec(c+dx))^{3/2}}\ldots$$

Mathematica [C] time = 0.22, size = 90, normalized size = 0.59

$$\frac{a\left(-9(\sec(c+dx)-1)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(\sec(c+dx)+1)\right)+8(\sec(c+dx)-1)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sec(c+dx)+1\right)\ldots\right.}{12d(\sec(c+dx)-1)(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (a*(-6 - 9*Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[c + d*x])/2]*(-1 + Sec
[c + d*x]) + 8*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]]*(-1 + Sec
[c + d*x])))/(12*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [B] time = 0.59, size = 546, normalized size = 3.59

$$\left[\frac{27\sqrt{2}\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+3a\cos(dx+c)+a}{\cos(dx+c)-1}\right)+48\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(27*sqrt(2)*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt
(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 48*(cos(d*x + c)^3 + cos
(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(
d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
- 8*a*cos(d*x + c) - a) + 4*(31*cos(d*x + c)^3 + 2*cos(d*x + c)^2 - 21*cos(
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*
d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d), -1/48*(27*sqrt(2)*(cos(d*x + c)
^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) -
48*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*arctan(2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
 c) + a)) - 2*(31*cos(d*x + c)^3 + 2*cos(d*x + c)^2 - 21*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)
^2 - a*d*cos(d*x + c) - a*d)]
```

giac [A] time = 7.09, size = 229, normalized size = 1.51

$$-\frac{\sqrt{2}\left(\frac{48\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{27\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{2\left(\left(-a\tan\left(\frac{1}{2}\right.\right.\right.}{}\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*sqrt(2)*(48*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 27*arctan(sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c
)^2 - 1)) - 3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a*sgn(tan(1/2*d*x + 1/2*
c)^2 - 1)*tan(1/2*d*x + 1/2*c)^2) + 2*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2
)*a^4 + 12*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^5)/(a^6*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))/d
```

maple [B] time = 1.52, size = 504, normalized size = 3.32

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))\,(-1+\cos(dx+c))^2\left(48\left(\cos^3(dx+c)\right)\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+}{1+\cos(dx-}}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/48/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c))^
2*(48*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+48*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)
*cos(d*x+c)^2+27*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1
/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-48*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2))+27*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2))*cos(d*x+c)^2-48*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcta
n(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+62*cos(d*x+c)^3
-27*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2))+4*cos(d*x+c)^2-27*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-42*cos(d*x+c))/sin(d*x+c
)^6/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^3}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)^3}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.176 $\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=214

$$\frac{87a^2}{160d(a\sec(c+dx)+a)^{5/2}}-\frac{17a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] 87/160*a^2/d/(a+a*sec(d*x+c))^(5/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x
+c))^(5/2)-17/16*a^2/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)+23/192*a/d/(a+
a*sec(d*x+c))^(3/2)+2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-151
/256*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-
105/128/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3880, 103, 151, 152, 156, 63, 207}

$$\frac{87a^2}{160d(a\sec(c+dx)+a)^{5/2}}-\frac{17a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (151*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[2]*Sqrt[a]*d) + (87*a^
2)/(160*d*(a + a*Sec[c + d*x])^(5/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
a*Sec[c + d*x])^(5/2)) - (17*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d
*x])^(5/2)) + (23*a)/(192*d*(a + a*Sec[c + d*x])^(3/2)) - 105/(128*d*Sqrt[a
 + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{a^6\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{a^3\,\text{Subst}\left(\int \frac{4a^2+\frac{9a^2x}{2}}{x(-a+ax)^2(a+ax)^{7/2}}\,dx, x, \sec(\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx)}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d(1-\sec}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d(1-\sec}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d(1-\sec}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d(1-\sec}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d(1-\sec}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{151\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d} + \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}}$$

Mathematica [C] time = 0.28, size = 102, normalized size = 0.48

$$\frac{\cot^4(c+dx)\left(151(\sec(c+dx)-1)^2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(\sec(c+dx)+1)\right)-2\left(32(\sec(c+dx)-1)^2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\text{se}\right.\right.}{160d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]^4*(-2*(105 + 32*Hypergeometric2F1[-5/2, 1, -3/2, 1 + Sec[c +
d*x]]*(-1 + Sec[c + d*x])^2 - 85*Sec[c + d*x]) + 151*Hypergeometric2F1[-5/2
, 1, -3/2, (1 + Sec[c + d*x])/2]*(-1 + Sec[c + d*x])^2))/(160*d*Sqrt[a*(1 +
 Sec[c + d*x])])
```

fricas [A] time = 0.68, size = 705, normalized size = 3.29

$$\left[\frac{2265\sqrt{2}\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}\,\sqrt{}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(2265*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 -
 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(co
s(d*x + c) - 1)) + 3840*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3
 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4
*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)) - 8*a*cos(d*x + c) - a) - 4*(2821*cos(d*x + c)^5 + 278*cos(d*x + c)
^4 - 3964*cos(d*x + c)^3 - 230*cos(d*x + c)^2 + 1575*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 -
 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d), 1/3
840*(2265*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*c
os(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 3840*(
cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos
(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(2821*cos(d*x + c)^5 + 278*
cos(d*x + c)^4 - 3964*cos(d*x + c)^3 - 230*cos(d*x + c)^2 + 1575*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^5 + a*d*cos(
d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)
 + a*d)]
```

giac [A] time = 8.55, size = 296, normalized size = 1.38

$$\frac{\sqrt{2}\left(\frac{3840\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{2265\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}+\frac{15\left(25\left(-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}-23\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\ldots\right.}\right.}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\ldots}\right.}{3840\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(2)*(3840*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2265*arctan
(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)) + 15*(25*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 23*sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*tan(1/2
*d*x + 1/2*c)^4) + 8*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a)*a^12 + 25*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^13 + 24
0*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^14)/(a^15*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)))/d
```

maple [B] time = 1.85, size = 746, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c
))^3*(3840*cos(d*x+c)^5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+2265*cos(d*x+c)^5*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
```

```
))+3840*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*2^(1/2)*arctan(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+2265*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^4*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-
7680*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+5642*cos(d*x+c)^5-4530*cos(d*x
+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2))-7680*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2+556*cos(d*x+c)^
4-4530*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2))*cos(d*x+c)^2+3840*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-792
8*cos(d*x+c)^3+2265*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+3840*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-460
*cos(d*x+c)^2+2265*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2))+3150*cos(d*x+c))/sin(d*x+c)^10/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^5}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^5/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^5}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**5/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.177 $\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=189

$$\frac{2a^4\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{6a^3\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2a\,t}{3d(a\sec}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+2*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)-2/3*a*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5*a
^2*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+6/7*a^3*tan(d*x+c)^7/d/(a+a*sec(d*
x+c))^(7/2)+2/9*a^4*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)
```

Rubi [A] time = 0.10, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 461, 203}

$$\frac{2a^4\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{6a^3\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a*Tan[c + d*x]^3)/(3*d*(
a + a*Sec[c + d*x])^(3/2)) + (2*a^2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x
])^(5/2)) + (6*a^3*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (2*a^
4*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^3)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+3x^6+ax^8-\frac{1}{a^3(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{6a^3}{7d(a+}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a}{5d(a+}$$

Mathematica [C] time = 19.22, size = 467, normalized size = 2.47

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\frac{1532}{315}\sin\left(\frac{1}{2}(c+dx)\right)+\frac{4}{9}\sin\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)-\frac{4}{63}\sin\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx\right.}{d\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]*((1532*Sin[(c + d*x)/2])/315 + (136*Sec[c +
d*x]*Sin[(c + d*x)/2])/315 - (176*Sec[c + d*x]^2*Sin[(c + d*x)/2])/105 - (4
*Sec[c + d*x]^3*Sin[(c + d*x)/2])/63 + (4*Sec[c + d*x]^4*Sin[(c + d*x)/2])/
9))/(d*Sqrt[a*(1 + Sec[c + d*x])]) + (16*(-3 - 2*Sqrt[2])*Cos[(c + d*x)/4]^
4*Cos[(c + d*x)/2]*Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[(c + d*x)/2])
/(1 + Cos[(c + d*x)/2])]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(c + d*x)/
2])/(1 + Cos[(c + d*x)/2])]*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[(c + d*x)/2])
*(EllipticF[ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]
- 2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]]
, 17 - 12*Sqrt[2]])*Sqrt[(-1 - Sqrt[2] + (2 + Sqrt[2])*Cos[(c + d*x)/2])*Se
c[(c + d*x)/4]^2]*Sec[c + d*x]^2*Sqrt[3 - 2*Sqrt[2] - Tan[(c + d*x)/4]^2])/
(d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.75, size = 355, normalized size = 1.88

$$\left[-\frac{315\left(\cos(dx+c)^5+\cos(dx+c)^4\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2(38}{315\left(ad\cos(dx+c)^5+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/315*(315*(cos(d*x + c)^5 + cos(d*x + c)^4)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(383*cos(d*x + c)^4
+ 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 - 5*cos(d*x + c) + 35)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^5 + a*d*cos(d*
x + c)^4), 2/315*(315*(cos(d*x + c)^5 + cos(d*x + c)^4)*sqrt(a)*arctan(sqrt
```

```
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
(383*cos(d*x + c)^4 + 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 - 5*cos(d*x +
c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x
 + c)^5 + a*d*cos(d*x + c)^4)]
```

giac [B] time = 9.45, size = 353, normalized size = 1.87

$$-\frac{\sqrt{2}\left(\frac{315\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+4\left(\frac{315a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\left(\frac{1470a^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\left(\cdots\right.\right.\right.\right.}{630d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/630*sqrt(2)*(315*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/(abs(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 4*(315*
a^4/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (1470*a^4/sgn(tan(1/2*d*x + 1/2*c)^2
- 1) - (2772*a^4/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + (257*a^4*tan(1/2*d*x + 1
/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 1314*a^4/sgn(tan(1/2*d*x + 1/2*c)
^2 - 1))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*
c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.40, size = 480, normalized size = 2.54

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(315\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{9}{2}}\left(\cos^4(dx+c)\right)\sin(dx+c)+1260\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/5040/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(315*2^(1/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)^4*sin(d*x+c)+1260*2^(1/2)*arctanh(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)^3*sin(d*x+c)+1890*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+1260*arctanh(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)*cos(d*x+c)*sin(d*x+c)+315*2^(1/2)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*sin(d*x+c)-12256*cos(d*x+c)^5+11168*c
os(d*x+c)^4+5312*cos(d*x+c)^3-4064*cos(d*x+c)^2-1280*cos(d*x+c)+1120)/sin(d
*x+c)/cos(d*x+c)^4/a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^6}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**6/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.178 $\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=125

$$\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-2*tan(d*x+c)/
d/(a+a*sec(d*x+c))^(1/2)+2/3*a*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5*a^
2*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 459, 302, 203}

$$\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Tan[c + d*x]^3)/(3*d*(a
 + a*Sec[c + d*x])^(3/2)) + (2*a^2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x]
)^(5/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx}}\right.}{d}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{2\,\text{Sub}}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2}{5d(a+}$$

Mathematica [C] time = 3.11, size = 238, normalized size = 1.90

$$\frac{16\sqrt{2}\tan^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{9/2}\left(-\frac{4}{9}\tan^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\,{}_2F_1\left(2,\frac{9}{2};\frac{11}{2};-2\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\right.}{5d\left(1-\tan^2\left(\frac{1}{2}(c+dx\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(9/2)*(-1/480*(Cos[c + d*x]*(9 + 5*Co
s[c + d*x])*Csc[(c + d*x)/2]^6*Sec[(c + d*x)/2]^2*(30*ArcTanh[Sqrt[1 - Sec[
c + d*x]]]*Cos[c + d*x]^2 + (-29 + 22*Cos[c + d*x] - 23*Cos[2*(c + d*x)])*S
qrt[1 - Sec[c + d*x]]))/Sqrt[1 - Sec[c + d*x]] - (4*Hypergeometric2F1[2, 9/
2, 11/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]^
2)/9)*Tan[c + d*x]^5)/(5*d*Sqrt[a*(1 + Sec[c + d*x])]*(1 - Tan[(c + d*x)/2]
^2)^(7/2))
```

fricas [A] time = 0.84, size = 311, normalized size = 2.49

$$\left[-\frac{15\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,(17}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(17*cos(d*x + c)^2 + c
os(d*x + c) - 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d
*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -2/15*(15*(cos(d*x + c)^3 + cos(d*x
+ c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) + (17*cos(d*x + c)^2 + cos(d*x + c) - 3)*sqrt((a*c
```

```
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(
d*x + c)^2)]
```

giac [B] time = 6.15, size = 283, normalized size = 2.26

$$\frac{\sqrt{2}\left(\frac{15\,\sqrt{2}\,\sqrt{-a}\,\log\left(\frac{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}+\frac{4\left(\left(\frac{13\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{40\,a^{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\cdots}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\right.\cdots}}\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(15*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*s
qrt(2)*abs(a) - 6*a))/(abs(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 4*((13*a^2
*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 40*a^2/sgn(tan(1/
2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 + 15*a^2/sgn(tan(1/2*d*x + 1/
2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.27, size = 231, normalized size = 1.85

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(15\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}+15\left(-\cdots\right.\right.}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/30/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*sin(d*x+c)*cos(d*x+c)^2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+15*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*cos(d*x+c)*2^(1/2)*sin(d*x+c)+68*cos(d*x+c)^3-64*cos(d*x+c)
^2-16*cos(d*x+c)+12)/sin(d*x+c)/cos(d*x+c)^2/a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan\left(c+d\,x\right)}^4}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.179 $\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=63

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+2*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 321, 203}

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.77, size = 119, normalized size = 1.89

$$\frac{16\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{5/2}\left(\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{\frac{1}{\cos(c+dx)+1}} - \cos(c+dx)\sin^{-1}\left(\frac{\tan\left(\frac{1}{2}\right.}{\sqrt{\cos(}}\right.\right.}{d\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*Cos[(c + d*x)/2]^6*Sec[c + d*x]^4*((1 + Sec[c + d*x])^(-1))^(5/2)*(-(Ar
cSin[Tan[(c + d*x)/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Cos[c + d*x]) + Sqrt[C
os[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sin[c + d*x]))/(d*Sqrt[a*(1 + Se
c[c + d*x])])
```

fricas [A] time = 0.82, size = 235, normalized size = 3.73

$$\left[-\frac{\sqrt{-a}\,(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(d\right.}{ad\cos(dx+c)+ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c)
 - a)/(cos(d*x + c) + 1)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d), 2*(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
 sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) +
a*d)]
```

giac [B] time = 1.91, size = 191, normalized size = 3.03

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|\text{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\text{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt
(2)*abs(a) - 6*a))/(abs(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 4*tan(1/2*d*x
 + 1/2*c)/(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)))/d
```

maple [B] time = 0.93, size = 116, normalized size = 1.84

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{2}\,\sin(dx+c)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}-2\cos(dx+c)+2\right)}{d\sin(dx+c)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2^(1/2)*sin(d*x+c)*arctanh(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)-2*cos(d*x+c)+2)/sin(d*x+c)/a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(c+d\,x)^2}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.180 $\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=165

$$-\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4ad}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}\,d}-\frac{\cos(c+dx)\cot(c+dx)\,\mathrm{se}}{}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+7/8*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-1/4
*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a/d-1/4*cos(d*x+c)*cot(d*x+c)*sec(1/2*d*
x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3887, 472, 583, 522, 203}

$$-\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4ad}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}\,d}-\frac{\cos(c+dx)\cot(c+dx)\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(7*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(4*Sq
rt[2]*Sqrt[a]*d) - (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*a*d) - (Cos[c
 + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d*x]])/(4*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= -\frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\text{Subst}\left(\int \frac{a-3}{x^2(1+ax^2}\right.}{}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+}}{4ad}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+}}{4ad}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}\,d} - \frac{\cot(c+dx)\sqrt{a+a\sec(c}}{4ad}$$

Mathematica [C] time = 24.30, size = 5534, normalized size = 33.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.58, size = 503, normalized size = 3.05

$$\left[-\frac{7\sqrt{2}\sqrt{-a}\,(\cos(dx+c)+1)\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx+c)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(7*sqrt(2)*sqrt(-a)*(cos(d*x + c) + 1)*log((2*sqrt(2)*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x
+ c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d
*x + c) + 8*sqrt(-a)*(cos(d*x + c) + 1)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos
```

```
(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4
*(3*cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))
/((a*d*cos(d*x + c) + a*d)*sin(d*x + c)), -1/8*(7*sqrt(2)*sqrt(a)*(cos(d*x
+ c) + 1)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 8*sqrt(a)*(cos(d*x + c) + 1)*arct
an(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(3*cos(d*x
+ c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d
*x + c) + a*d)*sin(d*x + c))]
```

giac [A] time = 1.42, size = 123, normalized size = 0.75

$$-\frac{\sqrt{2}\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{4\sqrt{-a}}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 4*sqrt(-a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)))/d
```

maple [B] time = 1.30, size = 374, normalized size = 2.27

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(8\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\;\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\;\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(\cos^2(dx+c)\right)\sin(dx+c)+7\sqrt{-\frac{2\cos}{1+\cos}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(8*2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)+7*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-8*2^(1/2)*sin(d*x+c)*arctanh(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)-6*cos(d*x+c)^3-7*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+4*cos(d*x+c)^2+2*cos(d*x+c))/sin(d*x+c)^3/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^2}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot (c+d\,x)^2}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2 (c+dx)}{\sqrt{a\,(\sec (c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.181 $\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=251

$$\frac{43\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{96a^2d}-\frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{16a^2d}-\frac{15\cos(c}{}$$

```
[Out] 43/96*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^2/d-15/32*cos(d*x+c)*cot(d*x+c)
^3*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^2/d-1/16*cos(d*x+c)^2*cot(
d*x+c)^3*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^2/d+2*arctan(a^(1/2)
*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-107/128*arctan(1/2*a^(1/2)*ta
n(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)+21/64*cot(d*x+c)
*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.23, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{43\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{96a^2d}-\frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{16a^2d}-\frac{15\cos(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
107*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(64*
Sqrt[2]*Sqrt[a]*d) + (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(64*a*d) +
(43*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(96*a^2*d) - (15*Cos[c + d*x
]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(32*a^2*d)
- (Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(3
/2))/(16*a^2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2 d}$$

$$= -\frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2 d} - \frac{\text{Subst}\left(\int \frac{}{x^4(1\cdots}\right.}{\cdots}$$

$$= -\frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{32a^2 d} - \frac{\cos^2(c+dx\cdots}{\cdots}$$

$$= \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2 d} - \frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx\cdots\right.}{32a^2 d}$$

$$= \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2 d} - \frac{15\,\text{co}\cdots}{\cdots}$$

$$= \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2 d} - \frac{15\,\text{co}\cdots}{\cdots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{107\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}\,d} + \frac{21\cot(c+dx)\sqrt{a+a\,\text{se}\cdots}}{64ad}$$

Mathematica [C] time = 24.18, size = 5574, normalized size = 22.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

fricas [A] time = 2.15, size = 666, normalized size = 2.65

$$\left[-\frac{321\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\sqrt{-a}\,\log\left(-\frac{2\,\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)-3\,a\cos(}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(321*sqrt(2)*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*s
qrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 384*(cos(d*x + c)^3 + cos(d
*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(
d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*
(205*cos(d*x + c)^4 + 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 - 63*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^3 + a*d*co
s(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c)), 1/384*(321*sqrt(2)*(c
os(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(a)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
*sin(d*x + c) + 384*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sq
rt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(
205*cos(d*x + c)^4 + 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 - 63*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^3 + a*d*cos
(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))]
```

giac [A] time = 1.56, size = 256, normalized size = 1.02

$$-\frac{\sqrt{2}\left(3\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{21}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{32\left(9\left(\sqrt{-a}\tan\left(\frac{1}{2}\right.\right.\right.}{}\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/384*sqrt(2)*(3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*tan(1/2*d*x + 1/2*
c)^2/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 21/(a*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)))*tan(1/2*d*x + 1/2*c) - 32*(9*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a) - 15*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a + 8*sqrt(-a)*a^2)/(
((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
a)^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.51, size = 722, normalized size = 2.88

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(1+\cos\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{2}\left(-384\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{arc}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c))
^2*(-384*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2
^(1/2))-321*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln
(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))-384*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d
*x+c)-321*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)
+384*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)
)+410*cos(d*x+c)^4+321*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))+384*2^(1/2)*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+142*cos(d*x+c)^3+321*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
-298*cos(d*x+c)^2-126*cos(d*x+c))/sin(d*x+c)^7/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(dx+c\right)^{4}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot\left(c+d\,x\right)^4}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.182 $\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=335

$$\frac{579\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{640a^3d}-\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{48a^3d}-\frac{23\cos^2(\ldots}{\ldots}$$

```
[Out] -323/768*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^2/d+579/640*cot(d*x+c)^5*(a+
a*sec(d*x+c))^(5/2)/a^3/d-101/128*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c
)^2*(a+a*sec(d*x+c))^(5/2)/a^3/d-23/192*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*d
*x+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^3/d-1/48*cos(d*x+c)^3*cot(d*x+c)^5*sec
(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^3/d-2*arctan(a^(1/2)*tan(d*x+c)/
(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+835/1024*arctan(1/2*a^(1/2)*tan(d*x+c)*2^
(1/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-189/512*cot(d*x+c)*(a+a*sec
(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.32, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{579\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{640a^3d}-\frac{323\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{768a^2d}-\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(\ldots\right.}{48a^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(835*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(51
2*Sqrt[2]*Sqrt[a]*d) - (189*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(512*a*d
) - (323*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(768*a^2*d) + (579*Cot[
c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(640*a^3*d) - (101*Cos[c + d*x]*Cot[
c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(128*a^3*d) - (23
*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(5/2
))/(192*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]^5*Sec[(c + d*x)/2]^6*(a + a*S
ec[c + d*x])^(5/2))/(48*a^3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3d}$$

$$= -\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d} - \frac{\text{Subst}\left(\int \frac{a-}{x^6(1+ax}\right.}{}$$

$$= -\frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} - \frac{\cos^3(c+dx)}{}$$

$$= -\frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3d} - \frac{23\cos^2(c+d}{}$$

$$= \frac{579\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3d} - \frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right.}{128a^3d}$$

$$= -\frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d} + \frac{579\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3d} - \frac{1}{}$$

$$= -\frac{189\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad} - \frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d} + \frac{579}{}$$

$$= -\frac{189\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad} - \frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d} + \frac{579}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{835\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{512\sqrt{2}\sqrt{a}\,d} - \frac{189\cot(c+dx)\sqrt{a+a\sec}}{512ad}$$

Mathematica [C] time = 24.15, size = 5618, normalized size = 16.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.91, size = 823, normalized size = 2.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(12525*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^
3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(
d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*s
in(d*x + c) + 15360*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2
*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(
2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c
) + 4*(9737*cos(d*x + c)^6 + 3451*cos(d*x + c)^5 - 14394*cos(d*x + c)^4 - 6
```

```
158*cos(d*x + c)^3 + 6065*cos(d*x + c)^2 + 2835*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 - 2*a
*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)*sin(d*x
+ c)), -1/15360*(12525*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x
 + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin
(d*x + c) + 15360*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*c
os(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*c
os(d*x + c) - a))*sin(d*x + c) + 2*(9737*cos(d*x + c)^6 + 3451*cos(d*x + c)
^5 - 14394*cos(d*x + c)^4 - 6158*cos(d*x + c)^3 + 6065*cos(d*x + c)^2 + 283
5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)
^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d
*cos(d*x + c) + a*d)*sin(d*x + c))]
```

giac [A] time = 1.84, size = 387, normalized size = 1.16

$$-\frac{\sqrt{2}\left(5\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(2\left(\frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{43}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\frac{}{a\mathrm{sgn}\left(\tan\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/15360*sqrt(2)*(5*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*tan(1/2*d*x +
 1/2*c)^2/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 43/(a*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 567/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)))*tan(1/2*d*x + 1/2*c) + 96*(145*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a) - 500*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a + 710*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^2 - 46
0*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*s
qrt(-a)*a^3 + 121*sqrt(-a)*a^4)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.50, size = 1068, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/15360/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+
c))^3*(-15360*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*
2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))-12525*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(d*
x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c))-15360*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c
)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))-12525*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(
d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))+30720*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))+19474*cos(d*x+c)^6+25050*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+30720*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)
```

```
))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*
x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)+6902*cos(d*x+c)^5+25050*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-15360*cos(d*x+c)*sin(d
*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-28788*cos(d*x+c)^4-
12525*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-15360
*2^(1/2)*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-12316*cos(d*x
+c)^3-12525*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+12130*cos(
d*x+c)^2+5670*cos(d*x+c))/sin(d*x+c)^11/a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^6}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**6/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.183 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^4d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{a^3d}+\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2*(a+a*sec(d*x+c))^(3/
2)/a^3/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^4/d+2*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3880, 88, 50, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^4d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{a^3d}+\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*Sqrt[a + a*
Sec[c + d*x]])/(a^2*d) - (2*(a + a*Sec[c + d*x])^(3/2))/(a^3*d) + (2*(a + a
*Sec[c + d*x])^(5/2))/(5*a^4*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
```

```
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2\sqrt{a+ax} + \frac{a^2\sqrt{a+ax}}{x} + a(a+ax)^{3/2}\right) dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$= -\frac{2(a+a\sec(c+dx))^{3/2}}{a^3 d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4 d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+\right.}{a^2 d}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d} - \frac{2(a+a\sec(c+dx))^{3/2}}{a^3 d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4 d} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d} - \frac{2(a+a\sec(c+dx))^{3/2}}{a^3 d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4 d} + \frac{2\,\text{Subst}}{}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2} d} + \frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d} - \frac{2(a+a\sec(c+dx))^{3/2}}{a^3 d} + \frac{2(a}{}$$

Mathematica [A] time = 0.18, size = 79, normalized size = 0.79

$$\frac{2\left(\sec^3(c+dx) - 2\sec^2(c+dx) - 2\sec(c+dx) - 5\sqrt{\sec(c+dx)+1}\,\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)+1\right)}{5ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(1 - 2*Sec[c + d*x] - 2*Sec[c + d*x]^2 + Sec[c + d*x]^3 - 5*ArcTanh[Sqrt
[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(5*a*d*Sqrt[a*(1 + Sec[c + d*x
])])
```

fricas [A] time = 0.59, size = 261, normalized size = 2.61

$$\left[\frac{5\sqrt{a}\cos(dx+c)^2\log\left(-8a\cos(dx+c)^2 + 4\left(2\cos(dx+c)^2 + \cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8a\cos(dx+\right.}{10a^2 d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/10*(5*sqrt(a)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)
^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*co
s(d*x + c) - a) + 4*(cos(d*x + c)^2 - 3*cos(d*x + c) + 1)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2), 1/5*(5*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + a))*cos(d*x + c)^2 + 2*(cos(d*x + c)^2 - 3*cos(d*x + c) + 1)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2)]
```

giac [A] time = 4.90, size = 168, normalized size = 1.68

$$-\frac{2\left(\frac{5\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}\left(5\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2+10\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a+4a^2\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(s
qrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*(5*(a*tan(1/2*d*x + 1/
2*c)^2 - a)^2 + 10*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 4*a^2)/((a*tan(1/2*d*
x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(tan(1/2*d*x +
 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.13, size = 224, normalized size = 2.24

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(5\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{2}\left(\cos^2(dx+c)\right)+10\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/20/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(5*arctan(1/2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*co
s(d*x+c)^2+10*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*cos(d*x+c)+5*2^(1/2)*arctan(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)+8*cos(d*x+c)^2-24*cos(d*x+c)+8)/cos(d*x+c)^2/a^2
```

maxima [A] time = 0.55, size = 110, normalized size = 1.10

$$\frac{\frac{5\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^4}-\frac{10\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^3}+\frac{10\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^2}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(5*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) +
 sqrt(a)))/a^(3/2) + 2*(a + a/cos(d*x + c))^(5/2)/a^4 - 10*(a + a/cos(d*x +
 c))^(3/2)/a^3 + 10*sqrt(a + a/cos(d*x + c))/a^2)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.184 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a+a*sec(d*x+c))^(1/2
)/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3880, 80, 63, 207}

$$\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*Sqrt[a + a*S
ec[c + d*x]])/(a^2*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-a+ax}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d}$$
$$= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d}$$

Mathematica [A] time = 0.08, size = 56, normalized size = 1.04

$$\frac{2\left(\sec(c+dx)+\sqrt{\sec(c+dx)+1}\,\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)+1\right)}{ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(1 + Sec[c + d*x] + ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x
]]))/(a*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.53, size = 191, normalized size = 3.54

$$\left[\frac{\sqrt{a}\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{2\,a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) +
4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d), -(sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c
) + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d)]
```

giac [B] time = 3.20, size = 99, normalized size = 1.83

$$\frac{2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)} - \frac{\sqrt{2}}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2*(arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-
a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)/(sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/(a*d)
```

maple [A] time = 1.05, size = 81, normalized size = 1.50

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sqrt{2}-2\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-2)/a^2
```

maxima [A] time = 0.81, size = 71, normalized size = 1.31

$$-\frac{\frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -(log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt
(a)))/a^(3/2) - 2*sqrt(a + a/cos(d*x + c))/a^2)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.185 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2}{ad\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2/a/d/(a+a*sec(d*x+c))
^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3880, 51, 63, 207}

$$\frac{2}{ad\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + 2/(a*d*Sqrt[a
+ a*Sec[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2}{ad\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{2}{ad\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2 d}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2}{ad\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.04, size = 38, normalized size = 0.70

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sec(c+dx)+1\right)}{ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]])/(a*d*Sqrt[a*(1 + Sec[
c + d*x])])
```

fricas [B] time = 0.77, size = 244, normalized size = 4.52

$$\left[\frac{\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(\right.\right.}{2\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/
(a^2*d*cos(d*x + c) + a^2*d), (sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
a)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a^2*d*cos(d*
x + c) + a^2*d)]
```

giac [B] time = 2.72, size = 102, normalized size = 1.89

$$-\frac{\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -(2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [A] time = 0.14, size = 45, normalized size = 0.83

$$\frac{\frac{2}{a\sqrt{a+a\sec(dx+c)}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(2/a/(a+a*sec(d*x+c))^(1/2)-2/a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^
(1/2)))
```

maxima [A] time = 0.59, size = 70, normalized size = 1.30

$$\frac{\frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{a}{\cos(dx+c)}}\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] (log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt(
a)))/a^(3/2) + 2/(sqrt(a + a/cos(d*x + c))*a))/d
```

mupad [B] time = 1.59, size = 50, normalized size = 0.93

$$\frac{2}{a\,d\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{a^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] 2/(a*d*(a + a/cos(c + d*x))^(1/2)) - (2*atanh((a + a/cos(c + d*x))^(1/2)/a^
(1/2)))/(a^(3/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.186 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{3}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{1}{3d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/3/d/(a+a*sec(d*x+c))^
(3/2)-1/4*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(
1/2)-3/2/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3880, 85, 152, 156, 63, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{3}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{1}{3d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) - 1/(3*d*(a + a*
Sec[c + d*x])^(3/2)) - 3/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{2ad}$$

$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-2a^4+\frac{3a^4x}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx, x\right.}{2a^4d}$$

$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x,\right.}{4d}$$

$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+\right.}{a^2d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{}{2a}$$

Mathematica [C] time = 0.06, size = 60, normalized size = 0.50

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(\sec(c+dx)+1)\right) - 2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sec(c+dx)+1\right)}{3d(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2
F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]])/(3*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [B] time = 0.71, size = 485, normalized size = 4.04

$$\left[\frac{3\sqrt{2}\left(\cos(dx+c)^2 + 2\cos(dx+c) + 1\right)\sqrt{a}\,\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c) - 3a\cos(dx+c) - a}{\cos(dx+c)-1}\right) + 12\left(\cos(dx+c)^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(2*sqrt
(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(
d*x + c) - a)/(cos(d*x + c) - 1)) + 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4*(1
1*cos(d*x + c)^2 + 9*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/12*(3*sqrt(2)*(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 12*(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(11*co
s(d*x + c)^2 + 9*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [A] time = 2.42, size = 185, normalized size = 1.54

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{24\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{\sqrt{2}\left(\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}a^{6}+9\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,a^{7}\right)}{a^{9}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/12*(3*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt
(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 24*arctan(1/2*sqrt(2)*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) - sqrt(2)*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^6 + 9*sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a)*a^7)/(a^9*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.29, size = 376, normalized size = 3.13

$$-\frac{\left(-1+\cos\left(dx+c\right)\right)^{2}\left(12\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sqrt{2}\,\left(\cos^{2}\left(dx+c\right)\right)+3\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{arcta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/12/d*(-1+cos(d*x+c))^2*(12*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1
/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2+3*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2))*cos(d*x+c)^2+24*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+6*cos(d*x+c)*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2))+12*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+22*cos(d*x+c)^2+3*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+18*cos(d*x+
c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^4/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(c+d\,x\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.187 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{11\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d}-\frac{3a}{20d(a\sec(c+dx)+a)^{5/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-3/20*a/d/(a+a*sec(d*x+
c))^(5/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)+5/24/d/(a+a*sec(d*x
+c))^(3/2)+11/32*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2
)/d*2^(1/2)+21/16/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3880, 103, 152, 156, 63, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{11\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d}-\frac{3a}{20d(a\sec(c+dx)+a)^{5/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (11*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*a^(3/2)*d) - (3*a)/(
20*d*(a + a*Sec[c + d*x])^(5/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(5/2)) + 5/(24*d*(a + a*Sec[c + d*x])^(3/2)) + 21/(16*a*d*Sqrt[a + a
*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a^4 \operatorname{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} - \frac{a \operatorname{Subst}\left(\int \frac{2a^2+\frac{7a^2x}{2}}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+\right.}{2d}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{}{24d(a+a}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{}{24d(a+a}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{}{24d(a+a}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{}{24d(a+a}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{11\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d} - \frac{3a}{20d(a+a\sec(c+dx))^{5/2}}$$

Mathematica [C] time = 0.18, size = 90, normalized size = 0.51

$$\frac{a\left(-11(\sec(c+dx)-1)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(\sec(c+dx)+1)\right)+8(\sec(c+dx)-1)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\sec(c+dx)+1\right)-\right.}{20d(\sec(c+dx)-1)(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*(-10 - 11*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2]*(-1 + S
ec[c + d*x]) + 8*Hypergeometric2F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]]*(-1 + S
ec[c + d*x])))/(20*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [B] time = 0.64, size = 592, normalized size = 3.36

$$\left[\frac{165\sqrt{2}\left(\cos\left(dx+c\right)^4+2\cos\left(dx+c\right)^3-2\cos\left(dx+c\right)-1\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+3a\cos(dx+c)+a}{\cos(dx+c)-1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/960*(165*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1
)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 480*(cos(d*x + c)^
4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2
+ 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(449*cos(d*x + c)^4 + 351*cos(d*x +
c)^3 - 365*cos(d*x + c)^2 - 315*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*
x + c) - a^2*d), -1/480*(165*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2
*cos(d*x + c) - 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 480*(cos(d*x + c)^4 +
 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(4
49*cos(d*x + c)^4 + 351*cos(d*x + c)^3 - 365*cos(d*x + c)^2 - 315*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^4 + 2*a^2
*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)]
```

giac [A] time = 1.63, size = 281, normalized size = 1.60

$$\frac{\frac{165\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{960\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{15\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}-\frac{2\sqrt{2}\left(3\left(a\tan\right.\right.}{}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/480*(165*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sq
rt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 960*arctan(1/2*sqrt(2)*sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)) + 15*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)*tan(1/2*d*x + 1/2*c)^2) - 2*sqrt(2)*(3*(a*tan(1/2*d*x +
1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^16 + 20*(-a*tan(1/2*d
*x + 1/2*c)^2 + a)^(3/2)*a^17 + 165*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^1
8)/(a^20*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.36, size = 514, normalized size = 2.92

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))\,(-1+\cos(dx+c))^3\left(480\left(\cos^4(dx+c)\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/480/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c))^
3*(480*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*2^(1/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+165*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^4*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+96
0*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+330*cos(d*x+c)^3*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+898*cos
(d*x+c)^4-960*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcta
n(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+702*cos(d*x+c)^3-330*co
s(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2))-480*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-730*cos(d*x+c)^2-165*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2))-630*cos(d*x+c))/sin(d*x+c)^8/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.188 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=238

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{203\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d}+\frac{139a^2}{224d(a\sec(c+dx)+a)^{7/2}}-\frac{19a^2}{16d(1-\sec(c+dx))(a\sec}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+139/224*a^2/d/(a+a*sec(
d*x+c))^(7/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(7/2)-19/16*a^2/d
/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(7/2)+15/64*a/d/(a+a*sec(d*x+c))^(5/2)-53/
384/d/(a+a*sec(d*x+c))^(3/2)-203/512*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(
1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)-309/256/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3880, 103, 151, 152, 156, 63, 207}

$$\frac{139a^2}{224d(a\sec(c+dx)+a)^{7/2}}-\frac{19a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (203*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(256*Sqrt[2]*a^(3/2)*d) + (139*a
^2)/(224*d*(a + a*Sec[c + d*x])^(7/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
 a*Sec[c + d*x])^(7/2)) - (19*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
d*x])^(7/2)) + (15*a)/(64*d*(a + a*Sec[c + d*x])^(5/2)) - 53/(384*d*(a + a*
Sec[c + d*x])^(3/2)) - 309/(256*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a^6\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{a^3\,\text{Subst}\left(\int \frac{4a^2+\frac{11a^2x}{2}}{x(-a+ax)^2(a+ax)^{9/2}}\,dx, x,\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{}{16d(1}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{203\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d} + \frac{139a^2}{224d(a+a\sec(c+dx}$$

Mathematica [C] time = 0.32, size = 99, normalized size = 0.42

$$\frac{\cot^4(c+dx)\left(203(\sec(c+dx)-1)^2\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{1}{2}(\sec(c+dx)+1)\right)-64(\sec(c+dx)-1)^2\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\text{se}\right.\right.}{224d(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cot[c + d*x]^4*(-322 + 203*Hypergeometric2F1[-7/2, 1, -5/2, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x])^2 - 64*Hypergeometric2F1[-7/2, 1, -5/2, 1 + Sec
[c + d*x]]*(-1 + Sec[c + d*x])^2 + 266*Sec[c + d*x]))/(224*d*(a*(1 + Sec[c
+ d*x]))^(3/2))
```

fricas [B] time = 0.64, size = 837, normalized size = 3.52

$$\left[\frac{4263\sqrt{2}\left(\cos(dx+c)^6+2\cos(dx+c)^5-\cos(dx+c)^4-4\cos(dx+c)^3-\cos(dx+c)^2+2\cos(dx+c)+1\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/21504*(4263*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4
- 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(2*s
qrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*c
os(d*x + c) - a)/(cos(d*x + c) - 1)) + 10752*(cos(d*x + c)^6 + 2*cos(d*x +
c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*
sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4
*(10363*cos(d*x + c)^6 + 8037*cos(d*x + c)^5 - 16538*cos(d*x + c)^4 - 14238
*cos(d*x + c)^3 + 7231*cos(d*x + c)^2 + 6489*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a
^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d), 1/10752*(4263*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x
 + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 10752*(cos(d*x + c)^6 + 2*cos(d*
x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(10363*cos(d*x + c)^6 + 8037*cos(
d*x + c)^5 - 16538*cos(d*x + c)^4 - 14238*cos(d*x + c)^3 + 7231*cos(d*x + c
)^2 + 6489*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*co
s(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(
d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [A] time = 1.90, size = 350, normalized size = 1.47

$$-\frac{\frac{4263\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{21504\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{21\left(29\sqrt{2}\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}-27\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\cdots\right.}\cdots\right)}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/10752*(4263*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))
/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 21504*arctan(1/2*sqrt(2)*sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1)) - 21*(29*sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 27*s
qrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^3*sgn(tan(1/2*d*x + 1/2*c)
^2 - 1)*tan(1/2*d*x + 1/2*c)^4) + 8*sqrt(2)*(3*(a*tan(1/2*d*x + 1/2*c)^2 -
a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^30 - 21*(a*tan(1/2*d*x + 1/2*c)^
2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^31 - 112*(-a*tan(1/2*d*x + 1
/2*c)^2 + a)^(3/2)*a^32 - 882*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^33)/(a^
35*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.38, size = 866, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/10752/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x
+c))^4*(10752*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^6*2^(1/2)+21504*cos(d*x+c)^5*2^
(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*2^(1/2))+4263*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1
/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^6-10752*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*2^(1/2))+8526*cos(d*x+c)^5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-43008*cos(d*x+c)^3*2^(1/2
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))-4263*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4
*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+20726*cos(d*x+c)^6-10752*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2-17052*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+16074*cos
(d*x+c)^5+21504*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-4263*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x
+c)^2-33076*cos(d*x+c)^4+10752*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+8526*cos(d*x+c)*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2))-28476*cos(d*x+c)^3+4263*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+14462*cos(d*x+c)^2+12978*cos(d*x+c
))/sin(d*x+c)^12/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cot\left(c+d\,x\right)}^5}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{5}{\left(c + d x \right)}}{\left(a \left(\sec{\left(c + d x \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.189 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2a^2\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\tan}{ad\sqrt{a\sec}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+2*tan(d*x+c)
/a/d/(a+a*sec(d*x+c))^(1/2)-2/3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5*a
*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^2*tan(d*x+c)^7/d/(a+a*sec(d*x+
c))^(7/2)
```

Rubi [A] time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 459, 302, 203}

$$\frac{2a^2\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2a\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\tan(c}{ad\sqrt{a\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(2*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Tan[c + d*x]^3)/(3*d*(
a + a*Sec[c + d*x])^(3/2)) + (2*a*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x])
^(5/2)) + (2*a^2*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(2a^2)\operatorname{Subst}\left(\int \frac{x^6(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} - \frac{(2a^2)\operatorname{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} - \frac{(2a^2)\operatorname{Subst}\left(\int \left(\frac{1}{a^3} - \frac{x^2}{a^2} + \frac{x^4}{a} - \frac{1}{a^3(1+ax^2)}\right)dx, x, -\frac{\text{ta}}{\sqrt{a+}}\right.}{d}$$
$$= \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \ldots$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \ldots$$

Mathematica [C] time = 2.88, size = 248, normalized size = 1.58

$$\frac{32\sqrt{2}\tan^7(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{11/2}\left(\frac{\cos(c+dx)(7\cos(c+dx)+11)\csc^8\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left((-198\cos(c+dx)+61\cos(2(c+dx))-44\ldots\right.}{3360\sqrt{1-\sec(c+dx)}}\right.}{7d\left(1-\tan\ldots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (32*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(11/2)*((Cos[c + d*x]*(11 + 7*Cos[c +
 d*x])*Csc[(c + d*x)/2]^8*Sec[(c + d*x)/2]^2*(105*ArcTanh[Sqrt[1 - Sec[c +
d*x]]]*Cos[c + d*x]^3 + (76 - 198*Cos[c + d*x] + 61*Cos[2*(c + d*x)] - 44*C
os[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]))/(3360*Sqrt[1 - Sec[c + d*x]]) - (
4*Hypergeometric2F1[2, 11/2, 13/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[
c + d*x]*Tan[(c + d*x)/2]^2)/11)*Tan[c + d*x]^7)/(7*d*(a*(1 + Sec[c + d*x])
)^(3/2)*(1 - Tan[(c + d*x)/2]^2)^(9/2))
```

fricas [A] time = 0.58, size = 343, normalized size = 2.18

$$\left[-\frac{105\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\ldots}{105\left(a^2d\cos(dx+c)^4+a^2d\cos\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(146*cos(d*x + c)^3
- 32*cos(d*x + c)^2 - 24*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3), 2/105
*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (146*cos(d*x + c
```

```
)^3 - 32*cos(d*x + c)^2 - 24*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)]
```

giac [B] time = 7.38, size = 338, normalized size = 2.15

$$-\frac{105\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a(2\sqrt{\right.\right.}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/105*(105*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^2*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1)) - log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 -
1))) + 2*(((139*sqrt(2)*a^2*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)
^2 - 1) - 539*sqrt(2)*a^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/
2*c)^2 + 385*sqrt(2)*a^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2
*c)^2 - 105*sqrt(2)*a^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*
c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/
d
```

maple [B] time = 1.37, size = 391, normalized size = 2.49

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(105\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\sqrt{2}\,\sin(dx+c)\left(\cos^3(dx+c)\right)+315\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(105*arctanh(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(7/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3+315*2^(1/2)*sin(d*x+c)*cos
(d*x+c)^2*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+315*2^(1/2)*sin(d*x+c)*
cos(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+105*2^(1/2)*arctanh(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)+2336*cos(d*x+c)^4-2848*cos(d*x+c
)^3+128*cos(d*x+c)^2+624*cos(d*x+c)-240)/sin(d*x+c)/cos(d*x+c)^3/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.190 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-2*tan(d*x+c)/
a/d/(a+a*sec(d*x+c))^(1/2)+2/3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3887, 302, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
2*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Tan[c + d*x]^3)/(3*d*(a
 + a*Sec[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\ldots\right)}{ad}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 4.83, size = 162, normalized size = 1.71

$$\frac{64\cos^6\left(\frac{1}{2}(c+dx)\right)\cot^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{7/2}\left((\sin(c+dx)-2\sin(2(c+dx)))\sqrt{\frac{1}{\cos(c+dx)+1}}\ldots\right.}{3d\left(\cot^2\left(\frac{1}{2}(c+dx)\right)-1\right)^2(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (64*Cos[(c + d*x)/2]^6*Cot[(c + d*x)/2]^4*Sec[c + d*x]^5*((1 + Sec[c + d*x]
)^(-1))^(7/2)*(3*ArcSin[Tan[(c + d*x)/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Cos
[c + d*x]^2 + Sqrt[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)]*(Sin[c + d*x
] - 2*Sin[2*(c + d*x)])))/(3*d*(-1 + Cot[(c + d*x)/2]^2)^2*(a*(1 + Sec[c +
d*x]))^(3/2))
```

fricas [A] time = 0.48, size = 295, normalized size = 3.11

$$\left[-\frac{3\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2+2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\sqrt{\ldots}}{3\left(a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 +
 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*(4*cos(d*x + c) - 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + a^
2*d*cos(d*x + c)), -2/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x + c) - 1)*sin(d*x + c
))/(a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

giac [B] time = 7.99, size = 258, normalized size = 2.72

$$\frac{3\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-a\left(2\sqrt{2}+3\right)\right|\right)}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+a\left(2\sqrt{2}-3\right)\right|\right)}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/3*(3*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))) + 2*(5*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*sqrt(2)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d

maple [A] time = 1.23, size = 142, normalized size = 1.49

$$-\frac{\left(3\cos\left(dx+c\right)\sin\left(dx+c\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\;\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)-8\left(\cos^{2}\left(dx+c\right)\right)+10\cos\left(d\right.\right.}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x)

[Out] -1/3/d*(3*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-8*cos(d*x+c)^2+10*cos(d*x+c)-2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)/a^2

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan\left(c+d\,x\right)}^4}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(3/2),x)

[Out] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}{\left(c + dx \right)}}{\left(a \left(\sec{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.191 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+2*arctan(1/2
*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 481, 203}

$$\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(2*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(a^(3/2)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}-\frac{4\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}\end{aligned}$$

Mathematica [C] time = 23.84, size = 4739, normalized size = 55.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-32*Cos[(c + d*x)/4]^2*Cos[(c + d*x)/2]^2*(-2/Sqrt[Sec[c + d*x]] + 2*Sqrt[
Sec[c + d*x]])*Sec[c + d*x]^2*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqr
t[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]],
1/2] - (2 + Sqrt[2])*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt
[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2
+ Sqrt[2])*EllipticPi[1/Sqrt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c
+ d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2]
 + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] +
 Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(
c + d*x)/4])^2*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c
 + d*x)/4])])/4 + EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(c + d*x)/4]/Sqrt[3
 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4^2]*S
qrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)]))/(d*(a*(1 + Se
c[c + d*x]))^(3/2)*(16*Cos[(c + d*x)/4]*Sqrt[Sec[c + d*x]]*Sin[(c + d*x)/4]
*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x
)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*Elliptic
Pi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 +
(1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[
2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])
*Tan[(c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sq
rt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt
[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sqrt[(1 + Sqrt
[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])])/4 + EllipticPi[
-3 + 2*Sqrt[2], ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[
2]]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*
Sqrt[2] + Tan[(c + d*x)/4]^2)]) - 16*Cos[(c + d*x)/4]^2*Sec[c + d*x]^(3/2)*
Sin[c + d*x]*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] +
Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[
2])*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*
x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*Ellipt
icPi[1/Sqrt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (
1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/
4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4]
)/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sq
rt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])])/4 +
 EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 1
7 - 12*Sqrt[2]]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*Sqrt[(-3 + 2*Sqrt[
2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)]) - 32*Cos[(c + d*x)/4]^2*Sqrt[Se
c[c + d*x]]*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + T
an[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2
])*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x
)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*Ellipti
cPi[1/Sqrt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1
 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2])*Sec[(c + d*x)/4]^2*Sqrt[(-1 - Sqrt[2
] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2]
+ Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[
(c + d*x)/4])*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c
+ d*x)/4])])/8 + ((-3 + 2*Sqrt[2])*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(c
 + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sec[(c + d*x)/4]^2*Tan[(c
 + d*x)/4]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2])/(4*Sqrt[(-3 + 2*Sqrt[2
])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)]) - (EllipticPi[-3 + 2*Sqrt[2], Ar
cSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sec[(c + d*x)/
```

```
4]^2*Tan[(c + d*x)/4]*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)
/4]^2)])/(4*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]) + (Sec[(c + d*x)/4]^2
*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqr
t[2] + Tan[(c + d*x)/4]^2)])/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(c + d*x)/
4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(c + d*x)/4]^2)/(3 -
2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*Tan[(c + d*x)/4]^2)/(3 - 2*Sqrt[2]))) +
(((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/
4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*EllipticPi
[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1
 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[2]
, ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*T
an[(c + d*x)/4])]], 1/2])*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[
2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sqrt[(1 + Sqrt[
2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1/4*(Sec[(c + d
*x)/4]^2*(-1 - Sqrt[2] + Tan[(c + d*x)/4]))/(-1 + Sqrt[2] + Tan[(c + d*x)/4
])^2 + Sec[(c + d*x)/4]^2/(4*(-1 + Sqrt[2] + Tan[(c + d*x)/4]))))/(8*Sqrt[(
-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]) + (((2
 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/
(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*EllipticPi[-(1
/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + S
qrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[2], Ar
cSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(
c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2]
+ Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sqrt[(1 + Sqrt[2]
+ Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1/4*(Sec[(c + d*x)
/4]^2*(1 - Sqrt[2] + Tan[(c + d*x)/4]))/(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2
 + Sec[(c + d*x)/4]^2/(4*(-1 + Sqrt[2] + Tan[(c + d*x)/4]))))/(8*Sqrt[(1 -
Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]) + (((2 + Sq
rt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 +
(1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*EllipticPi[-(1/Sqrt
[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2
])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[2], ArcSin[
2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d
*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan
[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(
c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*(Sec[(c + d*x)/4]^2/(4*(-
1 + Sqrt[2] + Tan[(c + d*x)/4])) - (Sec[(c + d*x)/4]^2*(1 + Sqrt[2] + Tan[(
c + d*x)/4]))/(4*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2)))/(8*Sqrt[(1 + Sqrt[2
] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]) + (Sqrt[(-1 - Sqr
t[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[
2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + T
an[(c + d*x)/4])^2*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Ta
n[(c + d*x)/4])]*(((2 + Sqrt[2])*(-1/4*((1 + Sqrt[2])*Sec[(c + d*x)/4]^2*(1
 + Sqrt[2] + Tan[(c + d*x)/4]))/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])^2 + Se
c[(c + d*x)/4]^2/(4*(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4]))))/(2^(3/4)*Sqrt[(
1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]*Sqrt[
1 - (1 + Sqrt[2] + Tan[(c + d*x)/4])/(Sqrt[2]*(1 + (1 + Sqrt[2])*Tan[(c + d
*x)/4]))]*Sqrt[1 - (Sqrt[2]*(1 + Sqrt[2] + Tan[(c + d*x)/4]))/(1 + (1 + Sqr
t[2])*Tan[(c + d*x)/4])]) + ((-2 + Sqrt[2])*(-1/4*((1 + Sqrt[2])*Sec[(c + d
*x)/4]^2*(1 + Sqrt[2] + Tan[(c + d*x)/4]))/(1 + (1 + Sqrt[2])*Tan[(c + d*x)
/4])^2 + Sec[(c + d*x)/4]^2/(4*(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4]))))/(2^(
3/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)
/4])]*(1 - (1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x
)/4]))*Sqrt[1 - (1 + Sqrt[2] + Tan[(c + d*x)/4])/(Sqrt[2]*(1 + (1 + Sqrt[2]
)*Tan[(c + d*x)/4]))]*Sqrt[1 - (Sqrt[2]*(1 + Sqrt[2] + Tan[(c + d*x)/4]))/(
1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]) - ((2 + Sqrt[2])*(-1/4*((1 + Sqrt[2])
*Sec[(c + d*x)/4]^2*(1 + Sqrt[2] + Tan[(c + d*x)/4]))/(1 + (1 + Sqrt[2])*Ta
n[(c + d*x)/4])^2 + Sec[(c + d*x)/4]^2/(4*(1 + (1 + Sqrt[2])*Tan[(c + d*x)/
4]))))/(2^(3/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Ta
```

```
n[(c + d*x)/4])]*(1 + (1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*T
an[(c + d*x)/4]))*Sqrt[1 - (1 + Sqrt[2] + Tan[(c + d*x)/4])/(Sqrt[2]*(1 + (
1 + Sqrt[2])*Tan[(c + d*x)/4]))]*Sqrt[1 - (Sqrt[2]*(1 + Sqrt[2] + Tan[(c +
d*x)/4]))/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]))))/4)))
```

fricas [A] time = 0.63, size = 295, normalized size = 3.47

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\,\sqrt{\cdots}}{\cdots}\right)}{a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c
) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a^2*d), -2*(sqrt(2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a^2*d)]
```

giac [A] time = 2.97, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2\arctan\left(\frac{\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sq
rt(a))/a^(3/2) - 2*arctan(sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sqrt(a))/a^(3
/2))/d
```

maple [B] time = 0.90, size = 142, normalized size = 1.67

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+2\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)\cdots}{\sin(dx+c)}\right.\right.}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))+2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c)))/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.192 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{71\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{32\sqrt{2}\,a^{3/2}d}+\frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{32a^2d}-\frac{\cos^2(c+dx)\cot(c+$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+71/64*arctan
(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+7
/32*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d-13/32*cos(d*x+c)*cot(d*x+c)*sec
(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a^2/d-1/16*cos(d*x+c)^2*cot(d*x+c)
*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{32a^2d}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{71\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{32\sqrt{2}\,a^{3/2}d}-\frac{\cos^2(c+dx)\cot(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(71*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(32*
Sqrt[2]*a^(3/2)*d) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(32*a^2*d) -
 (13*Cos[c + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d*x]])
/(32*a^2*d) - (Cos[c + d*x]^2*Cot[c + d*x]*Sec[(c + d*x)/2]^4*Sqrt[a + a*Se
c[c + d*x]])/(16*a^2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2 d}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{16a^2 d} - \frac{\text{Subst}\left(\int \frac{3a-5\ldots}{x^2(1+ax^2\ldots}\right.}{\ldots}$$

$$= -\frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{32a^2 d} - \frac{\cos^2(c+dx)\,\text{co}\ldots}{\ldots}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2 d} - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a\ldots}}{32a^2 d}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2 d} - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a\ldots}}{32a^2 d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{71\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32\sqrt{2}\,a^{3/2}d} + \frac{7\cot(c+dx)\sqrt{a+a\sec(\ldots}}{32a^2 d}$$

Mathematica [C] time = 24.65, size = 5578, normalized size = 25.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.66, size = 603, normalized size = 2.80

$$\left[-\frac{71\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^{2}+2a\cos(d}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(71*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*s
qrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos
(d*x + c) + 1))*sin(d*x + c) + 64*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqr
t(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)
+ a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(27*cos(d*x + c)^3 + 12*cos(d*x +
 c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^2*d*co
s(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c)), -1/64*(71*sqrt(
2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x +
 c) + 64*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x
 + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(27*cos(d*x + c)^3 + 12*cos
(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))]
```

giac [A] time = 1.49, size = 164, normalized size = 0.76

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{17\sqrt{2}}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 17*sqrt(2)/(a^2*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + 16*sqrt(2)/(((sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sqrt(-a)*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.33, size = 542, normalized size = 2.52

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(-1+\cos\left(dx+c\right)\right)^{2}\left(64\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(\cos^{2}\left(dx+c\right)\right)\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/64/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(64*2^(1/2)*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
```

```
))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)+128*cos(d*x
+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+71*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+64*2^(1/2)*sin(d
*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c
)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+142*cos(d*x+c)*sin(d*x+c)*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+71*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)-54*cos(d*x+c)^3-24*cos(d*x+c)^2+14*cos(d*x+c))/sin(d*
x+c)^5/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.193 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{533\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{256\sqrt{2}a^{3/2}d}+\frac{277\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{384a^3d}-\frac{\cos^3(c+dx)\mathrm{co}}{}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+277/384*cot(d
*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^3/d-81/128*cos(d*x+c)*cot(d*x+c)^3*sec(1/2
*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^3/d-7/64*cos(d*x+c)^2*cot(d*x+c)^3*s
ec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^3/d-1/48*cos(d*x+c)^3*cot(d*x+
c)^3*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(3/2)/a^3/d-533/512*arctan(1/2*a
^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-21/256*
cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.28, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{277\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{384a^3d}-\frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{256a^2d}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{533\tan^{-1}}{25}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
533*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(256
*Sqrt[2]*a^(3/2)*d) - (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(256*a^2*d
) + (277*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(384*a^3*d) - (81*Cos[c
 + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(128*
a^3*d) - (7*Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + a*Sec[c +
 d*x])^(3/2))/(64*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]^3*Sec[(c + d*x)/2]^
6*(a + a*Sec[c + d*x])^(3/2))/(48*a^3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3 d}$$

$$= -\frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3 d} - \frac{\mathrm{Subst}\left(\int \frac{3}{x^4(1+\cdots}\right.}{\cdots}$$

$$= -\frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3 d} - \frac{\cos^3(c+dx)\cdots}{\cdots}$$

$$= -\frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^3 d} - \frac{7\cos^2(c+d\cdots}{\cdots}$$

$$= \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3 d} - \frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx\cdots\right.}{128a^3 d}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2 d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3 d} - \frac{81\,\mathrm{c}\cdots}{\cdots}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2 d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3 d} - \frac{81\,\mathrm{c}\cdots}{\cdots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2} d} - \frac{533\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{256\sqrt{2}\,a^{3/2} d} - \frac{21\cot(c+dx)\sqrt{a+a\sec\cdots}}{256a^2 d}$$

Mathematica [C] time = 24.01, size = 5620, normalized size = 18.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.90, size = 712, normalized size = 2.35

$$\left[-\frac{1599\sqrt{2}\left(\cos\left(dx+c\right)^4+2\cos\left(dx+c\right)^3-2\cos\left(dx+c\right)-1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+\cdots}{\cos(dx+c)^2+2\cos\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3072*(1599*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)
- 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 1536*(cos(d*x + c)^4
+ 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3
+ 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*
x + c) - 4*(819*cos(d*x + c)^5 + 492*cos(d*x + c)^4 - 690*cos(d*x + c)^3 -
428*cos(d*x + c)^2 + 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
)))/((a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)
- a^2*d)*sin(d*x + c)), 1/1536*(1599*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x +
c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 1536*(co
s(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*arctan(2*sqrt
(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*
cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(819*cos(d*x + c)^5
+ 492*cos(d*x + c)^4 - 690*cos(d*x + c)^3 - 428*cos(d*x + c)^2 + 63*cos(d*x
 + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^2*d*cos(d*x + c)^4 + 2*
a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]
```

giac [A] time = 1.95, size = 289, normalized size = 0.95

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{37\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{417\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\cdots\right.\right.}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/1536*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*sqrt(2)*tan(1/2*d*x + 1/2
*c)^2/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 37*sqrt(2)/(a^2*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 417*sqrt(2)/(a^2*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) - 32*sqrt(2)*(21*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 36*(sqrt(-a)*tan
```

```
(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + 19*a^2)/(((s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^
3*sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.47, size = 732, normalized size = 2.42

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))\,(-1+\cos(dx+c))^3\left(-1536\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^4(dx+c)\right)\sin(dx+c)\sqrt{2}\ \mathrm{arct}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/1536/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c))
^3*(-1536*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*2^(1
/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))-1599*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*
ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x
+c))-3072*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))-3198*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x
+c))+1638*cos(d*x+c)^5+3072*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))+984*cos(d*x+c)^4+3198*cos(d*x+c)*sin(d*x+c)*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+1536*2^(1/2)*sin(d*x+c)*arctanh(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)-1380*cos(d*x+c)^3+1599*sin(d*x+c)*ln(-(-(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)-856*cos(d*x+c)^2+126*cos(d*x+c))/sin(d*x+c)^9/a
^2
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^4/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.194 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=387

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{16363\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{8192\sqrt{2}\,a^{3/2}d}+\frac{12267\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{10240a^4d}-\frac{\cos^4(c+dx}{}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-8171/12288*c
ot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^3/d+12267/10240*cot(d*x+c)^5*(a+a*sec(
d*x+c))^(5/2)/a^4/d-2045/2048*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*
(a+a*sec(d*x+c))^(5/2)/a^4/d-511/3072*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*d*x
+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^4/d-29/768*cos(d*x+c)^3*cot(d*x+c)^5*sec
(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^4/d-1/128*cos(d*x+c)^4*cot(d*x+c
)^5*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(5/2)/a^4/d+16363/16384*arctan(1/
2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-21/8
192*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.37, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{12267\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{10240a^4d}-\frac{8171\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12288a^3d}-\frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+}}{8192a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(16363*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
8192*Sqrt[2]*a^(3/2)*d) - (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8192*
a^2*d) - (8171*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12288*a^3*d) + (
12267*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(10240*a^4*d) - (2045*Cos[
c + d*x]*Cot[c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(204
8*a^4*d) - (511*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec
[c + d*x])^(5/2))/(3072*a^4*d) - (29*Cos[c + d*x]^3*Cot[c + d*x]^5*Sec[(c +
 d*x)/2]^6*(a + a*Sec[c + d*x])^(5/2))/(768*a^4*d) - (Cos[c + d*x]^4*Cot[c
+ d*x]^5*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(5/2))/(128*a^4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^5}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^4 d}$$

$$= -\frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} - \frac{\text{Subst}\left(\int \frac{3\ldots}{x^6(1+\ldots}\right.}{\ldots}$$

$$= -\frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} - \frac{\cos^4(c+dx\ldots}{\ldots}$$

$$= -\frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} - \frac{29\cos^3(c\ldots}{\ldots}$$

$$= -\frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} - \frac{511\cos^2(c\ldots}{\ldots}$$

$$= \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} - \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c\ldots\right.}{2048a^4d}$$

$$= -\frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} + \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} - \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} + \frac{122\ldots}{\ldots}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} - \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} + \frac{122\ldots}{\ldots}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{16363\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{8192\sqrt{2}\,a^{3/2}d} - \frac{21\cot(c+dx)\sqrt{a+a\ldots}}{8192a^2d}$$

Mathematica [C] time = 24.21, size = 5662, normalized size = 14.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
[Out] Result too large to show
```

fricas [A] time = 0.67, size = 955, normalized size = 2.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] [-1/491520*(245445*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c
)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(
(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*si
n(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1))*sin(d*x + c) + 245760*(cos(d*x + c)^6 + 2*cos(d*x + c)^
```

```
5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x
+ c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(151041*cos(d*x + c)^7 + 103
524*cos(d*x + c)^6 - 228999*cos(d*x + c)^5 - 181256*cos(d*x + c)^4 + 97611*
cos(d*x + c)^3 + 82340*cos(d*x + c)^2 + 315*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a
^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d)*sin(d*x + c)), -1/245760*(245445*sqrt(2)*(cos(d*x
+ c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 245760*(c
os(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*co
s(d*x + c) - a))*sin(d*x + c) + 2*(151041*cos(d*x + c)^7 + 103524*cos(d*x +
 c)^6 - 228999*cos(d*x + c)^5 - 181256*cos(d*x + c)^4 + 97611*cos(d*x + c)^
3 + 82340*cos(d*x + c)^2 + 315*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c)))/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x
+ c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x +
c) + a^2*d)*sin(d*x + c))]
```

giac [A] time = 2.51, size = 414, normalized size = 1.07

$$5\left(2\left(4\left(\frac{6\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{65\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{1451\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/245760*(5*(2*(4*(6*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)) - 65*sqrt(2)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*
d*x + 1/2*c)^2 + 1451*sqrt(2)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/
2*d*x + 1/2*c)^2 - 13503*sqrt(2)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) + 256*sqrt(2)*(555*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8 - 195
0*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a
 + 2780*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^4*a^2 - 1810*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^2*a^3 + 473*a^4)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
)/d
```

maple [B] time = 1.54, size = 1240, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/245760/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*
x+c))^4*(-245760*cos(d*x+c)^6*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))-245445*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
```

```
)/sin(d*x+c))-491520*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(
d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))-490890*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
5*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))+245760*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*
sin(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))+302082*cos(d*x+c)^7+245445*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+983040*cos(d*x+c)^3*sin(d*x+c)*2^
(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+207048*cos(d*x+c)^6+981780*
cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+245760*2^
(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)-457
998*cos(d*x+c)^5+245445*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c
)^2*sin(d*x+c)-491520*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/c
os(d*x+c)*2^(1/2))-362512*cos(d*x+c)^4-490890*cos(d*x+c)*sin(d*x+c)*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-245760*2^(1/2)*sin(d*x+c)*arctanh(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)+195222*cos(d*x+c)^3-245445*sin(d*x+c)*ln(-(-(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+164680*cos(d*x+c)^2+630*cos(d*x+c))/sin(d*
x+c)^13/a^2
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{6}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**6/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.195 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^4d}-\frac{6\sqrt{a\sec(c+dx)+a}}{a^3d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+2/3*(a+a*sec(d*x+c))^(
3/2)/a^4/d-6*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3880, 88, 63, 207}

$$\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^4d}-\frac{6\sqrt{a\sec(c+dx)+a}}{a^3d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - (6*Sqrt[a + a*
Sec[c + d*x]])/(a^3*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^4*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{3a^2}{\sqrt{a+ax}} + \frac{a^2}{x\sqrt{a+ax}} + a\sqrt{a+ax}\right) dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$= -\frac{6\sqrt{a+a\sec(c+dx)}}{a^3 d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^4 d} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+\right.}{a^2 d}$$

$$= -\frac{6\sqrt{a+a\sec(c+dx)}}{a^3 d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^4 d} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\,\text{s}\right.}{a^3 d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2} d} - \frac{6\sqrt{a+a\sec(c+dx)}}{a^3 d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^4 d}$$

Mathematica [A] time = 0.12, size = 69, normalized size = 0.88

$$\frac{2\left(\sec^2(c+dx) - 7\sec(c+dx) - 3\sqrt{\sec(c+dx)+1}\,\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right) - 8\right)}{3a^2 d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(-8 - 7*Sec[c + d*x] + Sec[c + d*x]^2 - 3*ArcTanh[Sqrt[1 + Sec[c + d*x]]
]*Sqrt[1 + Sec[c + d*x]]))/(3*a^2*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.48, size = 241, normalized size = 3.09

$$\left[\frac{3\sqrt{a}\cos(dx+c)\log\left(-8a\cos(dx+c)^2 + 4\left(2\cos(dx+c)^2 + \cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8a\cos(dx+c\right.}{6a^3 d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
*x + c) - a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(8*cos(d*x + c) -
1))/(a^3*d*cos(d*x + c)), 1/3*(3*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)
- 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(8*cos(d*x + c) - 1))/(a^3*d*co
s(d*x + c))]
```

giac [B] time = 4.51, size = 140, normalized size = 1.79

$$-\frac{2\left(\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} - \frac{\sqrt{2}\left(9a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(s
qrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(9*a*tan(1/2*d*x + 1
/2*c)^2 - 7*a)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.19, size = 155, normalized size = 1.99

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\cos\left(dx+c\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+3\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right.\right.}{6d\cos\left(dx+c\right)a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*cos(d*x+c)*2^(1/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)+3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+32*cos(d*x+c)-4)/cos(d*x+c)/a^3
```

maxima [B] time = 0.46, size = 163, normalized size = 2.09

$$\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^{4}}-\frac{18\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^{3}}+\frac{2\left(4a+\frac{3a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^{2}}-\frac{6}{\sqrt{a+\frac{a}{\cos(dx+c)}}\,a^{2}}-\frac{2}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) +
 sqrt(a)))/a^(5/2) + 2*(a + a/cos(d*x + c))^(3/2)/a^4 - 18*sqrt(a + a/cos(d
*x + c))/a^3 + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/2)*a^2)
- 6/(sqrt(a + a/cos(d*x + c))*a^2) - 2/((a + a/cos(d*x + c))^(3/2)*a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{tan}\left(c+d\,x\right)}^5}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}{\left(c+dx \right)}}{\left(a \left(\sec{\left(c+dx \right)}+1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.196 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}} dx$

Optimal. Leaf size=54

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{4}{a^2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-4/a^2/d/(a+a*sec(d*x+c)
)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3880, 78, 63, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{4}{a^2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - 4/(a^2*d*Sqrt[a
 + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-a+ax}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{a^2d}$$
$$= -\frac{4}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d}$$
$$= -\frac{4}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^3d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{4}{a^2d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.93

$$\frac{2\left(\sqrt{\sec(c+dx)+1}\,\tanh^{-1}\left(\sqrt{\sec(c+dx)+1}\right)-2\right)}{a^2d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(-2 + ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(a^2*d*Sq
rt[a*(1 + Sec[c + d*x])])
```

fricas [B] time = 0.67, size = 245, normalized size = 4.54

$$\left[\frac{\sqrt{a}\left(\cos(dx+c)+1\right)\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(\right.}{2\left(a^3d\cos(dx+c)+a^3d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) - 8*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/
(a^3*d*cos(d*x + c) + a^3*d), -(sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 a)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a^3*d*cos(d
*x + c) + a^3*d)]
```

giac [B] time = 4.92, size = 104, normalized size = 1.93

$$\frac{2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2*(arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-
a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/(a*d)
```

maple [B] time = 1.14, size = 154, normalized size = 2.85

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\cos(dx+c)\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)+\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{2}\right.\right.}{d\left(1+\cos(dx+c)\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2))+(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+4*cos(d*x+c))/(1+cos(d*x+c))/a^3
```

maxima [B] time = 0.45, size = 125, normalized size = 2.31

$$-\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2}+\frac{6}{\sqrt{a+\frac{a}{\cos(dx+c)}}\,a^2}-\frac{2}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
+ sqrt(a)))/a^(5/2) + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/2
)*a^2) + 6/(sqrt(a + a/cos(d*x + c))*a^2) - 2/((a + a/cos(d*x + c))^(3/2)*a
))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{\left(a\left(\sec(c+dx)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.197 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{2}{a^2d\sqrt{a\sec(c+dx)+a}}+\frac{2}{3ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+2/3/a/d/(a+a*sec(d*x+c
))^(3/2)+2/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3880, 51, 63, 207}

$$\frac{2}{a^2d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{2}{3ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + 2/(3*a*d*(a +
a*Sec[c + d*x])^(3/2)) + 2/(a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c\right.}{a^2d}$$

$$= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+\right.}{a^3d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.06, size = 40, normalized size = 0.51

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sec(c+dx)+1\right)}{3ad(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]])/(3*a*d*(a*(1 + Sec[c
 + d*x]))^(3/2))
```

fricas [B] time = 0.58, size = 321, normalized size = 4.12

$$\left[\frac{3\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-8a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(d}{\cos(d}}\right.}{6\left(a^3d\cos(dx+c)^2+2a^3d\cos(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)
^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(4*cos(d*x + c)^2 + 3*cos(d*x + c
))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d
*cos(d*x + c) + a^3*d), 1/3*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
2*a*cos(d*x + c) + a)) + 2*(4*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) +
 a^3*d)]
```

giac [A] time = 1.50, size = 130, normalized size = 1.67

$$-\frac{\frac{12\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}+\frac{\sqrt{2}\left(\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}a^{8}+6\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,a^{9}\right)}{a^{12}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(12*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(
sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*((-a*tan(1/2*d*x +
1/2*c)^2 + a)^(3/2)*a^8 + 6*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^9)/(a^12*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [A] time = 0.13, size = 62, normalized size = 0.79

$$\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{a+a\sec(dx+c)}}+\frac{2}{3a(a+a\sec(dx+c))^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/d*(-2/a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))+2/a^2/(a+a*sec(d*x+
c))^(1/2)+2/3/a/(a+a*sec(d*x+c))^(3/2))
```

maxima [A] time = 0.44, size = 87, normalized size = 1.12

$$\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) +
 sqrt(a)))/a^(5/2) + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/2)
*a^2))/d
```

mupad [B] time = 1.91, size = 69, normalized size = 0.88

$$\frac{\frac{2\left(a+\frac{a}{\cos(c+d\,x)}\right)}{a^{2}}+\frac{2}{3\,a}}{d\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{a^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] ((2*(a + a/cos(c + d*x)))/a^2 + 2/(3*a))/(d*(a + a/cos(c + d*x))^(3/2)) - (
2*atanh((a + a/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(5/2)*d)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan (c + dx)}{(a (\sec (c + dx) + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.198 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}-\frac{7}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{1}{2ad(a\sec(c+dx)+a)^{3/2}}-\frac{}{5d(a\,\mathrm{s}}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-1/5/d/(a+a*sec(d*x+c))^
(5/2)-1/2/a/d/(a+a*sec(d*x+c))^(3/2)-1/8*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-7/4/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3880, 85, 152, 156, 63, 207}

$$-\frac{7}{4a^2d\sqrt{a\sec(c+dx)+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}-\frac{1}{2ad(a\sec(c+dx)+a)^{3/2}}-\frac{}{5d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d) - 1/(5*d*(a + a*
Sec[c + d*x])^(5/2)) - 1/(2*a*d*(a + a*Sec[c + d*x])^(3/2)) - 7/(4*a^2*d*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{2ad}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{-6a^4+\frac{9a^4x}{2}}{x(-a+ax)(a+ax)^{3/2}}\,d\right.}{6a^4d}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}} -$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} - \frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{}{2a}$$

Mathematica [C] time = 0.07, size = 60, normalized size = 0.42

$$\frac{{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(\sec(c+dx)+1)\right) - 2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\sec(c+dx)+1\right)}{5d(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2
F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]])/(5*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [B] time = 0.64, size = 573, normalized size = 3.98

$$\left[\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3\,a\cos(dx+c)-a}{\cos(dx+c)-1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/80*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 40*(cos(d*x + c)^3 +
 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4
*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)) - 8*a*cos(d*x + c) - a) - 4*(49*cos(d*x + c)^3 + 80*cos(d*x + c)^2
+ 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/40*(5*sq
rt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arc
tan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
a*cos(d*x + c) + a)) - 40*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x +
c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(49*cos(d*x + c)^3 + 80*cos(d*x +
c)^2 + 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [A] time = 4.29, size = 227, normalized size = 1.58

$$-\frac{\frac{5\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{80\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\sqrt{2}\left(\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{20}+5\left(-a\right.\right.}{a^{25}\mathrm{sgn}\left(\tan\left(\frac{1}{2}d\right.\right.}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/40*(5*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt
(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 80*arctan(1/2*sqrt(2)*sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)
^2 - 1)) - sqrt(2)*((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a)*a^20 + 5*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^21 + 35*sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^22)/(a^25*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
))/d
```

maple [B] time = 1.33, size = 496, normalized size = 3.44

$$\frac{\left(-1+\cos\left(dx+c\right)\right)^3\left(40\left(\cos^3\left(dx+c\right)\right)\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)+5\left(\cos^3\left(dx+c\right)\right)\sqrt{-\frac{2}{1+}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/40/d*(-1+cos(d*x+c))^3*(40*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+5*cos(
d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2))+120*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2+15*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*c
os(d*x+c)^2+120*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+98*cos(d*x+c)^3+15*co
s(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2))+40*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+160*cos(d*x+c)^2+5*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))
+70*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^6/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.199 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d}+\frac{51}{32a^2d\sqrt{a\sec(c+dx)+a}}-\frac{5a}{28d(a\sec(c+dx)+a)^{7/2}}+$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-5/28*a/d/(a+a*sec(d*x+
c))^(7/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(7/2)+3/40/d/(a+a*sec(d*x
+c))^(5/2)+19/48/a/d/(a+a*sec(d*x+c))^(3/2)+13/64*arctanh(1/2*(a+a*sec(d*x+
c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)+51/32/a^2/d/(a+a*sec(d*x+c))^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3880, 103, 152, 156, 63, 207}

$$\frac{51}{32a^2d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d}-\frac{5a}{28d(a\sec(c+dx)+a)^{7/2}}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + (13*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*a^(5/2)*d) - (5*a)/(
28*d*(a + a*Sec[c + d*x])^(7/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(7/2)) + 3/(40*d*(a + a*Sec[c + d*x])^(5/2)) + 19/(48*a*d*(a + a*Sec
[c + d*x])^(3/2)) + 51/(32*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} - \frac{a\,\text{Subst}\left(\int \frac{2a^2+\frac{9a^2x}{2}}{x(-a+ax)(a+ax)^{9/2}}\,dx, x, \sec(c+\right.}{2d}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{}{40d(a+a}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{}{40d(a+a}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{}{40d(a+a}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{}{40d(a+a}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{}{40d(a+a}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{13\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d} - \frac{5a}{28d(a+a\sec(c+dx))^{7/2}}$$

Mathematica [C] time = 0.21, size = 90, normalized size = 0.45

$$\frac{a\left(-13(\sec(c+dx)-1)\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{1}{2}(\sec(c+dx)+1)\right)+8(\sec(c+dx)-1)\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\sec(c+dx)+1\right)\cdots\right.}{28d(\sec(c+dx)-1)(a(\sec(c+dx)+1))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a*(-14 - 13*Hypergeometric2F1[-7/2, 1, -5/2, (1 + Sec[c + d*x])/2]*(-1 + S
ec[c + d*x]) + 8*Hypergeometric2F1[-7/2, 1, -5/2, 1 + Sec[c + d*x]]*(-1 + S
ec[c + d*x])))/(28*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(7/2))
```

fricas [B] time = 0.61, size = 748, normalized size = 3.74

$$\left[\frac{1365\sqrt{2}\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)^2-3\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{2\cdots}{\cdots}\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/13440*(1365*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^
3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)
/(cos(d*x + c) - 1)) + 6720*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x
+ c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x +
c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(8017*cos(d*x + c)^5 + 12640*co
s(d*x + c)^4 - 1582*cos(d*x + c)^3 - 12040*cos(d*x + c)^2 - 5355*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^5 + 3*a^3*
d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*
d*cos(d*x + c) - a^3*d), -1/6720*(1365*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x
+ c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)
*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(a*cos(d*x + c) + a)) - 6720*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(
d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*arctan(2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c)
+ a)) - 2*(8017*cos(d*x + c)^5 + 12640*cos(d*x + c)^4 - 1582*cos(d*x + c)^3
 - 12040*cos(d*x + c)^2 - 5355*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c)))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x
 + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*cos(d*x + c) - a^3*d)]
```

giac [B] time = 2.02, size = 323, normalized size = 1.62

$$\frac{\frac{1365\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{13440\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{105\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{2\sqrt{2}\left(15\cdots\right.}{\cdots}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/6720*(1365*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(
sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 13440*arctan(1/2*sqrt(2)*sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x +
 1/2*c)^2 - 1)) + 105*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^3*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)*tan(1/2*d*x + 1/2*c)^2) + 2*sqrt(2)*(15*(a*tan(
1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^36 - 84*(a*
tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^37 - 38
5*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^38 - 2730*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)*a^39)/(a^42*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.57, size = 744, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/6720/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c)
)^4*(6720*cos(d*x+c)^5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+1365*cos(d*x+c)^5*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)+20160*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*2^(1/2)*arctan(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+4095*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^4*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+
13440*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+16034*cos(d*x+c)^5+2730*cos(d
*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2))-13440*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2+25280*cos(d*
x+c)^4-2730*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-20160*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2
))-3164*cos(d*x+c)^3-4095*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-6720*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/
2)-24080*cos(d*x+c)^2-1365*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-10710*cos(d*x+c))/sin(d*x+c)^10/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot\left(c+d\,x\right)^3}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.200 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=262

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{263\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,a^{5/2}d}+\frac{199a^2}{288d(a\sec(c+dx)+a)^{9/2}}-\frac{21a^2}{16d(1-\sec(c+dx))(a\sec(c+}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+199/288*a^2/d/(a+a*sec(
d*x+c))^(9/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(9/2)-21/16*a^2/d
/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(9/2)+135/448*a/d/(a+a*sec(d*x+c))^(7/2)+7
/640/d/(a+a*sec(d*x+c))^(5/2)-83/256/a/d/(a+a*sec(d*x+c))^(3/2)-263/1024*ar
ctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-761/512
/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3880, 103, 151, 152, 156, 63, 207}

$$\frac{199a^2}{288d(a\sec(c+dx)+a)^{9/2}}-\frac{21a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{9/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - (263*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(512*Sqrt[2]*a^(5/2)*d) + (199*a
^2)/(288*d*(a + a*Sec[c + d*x])^(9/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
 a*Sec[c + d*x])^(9/2)) - (21*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
d*x])^(9/2)) + (135*a)/(448*d*(a + a*Sec[c + d*x])^(7/2)) + 7/(640*d*(a + a
*Sec[c + d*x])^(5/2)) - 83/(256*a*d*(a + a*Sec[c + d*x])^(3/2)) - 761/(512*
a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a^6\,\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{11/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{a^3\,\text{Subst}\left(\int \frac{4a^2+\frac{13a^2x}{2}}{x(-a+ax)^2(a+ax)^{11/2}}\,dx, x, \text{se}\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+d}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\text{s}}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\text{s}}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\text{s}}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\text{s}}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\text{s}}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\text{s}}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16d(1-\text{s}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{263\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,a^{5/2}d} + \frac{199a^2}{288d(a+a\sec(c+dx))^{9/}}$$

Mathematica [C] time = 0.32, size = 99, normalized size = 0.38

$$\frac{\cot^4(c+dx)\left(263(\sec(c+dx)-1)^2\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};\frac{1}{2}(\sec(c+dx)+1)\right)-64(\sec(c+dx)-1)^2\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};\sec(\right.\right.}{288d(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cot[c + d*x]^4*(-450 + 263*Hypergeometric2F1[-9/2, 1, -7/2, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x])^2 - 64*Hypergeometric2F1[-9/2, 1, -7/2, 1 + Sec
[c + d*x]]*(-1 + Sec[c + d*x])^2 + 378*Sec[c + d*x]))/(288*d*(a*(1 + Sec[c
+ d*x]))^(5/2))
```

fricas [B] time = 0.69, size = 905, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/645120*(82845*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^
5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) +
 1)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 322560*(cos(d*x
 + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x
+ c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)
^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4*(382201*cos(d*x + c)^7 + 591520*c
os(d*x + c)^6 - 403607*cos(d*x + c)^5 - 1112040*cos(d*x + c)^4 - 189189*cos
(d*x + c)^3 + 531720*cos(d*x + c)^2 + 239715*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a
^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/322560*(82845*sqrt(2)*
(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*
cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x
 + c) + a)) - 322560*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 -
5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)/(2*a*cos(d*x + c) + a)) - 2*(382201*cos(d*x + c)^7 + 591520*cos(d*x +
c)^6 - 403607*cos(d*x + c)^5 - 1112040*cos(d*x + c)^4 - 189189*cos(d*x + c)
^3 + 531720*cos(d*x + c)^2 + 239715*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c)))/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(
d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [A] time = 2.67, size = 392, normalized size = 1.50

$$-\frac{\frac{82845\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{645120\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{315\left(33\sqrt{2}\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}-31\sqrt{2}\sqrt{-a\tan\cdots}\right)}{a^{4}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)\tan\left(\frac{1}{2}dx+\cdots\right)}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/322560*(82845*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a
))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 645120*arctan(1/2*sqrt(
2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)) - 315*(33*sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2
) - 31*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^4*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)*tan(1/2*d*x + 1/2*c)^4) - 8*sqrt(2)*(35*(a*tan(1/2*d*x + 1/
2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^56 - 225*(a*tan(1/2*d*x
 + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^57 + 1008*(a*tan(1
/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^58 + 4410*(-
a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^59 + 31185*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)*a^60)/(a^63*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.49, size = 986, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/322560/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x
+c))^5*(322560*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^7*2^(1/2)+82845*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d
*x+c)^7+967680*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^6*2^(1/2)+248535*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(
d*x+c)^6+322560*cos(d*x+c)^5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+82845*cos(d*x+c)^5*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2))-1612800*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*2^(1/2)*
arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+764402*cos(d*x+c)^
7-414225*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*arctan(1/(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2))-1612800*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/
2))+1183040*cos(d*x+c)^6-414225*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+322560*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2))*2^(1/2)*cos(d*x+c)^2-807214*cos(d*x+c)^5+82845*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2+9
67680*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-2224080*cos(d*x+c)^4+248535*cos
(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2))+322560*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-378378*cos(d*x+c)^3+8284
5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2))+1063440*cos(d*x+c)^2+479430*cos(d*x+c))/sin(d*x+c)^14/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.201 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^3(c+dx)}{3ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+2*tan(d*x+c)
/a^2/d/(a+a*sec(d*x+c))^(1/2)-2/3*tan(d*x+c)^3/a/d/(a+a*sec(d*x+c))^(3/2)+2
/5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3887, 302, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^3(c+dx)}{3ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(2*Tan[c + d*x])/(a^2*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Tan[c + d*x]^3)/(3*a
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x
])^(5/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}-\frac{1}{a^3(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{2}{\ldots}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3ad(a+a\sec(c+dx))^{3/2}} + \ldots$$

Mathematica [B] time = 6.08, size = 447, normalized size = 3.52

$$\frac{\sqrt{2}\sqrt{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\left(\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}-1\right)^3\tan^7(c+dx)\cot^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{\sec(c+dx)+1}\right)^{9/2}\left(\frac{2\tan^2\left(\frac{1}{2}(c+\ldots\right.}{\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\left(\frac{2\tan\ldots}{\tan^2\ldots}\right.}\right.}{d\left(1-\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+\ldots}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*Cot[(c + d*x)/2]^8*((1 + Sec[c + d*x])^(-1))^(9/2)*Sqrt[1 + Tan[(c
 + d*x)/2]^2]*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))^3*((Sq
rt[2]*ArcSin[(Sqrt[2]*Tan[(c + d*x)/2])/Sqrt[1 + Tan[(c + d*x)/2]^2]]*Tan[(
c + d*x)/2])/(Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 - (2*Tan[(c + d*x)/2]^2)/
(1 + Tan[(c + d*x)/2]^2)]) + (8*Tan[(c + d*x)/2]^6)/(5*(1 + Tan[(c + d*x)/2
]^2)^3*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))^3) + (4*Tan[(
c + d*x)/2]^4)/(3*(1 + Tan[(c + d*x)/2]^2)^2*(-1 + (2*Tan[(c + d*x)/2]^2)/(
1 + Tan[(c + d*x)/2]^2))^2) + (2*Tan[(c + d*x)/2]^2)/((1 + Tan[(c + d*x)/2]
^2)*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))))*Tan[c + d*x]^7
)/(d*(a*(1 + Sec[c + d*x]))^(5/2)*(1 - (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
 d*x)/2]^2))^(5/2))
```

fricas [A] time = 0.46, size = 323, normalized size = 2.54

$$\left[-\frac{15\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2(23\ldots}{15\left(a^3d\cos(dx+c)^3+a^3d\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(23*cos(d*x + c)^2 - 1
```

```
1*cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2), 2/15*(15*(cos(d*x + c)^3 + co
s(d*x + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(sqrt(a)*sin(d*x + c))) + (23*cos(d*x + c)^2 - 11*cos(d*x + c) + 3)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + a^3*d*cos(d*x + c)^2)]
```

giac [B] time = 6.43, size = 292, normalized size = 2.30

$$-\frac{15\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a(2\right.\right.}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/15*(15*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^3*sgn(tan(1/2*d*x + 1/2*c
)^2 - 1)) - log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
)) + 2*((37*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
- 40*sqrt(2)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 + 15*s
qrt(2)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*
x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.22, size = 302, normalized size = 2.38

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(15\sqrt{2}\,\sin(dx+c)\left(\cos^2(dx+c)\right)\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+30\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/60/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*2^(1/2)*sin(d*x+c)*cos(d*x+c
)^2*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+30*2^(1/2)*sin(d*x+c)*cos(d*x
+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+15*2^(1/2)*arctanh(1/2*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)-184*cos(d*x+c)^3+272*cos(d*x+c)^2-112*co
s(d*x+c)+24)/sin(d*x+c)/cos(d*x+c)^2/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(c+d\,x\right)^6}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(5/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{6}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.202 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-4*arctan(1/2*
a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+2*tan(
d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3887, 479, 522, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
4*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(a^(5/2)*d) + (2*Tan[c + d*x])/(a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{2+3ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}$$

$$= \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} + \frac{8\,\text{Subst}\left(\int \frac{1}{2+a\ldots}\right.}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{4\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 23.74, size = 5491, normalized size = 48.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.52, size = 414, normalized size = 3.66

$$\left[\frac{2\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right) - \sqrt{-a}\,(\cos(dx \ldots}{a^3d\cos(dx+c)\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [(2*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x +
c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(-a
)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos
(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^3*d*cos(d*x + c) + a^3*d), -2*(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*sq
rt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a) - sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c) + a^3*d)]
```

giac [A] time = 3.05, size = 73, normalized size = 0.65

$$\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a^2d\,\text{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/((a*tan(
1/2*d*x + 1/2*c)^2 - a)*a^2*d*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
```

maple [B] time = 1.21, size = 326, normalized size = 2.88

$$-\frac{\left(\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos(dx+c)+\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{2\cos}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/d*(2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)+2^(1/2)*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))*cos(d*x+c)+4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-2*sin(d*x
+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(1+cos(d*x+c))/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^4/(a*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.203 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\,a^{5/2}d}+\frac{\sin(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2a^2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+3/2*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+1/2
*sec(1/2*d*x+1/2*c)^2*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3887, 471, 522, 203}

$$\frac{\sin(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2a^2d\sqrt{a\sec(c+dx)+a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\,a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt
[2]*a^(5/2)*d) + (Sec[(c + d*x)/2]^2*Sin[c + d*x])/(2*a^2*d*Sqrt[a + a*Sec[
c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\mathrm{Subst}\left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}$$

$$= \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} - 3\,\mathrm{Subs}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c}{2a^2d\sqrt{a+a\sec(c+}}$$

Mathematica [C] time = 23.81, size = 5521, normalized size = 43.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
[Out] Result too large to show
```

fricas [A] time = 0.72, size = 492, normalized size = 3.87

$$\left[-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] [-1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*co
s(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
2 + 2*a^3*d*cos(d*x + c) + a^3*d), -1/2*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 4*(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [A] time = 10.29, size = 54, normalized size = 0.43

$$-\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{2a^3d\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^3*
d*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
```

maple [B] time = 0.92, size = 370, normalized size = 2.91

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(2\cos\left(dx+c\right)\sin\left(dx+c\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+3\cos\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*cos(d*x+c)*sin(d*x+c)*2^(1/2)*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+2*2^(1/2)*sin(d*x+c)*arctanh(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
-2*cos(d*x+c)^2+2*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(dx+c\right)^{2}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(c+d\,x\right)^2}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.204 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=265

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{319\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{128\sqrt{2}\,a^{5/2}d}+\frac{63\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{128a^3d}-\frac{\cos^3(c+dx)\cot(c}{}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+319/256*arct
an(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d
+63/128*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d-191/384*cos(d*x+c)*cot(d*x+
c)*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a^3/d-19/192*cos(d*x+c)^2*co
t(d*x+c)*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(1/2)/a^3/d-1/48*cos(d*x+c)^
3*cot(d*x+c)*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.23, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{63\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{128a^3d}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{319\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{128\sqrt{2}\,a^{5/2}d}-\frac{\cos^3(c+dx)\cot(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(319*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(12
8*Sqrt[2]*a^(5/2)*d) + (63*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(128*a^3*
d) - (191*Cos[c + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d
*x]])/(384*a^3*d) - (19*Cos[c + d*x]^2*Cot[c + d*x]*Sec[(c + d*x)/2]^4*Sqrt
[a + a*Sec[c + d*x]])/(192*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]*Sec[(c + d
*x)/2]^6*Sqrt[a + a*Sec[c + d*x]])/(48*a^3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3 d}$$

$$= -\frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3 d} - \frac{\text{Subst}\left(\int \frac{5a-7}{x^2(1+ax^2}\right.}{}$$

$$= -\frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{192a^3 d} - \frac{\cos^3(c+dx)\,c}{}$$

$$= -\frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{384a^3 d} - \frac{19\cos^2(c+dx}{}$$

$$= \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3 d} - \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{}}{384a^3 d}$$

$$= \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3 d} - \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{}}{384a^3 d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{319\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}\,a^{5/2}d} + \frac{63\cot(c+dx)\sqrt{a+a\,\text{se}}}{128a^3 d}$$

Mathematica [C] time = 23.93, size = 5604, normalized size = 21.15

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]

[Out] Result too large to show

fricas [A] time = 0.68, size = 691, normalized size = 2.61

$$-\frac{957\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+\cdots}{\cos(dx+c)^2+2\cos(dx\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

```
[Out] [-1/1536*(957*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 768*(cos(d*x + c)^3 + 3
*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4
*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x +
 c) + 4*(409*cos(d*x + c)^4 + 349*cos(d*x + c)^3 - 185*cos(d*x + c)^2 - 189
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)),
 -1/768*(957*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 768*(cos(d*x + c)^3 + 3*cos(d*x
+ c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*
x + c) - a))*sin(d*x + c) + 2*(409*cos(d*x + c)^4 + 349*cos(d*x + c)^3 - 18
5*cos(d*x + c)^2 - 189*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
))/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d)*sin(d*x + c))]
```

giac [A] time = 5.58, size = 205, normalized size = 0.77

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{31\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{291\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx\cdots\right.\right.}\right.}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

```
[Out] -1/768*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*sqrt(2)*tan(1/2*d*x + 1/2
*c)^2/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 31*sqrt(2)/(a^3*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 291*sqrt(2)/(a^3*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) - 96*sqrt(2)/(((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sqrt(-a)*a*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.40, size = 714, normalized size = 2.69

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^3\left(768\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+\cdots}{1+\cos(dx+\cdots}}}{2\cos\cdots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/768/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^3*(768*cos(d*x
+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+2304*2^(
1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)+957*
cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+2304*cos(
d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+2871*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+768*2^(1/2)
*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2871*cos(d*x+c)*sin(d
*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-818*cos(d*x+c)^4+957*sin(d*
x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-698*cos(d*x+c)^3+370*cos(d*x
+c)^2+378*cos(d*x+c))/sin(d*x+c)^7/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.205 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=355

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{9683\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4096\sqrt{2}\,a^{5/2}d}+\frac{5587\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{6144a^4d}-\frac{\cos^4(c+dx)}{}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+5587/6144*cot
(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^4/d-1527/2048*cos(d*x+c)*cot(d*x+c)^3*se
c(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^4/d-145/1024*cos(d*x+c)^2*cot(d
*x+c)^3*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^4/d-9/256*cos(d*x+c)^
3*cot(d*x+c)^3*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(3/2)/a^4/d-1/128*cos(
d*x+c)^4*cot(d*x+c)^3*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(3/2)/a^4/d-968
3/8192*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2
)/a^(5/2)/d-1491/4096*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.33, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{5587\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{6144a^4d}-\frac{1491\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4096a^3d}+\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{9683\,t}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
9683*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(40
96*Sqrt[2]*a^(5/2)*d) - (1491*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4096*
a^3*d) + (5587*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(6144*a^4*d) - (1
527*Cos[c + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/
2))/(2048*a^4*d) - (145*Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a
 + a*Sec[c + d*x])^(3/2))/(1024*a^4*d) - (9*Cos[c + d*x]^3*Cot[c + d*x]^3*S
ec[(c + d*x)/2]^6*(a + a*Sec[c + d*x])^(3/2))/(256*a^4*d) - (Cos[c + d*x]^4
*Cot[c + d*x]^3*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(3/2))/(128*a^4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^5}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^4d}$$

$$= -\frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d} - \frac{\mathrm{Subst}\left(\int -\right.}{}$$

$$= -\frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4d} - \frac{\cos^4(c+}{}$$

$$= -\frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4d} - \frac{9\cos^3(}{}$$

$$= -\frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{2048a^4d} - \frac{145\cos}{}$$

$$= \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4d} - \frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}\right.}{2048a^4}$$

$$= -\frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3d} + \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4d}$$

$$= -\frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3d} + \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4d}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{9683\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4096\sqrt{2}\,a^{5/2}d} - \frac{1491\cot(c+dx)\sqrt{a+}}{4096a^3}$$

Mathematica [C] time = 24.16, size = 5646, normalized size = 15.90

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.71, size = 868, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/49152*(29049*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c
)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a
*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1))*sin(d*x + c) + 24576*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c
)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c
)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*si
n(d*x + c) - 4*(14629*cos(d*x + c)^6 + 14233*cos(d*x + c)^5 - 14058*cos(d*x
```

```
 + c)^4 - 17426*cos(d*x + c)^3 + 2245*cos(d*x + c)^2 + 4473*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^5 + 3*a^3*d*co
s(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*co
s(d*x + c) - a^3*d)*sin(d*x + c)), 1/24576*(29049*sqrt(2)*(cos(d*x + c)^5 +
 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) -
1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 24576*(cos(d*x + c)^5 + 3*cos(d*
x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a
)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(1462
9*cos(d*x + c)^6 + 14233*cos(d*x + c)^5 - 14058*cos(d*x + c)^4 - 17426*cos(
d*x + c)^3 + 2245*cos(d*x + c)^2 + 4473*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 2*a^3
*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*cos(d*x + c) - a^3*d)*
sin(d*x + c))]
```

giac [A] time = 2.69, size = 331, normalized size = 0.93

$$-\frac{3\left(2\left(4\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{19\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{369\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*(3*(2*(4*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)) - 19*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*
d*x + 1/2*c)^2 + 369*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2
*d*x + 1/2*c)^2 - 2989*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) + 512*sqrt(2)*(12*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 21*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + 11
*a^2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a)^3*sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.56, size = 1066, normalized size = 3.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/24576/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c)
)^4*(24576*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*2^(
1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
*2^(1/2))+73728*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c
)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))+29049*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(
d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))+49152*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))+87147*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*si
n(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))-49152*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d
*x+c)^2*sin(d*x+c)+58098*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+
```

```
c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)
-1)/sin(d*x+c))-29258*cos(d*x+c)^6-73728*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-58098*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-28466*cos(d*x+c)^5-24576*2^(1/2)*sin(d*x
+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-87147*cos(d*x+c)*sin(d*x+c)*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+28116*cos(d*x+c)^4-29049*sin(d*x+c
)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+34852*cos(d*x+c)^3-4490*cos(d*x
+c)^2-8946*cos(d*x+c))/sin(d*x+c)^11/a^3
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(cot(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.206 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=439

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{74461\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{32768\sqrt{2}\,a^{5/2}d}+\frac{58077\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{40960a^5d}-\frac{\cos^5(c+dx}{}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-41693/49152*
cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^4/d+58077/40960*cot(d*x+c)^5*(a+a*sec
(d*x+c))^(5/2)/a^5/d-9467/8192*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2
*(a+a*sec(d*x+c))^(5/2)/a^5/d-2473/12288*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*
d*x+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^5/d-155/3072*cos(d*x+c)^3*cot(d*x+c)^
5*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^5/d-7/512*cos(d*x+c)^4*cot(
d*x+c)^5*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(5/2)/a^5/d-1/320*cos(d*x+c)
^5*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^10*(a+a*sec(d*x+c))^(5/2)/a^5/d+74461/65
536*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a
^(5/2)/d+8925/32768*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.42, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3887, 472, 579, 583, 522, 203}

$$\frac{58077\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{40960a^5d}-\frac{41693\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{49152a^4d}+\frac{8925\cot(c+dx)\sqrt{a\sec(c+a}}{32768a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(74461*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
32768*Sqrt[2]*a^(5/2)*d) + (8925*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(32
768*a^3*d) - (41693*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(49152*a^4*d
) + (58077*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(40960*a^5*d) - (9467
*Cos[c + d*x]*Cot[c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))
/(8192*a^5*d) - (2473*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a +
 a*Sec[c + d*x])^(5/2))/(12288*a^5*d) - (155*Cos[c + d*x]^3*Cot[c + d*x]^5*
Sec[(c + d*x)/2]^6*(a + a*Sec[c + d*x])^(5/2))/(3072*a^5*d) - (7*Cos[c + d*
x]^4*Cot[c + d*x]^5*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(5/2))/(512*a^5
*d) - (Cos[c + d*x]^5*Cot[c + d*x]^5*Sec[(c + d*x)/2]^10*(a + a*Sec[c + d*x
])^(5/2))/(320*a^5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^6}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^5 d}$$

$$= -\frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5 d} - \frac{\text{Subst}\left(\int \frac{5}{x^6(1\ldots}\right.}{\ldots}$$

$$= -\frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5 d} - \frac{\cos^5(c+dx)\ldots}{\ldots}$$

$$= -\frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5 d} - \frac{7\cos^4(c+\ldots}{\ldots}$$

$$= -\frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5 d} - \frac{155\cos^3(\ldots}{\ldots}$$

$$= -\frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5 d} - \frac{2473\cos^2\ldots}{\ldots}$$

$$= \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5 d} - \frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c\ldots\right.}{8192a^5 d}$$

$$= -\frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4 d} + \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{\ldots}}{40960a^5 d}$$

$$= \frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3 d} - \frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4 d} + \ldots$$

$$= \frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3 d} - \frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4 d} + \ldots$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{74461\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32768\sqrt{2}\,a^{5/2}d} + \frac{8925\cot(c+dx)\sqrt{a\ldots}}{32768a^{3}\ldots}$$

Mathematica [C] time = 24.27, size = 5688, normalized size = 12.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.81, size = 1023, normalized size = 2.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/1966080*(1116915*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x +
 c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x +
```

```
c) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 983040*(cos(d*x + c
)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)
^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3
 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d
*x + c) + 4*(639063*cos(d*x + c)^8 + 681555*cos(d*x + c)^7 - 986085*cos(d*x
 + c)^6 - 1360025*cos(d*x + c)^5 + 405445*cos(d*x + c)^4 + 836921*cos(d*x +
 c)^3 + 15305*cos(d*x + c)^2 - 133875*cos(d*x + c))*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*c
os(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos
(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)), -1/983040*(11169
15*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x
+ c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c)))*sin(d*x + c) + 983040*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + c
os(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*co
s(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*
sin(d*x + c) + 2*(639063*cos(d*x + c)^8 + 681555*cos(d*x + c)^7 - 986085*co
s(d*x + c)^6 - 1360025*cos(d*x + c)^5 + 405445*cos(d*x + c)^4 + 836921*cos(
d*x + c)^3 + 15305*cos(d*x + c)^2 - 133875*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^
3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*
d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))]
```

giac [A] time = 3.67, size = 454, normalized size = 1.03

$$-\frac{\left(2\left(4\left(6\left(\frac{8\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{91\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{3043\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/983040*((2*(4*(6*(8*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)) - 91*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/
2*d*x + 1/2*c)^2 + 3043*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(
1/2*d*x + 1/2*c)^2 - 47185*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*t
an(1/2*d*x + 1/2*c)^2 + 349965*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) - 1024*sqrt(2)*
(345*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
8 - 1230*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^6*a + 1760*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^4*a^2 - 1150*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2*a^3 + 299*a^4)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c
)^2 - 1)))/d
```

maple [B] time = 1.62, size = 1412, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/983040/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x
+c))^5*(-983040*cos(d*x+c)^7*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))-1116915*cos(d*x+c)^7*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))-2949120*cos(d*x+c)^6*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))-3350745*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))-983040*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
5*sin(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)/cos(d*x+c)*2^(1/2))+1278126*cos(d*x+c)^8-1116915*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+4915200*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+1363110*cos(d*x+c)^7+5
584575*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*ln(-(-(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+49
15200*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1
/2))-1972170*cos(d*x+c)^6+5584575*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))-983040*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^
(1/2))*cos(d*x+c)^2*sin(d*x+c)-2720050*cos(d*x+c)^5-1116915*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-2949120*cos(d*x+c)*sin(d
*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+810890*cos(d*x+c)^4
-3350745*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-98
3040*2^(1/2)*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1673842*c
os(d*x+c)^3-1116915*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+30
610*cos(d*x+c)^2-267750*cos(d*x+c))/sin(d*x+c)^15/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot\left(c+d\,x\right)^6}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{6}{\left(c + d x \right)}}{\left(a \left(\sec{\left(c + d x \right)} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**6/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.207 $\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{9/2}f}+\frac{91\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{32\sqrt{2}\,a^{9/2}f}+\frac{27\tan(e+fx)}{32a^3f(a\sec(e+fx)+a)^{3/2}}+\frac{11\tan(e+fx)}{24a^2f(a\sec(e+fx)+a)^{5/2}}$$

```
[Out] -2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(9/2)/f+91/64*arctan
(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(9/2)/f*2^(1/2)+1
/3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(7/2)+11/24*tan(f*x+e)/a^2/f/(a+a*sec(f*
x+e))^(5/2)+27/32*tan(f*x+e)/a^3/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 227, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3887, 471, 527, 522, 203}

$$\frac{27\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{64a^4f\sqrt{a\sec(e+fx)+a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{9/2}f}+\frac{91\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{32\sqrt{2}\,a^{9/2}f}+\frac{\sin(e+fx)\cos^2(e+fx)\,\mathrm{s}}{24a^4f\sqrt{a\sec(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sec[e + f*x])^(9/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(9/2)*f) +
(91*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(32*
Sqrt[2]*a^(9/2)*f) + (27*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(64*a^4*f*Sqrt[a
+ a*Sec[e + f*x]]) + (11*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f*x])/(96*
a^4*f*Sqrt[a + a*Sec[e + f*x]]) + (Cos[e + f*x]^2*Sec[(e + f*x)/2]^6*Sin[e
+ f*x])/(24*a^4*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(1+ax^2\right)\left(2+ax^2\right)^4}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3 f}$$

$$= \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{24a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1-5ax^2}{\left(1+ax^2\right)\left(2+ax^2\right)^3}\,dx, x, -\frac{\text{t}}{\sqrt{a+}}\right.}{3a^4 f}$$

$$= \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sin}{24a^4 f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{64a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4 f\sqrt{a+a\sec(e+fx)}} +$$

$$= \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{64a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4 f\sqrt{a+a\sec(e+fx)}} +$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{9/2} f} + \frac{91\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}\,a^{9/2} f} + \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\text{si}}{64a^4 f\sqrt{a+a\sec(e}}$$

Mathematica [C] time = 24.14, size = 5584, normalized size = 31.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sec[e + f*x])^(9/2),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.81, size = 674, normalized size = 3.81

$$\left[-\frac{273\sqrt{2}\left(\cos\left(fx+e\right)^4 + 4\cos\left(fx+e\right)^3 + 6\cos\left(fx+e\right)^2 + 4\cos\left(fx+e\right) + 1\right)\sqrt{-a}\,\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+}{\cos(fx}}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(273*sqrt(2)*(cos(f*x + e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2
+ 4*cos(f*x + e) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*co
s(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 384*(cos(f*x + e)^
4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2 + 4*cos(f*x + e) + 1)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(157*
cos(f*x + e)^3 + 206*cos(f*x + e)^2 + 81*cos(f*x + e))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 4*a^5*f*cos(f*x +
 e)^3 + 6*a^5*f*cos(f*x + e)^2 + 4*a^5*f*cos(f*x + e) + a^5*f), -1/192*(273
*sqrt(2)*(cos(f*x + e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2 + 4*cos(f*x
+ e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e))) - 384*(cos(f*x + e)^4 + 4*cos(f*x + e)^3
 + 6*cos(f*x + e)^2 + 4*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(157*cos(f*x
 + e)^3 + 206*cos(f*x + e)^2 + 81*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 4*a^5*f*cos(f*x + e)^3 +
 6*a^5*f*cos(f*x + e)^2 + 4*a^5*f*cos(f*x + e) + a^5*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*tan((f*x+exp(1))/
2)*sqrt(-a*tan((f*x+exp(1))/2)^2+a)*(-37/256*sqrt(2)/a^5/sign(tan((f*x+exp(
1))/2)^2-1)+tan((f*x+exp(1))/2)^2*(19/384*sqrt(2)/a^5/sign(tan((f*x+exp(1))
/2)^2-1)-1/96*sqrt(2)*tan((f*x+exp(1))/2)^2/a^5/sign(tan((f*x+exp(1))/2)^2-
1)))
```

maple [B] time = 1.17, size = 724, normalized size = 4.09

$$-\frac{\left(192\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\left(\cos^{4}\left(fx+e\right)\right)\sin\left(fx+e\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)\sqrt{2}}{2\cos\left(fx+e\right)}\right)+384\left(\cos^{3}\left(fx+e\right)\right)\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x)
```

```
[Out] -1/192/f*(192*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*
2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x
+e)*2^(1/2))+384*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(
f*x+e)*2^(1/2))+273*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f
*x+e)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/s
in(f*x+e))+546*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*
x+e))-384*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^
```

```
(1/2))-314*cos(f*x+e)^5-192*2^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)-546*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f
*x+e))+216*cos(f*x+e)^4-273*sin(f*x+e)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)+348*cos(f*x+e)^3-88*cos(f*x+e)^2-162*cos(f*x+e))*(a*(1+cos(f*x+e))/co
s(f*x+e))^(1/2)/sin(f*x+e)^3/(1+cos(f*x+e))^2/a^5
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + a/cos(e + f*x))^(9/2),x)
[Out] int(tan(e + f*x)^2/(a + a/cos(e + f*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sec(f*x+e))**(9/2),x)
[Out] Timed out
```

3.208 $\int (a + a\sec(c + dx))^n (e\tan(c + dx))^m \, dx$

Optimal. Leaf size=125

$$\frac{2^{m+n+1}(a\sec(c+dx)+a)^n(e\tan(c+dx))^{m+1}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+n+1}F_1\left(\frac{m+1}{2};m+n,1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

```
[Out] 2^(1+m+n)*AppellF1(1/2+1/2*m,m+n,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d*x
+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1+m+n)*(a+a*sec
(d*x+c))^n*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3889}

$$\frac{2^{m+n+1}(a\sec(c+dx)+a)^n(e\tan(c+dx))^{m+1}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+n+1}F_1\left(\frac{m+1}{2};m+n,1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

```
[Out] (2^(1 + m + n)*AppellF1[(1 + m)/2, m + n, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(1 + m + n)*(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^(1
 + m))/(d*e*(1 + m))
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \frac{2^{1+m+n}F_1\left(\frac{1+m}{2};m+n,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right.}{de(1+m)}$$

Mathematica [F] time = 1.29, size = 0, normalized size = 0.00

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n(e\tan(dx+c))^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n (e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

maple [F] time = 2.55, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n (e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n (e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\tan(c+d\,x))^m \left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^n,x)
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n (e\tan(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*(e*tan(d*x+c))**m,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*(e*tan(c + d*x))**m, x)
```

3.209 $\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=243

$$\frac{a^3(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{a^3\sec^3(c+dx)\cos^2(c+dx)^{\frac{m+4}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m-}{2}\right.}{de(m+1)}$$

```
[Out] 3*a^3*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2*
m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+3*a^3*(cos(d*x+c)^2)^(1+1/
2*m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e
*tan(d*x+c))^(1+m)/d/e/(1+m)+a^3*(cos(d*x+c)^2)^(2+1/2*m)*hypergeom([2+1/2*
m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)^3*(e*tan(d*x+c))^(1+m)/d
/e/(1+m)
```

Rubi [A] time = 0.23, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3886, 3476, 364, 2617, 2607, 32}

$$\frac{a^3(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{a^3\sec^3(c+dx)\cos^2(c+dx)^{\frac{m+4}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m-}{2}\right.}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m,x]
```

```
[Out] (3*a^3*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 +
m)) + (3*a^3*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2 +
 m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/(d
*e*(1 + m)) + (a^3*(Cos[c + d*x]^2)^((4 + m)/2)*Hypergeometric2F1[(1 + m)/2
, (4 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(e*Tan[c + d*x])^(1
+ m))/(d*e*(1 + m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
```

```
!IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx = \int \left(a^3(e\tan(c+dx))^m+3a^3\sec(c+dx)(e\tan(c+dx))^m+3a^3\sec\right.$$

$$= a^3\int (e\tan(c+dx))^m\,dx + a^3\int \sec^3(c+dx)(e\tan(c+dx))^m\,dx +$$

$$= \frac{3a^3\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{2+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+dx)}{de(1+m)}$$

$$= \frac{3a^3(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{a^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)(e\tan}{de(1+m)}$$

Mathematica [C] time = 6.23, size = 391, normalized size = 1.61

$$\frac{a^3e\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(-\tan^2(c+dx)\right)^{\frac{1-m}{2}}(e\tan(c+dx))^{m-1}\left({}_2F_1\left(\frac{3}{2},\frac{1-m}{2};\frac{5}{2};\sec^2(c+dx)\right)+\right.}{24d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a^3*e*(9*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (1 - m)/2, 3/2, Sec[c + d*x
]^2] + Hypergeometric2F1[3/2, (1 - m)/2, 5/2, Sec[c + d*x]^2])*Sec[(c + d*x
)/2]^6*(1 + Sec[c + d*x])^3*(e*Tan[c + d*x])^(-1 + m)*(-Tan[c + d*x]^2)^((1
 - m)/2))/(24*d) + (2^(-4 - m)*a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*
Sec[c + d*x]*(e*Tan[c + d*x])^m*(I*2^m*(((-I)*(-1 + E^((2*I)*(c + d*x))))/(
1 + E^((2*I)*(c + d*x))))^m*(1 + m)*Cos[c + d*x]*Hypergeometric2F1[1, m, 1
+ m, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))] - I*(((-I)*(-
1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c + d
*x)))^m*(1 + m)*Cos[c + d*x]*Hypergeometric2F1[m, m, 1 + m, (1 - E^((2*I)*(
c + d*x)))/2] + 3*2^(1 + m)*m*Sin[c + d*x]*Tan[c + d*x]^m))/(d*m*(1 + m)*Ta
n[c + d*x]^m)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3\right)(e\tan(dx+c))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*(e*tan(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3 (e\tan(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*tan(d*x + c))^m, x)
```

maple [F] time = 1.90, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^3 (e\tan(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3 (e\tan(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*tan(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\tan(c+d\,x))^m \left(a+\frac{a}{\cos(c+d\,x)}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int (e\tan(c+dx))^m \, dx + \int 3(e\tan(c+dx))^m \sec(c+dx)\, dx + \int 3(e\tan(c+dx))^m \sec^2(c+dx)\, dx + \int (e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(e*tan(d*x+c))**m,x)
```

```
[Out] a**3*(Integral((e*tan(c + d*x))**m, x) + Integral(3*(e*tan(c + d*x))**m*sec
(c + d*x), x) + Integral(3*(e*tan(c + d*x))**m*sec(c + d*x)**2, x) + Integr
al((e*tan(c + d*x))**m*sec(c + d*x)**3, x))
```

3.210 $\int (a+a\sec(c+dx))^2(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=161

$$\frac{a^2(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{2a^2\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1}{de(m+1)}$$

```
[Out] a^2*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m]
,-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+2*a^2*(cos(d*x+c)^2)^(1+1/2*
m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*t
an(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A] time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3886, 3476, 364, 2617, 2607, 32}

$$\frac{a^2(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{2a^2\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a^2*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^2*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)
) + (2*a^2*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2 + m
)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/(d*e
*(1 + m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a + a\sec(c + dx))^2(e\tan(c + dx))^m\,dx = \int \left(a^2(e\tan(c + dx))^m + 2a^2\sec(c + dx)(e\tan(c + dx))^m + a^2\sec^2(c\right.$$

$$= a^2\int (e\tan(c + dx))^m\,dx + a^2\int \sec^2(c + dx)(e\tan(c + dx))^m\,dx + \left(2\right.$$

$$= \frac{2a^2\cos^2(c + dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2}, \frac{2+m}{2}; \frac{3+m}{2}; \sin^2(c + dx)\right)\sec(c + dx)(e\,t}{de(1 + m)}$$

$$= \frac{a^2(e\tan(c + dx))^{1+m}}{de(1 + m)} + \frac{a^2\,{}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c + dx)\right)(e\tan(c +}{de(1 + m)}$$

Mathematica [C] time = 3.20, size = 358, normalized size = 2.22

$$\frac{a^2(\cos(c + dx) + 1)^2\csc(c + dx)\sec^4\left(\frac{1}{2}(c + dx)\right)\left(-\tan^2(c + dx)\right)^{\frac{1-m}{2}}(e\tan(c + dx))^m\,{}_2F_1\left(\frac{1}{2}, \frac{1-m}{2}; \frac{3}{2}; \sec^2(c + dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Csc[c + d*x]*Hypergeometric2F1[1/2, (1 - m)/2, 3/
2, Sec[c + d*x]^2]*Sec[(c + d*x)/2]^4*(e*Tan[c + d*x])^m*(-Tan[c + d*x]^2)^
((1 - m)/2))/(2*d) + (2^(-3 - m)*a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^
4*Sec[c + d*x]*(e*Tan[c + d*x])^m*(I*2^m*(((-I)*(-1 + E^((2*I)*(c + d*x))))
/(1 + E^((2*I)*(c + d*x))))^m*(1 + m)*Cos[c + d*x]*Hypergeometric2F1[1, m,
1 + m, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))] - I*(((-I)*
(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c +
 d*x)))^m*(1 + m)*Cos[c + d*x]*Hypergeometric2F1[m, m, 1 + m, (1 - E^((2*I)
*(c + d*x)))/2] + 2^(1 + m)*m*Sin[c + d*x]*Tan[c + d*x]^m))/(d*m*(1 + m)*Ta
n[c + d*x]^m)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\sec(dx + c)^2 + 2\,a^2\sec(dx + c) + a^2\right)(e\tan(dx + c))^m\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*(e*tan(d*x + c))^m
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx + c) + a)^2(e\tan(dx + c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^m, x)
```

maple [F] time = 2.37, size = 0, normalized size = 0.00

$$\int \left(a + a\sec\left(dx + c\right)\right)^{2} \left(e\tan\left(dx + c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx + c\right) + a\right)^{2} \left(e\tan\left(dx + c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\tan\left(c+d\,x\right)\right)}^m\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^2,x)
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2} \left(\int \left(e \tan{\left(c + dx \right)}\right)^{m}\, dx + \int 2 \left(e \tan{\left(c + dx \right)}\right)^{m} \sec{\left(c + dx \right)}\, dx + \int \left(e \tan{\left(c + dx \right)}\right)^{m} \sec^{2}{\left(c + dx \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**m,x)
[Out] a**2*(Integral((e*tan(c + d*x))**m, x) + Integral(2*(e*tan(c + d*x))**m*sec
(c + d*x), x) + Integral((e*tan(c + d*x))**m*sec(c + d*x)**2, x))
```

3.211 $\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=129

$$\frac{a(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{a\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\right.}{de(m+1)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/
d/e/(1+m)+a*(cos(d*x+c)^2)^(1+1/2*m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/
2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A] time = 0.08, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3884, 3476, 364, 2617}

$$\frac{a(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{a\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\right.}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (a*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometri
c2F1[(1 + m)/2, (2 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c
 + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx = a\int (e\tan(c+dx))^m\,dx + a\int \sec(c+dx)(e\tan(c+dx))^m\,dx$$
$$= \frac{a\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{2+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+dx)(e\tan}{de(1+m)}$$
$$= \frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{a\cos^2(c+}{}$$

Mathematica [A] time = 0.73, size = 105, normalized size = 0.81

$$\frac{a(e\tan(c+dx))^m\left(\frac{\tan(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{m+1}+\csc(c+dx)\left(-\tan^2(c+dx)\right)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3}{2};\sec^2(c+d\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a*(e*Tan[c + d*x])^m*((Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c +
 d*x]^2]*Tan[c + d*x])/(1 + m) + Csc[c + d*x]*Hypergeometric2F1[1/2, (1 - m
)/2, 3/2, Sec[c + d*x]^2]*(-Tan[c + d*x]^2)^((1 - m)/2)))/d
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)(e\tan(dx+c))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

maple [F] time = 2.45, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\tan(c+d\,x))^m\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (e\tan(c+dx))^m\,dx+\int (e\tan(c+dx))^m\sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**m,x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**m, x) + Integral((e*tan(c + d*x))**m*sec(c +
d*x), x))
```

3.212 $\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{e(e\tan(c+dx))^{m-1}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\tan^2(c+dx)\right)}{ad(1-m)}-\frac{e\sec(c+dx)\cos^2(c+dx)^{m/2}(e\tan(c+dx))^{m-1}\,{}_2F_1\left(\frac{m-1}{2}\right.}{ad(1-m)}$$

```
[Out] e*hypergeom([1, -1/2+1/2*m],[1/2+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-1+m
)/a/d/(1-m)-e*(cos(d*x+c)^2)^(1/2*m)*hypergeom([1/2*m, -1/2+1/2*m],[1/2+1/2
*m],sin(d*x+c)^2)*sec(d*x+c)*(e*tan(d*x+c))^(-1+m)/a/d/(1-m)
```

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3888, 3884, 3476, 364, 2617}

$$\frac{e(e\tan(c+dx))^{m-1}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\tan^2(c+dx)\right)}{ad(1-m)}-\frac{e\sec(c+dx)\cos^2(c+dx)^{m/2}(e\tan(c+dx))^{m-1}\,{}_2F_1\left(\frac{m-1}{2}\right.}{ad(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c +
d*x])^(-1 + m))/(a*d*(1 - m)) - (e*(Cos[c + d*x]^2)^(m/2)*Hypergeometric2F1
[(-1 + m)/2, m/2, (1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^
(-1 + m))/(a*d*(1 - m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
```

```
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{-2+m}\,dx}{a^2}$$
$$= -\frac{e^2\int(e\tan(c+dx))^{-2+m}\,dx}{a} + \frac{e^2\int\sec(c+dx)(e\tan(c+dx))^{-2+m}\,dx}{a}$$
$$= -\frac{e\cos^2(c+dx)^{m/2}\,{}_2F_1\left(\frac{1}{2}(-1+m),\frac{m}{2};\frac{1+m}{2};\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx))^{-1+m}}{ad(1-m)}$$
$$= \frac{e\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))^{-1+m}}{ad(1-m)} - \frac{e\cos^2(c+dx)^{m/2}\,{}_2F_1\left(\frac{1}{2}\right.}{}$$

Mathematica [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]),x]
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]), x]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

maple [F] time = 2.45, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x)
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(e\tan(c+d\,x)\right)^m}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^m)/(a*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^m}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x) + 1), x)/a
```

3.213 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=169

$$-\frac{e^3(e\tan(c+dx))^{m-3}\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\tan^2(c+dx)\right)}{a^2d(3-m)}+\frac{2e^3\sec(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-3}\,{}_2F_1\left(\frac{m}{}\right.}{a^2d(3-m)}$$

```
[Out] -e^3*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)-e^3*hypergeom([1, -3/2+1/2*m],[-1/2+
1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)+2*e^3*(cos(d*x+c)^2
)^(-1+1/2*m)*hypergeom([-1+1/2*m, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)*se
c(d*x+c)*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)
```

Rubi [A] time = 0.27, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3888, 3886, 3476, 364, 2617, 2607, 32}

$$-\frac{e^3(e\tan(c+dx))^{m-3}\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\tan^2(c+dx)\right)}{a^2d(3-m)}+\frac{2e^3\sec(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-3}\,{}_2F_1\left(\frac{m}{}\right.}{a^2d(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((e^3*(e*Tan[c + d*x])^(-3 + m))/(a^2*d*(3 - m))) - (e^3*Hypergeometric2F1
[1, (-3 + m)/2, (-1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-3 + m))/(a^
2*d*(3 - m)) + (2*e^3*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeometric2F1[(-3 +
 m)/2, (-2 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x]
)^(-3 + m))/(a^2*d*(3 - m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2(e\tan(c+dx))^{-4+m}\,dx}{a^4}$$

$$= \frac{e^4\int\left(a^2(e\tan(c+dx))^{-4+m}-2a^2\sec(c+dx)(e\tan(c+dx))^{-4+m}+a^2\sec^2(c+dx\right.}{a^4}$$

$$= \frac{e^4\int(e\tan(c+dx))^{-4+m}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)(e\tan(c+dx))^{-4+m}\,dx}{a^2}-\frac{\left(2e^4\right)\int s}{}$$

$$= \frac{2e^3\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,{}_2F_1\left(\frac{1}{2}(-3+m),\frac{1}{2}(-2+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)\sec(}{a^2d(3-m)}$$

$$= -\frac{e^3(e\tan(c+dx))^{-3+m}}{a^2d(3-m)}-\frac{e^3\,{}_2F_1\left(1,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\tan^2(c+dx)\right)(e\tan(}{a^2d(3-m)}$$

Mathematica [C] time = 6.92, size = 329, normalized size = 1.95

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(e\tan(c+dx))^m\left((m+1)\tan^2\left(\frac{1}{2}(c+dx)\right)\,{}_2F_1\left(m,\frac{m+3}{2};\frac{m+5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-3(m+3)\,{}_2F_1\right.}{2a^2d(m+1)(m+3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(-3*(3 + m)*Hypergeom
etric2F1[m, (1 + m)/2, (3 + m)/2, Tan[(c + d*x)/2]^2] + (1 + m)*Hypergeomet
ric2F1[m, (3 + m)/2, (5 + m)/2, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)*(e*
Tan[c + d*x])^m)/(2*a^2*d*(1 + m)*(3 + m)) + (I*2^(1 - m)*(((-I)*(-1 + E^((
2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m*Cos[c/2 + (d*x)/2]^4*(2^m*Hy
pergeometric2F1[1, m, 1 + m, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c
+ d*x))))] - (1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F1[m, m, 1 + m, (1
- E^((2*I)*(c + d*x)))/2])*Sec[c + d*x]^2*(e*Tan[c + d*x])^m)/(d*m*(a + a*S
ec[c + d*x])^2*Tan[c + d*x]^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\tan(dx+c))^m}{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

maple [F] time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,(e\tan(c+d\,x))^m}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^m)/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^m}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

3.214 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=252

$$\frac{e^5(e\tan(c+dx))^{m-5}\,{}_2F_1\left(1,\frac{m-5}{2};\frac{m-3}{2};-\tan^2(c+dx)\right)}{a^3d(5-m)}-\frac{e^5\sec^3(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-5}\,{}_2F_1\Big(}{a^3d(5-m)}$$

```
[Out] 3*e^5*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)+e^5*hypergeom([1, -5/2+1/2*m],[-3/2
+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)-3*e^5*(cos(d*x+c)^
2)^(-2+1/2*m)*hypergeom([-2+1/2*m, -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*s
ec(d*x+c)*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)-e^5*(cos(d*x+c)^2)^(-1+1/2*m)*h
ypergeom([-1+1/2*m, -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)^3*(e*
tan(d*x+c))^(-5+m)/a^3/d/(5-m)
```

Rubi [A] time = 0.34, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3888, 3886, 3476, 364, 2617, 2607, 32}

$$\frac{e^5(e\tan(c+dx))^{m-5}\,{}_2F_1\left(1,\frac{m-5}{2};\frac{m-3}{2};-\tan^2(c+dx)\right)}{a^3d(5-m)}-\frac{e^5\sec^3(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-5}\,{}_2F_1\Big(}{a^3d(5-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*e^5*(e*Tan[c + d*x])^(-5 + m))/(a^3*d*(5 - m)) + (e^5*Hypergeometric2F1[
1, (-5 + m)/2, (-3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-5 + m))/(a^3
*d*(5 - m)) - (3*e^5*(Cos[c + d*x]^2)^((-4 + m)/2)*Hypergeometric2F1[(-5 +
m)/2, (-4 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])
^(-5 + m))/(a^3*d*(5 - m)) - (e^5*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeomet
ric2F1[(-5 + m)/2, (-2 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(
e*Tan[c + d*x])^(-5 + m))/(a^3*d*(5 - m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
```

```
!IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx &= \frac{e^6\int(-a+a\sec(c+dx))^3(e\tan(c+dx))^{-6+m}\,dx}{a^6}\\
&= \frac{e^6\int\left(-a^3(e\tan(c+dx))^{-6+m}+3a^3\sec(c+dx)(e\tan(c+dx))^{-6+m}-3a^3\sec^2(c+dx\right.}{a^6}\\
&= -\frac{e^6\int(e\tan(c+dx))^{-6+m}\,dx}{a^3}+\frac{e^6\int\sec^3(c+dx)(e\tan(c+dx))^{-6+m}\,dx}{a^3}+\frac{\left(3e^6\right)\int\mathrm{se}}{}\\
&= -\frac{3e^5\cos^2(c+dx)^{\frac{1}{2}(-4+m)}\,{}_2F_1\left(\frac{1}{2}(-5+m),\frac{1}{2}(-4+m);\frac{1}{2}(-3+m);\sin^2(c+dx)\right)\sec(c}{a^3d(5-m)}\\
&= \frac{3e^5(e\tan(c+dx))^{-5+m}}{a^3d(5-m)}+\frac{e^5\,{}_2F_1\left(1,\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);-\tan^2(c+dx)\right)(e\tan(c+}{a^3d(5-m)}
\end{aligned}
$$

Mathematica [F] time = 11.60, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3, x]
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\tan(dx+c))^m}{a^3\sec(dx+c)^3+3a^3\sec(dx+c)^2+3a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

maple [F] time = 1.85, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,(e\tan(c+d\,x))^m}{a^3\,(\cos(c+d\,x)+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^3*(e*tan(c + d*x))^m)/(a^3*(cos(c + d*x) + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^m}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

3.215 $\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=131

$$\frac{2^{m+\frac{5}{2}}(a\sec(c+dx)+a)^{3/2}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{5}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m+\frac{3}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

```
[Out] 2^(5/2+m)*AppellF1(1/2+1/2*m,3/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d
*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5/2+m)*(a+a*s
ec(d*x+c))^(3/2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3889}

$$\frac{2^{m+\frac{5}{2}}(a\sec(c+dx)+a)^{3/2}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{5}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m+\frac{3}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

```
[Out] (2^(5/2 + m)*AppellF1[(1 + m)/2, 3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(5/2 + m)*(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^
(1 + m))/(d*e*(1 + m))
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx=\frac{2^{\frac{5}{2}+m}F_1\left(\frac{1+m}{2};\frac{3}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)}{de(1+m)}$$

Mathematica [F] time = 3.76, size = 0, normalized size = 0.00

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx + c) + a)^{\frac{3}{2}} (e \tan (dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

maple [F] time = 1.61, size = 0, normalized size = 0.00

$$\int (a + a \sec (dx + c))^{\frac{3}{2}} (e \tan (dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx + c) + a)^{\frac{3}{2}} (e \tan (dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e \tan (c + d\,x))^m \left(a + \frac{a}{\cos (c + d\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(3/2),x)
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sec (c + dx) + 1))^{\frac{3}{2}} (e \tan (c + dx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(e*tan(d*x+c))**m,x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(e*tan(c + d*x))**m, x)
```

3.216 $\int \sqrt{a+a\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=131

$$\frac{2^{m+\frac{3}{2}}\sqrt{a\sec(c+dx)+a}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{3}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m+\frac{1}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

```
[Out] 2^(3/2+m)*AppellF1(1/2+1/2*m,1/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d
*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3/2+m)*(a+a*s
ec(d*x+c))^(1/2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {3889}

$$\frac{2^{m+\frac{3}{2}}\sqrt{a\sec(c+dx)+a}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{3}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m+\frac{1}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

```
[Out] (2^(3/2 + m)*AppellF1[(1 + m)/2, 1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(3/2 + m)*Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^(1
 + m))/(d*e*(1 + m))
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,(e\tan(c+dx))^m\,dx = \frac{2^{\frac{3}{2}+m}F_1\left(\frac{1+m}{2};\frac{1}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}}}{de(1+m)}$$

Mathematica [F] time = 8.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sec(dx+c)+a}\,(e\tan(dx+c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(dx+c)+a}\,(e \tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

maple [F] time = 1.61, size = 0, normalized size = 0.00

$$\int \sqrt{a+a \sec(dx+c)}\,(e \tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(dx+c)+a}\,(e \tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e \tan(c+d\,x))^m \sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c+dx)+1)}\,(e \tan(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*(e*tan(d*x+c))**m,x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*tan(c + d*x))**m, x)
```

3.217 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=131

$$\frac{2^{m+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{1}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m-\frac{1}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2+1/2*m,-1/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(
d*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1/2+m)*(e*ta
n(d*x+c))^(1+m)/d/e/(1+m)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3889}

$$\frac{2^{m+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{1}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m-\frac{1}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2^(1/2 + m)*AppellF1[(1 + m)/2, -1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d
*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
+ Sec[c + d*x])^(-1))^(1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt
[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1+m}{2};-\frac{1}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+m}(e\tan(c+ }{de(1+m)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [F] time = 2.35, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]], x]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

maple [F] time = 1.67, size = 0, normalized size = 0.00

$$\int\frac{(e\tan(dx+c))^m}{\sqrt{a+a\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(e\tan(c+d\,x))^m}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\tan(c+dx))^m}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral((e*tan(c + d*x))**m/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.218 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=131

$$\frac{2^{m-\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m-\frac{1}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m-\frac{3}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] 2^(-1/2+m)*AppellF1(1/2+1/2*m,-3/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec
(d*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(-1/2+m)*(e*
tan(d*x+c))^(1+m)/d/e/(1+m)/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {3889}

$$\frac{2^{m-\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m-\frac{1}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m-\frac{3}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2^(-1/2 + m)*AppellF1[(1 + m)/2, -3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c +
d*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
 + Sec[c + d*x])^(-1))^(-1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*(a
 + a*Sec[c + d*x])^(3/2))
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{2^{-\frac{1}{2}+m}F_1\left(\frac{1+m}{2};-\frac{3}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+m}(e\tan(}{de(1+m)(a+a\sec(c+dx))^{3/2}}$$

Mathematica [F] time = 10.46, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2), x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{a\sec(dx+c)+a}\,(e\tan(dx+c))^m}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m/(a^2*sec(d*x + c)^2 +
2*a^2*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

maple [F] time = 1.55, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\tan(c+d\,x))^m}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.219 $\int (a+a\sec(c+dx))^n\tan^7(c+dx)\,dx$

Optimal. Leaf size=123

$$\frac{(a\sec(c+dx)+a)^{n+6}}{a^6d(n+6)}-\frac{5(a\sec(c+dx)+a)^{n+5}}{a^5d(n+5)}+\frac{(a\sec(c+dx)+a)^{n+4}\,{}_2F_1(1,n+4;n+5;\sec(c+dx)+1)}{a^4d(n+4)}+\frac{7(a\,}{}$$

```
[Out] 7*(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)+hypergeom([1, 4+n],[5+n],1+sec(d*x+c))
*(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)-5*(a+a*sec(d*x+c))^(5+n)/a^5/d/(5+n)+(a
+a*sec(d*x+c))^(6+n)/a^6/d/(6+n)
```

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3880, 88, 65}

$$\frac{(a\sec(c+dx)+a)^{n+4}\,{}_2F_1(1,n+4;n+5;\sec(c+dx)+1)}{a^4d(n+4)}+\frac{7(a\sec(c+dx)+a)^{n+4}}{a^4d(n+4)}-\frac{5(a\sec(c+dx)+a)^{n+5}}{a^5d(n+5)}+\frac{(a\,}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^7,x]
```

```
[Out] (7*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(4 + n)) + (Hypergeometric2F1[1, 4
+ n, 5 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(4 + n))
 - (5*(a + a*Sec[c + d*x])^(5 + n))/(a^5*d*(5 + n)) + (a + a*Sec[c + d*x])^
(6 + n)/(a^6*d*(6 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \tan^7(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^3(a+ax)^{3+n}}{x}\,dx, x, \sec(c+dx)\right)}{a^6 d}$$

$$= \frac{\text{Subst}\left(\int \left(7a^3(a+ax)^{3+n} - \frac{a^3(a+ax)^{3+n}}{x} - 5a^2(a+ax)^{4+n} + a(a+ax)^{5}\right.\right.}{a^6 d}$$

$$= \frac{7(a+a\sec(c+dx))^{4+n}}{a^4 d(4+n)} - \frac{5(a+a\sec(c+dx))^{5+n}}{a^5 d(5+n)} + \frac{(a+a\sec(c+d}{a^6 d(6+n)}$$

$$= \frac{7(a+a\sec(c+dx))^{4+n}}{a^4 d(4+n)} + \frac{{}_2F_1(1,4+n;5+n;1+\sec(c+dx))(a+a}{a^4 d(4+n)}$$

Mathematica [A] time = 0.46, size = 87, normalized size = 0.71

$$\frac{(\sec(c+dx)+1)^4(a(\sec(c+dx)+1))^n\left(\frac{{}_2F_1(1,n+4;n+5;\sec(c+dx)+1)}{n+4}+\frac{(\sec(c+dx)+1)^2}{n+6}-\frac{5(\sec(c+dx)+1)}{n+5}+\frac{7}{n+4}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^7,x]
```

```
[Out] ((1 + Sec[c + d*x])^4*(a*(1 + Sec[c + d*x]))^n*(7/(4 + n) + Hypergeometric2
F1[1, 4 + n, 5 + n, 1 + Sec[c + d*x]]/(4 + n) - (5*(1 + Sec[c + d*x]))/(5 +
 n) + (1 + Sec[c + d*x])^2/(6 + n)))/d
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\tan(dx+c)^7, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

maple [F] time = 1.39, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\tan^7(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(c+d\,x\right)^7\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**7,x)
```

```
[Out] Timed out
```

3.220 $\int (a+a\sec(c+dx))^n \tan^5(c+dx)\,dx$

Optimal. Leaf size=97

$$\frac{(a\sec(c+dx)+a)^{n+4}}{a^4d(n+4)}-\frac{(a\sec(c+dx)+a)^{n+3}\,{}_2F_1(1,n+3;n+4;\sec(c+dx)+1)}{a^3d(n+3)}-\frac{3(a\sec(c+dx)+a)^{n+3}}{a^3d(n+3)}$$

```
[Out] -3*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)-hypergeom([1, 3+n],[4+n],1+sec(d*x+c)
)*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)+(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3880, 88, 65}

$$-\frac{(a\sec(c+dx)+a)^{n+3}\,{}_2F_1(1,n+3;n+4;\sec(c+dx)+1)}{a^3d(n+3)}-\frac{3(a\sec(c+dx)+a)^{n+3}}{a^3d(n+3)}+\frac{(a\sec(c+dx)+a)^{n+4}}{a^4d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] (-3*(a + a*Sec[c + d*x])^(3 + n))/(a^3*d*(3 + n)) - (Hypergeometric2F1[1, 3
 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n))/(a^3*d*(3 + n)
) + (a + a*Sec[c + d*x])^(4 + n)/(a^4*d*(4 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{2+n}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{2+n}+\frac{a^2(a+ax)^{2+n}}{x}+a(a+ax)^{3+n}\right)dx, x, \sec(c\right.}{a^4d}$$

$$= -\frac{3(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)}+\frac{(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)}+\frac{\text{Subst}\left(\int \frac{(a+ax)^2}{x}\right.}{}$$

$$= -\frac{3(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)}-\frac{{}_2F_1(1,3+n;4+n;1+\sec(c+dx))(a+}{a^3d(3+n)}$$

Mathematica [A] time = 0.17, size = 72, normalized size = 0.74

$$\frac{(\sec(c+dx)+1)^3(a(\sec(c+dx)+1))^n(-(n+4)\,{}_2F_1(1,n+3;n+4;\sec(c+dx)+1)+(n+3)\sec(c+dx)-2n-}{d(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] ((1 + Sec[c + d*x])^3*(a*(1 + Sec[c + d*x]))^n*(-9 - 2*n - (4 + n)*Hypergeo
metric2F1[1, 3 + n, 4 + n, 1 + Sec[c + d*x]] + (3 + n)*Sec[c + d*x]))/(d*(3
 + n)*(4 + n))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\tan(dx+c)^5\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\tan^5(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^5\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec (c+dx)+1))^n \tan^5 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**5,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**5, x)
```

3.221 $\int (a + a\sec(c + dx))^n \tan^3(c + dx)\, dx$

Optimal. Leaf size=69

$$\frac{(a\sec(c+dx)+a)^{n+2}\,{}_2F_1(1,n+2;n+3;\sec(c+dx)+1)}{a^2d(n+2)}+\frac{(a\sec(c+dx)+a)^{n+2}}{a^2d(n+2)}$$

```
[Out] (a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)+hypergeom([1, 2+n],[3+n],1+sec(d*x+c))*(
a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3880, 80, 65}

$$\frac{(a\sec(c+dx)+a)^{n+2}\,{}_2F_1(1,n+2;n+3;\sec(c+dx)+1)}{a^2d(n+2)}+\frac{(a\sec(c+dx)+a)^{n+2}}{a^2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] (a + a*Sec[c + d*x])^(2 + n)/(a^2*d*(2 + n)) + (Hypergeometric2F1[1, 2 + n,
 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/(a^2*d*(2 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^n\tan^3(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{1+n}}{x}\,dx,x,\sec(c+dx)\right)}{a^2d}\\ &= \frac{(a+a\sec(c+dx))^{2+n}}{a^2d(2+n)}-\frac{\text{Subst}\left(\int \frac{(a+ax)^{1+n}}{x}\,dx,x,\sec(c+dx)\right)}{ad}\\ &= \frac{(a+a\sec(c+dx))^{2+n}}{a^2d(2+n)}+\frac{{}_2F_1(1,2+n;3+n;1+\sec(c+dx))(a+a\sec(c}{a^2d(2+n)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.71

$$\frac{(\sec(c+dx)+1)^2(a(\sec(c+dx)+1))^n({}_2F_1(1,n+2;n+3;\sec(c+dx)+1)+1)}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] ((1 + Hypergeometric2F1[1, 2 + n, 3 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*
x])^2*(a*(1 + Sec[c + d*x]))^n)/(d*(2 + n))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\tan(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

maple [F] time = 1.16, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\tan^3(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n \tan^3(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**3,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**3, x)
```

3.222 $\int (a+a\sec(c+dx))^n \tan(c+dx)\,dx$

Optimal. Leaf size=43

$$-\frac{(a\sec(c+dx)+a)^{n+1}\,{}_2F_1(1,n+1;n+2;\sec(c+dx)+1)}{ad(n+1)}$$

```
[Out] -hypergeom([1, 1+n],[2+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3880, 65}

$$-\frac{(a\sec(c+dx)+a)^{n+1}\,{}_2F_1(1,n+1;n+2;\sec(c+dx)+1)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x]
)^(1 + n))/(a*d*(1 + n)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^n\tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+ax)^n}{x}\,dx, x, \sec(c+dx)\right)}{d}\\ &= -\frac{{}_2F_1(1,1+n;2+n;1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 1.00

$$-\frac{(a(\sec(c+dx)+1))^{n+1}\,{}_2F_1(1,n+1;n+2;\sec(c+dx)+1)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x
]))^(1 + n))/(a*d*(1 + n)))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\tan(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(dx + c) + a)^n \tan(dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

maple **[F]** time = 1.15, size = 0, normalized size = 0.00

$$\int (a + a \sec(dx + c))^n \tan(dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c),x)
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(dx + c) + a)^n \tan(dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(c + d\,x) \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)*(a + a/cos(c + d*x))^n, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sec(c + dx) + 1))^n \tan(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x), x)
```

3.223 $\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=74

$$\frac{(a\sec(c+dx)+a)^n\,{}_2F_1(1,n;n+1;\sec(c+dx)+1)}{dn}-\frac{(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

```
[Out] -1/2*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n+hype
rgeom([1, n],[1+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3880, 86, 65, 68}

$$\frac{(a\sec(c+dx)+a)^n\,{}_2F_1(1,n;n+1;\sec(c+dx)+1)}{dn}-\frac{(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])
^n)/(2*d*n) + (Hypergeometric2F1[1, n, 1 + n, 1 + Sec[c + d*x]]*(a + a*Sec[
c + d*x])^n)/(d*n)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] && !IntegerQ[p]
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \frac{a^2\,\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{x}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\,\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{-a+ax}\,dx, x, \text{se}\right.}{d}$$
$$= -\frac{{}_2F_1\left(1, n; 1+n; \frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn} + \frac{{}_2F_1(1, n; 1+}{}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.77

$$-\frac{(a(\sec(c+dx)+1))^n\left({}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)-2\,{}_2F_1(1,n;n+1;\sec(c+dx)+1)\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^n,x]
[Out] -1/2*((Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2] - 2*Hypergeomet
ric2F1[1, n, 1 + n, 1 + Sec[c + d*x]])*(a*(1 + Sec[c + d*x]))^n)/(d*n)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\cot(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

maple [F] time = 1.44, size = 0, normalized size = 0.00

$$\int \cot(dx+c)(a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(c+d\,x\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec{\left(c + dx \right)} + 1\right)\right)^{n} \cot{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x), x)
```

3.224 $\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=127

$$-\frac{a(4-n)(a\sec(c+dx)+a)^{n-1}\,{}_2F_1\left(1,n-1;n;\frac{1}{2}(\sec(c+dx)+1)\right)}{4d(1-n)}+\frac{a(a\sec(c+dx)+a)^{n-1}\,{}_2F_1(1,n-1;n;\sec(c$$

```
[Out] -1/4*a*(4-n)*hypergeom([1, -1+n],[n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^(
-1+n)/d/(1-n)+a*hypergeom([1, -1+n],[n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(-1+
n)/d/(1-n)+1/2*a*(a+a*sec(d*x+c))^(-1+n)/d/(1-sec(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3880, 103, 156, 65, 68}

$$-\frac{a(4-n)(a\sec(c+dx)+a)^{n-1}\,{}_2F_1\left(1,n-1;n;\frac{1}{2}(\sec(c+dx)+1)\right)}{4d(1-n)}+\frac{a(a\sec(c+dx)+a)^{n-1}\,{}_2F_1(1,n-1;n;\sec(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(a*(4 - n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c + d*x])/2]*(a + a*Se
c[c + d*x])^(-1 + n))/(4*d*(1 - n)) + (a*Hypergeometric2F1[1, -1 + n, n, 1
+ Sec[c + d*x]]*(a + a*Sec[c + d*x])^(-1 + n))/(d*(1 - n)) + (a*(a + a*Sec[
c + d*x])^(-1 + n))/(2*d*(1 - Sec[c + d*x]))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 3880

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(d*b^(m - 1))^(-1), Subst[Int[((-a + b*x)^((m - 1)/2
)*(a + b*x)^((m - 1)/2 + n))/x, x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx = \frac{a^4\,\text{Subst}\left(\int \frac{(a+ax)^{-2+n}}{x(-a+ax)^2}\,dx,x,\sec(c+dx)\right)}{d}$$

$$= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{a\,\text{Subst}\left(\int \frac{(a+ax)^{-2+n}\left(2a^2+a^2(2-n)x\right)}{x(-a+ax)}\,dx,x,\text{sec}\right.}{2d}$$

$$= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} + \frac{a^2\,\text{Subst}\left(\int \frac{(a+ax)^{-2+n}}{x}\,dx,x,\sec(c+dx)\right)}{d}$$

$$= -\frac{a(4-n)\,{}_2F_1\left(1,-1+n;n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^{-1+}}{4d(1-n)}$$

Mathematica [A] time = 0.26, size = 96, normalized size = 0.76

$$-\frac{a(a(\sec(c+dx)+1))^{n-1}\left((n-4)(\sec(c+dx)-1)\,{}_2F_1\left(1,n-1;n;\frac{1}{2}(\sec(c+dx)+1)\right)+4(\sec(c+dx)-1)\,{}_2F\right.}{4d(n-1)(\sec(c+dx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/4*(a*(-2 + 2*n + (-4 + n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x]) + 4*Hypergeometric2F1[1, -1 + n, n, 1 + Sec[c +
 d*x]]*(-1 + Sec[c + d*x]))*(a*(1 + Sec[c + d*x]))^(-1 + n))/(d*(-1 + n)*(-
1 + Sec[c + d*x]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\cot(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int \left(\cot^3(dx+c)\right)(a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{n}\cot(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^{3}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{n}\cot^{3}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**n,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x)**3, x)
```

3.225 $\int (a + a\sec(c + dx))^n \tan^4(c + dx)\,dx$

Optimal. Leaf size=106

$$\frac{2^{n+5}\tan^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+5}(a\sec(c+dx)+a)^n F_1\left(\frac{5}{2};n+4,1;\frac{7}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

```
[Out] 1/5*2^(5+n)*AppellF1(5/2,4+n,1,7/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a*
sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5+n)*(a+a*sec(d*x+c))^n*t
an(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3889}

$$\frac{2^{n+5}\tan^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+5}(a\sec(c+dx)+a)^n F_1\left(\frac{5}{2};n+4,1;\frac{7}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

```
[Out] (2^(5 + n)*AppellF1[5/2, 4 + n, 1, 7/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c
 + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])^(
-1))^(5 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5)/(5*d)
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx = \frac{2^{5+n}F_1\left(\frac{5}{2};4+n,1;\frac{7}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{5+n}(a+}{5d}$$

Mathematica [F] time = 1.35, size = 0, normalized size = 0.00

$$\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\tan(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

maple [F] time = 1.04, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\tan^4(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x)
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n\tan^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**4,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**4, x)
```

3.226 $\int (a + a\sec(c + dx))^n \tan^2(c + dx)\,dx$

Optimal. Leaf size=106

$$\frac{2^{n+3}\tan^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+3}(a\sec(c+dx)+a)^n F_1\left(\frac{3}{2};n+2,1;\frac{5}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

```
[Out] 1/3*2^(3+n)*AppellF1(3/2,2+n,1,5/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a*
sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3+n)*(a+a*sec(d*x+c))^n*t
an(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3889}

$$\frac{2^{n+3}\tan^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+3}(a\sec(c+dx)+a)^n F_1\left(\frac{3}{2};n+2,1;\frac{5}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] (2^(3 + n)*AppellF1[3/2, 2 + n, 1, 5/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c
 + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])^(
-1))^(3 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3)/(3*d)
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx = \frac{2^{3+n}F_1\left(\frac{3}{2};2+n,1;\frac{5}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{3+n}(a+}{3d}$$

Mathematica [B] time = 11.33, size = 910, normalized size = 8.58

$$(a(\sec(c+dx)+1))^n\left(-\frac{{}_2F_1\left(1-n,n+2;2-n;\frac{1}{2}\left(1-\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^n\left(\tan\left(\frac{1}{2}(c+dx)\right)-1\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)+1\right)^n(\sec(c+}{n-1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^n*((-4*Hypergeometric2F1[-1 - n, n, -n, (1 - Tan[(c
 + d*x)/2])/2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(1 + Tan[(c + d*x)/2])^n
)/((1 + n)*(1 + Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])) - (Hypergeometric2
F1[1 - n, 2 + n, 2 - n, (1 - Tan[(c + d*x)/2])/2]*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^n*(-1 + Tan[(c + d*x)/2])*(1 + Tan[(c + d*x)/2])^n)/((-1 + n)*(1 +
```

```
 Sec[c + d*x])^n) - (120*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Sin[c + d*x]*(3*AppellF1[1/
2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, n
, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(45
*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c
 + d*x)/2]^2*(1 + 2*n - 2*n*Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*AppellF1[1
/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2*
(-5*AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 +
2*n - 2*(2 + n)*Cos[c + d*x] + Cos[2*(c + d*x)]) + 5*n*AppellF1[3/2, 1 + n,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[
c + d*x] + Cos[2*(c + d*x)]) - 48*(2*AppellF1[5/2, n, 3, 7/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1[5/2, 1 + n, 2, 7/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + n*(1 + n)*AppellF1[5/2, 2 + n, 1, 7/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cot[c + d*x]*Csc[c + d*x]*Sin[(c + d*x
)/2]^4) + 40*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)))/(4*d)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\tan(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sec(dx+c)+a)^n\tan(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int(a+a\sec(dx+c))^n\left(\tan^2(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x)
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sec(dx+c)+a)^n\tan(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tan(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sec (c + dx) + 1))^n \tan^2 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**2,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**2, x)
```

3.227 $\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=102

$$-\frac{2^{n-1}\cot(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-1}(a\sec(c+dx)+a)^n F_1\left(-\frac{1}{2};n-2,1;\frac{1}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

```
[Out] -2^(-1+n)*AppellF1(-1/2,-2+n,1,1/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a*
sec(d*x+c))/(a+a*sec(d*x+c)))*cot(d*x+c)*(1/(1+sec(d*x+c)))^(-1+n)*(a+a*sec
(d*x+c))^n/d
```

Rubi [A] time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3889}

$$-\frac{2^{n-1}\cot(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-1}(a\sec(c+dx)+a)^n F_1\left(-\frac{1}{2};n-2,1;\frac{1}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((2^(-1 + n)*AppellF1[-1/2, -2 + n, 1, 1/2, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]*((1
 + Sec[c + d*x])^(-1))^(-1 + n)*(a + a*Sec[c + d*x])^n)/d)
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{-1+n}F_1\left(-\frac{1}{2};-2+n,1;\frac{1}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot(c+dx)\left(\frac{1}{1+\sec(\right.}{d}$$

Mathematica [B] time = 4.18, size = 893, normalized size = 8.75

$$\frac{(a(\sec(c+dx)+1))^n\left(-2^n\cot\left(\frac{1}{2}(c+dx)\right)\,{}_2F_1\left(-\frac{1}{2},n;\frac{1}{2};\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(\cos^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^n*(-((2^n*Cot[(c + d*x)/2]*Hypergeometric2F1[-1/2,
n, 1/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d
*x)/2]^2*Sec[c + d*x])^n)/(1 + Sec[c + d*x])^n) + (2^n*Hypergeometric2F1[1/
2, n, 3/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c
+ d*x)/2]^2*Sec[c + d*x])^n*Tan[(c + d*x)/2])/(1 + Sec[c + d*x])^n - (60*Ap
```

```
pellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*
x)/2]^2*Cos[c + d*x]*Sin[c + d*x]*(3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(45*AppellF1[1/2, n, 1, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]^2*(1 + 2*n - 2*n
*Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2*(-5*AppellF1[3/2, n, 2, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[c + d*x
] + Cos[2*(c + d*x)]) + 5*n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[c + d*x] + Cos[2*(c + d*x)]
) - 48*(2*AppellF1[5/2, n, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 - 2*n*AppellF1[5/2, 1 + n, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] + n*(1 + n)*AppellF1[5/2, 2 + n, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])*Cot[c + d*x]*Csc[c + d*x]*Sin[(c + d*x)/2]^4) + 40*(AppellF1[3/2,
 n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 +
n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]^2)))/(2*d)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\cot(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

maple [F] time = 1.03, size = 0, normalized size = 0.00

$$\int \left(\cot^2(dx+c)\right)(a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n \cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x)**2, x)
```

3.228 $\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=106

$$-\frac{2^{n-3}\cot^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-3}(a\sec(c+dx)+a)^n F_1\left(-\frac{3}{2};n-4,1;-\frac{1}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

```
[Out] -1/3*2^(-3+n)*AppellF1(-3/2,-4+n,1,-1/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),
(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*cot(d*x+c)^3*(1/(1+sec(d*x+c)))^(-3+n)*(
a+a*sec(d*x+c))^n/d
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3889}

$$-\frac{2^{n-3}\cot^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-3}(a\sec(c+dx)+a)^n F_1\left(-\frac{3}{2};n-4,1;-\frac{1}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(2^(-3 + n)*AppellF1[-3/2, -4 + n, 1, -1/2, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]^3*(
(1 + Sec[c + d*x])^(-1))^(-3 + n)*(a + a*Sec[c + d*x])^n)/(3*d)
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{-3+n}F_1\left(-\frac{3}{2};-4+n,1;-\frac{1}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot^3(c+dx)\left(\right.}{3d}$$

Mathematica [F] time = 1.70, size = 0, normalized size = 0.00

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n, x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\cot(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \cot(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \left(\cot^4(dx+c)\right)(a+a\sec(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \cot(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^4 \left(a+\frac{a}{\cos(c+d\,x)}\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**n,x)
[Out] Timed out
```

3.229 $\int (a+a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{2^{n+\frac{7}{2}}\tan^{\frac{5}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{5}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{5}{4};n+\frac{3}{2},1;\frac{9}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

```
[Out] 1/5*2^(7/2+n)*AppellF1(5/4,3/2+n,1,9/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(
a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5/2+n)*(a+a*sec(d*x+c
))^n*tan(d*x+c)^(5/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3889}

$$\frac{2^{n+\frac{7}{2}}\tan^{\frac{5}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{5}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{5}{4};n+\frac{3}{2},1;\frac{9}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

```
[Out] (2^(7/2 + n)*AppellF1[5/4, 3/2 + n, 1, 9/4, -((a - a*Sec[c + d*x])/(a + a*S
ec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x
])^(-1))^(5/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(5/2))/(5*d)
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \frac{2^{\frac{7}{2}+n}F_1\left(\frac{5}{4};\frac{3}{2}+n,1;\frac{9}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+n}(a}{5d}$$

Mathematica [B] time = 19.02, size = 2072, normalized size = 18.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

```
[Out] (2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*(-1
 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 + n
)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2,
 Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2
, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-
```

```
1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Tan[c + d*x]^2)/(d*((2^n*Sec[c + d*x]^2*
(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*
AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n)
+ (AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]
] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2
]])*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n)))/
Sqrt[Tan[c + d*x]] + 2^n*(-1/2 - n)*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*
Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-3/2 - n)*(-2*AppellF1[1/4, 1/2 +
n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c +
d*x)/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2
 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/
2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*
x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Sqrt[Tan[c + d*x]] +
2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-1/2
 - n)*(-((1/2 + n)*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 +
 n)*(-1 + Tan[(c + d*x)/2])^(-1/2 + n)) - 2*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n)*(-1/5*(AppellF1[5/4, 1/2 + n
, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]) + ((1/2 + n)*AppellF1[5/4, 3/2 + n, 1, 9/4, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) - 2*(1/2 + n
)*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 +
n)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]) + (1/2 + n)*(AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c +
d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c +
 d*x)/2], -Tan[(c + d*x)/2]])*Sec[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2])^(1/
2 + n)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)^(-1/2 + n) - ((1/2 + n)*(
AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] +
 AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])
*Sec[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2])^(-1/2 + n)*(-1 + Tan[(c + d*x)/2
]^2)^(1/2 + n))/2 + (-1/6*((3/2 + n)*AppellF1[3/2, 1/2 + n, 5/2 + n, 5/2, T
an[(c + d*x)/2], -Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2) + ((3/2 + n)*Appell
F1[3/2, 5/2 + n, 1/2 + n, 5/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]]*Sec[(c
+ d*x)/2]^2)/6)*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^
(1/2 + n))*Sqrt[Tan[c + d*x]] + 2^(1 + n)*n*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(-1 + n)*(-1 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*AppellF1[1/4, 1/2 + n, 1
, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)
/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2 + n
, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 +
n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*x)/2
])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Sqrt[Tan[c + d*x]]*(-(Cos
[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d
*x]*Tan[c + d*x])))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

maple [F] time = 1.76, size = 0, normalized size = 0.00

$$\int (a + a \sec(dx + c))^n \left(\tan^{\frac{3}{2}}(dx + c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(dx + c) + a)^n \tan(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + d\,x)^{3/2} \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.230 $\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx$

Optimal. Leaf size=114

$$\frac{2^{n+\frac{5}{2}}\tan^{\frac{3}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{3}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{3}{4};n+\frac{1}{2},1;\frac{7}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

```
[Out] 1/3*2^(5/2+n)*AppellF1(3/4,1/2+n,1,7/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(
a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3/2+n)*(a+a*sec(d*x+c
))^n*tan(d*x+c)^(3/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3889}

$$\frac{2^{n+\frac{5}{2}}\tan^{\frac{3}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{3}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{3}{4};n+\frac{1}{2},1;\frac{7}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

```
[Out] (2^(5/2 + n)*AppellF1[3/4, 1/2 + n, 1, 7/4, -((a - a*Sec[c + d*x])/(a + a*S
ec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x
])^(-1))^(3/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2))/(3*d)
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx = \frac{2^{\frac{5}{2}+n}F_1\left(\frac{3}{4};\frac{1}{2}+n,1;\frac{7}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+n}(a+a}{3d}$$

Mathematica [B] time = 2.23, size = 238, normalized size = 2.09

$$\frac{56\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\tan(c+dx)}\,(a(\sec(c+dx}{d\left(6(\cos(c+dx)-1)\left(2F_1\left(\frac{7}{4};n+\frac{1}{2},2;\frac{11}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-(2n+1)F_1\left(\frac{7}{4};n+\frac{3}{2},1;\frac{11}{4};\tan^2\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

```
[Out] (56*AppellF1[3/4, 1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*x)/2]*Sqrt[Tan[c +
d*x]])/(d*(6*(2*AppellF1[7/4, 1/2 + n, 2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] - (1 + 2*n)*AppellF1[7/4, 3/2 + n, 1, 11/4, Tan[(c + d*x)/2]^
```

```
2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 21*AppellF1[3/4, 1/2 + n, 1,
 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\sqrt{\tan(dx+c)}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\sqrt{\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

maple [F] time = 1.73, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\sqrt{\tan}\,(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\sqrt{\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\tan(c+d\,x)}\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n\sqrt{\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**(1/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*sqrt(tan(c + d*x)), x)
```

3.231 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=111

$$\frac{2^{n+\frac{3}{2}}\sqrt{\tan(c+dx)}\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{1}{2}}(a\sec(c+dx)+a)^nF_1\left(\frac{1}{4};n-\frac{1}{2},1;\frac{5}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

```
[Out] 2^(3/2+n)*AppellF1(1/4,-1/2+n,1,5/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a
*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1/2+n)*(a+a*sec(d*x+c))^
n*tan(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3889}

$$\frac{2^{n+\frac{3}{2}}\sqrt{\tan(c+dx)}\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{1}{2}}(a\sec(c+dx)+a)^nF_1\left(\frac{1}{4};n-\frac{1}{2},1;\frac{5}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (2^(3/2 + n)*AppellF1[1/4, -1/2 + n, 1, 5/4, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*
x])^(-1))^(1/2 + n)*(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]])/d
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx=\frac{2^{\frac{3}{2}+n}F_1\left(\frac{1}{4};-\frac{1}{2}+n,1;\frac{5}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+n}(a+a\sec(c+dx}{d}$$

Mathematica [B] time = 1.61, size = 229, normalized size = 2.06

$$\frac{10\cos(c+dx)(\cos(c+dx)+1)\sqrt{\tan(c+dx)}\,(a(\sec(c+dx)+}{d\left(2(\cos(c+dx)-1)\left(2F_1\left(\frac{5}{4};n-\frac{1}{2},2;\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(1-2n)F_1\left(\frac{5}{4};n+\frac{1}{2},1;\frac{9}{4};\tan^2\left(\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (10*AppellF1[1/4, -1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Cos[c + d*x]*(1 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sqrt[Tan[c + d*x
]])/(d*(2*(2*AppellF1[5/4, -1/2 + n, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c +
```

```
d*x)/2]^2] + (1 - 2*n)*AppellF1[5/4, 1/2 + n, 1, 9/4, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 5*AppellF1[1/4, -1/2 + n, 1, 5/4
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sec(dx+c))^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
[Out] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sqrt{\tan(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(1/2),x)
[Out] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(c+dx)+1))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n/sqrt(tan(c + d*x)), x)
```

3.232 $\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=112

$$-\frac{2^{n+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{n-\frac{1}{2}}(a\sec(c+dx)+a)^n F_1\left(-\frac{1}{4};n-\frac{3}{2},1;\frac{3}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d\sqrt{\tan(c+dx)}}$$

```
[Out] -2^(1/2+n)*AppellF1(-1/4,-3/2+n,1,3/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a
-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(-1/2+n)*(a+a*sec(d*x+c
))^n/d/tan(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3889}

$$-\frac{2^{n+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{n-\frac{1}{2}}(a\sec(c+dx)+a)^n F_1\left(-\frac{1}{4};n-\frac{3}{2},1;\frac{3}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] -((2^(1/2 + n)*AppellF1[-1/4, -3/2 + n, 1, 3/4, -((a - a*Sec[c + d*x])/(a +
 a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c +
 d*x])^(-1))^(-1/2 + n)*(a + a*Sec[c + d*x])^n)/(d*Sqrt[Tan[c + d*x]]))
```

Rule 3889

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> -Simp[(2^(m + n + 1)*(e*Cot[c + d*x])^(m + 1)*(a +
b*Csc[c + d*x])^n*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*AppellF1[(m + 1)/2,
m + n, 1, (m + 3)/2, -((a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])), (a - b*C
sc[c + d*x])/(a + b*Csc[c + d*x])])/(d*e*(m + 1)), x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2^{\frac{1}{2}+n}F_1\left(-\frac{1}{4};-\frac{3}{2}+n,1;\frac{3}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+n}(a+a\sec}{d\sqrt{\tan(c+dx)}}$$

Mathematica [B] time = 15.86, size = 2164, normalized size = 19.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] -1/21*(2^(1/2 + n)*Cot[c + d*x]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*(21*Hypergeometric2
F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1,
7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hyperg
```

```
eometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*
AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*T
an[(c + d*x)/2]^4))/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((2^(-1/2 + n)
*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*Sec[c + d*x]^2*(Cos[(c + d*x)/2]^2*Sec
[c + d*x])^n*(21*Hypergeometric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2]
 + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*
x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^4))/(21*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Tan[c + d*x]^(3/2)) + (2^(-1/2 + n)*(Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*((Cos[c + d*x]*Sin[c +
d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*(21*Hypergeom
etric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 +
n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*
Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^
2 - 3*AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Tan[(c + d*x)/2]^4))/(21*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[
Tan[c + d*x]]) - (2^(1/2 + n)*n*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c
 + d*x)/2]^2*Sec[c + d*x])^(1 + n)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Co
s[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(21*Hypergeometric2F1[-1/4,
 -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometric
2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*AppellF1
[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c +
d*x)/2]^4))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Tan[c + d*x]]) -
 (2^(1/2 + n)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^n*(7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 7*Hypergeometric2F1[3/4,
-1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 6*
AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^3 + 7*Tan[(c + d*x)/2]^2*((-3*AppellF1[7
/4, -1/2 + n, 2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/7 + (3*(-1/2 + n)*AppellF1[7/4, 1/2 + n, 1, 11/4,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/7) - 3*Tan[(c + d*x)/2]^4*((-7*AppellF1[11/4, -1/2 + n, 2, 15/4, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/11
+ (7*(-1/2 + n)*AppellF1[11/4, 1/2 + n, 1, 15/4, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/11) + (21*Csc[(c + d*x)
/2]*Sec[(c + d*x)/2]*(Hypergeometric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/
2]^2] - (1 - Tan[(c + d*x)/2]^2)^(1/2 - n)))/4 + (21*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]*(-Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2] +
 (1 - Tan[(c + d*x)/2]^2)^(1/2 - n)))/4))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[Tan[c + d*x]]) - (2^(1/2 + n)*n*(Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*(21*Hypergeometric2F1[-1/
4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometr
ic2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*Appell
F1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c
+ d*x)/2]^4)*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[Tan[c + d*x]])))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

maple [F] time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sec(dx+c))^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
[Out] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\tan(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(3/2),x)
[Out] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(c+dx)+1))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/tan(d*x+c)**(3/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n/tan(c + d*x)**(3/2), x)
```

3.233 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=320

$$\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}{\sqrt{2}\,d}-\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx)}{\sqrt{2}\,d}$$

```
[Out] -2/3*(e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))*tan(d*x+c)/d+1/3*a*(e*cot(d*x+c)
)^(5/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*P
i+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)*tan(d*x+c)^2/d-1/2*a*arctan
(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/
2)-1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)
^(5/2)/d*2^(1/2)+1/4*a*(e*cot(d*x+c))^(5/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)-1/4*a*(e*cot(d*x+c))^(5/2)*ln(1+2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3900, 3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}{\sqrt{2}\,d}-\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])*Tan[c + d*x])/(3*d) - (a*(e
*Cot[c + d*x])^(5/2)*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c
 + 2*d*x]]*Tan[c + d*x]^2)/(3*d) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) - (a*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[
2]*d) + (a*(e*Cot[c + d*x])^(5/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt[2]*d) - (a*(e*Cot[c + d*x])^(5/2)*Log
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt
[2]*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \frac{a+a\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} + \frac{1}{3}\Big(2(e\cot(c+d$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{1}{3}\Big(a(e\cot(c+d$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{\Big(a(e\cot(c+dx)}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{1}{3}\Big(a(e\cot(c+d$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(c+dx))^5}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(c+dx))^5}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(c+dx))^5}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(c+dx))^5}{}$$

Mathematica [C] time = 3.80, size = 185, normalized size = 0.58

$$-\frac{a\sec(c+dx)(e\cot(c+dx))^{5/2}\left(\sqrt{\cot(c+dx)}\left(-3\sqrt{\sin(2(c+dx))}\sin^{-1}(\cos(c+dx)-\sin(c+dx))+4(\cos(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/6*(a*(e*Cot[c + d*x])^(5/2)*Sec[c + d*x]*(Sqrt[Cot[c + d*x]]*(4*(1 + Cos
[c + d*x])*Cot[c + d*x] - 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*
(c + d*x)]] + 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*S
qrt[Sin[2*(c + d*x)]]) + 2*(-1)^(1/4)*Sqrt[Csc[c + d*x]^2]*EllipticF[I*ArcS
```

```
inh[(-1)^(1/4)*Sqrt[Cot[c + d*x]]], -1]*Sin[2*(c + d*x)]))/(d*Cot[c + d*x]^
(5/2))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

maple [**C**] time = 2.48, size = 648, normalized size = 2.02

$$\frac{a(-1+\cos(dx+c))\left(3i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x)
[Out] 1/6*a/d*(-1+cos(d*x+c))*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*sin(d*x+c)-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*sin(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin
(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)
-4*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+2*cos(d*x+c)*2^(1
/2))*(1+cos(d*x+c))^2*(e*cos(d*x+c)/sin(d*x+c))^(5/2)/cos(d*x+c)^3/sin(d*x+
c)*2^(1/2)
```

maxima [**A**] time = 0.47, size = 186, normalized size = 0.58

$$\frac{\left(3e^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\sqrt{e}}+\cdots\right.\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*e^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/tan(d
*x + c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e)
 - 2*sqrt(e/tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(sqrt(2)*sqrt(e)*s
qrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e) + sqrt(2)*log(-sqrt(2)*sq
rt(e)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e)) - 8*(e/tan(d*x +
c))^(3/2))*a*e/d
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cot(c+d\,x))^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.234 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=346

$$\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)(e\,c}{\sqrt{2}d}$$

```
[Out] -2*(e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))*tan(d*x+c)/d+2*a*(e*cot(d*x+c))^(3
/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*
x),2^(1/2))*sin(d*x+c)*tan(d*x+c)/d/sin(2*d*x+2*c)^(1/2)-1/2*a*arctan(-1+2^
(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/d*2^(1/2)-1/2
*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)
/d*2^(1/2)-1/4*a*(e*cot(d*x+c))^(3/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x
+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+1/4*a*(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+2*a*(e*cot(d*x+c))^(3/
2)*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A] time = 0.28, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3900, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)(e\,c}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])*Tan[c + d*x])/d - (2*a*(e*C
ot[c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*x, 2]*Sin[c + d*x]*Tan[c + d*x])/
(d*Sqrt[Sin[2*c + 2*d*x]]) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*C
ot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d) - (a*ArcTan[1 + Sqrt[2]*
Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d)
- (a*(e*Cot[c + d*x])^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d) + (a*(e*Cot[c + d*x])^(3/2)*Log[1 + S
qrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d)
 + (2*a*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x]*Tan[c + d*x]^2)/d
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{a+a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \Big(2(e\cot(c+$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \Big(a(e\cot(c+$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \frac{2a(e\cot(c+}{}$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \frac{2a(e\cot(c+}{}$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \frac{2a(e\cot(c+}{}$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \frac{2a(e\cot(c+}{}$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \frac{2a(e\cot(c+}{}$$

$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \frac{2a(e\cot(c+}{}$$

Mathematica [C] time = 1.20, size = 191, normalized size = 0.55

$$\frac{ae(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\cot(c+dx)}\left(8\cot^2(c+dx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\cot^2(c+dx)\right)+3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*e*(1 + Cos[c + d*x])*Sqrt[e*Cot[c + d*x]]*Sec[(c + d*x)/2]^2*Sec[c + d*x
]*(8*Cot[c + d*x]^2*Hypergeometric2F1[3/4, 3/2, 7/4, -Cot[c + d*x]^2] + 3*S
qrt[Csc[c + d*x]^2]*(-4*Cos[c + d*x] - 4*Cos[c + d*x]^2 + ArcSin[Cos[c + d*
x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x]
 + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*Sqrt[Csc[c + d*x
]^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a), x)
```

maple [C] time = 2.35, size = 1390, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2*a/d*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+I*El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-I*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)-4*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2
*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*
x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
```

```
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)-4*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2*EllipticF((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)+4*cos(d*x+c)*2^(1/2))*sin(d*x+c)*(e*cos(d*
x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)^2*2^(1/2)
```

maxima [A] time = 0.50, size = 180, normalized size = 0.52

$$\frac{\left(2\sqrt{2}\sqrt{e}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)+2\sqrt{2}\sqrt{e}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)+\sqrt{2}\sqrt{e}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*sqrt(e)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/tan(d
*x + c)))/sqrt(e)) + 2*sqrt(2)*sqrt(e)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e)
 - 2*sqrt(e/tan(d*x + c)))/sqrt(e)) + sqrt(2)*sqrt(e)*log(sqrt(2)*sqrt(e)*s
qrt(e/tan(d*x + c)) + e + e/tan(d*x + c)) - sqrt(2)*sqrt(e)*log(-sqrt(2)*sq
rt(e)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c)) - 8*sqrt(e/tan(d*x + c)))*
a*e/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cot(c+d\,x))^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.235 $\int \sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=274

$$-\frac{a\sqrt{\tan(c+dx)}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}d}+\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}{\sqrt{2}d}$$

```
[Out] -a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x
),2^(1/2))*sec(d*x+c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/d+1/2*a*arc
tan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^
(1/2)+1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x
+c)^(1/2)/d*2^(1/2)-1/4*a*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(
d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(1/2
)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3900, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{a\sqrt{\tan(c+dx)}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}d}+\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sqrt[e*Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[
2*c + 2*d*x]])/d - (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) + (a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]]]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) - (a*Sqrt[e*Co
t[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c +
 d*x]])/(2*Sqrt[2]*d) + (a*Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

```
&& !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))\,dx = \left(\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \left(a\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx + \left(a\sqrt{e\cot(c+dx)}\right.$$
$$= \frac{\left(a\sqrt{e\cot(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(a\sqrt{e\cot(c\right.}{}$$
$$= \left(a\sqrt{e\cot(c+dx)}\,\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx +$$
$$= \frac{a\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} + \frac{\left(a\sqrt{e\,c\right.}{}$$
$$= \frac{a\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} + \frac{\left(a\sqrt{e\,c\right.}{}$$
$$= \frac{a\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} - \frac{a\sqrt{e\,c}}{}$$
$$= \frac{a\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} - \frac{a\tan^{-}}{}$$

Mathematica [C] time = 1.83, size = 169, normalized size = 0.62

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\cot(c+dx)}\left(\sqrt{\sin(2(c+dx))}\sqrt{\csc^2(c+dx)}\left(\log\left(\sin(c+dx)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sqrt[e*Cot[c + d*x]]*Sec[(c + d*x)/2]^2*Sec[c + d*x]*
(4*(-1)^(1/4)*Sqrt[Cot[c + d*x]]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Cot[c
+ d*x]]], -1] + Sqrt[Csc[c + d*x]^2]*(-ArcSin[Cos[c + d*x] - Sin[c + d*x]]
+ Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*Sqrt[Sin[2*(c
+ d*x)]]))/(4*d*Sqrt[Csc[c + d*x]^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cot(dx+c)}\,(a\sec(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a), x)
```

maple [C] time = 2.36, size = 284, normalized size = 1.04

$$-\frac{a\sqrt{\frac{e\cos(dx+c)}{\sin(dx+c)}}\,(-1+\cos(dx+c))\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\left(i\,\mathrm{EllipticPi}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a/d*(e*cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/sin(d*x+c)^2/cos(d*x
+c)*(1+cos(d*x+c))^2*2^(1/2)
```

maxima [A] time = 0.47, size = 166, normalized size = 0.61

$$-\frac{ae\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\sqrt{e}}+\frac{\sqrt{2}}{}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*e*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/tan(d*x
+ c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) -
2*sqrt(e/tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(sqrt(2)*sqrt(e)*sqrt
(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e) + sqrt(2)*log(-sqrt(2)*sqrt(
e)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\cot(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sqrt{e\cot(c+dx)}\,dx+\int \sqrt{e\cot(c+dx)}\sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(sqrt(e*cot(c + d*x))*sec(c
+ d*x), x))
```

3.236 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=299

$$\frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a\log\left(\tan(c+dx)-\sqrt{2}\right.}{2\sqrt{2}d\sqrt{\tan(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)/d/(e*cot(d*x+c))^(1/2)+2*a*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^
(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/d/(e*cot(d*x+c
))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*a*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2
^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*a*arctan(1+2^(1/2)*tan(d*x
+c)^(1/2))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*a*ln(1-2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(
1/2)-1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c
))^(1/2)/tan(d*x+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3900, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a\log\left(\tan(c+dx)-\sqrt{2}\right.}{2\sqrt{2}d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x]]) - (2*a*Cos[c + d*x]*EllipticE[c
 - Pi/4 + d*x, 2])/(d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - (a*Arc
Tan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[T
an[c + d*x]]) + (a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[
e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (a*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - (
a*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \frac{\int (a+a\sec(c+dx))\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{a\int \sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a\int \sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{(2a)\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+\right.}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a)\,\text{Subst}\left(\right.}{d\sqrt{e\cot}}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{(2a\cos(c+dx))\int \sqrt{\sin(2c+2dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} - \frac{a\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan}\right.}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c}}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan}\right.}{2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+}}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} + \frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\mathrm{ta}\right.}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(}}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} - \frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx}}$$

Mathematica [C] time = 1.60, size = 189, normalized size = 0.63

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(8\cot^3(c+dx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\cot^2(c+dx)\right)-3\cot(c+dx)\sqrt{\csc^2(}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]*(8*Cot[c + d*x]^3*Hyp
ergeometric2F1[3/4, 3/2, 7/4, -Cot[c + d*x]^2] - 3*Cot[c + d*x]*Sqrt[Csc[c
+ d*x]^2]*(-2 + 2*Cos[2*(c + d*x)] + ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sq
rt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*
x)]]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Csc[c + d*x
]^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sec(dx + c) + a}{\sqrt{e \cot(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

maple [C] time = 2.36, size = 1409, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x)
```

```
[Out] 1/2*a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(I*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1
/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-2*Elli
pticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+4*EllipticE(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)-2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+4*EllipticE(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-2*cos(d*
x+c)*2^(1/2)+2*2^(1/2))/sin(d*x+c)^5/(e*cos(d*x+c)/sin(d*x+c))^(1/2)*2^(1/2
)
```

maxima [A] time = 0.50, size = 166, normalized size = 0.56

$$-\frac{ae\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{e^{\frac{3}{2}}}-\frac{\sqrt{2}\log\cdots}{\cdots}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*e*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/tan(d*x
+ c)))/sqrt(e))/e^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) -
2*sqrt(e/tan(d*x + c)))/sqrt(e))/e^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(e)*sqrt
(e/tan(d*x + c)) + e + e/tan(d*x + c))/e^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(
e)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/e^(3/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{e\cot\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\sqrt{e\cot\left(c+dx\right)}}\,dx+\int \frac{\sec\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*cot(c
 + d*x)), x))
```

3.237 $\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=320

$$\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{2\cot(c+dx)(a\sec(c+dx)+3a)}{3d(e\cot(c+dx))^{3/2}}+\frac{a}{}$$

```
[Out] 2/3*cot(d*x+c)*(3*a+a*sec(d*x+c))/d/(e*cot(d*x+c))^(3/2)+1/3*a*cot(d*x+c)*c
sc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4
*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/d/(e*cot(d*x+c))^(3/2)-1/2*a*arctan(
-1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2
)-1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/t
an(d*x+c)^(3/2)+1/4*a*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*
x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A] time = 0.25, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3900, 3881, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{2\cot(c+dx)(a\sec(c+dx)+3a)}{3d(e\cot(c+dx))^{3/2}}+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (2*Cot[c + d*x]*(3*a + a*Sec[c + d*x]))/(3*d*(e*Cot[c + d*x])^(3/2)) - (a*C
ot[c + d*x]*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]
])/(3*d*(e*Cot[c + d*x])^(3/2)) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - (a*ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3
/2)) + (a*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*
(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - (a*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(
3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&& !IntegerQ[m]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{\int (a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a\int \frac{}{\sqrt{\mathrm{ta}}}}{(e\cot(c+dx)}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{\left(a\cos^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)} - \frac{a\,\mathrm{Su}}{}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{\left(a\cot(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{}{\sqrt{\mathrm{s}}}}{3(e\cot(c+dx))^{3/2}}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(}}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(}}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(}}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(}}{3d(e\cot(c+dx))^{3/2}}$$

Mathematica [C] time = 2.60, size = 224, normalized size = 0.70

$$\frac{a(\cos(c+dx)+1)\cos(2(c+dx))\csc(c+dx)\sqrt{\csc^2(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\csc^2(c+dx)}\left(12\cos(c+dx)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Cos[2*(c + d*x)]*Csc[c + d*x]*Sqrt[Csc[c + d*x]^2]*Se
c[(c + d*x)/2]^2*(-4*(-1)^(1/4)*Cot[c + d*x]^(3/2)*EllipticF[I*ArcSinh[(-1)
^(1/4)*Sqrt[Cot[c + d*x]]], -1] + Sqrt[Csc[c + d*x]^2]*(4 + 12*Cos[c + d*x]
 + 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Cot[c + d*x]*Sqrt[Sin[2*(c + d*x)]
] - 3*Cot[c + d*x]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]
]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*(e*Cot[c + d*x])^(3/2)*(-1 + Cot[c + d*x]
^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sec(dx + c) + a}{(e \cot(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

maple [C] time = 2.08, size = 688, normalized size = 2.15

$$a(-1+\cos(dx+c))\left(3i\cos(dx+c)\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\ \mathrm{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x)
```

```
[Out] 1/6*a/d*(-1+cos(d*x+c))*(3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*cos(d*x+c)*sin(d*x+c)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2
^(1/2)-2*2^(1/2))*(1+cos(d*x+c))^2/(e*cos(d*x+c)/sin(d*x+c))^(3/2)/sin(d*x+
c)^5*2^(1/2)
```

maxima [A] time = 0.45, size = 188, normalized size = 0.59

$$ae\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\sqrt{e}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan}}\right.}{\sqrt{}}}{e^2}\right.$$
$$\overline{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/tan(d*x
+ c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) -
2*sqrt(e/tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(sqrt(2)*sqrt(e)*sqrt
(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e) + sqrt(2)*log(-sqrt(2)*sqrt(
e)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e))/e^2 + 8/(e^2*sqrt(e/
tan(d*x + c))))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(e\cot\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{\sec\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*cot(c +
 d*x))**(3/2), x))
```

3.238 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=357

$$\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}{\sqrt{2}d}-\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{5}{2}}(c+dx)(e\cot$$

```
[Out] -4/3*a^2*(e*cot(d*x+c))^(5/2)*tan(d*x+c)/d-4/3*a^2*(e*cot(d*x+c))^(5/2)*sec
(d*x+c)*tan(d*x+c)/d+2/3*a^2*(e*cot(d*x+c))^(5/2)*(sin(c+1/4*Pi+d*x)^2)^(1/
2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*
d*x+2*c)^(1/2)*tan(d*x+c)^2/d-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(
e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/2)-1/2*a^2*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/2)+1/4*a^2*(e*c
ot(d*x+c))^(5/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(5/2)
/d*2^(1/2)-1/4*a^2*(e*cot(d*x+c))^(5/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d
*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3900, 3886, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2609, 2614, 2573, 2641, 2607, 30}

$$\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}{\sqrt{2}d}-\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{5}{2}}(c+dx)(e\cot$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x])/(3*d) - (4*a^2*(e*Cot[c + d*x]
)^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(3*d) - (2*a^2*(e*Cot[c + d*x])^(5/2)*El
lipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]]*Tan[c + d*x]
^2)/(3*d) + (a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(5
/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) + (a^2*(e*Cot
[c + d*x])^(5/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c +
 d*x]^(5/2))/(2*Sqrt[2]*d) - (a^2*(e*Cot[c + d*x])^(5/2)*Log[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt[2]*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \frac{(a+a\sec(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \left(\frac{a^2}{\tan^{\frac{5}{2}}(c+dx)} + \frac{2a^2\sec(c+\ldots}{\tan^{\frac{5}{2}}(c+d\ldots}\right.$$

$$= \left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx + \left(a^2(e\cot\ldots\right.$$

$$= -\frac{2a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+\ldots}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+\ldots}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+\ldots}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+\ldots}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+\ldots}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+\ldots}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+\ldots}{3d}$$

Mathematica [C] time = 2.33, size = 93, normalized size = 0.26

$$-\frac{2a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)(e\cot(c+dx))^{3/2}\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\left(2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\tan^2(c+dx)\right)-{}_2F_1\left(\frac{3}{4},\ldots\right.\right.}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*e*Cos[(c + d*x)/2]^4*(e*Cot[c + d*x])^(3/2)*(2 + 2*Hypergeometric2F
1[-3/4, 1/2, 1/4, -Tan[c + d*x]^2] - Hypergeometric2F1[3/4, 1, 7/4, -Cot[c
+ d*x]^2])*Sec[ArcCot[Cot[c + d*x]]/2]^4)/(3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 2.68, size = 650, normalized size = 1.82

$$\frac{a^2(-1+\cos(dx+c))\left(3i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{1\cdots}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*a^2/d*(-1+cos(d*x+c))*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*sin(d*x+c)-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*sin(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*s
in(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+
c)-2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+4*cos(d*x+c)*2^
(1/2))*(e*cos(d*x+c)/sin(d*x+c))^(5/2)*(1+cos(d*x+c))^2/cos(d*x+c)^3/sin(d*
x+c)*2^(1/2)
```

maxima [A] time = 0.50, size = 214, normalized size = 0.60

$$-\frac{8\,a^2\left(\frac{e}{\tan(dx+c)}\right)^{\frac{5}{2}}\tan(dx+c)-\left(3\,e^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log(\cdots}{}\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(8*a^2*(e/tan(d*x + c))^(5/2)*tan(d*x + c) - (3*e^2*(2*sqrt(2)*arctan
(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/tan(d*x + c)))/sqrt(e))/sqrt(e) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) - 2*sqrt(e/tan(d*x + c)))/sq
rt(e))/sqrt(e) - sqrt(2)*log(sqrt(2)*sqrt(e)*sqrt(e/tan(d*x + c)) + e + e/t
an(d*x + c))/sqrt(e) + sqrt(2)*log(-sqrt(2)*sqrt(e)*sqrt(e/tan(d*x + c)) +
e + e/tan(d*x + c))/sqrt(e)) - 8*(e/tan(d*x + c))^(3/2))*a^2*e)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cot(c+d\,x))^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
[Out] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.239 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=343

$$-\frac{4a^2\sin(c+dx)(e\cot(c+dx))^{3/2}}{d}+\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}\,d}-\frac{a^2\tan^{-1}\left(\sqrt{2}\right.}{}$$

```
[Out] -4*a^2*(e*cot(d*x+c))^(3/2)*sin(d*x+c)/d-4*a^2*(e*cot(d*x+c))^(3/2)*tan(d*x
+c)/d+4*a^2*(e*cot(d*x+c))^(3/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d
*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*sin(d*x+c)*tan(d*x+c)/d/sin(2*d*x+
2*c)^(1/2)-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)
*tan(d*x+c)^(3/2)/d*2^(1/2)-1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*c
ot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/d*2^(1/2)-1/4*a^2*(e*cot(d*x+c))^(3/2)*ln
(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+1/4*a^2*
(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(
3/2)/d*2^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3900, 3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2608, 2615, 2572, 2639, 2607, 30}

$$-\frac{4a^2\sin(c+dx)(e\cot(c+dx))^{3/2}}{d}+\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}\,d}-\frac{a^2\tan^{-1}\left(\sqrt{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x])/d - (4*a^2*(e*Cot[c + d*x])^(3
/2)*Tan[c + d*x])/d - (4*a^2*(e*Cot[c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*
x, 2]*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (a^2*ArcTan[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(
Sqrt[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(
3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d) - (a^2*(e*Cot[c + d*x])^(3/2)*Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d
) + (a^2*(e*Cot[c + d*x])^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
```

```
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{(a+a\sec(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \left(\frac{a^2}{\tan^{\frac{3}{2}}(c+dx)} + \frac{2a^2\sec(c+}{\tan^{\frac{3}{2}}(c+d}\right.$$

$$= \left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx + \left(a^2(e\cot\right.$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}\tan(c+}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+}{d}$$

Mathematica [C] time = 5.25, size = 220, normalized size = 0.64

$$-\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)(e\cot(c+dx))^{3/2}\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\left(16\sqrt{\cot(c+dx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\tan^2(c+dx\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*(a^2*Cos[(c + d*x)/2]^4*(e*Cot[c + d*x])^(3/2)*(2*Sqrt[2]*ArcTan[1 - S
qrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
]] + 16*Sqrt[Cot[c + d*x]] + 16*Sqrt[Cot[c + d*x]]*Hypergeometric2F1[-1/4,
1/2, 3/4, -Tan[c + d*x]^2] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*
Sec[ArcCot[Cot[c + d*x]]/2]^4)/(d*Cot[c + d*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 2.54, size = 1392, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*a^2/d*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x
+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+4*EllipticF(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-8*EllipticE(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)+4*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-8*EllipticE
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+8*cos(d*x+c)*2^(1/2))*sin(d*x+c)*(e*cos(
d*x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)^2*2^(1/2)
```

maxima [A] time = 0.46, size = 208, normalized size = 0.61

$$-\frac{8\,a^2\left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)-\left(2\sqrt{2}\sqrt{e}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)+2\sqrt{2}\sqrt{e}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+}}\right.}{2\sqrt{e}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*a^2*(e/tan(d*x + c))^(3/2)*tan(d*x + c) - (2*sqrt(2)*sqrt(e)*arctan
(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/tan(d*x + c)))/sqrt(e)) + 2*sqrt(2
)*sqrt(e)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) - 2*sqrt(e/tan(d*x + c)))/sq
rt(e)) + sqrt(2)*sqrt(e)*log(sqrt(2)*sqrt(e)*sqrt(e/tan(d*x + c)) + e + e/t
an(d*x + c)) - sqrt(2)*sqrt(e)*log(-sqrt(2)*sqrt(e)*sqrt(e/tan(d*x + c)) +
e + e/tan(d*x + c)) - 8*sqrt(e/tan(d*x + c)))*a^2*e)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cot(c+d\,x))^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.240 $\int \sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=311

$$-\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}{\sqrt{2}d}+\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{\tan(c+dx)}\sqrt{\cdot}}{\sqrt{2}d}$$

```
[Out] -2*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi
+d*x),2^(1/2))*sec(d*x+c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/d+1/2*a
^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2
)/d*2^(1/2)+1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)
*tan(d*x+c)^(1/2)/d*2^(1/2)-1/4*a^2*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c
))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+1/4*a^2*ln(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+2
*a^2*(e*cot(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.29, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3900, 3886, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 2607, 30}

$$-\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}{\sqrt{2}d}+\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{\tan(c+dx)}\sqrt{\cdot}}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sqrt[e*Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[
Sin[2*c + 2*d*x]])/d - (a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) - (a^2
*Sqrt[e*Cot[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sq
rt[Tan[c + d*x]])/(2*Sqrt[2]*d) + (a^2*Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*
a^2*Sqrt[e*Cot[c + d*x]]*Tan[c + d*x])/d
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&& !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))^2\,dx = \left(\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \left(\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \left(\frac{a^2}{\sqrt{\tan(c+dx)}}+\frac{2a^2\sec(c+dx)}{\sqrt{\tan(c+dx)}}+$$

$$= \left(a^2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx+\left(a^2\sqrt{e\cot(c+$$

$$= \frac{\left(2a^2\sqrt{e\cot(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{\sin(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}+\frac{\left(a^2\sqrt{e\,c}\right.}{}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,\tan(c+dx)}{d}+\left(2a^2\sqrt{e\cot(c+dx)}\,\sec(c+dx)\sqrt{\sin}\right.$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d}+\frac{2a^2}{}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d}+\frac{2a^2}{}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d}-\frac{a^2\sqrt{}}{}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d}-\frac{a^2\,\mathrm{t}}{}$$

Mathematica [C] time = 1.85, size = 118, normalized size = 0.38

$$\frac{a^2e(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\left(6\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)-2\cot^2(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\mathrm{co}\right.\right.}{6d\sqrt{e\cot(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*e*(1 + Cos[c + d*x])^2*(3*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x
]^2] + 6*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2] - 2*Cot[c + d*x]
```

```
^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*Sec[ArcCot[Cot[c + d*x]
]/2]^4)/(6*d*Sqrt[e*Cot[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e \cot(dx + c)}\, (a \sec(dx + c) + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 2.72, size = 655, normalized size = 2.11

$$\frac{a^2 (1 + \cos(dx + c))^2 \sqrt{\frac{e \cos(dx+c)}{\sin(dx+c)}}\, (-1 + \cos(dx + c)) \left(-i \operatorname{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}, \frac{1}{2} - \frac{i}{2}, \frac{\sqrt{2}}{2}\right) \sqrt{\frac{-1+c}{\sin}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x)
[Out] 1/2*a^2/d*(1+cos(d*x+c))^2*(e*cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*
(-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticP
i(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+I*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-2*EllipticF(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*sin(d*x+c)+2*cos(d*x+c)*2^(1/2)-2*2^(1/2))/cos(d*x+c)/sin(d
*x+c)^3*2^(1/2)
```

maxima [A] time = 0.47, size = 193, normalized size = 0.62

$$-\frac{a^2 e \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} + \frac{\sqrt{2}}{}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*e*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/tan(d
*x + c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e)
 - 2*sqrt(e/tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(sqrt(2)*sqrt(e)*s
qrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e) + sqrt(2)*log(-sqrt(2)*sq
rt(e)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e)) - 8*a^2*sqrt(e/ta
n(d*x + c))*tan(d*x + c))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\cot\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2} \left(\int \sqrt{e \cot{\left(c + d x \right)}}\, dx + \int 2 \sqrt{e \cot{\left(c + d x \right)}} \sec{\left(c + d x \right)}\, dx + \int \sqrt{e \cot{\left(c + d x \right)}} \sec^{2}{\left(c + d x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(2*sqrt(e*cot(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*cot(c + d*x))*sec(c + d*x)**2, x))
```

3.241 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=339

$$\frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}+$$

```
[Out] 4*a^2*sin(d*x+c)/d/(e*cot(d*x+c))^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)
^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/d/(e*cot(d
*x+c))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2
))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*a^2*arctan(1+2^(1/2)
*tan(d*x+c)^(1/2))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*a^2*
ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/ta
n(d*x+c)^(1/2)-1/4*a^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/
(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+2/3*a^2*tan(d*x+c)/d/(e*cot(d*x+c))^(
1/2)
```

Rubi [A] time = 0.32, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3900, 3886, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 2607, 30}

$$\frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (4*a^2*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x]]) - (4*a^2*Cos[c + d*x]*Ellipti
cE[c - Pi/4 + d*x, 2])/(d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - (a
^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*
Sqrt[Tan[c + d*x]]) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]
*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c +
d*x]]) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2
]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (2*a^2*Tan[c + d*x])/(3*d*Sq
rt[e*Cot[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
```

```
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \frac{\int (a+a\sec(c+dx))^2\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \left(a^2\sqrt{\tan(c+dx)}+2a^2\sec(c+dx)\sqrt{\tan(c+dx)}+a^2\sec^2(c+dx)\sqrt{\tan(c+dx)}\right.}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{a^2\int\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a^2\int\sec^2(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\left(2a^2\right)\int\sec}{\sqrt{e\cot(}}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{\left(4a^2\right)\int\cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a^2\,\mathrm{Subst}\left(\int\sqrt{x}\,dx,x,\right.}{d\sqrt{e\cot(c+dx)}\sqrt{\mathrm{ta}}}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} + \frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}} - \frac{\left(4a^2\sqrt{\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin}}{\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} + \frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}} - \frac{\left(4a^2\cos(c+dx)\right)\int\sqrt{\sin(2c+2dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} -$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} + \frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}} + \frac{a^2\,\mathrm{Su}}{}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} + \frac{a^2\log\left(1-\sqrt{2}\sqrt{\tan(c+d}\right.}{2\sqrt{2}\,d\sqrt{e\cot(c+dx)}}$$

$$= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} - \frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c}\right.}{\sqrt{2}\,d\sqrt{e\cot(c+dx)}\sqrt{\tan(}}$$

Mathematica [C] time = 11.46, size = 221, normalized size = 0.65

$$\frac{a^2 \sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\left(8\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+4\,{}_2F_1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^5*(4*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2]
 + 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*Cot[c +
d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] -
 Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))*Sec[c + d*x]*Sec[ArcC
ot[Cot[c + d*x]]/2]^4*Sin[(c + d*x)/2])/(3*d*Sqrt[e*Cot[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\cot(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

maple [C] time = 2.35, size = 1480, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x)
```

```
[Out] 1/6*a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(3*I*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-3*I*EllipticPi(((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+3*I*EllipticPi(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-3*I*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
```

```
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
+1/2*I,1/2*2^(1/2))+24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*c
os(d*x+c)^2*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ell
ipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-3*Elliptic
Pi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(
d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-3*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+24*EllipticE(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-12*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)-14*cos(d*x+c)^2*2^(1/2)+12*cos(d*x+c)*2^(1/2)+2*2^(1/2))/c
os(d*x+c)/sin(d*x+c)^5/(e*cos(d*x+c)/sin(d*x+c))^(1/2)*2^(1/2)
```

maxima [A] time = 0.51, size = 194, normalized size = 0.57

$$-\frac{3\,a^2e\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{e^{\frac{3}{2}}}-\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^2*e*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/ta
n(d*x + c)))/sqrt(e))/e^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(e) - 2*sqrt(e/tan(d*x + c)))/sqrt(e))/e^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(e
)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/e^(3/2) - sqrt(2)*log(-sqrt(2)
*sqrt(e)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/e^(3/2)) - 8*a^2*tan(d*
x + c)/sqrt(e/tan(d*x + c)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{e\cot\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{1}{\sqrt{e\cot\left(c+dx\right)}}\,dx+\int\frac{2\sec\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx+\int\frac{\sec^2\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
cot(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*cot(c + d*x)), x))
```

3.242 $\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=375

$$\frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}}+\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))}$$

```
[Out] 2*a^2*cot(d*x+c)/d/(e*cot(d*x+c))^(3/2)+4/3*a^2*csc(d*x+c)/d/(e*cot(d*x+c))
^(3/2)+2/3*a^2*cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*
Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/d/(e*cot(
d*x+c))^(3/2)-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^
(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d
/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*a^2*ln(1-2^(1/2)*tan(d*x
+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*a
^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)
/tan(d*x+c)^(3/2)+2/5*a^2*tan(d*x+c)/d/(e*cot(d*x+c))^(3/2)
```

Rubi [A] time = 0.33, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3900, 3886, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2611, 2614, 2573, 2641, 2607, 30}

$$\frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}}+\frac{a^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*Cot[c + d*x])/(d*(e*Cot[c + d*x])^(3/2)) + (4*a^2*Csc[c + d*x])/(3*d
*(e*Cot[c + d*x])^(3/2)) - (2*a^2*Cot[c + d*x]*Csc[c + d*x]*EllipticF[c - P
i/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*(e*Cot[c + d*x])^(3/2)) + (a^2*A
rcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Ta
n[c + d*x]^(3/2)) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d
*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x
]^(3/2)) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt
[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + (2*a^2*Tan[c + d*x])/(5*
d*(e*Cot[c + d*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{\int (a+a\sec(c+dx))^2\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \left(a^2\tan^{\frac{3}{2}}(c+dx)+2a^2\sec(c+dx)\tan^{\frac{3}{2}}(c+dx)+a^2\sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)\right) d}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2\int \tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{a^2\int \sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{(2a^2)\int \sec(c}{(e\cot(c+d}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{(2a^2)\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{}{(e\cot(}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}} - \frac{\left(2a^2\cos^{\frac{3}{2}}(c+dx)\right.}{3(e\cot(c+}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}} - \frac{\left(2a^2\cot(c+dx)\,c\right.}{}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\right.}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\right.}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\right.}{3d(e\cot(c+dx))^{3/2}}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\right.}{3d(e\cot(c+dx))^{3/2}}$$

Mathematica [C] time = 6.27, size = 127, normalized size = 0.34

$$\frac{a^2\sin^2(c+dx)(\sec(c+dx)+1)^2\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\left({}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)+2\left({}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-\right.\right.\right.}{10de\sqrt{e\cot(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + 2*(5*Cot[c + d*x]
^2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Hypergeometric2F1[1/2
, 5/4, 9/4, -Tan[c + d*x]^2]))*(1 + Sec[c + d*x])^2*Sec[ArcCot[Cot[c + d*x]
]/2]^4*Sin[c + d*x]^2)/(10*d*e*Sqrt[e*Cot[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)
```

maple [C] time = 2.28, size = 721, normalized size = 1.92

$$-\frac{a^2(-1+\cos(dx+c))\left(15i\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(\ldots}{\ldots}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x)
```

```
[Out] -1/30*a^2/d*(-1+cos(d*x+c))*(15*I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-15*I*sin(d*x+c)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-15*sin(d*x+c)*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+10*sin(d*x+c)*Ellipti
cF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-15*sin(d*x+c)*EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-24*2^(1/2)*cos(d*x
+c)^3+4*cos(d*x+c)^2*2^(1/2)+14*cos(d*x+c)*2^(1/2)+6*2^(1/2))*(1+cos(d*x+c)
)^2/(e*cos(d*x+c)/sin(d*x+c))^(3/2)/sin(d*x+c)^5/cos(d*x+c)*2^(1/2)
```

maxima [A] time = 0.49, size = 216, normalized size = 0.58

$$\frac{5a^2e\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\sqrt{e}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{e}\sqrt{\ldots}\right.}{}}{e^2}\right.}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(5*a^2*e*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/ta
n(d*x + c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(e) - 2*sqrt(e/tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*log(sqrt(2)*sqrt(e
```

```
)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e) + sqrt(2)*log(-sqrt(2)
*sqrt(e)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e))/e^2 + 8/(e^2*s
qrt(e/tan(d*x + c)))) + 8*a^2*tan(d*x + c)/(e/tan(d*x + c))^(3/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(e\cot\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{2\sec\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{\sec^2\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*co
t(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*cot(c + d*x))**(3/2),
x))
```

3.243 $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=405

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}ad}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(}{\sqrt{2}ad}$$

```
[Out] 2/5*cot(d*x+c)*(e*cot(d*x+c))^(3/2)*(1-sec(d*x+c))/a/d-2/5*(e*cot(d*x+c))^(
3/2)*(5-3*sec(d*x+c))*tan(d*x+c)/a/d-6/5*(e*cot(d*x+c))^(3/2)*(sin(c+1/4*Pi
+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*sin(d
*x+c)*tan(d*x+c)/a/d/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^
(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/a/d*2^(1/2)-1/2*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/a/d*2^(1/2)-1/4*
(e*cot(d*x+c))^(3/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(
3/2)/a/d*2^(1/2)+1/4*(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan
(d*x+c))*tan(d*x+c)^(3/2)/a/d*2^(1/2)-6/5*(e*cot(d*x+c))^(3/2)*sin(d*x+c)*t
an(d*x+c)^2/a/d
```

Rubi [A] time = 0.43, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3900, 3888, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}{\sqrt{2}ad}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)(e\cot(}{\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(e*Cot[c + d*x])^(3/2)*(1 - Sec[c + d*x]))/(5*a*d) - (2*(e*
Cot[c + d*x])^(3/2)*(5 - 3*Sec[c + d*x])*Tan[c + d*x])/(5*a*d) + (6*(e*Cot[
c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*x, 2]*Sin[c + d*x]*Tan[c + d*x])/(5*
a*d*Sqrt[Sin[2*c + 2*d*x]]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Co
t[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*a*d) - (ArcTan[1 + Sqrt[2]*S
qrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*a*d)
 - ((e*Cot[c + d*x])^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*a*d) + ((e*Cot[c + d*x])^(3/2)*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*a*d
) - (6*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x]*Tan[c + d*x]^2)/(5*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} + \frac{\left(2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int}{5a^2}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))}{5ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))}{5ad}$$

Mathematica [C] time = 4.05, size = 316, normalized size = 0.78

$$-\frac{e\sin^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\cot(c+dx)}\left(-40\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+24\cot^4(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/30*(e*Sqrt[e*Cot[c + d*x]]*(30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d
*x]]]*Cot[c + d*x]^(3/2) - 30*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
]*Cot[c + d*x]^(3/2) + 120*Cot[c + d*x]^2 - 24*Cot[c + d*x]^4 + 24*Cot[c +
d*x]^4*Hypergeometric2F1[-5/4, -1/2, -1/4, -Tan[c + d*x]^2] - 120*Cot[c + d
*x]^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -Tan[c + d*x]^2] - 40*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 15*Sqrt[2]*Cot[c + d*x]^(3/2)*Log[1
 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 15*Sqrt[2]*Cot[c + d*x]^(3/
2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[c + d*x]*(1 + Sq
rt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

maple [C] time = 2.00, size = 2113, normalized size = 5.22

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/10/a/d*(-1+cos(d*x+c))*(10*I*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))+5*I*cos(d*x+c)^2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
+1/2*I,1/2*2^(1/2))+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*co
s(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x
+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^
2*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-5*I*cos(d*x+c)^2*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+10*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)+10*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)-24*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)+12*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+5*I*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
```

```
))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*I*((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+5*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)+5*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)-12*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/
2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+6*EllipticF(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)-6*cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2^(1/2))*(e*c
os(d*x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)^2/sin(d*x+c)*2^(1/2)
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)^{3/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*cot(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
[Out] Integral((e*cot(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.244 $\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=325

$$-\frac{\sqrt{\tan(c+dx)}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}ad}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}{\sqrt{2}ad}$$

```
[Out] 2/3*cot(d*x+c)*(1-sec(d*x+c))*(e*cot(d*x+c))^(1/2)/a/d+1/3*(sin(c+1/4*Pi+d*
x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+
c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/a/d+1/2*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/2)+1/2*arctan(
1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/
2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d
*x+c)^(1/2)/a/d*2^(1/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*co
t(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3900, 3888, 3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{\sqrt{\tan(c+dx)}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}ad}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}{\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*Sqrt[e*Cot[c + d*x]]*(1 - Sec[c + d*x]))/(3*a*d) - (Sqrt[e*
Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*
x]])/(3*a*d) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*Cot[c + d*x]]
*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d) - (Sqrt[e*Cot[c +
d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]]
)/(2*Sqrt[2]*a*d) + (Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&& !IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} + \frac{\left(2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \cdots}{3a^2}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \cdots}{3a}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}\right)\int \cdots}{3a\sqrt{\cos(c+dx)}}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sec(c+dx)\sqrt{\sin(2c\cdots}}{3a}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec\cdots}{3ad}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec\cdots}{3ad}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec\cdots}{3ad}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec\cdots}{3ad}$$

Mathematica [C] time = 1.71, size = 135, normalized size = 0.42

$$-\frac{4\sin^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\cot(c+dx)}\left(3\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)+\cot^2(c+\cdots\right.}{3ad}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(Cot[c + d*x]^2*Hypergeometric2F1[-3/
4, -1/2, 1/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c
+ d*x]^2] + Cot[c + d*x]^2*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*
x]^2]))*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(dx+c)}}{a \sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(a*sec(d*x + c) + a), x)
```

maple [C] time = 2.12, size = 1269, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/6/a/d*(e*cos(d*x+c)/sin(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*
(3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))+3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin
(d*x+c)-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+
c)+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))-8*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/
2))+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
```

```
))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-8*Elliptic
F(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2*2^(1/2)-2*cos
(d*x+c)*2^(1/2))/sin(d*x+c)^5/cos(d*x+c)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(dx+c)}}{a \sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(e*cot(d*x + c))/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\sqrt{e \cot(c+d\,x)}}{a\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*cot(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e \cot(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
[Out] Integral(sqrt(e*cot(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.245 $\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=347

$$\frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)(1-\text{se}}{ad\sqrt{e\cot(c+}}$$

```
[Out] 2*cot(d*x+c)*(1-sec(d*x+c))/a/d/(e*cot(d*x+c))^(1/2)+2*sin(d*x+c)/a/d/(e*co
t(d*x+c))^(1/2)+2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*
EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a/d/(e*cot(d*x+c))^(1/2)/sin(2*d*x+2*c
)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d*2^(1/2)/(e*cot(d*x+c))^
(1/2)/tan(d*x+c)^(1/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d*2^(1/2)/(
e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(
d*x+c))/a/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)-1/4*ln(1+2^(1/2)*
tan(d*x+c)^(1/2)+tan(d*x+c))/a/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1
/2)
```

Rubi [A] time = 0.36, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3900, 3888, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)(1-\text{se}}{ad\sqrt{e\cot(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cot[c + d*x]*(1 - Sec[c + d*x]))/(a*d*Sqrt[e*Cot[c + d*x]]) + (2*Sin[c +
 d*x])/(a*d*Sqrt[e*Cot[c + d*x]]) - (2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*
x, 2])/(a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]
) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]
]*Sqrt[Tan[c + d*x]]) + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/
(2*Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - Log[1 + Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[
Tan[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

IntegerQ[n]

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\tan(c+dx)}}{a+a\sec(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\int\left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{\tan(c+}}{a^2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+d}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{\int\sqrt{\tan(c+dx)}\,dx}{a\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}} +$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\int\cos(c+d}{a\sqrt{e\cot(c+}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{\left(2\sqrt{\cos(c+d}\right.}{a\sqrt{}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{(2\cos(c+dx)}{a\sqrt{e\cot(c+}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(c+dx}{ad\sqrt{e\cot(c+}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(c+dx}{ad\sqrt{e\cot(c+}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(c+dx}{ad\sqrt{e\cot(c+}}$$

Mathematica [C] time = 3.71, size = 249, normalized size = 0.72

$$-\frac{\sin^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\left(8\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+24\cot^2(c+dx)\,{}_2F_1\left(-\frac{1}{2},-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] -1/6*(Csc[c + d*x]*(24*Cot[c + d*x]^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -T
an[c + d*x]^2] + 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 3*Co
t[c + d*x]^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt
[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]
*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[
2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c +
 d*x)/2]^2)/(a*d*Sqrt[e*Cot[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cot\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)), x)
```

maple [C] time = 2.07, size = 352, normalized size = 1.01

$$-\frac{\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\left(1+\cos\left(dx+c\right)\right)^{2}\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx}{\sin(dx+c)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x)
```

```
[Out] -1/2/a/d*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(1+cos(d*x+c))^
2*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*
2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))+4*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2
*2^(1/2))-2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*
2^(1/2))-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2)))*(-1+cos(d*x+c))/sin(d*x+c)^3/(e*cos(d*x+c)/sin(d*x+c))^(1/2)*
2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cot\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{a\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\cot\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{e\cot\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*sec(c + d*x) + sqrt(e*cot(c + d*x))), x)/a
```

3.246 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=290

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}\right.}{2\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))}$$

```
[Out] -cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellipt
icF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a/d/(e*cot(d*x+c))^(3/2
)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/
tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^
(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)
)/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*tan(d*
x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A] time = 0.29, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3900, 3888, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}\right.}{2\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cot[c + d*x]*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*
x]])/(a*d*(e*Cot[c + d*x])^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/
(Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - ArcTan[1 + Sqrt[2
]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2
)) + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*C
ot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \frac{-a+a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\cos^{\frac{3}{2}}(c+dx)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx, x, \tan\right.}{ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(}$$

$$= \frac{\left(\cot(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{a(e\cot(c+dx))^{3/2}} - \frac{2\,\text{Subs}}{ad(e}$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int\right.}{ad(e\,\text{co}}$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int\right.}{2ad(e}$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} + \frac{\log\left(1-\right.}{2\sqrt{2}\,a}$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} + \frac{\tan^{-1}}{\sqrt{2}\,ad(e}$$

Mathematica [C] time = 8.86, size = 112, normalized size = 0.39

$$\frac{4\sin^2\left(\frac{1}{2}(c+dx)\right)\cot^2(c+dx)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\left(3\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)+\cot^2(c+dx)\,{}_2F_1\right.}{3ad(e\cot(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (4*Cot[c + d*x]^2*Csc[c + d*x]*(3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c +
 d*x]^2] + Cot[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*
```

```
(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d*(e*Cot[c + d*x])^(3/2
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e \cot\left(dx+c\right)\right)^{\frac{3}{2}} \left(a \sec\left(dx+c\right)+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

maple [C] time = 1.97, size = 319, normalized size = 1.10

$$\frac{\left(1+\cos\left(dx+c\right)\right)^{2} \left(i \operatorname{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}, \frac{1}{2}-\frac{i}{2}, \frac{\sqrt{2}}{2}\right) - i \operatorname{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}, \frac{1}{2}+\frac{i}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
[Out] 1/2/a/d*(1+cos(d*x+c))^2*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticF(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)/(e*cos(d*x+c)/sin(d*x+c))^(3/2
)/sin(d*x+c)^4*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e \cot\left(dx+c\right)\right)^{\frac{3}{2}} \left(a \sec\left(dx+c\right)+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{a\,{\left(e\,\cot\left(c+d\,x\right)\right)}^{3/2}\,\left(\cos\left(c+d\,x\right)+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*sec(c + d*x) + (e*cot(c + d*x))**(3/2))
, x)/a
```

3.247 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=325

$$\frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\log\left(\mathrm{ta}\right.}{2\sqrt{2}}$$

```
[Out] 2*cos(d*x+c)*cot(d*x+c)/a/d/(e*cot(d*x+c))^(5/2)+2*cos(d*x+c)*cot(d*x+c)^2*
(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2
^(1/2))/a/d/(e*cot(d*x+c))^(5/2)/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)
*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/2*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c
)^(5/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(5
/2)*2^(1/2)/tan(d*x+c)^(5/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/
a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)
```

Rubi [A] time = 0.32, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3900, 3888, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$\frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\log\left(\mathrm{ta}\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cos[c + d*x]*Cot[c + d*x])/(a*d*(e*Cot[c + d*x])^(5/2)) - (2*Cos[c + d*x
]*Cot[c + d*x]^2*EllipticE[c - Pi/4 + d*x, 2])/(a*d*(e*Cot[c + d*x])^(5/2)*
Sqrt[Sin[2*c + 2*d*x]]) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a
*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - Log[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x
])^(5/2)*Tan[c + d*x]^(5/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$
$$= \frac{\int(-a+a\sec(c+dx))\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$
$$= -\frac{\int\sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} + \frac{\int\sec(c+dx)\sqrt{\tan(c+dx)}\,d}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+d}$$
$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\int\cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\text{Su}}{ac}$$
$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{\left(2\cos^{\frac{5}{2}}(c+dx)\right)\int\sqrt{\cos(c+dx)}\sqrt{\text{si}}}{a(e\cot(c+dx))^{5/2}\sin^{\frac{5}{2}}(c+}$$
$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{\left(2\cos(c+dx)\cot^2(c+dx)\right)\int\sqrt{\sin(2}}{a(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2}}$$
$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+\right.}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx}}$$
$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+\right.}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx}}$$
$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+\right.}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx}}$$

Mathematica [C] time = 62.67, size = 194, normalized size = 0.60

$$-\frac{\sin^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\cot(c+dx)}\left(3\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\left(\log\left(\cot(c+dx)-\sqrt{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] -1/6*(Sqrt[e*Cot[c + d*x]]*(-8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*
x]^2] + 3*Sqrt[2]*Cot[c + d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x
]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c
 + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]]))*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(a*d*e^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

maple [C] time = 2.40, size = 1419, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2/a/d*(-1+cos(d*x+c))^2*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)+I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/
2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-I*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)-4*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-E
```

```
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)-4*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2*cos(d*x+c)*2^(1/2)-2*2
^(1/2))*cos(d*x+c)^2*(1+cos(d*x+c))^2/sin(d*x+c)^7/(e*cos(d*x+c)/sin(d*x+c)
)^(5/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}\,(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{a\,(e\cot(c+d\,x))^{5/2}\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.248 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=335

$$-\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}-\frac{\log}{2}$$

```
[Out] -2/3*cot(d*x+c)^3*(3-sec(d*x+c))/a/d/(e*cot(d*x+c))^(7/2)+1/3*cot(d*x+c)^3*
csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/
4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a/d/(e*cot(d*x+c))^(7/2)+1/2*arctan
(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(
7/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2
)/tan(d*x+c)^(7/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot
(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)
```

Rubi [A] time = 0.34, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3900, 3888, 3881, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641}

$$-\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}-\frac{\log}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-2*Cot[c + d*x]^3*(3 - Sec[c + d*x]))/(3*a*d*(e*Cot[c + d*x])^(7/2)) - (Co
t[c + d*x]^3*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x
]])/(3*a*d*(e*Cot[c + d*x])^(7/2)) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + ArcTan[1 + Sqrt[
2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/
2)) - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*
Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int(-a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cos^{\frac{7}{2}}(c+dx)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx}}}{3a(e\cot(c+dx))^{7/2}\sin^{\frac{7}{2}}(c+d}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\left(\cot^3(c+dx)\csc(c+dx)\sqrt{\sin(2c}\right.}{3a(e\cot(c+}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}\right.}{3ad(e\cot(c+}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}\right.}{3ad(e\cot(c+}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}\right.}{3ad(e\cot(c+}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}\right.}{3ad(e\cot(c+}$$

Mathematica [C] time = 15.93, size = 130, normalized size = 0.39

$$-\frac{4\sin^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\cot(c+dx)}\left(-3\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\tan^2(c+dx)\right)+3\,{}_2F_1\right.}{3ade^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]

[Out] (-4*Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(3 - 3*Hypergeometric2F1[-1/2, 1/4, 5/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2] + Cot[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d*e^4)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(dx+c))^{\frac{7}{2}}\,(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)

maple [C] time = 2.38, size = 698, normalized size = 2.08

$$-\frac{(-1+\cos(dx+c))\left(3i\cos(dx+c)\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x)

[Out] -1/6/a/d*(-1+cos(d*x+c))*(3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/si

```
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+6*cos(d*x+c)^2*2^(1/2)-8*cos(d*x+c)*
2^(1/2)+2*2^(1/2))*cos(d*x+c)^2*(1+cos(d*x+c))^2/(e*cos(d*x+c)/sin(d*x+c))^
(7/2)/sin(d*x+c)^7*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e \cot\left(dx+c\right)\right)^{\frac{7}{2}} \left(a \sec\left(dx+c\right)+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{a\,{\left(e\,\cot\left(c+d\,x\right)\right)}^{7/2}\,\left(\cos\left(c+d\,x\right)+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(7/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(7/2)*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

3.249 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=371

$$-\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}}-\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\tan^{-}}{\sqrt{2}\,ad\,\mathrm{ta}}$$

```
[Out] -6/5*cos(d*x+c)*cot(d*x+c)^3/a/d/(e*cot(d*x+c))^(9/2)-2/15*cot(d*x+c)^3*(5-
3*sec(d*x+c))/a/d/(e*cot(d*x+c))^(9/2)-6/5*cos(d*x+c)*cot(d*x+c)^4*(sin(c+1
/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/
a/d/(e*cot(d*x+c))^(9/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x
+c)^(1/2))/a/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/2*arctan(1+2
^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+
1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(9/2)*2^(1
/2)/tan(d*x+c)^(9/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*c
ot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)
```

Rubi [A] time = 0.37, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3900, 3888, 3881, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639}

$$-\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}}-\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\tan^{-}}{\sqrt{2}\,ad\,\mathrm{ta}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-6*Cos[c + d*x]*Cot[c + d*x]^3)/(5*a*d*(e*Cot[c + d*x])^(9/2)) - (2*Cot[c
+ d*x]^3*(5 - 3*Sec[c + d*x]))/(15*a*d*(e*Cot[c + d*x])^(9/2)) + (6*Cos[c +
 d*x]*Cot[c + d*x]^4*EllipticE[c - Pi/4 + d*x, 2])/(5*a*d*(e*Cot[c + d*x])^
(9/2)*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqr
t[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) +
 Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c
 + d*x])^(9/2)*Tan[c + d*x]^(9/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{9}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$

$$= \frac{\int(-a+a\sec(c+dx))\tan^{\frac{5}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} - \frac{2\int\left(-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)\right)\sqrt{\text{ta}}}{5a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(}$$

$$= -\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} - \frac{3\int\sec(c+dx)\sqrt{\tan(c+dx)}\,d}{5a(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{6}{5a}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{(6}{}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{(6}{}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{6}{}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{6}{}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{6}{}$$

Mathematica [C] time = 19.74, size = 261, normalized size = 0.70

$$\frac{\sin^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sqrt{\sec^2(c+dx)}+1\right)\sqrt{e\cot(c+dx)}\left(8\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)-8\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Sqrt[e*Cot[c + d*x]]*(-8 + 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
]*Cot[c + d*x]^(3/2) - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot
[c + d*x]^(3/2) + 8*Hypergeometric2F1[-1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 8*
Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*Cot[c + d*x]^
(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*Cot[c
+ d*x]^(3/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[c + d*
x]*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(6*a*d*e^5)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)), x)
```

maple [C] time = 2.09, size = 1505, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/30/a/d*(-1+cos(d*x+c))^2*(-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))-15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))-15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
cos(d*x+c)^3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))-36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(
d*x+c)^3*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)+18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*Ellipt
icF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+15*I*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+15*I*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-36*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2),1/2*2^(1/2))+28*2^(1/2)*cos(d*x+c)^3-24*cos(d*x+c)^2*2^(1/2)-10*cos(
d*x+c)*2^(1/2)+6*2^(1/2))*cos(d*x+c)^2*(1+cos(d*x+c))^2/(e*cos(d*x+c)/sin(d
*x+c))^(9/2)/sin(d*x+c)^9*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e \cot\left(dx + c\right)\right)^{\frac{9}{2}} \left(a \sec\left(dx + c\right) + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c + d\,x\right)}{a\,{\left(e\,\cot\left(c + d\,x\right)\right)}^{9/2}\,\left(\cos\left(c + d\,x\right) + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(9/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(9/2)*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(9/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

3.250 $\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=413

$$-\frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}-\frac{\tan^{-1}(}{\sqrt{2}\,a^2d\sqrt{\mathrm{ta}}}$$

```
[Out] 2*cot(d*x+c)/a^2/d/(e*cot(d*x+c))^(1/2)-12/5*cos(d*x+c)*cot(d*x+c)/a^2/d/(e
*cot(d*x+c))^(1/2)-4/5*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(1/2)+4/5*cot(d*x+
c)^2*csc(d*x+c)/a^2/d/(e*cot(d*x+c))^(1/2)+12/5*cos(d*x+c)*(sin(c+1/4*Pi+d*
x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e
*cot(d*x+c))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1
/2))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/
4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1
/2)/tan(d*x+c)^(1/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d*2^
(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 19, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {3900, 3888, 3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2609, 2608, 2615, 2572, 2639, 2607, 30}

$$-\frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}-\frac{\tan^{-1}(}{\sqrt{2}\,a^2d\sqrt{\mathrm{ta}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x])/(a^2*d*Sqrt[e*Cot[c + d*x]]) - (12*Cos[c + d*x]*Cot[c + d*
x])/(5*a^2*d*Sqrt[e*Cot[c + d*x]]) - (4*Cot[c + d*x]^3)/(5*a^2*d*Sqrt[e*Cot
[c + d*x]]) + (4*Cot[c + d*x]^2*Csc[c + d*x])/(5*a^2*d*Sqrt[e*Cot[c + d*x]]
) - (12*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2])/(5*a^2*d*Sqrt[e*Cot[c +
d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqr
t[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) +
 Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*Sqrt[e
*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx}{\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^4\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2}{\tan^{\frac{7}{2}}(c+dx)} - \frac{2a^2\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)} + \frac{a^2\sec^2(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\right)dx}{a^4\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}} + \frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}} - \ldots$$

$$= -\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{\int \frac{1}{\tan^{\frac{3}{2}}(c}}{a^2\sqrt{e\cot(c+dx}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx}}$$

Mathematica [F] time = 13.85, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cot\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

maple [C] time = 2.32, size = 2117, normalized size = 5.13

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x)
```

```
[Out] -1/10/a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^3*(-10*I*cos(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))+24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*cos(d*x+c)^2*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2*2^(1/2))-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(
d*x+c)^2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)+5*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellip
ticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+5
*I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-10*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-10*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+48*EllipticE(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)-24*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)-5*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*I
*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+
```

```
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)-5*Ellipt
icPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-5*EllipticPi(((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)+24*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)-12*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+2*cos(d*x+c)^2*2^(1/2)-2*
cos(d*x+c)*2^(1/2))/sin(d*x+c)^7/(e*cos(d*x+c)/sin(d*x+c))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cot\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{2}}{a^{2}\,\sqrt{e\cot\left(c+d\,x\right)}\,\left(\cos\left(c+d\,x\right)+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\cot\left(c+dx\right)}\sec^{2}\left(c+dx\right)+2\sqrt{e\cot\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{e\cot\left(c+dx\right)}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*cot(c + d*x))*s
ec(c + d*x) + sqrt(e*cot(c + d*x))), x)/a**2
```

3.251 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=359

$$-\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\mathrm{ta}\right.}{\sqrt{2}\,a^2d\tan^{\frac{3}{2}}(c+d}$$

```
[Out] -4/3*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(3/2)+4/3*cot(d*x+c)^2*csc(d*x+c)/a^
2/d/(e*cot(d*x+c))^(3/2)-2/3*cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1
/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(
1/2)/a^2/d/(e*cot(d*x+c))^(3/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2
/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*ln(1-2
^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(
d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x
+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A] time = 0.39, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3900, 3888, 3886, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2609, 2614, 2573, 2641, 2607, 30}

$$-\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\mathrm{ta}\right.}{\sqrt{2}\,a^2d\tan^{\frac{3}{2}}(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-4*Cot[c + d*x]^3)/(3*a^2*d*(e*Cot[c + d*x])^(3/2)) + (4*Cot[c + d*x]^2*Cs
c[c + d*x])/(3*a^2*d*(e*Cot[c + d*x])^(3/2)) + (2*Cot[c + d*x]*Csc[c + d*x]
*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*(e*Cot[c + d
*x])^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[
c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + Log[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/
2)*Tan[c + d*x]^(3/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^4(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \left(\frac{a^2}{\tan^{\frac{5}{2}}(c+dx)} - \frac{2a^2\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)} + \frac{a^2\sec^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\right)dx}{a^4(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\int}{3a^2(e\cot(c}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{\left(2\cos^{\frac{3}{2}}(c+\right.}{3a^2(e\,co}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{\left(2\cot(c+d\right.}{}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx}{}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx}{}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx}{}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx}{}$$

Mathematica [F] time = 10.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)

maple [C] time = 2.04, size = 1267, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x)

[Out] -1/6/a^2/d*(-1+cos(d*x+c))^2*(3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-10*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-10*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2^(1/2))*cos(d*x+c)*(1+cos(d*x+c))^2/(e*cos(d*x+c)/sin(d*x+c))^(3/2)/sin(d*x+c)^7*2^(1/2)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^2}{a^2 (e \cot(c+dx))^{3/2} (\cos(c+dx)+1)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.252 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=355

$$-\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan\right.}}{\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx}$$

```
[Out] -4*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(5/2)+4*cos(d*x+c)*cot(d*x+c)^3/a^2/d/
(e*cot(d*x+c))^(5/2)-4*cos(d*x+c)*cot(d*x+c)^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/
sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e*cot(d*x+c))
^(5/2)/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(
e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))/a^2/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/4*ln(1-2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+
c)^(5/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))
^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)
```

Rubi [A] time = 0.42, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3900, 3888, 3886, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2608, 2615, 2572, 2639, 2607, 30}

$$-\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan\right.}}{\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-4*Cot[c + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(5/2)) + (4*Cos[c + d*x]*Cot[c
+ d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(5/2)) + (4*Cos[c + d*x]*Cot[c + d*x]^2*E
llipticE[c - Pi/4 + d*x, 2])/(a^2*d*(e*Cot[c + d*x])^(5/2)*Sqrt[Sin[2*c + 2
*d*x]]) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c +
d*x])^(5/2)*Tan[c + d*x]^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(S
qrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - Log[1 - Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*T
an[c + d*x]^(5/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*
Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
```

```
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^4(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int \left(\frac{a^2}{\tan^{\frac{3}{2}}(c+dx)} - \frac{2a^2\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)} + \frac{a^2\sec^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\right)dx}{a^4(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} + \frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} - \frac{\int \sqrt{\tan}}{a^2(e\cot(c+}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{\left(4\cos^{\frac{5}{2}}(c+\right.}{a^2(e}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{\left(4\cos(c+dx\right.}{a^2(e\,c}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)}{a^2d(e\cot(}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)}{a^2d(e\cot(}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)}{a^2d(e\cot(}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)}{a^2d(e\cot(}$$

Mathematica [F] time = 5.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

maple [C] time = 2.14, size = 360, normalized size = 1.01

$$\frac{(1+\cos(dx+c))^2\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x)
[Out] 1/2/a^2/d*(1+cos(d*x+c))^2*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticF(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+8*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^2/(e*cos(d*x+c)/sin(d*x+c))^(5/2)
/sin(d*x+c)^5*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{a^2\,(e\cot(c+d\,x))^{5/2}\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.253 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=321

$$\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}-\frac{\log\left(\tan(\right.}{2\sqrt{2}a^2d}$$

```
[Out] 2*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(7/2)+2*cot(d*x+c)^3*csc(d*x+c)*(sin(c+
1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))
*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*cot(d*x+c))^(7/2)+1/2*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/2*arcta
n(1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)
^(7/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(
7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))
/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)
```

Rubi [A] time = 0.37, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3900, 3888, 3886, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 2607, 30}

$$\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}-\frac{\log\left(\tan(\right.}{2\sqrt{2}a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(7/2)) - (2*Cot[c + d*x]^3*Csc[c
 + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(a^2*d*(e*Cot[
c + d*x])^(7/2)) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e
*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) - Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x]
)^(7/2)*Tan[c + d*x]^(7/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}dx = \frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}dx}{(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\sqrt{\tan(c+dx)}}dx}{a^4(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int \left(\frac{a^2}{\sqrt{\tan(c+dx)}} - \frac{2a^2\sec(c+dx)}{\sqrt{\tan(c+dx)}} + \frac{a^2\sec^2(c+dx)}{\sqrt{\tan(c+dx)}}\right)dx}{a^4(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int \frac{1}{\sqrt{\tan(c+dx)}}dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\int \frac{\sec^2(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{\left(2\cos^{\frac{7}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{a^2(e\cot(c+dx))^{7/2}\sin^{\frac{7}{2}}(c+dx)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}dx, x, t\right.}{a^2d(e\cot(c+dx))^{7/2}}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{\left(2\cot^3(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)}{a^2(e\cot(c+dx))^{7/2}}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{7/2}}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{7/2}}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{7/2}}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{7/2}}$$

Mathematica [F] time = 5.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cot\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

maple [C] time = 2.23, size = 653, normalized size = 2.03

$$-\frac{\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\ \mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2/a^2/d*(I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)-I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-6*EllipticF(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-2*cos(d*x+c)*2^(1/2)+2*2^(1/2))*(-1+co
s(d*x+c))*cos(d*x+c)^3*(1+cos(d*x+c))^2/sin(d*x+c)^7/(e*cos(d*x+c)/sin(d*x+
c))^(7/2)*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{7/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(7/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(7/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.254 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=357

$$\frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}}-\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}\,a^2d\tan^{\frac{9}{2}}(c+dx}$$

```
[Out] 2/3*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(9/2)-4*cos(d*x+c)*cot(d*x+c)^3/a^2/d
/(e*cot(d*x+c))^(9/2)-4*cos(d*x+c)*cot(d*x+c)^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)
/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e*cot(d*x+c)
)^(9/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/
(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/2*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))/a^2/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/4*ln(1-2^(1
/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x
+c)^(9/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c)
)^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)
```

Rubi [A] time = 0.39, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3900, 3888, 3886, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 2607, 30}

$$\frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}}-\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}\,a^2d\tan^{\frac{9}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x]^3)/(3*a^2*d*(e*Cot[c + d*x])^(9/2)) - (4*Cos[c + d*x]*Cot[c
 + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(9/2)) + (4*Cos[c + d*x]*Cot[c + d*x]^4*
EllipticE[c - Pi/4 + d*x, 2])/(a^2*d*(e*Cot[c + d*x])^(9/2)*Sqrt[Sin[2*c +
2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c +
 d*x])^(9/2)*Tan[c + d*x]^(9/2)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(
Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) + Log[1 - Sqrt[2]*
Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*
Tan[c + d*x]^(9/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2
*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
```

```
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$
$$= \frac{\int(-a+a\sec(c+dx))^2\sqrt{\tan(c+dx)}\,dx}{a^4(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$
$$= \frac{\int\left(a^2\sqrt{\tan(c+dx)}-2a^2\sec(c+dx)\sqrt{\tan(c+dx)}+a^2\sec^2(c+dx\right.}{a^4(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$
$$= \frac{\int\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\int\sec^2(c+dx)\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx}$$
$$= -\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\int\cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + $$
$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{\left(4\cos^{\frac{9}{2}}(c+d\right.}{a^2(e}$$
$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{\left(4\cos(c+dx\right.}{a^2(e\cot}$$
$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c+dx)}{a^2d(e\cot(}$$
$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c+dx)}{a^2d(e\cot(}$$
$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c+dx)}{a^2d(e\cot(}$$

Mathematica [F] time = 55.44, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)^2), x)
```

maple [C] time = 2.09, size = 1480, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2/d*(-1+cos(d*x+c))^2*(3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*cos(d*x+c)^2-3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^2+3*I*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)-3*I*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c
)^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))+12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellip
ticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-24*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-3*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)-3*EllipticPi(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+12*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)-24*EllipticE(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)+
10*cos(d*x+c)^2*2^(1/2)-12*cos(d*x+c)*2^(1/2)+2*2^(1/2))*cos(d*x+c)^3*(1+co
s(d*x+c))^2/(e*cos(d*x+c)/sin(d*x+c))^(9/2)/sin(d*x+c)^9*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cot\left(dx+c\right)\right)^{\frac{9}{2}}\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d\,x)^2}{a^2\,{\left(e\,\cot (c+d\,x)\right)}^{9/2}\,{\left(\cos (c+d\,x)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(9/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(9/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(9/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.255 $\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=389

$$\frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}}+\frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}}-\frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx)}$$

```
[Out] 2/5*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(11/2)+2*cot(d*x+c)^5/a^2/d/(e*cot(d*
x+c))^(11/2)-4/3*cot(d*x+c)^4*csc(d*x+c)/a^2/d/(e*cot(d*x+c))^(11/2)-2/3*co
t(d*x+c)^5*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellipti
cF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*cot(d*x+c))^(11
/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(11/2)*2^(
1/2)/tan(d*x+c)^(11/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(
d*x+c))^(11/2)*2^(1/2)/tan(d*x+c)^(11/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+
tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(11/2)*2^(1/2)/tan(d*x+c)^(11/2)-1/4*ln(1+
2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(11/2)*2^(1/2)/ta
n(d*x+c)^(11/2)
```

Rubi [A] time = 0.48, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3900, 3888, 3886, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2611, 2614, 2573, 2641, 2607, 30}

$$\frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}}+\frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}}-\frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x]^3)/(5*a^2*d*(e*Cot[c + d*x])^(11/2)) + (2*Cot[c + d*x]^5)/(
a^2*d*(e*Cot[c + d*x])^(11/2)) - (4*Cot[c + d*x]^4*Csc[c + d*x])/(3*a^2*d*(
e*Cot[c + d*x])^(11/2)) + (2*Cot[c + d*x]^5*Csc[c + d*x]*EllipticF[c - Pi/4
 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*(e*Cot[c + d*x])^(11/2)) + ArcT
an[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(11/2)*T
an[c + d*x]^(11/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d
*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x]^(11/2)) + Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(11/2)*Tan[c + d*
x]^(11/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*
a^2*d*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x]^(11/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3900

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

$$= \frac{\int(-a+a\sec(c+dx))^2\tan^{\frac{3}{2}}(c+dx)\,dx}{a^4(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

$$= \frac{\int\left(a^2\tan^{\frac{3}{2}}(c+dx) - 2a^2\sec(c+dx)\tan^{\frac{3}{2}}(c+dx) + a^2\sec^2(c+dx\right.}{a^4(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

$$= \frac{\int\tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} + \frac{\int\sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)}{a^2(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+}$$

$$= \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}} + \frac{2\int}{3a^2(e\cot(c+}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\,\mathrm{cs}}{3a^2d(e\cot(c+}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\,\mathrm{cs}}{3a^2d(e\cot(c+}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\,\mathrm{cs}}{3a^2d(e\cot(c+}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\,\mathrm{cs}}{3a^2d(e\cot(c+}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\,\mathrm{cs}}{3a^2d(e\cot(c+}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\,\mathrm{cs}}{3a^2d(e\cot(c+}$$

Mathematica [F] time = 13.53, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(dx+c))^{\frac{11}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(11/2)*(a*sec(d*x + c) + a)^2), x)
```

maple [C] time = 2.11, size = 721, normalized size = 1.85

$$-\frac{(-1+\cos(dx+c))\left(15i\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+}{\sin(}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x)
[Out] -1/30/a^2/d*(-1+cos(d*x+c))*(15*I*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-15*I*sin(d*x+c)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-15*sin(d*x+c)*EllipticPi(((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-15*sin(d*x+c)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+50*sin(d*x+c)*
EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-24*2^(1/2)*cos(d*x
+c)^3+44*cos(d*x+c)^2*2^(1/2)-26*cos(d*x+c)*2^(1/2)+6*2^(1/2))*cos(d*x+c)^3
*(1+cos(d*x+c))^2/(e*cos(d*x+c)/sin(d*x+c))^(11/2)/sin(d*x+c)^9*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{a^2\,(e\cot(c+d\,x))^{11/2}\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(11/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(11/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(11/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.256 $\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}-\frac{\tan^4(c+dx)(35a+24b\sec(c+dx))}{140d}+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{70d}$$

```
[Out] a*ln(cos(d*x+c))/d-16/35*b*sec(d*x+c)/d+1/70*(35*a+16*b*sec(d*x+c))*tan(d*x
+c)^2/d-1/140*(35*a+24*b*sec(d*x+c))*tan(d*x+c)^4/d+1/42*(7*a+6*b*sec(d*x+c
))*tan(d*x+c)^6/d
```

Rubi [A] time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3881, 3884, 3475, 2606, 8}

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}-\frac{\tan^4(c+dx)(35a+24b\sec(c+dx))}{140d}+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{70d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (16*b*Sec[c + d*x])/(35*d) + ((35*a + 16*b*Sec[c
+ d*x])*Tan[c + d*x]^2)/(70*d) - ((35*a + 24*b*Sec[c + d*x])*Tan[c + d*x]^4
)/(140*d) + ((7*a + 6*b*Sec[c + d*x])*Tan[c + d*x]^6)/(42*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx = \frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d} - \frac{1}{7}\int (7a+6b\sec(c+dx))\tan^5(c+
$$
$$= -\frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d} + \frac{(7a+6b\sec(c+dx))\tan^6(c+
$$
$$= \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} - \frac{(35a+24b\sec(c+dx))\tan^4(c+
$$
$$= \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} - \frac{(35a+24b\sec(c+dx))\tan^4(c+
$$
$$= \frac{a\log(\cos(c+dx))}{d} + \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} - \frac{(35a+24b
$$
$$= \frac{a\log(\cos(c+dx))}{d} - \frac{16b\sec(c+dx)}{35d} + \frac{(35a+16b\sec(c+dx))\tan^2(c+
$$

Mathematica [A] time = 0.47, size = 106, normalized size = 0.95

$$\frac{a\left(2\tan^6(c+dx)-3\tan^4(c+dx)+6\tan^2(c+dx)+12\log(\cos(c+dx))\right)}{12d}+\frac{b\sec^7(c+dx)}{7d}-\frac{3b\sec^5(c+dx)}{5d}+\frac{b\,\mathrm{se}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] -((b*Sec[c + d*x])/d) + (b*Sec[c + d*x]^3)/d - (3*b*Sec[c + d*x]^5)/(5*d) +
 (b*Sec[c + d*x]^7)/(7*d) + (a*(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3
*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

fricas [A] time = 1.29, size = 101, normalized size = 0.91

$$\frac{420\,a\cos(dx+c)^7\log(-\cos(dx+c)) - 420\,b\cos(dx+c)^6 + 630\,a\cos(dx+c)^5 + 420\,b\cos(dx+c)^4 - 315\,a\,\mathrm{co}}{420\,d\cos(dx+c)^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(420*a*cos(d*x + c)^7*log(-cos(d*x + c)) - 420*b*cos(d*x + c)^6 + 630
*a*cos(d*x + c)^5 + 420*b*cos(d*x + c)^4 - 315*a*cos(d*x + c)^3 - 252*b*cos
(d*x + c)^2 + 70*a*cos(d*x + c) + 60*b)/(d*cos(d*x + c)^7)
```

giac [B] time = 11.58, size = 317, normalized size = 2.86

$$-\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{1089\,a+384\,b+\frac{8463\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2688\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{28749\,a(\cos(}{(\cos(dx+}}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1089*a + 384*b + 84
63*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2688*b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 28749*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 8064*b
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 56035*a*(cos(d*x + c) - 1)^3/(
cos(d*x + c) + 1)^3 + 13440*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 5
6035*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a*(cos(d*x + c) -
```

```
1)^5/(cos(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6 + 1089*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 1)^7)/d
```

maple [B] time = 0.62, size = 216, normalized size = 1.95

$$\frac{\left(\tan^{6}(dx+c)\right)a}{6d}-\frac{a\left(\tan^{4}(dx+c)\right)}{4d}+\frac{a\left(\tan^{2}(dx+c)\right)}{2d}+\frac{a\ln(\cos(dx+c))}{d}+\frac{b\left(\sin^{8}(dx+c)\right)}{7d\cos(dx+c)^{7}}-\frac{b\left(\sin^{8}(dx+c)\right)}{35d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^7,x)
```

```
[Out] 1/6/d*tan(d*x+c)^6*a-1/4*a*tan(d*x+c)^4/d+1/2*a*tan(d*x+c)^2/d+a*ln(cos(d*x
+c))/d+1/7/d*b*sin(d*x+c)^8/cos(d*x+c)^7-1/35/d*b*sin(d*x+c)^8/cos(d*x+c)^5
+1/35/d*b*sin(d*x+c)^8/cos(d*x+c)^3-1/7/d*b*sin(d*x+c)^8/cos(d*x+c)-16/35*b
*cos(d*x+c)/d-1/7/d*b*cos(d*x+c)*sin(d*x+c)^6-6/35/d*b*cos(d*x+c)*sin(d*x+c
)^4-8/35/d*b*cos(d*x+c)*sin(d*x+c)^2
```

maxima [A] time = 0.32, size = 94, normalized size = 0.85

$$\frac{420\,a\log(\cos(dx+c))-\frac{420\,b\cos(dx+c)^{6}-630\,a\cos(dx+c)^{5}-420\,b\cos(dx+c)^{4}+315\,a\cos(dx+c)^{3}+252\,b\cos(dx+c)^{2}-70\,a\cos(dx+c)-60\,b}{\cos(dx+c)^{7}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(420*a*log(cos(d*x + c)) - (420*b*cos(d*x + c)^6 - 630*a*cos(d*x + c)
^5 - 420*b*cos(d*x + c)^4 + 315*a*cos(d*x + c)^3 + 252*b*cos(d*x + c)^2 - 7
0*a*cos(d*x + c) - 60*b)/cos(d*x + c)^7)/d
```

mupad [B] time = 5.24, size = 221, normalized size = 1.99

$$\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}+\left(-\frac{128\,a}{3}-32\,b\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\left(14\,a+\frac{96\,b}{5}\right)\tan\left(\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-21\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + b/cos(c + d*x)),x)
```

```
[Out] ((32*b)/35 - tan(c/2 + (d*x)/2)^2*(2*a + (32*b)/5) + tan(c/2 + (d*x)/2)^4*(
14*a + (96*b)/5) - tan(c/2 + (d*x)/2)^6*((128*a)/3 + 32*b) + (128*a*tan(c/2
 + (d*x)/2)^8)/3 - 14*a*tan(c/2 + (d*x)/2)^10 + 2*a*tan(c/2 + (d*x)/2)^12)/
(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2
)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x
)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 8.48, size = 148, normalized size = 1.33

$$\begin{cases}-\frac{a\log\left(\tan^{2}(c+dx)+1\right)}{2d}+\frac{a\tan^{6}(c+dx)}{6d}-\frac{a\tan^{4}(c+dx)}{4d}+\frac{a\tan^{2}(c+dx)}{2d}+\frac{b\tan^{6}(c+dx)\sec(c+dx)}{7d}-\frac{6b\tan^{4}(c+dx)\sec(c+dx)}{35d}+\frac{8b\tan}{} \\ x\,(a+b\sec(c))\tan^{7}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**6/(6*d) - a*
tan(c + d*x)**4/(4*d) + a*tan(c + d*x)**2/(2*d) + b*tan(c + d*x)**6*sec(c +
 d*x)/(7*d) - 6*b*tan(c + d*x)**4*sec(c + d*x)/(35*d) + 8*b*tan(c + d*x)**2
*sec(c + d*x)/(35*d) - 16*b*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a + b*sec(c
))*tan(c)**7, True))
```

3.257 $\int (a + b\sec(c + dx))\tan^5(c + dx)\,dx$

Optimal. Leaf size=84

$$\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{30d}-\frac{a\log(\cos(c+dx))}{d}+\frac{8b\sec(c+dx)}{15d}$$

```
[Out] -a*ln(cos(d*x+c))/d+8/15*b*sec(d*x+c)/d-1/30*(15*a+8*b*sec(d*x+c))*tan(d*x+
c)^2/d+1/20*(5*a+4*b*sec(d*x+c))*tan(d*x+c)^4/d
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3881, 3884, 3475, 2606, 8}

$$\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{30d}-\frac{a\log(\cos(c+dx))}{d}+\frac{8b\sec(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (8*b*Sec[c + d*x])/(15*d) - ((15*a + 8*b*Sec[c
 + d*x])*Tan[c + d*x]^2)/(30*d) + ((5*a + 4*b*Sec[c + d*x])*Tan[c + d*x]^4)
/(20*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx = \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d} - \frac{1}{5}\int (5a+4b\sec(c+dx))\tan^3(c+
$$
$$= -\frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} + \frac{(5a+4b\sec(c+dx))\tan^4(c+dx
$$
$$= -\frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} + \frac{(5a+4b\sec(c+dx))\tan^4(c+dx
$$
$$= -\frac{a\log(\cos(c+dx))}{d} - \frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} + \frac{(5a+4b\sec
$$
$$= -\frac{a\log(\cos(c+dx))}{d} + \frac{8b\sec(c+dx)}{15d} - \frac{(15a+8b\sec(c+dx))\tan^2(c+
$$

Mathematica [A] time = 0.20, size = 82, normalized size = 0.98

$$-\frac{a\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d}+\frac{b\sec^5(c+dx)}{5d}-\frac{2b\sec^3(c+dx)}{3d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (b*Sec[c + d*x])/d - (2*b*Sec[c + d*x]^3)/(3*d) + (b*Sec[c + d*x]^5)/(5*d)
- (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

fricas [A] time = 1.13, size = 79, normalized size = 0.94

$$-\frac{60\,a\cos(dx+c)^5\log(-\cos(dx+c)) - 60\,b\cos(dx+c)^4 + 60\,a\cos(dx+c)^3 + 40\,b\cos(dx+c)^2 - 15\,a\cos(d
}{60\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^5*log(-cos(d*x + c)) - 60*b*cos(d*x + c)^4 + 60*a*
cos(d*x + c)^3 + 40*b*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 12*b)/(d*cos(d*x
 + c)^5)
```

giac [B] time = 3.01, size = 248, normalized size = 2.95

$$\frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{137\,a+64\,b+\frac{805\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{320\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1970\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+
}{\left(\frac{\cos(
}{\cos(
}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (137*a + 64*b + 805*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1970*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 640*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 805*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)^5)/d
```

maple [B] time = 0.64, size = 161, normalized size = 1.92

$$\frac{a\left(\tan^4(dx+c)\right)}{4d}-\frac{a\left(\tan^2(dx+c)\right)}{2d}-\frac{a\ln(\cos(dx+c))}{d}+\frac{b\left(\sin^6(dx+c)\right)}{5d\cos(dx+c)^5}-\frac{b\left(\sin^6(dx+c)\right)}{15d\cos(dx+c)^3}+\frac{b\left(\sin^6(dx+c)\right)}{5d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^5,x)
```

```
[Out] 1/4*a*tan(d*x+c)^4/d-1/2*a*tan(d*x+c)^2/d-a*ln(cos(d*x+c))/d+1/5/d*b*sin(d*
x+c)^6/cos(d*x+c)^5-1/15/d*b*sin(d*x+c)^6/cos(d*x+c)^3+1/5/d*b*sin(d*x+c)^6
/cos(d*x+c)+8/15*b*cos(d*x+c)/d+1/5/d*b*cos(d*x+c)*sin(d*x+c)^4+4/15/d*b*co
s(d*x+c)*sin(d*x+c)^2
```

maxima [A] time = 0.50, size = 72, normalized size = 0.86

$$-\frac{60\,a\log(\cos(dx+c))-\frac{60\,b\cos(dx+c)^4-60\,a\cos(dx+c)^3-40\,b\cos(dx+c)^2+15\,a\cos(dx+c)+12\,b}{\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(cos(d*x + c)) - (60*b*cos(d*x + c)^4 - 60*a*cos(d*x + c)^3
- 40*b*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 12*b)/cos(d*x + c)^5)/d
```

mupad [B] time = 5.85, size = 162, normalized size = 1.93

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(10\,a+\frac{32\,b}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(-2\,a-\frac{16\,b}{3}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((16*b)/15 - tan(c/2 + (d*x)/2)^2*(2*
a + (16*b)/3) + tan(c/2 + (d*x)/2)^4*(10*a + (32*b)/3) - 10*a*tan(c/2 + (d*
x)/2)^6 + 2*a*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2
 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2
+ (d*x)/2)^10 - 1))
```

sympy [A] time = 3.13, size = 112, normalized size = 1.33

$$\begin{cases}\frac{a\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a\tan^4(c+dx)}{4d}-\frac{a\tan^2(c+dx)}{2d}+\frac{b\tan^4(c+dx)\sec(c+dx)}{5d}-\frac{4b\tan^2(c+dx)\sec(c+dx)}{15d}+\frac{8b\sec(c+dx)}{15d} & \text{for } d \neq \\ x\left(a+b\sec(c)\right)\tan^5(c) & \text{otherv}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4/(4*d) - a*t
an(c + d*x)**2/(2*d) + b*tan(c + d*x)**4*sec(c + d*x)/(5*d) - 4*b*tan(c + d
*x)**2*sec(c + d*x)/(15*d) + 8*b*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a + b*
sec(c))*tan(c)**5, True))
```

3.258 $\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=55

$$\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}+\frac{a\log(\cos(c+dx))}{d}-\frac{2b\sec(c+dx)}{3d}$$

```
[Out] a*ln(cos(d*x+c))/d-2/3*b*sec(d*x+c)/d+1/6*(3*a+2*b*sec(d*x+c))*tan(d*x+c)^2
/d
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3881, 3884, 3475, 2606, 8}

$$\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}+\frac{a\log(\cos(c+dx))}{d}-\frac{2b\sec(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (2*b*Sec[c + d*x])/(3*d) + ((3*a + 2*b*Sec[c + d*
x])*Tan[c + d*x]^2)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a + b\sec(c+dx))\tan^3(c+dx)\,dx = \frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d} - \frac{1}{3}\int (3a+2b\sec(c+dx))\tan(c+$$

$$= \frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d} - a\int \tan(c+dx)\,dx - \frac{1}{3}(2b)\int \text{se}$$

$$= \frac{a\log(\cos(c+dx))}{d} + \frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d} - \frac{(2b)\,\text{Subst}(}{}$$

$$= \frac{a\log(\cos(c+dx))}{d} - \frac{2b\sec(c+dx)}{3d} + \frac{(3a+2b\sec(c+dx))\tan^2(c+}{6d}$$

Mathematica [**A**] time = 0.14, size = 55, normalized size = 1.00

$$\frac{a\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d} + \frac{b\sec^3(c+dx)}{3d} - \frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((b*Sec[c + d*x])/d) + (b*Sec[c + d*x]^3)/(3*d) + (a*(2*Log[Cos[c + d*x]]
+ Tan[c + d*x]^2))/(2*d)
```

fricas [**A**] time = 0.56, size = 57, normalized size = 1.04

$$\frac{6\,a\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right) - 6\,b\cos\left(dx+c\right)^2 + 3\,a\cos\left(dx+c\right) + 2\,b}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*cos(d*x + c)^3*log(-cos(d*x + c)) - 6*b*cos(d*x + c)^2 + 3*a*cos(d
*x + c) + 2*b)/(d*cos(d*x + c)^3)
```

giac [**B**] time = 0.95, size = 179, normalized size = 3.25

$$-\frac{6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{11\,a+8\,b+\frac{45\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{24\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a}{(\mathrm{c}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11*a + 8*b + 45*a*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 24*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*a*(cos(d*x + c) - 1)
^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3)/d
```

maple [**B**] time = 0.64, size = 104, normalized size = 1.89

$$\frac{a\left(\tan^2\left(dx+c\right)\right)}{2d} + \frac{a\ln\left(\cos\left(dx+c\right)\right)}{d} + \frac{b\left(\sin^4\left(dx+c\right)\right)}{3d\cos\left(dx+c\right)^3} - \frac{b\left(\sin^4\left(dx+c\right)\right)}{3d\cos\left(dx+c\right)} - \frac{b\cos\left(dx+c\right)\left(\sin^2\left(dx+c\right)\right)}{3d} - \frac{2b\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^3,x)
```

```
[Out] 1/2*a*tan(d*x+c)^2/d+a*ln(cos(d*x+c))/d+1/3/d*b*sin(d*x+c)^4/cos(d*x+c)^3-1
/3/d*b*sin(d*x+c)^4/cos(d*x+c)-1/3/d*b*cos(d*x+c)*sin(d*x+c)^2-2/3*b*cos(d*
x+c)/d
```

maxima [A] time = 0.51, size = 50, normalized size = 0.91

$$\frac{6\,a\log\left(\cos\left(dx+c\right)\right)-\frac{6\,b\cos\left(dx+c\right)^2-3\,a\cos\left(dx+c\right)-2\,b}{\cos\left(dx+c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a*log(cos(d*x + c)) - (6*b*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 2*b)/
cos(d*x + c)^3)/d
```

mupad [B] time = 2.23, size = 102, normalized size = 1.85

$$\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(-2\,a-4\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{4\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

```
[Out] ((4*b)/3 - tan(c/2 + (d*x)/2)^2*(2*a + 4*b) + 2*a*tan(c/2 + (d*x)/2)^4)/(d*
(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1
)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 0.92, size = 76, normalized size = 1.38

$$\begin{cases} -\frac{a\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^2\left(c+dx\right)}{2d}+\frac{b\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{3d}-\frac{2b\sec\left(c+dx\right)}{3d} & \text{for}\ d\neq 0 \\ x\left(a+b\sec\left(c\right)\right)\tan^3\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2/(2*d) + b*
tan(c + d*x)**2*sec(c + d*x)/(3*d) - 2*b*sec(c + d*x)/(3*d), Ne(d, 0)), (x*
(a + b*sec(c))*tan(c)**3, True))
```

3.259 $\int (a+b\sec(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=25

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*ln(cos(d*x+c))/d+b*sec(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3884, 3475, 2606, 8}

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))\tan(c+dx)\,dx &= a\int \tan(c+dx)\,dx+b\int \sec(c+dx)\tan(c+dx)\,dx\\
&= -\frac{a\log(\cos(c+dx))}{d}+\frac{b\,\text{Subst}\left(\int 1\,dx,x,\sec(c+dx)\right)}{d}\\
&= -\frac{a\log(\cos(c+dx))}{d}+\frac{b\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x],x]
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

fricas [A] time = 1.02, size = 34, normalized size = 1.36

$$-\frac{a\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-b}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="fricas")
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - b)/(d*cos(d*x + c))
```

giac [B] time = 1.03, size = 107, normalized size = 4.28

$$\frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a+2\,b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="giac")
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + 2*b + a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

maple [A] time = 0.18, size = 25, normalized size = 1.00

$$\frac{b\sec\left(dx+c\right)}{d}+\frac{a\ln\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c),x)
[Out] b*sec(d*x+c)/d+a/d*ln(sec(d*x+c))
```

maxima [A] time = 0.56, size = 26, normalized size = 1.04

$$-\frac{a\log\left(\cos\left(dx+c\right)\right)-\frac{b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="maxima")
[Out] -(a*log(cos(d*x + c)) - b/cos(d*x + c))/d
```

mupad [B] time = 1.30, size = 40, normalized size = 1.60

$$\frac{2\,a\,\mathrm{atanh}\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d}-\frac{2\,b}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x)),x)
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [A] time = 0.26, size = 37, normalized size = 1.48

$$\begin{cases} \frac{a\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{b\sec(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a+b\sec(c)\right)\tan(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + b*sec(c + d*x)/d, Ne(d, 0)),
(x*(a + b*sec(c))*tan(c), True))
```

3.260 $\int \cot(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{(a+b)\log(1-\cos(c+dx))}{2d}+\frac{(a-b)\log(\cos(c+dx)+1)}{2d}$$

```
[Out] 1/2*(a+b)*ln(1-cos(d*x+c))/d+1/2*(a-b)*ln(1+cos(d*x+c))/d
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3883, 2668, 633, 31}

$$\frac{(a+b)\log(1-\cos(c+dx))}{2d}+\frac{(a-b)\log(\cos(c+dx)+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a + b)*Log[1 - Cos[c + d*x]])/(2*d) + ((a - b)*Log[1 + Cos[c + d*x]])/(2*
d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3883

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)(a+b\sec(c+dx))\,dx &= \int (b+a\cos(c+dx))\csc(c+dx)\,dx\\
&= -\frac{a\,\text{Subst}\left(\int \frac{b+x}{a^2-x^2}\,dx, x, a\cos(c+dx)\right)}{d}\\
&= -\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a-x}\,dx, x, a\cos(c+dx)\right)}{2d}-\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, a\cos(c+dx)\right)}{2d}\\
&= \frac{(a+b)\log(1-\cos(c+dx))}{2d}+\frac{(a-b)\log(1+\cos(c+dx))}{2d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 1.40

$$\frac{a(\log(\tan(c+dx))+\log(\cos(c+dx)))}{d}+\frac{b\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x]),x]
[Out] -((b*Log[Cos[c/2 + (d*x)/2]])/d) + (b*Log[Sin[c/2 + (d*x)/2]])/d + (a*(Log[
Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

fricas [A] time = 0.82, size = 38, normalized size = 0.88

$$\frac{(a-b)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+(a+b)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/2*((a - b)*log(1/2*cos(d*x + c) + 1/2) + (a + b)*log(-1/2*cos(d*x + c) +
1/2))/d
```

giac [A] time = 0.26, size = 61, normalized size = 1.42

$$\frac{(a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] 1/2*((a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

maple [A] time = 0.46, size = 35, normalized size = 0.81

$$\frac{a\ln(\sin(dx+c))}{d}+\frac{b\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c)),x)
[Out] a*ln(sin(d*x+c))/d+1/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.52, size = 34, normalized size = 0.79

$$\frac{(a-b)\log(\cos(dx+c)+1)+(a+b)\log(\cos(dx+c)-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((a - b)*log(cos(d*x + c) + 1) + (a + b)*log(cos(d*x + c) - 1))/d
```

mupad [B] time = 1.31, size = 51, normalized size = 1.19

$$\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}+\frac{b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (b*lo
g(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))\cot (c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x), x)
```

3.261 $\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=72

$$-\frac{(2a+b)\log(1-\cos(c+dx))}{4d}-\frac{(2a-b)\log(\cos(c+dx)+1)}{4d}-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

```
[Out] -1/4*(2*a+b)*ln(1-cos(d*x+c))/d-1/4*(2*a-b)*ln(1+cos(d*x+c))/d-1/2*cot(d*x+
c)^2*(a+b*sec(d*x+c))/d
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3882, 3883, 2668, 633, 31}

$$-\frac{(2a+b)\log(1-\cos(c+dx))}{4d}-\frac{(2a-b)\log(\cos(c+dx)+1)}{4d}-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((2*a + b)*Log[1 - Cos[c + d*x]])/(4*d) - ((2*a - b)*Log[1 + Cos[c + d*x]]
)/(4*d) - (Cot[c + d*x]^2*(a + b*Sec[c + d*x]))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3883

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{1}{2}\int \cot(c+dx)(-2a-b\sec(c+dx))$$
$$= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{1}{2}\int (-b-2a\cos(c+dx))\csc(c+dx)$$
$$= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{a\,\text{Subst}\left(\int \frac{-b+x}{4a^2-x^2}\,dx,x,-2a\cos(c+d\right.}{d}$$
$$= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{2a-x}\,dx,x,-2a\cos(\right.}{4d}$$
$$= -\frac{(2a+b)\log(1-\cos(c+dx))}{4d}-\frac{(2a-b)\log(1+\cos(c+dx))}{4d}-\frac{\cot^2(c+}{}$$

Mathematica [A] time = 1.35, size = 114, normalized size = 1.58

$$-\frac{a\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d}-\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\log\left(\sin\left(\right.\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x]),x]
[Out] -1/8*(b*Csc[(c + d*x)/2]^2)/d + (b*Log[Cos[(c + d*x)/2]])/(2*d) - (b*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[T
an[c + d*x]]))/(2*d) + (b*Sec[(c + d*x)/2]^2)/(8*d)
```

fricas [A] time = 0.49, size = 99, normalized size = 1.38

$$\frac{2\,b\cos(dx+c)-\left((2\,a-b)\cos(dx+c)^2-2\,a+b\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left((2\,a+b)\cos(dx+c)^2-2\,a-b\right)\mathrm{l}}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/4*(2*b*cos(d*x + c) - ((2*a - b)*cos(d*x + c)^2 - 2*a + b)*log(1/2*cos(d*
x + c) + 1/2) - ((2*a + b)*cos(d*x + c)^2 - 2*a - b)*log(-1/2*cos(d*x + c)
+ 1/2) + 2*a)/(d*cos(d*x + c)^2 - d)
```

giac [B] time = 2.50, size = 170, normalized size = 2.36

$$-\frac{2\,(2\,a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-8\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{4\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}-\frac{a(\cos(dx+}{\cos(dx+}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] -1/8*(2*(2*a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*a*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 4*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*
(cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/d
```

maple [A] time = 0.65, size = 85, normalized size = 1.18

$$-\frac{a\left(\cot^2(dx+c)\right)}{2d}-\frac{a\ln(\sin(dx+c))}{d}-\frac{b\left(\cos^3(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{b\cos(dx+c)}{2d}-\frac{b\ln(\csc(dx+c)-\cot(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/2*a*cot(d*x+c)^2/d-a*ln(sin(d*x+c))/d-1/2/d*b/sin(d*x+c)^2*cos(d*x+c)^3-
1/2*b*cos(d*x+c)/d-1/2/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.35, size = 62, normalized size = 0.86

$$-\frac{(2\,a-b)\log\left(\cos\left(dx+c\right)+1\right)+(2\,a+b)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,(b\cos(dx+c)+a)}{\cos(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*((2*a - b)*log(cos(d*x + c) + 1) + (2*a + b)*log(cos(d*x + c) - 1) - 2
*(b*cos(d*x + c) + a)/(cos(d*x + c)^2 - 1))/d
```

mupad [B] time = 1.36, size = 86, normalized size = 1.19

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a}{8}-\frac{b}{8}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a}{8}+\frac{b}{8}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a+\frac{b}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (tan(c/2 + (d*x)/2)^2*(a/8 - b/8))/d
- (cot(c/2 + (d*x)/2)^2*(a/8 + b/8))/d - (log(tan(c/2 + (d*x)/2))*(a + b/2)
)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**3, x)
```

3.262 $\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=102

$$\frac{(8a+3b)\log(1-\cos(c+dx))}{16d}+\frac{(8a-3b)\log(\cos(c+dx)+1)}{16d}-\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+dx)(4a$$

[Out] 1/16*(8*a+3*b)*ln(1-cos(d*x+c))/d+1/16*(8*a-3*b)*ln(1+cos(d*x+c))/d-1/4*cot(d*x+c)^4*(a+b*sec(d*x+c))/d+1/8*cot(d*x+c)^2*(4*a+3*b*sec(d*x+c))/d

Rubi [A] time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3882, 3883, 2668, 633, 31}

$$\frac{(8a+3b)\log(1-\cos(c+dx))}{16d}+\frac{(8a-3b)\log(\cos(c+dx)+1)}{16d}-\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+dx)(4a$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^5*(a + b*Sec[c + d*x]),x]

[Out] ((8*a + 3*b)*Log[1 - Cos[c + d*x]])/(16*d) + ((8*a - 3*b)*Log[1 + Cos[c + d*x]])/(16*d) - (Cot[c + d*x]^4*(a + b*Sec[c + d*x]))/(4*d) + (Cot[c + d*x]^2*(4*a + 3*b*Sec[c + d*x]))/(8*d)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[m, -1]
```

Rule 3883

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{1}{4}\int \cot^3(c+dx)(-4a-3b\sec(c+$$
$$= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d}$$
$$= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d}$$
$$= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d}$$
$$= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d}$$
$$= \frac{(8a+3b)\log(1-\cos(c+dx))}{16d} + \frac{(8a-3b)\log(1+\cos(c+dx))}{16d} - \cot$$

Mathematica [A] time = 0.40, size = 166, normalized size = 1.63

$$\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos(c+dx))\right)}{4d} - \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5b\csc^2\left(\frac{1}{2}(\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (5*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x)/2]^4)/(64*d) - (3*b*Log[
Cos[(c + d*x)/2]])/(8*d) + (3*b*Log[Sin[(c + d*x)/2]])/(8*d) + (a*(2*Cot[c
+ d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*
d) - (5*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(64*d)
```

fricas [A] time = 0.48, size = 168, normalized size = 1.65

$$-\frac{10\,b\cos(dx+c)^3 + 16\,a\cos(dx+c)^2 - 6\,b\cos(dx+c) - \left((8\,a-3\,b)\cos(dx+c)^4 - 2\,(8\,a-3\,b)\cos(dx+c\right.}{16\left(d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*b*cos(d*x + c)^3 + 16*a*cos(d*x + c)^2 - 6*b*cos(d*x + c) - ((8*a
 - 3*b)*cos(d*x + c)^4 - 2*(8*a - 3*b)*cos(d*x + c)^2 + 8*a - 3*b)*log(1/2*
cos(d*x + c) + 1/2) - ((8*a + 3*b)*cos(d*x + c)^4 - 2*(8*a + 3*b)*cos(d*x +
 c)^2 + 8*a + 3*b)*log(-1/2*cos(d*x + c) + 1/2) - 12*a)/(d*cos(d*x + c)^4 -
 2*d*cos(d*x + c)^2 + d)
```

giac [B] time = 0.97, size = 266, normalized size = 2.61

$$\frac{4\,(8\,a+3\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 64\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a+b+\frac{12\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{18}{(\right.}{(\cos(dx+c)-1)^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(4*(8*a + 3*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 64*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 12*a*(cos
```

```
(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 48*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 12*a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2)/d
```

maple [A] time = 0.54, size = 134, normalized size = 1.31

$$-\frac{a\left(\cot^{4}(dx+c)\right)}{4d}+\frac{a\left(\cot^{2}(dx+c)\right)}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{b\left(\cos^{5}(dx+c)\right)}{4d\sin(dx+c)^{4}}+\frac{b\left(\cos^{5}(dx+c)\right)}{8d\sin(dx+c)^{2}}+\frac{b\left(\cos^{3}(dx+c)\right)}{8d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/4*a*cot(d*x+c)^4/d+1/2*a*cot(d*x+c)^2/d+a*ln(sin(d*x+c))/d-1/4/d*b/sin(d
*x+c)^4*cos(d*x+c)^5+1/8/d*b/sin(d*x+c)^2*cos(d*x+c)^5+1/8*b*cos(d*x+c)^3/d
+3/8*b*cos(d*x+c)/d+3/8/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.50, size = 99, normalized size = 0.97

$$\frac{(8a-3b)\log(\cos(dx+c)+1)+(8a+3b)\log(\cos(dx+c)-1)-\frac{2\left(5b\cos(dx+c)^{3}+8a\cos(dx+c)^{2}-3b\cos(dx+c)-6a\right)}{\cos(dx+c)^{4}-2\cos(dx+c)^{2}+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*((8*a - 3*b)*log(cos(d*x + c) + 1) + (8*a + 3*b)*log(cos(d*x + c) - 1)
 - 2*(5*b*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 - 3*b*cos(d*x + c) - 6*a)/(co
s(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

mupad [B] time = 1.34, size = 128, normalized size = 1.25

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\left(\frac{3\,a}{16}-\frac{b}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\left(\frac{a}{64}-\frac{b}{64}\right)}{d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\left((-3\,a-2\,b)\,\tan\left(\frac{c}{2}\right.\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*((3*a)/16 - b/8))/d - (tan(c/2 + (d*x)/2)^4*(a/64 - b
/64))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2)^4*(a/4
+ b/4 - tan(c/2 + (d*x)/2)^2*(3*a + 2*b)))/(16*d) + (log(tan(c/2 + (d*x)/2)
)*(a + (3*b)/8))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))\cot^{5}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**5, x)
```

3.263 $\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=130

$$-\frac{(16a+5b)\log(1-\cos(c+dx))}{32d}-\frac{(16a-5b)\log(\cos(c+dx)+1)}{32d}-\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+}{}$$

```
[Out] -1/32*(16*a+5*b)*ln(1-cos(d*x+c))/d-1/32*(16*a-5*b)*ln(1+cos(d*x+c))/d-1/6*
cot(d*x+c)^6*(a+b*sec(d*x+c))/d+1/24*cot(d*x+c)^4*(6*a+5*b*sec(d*x+c))/d-1/
16*cot(d*x+c)^2*(8*a+5*b*sec(d*x+c))/d
```

Rubi [A] time = 0.18, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3882, 3883, 2668, 633, 31}

$$-\frac{(16a+5b)\log(1-\cos(c+dx))}{32d}-\frac{(16a-5b)\log(\cos(c+dx)+1)}{32d}-\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((16*a + 5*b)*Log[1 - Cos[c + d*x]])/(32*d) - ((16*a - 5*b)*Log[1 + Cos[c
+ d*x]])/(32*d) - (Cot[c + d*x]^6*(a + b*Sec[c + d*x]))/(6*d) + (Cot[c + d*
x]^4*(6*a + 5*b*Sec[c + d*x]))/(24*d) - (Cot[c + d*x]^2*(8*a + 5*b*Sec[c +
d*x]))/(16*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3883

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{1}{6}\int \cot^5(c+dx)(-6a-5b\sec(c+dx$$

$$= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} +$$

$$= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} -$$

$$= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} -$$

$$= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} -$$

$$= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} -$$

$$= -\frac{(16a+5b)\log(1-\cos(c+dx))}{32d} - \frac{(16a-5b)\log(1+\cos(c+dx))}{32d} -$$

Mathematica [A] time = 0.62, size = 216, normalized size = 1.66

$$-\frac{a\left(2\cot^6(c+dx)-3\cot^4(c+dx)+6\cot^2(c+dx)+12\log(\tan(c+dx))+12\log(\cos(c+dx))\right)}{12d} - \frac{b\csc^6\left(\frac{1}{2}(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-11*b*Csc[(c + d*x)/2]^2)/(64*d) + (b*Csc[(c + d*x)/2]^4)/(32*d) - (b*Csc[
(c + d*x)/2]^6)/(384*d) + (5*b*Log[Cos[(c + d*x)/2]])/(16*d) - (5*b*Log[Sin
[(c + d*x)/2]])/(16*d) - (a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c
+ d*x]^6 + 12*Log[Cos[c + d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) + (11*b*Sec
[(c + d*x)/2]^2)/(64*d) - (b*Sec[(c + d*x)/2]^4)/(32*d) + (b*Sec[(c + d*x)/
2]^6)/(384*d)
```

fricas [A] time = 0.53, size = 237, normalized size = 1.82

$$\frac{66\,b\cos(dx+c)^5 + 144\,a\cos(dx+c)^4 - 80\,b\cos(dx+c)^3 - 216\,a\cos(dx+c)^2 + 30\,b\cos(dx+c) - 3\left((16\,a-5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*b*cos(d*x + c)^5 + 144*a*cos(d*x + c)^4 - 80*b*cos(d*x + c)^3 - 21
6*a*cos(d*x + c)^2 + 30*b*cos(d*x + c) - 3*((16*a - 5*b)*cos(d*x + c)^6 - 3
*(16*a - 5*b)*cos(d*x + c)^4 + 3*(16*a - 5*b)*cos(d*x + c)^2 - 16*a + 5*b)*
log(1/2*cos(d*x + c) + 1/2) - 3*((16*a + 5*b)*cos(d*x + c)^6 - 3*(16*a + 5*
b)*cos(d*x + c)^4 + 3*(16*a + 5*b)*cos(d*x + c)^2 - 16*a - 5*b)*log(-1/2*co
s(d*x + c) + 1/2) + 88*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(
d*x + c)^2 - d)
```

giac [B] time = 0.57, size = 358, normalized size = 2.75

$$-\frac{12\,(16\,a+5\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a+b+\frac{12\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{87\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/384*(12*(16*a + 5*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) -
 384*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 12*a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + 87*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 45*b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 + 352*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 + 110*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)
^3/(cos(d*x + c) - 1)^3 - 87*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 45*b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 + 9*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a*(cos(d*x +
 c) - 1)^3/(cos(d*x + c) + 1)^3 + b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)
^3)/d
```

maple [A] time = 0.62, size = 185, normalized size = 1.42

$$-\frac{a\left(\cot^{6}(dx+c)\right)}{6d}+\frac{a\left(\cot^{4}(dx+c)\right)}{4d}-\frac{a\left(\cot^{2}(dx+c)\right)}{2d}-\frac{a\ln(\sin(dx+c))}{d}-\frac{b\left(\cos^{7}(dx+c)\right)}{6d\sin(dx+c)^{6}}+\frac{b\left(\cos^{7}(dx+c\right.}{24d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/6/d*a*cot(d*x+c)^6+1/4*a*cot(d*x+c)^4/d-1/2*a*cot(d*x+c)^2/d-a*ln(sin(d*
x+c))/d-1/6/d*b/sin(d*x+c)^6*cos(d*x+c)^7+1/24/d*b/sin(d*x+c)^4*cos(d*x+c)^
7-1/16/d*b/sin(d*x+c)^2*cos(d*x+c)^7-1/16/d*b*cos(d*x+c)^5-5/48*b*cos(d*x+c
)^3/d-5/16*b*cos(d*x+c)/d-5/16/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.37, size = 133, normalized size = 1.02

$$-\frac{3(16a-5b)\log(\cos(dx+c)+1)+3(16a+5b)\log(\cos(dx+c)-1)-\frac{2\left(33b\cos(dx+c)^{5}+72a\cos(dx+c)^{4}-40b\cos\right.}{\cos(dx+c)^{6}-3\cos(}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(3*(16*a - 5*b)*log(cos(d*x + c) + 1) + 3*(16*a + 5*b)*log(cos(d*x +
c) - 1) - 2*(33*b*cos(d*x + c)^5 + 72*a*cos(d*x + c)^4 - 40*b*cos(d*x + c)^
3 - 108*a*cos(d*x + c)^2 + 15*b*cos(d*x + c) + 44*a)/(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/d
```

mupad [B] time = 1.50, size = 170, normalized size = 1.31

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(\frac{a}{32}-\frac{3b}{128}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(\left(\frac{29a}{2}+\frac{15b}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+\left(-2a-\frac{3b}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+\frac{a}{6}+\frac{b}{6}\right)}{64d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(a/32 - (3*b)/128))/d - (cot(c/2 + (d*x)/2)^6*(a/6 +
b/6 - tan(c/2 + (d*x)/2)^2*(2*a + (3*b)/2) + tan(c/2 + (d*x)/2)^4*((29*a)/2
 + (15*b)/2)))/(64*d) - (tan(c/2 + (d*x)/2)^2*((29*a)/128 - (15*b)/128))/d
- (tan(c/2 + (d*x)/2)^6*(a/384 - b/384))/d + (a*log(tan(c/2 + (d*x)/2)^2 +
1))/d - (log(tan(c/2 + (d*x)/2))*(a + (5*b)/16))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))\cot^{7}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**7, x)
```

3.264 $\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx$

Optimal. Leaf size=102

$$\frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}-\frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{24d}+\frac{\tan(c+dx)(16a+5b\sec(c+dx))}{16d}-ax-$$

```
[Out] -a*x-5/16*b*arctanh(sin(d*x+c))/d+1/16*(16*a+5*b*sec(d*x+c))*tan(d*x+c)/d-1
/24*(8*a+5*b*sec(d*x+c))*tan(d*x+c)^3/d+1/30*(6*a+5*b*sec(d*x+c))*tan(d*x+c
)^5/d
```

Rubi [A] time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{3881, 3770\}$

$$\frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}-\frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{24d}+\frac{\tan(c+dx)(16a+5b\sec(c+dx))}{16d}-ax-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*x) - (5*b*ArcTanh[Sin[c + d*x]])/(16*d) + ((16*a + 5*b*Sec[c + d*x])*Ta
n[c + d*x])/(16*d) - ((8*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^3)/(24*d) + ((6
*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx &= \frac{(6a+5b\sec(c+dx))\tan^5(c+dx)}{30d}-\frac{1}{6}\int (6a+5b\sec(c+dx))\tan^4(c \\
&= -\frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d}+\frac{(6a+5b\sec(c+dx))\tan^5(c+ \\
&= \frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5b\sec(c+dx))\tan^3(c+d \\
&= -ax+\frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5b\sec(c+dx))\tan^3 \\
&= -ax-\frac{5b\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d}-
\end{aligned}$$

Mathematica [A] time = 1.07, size = 103, normalized size = 1.01

$$\frac{\frac{1}{8}\tan(c+dx)\sec^5(c+dx)(1168a\cos(c+dx)+568a\cos(3(c+dx))+184a\cos(5(c+dx))+140b\cos(2(c+dx}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] (-240*a*ArcTan[Tan[c + d*x]] - 75*b*ArcTanh[Sin[c + d*x]] + ((295*b + 1168*
a*Cos[c + d*x] + 140*b*Cos[2*(c + d*x)] + 568*a*Cos[3*(c + d*x)] + 165*b*Co
s[4*(c + d*x)] + 184*a*Cos[5*(c + d*x)])*Sec[c + d*x]^5*Tan[c + d*x])/8)/(2
40*d)
```

fricas [A] time = 0.56, size = 134, normalized size = 1.31

$$-\frac{480\,adx\cos\left(dx+c\right)^{6}+75\,b\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-75\,b\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)-2\,(}{480}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 + 75*b*cos(d*x + c)^6*log(sin(d*x + c) + 1
) - 75*b*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*a*cos(d*x + c)^5 +
165*b*cos(d*x + c)^4 - 176*a*cos(d*x + c)^3 - 130*b*cos(d*x + c)^2 + 48*a*c
os(d*x + c) + 40*b)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

giac [B] time = 4.54, size = 228, normalized size = 2.24

$$-\frac{240\,(dx+c)a+75\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-75\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-75\,b\tan(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)*a + 75*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 75*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(240*a*tan(1/2*d*x + 1/2*c)^11 - 75*b*
tan(1/2*d*x + 1/2*c)^11 - 1520*a*tan(1/2*d*x + 1/2*c)^9 + 425*b*tan(1/2*d*x
 + 1/2*c)^9 + 4128*a*tan(1/2*d*x + 1/2*c)^7 - 990*b*tan(1/2*d*x + 1/2*c)^7
- 4128*a*tan(1/2*d*x + 1/2*c)^5 - 990*b*tan(1/2*d*x + 1/2*c)^5 + 1520*a*tan
(1/2*d*x + 1/2*c)^3 + 425*b*tan(1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/
2*c) - 75*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 0.43, size = 178, normalized size = 1.75

$$\frac{a\left(\tan^{5}\left(dx+c\right)\right)}{5d}-\frac{a\left(\tan^{3}\left(dx+c\right)\right)}{3d}+\frac{a\tan\left(dx+c\right)}{d}-ax-\frac{ca}{d}+\frac{b\left(\sin^{7}\left(dx+c\right)\right)}{6d\cos\left(dx+c\right)^{6}}-\frac{b\left(\sin^{7}\left(dx+c\right)\right)}{24d\cos\left(dx+c\right)^{4}}+\frac{b\left(\sin^{7}\left(dx+c\right)\right.}{16d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^6,x)
```

```
[Out] 1/5*a*tan(d*x+c)^5/d-1/3*a*tan(d*x+c)^3/d+a*tan(d*x+c)/d-a*x-1/d*c*a+1/6/d*
b*sin(d*x+c)^7/cos(d*x+c)^6-1/24/d*b*sin(d*x+c)^7/cos(d*x+c)^4+1/16/d*b*sin
(d*x+c)^7/cos(d*x+c)^2+1/16*b*sin(d*x+c)^5/d+5/48*b*sin(d*x+c)^3/d+5/16*b*s
in(d*x+c)/d-5/16/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.87, size = 134, normalized size = 1.31

$$\frac{32\left(3\,\tan\left(dx+c\right)^{5}-5\,\tan\left(dx+c\right)^{3}-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a-5\,b\left(\frac{2\left(33\,\sin(dx+c)^{5}-40\,\sin(dx+c)^{3}+15\,\sin(dx+c)\right.}{\sin(dx+c)^{6}-3\,\sin(dx+c)^{4}+3\,\sin(dx+c)^{2}-1}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x
 + c))*a - 5*b*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d*
x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.51, size = 331, normalized size = 3.25

$$\frac{\left(\frac{5b}{8}-2a\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\left(\frac{38a}{3}-\frac{85b}{24}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+\left(\frac{33b}{4}-\frac{172a}{5}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\left(\frac{172a}{5}+\frac{33b}{4}\right)\tan\left(\frac{c}{2}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+15\,\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(2*a + (5*b)/8) - tan(c/2 + (d*x)/2)^11*(2*a - (5*b)/8)
 - tan(c/2 + (d*x)/2)^3*((38*a)/3 + (85*b)/24) + tan(c/2 + (d*x)/2)^9*((38*
a)/3 - (85*b)/24) + tan(c/2 + (d*x)/2)^5*((172*a)/5 + (33*b)/4) - tan(c/2 +
 (d*x)/2)^7*((172*a)/5 - (33*b)/4))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2
 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c
/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (5*b*atanh((125*b^3*tan(c/
2 + (d*x)/2))/(64*(20*a^2*b + (125*b^3)/64)) + (20*a^2*b*tan(c/2 + (d*x)/2)
)/(20*a^2*b + (125*b^3)/64)))/(8*d) - (2*a*atan((64*a^3*tan(c/2 + (d*x)/2))
/((25*a*b^2)/4 + 64*a^3) + (25*a*b^2*tan(c/2 + (d*x)/2))/(4*((25*a*b^2)/4 +
 64*a^3))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**6,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**6, x)
```

3.265 $\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{\tan^3(c+dx)(4a+3b\sec(c+dx))}{12d}-\frac{\tan(c+dx)(8a+3b\sec(c+dx))}{8d}+ax+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] a*x+3/8*b*arctanh(sin(d*x+c))/d-1/8*(8*a+3*b*sec(d*x+c))*tan(d*x+c)/d+1/12*
(4*a+3*b*sec(d*x+c))*tan(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3770}

$$\frac{\tan^3(c+dx)(4a+3b\sec(c+dx))}{12d}-\frac{\tan(c+dx)(8a+3b\sec(c+dx))}{8d}+ax+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x + (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - ((8*a + 3*b*Sec[c + d*x])*Tan[c +
 d*x])/(8*d) + ((4*a + 3*b*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx &= \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d}-\frac{1}{4}\int (4a+3b\sec(c+dx))\tan^2(c+ \\ &= -\frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d} \\ &= ax-\frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+3b\sec(c+dx))\tan^3(c+ \\ &= ax+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+\end{aligned}$$

Mathematica [A] time = 0.60, size = 79, normalized size = 1.08

$$\frac{-\left(\tan(c+dx)\sec^3(c+dx)(32a\cos(c+dx)+16a\cos(3(c+dx))+15b\cos(2(c+dx))+3b)\right)+48a\tan^{-1}(\tan(c-}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (48*a*ArcTan[Tan[c + d*x]] + 18*b*ArcTanh[Sin[c + d*x]] - (3*b + 32*a*Cos[c
 + d*x] + 15*b*Cos[2*(c + d*x)] + 16*a*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Tan
[c + d*x])/(48*d)
```

fricas [A] time = 0.52, size = 112, normalized size = 1.53

$$\frac{48\,adx\cos\left(dx+c\right)^{4}+9\,b\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-9\,b\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)-2\,(32}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 + 9*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
9*b*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*a*cos(d*x + c)^3 + 15*b*c
os(d*x + c)^2 - 8*a*cos(d*x + c) - 6*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 1.58, size = 172, normalized size = 2.36

$$\frac{24\,(dx+c)a+9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-9\,b\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a + 9*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*b*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) + 2*(24*a*tan(1/2*d*x + 1/2*c)^7 - 9*b*tan(1/2*
d*x + 1/2*c)^7 - 104*a*tan(1/2*d*x + 1/2*c)^5 + 33*b*tan(1/2*d*x + 1/2*c)^5
 + 104*a*tan(1/2*d*x + 1/2*c)^3 + 33*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1/
2*d*x + 1/2*c) - 9*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/
d
```

maple [A] time = 0.43, size = 127, normalized size = 1.74

$$\frac{a\left(\tan^{3}\left(dx+c\right)\right)}{3d}-\frac{a\tan\left(dx+c\right)}{d}+ax+\frac{ca}{d}+\frac{b\left(\sin^{5}\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^{4}}-\frac{b\left(\sin^{5}\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^{2}}-\frac{b\left(\sin^{3}\left(dx+c\right)\right)}{8d}-\frac{3b\sin\left(dx+\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^4,x)
```

```
[Out] 1/3*a*tan(d*x+c)^3/d-a*tan(d*x+c)/d+a*x+1/d*c*a+1/4/d*b*sin(d*x+c)^5/cos(d*
x+c)^4-1/8/d*b*sin(d*x+c)^5/cos(d*x+c)^2-1/8*b*sin(d*x+c)^3/d-3/8*b*sin(d*x
+c)/d+3/8/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.62, size = 102, normalized size = 1.40

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a+3\,b\left(\frac{2\left(5\,\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\mathrm{lo}\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a + 3*b*(2*(5*sin(
d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*lo
g(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.23, size = 267, normalized size = 3.66

$$\frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^3+9\,a\,b^2}+\frac{9\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^3+9\,a\,b^2}\right)}{d}+\frac{3\,b\,\mathrm{atanh}\left(\frac{27\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\left(24\,a^2\,b+\frac{27\,b^3}{8}\right)}+\frac{24\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{24\,a^2\,b+\frac{27\,b^3}{8}}\right)}{4\,d}-\frac{\left(\frac{3\,b}{4}-2\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{d\,\left(\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*atan((64*a^3*tan(c/2 + (d*x)/2))/(9*a*b^2 + 64*a^3) + (9*a*b^2*tan(c/2
 + (d*x)/2))/(9*a*b^2 + 64*a^3)))/d + (3*b*atanh((27*b^3*tan(c/2 + (d*x)/2)
)/(8*(24*a^2*b + (27*b^3)/8)) + (24*a^2*b*tan(c/2 + (d*x)/2))/(24*a^2*b + (
27*b^3)/8)))/(4*d) - (tan(c/2 + (d*x)/2)*(2*a + (3*b)/4) - tan(c/2 + (d*x)/
2)^7*(2*a - (3*b)/4) - tan(c/2 + (d*x)/2)^3*((26*a)/3 + (11*b)/4) + tan(c/2
 + (d*x)/2)^5*((26*a)/3 - (11*b)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2
 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx)) \tan^4 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**4, x)
```

3.266 $\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx$

Optimal. Leaf size=45

$$\frac{\tan(c+dx)(2a+b\sec(c+dx))}{2d} - ax - \frac{b\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] -a*x-1/2*b*arctanh(sin(d*x+c))/d+1/2*(2*a+b*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{3881, 3770\}$

$$\frac{\tan(c+dx)(2a+b\sec(c+dx))}{2d} - ax - \frac{b\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) - (b*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a + b*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))\tan^2(c+dx)\,dx &= \frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d} - \frac{1}{2}\int (2a+b\sec(c+dx))\,dx\\
&= -ax + \frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d} - \frac{1}{2}b\int \sec(c+dx)\,dx\\
&= -ax - \frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.33

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d} + \frac{a\tan(c+dx)}{d} - \frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) - (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

fricas [B] time = 0.57, size = 87, normalized size = 1.93

$$-\frac{4\,adx\cos\left(dx+c\right)^{2}+b\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-b\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(2\,a\cos\left(\right.\right.}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 + b*cos(d*x + c)^2*log(sin(d*x + c) + 1) - b*c
os(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*cos(d*x + c) + b)*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

giac [B] time = 1.52, size = 115, normalized size = 2.56

$$-\frac{2\,(dx+c)a+b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,a\,t}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a + b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 2*(2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/
2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.41, size = 78, normalized size = 1.73

$$-ax+\frac{a\tan\left(dx+c\right)}{d}-\frac{ca}{d}+\frac{b\left(\sin^{3}\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^{2}}+\frac{b\sin\left(dx+c\right)}{2d}-\frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^2,x)
```

```
[Out] -a*x+a*tan(d*x+c)/d-1/d*c*a+1/2/d*b*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b*sin(d*x
+c)/d-1/2/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.74, size = 65, normalized size = 1.44

$$-\frac{4\,(dx+c-\tan\left(dx+c\right))a+b\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c - tan(d*x + c))*a + b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 1.39, size = 96, normalized size = 2.13

$$\frac{a\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}-\frac{b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b\,\sin\left(c+d\,x\right)}{2\,d\,\cos\left(c+d\,x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x))/(d*cos(c + d*x)) - (b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/d - (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b*sin(
c + d*x))/(2*d*cos(c + d*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx)) \tan^2 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**2, x)
```

3.267 $\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=26

$$-\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}-ax$$

[Out] -a*x-cot(d*x+c)*(a+b*sec(d*x+c))/d

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x]),x]

[Out] -(a*x) - (Cot[c + d*x]*(a + b*Sec[c + d*x]))/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}-\int a\,dx\\ &= -ax-\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}\end{aligned}$$

Mathematica [C] time = 0.02, size = 43, normalized size = 1.65

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x]),x]

[Out] -((b*Csc[c + d*x])/d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[c + d*x]^2])/d

fricas [A] time = 0.64, size = 33, normalized size = 1.27

$$-\frac{adx\sin(dx+c)+a\cos(dx+c)+b}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*d*x*sin(d*x + c) + a*cos(d*x + c) + b)/(d*sin(d*x + c))
```

giac [A] time = 0.21, size = 52, normalized size = 2.00

$$-\frac{2\,(dx+c)a - a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \frac{a+b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a - a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c) + (a
+ b)/tan(1/2*d*x + 1/2*c))/d
```

maple [A] time = 0.45, size = 35, normalized size = 1.35

$$\frac{a\,(-\cot\,(dx+c) - dx - c) - \frac{b}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-cot(d*x+c)-d*x-c)-b/sin(d*x+c))
```

maxima [A] time = 0.54, size = 31, normalized size = 1.19

$$-\frac{\left(dx + c + \frac{1}{\tan(dx+c)}\right)a + \frac{b}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c + 1/tan(d*x + c))*a + b/sin(d*x + c))/d
```

mupad [B] time = 1.30, size = 48, normalized size = 1.85

$$\frac{\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\,\left(\frac{a}{2} - \frac{b}{2}\right)}{d} - \frac{\frac{a}{2} + \frac{b}{2}}{d\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)} - a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(a/2 - b/2))/d - (a/2 + b/2)/(d*tan(c/2 + (d*x)/2)) - a
*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec\,(c+dx))\cot^2\,(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**2, x)
```

3.268 $\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=55

$$-\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}+ax$$

```
[Out] a*x-1/3*cot(d*x+c)^3*(a+b*sec(d*x+c))/d+1/3*cot(d*x+c)*(3*a+2*b*sec(d*x+c))
/d
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^3*(a + b*Sec[c + d*x]))/(3*d) + (Cot[c + d*x]*(3*a + 2*
b*Sec[c + d*x]))/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{1}{3}\int \cot^2(c+dx)(-3a-2b\sec(c+dx\\ &= -\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}+\frac{1}{3}\\ &= ax-\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}\end{aligned}$$

Mathematica [C] time = 0.03, size = 62, normalized size = 1.13

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}-\frac{b\csc^3(c+dx)}{3d}+\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]^3*Hypergeom
etric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d)
```

fricas [A] time = 0.51, size = 87, normalized size = 1.58

$$\frac{4\,a\cos\left(dx+c\right)^{3}+3\,b\cos\left(dx+c\right)^{2}-3\,a\cos\left(dx+c\right)+3\left(adx\cos\left(dx+c\right)^{2}-adx\right)\sin\left(dx+c\right)-2\,b}{3\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(4*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 3*a*cos(d*x + c) + 3*(a*d*x*
cos(d*x + c)^2 - a*d*x)*sin(d*x + c) - 2*b)/((d*cos(d*x + c)^2 - d)*sin(d*x
 + c))
```

giac [B] time = 0.26, size = 112, normalized size = 2.04

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+24\left(dx+c\right)a-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{15\,a\tan(}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 + 24*(d*x + c)*a
- 15*a*tan(1/2*d*x + 1/2*c) + 9*b*tan(1/2*d*x + 1/2*c) + (15*a*tan(1/2*d*x
+ 1/2*c)^2 + 9*b*tan(1/2*d*x + 1/2*c)^2 - a - b)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.81, size = 86, normalized size = 1.56

$$\frac{a\left(-\frac{\left(\cot^{3}(dx+c)\right)}{3}+\cot\left(dx+c\right)+dx+c\right)+b\left(-\frac{\cos^{4}(dx+c)}{3\sin(dx+c)^{3}}+\frac{\cos^{4}(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+b*(-1/3/sin(d*x+c)^3*cos(d*x+c)
^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.57, size = 59, normalized size = 1.07

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^{2}-1}{\tan(dx+c)^{3}}\right)a+\frac{\left(3\,\sin(dx+c)^{2}-1\right)b}{\sin(dx+c)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + (3*sin(d*x +
 c)^2 - 1)*b/sin(d*x + c)^3)/d
```

mupad [B] time = 1.56, size = 90, normalized size = 1.64

$$a\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,\left(\frac{a}{24}-\frac{b}{24}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,\left(\left(-5\,a-3\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+\frac{a}{3}+\frac{b}{3}\right)}{8\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a}{8}-\frac{3\,b}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

```
[Out] a*x + (tan(c/2 + (d*x)/2)^3*(a/24 - b/24))/d - (cot(c/2 + (d*x)/2)^3*(a/3 +
 b/3 - tan(c/2 + (d*x)/2)^2*(5*a + 3*b)))/(8*d) - (tan(c/2 + (d*x)/2)*((5*a
)/8 - (3*b)/8))/d
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\cot^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**4, x)
```

3.269 $\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=84

$$-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{15d}-ax$$

```
[Out] -a*x-1/5*cot(d*x+c)^5*(a+b*sec(d*x+c))/d+1/15*cot(d*x+c)^3*(5*a+4*b*sec(d*x
+c))/d-1/15*cot(d*x+c)*(15*a+8*b*sec(d*x+c))/d
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3882, 8}

$$-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{15d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]^5*(a + b*Sec[c + d*x]))/(5*d) + (Cot[c + d*x]^3*(5*a
 + 4*b*Sec[c + d*x]))/(15*d) - (Cot[c + d*x]*(15*a + 8*b*Sec[c + d*x]))/(15
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{1}{5}\int \cot^4(c+dx)(-5a-4b\sec(c+\\ &= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}\\ &= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}\\ &= -ax-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+\end{aligned}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.94

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d}-\frac{b\csc^5(c+dx)}{5d}+\frac{2b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) + (2*b*Csc[c + d*x]^3)/(3*d) - (b*Csc[c + d*x]^5)/(5*
d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(
5*d)
```

fricas [A] time = 0.60, size = 130, normalized size = 1.55

$$-\frac{23\,a\cos\left(dx+c\right)^{5}+15\,b\cos\left(dx+c\right)^{4}-35\,a\cos\left(dx+c\right)^{3}-20\,b\cos\left(dx+c\right)^{2}+15\,a\cos\left(dx+c\right)+15\left(adx\cos\right.}{15\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(23*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 35*a*cos(d*x + c)^3 - 20
*b*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 15*(a*d*x*cos(d*x + c)^4 - 2*a*d*x*
cos(d*x + c)^2 + a*d*x)*sin(d*x + c) + 8*b)/((d*cos(d*x + c)^4 - 2*d*cos(d*
x + c)^2 + d)*sin(d*x + c))
```

giac [B] time = 0.37, size = 170, normalized size = 2.02

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+25\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-480\,(dx+c)a+33}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 - 35*a*tan(1
/2*d*x + 1/2*c)^3 + 25*b*tan(1/2*d*x + 1/2*c)^3 - 480*(d*x + c)*a + 330*a*t
an(1/2*d*x + 1/2*c) - 150*b*tan(1/2*d*x + 1/2*c) - (330*a*tan(1/2*d*x + 1/2
*c)^4 + 150*b*tan(1/2*d*x + 1/2*c)^4 - 35*a*tan(1/2*d*x + 1/2*c)^2 - 25*b*t
an(1/2*d*x + 1/2*c)^2 + 3*a + 3*b)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.82, size = 129, normalized size = 1.54

$$\frac{a\left(-\frac{\left(\cot^{5}(dx+c)\right)}{5}+\frac{\left(\cot^{3}(dx+c)\right)}{3}-\cot\left(dx+c\right)-dx-c\right)+b\left(-\frac{\cos^{6}(dx+c)}{5\sin(dx+c)^{5}}+\frac{\cos^{6}(dx+c)}{15\sin(dx+c)^{3}}-\frac{\cos^{6}(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4(\cos}{}\right.}{5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+b*(-1/5/sin(d*
x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c
)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.69, size = 79, normalized size = 0.94

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^{4}-5\,\tan(dx+c)^{2}+3}{\tan(dx+c)^{5}}\right)a+\frac{\left(15\,\sin(dx+c)^{4}-10\,\sin(dx+c)^{2}+3\right)b}{\sin(dx+c)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a + (15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*b/sin(d*x + c)^5)/d
```

mupad [B] time = 1.39, size = 132, normalized size = 1.57

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{a}{160}-\frac{b}{160}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left((22a+10b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(-\frac{7a}{3}-\frac{5b}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{5}+\frac{b}{5}\right)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a/160 - b/160))/d - (cot(c/2 + (d*x)/2)^5*(a/5 + b/5
 - tan(c/2 + (d*x)/2)^2*((7*a)/3 + (5*b)/3) + tan(c/2 + (d*x)/2)^4*(22*a +
10*b)))/(32*d) - (tan(c/2 + (d*x)/2)^3*((7*a)/96 - (5*b)/96))/d - a*x + (ta
n(c/2 + (d*x)/2)*((11*a)/16 - (5*b)/16))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\cot^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**6, x)
```

3.270 $\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d}-\frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d}+\frac{\cot(c$$

```
[Out] a*x-1/7*cot(d*x+c)^7*(a+b*sec(d*x+c))/d+1/35*cot(d*x+c)^5*(7*a+6*b*sec(d*x+
c))/d+1/35*cot(d*x+c)*(35*a+16*b*sec(d*x+c))/d-1/105*cot(d*x+c)^3*(35*a+24*
b*sec(d*x+c))/d
```

Rubi [A] time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3882, 8}

$$-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d}-\frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d}+\frac{\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + b*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^7*(a + b*Sec[c + d*x]))/(7*d) + (Cot[c + d*x]^5*(7*a +
6*b*Sec[c + d*x]))/(35*d) + (Cot[c + d*x]*(35*a + 16*b*Sec[c + d*x]))/(35*d
) - (Cot[c + d*x]^3*(35*a + 24*b*Sec[c + d*x]))/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{1}{7}\int \cot^6(c+dx)(-7a-6b\sec(c+dx\\ &= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d}+\frac{1}{3}\\ &= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d}-\text{c}\\ &= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d}+\text{c}\\ &= ax-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d}\end{aligned}$$

Mathematica [C] time = 0.05, size = 92, normalized size = 0.83

$$-\frac{a\cot^7(c+dx)\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\tan^2(c+dx)\right)}{7d}-\frac{b\csc^7(c+dx)}{7d}+\frac{3b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{d}+\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/d + (3*b*Csc[c + d*x]^5)/(5*d) - (b
*Csc[c + d*x]^7)/(7*d) - (a*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2,
 -Tan[c + d*x]^2])/(7*d)
```

fricas [A] time = 0.51, size = 179, normalized size = 1.61

$$\frac{176\,a\cos\left(dx+c\right)^{7}+105\,b\cos\left(dx+c\right)^{6}-406\,a\cos\left(dx+c\right)^{5}-210\,b\cos\left(dx+c\right)^{4}+350\,a\cos\left(dx+c\right)^{3}+168\cdots}{105\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(176*a*cos(d*x + c)^7 + 105*b*cos(d*x + c)^6 - 406*a*cos(d*x + c)^5 -
 210*b*cos(d*x + c)^4 + 350*a*cos(d*x + c)^3 + 168*b*cos(d*x + c)^2 - 105*a
*cos(d*x + c) + 105*(a*d*x*cos(d*x + c)^6 - 3*a*d*x*cos(d*x + c)^4 + 3*a*d*
x*cos(d*x + c)^2 - a*d*x)*sin(d*x + c) - 48*b)/((d*cos(d*x + c)^6 - 3*d*cos
(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.35, size = 225, normalized size = 2.03

$$15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-189\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+147\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1295\,a\tan\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 - 15*b*tan(1/2*d*x + 1/2*c)^7 - 189*a*
tan(1/2*d*x + 1/2*c)^5 + 147*b*tan(1/2*d*x + 1/2*c)^5 + 1295*a*tan(1/2*d*x
+ 1/2*c)^3 - 735*b*tan(1/2*d*x + 1/2*c)^3 + 13440*(d*x + c)*a - 9765*a*tan(
1/2*d*x + 1/2*c) + 3675*b*tan(1/2*d*x + 1/2*c) + (9765*a*tan(1/2*d*x + 1/2*
c)^6 + 3675*b*tan(1/2*d*x + 1/2*c)^6 - 1295*a*tan(1/2*d*x + 1/2*c)^4 - 735*
b*tan(1/2*d*x + 1/2*c)^4 + 189*a*tan(1/2*d*x + 1/2*c)^2 + 147*b*tan(1/2*d*x
 + 1/2*c)^2 - 15*a - 15*b)/tan(1/2*d*x + 1/2*c)^7)/d
```

maple [A] time = 0.91, size = 162, normalized size = 1.46

$$\frac{a\left(-\frac{\left(\cot^{7}(dx+c)\right)}{7}+\frac{\left(\cot^{5}(dx+c)\right)}{5}-\frac{\left(\cot^{3}(dx+c)\right)}{3}+\cot\left(dx+c\right)+dx+c\right)+b\left(-\frac{\cos^{8}(dx+c)}{7\sin(dx+c)^{7}}+\frac{\cos^{8}(dx+c)}{35\sin(dx+c)^{5}}-\frac{\cos^{8}(dx+c)}{35\sin(dx+c)^{3}}\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+
c)+b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/si
n(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^6+
6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.66, size = 100, normalized size = 0.90

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan\left(dx+c\right)^{6}-35\,\tan\left(dx+c\right)^{4}+21\,\tan\left(dx+c\right)^{2}-15}{\tan\left(dx+c\right)^{7}}\right)a+\frac{3\left(35\,\sin\left(dx+c\right)^{6}-35\,\sin\left(dx+c\right)^{4}+21\,\sin\left(dx+c\right)^{2}-5\right)b}{\sin\left(dx+c\right)^{7}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a + 3*(35*sin(d*x + c)^6 - 35*sin(d*x + c)
^4 + 21*sin(d*x + c)^2 - 5)*b/sin(d*x + c)^7)/d
```

mupad [B] time = 1.62, size = 174, normalized size = 1.57

$$a\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{37\,a}{384}-\frac{7\,b}{128}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{9\,a}{640}-\frac{7\,b}{640}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{a}{896}-\frac{b}{896}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\left(-93\,a-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8*(a + b/cos(c + d*x)),x)
```

```
[Out] a*x + (tan(c/2 + (d*x)/2)^3*((37*a)/384 - (7*b)/128))/d - (tan(c/2 + (d*x)/
2)^5*((9*a)/640 - (7*b)/640))/d + (tan(c/2 + (d*x)/2)^7*(a/896 - b/896))/d
- (cot(c/2 + (d*x)/2)^7*(a/7 + b/7 - tan(c/2 + (d*x)/2)^2*((9*a)/5 + (7*b)/
5) + tan(c/2 + (d*x)/2)^4*((37*a)/3 + 7*b) - tan(c/2 + (d*x)/2)^6*(93*a + 3
5*b)))/(128*d) - (tan(c/2 + (d*x)/2)*((93*a)/128 - (35*b)/128))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.271 $\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx$

Optimal. Leaf size=185

$$\frac{a^2\sec^8(c+dx)}{8d}-\frac{2a^2\sec^6(c+dx)}{3d}+\frac{3a^2\sec^4(c+dx)}{2d}-\frac{2a^2\sec^2(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^9(c+dx)}{9d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d-2*a^2*sec(d*x+c)^2/d-8/3*a*b*sec(d
*x+c)^3/d+3/2*a^2*sec(d*x+c)^4/d+12/5*a*b*sec(d*x+c)^5/d-2/3*a^2*sec(d*x+c)
^6/d-8/7*a*b*sec(d*x+c)^7/d+1/8*a^2*sec(d*x+c)^8/d+2/9*a*b*sec(d*x+c)^9/d+1
/10*b^2*tan(d*x+c)^10/d
```

Rubi [A] time = 0.13, antiderivative size = 217, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3885, 948}

$$\frac{(a^2-4b^2)\sec^8(c+dx)}{8d}-\frac{(2a^2-3b^2)\sec^6(c+dx)}{3d}+\frac{(3a^2-2b^2)\sec^4(c+dx)}{2d}-\frac{(4a^2-b^2)\sec^2(c+dx)}{2d}-\frac{a^2\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d - ((4*a^2 - b^2)*Sec[
c + d*x]^2)/(2*d) - (8*a*b*Sec[c + d*x]^3)/(3*d) + ((3*a^2 - 2*b^2)*Sec[c +
 d*x]^4)/(2*d) + (12*a*b*Sec[c + d*x]^5)/(5*d) - ((2*a^2 - 3*b^2)*Sec[c + d
*x]^6)/(3*d) - (8*a*b*Sec[c + d*x]^7)/(7*d) + ((a^2 - 4*b^2)*Sec[c + d*x]^8
)/(8*d) + (2*a*b*Sec[c + d*x]^9)/(9*d) + (b^2*Sec[c + d*x]^10)/(10*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^4}{x}\,dx, x, b\sec(c+dx)\right)}{b^8 d}\\ &= \frac{\text{Subst}\left(\int \left(2ab^8+\frac{a^2b^8}{x}-b^6(4a^2-b^2)x-8ab^6x^2+2b^4(3a^2-2b^2)x^3\right.\right.}{}\\ &= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{(4a^2-b^2)\sec^2(c+dx)}{2d}-\end{aligned}$$

Mathematica [A] time = 0.42, size = 173, normalized size = 0.94

$$\frac{315(a^2-4b^2)\sec^8(c+dx)-840(2a^2-3b^2)\sec^6(c+dx)+1260(3a^2-2b^2)\sec^4(c+dx)-1260(4a^2-b^2)\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] (-2520*a^2*Log[Cos[c + d*x]] + 5040*a*b*Sec[c + d*x] - 1260*(4*a^2 - b^2)*S
ec[c + d*x]^2 - 6720*a*b*Sec[c + d*x]^3 + 1260*(3*a^2 - 2*b^2)*Sec[c + d*x]
^4 + 6048*a*b*Sec[c + d*x]^5 - 840*(2*a^2 - 3*b^2)*Sec[c + d*x]^6 - 2880*a*
b*Sec[c + d*x]^7 + 315*(a^2 - 4*b^2)*Sec[c + d*x]^8 + 560*a*b*Sec[c + d*x]^
9 + 252*b^2*Sec[c + d*x]^10)/(2520*d)
```

fricas [A] time = 0.61, size = 181, normalized size = 0.98

$$-\frac{2520\,a^2\cos\left(dx+c\right)^{10}\log\left(-\cos\left(dx+c\right)\right)-5040\,ab\cos\left(dx+c\right)^9+6720\,ab\cos\left(dx+c\right)^7+1260\left(4\,a^2-b^2\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a^2*cos(d*x + c)^10*log(-cos(d*x + c)) - 5040*a*b*cos(d*x + c
)^9 + 6720*a*b*cos(d*x + c)^7 + 1260*(4*a^2 - b^2)*cos(d*x + c)^8 - 6048*a*
b*cos(d*x + c)^5 - 1260*(3*a^2 - 2*b^2)*cos(d*x + c)^6 + 2880*a*b*cos(d*x +
 c)^3 + 840*(2*a^2 - 3*b^2)*cos(d*x + c)^4 - 560*a*b*cos(d*x + c) - 315*(a^
2 - 4*b^2)*cos(d*x + c)^2 - 252*b^2)/(d*cos(d*x + c)^10)
```

giac [B] time = 19.61, size = 489, normalized size = 2.64

$$\frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{7381\,a^2+4096\,ab+\frac{78850\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{40960\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (7381*a^2 + 40
96*a*b + 78850*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 40960*a*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 382545*a^2*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + 184320*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 111420
0*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 491520*a*b*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3 + 2171610*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c
) + 1)^4 + 860160*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 2736972*a
^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 516096*a*b*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 - 258048*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) +
1)^5 + 2171610*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1114200*a^2*
(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 382545*a^2*(cos(d*x + c) - 1)^8
/(cos(d*x + c) + 1)^8 + 78850*a^2*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9
 + 7381*a^2*(cos(d*x + c) - 1)^10/(cos(d*x + c) + 1)^10)/((cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 1)^10)/d
```

maple [A] time = 0.77, size = 317, normalized size = 1.71

$$\frac{\left(\tan^8\left(dx+c\right)\right)a^2}{8d}-\frac{a^2\left(\tan^6\left(dx+c\right)\right)}{6d}+\frac{a^2\left(\tan^4\left(dx+c\right)\right)}{4d}-\frac{a^2\left(\tan^2\left(dx+c\right)\right)}{2d}-\frac{a^2\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{2ab\left(\sin^{10}\left(dx\right.\right.}{9d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x)
```

```
[Out] 1/8/d*tan(d*x+c)^8*a^2-1/6/d*a^2*tan(d*x+c)^6+1/4*a^2*tan(d*x+c)^4/d-1/2*a^
2*tan(d*x+c)^2/d-a^2*ln(cos(d*x+c))/d+2/9/d*a*b*sin(d*x+c)^10/cos(d*x+c)^9-
2/63/d*a*b*sin(d*x+c)^10/cos(d*x+c)^7+2/105/d*a*b*sin(d*x+c)^10/cos(d*x+c)^
5-2/63/d*a*b*sin(d*x+c)^10/cos(d*x+c)^3+2/9/d*a*b*sin(d*x+c)^10/cos(d*x+c)+
256/315*a*b*cos(d*x+c)/d+2/9/d*a*b*cos(d*x+c)*sin(d*x+c)^8+16/63/d*a*b*cos(
d*x+c)*sin(d*x+c)^6+32/105/d*a*b*cos(d*x+c)*sin(d*x+c)^4+128/315/d*a*b*cos(
d*x+c)*sin(d*x+c)^2+1/10/d*b^2*sin(d*x+c)^10/cos(d*x+c)^10
```

maxima [A] time = 0.71, size = 174, normalized size = 0.94

$$-\frac{2520\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{5040\,ab\cos(dx+c)^9-6720\,ab\cos(dx+c)^7-1260\left(4\,a^2-b^2\right)\cos(dx+c)^8+6048\,ab\cos(dx+c)^5+1260\left(3\,a^2-2\,b^2\right)}{\text{co}}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a^2*log(cos(d*x + c)) - (5040*a*b*cos(d*x + c)^9 - 6720*a*b*c
os(d*x + c)^7 - 1260*(4*a^2 - b^2)*cos(d*x + c)^8 + 6048*a*b*cos(d*x + c)^5
 + 1260*(3*a^2 - 2*b^2)*cos(d*x + c)^6 - 2880*a*b*cos(d*x + c)^3 - 840*(2*a
^2 - 3*b^2)*cos(d*x + c)^4 + 560*a*b*cos(d*x + c) + 315*(a^2 - 4*b^2)*cos(d
*x + c)^2 + 252*b^2)/cos(d*x + c)^10)/d
```

mupad [B] time = 5.07, size = 344, normalized size = 1.86

$$\frac{\frac{512\,a\,b}{315}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(20\,a^2+\frac{512\,b\,a}{7}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+\frac{1024\,b\,a}{63}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(\frac{740\,a^2}{3}+\frac{1024\,b\,a}{3}\right)-\text{ta}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+45\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-120\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9*(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((512*a*b)/315 + tan(c/2 + (d*x)/2)^4*((512*a*b)/7 + 20*a^2) - tan(c/2 + (d
*x)/2)^2*((1024*a*b)/63 + 2*a^2) + tan(c/2 + (d*x)/2)^8*((1024*a*b)/3 + (74
0*a^2)/3) - tan(c/2 + (d*x)/2)^6*((4096*a*b)/21 + (272*a^2)/3) + (740*a^2*t
an(c/2 + (d*x)/2)^12)/3 - (272*a^2*tan(c/2 + (d*x)/2)^14)/3 + 20*a^2*tan(c/
2 + (d*x)/2)^16 - 2*a^2*tan(c/2 + (d*x)/2)^18 - tan(c/2 + (d*x)/2)^10*((102
4*a*b)/5 + 348*a^2 - (512*b^2)/5))/(d*(45*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2
 + (d*x)/2)^2 - 120*tan(c/2 + (d*x)/2)^6 + 210*tan(c/2 + (d*x)/2)^8 - 252*t
an(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^12 - 120*tan(c/2 + (d*x)/2)^1
4 + 45*tan(c/2 + (d*x)/2)^16 - 10*tan(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2
)^20 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 34.47, size = 314, normalized size = 1.70

$$\begin{cases}\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^8\left(c+dx\right)}{8d}-\frac{a^2\tan^6\left(c+dx\right)}{6d}+\frac{a^2\tan^4\left(c+dx\right)}{4d}-\frac{a^2\tan^2\left(c+dx\right)}{2d}+\frac{2ab\tan^8\left(c+dx\right)\sec\left(c+dx\right)}{9d}-\frac{16ab\tan^6\left(c\right.}{6} \\ x\left(a+b\sec\left(c\right)\right)^2\tan^9\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**8/(8*d)
 - a**2*tan(c + d*x)**6/(6*d) + a**2*tan(c + d*x)**4/(4*d) - a**2*tan(c + d
*x)**2/(2*d) + 2*a*b*tan(c + d*x)**8*sec(c + d*x)/(9*d) - 16*a*b*tan(c + d*
x)**6*sec(c + d*x)/(63*d) + 32*a*b*tan(c + d*x)**4*sec(c + d*x)/(105*d) - 1
```

```
28*a*b*tan(c + d*x)**2*sec(c + d*x)/(315*d) + 256*a*b*sec(c + d*x)/(315*d)
+ b**2*tan(c + d*x)**8*sec(c + d*x)**2/(10*d) - b**2*tan(c + d*x)**6*sec(c
+ d*x)**2/(10*d) + b**2*tan(c + d*x)**4*sec(c + d*x)**2/(10*d) - b**2*tan(c
 + d*x)**2*sec(c + d*x)**2/(10*d) + b**2*sec(c + d*x)**2/(10*d), Ne(d, 0)),
 (x*(a + b*sec(c))**2*tan(c)**9, True))
```

3.272 $\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$

Optimal. Leaf size=149

$$\frac{a^2\sec^6(c+dx)}{6d}-\frac{3a^2\sec^4(c+dx)}{4d}+\frac{3a^2\sec^2(c+dx)}{2d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^7(c+dx)}{7d}-\frac{6ab\sec^5(c+dx)}{5d}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a*b*sec(d*x+c)/d+3/2*a^2*sec(d*x+c)^2/d+2*a*b*sec(d*
x+c)^3/d-3/4*a^2*sec(d*x+c)^4/d-6/5*a*b*sec(d*x+c)^5/d+1/6*a^2*sec(d*x+c)^6
/d+2/7*a*b*sec(d*x+c)^7/d+1/8*b^2*tan(d*x+c)^8/d
```

Rubi [A] time = 0.11, antiderivative size = 169, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 948}

$$\frac{(a^2-3b^2)\sec^6(c+dx)}{6d}-\frac{3(a^2-b^2)\sec^4(c+dx)}{4d}+\frac{(3a^2-b^2)\sec^2(c+dx)}{2d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^7(c+}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a*b*Sec[c + d*x])/d + ((3*a^2 - b^2)*Sec[c +
 d*x]^2)/(2*d) + (2*a*b*Sec[c + d*x]^3)/d - (3*(a^2 - b^2)*Sec[c + d*x]^4)/
(4*d) - (6*a*b*Sec[c + d*x]^5)/(5*d) + ((a^2 - 3*b^2)*Sec[c + d*x]^6)/(6*d)
 + (2*a*b*Sec[c + d*x]^7)/(7*d) + (b^2*Sec[c + d*x]^8)/(8*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^3}{x}\,dx,x,b\sec(c+dx)\right)}{b^6d}\\ &= -\frac{\text{Subst}\left(\int \left(2ab^6+\frac{a^2b^6}{x}-b^4\left(3a^2-b^2\right)x-6ab^4x^2+3b^2\left(a^2-b^2\right)x^3\right.\right.}{b^6d}\\ &= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2ab\sec(c+dx)}{d}+\frac{\left(3a^2-b^2\right)\sec^2(c+dx)}{2d}+\frac{2}{}\end{aligned}$$

Mathematica [A] time = 0.35, size = 138, normalized size = 0.93

$$\frac{140\left(a^2-3b^2\right)\sec^6(c+dx)-630\left(a^2-b^2\right)\sec^4(c+dx)+420\left(3a^2-b^2\right)\sec^2(c+dx)+840a^2\log(\cos(c+dx}{84}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (840*a^2*Log[Cos[c + d*x]] - 1680*a*b*Sec[c + d*x] + 420*(3*a^2 - b^2)*Sec[
c + d*x]^2 + 1680*a*b*Sec[c + d*x]^3 - 630*(a^2 - b^2)*Sec[c + d*x]^4 - 100
8*a*b*Sec[c + d*x]^5 + 140*(a^2 - 3*b^2)*Sec[c + d*x]^6 + 240*a*b*Sec[c + d
*x]^7 + 105*b^2*Sec[c + d*x]^8)/(840*d)
```

fricas [A] time = 0.57, size = 146, normalized size = 0.98

$$\frac{840\,a^2\cos\left(dx+c\right)^8\log\left(-\cos\left(dx+c\right)\right)-1680\,ab\cos\left(dx+c\right)^7+1680\,ab\cos\left(dx+c\right)^5+420\left(3\,a^2-b^2\right)\cos\left(dx\right.}{840\,d\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(840*a^2*cos(d*x + c)^8*log(-cos(d*x + c)) - 1680*a*b*cos(d*x + c)^7
+ 1680*a*b*cos(d*x + c)^5 + 420*(3*a^2 - b^2)*cos(d*x + c)^6 - 1008*a*b*cos
(d*x + c)^3 - 630*(a^2 - b^2)*cos(d*x + c)^4 + 240*a*b*cos(d*x + c) + 140*(
a^2 - 3*b^2)*cos(d*x + c)^2 + 105*b^2)/(d*cos(d*x + c)^8)
```

giac [B] time = 8.89, size = 415, normalized size = 2.79

$$-\frac{840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2283\,a^2+1536\,ab+\frac{19944\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12288\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/840*(840*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 840*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2283*a^2 + 1536
*a*b + 19944*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12288*a*b*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 77364*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + 43008*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 175448*a^2
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 86016*a*b*(cos(d*x + c) - 1)^3
/(cos(d*x + c) + 1)^3 + 231490*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^
4 + 53760*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 26880*b^2*(cos(d*
x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 175448*a^2*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5 + 77364*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1994
4*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 2283*a^2*(cos(d*x + c) -
1)^8/(cos(d*x + c) + 1)^8)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^8)/d
```

maple [A] time = 0.64, size = 256, normalized size = 1.72

$$\frac{a^2\left(\tan^6\left(dx+c\right)\right)}{6d}-\frac{a^2\left(\tan^4\left(dx+c\right)\right)}{4d}+\frac{a^2\left(\tan^2\left(dx+c\right)\right)}{2d}+\frac{a^2\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{2ab\left(\sin^8\left(dx+c\right)\right)}{7d\cos\left(dx+c\right)^7}-\frac{2ab\left(\sin^8\left(d\cdots\right.\right.}{35d\cos\left(dx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x)
```

```
[Out] 1/6/d*a^2*tan(d*x+c)^6-1/4*a^2*tan(d*x+c)^4/d+1/2*a^2*tan(d*x+c)^2/d+a^2*ln
(cos(d*x+c))/d+2/7/d*a*b*sin(d*x+c)^8/cos(d*x+c)^7-2/35/d*a*b*sin(d*x+c)^8/
cos(d*x+c)^5+2/35/d*a*b*sin(d*x+c)^8/cos(d*x+c)^3-2/7/d*a*b*sin(d*x+c)^8/co
s(d*x+c)-32/35*a*b*cos(d*x+c)/d-2/7/d*a*b*cos(d*x+c)*sin(d*x+c)^6-12/35/d*a
*b*cos(d*x+c)*sin(d*x+c)^4-16/35/d*a*b*cos(d*x+c)*sin(d*x+c)^2+1/8/d*b^2*si
n(d*x+c)^8/cos(d*x+c)^8
```

maxima [A] time = 0.44, size = 139, normalized size = 0.93

$$\frac{840\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{1680\,ab\cos(dx+c)^7-1680\,ab\cos(dx+c)^5-420\left(3\,a^2-b^2\right)\cos(dx+c)^6+1008\,ab\cos(dx+c)^3+630\left(a^2-b^2\right)\cos(dx+c}{\cos(dx+c)^8}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/840*(840*a^2*log(cos(d*x + c)) - (1680*a*b*cos(d*x + c)^7 - 1680*a*b*cos(
d*x + c)^5 - 420*(3*a^2 - b^2)*cos(d*x + c)^6 + 1008*a*b*cos(d*x + c)^3 + 6
30*(a^2 - b^2)*cos(d*x + c)^4 - 240*a*b*cos(d*x + c) - 140*(a^2 - 3*b^2)*co
s(d*x + c)^2 - 105*b^2)/cos(d*x + c)^8)/d
```

mupad [B] time = 5.17, size = 280, normalized size = 1.88

$$-\frac{\frac{64\,a\,b}{35}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(16\,a^2+\frac{256\,b\,a}{5}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^2+\frac{512\,b\,a}{35}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{170\,a^2}{3}+\frac{512\,b\,a}{5}\right)-\frac{170}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-56\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+70\,\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + b/cos(c + d*x))^2,x)
```

```
[Out] - ((64*a*b)/35 + tan(c/2 + (d*x)/2)^4*((256*a*b)/5 + 16*a^2) - tan(c/2 + (d
*x)/2)^2*((512*a*b)/35 + 2*a^2) - tan(c/2 + (d*x)/2)^6*((512*a*b)/5 + (170*
a^2)/3) - (170*a^2*tan(c/2 + (d*x)/2)^10)/3 + 16*a^2*tan(c/2 + (d*x)/2)^12
- 2*a^2*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^8*(64*a*b + (256*a^2)/3
- 32*b^2))/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/
2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*ta
n(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
 - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 14.24, size = 252, normalized size = 1.69

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^6\left(c+dx\right)}{6d}-\frac{a^2\tan^4\left(c+dx\right)}{4d}+\frac{a^2\tan^2\left(c+dx\right)}{2d}+\frac{2ab\tan^6\left(c+dx\right)\sec\left(c+dx\right)}{7d}-\frac{12ab\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{35d} \\ x\left(a+b\sec\left(c\right)\right)^2\tan^7\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**6/(6*d
) - a**2*tan(c + d*x)**4/(4*d) + a**2*tan(c + d*x)**2/(2*d) + 2*a*b*tan(c +
 d*x)**6*sec(c + d*x)/(7*d) - 12*a*b*tan(c + d*x)**4*sec(c + d*x)/(35*d) +
16*a*b*tan(c + d*x)**2*sec(c + d*x)/(35*d) - 32*a*b*sec(c + d*x)/(35*d) + b
**2*tan(c + d*x)**6*sec(c + d*x)**2/(8*d) - b**2*tan(c + d*x)**4*sec(c + d*
x)**2/(8*d) + b**2*tan(c + d*x)**2*sec(c + d*x)**2/(8*d) - b**2*sec(c + d*x
)**2/(8*d), Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c)**7, True))
```

3.273 $\int (a+b\sec(c+dx))^2 \tan^5(c+dx)\,dx$

Optimal. Leaf size=115

$$\frac{a^2\sec^4(c+dx)}{4d}-\frac{a^2\sec^2(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{4ab\sec^3(c+dx)}{3d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2 \ldots}{}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d-a^2*sec(d*x+c)^2/d-4/3*a*b*sec(d*x
+c)^3/d+1/4*a^2*sec(d*x+c)^4/d+2/5*a*b*sec(d*x+c)^5/d+1/6*b^2*tan(d*x+c)^6/
d
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 948}

$$\frac{(a^2-2b^2)\sec^4(c+dx)}{4d}-\frac{(2a^2-b^2)\sec^2(c+dx)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{4ab\sec^3(c+dx)}{3d}+\frac{2ab \ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d - ((2*a^2 - b^2)*Sec[
c + d*x]^2)/(2*d) - (4*a*b*Sec[c + d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sec[c + d
*x]^4)/(4*d) + (2*a*b*Sec[c + d*x]^5)/(5*d) + (b^2*Sec[c + d*x]^6)/(6*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x}\,dx, x, b\sec(c+dx)\right)}{b^4 d}\\ &= \frac{\text{Subst}\left(\int \left(2ab^4+\frac{a^2b^4}{x}-b^2(2a^2-b^2)x-4ab^2x^2+(a^2-2b^2)x^3+2ax^4 \ldots\right.\right.}{b^4 d}\\ &= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{(2a^2-b^2)\sec^2(c+dx)}{2d}-\frac{4ab \ldots}{}\end{aligned}$$

Mathematica [A] time = 0.27, size = 105, normalized size = 0.91

$$\frac{15(a^2-2b^2)\sec^4(c+dx)+30(b^2-2a^2)\sec^2(c+dx)-60a^2\log(\cos(c+dx))+24ab\sec^5(c+dx)-80ab\sec^3(\ldots}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-60*a^2*Log[Cos[c + d*x]] + 120*a*b*Sec[c + d*x] + 30*(-2*a^2 + b^2)*Sec[c
 + d*x]^2 - 80*a*b*Sec[c + d*x]^3 + 15*(a^2 - 2*b^2)*Sec[c + d*x]^4 + 24*a*
b*Sec[c + d*x]^5 + 10*b^2*Sec[c + d*x]^6)/(60*d)
```

fricas [A] time = 0.56, size = 115, normalized size = 1.00

$$-\frac{60\,a^2\cos(dx+c)^6\log(-\cos(dx+c)) - 120\,ab\cos(dx+c)^5 + 80\,ab\cos(dx+c)^3 + 30\left(2\,a^2-b^2\right)\cos(dx+}{60\,d\cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a^2*cos(d*x + c)^6*log(-cos(d*x + c)) - 120*a*b*cos(d*x + c)^5 +
80*a*b*cos(d*x + c)^3 + 30*(2*a^2 - b^2)*cos(d*x + c)^4 - 24*a*b*cos(d*x +
c) - 15*(a^2 - 2*b^2)*cos(d*x + c)^2 - 10*b^2)/(d*cos(d*x + c)^6)
```

giac [B] time = 5.39, size = 341, normalized size = 2.97

$$60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{147\,a^2+128\,ab+\frac{1002\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{768\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2925\,a^2(}{(\cos}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (147*a^2 + 128*a*b +
 1002*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 768*a*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 2925*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 1920*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4140*a^2*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3 + 1280*a*b*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 - 640*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*a^2*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^2*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 147*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^6)/d
```

maple [A] time = 0.66, size = 197, normalized size = 1.71

$$\frac{a^2\left(\tan^4(dx+c)\right)}{4d} - \frac{a^2\left(\tan^2(dx+c)\right)}{2d} - \frac{a^2\ln(\cos(dx+c))}{d} + \frac{2ab\left(\sin^6(dx+c)\right)}{5d\cos(dx+c)^5} - \frac{2ab\left(\sin^6(dx+c)\right)}{15d\cos(dx+c)^3} + \frac{2ab\left(\sin\right.}{5d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x)
```

```
[Out] 1/4*a^2*tan(d*x+c)^4/d-1/2*a^2*tan(d*x+c)^2/d-a^2*ln(cos(d*x+c))/d+2/5/d*a*
b*sin(d*x+c)^6/cos(d*x+c)^5-2/15/d*a*b*sin(d*x+c)^6/cos(d*x+c)^3+2/5/d*a*b*
sin(d*x+c)^6/cos(d*x+c)+16/15*a*b*cos(d*x+c)/d+2/5/d*a*b*cos(d*x+c)*sin(d*x
+c)^4+8/15/d*a*b*cos(d*x+c)*sin(d*x+c)^2+1/6/d*b^2*sin(d*x+c)^6/cos(d*x+c)^
6
```

maxima [A] time = 0.33, size = 108, normalized size = 0.94

$$-\frac{60\,a^2\log(\cos(dx+c)) - \frac{120\,ab\cos(dx+c)^5-80\,ab\cos(dx+c)^3-30\left(2\,a^2-b^2\right)\cos(dx+c)^4+24\,ab\cos(dx+c)+15\left(a^2-2\,b^2\right)\cos(dx+c)^2+10\,b}{\cos(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a^2*log(cos(d*x + c)) - (120*a*b*cos(d*x + c)^5 - 80*a*b*cos(d*x
+ c)^3 - 30*(2*a^2 - b^2)*cos(d*x + c)^4 + 24*a*b*cos(d*x + c) + 15*(a^2 -
2*b^2)*cos(d*x + c)^2 + 10*b^2)/cos(d*x + c)^6)/d
```

mupad [B] time = 4.97, size = 215, normalized size = 1.87

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}+\frac{\frac{32\,a\,b}{15}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(12\,a^2+32\,b\,a\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+\frac{64\,b\,a}{5}\right)+12\,a^2\,\tan\left(\frac{c}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-20\,\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d + ((32*a*b)/15 + tan(c/2 + (d*x)/2)^4
*(32*a*b + 12*a^2) - tan(c/2 + (d*x)/2)^2*((64*a*b)/5 + 2*a^2) + 12*a^2*tan
(c/2 + (d*x)/2)^8 - 2*a^2*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^6*((64
*a*b)/3 + 20*a^2 - (32*b^2)/3))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (
d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 +
 (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [A] time = 5.44, size = 189, normalized size = 1.64

$$\begin{cases}\frac{a^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^2\tan^4(c+dx)}{4d}-\frac{a^2\tan^2(c+dx)}{2d}+\frac{2ab\tan^4(c+dx)\sec(c+dx)}{5d}-\frac{8ab\tan^2(c+dx)\sec(c+dx)}{15d}+\frac{16ab\sec(c+dx)}{15d}+\frac{b^2\,\mathrm{t}}{} \\ x\,(a+b\sec(c))^2\tan^5(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**4/(4*d)
 - a**2*tan(c + d*x)**2/(2*d) + 2*a*b*tan(c + d*x)**4*sec(c + d*x)/(5*d) -
8*a*b*tan(c + d*x)**2*sec(c + d*x)/(15*d) + 16*a*b*sec(c + d*x)/(15*d) + b*
*2*tan(c + d*x)**4*sec(c + d*x)**2/(6*d) - b**2*tan(c + d*x)**2*sec(c + d*x
)**2/(6*d) + b**2*sec(c + d*x)**2/(6*d), Ne(d, 0)), (x*(a + b*sec(c))**2*ta
n(c)**5, True))
```

3.274 $\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=87

$$\frac{(a^2-b^2)\sec^2(c+dx)}{2d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^4(c+dx)}{4d}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a*b*sec(d*x+c)/d+1/2*(a^2-b^2)*sec(d*x+c)^2/d+2/3*a*
b*sec(d*x+c)^3/d+1/4*b^2*sec(d*x+c)^4/d
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{(a^2-b^2)\sec^2(c+dx)}{2d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a*b*Sec[c + d*x])/d + ((a^2 - b^2)*Sec[c + d
*x]^2)/(2*d) + (2*a*b*Sec[c + d*x]^3)/(3*d) + (b^2*Sec[c + d*x]^4)/(4*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)}{x}\,dx,x,b\sec(c+dx)\right)}{b^2d}\\ &= -\frac{\text{Subst}\left(\int \left(2ab^2+\frac{a^2b^2}{x}-(a^2-b^2)x-2ax^2-x^3\right)dx,x,b\sec(c+dx\right.}{b^2d}\\ &= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2ab\sec(c+dx)}{d}+\frac{(a^2-b^2)\sec^2(c+dx)}{2d}+\frac{2a}{}\end{aligned}$$

Mathematica [A] time = 0.43, size = 74, normalized size = 0.85

$$\frac{6(a^2-b^2)\sec^2(c+dx)+12a^2\log(\cos(c+dx))+8ab\sec^3(c+dx)-24ab\sec(c+dx)+3b^2\sec^4(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (12*a^2*Log[Cos[c + d*x]] - 24*a*b*Sec[c + d*x] + 6*(a^2 - b^2)*Sec[c + d*x
]^2 + 8*a*b*Sec[c + d*x]^3 + 3*b^2*Sec[c + d*x]^4)/(12*d)
```

fricas [A] time = 0.53, size = 82, normalized size = 0.94

$$\frac{12\,a^2\cos(dx+c)^4\log(-\cos(dx+c)) - 24\,ab\cos(dx+c)^3 + 8\,ab\cos(dx+c) + 6\left(a^2-b^2\right)\cos(dx+c)^2 + 3\,b^2}{12\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) - 24*a*b*cos(d*x + c)^3 + 8*
a*b*cos(d*x + c) + 6*(a^2 - b^2)*cos(d*x + c)^2 + 3*b^2)/(d*cos(d*x + c)^4)
```

giac [B] time = 3.75, size = 267, normalized size = 3.07

$$-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{25\,a^2+32\,ab+\frac{124\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{128\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{198\,a^2(\cos(dx+}{(\cos(dx+c)+}}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (25*a^2 + 32*a*b +
124*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 128*a*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 198*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 96
*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 48*b^2*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
+ 25*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 1)^4)/d
```

maple [A] time = 0.66, size = 136, normalized size = 1.56

$$\frac{a^2\left(\tan^2(dx+c)\right)}{2d}+\frac{a^2\ln(\cos(dx+c))}{d}+\frac{2ab\left(\sin^4(dx+c)\right)}{3d\cos(dx+c)^3}-\frac{2ab\left(\sin^4(dx+c)\right)}{3d\cos(dx+c)}-\frac{2ab\cos(dx+c)\left(\sin^2(dx+c)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x)
```

```
[Out] 1/2*a^2*tan(d*x+c)^2/d+a^2*ln(cos(d*x+c))/d+2/3/d*a*b*sin(d*x+c)^4/cos(d*x+
c)^3-2/3/d*a*b*sin(d*x+c)^4/cos(d*x+c)-2/3/d*a*b*cos(d*x+c)*sin(d*x+c)^2-4/
3*a*b*cos(d*x+c)/d+1/4/d*b^2*sin(d*x+c)^4/cos(d*x+c)^4
```

maxima [A] time = 0.50, size = 75, normalized size = 0.86

$$\frac{12\,a^2\log(\cos(dx+c)) - \frac{24\,ab\cos(dx+c)^3-8\,ab\cos(dx+c)-6\left(a^2-b^2\right)\cos(dx+c)^2-3\,b^2}{\cos(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*a^2*log(cos(d*x + c)) - (24*a*b*cos(d*x + c)^3 - 8*a*b*cos(d*x + c
) - 6*(a^2 - b^2)*cos(d*x + c)^2 - 3*b^2)/cos(d*x + c)^4)/d
```

mupad [B] time = 3.73, size = 151, normalized size = 1.74

$$-\frac{\frac{8ab}{3}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2+\frac{32ba}{3}\right)-2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(4a^2+8ab-4b^2\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{2a^2\,\mathrm{atanh}\left(\mathrm{ta}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

```
[Out] - ((8*a*b)/3 - tan(c/2 + (d*x)/2)^2*((32*a*b)/3 + 2*a^2) - 2*a^2*tan(c/2 +
(d*x)/2)^6 + tan(c/2 + (d*x)/2)^4*(8*a*b + 4*a^2 - 4*b^2))/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 1.86, size = 126, normalized size = 1.45

$$\begin{cases} -\frac{a^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a^2\tan^2(c+dx)}{2d}+\frac{2ab\tan^2(c+dx)\sec(c+dx)}{3d}-\frac{4ab\sec(c+dx)}{3d}+\frac{b^2\tan^2(c+dx)\sec^2(c+dx)}{4d}-\frac{b^2\sec^2(c+dx)}{4d} \\ x\left(a+b\sec(c)\right)^2\tan^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2/(2*d
) + 2*a*b*tan(c + d*x)**2*sec(c + d*x)/(3*d) - 4*a*b*sec(c + d*x)/(3*d) + b
**2*tan(c + d*x)**2*sec(c + d*x)**2/(4*d) - b**2*sec(c + d*x)**2/(4*d), Ne(
d, 0)), (x*(a + b*sec(c))**2*tan(c)**3, True))
```

3.275 $\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=47

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^2(c+dx)}{2d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d+1/2*b^2*sec(d*x+c)^2/d
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 43}

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d + (b^2*Sec[c + d*x]^2
)/(2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^2}{x}\,dx,x,b\sec(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2}{x}+x\right)\,dx,x,b\sec(c+dx)\right)}{d}\\ &= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 42, normalized size = 0.89

$$\frac{-2a^2\log(\cos(c+dx))+4ab\sec(c+dx)+b^2\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (-2*a^2*Log[Cos[c + d*x]] + 4*a*b*Sec[c + d*x] + b^2*Sec[c + d*x]^2)/(2*d)
```

fricas [A] time = 0.56, size = 51, normalized size = 1.09

$$-\frac{2\,a^2\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-4\,ab\cos\left(dx+c\right)-b^2}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*cos(d*x + c)^2*log(-cos(d*x + c)) - 4*a*b*cos(d*x + c) - b^2)/(
d*cos(d*x + c)^2)
```

giac [B] time = 0.87, size = 191, normalized size = 4.06

$$\frac{2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{3\,a^2+8\,ab+\frac{6\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2 \cdots}{(\cdots}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3*a^2 + 8*a*b + 6*a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^
2)/d
```

maple [A] time = 0.19, size = 45, normalized size = 0.96

$$\frac{b^2\left(\sec^2\left(dx+c\right)\right)}{2d}+\frac{2ab\sec\left(dx+c\right)}{d}+\frac{a^2\ln\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c),x)
```

```
[Out] 1/2*b^2*sec(d*x+c)^2/d+2*a*b*sec(d*x+c)/d+1/d*a^2*ln(sec(d*x+c))
```

maxima [A] time = 0.38, size = 42, normalized size = 0.89

$$-\frac{2\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{4\,ab\cos(dx+c)+b^2}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*log(cos(d*x + c)) - (4*a*b*cos(d*x + c) + b^2)/cos(d*x + c)^2)/
d
```

mupad [B] time = 1.53, size = 81, normalized size = 1.72

$$\frac{4\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,a\,b-2\,b^2\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (4*a*b - tan(c/2 + (d*x)/2)^2*(4*a*b - 2*b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2
*tan(c/2 + (d*x)/2)^2 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

sympy [A] time = 0.49, size = 60, normalized size = 1.28

$$\begin{cases} \frac{a^2 \log \left(\tan^2(c+dx)+1\right)}{2d} + \frac{2ab \sec (c+dx)}{d} + \frac{b^2 \sec^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x (a + b \sec (c))^2 \tan (c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + 2*a*b*sec(c + d*x)/d + b**
2*sec(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c), True))
```

3.276 $\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=61

$$\frac{a^2\log(\cos(c+dx))}{d}+\frac{(a+b)^2\log(1-\sec(c+dx))}{2d}+\frac{(a-b)^2\log(\sec(c+dx)+1)}{2d}$$

```
[Out] a^2*ln(cos(d*x+c))/d+1/2*(a+b)^2*ln(1-sec(d*x+c))/d+1/2*(a-b)^2*ln(1+sec(d*
x+c))/d
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 1802}

$$\frac{a^2\log(\cos(c+dx))}{d}+\frac{(a+b)^2\log(1-\sec(c+dx))}{2d}+\frac{(a-b)^2\log(\sec(c+dx)+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d + ((a + b)^2*Log[1 - Sec[c + d*x]])/(2*d) + ((a -
 b)^2*Log[1 + Sec[c + d*x]])/(2*d)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx &= -\frac{b^2\,\text{Subst}\left(\int \frac{(a+x)^2}{x\left(b^2-x^2\right)}\,dx,x,b\sec(c+dx)\right)}{d}\\ &= -\frac{b^2\,\text{Subst}\left(\int \left(\frac{(a+b)^2}{2b^2(b-x)}+\frac{a^2}{b^2x}-\frac{(a-b)^2}{2b^2(b+x)}\right)dx,x,b\sec(c+dx)\right)}{d}\\ &= \frac{a^2\log(\cos(c+dx))}{d}+\frac{(a+b)^2\log(1-\sec(c+dx))}{2d}+\frac{(a-b)^2\log(1+}{2d}\end{aligned}$$

Mathematica [A] time = 0.11, size = 53, normalized size = 0.87

$$\frac{(a+b)^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+(a-b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-b^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

[Out] ((a - b)^2*Log[Cos[(c + d*x)/2]] - b^2*Log[Cos[c + d*x]] + (a + b)^2*Log[Si
n[(c + d*x)/2]])/d

fricas [A] time = 0.90, size = 68, normalized size = 1.11

$$-\frac{2 b^2 \log\left(-\cos\left(dx+c\right)\right) - \left(a^2 - 2 ab + b^2\right) \log\left(\frac{1}{2} \cos\left(dx+c\right) + \frac{1}{2}\right) - \left(a^2 + 2 ab + b^2\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right) + \frac{1}{2}\right)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*log(-cos(d*x + c)) - (a^2 - 2*a*b + b^2)*log(1/2*cos(d*x + c) +
 1/2) - (a^2 + 2*a*b + b^2)*log(-1/2*cos(d*x + c) + 1/2))/d
```

giac [A] time = 0.27, size = 101, normalized size = 1.66

$$-\frac{2 a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) + 2 b^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) - \left(a^2 + 2 ab + b^2\right) \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + 2*b^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 + 2*a*b + b^2)*lo
g(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)))/d
```

maple [A] time = 0.55, size = 53, normalized size = 0.87

$$\frac{b^2 \ln\left(\tan\left(dx+c\right)\right)}{d} + \frac{2ab \ln\left(\csc\left(dx+c\right) - \cot\left(dx+c\right)\right)}{d} + \frac{a^2 \ln\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*b^2*ln(tan(d*x+c))+2/d*a*b*ln(csc(d*x+c)-cot(d*x+c))+a^2*ln(sin(d*x+c))
/d
```

maxima [A] time = 0.42, size = 62, normalized size = 1.02

$$-\frac{2 b^2 \log\left(\cos\left(dx+c\right)\right) - \left(a^2 - 2 ab + b^2\right) \log\left(\cos\left(dx+c\right) + 1\right) - \left(a^2 + 2 ab + b^2\right) \log\left(\cos\left(dx+c\right) - 1\right)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*log(cos(d*x + c)) - (a^2 - 2*a*b + b^2)*log(cos(d*x + c) + 1) -
 (a^2 + 2*a*b + b^2)*log(cos(d*x + c) - 1))/d
```

mupad [B] time = 1.43, size = 96, normalized size = 1.57

$$\frac{a^2 \ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d} + \frac{b^2 \ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d} - \frac{a^2 \ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)}{d} - \frac{b^2 \ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 - 1\right)}{d} + \frac{2\,a\,b \ln\left(\tan\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (b^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*lo
g(tan(c/2 + (d*x)/2)^2 + 1))/d - (b^2*log(tan(c/2 + (d*x)/2)^2 - 1))/d + (2
*a*b*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \cot(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x), x)
```

3.277 $\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=92

$$-\frac{\cot^2(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}-\frac{a(a+b)\log(1-\sec(c+dx))}{2d}-\frac{a(a-b)\log(\sec(c$$

```
[Out] -a^2*ln(cos(d*x+c))/d-1/2*a*(a+b)*ln(1-sec(d*x+c))/d-1/2*a*(a-b)*ln(1+sec(d
*x+c))/d-1/2*cot(d*x+c)^2*(a^2+b^2+2*a*b*sec(d*x+c))/d
```

Rubi [A] time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3885, 1805, 801}

$$-\frac{\cot^2(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}-\frac{a(a+b)\log(1-\sec(c+dx))}{2d}-\frac{a(a-b)\log(\sec(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) - (a*(a + b)*Log[1 - Sec[c + d*x]])/(2*d) - (a
*(a - b)*Log[1 + Sec[c + d*x]])/(2*d) - (Cot[c + d*x]^2*(a^2 + b^2 + 2*a*b*
Sec[c + d*x]))/(2*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{(a+x)^2}{x\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d} - \frac{b^2\,\text{Subst}\left(\int \frac{-2a^2-2ax}{x\left(b^2-x^2\right)}\,dx,\right.}{2d}$$

$$= -\frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d} - \frac{b^2\,\text{Subst}\left(\int\left(-\frac{a(a+b)}{b^2(b-x)} - \right.\right.}{}$$

$$= -\frac{a^2\log(\cos(c+dx))}{d} - \frac{a(a+b)\log(1-\sec(c+dx))}{2d} - \frac{a(a-b)\log(1}{2}$$

Mathematica [A] time = 0.47, size = 82, normalized size = 0.89

$$-\frac{(a+b)^2\csc^2\left(\frac{1}{2}(c+dx)\right)+(a-b)^2\sec^2\left(\frac{1}{2}(c+dx)\right)+8a\left((a+b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+(a-b)\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/8*((a + b)^2*Csc[(c + d*x)/2]^2 + 8*a*((a - b)*Log[Cos[(c + d*x)/2]] + (
a + b)*Log[Sin[(c + d*x)/2]]) + (a - b)^2*Sec[(c + d*x)/2]^2)/d
```

fricas [A] time = 0.51, size = 113, normalized size = 1.23

$$\frac{2ab\cos(dx+c)+a^2+b^2-\left(\left(a^2-ab\right)\cos(dx+c)^2-a^2+ab\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left(\left(a^2+ab\right)\cos(dx\right.}{2\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cos(d*x + c) + a^2 + b^2 - ((a^2 - a*b)*cos(d*x + c)^2 - a^2 + a
*b)*log(1/2*cos(d*x + c) + 1/2) - ((a^2 + a*b)*cos(d*x + c)^2 - a^2 - a*b)*
log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

giac [B] time = 1.57, size = 209, normalized size = 2.27

$$\frac{8a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-4\left(a^2+ab\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)+}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + a^2*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*(a^2 + a*b)*log(abs(-co
s(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a^2 + 2*a*b + b^2 + 4*a^2*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 4*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

maple [A] time = 0.69, size = 108, normalized size = 1.17

$$-\frac{a^2\left(\cot^2(dx+c)\right)}{2d}-\frac{a^2\ln(\sin(dx+c))}{d}-\frac{ab\left(\cos^3(dx+c)\right)}{d\sin(dx+c)^2}-\frac{ab\cos(dx+c)}{d}-\frac{ab\ln(\csc(dx+c)-\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/2*a^2*cot(d*x+c)^2/d-a^2*ln(sin(d*x+c))/d-1/d*a*b/sin(d*x+c)^2*cos(d*x+c
)^3-a*b*cos(d*x+c)/d-1/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/2/d*b^2/sin(d*x+c)
^2
```

maxima [A] time = 0.50, size = 72, normalized size = 0.78

$$-\frac{\left(a^2-ab\right)\log\left(\cos\left(dx+c\right)+1\right)+\left(a^2+ab\right)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,ab\cos(dx+c)+a^2+b^2}{\cos(dx+c)^2-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a^2 - a*b)*log(cos(d*x + c) + 1) + (a^2 + a*b)*log(cos(d*x + c) - 1)
 - (2*a*b*cos(d*x + c) + a^2 + b^2)/(cos(d*x + c)^2 - 1))/d
```

mupad [B] time = 1.36, size = 98, normalized size = 1.07

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a-b\right)^2}{8\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2+b\,a\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{8}+\frac{a\,b}{4}+\frac{b^2}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (tan(c/2 + (d*x)/2)^2*(a - b)^2)/(8
*d) - (log(tan(c/2 + (d*x)/2))*(a*b + a^2))/d - (cot(c/2 + (d*x)/2)^2*((a*b
)/4 + a^2/8 + b^2/8))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**3, x)
```

3.278 $\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=126

$$-\frac{\cot^4(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{4d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{a(4a+3b)\log(1-\sec(c+dx))}{8d}+\frac{a(4a-3b)\,\mathrm{l}}{}$$

```
[Out] a^2*ln(cos(d*x+c))/d+1/8*a*(4*a+3*b)*ln(1-sec(d*x+c))/d+1/8*a*(4*a-3*b)*ln(
1+sec(d*x+c))/d+1/4*a*cot(d*x+c)^2*(2*a+3*b*sec(d*x+c))/d-1/4*cot(d*x+c)^4*
(a^2+b^2+2*a*b*sec(d*x+c))/d
```

Rubi [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3885, 1805, 823, 801}

$$-\frac{\cot^4(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{4d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{a(4a+3b)\log(1-\sec(c+dx))}{8d}+\frac{a(4a-3b)\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d + (a*(4*a + 3*b)*Log[1 - Sec[c + d*x]])/(8*d) + (
a*(4*a - 3*b)*Log[1 + Sec[c + d*x]])/(8*d) + (a*Cot[c + d*x]^2*(2*a + 3*b*S
ec[c + d*x]))/(4*d) - (Cot[c + d*x]^4*(a^2 + b^2 + 2*a*b*Sec[c + d*x]))/(4*
d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{b^6\operatorname{Subst}\left(\int \frac{(a+x)^2}{x(b^2-x^2)^3}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= -\frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d} + \frac{b^4\operatorname{Subst}\left(\int \frac{-4a^2-6ax}{x(b^2-x^2)^2}\,dx, x, b\right.}{4d}$$
$$= \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d} - \frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c\right.}{4d}$$
$$= \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d} - \frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c\right.}{4d}$$
$$= \frac{a^2\log(\cos(c+dx))}{d} + \frac{a(4a+3b)\log(1-\sec(c+dx))}{8d} + \frac{a(4a-3b)\log(}{8}$$

Mathematica [A] time = 3.19, size = 148, normalized size = 1.17

$$\frac{2\left(7a^2+10ab+3b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+2\left(7a^2-10ab+3b^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)-(a+b)^2\csc^4\left(\frac{1}{2}(c+dx)\right)-(a-b}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(7*a^2 + 10*a*b + 3*b^2)*Csc[(c + d*x)/2]^2 - (a + b)^2*Csc[(c + d*x)/2]
^4 + 16*a*((4*a - 3*b)*Log[Cos[(c + d*x)/2]] + (4*a + 3*b)*Log[Sin[(c + d*x
)/2]]) + 2*(7*a^2 - 10*a*b + 3*b^2)*Sec[(c + d*x)/2]^2 - (a - b)^2*Sec[(c +
 d*x)/2]^4)/(64*d)
```

fricas [A] time = 0.51, size = 203, normalized size = 1.61

$$-\frac{10\,ab\cos\left(dx+c\right)^3-6\,ab\cos\left(dx+c\right)+4\left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2-6\,a^2-2\,b^2-\left(\left(4\,a^2-3\,ab\right)\cos\left(dx+c\right)^4-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(10*a*b*cos(d*x + c)^3 - 6*a*b*cos(d*x + c) + 4*(2*a^2 + b^2)*cos(d*x
+ c)^2 - 6*a^2 - 2*b^2 - ((4*a^2 - 3*a*b)*cos(d*x + c)^4 - 2*(4*a^2 - 3*a*b
)*cos(d*x + c)^2 + 4*a^2 - 3*a*b)*log(1/2*cos(d*x + c) + 1/2) - ((4*a^2 + 3
*a*b)*cos(d*x + c)^4 - 2*(4*a^2 + 3*a*b)*cos(d*x + c)^2 + 4*a^2 + 3*a*b)*lo
g(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [B] time = 0.36, size = 360, normalized size = 2.86

$$-\frac{64\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{2\,ab(\cos(dx+c}{(\cos(dx+c)+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(64*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + 12*a^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*a*b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 8*(4*a^2 + 3*a*b)*lo
g(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a^2 + 2*a*b + b^2 + 12*a
^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16*a*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 36*a*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

maple [A] time = 0.58, size = 169, normalized size = 1.34

$$-\frac{a^2\left(\cot^4(dx+c)\right)}{4d}+\frac{a^2\left(\cot^2(dx+c)\right)}{2d}+\frac{a^2\ln(\sin(dx+c))}{d}-\frac{ab\left(\cos^5(dx+c)\right)}{2d\sin(dx+c)^4}+\frac{ab\left(\cos^5(dx+c)\right)}{4d\sin(dx+c)^2}+\frac{ab\left(\cos^3(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/4*a^2*cot(d*x+c)^4/d+1/2*a^2*cot(d*x+c)^2/d+a^2*ln(sin(d*x+c))/d-1/2/d*a
*b/sin(d*x+c)^4*cos(d*x+c)^5+1/4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^5+1/4*a*b*co
s(d*x+c)^3/d+3/4*a*b*cos(d*x+c)/d+3/4/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/4/d
*b^2/sin(d*x+c)^4*cos(d*x+c)^4
```

maxima [A] time = 0.43, size = 122, normalized size = 0.97

$$\frac{\left(4a^2-3ab\right)\log(\cos(dx+c)+1)+\left(4a^2+3ab\right)\log(\cos(dx+c)-1)-\frac{2\left(5ab\cos(dx+c)^3-3ab\cos(dx+c)+2\left(2a^2+b^2\right)\right.}{\cos(dx+c)^4-2\cos(dx+c)^2+}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((4*a^2 - 3*a*b)*log(cos(d*x + c) + 1) + (4*a^2 + 3*a*b)*log(cos(d*x +
c) - 1) - 2*(5*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c) + 2*(2*a^2 + b^2)*co
s(d*x + c)^2 - 3*a^2 - b^2)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

mupad [B] time = 1.45, size = 164, normalized size = 1.30

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{5a^2}{32}-\frac{3ab}{16}+\frac{b^2}{32}+\frac{(a-b)^2}{32}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a-b)^2}{64d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2+\frac{3ba}{4}\right)}{d}-\frac{a^2\ln\left(\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*((5*a^2)/32 - (3*a*b)/16 + b^2/32 + (a - b)^2/32))/d
- (tan(c/2 + (d*x)/2)^4*(a - b)^2)/(64*d) + (log(tan(c/2 + (d*x)/2))*((3*a*
b)/4 + a^2))/d - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2
)^4*((a*b)/2 + a^2/4 + b^2/4 - tan(c/2 + (d*x)/2)^2*(4*a*b + 3*a^2 + b^2)))
/(16*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\cot^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**5, x)
```

3.279 $\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$

Optimal. Leaf size=157

$$\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{5ab\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ab\tan^5(c+dx)\sec(c+dx)}{3d}-\frac{5a}{}$$

[Out] -a^2*x-5/8*a*b*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+5/8*a*b*sec(d*x+c)*tan(d*x+c)/d-1/3*a^2*tan(d*x+c)^3/d-5/12*a*b*sec(d*x+c)*tan(d*x+c)^3/d+1/5*a^2*tan(d*x+c)^5/d+1/3*a*b*sec(d*x+c)*tan(d*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d

Rubi [A] time = 0.20, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{5ab\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ab\tan^5(c+dx)\sec(c+dx)}{3d}-\frac{5a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -(a^2*x) - (5*a*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5*
a*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a*b*
Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a*b*Sec
[c + d*x]*Tan[c + d*x]^5)/(3*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx = \int \left(a^2\tan^6(c+dx)+2ab\sec(c+dx)\tan^6(c+dx)+b^2\sec^2(c+dx)\,\mathrm{t}\right.$$
$$= a^2\int \tan^6(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^6(c+dx)\,dx+b^2\int \mathrm{s}$$
$$= \frac{a^2\tan^5(c+dx)}{5d}+\frac{ab\sec(c+dx)\tan^5(c+dx)}{3d}-a^2\int \tan^4(c+dx)\,dx$$
$$= -\frac{a^2\tan^3(c+dx)}{3d}-\frac{5ab\sec(c+dx)\tan^3(c+dx)}{12d}+\frac{a^2\tan^5(c+dx)}{5d}+$$
$$= \frac{a^2\tan(c+dx)}{d}+\frac{5ab\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\tan^3(c+dx)}{3d}-\frac{5a}{}$$
$$= -a^2x-\frac{5ab\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)}{d}+\frac{5ab\sec(c+dx)\,\mathrm{t}}{8d}$$

Mathematica [A] time = 1.35, size = 293, normalized size = 1.87

$$\frac{-2100\sec^6(c+dx)\left(7a^2(c+dx)-\left(a^2+b^2\right)\tan(c+dx)\right)-\left(\sec^7(c+dx)\left(-3444a^2\sin(3(c+dx))-1988a^2\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] (16800*a*b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) - Sec[c + d*x]^7*(8820*a^2*(c + d*x)*Cos[3*(c + d*x)]
 + 2940*a^2*(c + d*x)*Cos[5*(c + d*x)] + 420*a^2*c*Cos[7*(c + d*x)] + 420*a
^2*d*x*Cos[7*(c + d*x)] - 3444*a^2*Sin[3*(c + d*x)] + 1260*b^2*Sin[3*(c + d
*x)] - 980*a*b*Sin[4*(c + d*x)] - 1988*a^2*Sin[5*(c + d*x)] - 420*b^2*Sin[5
*(c + d*x)] - 1155*a*b*Sin[6*(c + d*x)] - 644*a^2*Sin[7*(c + d*x)] + 60*b^2
*Sin[7*(c + d*x)]) + 5950*a*b*Sec[c + d*x]^5*Tan[c + d*x] - 2100*Sec[c + d*
x]^6*(7*a^2*(c + d*x) - (a^2 + b^2)*Tan[c + d*x]))/(26880*d)
```

fricas [A] time = 0.51, size = 184, normalized size = 1.17

$$-\frac{1680\,a^2dx\cos\left(dx+c\right)^7+525\,ab\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right)-525\,ab\cos\left(dx+c\right)^7\log\left(-\sin\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/1680*(1680*a^2*d*x*cos(d*x + c)^7 + 525*a*b*cos(d*x + c)^7*log(sin(d*x +
 c) + 1) - 525*a*b*cos(d*x + c)^7*log(-sin(d*x + c) + 1) - 2*(1155*a*b*cos(
d*x + c)^5 + 8*(161*a^2 - 15*b^2)*cos(d*x + c)^6 - 910*a*b*cos(d*x + c)^3 -
 8*(77*a^2 - 45*b^2)*cos(d*x + c)^4 + 280*a*b*cos(d*x + c) + 24*(7*a^2 - 15
*b^2)*cos(d*x + c)^2 + 120*b^2)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

giac [A] time = 9.20, size = 282, normalized size = 1.80

$$840\,(dx+c)a^2 + 525\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 525\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(840\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)*a^2 + 525*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 52
5*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(840*a^2*tan(1/2*d*x + 1/2*c)^
13 - 525*a*b*tan(1/2*d*x + 1/2*c)^13 - 6160*a^2*tan(1/2*d*x + 1/2*c)^11 + 3
500*a*b*tan(1/2*d*x + 1/2*c)^11 + 19768*a^2*tan(1/2*d*x + 1/2*c)^9 - 9905*a
*b*tan(1/2*d*x + 1/2*c)^9 - 28896*a^2*tan(1/2*d*x + 1/2*c)^7 + 7680*b^2*tan
(1/2*d*x + 1/2*c)^7 + 19768*a^2*tan(1/2*d*x + 1/2*c)^5 + 9905*a*b*tan(1/2*d
*x + 1/2*c)^5 - 6160*a^2*tan(1/2*d*x + 1/2*c)^3 - 3500*a*b*tan(1/2*d*x + 1/
2*c)^3 + 840*a^2*tan(1/2*d*x + 1/2*c) + 525*a*b*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

maple [A] time = 0.41, size = 219, normalized size = 1.39

$$\frac{a^2\left(\tan^5(dx+c)\right)}{5d} - \frac{a^2\left(\tan^3(dx+c)\right)}{3d} + \frac{a^2\tan(dx+c)}{d} - a^2x - \frac{a^2c}{d} + \frac{ab\left(\sin^7(dx+c)\right)}{3d\cos(dx+c)^6} - \frac{ab\left(\sin^7(dx+c)\right)}{12d\cos(dx+c)^4} + \frac{ab\left(\sin^7\right.}{8d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x)
```

```
[Out] 1/5*a^2*tan(d*x+c)^5/d-1/3*a^2*tan(d*x+c)^3/d+a^2*tan(d*x+c)/d-a^2*x-1/d*a^
2*c+1/3/d*a*b*sin(d*x+c)^7/cos(d*x+c)^6-1/12/d*a*b*sin(d*x+c)^7/cos(d*x+c)^
4+1/8/d*a*b*sin(d*x+c)^7/cos(d*x+c)^2+1/8*a*b*sin(d*x+c)^5/d+5/24*a*b*sin(d
*x+c)^3/d+5/8*a*b*sin(d*x+c)/d-5/8/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/7/d*b^
2*sin(d*x+c)^7/cos(d*x+c)^7
```

maxima [A] time = 0.69, size = 150, normalized size = 0.96

$$\frac{240\,b^2\tan(dx+c)^7 + 112\left(3\,\tan(dx+c)^5 - 5\,\tan(dx+c)^3 - 15\,dx - 15\,c + 15\,\tan(dx+c)\right)a^2 - 35\,ab\left(\frac{2\left(33\,\sin(d\right.}{\sin(dx+}\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/1680*(240*b^2*tan(d*x + c)^7 + 112*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 -
 15*d*x - 15*c + 15*tan(d*x + c))*a^2 - 35*a*b*(2*(33*sin(d*x + c)^5 - 40*s
in(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin
(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.64, size = 403, normalized size = 2.57

$$\frac{\left(\frac{5\,a\,b}{4}-2\,a^2\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(\frac{44\,a^2}{3}-\frac{25\,a\,b}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{283\,a\,b}{12}-\frac{706\,a^2}{15}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{344\,a^2}{5}-\frac{128\,b^2}{7}\right)\mathrm{t}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^7*((344*a^2)/5 - (128*b^2)/7) + tan(c/2 + (d*x)/2)^13*(
(5*a*b)/4 - 2*a^2) + tan(c/2 + (d*x)/2)^3*((25*a*b)/3 + (44*a^2)/3) - tan(c
/2 + (d*x)/2)^11*((25*a*b)/3 - (44*a^2)/3) - tan(c/2 + (d*x)/2)^5*((283*a*b
)/12 + (706*a^2)/15) + tan(c/2 + (d*x)/2)^9*((283*a*b)/12 - (706*a^2)/15) -
 tan(c/2 + (d*x)/2)*((5*a*b)/4 + 2*a^2))/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*ta
n(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21
*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 -
1)) - (2*a^2*atan((64*a^6*tan(c/2 + (d*x)/2))/(64*a^6 + 25*a^4*b^2) + (25*a
^4*b^2*tan(c/2 + (d*x)/2))/(64*a^6 + 25*a^4*b^2)))/d - (5*a*b*atanh((40*a^5
*b*tan(c/2 + (d*x)/2))/(40*a^5*b + (125*a^3*b^3)/8) + (125*a^3*b^3*tan(c/2
+ (d*x)/2))/(8*(40*a^5*b + (125*a^3*b^3)/8))))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \tan^6(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**6, x)
```

3.280 $\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3ab\tan(c+dx)\sec}{4d}$$

```
[Out] a^2*x+3/4*a*b*arctanh(sin(d*x+c))/d-a^2*tan(d*x+c)/d-3/4*a*b*sec(d*x+c)*tan
(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+1/2*a*b*sec(d*x+c)*tan(d*x+c)^3/d+1/5*b^2*
tan(d*x+c)^5/d
```

Rubi [A] time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3ab\tan(c+dx)\sec}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] a^2*x + (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a*b
*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a*b*Sec[c
 + d*x]*Tan[c + d*x]^3)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx = \int \left(a^2\tan^4(c+dx)+2ab\sec(c+dx)\tan^4(c+dx)+b^2\sec^2(c+dx)\,\text{t}\right.$$
$$= a^2\int \tan^4(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^4(c+dx)\,dx+b^2\int \text{s}$$
$$= \frac{a^2\tan^3(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan^3(c+dx)}{2d}-a^2\int \tan^2(c+dx)\,dx$$
$$= -\frac{a^2\tan(c+dx)}{d}-\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{a}{}$$
$$= a^2x+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3ab\sec(c+dx)\tan}{4d}$$

Mathematica [B] time = 0.97, size = 355, normalized size = 3.06

$$\sec^5(c+dx)\left(-80a^2\sin(c+dx)-160a^2\sin(3(c+dx))-80a^2\sin(5(c+dx))+60a^2c\cos(5(c+dx))+60a^2dx\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (Sec[c + d*x]^5*(60*a^2*c*Cos[5*(c + d*x)] + 60*a^2*d*x*Cos[5*(c + d*x)] -
45*a*b*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b*C
os[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 150*a*Cos[c + d*
x]*(4*a*(c + d*x) - 3*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*b*Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 75*a*Cos[3*(c + d*x)]*(4*a*(c + d*x
) - 3*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*b*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) - 80*a^2*Sin[c + d*x] + 120*b^2*Sin[c + d*x] - 60*a*b
*Sin[2*(c + d*x)] - 160*a^2*Sin[3*(c + d*x)] - 60*b^2*Sin[3*(c + d*x)] - 15
0*a*b*Sin[4*(c + d*x)] - 80*a^2*Sin[5*(c + d*x)] + 12*b^2*Sin[5*(c + d*x)])
)/(960*d)
```

fricas [A] time = 0.55, size = 151, normalized size = 1.30

$$120\,a^2dx\cos(dx+c)^5+45\,ab\cos(dx+c)^5\log(\sin(dx+c)+1)-45\,ab\cos(dx+c)^5\log(-\sin(dx+c)+1)$$

1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*d*x*cos(d*x + c)^5 + 45*a*b*cos(d*x + c)^5*log(sin(d*x + c)
+ 1) - 45*a*b*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(75*a*b*cos(d*x + c
)^3 + 4*(20*a^2 - 3*b^2)*cos(d*x + c)^4 - 30*a*b*cos(d*x + c) - 4*(5*a^2 -
6*b^2)*cos(d*x + c)^2 - 12*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [B] time = 2.39, size = 220, normalized size = 1.90

$$60\,(dx+c)a^2+45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-45\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*a^2 + 45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 45
*a*b*tan(1/2*d*x + 1/2*c)^9 - 320*a^2*tan(1/2*d*x + 1/2*c)^7 + 210*a*b*tan(
1/2*d*x + 1/2*c)^7 + 520*a^2*tan(1/2*d*x + 1/2*c)^5 - 192*b^2*tan(1/2*d*x +
 1/2*c)^5 - 320*a^2*tan(1/2*d*x + 1/2*c)^3 - 210*a*b*tan(1/2*d*x + 1/2*c)^3
 + 60*a^2*tan(1/2*d*x + 1/2*c) + 45*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.39, size = 164, normalized size = 1.41

$$\frac{a^2\left(\tan^3(dx+c)\right)}{3d}-\frac{a^2\tan(dx+c)}{d}+a^2x+\frac{a^2c}{d}+\frac{ab\left(\sin^5(dx+c)\right)}{2d\cos(dx+c)^4}-\frac{ab\left(\sin^5(dx+c)\right)}{4d\cos(dx+c)^2}-\frac{ab\left(\sin^3(dx+c)\right)}{4d}-\frac{3ab\sin}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x)
```

```
[Out] 1/3*a^2*tan(d*x+c)^3/d-a^2*tan(d*x+c)/d+a^2*x+1/d*a^2*c+1/2/d*a*b*sin(d*x+c
)^5/cos(d*x+c)^4-1/4/d*a*b*sin(d*x+c)^5/cos(d*x+c)^2-1/4*a*b*sin(d*x+c)^3/d
-3/4*a*b*sin(d*x+c)/d+3/4/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/5/d*b^2*sin(d*x
+c)^5/cos(d*x+c)^5
```

maxima [A] time = 0.60, size = 118, normalized size = 1.02

$$\frac{24\,b^2\tan(dx+c)^5+40\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^2+15\,ab\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log(s\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/120*(24*b^2*tan(d*x + c)^5 + 40*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^2 + 15*a*b*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4
- 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)
))/d
```

mupad [B] time = 2.56, size = 332, normalized size = 2.86

$$\frac{\left(2\,a^2-\frac{3\,a\,b}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(7\,a\,b-\frac{32\,a^2}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{52\,a^2}{3}-\frac{32\,b^2}{5}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{32\,a^2}{3}-7\,b\,a\right)\tan\left(\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*((52*a^2)/3 - (32*b^2)/5) - tan(c/2 + (d*x)/2)^9*((3*
a*b)/2 - 2*a^2) - tan(c/2 + (d*x)/2)^3*(7*a*b + (32*a^2)/3) + tan(c/2 + (d*
x)/2)^7*(7*a*b - (32*a^2)/3) + tan(c/2 + (d*x)/2)*((3*a*b)/2 + 2*a^2))/(d*(
5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6
- 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) + (2*a^2*atan((64*a^
6*tan(c/2 + (d*x)/2))/(64*a^6 + 36*a^4*b^2) + (36*a^4*b^2*tan(c/2 + (d*x)/2
))/(64*a^6 + 36*a^4*b^2)))/d + (3*a*b*atanh((48*a^5*b*tan(c/2 + (d*x)/2))/(
48*a^5*b + 27*a^3*b^3) + (27*a^3*b^3*tan(c/2 + (d*x)/2))/(48*a^5*b + 27*a^3
*b^3)))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^2 \tan^4 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**4, x)
```

3.281 $\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=70

$$\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

```
[Out] -a^2*x-a*b*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+a*b*sec(d*x+c)*tan(d*x+c)
/d+1/3*b^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec[
c + d*x]*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a^2\tan^2(c+dx)+2ab\sec(c+dx)\tan^2(c+dx)+b^2\sec^2(c+dx)\,t\right.$$
$$= a^2\int \tan^2(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^2(c+dx)\,dx+b^2\int s$$
$$= \frac{a^2\tan(c+dx)}{d}+\frac{ab\sec(c+dx)\tan(c+dx)}{d}-a^2\int 1\,dx-(ab)\int \sec$$
$$= -a^2x-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec(c+dx)\tan}{d}$$

Mathematica [B] time = 1.20, size = 201, normalized size = 2.87

$$\frac{\sec^3(c+dx)\left(2\sin(c+dx)\left(\left(3a^2-b^2\right)\cos(2(c+dx))+3a^2+6ab\cos(c+dx)+b^2\right)-9a\cos(c+dx)\left(a(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]^3*(-9*a*Cos[c + d*x]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*a*Cos[3
*(c + d*x)]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*(3*a^2 + b^2 + 6*a*b*Cos[c + d
*x] + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*d)
```

fricas [A] time = 0.63, size = 115, normalized size = 1.64

$$-\frac{6\,a^2dx\cos(dx+c)^3+3\,ab\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,ab\cos(dx+c)^3\log(-\sin(dx+c)+1)-2}{6\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(3*a*b*cos(d*x + c) + (3
*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [B] time = 1.67, size = 158, normalized size = 2.26

$$-\frac{3\,(dx+c)a^2+3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan(\right.}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*
tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x +
```

```
1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.39, size = 109, normalized size = 1.56

$$-a^2x+\frac{a^2\tan(dx+c)}{d}-\frac{a^2c}{d}+\frac{ab\left(\sin^3(dx+c)\right)}{d\cos(dx+c)^2}+\frac{ab\sin(dx+c)}{d}-\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\left(\sin^3(dx\right.}{3d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x)
```

```
[Out] -a^2*x+a^2*tan(d*x+c)/d-1/d*a^2*c+1/d*a*b*sin(d*x+c)^3/cos(d*x+c)^2+a*b*sin
(d*x+c)/d-1/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*b^2*sin(d*x+c)^3/cos(d*x+
c)^3
```

maxima [A] time = 0.66, size = 82, normalized size = 1.17

$$\frac{2\,b^2\tan(dx+c)^3-6\,(dx+c-\tan(dx+c))a^2-3\,ab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a*b*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

mupad [B] time = 1.67, size = 227, normalized size = 3.24

$$-\frac{\frac{b^2\sin(3c+3dx)}{12}-\frac{b^2\sin(c+dx)}{4}-\frac{a^2\sin(3c+3dx)}{4}-\frac{a^2\sin(c+dx)}{4}+\frac{3a^2\cos(c+dx)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}+\frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(3c+3dx}{2}}{d\left(\frac{3\cos(c+dx)}{4}+\frac{\cos(3c+3dx)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

```
[Out] -((b^2*sin(3*c + 3*d*x))/12 - (b^2*sin(c + d*x))/4 - (a^2*sin(3*c + 3*d*x))
/4 - (a^2*sin(c + d*x))/4 + (3*a^2*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/2 + (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(
3*c + 3*d*x))/2 - (a*b*sin(2*c + 2*d*x))/2 + (3*a*b*cos(c + d*x)*atanh(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (a*b*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2))*cos(3*c + 3*d*x))/2)/(d*((3*cos(c + d*x))/4 + cos(3*c + 3*d*
x)/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**2, x)
```

3.282 $\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=48

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d-b^2*cot(d*x+c)/d-2*a*b*csc(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3886, 3473, 8, 2606, 3767}

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d - (b^2*Cot[c + d*x])/d - (2*a*b*Csc[c + d*x
])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a^2\cot^2(c+dx)+2ab\cot(c+dx)\csc(c+dx)+b^2\csc^2(c+dx)\right)dx$$
$$= a^2\int \cot^2(c+dx)\,dx+(2ab)\int \cot(c+dx)\csc(c+dx)\,dx+b^2\int \csc^2(c$$
$$= -\frac{a^2\cot(c+dx)}{d}-a^2\int 1\,dx-\frac{(2ab)\operatorname{Subst}(\int 1\,dx,x,\csc(c+dx))}{d}-\frac{b^2\,\mathrm{S}}{}$$
$$= -a^2x-\frac{a^2\cot(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}$$

Mathematica [A] time = 0.38, size = 39, normalized size = 0.81

$$-\frac{\left(a^2+b^2\right)\cot(c+dx)+a(a(c+dx)+2b\csc(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
[Out] -(((a^2 + b^2)*Cot[c + d*x] + a*(a*(c + d*x) + 2*b*Csc[c + d*x]))/d)
```

fricas [A] time = 0.56, size = 44, normalized size = 0.92

$$-\frac{a^2dx\sin(dx+c)+2\,ab+\left(a^2+b^2\right)\cos(dx+c)}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(a^2*d*x*sin(d*x + c) + 2*a*b + (a^2 + b^2)*cos(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 1.03, size = 80, normalized size = 1.67

$$-\frac{2\,(dx+c)a^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{a^2+2\,ab+b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] -1/2*(2*(d*x + c)*a^2 - a^2*tan(1/2*d*x + 1/2*c) + 2*a*b*tan(1/2*d*x + 1/2*
c) - b^2*tan(1/2*d*x + 1/2*c) + (a^2 + 2*a*b + b^2)/tan(1/2*d*x + 1/2*c))/d
```

maple [A] time = 0.68, size = 49, normalized size = 1.02

$$\frac{a^2\left(-\cot(dx+c)-dx-c\right)-\frac{2ab}{\sin(dx+c)}-b^2\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x)
[Out] 1/d*(a^2*(-cot(d*x+c)-d*x-c)-2*a*b/sin(d*x+c)-b^2*cot(d*x+c))
```

maxima [A] time = 0.56, size = 47, normalized size = 0.98

$$-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2+\frac{2\,ab}{\sin(dx+c)}+\frac{b^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c + 1/tan(d*x + c))*a^2 + 2*a*b/sin(d*x + c) + b^2/tan(d*x + c))/d
```

mupad [B] time = 1.43, size = 58, normalized size = 1.21

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a-b\right)}^2}{2\,d}-\frac{\frac{a^2}{2}+a\,b+\frac{b^2}{2}}{d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(a - b)^2)/(2*d) - (a*b + a^2/2 + b^2/2)/(d*tan(c/2 + (
d*x)/2)) - a^2*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec{\left(c + d x \right)}\right)^{2} \cot^{2}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**2, x)
```

3.283 $\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=85

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d-1/3*b^2*cot(d*x+c)^3/d+2*a*b*
csc(d*x+c)/d-2/3*a*b*csc(d*x+c)^3/d
```

Rubi [A] time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3886, 3473, 8, 2606, 2607, 30}

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) - (b^2*Cot[c + d*
x]^3)/(3*d) + (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)+2ab\cot^3(c+dx)\csc(c+dx)+b^2\cot^2(c+dx)\,\mathrm{cs}\right.$$
$$= a^2\int \cot^4(c+dx)\,dx+(2ab)\int \cot^3(c+dx)\csc(c+dx)\,dx+b^2\int \mathrm{co}$$
$$= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx-\frac{(2ab)\,\mathrm{Subst}\left(\int\left(-1+x^2\right)\right.}{d}$$
$$= \frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{b^2\cot^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}$$
$$= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{b^2\cot^3(c+dx)}{3d}+\frac{2ab\csc(c}{d}$$

Mathematica [A] time = 0.48, size = 122, normalized size = 1.44

$$-\frac{\csc^3(c+dx)\left(-9a^2c\sin(c+dx)-9a^2dx\sin(c+dx)+3a^2c\sin(3(c+dx))+3a^2dx\sin(3(c+dx))+4a^2\cos(3\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/12*(Csc[c + d*x]^3*(-4*a*b + 3*b^2*Cos[c + d*x] + 12*a*b*Cos[2*(c + d*x)
] + 4*a^2*Cos[3*(c + d*x)] + b^2*Cos[3*(c + d*x)] - 9*a^2*c*Sin[c + d*x] -
9*a^2*d*x*Sin[c + d*x] + 3*a^2*c*Sin[3*(c + d*x)] + 3*a^2*d*x*Sin[3*(c + d*
x)]))/d
```

fricas [A] time = 0.50, size = 102, normalized size = 1.20

$$\frac{6\,ab\cos\left(dx+c\right)^2+\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^3-3\,a^2\cos\left(dx+c\right)-4\,ab+3\left(a^2dx\cos\left(dx+c\right)^2-a^2dx\right)\sin\left(dx\right.}{3\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*a*b*cos(d*x + c)^2 + (4*a^2 + b^2)*cos(d*x + c)^3 - 3*a^2*cos(d*x +
c) - 4*a*b + 3*(a^2*d*x*cos(d*x + c)^2 - a^2*d*x)*sin(d*x + c))/((d*cos(d*x
 + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.32, size = 176, normalized size = 2.07

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,(dx+c)a^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1
/2*d*x + 1/2*c)^3 + 24*(d*x + c)*a^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 18*a*b
*tan(1/2*d*x + 1/2*c) - 3*b^2*tan(1/2*d*x + 1/2*c) + (15*a^2*tan(1/2*d*x +
1/2*c)^2 + 18*a*b*tan(1/2*d*x + 1/2*c)^2 + 3*b^2*tan(1/2*d*x + 1/2*c)^2 - a
^2 - 2*a*b - b^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.82, size = 111, normalized size = 1.31

$$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot\left(dx+c\right)+dx+c\right)+2ab\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}\right)-\frac{b^2\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a*b*(-1/3/sin(d*x+c)^3*cos(
d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c))-1/3*b
^2/sin(d*x+c)^3*cos(d*x+c)^3)
```

maxima [A] time = 0.57, size = 76, normalized size = 0.89

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^2-1}{\tan\left(dx+c\right)^3}\right)a^2+\frac{2\left(3\,\sin\left(dx+c\right)^2-1\right)ab}{\sin\left(dx+c\right)^3}-\frac{b^2}{\tan\left(dx+c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 + 2*(3*sin(d
*x + c)^2 - 1)*a*b/sin(d*x + c)^3 - b^2/tan(d*x + c)^3)/d
```

mupad [B] time = 1.46, size = 118, normalized size = 1.39

$$a^2\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,{\left(a-b\right)}^2}{24\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a\,\left(a-b\right)}{2}+\frac{{\left(a-b\right)}^2}{8}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{2\,a\,b}{3}+\frac{a^2}{3}+\frac{b^2}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(5\,a^2\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (tan(c/2 + (d*x)/2)^3*(a - b)^2)/(24*d) - (tan(c/2 + (d*x)/2)*((a*(
a - b))/2 + (a - b)^2/8))/d - (cot(c/2 + (d*x)/2)^3*((2*a*b)/3 + a^2/3 + b^
2/3 - tan(c/2 + (d*x)/2)^2*(6*a*b + 5*a^2 + b^2)))/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^2\cot^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**4, x)
```

3.284 $\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=122

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}-$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-1/5*a^2*cot(d*x+c)^5/d-1/5*b
^2*cot(d*x+c)^5/d-2*a*b*csc(d*x+c)/d+4/3*a*b*csc(d*x+c)^3/d-2/5*a*b*csc(d*x
+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c +
 d*x]^5)/(5*d) - (b^2*Cot[c + d*x]^5)/(5*d) - (2*a*b*Csc[c + d*x])/d + (4*a
*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)+2ab\cot^5(c+dx)\csc(c+dx)+b^2\cot^4(c+dx)\csc^2(\right.$$
$$= a^2\int \cot^6(c+dx)\,dx+(2ab)\int \cot^5(c+dx)\csc(c+dx)\,dx+b^2\int \cot^4($$
$$= -\frac{a^2\cot^5(c+dx)}{5d}-a^2\int \cot^4(c+dx)\,dx-\frac{(2ab)\,\text{Subst}\left(\int \left(-1+x^2\right)^2 dx\right.}{d}$$
$$= \frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}+a^2\int \cot^2(c+dx)$$
$$= -\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}-$$
$$= -a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+}{5d}$$

Mathematica [A] time = 0.59, size = 198, normalized size = 1.62

$$-\frac{\csc^5(c+dx)\left(10\left(5a^2+3b^2\right)\cos(c+dx)+150a^2c\sin(c+dx)+150a^2dx\sin(c+dx)-75a^2c\sin(3(c+dx))-75}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/240*(Csc[c + d*x]^5*(116*a*b + 10*(5*a^2 + 3*b^2)*Cos[c + d*x] - 80*a*b*
Cos[2*(c + d*x)] - 25*a^2*Cos[3*(c + d*x)] + 15*b^2*Cos[3*(c + d*x)] + 60*a
*b*Cos[4*(c + d*x)] + 23*a^2*Cos[5*(c + d*x)] + 3*b^2*Cos[5*(c + d*x)] + 15
0*a^2*c*Sin[c + d*x] + 150*a^2*d*x*Sin[c + d*x] - 75*a^2*c*Sin[3*(c + d*x)]
 - 75*a^2*d*x*Sin[3*(c + d*x)] + 15*a^2*c*Sin[5*(c + d*x)] + 15*a^2*d*x*Sin
[5*(c + d*x)]))/d
```

fricas [A] time = 0.48, size = 152, normalized size = 1.25

$$-\frac{30\,ab\cos\left(dx+c\right)^4+\left(23\,a^2+3\,b^2\right)\cos\left(dx+c\right)^5-35\,a^2\cos\left(dx+c\right)^3-40\,ab\cos\left(dx+c\right)^2+15\,a^2\cos\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(30*a*b*cos(d*x + c)^4 + (23*a^2 + 3*b^2)*cos(d*x + c)^5 - 35*a^2*cos
(d*x + c)^3 - 40*a*b*cos(d*x + c)^2 + 15*a^2*cos(d*x + c) + 16*a*b + 15*(a^
2*d*x*cos(d*x + c)^4 - 2*a^2*d*x*cos(d*x + c)^2 + a^2*d*x)*sin(d*x + c))/((
d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [B] time = 1.21, size = 273, normalized size = 2.24

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+50\,ab\tan\left(\frac{1}{2}\,dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*
tan(1/2*d*x + 1/2*c)^5 - 35*a^2*tan(1/2*d*x + 1/2*c)^3 + 50*a*b*tan(1/2*d*x
 + 1/2*c)^3 - 15*b^2*tan(1/2*d*x + 1/2*c)^3 - 480*(d*x + c)*a^2 + 330*a^2*t
an(1/2*d*x + 1/2*c) - 300*a*b*tan(1/2*d*x + 1/2*c) + 30*b^2*tan(1/2*d*x + 1
/2*c) - (330*a^2*tan(1/2*d*x + 1/2*c)^4 + 300*a*b*tan(1/2*d*x + 1/2*c)^4 +
30*b^2*tan(1/2*d*x + 1/2*c)^4 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 - 50*a*b*tan(
1/2*d*x + 1/2*c)^2 - 15*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b^2)
/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.81, size = 154, normalized size = 1.26

$$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a*b*(-1/5/
sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos
(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/5*b^2/sin(d
*x+c)^5*cos(d*x+c)^5)
```

maxima [A] time = 0.66, size = 96, normalized size = 0.79

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2+\frac{2\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)ab}{\sin(dx+c)^5}+\frac{3\,b^2}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^2 + 2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a*b/sin(d*x + c
)^5 + 3*b^2/tan(d*x + c)^5)/d
```

mupad [B] time = 1.50, size = 191, normalized size = 1.57

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(a-b)^2}{160\,d}-a^2\,x-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{a^2}{16}-\frac{a\,b}{12}+\frac{b^2}{48}+\frac{(a-b)^2}{96}\right)}{d}-\frac{\frac{2\,a\,b}{5}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(22\,a^2+20\,a\,b+2\,b\right.}{32\,d\,t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a - b)^2)/(160*d) - a^2*x - (tan(c/2 + (d*x)/2)^3*(a
^2/16 - (a*b)/12 + b^2/48 + (a - b)^2/96))/d - ((2*a*b)/5 + tan(c/2 + (d*x)
/2)^4*(20*a*b + 22*a^2 + 2*b^2) + a^2/5 + b^2/5 - tan(c/2 + (d*x)/2)^2*((10
*a*b)/3 + (7*a^2)/3 + b^2))/(32*d*tan(c/2 + (d*x)/2)^5) + (tan(c/2 + (d*x)/
2)*((21*a^2)/32 - (9*a*b)/16 + b^2/32 + (a - b)^2/32))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\cot^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sec(d*x+c))**2,x)
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**6, x)
```

3.285 $\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=153

$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^7(c+dx)}{7d}+\frac{6ab\csc^5(c+dx)}{5d}-$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d+1/5*a^2*cot(d*x+c)^5/d-1/7*a^
2*cot(d*x+c)^7/d-1/7*b^2*cot(d*x+c)^7/d+2*a*b*csc(d*x+c)/d-2*a*b*csc(d*x+c)
^3/d+6/5*a*b*csc(d*x+c)^5/d-2/7*a*b*csc(d*x+c)^7/d
```

Rubi [A] time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^7(c+dx)}{7d}+\frac{6ab\csc^5(c+dx)}{5d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*
x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)/(7*d) - (b^2*Cot[c + d*x]^7)/(7*d) + (2*
a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/d + (6*a*b*Csc[c + d*x]^5)/(5*
d) - (2*a*b*Csc[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a^2\cot^8(c+dx)+2ab\cot^7(c+dx)\csc(c+dx)+b^2\cot^6(c+dx)\csc^2(\right.$$
$$= a^2\int \cot^8(c+dx)\,dx+(2ab)\int \cot^7(c+dx)\csc(c+dx)\,dx+b^2\int \cot^6($$
$$= -\frac{a^2\cot^7(c+dx)}{7d}-a^2\int \cot^6(c+dx)\,dx-\frac{(2ab)\,\mathrm{Subst}\left(\int \left(-1+x^2\right)^3 dx\right.}{d}$$
$$= \frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}+a^2\int \cot^4(c+dx)$$
$$= -\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}+$$
$$= \frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b}{}$$
$$= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+d}{7d}$$

Mathematica [A] time = 0.84, size = 257, normalized size = 1.68

$$-\frac{\csc^7(c+dx)\left(-3675a^2c\sin(c+dx)-3675a^2dx\sin(c+dx)+2205a^2c\sin(3(c+dx))+2205a^2dx\sin(3(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/6720*(Csc[c + d*x]^7*(-1272*a*b + 525*b^2*Cos[c + d*x] + 3612*a*b*Cos[2*
(c + d*x)] + 1176*a^2*Cos[3*(c + d*x)] + 315*b^2*Cos[3*(c + d*x)] - 840*a*b
*Cos[4*(c + d*x)] - 392*a^2*Cos[5*(c + d*x)] + 105*b^2*Cos[5*(c + d*x)] + 4
20*a*b*Cos[6*(c + d*x)] + 176*a^2*Cos[7*(c + d*x)] + 15*b^2*Cos[7*(c + d*x)
] - 3675*a^2*c*Sin[c + d*x] - 3675*a^2*d*x*Sin[c + d*x] + 2205*a^2*c*Sin[3*
(c + d*x)] + 2205*a^2*d*x*Sin[3*(c + d*x)] - 735*a^2*c*Sin[5*(c + d*x)] - 7
35*a^2*d*x*Sin[5*(c + d*x)] + 105*a^2*c*Sin[7*(c + d*x)] + 105*a^2*d*x*Sin[
7*(c + d*x)]))/d
```

fricas [A] time = 0.50, size = 206, normalized size = 1.35

$$\frac{210\,ab\cos(dx+c)^6+\left(176\,a^2+15\,b^2\right)\cos(dx+c)^7-406\,a^2\cos(dx+c)^5-420\,ab\cos(dx+c)^4+350\,a^2\cos(d}{105\left(d\cos(dx+c)^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(210*a*b*cos(d*x + c)^6 + (176*a^2 + 15*b^2)*cos(d*x + c)^7 - 406*a^2
*cos(d*x + c)^5 - 420*a*b*cos(d*x + c)^4 + 350*a^2*cos(d*x + c)^3 + 336*a*b
*cos(d*x + c)^2 - 105*a^2*cos(d*x + c) - 96*a*b + 105*(a^2*d*x*cos(d*x + c)
^6 - 3*a^2*d*x*cos(d*x + c)^4 + 3*a^2*d*x*cos(d*x + c)^2 - a^2*d*x)*sin(d*x
 + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

giac [B] time = 0.43, size = 366, normalized size = 2.39

$$\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-189\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+294\,ab\,\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 30*a*b*tan(1/2*d*x + 1/2*c)^7 + 15
*b^2*tan(1/2*d*x + 1/2*c)^7 - 189*a^2*tan(1/2*d*x + 1/2*c)^5 + 294*a*b*tan(
1/2*d*x + 1/2*c)^5 - 105*b^2*tan(1/2*d*x + 1/2*c)^5 + 1295*a^2*tan(1/2*d*x
+ 1/2*c)^3 - 1470*a*b*tan(1/2*d*x + 1/2*c)^3 + 315*b^2*tan(1/2*d*x + 1/2*c)
^3 + 13440*(d*x + c)*a^2 - 9765*a^2*tan(1/2*d*x + 1/2*c) + 7350*a*b*tan(1/2
*d*x + 1/2*c) - 525*b^2*tan(1/2*d*x + 1/2*c) + (9765*a^2*tan(1/2*d*x + 1/2*
c)^6 + 7350*a*b*tan(1/2*d*x + 1/2*c)^6 + 525*b^2*tan(1/2*d*x + 1/2*c)^6 - 1
295*a^2*tan(1/2*d*x + 1/2*c)^4 - 1470*a*b*tan(1/2*d*x + 1/2*c)^4 - 315*b^2*
tan(1/2*d*x + 1/2*c)^4 + 189*a^2*tan(1/2*d*x + 1/2*c)^2 + 294*a*b*tan(1/2*d
*x + 1/2*c)^2 + 105*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2 - 30*a*b - 15*b^2)/
tan(1/2*d*x + 1/2*c)^7)/d
```

maple [A] time = 0.95, size = 187, normalized size = 1.22

$$\frac{a^2\left(-\frac{\left(\cot^7(dx+c)\right)}{7}+\frac{\left(\cot^5(dx+c)\right)}{5}-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*
x+c)+2*a*b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1
/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/7*b^2/sin(d*x+c)^7*c
os(d*x+c)^7)
```

maxima [A] time = 0.53, size = 116, normalized size = 0.76

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a^2+\frac{6\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)ab}{\sin(dx+c)^7}-\frac{}{\tan}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^2 + 6*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a*b/sin(d*x + c)^7 - 15*b^2/tan(d*x + c)^7)/d
```

mupad [B] time = 1.50, size = 258, normalized size = 1.69

$$a^2\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(a-b)^2}{896\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{3\,a^2}{32}-\frac{5\,a\,b}{48}+\frac{b^2}{48}+\frac{(a-b)^2}{384}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{a^2}{80}-\frac{3\,a\,b}{160}+\frac{b^2}{160}+\frac{(a-b)^2}{640}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8*(a + b/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (tan(c/2 + (d*x)/2)^7*(a - b)^2)/(896*d) + (tan(c/2 + (d*x)/2)^3*((
3*a^2)/32 - (5*a*b)/48 + b^2/48 + (a - b)^2/384))/d - (tan(c/2 + (d*x)/2)^5
*(a^2/80 - (3*a*b)/160 + b^2/160 + (a - b)^2/640))/d - ((2*a*b)/7 + tan(c/2
 + (d*x)/2)^4*(14*a*b + (37*a^2)/3 + 3*b^2) - tan(c/2 + (d*x)/2)^6*(70*a*b
+ 93*a^2 + 5*b^2) + a^2/7 + b^2/7 - tan(c/2 + (d*x)/2)^2*((14*a*b)/5 + (9*a
^2)/5 + b^2))/(128*d*tan(c/2 + (d*x)/2)^7) - (tan(c/2 + (d*x)/2)*((23*a^2)/
32 - (17*a*b)/32 + b^2/32 + (a - b)^2/128))/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.286 $\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=250

$$-\frac{\left(a^2-b^2\right)^4\log(a+b\sec(c+dx))}{ab^8d}-\frac{a\left(a^2-4b^2\right)\sec^4(c+dx)}{4b^4d}+\frac{\left(a^2-4b^2\right)\sec^5(c+dx)}{5b^3d}-\frac{a\left(a^4-4a^2b^2+6b^4\right)\text{se}}{2b^6d}$$

```
[Out] -ln(cos(d*x+c))/a/d-(a^2-b^2)^4*ln(a+b*sec(d*x+c))/a/b^8/d+(a^6-4*a^4*b^2+6
*a^2*b^4-4*b^6)*sec(d*x+c)/b^7/d-1/2*a*(a^4-4*a^2*b^2+6*b^4)*sec(d*x+c)^2/b
^6/d+1/3*(a^4-4*a^2*b^2+6*b^4)*sec(d*x+c)^3/b^5/d-1/4*a*(a^2-4*b^2)*sec(d*x
+c)^4/b^4/d+1/5*(a^2-4*b^2)*sec(d*x+c)^5/b^3/d-1/6*a*sec(d*x+c)^6/b^2/d+1/7
*sec(d*x+c)^7/b/d
```

Rubi [A] time = 0.20, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3885, 894}

$$\frac{\left(a^2-4b^2\right)\sec^5(c+dx)}{5b^3d}-\frac{a\left(a^2-4b^2\right)\sec^4(c+dx)}{4b^4d}+\frac{\left(-4a^2b^2+a^4+6b^4\right)\sec^3(c+dx)}{3b^5d}-\frac{a\left(-4a^2b^2+a^4+6b^4\right)\text{s}}{2b^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((a^2 - b^2)^4*Log[a + b*Sec[c + d*x]])/(a*b^8
*d) + ((a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^6)*Sec[c + d*x])/(b^7*d) - (a*(a^
4 - 4*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/(2*b^6*d) + ((a^4 - 4*a^2*b^2 + 6*b^
4)*Sec[c + d*x]^3)/(3*b^5*d) - (a*(a^2 - 4*b^2)*Sec[c + d*x]^4)/(4*b^4*d) +
 ((a^2 - 4*b^2)*Sec[c + d*x]^5)/(5*b^3*d) - (a*Sec[c + d*x]^6)/(6*b^2*d) +
Sec[c + d*x]^7/(7*b*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^4}{x(a+x)}\,dx,x,b\sec(c+dx)\right)}{b^8d}$$

$$=\frac{\text{Subst}\left(\int \left(a^6\left(1+\frac{-4a^4b^2+6a^2b^4-4b^6}{a^6}\right)+\frac{b^8}{ax}-a\left(a^4-4a^2b^2+6b^4\right)x+\left(a^4-4a^2b^2+6b\right.\right.\right.}{b^8d}$$

$$=-\frac{\log(\cos(c+dx))}{ad}-\frac{\left(a^2-b^2\right)^4\log(a+b\sec(c+dx))}{ab^8d}+\frac{\left(a^6-4a^4b^2+6a^2b^4-4b^6\right)}{b^7d}$$

Mathematica [B] time = 6.24, size = 520, normalized size = 2.08

$$-\frac{\left(2b^2-a^2\right)\left(a^4-2a^2b^2+2b^4\right)\sec^2(c+dx)(a\cos(c+dx)+b)}{b^7d(a+b\sec(c+dx))}-\frac{a\left(a^4-4a^2b^2+6b^4\right)\sec^3(c+dx)(a\cos(c+dx)+}{2b^6d(a+b\sec(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^9/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*(b + a*Cos[c + d*x])*Log[Cos[c + d
*x]]*Sec[c + d*x])/(b^8*d*(a + b*Sec[c + d*x])) + ((-a^8 + 4*a^6*b^2 - 6*a^
4*b^4 + 4*a^2*b^6 - b^8)*(b + a*Cos[c + d*x])*Log[b + a*Cos[c + d*x]]*Sec[c
 + d*x])/(a*b^8*d*(a + b*Sec[c + d*x])) - ((-a^2 + 2*b^2)*(a^4 - 2*a^2*b^2
+ 2*b^4)*(b + a*Cos[c + d*x])*Sec[c + d*x]^2)/(b^7*d*(a + b*Sec[c + d*x]))
- (a*(a^4 - 4*a^2*b^2 + 6*b^4)*(b + a*Cos[c + d*x])*Sec[c + d*x]^3)/(2*b^6*
d*(a + b*Sec[c + d*x])) + ((a^4 - 4*a^2*b^2 + 6*b^4)*(b + a*Cos[c + d*x])*S
ec[c + d*x]^4)/(3*b^5*d*(a + b*Sec[c + d*x])) + (a*(-a + 2*b)*(a + 2*b)*(b
+ a*Cos[c + d*x])*Sec[c + d*x]^5)/(4*b^4*d*(a + b*Sec[c + d*x])) - ((-a + 2
*b)*(a + 2*b)*(b + a*Cos[c + d*x])*Sec[c + d*x]^6)/(5*b^3*d*(a + b*Sec[c +
d*x])) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]^7)/(6*b^2*d*(a + b*Sec[c + d*
x])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]^8)/(7*b*d*(a + b*Sec[c + d*x]))
```

fricas [A] time = 0.65, size = 293, normalized size = 1.17

$$-\frac{70\,a^2b^6\cos(dx+c)+420\left(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8\right)\cos(dx+c)^7\log(a\cos(dx+c)+b)-420\left(a^8-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/420*(70*a^2*b^6*cos(d*x + c) + 420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*
b^6 + b^8)*cos(d*x + c)^7*log(a*cos(d*x + c) + b) - 420*(a^8 - 4*a^6*b^2 +
6*a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^7*log(-cos(d*x + c)) - 60*a*b^7 - 420*(
a^7*b - 4*a^5*b^3 + 6*a^3*b^5 - 4*a*b^7)*cos(d*x + c)^6 + 210*(a^6*b^2 - 4*
a^4*b^4 + 6*a^2*b^6)*cos(d*x + c)^5 - 140*(a^5*b^3 - 4*a^3*b^5 + 6*a*b^7)*c
os(d*x + c)^4 + 105*(a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^3 - 84*(a^3*b^5 - 4*
a*b^7)*cos(d*x + c)^2)/(a*b^8*d*cos(d*x + c)^7)
```

giac [B] time = 17.34, size = 1768, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(210*(a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(abs(a + b - 2*b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))/b^8 - 420*(a^7 - 4*a^
5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 1))/b^8 - 210*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + 2*b^8)*log(abs(
2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(b^8*abs(a))
 + (1089*a^7 - 840*a^6*b - 4356*a^5*b^2 + 3080*a^4*b^3 + 6534*a^3*b^4 - 408
8*a^2*b^5 - 4356*a*b^6 + 2232*b^7 + 7623*a^7*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - 5040*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 31332*a^5*b^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 19040*a^4*b^3*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 48258*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 26
096*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 33012*a*b^6*(cos(d*x +
```

```
c) - 1)/(cos(d*x + c) + 1) + 14784*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 22869*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12600*a^6*b*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 95676*a^5*b^2*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 47880*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 151494*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 67368*a^2*b
^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 107436*a*b^6*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 40152*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 38115*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 16800*a^6*b*(
cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 160860*a^5*b^2*(cos(d*x + c) - 1
)^3/(cos(d*x + c) + 1)^3 + 62720*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 + 258930*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 86240*
a^2*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 192220*a*b^6*(cos(d*x +
 c) - 1)^3/(cos(d*x + c) + 1)^3 + 53760*b^7*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 + 38115*a^7*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 12600*a^
6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 160860*a^5*b^2*(cos(d*x + c
) - 1)^4/(cos(d*x + c) + 1)^4 + 45080*a^4*b^3*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4 + 258930*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 5
6840*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 192220*a*b^6*(cos(
d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 24360*b^7*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 + 22869*a^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 504
0*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 95676*a^5*b^2*(cos(d*x
+ c) - 1)^5/(cos(d*x + c) + 1)^5 + 16800*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 151494*a^3*b^4*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
- 18480*a^2*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 107436*a*b^6*(c
os(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 6720*b^7*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 + 7623*a^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 84
0*a^6*b*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 31332*a^5*b^2*(cos(d*x
+ c) - 1)^6/(cos(d*x + c) + 1)^6 + 2520*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d
*x + c) + 1)^6 + 48258*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 -
2520*a^2*b^5*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 33012*a*b^6*(cos(d
*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 840*b^7*(cos(d*x + c) - 1)^6/(cos(d*x
 + c) + 1)^6 + 1089*a^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 4356*a^
5*b^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 6534*a^3*b^4*(cos(d*x + c
) - 1)^7/(cos(d*x + c) + 1)^7 - 4356*a*b^6*(cos(d*x + c) - 1)^7/(cos(d*x +
c) + 1)^7)/(b^8*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^7))/d
```

maple [A] time = 0.55, size = 460, normalized size = 1.84

$$-\frac{\ln(b+a\cos(dx+c))}{da}-\frac{4}{5db\cos(dx+c)^5}+\frac{2}{db\cos(dx+c)^3}-\frac{4}{db\cos(dx+c)}+\frac{1}{7db\cos(dx+c)^7}-\frac{a}{6d\,b^2\cos(d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d/a*ln(b+a*cos(d*x+c))-4/5/d/b/cos(d*x+c)^5+2/d/b/cos(d*x+c)^3-4/d/b/cos
(d*x+c)+1/7/d/b/cos(d*x+c)^7-1/6/d*a/b^2/cos(d*x+c)^6+1/5/d/b^3/cos(d*x+c)^
5*a^2+1/3/d/b^5/cos(d*x+c)^3*a^4-4/3/d/b^3/cos(d*x+c)^3*a^2+1/d/b^7/cos(d*x
+c)*a^6-4/d/b^5/cos(d*x+c)*a^4-3/d/b^2*a/cos(d*x+c)^2+1/d/b^8*a^7*ln(cos(d*
x+c))-4/d/b^6*a^5*ln(cos(d*x+c))+6/d/b^4*a^3*ln(cos(d*x+c))-4/d/b^2*a*ln(co
s(d*x+c))-1/d/b^8*a^7*ln(b+a*cos(d*x+c))+4/d/b^6*a^5*ln(b+a*cos(d*x+c))-6/d
/b^4*a^3*ln(b+a*cos(d*x+c))+4/d/b^2*a*ln(b+a*cos(d*x+c))-1/2/d/b^6*a^5/cos(
d*x+c)^2+2/d/b^4*a^3/cos(d*x+c)^2+6/d/b^3/cos(d*x+c)*a^2-1/4/d/b^4*a^3/cos(
d*x+c)^4+1/d/b^2*a/cos(d*x+c)^4
```

maxima [A] time = 0.62, size = 268, normalized size = 1.07

$$\frac{\frac{420\left(a^7-4a^5b^2+6a^3b^4-4ab^6\right)\log(\cos(dx+c))}{b^8}-\frac{420\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)\log(a\cos(dx+c)+b)}{ab^8}-\frac{70ab^5\cos(dx+c)-420\left(a^6-4a^4b^2+6a\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*(420*(a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(cos(d*x + c))/b^8 -
420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*log(a*cos(d*x + c) + b)
/(a*b^8) - (70*a*b^5*cos(d*x + c) - 420*(a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^
6)*cos(d*x + c)^6 - 60*b^6 + 210*(a^5*b - 4*a^3*b^3 + 6*a*b^5)*cos(d*x + c)
^5 - 140*(a^4*b^2 - 4*a^2*b^4 + 6*b^6)*cos(d*x + c)^4 + 105*(a^3*b^3 - 4*a*
b^5)*cos(d*x + c)^3 - 84*(a^2*b^4 - 4*b^6)*cos(d*x + c)^2)/(b^7*cos(d*x + c
)^7))/d
```

mupad [B] time = 2.97, size = 631, normalized size = 2.52

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}-\frac{\frac{2\left(105\,a^6-385\,a^4\,b^2+511\,a^2\,b^4-279\,b^6\right)}{105\,b^7}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(a^6+a^5\,b-3\,a^4\,b^2-3\,a^3\,b^3+3\,a^2\,b^4+3\,a\,b^5-b^6\right)}{b^7}-\frac{2\tan\left(\frac{c}{2}+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - ((2*(105*a^6 - 279*b^6 + 511*a^2*b^4
- 385*a^4*b^2))/(105*b^7) + (2*tan(c/2 + (d*x)/2)^12*(3*a*b^5 + a^5*b + a^6
 - b^6 + 3*a^2*b^4 - 3*a^3*b^3 - 3*a^4*b^2))/b^7 - (2*tan(c/2 + (d*x)/2)^10
*(19*a*b^5 + 5*a^5*b + 6*a^6 - 8*b^6 + 22*a^2*b^4 - 17*a^3*b^3 - 20*a^4*b^2
))/b^7 - (4*tan(c/2 + (d*x)/2)^6*(71*a*b^5 + 15*a^5*b + 30*a^6 - 96*b^6 + 1
54*a^2*b^4 - 54*a^3*b^3 - 112*a^4*b^2))/(3*b^7) + (2*tan(c/2 + (d*x)/2)^8*(
142*a*b^5 + 30*a^5*b + 45*a^6 - 87*b^6 + 203*a^2*b^4 - 108*a^3*b^3 - 161*a^
4*b^2))/(3*b^7) + (2*tan(c/2 + (d*x)/2)^4*(95*a*b^5 + 25*a^5*b + 75*a^6 - 2
39*b^6 + 401*a^2*b^4 - 85*a^3*b^3 - 285*a^4*b^2))/(5*b^7) - (2*tan(c/2 + (d
*x)/2)^2*(45*a*b^5 + 15*a^5*b + 90*a^6 - 264*b^6 + 466*a^2*b^4 - 45*a^3*b^3
 - 340*a^4*b^2))/(15*b^7))/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/
2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*
x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)) - (log(tan
(c/2 + (d*x)/2)^2 - 1)*(4*a*b^6 - a^7 - 6*a^3*b^4 + 4*a^5*b^2))/(b^8*d) - (
log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^4)
/(a*b^8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**9/(a + b*sec(c + d*x)), x)
```

3.287 $\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=170

$$-\frac{\left(a^2-b^2\right)^3\log(a+b\sec(c+dx))}{ab^6d}-\frac{a\left(a^2-3b^2\right)\sec^2(c+dx)}{2b^4d}+\frac{\left(a^2-3b^2\right)\sec^3(c+dx)}{3b^3d}+\frac{\left(a^4-3a^2b^2+3b^4\right)\sec}{b^5d}$$

```
[Out] ln(cos(d*x+c))/a/d-(a^2-b^2)^3*ln(a+b*sec(d*x+c))/a/b^6/d+(a^4-3*a^2*b^2+3*
b^4)*sec(d*x+c)/b^5/d-1/2*a*(a^2-3*b^2)*sec(d*x+c)^2/b^4/d+1/3*(a^2-3*b^2)*
sec(d*x+c)^3/b^3/d-1/4*a*sec(d*x+c)^4/b^2/d+1/5*sec(d*x+c)^5/b/d
```

Rubi [A] time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{\left(a^2-3b^2\right)\sec^3(c+dx)}{3b^3d}-\frac{a\left(a^2-3b^2\right)\sec^2(c+dx)}{2b^4d}+\frac{\left(-3a^2b^2+a^4+3b^4\right)\sec(c+dx)}{b^5d}-\frac{\left(a^2-b^2\right)^3\log(a+b\sec}{ab^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) - ((a^2 - b^2)^3*Log[a + b*Sec[c + d*x]])/(a*b^6*d)
 + ((a^4 - 3*a^2*b^2 + 3*b^4)*Sec[c + d*x])/(b^5*d) - (a*(a^2 - 3*b^2)*Sec[
c + d*x]^2)/(2*b^4*d) + ((a^2 - 3*b^2)*Sec[c + d*x]^3)/(3*b^3*d) - (a*Sec[c
 + d*x]^4)/(4*b^2*d) + Sec[c + d*x]^5/(5*b*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx=-\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{x(a+x)}\,dx,x,b\sec(c+dx)\right)}{b^6d}$$

$$=-\frac{\text{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2\left(-a^2+b^2\right)}{a^4}\right)+\frac{b^6}{ax}+a\left(a^2-3b^2\right)x-\left(a^2-3b^2\right)x^2+ax^3-x^4+\right.\right.}{b^6d}$$

$$=\frac{\log(\cos(c+dx))}{ad}-\frac{\left(a^2-b^2\right)^3\log(a+b\sec(c+dx))}{ab^6d}+\frac{\left(a^4-3a^2b^2+3b^4\right)\sec(c+a}{b^5d}$$

Mathematica [B] time = 6.19, size = 371, normalized size = 2.18

$$\frac{a\left(3b^2-a^2\right)\sec^3(c+dx)(a\cos(c+dx)+b)}{2b^4d(a+b\sec(c+dx))}+\frac{\left(a^2-3b^2\right)\sec^4(c+dx)(a\cos(c+dx)+b)}{3b^3d(a+b\sec(c+dx))}+\frac{\left(a^5-3a^3b^2+3ab^4\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^5 - 3*a^3*b^2 + 3*a*b^4)*(b + a*Cos[c + d*x])*Log[Cos[c + d*x]]*Sec[c +
 d*x])/(b^6*d*(a + b*Sec[c + d*x])) + ((-a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6)
*(b + a*Cos[c + d*x])*Log[b + a*Cos[c + d*x]]*Sec[c + d*x])/(a*b^6*d*(a + b
*Sec[c + d*x])) + ((a^4 - 3*a^2*b^2 + 3*b^4)*(b + a*Cos[c + d*x])*Sec[c + d
*x]^2)/(b^5*d*(a + b*Sec[c + d*x])) + (a*(-a^2 + 3*b^2)*(b + a*Cos[c + d*x]
)*Sec[c + d*x]^3)/(2*b^4*d*(a + b*Sec[c + d*x])) + ((a^2 - 3*b^2)*(b + a*Co
s[c + d*x])*Sec[c + d*x]^4)/(3*b^3*d*(a + b*Sec[c + d*x])) - (a*(b + a*Cos[
c + d*x])*Sec[c + d*x]^5)/(4*b^2*d*(a + b*Sec[c + d*x])) + ((b + a*Cos[c +
d*x])*Sec[c + d*x]^6)/(5*b*d*(a + b*Sec[c + d*x]))
```

fricas [A] time = 0.58, size = 205, normalized size = 1.21

$$-\frac{15\,a^2b^4\cos\left(dx+c\right)+60\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\cos\left(dx+c\right)^5\log\left(a\cos\left(dx+c\right)+b\right)-60\left(a^6-3\,a^4b^2+3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*a^2*b^4*cos(d*x + c) + 60*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
(d*x + c)^5*log(a*cos(d*x + c) + b) - 60*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cos(
d*x + c)^5*log(-cos(d*x + c)) - 12*a*b^5 - 60*(a^5*b - 3*a^3*b^3 + 3*a*b^5)
*cos(d*x + c)^4 + 30*(a^4*b^2 - 3*a^2*b^4)*cos(d*x + c)^3 - 20*(a^3*b^3 - 3
*a*b^5)*cos(d*x + c)^2)/(a*b^6*d*cos(d*x + c)^5)
```

giac [B] time = 11.52, size = 1052, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(30*(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(abs(a + b - 2*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))/b^6 - 60*(a^5 - 3*a^3*b^2 + 3*a*b^4
)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^6 - 30*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - 2*b^6)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2
*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 2*abs(a)))/(b^6*abs(a)) + (137*a^5 - 120*a^4*b - 411*a^3*b^2 +
320*a^2*b^3 + 411*a*b^4 - 264*b^5 + 685*a^5*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 480*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2175*a^3*b^2*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 1360*a^2*b^3*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 2295*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1200*b^5*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1370*a^5*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 720*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 447
0*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2000*a^2*b^3*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5070*a*b^4*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 - 1920*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1370*a
^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 480*a^4*b*(cos(d*x + c) - 1)
^3/(cos(d*x + c) + 1)^3 - 4470*a^3*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 + 1200*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 5070*a*b^4
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 720*b^5*(cos(d*x + c) - 1)^3/(
cos(d*x + c) + 1)^3 + 685*a^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1
20*a^4*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2175*a^3*b^2*(cos(d*x
+ c) - 1)^4/(cos(d*x + c) + 1)^4 + 240*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 + 2295*a*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 120*
```

```
b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^5*(cos(d*x + c) - 1)^
5/(cos(d*x + c) + 1)^5 - 411*a^3*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1
)^5 + 411*a*b^4*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/(b^6*((cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

maple [A] time = 0.48, size = 292, normalized size = 1.72

$$-\frac{a^5 \ln(b+a\cos(dx+c))}{d\,b^6}+\frac{3a^3\ln(b+a\cos(dx+c))}{d\,b^4}-\frac{3a\ln(b+a\cos(dx+c))}{d\,b^2}+\frac{\ln(b+a\cos(dx+c))}{da}-\frac{}{4d\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d/b^6*a^5*ln(b+a*cos(d*x+c))+3/d/b^4*a^3*ln(b+a*cos(d*x+c))-3/d/b^2*a*ln
(b+a*cos(d*x+c))+1/d/a*ln(b+a*cos(d*x+c))-1/4/d/b^2*a/cos(d*x+c)^4+1/3/d/b^
3/cos(d*x+c)^3*a^2-1/d/b/cos(d*x+c)^3+1/d/b^5/cos(d*x+c)*a^4-3/d/b^3/cos(d*
x+c)*a^2+3/d/b/cos(d*x+c)-1/2/d/b^4*a^3/cos(d*x+c)^2+3/2/d/b^2*a/cos(d*x+c)
^2+1/d/b^6*a^5*ln(cos(d*x+c))-3/d/b^4*a^3*ln(cos(d*x+c))+3/d/b^2*a*ln(cos(d
*x+c))+1/5/d/b/cos(d*x+c)^5
```

maxima [A] time = 0.60, size = 183, normalized size = 1.08

$$\frac{\frac{60\left(a^5-3\,a^3b^2+3\,ab^4\right)\log(\cos(dx+c))}{b^6}-\frac{60\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\log(a\cos(dx+c)+b)}{ab^6}-\frac{15\,ab^3\cos(dx+c)-60\left(a^4-3\,a^2b^2+3\,b^4\right)\cos(dx+c)^4-12\,b}{b^5\cos}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(cos(d*x + c))/b^6 - 60*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*log(a*cos(d*x + c) + b)/(a*b^6) - (15*a*b^3*cos(d*
x + c) - 60*(a^4 - 3*a^2*b^2 + 3*b^4)*cos(d*x + c)^4 - 12*b^4 + 30*(a^3*b -
 3*a*b^3)*cos(d*x + c)^3 - 20*(a^2*b^2 - 3*b^4)*cos(d*x + c)^2)/(b^5*cos(d*
x + c)^5))/d
```

mupad [B] time = 2.40, size = 395, normalized size = 2.32

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\left(a^4-3\,a^2\,b^2+3\,b^4\right)}{b^6\,d}-\frac{\frac{2\left(15\,a^4-40\,a^2\,b^2+33\,b^4\right)}{15\,b^5}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(a^4+a^3\,b-2\,a^2\,b^2-2\,a\,b^3+b^4\right)}{b^5}-\frac{2\tan(}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + b/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 - 1)*(a^4 + 3*b^4 - 3*a^2*b^2))/(b^6*d) - ((2*(
15*a^4 + 33*b^4 - 40*a^2*b^2))/(15*b^5) + (2*tan(c/2 + (d*x)/2)^8*(a^3*b -
2*a*b^3 + a^4 + b^4 - 2*a^2*b^2))/b^5 - (2*tan(c/2 + (d*x)/2)^6*(3*a^3*b -
8*a*b^3 + 4*a^4 + 6*b^4 - 10*a^2*b^2))/b^5 - (2*tan(c/2 + (d*x)/2)^2*(3*a^3
*b - 6*a*b^3 + 12*a^4 + 30*b^4 - 34*a^2*b^2))/(3*b^5) + (2*tan(c/2 + (d*x)/
2)^4*(9*a^3*b - 24*a*b^3 + 18*a^4 + 48*b^4 - 50*a^2*b^2))/(3*b^5))/(d*(5*ta
n(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*
tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - log(tan(c/2 + (d*x)/2)
^2 + 1)/(a*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^
2)*(a^2 - b^2)^3)/(a*b^6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**7/(a + b*sec(c + d*x)), x)
```

3.288 $\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=108

$$-\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4d}+\frac{(a^2-2b^2)\sec(c+dx)}{b^3d}-\frac{a\sec^2(c+dx)}{2b^2d}-\frac{\log(\cos(c+dx))}{ad}+\frac{\sec^3(c+dx)}{3bd}$$

```
[Out] -ln(cos(d*x+c))/a/d-(a^2-b^2)^2*ln(a+b*sec(d*x+c))/a/b^4/d+(a^2-2*b^2)*sec(
d*x+c)/b^3/d-1/2*a*sec(d*x+c)^2/b^2/d+1/3*sec(d*x+c)^3/b/d
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{(a^2-2b^2)\sec(c+dx)}{b^3d}-\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4d}-\frac{a\sec^2(c+dx)}{2b^2d}-\frac{\log(\cos(c+dx))}{ad}+\frac{\sec^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((a^2 - b^2)^2*Log[a + b*Sec[c + d*x]])/(a*b^4
*d) + ((a^2 - 2*b^2)*Sec[c + d*x])/(b^3*d) - (a*Sec[c + d*x]^2)/(2*b^2*d) +
 Sec[c + d*x]^3/(3*b*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)}\,dx,x,b\sec(c+dx)\right)}{b^4d}\\ &= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{(a^2-b^2)^2}{a(a+x)}\right)dx,x,b\sec(c+dx)\right)}{b^4d}\\ &= -\frac{\log(\cos(c+dx))}{ad}-\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4d}+\frac{(a^2-2b^2)\sec(c+dx)}{b^3d}-\frac{a}{}\end{aligned}$$

Mathematica [A] time = 0.40, size = 108, normalized size = 1.00

$$\frac{-3a^2b^2\sec^2(c+dx)+6ab(a^2-2b^2)\sec(c+dx)+6a^2(a^2-2b^2)\log(\cos(c+dx))-6(a^2-b^2)^2\log(a\cos(c}{6ab^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] (6*a^2*(a^2 - 2*b^2)*Log[Cos[c + d*x]] - 6*(a^2 - b^2)^2*Log[b + a*Cos[c +
d*x]] + 6*a*b*(a^2 - 2*b^2)*Sec[c + d*x] - 3*a^2*b^2*Sec[c + d*x]^2 + 2*a*b
^3*Sec[c + d*x]^3)/(6*a*b^4*d)
```

fricas [A] time = 0.55, size = 129, normalized size = 1.19

$$-\frac{3\,a^2b^2\cos\left(dx+c\right)+6\left(a^4-2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^3\log\left(a\cos\left(dx+c\right)+b\right)-6\left(a^4-2\,a^2b^2\right)\cos\left(dx+c\right)^3\log}{6\,ab^4d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^2*b^2*cos(d*x + c) + 6*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^3*log
(a*cos(d*x + c) + b) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c)^3*log(-cos(d*x + c)
) - 2*a*b^3 - 6*(a^3*b - 2*a*b^3)*cos(d*x + c)^2)/(a*b^4*d*cos(d*x + c)^3)
```

giac [B] time = 3.14, size = 560, normalized size = 5.19

$$-\frac{\frac{3\left(a^3-2\,ab^2\right)\log\left(\left|a+b-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{b^4}-\frac{6\left(a^3-2\,ab^2\right)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^4}-\frac{3\left(a^4-2\,a^2b^2+2\,b^4\right)\log\left(\frac{\left|2\,b+\frac{2\,a(\cos}{\cos}\right.}{\left|2\,b+\frac{2\,a(\cos}{\cos}\right.}\right.}{b^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(a^3 - 2*a*b^2)*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2))/b^4 - 6*(a^3 - 2*a*b^2)*log(abs(-(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 1))/b^4 - 3*(a^4 - 2*a^2*b^2 + 2*b^4)*log(abs(2*b +
 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(b^4*abs(a)) + (1
1*a^3 - 12*a^2*b - 22*a*b^2 + 20*b^3 + 33*a^3*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 24*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 78*a*b^2*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 48*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 33*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12*a^2*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 - 78*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 + 12*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*a^3*(cos
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 22*a*b^2*(cos(d*x + c) - 1)^3/(cos(
d*x + c) + 1)^3)/(b^4*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

maple [A] time = 0.44, size = 163, normalized size = 1.51

$$-\frac{a^3\ln\left(b+a\cos\left(dx+c\right)\right)}{d\,b^4}+\frac{2a\ln\left(b+a\cos\left(dx+c\right)\right)}{d\,b^2}-\frac{\ln\left(b+a\cos\left(dx+c\right)\right)}{da}-\frac{a}{2d\,b^2\cos\left(dx+c\right)^2}+\frac{a^2}{d\,b^3\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d/b^4*a^3*ln(b+a*cos(d*x+c))+2/d/b^2*a*ln(b+a*cos(d*x+c))-1/d/a*ln(b+a*c
os(d*x+c))-1/2/d/b^2*a/cos(d*x+c)^2+1/d/b^3/cos(d*x+c)*a^2-2/d/b/cos(d*x+c)
+1/d/b^4*a^3*ln(cos(d*x+c))-2/d/b^2*a*ln(cos(d*x+c))+1/3/d/b/cos(d*x+c)^3
```

maxima [A] time = 0.40, size = 110, normalized size = 1.02

$$\frac{\frac{6\left(a^{3}-2ab^{2}\right)\log(\cos(dx+c))}{b^{4}}-\frac{6\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\log(a\cos(dx+c)+b)}{ab^{4}}-\frac{3ab\cos(dx+c)-6\left(a^{2}-2b^{2}\right)\cos(dx+c)^{2}-2b^{2}}{b^{3}\cos(dx+c)^{3}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(a^3 - 2*a*b^2)*log(cos(d*x + c))/b^4 - 6*(a^4 - 2*a^2*b^2 + b^4)*lo
g(a*cos(d*x + c) + b)/(a*b^4) - (3*a*b*cos(d*x + c) - 6*(a^2 - 2*b^2)*cos(d
*x + c)^2 - 2*b^2)/(b^3*cos(d*x + c)^3))/d
```

mupad [B] time = 1.88, size = 227, normalized size = 2.10

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}-\frac{\frac{2\left(3\,a^2-5\,b^2\right)}{3\,b^3}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^2+a\,b-4\,b^2\right)}{b^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(a^2+a\,b-b^2\right)}{b^3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right.}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - ((2*(3*a^2 - 5*b^2))/(3*b^3) - (2*tan
(c/2 + (d*x)/2)^2*(a*b + 2*a^2 - 4*b^2))/b^3 + (2*tan(c/2 + (d*x)/2)^4*(a*b
 + a^2 - b^2))/b^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + t
an(c/2 + (d*x)/2)^6 - 1)) + (a*log(tan(c/2 + (d*x)/2)^2 - 1)*(a^2 - 2*b^2))
/(b^4*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a
^2 - b^2)^2)/(a*b^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

3.289 $\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=59

$$-\frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d}+\frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{bd}$$

```
[Out] ln(cos(d*x+c))/a/d-(a^2-b^2)*ln(a+b*sec(d*x+c))/a/b^2/d+sec(d*x+c)/b/d
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$-\frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d}+\frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) - ((a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a*b^2*d) +
 Sec[c + d*x]/(b*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^2d}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)dx, x, b\sec(c+dx)\right)}{b^2d}\\ &= \frac{\log(\cos(c+dx))}{ad}-\frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d}+\frac{\sec(c+dx)}{bd}\end{aligned}$$

Mathematica [A] time = 0.13, size = 52, normalized size = 0.88

$$\frac{(b^2-a^2)\log(a\cos(c+dx)+b)+a^2\log(\cos(c+dx))+ab\sec(c+dx)}{ab^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

[Out] (a^2*Log[Cos[c + d*x]] + (-a^2 + b^2)*Log[b + a*Cos[c + d*x]] + a*b*Sec[c + d*x])/(a*b^2*d)

fricas [A] time = 0.51, size = 69, normalized size = 1.17

$$\frac{a^2 \cos(dx+c) \log(-\cos(dx+c)) - \left(a^2-b^2\right) \cos(dx+c) \log(a\cos(dx+c)+b) + ab}{ab^2 d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a^2*cos(d*x + c)*log(-cos(d*x + c)) - (a^2 - b^2)*cos(d*x + c)*log(a*cos(d
*x + c) + b) + a*b)/(a*b^2*d*cos(d*x + c))
```

giac [B] time = 2.07, size = 289, normalized size = 4.90

$$-\frac{\frac{a \log\left(\left|a+b-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{b^2}-\frac{2\,a \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2}-\frac{\left(a^2-2\,b^2\right)\log\left(\frac{\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right.}{\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right.}\right.}{b^2|a|}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2))/b^2 - 2*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^
2 - (a^2 - 2*b^2)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 2*abs(a)))/(b^2*abs(a)) + 2*(a - 2*b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/(b^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

maple [A] time = 0.39, size = 70, normalized size = 1.19

$$-\frac{a \ln(b+a\cos(dx+c))}{d\,b^2}+\frac{\ln(b+a\cos(dx+c))}{da}+\frac{a \ln(\cos(dx+c))}{d\,b^2}+\frac{1}{db\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d/b^2*a*ln(b+a*cos(d*x+c))+1/d/a*ln(b+a*cos(d*x+c))+1/d/b^2*a*ln(cos(d*x
+c))+1/d/b/cos(d*x+c)
```

maxima [A] time = 0.42, size = 57, normalized size = 0.97

$$\frac{\frac{a\log(\cos(dx+c))}{b^2}-\frac{\left(a^2-b^2\right)\log(a\cos(dx+c)+b)}{ab^2}+\frac{1}{b\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(cos(d*x + c))/b^2 - (a^2 - b^2)*log(a*cos(d*x + c) + b)/(a*b^2) + 1/
(b*cos(d*x + c)))/d
```

mupad [B] time = 1.54, size = 115, normalized size = 1.95

$$\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{b^2 d}-\frac{2}{bd\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{ad}-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 - 1))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 -
1)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - (log(a + b - a*tan(c/2 + (d*x)/
2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a/b^2 - 1/a))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

3.290 $\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=35

$$-\frac{\log(a+b\sec(c+dx))}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

```
[Out] -ln(cos(d*x+c))/a/d-ln(a+b*sec(d*x+c))/a/d
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3885, 36, 29, 31}

$$-\frac{\log(a+b\sec(c+dx))}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Log[a + b*Sec[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sec(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sec(c+dx)\right)}{ad}\\
&= -\frac{\log(\cos(c+dx))}{ad}-\frac{\log(a+b\sec(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 19, normalized size = 0.54

$$-\frac{\log(a\cos(c+dx)+b)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[b + a*Cos[c + d*x]]/(a*d))
```

fricas [A] time = 0.48, size = 19, normalized size = 0.54

$$-\frac{\log\left(a\cos\left(dx+c\right)+b\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -log(a*cos(d*x + c) + b)/(a*d)
```

giac [B] time = 0.35, size = 114, normalized size = 3.26

$$\frac{\log\left(\frac{\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|\right|}{\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|\right|}\right)}{d|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(d*a
bs(a))
```

maple [A] time = 0.13, size = 35, normalized size = 1.00

$$-\frac{\ln\left(a+b\sec\left(dx+c\right)\right)}{da}+\frac{\ln\left(\sec\left(dx+c\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] -ln(a+b*sec(d*x+c))/d/a+1/d/a*ln(sec(d*x+c))
```

maxima [A] time = 0.44, size = 19, normalized size = 0.54

$$-\frac{\log\left(a\cos\left(dx+c\right)+b\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log(a*cos(d*x + c) + b)/(a*d)
```

mupad [B] time = 1.47, size = 71, normalized size = 2.03

$$\frac{\mathrm{atan}\left(\frac{a\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1{}\mathrm{i}+b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1{}\mathrm{i}+b\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1{}\mathrm{i}}\right)\,2{}\mathrm{i}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan((a*sin(c/2 + (d*x)/2)^2)/(a*cos(c/2 + (d*x)/2)^2*1i + b*cos(c/2 + (d*
x)/2)^2*1i + b*sin(c/2 + (d*x)/2)^2*1i))*2i)/(a*d)
```

sympy [A] time = 5.73, size = 82, normalized size = 2.34

$$\begin{cases} \frac{\tilde{\infty} x \tan{\left(c \right)}}{\sec{\left(c \right)}} & \text{for}\: a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 a d} & \text{for}\: b = 0 \\ - \frac{1}{b d \sec{\left(c + d x \right)}} & \text{for}\: a = 0 \\ \frac{x \tan{\left(c \right)}}{a + b \sec{\left(c \right)}} & \text{for}\: d = 0 \\ - \frac{\log{\left(\frac{a}{b} + \sec{\left(c + d x \right)} \right)}}{a d} + \frac{\log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 a d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*tan(c)/sec(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c
 + d*x)**2 + 1)/(2*a*d), Eq(b, 0)), (-1/(b*d*sec(c + d*x)), Eq(a, 0)), (x*t
an(c)/(a + b*sec(c)), Eq(d, 0)), (-log(a/b + sec(c + d*x))/(a*d) + log(tan(
c + d*x)**2 + 1)/(2*a*d), True))
```

3.291 $\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{b^2\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)}+\frac{\log(1-\sec(c+dx))}{2d(a+b)}+\frac{\log(\sec(c+dx)+1)}{2d(a-b)}+\frac{\log(\cos(c+dx))}{ad}$$

```
[Out] ln(cos(d*x+c))/a/d+1/2*ln(1-sec(d*x+c))/(a+b)/d+1/2*ln(1+sec(d*x+c))/(a-b)/
d-b^2*ln(a+b*sec(d*x+c))/a/(a^2-b^2)/d
```

Rubi [A] time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3885, 894}

$$-\frac{b^2\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)}+\frac{\log(1-\sec(c+dx))}{2d(a+b)}+\frac{\log(\sec(c+dx)+1)}{2d(a-b)}+\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Log[1 - Sec[c + d*x]]/(2*(a + b)*d) + Log[1 + Sec
[c + d*x]]/(2*(a - b)*d) - (b^2*Log[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx &= -\frac{b^2\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= -\frac{b^2\,\text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)}+\frac{1}{ab^2x}+\frac{1}{a(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)b^2(b+x)}\right)dx, x, b\sec(c+dx)\right)}{d}\\ &= \frac{\log(\cos(c+dx))}{ad}+\frac{\log(1-\sec(c+dx))}{2(a+b)d}+\frac{\log(1+\sec(c+dx))}{2(a-b)d}-\frac{b^2\log(a+b\sec(c+dx))}{a\left(a^2-b^2\right)d}\end{aligned}$$

Mathematica [A] time = 0.11, size = 70, normalized size = 0.74

$$\frac{b^2(-\log(a\cos(c+dx)+b))+a(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*(a + b)*Log[Cos[(c + d*x)/2]] - b^2*Log[b + a*Cos[c + d*x]] + a*(a - b)*
Log[Sin[(c + d*x)/2]])/(a*(a - b)*(a + b)*d)
```

fricas [A] time = 0.52, size = 75, normalized size = 0.80

$$-\frac{2\,b^2\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^2+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^3-ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b) - (a^2 + a*b)*log(1/2*cos(d*x + c) + 1/
2) - (a^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)
```

giac [B] time = 0.31, size = 257, normalized size = 2.73

$$-\frac{\frac{a\log\left(\left|-a-b+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^2-b^2}-\frac{\left(a^2-2\,b^2\right)\log\left(\frac{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|\right|}{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|\right|}\right)}{\left(a^2-b^2\right)|a|}-\frac{\log\left(\frac{|-\cos(dx+c)+}{|\cos(dx+c)+1}\right.}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(-a - b + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2))/(a^2 - b^2) - (a^2 - 2*b^2)*log(abs(-2*b - 2*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a)
)/abs(-2*b - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 2*abs(a)))/((a^2 - b^2)*abs(a)) - log(abs(-cos(d*
x + c) + 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

maple [A] time = 0.71, size = 80, normalized size = 0.85

$$-\frac{b^2\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)\left(a-b\right)a}+\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{d\left(2a+2b\right)}+\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d*b^2/(a+b)/(a-b)/a*ln(b+a*cos(d*x+c))+1/d/(2*a+2*b)*ln(-1+cos(d*x+c))+1
/d/(2*a-2*b)*ln(1+cos(d*x+c))
```

maxima [A] time = 0.50, size = 68, normalized size = 0.72

$$-\frac{\frac{2\,b^2\log(a\cos(dx+c)+b)}{a^3-ab^2}-\frac{\log(\cos(dx+c)+1)}{a-b}-\frac{\log(\cos(dx+c)-1)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b)/(a^3 - a*b^2) - log(cos(d*x + c) + 1)/(
a - b) - log(cos(d*x + c) - 1)/(a + b))/d
```

mupad [B] time = 1.75, size = 93, normalized size = 0.99

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a\,d}+\frac{b^2\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d\,\left(a\,b^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) +
 (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a*b
^2 - a^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x)), x)
```

3.292 $\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=157

$$-\frac{b^4\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sec(c+dx))}+\frac{1}{4d(a-b)(\sec(c+dx)+1)}-\frac{(2a+3b)\log(1-\sec(c+}{4d(a+b)^2}$$

```
[Out] -ln(cos(d*x+c))/a/d-1/4*(2*a+3*b)*ln(1-sec(d*x+c))/(a+b)^2/d-1/4*(2*a-3*b)*
ln(1+sec(d*x+c))/(a-b)^2/d-b^4*ln(a+b*sec(d*x+c))/a/(a^2-b^2)^2/d+1/4/(a+b)
/d/(1-sec(d*x+c))+1/4/(a-b)/d/(1+sec(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$-\frac{b^4\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sec(c+dx))}+\frac{1}{4d(a-b)(\sec(c+dx)+1)}-\frac{(2a+3b)\log(1-\sec(c+}{4d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((2*a + 3*b)*Log[1 - Sec[c + d*x]])/(4*(a + b)
^2*d) - ((2*a - 3*b)*Log[1 + Sec[c + d*x]])/(4*(a - b)^2*d) - (b^4*Log[a +
b*Sec[c + d*x]])/(a*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sec[c + d*x])) +
 1/(4*(a - b)*d*(1 + Sec[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{b^4\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^2}\,dx,x,b\sec(c+dx)\right)}{d}\\ &= \frac{b^4\,\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2}+\frac{2a+3b}{4b^4(a+b)^2(b-x)}+\frac{1}{ab^4x}-\frac{1}{a(a-b)^2(a+b)^2(a+x)}-\frac{1}{4(a-b)b^3(b+x)^2}+\frac{}{4}\right.\right.}{d}\\ &= -\frac{\log(\cos(c+dx))}{ad}-\frac{(2a+3b)\log(1-\sec(c+dx))}{4(a+b)^2d}-\frac{(2a-3b)\log(1+\sec(c+dx)}{4(a-b)^2d}\end{aligned}$$

Mathematica [A] time = 1.05, size = 141, normalized size = 0.90

$$-\frac{8b^4\log(a\cos(c+dx)+b)+a(a-b)^2(a+b)\csc^2\left(\frac{1}{2}(c+dx)\right)+a(a-b)(a+b)^2\sec^2\left(\frac{1}{2}(c+dx)\right)+4a(a-b)^2(2}{8ad(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/8*(a*(a - b)^2*(a + b)*Csc[(c + d*x)/2]^2 + 4*a*(2*a - 3*b)*(a + b)^2*Lo
g[Cos[(c + d*x)/2]] + 8*b^4*Log[b + a*Cos[c + d*x]] + 4*a*(a - b)^2*(2*a +
3*b)*Log[Sin[(c + d*x)/2]] + a*(a - b)*(a + b)^2*Sec[(c + d*x)/2]^2)/(a*(a
- b)^2*(a + b)^2*d)
```

fricas [A] time = 0.60, size = 263, normalized size = 1.68

$$\frac{2\,a^4-2\,a^2b^2-2\left(a^3b-ab^3\right)\cos\left(dx+c\right)-4\left(b^4\cos\left(dx+c\right)^2-b^4\right)\log\left(a\cos\left(dx+c\right)+b\right)+\left(2\,a^4+a^3b-4\,a^2b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^4 - 2*a^2*b^2 - 2*(a^3*b - a*b^3)*cos(d*x + c) - 4*(b^4*cos(d*x +
c)^2 - b^4)*log(a*cos(d*x + c) + b) + (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3
- (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c
) + 1/2) + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 - (2*a^4 - a^3*b - 4*a^2*b^
2 + 3*a*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^5 - 2*a^3*b^
2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

giac [B] time = 0.86, size = 403, normalized size = 2.57

$$-\frac{\frac{2\,(2\,a+3\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2}-\frac{4\left(a^3-2\,ab^2\right)\log\left(\left|a+b-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^4-2\,a^2b^2+b^4}-\frac{\left(a+b+\frac{4\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\left(a^2+2\,ab+b^2\right)(\cos(dx+c)-}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*a + 3*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2
+ 2*a*b + b^2) - 4*(a^3 - 2*a*b^2)*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2))/(a^4 - 2*a^2*b^2 + b^4) - (a + b + 4*a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1))*(cos(d*x + c) + 1)/((a^2 + 2*a*b + b^2)*(cos(d*x + c) - 1)) - 4*(a
^4 - 2*a^2*b^2 + 2*b^4)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 2*abs(a)))/((a^4 - 2*a^2*b^2 + b^4)*abs(a)) - (cos(d*x + c) - 1)/(
(a - b)*(cos(d*x + c) + 1)))/d
```

maple [A] time = 0.88, size = 167, normalized size = 1.06

$$-\frac{b^4\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)^2\left(a-b\right)^2a}+\frac{1}{d\left(4a+4b\right)\left(-1+\cos\left(dx+c\right)\right)}-\frac{\ln\left(-1+\cos\left(dx+c\right)\right)a}{2d\left(a+b\right)^2}-\frac{3\ln\left(-1+\cos\left(dx+c\right)\right)b}{4d\left(a+b\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c)),x)

[Out] -1/d*b^4/(a+b)^2/(a-b)^2/a*ln(b+a*cos(d*x+c))+1/d/(4*a+4*b)/(-1+cos(d*x+c))-1/2/d/(a+b)^2*ln(-1+cos(d*x+c))*a-3/4/d/(a+b)^2*ln(-1+cos(d*x+c))*b-1/d/(4*a-4*b)/(1+cos(d*x+c))+3/4/d/(a-b)^2*ln(1+cos(d*x+c))*b-1/2*a*ln(1+cos(d*x+c))/(a-b)^2/d

maxima [A] time = 0.34, size = 144, normalized size = 0.92

$$-\frac{\frac{4\,b^4\log(a\cos(dx+c)+b)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{(2\,a+3\,b)\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2}+\frac{2\,(b\cos(dx+c)-a)}{\left(a^2-b^2\right)\cos(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/4*(4*b^4*log(a*cos(d*x + c) + b)/(a^5 - 2*a^3*b^2 + a*b^4) + (2*a - 3*b)*log(cos(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2) + 2*(b*cos(d*x + c) - a)/((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2))/d

mupad [B] time = 1.85, size = 174, normalized size = 1.11

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d\,\left(4\,a-4\,b\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a+3\,b\right)}{d\,\left(2\,a^2+4\,a\,b+2\,b^2\right)}-\frac{a-b}{2\,d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a+b\right)\,\left(4\,a-4\,b\right)}-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x)),x)

[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - tan(c/2 + (d*x)/2)^2/(2*d*(4*a - 4*b)) - (log(tan(c/2 + (d*x)/2))*(2*a + 3*b))/(d*(4*a*b + 2*a^2 + 2*b^2)) - (a - b)/(2*d*tan(c/2 + (d*x)/2)^2*(a + b)*(4*a - 4*b)) - (b^4*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(a*d*(a^2 - b^2)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c)),x)

[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x)), x)

3.293 $\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{16d(a+b)^3}+\frac{(8a^2-21ab+15b^2)\log(\sec(c+dx)+1)}{16d(a-b)^3}-\frac{b^6\log(a+b\sec(c+dx))}{ad(a^2-b^2)^3}-$$

```
[Out] ln(cos(d*x+c))/a/d+1/16*(8*a^2+21*a*b+15*b^2)*ln(1-sec(d*x+c))/(a+b)^3/d+1/
16*(8*a^2-21*a*b+15*b^2)*ln(1+sec(d*x+c))/(a-b)^3/d-b^6*ln(a+b*sec(d*x+c))/
a/(a^2-b^2)^3/d-1/16/(a+b)/d/(1-sec(d*x+c))^2+1/16*(-5*a-7*b)/(a+b)^2/d/(1-
sec(d*x+c))-1/16/(a-b)/d/(1+sec(d*x+c))^2+1/16*(-5*a+7*b)/(a-b)^2/d/(1+sec(
d*x+c))
```

Rubi [A] time = 0.29, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3885, 894}

$$-\frac{b^6\log(a+b\sec(c+dx))}{ad(a^2-b^2)^3}+\frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{16d(a+b)^3}+\frac{(8a^2-21ab+15b^2)\log(\sec(c+dx)+1)}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sec[c + d*x]])
/(16*(a + b)^3*d) + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sec[c + d*x]])/(16*(
a - b)^3*d) - (b^6*Log[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)^3*d) - 1/(16*(a
+ b)*d*(1 - Sec[c + d*x])^2) - (5*a + 7*b)/(16*(a + b)^2*d*(1 - Sec[c + d*x
])) - 1/(16*(a - b)*d*(1 + Sec[c + d*x])^2) - (5*a - 7*b)/(16*(a - b)^2*d*(
1 + Sec[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx=-\frac{b^6\,\text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)^3}\,dx,x,b\sec(c+dx)\right)}{d}$$

$$=-\frac{b^6\,\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)(b-x)^3}+\frac{5a+7b}{16b^5(a+b)^2(b-x)^2}+\frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)}+\frac{1}{ab^6x}+\frac{1}{a(a-b)^3(a+b)^3(a+x)}+\right.\right.}{d}$$

$$=\frac{\log(\cos(c+dx))}{ad}+\frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{16(a+b)^3d}+\frac{(8a^2-21ab+15b^2)\,\mathrm{lo}}{16(a-b}$$

Mathematica [C] time = 6.25, size = 625, normalized size = 2.67

$$\frac{\left(-8a^2+21ab-15b^2\right)\sec(c+dx)\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b)}{16d(b-a)^3(a+b\sec(c+dx))}-\frac{i\left(-8a^2+21ab-15b^2\right)\tan^{-1}(\tan}{8d(b-a)^3(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*I)*(a^5 - 3*a^3*b^2 + 3*a*b^4)*(c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d
*x])/((a - b)^3*(a + b)^3*d*(a + b*Sec[c + d*x])) - ((I/8)*(-8*a^2 + 21*a*b
 - 15*b^2)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x])/((-a + b
)^3*d*(a + b*Sec[c + d*x])) - ((I/8)*(8*a^2 + 21*a*b + 15*b^2)*ArcTan[Tan[c
 + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x])/((a + b)^3*d*(a + b*Sec[c + d*x
])) + ((7*a + 9*b)*(b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^2*Sec[c + d*x])/(3
2*(a + b)^2*d*(a + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*Csc[(c + d*x)/2
]^4*Sec[c + d*x])/(64*(a + b)*d*(a + b*Sec[c + d*x])) + ((-8*a^2 + 21*a*b -
 15*b^2)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2]^2]*Sec[c + d*x])/(16*(-a
 + b)^3*d*(a + b*Sec[c + d*x])) + (b^6*(b + a*Cos[c + d*x])*Log[b + a*Cos[c
 + d*x]]*Sec[c + d*x])/(a*(-a^2 + b^2)^3*d*(a + b*Sec[c + d*x])) + ((8*a^2
+ 21*a*b + 15*b^2)*(b + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]^2]*Sec[c + d*x
])/(16*(a + b)^3*d*(a + b*Sec[c + d*x])) + ((7*a - 9*b)*(b + a*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Sec[c + d*x])/(32*(-a + b)^2*d*(a + b*Sec[c + d*x])) +
 ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sec[c + d*x])/(64*(-a + b)*d*(a +
 b*Sec[c + d*x]))
```

fricas [B] time = 0.77, size = 576, normalized size = 2.46

$$\frac{12\,a^6-32\,a^4b^2+20\,a^2b^4+2\left(5\,a^5b-14\,a^3b^3+9\,ab^5\right)\cos\left(dx+c\right)^3-8\left(2\,a^6-5\,a^4b^2+3\,a^2b^4\right)\cos\left(dx+c\right)^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^6 - 32*a^4*b^2 + 20*a^2*b^4 + 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)
*cos(d*x + c)^3 - 8*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 - 2*(3*a
^5*b - 10*a^3*b^3 + 7*a*b^5)*cos(d*x + c) - 16*(b^6*cos(d*x + c)^4 - 2*b^6*
cos(d*x + c)^2 + b^6)*log(a*cos(d*x + c) + b) + (8*a^6 + 3*a^5*b - 24*a^4*b
^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 + (8*a^6 + 3*a^5*b - 24*a^4*b^2 - 1
0*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^4 - 2*(8*a^6 + 3*a^5*b - 24
*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^2)*log(1/2*cos(
d*x + c) + 1/2) + (8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 -
 15*a*b^5 + (8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*
b^5)*cos(d*x + c)^4 - 2*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2
*b^4 - 15*a*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^4 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^
4 - a*b^6)*d*cos(d*x + c)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)
```

giac [B] time = 1.14, size = 649, normalized size = 2.77

$$\frac{\frac{4\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{32\left(a^5-3\,a^3b^2+3\,ab^4\right)\log\left(\left|a+b-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\frac{12\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(4*(8*a^2 + 21*a*b + 15*b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x +
c) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 32*(a^5 - 3*a^3*b^2 + 3*a*b^4)*l
og(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))
/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (12*a*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - 16*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^2
 - 2*a*b + b^2) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 28*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16*b^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 126*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 90*b^2*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*(cos(d*x + c) - 1)^2) - 32*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - 2*b^6
)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*abs(a)))/d
```

maple [A] time = 0.68, size = 308, normalized size = 1.32

$$-\frac{b^6 \ln (b + a\cos (dx + c))}{d (a + b)^3 (a - b)^3 a} - \frac{1}{2d (8a + 8b) (-1 + \cos (dx + c))^2} - \frac{7a}{16d (a + b)^2 (-1 + \cos (dx + c))} - \frac{9b}{16d (a + b)^2 (-1 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d*b^6/(a+b)^3/(a-b)^3/a*ln(b+a*cos(d*x+c))-1/2/d/(8*a+8*b)/(-1+cos(d*x+c
))^2-7/16/d/(a+b)^2/(-1+cos(d*x+c))*a-9/16/d/(a+b)^2/(-1+cos(d*x+c))*b+1/2/
d/(a+b)^3*ln(-1+cos(d*x+c))*a^2+21/16/d/(a+b)^3*ln(-1+cos(d*x+c))*a*b+15/16
/d/(a+b)^3*ln(-1+cos(d*x+c))*b^2-1/2/d/(8*a-8*b)/(1+cos(d*x+c))^2+7/16/d/(a
-b)^2/(1+cos(d*x+c))*a-9/16/d/(a-b)^2/(1+cos(d*x+c))*b+1/2/d/(a-b)^3*ln(1+c
os(d*x+c))*a^2-21/16/d/(a-b)^3*ln(1+cos(d*x+c))*a*b+15/16/d/(a-b)^3*ln(1+co
s(d*x+c))*b^2
```

maxima [A] time = 0.52, size = 289, normalized size = 1.24

$$-\frac{\frac{16\,b^6 \log(a\cos(dx+c)+b)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6} - \frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\left(\left(5\,a^2b-9\,b^3\right)\cos(dx+c)^3+6\,a^3-10}{\left(a^4-2\,a^2b^2+b^4\right)\cos(dx+c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*b^6*log(a*cos(d*x + c) + b)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 - (8*a^2 - 21*a*b + 15*b^2)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b +
3*a*b^2 + b^3) - 2*((5*a^2*b - 9*b^3)*cos(d*x + c)^3 + 6*a^3 - 10*a*b^2 - 4
*(2*a^3 - 3*a*b^2)*cos(d*x + c)^2 - (3*a^2*b - 7*b^3)*cos(d*x + c))/((a^4 -
 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b
^2 + b^4)*cos(d*x + c)^2))/d
```

mupad [B] time = 2.27, size = 290, normalized size = 1.24

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(8\,a^2+21\,a\,b+15\,b^2\right)}{d\,\left(8\,a^3+24\,a^2\,b+24\,a\,b^2+8\,b^3\right)} - \frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{4\,d\,\left(16\,a-16\,b\right)} - \frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a\,d} - \frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{16\,b}{{\left(16\,a-16\,b\right)}^2} - \right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*(21*a*b + 8*a^2 + 15*b^2))/(d*(24*a*b^2 + 24*a^2*b
 + 8*a^3 + 8*b^3)) - tan(c/2 + (d*x)/2)^4/(4*d*(16*a - 16*b)) - log(tan(c/2
 + (d*x)/2)^2 + 1)/(a*d) - (tan(c/2 + (d*x)/2)^2*((16*b)/(16*a - 16*b)^2 -
3/(16*a - 16*b)))/d - ((a^2 - 2*a*b + b^2)/(4*(a + b)) + (tan(c/2 + (d*x)/2
)^2*(5*a*b^2 + 2*a^2*b - 3*a^3 - 4*b^3))/(a + b)^2)/(d*tan(c/2 + (d*x)/2)^4
*(16*a^2 - 32*a*b + 16*b^2)) - (b^6*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*
tan(c/2 + (d*x)/2)^2))/(a*d*(a^2 - b^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

3.294 $\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=198

$$-\frac{a\left(a^2-2b^2\right)\tan(c+dx)}{b^4d}+\frac{\left(4a^2-7b^2\right)\tan(c+dx)\sec(c+dx)}{8b^3d}+\frac{\left(8a^4-20a^2b^2+15b^4\right)\tanh^{-1}(\sin(c+dx))}{8b^5d}-\frac{2(a}{}$$

```
[Out] -x/a+1/8*(8*a^4-20*a^2*b^2+15*b^4)*arctanh(sin(d*x+c))/b^5/d-2*(a-b)^(5/2)*
(a+b)^(5/2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/b^5/d-a*(
a^2-2*b^2)*tan(d*x+c)/b^4/d+1/8*(4*a^2-7*b^2)*sec(d*x+c)*tan(d*x+c)/b^3/d-1
/3*a*tan(d*x+c)^3/b^2/d+1/4*sec(d*x+c)*tan(d*x+c)^3/b/d
```

Rubi [A] time = 0.37, antiderivative size = 271, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3898, 2897, 2659, 208, 3770, 3767, 8, 3768}

$$-\frac{a\left(a^2-3b^2\right)\tan(c+dx)}{b^4d}+\frac{\left(a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{\left(-3a^2b^2+a^4+3b^4\right)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{\left(a^2-3b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (3*ArcTanh[Sin[c + d*x]])/(8*b*d) + ((a^2 - 3*b^2)*ArcTanh[Sin[c +
 d*x]])/(2*b^3*d) + ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(b^5*
d) - (2*(a - b)^(5/2)*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/(a*b^5*d) - (a*Tan[c + d*x])/(b^2*d) - (a*(a^2 - 3*b^2)*Tan[c
 + d*x])/(b^4*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(8*b*d) + ((a^2 - 3*b^2)*S
ec[c + d*x]*Tan[c + d*x])/(2*b^3*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*b*d)
 - (a*Tan[c + d*x]^3)/(3*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin(c+dx)\tan^5(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= \int \left(-\frac{1}{a} - \frac{\left(a^2-b^2\right)^3}{ab^5(b+a\cos(c+dx))} + \frac{\left(a^4-3a^2b^2+3b^4\right)\sec(c+dx)}{b^5} + \frac{\left(-a^3+3ab^2\right)\text{s}}{b^4}\right.$$

$$= -\frac{x}{a} - \frac{a\int \sec^4(c+dx)\,dx}{b^2} + \frac{\int \sec^5(c+dx)\,dx}{b} - \frac{\left(a\left(a^2-3b^2\right)\right)\int \sec^2(c+dx)\,dx}{b^4} +$$

$$= -\frac{x}{a} + \frac{\left(a^4-3a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{\left(a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{2b^3d}$$

$$= -\frac{x}{a} + \frac{\left(a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{\left(a^4-3a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{2}{\;}$$

$$= -\frac{x}{a} + \frac{3\tanh^{-1}(\sin(c+dx))}{8bd} + \frac{\left(a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{\left(a^4-3a^2b^2+3b^4\right)}{b}$$

Mathematica [B] time = 6.20, size = 907, normalized size = 4.58

$$-\frac{2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))\sec(c+dx)\left(b^2-a^2\right)^3}{ab^5\sqrt{a^2-b^2}\,d(a+b\sec(c+dx))} - \frac{a(b+a\cos(c+dx))\sec(c+dx)}{6b^2d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*d*(a + b*Sec[c + d*x])))
 - (2*(-a^2 + b^2)^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(
b + a*Cos[c + d*x])*Sec[c + d*x])/(a*b^5*Sqrt[a^2 - b^2]*d*(a + b*Sec[c + d
*x])) + ((-8*a^4 + 20*a^2*b^2 - 15*b^4)*(b + a*Cos[c + d*x])*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x])/(8*b^5*d*(a + b*Sec[c + d*x])) + (
(8*a^4 - 20*a^2*b^2 + 15*b^4)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]*Sec[c + d*x])/(8*b^5*d*(a + b*Sec[c + d*x])) + ((b + a*Cos
[c + d*x])*Sec[c + d*x])/(16*b*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^4) + ((12*a^2 - 4*a*b - 27*b^2)*(b + a*Cos[c + d*x])*Sec[c
 + d*x])/(48*b^3*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^2) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[(c + d*x)/2])/(6*b^2*d*(a
+ b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) - ((b + a*Cos[c
+ d*x])*Sec[c + d*x])/(16*b*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^4) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[(c + d*x)/2])/(
6*b^2*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((-
12*a^2 + 4*a*b + 27*b^2)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(48*b^3*d*(a +
b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + ((b + a*Cos[c +
d*x])*Sec[c + d*x]*(-3*a^3*Sin[(c + d*x)/2] + 7*a*b^2*Sin[(c + d*x)/2]))/(3
*b^4*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((b +
a*Cos[c + d*x])*Sec[c + d*x]*(-3*a^3*Sin[(c + d*x)/2] + 7*a*b^2*Sin[(c + d*
x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
)
```

fricas [A] time = 1.05, size = 603, normalized size = 3.05

$$\left[-\frac{48\,b^5dx\cos\left(dx+c\right)^4-24\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}\,\cos\left(dx+c\right)^4\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b}{a^2\cos(dx+c)^2+2\,ab\cos(d}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(48*b^5*d*x*cos(d*x + c)^4 - 24*(a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b
^2)*cos(d*x + c)^4*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^
4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2*b^3*cos(d*x + c) - 6*a*b^
4 + 8*(3*a^4*b - 7*a^2*b^3)*cos(d*x + c)^3 - 3*(4*a^3*b^2 - 9*a*b^4)*cos(d*
x + c)^2)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4), -1/48*(48*b^5*d*x*cos(d*x
 + c)^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^4 - 3*(8*
a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 3*(8*a^
5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2
*b^3*cos(d*x + c) - 6*a*b^4 + 8*(3*a^4*b - 7*a^2*b^3)*cos(d*x + c)^3 - 3*(4
*a^3*b^2 - 9*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4)]
```

giac [B] time = 4.21, size = 746, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(24*((a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + b^4)*sqrt(-a^2 + b^2)*abs(a
)*abs(-a + b)*abs(b) + (a^5*b + a^4*b^2 - 2*a^3*b^3 - 2*a^2*b^4 + a*b^5 + 2
*b^6)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arc
tan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^6 + sqrt(b^12 + (a*b^5 + b^6)*(a*b^5 - b^
```

```
6)))/(a*b^5 - b^6))))/((a*b^4 - b^5)*a^2*b^2 + (a*b^6 - b^7)*abs(a)*abs(b))
 + 24*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 + a*b^6 - 2*b^7 - a^5*abs(a)*abs(b) +
3*a^3*b^2*abs(a)*abs(b) - 3*a*b^4*abs(a)*abs(b) + b^5*abs(a)*abs(b))*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^6 - sqrt
(b^12 + (a*b^5 + b^6)*(a*b^5 - b^6)))/(a*b^5 - b^6))))/(a^2*b^6 - b^6*abs(a
)*abs(b)) - 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)*log(abs(tan(1/2*d*x + 1/2*c) +
1))/b^5 + 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
)/b^5 - 2*(24*a^3*tan(1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^7
- 48*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 21*b^3*tan(1/2*d*x + 1/2*c)^7 - 72*a^3*
tan(1/2*d*x + 1/2*c)^5 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 176*a*b^2*tan(1/
2*d*x + 1/2*c)^5 + 45*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2
*c)^3 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 176*a*b^2*tan(1/2*d*x + 1/2*c)^3
+ 45*b^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c) + 12*a^2*b*ta
n(1/2*d*x + 1/2*c) + 48*a*b^2*tan(1/2*d*x + 1/2*c) - 21*b^3*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*b^4))/d
```

maple [B] time = 0.46, size = 785, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/2/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a^2-2/d/b^2/(tan(1/2*d*x+1/2*c)+1)*a-1/2/d
/b^2/(tan(1/2*d*x+1/2*c)+1)^2*a+1/d/b^5*ln(tan(1/2*d*x+1/2*c)+1)*a^4-5/2/d/
b^3*ln(tan(1/2*d*x+1/2*c)+1)*a^2+1/2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2-2/d/b
^2/(tan(1/2*d*x+1/2*c)-1)*a+1/3/d/b^2/(tan(1/2*d*x+1/2*c)+1)^3*a-1/2/d/b^3/
(tan(1/2*d*x+1/2*c)+1)^2*a^2+1/3/d/b^2/(tan(1/2*d*x+1/2*c)-1)^3*a+1/2/d/b^3
/(tan(1/2*d*x+1/2*c)-1)^2*a^2+1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*a-1/d/b^5*
ln(tan(1/2*d*x+1/2*c)-1)*a^4+5/2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*a^2+1/d/b^4
/(tan(1/2*d*x+1/2*c)-1)*a^3+1/d/b^4/(tan(1/2*d*x+1/2*c)+1)*a^3-6/d/b*a/((a-
b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*b
/a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)
)+15/8/d/b*ln(tan(1/2*d*x+1/2*c)+1)-7/8/d/b/(tan(1/2*d*x+1/2*c)+1)+1/4/d/b/
(tan(1/2*d*x+1/2*c)-1)^4+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^3-5/8/d/b/(tan(1/2*
d*x+1/2*c)-1)^2-15/8/d/b*ln(tan(1/2*d*x+1/2*c)-1)-7/8/d/b/(tan(1/2*d*x+1/2*
c)-1)-1/4/d/b/(tan(1/2*d*x+1/2*c)+1)^4-2/d/a*arctan(tan(1/2*d*x+1/2*c))+1/2
/d/b/(tan(1/2*d*x+1/2*c)+1)^3+5/8/d/b/(tan(1/2*d*x+1/2*c)+1)^2-2/d/b^5*a^5/
((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+6
/d/b^3*a^3/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 4.13, size = 9148, normalized size = 46.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^6/(a + b/cos(c + d*x)),x)

[Out] ((tan(c/2 + (d*x)/2)*(16*a*b^2 + 4*a^2*b - 8*a^3 - 7*b^3))/(4*b^4) - (tan(c/2 + (d*x)/2)^7*(16*a*b^2 - 4*a^2*b - 8*a^3 + 7*b^3))/(4*b^4) - (tan(c/2 + (d*x)/2)^3*(176*a*b^2 + 12*a^2*b - 72*a^3 - 45*b^3))/(12*b^4) + (tan(c/2 + (d*x)/2)^5*(176*a*b^2 - 12*a^2*b - 72*a^3 + 45*b^3))/(12*b^4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (2*atan((((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 - (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*128i)/(a*b^16))*1i)/a - (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a - (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16)/a - ((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*128i)/(a*b^16))*1i)/a + (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a + (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16)/a)/((((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 - (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*128i)/(a*b^16))*1i)/a - (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a - (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16)*1i)/a + (((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16

+ 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18) *128i)/(a*b^16))*1i)/a + (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a + (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16)*1i)/a - (256*(64*a^19*b - 2145*a*b^19 - 64*a^20 + 795*b^20 - 3130*a^2*b^18 + 12805*a^3*b^17 + 2569*a^4*b^16 - 33634*a^5*b^15 + 7876*a^6*b^14 + 51074*a^7*b^13 - 23883*a^8*b^12 - 49501*a^9*b^11 + 30942*a^10*b^10 + 31881*a^11*b^9 - 23865*a^12*b^8 - 13776*a^13*b^7 + 11768*a^14*b^6 + 3936*a^15*b^5 - 3712*a^16*b^4 - 704*a^17*b^3 + 704*a^18*b^2))/b^16)))/(a*d) + (atan(((((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 + (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 + (((128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 - (((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 - (128*tan(c/2 + (d*x)/2)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2) *(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/b^21)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2)*1i)/b^5 + (((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 - (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 - (((128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 + (((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (128*tan(c/2 + (d*x)/2)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/b^21)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2)*1i)/b^5)/((256*(64*a^

19*b - 2145*a*b^19 - 64*a^20 + 795*b^20 - 3130*a^2*b^18 + 12805*a^3*b^17 + 2569*a^4*b^16 - 33634*a^5*b^15 + 7876*a^6*b^14 + 51074*a^7*b^13 - 23883*a^8 *b^12 - 49501*a^9*b^11 + 30942*a^10*b^10 + 31881*a^11*b^9 - 23865*a^12*b^8 - 13776*a^13*b^7 + 11768*a^14*b^6 + 3936*a^15*b^5 - 3712*a^16*b^4 - 704*a^1 7*b^3 + 704*a^18*b^2))/b^16 + (((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a ^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a ^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b ^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704* a^18*b^3 - 704*a^19*b^2))/b^16 + (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b ^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310 *a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^ 11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248 *a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 + (((128*tan(c/2 + (d*x)/2)* (128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047 *a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 17 92*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^ 9 + 128*a^15*b^8))/b^16 - (((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^2 0 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b ^15 - 192*a^10*b^14))/b^16 - (128*tan(c/2 + (d*x)/2)*(a^4 + (15*b^4)/8 - (5 *a^2*b^2)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 3 84*a^6*b^19 - 128*a^7*b^18))/b^21)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5) *(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5 - (((128*tan(c/2 + (d*x)/2) *(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^ 18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 2798 3*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^1 2*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 - (((128*(576*a*b^21 - 1 92*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7 376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10* b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 - (((128 *tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^1 3*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 + (((128*(192*a^2*b^22 - 256*a^ 3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288* a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (128*tan(c/2 + (d*x)/2)*(a ^4 + (15*b^4)/8 - (5*a^2*b^2)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^2 1 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/b^21)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^ 4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5))*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2)*2i)/(b^5*d) + (atan(((((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b ^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 279 83*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^ 12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 + (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10 *b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 + (((a + b)^5*(a - b)^5)^(1/2)*((128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 3 22*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^ 12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 - (((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^ 6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16

- (128*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/(a*b^21))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5)))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2)*1i)/(a*b^5) + (((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 - (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 - (((a + b)^5*(a - b)^5)^(1/2)*((128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 + (((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (128*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/(a*b^21))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5)))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2)*1i)/(a*b^5))/((256*(64*a^19*b - 2145*a*b^19 - 64*a^20 + 795*b^20 - 3130*a^2*b^18 + 12805*a^3*b^17 + 2569*a^4*b^16 - 33634*a^5*b^15 + 7876*a^6*b^14 + 51074*a^7*b^13 - 23883*a^8*b^12 - 49501*a^9*b^11 + 30942*a^10*b^10 + 31881*a^11*b^9 - 23865*a^12*b^8 - 13776*a^13*b^7 + 11768*a^14*b^6 + 3936*a^15*b^5 - 3712*a^16*b^4 - 704*a^17*b^3 + 704*a^18*b^2))/b^16 + (((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 + (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 + (((a + b)^5*(a - b)^5)^(1/2)*((128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 - (((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 - (128*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/(a*b^21))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5)))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5) - (((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 - (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 - (((a + b)^5*(a - b)^5)^(1/2)*((128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^1

```
2*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 + (((128*(192*a
^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176
*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (128*tan(c
/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^23 - 384*a^3*b^22 + 51
2*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/(a*b^21))*((a + b
)^5*(a - b)^5)^(1/2))/(a*b^5)))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^
5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5)))*((a + b)^5*(a - b)^5)^(1/2)*2i)/
(a*b^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

3.295 $\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{x}{a}+\frac{\tan(c+d\cdots}{\cdots}$$

```
[Out] x/a+1/2*(2*a^2-3*b^2)*arctanh(sin(d*x+c))/b^3/d-2*(a-b)^(3/2)*(a+b)^(3/2)*a
rctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/b^3/d-a*tan(d*x+c)/b^2
/d+1/2*sec(d*x+c)*tan(d*x+c)/b/d
```

Rubi [A] time = 0.34, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3898, 2893, 3057, 2659, 208, 3770}

$$\frac{(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{a\tan(c+dx)}{b^2d}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d}+\frac{x}{a}+\frac{\tan(c+d\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] x/a + ((2*a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*(a - b)^(3/2)*
(a + b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b^3*d
) - (a*Tan[c + d*x])/(b^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
```

```
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin(c+dx)\tan^3(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\int \frac{\left(-2a^2+3b^2-ab\cos(c+dx)-2b^2\cos^2(c+dx)\right)\sec(c+dx)}{b+a\cos(c+dx)}}{2b^2}$$

$$= \frac{x}{a} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\left(a^2-b^2\right)^2\int \frac{1}{b+a\cos(c+dx)}\,dx}{ab^3} - \left(-2a^2+3\right.$$

$$= \frac{x}{a} + \frac{\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \left(2\right.$$

$$= \frac{x}{a} + \frac{\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d} -$$

Mathematica [B] time = 2.32, size = 287, normalized size = 2.28

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(-\frac{4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^3} + \frac{4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{b^3} + \frac{8\left(a^2-b^2\right)^{3/2}\tanh^{-1}\left(\right.}{ab^3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((4*c)/a + (4*d*x)/a + (8*(a^2 - b^2)^(3
/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3) - (4*a^2*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^3 + (6*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]])/b + (4*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b^3
- (6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b + 1/(b*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2) - 1/(b*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (4*
a*Tan[c + d*x])/b^2))/(4*d*(a + b*Sec[c + d*x]))
```

fricas [A] time = 0.93, size = 444, normalized size = 3.52

$$\left[\frac{4\,b^3dx\cos\left(dx+c\right)^2-2\left(a^2-b^2\right)^{\frac{3}{2}}\cos\left(dx+c\right)^2\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^3*d*x*cos(d*x + c)^2 - 2*(a^2 - b^2)^(3/2)*cos(d*x + c)^2*log((2*
a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(
d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d
*x + c) + b^2)) + (2*a^3 - 3*a*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) -
(2*a^3 - 3*a*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a^2*b*cos(d*
x + c) - a*b^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^2), 1/4*(4*b^3*d*x*cos(
d*x + c)^2 - 4*(a^2 - b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^3 - 3*a*b^
2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a*b^2)*cos(d*x + c)^2*
log(-sin(d*x + c) + 1) - 2*(2*a^2*b*cos(d*x + c) - a*b^2)*sin(d*x + c))/(a*
b^3*d*cos(d*x + c)^2)]
```

giac [B] time = 1.48, size = 476, normalized size = 3.78

$$-\frac{\frac{2\left(\left(a^2+ab-b^2\right)\sqrt{-a^2+b^2}\,|a||-a+b||b|+\left(a^3b+a^2b^2-ab^3-2\,b^4\right)\sqrt{-a^2+b^2}\,|-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{b^4+\sqrt{b^8+\left(ab^3+b^4\right)\left(ab^3-b^4\right)}}{ab^3-b^4}}}\right)\right)}{\left(ab^2-b^3\right)a^2b^2+\left(ab^4-b^5\right)|a||b|}+\frac{2\left(a^4b-2\,a\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*((a^2 + a*b - b^2)*sqrt(-a^2 + b^2)*abs(a)*abs(-a + b)*abs(b) + (a^
3*b + a^2*b^2 - a*b^3 - 2*b^4)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*
(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^4 + sqrt(b^8 + (
a*b^3 + b^4)*(a*b^3 - b^4)))/(a*b^3 - b^4))))/((a*b^2 - b^3)*a^2*b^2 + (a*b
^4 - b^5)*abs(a)*abs(b)) + 2*(a^4*b - 2*a^2*b^3 - a*b^4 + 2*b^5 - a^3*abs(a
)*abs(b) + 2*a*b^2*abs(a)*abs(b) - b^3*abs(a)*abs(b))*(pi*floor(1/2*(d*x +
c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^4 - sqrt(b^8 + (a*b^3 +
 b^4)*(a*b^3 - b^4)))/(a*b^3 - b^4))))/(a^2*b^4 - b^4*abs(a)*abs(b)) - (2*a
^2 - 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + (2*a^2 - 3*b^2)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(1/
2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 - 1)^2*b^2))/d
```

maple [B] time = 0.44, size = 374, normalized size = 2.97

$$-\frac{2a^3\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^3\sqrt{(a-b)\,(a+b)}}+\frac{4a\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{db\sqrt{(a-b)\,(a+b)}}-\frac{2b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{da\sqrt{(a-b)\,(a+b)}}+\frac{1}{2db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d/b^3*a^3/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a
+b))^(1/2))+4/d/b*a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((
a-b)*(a+b))^(1/2))-2/d*b/a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(
a-b)/((a-b)*(a+b))^(1/2))+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^2+1/d/b^2/(tan(1/2
*d*x+1/2*c)-1)*a+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)-1/d/b^3*ln(tan(1/2*d*x+1/2*
c)-1)*a^2+3/2/d/b*ln(tan(1/2*d*x+1/2*c)-1)-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2
+1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*a+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)+1/d/b^3*ln
(tan(1/2*d*x+1/2*c)+1)*a^2-3/2/d/b*ln(tan(1/2*d*x+1/2*c)+1)+2/d/a*arctan(ta
n(1/2*d*x+1/2*c))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [**B**] time = 3.29, size = 6062, normalized size = 48.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*3i)/
(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*
sin(c/2 + (d*x)/2)^2)) + (atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*
sin(c/2 + (d*x)/2)^4*3i)/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4
- 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (cos(c/2 + (d*x)/2)*sin(c
/2 + (d*x)/2)^3)/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (cos(c/2 + (d*x)/2)^3*sin(c/2 + (
d*x)/2))/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (2*atan((4*a^13*sin(c/2 + (d*x)/2) + 4*a^
12*b*sin(c/2 + (d*x)/2) + 12*a^2*b^11*sin(c/2 + (d*x)/2) + 12*a^3*b^10*sin(
c/2 + (d*x)/2) + 15*a^4*b^9*sin(c/2 + (d*x)/2) + 15*a^5*b^8*sin(c/2 + (d*x)
/2) - 59*a^6*b^7*sin(c/2 + (d*x)/2) - 59*a^7*b^6*sin(c/2 + (d*x)/2) + 57*a^
8*b^5*sin(c/2 + (d*x)/2) + 57*a^9*b^4*sin(c/2 + (d*x)/2) - 24*a^10*b^3*sin(
c/2 + (d*x)/2) - 24*a^11*b^2*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2)*(12*
a*b^11 + 4*a^11*b + 4*a^12 + 12*a^2*b^10 + 15*a^3*b^9 + 15*a^4*b^8 - 59*a^5
*b^7 - 59*a^6*b^6 + 57*a^7*b^5 + 57*a^8*b^4 - 24*a^9*b^3 - 24*a^10*b^2)))*c
os(c/2 + (d*x)/2)^4)/(a*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*
cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (2*atan((4*a^13*sin(c/2 + (d*
x)/2) + 4*a^12*b*sin(c/2 + (d*x)/2) + 12*a^2*b^11*sin(c/2 + (d*x)/2) + 12*a
^3*b^10*sin(c/2 + (d*x)/2) + 15*a^4*b^9*sin(c/2 + (d*x)/2) + 15*a^5*b^8*sin
(c/2 + (d*x)/2) - 59*a^6*b^7*sin(c/2 + (d*x)/2) - 59*a^7*b^6*sin(c/2 + (d*x
)/2) + 57*a^8*b^5*sin(c/2 + (d*x)/2) + 57*a^9*b^4*sin(c/2 + (d*x)/2) - 24*a
^10*b^3*sin(c/2 + (d*x)/2) - 24*a^11*b^2*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (
d*x)/2)*(12*a*b^11 + 4*a^11*b + 4*a^12 + 12*a^2*b^10 + 15*a^3*b^9 + 15*a^4*
b^8 - 59*a^5*b^7 - 59*a^6*b^6 + 57*a^7*b^5 + 57*a^8*b^4 - 24*a^9*b^3 - 24*a
^10*b^2)))*sin(c/2 + (d*x)/2)^4)/(a*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*
x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (a^2*atan((sin(c/
2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*2i)/(b^3*d*(cos(c
/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (
d*x)/2)^2)) - (a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(c/2
 + (d*x)/2)^4*2i)/(b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*c
```

os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (atan((sin(c/2 + (d*x)/2)*1i)/
cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2*6i)/(b*d*(cos
(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 +
 (d*x)/2)^2)) + (2*a*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3)/(b^2*d*(cos(c
/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (
d*x)/2)^2)) - (2*a*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2))/(b^2*d*(cos(c/2
 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*
x)/2)^2)) - (4*atan((4*a^13*sin(c/2 + (d*x)/2) + 4*a^12*b*sin(c/2 + (d*x)/2
) + 12*a^2*b^11*sin(c/2 + (d*x)/2) + 12*a^3*b^10*sin(c/2 + (d*x)/2) + 15*a^
4*b^9*sin(c/2 + (d*x)/2) + 15*a^5*b^8*sin(c/2 + (d*x)/2) - 59*a^6*b^7*sin(c
/2 + (d*x)/2) - 59*a^7*b^6*sin(c/2 + (d*x)/2) + 57*a^8*b^5*sin(c/2 + (d*x)/
2) + 57*a^9*b^4*sin(c/2 + (d*x)/2) - 24*a^10*b^3*sin(c/2 + (d*x)/2) - 24*a^
11*b^2*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2)*(12*a*b^11 + 4*a^11*b + 4*
a^12 + 12*a^2*b^10 + 15*a^3*b^9 + 15*a^4*b^8 - 59*a^5*b^7 - 59*a^6*b^6 + 57
*a^7*b^5 + 57*a^8*b^4 - 24*a^9*b^3 - 24*a^10*b^2)))*cos(c/2 + (d*x)/2)^2*si
n(c/2 + (d*x)/2)^2)/(a*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*c
os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (a^2*atan((sin(c/2 + (d*x)/2)*
1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2*4i)/(b^3*
d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin
(c/2 + (d*x)/2)^2)) + (atan(((8*a^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b
^4 - 3*a^4*b^2)^(3/2) - 8*a^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2)^(5/2) + 8*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b
^2)^(5/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3
/2) - 26*a^2*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/
2) - 6*a^3*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2)
 + 21*a^4*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2)
+ 9*a^5*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) +
12*a^6*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 2
0*a^7*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 22
*a^2*b^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 14
*a^3*b^12*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 36
*a^4*b^11*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 24
*a^5*b^10*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 47
*a^6*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 79*
a^7*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 49*a
^8*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 65*a^
9*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 24*a^1
0*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 24*a^1
1*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 4*a^12
*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 4*a^13*
b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 8*a*b^2*
sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*a^2*b*sin(
c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) - 8*a*b^8*sin(c/2
+ (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 8*a^8*b*sin(c/2 + (d
x)/2)(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2))*1i)/(a*b*cos(c/2 + (d*x)/
2)*(12*a*b^15 - 24*a^2*b^14 - 33*a^3*b^13 + 90*a^4*b^12 + 22*a^5*b^11 - 134
*a^6*b^10 + 17*a^7*b^9 + 100*a^8*b^8 - 31*a^9*b^7 - 38*a^10*b^6 + 16*a^11*b
^5 + 6*a^12*b^4 - 3*a^13*b^3)))*cos(c/2 + (d*x)/2)^4*((a + b)^3*(a - b)^3)^
(1/2)*2i)/(a*b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2
 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (atan(((8*a^9*sin(c/2 + (d*x)/2)*(a^
6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 8*a^3*sin(c/2 + (d*x)/2)*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2)^(3/2) - 26*a^2*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2)^(3/2) - 6*a^3*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2)^(3/2) + 21*a^4*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 -
3*a^4*b^2)^(3/2) + 9*a^5*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*
a^4*b^2)^(3/2) + 12*a^6*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2)^(3/2) - 20*a^7*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^

```
4*b^2)^(3/2) - 22*a^2*b^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)^(1/2) + 14*a^3*b^12*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)^(1/2) + 36*a^4*b^11*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)^(1/2) + 24*a^5*b^10*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)^(1/2) - 47*a^6*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4
*b^2)^(1/2) - 79*a^7*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2)^(1/2) + 49*a^8*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b
^2)^(1/2) + 65*a^9*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2)^(1/2) - 24*a^10*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2)^(1/2) - 24*a^11*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2)^(1/2) + 4*a^12*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
)^(1/2) + 4*a^13*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)
^(1/2) - 8*a*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/
2) + 8*a^2*b*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) -
 8*a*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 8*a
^8*b*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2))*1i)/(a*b
*cos(c/2 + (d*x)/2)*(12*a*b^15 - 24*a^2*b^14 - 33*a^3*b^13 + 90*a^4*b^12 +
22*a^5*b^11 - 134*a^6*b^10 + 17*a^7*b^9 + 100*a^8*b^8 - 31*a^9*b^7 - 38*a^1
0*b^6 + 16*a^11*b^5 + 6*a^12*b^4 - 3*a^13*b^3)))*sin(c/2 + (d*x)/2)^4*((a +
 b)^3*(a - b)^3)^(1/2)*2i)/(a*b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)
/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (atan(((8*a^9*sin(c
/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 8*a^3*sin(c/2 + (
d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^3*sin(c/2 + (d*x)/2
)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 26*a^2*b^7*sin(c/2 + (d*x)/2)*(a^6
- b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 6*a^3*b^6*sin(c/2 + (d*x)/2)*(a^6 -
b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) + 21*a^4*b^5*sin(c/2 + (d*x)/2)*(a^6 - b
^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) + 9*a^5*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6
 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) + 12*a^6*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6
+ 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 20*a^7*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 22*a^2*b^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 14*a^3*b^12*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 36*a^4*b^11*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 24*a^5*b^10*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 47*a^6*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 79*a^7*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3
*a^2*b^4 - 3*a^4*b^2)^(1/2) + 49*a^8*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2)^(1/2) + 65*a^9*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a
^2*b^4 - 3*a^4*b^2)^(1/2) - 24*a^10*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a
^2*b^4 - 3*a^4*b^2)^(1/2) - 24*a^11*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a
^2*b^4 - 3*a^4*b^2)^(1/2) + 4*a^12*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^
2*b^4 - 3*a^4*b^2)^(1/2) + 4*a^13*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2
*b^4 - 3*a^4*b^2)^(1/2) - 8*a*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2)^(5/2) + 8*a^2*b*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2)^(5/2) - 8*a*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4
*b^2)^(3/2) - 8*a^8*b*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
)^(3/2))*1i)/(a*b*cos(c/2 + (d*x)/2)*(12*a*b^15 - 24*a^2*b^14 - 33*a^3*b^13
 + 90*a^4*b^12 + 22*a^5*b^11 - 134*a^6*b^10 + 17*a^7*b^9 + 100*a^8*b^8 - 31
*a^9*b^7 - 38*a^10*b^6 + 16*a^11*b^5 + 6*a^12*b^4 - 3*a^13*b^3)))*cos(c/2 +
 (d*x)/2)^2*sin(c/2 + (d*x)/2)^2*((a + b)^3*(a - b)^3)^(1/2)*4i)/(a*b^3*d*(
cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/
2 + (d*x)/2)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c)),x)
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.296 $\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}-\frac{x}{a}+\frac{\tanh^{-1}(\sin(c+dx))}{bd}$$

```
[Out] -x/a+arctanh(sin(d*x+c))/b/d-2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a/b/d
```

Rubi [A] time = 0.18, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3894, 4051, 3770, 3919, 3831, 2659, 208}

$$-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}-\frac{x}{a}+\frac{\tanh^{-1}(\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + ArcTanh[Sin[c + d*x]]/(b*d) - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
```

```
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4051

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \int \frac{-1+\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx \\
&= \frac{\int \sec(c+dx)\,dx}{b} + \frac{\int \frac{-b-a\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b} \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b} \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd} \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 115, normalized size = 1.51

$$-\frac{-2\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\right.}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*c + b*d*x - 2*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2]])/Sqrt
[a^2 - b^2]] + a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - a*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]])/(a*b*d))
```

fricas [A] time = 0.55, size = 253, normalized size = 3.33

$$\left[-\frac{2\,bdx - a\log\left(\sin\left(dx+c\right)+1\right) + a\log\left(-\sin\left(dx+c\right)+1\right) - \sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{}}{a^2\cos(dx+c)^2+2}\right.}{2\,abd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*b*d*x - a*log(sin(d*x + c) + 1) + a*log(-sin(d*x + c) + 1) - sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(
```

```
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + b^2)))/(a*b*d), -1/2*(2*b*d*x - a*log(sin(d*x +
 c) + 1) + a*log(-sin(d*x + c) + 1) + 2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/(a*b*d)]
```

giac [B] time = 1.47, size = 140, normalized size = 1.84

$$-\frac{\frac{dx+c}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)\left(a^2-b^2\right)}{\sqrt{-a^2+b^2}\,ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/b + log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/b + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + ar
ctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*
(a^2 - b^2)/(sqrt(-a^2 + b^2)*a*b))/d
```

maple [B] time = 0.42, size = 153, normalized size = 2.01

$$-\frac{2a\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{db\sqrt{(a-b)\,(a+b)}}+\frac{2b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{da\sqrt{(a-b)\,(a+b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{db}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{db}-\frac{2\,\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d/b*a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))
^(1/2))+2/d*b/a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))-1/d/b*ln(tan(1/2*d*x+1/2*c)-1)+1/d/b*ln(tan(1/2*d*x+1/2*c)+1
)-2/d/a*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.63, size = 121, normalized size = 1.59

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}-\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}-\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a+b)}\right)\sqrt{a^2-b^2}}{a\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) - (2*atan(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) - (2*atanh((sin(c/2 + (d*x)/2)*(a^2 - b
^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a + b)))*(a^2 - b^2)^(1/2))/(a*b*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.297 $\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{a\cot(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a^2-b^2}\tan\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)}{ad\left(a^2-b^2\right)^{3/2}}-\frac{x}{a}$$

```
[Out] -x/a-2*b^3*arctanh((a^2-b^2)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b))/a/(a^2-b^2)^(3
/2)/d-a*cot(d*x+c)/(a^2-b^2)/d+b*csc(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.24, antiderivative size = 135, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3898, 2902, 2606, 8, 3473, 2735, 2659, 208}

$$-\frac{a\cot(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}+\frac{b^2x}{a\left(a^2-b^2\right)}-\frac{ax}{a^2-b^2}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*x)/(a^2 - b^2)) + (b^2*x)/(a*(a^2 - b^2)) - (2*b^3*ArcTanh[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d) - (a*Co
t[c + d*x])/((a^2 - b^2)*d) + (b*Csc[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \int \frac{\cos(c+dx)\cot^2(c+dx)}{b+a\cos(c+dx)}\,dx \\
&= \frac{a\int \cot^2(c+dx)\,dx}{a^2-b^2} - \frac{b\int \cot(c+dx)\csc(c+dx)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\cos(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2} \\
&= \frac{b^2x}{a\left(a^2-b^2\right)} - \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} - \frac{a\int 1\,dx}{a^2-b^2} - \frac{b^3\int \frac{1}{b+a\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)} + \frac{b\,\mathrm{Subst}(\int 1\,dx, x, \mathrm{csc}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ax}{a^2-b^2} + \frac{b^2x}{a\left(a^2-b^2\right)} - \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} + \frac{b\csc(c+dx)}{\left(a^2-b^2\right)d} - \frac{\left(2b^3\right)\mathrm{Subst}\left(\int \frac{1}{a+b+(-a+b)x}\right.}{a\left(a^2-\right.} \\
&= -\frac{ax}{a^2-b^2} + \frac{b^2x}{a\left(a^2-b^2\right)} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}d} - \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} + \frac{b\csc(c+}{\left(a^2-b^2\right.}
\end{aligned}
$$

Mathematica [A] time = 0.42, size = 147, normalized size = 1.39

$$
-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{a^2-b^2}\left(\left(a^2-b^2\right)(c+dx)\sin(c+dx)+a^2\cos(c+dx)-ab\right)-2b^3\sin(c+\right.}{2ad(a-b)(a+b)\sqrt{a^2-b^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/2*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-2*b^3*ArcTanh[((-a + b)*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]]*Sin[c + d*x] + Sqrt[a^2 - b^2]*(-(a*b) + a^2*Cos[
c + d*x] + (a^2 - b^2)*(c + d*x)*Sin[c + d*x])))/(a*(a - b)*(a + b)*Sqrt[a^
2 - b^2]*d)
```

fricas [A] time = 0.51, size = 362, normalized size = 3.42

$$\left[-\frac{\sqrt{a^2-b^2}\,b^3\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin\left(dx+c\right)-2\,a^3b+2\,ab^3}{2\left(a^5-2\,a^3b^2+ab^4\right)d\sin\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - b^2)*b^3*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)
/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*a^3*b +
2*a*b^3 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*x*sin(d*x + c) + 2*(a^4 - a^2*b^2)*co
s(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*sin(d*x + c)), -(sqrt(-a^2 + b^2)*
b^3*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)
))*sin(d*x + c) - a^3*b + a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*d*x*sin(d*x + c)
+ (a^4 - a^2*b^2)*cos(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*sin(d*x + c))]
```

giac [B] time = 0.45, size = 582, normalized size = 5.49

$$-\frac{\frac{2\left(a^5-a^4b-2\,a^3b^2+3\,a^2b^3+ab^4-2\,b^5-a^2|-a^3+ab^2|+ab|-a^3+ab^2|+b^2|-a^3+ab^2|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{a^2b-b^3+\sqrt{\left(a^3+a^2b-ab^2-b^3\right)\left(a^3-a^2b-ab^2+b^3\right)+(a^2}}{a^3-a^2b-ab^2+b^3}}}\right.\right.}{a^2b|-a^3+ab^2|-b^3|-a^3+ab^2|+\left(a^3-ab^2\right)^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^5 - a^4*b - 2*a^3*b^2 + 3*a^2*b^3 + a*b^4 - 2*b^5 - a^2*abs(-a^3
 + a*b^2) + a*b*abs(-a^3 + a*b^2) + b^2*abs(-a^3 + a*b^2))*(pi*floor(1/2*(d
*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^2*b - b^3 + sqrt((
a^3 + a^2*b - a*b^2 - b^3)*(a^3 - a^2*b - a*b^2 + b^3) + (a^2*b - b^3)^2))/
(a^3 - a^2*b - a*b^2 + b^3))))/(a^2*b*abs(-a^3 + a*b^2) - b^3*abs(-a^3 + a*
b^2) + (a^3 - a*b^2)^2) + 2*((a^2 - a*b - b^2)*sqrt(-a^2 + b^2)*abs(-a^3 +
a*b^2)*abs(-a + b) + (a^5 - a^4*b - 2*a^3*b^2 + 3*a^2*b^3 + a*b^4 - 2*b^5)*
sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(ta
n(1/2*d*x + 1/2*c)/sqrt(-(a^2*b - b^3 - sqrt((a^3 + a^2*b - a*b^2 - b^3)*(a
^3 - a^2*b - a*b^2 + b^3) + (a^2*b - b^3)^2))/(a^3 - a^2*b - a*b^2 + b^3)))
)/((a^3 - a*b^2)^2*(a^2 - 2*a*b + b^2) - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5
)*abs(-a^3 + a*b^2)) - tan(1/2*d*x + 1/2*c)/(a - b) + 1/((a + b)*tan(1/2*d*
x + 1/2*c)))/d
```

maple [A] time = 0.62, size = 123, normalized size = 1.16

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\left(a-b\right)}-\frac{2b^3\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\left(a-b\right)\left(a+b\right)a\sqrt{\left(a-b\right)\left(a+b\right)}}-\frac{1}{2d\left(a+b\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/2/d/(a-b)*tan(1/2*d*x+1/2*c)-2/d/(a-b)/(a+b)*b^3/a/((a-b)*(a+b))^(1/2)*ar
ctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/2/d/(a+b)/tan(1/2*d*x
+1/2*c)-2/d/a*arctan(tan(1/2*d*x+1/2*c))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [**B**] time = 3.94, size = 1002, normalized size = 9.45

$$-\frac{1\mathrm{i}\cos(c+d\,x)\,a^6-a^5\,b\,1\mathrm{i}-2\mathrm{i}\cos(c+d\,x)\,a^4\,b^2+a^3\,b^3\,2\mathrm{i}+1\mathrm{i}\cos(c+d\,x)\,a^2\,b^4-a\,b^5\,1\mathrm{i}}{1\mathrm{i}\,d\sin(c+d\,x)\,a^7-3\mathrm{i}\,d\sin(c+d\,x)\,a^5\,b^2+3\mathrm{i}\,d\sin(c+d\,x)\,a^3\,b^4-1\mathrm{i}\,d\sin(c+d\,x)\,a\,b^6}+\frac{-a^6\,\mathrm{atan}\left(\frac{\sin(}{\cos(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (b^6*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*2i - a^6*atan(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2))*2i - a^2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*6i + a^4*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*6i + b^3*
atanh((2*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) -
 a^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 2*b^13
*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 9*a^2*b^11*
sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 3*a^3*b^10*s
in(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 18*a^4*b^9*si
n(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 12*a^5*b^8*sin
(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 21*a^6*b^7*sin(
c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 19*a^7*b^6*sin(c
/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 15*a^8*b^5*sin(c/
2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 15*a^9*b^4*sin(c/2
 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 6*a^10*b^3*sin(c/2
+ (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 6*a^11*b^2*sin(c/2 +
 (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + a^12*b*sin(c/2 + (d*x
)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a^16 -
 3*a^2*b^14 + 18*a^4*b^12 - 46*a^6*b^10 + 65*a^8*b^8 - 55*a^10*b^6 + 28*a^1
2*b^4 - 8*a^14*b^2)))*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)*2i)/(a^7*d*
1i + a^3*b^4*d*3i - a^5*b^2*d*3i - a*b^6*d*1i) - (a^6*cos(c + d*x)*1i - a*b
^5*1i - a^5*b*1i + a^3*b^3*2i + a^2*b^4*cos(c + d*x)*1i - a^4*b^2*cos(c + d
*x)*2i)/(a^7*d*sin(c + d*x)*1i - a*b^6*d*sin(c + d*x)*1i + a^3*b^4*d*sin(c
+ d*x)*3i - a^5*b^2*d*sin(c + d*x)*3i)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.298 $\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=177

$$-\frac{a\cot^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a\left(a^2-2b^2\right)\cot(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{b\csc^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(a^2-2b^2\right)\csc(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a^2-b^2}\tan\left(\frac{1}{2}(c+dx\right.}{a+b}\right.}{ad\left(a^2-b^2\right)^{5/2}}$$

```
[Out] x/a-2*b^5*arctanh((a^2-b^2)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b))/a/(a^2-b^2)^(5/
2)/d+a*(a^2-2*b^2)*cot(d*x+c)/(a^2-b^2)^2/d-1/3*a*cot(d*x+c)^3/(a^2-b^2)/d-
b*(a^2-2*b^2)*csc(d*x+c)/(a^2-b^2)^2/d+1/3*b*csc(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A] time = 0.39, antiderivative size = 256, normalized size of antiderivative = 1.45, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3898, 2902, 2606, 3473, 8, 2735, 2659, 208}

$$-\frac{a\cot^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{ab^2\cot(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{a\cot(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\csc^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{b^3\csc(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}+\frac{b^4x}{a\left(a^2-b^2\right)^2}-\frac{}{\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (b^4*x)/(a*(a^2 - b^2)^2) + (a*x)/(a^2 - b^2)
- (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(5
/2)*(a + b)^(5/2)*d) - (a*b^2*Cot[c + d*x])/((a^2 - b^2)^2*d) + (a*Cot[c +
d*x])/((a^2 - b^2)*d) - (a*Cot[c + d*x]^3)/(3*(a^2 - b^2)*d) + (b^3*Csc[c +
 d*x])/((a^2 - b^2)^2*d) - (b*Csc[c + d*x])/((a^2 - b^2)*d) + (b*Csc[c + d*
x]^3)/(3*(a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\cot^4(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= \frac{a\int \cot^4(c+dx)\,dx}{a^2-b^2} - \frac{b\int \cot^3(c+dx)\csc(c+dx)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\cos(c+dx)\cot^2(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{a\cot^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\left(ab^2\right)\int \cot^2(c+dx)\,dx}{\left(a^2-b^2\right)^2} - \frac{b^3\int \cot(c+dx)\csc(c+dx)\,dx}{\left(a^2-b^2\right)^2} + \frac{b^4\int}{(}$$

$$= \frac{b^4x}{a\left(a^2-b^2\right)^2} - \frac{ab^2\cot(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} - \frac{a\cot^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{b\csc(c+dx)}{\left(a^2-b^2\right)d} + \frac{b}{3}$$

$$= -\frac{ab^2x}{\left(a^2-b^2\right)^2} + \frac{b^4x}{a\left(a^2-b^2\right)^2} + \frac{ax}{a^2-b^2} - \frac{ab^2\cot(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} - \frac{a\cot^3(c+}{3\left(a^2-b\right.}$$

$$= -\frac{ab^2x}{\left(a^2-b^2\right)^2} + \frac{b^4x}{a\left(a^2-b^2\right)^2} + \frac{ax}{a^2-b^2} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{ab^2\cot(c+}{\left(a^2-b^2\right)^2}$$

Mathematica [B] time = 6.20, size = 416, normalized size = 2.35

$$\frac{2b^5\sec(c+dx)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}\left(b^2-a^2\right)^2(a+b\sec(c+dx))} + \frac{(c+dx)\sec(c+dx)(a\cos(c+dx)+b)}{ad(a+b\sec(c+dx))} + \frac{\csc\left(\frac{1}{2}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*d*(a + b*Sec[c + d*x])) +
(2*b^5*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c +
d*x])*Sec[c + d*x])/(a*Sqrt[a^2 - b^2]*(-a^2 + b^2)^2*d*(a + b*Sec[c + d*x]
)) + ((8*a*Cos[(c + d*x)/2] + 11*b*Cos[(c + d*x)/2])*(b + a*Cos[c + d*x])*C
sc[(c + d*x)/2]*Sec[c + d*x])/(12*(a + b)^2*d*(a + b*Sec[c + d*x])) - ((b +
 a*Cos[c + d*x])*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*Sec[c + d*x])/(24*(a +
 b)*d*(a + b*Sec[c + d*x])) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]*Sec[c
+ d*x]*(-8*a*Sin[(c + d*x)/2] + 11*b*Sin[(c + d*x)/2]))/(12*(-a + b)^2*d*(a
 + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]
*Tan[(c + d*x)/2])/(24*(-a + b)*d*(a + b*Sec[c + d*x]))
```

fricas [B] time = 0.53, size = 742, normalized size = 4.19

$$\left[\frac{4\,a^5b - 14\,a^3b^3 + 10\,ab^5 + 2\left(4\,a^6 - 11\,a^4b^2 + 7\,a^2b^4\right)\cos\left(dx+c\right)^3 + 3\left(b^5\cos\left(dx+c\right)^2 - b^5\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\,c\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(4*a^5*b - 14*a^3*b^3 + 10*a*b^5 + 2*(4*a^6 - 11*a^4*b^2 + 7*a^2*b^4)*
cos(d*x + c)^3 + 3*(b^5*cos(d*x + c)^2 - b^5)*sqrt(a^2 - b^2)*log((2*a*b*co
s(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x +
c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + b^2))*sin(d*x + c) - 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 - 6
*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c) + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*d*x*cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*x)*sin
(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2 - (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)*sin(d*x + c)), 1/3*(2*a^5*b - 7*a^3*b^3
 + 5*a*b^5 + (4*a^6 - 11*a^4*b^2 + 7*a^2*b^4)*cos(d*x + c)^3 - 3*(b^5*cos(d
*x + c)^2 - b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^5*b - 3*a^3*b^3 + 2*a*
b^5)*cos(d*x + c)^2 - 3*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c) + 3*((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*x*cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*d*x)*sin(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d
*cos(d*x + c)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)*sin(d*x + c))]
```

giac [B] time = 0.61, size = 1073, normalized size = 6.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(24*((a^4 - a^3*b - 2*a^2*b^2 + 2*a*b^3 + b^4)*sqrt(-a^2 + b^2)*abs(a
^5 - 2*a^3*b^2 + a*b^4)*abs(-a + b) - (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3
+ 6*a^5*b^4 - 7*a^4*b^5 - 4*a^3*b^6 + 6*a^2*b^7 + a*b^8 - 2*b^9)*sqrt(-a^2
+ b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x
+ 1/2*c)/sqrt(-(a^4*b - 2*a^2*b^3 + b^5 + sqrt((a^5 + a^4*b - 2*a^3*b^2 - 2
*a^2*b^3 + a*b^4 + b^5)*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)
 + (a^4*b - 2*a^2*b^3 + b^5)^2))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b
^4 - b^5))))/((a^5 - 2*a^3*b^2 + a*b^4)^2*(a^2 - 2*a*b + b^2) + (a^6*b - 2*
a^5*b^2 - a^4*b^3 + 4*a^3*b^4 - a^2*b^5 - 2*a*b^6 + b^7)*abs(a^5 - 2*a^3*b^
2 + a*b^4)) + 24*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 7*a^4*b
^5 - 4*a^3*b^6 + 6*a^2*b^7 + a*b^8 - 2*b^9 + a^4*abs(a^5 - 2*a^3*b^2 + a*b^
4) - a^3*b*abs(a^5 - 2*a^3*b^2 + a*b^4) - 2*a^2*b^2*abs(a^5 - 2*a^3*b^2 + a
*b^4) + 2*a*b^3*abs(a^5 - 2*a^3*b^2 + a*b^4) + b^4*abs(a^5 - 2*a^3*b^2 + a*
```

```
b^4))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(
-(a^4*b - 2*a^2*b^3 + b^5 - sqrt((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b
^4 + b^5)*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5) + (a^4*b - 2*
a^2*b^3 + b^5)^2))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5))))/(
a^4*b*abs(a^5 - 2*a^3*b^2 + a*b^4) - 2*a^2*b^3*abs(a^5 - 2*a^3*b^2 + a*b^4)
 + b^5*abs(a^5 - 2*a^3*b^2 + a*b^4) - (a^5 - 2*a^3*b^2 + a*b^4)^2) - (a^2*t
an(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1/2*d*x + 1/
2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c) + 36*a*b*tan(1/2*d*x + 1/2*c) - 21*b^2
*tan(1/2*d*x + 1/2*c))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (15*a*tan(1/2*d*x
+ 1/2*c)^2 + 21*b*tan(1/2*d*x + 1/2*c)^2 - a - b)/((a^2 + 2*a*b + b^2)*tan(
1/2*d*x + 1/2*c)^3))/d
```

maple [A] time = 0.70, size = 238, normalized size = 1.34

$$\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,(a-b)^2}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{24d\,(a-b)^2}-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,(a-b)^2}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8d\,(a-b)^2}-\frac{2b^5\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,(a+b)^2\,(a-b)^2\,a\sqrt{(a-b)\,(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/24/d/(a-b)^2*a*tan(1/2*d*x+1/2*c)^3-1/24/d/(a-b)^2*tan(1/2*d*x+1/2*c)^3*b
-5/8/d/(a-b)^2*a*tan(1/2*d*x+1/2*c)+7/8/d/(a-b)^2*tan(1/2*d*x+1/2*c)*b-2/d/
(a+b)^2/(a-b)^2*b^5/a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/
((a-b)*(a+b))^(1/2))-1/24/d/(a+b)/tan(1/2*d*x+1/2*c)^3+5/8/d/(a+b)^2/tan(1/
2*d*x+1/2*c)*a+7/8/d/(a+b)^2/tan(1/2*d*x+1/2*c)*b+2/d/a*arctan(tan(1/2*d*x+
1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 11.11, size = 3859, normalized size = 21.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

```
[Out] (a^10*((cos(3*c + 3*d*x)*4i)/3 - sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2))*6i + atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3
*d*x)*2i) + a*((b^9*8i)/3 - b^9*cos(2*c + 2*d*x)*4i) - a^7*((b^3*14i)/3 - b
^3*cos(2*c + 2*d*x)*10i) + a^5*(b^5*10i - b^5*cos(2*c + 2*d*x)*18i) - a^3*(
(b^7*26i)/3 - b^7*cos(2*c + 2*d*x)*14i) + a^9*((b*2i)/3 - b*cos(2*c + 2*d*x
)*2i) + a^8*(b^2*cos(c + d*x)*1i - (b^2*cos(3*c + 3*d*x)*19i)/3 + b^2*sin(c
 + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*30i - b^2*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*10i) - a^2*(b^8*cos(c + d*x
)*1i - (b^8*cos(3*c + 3*d*x)*7i)/3 + b^8*sin(c + d*x)*atan(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2))*30i - b^8*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
```

)*sin(3*c + 3*d*x)*10i) - a^6*(b^4*cos(c + d*x)*3i - b^4*cos(3*c + 3*d*x)*1
1i + b^4*sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*60i - b^4
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*20i) + a^4*(b
^6*cos(c + d*x)*3i - (b^6*cos(3*c + 3*d*x)*25i)/3 + b^6*sin(c + d*x)*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*60i - b^6*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*20i) + b^10*sin(c + d*x)*atan(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2))*6i - b^10*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*sin(3*c + 3*d*x)*2i + b^5*atanh((2*b^11*sin(c/2 + (d*x)/2)*(a^10 - b
^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(3/2) - a^21*sin(c/2
 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)
^(1/2) + 2*b^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 +
10*a^6*b^4 - 5*a^8*b^2)^(1/2) + a^20*b*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*
a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 15*a^2*b^19*sin(c/2
+ (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^
(1/2) + 5*a^3*b^18*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6
 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 55*a^4*b^17*sin(c/2 + (d*x)/2)*(a^10 - b
^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 35*a^5*b^16*
sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a
^8*b^2)^(1/2) - 130*a^6*b^15*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 -
10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 110*a^7*b^14*sin(c/2 + (d*x)/2
)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 2
15*a^8*b^13*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a
^6*b^4 - 5*a^8*b^2)^(1/2) - 205*a^9*b^12*sin(c/2 + (d*x)/2)*(a^10 - b^10 +
5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 253*a^10*b^11*sin(
c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b
^2)^(1/2) + 251*a^11*b^10*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*
a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 210*a^12*b^9*sin(c/2 + (d*x)/2)*(
a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 210*
a^13*b^8*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*
b^4 - 5*a^8*b^2)^(1/2) - 120*a^14*b^7*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a
^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 120*a^15*b^6*sin(c/2
+ (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^
(1/2) + 45*a^16*b^5*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^
6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 45*a^17*b^4*sin(c/2 + (d*x)/2)*(a^10 -
b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 10*a^18*b^3
*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*
a^8*b^2)^(1/2) + 10*a^19*b^2*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 -
10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a^26 + 5*a
^2*b^24 - 50*a^4*b^22 + 230*a^6*b^20 - 645*a^8*b^18 + 1231*a^10*b^16 - 1688
*a^12*b^14 + 1708*a^14*b^12 - 1286*a^16*b^10 + 715*a^18*b^8 - 286*a^20*b^6
+ 78*a^22*b^4 - 13*a^24*b^2)))*sin(3*c + 3*d*x)*(a^10 - b^10 + 5*a^2*b^8 -
10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)*2i - b^5*atanh((2*b^11*sin(c/2 +
 (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(
3/2) - a^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a
^6*b^4 - 5*a^8*b^2)^(1/2) + 2*b^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*
b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + a^20*b*sin(c/2 + (d*x)/2
)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 1
5*a^2*b^19*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^
6*b^4 - 5*a^8*b^2)^(1/2) + 5*a^3*b^18*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a
^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 55*a^4*b^17*sin(c/2 +
 (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(
1/2) - 35*a^5*b^16*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6
 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 130*a^6*b^15*sin(c/2 + (d*x)/2)*(a^10 -
b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 110*a^7*b^1
4*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5
*a^8*b^2)^(1/2) + 215*a^8*b^13*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8
- 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 205*a^9*b^12*sin(c/2 + (d*x)
/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) -
 253*a^10*b^11*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 1

```
0*a^6*b^4 - 5*a^8*b^2)^(1/2) + 251*a^11*b^10*sin(c/2 + (d*x)/2)*(a^10 - b^1
0 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 210*a^12*b^9*s
in(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^
8*b^2)^(1/2) - 210*a^13*b^8*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 1
0*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 120*a^14*b^7*sin(c/2 + (d*x)/2)
*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 12
0*a^15*b^6*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^
6*b^4 - 5*a^8*b^2)^(1/2) + 45*a^16*b^5*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*
a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 45*a^17*b^4*sin(c/2
+ (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^
(1/2) - 10*a^18*b^3*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^
6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 10*a^19*b^2*sin(c/2 + (d*x)/2)*(a^10 -
b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(cos(c/2 + (
d*x)/2)*(a^26 + 5*a^2*b^24 - 50*a^4*b^22 + 230*a^6*b^20 - 645*a^8*b^18 + 12
31*a^10*b^16 - 1688*a^12*b^14 + 1708*a^14*b^12 - 1286*a^16*b^10 + 715*a^18*
b^8 - 286*a^20*b^6 + 78*a^22*b^4 - 13*a^24*b^2)))*sin(c + d*x)*(a^10 - b^10
 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)*6i)/(a^11*d*sin(3
*c + 3*d*x)*1i - a^11*d*sin(c + d*x)*3i + a^3*b^8*d*sin(3*c + 3*d*x)*5i - a
^5*b^6*d*sin(3*c + 3*d*x)*10i + a^7*b^4*d*sin(3*c + 3*d*x)*10i - a^9*b^2*d*
sin(3*c + 3*d*x)*5i + a*b^10*d*sin(c + d*x)*3i - a*b^10*d*sin(3*c + 3*d*x)*
1i - a^3*b^8*d*sin(c + d*x)*15i + a^5*b^6*d*sin(c + d*x)*30i - a^7*b^4*d*si
n(c + d*x)*30i + a^9*b^2*d*sin(c + d*x)*15i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.299 $\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=255

$$\frac{(a^2-b^2)^4}{ab^8d(a+b\sec(c+dx))}+\frac{(a^2-b^2)^3(7a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^8d}-\frac{4a(a^2-2b^2)\sec^3(c+dx)}{3b^5d}+\frac{(3a^2-4b^2)\text{se}}{4b^4d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d+(a^2-b^2)^3*(7*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^8/d-
2*a*(3*a^4-8*a^2*b^2+6*b^4)*sec(d*x+c)/b^7/d+1/2*(5*a^4-12*a^2*b^2+6*b^4)*s
ec(d*x+c)^2/b^6/d-4/3*a*(a^2-2*b^2)*sec(d*x+c)^3/b^5/d+1/4*(3*a^2-4*b^2)*se
c(d*x+c)^4/b^4/d-2/5*a*sec(d*x+c)^5/b^3/d+1/6*sec(d*x+c)^6/b^2/d+(a^2-b^2)^
4/a/b^8/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3885, 894}

$$\frac{(3a^2-4b^2)\sec^4(c+dx)}{4b^4d}-\frac{4a(a^2-2b^2)\sec^3(c+dx)}{3b^5d}+\frac{(-12a^2b^2+5a^4+6b^4)\sec^2(c+dx)}{2b^6d}-\frac{2a(-8a^2b^2+3a^4+6}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + ((a^2 - b^2)^3*(7*a^2 + b^2)*Log[a + b*Sec[c
 + d*x]])/(a^2*b^8*d) - (2*a*(3*a^4 - 8*a^2*b^2 + 6*b^4)*Sec[c + d*x])/(b^7
*d) + ((5*a^4 - 12*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/(2*b^6*d) - (4*a*(a^2 -
 2*b^2)*Sec[c + d*x]^3)/(3*b^5*d) + ((3*a^2 - 4*b^2)*Sec[c + d*x]^4)/(4*b^4
*d) - (2*a*Sec[c + d*x]^5)/(5*b^3*d) + Sec[c + d*x]^6/(6*b^2*d) + (a^2 - b^
2)^4/(a*b^8*d*(a + b*Sec[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^4}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^8d}\\ &= \frac{\text{Subst}\left(\int \left(-2a(3a^4-8a^2b^2+6b^4)+\frac{b^8}{a^2x}+(5a^4-12a^2b^2+6b^4)x-4a(a^2-2b^2)x\right.\right.}{b^8d}\\ &= -\frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2-b^2)^3(7a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^8d}-\frac{2a(3a^4-8a^2b^2}{b}\end{aligned}$$

Mathematica [B] time = 6.31, size = 528, normalized size = 2.07

$$-\frac{(b-a)^4(a+b)^4\sec^2(c+dx)(a\cos(c+dx)+b)}{a^2b^7d(a+b\sec(c+dx))^2}+\frac{4a\left(2b^2-a^2\right)\sec^5(c+dx)(a\cos(c+dx)+b)^2}{3b^5d(a+b\sec(c+dx))^2}+\frac{\left(3a^2-4b^2\right)}{4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^9/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((-a + b)^4*(a + b)^4*(b + a*Cos[c + d*x])*Sec[c + d*x]^2)/(a^2*b^7*d*(a
+ b*Sec[c + d*x])^2)) + ((-7*a^6 + 20*a^4*b^2 - 18*a^2*b^4 + 4*b^6)*(b + a*
Cos[c + d*x])^2*Log[Cos[c + d*x]]*Sec[c + d*x]^2)/(b^8*d*(a + b*Sec[c + d*x
])^2) + ((7*a^8 - 20*a^6*b^2 + 18*a^4*b^4 - 4*a^2*b^6 - b^8)*(b + a*Cos[c +
 d*x])^2*Log[b + a*Cos[c + d*x]]*Sec[c + d*x]^2)/(a^2*b^8*d*(a + b*Sec[c +
d*x])^2) - (2*a*(3*a^4 - 8*a^2*b^2 + 6*b^4)*(b + a*Cos[c + d*x])^2*Sec[c +
d*x]^3)/(b^7*d*(a + b*Sec[c + d*x])^2) + ((5*a^4 - 12*a^2*b^2 + 6*b^4)*(b +
 a*Cos[c + d*x])^2*Sec[c + d*x]^4)/(2*b^6*d*(a + b*Sec[c + d*x])^2) + (4*a*
(-a^2 + 2*b^2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^5)/(3*b^5*d*(a + b*Sec[c
 + d*x])^2) + ((3*a^2 - 4*b^2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^6)/(4*b^
4*d*(a + b*Sec[c + d*x])^2) - (2*a*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^7)/(
5*b^3*d*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^8)/(
6*b^2*d*(a + b*Sec[c + d*x])^2)
```

fricas [A] time = 0.64, size = 423, normalized size = 1.66

$$-\frac{14\,a^3b^6\cos\left(dx+c\right)-10\,a^2b^7+60\left(7\,a^8b-20\,a^6b^3+18\,a^4b^5-4\,a^2b^7+b^9\right)\cos\left(dx+c\right)^6+30\left(7\,a^7b^2-20\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(14*a^3*b^6*cos(d*x + c) - 10*a^2*b^7 + 60*(7*a^8*b - 20*a^6*b^3 + 18
*a^4*b^5 - 4*a^2*b^7 + b^9)*cos(d*x + c)^6 + 30*(7*a^7*b^2 - 20*a^5*b^4 + 1
8*a^3*b^6)*cos(d*x + c)^5 - 10*(7*a^6*b^3 - 20*a^4*b^5 + 18*a^2*b^7)*cos(d*
x + c)^4 + 5*(7*a^5*b^4 - 20*a^3*b^6)*cos(d*x + c)^3 - 3*(7*a^4*b^5 - 20*a^
2*b^7)*cos(d*x + c)^2 - 60*((7*a^9 - 20*a^7*b^2 + 18*a^5*b^4 - 4*a^3*b^6 -
a*b^8)*cos(d*x + c)^7 + (7*a^8*b - 20*a^6*b^3 + 18*a^4*b^5 - 4*a^2*b^7 - b^
9)*cos(d*x + c)^6)*log(a*cos(d*x + c) + b) + 60*((7*a^9 - 20*a^7*b^2 + 18*a
^5*b^4 - 4*a^3*b^6)*cos(d*x + c)^7 + (7*a^8*b - 20*a^6*b^3 + 18*a^4*b^5 - 4
*a^2*b^7)*cos(d*x + c)^6)*log(-cos(d*x + c)))/(a^3*b^8*d*cos(d*x + c)^7 + a
^2*b^9*d*cos(d*x + c)^6)
```

giac [B] time = 21.61, size = 1696, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*(7*a^9 - 7*a^8*b - 20*a^7*b^2 + 20*a^6*b^3 + 18*a^5*b^4 - 18*a^4*b
^5 - 4*a^3*b^6 + 4*a^2*b^7 - a*b^8 + b^9)*log(abs(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^8
 - a^2*b^9) + 60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 -
 60*(7*a^6 - 20*a^4*b^2 + 18*a^2*b^4 - 4*b^6)*log(abs(-(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 1))/b^8 - 60*(7*a^9 + 9*a^8*b - 18*a^7*b^2 - 26*a^6*b^3
 + 12*a^5*b^4 + 24*a^4*b^5 + 2*a^3*b^6 - 6*a^2*b^7 - 3*a*b^8 - b^9 + 7*a^9*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7*a^8*b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 20*a^7*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a^6*b^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^5*b^4*(cos(d*x + c) - 1)/(cos(
```

```
d*x + c) + 1) - 18*a^4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^
6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a^2*b^7*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - a*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b^9*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^8) + (1029*a^6 - 720
*a^5*b - 2940*a^4*b^2 + 1760*a^3*b^3 + 2646*a^2*b^4 - 1168*a*b^5 - 588*b^6
+ 6174*a^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3600*a^5*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - 18240*a^4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 9120*a^3*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16956*a^2*b^4*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 6288*a*b^5*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 3888*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 15435*a^6*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 7200*a^5*b*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 46500*a^4*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 18240*a^3*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 44730*a^2*b^
4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12960*a*b^5*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 - 10740*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 20580*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 7200*a^5*b*(cos
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 62400*a^4*b^2*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 17600*a^3*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 + 60840*a^2*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 11680*a*b^5
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 15520*b^6*(cos(d*x + c) - 1)^3
/(cos(d*x + c) + 1)^3 + 15435*a^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 - 3600*a^5*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 46500*a^4*b^2*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 8160*a^3*b^3*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 + 44730*a^2*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1
)^4 - 4560*a*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 10740*b^6*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 6174*a^6*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 - 720*a^5*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 182
40*a^4*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1440*a^3*b^3*(cos(d*
x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 16956*a^2*b^4*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 - 720*a*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 3
888*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1029*a^6*(cos(d*x + c)
- 1)^6/(cos(d*x + c) + 1)^6 - 2940*a^4*b^2*(cos(d*x + c) - 1)^6/(cos(d*x +
c) + 1)^6 + 2646*a^2*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 588*b^
6*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/(b^8*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 1)^6))/d
```

maple [B] time = 0.56, size = 498, normalized size = 1.95

$$\frac{3a^2}{4d\,b^4\cos\left(dx+c\right)^4}-\frac{6a^2}{d\,b^4\cos\left(dx+c\right)^2}-\frac{a^6}{d\,b^7\left(b+a\cos\left(dx+c\right)\right)}+\frac{4a^4}{d\,b^5\left(b+a\cos\left(dx+c\right)\right)}-\frac{6a^2}{d\,b^3\left(b+a\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 3/4/d/b^4/cos(d*x+c)^4*a^2-6/d/b^4/cos(d*x+c)^2*a^2-1/d*a^6/b^7/(b+a*cos(d*
x+c))+4/d*a^4/b^5/(b+a*cos(d*x+c))-6/d*a^2/b^3/(b+a*cos(d*x+c))-1/d/a^2*b/(
b+a*cos(d*x+c))+7/d/b^8*a^6*ln(b+a*cos(d*x+c))-20/d/b^6*a^4*ln(b+a*cos(d*x+
c))+18/d/b^4*a^2*ln(b+a*cos(d*x+c))-7/d/b^8*ln(cos(d*x+c))*a^6-4/3/d*a^3/b^
5/cos(d*x+c)^3+8/3/d*a/b^3/cos(d*x+c)^3-6/d*a^5/b^7/cos(d*x+c)+16/d*a^3/b^5
/cos(d*x+c)+20/d/b^6*ln(cos(d*x+c))*a^4-18/d/b^4*ln(cos(d*x+c))*a^2-2/5/d/b
^3*a/cos(d*x+c)^5+5/2/d/b^6/cos(d*x+c)^2*a^4-4/d/b^2*ln(b+a*cos(d*x+c))-1/d
/a^2*ln(b+a*cos(d*x+c))-1/d/b^2/cos(d*x+c)^4+3/d/b^2/cos(d*x+c)^2+4/d/b^2*l
n(cos(d*x+c))+1/6/d/b^2/cos(d*x+c)^6+4/d/b/(b+a*cos(d*x+c))-12/d*a/b^3/cos(
d*x+c)
```

maxima [A] time = 0.43, size = 321, normalized size = 1.26

$$-\frac{\frac{14\,a^3b^5\cos(dx+c)-10\,a^2b^6+60\left(7\,a^8-20\,a^6b^2+18\,a^4b^4-4\,a^2b^6+b^8\right)\cos(dx+c)^6+30\left(7\,a^7b-20\,a^5b^3+18\,a^3b^5\right)\cos(dx+c)^5-10\left(7\,a^6b^2-20\,a^4b^4+18\,a^2b^6\right.}{a^3b^7\cos(dx+c)^7+a^2b^8\cos(dx+c)^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*((14*a^3*b^5*cos(d*x + c) - 10*a^2*b^6 + 60*(7*a^8 - 20*a^6*b^2 + 18*
a^4*b^4 - 4*a^2*b^6 + b^8)*cos(d*x + c)^6 + 30*(7*a^7*b - 20*a^5*b^3 + 18*a
^3*b^5)*cos(d*x + c)^5 - 10*(7*a^6*b^2 - 20*a^4*b^4 + 18*a^2*b^6)*cos(d*x +
 c)^4 + 5*(7*a^5*b^3 - 20*a^3*b^5)*cos(d*x + c)^3 - 3*(7*a^4*b^4 - 20*a^2*b
^6)*cos(d*x + c)^2)/(a^3*b^7*cos(d*x + c)^7 + a^2*b^8*cos(d*x + c)^6) + 60*
(7*a^6 - 20*a^4*b^2 + 18*a^2*b^4 - 4*b^6)*log(cos(d*x + c))/b^8 - 60*(7*a^8
 - 20*a^6*b^2 + 18*a^4*b^4 - 4*a^2*b^6 - b^8)*log(a*cos(d*x + c) + b)/(a^2*
b^8))/d
```

mupad [B] time = 4.77, size = 760, normalized size = 2.98

$$\frac{\frac{2\left(-105\,a^7-105\,a^6\,b+265\,a^5\,b^2+265\,a^4\,b^3-191\,a^3\,b^4-191\,a^2\,b^5+15\,a\,b^6+15\,b^7\right)}{15\,a\,b^7}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(-42\,a^7-7\,a^6\,b+113\,a^5\,b^2+13\,a^4\,b^3-95\,a^3\,b^4-5\,a^2}{a\,b^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*(15*a*b^6 - 105*a^6*b - 105*a^7 + 15*b^7 - 191*a^2*b^5 - 191*a^3*b^4 +
265*a^4*b^3 + 265*a^5*b^2))/(15*a*b^7) - (2*tan(c/2 + (d*x)/2)^10*(19*a*b^6
 - 7*a^6*b - 42*a^7 + 6*b^7 - 5*a^2*b^5 - 95*a^3*b^4 + 13*a^4*b^3 + 113*a^5
*b^2))/(a*b^7) - (4*tan(c/2 + (d*x)/2)^6*(7*a*b^6 - 105*a^6*b - 210*a^7 + 3
0*b^7 - 145*a^2*b^5 - 362*a^3*b^4 + 244*a^4*b^3 + 523*a^5*b^2))/(3*a*b^7) +
 (2*tan(c/2 + (d*x)/2)^8*(91*a*b^6 - 105*a^6*b - 315*a^7 + 45*b^7 - 99*a^2*
b^5 - 613*a^3*b^4 + 223*a^4*b^3 + 809*a^5*b^2))/(3*a*b^7) + (2*tan(c/2 + (d
*x)/2)^4*(10*a*b^6 - 350*a^6*b - 525*a^7 + 75*b^7 - 598*a^2*b^5 - 862*a^3*b
^4 + 860*a^4*b^3 + 1290*a^5*b^2))/(5*a*b^7) - (2*tan(c/2 + (d*x)/2)^2*(45*a
*b^6 - 525*a^6*b - 630*a^7 + 90*b^7 - 955*a^2*b^5 - 1067*a^3*b^4 + 1325*a^4
*b^3 + 1555*a^5*b^2))/(15*a*b^7) + (2*tan(c/2 + (d*x)/2)^12*(4*a*b^6 - 7*a^
7 + b^7 - 18*a^3*b^4 + 20*a^5*b^2))/(a*b^7))/(d*(a + b - tan(c/2 + (d*x)/2)
^14*(a - b) - tan(c/2 + (d*x)/2)^2*(7*a + 5*b) + tan(c/2 + (d*x)/2)^12*(7*a
 - 5*b) + tan(c/2 + (d*x)/2)^4*(21*a + 9*b) - tan(c/2 + (d*x)/2)^10*(21*a -
 9*b) - tan(c/2 + (d*x)/2)^6*(35*a + 5*b) + tan(c/2 + (d*x)/2)^8*(35*a - 5*
b))) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2)^2 -
1)*(7*a^6 - 4*b^6 + 18*a^2*b^4 - 20*a^4*b^2))/(b^8*d) + (log(a + b - a*tan(
c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^3*(7*a^2 + b^2))/(a^
2*b^8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{9}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**9/(a + b*sec(c + d*x))**2, x)
```

3.300 $\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$\frac{\left(a^2-b^2\right)^3}{ab^6d(a+b\sec(c+dx))}+\frac{\left(a^2-b^2\right)^2\left(5a^2+b^2\right)\log(a+b\sec(c+dx))}{a^2b^6d}-\frac{2a\left(2a^2-3b^2\right)\sec(c+dx)}{b^5d}+\frac{3\left(a^2-b^2\right)\sec}{2b^4d}$$

```
[Out] ln(cos(d*x+c))/a^2/d+(a^2-b^2)^2*(5*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^6/d-2
*a*(2*a^2-3*b^2)*sec(d*x+c)/b^5/d+3/2*(a^2-b^2)*sec(d*x+c)^2/b^4/d-2/3*a*se
c(d*x+c)^3/b^3/d+1/4*sec(d*x+c)^4/b^2/d+(a^2-b^2)^3/a/b^6/d/(a+b*sec(d*x+c)
)
```

Rubi [A] time = 0.15, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3885, 894}

$$\frac{3\left(a^2-b^2\right)\sec^2(c+dx)}{2b^4d}-\frac{2a\left(2a^2-3b^2\right)\sec(c+dx)}{b^5d}+\frac{\left(a^2-b^2\right)^3}{ab^6d(a+b\sec(c+dx))}+\frac{\left(a^2-b^2\right)^2\left(5a^2+b^2\right)\log(a+b\sec}{a^2b^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((a^2 - b^2)^2*(5*a^2 + b^2)*Log[a + b*Sec[c +
d*x]])/(a^2*b^6*d) - (2*a*(2*a^2 - 3*b^2)*Sec[c + d*x])/(b^5*d) + (3*(a^2 -
 b^2)*Sec[c + d*x]^2)/(2*b^4*d) - (2*a*Sec[c + d*x]^3)/(3*b^3*d) + Sec[c +
d*x]^4/(4*b^2*d) + (a^2 - b^2)^3/(a*b^6*d*(a + b*Sec[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{x(a+x)^2}\,dx,x,b\sec(c+dx)\right)}{b^6d}\\ &= -\frac{\text{Subst}\left(\int \left(2\left(2a^3-3ab^2\right)+\frac{b^6}{a^2x}-3\left(a^2-b^2\right)x+2ax^2-x^3+\frac{\left(a^2-b^2\right)^3}{a(a+x)^2}-\frac{\left(a^2-b^2\right)^2\left(5a^2+\right.}{a^2(a+x)}\right.}{b^6d}\\ &= \frac{\log(\cos(c+dx))}{a^2d}+\frac{\left(a^2-b^2\right)^2\left(5a^2+b^2\right)\log(a+b\sec(c+dx))}{a^2b^6d}-\frac{2a\left(2a^2-3b^2\right)\sec}{b^5d}\end{aligned}$$

Mathematica [B] time = 6.22, size = 383, normalized size = 2.14

$$\frac{(b-a)^3(a+b)^3\sec^2(c+dx)(a\cos(c+dx)+b)}{a^2b^5d(a+b\sec(c+dx))^2}+\frac{2a\left(3b^2-2a^2\right)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{b^5d(a+b\sec(c+dx))^2}+\frac{\left(-5a^4+9a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-a + b)^3*(a + b)^3*(b + a*Cos[c + d*x])*Sec[c + d*x]^2)/(a^2*b^5*d*(a +
b*Sec[c + d*x])^2) + ((-5*a^4 + 9*a^2*b^2 - 3*b^4)*(b + a*Cos[c + d*x])^2*L
og[Cos[c + d*x]]*Sec[c + d*x]^2)/(b^6*d*(a + b*Sec[c + d*x])^2) + ((5*a^6 -
 9*a^4*b^2 + 3*a^2*b^4 + b^6)*(b + a*Cos[c + d*x])^2*Log[b + a*Cos[c + d*x]
]*Sec[c + d*x]^2)/(a^2*b^6*d*(a + b*Sec[c + d*x])^2) + (2*a*(-2*a^2 + 3*b^2
)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^3)/(b^5*d*(a + b*Sec[c + d*x])^2) - (
3*(-a + b)*(a + b)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^4)/(2*b^4*d*(a + b*S
ec[c + d*x])^2) - (2*a*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^5)/(3*b^3*d*(a +
 b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^6)/(4*b^2*d*(a +
 b*Sec[c + d*x])^2)
```

fricas [A] time = 0.58, size = 312, normalized size = 1.74

$$-\frac{5\,a^3b^4\cos(dx+c)-3\,a^2b^5+12\left(5\,a^6b-9\,a^4b^3+3\,a^2b^5-b^7\right)\cos(dx+c)^4+6\left(5\,a^5b^2-9\,a^3b^4\right)\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(5*a^3*b^4*cos(d*x + c) - 3*a^2*b^5 + 12*(5*a^6*b - 9*a^4*b^3 + 3*a^2
*b^5 - b^7)*cos(d*x + c)^4 + 6*(5*a^5*b^2 - 9*a^3*b^4)*cos(d*x + c)^3 - 2*(
5*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c)^2 - 12*((5*a^7 - 9*a^5*b^2 + 3*a^3*b^4
+ a*b^6)*cos(d*x + c)^5 + (5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x +
 c)^4)*log(a*cos(d*x + c) + b) + 12*((5*a^7 - 9*a^5*b^2 + 3*a^3*b^4)*cos(d*
x + c)^5 + (5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^4)*log(-cos(d*x +
 c)))/(a^3*b^6*d*cos(d*x + c)^5 + a^2*b^7*d*cos(d*x + c)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
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5536,[7,3,13]%%%}+%%%{7516192768,[7,2,14]%%%}+%%%{-2147483648,[7,1,15]%%%}+
%%%{268435456,[7,0,16]%%%}+%%%{-805306368,[6,8,8]%%%}+%%%{7516192768,[6,7,9
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111669149696,[5,5,11]%%%}+%%%{185220464640,[5,4,12]%%%}+%%%{-186831077376,[
5,3,13]%%%}+%%%{113816633344,[5,2,14]%%%}+%%%{-38654705664,[5,1,15]%%%}+%%%
{5637144576,[5,0,16]%%%}+%%%{1342177280,[4,8,8]%%%}+%%%{-5368709120,[4,7,9]
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]%%%}+%%%{19327352832,[2,6,10]%%%}+%%%{-33285996544,[2,5,11]%%%}+%%%{805306
3680,[2,4,12]%%%}+%%%{41875931136,[2,3,13]%%%}+%%%{-55834574848,[2,2,14]%%%
}+%%%{28991029248,[2,1,15]%%%}+%%%{-5637144576,[2,0,16]%%%}+%%%{805306368,[
1,8,8]%%%}+%%%{-2147483648,[1,7,9]%%%}+%%%{-2147483648,[1,6,10]%%%}+%%%{128
84901888,[1,5,11]%%%}+%%%{-13421772800,[1,4,12]%%%}+%%%{-2147483648,[1,3,13
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,1,1]%%%}+%%%{1,[7,0,2]%%%}+%%%{-3,[6,2,0]%%%}+%%%{10,[6,1,1]%%%}+%%%{-7,[6
,0,2]%%%}+%%%{1,[5,2,0]%%%}+%%%{-18,[5,1,1]%%%}+%%%{21,[5,0,2]%%%}+%%%{5,[4
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21,[2,0,2]%%%}+%%%{3,[1,2,0]%%%}+%%%{10,[1,1,1]%%%}+%%%{7,[1,0,2]%%%}+%%%{-
1,[0,2,0]%%%}+%%%{-2,[0,1,1]%%%}+%%%{-1,[0,0,2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.49, size = 324, normalized size = 1.81

$$-\frac{a^4}{d\,b^5\left(b+a\cos\left(dx+c\right)\right)}+\frac{3a^2}{d\,b^3\left(b+a\cos\left(dx+c\right)\right)}-\frac{3}{db\left(b+a\cos\left(dx+c\right)\right)}+\frac{b}{d\,a^2\left(b+a\cos\left(dx+c\right)\right)}+\frac{5a^4\ln\left(b+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*a^4/b^5/(b+a*cos(d*x+c))+3/d*a^2/b^3/(b+a*cos(d*x+c))-3/d/b/(b+a*cos(d
*x+c))+1/d/a^2*b/(b+a*cos(d*x+c))+5/d/b^6*a^4*ln(b+a*cos(d*x+c))-9/d/b^4*a^
2*ln(b+a*cos(d*x+c))+3/d/b^2*ln(b+a*cos(d*x+c))+1/d/a^2*ln(b+a*cos(d*x+c))+
3/2/d/b^4/cos(d*x+c)^2*a^2-3/2/d/b^2/cos(d*x+c)^2-5/d/b^6*ln(cos(d*x+c))*a^
4+9/d/b^4*ln(cos(d*x+c))*a^2-3/d/b^2*ln(cos(d*x+c))+1/4/d/b^2/cos(d*x+c)^4-
2/3/d*a/b^3/cos(d*x+c)^3-4/d*a^3/b^5/cos(d*x+c)+6/d*a/b^3/cos(d*x+c)
```

maxima [A] time = 0.51, size = 227, normalized size = 1.27

$$-\frac{\frac{5\,a^3b^3\cos(dx+c)-3\,a^2b^4+12\left(5\,a^6-9\,a^4b^2+3\,a^2b^4-b^6\right)\cos(dx+c)^4+6\left(5\,a^5b-9\,a^3b^3\right)\cos(dx+c)^3-2\left(5\,a^4b^2-9\,a^2b^4\right)\cos(dx+c)^2}{a^3b^5\cos(dx+c)^5+a^2b^6\cos(dx+c)^4}+\frac{12\left(5\,a^4-9\,a^2b^2+3\right.}{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*((5*a^3*b^3*cos(d*x + c) - 3*a^2*b^4 + 12*(5*a^6 - 9*a^4*b^2 + 3*a^2*
b^4 - b^6)*cos(d*x + c)^4 + 6*(5*a^5*b - 9*a^3*b^3)*cos(d*x + c)^3 - 2*(5*a
^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^2)/(a^3*b^5*cos(d*x + c)^5 + a^2*b^6*cos(d
```

```
*x + c)^4) + 12*(5*a^4 - 9*a^2*b^2 + 3*b^4)*log(cos(d*x + c))/b^6 - 12*(5*a
^6 - 9*a^4*b^2 + 3*a^2*b^4 + b^6)*log(a*cos(d*x + c) + b)/(a^2*b^6))/d
```

mupad [B] time = 3.09, size = 505, normalized size = 2.82

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(20\,a^5+5\,a^4\,b-31\,a^3\,b^2-4\,a^2\,b^3+8\,a\,b^4+4\,b^5\right)}{a\,b^5}-\frac{2\,\left(15\,a^5+15\,a^4\,b-22\,a^3\,b^2-22\,a^2\,b^3+3\,a\,b^4+3\,b^5\right)}{3\,a\,b^5}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(60\,a^5+45\,a^4\,b-\right.}{3\,a}}{d\,\left(\left(b-a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\left(5\,a-3\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(2\,b-10\,a\right)\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^6*(8*a*b^4 + 5*a^4*b + 20*a^5 + 4*b^5 - 4*a^2*b^3 -
31*a^3*b^2))/(a*b^5) - (2*(3*a*b^4 + 15*a^4*b + 15*a^5 + 3*b^5 - 22*a^2*b^3
 - 22*a^3*b^2))/(3*a*b^5) + (2*tan(c/2 + (d*x)/2)^2*(6*a*b^4 + 45*a^4*b + 6
0*a^5 + 12*b^5 - 66*a^2*b^3 - 83*a^3*b^2))/(3*a*b^5) - (2*tan(c/2 + (d*x)/2
)^4*(6*a*b^4 + 45*a^4*b + 90*a^5 + 18*b^5 - 56*a^2*b^3 - 127*a^3*b^2))/(3*a
*b^5) + (2*tan(c/2 + (d*x)/2)^8*(a - b)*(4*a*b^3 - 5*a^3*b - 5*a^4 + b^4 +
4*a^2*b^2))/(a*b^5))/(d*(a + b - tan(c/2 + (d*x)/2)^10*(a - b) - tan(c/2 +
(d*x)/2)^2*(5*a + 3*b) + tan(c/2 + (d*x)/2)^4*(10*a + 2*b) + tan(c/2 + (d*x
)/2)^8*(5*a - 3*b) - tan(c/2 + (d*x)/2)^6*(10*a - 2*b))) - log(tan(c/2 + (d
*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2)^2 - 1)*(5*a^4 + 3*b^4 - 9*a
^2*b^2))/(b^6*d) + (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/
2)^2)*(a^2 - b^2)^2*(5*a^2 + b^2))/(a^2*b^6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{7}{\left(c + dx \right)}}{\left(a + b \sec{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**7/(a + b*sec(c + d*x))**2, x)
```

3.301 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$\frac{(a^2-b^2)^2}{ab^4d(a+b\sec(c+dx))}+\frac{(3a^2+b^2)(a^2-b^2)\log(a+b\sec(c+dx))}{a^2b^4d}-\frac{\log(\cos(c+dx))}{a^2d}-\frac{2a\sec(c+dx)}{b^3d}+\frac{\sec^2(c+$$

```
[Out] -ln(cos(d*x+c))/a^2/d+(a^2-b^2)*(3*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^4/d-2*
a*sec(d*x+c)/b^3/d+1/2*sec(d*x+c)^2/b^2/d+(a^2-b^2)^2/a/b^4/d/(a+b*sec(d*x+
c))
```

Rubi [A] time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{(a^2-b^2)^2}{ab^4d(a+b\sec(c+dx))}+\frac{(3a^2+b^2)(a^2-b^2)\log(a+b\sec(c+dx))}{a^2b^4d}-\frac{\log(\cos(c+dx))}{a^2d}-\frac{2a\sec(c+dx)}{b^3d}+\frac{\sec^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + ((a^2 - b^2)*(3*a^2 + b^2)*Log[a + b*Sec[c +
 d*x]])/(a^2*b^4*d) - (2*a*Sec[c + d*x])/(b^3*d) + Sec[c + d*x]^2/(2*b^2*d)
 + (a^2 - b^2)^2/(a*b^4*d*(a + b*Sec[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)^2}\,dx,x,b\sec(c+dx)\right)}{b^4d}\\ &= \frac{\text{Subst}\left(\int \left(-2a+\frac{b^4}{a^2x}+x-\frac{(a^2-b^2)^2}{a(a+x)^2}+\frac{(a^2-b^2)(3a^2+b^2)}{a^2(a+x)}\right)dx,x,b\sec(c+dx)\right)}{b^4d}\\ &= -\frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2-b^2)(3a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^4d}-\frac{2a\sec(c+dx)}{b^3d}+\end{aligned}$$

Mathematica [A] time = 0.60, size = 187, normalized size = 1.55

$$\frac{b\left(-3a^3b\sec(c+dx)+a^2b^2\sec^2(c+dx)-2\left(3a^4+a^2\left(3a^2-2b^2\right)\log(\cos(c+dx))-2a^2b^2+\left(-3a^4+2a^2b^2+b^4\right.\right.\right.}{2a^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a*Cos[c + d*x]*(a^2*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]] + (-3*a^4 + 2*a^2
*b^2 + b^4)*Log[b + a*Cos[c + d*x]]) + b*(-2*(3*a^4 - 2*a^2*b^2 + b^4 + a^2
*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]] + (-3*a^4 + 2*a^2*b^2 + b^4)*Log[b + a*C
os[c + d*x]]) - 3*a^3*b*Sec[c + d*x] + a^2*b^2*Sec[c + d*x]^2))/(2*a^2*b^4*
d*(b + a*Cos[c + d*x]))
```

fricas [A] time = 0.55, size = 219, normalized size = 1.81

$$-\frac{3\,a^{3}b^{2}\cos\left(dx+c\right)-a^{2}b^{3}+2\left(3\,a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{2}-2\left(\left(3\,a^{5}-2\,a^{3}b^{2}-ab^{4}\right)\cos\left(dx+c\right)^{3}+\left(3\right.\right.}{2\left(a^{3}b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^3*b^2*cos(d*x + c) - a^2*b^3 + 2*(3*a^4*b - 2*a^2*b^3 + b^5)*cos(
d*x + c)^2 - 2*((3*a^5 - 2*a^3*b^2 - a*b^4)*cos(d*x + c)^3 + (3*a^4*b - 2*a
^2*b^3 - b^5)*cos(d*x + c)^2)*log(a*cos(d*x + c) + b) + 2*((3*a^5 - 2*a^3*b
^2)*cos(d*x + c)^3 + (3*a^4*b - 2*a^2*b^3)*cos(d*x + c)^2)*log(-cos(d*x + c
)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2)
```

giac [B] time = 6.54, size = 568, normalized size = 4.69

$$\frac{\frac{2\left(3\,a^{5}-3\,a^{4}b-2\,a^{3}b^{2}+2\,a^{2}b^{3}-ab^{4}+b^{5}\right)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^{3}b^{4}-a^{2}b^{5}}+\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{2}}-\frac{2\left(3\,a^{2}-2\,b^{2}\right)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right.\right.}{b^{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(3*a^5 - 3*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 + b^5)*log(abs(a +
b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1)))/(a^3*b^4 - a^2*b^5) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 1))/a^2 - 2*(3*a^2 - 2*b^2)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 1))/b^4 + (9*a^2 - 8*a*b - 6*b^2 + 18*a^2*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*b^2*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 6*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(b^4*((cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 1)^2) - 2*(3*a^5 + 5*a^4*b - 4*a^2*b^3 - 3
*a*b^4 - b^5 + 3*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^4*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 2*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*b^4*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((
a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1))*a^2*b^4))/d
```

maple [A] time = 0.48, size = 192, normalized size = 1.59

$$-\frac{a^{2}}{d\,b^{3}\left(b+a\cos\left(dx+c\right)\right)}+\frac{2}{db\left(b+a\cos\left(dx+c\right)\right)}-\frac{b}{d\,a^{2}\left(b+a\cos\left(dx+c\right)\right)}+\frac{3a^{2}\ln\left(b+a\cos\left(dx+c\right)\right)}{d\,b^{4}}-\frac{2\ln\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/b^3/(b+a*cos(d*x+c))+2/d/b/(b+a*cos(d*x+c))-1/d/a^2*b/(b+a*cos(d*x
+c))+3/d/b^4*a^2*ln(b+a*cos(d*x+c))-2/d/b^2*ln(b+a*cos(d*x+c))-1/d/a^2*ln(b
+a*cos(d*x+c))-3/d/b^4*ln(cos(d*x+c))*a^2+2/d/b^2*ln(cos(d*x+c))+1/2/d/b^2/
cos(d*x+c)^2-2/d*a/b^3/cos(d*x+c)
```

maxima [A] time = 0.34, size = 149, normalized size = 1.23

$$-\frac{\frac{3a^3b\cos(dx+c)-a^2b^2+2\left(3a^4-2a^2b^2+b^4\right)\cos(dx+c)^2}{a^3b^3\cos(dx+c)^3+a^2b^4\cos(dx+c)^2}+\frac{2\left(3a^2-2b^2\right)\log(\cos(dx+c))}{b^4}-\frac{2\left(3a^4-2a^2b^2-b^4\right)\log(a\cos(dx+c)+b)}{a^2b^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a^3*b*cos(d*x + c) - a^2*b^2 + 2*(3*a^4 - 2*a^2*b^2 + b^4)*cos(d*x
 + c)^2)/(a^3*b^3*cos(d*x + c)^3 + a^2*b^4*cos(d*x + c)^2) + 2*(3*a^2 - 2*b
^2)*log(cos(d*x + c))/b^4 - 2*(3*a^4 - 2*a^2*b^2 - b^4)*log(a*cos(d*x + c)
+ b)/(a^2*b^4))/d
```

mupad [B] time = 2.13, size = 286, normalized size = 2.36

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(-6\,a^3-3\,a^2\,b+a\,b^2+2\,b^3\right)}{a\,b^3}-\frac{2\left(-3\,a^3-3\,a^2\,b+a\,b^2+b^3\right)}{a\,b^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4(a-b)\left(3\,a^2+3\,a\,b+b^2\right)}{a\,b^3}}{d\left((b-a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+(3\,a-b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+(-3\,a-b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - ((2*tan(c/2 + (d*x)/2)^2*(a*b^2 - 3
*a^2*b - 6*a^3 + 2*b^3))/(a*b^3) - (2*(a*b^2 - 3*a^2*b - 3*a^3 + b^3))/(a*b
^3) + (2*tan(c/2 + (d*x)/2)^4*(a - b)*(3*a*b + 3*a^2 + b^2))/(a*b^3))/(d*(a
 + b - tan(c/2 + (d*x)/2)^2*(3*a + b) - tan(c/2 + (d*x)/2)^6*(a - b) + tan(
c/2 + (d*x)/2)^4*(3*a - b))) - (log(tan(c/2 + (d*x)/2)^2 - 1)*(3*a^2 - 2*b^
2))/(b^4*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)
*(b^4 - 3*a^4 + 2*a^2*b^2))/(a^2*b^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

3.302 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$\frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d}+\frac{\log(\cos(c+dx))}{a^2d}$$

```
[Out] ln(cos(d*x+c))/a^2/d+(a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^2/d+(a^2-b^2)/a/b^2
/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d}+\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((a^2 + b^2)*Log[a + b*Sec[c + d*x]])/(a^2*b^2*
d) + (a^2 - b^2)/(a*b^2*d*(a + b*Sec[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^2}\,dx,x,b\sec(c+dx)\right)}{b^2d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2x}+\frac{a^2-b^2}{a(a+x)^2}+\frac{-a^2-b^2}{a^2(a+x)}\right)dx,x,b\sec(c+dx)\right)}{b^2d}\\ &= \frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d}+\frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.28, size = 62, normalized size = 0.84

$$-\frac{\frac{b-\frac{b^3}{a^2}}{a\cos(c+dx)+b}-\frac{(a^2+b^2)\log(a\cos(c+dx)+b)}{a^2}+\log(\cos(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((b - b^3/a^2)/(b + a*Cos[c + d*x]) + Log[Cos[c + d*x]] - ((a^2 + b^2)*Lo
g[b + a*Cos[c + d*x]])/a^2)/(b^2*d))
```

fricas [A] time = 0.51, size = 102, normalized size = 1.38

$$-\frac{a^2b - b^3 - \left(a^2b + b^3 + \left(a^3 + ab^2\right)\cos\left(dx + c\right)\right)\log\left(a\cos\left(dx + c\right) + b\right) + \left(a^3\cos\left(dx + c\right) + a^2b\right)\log\left(-\cos\left(dx + \right.\right.}{a^3b^2d\cos\left(dx + c\right) + a^2b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b - b^3 - (a^2*b + b^3 + (a^3 + a*b^2)*cos(d*x + c))*log(a*cos(d*x +
c) + b) + (a^3*cos(d*x + c) + a^2*b)*log(-cos(d*x + c)))/(a^3*b^2*d*cos(d*x
 + c) + a^2*b^3*d)
```

giac [B] time = 1.03, size = 313, normalized size = 4.23

$$\frac{\frac{\left(a^3-a^2b+ab^2-b^3\right)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3b^2-a^2b^3} - \frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} - \frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2} - \frac{a^3+3a^2b+3ab^2+b^3+\frac{a^3(\cos(dx+c)-}{\cos(dx+c)+1}}{\left(a+b+\frac{a(\cos}{\cos(}\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((a^3 - a^2*b + a*b^2 - b^3)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^2 - a^2*b^3) -
 log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3
+ a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^2*b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^2))/d
```

maple [A] time = 0.53, size = 93, normalized size = 1.26

$$-\frac{1}{db\left(b + a\cos\left(dx + c\right)\right)} + \frac{b}{d\,a^2\left(b + a\cos\left(dx + c\right)\right)} + \frac{\ln\left(b + a\cos\left(dx + c\right)\right)}{d\,b^2} + \frac{\ln\left(b + a\cos\left(dx + c\right)\right)}{d\,a^2} - \frac{\ln\left(\cos\left(dx + c\right.\right.}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/b/(b+a*cos(d*x+c))+1/d/a^2*b/(b+a*cos(d*x+c))+1/d/b^2*ln(b+a*cos(d*x+c
))+1/d/a^2*ln(b+a*cos(d*x+c))-1/d/b^2*ln(cos(d*x+c))
```

maxima [A] time = 0.39, size = 74, normalized size = 1.00

$$-\frac{\frac{a^2-b^2}{a^3b\cos(dx+c)+a^2b^2} + \frac{\log(\cos(dx+c))}{b^2} - \frac{\left(a^2+b^2\right)\log(a\cos(dx+c)+b)}{a^2b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((a^2 - b^2)/(a^3*b*cos(d*x + c) + a^2*b^2) + log(cos(d*x + c))/b^2 - (a^2
 + b^2)*log(a*cos(d*x + c) + b)/(a^2*b^2))/d
```

mupad [B] time = 1.65, size = 124, normalized size = 1.68

$$\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{b^2\,d}-\frac{\cdots}{a\,b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(1/a^2 + 1/b^
2))/d - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - log(tan(c/2 + (d*x)/2)^2 -
1)/(b^2*d) - (2*(a + b))/(a*b*d*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

3.303 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=54

$$-\frac{\log(a+b\sec(c+dx))}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}+\frac{1}{ad(a+b\sec(c+dx))}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-ln(a+b*sec(d*x+c))/a^2/d+1/a/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 44}

$$-\frac{\log(a+b\sec(c+dx))}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}+\frac{1}{ad(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - Log[a + b*Sec[c + d*x]]/(a^2*d) + 1/(a*d*(a
+ b*Sec[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{1}{a(a+x)^2}-\frac{1}{a^2(a+x)}\right)dx, x, b\sec(c+dx)\right)}{d}\\ &= -\frac{\log(\cos(c+dx))}{a^2d}-\frac{\log(a+b\sec(c+dx))}{a^2d}+\frac{1}{ad(a+b\sec(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 1.00

$$-\frac{b\log(a\cos(c+dx)+b)+a\cos(c+dx)\log(a\cos(c+dx)+b)+b}{a^2d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b + b*Log[b + a*Cos[c + d*x]] + a*Cos[c + d*x]*Log[b + a*Cos[c + d*x]])/
(a^2*d*(b + a*Cos[c + d*x])))
```

fricas [A] time = 0.51, size = 46, normalized size = 0.85

$$-\frac{(a\cos(dx+c)+b)\log(a\cos(dx+c)+b)+b}{a^3d\cos(dx+c)+a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((a*cos(d*x + c) + b)*log(a*cos(d*x + c) + b) + b)/(a^3*d*cos(d*x + c) + a
^2*b*d)
```

giac [B] time = 1.41, size = 238, normalized size = 4.41

$$-\frac{\frac{(a-b)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3-a^2b}-\frac{a^2-2\,ab-b^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\left(a^3-a^2b\right)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((a - b)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3 - a^2*b) - (a^2 - 2*a*b - b^2 + a^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^3 - a^2*b)*(a +
b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1))) - log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2)/d
```

maple [A] time = 0.13, size = 54, normalized size = 1.00

$$\frac{1}{ad\left(a+b\sec(dx+c)\right)}-\frac{\ln\left(a+b\sec(dx+c)\right)}{a^2d}+\frac{\ln\left(\sec(dx+c)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/a/d/(a+b*sec(d*x+c))-ln(a+b*sec(d*x+c))/a^2/d+1/d/a^2*ln(sec(d*x+c))
```

maxima [A] time = 0.44, size = 41, normalized size = 0.76

$$-\frac{\frac{b}{a^3\cos(dx+c)+a^2b}+\frac{\log(a\cos(dx+c)+b)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(b/(a^3*cos(d*x + c) + a^2*b) + log(a*cos(d*x + c) + b)/a^2)/d
```

mupad [B] time = 1.48, size = 257, normalized size = 4.76

$$\frac{2\,\mathrm{atanh}\left(\frac{a}{2\left(\frac{a}{2}+b+\frac{a\,\cos(c+d\,x)}{2}\right)}-\frac{a\,\cos(c+d\,x)}{2\left(\frac{a}{2}+b+\frac{a\,\cos(c+d\,x)}{2}\right)}\right)}{a^2\,d}-\frac{b\left(a+a\,\cos(c+d\,x)-2\,a\,\mathrm{atanh}\left(\frac{a}{2\left(\frac{a}{2}+b+\frac{a\,\cos(c+d\,x)}{2}\right)}-\frac{a\,\cos(c}{2\left(\frac{a}{2}+b+\frac{a\,c}{}\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*atanh(a/(2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 +
b + (a*cos(c + d*x))/2))))/(a^2*d) - (b*(a + a*cos(c + d*x) - 2*a*atanh(a/(
2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c
 + d*x))/2))) + 2*a*cos(c + d*x)*atanh(a/(2*(a/2 + b + (a*cos(c + d*x))/2))
 - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c + d*x))/2))) + (2*atanh(a/(2*(a/
2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c + d*
x))/2)))*(a^3*d - a^3*d*cos(c + d*x)))/(a^2*d)))/(a^2*d*(a - b)*(b + a*cos(
c + d*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\tilde{\infty} x \tan(c)}{\sec^2(c)} \\ -\frac{1}{2b^2 d \sec^2(c+dx)} \\ \frac{\int \frac{\tan(c+dx)}{\cos^2(c+dx)\sec^2(c+dx)-2\cos(c+dx)\sec(c+dx)+1}\,dx}{a^2} \\ \frac{x\tan(c)}{(a+b\sec(c))^2} \\ \frac{\log\left(\tan^2(c+dx)+1\right)}{2a^2 d} \\ -\frac{2a\log\left(\frac{a}{b}+\sec(c+dx)\right)}{2a^3d+2a^2bd\sec(c+dx)} + \frac{a\log\left(\tan^2(c+dx)+1\right)}{2a^3d+2a^2bd\sec(c+dx)} + \frac{2a}{2a^3d+2a^2bd\sec(c+dx)} - \frac{2b\log\left(\frac{a}{b}+\sec(c+dx)\right)\sec(c+dx)}{2a^3d+2a^2bd\sec(c+dx)} + \frac{b\log\left(\tan^2(c+dx)+1\right)\sec(c+dx)}{2a^3d+2a^2bd\sec(c+dx)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*tan(c)/sec(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-1/(2*
b**2*d*sec(c + d*x)**2), Eq(a, 0)), (Integral(tan(c + d*x)/(cos(c + d*x)**2
*sec(c + d*x)**2 - 2*cos(c + d*x)*sec(c + d*x) + 1), x)/a**2, Eq(b, -a*cos(
c + d*x))), (x*tan(c)/(a + b*sec(c))**2, Eq(d, 0)), (log(tan(c + d*x)**2 +
1)/(2*a**2*d), Eq(b, 0)), (-2*a*log(a/b + sec(c + d*x))/(2*a**3*d + 2*a**2*
b*d*sec(c + d*x)) + a*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2*a**2*b*d*sec(c
 + d*x)) + 2*a/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)) - 2*b*log(a/b + sec(c +
 d*x))*sec(c + d*x)/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)) + b*log(tan(c + d*
x)**2 + 1)*sec(c + d*x)/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)), True))
```

3.304 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{b^2}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b^2\left(3a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^2}+\frac{\log(\cos(c+dx))}{a^2d}+\frac{\log(1-\sec(c+dx))}{2d(a+b)^2}$$

```
[Out] ln(cos(d*x+c))/a^2/d+1/2*ln(1-sec(d*x+c))/(a+b)^2/d+1/2*ln(1+sec(d*x+c))/(a
-b)^2/d-b^2*(3*a^2-b^2)*ln(a+b*sec(d*x+c))/a^2/(a^2-b^2)^2/d+b^2/a/(a^2-b^2
)/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 894}

$$\frac{b^2}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b^2\left(3a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^2}+\frac{\log(\cos(c+dx))}{a^2d}+\frac{\log(1-\sec(c+dx))}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + Log[1 - Sec[c + d*x]]/(2*(a + b)^2*d) + Log[1 +
 Sec[c + d*x]]/(2*(a - b)^2*d) - (b^2*(3*a^2 - b^2)*Log[a + b*Sec[c + d*x]]
)/(a^2*(a^2 - b^2)^2*d) + b^2/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{b^2\operatorname{Subst}\left(\int \frac{1}{x(a+x)^2\left(b^2-x^2\right)}\,dx,x,b\sec(c+dx)\right)}{d}\\ &= -\frac{b^2\operatorname{Subst}\left(\int \left(\frac{1}{2b^2(a+b)^2(b-x)}+\frac{1}{a^2b^2x}+\frac{1}{a(a-b)(a+b)(a+x)^2}+\frac{3a^2-b^2}{a^2(a-b)^2(a+b)^2(a+x)}-\frac{1}{2(a-b)^2b^2}\right.\right.}{d}\\ &= \frac{\log(\cos(c+dx))}{a^2d}+\frac{\log(1-\sec(c+dx))}{2(a+b)^2d}+\frac{\log(1+\sec(c+dx))}{2(a-b)^2d}-\frac{b^2\left(3a^2-b^2\right)\mathrm{l}}{a^2\left(\right.}\end{aligned}$$

Mathematica [A] time = 0.36, size = 189, normalized size = 1.37

$$\frac{a\cos(c+dx)\left(\left(b^4-3a^2b^2\right)\log(a\cos(c+dx)+b)+a^2(a-b)^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2(a+b)^2\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*Cos[c + d*x]*(a^2*(a + b)^2*Log[Cos[(c + d*x)/2]] + (-3*a^2*b^2 + b^4)*L
og[b + a*Cos[c + d*x]] + a^2*(a - b)^2*Log[Sin[(c + d*x)/2]]) + b*(a^2*(a +
 b)^2*Log[Cos[(c + d*x)/2]] + (-3*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]] +
(a - b)*(-(b^2*(a + b)) + a^2*(a - b)*Log[Sin[(c + d*x)/2]])))/(a^2*(a - b)
^2*(a + b)^2*d*(b + a*Cos[c + d*x]))
```

fricas [A] time = 0.60, size = 234, normalized size = 1.70

$$-\frac{2\,a^2b^3-2\,b^5+2\left(3\,a^2b^3-b^5+\left(3\,a^3b^2-ab^4\right)\cos\left(dx+c\right)\right)\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^4b+2\,a^3b^2+a^2b^3+\left(a^5\right.\right.}{2\left(\left(a^7-2\,a^5b^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b^3 - 2*b^5 + 2*(3*a^2*b^3 - b^5 + (3*a^3*b^2 - a*b^4)*cos(d*x
+ c))*log(a*cos(d*x + c) + b) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4
*b + a^3*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^4*b - 2*a^3*b^
2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*log(-1/2*cos(d*x + c)
 + 1/2))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 +
 a^2*b^5)*d)
```

giac [B] time = 0.28, size = 303, normalized size = 2.20

$$-\frac{\frac{2\left(3\,a^2b^2-b^4\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-2\,a^4b^2+a^2b^4}-\frac{\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right)}{a^2+2\,ab+b^2}-\frac{2\left(3\,a^2b^2+4\,ab^3+b^4+\frac{3\,a^2b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(a^5+a^4b-a^3b^2-a^2b^3\right)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}+\frac{2\,\mathrm{l}}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*a^2*b^2 - b^4)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 2*a^4*b^2 + a^2*
b^4) - log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2
) - 2*(3*a^2*b^2 + 4*a*b^3 + b^4 + 3*a^2*b^2*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^5 + a^4*b - a^3*b^
2 - a^2*b^3)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1))/a^2)/d
```

maple [A] time = 0.61, size = 141, normalized size = 1.02

$$-\frac{b^3}{d\,a^2\left(a+b\right)\left(a-b\right)\left(b+a\cos\left(dx+c\right)\right)}-\frac{3b^2\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)^2\left(a-b\right)^2}+\frac{b^4\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)^2\left(a-b\right)^2a^2}+\frac{\ln\left(-1+\cos\left(dx\right.\right.}{2d\left(a+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*b^3/(a+b)/(a-b)/(b+a*cos(d*x+c))-3/d*b^2/(a+b)^2/(a-b)^2*ln(b+a*co
s(d*x+c))+1/d*b^4/(a+b)^2/(a-b)^2/a^2*ln(b+a*cos(d*x+c))+1/2/d/(a+b)^2*ln(-
1+cos(d*x+c))+1/2*ln(1+cos(d*x+c))/(a-b)^2/d
```

maxima [A] time = 0.32, size = 142, normalized size = 1.03

$$-\frac{\frac{2\,b^3}{a^4b-a^2b^3+\left(a^5-a^3b^2\right)\cos\left(dx+c\right)}+\frac{2\left(3\,a^2b^2-b^4\right)\log\left(a\cos\left(dx+c\right)+b\right)}{a^6-2\,a^4b^2+a^2b^4}-\frac{\log\left(\cos\left(dx+c\right)+1\right)}{a^2-2\,ab+b^2}-\frac{\log\left(\cos\left(dx+c\right)-1\right)}{a^2+2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(d*x + c)) + 2*(3*a^2*b^2
 - b^4)*log(a*cos(d*x + c) + b)/(a^6 - 2*a^4*b^2 + a^2*b^4) - log(cos(d*x +
 c) + 1)/(a^2 - 2*a*b + b^2) - log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

mupad [B] time = 1.98, size = 160, normalized size = 1.16

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a^2+2\,a\,b+b^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}-\frac{b^2\,\ln\left(a+b-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(3\,a^2-b^2\right)}{a^2\,d\,\left(a^2-b^2\right)^2}-\frac{}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(2*a*b + a^2 + b^2)) - log(tan(c/2 + (d*x)/2)^2
+ 1)/(a^2*d) - (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/
2)^2)*(3*a^2 - b^2))/(a^2*d*(a^2 - b^2)^2) - (2*b^3)/(a*d*(a + b)*(a - b)^2
*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

3.305 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=197

$$\frac{b^4}{ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{b^4\left(5a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^3}-\frac{\log(\cos(c+dx))}{a^2d}+\frac{1}{4d(a+b)^2(1-\sec(c+}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-1/2*(a+2*b)*ln(1-sec(d*x+c))/(a+b)^3/d-1/2*(a-2*b)*ln
(1+sec(d*x+c))/(a-b)^3/d-b^4*(5*a^2-b^2)*ln(a+b*sec(d*x+c))/a^2/(a^2-b^2)^3
/d+1/4/(a+b)^2/d/(1-sec(d*x+c))+1/4/(a-b)^2/d/(1+sec(d*x+c))+b^4/a/(a^2-b^2
)^2/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.23, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3885, 894}

$$\frac{b^4}{ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{b^4\left(5a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^3}-\frac{\log(\cos(c+dx))}{a^2d}+\frac{1}{4d(a+b)^2(1-\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - ((a + 2*b)*Log[1 - Sec[c + d*x]])/(2*(a + b)
^3*d) - ((a - 2*b)*Log[1 + Sec[c + d*x]])/(2*(a - b)^3*d) - (b^4*(5*a^2 - b
^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^3*d) + 1/(4*(a + b)^2*d*(1 -
Sec[c + d*x])) + 1/(4*(a - b)^2*d*(1 + Sec[c + d*x])) + b^4/(a*(a^2 - b^2)^
2*d*(a + b*Sec[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\frac{b^4\,\text{Subst}\left(\int \frac{1}{x(a+x)^2\left(b^2-x^2\right)^2}\,dx,x,b\sec(c+dx)\right)}{d}$$

$$=\frac{b^4\,\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)^2(b-x)^2}+\frac{a+2b}{2b^4(a+b)^3(b-x)}+\frac{1}{a^2b^4x}-\frac{1}{a(a-b)^2(a+b)^2(a+x)^2}+\frac{-5a^2+b^2}{a^2(a-b)^3(a+b)^3(a}\right.\right.}{d}$$

$$=-\frac{\log(\cos(c+dx))}{a^2d}-\frac{(a+2b)\log(1-\sec(c+dx))}{2(a+b)^3d}-\frac{(a-2b)\log(1+\sec(c+dx))}{2(a-b)^3d}-$$

Mathematica [C] time = 2.27, size = 351, normalized size = 1.78

$$\sec^2(c+dx)(a\cos(c+dx)+b)\left(-\frac{8b^5}{a^2(a-b)^2(a+b)^2}+\frac{8b^4\left(b^2-5a^2\right)(a\cos(c+dx)+b)\log(a\cos(c+dx)+b)}{a^2\left(a^2-b^2\right)^3}-\frac{16i\left(a^4-3a^2b^2-2b^4\right)(c+dx)(a}{(a-b)^3(a+b)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((-8*b^5)/(a^2*(a - b)^2*(a + b)^2) - ((16*I)*(a^4 -
3*a^2*b^2 - 2*b^4)*(c + d*x)*(b + a*Cos[c + d*x]))/((a - b)^3*(a + b)^3) +
((8*I)*(a - 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x]))/(a - b)^3 + ((8
*I)*(a + 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x]))/(a + b)^3 - ((b +
a*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/(a + b)^2 + (4*(a - 2*b)*(b + a*Cos[c +
 d*x])*Log[Cos[(c + d*x)/2]^2])/(-a + b)^3 + (8*b^4*(-5*a^2 + b^2)*(b + a*C
os[c + d*x])*Log[b + a*Cos[c + d*x]])/(a^2*(a^2 - b^2)^3) - (4*(a + 2*b)*(b
 + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]^2])/(a + b)^3 - ((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a - b)^2)*Sec[c + d*x]^2)/(8*d*(a + b*Sec[c + d*x])
^2)
```

fricas [B] time = 0.77, size = 693, normalized size = 3.52

$$a^6b+a^2b^5-2\,b^7-2\left(a^6b-a^4b^3+a^2b^5-b^7\right)\cos\left(dx+c\right)^2+\left(a^7-2\,a^5b^2+a^3b^4\right)\cos\left(dx+c\right)+2\left(5\,a^2b^5-b^7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^6*b + a^2*b^5 - 2*b^7 - 2*(a^6*b - a^4*b^3 + a^2*b^5 - b^7)*cos(d*x
+ c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*cos(d*x + c) + 2*(5*a^2*b^5 - b^7 - (5
*a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (5*a^2*b^5 - b^7)*cos(d*x + c)^2 + (5*a^
3*b^4 - a*b^6)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (a^6*b + a^5*b^2 - 3
*a^4*b^3 - 5*a^3*b^4 - 2*a^2*b^5 - (a^7 + a^6*b - 3*a^5*b^2 - 5*a^4*b^3 - 2
*a^3*b^4)*cos(d*x + c)^3 - (a^6*b + a^5*b^2 - 3*a^4*b^3 - 5*a^3*b^4 - 2*a^2
*b^5)*cos(d*x + c)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 5*a^4*b^3 - 2*a^3*b^4)*co
s(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^6*b - a^5*b^2 - 3*a^4*b^3 + 5*
a^3*b^4 - 2*a^2*b^5 - (a^7 - a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*cos
(d*x + c)^3 - (a^6*b - a^5*b^2 - 3*a^4*b^3 + 5*a^3*b^4 - 2*a^2*b^5)*cos(d*x
 + c)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*cos(d*x + c))*l
og(-1/2*cos(d*x + c) + 1/2))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos
(d*x + c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x + c)^2 -
(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c) - (a^8*b - 3*a^6*b^3
 + 3*a^4*b^5 - a^2*b^7)*d)
```

giac [B] time = 1.93, size = 656, normalized size = 3.33

$$-\frac{\frac{4\,(a+2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{8\left(5\,a^2b^4-b^6\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6}-\frac{a^5-a^4b-a^3b^2+a^2b^3+\frac{3\,a^5(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a^4b(\cos(dx+c}{\cos(dx+c)+}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 +
3*a^2*b + 3*a*b^2 + b^3) + 8*(5*a^2*b^4 - b^6)*log(abs(-a - b - a*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a
```

```
^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6) - (a^5 - a^4*b - a^3*b^2 + a^2*b^3 +
3*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^4*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 3*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*
b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a*b^4*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*a^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 - 2*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 12*a^
2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b^4*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 - 4*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)) - (cos(d*x + c) -
1)/((a^2 - 2*a*b + b^2)*(cos(d*x + c) + 1)) - 8*log(abs(-(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1))/a^2)/d
```

maple [A] time = 0.72, size = 226, normalized size = 1.15

$$-\frac{b^5}{d\,a^2\,(a+b)^2\,(a-b)^2\,(b+a\cos(dx+c))}-\frac{5b^4\ln(b+a\cos(dx+c))}{d\,(a+b)^3\,(a-b)^3}+\frac{b^6\ln(b+a\cos(dx+c))}{d\,(a+b)^3\,(a-b)^3\,a^2}+\frac{1}{4d\,(a+b)^2\,(-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*b^5/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))-5/d*b^4/(a+b)^3/(a-b)^3*ln(b+
a*cos(d*x+c))+1/d*b^6/(a+b)^3/(a-b)^3/a^2*ln(b+a*cos(d*x+c))+1/4/d/(a+b)^2/
(-1+cos(d*x+c))-1/2/d/(a+b)^3*ln(-1+cos(d*x+c))*a-1/d/(a+b)^3*ln(-1+cos(d*x
+c))*b-1/4/d/(a-b)^2/(1+cos(d*x+c))-1/2/d/(a-b)^3*ln(1+cos(d*x+c))*a+1/d/(a
-b)^3*ln(1+cos(d*x+c))*b
```

maxima [A] time = 0.54, size = 303, normalized size = 1.54

$$-\frac{\frac{2\left(5a^2b^4-b^6\right)\log(a\cos(dx+c)+b)}{a^8-3a^6b^2+3a^4b^4-a^2b^6}+\frac{(a-2b)\log(\cos(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}+\frac{(a+2b)\log(\cos(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3}+\frac{a^4b+a^2b^3+2b^5-2\left(a^4b+b^5\right.}{a^6b-2a^4b^3+a^2b^5-\left(a^7-2a^5b^2+a^3b^4\right)\cos(dx+c)^3-(}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(5*a^2*b^4 - b^6)*log(a*cos(d*x + c) + b)/(a^8 - 3*a^6*b^2 + 3*a^4*
b^4 - a^2*b^6) + (a - 2*b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) + (a + 2*b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (
a^4*b + a^2*b^3 + 2*b^5 - 2*(a^4*b + b^5)*cos(d*x + c)^2 + (a^5 - a^3*b^2)*
cos(d*x + c))/(a^6*b - 2*a^4*b^3 + a^2*b^5 - (a^7 - 2*a^5*b^2 + a^3*b^4)*co
s(d*x + c)^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 + (a^7 - 2*a^5*
b^2 + a^3*b^4)*cos(d*x + c)))/d
```

mupad [B] time = 2.48, size = 313, normalized size = 1.59

$$\frac{\frac{a^2-2ab+b^2}{2(a+b)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4-16b^5\right)}{2a(a+b)^2(a-b)}}{d\left(\left(4a^3-12a^2b+12ab^2-4b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(-4a^3+4a^2b+4ab^2-4b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(2*(a + b)) - (tan(c/2 + (d*x)/2)^2*(a*b^4 - 4*a^4*b +
 a^5 - 16*b^5 - 4*a^2*b^3 + 6*a^3*b^2))/(2*a*(a + b)^2*(a - b)))/(d*(tan(c/
```

```
2 + (d*x)/2)^2*(4*a*b^2 + 4*a^2*b - 4*a^3 - 4*b^3) + tan(c/2 + (d*x)/2)^4*(
12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^
2) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2))*(a +
2*b))/(d*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - (b^4*log(a + b - a*tan(c/2 + (d
*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(5*a^2 - b^2))/(a^2*d*(a^2 - b^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

3.306 $\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=278

$$\frac{\left(4a^2+13ab+12b^2\right)\log(1-\sec(c+dx))}{8d(a+b)^4}+\frac{\left(4a^2-13ab+12b^2\right)\log(\sec(c+dx)+1)}{8d(a-b)^4}+\frac{b^6}{ad\left(a^2-b^2\right)^3(a+b\sec(c+$$

```
[Out] ln(cos(d*x+c))/a^2/d+1/8*(4*a^2+13*a*b+12*b^2)*ln(1-sec(d*x+c))/(a+b)^4/d+1
/8*(4*a^2-13*a*b+12*b^2)*ln(1+sec(d*x+c))/(a-b)^4/d-b^6*(7*a^2-b^2)*ln(a+b*
sec(d*x+c))/a^2/(a^2-b^2)^4/d-1/16/(a+b)^2/d/(1-sec(d*x+c))^2+1/16*(-5*a-9*
b)/(a+b)^3/d/(1-sec(d*x+c))-1/16/(a-b)^2/d/(1+sec(d*x+c))^2+1/16*(-5*a+9*b)
/(a-b)^3/d/(1+sec(d*x+c))+b^6/a/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.37, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3885, 894}

$$\frac{b^6}{ad\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{b^6\left(7a^2-b^2\right)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^4}+\frac{\left(4a^2+13ab+12b^2\right)\log(1-\sec(c+dx))}{8d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((4*a^2 + 13*a*b + 12*b^2)*Log[1 - Sec[c + d*x]
])/(8*(a + b)^4*d) + ((4*a^2 - 13*a*b + 12*b^2)*Log[1 + Sec[c + d*x]])/(8*(
a - b)^4*d) - (b^6*(7*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^
4*d) - 1/(16*(a + b)^2*d*(1 - Sec[c + d*x])^2) - (5*a + 9*b)/(16*(a + b)^3*
d*(1 - Sec[c + d*x])) - 1/(16*(a - b)^2*d*(1 + Sec[c + d*x])^2) - (5*a - 9*
b)/(16*(a - b)^3*d*(1 + Sec[c + d*x])) + b^6/(a*(a^2 - b^2)^3*d*(a + b*Sec[
c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b^6\,\text{Subst}\left(\int \frac{1}{x(a+x)^2(b^2-x^2)^3}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{b^6\,\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)^2(b-x)^3} + \frac{5a+9b}{16b^5(a+b)^3(b-x)^2} + \frac{4a^2+13ab+12b^2}{8b^6(a+b)^4(b-x)} + \frac{1}{a^2b^6x} + \frac{1}{a(a-b)^3(a+b)^3}\right.\right.}{}$$

$$= \frac{\log(\cos(c+dx))}{a^2d} + \frac{(4a^2+13ab+12b^2)\log(1-\sec(c+dx))}{8(a+b)^4d} + \frac{(4a^2-13ab+12}{8}$$

Mathematica [C] time = 3.14, size = 473, normalized size = 1.70

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{64b^7}{a^2(b-a)^3(a+b)^3} + \frac{8(4a^2-13ab+12b^2)\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b)}{(a-b)^4} - \frac{16i(4a^2-13ab+12b^2)\,\text{ta}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((64*b^7)/(a^2*(-a + b)^3*(a + b)^3) + ((128*I)*(a^6
- 4*a^4*b^2 + 6*a^2*b^4 + 3*b^6)*(c + d*x)*(b + a*Cos[c + d*x]))/((a - b)^4
*(a + b)^4) - ((16*I)*(4*a^2 - 13*a*b + 12*b^2)*ArcTan[Tan[c + d*x]]*(b + a
*Cos[c + d*x]))/(a - b)^4 - ((16*I)*(4*a^2 + 13*a*b + 12*b^2)*ArcTan[Tan[c
+ d*x]]*(b + a*Cos[c + d*x]))/(a + b)^4 + (2*(7*a + 11*b)*(b + a*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/(a + b)^3 - ((b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^
4)/(a + b)^2 + (8*(4*a^2 - 13*a*b + 12*b^2)*(b + a*Cos[c + d*x])*Log[Cos[(c
 + d*x)/2]^2])/(a - b)^4 + (64*(-7*a^2*b^6 + b^8)*(b + a*Cos[c + d*x])*Log[
b + a*Cos[c + d*x]])/(a^2*(a^2 - b^2)^4) + (8*(4*a^2 + 13*a*b + 12*b^2)*(b
+ a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]^2])/(a + b)^4 + (2*(7*a - 11*b)*(b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a - b)^3 - ((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^4)/(a - b)^2)*Sec[c + d*x]^2)/(64*d*(a + b*Sec[c + d*x])^2)
```

fricas [B] time = 1.10, size = 1378, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a^8*b - 18*a^6*b^3 + 2*a^4*b^5 + 2*a^2*b^7 + 8*b^9 + 2*(5*a^8*b - 18
*a^6*b^3 + 13*a^4*b^5 - 4*a^2*b^7 + 4*b^9)*cos(d*x + c)^4 - 2*(4*a^9 - 15*a
^7*b^2 + 18*a^5*b^4 - 7*a^3*b^6)*cos(d*x + c)^3 - 2*(7*a^8*b - 24*a^6*b^3 +
 11*a^4*b^5 - 2*a^2*b^7 + 8*b^9)*cos(d*x + c)^2 + 6*(a^9 - 4*a^7*b^2 + 5*a^
5*b^4 - 2*a^3*b^6)*cos(d*x + c) - 8*(7*a^2*b^7 - b^9 + (7*a^3*b^6 - a*b^8)*
cos(d*x + c)^5 + (7*a^2*b^7 - b^9)*cos(d*x + c)^4 - 2*(7*a^3*b^6 - a*b^8)*c
os(d*x + c)^3 - 2*(7*a^2*b^7 - b^9)*cos(d*x + c)^2 + (7*a^3*b^6 - a*b^8)*co
s(d*x + c))*log(a*cos(d*x + c) + b) + (4*a^8*b + 3*a^7*b^2 - 16*a^6*b^3 - 1
4*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 + 12*a^2*b^7 + (4*a^9 + 3*a^8*b - 16*a^
7*b^2 - 14*a^6*b^3 + 24*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c)^5 +
 (4*a^8*b + 3*a^7*b^2 - 16*a^6*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 +
 12*a^2*b^7)*cos(d*x + c)^4 - 2*(4*a^9 + 3*a^8*b - 16*a^7*b^2 - 14*a^6*b^3
+ 24*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c)^3 - 2*(4*a^8*b + 3*a^7
*b^2 - 16*a^6*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 + 12*a^2*b^7)*cos(
d*x + c)^2 + (4*a^9 + 3*a^8*b - 16*a^7*b^2 - 14*a^6*b^3 + 24*a^5*b^4 + 35*a
^4*b^5 + 12*a^3*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (4*a^8*b -
```

```
3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4 + 24*a^4*b^5 - 35*a^3*b^6 + 12*a^2*b^7
 + (4*a^9 - 3*a^8*b - 16*a^7*b^2 + 14*a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 1
2*a^3*b^6)*cos(d*x + c)^5 + (4*a^8*b - 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4
+ 24*a^4*b^5 - 35*a^3*b^6 + 12*a^2*b^7)*cos(d*x + c)^4 - 2*(4*a^9 - 3*a^8*b
 - 16*a^7*b^2 + 14*a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x
+ c)^3 - 2*(4*a^8*b - 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4 + 24*a^4*b^5 - 35
*a^3*b^6 + 12*a^2*b^7)*cos(d*x + c)^2 + (4*a^9 - 3*a^8*b - 16*a^7*b^2 + 14*
a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c))*log(-1/2*cos(
d*x + c) + 1/2))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*co
s(d*x + c)^5 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos
(d*x + c)^4 - 2*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(
d*x + c)^3 - 2*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos
(d*x + c)^2 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*
x + c) + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d)
```

giac [B] time = 1.19, size = 795, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*(4*a^2 + 13*a*b + 12*b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x +
c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 64*(7*a^2*b^6 - b^8)
*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1)))/(a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*
b^8) - (12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 32*a*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 20*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a
^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a*b*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^4 -
 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 32*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 20*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 156*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 144*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^
2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)^2) + 64*(
7*a^3*b^6 + 5*a^2*b^7 - 3*a*b^8 - b^9 + 7*a^3*b^6*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 7*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*b^8*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + b^9*(cos(d*x + c) - 1)/(cos(d*x + c) +
1))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(a + b + a*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))
 - 64*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2)/d
```

maple [A] time = 0.74, size = 367, normalized size = 1.32

$$-\frac{b^{7}}{d\,a^{2}\left(a+b\right)^{3}\left(a-b\right)^{3}\left(b+a\cos\left(dx+c\right)\right)}-\frac{7b^{6}\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)^{4}\left(a-b\right)^{4}}+\frac{b^{8}\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)^{4}\left(a-b\right)^{4}a^{2}}-\frac{}{16d\left(a+b\right)^{2}(-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*b^7/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))-7/d*b^6/(a+b)^4/(a-b)^4*ln(b+
a*cos(d*x+c))+1/d*b^8/(a+b)^4/(a-b)^4/a^2*ln(b+a*cos(d*x+c))-1/16/d/(a+b)^2
/(-1+cos(d*x+c))^2-7/16/d/(a+b)^3/(-1+cos(d*x+c))*a-11/16/d/(a+b)^3/(-1+cos
(d*x+c))*b+1/2/d/(a+b)^4*ln(-1+cos(d*x+c))*a^2+13/8/d/(a+b)^4*ln(-1+cos(d*x
+c))*a*b+3/2/d/(a+b)^4*ln(-1+cos(d*x+c))*b^2-1/16/d/(a-b)^2/(1+cos(d*x+c))^
2+7/16/d/(a-b)^3/(1+cos(d*x+c))*a-11/16/d/(a-b)^3/(1+cos(d*x+c))*b+1/2/d/(a
-b)^4*ln(1+cos(d*x+c))*a^2-13/8/d/(a-b)^4*ln(1+cos(d*x+c))*a*b+3/2/d/(a-b)^
4*ln(1+cos(d*x+c))*b^2
```

maxima [B] time = 0.38, size = 558, normalized size = 2.01

$$-\frac{\frac{8\left(7a^2b^6-b^8\right)\log(a\cos(dx+c)+b)}{a^{10}-4a^8b^2+6a^6b^4-4a^4b^6+a^2b^8}-\frac{\left(4a^2-13ab+12b^2\right)\log(\cos(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{\left(4a^2+13ab+12b^2\right)\log(\cos(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{\cdots}{a^8b-3a^6b^3+3a^4b^5-a^2b\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(8*(7*a^2*b^6 - b^8)*log(a*cos(d*x + c) + b)/(a^10 - 4*a^8*b^2 + 6*a^6
*b^4 - 4*a^4*b^6 + a^2*b^8) - (4*a^2 - 13*a*b + 12*b^2)*log(cos(d*x + c) +
1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + 13*a*b + 12*b^2)*
log(cos(d*x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(3*a^
6*b - 6*a^4*b^3 - 5*a^2*b^5 - 4*b^7 + (5*a^6*b - 13*a^4*b^3 - 4*b^7)*cos(d*
x + c)^4 - (4*a^7 - 11*a^5*b^2 + 7*a^3*b^4)*cos(d*x + c)^3 - (7*a^6*b - 17*
a^4*b^3 - 6*a^2*b^5 - 8*b^7)*cos(d*x + c)^2 + 3*(a^7 - 3*a^5*b^2 + 2*a^3*b^
4)*cos(d*x + c))/(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7 + (a^9 - 3*a^7*b^
2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^5 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 -
a^2*b^7)*cos(d*x + c)^4 - 2*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x
 + c)^3 - 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (a^9
 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)))/d
```

mupad [B] time = 2.97, size = 471, normalized size = 1.69

$$\frac{\frac{a^3-3a^2b+3ab^2-b^3}{4(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-13a^4+20a^3b+18a^2b^2-44ab^3+19b^4\right)}{4(a+b)^2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(3a^7-10a^6b+5a^5b^2+20a^4b^3-35a^3b^4+22a^2b^5-\cdots\right)}{a(a+b)^3(a-b)}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(16a^4-64a^3b+96a^2b^2-64ab^3+16b^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(16a^4-32a^3b+32ab^3-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(4*(a + b)) + (tan(c/2 + (d*x)/2)^2*(20*a^
3*b - 44*a*b^3 - 13*a^4 + 19*b^4 + 18*a^2*b^2))/(4*(a + b)^2) + (tan(c/2 +
(d*x)/2)^4*(3*a^7 - 10*a^6*b - 5*a*b^6 + 32*b^7 + 22*a^2*b^5 - 35*a^3*b^4 +
 20*a^4*b^3 + 5*a^5*b^2))/(a*(a + b)^3*(a - b)))/(d*(tan(c/2 + (d*x)/2)^6*(
16*a^4 - 64*a^3*b - 64*a*b^3 + 16*b^4 + 96*a^2*b^2) - tan(c/2 + (d*x)/2)^4*
(32*a*b^3 - 32*a^3*b + 16*a^4 - 16*b^4))) - tan(c/2 + (d*x)/2)^4/(64*d*(a -
 b)^2) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (tan(c/2 + (d*x)/2)^2*((32
*a*b + 16*a^2 - 48*b^2)/(512*(a - b)^4) - 7/(32*(a - b)^2)))/d + (log(tan(c
/2 + (d*x)/2))*(13*a*b + 4*a^2 + 12*b^2))/(d*(16*a*b^3 + 16*a^3*b + 4*a^4 +
 4*b^4 + 24*a^2*b^2)) - (b^6*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2
 + (d*x)/2)^2)*(7*a^2 - b^2))/(a^2*d*(a^2 - b^2)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

3.307 $\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal. Leaf size=200

$$-\frac{a\left(4a^2-5b^2\right)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^5d}+\frac{\left(3a^2-2b^2\right)\tan(c}{b^4d}$$

```
[Out] -x/a^2-a*(4*a^2-5*b^2)*arctanh(sin(d*x+c))/b^5/d+2*(a-b)^(3/2)*(a+b)^(3/2)*
(4*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/b^5/d+(
a^2-b^2)^2*sin(d*x+c)/a/b^4/d/(b+a*cos(d*x+c))+(3*a^2-2*b^2)*tan(d*x+c)/b^4
/d-a*sec(d*x+c)*tan(d*x+c)/b^3/d+1/3*tan(d*x+c)^3/b^2/d
```

Rubi [A] time = 0.43, antiderivative size = 283, normalized size of antiderivative = 1.42, number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3898, 2897, 2664, 12, 2659, 208, 3770, 3767, 8, 3768}

$$\frac{3\left(a^2-b^2\right)\tan(c+dx)}{b^4d}-\frac{2a\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{\left(a^2-b^2\right)^2\sin(c+dx)}{ab^4d(a\cos(c+dx)+b)}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-}}{a^2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (a*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*a*(2*a^2 - 3*b^2)*ArcTanh
[Sin[c + d*x]])/(b^5*d) - (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*b^3*d) + (4*(a - b)^(3/2)*(a + b)^(
3/2)*(2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
2*b^5*d) + ((a^2 - b^2)^2*Sin[c + d*x])/(a*b^4*d*(b + a*Cos[c + d*x])) + Ta
n[c + d*x]/(b^2*d) + (3*(a^2 - b^2)*Tan[c + d*x])/(b^4*d) - (a*Sec[c + d*x]
*Tan[c + d*x])/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
```

```
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{a^2} + \frac{\left(a^2-b^2\right)^3}{a^2b^4(b+a\cos(c+dx))^2} + \frac{2\left(2a^6-3a^4b^2+b^6\right)}{a^2b^5(b+a\cos(c+dx))} + \frac{2\left(-2a^3+3ab^2\right)\mathrm{sec}}{b^5}\right.$$

$$= -\frac{x}{a^2} - \frac{(2a)\int \sec^3(c+dx)\,dx}{b^3} + \frac{\int \sec^4(c+dx)\,dx}{b^2} - \frac{\left(2a\left(2a^2-3b^2\right)\right)\int \sec(c+dx)\,d}{b^5}$$

$$= -\frac{x}{a^2} - \frac{2a\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{\left(a^2-b^2\right)^2\sin(c+dx)}{ab^4d(b+a\cos(c+dx))} - \frac{a\sec(c+dx}{b^3}$$

$$= -\frac{x}{a^2} - \frac{a\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2a\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{4(a-b)^{3/2}(a+}{}$$

$$= -\frac{x}{a^2} - \frac{a\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2a\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{4(a-b)^{3/2}(a+}{}$$

$$= -\frac{x}{a^2} - \frac{a\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2a\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{2(a-b)^{3/2}(a+}{}$$

Mathematica [B] time = 6.25, size = 865, normalized size = 4.32

$$\frac{(b+a\cos(c+dx))^2\sin\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)}{6b^2d(a+b\sec(c+dx))^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{(b+a\cos(c+dx))^2\left(9a^2\sin\left(\frac{1}{2}(c+dx)\right)-7b^2\sin\right.}{3b^4d(a+b\sec(c+dx))^2\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((c + d*x)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(a^2*d*(a + b*Sec[c + d
*x])^2)) - (2*(-a^2 + b^2)^2*(4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(a^2*b^5*Sqrt[a
^2 - b^2]*d*(a + b*Sec[c + d*x])^2) + ((4*a^3 - 5*a*b^2)*(b + a*Cos[c + d*x
])^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2)/(b^5*d*(a + b
*Sec[c + d*x])^2) + ((-4*a^3 + 5*a*b^2)*(b + a*Cos[c + d*x])^2*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2)/(b^5*d*(a + b*Sec[c + d*x])^2)
 + ((-6*a + b)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(12*b^3*d*(a + b*Sec[
c + d*x])^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + ((b + a*Cos[c + d*x]
)^2*Sec[c + d*x]^2*Sin[(c + d*x)/2])/(6*b^2*d*(a + b*Sec[c + d*x])^2*(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])^3) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2
*Sin[(c + d*x)/2])/(6*b^2*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^3) + ((6*a - b)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(12*b^
3*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + ((b +
 a*Cos[c + d*x])^2*Sec[c + d*x]^2*(9*a^2*Sin[(c + d*x)/2] - 7*b^2*Sin[(c +
d*x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(9*a^2*Sin[(c + d*x)/2] - 7*
b^2*Sin[(c + d*x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(a^4*Sin[c + d*x]
 - 2*a^2*b^2*Sin[c + d*x] + b^4*Sin[c + d*x]))/(a*b^4*d*(a + b*Sec[c + d*x]
)^2)
```

fricas [B] time = 0.92, size = 843, normalized size = 4.22

$$\left[-\frac{6\,ab^5dx\cos\left(dx+c\right)^4+6\,b^6dx\cos\left(dx+c\right)^3+3\left(\left(4\,a^5-3\,a^3b^2-ab^4\right)\cos\left(dx+c\right)^4+\left(4\,a^4b-3\,a^2b^3-b^5\right)c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*a*b^5*d*x*cos(d*x + c)^4 + 6*b^6*d*x*cos(d*x + c)^3 + 3*((4*a^5 -
3*a^3*b^2 - a*b^4)*cos(d*x + c)^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*cos(d*x + c
)^3)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 3*((4*a^6 - 5*a^4*b^2)*cos(d*
x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*
((4*a^6 - 5*a^4*b^2)*cos(d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)
*log(-sin(d*x + c) + 1) + 2*(2*a^3*b^3*cos(d*x + c) - a^2*b^4 - (12*a^5*b -
 13*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - (6*a^4*b^2 - 7*a^2*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/(a^3*b^5*d*cos(d*x + c)^4 + a^2*b^6*d*cos(d*x + c)^3), -
1/6*(6*a*b^5*d*x*cos(d*x + c)^4 + 6*b^6*d*x*cos(d*x + c)^3 - 6*((4*a^5 - 3*
a^3*b^2 - a*b^4)*cos(d*x + c)^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*cos(d*x + c)^
3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) + 3*((4*a^6 - 5*a^4*b^2)*cos(d*x + c)^4 + (4*a^5*b - 5*a
^3*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((4*a^6 - 5*a^4*b^2)*cos(
d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) +
 2*(2*a^3*b^3*cos(d*x + c) - a^2*b^4 - (12*a^5*b - 13*a^3*b^3 + 3*a*b^5)*co
s(d*x + c)^3 - (6*a^4*b^2 - 7*a^2*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a^3*b
^5*d*cos(d*x + c)^4 + a^2*b^6*d*cos(d*x + c)^3)]
```

giac [B] time = 4.34, size = 411, normalized size = 2.06

$$-\frac{\frac{3\,(dx+c)}{a^2}+\frac{3\left(4\,a^3-5\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^5}-\frac{3\left(4\,a^3-5\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^5}+\frac{6\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 + 3*(4*a^3 - 5*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) +
1))/b^5 - 3*(4*a^3 - 5*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^5 + 6*(a
^4*tan(1/2*d*x + 1/2*c) - 2*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x
+ 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a*
b^4) - 6*(4*a^6 - 7*a^4*b^2 + 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2*b^5) + 2*(9*a^2*tan(1/2*d*x
 + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*b^2*tan(1/2*d*x + 1/2*c)^5 -
 18*a^2*tan(1/2*d*x + 1/2*c)^3 + 16*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(
1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) - 6*b^2*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^4))/d
```

maple [B] time = 0.51, size = 723, normalized size = 3.62

$$-\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^4\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}+\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d/b^4*a^3*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b-a-b)+4/d/b^2*a*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b-a-b)-2/d/a*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b-a-b)+8/d/b^5*a^4/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)
*(a-b)/((a-b)*(a+b))^(1/2))-14/d/b^3*a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+4/d/b/((a-b)*(a+b))^(1/2)*arctanh(t
an(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*b/a^2/((a-b)*(a+b))^(1/2)*
arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/3/d/b^2/(tan(1/2*d*
x+1/2*c)-1)^3-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)^2*a-1/2/d/b^2/(tan(1/2*d*x+1/2
*c)-1)^2-3/d/b^4/(tan(1/2*d*x+1/2*c)-1)*a^2-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*
a+2/d/b^2/(tan(1/2*d*x+1/2*c)-1)+4/d*a^3/b^5*ln(tan(1/2*d*x+1/2*c)-1)-5/d*a
/b^3*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/b^2/(tan(1/2*d*x+1/2*c)+1)^3+1/d/b^3/(t
an(1/2*d*x+1/2*c)+1)^2*a+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2-3/d/b^4/(tan(1/
2*d*x+1/2*c)+1)*a^2-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a+2/d/b^2/(tan(1/2*d*x+1
/2*c)+1)-4/d*a^3/b^5*ln(tan(1/2*d*x+1/2*c)+1)+5/d*a/b^3*ln(tan(1/2*d*x+1/2*
c)+1)-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 4.82, size = 9452, normalized size = 47.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5*(10*a*b^3 - 6*a^3*b + 36*a^4 + 9*b^4 - 37*a^2*b^2)
)/(3*a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(6*a^3*b - 10*a*b^3 + 36*a^4 + 9*b^4
- 37*a^2*b^2))/(3*a*b^4) + (2*tan(c/2 + (d*x)/2)*(2*a^3*b - 2*a*b^3 + 4*a^4
 + b^4 - 5*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^7*(a - b)*(3*a*b^2 - 2
*a^2*b - 4*a^3 + b^3))/(a*b^4))/(d*(a + b + tan(c/2 + (d*x)/2)^8*(a - b) -
tan(c/2 + (d*x)/2)^2*(4*a + 2*b) - tan(c/2 + (d*x)/2)^6*(4*a - 2*b) + 6*a*t
an(c/2 + (d*x)/2)^4)) - (2*atan(((((((((((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13
*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*
b^15 - 12*a^12*b^14))/(a^3*b^16) - (tan(c/2 + (d*x)/2)*(2*a^6*b^23 - 6*a^7*
b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18)*8192i)/(a^6*b^1
6))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^
21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 -
60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11
 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(
4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 +
74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 +
 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*
b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16))*1i)/a^2 - (8
192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19
+ 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 58
```

8*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16))/a^2 - ((((((((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) + (tan(c/2 + (d*x)/2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18)*8192i)/(a^6*b^16))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16))/a^2)/(((((((((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) - (tan(c/2 + (d*x)/2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18)*8192i)/(a^6*b^16))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16))*1i)/a^2 + ((((((((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) + (tan(c/2 + (d*x)/2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18)*8192i)/(a^6*b^16))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16))*1i)/a^2 - (16384*(5*a*b^17 - 256*a^17*b + 256*a^18 - 5*b^18 - 91*a^2*b^16 + 116*a^3*b^15 - 14*a^4*b^14 + 174*a^5*b^13 + 582*a^6*b^12 - 1036*a^7*b^11 - 1133*a^8*b^10 + 101*a^9*b^9 + 2245*a^10*b^8 + 2624*a^11*b^7 - 3792*a^12*b^6 - 3264*a^13*b^5 + 3488*a^14*b^4 + 1536*a^15*b^3 - 1536*a^16*b^2))/(a^3*b^16))))/(a^2*d) - (a*atan(((a*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9

+ 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^1
7*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16) + (a*((8192*(4*a*b^21 -
3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16
- 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*
b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a
^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) + (a*((8192*tan(c/2 + (d
x)/2)(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 +
12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 8
2*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 +
32*a^17*b^8))/(a^4*b^16) + (a*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20
+ 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12
*a^12*b^14))/(a^3*b^16) + (8192*tan(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*(2*a^6*b
^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18))/(a
^3*b^21))*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)
*(4*a^2 - 5*b^2)*1i)/b^5 + (a*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*
b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16
- 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11
+ 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*
b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*
b^16) - (a*((8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^
18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13
- 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^
14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*
b^16) - (a*((8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21
- 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60
*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 +
32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) - (a*((8192*(9*a^5*b
^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3
*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) - (8192*tan(c/2 + (d*
x)/2)*(4*a^2 - 5*b^2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 +
6*a^10*b^19 - 2*a^11*b^18))/(a^3*b^21))*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^
2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2)*1i)/b^5)/((16384*(5*a*b^17 -
256*a^17*b + 256*a^18 - 5*b^18 - 91*a^2*b^16 + 116*a^3*b^15 - 14*a^4*b^14
+ 174*a^5*b^13 + 582*a^6*b^12 - 1036*a^7*b^11 - 1133*a^8*b^10 + 101*a^9*b^9
+ 2245*a^10*b^8 + 2624*a^11*b^7 - 3792*a^12*b^6 - 3264*a^13*b^5 + 3488*a^1
4*b^4 + 1536*a^15*b^3 - 1536*a^16*b^2))/(a^3*b^16) - (a*((8192*tan(c/2 + (d
x)/2)(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 4
0*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624
*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 +
4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b
^3 - 1536*a^19*b^2))/(a^4*b^16) + (a*((8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20
+ 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 1
92*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^
10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*
b^5 + 192*a^18*b^4))/(a^3*b^16) + (a*((8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23
- 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^
8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a
^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*
b^16) + (a*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*
a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b
^16) + (8192*tan(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*(2*a^6*b^23 - 6*a^7*b^22 +
8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18))/(a^3*b^21))*(4*a^2 -
5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b
^5 + (a*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 -
4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^
7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*
a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 +
3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16) - (a*((8192*(4*a
*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*

a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 10
40*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7
 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) - (a*((8192*tan(
c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6
*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*
b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^1
6*b^9 + 32*a^17*b^8))/(a^4*b^16) - (a*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*
a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b
^15 - 12*a^12*b^14))/(a^3*b^16) - (8192*tan(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*
(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b
^18))/(a^3*b^21))*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^
2))/b^5)*(4*a^2 - 5*b^2))/b^5))*(4*a^2 - 5*b^2)*2i)/(b^5*d) - (atan((((4*a^
2 + b^2)*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 -
 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a
^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361
*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 +
 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16) + ((4*a^2 + b^2
)*((8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a
^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a
^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 +
1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) + (
((8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b
^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15
- 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^
10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) + ((4*a^2 + b^2)*((8192*(9*a^5*
b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 +
3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) + (8192*tan(c/2 + (d
x)/2)(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^6*b^23 - 6*a^7*b^22 +
 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18))/(a^6*b^21))*((a + b)
^3*(a - b)^3)^(1/2))/(a^2*b^5))*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2))/
(a^2*b^5))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5))*((a + b)^3*(a - b)^3)^(1
/2)*1i)/(a^2*b^5) + ((4*a^2 + b^2)*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*
a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*
b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*
b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*
a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/
(a^4*b^16) - ((4*a^2 + b^2)*((8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3
*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^
14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366
*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192
*a^18*b^4))/(a^3*b^16) - (((8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^2
2 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 16
0*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 +
128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) - ((4
a^2 + b^2)((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25
*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*
b^16) - (8192*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2)*
(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b
^18))/(a^6*b^21))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5))*(4*a^2 + b^2)*((a
 + b)^3*(a - b)^3)^(1/2))/(a^2*b^5))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5)
)*((a + b)^3*(a - b)^3)^(1/2)*1i)/(a^2*b^5))/((16384*(5*a*b^17 - 256*a^17*b
 + 256*a^18 - 5*b^18 - 91*a^2*b^16 + 116*a^3*b^15 - 14*a^4*b^14 + 174*a^5*b
^13 + 582*a^6*b^12 - 1036*a^7*b^11 - 1133*a^8*b^10 + 101*a^9*b^9 + 2245*a^1
0*b^8 + 2624*a^11*b^7 - 3792*a^12*b^6 - 3264*a^13*b^5 + 3488*a^14*b^4 + 153
6*a^15*b^3 - 1536*a^16*b^2))/(a^3*b^16) - ((4*a^2 + b^2)*((8192*tan(c/2 + (
d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 -
40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 62
4*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 +
 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*

```
b^3 - 1536*a^19*b^2))/(a^4*b^16) + ((4*a^2 + b^2)*((8192*(4*a*b^21 - 3*b^22
 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280
*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 +
 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^
6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) + (((8192*tan(c/2 + (d*x)/2)*(
2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b
^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b
^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*
b^8))/(a^4*b^16) + ((4*a^2 + b^2)*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*
b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15
- 12*a^12*b^14))/(a^3*b^16) + (8192*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*((a +
b)^3*(a - b)^3)^(1/2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 +
6*a^10*b^19 - 2*a^11*b^18))/(a^6*b^21))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b
^5))*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5))*((a + b)^3*(a -
b)^3)^(1/2))/(a^2*b^5))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5) + ((4*a^2 +
b^2)*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a
^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b
^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^1
2*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 368
0*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16) - ((4*a^2 + b^2)*((
8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b
^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*
b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696
*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) - (((81
92*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20
- 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 10
0*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 -
 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) - ((4*a^2 + b^2)*((8192*(9*a^5*b^21
 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^
10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) - (8192*tan(c/2 + (d*x)/
2)*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a
^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18))/(a^6*b^21))*((a + b)^3*(
a - b)^3)^(1/2))/(a^2*b^5))*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^2
*b^5))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5))*((a + b)^3*(a - b)^3)^(1/2))
/(a^2*b^5)))*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/(a^2*b^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**6/(a + b*sec(c + d*x))**2, x)
```

3.308 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d}+\frac{x}{a^2}-\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d}+$$

```
[Out] x/a^2-2*a*arctanh(sin(d*x+c))/b^3/d+(2*a^2-b^2)*sin(d*x+c)/a/b^2/d/(b+a*cos
(d*x+c))+2*(2*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*
(a-b)^(1/2)*(a+b)^(1/2)/a^2/b^3/d+tan(d*x+c)/b/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.33, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3898, 2890, 3057, 2659, 208, 3770}

$$\frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d}+\frac{x}{a^2}-\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*a*ArcTanh[Sin[c + d*x]])/(b^3*d) + (2*Sqrt[a - b]*Sqrt[a + b]*(2
*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*b^3*d
) + ((2*a^2 - b^2)*Sin[c + d*x])/(a*b^2*d*(b + a*Cos[c + d*x])) + Tan[c + d
*x]/(b*d*(b + a*Cos[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
```

```
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx))} + \frac{\int \frac{(-2a^2-ab\cos(c+dx)+b^2\cos^2(c+dx))\sec}{b+a\cos(c+dx)}}{ab^2}$$

$$= \frac{x}{a^2} + \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx))} - \frac{(2a)\int \sec(c+dx)\,dx}{b^3} - \frac{(-2}{}$$

$$= \frac{x}{a^2} - \frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx))} - \frac{(2}{}$$

$$= \frac{x}{a^2} - \frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{2\sqrt{a-b}\sqrt{a+b}\,(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d}$$

Mathematica [B] time = 1.58, size = 327, normalized size = 2.18

$$\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{(a^2-b^2)\sin(c+dx)}{ab^2} + \frac{(c+dx)(a\cos(c+dx)+b)}{a^2} + \frac{2(-2a^4+a^2b^2+b^4)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a^2-b^2}}\right.}{a^2b^3\sqrt{a^2-b^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(((c + d*x)*(b + a*Cos[c + d*x]))/a^2
+ (2*(-2*a^4 + a^2*b^2 + b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]]*(b + a*Cos[c + d*x]))/(a^2*b^3*Sqrt[a^2 - b^2]) + (2*a*(b + a*Cos[c
 + d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^3 - (2*a*(b + a*Cos[c
+ d*x])*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b^3 + ((b + a*Cos[c + d*x
])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((b + a*
Cos[c + d*x])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
 + ((a^2 - b^2)*Sin[c + d*x])/(a*b^2)))/(d*(a + b*Sec[c + d*x])^2)
```

fricas [A] time = 0.65, size = 584, normalized size = 3.89

$$\left[\frac{2\,ab^3dx\cos\left(dx+c\right)^2+2\,b^4dx\cos\left(dx+c\right)+\left(\left(2\,a^3+ab^2\right)\cos\left(dx+c\right)^2+\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^3*d*x*cos(d*x + c)^2 + 2*b^4*d*x*cos(d*x + c) + ((2*a^3 + a*b^2
)*cos(d*x + c)^2 + (2*a^2*b + b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) - 2*(a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(sin(d*x + c)
 + 1) + 2*(a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(-sin(d*x + c) + 1)
+ 2*(a^2*b^2 + (2*a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos
(d*x + c)^2 + a^2*b^4*d*cos(d*x + c)), (a*b^3*d*x*cos(d*x + c)^2 + b^4*d*x*
cos(d*x + c) + ((2*a^3 + a*b^2)*cos(d*x + c)^2 + (2*a^2*b + b^3)*cos(d*x +
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c))) - (a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(sin(d*x
 + c) + 1) + (a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(-sin(d*x + c) +
1) + (a^2*b^2 + (2*a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*co
s(d*x + c)^2 + a^2*b^4*d*cos(d*x + c))]
```

giac [B] time = 1.55, size = 294, normalized size = 1.96

$$\frac{\frac{dx+c}{a^2}-\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^3}+\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^3}-\frac{2\left(2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,a^2\tan\left(\right.\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^2 - 2*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + 2*a*log(abs(t
an(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a^2*tan(1/2*d*x + 1/2*c)^3 - a*b*tan(1
/2*d*x + 1/2*c)^3 - b^2*tan(1/2*d*x + 1/2*c)^3 - 2*a^2*tan(1/2*d*x + 1/2*c)
 - a*b*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*a*
b^2) + 2*(2*a^4 - a^2*b^2 - b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
 + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a
^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2*b^3))/d
```

maple [B] time = 0.40, size = 353, normalized size = 2.35

$$-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}+\frac{4a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d/b^2*a*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b-a-b)+2/d/a*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b-a-b)+4/d/b^3*a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/(
```

```
(a-b)*(a+b))^(1/2))-2/d/b/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a
-b)/((a-b)*(a+b))^(1/2))-2/d*b/a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+
1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/b^2/(tan(1/2*d*x+1/2*c)-1)+2/d*a/b^3*
ln(tan(1/2*d*x+1/2*c)-1)-1/d/b^2/(tan(1/2*d*x+1/2*c)+1)-2/d*a/b^3*ln(tan(1/
2*d*x+1/2*c)+1)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.47, size = 7044, normalized size = 46.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*atan(((((((((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 +
6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (tan(c/2 + (d*x)/2)*(2*a
^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10)
*8192i)/(a^6*b^8))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b
^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^
9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8))*1
i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 +
11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^
4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 + (8192*tan(c/2 + (d*x)/
2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b
^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^1
0*b^3 + 16*a^11*b^2))/(a^4*b^8))/a^2 - ((((((((8192*(8*a^5*b^13 - 3*a^4*b^1
4 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b
^8) + (tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^1
2 + 6*a^10*b^11 - 2*a^11*b^10)*8192i)/(a^6*b^8))*1i)/a^2 + (8192*tan(c/2 +
(d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 +
3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b
^5 + 8*a^13*b^4))/(a^4*b^8))*1i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^1
2 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8
*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i
)/a^2 - (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a
^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 1
2*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))/a^2)/((((((
(((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 1
3*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^
7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10)*8192i)/(a^6*b
^8))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b
^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^1
0*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8))*1i)/a^2 - (8192
*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 1
8*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^
3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b
- a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^
8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^1
1*b^2))/(a^4*b^8))*1i)/a^2 + ((((((((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6
```

```
*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) + (ta
n(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^1
0*b^11 - 2*a^11*b^10)*8192i)/(a^6*b^8))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*
(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^1
0 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^
13*b^4))/(a^4*b^8))*1i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3
*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14
*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 - (
8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 -
 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5
 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))*1i)/a^2 - (16384*(2*a
*b^9 - 16*a^9*b + 16*a^10 - 2*b^10 - 16*a^2*b^8 + 24*a^3*b^7 - 18*a^4*b^6 +
 26*a^5*b^5 + 12*a^6*b^4 - 36*a^7*b^3 + 8*a^8*b^2))/(a^3*b^8))))/(a^2*d) +
((2*tan(c/2 + (d*x)/2)*(a*b + 2*a^2 - b^2))/(a*b^2) - (2*tan(c/2 + (d*x)/2)
^3*(a - b)*(2*a + b))/(a*b^2))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) - 2
*a*tan(c/2 + (d*x)/2)^2)) + (atan((((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)
^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*((
(2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/
2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^
6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (8
192*tan(c/2 + (d*x)/2)*((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*
b))*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^
11*b^10))/(a^4*b^8)) - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 +
10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8
- 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8)) - (8192
*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 1
8*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^
3 + 24*a^12*b^2))/(a^3*b^8)) + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12
 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a
^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/
(a^4*b^8))*1i - ((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((
2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2
))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^
2)^(1/2)/(a^2*b))*((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11
 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) + (8192*tan(c/2 + (d*x)
/2)*((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(2*a^6*b^15 - 6
*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10))/(a^4*b^8)
) + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^
5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*
a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8)) - (8192*(3*b^14 - 5*a*b^13
 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*
b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(
a^3*b^8)) - (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 +
 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6
 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))*1i)/(((2
*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2)
)/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2
)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(
(8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*
a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (8192*tan(c/2 + (d*x)/2)*((2*(a^2 - b^2)
^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b
^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10))/(a^4*b^8)) - (8192*tan(c/2 +
 (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 +
 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*
b^5 + 8*a^13*b^4))/(a^4*b^8)) - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a
^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 +
14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8)) + (8192*t
an(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3
*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a
```

```
^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8)) + ((2*(a^2 - b^2)^(1/2))/b^
3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1
/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*
(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*((8192*(8*a^5*b^13 - 3*
a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))
/(a^3*b^8) + (8192*tan(c/2 + (d*x)/2)*((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b
^2)^(1/2)/(a^2*b))*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a
^10*b^11 - 2*a^11*b^10))/(a^4*b^8)) + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15
- 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^
9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^
4*b^8)) - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 +
 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b
^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8)) - (8192*tan(c/2 + (d*x)/2)*(16*
a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21
*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 +
 16*a^11*b^2))/(a^4*b^8)) - (16384*(2*a*b^9 - 16*a^9*b + 16*a^10 - 2*b^10 -
 16*a^2*b^8 + 24*a^3*b^7 - 18*a^4*b^6 + 26*a^5*b^5 + 12*a^6*b^4 - 36*a^7*b^
3 + 8*a^8*b^2))/(a^3*b^8)))*(((a^2 - b^2)^(1/2)*4i)/b^3 + ((a^2 - b^2)^(1/2
)*2i)/(a^2*b)))/d + (a*atan(((a*((2*a*((2*a*((2*a*((8192*(8*a^5*b^13 - 3*a^
4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(
a^3*b^8) - (16384*tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13
- 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10))/(a^3*b^11)))/b^3 - (8192*tan(c/2
 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11
 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^1
2*b^5 + 8*a^13*b^4))/(a^4*b^8)))/b^3 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^1
2 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8
*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8)))/b
^3 + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*
b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a
^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))*2i)/b^3 - (a*((
2*a*((2*a*((2*a*((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 +
 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) + (16384*tan(c/2 + (d*x)/
2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^1
1*b^10))/(a^3*b^11)))/b^3 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^
14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9
*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8)))/b
^3 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a
^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 -
52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8)))/b^3 - (8192*tan(c/2 + (d*x)/2)*(16*
a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21
*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 +
 16*a^11*b^2))/(a^4*b^8))*2i)/b^3)/((2*a*((2*a*((2*a*((2*a*((8192*(8*a^5*b^
13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^1
0*b^8))/(a^3*b^8) - (16384*tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*
a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10))/(a^3*b^11)))/b^3 - (819
2*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 +
 a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6
 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8)))/b^3 - (8192*(3*b^14 - 5*a*b^13 +
6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7
 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3
*b^8)))/b^3 + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13
 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b
^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8)))/b^3 +
 (2*a*((2*a*((2*a*((2*a*((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^
7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) + (16384*tan(c/2
+ (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11
 - 2*a^11*b^10))/(a^3*b^11)))/b^3 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 -
6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9
+ 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*
```

```
b^8)))/b^3 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^1
0 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^1
0*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8)))/b^3 - (8192*tan(c/2 + (d*x)
/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*
b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^
10*b^3 + 16*a^11*b^2))/(a^4*b^8)))/b^3 - (16384*(2*a*b^9 - 16*a^9*b + 16*a^
10 - 2*b^10 - 16*a^2*b^8 + 24*a^3*b^7 - 18*a^4*b^6 + 26*a^5*b^5 + 12*a^6*b^
4 - 36*a^7*b^3 + 8*a^8*b^2))/(a^3*b^8)))*4i)/(b^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

3.309 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

```
[Out] -x/a^2+2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/d/(a-b)^
(1/2)/(a+b)^(1/2)+tan(d*x+c)/a/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3894, 4061, 12, 3783, 2659, 208}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) + (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) + Tan[c + d*x]/(a*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
```

```
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{-1+\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\int \frac{a^2-b^2}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$
$$= \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\int \frac{1}{a+b\sec(c+dx)}\,dx}{a}$$
$$= -\frac{x}{a^2} + \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} + \frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2}$$
$$= -\frac{x}{a^2} + \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} + \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}$$
$$= -\frac{x}{a^2} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}\,d} + \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

Mathematica [A] time = 0.26, size = 80, normalized size = 0.94

$$-\frac{\frac{2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{a\sin(c+dx)}{a\cos(c+dx)+b} + c + dx}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((c + d*x + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqr
t[a^2 - b^2] - (a*Sin[c + d*x])/(b + a*Cos[c + d*x]))/(a^2*d))
```

fricas [B] time = 0.53, size = 383, normalized size = 4.51

$$\left[-\frac{2\left(a^3-ab^2\right)dx\cos(dx+c) + 2\left(a^2b-b^3\right)dx - \left(ab\cos(dx+c)+b^2\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx\right.}{a^2\cos(\right.}}{2\left(\left(a^5-a^3b^2\right)d\cos(dx+c)+\left(a^4b-a^2b^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3 - a*b^2)*d*x*cos(d*x + c) + 2*(a^2*b - b^3)*d*x - (a*b*cos(d*
x + c) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d
*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b
^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^3 - a*b^2)*sin(
```

```
d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c) + (a^4*b - a^2*b^3)*d), -((a^3 -
a*b^2)*d*x*cos(d*x + c) + (a^2*b - b^3)*d*x - (a*b*cos(d*x + c) + b^2)*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) - (a^3 - a*b^2)*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c) +
 (a^4*b - a^2*b^3)*d)]
```

giac [A] time = 0.66, size = 144, normalized size = 1.69

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}\,a^2}-\frac{dx+c}{a^2}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2
)*a^2) - (d*x + c)/a^2 - 2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a))/d
```

maple [A] time = 0.43, size = 120, normalized size = 1.41

$$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}+\frac{2b\,\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^2\sqrt{(a-b)\,(a+b)}}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d/a*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-
b)+2/d*b/a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a
+b))^(1/2))-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.14, size = 551, normalized size = 6.48

$$-\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^2\,d}-\frac{b^2\left(a\,\sin\left(c+d\,x\right)+2\,\mathrm{atanh}\left(\frac{2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^2-b^2\right)^{3/2}-a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}+2\,b^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}-\cdots}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a\,b^2-a^3\right)\left(b\left(\cdots\right.\right.}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

```
[Out] - (2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*d) - (b^2*(a*sin(c +
 d*x) + 2*atanh((2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - a^5*sin(c/2 +
 (d*x)/2)*(a^2 - b^2)^(1/2) + 2*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) -
3*a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^3*b^2*sin(c/2 + (d*x)/2)
*(a^2 - b^2)^(1/2) + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 +
 (d*x)/2)*(a*b^2 - a^3)*(b*(a^2 - b^2) + a*b^2 - a^2*b - a^3 + b^3)))*(a^2
- b^2)^(1/2)) - a^3*sin(c + d*x) + 2*a*b*cos(c + d*x)*atanh((2*b^3*sin(c/2
+ (d*x)/2)*(a^2 - b^2)^(3/2) - a^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2
*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*a^2*b^3*sin(c/2 + (d*x)/2)*(a
^2 - b^2)^(1/2) + a^3*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^4*b*sin(
c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(b*(a^2
 - b^2) + a*b^2 - a^2*b - a^3 + b^3)))*(a^2 - b^2)^(1/2))/(a^2*d*(a^2 - b^2
)*(b + a*cos(c + d*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.310 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=227

$$-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{x}{a^2}-$$

```
[Out] -x/a^2-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^
(5/2)/(a+b)^(5/2)/d-4*b^3*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c
)/(a+b)^(1/2))/a^2/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-co
s(d*x+c))+1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))+b^4*sin(d*x+c)/a/(a^2-b^2
)^2/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.41, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3898, 2897, 2648, 2664, 12, 2659, 208}

$$\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{x}{a^2}-2$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]
*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[
c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(
1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[c + d
*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{a^2} - \frac{1}{2(a-b)^2(-1-\cos(c+dx))} + \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \frac{\cdots}{a^2\left(a^2-b^2\right.\cdots}\right.$$

$$= -\frac{x}{a^2} - \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} + \frac{b^4\int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{a^2\left(a^2-b^2\right)} + \frac{\left(2b^3\left(2a^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{x}{a^2} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{b^4\,\mathrm{s}\cdots}{a\left(a^2-b^2\right)^2d\cdots}$$

$$= -\frac{x}{a^2} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \cdots$$

$$= -\frac{x}{a^2} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \cdots$$

$$= -\frac{x}{a^2} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \cdots$$

Mathematica [A] time = 1.73, size = 209, normalized size = 0.92

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(-\frac{4b^3\left(b^2-4a^2\right)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}} - \frac{2(c+dx)(a\cos(c+dx)+b)}{a^2} + \frac{2b^4\sin(c+dx\cdots}{a(a-b)^2(a+b)\cdots}\cdots\right.}{2d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((-2*(c + d*x)*(b + a*Cos[c + d*x]))/a
^2 - (4*b^3*(-4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]]*(b + a*Cos[c + d*x]))/(a^2*(a^2 - b^2)^(5/2)) - ((b + a*Cos[c + d*x])*
Cot[(c + d*x)/2])/(a + b)^2 + (2*b^4*Sin[c + d*x])/(a*(a - b)^2*(a + b)^2)
+ ((b + a*Cos[c + d*x])*Tan[(c + d*x)/2])/(a - b)^2))/(2*d*(a + b*Sec[c + d
*x])^2)
```

fricas [A] time = 0.57, size = 705, normalized size = 3.11

$$\left[\frac{4\,a^5b^2 - 2\,a^3b^4 - 2\,ab^6 - \left(4\,a^2b^4 - b^6 + \left(4\,a^3b^3 - ab^5\right)\cos\left(dx + c\right)\right)\sqrt{a^2 - b^2}\,\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2-2\,b^2\right)\cos(dx+c)^2+2}{a^2\cos(dx+c)^2+}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*a^5*b^2 - 2*a^3*b^4 - 2*a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 - a*b
^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*c
os(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(
a^7 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)
- 2*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^
4*b^3 + 3*a^2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4
- a^3*b^6)*d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*si
n(d*x + c)), (2*a^5*b^2 - a^3*b^4 - a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 -
 a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^7 - a*b^6)*cos(d*x
+ c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c) - ((a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*
d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)
 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*sin(d*x + c))]
```

giac [A] time = 0.31, size = 332, normalized size = 1.46

$$-\frac{\frac{4\left(4\,a^2b^3-b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-2\,a^4b^2+a^2b^4\right)\sqrt{-a^2+b^2}} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+}{\left(a^5-2\,a^3b^2+ab^4\right)\left(a\tan\left(\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/
(a^2 - 2*a*b + b^2) + (a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b*tan(1/2*d*x + 1
/2*c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - a*b^3*tan(1/2*d*x + 1/2*c)^2 +
 4*b^4*tan(1/2*d*x + 1/2*c)^2 - a^4 + a^3*b + a^2*b^2 - a*b^3)/((a^5 - 2*a^
3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) + 2*(d*x + c)/a^2)/d
```

maple [A] time = 0.62, size = 255, normalized size = 1.12

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\left(a^2-2ab+b^2\right)}-\frac{2b^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a+b\right)^2\left(a-b\right)^2 a\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}-\frac{8b^3\operatorname{arctanh}\left(\frac{\tan(}{\sqrt{(}}\right.}{d\left(a+b\right)^2\left(a-b\right)^2\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/2/d/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d*b^4/(a+b)^2/(a-b)^2/a*tan(1/2*
d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)-8/d*b^3/(a+b
)^2/(a-b)^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))+2/d*b^5/(a+b)^2/(a-b)^2/a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*
d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/2/d/(a+b)^2/tan(1/2*d*x+1/2*c)-2/d/
a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 6.46, size = 6093, normalized size = 26.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(a^4 - 3*a^3*b - a*b^3
 + 4*b^4 + 3*a^2*b^2))/(a*(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6
*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2
*b^3))) - (2*atan(-((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23
+ 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^
18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2
176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*
a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 4
80*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) - ((32*a^28 - 32*a^27*b + 32*a^6
*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 547
2*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16
*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 +
 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b
^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^
29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 48
00*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^
17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^
9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a
^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)/a^2 + (tan(c/2 + (
d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 -
1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*
b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12
```

```
+ 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*
a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224
*a^24*b^2) + ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b
^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8
256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a
^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 -
 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (tan(c/2 + (
d*x)/2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*
b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 -
 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688
*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b
^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^2
8*b^2)*1i)/a^2)*1i)/a^2)/a^2)/(64*a^2*b^22 - 192*a^3*b^21 - 640*a^4*b^20 +
1984*a^5*b^19 + 2624*a^6*b^18 - 8192*a^7*b^17 - 6400*a^8*b^16 + 18496*a^9*b
^15 + 11072*a^10*b^14 - 25856*a^11*b^13 - 14464*a^12*b^12 + 23872*a^13*b^11
 + 13760*a^14*b^10 - 15104*a^15*b^9 - 8704*a^16*b^8 + 6592*a^17*b^7 + 3200*
a^18*b^6 - 1856*a^19*b^5 - 512*a^20*b^4 + 256*a^21*b^3 - ((tan(c/2 + (d*x)/
2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*
a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16
- 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 822
4*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*
b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24
*b^2) - ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 +
 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a
^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b
^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688
*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/
2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20
- 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760
*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19
*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 +
5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2
)*1i)/a^2)*1i)/a^2)*1i)/a^2 + ((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 6
4*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 +
6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^
12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^
10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a
^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) + ((32*a^28 - 32*a^27
*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11
*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13
- 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880
*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 +
 480*a^25*b^3 - 352*a^26*b^2 + (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b
^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^1
2*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^1
4 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5
760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b
^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)*1i)/a^2
)))/(a^2*d) + tan(c/2 + (d*x)/2)/(2*d*(a - b)^2) + (b^3*atan(((b^3*(2*a + b
)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 6
72*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17
 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31
744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^
18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*
a^23*b^3 - 224*a^24*b^2) + (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a
- b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 208
0*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*
b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10
- 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^2
```

3*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (b^3*tan(c/2 + (d*x)/2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*1i)/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2) + (b^3*(2*a + b)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) - (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (b^3*tan(c/2 + (d*x)/2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*1i)/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))/(64*a^2*b^22 - 192*a^3*b^21 - 640*a^4*b^20 + 1984*a^5*b^19 + 2624*a^6*b^18 - 8192*a^7*b^17 - 6400*a^8*b^16 + 18496*a^9*b^15 + 11072*a^10*b^14 - 25856*a^11*b^13 - 14464*a^12*b^12 + 23872*a^13*b^11 + 13760*a^14*b^10 - 15104*a^15*b^9 - 8704*a^16*b^8 + 6592*a^17*b^7 + 3200*a^18*b^6 - 1856*a^19*b^5 - 512*a^20*b^4 + 256*a^21*b^3 + (b^3*(2*a + b)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) + (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (b^3*tan(c/2 + (d*x)/2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2) - (b^3*(2*a + b)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*

```
a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 92
8*a^23*b^3 - 224*a^24*b^2) - (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*
a - b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2
080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^1
4*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^1
0 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a
^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (b^3*tan(c/2 + (d*x
)/2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7
*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a
^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b
^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 -
 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25
*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a
^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b
^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*
a - b))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2
)))*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*2i)/(d*(a^12 - a^2*b^10
 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.311 $\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=360

$$-\frac{2b^7\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^6\sin(c+dx)}{ad\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{4b^5\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}+\cdots$$

```
[Out] x/a^2-2*b^7*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^(
7/2)/(a+b)^(7/2)/d-4*b^5*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)
/(a+b)^(1/2))/a^2/(a-b)^(7/2)/(a+b)^(7/2)/d-1/12*sin(d*x+c)/(a+b)^2/d/(1-co
s(d*x+c))^2-1/12*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/4*(3*a+5*b)*sin(d*x+
c)/(a+b)^3/d/(1-cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))^2-1/4*
(3*a-5*b)*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+
cos(d*x+c))+b^6*sin(d*x+c)/a/(a^2-b^2)^3/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.57, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3898, 2897, 2650, 2648, 2664, 12, 2659, 208}

$$\frac{b^6\sin(c+dx)}{ad\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{2b^7\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}-\frac{4b^5\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}+\frac{x}{a^2}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*b^7*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a
 - b)^(7/2)*(a + b)^(7/2)*d) - (4*b^5*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c +
 d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])^2) - Sin[c + d*x]/(12*(a + b)^2*d*
(1 - Cos[c + d*x])) + ((3*a + 5*b)*Sin[c + d*x])/(4*(a + b)^3*d*(1 - Cos[c
+ d*x])) + Sin[c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) - ((3*a - 5*b
)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c + d*x])) + Sin[c + d*x]/(12*(a -
b)^2*d*(1 + Cos[c + d*x])) + (b^6*Sin[c + d*x])/(a*(a^2 - b^2)^3*d*(b + a*C
os[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$
$$= \int \left(\frac{1}{a^2} + \frac{1}{4(a-b)^2(-1-\cos(c+dx))^2} + \frac{3a-5b}{4(a-b)^3(-1-\cos(c+dx))} + \frac{}{4(a+b)^2}\right.$$
$$= \frac{x}{a^2} + \frac{(3a-5b)\int \frac{1}{-1-\cos(c+dx)}\,dx}{4(a-b)^3} + \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^2} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^2} - \frac{(3a}{}$$
$$= \frac{x}{a^2} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} + \frac{(3a+5b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin}{12(a-b)^2d(}$$
$$= \frac{x}{a^2} - \frac{4b^5\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} -$$
$$= \frac{x}{a^2} - \frac{4b^5\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} -$$
$$= \frac{x}{a^2} - \frac{2b^7\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4b^5\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} -$$

Mathematica [A] time = 2.44, size = 303, normalized size = 0.84

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(-\frac{48b^5\left(b^2-6a^2\right)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{7/2}} + \frac{24(c+dx)(a\cos(c+dx)+b)}{a^2} + \frac{24b^6\sin(c}{a(a-b)^3(a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((24*(c + d*x)*(b + a*Cos[c + d*x]))/a
^2 - (48*b^5*(-6*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]]*(b + a*Cos[c + d*x]))/(a^2*(a^2 - b^2)^(7/2)) + (4*(4*a + 7*b)*(b + a
*Cos[c + d*x])*Cot[(c + d*x)/2])/(a + b)^3 - ((b + a*Cos[c + d*x])*Cot[(c +
 d*x)/2]*Csc[(c + d*x)/2]^2)/(a + b)^2 + (24*b^6*Sin[c + d*x])/(a*(a - b)^3
*(a + b)^3) + (4*(-4*a + 7*b)*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])/(a - b
)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a - b)^2)
)/(24*d*(a + b*Sec[c + d*x])^2)
```

fricas [B] time = 0.66, size = 1481, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(8*a^7*b^2 - 40*a^5*b^4 + 26*a^3*b^6 + 6*a*b^8 + 2*(4*a^9 - 13*a^7*b^2
 + 2*a^5*b^4 + 4*a^3*b^6 + 3*a*b^8)*cos(d*x + c)^4 - 2*(2*a^8*b - 11*a^6*b^
```

```
3 + 16*a^4*b^5 - 7*a^2*b^7)*cos(d*x + c)^3 - 3*(6*a^2*b^6 - b^8 - (6*a^3*b^
5 - a*b^7)*cos(d*x + c)^3 - (6*a^2*b^6 - b^8)*cos(d*x + c)^2 + (6*a^3*b^5 -
 a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^
2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) -
 6*(a^9 - 2*a^7*b^2 - 7*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2 + 2*(
a^8*b - 8*a^6*b^3 + 13*a^4*b^5 - 6*a^2*b^7)*cos(d*x + c) + 6*((a^9 - 4*a^7*
b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c)^3 + (a^8*b - 4*a^6*b^
3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*x*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a^4*
b^5 - 4*a^2*b^7 + b^9)*d*x)*sin(d*x + c))/(((a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 - (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a
^5*b^6 + a^3*b^8)*d*cos(d*x + c) - (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*
b^7 + a^2*b^9)*d)*sin(d*x + c)), 1/3*(4*a^7*b^2 - 20*a^5*b^4 + 13*a^3*b^6 +
 3*a*b^8 + (4*a^9 - 13*a^7*b^2 + 2*a^5*b^4 + 4*a^3*b^6 + 3*a*b^8)*cos(d*x +
 c)^4 - (2*a^8*b - 11*a^6*b^3 + 16*a^4*b^5 - 7*a^2*b^7)*cos(d*x + c)^3 + 3*
(6*a^2*b^6 - b^8 - (6*a^3*b^5 - a*b^7)*cos(d*x + c)^3 - (6*a^2*b^6 - b^8)*c
os(d*x + c)^2 + (6*a^3*b^5 - a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-
sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x +
 c) - 3*(a^9 - 2*a^7*b^2 - 7*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2
+ (a^8*b - 8*a^6*b^3 + 13*a^4*b^5 - 6*a^2*b^7)*cos(d*x + c) + 3*((a^9 - 4*a
^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c)^3 + (a^8*b - 4*a^6
*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*x*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 +
 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a
^4*b^5 - 4*a^2*b^7 + b^9)*d*x)*sin(d*x + c))/(((a^11 - 4*a^9*b^2 + 6*a^7*b^
4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5
 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 - (a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c) - (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a
^4*b^7 + a^2*b^9)*d)*sin(d*x + c))]
```

giac [A] time = 0.36, size = 487, normalized size = 1.35

$$-\frac{\frac{48\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{48\left(6\,a^2b^5-b^7\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\sqrt{-a^2+b^2}}\right.\right.}{\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)\sqrt{-a^2+b^2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(48*b^6*tan(1/2*d*x + 1/2*c)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*(
a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 48*(6*a^2*b
^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^8 - 3*a^6
*b^2 + 3*a^4*b^4 - a^2*b^6)*sqrt(-a^2 + b^2)) - (a^4*tan(1/2*d*x + 1/2*c)^3
 - 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*
b^3*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*
x + 1/2*c) + 72*a^3*b*tan(1/2*d*x + 1/2*c) - 126*a^2*b^2*tan(1/2*d*x + 1/2*
c) + 96*a*b^3*tan(1/2*d*x + 1/2*c) - 27*b^4*tan(1/2*d*x + 1/2*c))/(a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6) - 24*(d*x + c
)/a^2 - (15*a*tan(1/2*d*x + 1/2*c)^2 + 27*b*tan(1/2*d*x + 1/2*c)^2 - a - b)
/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)^3))/d
```

maple [A] time = 0.69, size = 416, normalized size = 1.16

$$\frac{a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\left(a-b\right)\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{24d\left(a-b\right)\left(a^{2}-2ab+b^{2}\right)}-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\left(a-b\right)\left(a^{2}-2ab+b^{2}\right)}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8d\left(a-b\right)\left(a^{2}-2ab+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/24/d/(a-b)/(a^2-2*a*b+b^2)*a*tan(1/2*d*x+1/2*c)^3-1/24/d/(a-b)/(a^2-2*a*b
+b^2)*tan(1/2*d*x+1/2*c)^3*b-5/8/d/(a-b)/(a^2-2*a*b+b^2)*a*tan(1/2*d*x+1/2*
c)+9/8/d/(a-b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*b-2/d*b^6/(a+b)^3/(a-b)^3
/a*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)-1
2/d*b^5/(a+b)^3/(a-b)^3/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))+2/d*b^7/(a+b)^3/(a-b)^3/a^2/((a-b)*(a+b))^(1/2)*arct
anh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/24/d/(a+b)^2/tan(1/2*d*
x+1/2*c)^3+5/8/d/(a+b)^3/tan(1/2*d*x+1/2*c)*a+9/8/d/(a+b)^3/tan(1/2*d*x+1/2
*c)*b+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 6.97, size = 8348, normalized size = 23.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b/cos(c + d*x))^2,x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^2) + ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(3
*(a + b)) + (2*tan(c/2 + (d*x)/2)^2*(11*a^3*b - 31*a*b^3 - 8*a^4 + 13*b^4 +
 15*a^2*b^2))/(3*(a + b)^2) - (tan(c/2 + (d*x)/2)^4*(11*a^5*b - 9*a*b^5 - 5
*a^6 + 16*b^6 + 31*a^2*b^4 - 34*a^3*b^3 + 6*a^4*b^2))/(a*(a + b)^3))/(d*(ta
n(c/2 + (d*x)/2)^5*(8*a^4 - 32*a^3*b - 32*a*b^3 + 8*b^4 + 48*a^2*b^2) - tan
(c/2 + (d*x)/2)^3*(16*a*b^3 - 16*a^3*b + 8*a^4 - 8*b^4))) + (tan(c/2 + (d*x
)/2)*((16*a*b + 8*a^2 - 24*b^2)/(64*(a - b)^4) - 3/(4*(a - b)^2)))/d + (2*a
tan(((tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3*b^33 - 128*a^4*b^32
- 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 15360*a^8*b^28 - 31360*a^
9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^12*b^24 - 114240*a^13
*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^16*b^20 + 96000*a^17*
b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^20*b^16 + 412640*a^21
*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a^24*b^12 + 243456*a^2
5*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^29*b^7 - 4480*a^30*b^6
 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384*a^34*b^2) - ((32*a^3
8 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^30 - 4992*a^10*b^28 +
 2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 - 57920*a^14*b^24 + 774
72*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 107008*a^18*b^20 + 344
960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 76032*a^22*b^16 + 4319
```

04*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 + 167552*a^26*b^12 + 171
072*a^27*b^11 - 113664*a^28*b^10 - 68960*a^29*b^9 + 53568*a^30*b^8 + 20064*
a^31*b^7 - 17536*a^32*b^6 - 4032*a^33*b^5 + 3840*a^34*b^4 + 512*a^35*b^3 -
512*a^36*b^2 + (tan(c/2 + (d*x)/2)*(64*a^39*b - 64*a^7*b^33 + 128*a^8*b^32
+ 896*a^9*b^31 - 1920*a^10*b^30 - 5760*a^11*b^29 + 13440*a^12*b^28 + 22400*
a^13*b^27 - 58240*a^14*b^26 - 58240*a^15*b^25 + 174720*a^16*b^24 + 104832*a
^17*b^23 - 384384*a^18*b^22 - 128128*a^19*b^21 + 640640*a^20*b^20 + 91520*a
^21*b^19 - 823680*a^22*b^18 + 823680*a^24*b^16 - 91520*a^25*b^15 - 640640*a
^26*b^14 + 128128*a^27*b^13 + 384384*a^28*b^12 - 104832*a^29*b^11 - 174720*
a^30*b^10 + 58240*a^31*b^9 + 58240*a^32*b^8 - 22400*a^33*b^7 - 13440*a^34*b
^6 + 5760*a^35*b^5 + 1920*a^36*b^4 - 896*a^37*b^3 - 128*a^38*b^2)*1i)/a^2)*
1i)/a^2)/a^2 + (tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3*b^33 - 128
*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 15360*a^8*b^28
- 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^12*b^24 - 1
14240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^16*b^20 + 9
6000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^20*b^16 + 4
12640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a^24*b^12 +
243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^29*b^7 - 448
0*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384*a^34*b^2)
- ((32*a^37*b - 32*a^38 + 32*a^6*b^32 + 32*a^7*b^31 - 640*a^8*b^30 + 4992*a
^10*b^28 - 2624*a^11*b^27 - 21504*a^12*b^26 + 19872*a^13*b^25 + 57920*a^14*
b^24 - 77472*a^15*b^23 - 100992*a^16*b^22 + 195008*a^17*b^21 + 107008*a^18*
b^20 - 344960*a^19*b^19 - 39424*a^20*b^18 + 446688*a^21*b^17 - 76032*a^22*b
^16 - 431904*a^23*b^15 + 161920*a^24*b^14 + 313984*a^25*b^13 - 167552*a^26*
b^12 - 171072*a^27*b^11 + 113664*a^28*b^10 + 68960*a^29*b^9 - 53568*a^30*b^
8 - 20064*a^31*b^7 + 17536*a^32*b^6 + 4032*a^33*b^5 - 3840*a^34*b^4 - 512*a
^35*b^3 + 512*a^36*b^2 + (tan(c/2 + (d*x)/2)*(64*a^39*b - 64*a^7*b^33 + 128
*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 5760*a^11*b^29 + 13440*a^12*b^2
8 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*a^15*b^25 + 174720*a^16*b^24
+ 104832*a^17*b^23 - 384384*a^18*b^22 - 128128*a^19*b^21 + 640640*a^20*b^20
+ 91520*a^21*b^19 - 823680*a^22*b^18 + 823680*a^24*b^16 - 91520*a^25*b^15
- 640640*a^26*b^14 + 128128*a^27*b^13 + 384384*a^28*b^12 - 104832*a^29*b^11
- 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a^32*b^8 - 22400*a^33*b^7 - 13
440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 - 896*a^37*b^3 - 128*a^38*b^2)
*1i)/a^2)*1i)/a^2)/a^2)/(((tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3
*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 1536
0*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^
12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^
16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^
20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a
^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^2
9*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384
*a^34*b^2) - ((32*a^38 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^
30 - 4992*a^10*b^28 + 2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 -
57920*a^14*b^24 + 77472*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 1
07008*a^18*b^20 + 344960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 7
6032*a^22*b^16 + 431904*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 + 1
67552*a^26*b^12 + 171072*a^27*b^11 - 113664*a^28*b^10 - 68960*a^29*b^9 + 53
568*a^30*b^8 + 20064*a^31*b^7 - 17536*a^32*b^6 - 4032*a^33*b^5 + 3840*a^34*
b^4 + 512*a^35*b^3 - 512*a^36*b^2 + (tan(c/2 + (d*x)/2)*(64*a^39*b - 64*a^7
*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 5760*a^11*b^29 + 134
40*a^12*b^28 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*a^15*b^25 + 174720
*a^16*b^24 + 104832*a^17*b^23 - 384384*a^18*b^22 - 128128*a^19*b^21 + 64064
0*a^20*b^20 + 91520*a^21*b^19 - 823680*a^22*b^18 + 823680*a^24*b^16 - 91520
*a^25*b^15 - 640640*a^26*b^14 + 128128*a^27*b^13 + 384384*a^28*b^12 - 10483
2*a^29*b^11 - 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a^32*b^8 - 22400*a^
33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 - 896*a^37*b^3 - 12
8*a^38*b^2)*1i)/a^2)*1i)/a^2)*1i)/a^2 - ((tan(c/2 + (d*x)/2)*(32*a^36 - 96*
a^35*b + 64*a^3*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*

a^7*b^29 - 15360*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b
^25 - 194240*a^12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b
^21 - 580608*a^16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b
^17 - 522720*a^20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*
b^13 - 138880*a^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*
b^9 + 40320*a^29*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408
*a^33*b^3 - 384*a^34*b^2) - ((32*a^37*b - 32*a^38 + 32*a^6*b^32 + 32*a^7*b^
31 - 640*a^8*b^30 + 4992*a^10*b^28 - 2624*a^11*b^27 - 21504*a^12*b^26 + 198
72*a^13*b^25 + 57920*a^14*b^24 - 77472*a^15*b^23 - 100992*a^16*b^22 + 19500
8*a^17*b^21 + 107008*a^18*b^20 - 344960*a^19*b^19 - 39424*a^20*b^18 + 44668
8*a^21*b^17 - 76032*a^22*b^16 - 431904*a^23*b^15 + 161920*a^24*b^14 + 31398
4*a^25*b^13 - 167552*a^26*b^12 - 171072*a^27*b^11 + 113664*a^28*b^10 + 6896
0*a^29*b^9 - 53568*a^30*b^8 - 20064*a^31*b^7 + 17536*a^32*b^6 + 4032*a^33*b
^5 - 3840*a^34*b^4 - 512*a^35*b^3 + 512*a^36*b^2 + (tan(c/2 + (d*x)/2)*(64*
a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 5760*
a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*a^1
5*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 384384*a^18*b^22 - 128128*a^
19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 823680*a^22*b^18 + 823680*a^
24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 128128*a^27*b^13 + 384384*a^
28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a^32
*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 - 89
6*a^37*b^3 - 128*a^38*b^2)*1i)/a^2)*1i)/a^2)*1i)/a^2 + 64*a^2*b^32 - 256*a^
3*b^31 - 960*a^4*b^30 + 3840*a^5*b^29 + 6144*a^6*b^28 - 23168*a^7*b^27 - 25
088*a^8*b^26 + 78784*a^9*b^25 + 76800*a^10*b^24 - 173760*a^11*b^23 - 183168
*a^12*b^22 + 269952*a^13*b^21 + 334080*a^14*b^20 - 314880*a^15*b^19 - 45388
8*a^16*b^18 + 291456*a^17*b^17 + 449856*a^18*b^16 - 221568*a^19*b^15 - 3184
00*a^20*b^14 + 136960*a^21*b^13 + 155904*a^22*b^12 - 64896*a^23*b^11 - 4992
0*a^24*b^10 + 21440*a^25*b^9 + 9344*a^26*b^8 - 4288*a^27*b^7 - 768*a^28*b^6
 + 384*a^29*b^5)))/(a^2*d) + (b^5*atan(((b^5*(tan(c/2 + (d*x)/2)*(32*a^36 -
 96*a^35*b + 64*a^3*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7
680*a^7*b^29 - 15360*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^
11*b^25 - 194240*a^12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^
15*b^21 - 580608*a^16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^
19*b^17 - 522720*a^20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a
^23*b^13 - 138880*a^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a
^27*b^9 + 40320*a^29*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 +
1408*a^33*b^3 - 384*a^34*b^2) - (b^5*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1
/2)*(32*a^38 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^30 - 4992*
a^10*b^28 + 2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 - 57920*a^14
*b^24 + 77472*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 107008*a^18
*b^20 + 344960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 76032*a^22*
b^16 + 431904*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 + 167552*a^26
*b^12 + 171072*a^27*b^11 - 113664*a^28*b^10 - 68960*a^29*b^9 + 53568*a^30*b
^8 + 20064*a^31*b^7 - 17536*a^32*b^6 - 4032*a^33*b^5 + 3840*a^34*b^4 + 512*
a^35*b^3 - 512*a^36*b^2 + (b^5*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*((a + b)^7*
(a - b)^7)^(1/2)*(64*a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1
920*a^10*b^30 - 5760*a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 58240*
a^14*b^26 - 58240*a^15*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 384384*
a^18*b^22 - 128128*a^19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 823680*
a^22*b^18 + 823680*a^24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 128128*
a^27*b^13 + 384384*a^28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 58240*
a^31*b^9 + 58240*a^32*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5
 + 1920*a^36*b^4 - 896*a^37*b^3 - 128*a^38*b^2))/(a^16 - a^2*b^14 + 7*a^4*b
^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2)))/
(a^16 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21
*a^12*b^4 - 7*a^14*b^2))*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*1i)/(a^1
6 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^1
2*b^4 - 7*a^14*b^2) + (b^5*(tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^
3*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 153

60*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a
^12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a
^16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a
^20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*
a^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^
29*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 38
4*a^34*b^2) - (b^5*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*(32*a^37*b - 3
2*a^38 + 32*a^6*b^32 + 32*a^7*b^31 - 640*a^8*b^30 + 4992*a^10*b^28 - 2624*a
^11*b^27 - 21504*a^12*b^26 + 19872*a^13*b^25 + 57920*a^14*b^24 - 77472*a^15
*b^23 - 100992*a^16*b^22 + 195008*a^17*b^21 + 107008*a^18*b^20 - 344960*a^1
9*b^19 - 39424*a^20*b^18 + 446688*a^21*b^17 - 76032*a^22*b^16 - 431904*a^23
*b^15 + 161920*a^24*b^14 + 313984*a^25*b^13 - 167552*a^26*b^12 - 171072*a^2
7*b^11 + 113664*a^28*b^10 + 68960*a^29*b^9 - 53568*a^30*b^8 - 20064*a^31*b^
7 + 17536*a^32*b^6 + 4032*a^33*b^5 - 3840*a^34*b^4 - 512*a^35*b^3 + 512*a^3
6*b^2 + (b^5*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*(
64*a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 57
60*a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*
a^15*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 384384*a^18*b^22 - 128128
*a^19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 823680*a^22*b^18 + 823680
*a^24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 128128*a^27*b^13 + 384384
*a^28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a
^32*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 -
896*a^37*b^3 - 128*a^38*b^2))/(a^16 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10
+ 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2)))/(a^16 - a^2*b^14 +
7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14
b^2))(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*1i)/(a^16 - a^2*b^14 + 7*a
^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2
))/(64*a^2*b^32 - 256*a^3*b^31 - 960*a^4*b^30 + 3840*a^5*b^29 + 6144*a^6*b^
28 - 23168*a^7*b^27 - 25088*a^8*b^26 + 78784*a^9*b^25 + 76800*a^10*b^24 - 1
73760*a^11*b^23 - 183168*a^12*b^22 + 269952*a^13*b^21 + 334080*a^14*b^20 -
314880*a^15*b^19 - 453888*a^16*b^18 + 291456*a^17*b^17 + 449856*a^18*b^16 -
221568*a^19*b^15 - 318400*a^20*b^14 + 136960*a^21*b^13 + 155904*a^22*b^12
- 64896*a^23*b^11 - 49920*a^24*b^10 + 21440*a^25*b^9 + 9344*a^26*b^8 - 4288
*a^27*b^7 - 768*a^28*b^6 + 384*a^29*b^5 + (b^5*(tan(c/2 + (d*x)/2)*(32*a^36
- 96*a^35*b + 64*a^3*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 +
7680*a^7*b^29 - 15360*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*
a^11*b^25 - 194240*a^12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*
a^15*b^21 - 580608*a^16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*
a^19*b^17 - 522720*a^20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632
*a^23*b^13 - 138880*a^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480
*a^27*b^9 + 40320*a^29*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4
+ 1408*a^33*b^3 - 384*a^34*b^2) - (b^5*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^
(1/2)*(32*a^38 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^30 - 499
2*a^10*b^28 + 2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 - 57920*a^
14*b^24 + 77472*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 107008*a^
18*b^20 + 344960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 76032*a^2
2*b^16 + 431904*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 + 167552*a^
26*b^12 + 171072*a^27*b^11 - 113664*a^28*b^10 - 68960*a^29*b^9 + 53568*a^30
*b^8 + 20064*a^31*b^7 - 17536*a^32*b^6 - 4032*a^33*b^5 + 3840*a^34*b^4 + 51
2*a^35*b^3 - 512*a^36*b^2 + (b^5*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*((a + b)^
7*(a - b)^7)^(1/2)*(64*a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 -
1920*a^10*b^30 - 5760*a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 5824
0*a^14*b^26 - 58240*a^15*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 38438
4*a^18*b^22 - 128128*a^19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 82368
0*a^22*b^18 + 823680*a^24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 12812
8*a^27*b^13 + 384384*a^28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 5824
0*a^31*b^9 + 58240*a^32*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b
^5 + 1920*a^36*b^4 - 896*a^37*b^3 - 128*a^38*b^2))/(a^16 - a^2*b^14 + 7*a^4
*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2))

```
)/(a^16 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 +
21*a^12*b^4 - 7*a^14*b^2))*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2))/(a^16
 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12
*b^4 - 7*a^14*b^2) - (b^5*(tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3
*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 1536
0*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^
12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^
16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^
20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a
^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^2
9*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384
*a^34*b^2) - (b^5*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*(32*a^37*b - 32
*a^38 + 32*a^6*b^32 + 32*a^7*b^31 - 640*a^8*b^30 + 4992*a^10*b^28 - 2624*a^
11*b^27 - 21504*a^12*b^26 + 19872*a^13*b^25 + 57920*a^14*b^24 - 77472*a^15*
b^23 - 100992*a^16*b^22 + 195008*a^17*b^21 + 107008*a^18*b^20 - 344960*a^19
*b^19 - 39424*a^20*b^18 + 446688*a^21*b^17 - 76032*a^22*b^16 - 431904*a^23*
b^15 + 161920*a^24*b^14 + 313984*a^25*b^13 - 167552*a^26*b^12 - 171072*a^27
*b^11 + 113664*a^28*b^10 + 68960*a^29*b^9 - 53568*a^30*b^8 - 20064*a^31*b^7
 + 17536*a^32*b^6 + 4032*a^33*b^5 - 3840*a^34*b^4 - 512*a^35*b^3 + 512*a^36
*b^2 + (b^5*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*(6
4*a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 576
0*a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*a
^15*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 384384*a^18*b^22 - 128128*
a^19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 823680*a^22*b^18 + 823680*
a^24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 128128*a^27*b^13 + 384384*
a^28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a^
32*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 -
896*a^37*b^3 - 128*a^38*b^2))/(a^16 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10 +
 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2)))/(a^16 - a^2*b^14 +
7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*
b^2))*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2))/(a^16 - a^2*b^14 + 7*a^4*b
^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2)))*
(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*2i)/(d*(a^16 - a^2*b^14 + 7*a^4*b
^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

3.312 $\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=761

$$-\frac{e^{5/2}\left(a^2-b^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ab^2d}+\frac{e^{5/2}\left(a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ab^2d}+\frac{e^{5/2}\left(a^2-b^2\right)\log\left(\sqrt{e}\tan(c+\right.}{2\sqrt{}}$$

```
[Out] 1/2*a*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/d*2^(1/2)-
1/2*(a^2-b^2)*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/b^2/
d*2^(1/2)-1/2*a*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/
d*2^(1/2)+1/2*(a^2-b^2)*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/
2))/a/b^2/d*2^(1/2)-1/4*a*e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e
^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)+1/4*(a^2-b^2)*e^(5/2)*ln(e^(1/2)-2^(1/2)*(
e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/b^2/d*2^(1/2)+1/4*a*e^(5/2)*ln(e^
(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)-1/4*(a
^2-b^2)*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/a/b^2/d*2^(1/2)+2*e^2*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),-(a
-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(a-b)^(1/2)*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(
e*tan(d*x+c))^(1/2)/a/b/d/sin(d*x+c)^(1/2)-2*e^2*EllipticPi(sin(d*x+c)^(1/2
)/(1+cos(d*x+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(a-b)^(1/2)*(a+b)
^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a/b/d/sin(d*x+c)^(1/2)+2*e^2*c
os(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4
*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/b/d/sin(2*d*x+2*c)^(1/2)+2*e*cos(d*x
+c)*(e*tan(d*x+c))^(3/2)/b/d
```

Rubi [A] time = 1.17, antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 22, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.880, Rules used = {3891, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 3890, 2733, 2730, 2906, 2905, 490, 1213, 537}

$$-\frac{e^{5/2}\left(a^2-b^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ab^2d}+\frac{e^{5/2}\left(a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ab^2d}+\frac{e^{5/2}\left(a^2-b^2\right)\log\left(\sqrt{e}\tan(c+\right.}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*b^2
*d) - ((a^2 - b^2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e
]])/(Sqrt[2]*a*b^2*d) - (a*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*b^2*d) - (a*e^(5/2)*Log[Sqrt[e] + Sq
rt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) + ((a
^2 - b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*a*b^2*d) + (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d
*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) - ((a^2 - b^2)*e^(5/
2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*S
qrt[2]*a*b^2*d) + (2*Sqrt[2]*Sqrt[a - b]*Sqrt[a + b]*e^2*Sqrt[Cos[c + d*x]]
*EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 +
Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*b*d*Sqrt[Sin[c + d*x]]) - (2*S
qrt[2]*Sqrt[a - b]*Sqrt[a + b]*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[a - b
]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[
e*Tan[c + d*x]])/(a*b*d*Sqrt[Sin[c + d*x]]) - (2*e^2*Cos[c + d*x]*EllipticE
[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(b*d*Sqrt[Sin[2*c + 2*d*x]]) + (2
*e*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2730

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3890

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3891

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Dist[e^2/b^2, Int[(e*Cot[c + d*x])^(m - 2)*(a - b*Csc[c
 + d*x]), x], x] + Dist[(e^2*(a^2 - b^2))/b^2, Int[(e*Cot[c + d*x])^(m - 2)
/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2
, 0] && IGtQ[m - 1/2, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = -\frac{e^2\int (a-b\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx}{b^2} + \frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx}{b^2}$$

$$= -\frac{\left(ae^2\right)\int \sqrt{e\tan(c+dx)}\,dx}{b^2} + \frac{e^2\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{b} + \frac{\left(\left(a^2-b^2\right)e^2\right)\int \sqrt{e}}{ab^2}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} - \frac{\left(2e^2\right)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{b} - \frac{\left(ae^3\right)\text{Subst}}{}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} - \frac{\left(2ae^3\right)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{b^2d} + \frac{\left(2\left(a\right.\right.}{}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} + \frac{\left(ae^3\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{b^2d} - \frac{\left(ae^3\right)}{}$$

$$= -\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{bd\sqrt{\sin(2c+2dx)}} + \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} -$$

$$= -\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,b^2d} + \frac{\left(a^2-b^2\right)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{2}$$

$$= \frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,b^2d} - \frac{\left(a^2-b^2\right)e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ab^2d} - \frac{ae^{5/2}\tan^{-1}}{}$$

$$= \frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,b^2d} - \frac{\left(a^2-b^2\right)e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ab^2d} - \frac{ae^{5/2}\tan^{-1}}{}$$

Mathematica [C] time = 26.44, size = 1846, normalized size = 2.43

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Cot[c + d*x]*(e*Tan[c + d*x])^(5/2))/(b*d*(a + b*Se
c[c + d*x])) - ((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Tan[c + d*x])^(5/2)*((
4*a*Sec[c + d*x]^2*((-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a
^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2
+ b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4)*S
qrt[Tan[c + d*x]] + b*Tan[c + d*x]] - Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[b
]*(-a^2 + b^2)^(1/4)*Sqrt[Tan[c + d*x]] + b*Tan[c + d*x]])/(4*Sqrt[2]*Sqrt[
b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(3*a^2 - 3*b^2))*(a + b*
Sqrt[1 + Tan[c + d*x]^2]))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2)
) - (b*Sec[c + d*x]*(6*Sqrt[2]*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - 6*Sqrt[2]*a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b
^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(
3/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + (
6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c +
 d*x]])/(a^2 - b^2)^(1/4)] - 3*Sqrt[2]*a^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] + 3*Sqrt[2]*b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]] + 3*Sqrt[2]*a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] - 3*Sqrt[2]*b^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + (3
```

```
+ 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - (3 + 3*I)*Sqrt[b]*(a
^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[c + d*x]] + I*b*Tan[c + d*x]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan
[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))*(a + b*S
qrt[1 + Tan[c + d*x]^2]))/(4*(a^3 - a*b^2)*(b + a*Cos[c + d*x])*(1 + Tan[c
+ d*x]^2)) + (Cos[2*(c + d*x)]*Sec[c + d*x]^2*(-84*Sqrt[2]*b*ArcTan[1 - Sqr
t[2]*Sqrt[Tan[c + d*x]]] + 84*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]]] + ((42 + 42*I)*(-a^2 + 2*b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c +
d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + ((42 + 42*I)*(a^2
- 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)]
)/(Sqrt[b]*(a^2 - b^2)^(1/4)) + 42*Sqrt[2]*b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]] - 42*Sqrt[2]*b*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]] + ((21 + 21*I)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2
 - b^2)^(1/4)) + ((21 + 21*I)*(-a^2 + 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*
(a^2 - b^2)^(1/4)) + (112*a^3*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (168*a*b
^2*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (24*a*b^2*AppellF1[7/4, 1/2, 1, 11/
4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(7/2))/(
a^2 - b^2) - (168*a*Tan[c + d*x]^(3/2))/Sqrt[1 + Tan[c + d*x]^2])*(a + b*Sq
rt[1 + Tan[c + d*x]^2]))/(84*a*(b + a*Cos[c + d*x])*(-1 + Tan[c + d*x]^2)*S
qrt[1 + Tan[c + d*x]^2])))/(b*d*(a + b*Sec[c + d*x])*Tan[c + d*x]^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.76, size = 3747, normalized size = 4.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x)
[Out] -1/d*(a-b)*(-1+cos(d*x+c))^2*(-I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*b^2+2*cos(d*x+c)*2^(1/2)*a*b-(a^2-b^2)^(1/2)*cos(d*x+c)*((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b+
```

```
2*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b-4*cos(d*x+c)*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b-I*cos(d*x+c)*((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2-cos(d*x+c)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^2+cos(d*x+c)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b
^2-cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1
/2)),1/2*2^(1/2))*a^2+cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a
+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^2-cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2+I*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*b^2-cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*b^2+(a^2-b^2)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b
+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a+(a^2-b^2)^(1/2)*((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b-(a^2-b^2)^(1/2)*((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2)
)*a-(a^2-b^2)^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+
b))^(1/2)),1/2*2^(1/2))*b+2*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b
-4*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b+I*cos(d*x+c)*((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2+(a^2-b^2)^(1/2)*cos(d*x+c)*
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2)
)*a+(a^2-b^2)^(1/2)*cos(d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+(
(a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b-(a^2-b^2)^(1/2)*cos(d*x+c)*((1-cos(d*x+c
```

```
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a-2*2^(1/
2)*a*b-((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2-((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^2+((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b
^2-((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^
(1/2))*a^2+((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)
),1/2*2^(1/2))*b^2-((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b
^2)*cos(d*x+c)^2*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(5/2)/sin(d*x+c
)^7*2^(1/2)/b/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/a
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(e\tan(c+d\,x)\right)^{5/2}}{b+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(5/2))/(b + a*cos(c + d*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{\frac{5}{2}}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((e*tan(c + d*x))**(5/2)/(a + b*sec(c + d*x)), x)
```

3.313 $\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=740

$$-\frac{e^{3/2}\left(a^2-b^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ab^2d}+\frac{e^{3/2}\left(a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ab^2d}-\frac{e^{3/2}\left(a^2-b^2\right)\log\left(\sqrt{e}\tan(c+\right.}{2\sqrt{}}$$

```
[Out] 1/2*a*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/d*2^(1/2)-
1/2*(a^2-b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/b^2/
d*2^(1/2)-1/2*a*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/
d*2^(1/2)+1/2*(a^2-b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/
2))/a/b^2/d*2^(1/2)+1/4*a*e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e
^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)-1/4*(a^2-b^2)*e^(3/2)*ln(e^(1/2)-2^(1/2)*(
e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/b^2/d*2^(1/2)-1/4*a*e^(3/2)*ln(e^
(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)+1/4*(a
^2-b^2)*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/a/b^2/d*2^(1/2)-2*e^2*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),
b/(a-(a^2-b^2)^(1/2)),I)*2^(1/2)*(a^2-b^2)^(1/2)*sin(d*x+c)^(1/2)/a/b/d/(-c
os(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)+2*e^2*EllipticPi((-cos(d*x+c))^(1/2)/
(1+sin(d*x+c))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(1/2)*(a^2-b^2)^(1/2)*sin(d
*x+c)^(1/2)/a/b/d/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)-e^2*(sin(c+1/4*P
i+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(
d*x+c)*sin(2*d*x+2*c)^(1/2)/b/d/(e*tan(d*x+c))^(1/2)
```

Rubi [A] time = 1.01, antiderivative size = 740, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 19, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {3891, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 3892, 2733, 2729, 2907, 1213, 537}

$$-\frac{e^{3/2}\left(a^2-b^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ab^2d}+\frac{e^{3/2}\left(a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ab^2d}-\frac{e^{3/2}\left(a^2-b^2\right)\log\left(\sqrt{e}\tan(c+\right.}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*b^2
*d) - ((a^2 - b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e
]])/(Sqrt[2]*a*b^2*d) - (a*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*b^2*d) + (a*e^(3/2)*Log[Sqrt[e] + Sq
rt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) - ((a
^2 - b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*a*b^2*d) - (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d
*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(3/
2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*S
qrt[2]*a*b^2*d) - (2*Sqrt[2]*Sqrt[a^2 - b^2]*e^2*EllipticPi[b/(a - Sqrt[a^2
 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[
c + d*x]])/(a*b*d*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt[2]*Sq
rt[a^2 - b^2]*e^2*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c +
d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*b*d*Sqrt[-Cos[c +
 d*x]]*Sqrt[e*Tan[c + d*x]]) + (e^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*
x]*Sqrt[Sin[2*c + 2*d*x]])/(b*d*Sqrt[e*Tan[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2729

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan[e + f*x]])/Sqrt[Sin[e
 + f*x]], Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3891

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Dist[e^2/b^2, Int[(e*Cot[c + d*x])^(m - 2)*(a - b*Csc[c
 + d*x]), x], x] + Dist[(e^2*(a^2 - b^2))/b^2, Int[(e*Cot[c + d*x])^(m - 2)
/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2
```

```
, 0] && IGtQ[m - 1/2, 0]
```

Rule 3892

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = -\frac{e^2\int \frac{a-b\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{b^2}+\frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{b^2}$$

$$= -\frac{\left(ae^2\right)\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{b^2}+\frac{e^2\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{b}+\frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{ab^2}-\frac{((}{}$$

$$= -\frac{\left(ae^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx,x,e\tan(c+dx)\right)}{b^2d}+\frac{\left(\left(a^2-b^2\right)e^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,a\right.}{ab^2d}$$

$$= -\frac{\left(2ae^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{b^2d}+\frac{\left(2\left(a^2-b^2\right)e^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,\right.}{ab^2d}$$

$$= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}}-\frac{\left(ae^2\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan}\right.}{b^2d}$$

$$= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}}+\frac{\left(ae^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,\right.}{2\sqrt{2}b^2d}$$

$$= \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}-\frac{\left(a^2-b^2\right)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{2}$$

$$= \frac{ae^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d}-\frac{\left(a^2-b^2\right)e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}-\frac{ae^{3/2}\tan}{}$$

Mathematica [C] time = 6.91, size = 202, normalized size = 0.27

$$2e\sqrt{1-\tan\left(\frac{1}{2}(c+dx)\right)}\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\sqrt{\frac{-\sin(c+dx)+\cos(c+dx)-1}{\cos(c+dx)+1}}\left(-\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\sqrt{-\tan\left(\right.}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*e*Cot[(c + d*x)/2]*(EllipticPi[-I, ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1]
+ EllipticPi[I, ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1] - EllipticPi[-(Sqrt[a
- b]/Sqrt[a + b]), ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1] - EllipticPi[Sqrt[a
 - b]/Sqrt[a + b], ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1])*Sqrt[(-1 + Cos[c +
 d*x] - Sin[c + d*x])/(1 + Cos[c + d*x])]*Sqrt[1 - Tan[(c + d*x)/2]]*Sqrt[e
*Tan[c + d*x]])/(a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.64, size = 1801, normalized size = 2.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/2/d*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(3/2)*(-1+cos(d*x+c))*cos(
d*x+c)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-I*(a^2-b^2)^(1/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))*b^2+I*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2+I*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2-I*(a^2-b^2)^
(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))*a^2+2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b+2*(a^2-b^2)^(1/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*b+I*(a^2-b^
2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))-I*(a^2-b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2-(a^2-b^2)^(1/2)*
EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*b^2-2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-
b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^2*b-2*EllipticPi(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^2
+2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a
-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^2-2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(
1/2))*a^2+2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^2-2*(a^2-b^2)^(1/2)
*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b
)*(a+b))^(1/2)),1/2*2^(1/2))*a^2+2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/
2))*b^2-(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2-1/2*I,1/2*2^(1/2))*a^2-(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2+2*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/
2*2^(1/2))*a^2*b+2*I*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b-2*I*(a^2-b^2)^(1/2)*EllipticPi
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*b+(a
^2-b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))+(a^2-b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
```

```
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^3+2*Ell
ipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b
))^(1/2)),1/2*2^(1/2))*b^3-2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^3-2*EllipticPi((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2
)),1/2*2^(1/2))*b^3)/sin(d*x+c)^4*2^(1/2)/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^
(1/2)+a-b)/(a^2-b^2)^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(e\tan(c+d\,x)\right)^{3/2}}{b+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(3/2))/(b + a*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+b*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(3/2)/(a + b*sec(c + d*x)), x)
```

3.314 $\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=415

$$\frac{2\sqrt{2}\,b\sqrt{\cos(c+dx)}\,\sqrt{e\tan(c+dx)}\,\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a-b}\,\sqrt{a+b}\,\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}\,b\sqrt{\cos(c+dx)}\,\sqrt{e\tan(c+dx)}\,\Pi\left(\right.}{ad\sqrt{a-b}\,\sqrt{a+b}\,\sqrt{}}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a/d*2^(1/2
)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a
/d*2^(1/2)+2*b*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),-(a-b)^(1/2
)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a/d/(a-b)^(1
/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2)-2*b*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x
+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c
))^(1/2)/a/d/(a-b)^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3890, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2733, 2730, 2906, 2905, 490, 1213, 537}

$$\frac{2\sqrt{2}\,b\sqrt{\cos(c+dx)}\,\sqrt{e\tan(c+dx)}\,\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a-b}\,\sqrt{a+b}\,\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}\,b\sqrt{\cos(c+dx)}\,\sqrt{e\tan(c+dx)}\,\Pi\left(\right.}{ad\sqrt{a-b}\,\sqrt{a+b}\,\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (2*Sqrt[2]*b*Sqrt[Cos[c + d*
x]]*EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1
 + Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*S
qrt[Sin[c + d*x]]) - (2*Sqrt[2]*b*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[a - b]
/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[e
*Tan[c + d*x]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*Sqrt[Sin[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2730

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3890

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{\int \sqrt{e\tan(c+dx)}\,dx}{a} - \frac{b\int \frac{\sqrt{e\tan(c+dx)}}{b+a\cos(c+dx)}\,dx}{a}$$

$$= \frac{e\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{ad} - \frac{\left(b\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{}{(b+a\cos(c}}{a}$$

$$= \frac{(2e)\,\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{\left(b\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{}{\sqrt{-}}}{a\sqrt{\sin(c+dx)}}$$

$$= -\frac{e\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{e\,\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} -$$

$$= \frac{\sqrt{e}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{\sqrt{e}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x,\right.}{2\sqrt{2}ad}$$

$$= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+d\right.}{2\sqrt{2}a}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{\right.}}{}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{\right.}}{}$$

Mathematica [C] time = 5.48, size = 224, normalized size = 0.54

$$-\frac{4\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{e\tan(c+dx)}\,(a\cos(c+dx)+b)\left(\frac{b\left(\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-1\right)-\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\mathrm{s}\right.\right.}{\sqrt{a-b}\sqrt{a+b}}\right.}{ad\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-4*(b + a*Cos[c + d*x])*Csc[c + d*x]*((-I)*EllipticPi[-I, ArcSin[Sqrt[Tan[
(c + d*x)/2]]], -1] + I*EllipticPi[I, ArcSin[Sqrt[Tan[(c + d*x)/2]]], -1] +
 (b*(-EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Tan[(c + d*x)/2]]]
, -1] + EllipticPi[Sqrt[a - b]/Sqrt[a + b], ArcSin[Sqrt[Tan[(c + d*x)/2]]],
 -1]))/(Sqrt[a - b]*Sqrt[a + b]))*Sqrt[Tan[(c + d*x)/2]]*Sqrt[e*Tan[c + d*x
]])/(a*d*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \tan(dx+c)}}{b \sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.75, size = 859, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(e*sin(d*x+c)/cos(d
*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(I*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-I*EllipticPi(((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-I*EllipticPi
(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+I*El
lipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*b-(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
,(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))+(a^2-b^2)^(1/2)*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)
),1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b
)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a-b*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))+Elli
pticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+
b))^(1/2)),1/2*2^(1/2))*a-b*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))-EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a+EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+E
llipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*b)/sin(d*x+c)^3*2^(1/2)*b/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \tan(dx+c)}}{b \sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\sqrt{e\tan(c+d\,x)}}{b+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(1/2))/(b + a*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*tan(c + d*x))/(a + b*sec(c + d*x)), x)
```

3.315 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=422

$$-\frac{2\sqrt{2}\,b\sqrt{\sin(c+dx)}\,\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a^2-b^2}\,\sqrt{-\cos(c+dx)}\,\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}\,b\sqrt{\sin(c+dx)}\,\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right.\right.}{ad\sqrt{a^2-b^2}\,\sqrt{-\cos(c+dx)}\,\sqrt{e\tan(c+dx)}}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)-1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)/e^(1/2
)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/
2)/e^(1/2)-2*b*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),b/(a-(a^
2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a/d/(a^2-b^2)^(1/2)/(-cos(d*x+c))
^(1/2)/(e*tan(d*x+c))^(1/2)+2*b*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c
))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a/d/(a^2-b^2)^(1
/2)/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3892, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2733, 2729, 2907, 1213, 537}

$$-\frac{2\sqrt{2}\,b\sqrt{\sin(c+dx)}\,\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a^2-b^2}\,\sqrt{-\cos(c+dx)}\,\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}\,b\sqrt{\sin(c+dx)}\,\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right.\right.}{ad\sqrt{a^2-b^2}\,\sqrt{-\cos(c+dx)}\,\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e]))
 + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e])
 - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sq
rt[2]*a*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]]/(2*Sqrt[2]*a*d*Sqrt[e]) - (2*Sqrt[2]*b*EllipticPi[b/(a - Sqrt[
a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[S
in[c + d*x]])/(a*Sqrt[a^2 - b^2]*d*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]
) + (2*Sqrt[2]*b*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d
*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*Sqrt[a^2 - b^2]*d*
Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2729

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan[e + f*x]])/Sqrt[Sin[e
 + f*x]], Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3892

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} - \frac{b\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= \frac{e\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx, x, e\tan(c+dx)\right)}{ad} - \frac{b\int \frac{\sqrt{e\cot(c+dx)}}{b+a\cos(c+dx)}\,dx}{a\sqrt{e\cot(c+dx)}\,\sqrt{e\tan(c}}$$

$$= \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{\left(b\sqrt{\sin(c+dx)}\right)\int \frac{}{(b+a\,\mathrm{co}}}{a\sqrt{-\cos(c+dx)}\,\sqrt{}}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(}\right.}{ad}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad} + \frac{\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,d\right.}{2a}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad\sqrt{e}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\,\mathrm{ta}\right.}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}} - \frac{\log\left(\sqrt{e}+\right.}{}$$

Mathematica [C] time = 12.80, size = 246, normalized size = 0.58

$$\frac{4\left(\left(b^2-a^2\right)\Pi\left(-i;\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right)\middle|-1\right)+a^2\left(-\Pi\left(i;\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right)\middle|-1\right)\right)-b^2\Pi\left(-\frac{\sqrt{a+b}}{\sqrt{a-b}};\sin^{-1}\left(\frac{}{\sqrt{\mathrm{ta}}}\right.\right.\right.}{ad\left(a^2-b^2\right)\sqrt{\tan\left(\frac{1}{2}(c\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (4*(a*(a - b)*EllipticF[ArcSin[1/Sqrt[Tan[(c + d*x)/2]]], -1] + (-a^2 + b^2
)*EllipticPi[-I, ArcSin[1/Sqrt[Tan[(c + d*x)/2]]], -1] - a^2*EllipticPi[I,
ArcSin[1/Sqrt[Tan[(c + d*x)/2]]], -1] + b^2*EllipticPi[I, ArcSin[1/Sqrt[Tan
[(c + d*x)/2]]], -1] - b^2*EllipticPi[-(Sqrt[a + b]/Sqrt[a - b]), ArcSin[1/
Sqrt[Tan[(c + d*x)/2]]], -1] - b^2*EllipticPi[Sqrt[a + b]/Sqrt[a - b], ArcS
in[1/Sqrt[Tan[(c + d*x)/2]]], -1]))/(a*(a^2 - b^2)*d*Sqrt[1 + Cot[(c + d*x)
/2]]*Sqrt[-1 + Tan[(c + d*x)/2]]*Sqrt[e*Tan[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sec\left(dx+c\right)+a\right)\sqrt{e \tan\left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

maple [B] time = 1.73, size = 2313, normalized size = 5.48

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)*(a^2-b^2)^(3/2)*a-(a^2-b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a+(a^2-b^2)^(3/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-(a^2-b^2)^(
3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*a+(a^2-b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*b+2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2)
)*b^3+2*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^3+(a^2-b^2)^(1/2)*Ell
ipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))
*a^3-(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))*b^3+(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^3+2*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(
1/2))*a^2*b^2-4*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-
b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^3-2*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2
))*a^2*b^2+4*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)
/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^3-2*EllipticF(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^3-(a^2-b^2)^(1/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
```

```
(1/2))*b^3+4*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(
1/2))*(a^2-b^2)^(1/2)*a^2*b-2*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2-2*(a^2-b^2)^(1/2)*EllipticPi((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2))
,1/2*2^(1/2))*a*b^2-I*(a^2-b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-I*(a^2-b^2)^(1/2)*EllipticPi(((
1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^3-I*(a^
2-b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*a+I*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^3-I*(a^2-b^2)^(1/2)*EllipticPi((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^3-2*(a
^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b
)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^2-3*(a^2-b^2)^(1/2)*EllipticP
i(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2*b
+3*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*a*b^2-3*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*b+3*(a^2-b^2)^(1/2)*
EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*a*b^2+I*(a^2-b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+I*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^3+I*(a^2-b^2)^(3/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))*b+3*I*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*b-3*I*(a^2-b^2)^(1/2)*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*b^2-3*I*(a
^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*a^2*b+3*I*(a^2-b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b^2+2*EllipticPi(((1-cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^
(1/2))*b^4-2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)
/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^4)*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))/sin(d*x+c)^2/cos(d*x+c)*(1+cos(d*x+c)
)^2/(e*sin(d*x+c)/cos(d*x+c))^(1/2)*2^(1/2)/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2
)^(1/2)+a-b)/(a^2-b^2)^(1/2)/(a-b)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{\sqrt{e\tan(c+d\,x)}\,\left(b+a\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(1/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*tan(c + d*x))^(1/2)*(b + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\tan(c+dx)}\,(a+b\sec(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*tan(c + d*x))*(a + b*sec(c + d*x))), x)
```

3.316 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=863

$$\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\,\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)\sqrt{e\tan(c+dx)}\,b^3}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\,\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\right.\right.}{a(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b^2)/d/e^(3/2)*2^
(1/2)-1/2*b^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/(a^2-b^2)/d/
e^(3/2)*2^(1/2)-1/2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b
^2)/d/e^(3/2)*2^(1/2)+1/2*b^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2)
)/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)-1/4*a*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/
2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/d/e^(3/2)*2^(1/2)+1/4*b^2*ln(e^(1/2)-2^(1/
2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)+1
/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/
d/e^(3/2)*2^(1/2)-1/4*b^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*t
an(d*x+c))/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)-2*(a-b*sec(d*x+c))/(a^2-b^2)/d/e/(
e*tan(d*x+c))^(1/2)+2*b^3*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),
-(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a
/(a-b)^(3/2)/(a+b)^(3/2)/d/e^2/sin(d*x+c)^(1/2)-2*b^3*EllipticPi(sin(d*x+c)
^(1/2)/(1+cos(d*x+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(
1/2)*(e*tan(d*x+c))^(1/2)/a/(a-b)^(3/2)/(a+b)^(3/2)/d/e^2/sin(d*x+c)^(1/2)-
2*b*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(
c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/(a^2-b^2)/d/e^2/sin(2*d*x+2*c)^
(1/2)-2*b*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/(a^2-b^2)/d/e^3
```

Rubi [A] time = 1.25, antiderivative size = 863, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 23, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.920, Rules used = {3893, 3882, 3884, 3476, 329, 297, 1162, 617, 204, 1165, 628, 2613, 2615, 2572, 2639, 3890, 2733, 2730, 2906, 2905, 490, 1213, 537}

$$\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\,\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)\sqrt{e\tan(c+dx)}\,b^3}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\,\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};\sin^{-1}\left(\right.\right.}{a(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)
*d*e^(3/2)) - (b^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*a*(a^2 - b^2)*d*e^(3/2)) - (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)*d*e^(3/2)) + (b^2*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) - (a*Log[Sqrt
[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2
 - b^2)*d*e^(3/2)) + (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt
[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) + (a*Log[Sqrt[e] + S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2 - b^2)
*d*e^(3/2)) - (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) - (2*(a - b*Sec[c + d*x]))/
((a^2 - b^2)*d*e*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt[2]*b^3*Sqrt[Cos[c + d*x]]*
EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + C
os[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d*e
^2*Sqrt[Sin[c + d*x]]) - (2*Sqrt[2]*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[
a - b]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*
Sqrt[e*Tan[c + d*x]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d*e^2*Sqrt[Sin[c + d*x
]]) + (2*b*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/
```

```
((a^2 - b^2)*d*e^2*Sqrt[Sin[2*c + 2*d*x]]) - (2*b*Cos[c + d*x]*(e*Tan[c + d
*x])^(3/2))/((a^2 - b^2)*d*e^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2730

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
```

```
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3890

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3893

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[1/(a^2 - b^2), Int[(e*Cot[c + d*x])^m*(a - b*Csc[c
+ d*x]), x], x] + Dist[b^2/(e^2*(a^2 - b^2)), Int[(e*Cot[c + d*x])^(m + 2)/
(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2,
 0] && ILtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{a-b\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2-b^2} + \frac{b^2\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx}{\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} + \frac{2\int\left(-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)\right)\sqrt{e\tan(c}{\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} - \frac{a\int\sqrt{e\tan(c+dx)}\,dx}{\left(a^2-b^2\right)e^2} - \frac{b\int\sec(c+}{(}$$

$$= -\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{\left(a^2-b^2\right)de^3} + (2$$

$$= -\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{\left(a^2-b^2\right)de^3} - (2$$

$$= -\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{\left(a^2-b^2\right)de^3} + b^2$$

$$= \frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{3/2}} - b^2\log\left(\sqrt{e}+\right.$$

$$= -\frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}} + \frac{b^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}} - a\,\mathrm{lo}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right)de^{3/2}} - \frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}} - a\tan^{-}$$

Mathematica [C] time = 27.53, size = 1571, normalized size = 1.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-2*(b - a*Cos[c + d*x])*Csc[c + d*x])/
(-a^2 + b^2) + (2*b*Sin[c + d*x])/(-a^2 + b^2))*Tan[c + d*x]^2)/(d*(a + b*S
ec[c + d*x])*(e*Tan[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*T
an[c + d*x]^(3/2)*(-1/12*((-a^2 + 3*b^2)*Sec[c + d*x]*(6*Sqrt[2]*(a^2 - b^2
)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*a^2*ArcTan[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
 - (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan
[c + d*x]])/(a^2 - b^2)^(1/4)] + (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan
[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] - 3*Sqrt[2]*a^
2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*b^2*Log[1
- Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*a^2*Log[1 + Sqrt[2
]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 3*Sqrt[2]*b^2*Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]] + (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt
[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Ta
n[c + d*x]] - (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + (1
+ I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] + 8*a
```

```
*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Tan[c + d*x]^(3/2))*(a + b*Sqrt[1 + Tan[c + d*x]^2]))/((a^3 - a*b^2)*
(b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)) + (b*Cos[2*(c + d*x)]*Sec[c + d*
x]^2*(-84*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 84*Sqrt[2]*b*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + ((42 + 42*I)*(-a^2 + 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2
- b^2)^(1/4)) + ((42 + 42*I)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt
[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + 42*Sqrt[2
]*b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 42*Sqrt[2]*b*Log[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + ((21 + 21*I)*(a^2 - 2*b^2)*
Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]]
+ I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + ((21 + 21*I)*(-a^2 + 2*b
^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*
x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + (112*a^3*AppellF1[3/
4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c +
d*x]^(3/2))/(a^2 - b^2) - (168*a*b^2*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*
x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (
24*a*b^2*AppellF1[7/4, 1/2, 1, 11/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/
(a^2 - b^2)]*Tan[c + d*x]^(7/2))/(a^2 - b^2) - (168*a*Tan[c + d*x]^(3/2))/S
qrt[1 + Tan[c + d*x]^2])*(a + b*Sqrt[1 + Tan[c + d*x]^2]))/(84*a*(b + a*Cos
[c + d*x])*(-1 + Tan[c + d*x]^2)*Sqrt[1 + Tan[c + d*x]^2])))/((a - b)*(a +
b)*d*(a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

maple [B] time = 1.71, size = 6426, normalized size = 7.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d x)}{(e \tan (c+d x))^{3/2} (b+a \cos (c+d x))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(3/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/((e*tan(c + d*x))^(3/2)*(b + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e \tan (c+dx))^{\frac{3}{2}} (a+b \sec (c+dx))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*tan(c + d*x))**(3/2)*(a + b*sec(c + d*x))), x)
```

3.317 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=836

$$-\frac{2\sqrt{2}\,\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)\sqrt{\sin(c+dx)}\,b^3}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}\,\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)\sqrt{s}}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(}}$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b^2)/d/e^(5/2)*2^
(1/2)-1/2*b^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/(a^2-b^2)/d/
e^(5/2)*2^(1/2)-1/2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b
^2)/d/e^(5/2)*2^(1/2)+1/2*b^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2)
)/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)+1/4*a*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/
2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/d/e^(5/2)*2^(1/2)-1/4*b^2*ln(e^(1/2)-2^(1/
2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)-1
/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/
d/e^(5/2)*2^(1/2)+1/4*b^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*t
an(d*x+c))/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)-2*b^3*EllipticPi((-cos(d*x+c))^(1/
2)/(1+sin(d*x+c))^(1/2),b/(a-(a^2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a
/(a^2-b^2)^(3/2)/d/e^2/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)+2*b^3*Ellip
ticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(
1/2)*sin(d*x+c)^(1/2)/a/(a^2-b^2)^(3/2)/d/e^2/(-cos(d*x+c))^(1/2)/(e*tan(d*
x+c))^(1/2)-1/3*b*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(c
os(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/(a^2-b^2)/d/e^2/(
e*tan(d*x+c))^(1/2)-2/3*(a-b*sec(d*x+c))/(a^2-b^2)/d/e/(e*tan(d*x+c))^(3/2)
```

Rubi [A] time = 1.06, antiderivative size = 836, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 20, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3893, 3882, 3884, 3476, 329, 211, 1165, 628, 1162, 617, 204, 2614, 2573, 2641, 3892, 2733, 2729, 2907, 1213, 537}

$$-\frac{2\sqrt{2}\,\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)\sqrt{\sin(c+dx)}\,b^3}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}\,\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\sin^{-1}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)\sqrt{s}}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)
*d*e^(5/2)) - (b^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*a*(a^2 - b^2)*d*e^(5/2)) - (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)*d*e^(5/2)) + (b^2*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) + (a*Log[Sqrt
[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2
 - b^2)*d*e^(5/2)) - (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt
[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) - (a*Log[Sqrt[e] + S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2 - b^2)
*d*e^(5/2)) + (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) - (2*(a - b*Sec[c + d*x]))/
(3*(a^2 - b^2)*d*e*(e*Tan[c + d*x])^(3/2)) - (2*Sqrt[2]*b^3*EllipticPi[b/(a
 - Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1
]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)^(3/2)*d*e^2*Sqrt[-Cos[c + d*x]]*Sqrt[e
*Tan[c + d*x]]) + (2*Sqrt[2]*b^3*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin
[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*(a
^2 - b^2)^(3/2)*d*e^2*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]) + (b*Ellipt
icF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*(a^2 - b^2)*
d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
```

```
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2729

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan[e + f*x]])/Sqrt[Sin[e
 + f*x]], Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3892

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3893

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[1/(a^2 - b^2), Int[(e*Cot[c + d*x])^m*(a - b*Csc[c
+ d*x]), x], x] + Dist[b^2/(e^2*(a^2 - b^2)), Int[(e*Cot[c + d*x])^(m + 2)/
(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2,
 0] && ILtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{\int \frac{a-b\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2-b^2} + \frac{b^2\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2} + \frac{b^2\int \frac{}{\sqrt{e\,\mathrm{t}}}}{a\left(a^2\right.}$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} - \frac{a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{\left(a^2-b^2\right)e^2} + \frac{b\int \frac{\sec(c+dx}{\sqrt{e\tan(c+d}}}{3\left(a^2-b^2\right.}$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx, x, e\tan(c\right.}{\left(a^2-b^2\right)de}$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c\right.}{a\left(a^2-b^2\right)de^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} + \frac{bF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{s}}{3\left(a^2-b^2\right)de^2\sqrt{e\tan(c+}}$$

$$= -\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a\left(a^2-b^2\right)de^{5/2}} + \frac{b^2\log\left(\sqrt{e}\right.}{}$$

$$= -\frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a\left(a^2-b^2\right)de^{5/2}} + \frac{b^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a\left(a^2-b^2\right)de^{5/2}} + \frac{a\,\mathrm{lc}}{}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right)de^{5/2}} - \frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a\left(a^2-b^2\right)de^{5/2}} - \frac{a\tan^{-}}{\sqrt{}}$$

Mathematica [C] time = 23.90, size = 2169, normalized size = 2.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*((2*a)/(3*(a^2 - b^2)) - (2*(-a + b*Cos[c + d*x])*Csc
[c + d*x]^2)/(3*(-a^2 + b^2)))*Sec[c + d*x]*Tan[c + d*x]^3)/(d*(a + b*Sec[c
 + d*x])*(e*Tan[c + d*x])^(5/2)) - ((b + a*Cos[c + d*x])*Sec[c + d*x]*Tan[c
 + d*x]^(5/2)*((2*(3*a^2 - 5*b^2)*Sec[c + d*x]^3*(a + b*Sqrt[1 + Tan[c + d*
x]^2])*(((-1/8 + I/8)*a*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/
(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2
 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]]))/(Sqrt[b]*(a^2
- b^2)^(3/4)) + (5*b*(-a^2 + b^2)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[c + d*x]
^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]]*Sqrt[1 + Tan[c + d
*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*T
an[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2
)*(a^2 - b^2*(1 + Tan[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*
x]^2)^2) + (8*a*b*Sec[c + d*x]^2*(a + b*Sqrt[1 + Tan[c + d*x]^2])*((Sqrt[b]
*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2 + b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Tan[c + d*x]] +
 b*Tan[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Tan[c + d*x]] + b*Tan[c + d*x]]))/(4*Sqrt[2]*(-a^2 + b^2)^(3/4)) + (
5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]])/(Sqrt[1 + Tan[c + d*x]^2]*(-5*(a^2 -
 b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
 - b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, -Tan[c + d
*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)*(-a^2 + b^2*(1 +
Tan[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2)) + ((3
*a^2 - 3*b^2)*Cos[2*(c + d*x)]*Sec[c + d*x]^3*(a + b*Sqrt[1 + Tan[c + d*x]^
2])*((-20*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/a + (20*Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/a + ((10 - 10*I)*(a^2 - 2*b^2)*ArcTa
n[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(
a^2 - b^2)^(3/4)) - ((10 - 10*I)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (10
*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/a + (10*Sqrt[2
]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/a + ((5 - 5*I)*(a^2 -
 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c
+ d*x]] + I*b*Tan[c + d*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - ((5 - 5*I)*(a^
2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan
[c + d*x]] + I*b*Tan[c + d*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (8*b*Appell
F1[5/4, 1/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan
[c + d*x]^(5/2))/(-a^2 + b^2) - (200*b*(-a^2 + b^2)*AppellF1[1/4, 1/2, 1, 5
/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]])/
(Sqrt[1 + Tan[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*AppellF1[5/4, 1/2, 2
, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*Ap
pellF1[5/4, 3/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
)*Tan[c + d*x]^2)*(-a^2 + b^2*(1 + Tan[c + d*x]^2)))))/(20*(b + a*Cos[c + d
*x])*(1 - Tan[c + d*x]^2)*(1 + Tan[c + d*x]^2))))/(6*(a - b)*(a + b)*d*(a +
 b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

maple [B] time = 1.93, size = 16178, normalized size = 19.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{(e\tan(c+d\,x))^{5/2}\,(b+a\,\cos(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(5/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*tan(c + d*x))^(5/2)*(b + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\tan(c+dx))^{\frac{5}{2}}\,(a+b\sec(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
[Out] Integral(1/((e*tan(c + d*x))**(5/2)*(a + b*sec(c + d*x))), x)
```

3.318 $\int \sqrt{a+b\sec(c+dx)}\,\tan^5(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{2\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4d}+\frac{2(a+b\sec(c+dx))^{9/2}}{9b^4d}-\frac{6a(a+b\sec}{7b^4}$$

```
[Out] -2/3*a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(3/2)/b^4/d+2/5*(3*a^2-2*b^2)*(a+b*sec(
d*x+c))^(5/2)/b^4/d-6/7*a*(a+b*sec(d*x+c))^(7/2)/b^4/d+2/9*(a+b*sec(d*x+c))
^(9/2)/b^4/d-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+b*sec
(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 207}

$$\frac{2\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4d}+\frac{2(a+b\sec(c+dx))^{9/2}}{9b^4d}-\frac{6a(a+b\sec}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + b*Se
c[c + d*x]])/d - (2*a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(3/2))/(3*b^4*d) +
 (2*(3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d) - (6*a*(a + b*Sec
[c + d*x])^(7/2))/(7*b^4*d) + (2*(a + b*Sec[c + d*x])^(9/2))/(9*b^4*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\left(b^2-x^2\right)^2}{x}dx, x, b\sec(c+dx)\right)}{b^4 d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-a^2+b^2+2ax^2-x^4\right)^2}{-a+x^2}dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(b^4-a\left(a^2-2b^2\right)x^2+\left(3a^2-2b^2\right)x^4-3ax^6+x^8+\frac{ab^4}{-a+x^2}\right) c$$
$$= \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4 d} + \frac{2\left(3a^2-2b\right.}{}$$
$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a\left(a^2-2b^2\right)(a}{3}$$

Mathematica [A] time = 6.31, size = 254, normalized size = 1.50

$$\frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{4\left(a^2+21b^2\right)\sec^2(c+dx)}{105b^2}-\frac{4a\left(21b^2-4a^2\right)\sec(c+dx)}{315b^3}+\frac{2\left(-16a^4+84a^2b^2+315b^4\right)}{315b^4}+\frac{2a\sec^3(c+dx)}{63b}+\frac{2}{9}\sec^4(c+d\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(-16*a^4 + 84*a^2*b^2 + 315*b^4))/(315*b^4) -
 (4*a*(-4*a^2 + 21*b^2)*Sec[c + d*x])/(315*b^3) - (4*(a^2 + 21*b^2)*Sec[c +
 d*x]^2)/(105*b^2) + (2*a*Sec[c + d*x]^3)/(63*b) + (2*Sec[c + d*x]^4)/9))/d
 - (Sqrt[a*Cos[c + d*x]]*(-Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c +
d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x]^2)/(d*Sqrt[b + a*Cos[c + d*x]]*(1 - Cos[c + d*x
]^2))
```

fricas [A] time = 1.64, size = 425, normalized size = 2.51

$$\left[\frac{315\sqrt{a}b^4\cos(dx+c)^4\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2+4\left(2a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*sqrt(a)*b^4*cos(d*x + c)^4*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) + 4*(5*a*b^3*cos(d*x + c) - (16*a^4 - 84*a
^2*b^2 - 315*b^4)*cos(d*x + c)^4 + 35*b^4 + 2*(4*a^3*b - 21*a*b^3)*cos(d*x
+ c)^3 - 6*(a^2*b^2 + 21*b^4)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c)))/(b^4*d*cos(d*x + c)^4), 1/315*(315*sqrt(-a)*b^4*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 b))*cos(d*x + c)^4 + 2*(5*a*b^3*cos(d*x + c) - (16*a^4 - 84*a^2*b^2 - 315*
b^4)*cos(d*x + c)^4 + 35*b^4 + 2*(4*a^3*b - 21*a*b^3)*cos(d*x + c)^3 - 6*(a
```

```
^2*b^2 + 21*b^4)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
b^4*d*cos(d*x + c)^4)]
```

giac [B] time = 3.76, size = 966, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 2/315*(315*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1
/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
 a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(315*(sqrt(a - b)*tan(1/2*d*x
 + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*
a*tan(1/2*d*x + 1/2*c)^2 + a + b))^8*a - 3150*(sqrt(a - b)*tan(1/2*d*x + 1/
2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan
(1/2*d*x + 1/2*c)^2 + a + b))^7*sqrt(a - b)*a + 210*(sqrt(a - b)*tan(1/2*d*
x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2
*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^6*(39*a^2 - 5*a*b - 32*b^2) - 630*(sqrt
(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*
d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^5*(9*a^2 + 15*a*b - 1
6*b^2)*sqrt(a - b) - 252*(25*a^3 - 37*a^2*b + 80*a*b^2 - 72*b^3)*(sqrt(a -
b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x +
 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^4 + 945*a^5 + 3864*a^4*b +
 2562*a^3*b^2 + 2448*a^2*b^3 - 1083*a*b^4 + 224*b^5 + 42*(255*a^3 + 2*a^2*b
 + 655*a*b^2 - 288*b^3)*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b))^3*sqrt(a - b) - 18*(175*a^4 - 483*a^3*b + 1113*a^2*b^2 - 773*a*b^3
+ 448*b^4)*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c
)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2 - 1
8*(105*a^4 + 637*a^3*b + 203*a^2*b^2 + 447*a*b^3 - 112*b^4)*(sqrt(a - b)*ta
n(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*
c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*sqrt(a - b))/(sqrt(a - b)*tan(1
/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^
4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^9)*sgn(cos(d*x + c))
/d
```

maple [B] time = 2.05, size = 3268, normalized size = 19.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x)
```

```
[Out] -1/2520/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+co
s(d*x+c))^4*(630*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(
d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((
b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c
)^2/(a-b)^(1/2))*cos(d*x+c)^6*b^6-280*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d
*x+c))^2)^(3/2)*(a-b)^(1/2)*b^4-630*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)
^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+
b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(
1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*b^6+630*ln(-2*(-1+cos(d*x+c)
)*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^7*a*b^5-1
197*cos(d*x+c)^6*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(3/2)*b^4+1260*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a
```

```
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2
/(a-b)^(1/2))*cos(d*x+c)^6*a*b^5-1260*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(
a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x
+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-
b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*a*b^5+2625*cos(d*x+c)^4*
(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*b^4+168*co
s(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)
*b^4-2121*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
*(a-b)^(1/2)*b^5-2121*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)*(a-b)^(1/2)*b^5-189*cos(d*x+c)^5*(a-b)^(1/2)*((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*b^4+2744*cos(d*x+c)^3*((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*b^4-840*cos(d*x+c)*((b+a*co
s(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*b^4-399*cos(d*x+c)
^7*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*b^4+128
*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1
/2)*a^5+128*cos(d*x+c)^7*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/
2)*(a-b)^(1/2)*a^5+1260*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)
+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(
d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^7*a^2*b^4-630*ln(-(-1+cos(d*x+c))*(2*cos(d
*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*
cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1
/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^7*a*b^5-1260*ln(-2*
(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x
+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*
x+c)^7*a^2*b^4-648*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(1/2)*(a-b)^(1/2)*a^2*b^3+128*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)*a^4*b-648*cos(d*x+c)^6*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)*a^3*b^2-2121*cos(d*x+c)^6
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)*a*b^4-212
1*cos(d*x+c)^7*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(
1/2)*a*b^4-648*cos(d*x+c)^7*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(1/2)*(a-b)^(1/2)*a^3*b^2+128*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(1+
cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)*a^4*b-648*cos(d*x+c)^5*((b+a*cos(d*x+c))*c
os(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)*a^2*b^3+1260*ln(4*a^(1/2)*cos
(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+
a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*a^(3/2
)*cos(d*x+c)^7*(a-b)^(1/2)*b^4+1260*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c
)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*a^(1/2)*cos(d*x+c)^6*(a-b)^(1
/2)*b^5-72*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2
)*(a-b)^(1/2)*a^2*b^2-192*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(3/2)*(a-b)^(1/2)*a^3*b+1008*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(
d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*a*b^3+120*cos(d*x+c)^4*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*a^2*b^2-64*cos(d*x+
c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*a^3*b
+216*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b
)^(1/2)*a*b^3+48*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(3/2)*(a-b)^(1/2)*a^2*b^2-40*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a*b^3-24*cos(d*x+c)^7*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*a^2*b^2-64*cos(d*x+c)^6*((b+a*c
os(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*a^3*b+336*cos(d*x
+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*a*b^
3-24*cos(d*x+c)^5*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(3/2)*a^2*b^2-192*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(3/2)*(a-b)^(1/2)*a^3*b+968*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*a*b^3+144*cos(d*x+c)^3*((b+a*cos(d*x+c)
)*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*a^2*b^2-120*cos(d*x+c)^2*(
```

```
(b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*a*b^3)/sin(
d*x+c)^8/cos(d*x+c)^4/((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)/
(a-b)^(1/2)/b^4
```

maxima [A] time = 0.43, size = 191, normalized size = 1.13

$$\frac{315\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+630\sqrt{a+\frac{b}{\cos(dx+c)}}+\frac{70\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{9}{2}}}{b^4}-\frac{270\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{7}{2}}a}{b^4}+\frac{378\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}a^2}{b^4}-}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/315*(315*sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos
(d*x + c)) + sqrt(a))) + 630*sqrt(a + b/cos(d*x + c)) + 70*(a + b/cos(d*x +
 c))^(9/2)/b^4 - 270*(a + b/cos(d*x + c))^(7/2)*a/b^4 + 378*(a + b/cos(d*x
+ c))^(5/2)*a^2/b^4 - 210*(a + b/cos(d*x + c))^(3/2)*a^3/b^4 - 252*(a + b/c
os(d*x + c))^(5/2)/b^2 + 420*(a + b/cos(d*x + c))^(3/2)*a/b^2)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^5\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,\tan^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**5, x)
```

3.319 $\int \sqrt{a+b\sec(c+dx)}\,\tan^3(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d}-\frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2\sqrt{a+b\sec(c+dx)}}{d}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

```
[Out] -2/3*a*(a+b*sec(d*x+c))^(3/2)/b^2/d+2/5*(a+b*sec(d*x+c))^(5/2)/b^2/d+2*arct
anh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 207}

$$\frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d}-\frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2\sqrt{a+b\sec(c+dx)}}{d}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Sec
[c + d*x]])/d - (2*a*(a + b*Sec[c + d*x])^(3/2))/(3*b^2*d) + (2*(a + b*Sec[
c + d*x])^(5/2))/(5*b^2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\,(b^2-x^2)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2(-a^2+b^2+2ax^2-x^4)}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d}$$

$$= -\frac{2\,\text{Subst}\left(\int \left(b^2+ax^2-x^4+\frac{ab^2}{-a+x^2}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d}$$

$$= -\frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d} + \frac{2(a+b\sec(c+d}{5b^2d}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a+b\sec(c}{3b^2d}$$

Mathematica [A] time = 6.22, size = 194, normalized size = 1.94

$$\frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{2(2a^2+15b^2)}{15b^2}+\frac{2a\sec(c+dx)}{15b}+\frac{2}{5}\sec^2(c+dx)\right)}{d}+\frac{\sin^2(c+dx)\sqrt{a\cos(c+dx)}\sqrt{a+b\sec(c+d}{d\left(1-\cos^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((-2*(2*a^2 + 15*b^2))/(15*b^2) + (2*a*Sec[c + d*
x])/(15*b) + (2*Sec[c + d*x]^2)/5))/d + (Sqrt[a*Cos[c + d*x]]*(-Log[1 - Sqr
t[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*
x]]/Sqrt[a*Cos[c + d*x]]])*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]^2)/(d*Sqrt
[b + a*Cos[c + d*x]]*(1 - Cos[c + d*x]^2))
```

fricas [A] time = 1.45, size = 311, normalized size = 3.11

$$\left[\frac{15\sqrt{a}\,b^2\cos(dx+c)^2\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2-4\left(2a\cos(dx+c)^2+b\cos(dx+c)\right)\right.}{30\,b^2d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*b^2*cos(d*x + c)^2*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(
d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c))) + 4*(a*b*cos(d*x + c) - (2*a^2 + 15*b^2)*cos
(d*x + c)^2 + 3*b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(b^2*d*cos(d*
x + c)^2), -1/15*(15*sqrt(-a)*b^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))*cos(d*x + c)^2 - 2*(a
*b*cos(d*x + c) - (2*a^2 + 15*b^2)*cos(d*x + c)^2 + 3*b^2)*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c)))/(b^2*d*cos(d*x + c)^2)]
```

giac [B] time = 2.17, size = 539, normalized size = 5.39

$$-\frac{2\left(\frac{15\,a\arctan\left(-\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\left(15\left(\sqrt{a-b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,a\cdots\right.}\right.\right.}{\cdots}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(15*(sqrt(a - b)*tan(1/2*d*x +
 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*
tan(1/2*d*x + 1/2*c)^2 + a + b))^4*a - 30*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)
^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2
*d*x + 1/2*c)^2 + a + b))^3*(a + 2*b)*sqrt(a - b) + 20*(sqrt(a - b)*tan(1/2
*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4
- 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2*(4*a*b - 3*b^2) - 15*a^3 - 10*a^2*
b - 35*a*b^2 + 12*b^3 + 10*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan
(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2
 + a + b))*(3*a^2 - a*b + 6*b^2)*sqrt(a - b))/(sqrt(a - b)*tan(1/2*d*x + 1/
2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan
(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^5)*sgn(cos(d*x + c))/d
```

maple [B] time = 1.72, size = 2342, normalized size = 23.42

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x)
```

```
[Out] -1/60/d*(1+cos(d*x+c))*(-1+cos(d*x+c))^4*(-36*(a-b)^(1/2)*cos(d*x+c)*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*b^2+6*(a-b)^(1/2)*cos(d*x+c)
^5*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*b^2+6*(a-b)^(1/2)*c
os(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*b^2+54*(a-
b)^(1/2)*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*
b^3+8*(a-b)^(1/2)*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(1/2)*a^3+8*(a-b)^(1/2)*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos
(d*x+c))^2)^(1/2)*a^3+54*(a-b)^(1/2)*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x
+c)/(1+cos(d*x+c))^2)^(1/2)*b^3+18*(a-b)^(1/2)*cos(d*x+c)^4*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*b^2+30*cos(d*x+c)^5*ln(-2*(-1+cos(d*x
+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^2-15*cos(d*
x+c)^5*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^
(1/2))*a*b^3-30*cos(d*x+c)^5*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*
((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d
*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)
/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^2+15*cos(d*x+c)^5*ln(-(-1+cos(d*x+c))*(2*c
os(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-
2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a*b^3+30*cos(d*x+c)^4*ln(-
```

```
2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1
+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a*b^
3-30*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d
*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)
^2/(a-b)^(1/2))*a*b^3-30*(a-b)^(1/2)*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x
+c)/(1+cos(d*x+c))^2)^(3/2)*b^2-15*cos(d*x+c)^4*ln(-2*(-1+cos(d*x+c))*(2*co
s(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2
*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*b^4+15*cos(d*x+c)^4*ln(-(-1
+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*b^4-12*((
b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*b^2+54*(a-b)
^(1/2)*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a*
b^2+8*(a-b)^(1/2)*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(1/2)*a^2*b+8*(a-b)^(1/2)*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)*a^2*b+54*(a-b)^(1/2)*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a*b^2-30*(a-b)^(1/2)*a^(3/2)*cos(d*x+c)^5*l
n(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x
+c)+2*b)*b^2-30*(a-b)^(1/2)*a^(1/2)*cos(d*x+c)^4*ln(4*a^(1/2)*cos(d*x+c)*((
b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+
c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^3-12*(a-b)^(1/
2)*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a*b-4*
(a-b)^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)
*a*b-4*(a-b)^(1/2)*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(3/2)*a*b-12*(a-b)^(1/2)*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(3/2)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)/((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)/sin(d*x+c)^8/cos(d*x+c)^2/b^
2/(a-b)^(1/2)
```

maxima [A] time = 0.42, size = 108, normalized size = 1.08

$$-\frac{15\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+30\sqrt{a+\frac{b}{\cos(dx+c)}}-\frac{6\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{b^2}+\frac{10\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{b^2}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(
d*x + c)) + sqrt(a))) + 30*sqrt(a + b/cos(d*x + c)) - 6*(a + b/cos(d*x + c)
)^(5/2)/b^2 + 10*(a + b/cos(d*x + c))^(3/2)*a/b^2)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^3\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,\tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**3, x)
```

3.320 $\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a+b\sec(c+dx)}}{d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+b*sec(d*x+c))^(1/
2)/d
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3885, 50, 63, 207}

$$\frac{2\sqrt{a+b\sec(c+dx)}}{d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + b*Se
c[c + d*x]])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{2\sqrt{a+b\sec(c+dx)}}{d} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{2\sqrt{a+b\sec(c+dx)}}{d} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$
$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+b\sec(c+dx)}}{d}$$

Mathematica [B] time = 0.28, size = 137, normalized size = 2.69

$$\frac{\sqrt{a+b\sec(c+dx)}\left(2\sqrt{a\cos(c+dx)+b} + \sqrt{a\cos(c+dx)}\,\log\left(1-\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}\right) - \sqrt{a\cos(c+dx)}\,\log\left(\frac{\sqrt{a\cos(c}}{\sqrt{a\cos}}\right.\right.}{d\sqrt{a\cos(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] ((2*Sqrt[b + a*Cos[c + d*x]] + Sqrt[a*Cos[c + d*x]]*Log[1 - Sqrt[b + a*Cos[
c + d*x]]/Sqrt[a*Cos[c + d*x]]] - Sqrt[a*Cos[c + d*x]]*Log[1 + Sqrt[b + a*C
os[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[b + a
*Cos[c + d*x]])
```

fricas [A] time = 0.71, size = 192, normalized size = 3.76

$$\left[\frac{\sqrt{a}\,\log\left(-8\,a^2\cos\left(dx+c\right)^2 - 8\,ab\cos\left(dx+c\right) - b^2 + 4\left(2\,a\cos\left(dx+c\right)^2 + b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right) + \right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a
*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))) + 4*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/d, (sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x
+ c) + b)) + 2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/d]
```

giac [B] time = 0.41, size = 185, normalized size = 3.63

$$\frac{2\left(\frac{a\arctan\left(-\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - \sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + a + b} + \sqrt{a-b}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{}{\sqrt{a-b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - \sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - b}}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 2*(a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x +
 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)
+ sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*b/(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2
 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d
*x + 1/2*c)^2 + a + b) - sqrt(a - b)))*sgn(cos(d*x + c))/d
```

maple [A] time = 0.24, size = 42, normalized size = 0.82

$$\frac{2\sqrt{a+b\sec(dx+c)}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x)
```

```
[Out] 1/d*(2*(a+b*sec(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1
/2)))
```

maxima [A] time = 0.42, size = 67, normalized size = 1.31

$$\frac{\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{\cos(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] (sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c))
 + sqrt(a))) + 2*sqrt(a + b/cos(d*x + c)))/d
```

mupad [B] time = 1.86, size = 47, normalized size = 0.92

$$\frac{2\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{d}-\frac{2\sqrt{a}\ \mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(a + b/cos(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + b/cos(c + d*x))^(1
/2)/a^(1/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sec(c+dx)}\ \tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x), x)
```

3.321 $\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh((a+b*sec(d*x+c)
)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2))*(a+b)^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3885, 898, 1287, 206, 207}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d - (Sqrt[a - b]*ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/d - (Sqrt[a + b]*ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(-\frac{a}{b^2(a-x^2)}+\frac{a+b}{2b^2(a+b-x^2)}+\frac{-a+b}{2b^2(-a+b+x^2)}\right)dx, x, \sqrt{a+b\text{s}}\right.}{d}$$

$$= \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a+b+}\right.}{}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+}}{}$$

Mathematica [B] time = 4.38, size = 333, normalized size = 3.14

$$\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\sec(c+dx)}\left(\sqrt{b}\sqrt{b-a}\sqrt{\frac{a\cos(c+dx)+b}{b\cos(c+dx)+b}}\sin^{-1}\left(\frac{\sqrt{b-a}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}}{\sqrt{b}}\right)+2\sqrt{a}\right.}{d(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(Sqrt[-a - b]*A
rcTanh[(Sqrt[-a - b]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2])/Sqrt[-((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)]]*Sqrt[(-b - a*Cos[c + d*x])/(1 + Cos[c +
d*x])] + 2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])])/
Sqrt[(b + a*Cos[c + d*x])/(1 + Cos[c + d*x])]]*Sqrt[(b + a*Cos[c + d*x])/(1
 + Cos[c + d*x])] + Sqrt[b]*Sqrt[-a + b]*ArcSin[(Sqrt[-a + b]*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])])/Sqrt[b]]*Sqrt[(b + a*Cos[c + d*x])/(b + b*Cos[c +
 d*x])])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

fricas [B] time = 1.08, size = 2132, normalized size = 20.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2
*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d
*x + c))) + sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 -
4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 +
2*cos(d*x + c) + 1)) + sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)
^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*
x + c)^2 - 2*cos(d*x + c) + 1)))/d, 1/4*(2*sqrt(-a - b)*arctan(2*sqrt(-a -
```

```
b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x
+ c) + b)) + 2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2
 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b
)/cos(d*x + c))) + sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 +
 b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)))/d, -1/4*(2*sqrt(-a + b)*arctan(-2*sqrt(-a + b)
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x +
c) + b)) - 2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 -
 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b
^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)
^2 - 2*cos(d*x + c) + 1)))/d, -1/2*(sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) +
 b)) - sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - sqrt(a)*log(-8*a^2*cos(
d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x +
 c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/d, -1/4*(4*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a
*cos(d*x + c) + b)) - sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^
2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a
*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a +
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/d, -1/4*(4*sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c
) + b)) - 2*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - sqrt(a - b)*log(-(
(8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 +
b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*
a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/d, -1/4*
(4*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d
*x + c)/(2*a*cos(d*x + c) + b)) + 2*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) +
 b)) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2
*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)))/d, -1/2*(2*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + sqrt(-a + b)*arc
tan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((
2*a - b)*cos(d*x + c) + b)) - sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)))/d
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 1.52, size = 575, normalized size = 5.42

$$\frac{\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,\sqrt{4}\,\cos(dx+c)\,(-1+\cos(dx+c))\left(\sqrt{a+b}\,\ln\left(-\frac{2\left(2\cos(dx+c)\sqrt{a+b}\,\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(1+\cos(dx+c))^2}}+2a\cos(dx+\right.}{-1+\cos(d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)*cos(d*x+c)*(-1+cos(d*x+c)
)*((a+b)^(1/2)*ln(-2*(2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*co
s(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*(a-b)^(1/2
)-2*a^(1/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2
)+4*a*cos(d*x+c)+2*b)*(a-b)^(1/2)-a*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)
^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+
b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(
1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))+ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(
1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*
cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/
2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*b)/sin(d*x+c)^2/((b+a*cos(d*x+c))*cos(d*x+c
)/(1+cos(d*x+c))^2)^(1/2)/(a-b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x), x)
```

3.322 $\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=215

$$-\frac{\cot^2(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+a*arctanh((a+b*sec(d*x
+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-3/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/
(a-b)^(1/2))/d/(a-b)^(1/2)+a*arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/
(a+b)^(1/2)+3/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
-1/2*cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3885, 898, 1315, 1178, 12, 1093, 206, 1170, 207}

$$-\frac{\cot^2(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (a*ArcTanh[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]*d) - (3*b*ArcTanh[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d) + (a*ArcTanh[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]])/(Sqrt[a + b]*d) + (3*b*ArcTanh[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]])/(4*Sqrt[a + b]*d) - (Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d
*x]])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1315

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[(d*e*f^2)/(c*d^
2 - b*d*e + a*e^2), Int[((f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1))/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{(2b^4)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{(2b^2)\,\text{Subst}\left(\int \frac{-a^2+b^2+ax^2}{(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} + \frac{(2ab^2)\,\text{Sub}}{}$$

$$= -\frac{b^2\sqrt{a+b\sec(c+dx)}}{2d\left(a^2-b^2-2a(a+b\sec(c+dx))+(a+b\sec(c+dx))^2\right)} + \frac{(2ab^2)\,\text{Su}}{}$$

$$= -\frac{b^2\sqrt{a+b\sec(c+dx)}}{2d\left(a^2-b^2-2a(a+b\sec(c+dx))+(a+b\sec(c+dx))^2\right)} + \frac{a\,\text{Subst}\big(}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{}}{}\right)}{\sqrt{a+}}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} - \frac{3b\tanh^{-1}\left(\right)}{4\sqrt{a}}$$

Mathematica [B] time = 18.95, size = 937, normalized size = 4.36

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{1}{2}-\frac{1}{2}\csc^2(c+dx)\right)}{d} + \frac{\left(\frac{2\left(4\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{-a\cos(c+dx)}}\right)-\sqrt{a}\left(\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b+a\cos(c+dx)}}{\sqrt{a-b}\sqrt{-a\cos(c+dx)}}\right)+\sqrt{a-b}\tan^{-1}\right.\right.}{\sqrt{a-b}\sqrt{a+b}\left(a^2-2b^2-2(b+a\cos(c+dx))^2+4\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((1/2 - Csc[c + d*x]^2/2)*Sqrt[a + b*Sec[c + d*x]])/d + (((3*a*b*(-(Sqrt[-a
^2]*Sqrt[a + b]*Log[-Sqrt[-a + b] + Sqrt[b + a*Cos[c + d*x]]]) + Sqrt[-a^2]
*Sqrt[a + b]*Log[Sqrt[-a + b] + Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*
Log[-Sqrt[a + b] + Sqrt[b + a*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[Sqrt[a +
b] + Sqrt[b + a*Cos[c + d*x]]] + Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt[a]*Sqr
t[-(a*Cos[c + d*x])] - Sqrt[-a + b]*Sqrt[b + a*Cos[c + d*x]]] - Sqrt[-a^2]*
Sqrt[a + b]*Log[b + Sqrt[a]*Sqrt[-(a*Cos[c + d*x])] + Sqrt[-a + b]*Sqrt[b +
 a*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])]
 - Sqrt[a + b]*Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[b + Sqrt[-a]*
Sqrt[-(a*Cos[c + d*x])] + Sqrt[a + b]*Sqrt[b + a*Cos[c + d*x]]]))/(2*(-a)^(
3/2)*Sqrt[-a + b]*Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]]) +
 (2*Sqrt[a]*(Sqrt[a - b]*(a + b)*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/
(Sqrt[a - b]*Sqrt[-(a*Cos[c + d*x])])] + (a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a
]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])])*Sqrt[-(
a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]])/((a - b)*(a + b)) + (2*a^2*(4*Sqrt[a -
 b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]] -
Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]
*Sqrt[-(a*Cos[c + d*x])])] + Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c +
 d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])]))*Sqrt[-(a*Cos[c + d*x])]*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - 2*b^2 +
```

```
4*b*(b + a*Cos[c + d*x]) - 2*(b + a*Cos[c + d*x])^2)))*Sqrt[a + b*Sec[c + d
*x]])/(4*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

fricas [B] time = 3.14, size = 3523, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2
 + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x +
c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - ((4*a^2 + a*b - 3*b^2)*co
s(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)
*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x
+ c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((4*a^2 - a*b - 3*b^2)*cos(d
*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a +
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 -
(a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
*cos(d*x + c)^2 - 2*((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3
*b^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) + 8*((a^2 - b^2)*cos(d*x +
 c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) -
 b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))) - ((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b
+ 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*(
(2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*
(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + 2*((4*
a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(-a + b)*arcta
n(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*
a - b)*cos(d*x + c) + b)) + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt
(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x +
 c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) +
((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*lo
g(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^
2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2
*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^
2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/8*(4*(a^2 - b^2)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + ((4*a^2 + a*b - 3*b^2)*cos(d*x
 + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b))
- ((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(-a - b)
*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)
/((2*a + b)*cos(d*x + c) + b)) + 4*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)
*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(
d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d
*x + c)^2 - a^2 + b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - ((4*a^2 + a*b - 3*b^2
)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b +
b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(
d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((4*a^2 - a*b - 3*b^2)*c
```

```
os(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2
)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt
(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x
 + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^
2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a
*cos(d*x + c) + b)) - 2*((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b
 + 3*b^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - ((4*a^2 + a*b - 3*b^
2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b +
 b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*
sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos
(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x +
 c)^2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d
*x + c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(
-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/
(2*a*cos(d*x + c) + b)) + 2*((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 -
 a*b + 3*b^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) + ((4*a^2 - a*b -
 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*
a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x +
 c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2
)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(
d*x + c)^2 - (a^2 - b^2)*d), 1/8*(4*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/c
os(d*x + c))*cos(d*x + c)^2 + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sq
rt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
c)/(2*a*cos(d*x + c) + b)) + ((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2
- a*b + 3*b^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b
)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - ((4*a^2 - a*b
- 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(-a - b)*arctan(2*sqrt(-
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(
d*x + c) + b)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d)]
```

giac [B] time = 1.89, size = 514, normalized size = 2.39

$$\left(\frac{16\,a\arctan\left(-\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}+\sqrt{a-b}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\,(4\,a+3\,b)\arctan\left(-\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\ldots}}{\ldots}\right)}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(16*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*
d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a
+ b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(4*a + 3*b)*arctan(-(sqrt(a - b)
*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1
/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) +
 (4*a - 3*b)*log(abs((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d
*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a +
 b))*(a - b) - sqrt(a - b)*a))/sqrt(a - b) + sqrt(a*tan(1/2*d*x + 1/2*c)^4
- b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - 2*((sqrt
(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*
d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*a - (a + b)*sqrt(a -
b))/((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 -
```

```
b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2 - a - b))
*sgn(cos(d*x + c))/d
```

maple [B] time = 1.48, size = 2844, normalized size = 13.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))*(-4*(a-b)^(3/2)*(a+b)^(1/2)*cos(d*x+c)*4^(1/2)*((b+
a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a-16*(a-b)^(3/2)*(a+b)^(1/
2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)+4*(a+b)^(1/2)*cos(d
*x+c)^2*4^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2
/(a-b)^(1/2))*a^3+3*(a+b)^(1/2)*cos(d*x+c)^2*4^(1/2)*ln(-(-1+cos(d*x+c))*(2
*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2
)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*b^3+7*(a-b)^(3/2)*4^(1/2
)*ln(-2*(2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c
))^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*a*b+3*(a+b)^(1/2)*4^(1/
2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+
c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*
cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))
*a^2*b-16*(a-b)^(3/2)*(a+b)^(1/2)*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(3/2)-32*(a-b)^(3/2)*(a+b)^(1/2)*cos(d*x+c)*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)+4*(a-b)^(3/2)*4^(1/2)*ln(-2*(2*cos
(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*
a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos
(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*a^2+3*(a-b)^(3/2)*4^(1/2)*ln(-2*(2*co
s(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2
*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+co
s(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*b^2-4*(a+b)^(1/2)*4^(1/2)*ln(-(-1+co
s(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1
+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3-3*(a+b)^
(1/2)*4^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*c
os(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(
a-b)^(1/2))*b^3+4*(a-b)^(3/2)*(a+b)^(1/2)*cos(d*x+c)*4^(1/2)*((b+a*cos(d*x+
c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*b+8*(a-b)^(3/2)*(a+b)^(1/2)*a^(1/2)*
cos(d*x+c)^2*4^(1/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(
1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b+4*(a-b)^(3/2)*(a+b)^(1/2)*cos(d*x+c)^2*4^(
1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a-4*(a-b)^(3/2)*(
a+b)^(1/2)*cos(d*x+c)^2*4^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(1/2)*b+8*(a-b)^(3/2)*(a+b)^(1/2)*a^(3/2)*cos(d*x+c)^2*4^(1/2)*ln(4*a^(
1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1
/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b
)+4*(a+b)^(1/2)*4^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)
+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(
d*x+c)^2/(a-b)^(1/2))*a*b^2-8*(a-b)^(3/2)*(a+b)^(1/2)*a^(3/2)*4^(1/2)*ln(4*
a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a
^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+
2*b)-4*(a-b)^(3/2)*cos(d*x+c)^2*4^(1/2)*ln(-2*(2*cos(d*x+c)*(a+b)^(1/2)*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+
c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-
```

```
1+cos(d*x+c)))*a^2-3*(a-b)^(3/2)*cos(d*x+c)^2*4^(1/2)*ln(-2*(2*cos(d*x+c)*(
a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*a*cos(d*x
+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(1/2)+b)/(-1+cos(d*x+c)))*b^2-7*(a-b)^(3/2)*cos(d*x+c)^2*4^(1/2)*ln(-2*(
2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/
2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(
1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*a*b-8*(a-b)^(3/2)*(a+b)^(1/2)*a^
(1/2)*4^(1/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1
/2)+4*a*cos(d*x+c)+2*b)*b-3*(a+b)^(1/2)*cos(d*x+c)^2*4^(1/2)*ln(-(-1+cos(d*
x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos
(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b-4*(a+b)^(1
/2)*cos(d*x+c)^2*4^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+
a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c
)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin
(d*x+c)^2/(a-b)^(1/2))*a*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c
)/sin(d*x+c)^4/((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)/(a+b)^(
3/2)/(a-b)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(dx + c\right) + a} \cot\left(dx + c\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{cot}\left(c+d\,x\right)^3\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec{\left(c + dx \right)}} \cot^{3}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x)**3, x)
```

3.323 $\int \sqrt{a+b\sec(c+dx)}\,\tan^2(c+dx)\,dx$

Optimal. Leaf size=344

$$-\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2\tan(c+dx)\sqrt{a+}}{3d}$$

```
[Out] -2/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/b^2/d-2/3*(a+2*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d
*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/3*(a+b*sec(d*x+c))^
(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.39, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3894, 4057, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2\tan(c+dx)\sqrt{a+}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) - (2*Sqrt[a + b]*(a + 2*b
)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/(3*b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4057

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*
A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,\tan^2(c+dx)\,dx = \int \sqrt{a+b\sec(c+dx)}\,\left(-1+\sec^2(c+dx)\right)dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{-\frac{3a}{2} - b\sec(c+dx) + \frac{1}{2}a\sec^2(c$$

$$\sqrt{a+b\sec(c+dx)}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{-\frac{3a}{2} + \left(-\frac{a}{2}-b\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{3b^2d}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{3b^2d}$$

Mathematica [C] time = 17.88, size = 692, normalized size = 2.01

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{3b}+\frac{2}{3}\tan(c+dx)\right)}{d} - \frac{2\sqrt{\frac{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\,\sqrt{a+b\sec(c+dx)}\left(a\sqrt{\frac{b-a}{a+b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*Sqrt[a + b*Sec[c + d*x]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*((-I)*a*(a - b)*EllipticE[I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a - b)*b*EllipticF[I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] - (6*I)*a*b*EllipticPi[-((a + b)/(a - b)), I*ArcSi
nh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] + a*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]*(b - b*Tan[(c + d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^2)^2)))/(3*b*Sqrt[(
-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 + Ta
n[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b - b*Tan[(c + d*x)/2]^4 + a*(
-1 + Tan[(c + d*x)/2]^2)^2)) + (Sqrt[a + b*Sec[c + d*x]]*((2*a*Sin[c + d*x]
)/(3*b) + (2*Tan[c + d*x])/3))/d
```

fricas [F] time = 22.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)+a}\,\tan\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(dx+c\right)+a}\,\tan\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

maple [B] time = 1.40, size = 1106, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))^2*(cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+6*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a*b-4*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a*b+2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
```

```
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+6*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*si
n(d*x+c)*a*b-4*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*a*b+2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-cos(d*x+c)^3*a^2-cos(d*x+c)^3*
a*b+cos(d*x+c)^2*a^2-cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2+2*a*b*cos(d*x+c)+b^2
)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/cos
(d*x+c)/sin(d*x+c)^5/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(dx+c) + a} \tan(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+dx)^2 \sqrt{a + \frac{b}{\cos(c+dx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec(c+dx)} \tan^2(c+dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**2, x)
```

3.324 $\int \sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-b)/
(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)*(
b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3780}

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
+ d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x
]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x])
)/(Sqrt[a + b]*d)
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(}{\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.26, size = 151, normalized size = 1.21

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\sqrt{a+b\sec(c+dx)}\left((b-a)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\right.}{d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(dx + c\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

maple [A] time = 1.14, size = 215, normalized size = 1.72

$$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\left(1+\cos\left(dx+c\right)\right)^2\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)a-\text{Elliptic}\right.}{d\left(b+a\cos\left(dx+c\right)\right)\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2),x)
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1+cos(d*x+c))^2*(EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,((a-b)/(a+b))^(1/2)))*(-1+cos(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(dx + c\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{b}{\cos\left(c + d\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x)), x)
```

3.325 $\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=246

$$-\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a}{a}\right.}{d}$$

```
[Out] cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2
)/d+2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-
b)/(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2
)*(b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)-cot(d*x+c)*(a+b*s
ec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.21, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3896, 3780, 3875, 3832}

$$-\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a}{a}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/d - (Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]])/d +
(2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c +
 d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x]
))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x]))
/(Sqrt[a + b]*d)
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rubi steps

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \left(-\sqrt{a+b\sec(c+dx)}+\csc^2(c+dx)\sqrt{a+b\sec(c+dx)}\right)dx$$
$$= -\int \sqrt{a+b\sec(c+dx)}\,dx + \int \csc^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= -\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d} + \frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+}}\right.\right.}{}$$
$$= \frac{\sqrt{a+b}\,\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{se}}{a}}}{d}$$

Mathematica [A] time = 3.58, size = 154, normalized size = 0.63

$$\frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(\sec(c+dx)+1)}}\left((b-2a)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+4a\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\right.\right.}{a\cos(c+dx)+b}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(-Cot[c + d*x] - (2*Cos[(c + d*x)/2]^2*((-2*a + b
)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*a*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sqrt[(1 + Sec[c + d*x])^(-1)]*
Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))])/(b + a*Cos[c + d*x
])))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

maple [B] time = 1.32, size = 628, normalized size = 2.55

$$-\frac{(-1+\cos(dx+c))^2\left(2a\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\ \mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)\sin(dx+c)\cos(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x)

[Out] -1/d*(-1+cos(d*x+c))^2*(2*a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)*cos(d*x+c)-4*a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)+2*a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)-EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b-4*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a+a*cos(d*x+c)^2+b*cos(d*x+c))*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(dx + c\right) + a} \cot\left(dx + c\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\mathrm{cot}\left(c+d\,x\right)}^2\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)

[Out] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec{\left(c + dx \right)}} \cot^{2}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x)**2, x)

3.326 $\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=148

$$\frac{2\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4d}-\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2(a+b\sec(c+dx))^{7/2}}{7b^4d}-\frac{6a(a+b\sec(c+$$

```
[Out] 2/3*(3*a^2-2*b^2)*(a+b*sec(d*x+c))^(3/2)/b^4/d-6/5*a*(a+b*sec(d*x+c))^(5/2)
/b^4/d+2/7*(a+b*sec(d*x+c))^(7/2)/b^4/d-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^
(1/2))/d/a^(1/2)-2*a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(1/2)/b^4/d
```

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3885, 898, 1153, 207}

$$\frac{2\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4d}-\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2(a+b\sec(c+dx))^{7/2}}{7b^4d}-\frac{6a(a+b\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sec[c + d*x]])/(b^4*d) + (2*(3*a^2 - 2*b^2)*(a + b*Sec[c +
d*x])^(3/2))/(3*b^4*d) - (6*a*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d) + (2*(a
 + b*Sec[c + d*x])^(7/2))/(7*b^4*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\left(-a^2+b^2+2ax^2-x^4\right)^2}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-a^3+2ab^2+\left(3a^2-2b^2\right)x^2-3ax^4+x^6+\frac{b^4}{-a+x^2}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}$$

$$= -\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4 d} + \frac{2\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{3/2}}{3b^4 d} - \frac{6a(a+\cdots}{\cdots}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{2a\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4 d} + \frac{2\left(3a^2-2b^2\right)(a\cdots}{3\cdots}$$

Mathematica [A] time = 6.32, size = 248, normalized size = 1.68

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(\frac{8a\left(35b^2-12a^2\right)}{105b^4} - \frac{4\left(35b^2-12a^2\right)\sec(c+dx)}{105b^3} - \frac{12a\sec^2(c+dx)}{35b^2} + \frac{2\sec^3(c+dx)}{7b}\right)}{d\sqrt{a+b\sec(c+dx)}} - \frac{\sin(c+dx)\tan\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((8*a*(-12*a^2 + 35*b^2))/(105*b^4) - (4
*(-12*a^2 + 35*b^2)*Sec[c + d*x])/(105*b^3) - (12*a*Sec[c + d*x]^2)/(35*b^2
) + (2*Sec[c + d*x]^3)/(7*b)))/(d*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[a*Cos[c
 + d*x]]*Sqrt[b + a*Cos[c + d*x]]*(-Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a
*Cos[c + d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*S
in[c + d*x]*Tan[c + d*x])/(a*d*(1 - Cos[c + d*x]^2)*Sqrt[a + b*Sec[c + d*x]
])
```

fricas [A] time = 2.01, size = 381, normalized size = 2.57

$$\left[\frac{105\sqrt{a}\,b^4\cos(dx+c)^3\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2+4\left(2a\cos(dx+c)^2+b\cos(dx+c)\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*sqrt(a)*b^4*cos(d*x + c)^3*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) - 4*(18*a^2*b^2*cos(d*x + c) - 15*a*b^3 +
4*(12*a^4 - 35*a^2*b^2)*cos(d*x + c)^3 - 2*(12*a^3*b - 35*a*b^3)*cos(d*x +
c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a*b^4*d*cos(d*x + c)^3), 1/
105*(105*sqrt(-a)*b^4*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))*cos(d*x + c)^3 - 2*(18*a^2*b^2*co
s(d*x + c) - 15*a*b^3 + 4*(12*a^4 - 35*a^2*b^2)*cos(d*x + c)^3 - 2*(12*a^3*
```

```
b - 35*a*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a*b
^4*d*cos(d*x + c)^3)]
```

giac [B] time = 6.08, size = 722, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/105*(105*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b) + sqrt(a - b))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
- 2*(105*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^
4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^6 - 840
*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*ta
n(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^5*sqrt(a - b) +
 35*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b
*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^4*(27*a - 23
*b) + 280*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)
^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^3*(3*a
 + 4*b)*sqrt(a - b) - 21*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1
/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
 a + b))^2*(65*a^2 - 2*a*b - 15*b^2) + 315*a^3 + 707*a^2*b - 7*a*b^2 - 55*b
^3 - 56*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4
 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*(19*a*b
+ 5*b^2)*sqrt(a - b))/((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2
*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a
 + b) - sqrt(a - b))^7*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.75, size = 4997, normalized size = 33.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/420/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos
(d*x+c))^4*(192*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(1/2)*cos(d*x+c)^6*a^5+192*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(1/2)*cos(d*x+c)^5*a^5-210*cos(d*x+c)^6*ln(-2*(-1+cos(d*x+c))*(2
*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2
)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^6*b-105*cos(d*x+c)^6*l
n(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a
^5*b^2+420*cos(d*x+c)^6*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+
c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/si
n(d*x+c)^2/(a-b)^(1/2))*a^4*b^3-315*cos(d*x+c)^6*ln(-2*(-1+cos(d*x+c))*(2*c
os(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-
2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^4+105*cos(d*x+c)^6*l
n(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a
^2*b^5+210*cos(d*x+c)^6*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)
```

```
+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(
d*x+c)^2/(a-b)^(1/2))*a^6*b+105*cos(d*x+c)^6*ln(-(-1+cos(d*x+c))*(2*cos(d*x
+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*co
s(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2
)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^5*b^2-420*cos(d*x+c)^6*ln(-(-1
+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4*b^3+3
15*cos(d*x+c)^6*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2
/(a-b)^(1/2))*a^3*b^4-105*cos(d*x+c)^6*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a
-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+
c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b
)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^5+105*cos(d*x+c)^5*ln(-2*(-1+cos
(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+
cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^6*b-210*cos
(d*x+c)^5*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-
b)^(1/2))*a^5*b^2-105*cos(d*x+c)^5*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b
)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)
+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^
(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4*b^3+420*cos(d*x+c)^5*ln(-2*(-1+cos(d
*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+co
s(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^4-315*cos
(d*x+c)^5*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-
b)^(1/2))*a^2*b^5+105*cos(d*x+c)^5*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b
)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)
+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^
(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a*b^6-105*cos(d*x+c)^5*ln(-(-1+cos(d*x+c
))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*
x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^6*b+210*cos(d*x+c
)^5*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x
+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2)
)*a^5*b^2+105*cos(d*x+c)^5*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((
b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x
+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/s
in(d*x+c)^2/(a-b)^(1/2))*a^4*b^3-420*cos(d*x+c)^5*ln(-(-1+cos(d*x+c))*(2*co
s(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2
*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^4+315*cos(d*x+c)^5*ln
(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1
+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*
b^5-105*cos(d*x+c)^5*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*co
s(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*
((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x
+c)^2/(a-b)^(1/2))*a*b^6-60*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(3/2)*(a-b)^(3/2)*a*b^3+105*cos(d*x+c)^6*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c
)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(
d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*
(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^7-105*cos(d*x+c)^6*ln(-(-1+cos(d
```

```
*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+co
s(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^7-524*(a-b)^(
3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^5*a^3*
b^2-524*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*co
s(d*x+c)^6*a^3*b^2+192*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)*cos(d*x+c)^4*a^4*b-524*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c
)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^4*a^2*b^3+210*(a-b)^(3/2)*a^(3/2)*cos(
d*x+c)^6*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4
*a*cos(d*x+c)+2*b)*b^4+210*(a-b)^(3/2)*a^(1/2)*cos(d*x+c)^5*ln(4*a^(1/2)*co
s(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^5-1
08*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x
+c)^5*a^2*b^2-288*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(3/2)*cos(d*x+c)^4*a^3*b+840*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+
cos(d*x+c))^2)^(3/2)*cos(d*x+c)^4*a*b^3+180*(a-b)^(3/2)*((b+a*cos(d*x+c))*c
os(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^3*a^2*b^2-96*(a-b)^(3/2)*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^2*a^3*b+100*(a-b
)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^2*a
*b^3+72*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*co
s(d*x+c)*a^2*b^2+192*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(1/2)*cos(d*x+c)^5*a^4*b-524*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^5*a^2*b^3-36*(a-b)^(3/2)*cos(d*x+c)^6*((
b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a^2*b^2-96*(a-b)^(3/2)*(
(b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^5*a^3*b+280*
(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)
^5*a*b^3-36*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2
)*cos(d*x+c)^4*a^2*b^2-288*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(3/2)*cos(d*x+c)^3*a^3*b+780*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^3*a*b^3+216*(a-b)^(3/2)*((b+a*cos(
d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^2*a^2*b^2-180*(a-b)^(
3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)*a*b^3)
/cos(d*x+c)^3/sin(d*x+c)^8/((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(
3/2)/b^4/a/(a-b)^(3/2)
```

maxima [A] time = 0.43, size = 175, normalized size = 1.18

$$\frac{\frac{105\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{30\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{7}{2}}}{b^4}-\frac{126\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}a}{b^4}+\frac{210\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2}{b^4}-\frac{210\sqrt{a+\frac{b}{\cos(dx+c)}}\,a^3}{b^4}-\frac{140\left(a+\frac{b}{\cos(dx+c)}\right)}{b^2}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c
)) + sqrt(a)))/sqrt(a) + 30*(a + b/cos(d*x + c))^(7/2)/b^4 - 126*(a + b/cos
(d*x + c))^(5/2)*a/b^4 + 210*(a + b/cos(d*x + c))^(3/2)*a^2/b^4 - 210*sqrt(
a + b/cos(d*x + c))*a^3/b^4 - 140*(a + b/cos(d*x + c))^(3/2)/b^2 + 420*sqrt
(a + b/cos(d*x + c))*a/b^2)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^5}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**5/sqrt(a + b*sec(c + d*x)), x)
```

3.327 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2/3*(a+b*sec(d*x+c))^(3/2)/b^2/d+2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/
d/a^(1/2)-2*a*(a+b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1153, 207}

$$\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*Sqrt[a + b
*Sec[c + d*x]])/(b^2*d) + (2*(a + b*Sec[c + d*x])^(3/2))/(3*b^2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{b^2 d}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2 d}$$
$$= -\frac{2\,\text{Subst}\left(\int \left(a-x^2+\frac{b^2}{-a+x^2}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2 d}$$
$$= -\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2 d}+\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2 d}-\frac{2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2 d}+\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2 d}$$

Mathematica [B] time = 1.23, size = 194, normalized size = 2.46

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(\frac{2\sec(c+dx)}{3b}-\frac{4a}{3b^2}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\tan(c+dx)\sqrt{a\cos(c+dx)}\sqrt{a\cos(c+dx)+b}\,(1\ldots}{ad\left(1-\cos^2(c+dx)\right)\sqrt{a+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-4*a)/(3*b^2) + (2*Sec[c + d*x])/(3*b)
))/(d*Sqrt[a + b*Sec[c + d*x]]) + (Sqrt[a*Cos[c + d*x]]*Sqrt[b + a*Cos[c +
d*x]]*(-Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] + Log[1 + Sq
rt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*Sin[c + d*x]*Tan[c + d*x])/(a
*d*(1 - Cos[c + d*x]^2)*Sqrt[a + b*Sec[c + d*x]])
```

fricas [A] time = 1.19, size = 273, normalized size = 3.46

$$\left[\frac{3\sqrt{a}\,b^2\cos(dx+c)\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2-4\left(2a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\ldots\right.}{6ab^2d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*b^2*cos(d*x + c)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x
+ c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c))) - 4*(2*a^2*cos(d*x + c) - a*b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)))/(a*b^2*d*cos(d*x + c)), -1/3*(3*sqrt(-a)*b^2*arctan
(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d
*x + c) + b))*cos(d*x + c) + 2*(2*a^2*cos(d*x + c) - a*b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)))/(a*b^2*d*cos(d*x + c))]
```

giac [B] time = 1.44, size = 288, normalized size = 3.65

$$\frac{2\left(\frac{3\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{2\left(3\left(\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\sqrt{a\tan\cdots}\right.\right.}{\left(\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-\cdots}\right.}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x
 + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b
) + sqrt(a - b))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*(
3*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*t
an(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2 - 3*a - b)/(
(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan
(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^3*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.68, size = 3003, normalized size = 38.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/12/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^3*(3*co
s(d*x+c)^4*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*
cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(
d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b
)^(1/2))*a^5-3*cos(d*x+c)^4*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(
d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b
)/sin(d*x+c)^2/(a-b)^(1/2))*a^5+6*cos(d*x+c)^3*ln(-(-1+cos(d*x+c))*(2*cos(d
*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*
cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1
/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^3-3*cos(d*x+c)^3*ln(-(-1
+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a*b^4-3*c
os(d*x+c)^3*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*c
os(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(
a-b)^(1/2))*a^4*b+6*cos(d*x+c)^3*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^
(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b
*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1
/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^2-6*cos(d*x+c)^3*ln(-2*(-1+cos(d*x+c
))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*
x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^3+3*cos(d*x+c
)^3*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/
2))*a*b^4+4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2
)*a*b-8*cos(d*x+c)^3*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
```

```
)^2)^(1/2)*a^3-8*cos(d*x+c)^4*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)*a^3-6*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a
-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+
c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b
)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4*b+6*cos(d*x+c)^4*ln(-(-1+cos(d*x+c
))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*
x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^2-3*cos(d*x+c
)^4*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x
+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2)
)*a^2*b^3+6*cos(d*x+c)^4*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((
b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x
+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/s
in(d*x+c)^2/(a-b)^(1/2))*a^4*b-6*cos(d*x+c)^4*ln(-2*(-1+cos(d*x+c))*(2*cos(
d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a
*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(
1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^2+3*cos(d*x+c)^4*ln(-2*
(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x
+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^
3+3*cos(d*x+c)^3*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+
a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^
2/(a-b)^(1/2))*a^4*b-6*cos(d*x+c)^3*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)
^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+
b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(
1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^2-8*cos(d*x+c)^2*(a-b)^(3/2)*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^2*b+6*cos(d*x+c)^4*ln(4*a^
(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(
1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*
b)*(a-b)^(3/2)*a^(3/2)*b^2+6*cos(d*x+c)^3*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*(a-b)^(3/2)*a^(1/2)*b^3
+12*cos(d*x+c)^2*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(3/2)*a*b+4*cos(d*x+c)^3*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d
*x+c))^2)^(3/2)*a*b+12*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c
))^2)^(3/2)*(a-b)^(3/2)*a*b-8*cos(d*x+c)^3*(a-b)^(3/2)*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^2*b)/sin(d*x+c)^6/cos(d*x+c)/((b+a*cos(d
*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)/b^2/a/(a-b)^(3/2)
```

maxima [A] time = 0.43, size = 92, normalized size = 1.16

$$-\frac{\frac{3 \log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}{b^2}+\frac{6\sqrt{a+\frac{b}{\cos(dx+c)}}\,a}{b^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c))
+ sqrt(a)))/sqrt(a) - 2*(a + b/cos(d*x + c))^(3/2)/b^2 + 6*sqrt(a + b/cos(d
*x + c))*a/b^2)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^3}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

3.328 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3885, 63, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}
\end{aligned}$$

Mathematica [B] time = 0.20, size = 108, normalized size = 3.48

$$\frac{\sqrt{a\cos(c+dx)+b}\left(\log\left(1-\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}\right)-\log\left(\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}+1\right)\right)}{d\sqrt{a\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cos[c + d*x]]*(Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c +
d*x]]] - Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]]))/(d*Sqrt[a
*Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

fricas [B] time = 1.61, size = 145, normalized size = 4.68

$$\left[\frac{\log\left(-8\,a^2\cos\left(dx+c\right)^2-8\,ab\cos\left(dx+c\right)-b^2+4\left(2\,a\cos\left(dx+c\right)^2+b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right)}{2\,\sqrt{a}\,d}, \frac{\sqrt{-a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x
+ c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
sqrt(a)*d), sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))/(a*d)]
```

giac [B] time = 1.61, size = 109, normalized size = 3.52

$$-\frac{2\,\arctan\left(-\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}+\sqrt{a-b}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,d\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) +
sqrt(a - b))/sqrt(-a))/(sqrt(-a)*d*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
```

maple [A] time = 0.14, size = 26, normalized size = 0.84

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

maxima [A] time = 0.41, size = 49, normalized size = 1.58

$$\frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt(a
)))/(sqrt(a)*d)
```

mupad [B] time = 1.63, size = 27, normalized size = 0.87

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*atanh((a + b/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

3.329 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-arctanh((a+b*sec(d*x+c)
)^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2))/d/(a+b)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3885, 898, 1170, 206, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - ArcTanh[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
```

```
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= -\frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}\left(b^2-x^2\right)}\,dx, x, b\sec(c+dx)\right)}{d} \\
&= -\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= -\frac{\left(2b^2\right)\text{Subst}\left(\int \left(-\frac{1}{b^2\left(a-x^2\right)}+\frac{1}{2b^2\left(a+b-x^2\right)}-\frac{1}{2b^2\left(-a+b+x^2\right)}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right.}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}+\frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\sec\right.}{d} \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}
\end{aligned}$$

Mathematica [B] time = 5.98, size = 218, normalized size = 2.06

$$\frac{\sqrt{a\cos(c+dx)+b}\left(\sqrt{a}\left(\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{a\cos(c+dx)+b}}{\sqrt{a-b}\sqrt{-a\cos(c+dx)}}\right)+\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{a\cos(c+dx)+b}}{\sqrt{a+b}\sqrt{-a\cos(c+dx)}}\right)\right)-2\sqrt{a-b}\sqrt{a}\right.}{d\sqrt{a-b}\sqrt{a+b}\sqrt{-a\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((-2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c
 + d*x])]] + Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])
/(Sqrt[a - b]*Sqrt[-(a*Cos[c + d*x])])] + Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[
b + a*Cos[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])]))*Sqrt[b + a*Co
s[c + d*x]])/(Sqrt[a - b]*Sqrt[a + b]*d*Sqrt[-(a*Cos[c + d*x])]*Sqrt[a + b*
Sec[c + d*x]])
```

fricas [B] time = 4.41, size = 2420, normalized size = 22.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c)
- b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))) + (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)
*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x
+ c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (a^2 - a*b)*sqrt(a + b)*log(
-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2
+ b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(
4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^3
- a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - (a^2 + a*b)
```

```
*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b
)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)) - (a^2 - a*b)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c
)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d
*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*d), -1/4*(2*(a^2 + a*b)*sq
rt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*c
os(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 2*(a^2 - b^2)*sqrt(a)*log(-8*a^
2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos
(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (a^2 - a*b)*s
qrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*
cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c
) + 1)))/((a^3 - a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)
) + 2*(a^2 + a*b)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c)
+ b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 - a*b)
*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b
)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x +
 c) + 1)))/((a^3 - a*b^2)*d), 1/4*(2*(a^2 - a*b)*sqrt(-a - b)*arctan(2*sqrt
(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*co
s(d*x + c) + b)) + 2*(a^2 - b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) + (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 -
 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*
x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*
b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*d
), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(a^2 - a*b)*sqrt(-a
 - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
 + c)/((2*a + b)*cos(d*x + c) + b)) - (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2
- 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d
*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3
*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*
d), -1/2*((a^2 + a*b)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 -
a*b)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - (a^2 - b^2)*sqrt(a)*log(-
8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b
*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/((a^3 - a*
b^2)*d), -1/2*(2*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (a^2 + a*b)*sqr
t(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*co
s(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 - a*b)*sqrt(-a - b)*arctan(
2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a +
 b)*cos(d*x + c) + b)))/((a^3 - a*b^2)*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
```

```
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, i
ntegration of abs or sign assumes constant sign by intervals (correct if th
e argument is real):Check [abs(cos(d*t_nostep+c))]Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Discontinui
ties at zeroes of cos(d*t_nostep+c) were not checkedWarning, integration of
 abs or sign assumes constant sign by intervals (correct if the argument is
 real):Check [abs(t_nostep^2-1)]Evaluation time: 0.58Error: Bad Argument Ty
pe
```

maple [B] time = 1.54, size = 690, normalized size = 6.51

$$\frac{\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,\sqrt{4}\,\cos(dx+c)\left(-2(a-b)^{\frac{3}{2}}a^{\frac{3}{2}}\ln\left(4\sqrt{a}\,\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(1+\cos(dx+c))^2}}+4\sqrt{a}\,\sqrt{\frac{(b+a\cos(}{(1+}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)*cos(d*x+c)*(-2*(a-b)^(3/2
)*a^(3/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c
))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+
4*a*cos(d*x+c)+2*b)+(a-b)^(3/2)*(a+b)^(1/2)*ln(-2*(2*cos(d*x+c)*(a+b)^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*a*cos(d*x+c)+b*cos(
d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b
)/(-1+cos(d*x+c)))*a-2*(a-b)^(3/2)*a^(1/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*co
s(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b-a^3*ln(-(-1+cos(d*x+
c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))+ln(-(-1+cos(d*x+c)
)*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a*b^2)*(-1+cos(d*x+c
))/sin(d*x+c)^2/((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)/(a-b)^
(3/2)/a/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

3.330 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=260

$$\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)(\sec(c+dx)+1)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}+\cdots$$

```
[Out] -1/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d+1/4*b*arct
anh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d-2*arctanh((a+b*sec(d*
x+c))^(1/2)/a^(1/2))/d/a^(1/2)+arctanh((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/
d/(a-b)^(1/2)+arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+1/4
*(a+b*sec(d*x+c))^(1/2)/(a+b)/d/(1-sec(d*x+c))+1/4*(a+b*sec(d*x+c))^(1/2)/(
a-b)/d/(1+sec(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3885, 898, 1238, 206, 199, 207}

$$\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)(\sec(c+dx)+1)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + ArcTanh[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - (b*ArcTanh[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/2)*d) + (b*ArcTanh[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]])/(4*(a + b)^(3/2)*d) + ArcTanh[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]]/(Sqrt[a + b]*d) + Sqrt[a + b*Sec[c + d*x]]/(4*(a + b)*d*(1 -
 Sec[c + d*x])) + Sqrt[a + b*Sec[c + d*x]]/(4*(a - b)*d*(1 + Sec[c + d*x]))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
```

```
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{\left(2b^4\right)\text{Subst}\left(\int \frac{1}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{\left(2b^4\right)\text{Subst}\left(\int \left(-\frac{1}{b^4\left(a-x^2\right)} + \frac{1}{4b^3\left(a+b-x^2\right)^2} + \frac{1}{2b^4\left(a+b-x^2\right)} - \frac{1}{4b^3\left(-a+b+x^2\right)^2} - \frac{1}{2b^4\left(-a+b+x^2\right)}\right) c\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} - \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\sec(c+\right.}{d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d} + \frac{}{4(a}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} + \frac{b}{}$$

Mathematica [B] time = 6.89, size = 1022, normalized size = 3.93

$$\frac{(b+a\cos(c+dx))\left(\frac{(a-b\cos(c+dx))\csc^2(c+dx)}{2\left(b^2-a^2\right)} + \frac{a}{2\left(a^2-b^2\right)}\right)\sec(c+dx)}{d\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{b+a\cos(c+dx)}\left(-\frac{b\left(-\sqrt{-a^2}\sqrt{a+b}\log\left(\sqrt{b+a\cos(c\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] -1/4*(Sqrt[b + a*Cos[c + d*x]]*(-1/2*(a^2*b*(-(Sqrt[-a^2]*Sqrt[a + b]*Log[-
Sqrt[-a + b] + Sqrt[b + a*Cos[c + d*x]]]) + Sqrt[-a^2]*Sqrt[a + b]*Log[Sqrt
[-a + b] + Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[-Sqrt[a + b] + Sq
rt[b + a*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[Sqrt[a + b] + Sqrt[b + a*Cos[c
 + d*x]]] + Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt[a]*Sqrt[-(a*Cos[c + d*x])]
```

```
- Sqrt[-a + b]*Sqrt[b + a*Cos[c + d*x]]] - Sqrt[-a^2]*Sqrt[a + b]*Log[b + S
qrt[a]*Sqrt[-(a*Cos[c + d*x])] + Sqrt[-a + b]*Sqrt[b + a*Cos[c + d*x]]] + a
*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])] - Sqrt[a + b]*Sqrt[b
 + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x]
)] + Sqrt[a + b]*Sqrt[b + a*Cos[c + d*x]]]))/((-a)^(3/2)*Sqrt[-a + b]*Sqrt[
a + b]*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]]) - ((2*a^2 - 3*b^2)*(Sqrt
[a - b]*(a + b)*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]*Sqrt
[-(a*Cos[c + d*x])])] + (a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[
c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])])*Sqrt[-(a*Cos[c + d*x])]*
Sqrt[Sec[c + d*x]])/(Sqrt[a]*(a - b)*(a + b)) - (a*(2*a^2 - 2*b^2)*(4*Sqrt[
a - b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]]
 - Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a -
 b]*Sqrt[-(a*Cos[c + d*x])])] + Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[
c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])]))*Sqrt[-(a*Cos[c + d*x])]
*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - 2*b^2
 + 4*b*(b + a*Cos[c + d*x]) - 2*(b + a*Cos[c + d*x])^2)))*Sqrt[Sec[c + d*x]
])/((a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]]) + ((b + a*Cos[c + d*x])*(a/
(2*(a^2 - b^2)) + ((a - b*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(-a^2 + b^2)))*S
ec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [B] time = 42.44, size = 4336, normalized size = 16.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
) + (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 -
 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x +
 c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3
 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(
-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2
+ b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(
4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a
^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a
^5 - 2*a^3*b^2 + a*b^4)*d), -1/16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2
*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (4*a^4 + 3*a^3*b
 - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x +
c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*
a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b
- 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b
^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sq
rt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d
*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*cos(d*
x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b
^4)*d), -1/16*(2*(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b
- 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c
) + b)) + 8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2
)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos
```

(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c
))) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2
 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x
 + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*s
qrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(c
os(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*cos(d*x + c)^2 -
(a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^
5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/
16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c
)/(2*a*cos(d*x + c) + b)) + 2*(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a
^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*
sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b
)*cos(d*x + c) + b)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*
a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a
*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x +
c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)
*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*
cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2
 + a*b^4)*d), 1/16*(2*(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a
^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a -
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x
 + c) + b)) - 8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x +
c)^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a
*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))) - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2
*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos
(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a -
b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c)
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 8*((a^4 - a^2*b^2)*cos(d*x + c)^
2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/
((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d),
 -1/16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)
*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
 + c)/(2*a*cos(d*x + c) + b)) - 2*(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 -
(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a - b)*arctan
(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a
+ b)*cos(d*x + c) + b)) + (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 +
 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 -
8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x
 + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b
^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^
2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*
b^2 + a*b^4)*d), -1/8*((4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*
a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a
 + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d
*x + c) + b)) - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b -
 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sq
rt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c)
+ b)) + 4*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
) - 4*((a^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x +
c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/8*(8*(a^4 - 2*a^2*b^2 + b^4 - (a^4
- 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (4*a^4 +

```
3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos
(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (4*a^4 - 3*a^3*b
 - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x +
c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 4*((a^4 - a^2*b^2)*cos(d
*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*
b^4)*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign b
y intervals (correct if the argument is real):Check [abs(cos(d*t_nostep+c))
]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Discontinuities at zeroes of cos(d*t_nostep+c) were not chec
kedWarning, integration of abs or sign assumes constant sign by intervals (
correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time:
1.26Error: Bad Argument Type
```

maple [B] time = 1.73, size = 4203, normalized size = 16.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))*(8*cos(d*x+c)^2*(a-b)^(3/2)*(a+b)^(1/2)*a^(7/2)*4^(
1/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*c
os(d*x+c)+2*b)-5*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+
c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^
(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c
)))*4^(1/2)*a*b^3+(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(
d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b
)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*a^4*b+9*(a+b)^(1/2)*ln(-(-1+cos(d*x+c))
*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(
1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
```

```
c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*a^3*b^2-(a+b)
^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(
d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+
c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1
/2))*4^(1/2)*a^2*b^3-5*(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^
(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b
*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1
/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*a*b^4-8*(a-b)^(3/2)*(a+b)^(1/2)*a^
(7/2)*4^(1/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1
/2)+4*a*cos(d*x+c)+2*b)-16*cos(d*x+c)^2*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d
*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a^2-4*cos(d*x+c)^2*(a-b)^(3/2)*ln
(-2*(2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x
+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^4+4*cos(d*x+c)^2*
(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-
b)^(1/2))*4^(1/2)*a^5-32*cos(d*x+c)*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a^2+16*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a*b+5*(a-b)^(3/2)*ln(-2*(2*c
os(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+
2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^3*b-4*(a-b)^(3/2)*ln(-2*(
2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/
2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(
1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^2*b^2-16*(a-b)^(3/2)*(
a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a^2+4*(a-b)
^(3/2)*ln(-2*(2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c)
)*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^4-4*(a+b
)^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(
1/2))*4^(1/2)*a^5+4*cos(d*x+c)^2*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*
cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*4^(1/2)*a^3-12*cos(d*x+c)^2*(a-b)^(3/2)*
(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*4^(1/2)*a^
2*b+8*cos(d*x+c)^2*(a-b)^(3/2)*(a+b)^(1/2)*a^(5/2)*4^(1/2)*ln(4*a^(1/2)*cos
(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+
a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b-8*co
s(d*x+c)^2*(a-b)^(3/2)*(a+b)^(1/2)*a^(3/2)*4^(1/2)*ln(4*a^(1/2)*cos(d*x+c)*
((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^2+8*cos(d*x+
c)*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(
1/2)*4^(1/2)*a^2*b-8*cos(d*x+c)^2*(a-b)^(3/2)*(a+b)^(1/2)*a^(1/2)*4^(1/2)*l
n(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x
+c)+2*b)*b^3-4*cos(d*x+c)*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x
+c)/(1+cos(d*x+c))^2)^(1/2)*4^(1/2)*a*b^2-4*cos(d*x+c)*(a-b)^(3/2)*(a+b)^(1
/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*4^(1/2)*a^3+4*(a-b
)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*4^
(1/2)*a^2*b+4*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)*4^(1/2)*a*b^2-8*(a-b)^(3/2)*(a+b)^(1/2)*a^(5/2)*4^(1/2)*ln(
4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4
*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c
)+2*b)*b+16*cos(d*x+c)^2*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+
c)/(1+cos(d*x+c))^2)^(3/2)*a*b-5*cos(d*x+c)^2*(a-b)^(3/2)*ln(-2*(2*cos(d*x+
c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*a*cos
(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
```

```
c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^3*b+4*cos(d*x+c)^2*(a-b)^(3/2)*l
n(-2*(2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*
x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^2*b^2+5*cos(d*x+
c)^2*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+
c)/(1+cos(d*x+c))^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*
a*b^3+8*(a-b)^(3/2)*(a+b)^(1/2)*a^(3/2)*4^(1/2)*ln(4*a^(1/2)*cos(d*x+c)*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^2-cos(d*x+c)^2*
(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-
b)^(1/2))*4^(1/2)*a^4*b-9*cos(d*x+c)^2*(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*c
os(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-
2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*a^3*b^2+cos(d*x+c)
^2*(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+
c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/
(a-b)^(1/2))*4^(1/2)*a^2*b^3+5*cos(d*x+c)^2*(a+b)^(1/2)*ln(-(-1+cos(d*x+c))
*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(
1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*a*b^4+32*cos(
d*x+c)*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(3/2)*a*b+8*(a-b)^(3/2)*(a+b)^(1/2)*a^(1/2)*4^(1/2)*ln(4*a^(1/2)*cos(d*x
+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*co
s(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^3)*((b+a
*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)/((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)/sin(d*x+c)^4/(a+b)^(5/2)/(a-b)^(5/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^3}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

3.331 $\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=404

$$-\frac{2(a-b)\sqrt{a+b}\left(8a^2-21b^2\right)\cot(c+dx)\sqrt{-\frac{b(\sec(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}+\frac{2\sqrt{a}}{}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d-2/15*(a-b)*(8*a^2-21*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d
*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(-b*(-1+sec(d*x+c
))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d+2/15*(-8*a^2+2*a*b+21
*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(a+b)^(1/2)*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(b*(1+sec(d*x+c))/(-a+
b))^(1/2)/b^3/d-8/15*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/5*sec(d*x+
c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A] time = 0.76, antiderivative size = 610, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3895, 3784, 3837, 3832, 4004, 3860, 4082, 4005}

$$-\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}-\frac{2(a-b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(8*a^
2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(15*b^4*d) + (4*Sqrt[a + b]*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) -
(2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) - (2*Sqrt[a
 + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - (8*a*Sqrt[a + b*Sec[c + d*x]]*Tan[c
 + d*x])/(15*b^2*d) + (2*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(5*b*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
```

```
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3837

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[(Csc[e + f
*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3895

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \left(\frac{1}{\sqrt{a+b\sec(c+dx)}} - \frac{2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}} + \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\right) dx$$

$$= -\left(2\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\right) + \int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+}$$

$$= -\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{ad}$$

$$= \frac{4(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{b^2d}$$

$$= \frac{4(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{b^2d}$$

$$= \frac{4(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{b^2d}$$

Mathematica [B] time = 16.94, size = 835, normalized size = 2.07

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\left(-\frac{2(21b^2-8a^2)\sin(c+dx)}{15b^3} + \frac{2\sec(c+dx)\tan(c+dx)}{5b} - \frac{8a\tan(c+dx)}{15b^2}\right)}{d\sqrt{a+b\sec(c+dx)}} - \frac{2\sqrt{b+a\cos(c+dx)}\,\sqrt{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^
2)^(-1)]*(8*a^3*Tan[(c + d*x)/2] + 8*a^2*b*Tan[(c + d*x)/2] - 21*a*b^2*Tan[
(c + d*x)/2] - 21*b^3*Tan[(c + d*x)/2] - 16*a^3*Tan[(c + d*x)/2]^3 + 42*a*b
^2*Tan[(c + d*x)/2]^3 + 8*a^3*Tan[(c + d*x)/2]^5 - 8*a^2*b*Tan[(c + d*x)/2]
^5 - 21*a*b^2*Tan[(c + d*x)/2]^5 + 21*b^3*Tan[(c + d*x)/2]^5 - 30*b^3*Ellip
ticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
- 30*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c +
 d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*a^3 + 8*a^2*b - 21*a*b^2 - 21*b^3)*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] - 2*b*(4*a^2 + a*b - 18*b^2)*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*
x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
]))/(15*b^3*d*Sqrt[a + b*Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-2*(-8*a^2 + 21*b^2)*Sin[c + d
*x])/(15*b^3) - (8*a*Tan[c + d*x])/(15*b^2) + (2*Sec[c + d*x]*Tan[c + d*x])
/(5*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan (dx+c)^4}{\sqrt{b \sec (dx+c)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 1.80, size = 1780, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x)

[Out] 2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(-21*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+4*cos(d*x+c)^2*a^2*b+3*b^3-30*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3-20*a*b^2*cos(d*x+c)^3+36*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-21*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-30*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+36*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+8*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-cos(d*x+c)*a*b^2+4*cos(d*x+c)^4*a^2*b+21*cos(d*x+c)^4*a*b^2-8*a^2*cos(d*x+c)^3*b+8*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-21*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-8*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-2*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-21*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli

```
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-2*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^2+8*cos(d*x+c)^3*a^3-24*cos(d*x+c)^2*b^3+21*cos(d*x+c)^3*b^3-8*cos(d*x+c
)^4*a^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(dx+c\right)^{4}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan\left(c+d\,x\right)^{4}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

3.332 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=310

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{}}{}$$

[Out] -2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d

Rubi [A] time = 0.25, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3894, 4059, 3921, 3784, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{}}{}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]

[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= \int \frac{-1+\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= \int \frac{-1-\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{b^2d} \\
&= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{b^2d}
\end{aligned}
$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
[Out] $Aborted
```

fricas [F] time = 22.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.39, size = 823, normalized size = 2.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2
*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*
x+c)*cos(d*x+c)-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+2*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b-(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+
c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d
*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+a*cos
(d*x+c)^2-a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^5/(b+a*cos(d*x+c))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^2}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

3.333 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3784}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+d}{a-b}}}{ad}$$

Mathematica [A] time = 0.23, size = 138, normalized size = 1.30

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sec(c+dx)\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-2\Pi\left(-1;\sin^{-1}\right.\right.}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 24.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

maple [A] time = 1.25, size = 180, normalized size = 1.70

$$\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\left(1+\cos\left(dx+c\right)\right)^{2}\left(-1+\cos\left(dx+c\right)\right)\left(-\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.}{d\left(b+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c
))*(-EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+2*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)))/(b+a*cos(d*x+c))/sin(d
*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(c + d*x)), x)
```

3.334 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=361

$$\frac{b^2\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\sqrt{a+\cdots}\right.\right.}{d\sqrt{a+b}}$$

```
[Out] cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d/(a+b)^(1
/2)-cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d/(a+b
)^(1/2)+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,
((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(
d*x+c))/(a-b))^(1/2)/a/d-cot(d*x+c)/d/(a+b*sec(d*x+c))^(1/2)+b^2*tan(d*x+c)
/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3896, 3784, 3875, 3833, 21, 3829, 3832, 4004}

$$\frac{b^2\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\sqrt{a+\cdots}\right.\right.}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(Sqrt[a + b]*d) - (Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (2*Sqrt[a
 + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - Cot[c + d*x]/(d*Sqrt[a + b*Sec[c +
d*x]]) + (b^2*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
```

```
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int\left(-\frac{1}{\sqrt{a+b\sec(c+dx)}}+\frac{\csc^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\right)dx$$

$$= -\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\csc^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{ad}$$

$$= \frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{ad}$$

$$= \frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{ad}$$

$$= \frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{ad}$$

$$= \frac{\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}\,d} - \frac{\cot(c}{}$$

Mathematica [C] time = 19.02, size = 1198, normalized size = 3.32

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\left(\frac{(a\cos(c+dx)-b)\csc(c+dx)}{b^2-a^2}+\frac{b\sin(c+dx)}{b^2-a^2}\right)}{d\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-b^2\sqrt{\frac{b-}{a+}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(((-b + a*Cos[c + d*x])*Csc[c + d*x])/(-
a^2 + b^2) + (b*Sin[c + d*x])/(-a^2 + b^2)))/(d*Sqrt[a + b*Sec[c + d*x]]) -
 (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(a*b*Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c +
d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + (4*I)*a^2*Ellip
ticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*b^2*EllipticPi[-((a + b)/
(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a -
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] + (4*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcS
inh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
```

```
+ d*x)/2]^2)/(a + b)] - I*(2*a^2 - a*b - b^2)*EllipticF[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*Sqrt[a + b*Se
c[c + d*x]]*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2
)]*(-1 + Tan[(c + d*x)/2]^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.71, size = 1409, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(-1+cos(d*x+c))^2*(4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-
b)/(a+b))^(1/2))*a^2*sin(d*x+c)*cos(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)*cos(d*x+c)-2*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x
+c)+cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*sin(d*x+c)*a*b+3*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+4*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*
x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*s
in(d*x+c)+EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*si
n(d*x+c)*a*b+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
```

```
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b^2*sin(d*x+c)-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*sin(d*x+c)*a*b-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*sin(d*x+c)-cos(d*x+c)^2*a^2+cos(d*x+c)^2*a*b-a*b*cos(d*x+c)+co
s(d*x+c)*b^2)*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos
(d*x+c))/sin(d*x+c)^5/(a-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^2}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

3.335 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(a+b\sec(c+dx))^{5/2}}{5b^4d}-$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2*a*(a+b*sec(d*x+c))^(
3/2)/b^4/d+2/5*(a+b*sec(d*x+c))^(5/2)/b^4/d+2*(a^2-b^2)^2/a/b^4/d/(a+b*sec(
d*x+c))^(1/2)+2*(3*a^2-2*b^2)*(a+b*sec(d*x+c))^(1/2)/b^4/d
```

Rubi [A] time = 0.17, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 206}

$$\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2)
^2)/(a*b^4*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 - 2*b^2)*Sqrt[a + b*Sec[
c + d*x]])/(b^4*d) - (2*a*(a + b*Sec[c + d*x])^(3/2))/(b^4*d) + (2*(a + b*S
ec[c + d*x])^(5/2))/(5*b^4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{b^4d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{(-a^2+b^2+2ax^2-x^4)^2}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-\frac{(a^2-b^2)^2}{ax^2}-3ax^2+x^4-\frac{b^4}{a(a-x^2)}\right)dx, x, \sqrt{a+b\sec(c+\right.}{b^4d}$$

$$= \frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d} - \frac{2a(a+b\sec(c+}{b^4d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^2-2b^2)\sqrt{a+b\,\text{se}}}{b^4d}$$

Mathematica [A] time = 6.39, size = 263, normalized size = 1.78

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(-\frac{2(b^2-a^2)^2}{a^2b^3(a\cos(c+dx)+b)}+\frac{2(16a^4-20a^2b^2+5b^4)}{5a^2b^4}-\frac{6a\sec(c+dx)}{5b^3}+\frac{2\sec^2(c+dx)}{5b^2}\right)}{d(a+b\sec(c+dx))^{3/2}} - \tan^2(c+dx$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(16*a^4 - 20*a^2*b^2 + 5*b^4))/(
5*a^2*b^4) - (2*(-a^2 + b^2)^2)/(a^2*b^3*(b + a*Cos[c + d*x])) - (6*a*Sec[c
 + d*x])/(5*b^3) + (2*Sec[c + d*x]^2)/(5*b^2)))/(d*(a + b*Sec[c + d*x])^(3/
2)) - (Sqrt[a*Cos[c + d*x]]*(b + a*Cos[c + d*x])^(3/2)*(-Log[1 - Sqrt[b + a
*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqr
t[a*Cos[c + d*x]]])*Tan[c + d*x]^2)/(a^2*d*(1 - Cos[c + d*x]^2)*(a + b*Sec[
c + d*x])^(3/2))
```

fricas [A] time = 1.42, size = 467, normalized size = 3.16

$$\left[\frac{5\left(ab^4\cos(dx+c)^3+b^5\cos(dx+c)^2\right)\sqrt{a}\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2+4\left(2a\cos(dx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/10*(5*(a*b^4*cos(d*x + c)^3 + b^5*cos(d*x + c)^2)*sqrt(a)*log(-8*a^2*cos
(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - 4*(2*a^3*b^2*cos(d
*x + c) - a^2*b^3 - (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 - 2*(4*a
^4*b - 5*a^2*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/
(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2), 1/5*(5*(a*b^4*cos(d*
x + c)^3 + b^5*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(2*a^3*b^2*
```

```
cos(d*x + c) - a^2*b^3 - (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 - 2
*(4*a^4*b - 5*a^2*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(dx+c)^5}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^5/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 2.39, size = 6612, normalized size = 44.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.43, size = 194, normalized size = 1.31

$$\frac{\frac{5\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{10}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,a}+\frac{2\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{b^4}-\frac{10\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{b^4}+\frac{30\sqrt{a+\frac{b}{\cos(dx+c)}}\,a^2}{b^4}+\frac{10\,a^3}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,b^4}-\frac{20\sqrt{a+}}{}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(5*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) +
 sqrt(a)))/a^(3/2) + 10/(sqrt(a + b/cos(d*x + c))*a) + 2*(a + b/cos(d*x + c
))^(5/2)/b^4 - 10*(a + b/cos(d*x + c))^(3/2)*a/b^4 + 30*sqrt(a + b/cos(d*x
+ c))*a^2/b^4 + 10*a^3/(sqrt(a + b/cos(d*x + c))*b^4) - 20*sqrt(a + b/cos(d
*x + c))/b^2 - 20*a/(sqrt(a + b/cos(d*x + c))*b^2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^5}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x))**(3/2), x)
```

3.336 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\left(a^2-b^2\right)}{ab^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a^2-b^2)/a/b^2/d/(a+
b*sec(d*x+c))^(1/2)+2*(a+b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 206}

$$\frac{2\left(a^2-b^2\right)}{ab^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2))
/(a*b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*x]])/(b^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{b^2d}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d}$$
$$= -\frac{2\,\text{Subst}\left(\int \left(-1+\frac{a^2-b^2}{ax^2}-\frac{b^2}{a(a-x^2)}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d}$$
$$= \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{a+b\sec(c+dx)}}{b^2d} + \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\,\text{se}\right.}{ad}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}$$

Mathematica [A] time = 1.03, size = 167, normalized size = 1.90

$$\frac{4a^2-\frac{b^2\sqrt{a\cos(c+dx)+b}\,\log\left(1-\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}\right)}{\sqrt{a\cos(c+dx)}}+\frac{b^2\sqrt{a\cos(c+dx)+b}\,\log\left(\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}+1\right)}{\sqrt{a\cos(c+dx)}}+2ab\sec(c+dx)-2b^2}{ab^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*a^2 - 2*b^2 - (b^2*Sqrt[b + a*Cos[c + d*x]]*Log[1 - Sqrt[b + a*Cos[c + d
*x]]/Sqrt[a*Cos[c + d*x]]])/Sqrt[a*Cos[c + d*x]] + (b^2*Sqrt[b + a*Cos[c +
d*x]]*Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])/Sqrt[a*Cos[c
+ d*x]] + 2*a*b*Sec[c + d*x])/(a*b^2*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [A] time = 1.17, size = 317, normalized size = 3.60

$$\left[\frac{(ab^2\cos(dx+c)+b^3)\sqrt{a}\,\log\left(-8\,a^2\cos(dx+c)^2-8\,ab\cos(dx+c)-b^2-4\left(2\,a\cos(dx+c)^2+b\cos(dx+c)\right.\right.}{2\left(a^3b^2d\cos(dx+c)+a^2b^3d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^2*cos(d*x + c) + b^3)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) + 4*(a^2*b + (2*a^3 - a*b^2)*cos(d*x + c
))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a^3*b^2*d*cos(d*x + c) + a^2*b
^3*d), -((a*b^2*cos(d*x + c) + b^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(a^2*b
 + (2*a^3 - a*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
a^3*b^2*d*cos(d*x + c) + a^2*b^3*d)]
```

giac [B] time = 2.06, size = 307, normalized size = 3.49

$$2\left(\frac{\frac{\left(2\,a^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)-a^{2}b\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)-ab^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{2}b^{2}}-\frac{2\,a^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)+a^{2}b\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)-\cdots}{a^{2}b^{2}}}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}}\cdots\right.$$
$$\overline{\qquad\qquad\qquad\qquad d\qquad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2*(((2*a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - a^2*b*sgn(tan(1/2*d*x + 1/2*c)
^2 - 1) - a*b^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/(a^
2*b^2) - (2*a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + a^2*b*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1) - a*b^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/(a^2*b^2))/sqrt(a*tan
(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2
 + a + b) + arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b) + sqrt(a - b))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
))/d
```

maple [B] time = 1.66, size = 2830, normalized size = 32.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^3*(2*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*
(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*
x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a
-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4*b^2+4*(a-b)^(3/2)*cos(d*x+c)^3*(
(b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a^3+12*(a-b)^(3/2)*cos(
d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a^3+4*(a-b)^(3/
2)*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^4-2*
cos(d*x+c)^2*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c)
)*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*co
s(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a
-b)^(1/2))*a^3*b^3-2*cos(d*x+c)^2*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)
^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+
b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(
1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4*b^2+2*cos(d*x+c)^2*ln(-2*(-1+cos(d*x+
c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^3+cos(d*x+c)
*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a
^3*b^3-cos(d*x+c)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d
*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)
^2/(a-b)^(1/2))*a^2*b^4-cos(d*x+c)*ln(-2*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b
)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)
+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^
(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^3+cos(d*x+c)*ln(-2*(-1+cos(d*x+c))
*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(
```

```
1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^4+12*(a-b)^(3/2
)*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a^3+cos
(d*x+c)^3*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*c
os(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d
*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)
^(1/2))*a^5*b-cos(d*x+c)^3*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((
b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x
+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/s
in(d*x+c)^2/(a-b)^(1/2))*a^4*b^2-cos(d*x+c)^3*ln(-2*(-1+cos(d*x+c))*(2*cos(
d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a
*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(
1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^5*b+cos(d*x+c)^3*ln(-2*(-1+
cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4*b^2+8*
(a-b)^(3/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
*a^3*b+2*(a-b)^(3/2)*a^(5/2)*cos(d*x+c)^3*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^2+4*(a-b)^(3/2)*a^(3/
2)*cos(d*x+c)^2*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos
(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(1/2)+4*a*cos(d*x+c)+2*b)*b^3-12*(a-b)^(3/2)*cos(d*x+c)^2*((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a*b^2+2*(a-b)^(3/2)*a^(1/2)*cos(d*x+c)*
ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2
)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*
x+c)+2*b)*b^4-4*(a-b)^(3/2)*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(1+co
s(d*x+c))^2)^(3/2)*a*b^2-12*(a-b)^(3/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*
x+c)/(1+cos(d*x+c))^2)^(3/2)*a*b^2+8*(a-b)^(3/2)*cos(d*x+c)^2*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^3*b+4*(a-b)^(3/2)*cos(d*x+c)*((b+
a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^2*b^2+4*(a-b)^(3/2)*((b+
a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^2*b^2+4*(a-b)^(3/2)*cos(
d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^4-4*((b+a*c
os(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a*b^2+4*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a^3)*cos(d*x+c)*((b
+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)/(b+a*cos(d*x+c))/((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)/sin(d*x+c)^6/b^2/(a-b)^(3/2)/a^2
```

maxima [A] time = 0.42, size = 110, normalized size = 1.25

$$-\frac{\frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,a}-\frac{2\sqrt{a+\frac{b}{\cos(dx+c)}}}{b^2}-\frac{2a}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -(log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt
(a)))/a^(3/2) + 2/(sqrt(a + b/cos(d*x + c))*a) - 2*sqrt(a + b/cos(d*x + c))
/b^2 - 2*a/(sqrt(a + b/cos(d*x + c))*b^2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^3}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

3.337 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2}{ad\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2/a/d/(a+b*sec(d*x+c))
^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3885, 51, 63, 207}

$$\frac{2}{ad\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + 2/(a*d*Sqrt[a
+ b*Sec[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{2}{ad\sqrt{a+b\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{ad}$$
$$= \frac{2}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2}{ad\sqrt{a+b\sec(c+dx)}}$$

Mathematica [B] time = 0.39, size = 128, normalized size = 2.37

$$\frac{\sec(c+dx)\left(\sqrt{a\cos(c+dx)}\sqrt{a\cos(c+dx)+b}\left(\log\left(1-\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}\right)-\log\left(\frac{\sqrt{a\cos(c+dx)+b}}{\sqrt{a\cos(c+dx)}}+1\right)\right)+2a\cos(c}{a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*a*Cos[c + d*x] + Sqrt[a*Cos[c + d*x]]*Sqrt[b + a*Cos[c + d*x]]*(Log[1 -
 Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] - Log[1 + Sqrt[b + a*Cos[c
+ d*x]]/Sqrt[a*Cos[c + d*x]]]))*Sec[c + d*x])/(a^2*d*Sqrt[a + b*Sec[c + d*x
]])
```

fricas [B] time = 1.04, size = 260, normalized size = 4.81

$$\left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)+(a\cos(dx+c)+b)\sqrt{a}\log\left(-8\,a^2\cos(dx+c)^2-8\,ab\cos(dx+c)-b^2+4\,(2}{2\left(a^3d\cos(dx+c)+a^2bd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*a*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a*cos(d*x
 + c) + b)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4
*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))))/(a^3*d*cos(d*x + c) + a^2*b*d), ((a*cos(d*x + c) + b)*sqrt(-a
)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2
*a*cos(d*x + c) + b)) + 2*a*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
 + c))/(a^3*d*cos(d*x + c) + a^2*b*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(cos
(d*t_nostep+c))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Discontinuities at zeroes of cos(d*t_nostep+c
) were not checkedWarning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Ev
aluation time: 0.8Unable to divide, perhaps due to rounding error%%%{%%%{4,
[2,2]%%%}+%%%{-4,[1,3]%%%},[2,1]%%%}+%%%{%%{[%%%{8,[2,2]%%%}+%%%{8,[1,3]%%%
},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[1,1]%%%}+%%%{%%%{4,[3,2]%%%
}+%%%{8,[2,3]%%%}+%%%{4,[1,4]%%%},[0,1]%%%} / %%%{%%%{1,[2,0]%%%}+%%%{-2,[1
,1]%%%}+%%%{1,[0,2]%%%},[2,0]%%%}+%%%{%%{[%%%{2,[2,0]%%%}+%%%{-2,[0,2]%%%},
0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[1,0]%%%}+%%%{%%%{1,[3,0]%%%}+
%%%{1,[2,1]%%%}+%%%{-1,[1,2]%%%}+%%%{-1,[0,3]%%%},[0,0]%%%} Error: Bad Argu
ment Value
```

maple [A] time = 0.14, size = 45, normalized size = 0.83

$$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b\sec(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(-2/a^(3/2)*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))+2/a/(a+b*sec(d*x+c)
)^(1/2))
```

maxima [A] time = 0.42, size = 70, normalized size = 1.30

$$\frac{\frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] (log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt(
a)))/a^(3/2) + 2/(sqrt(a + b/cos(d*x + c))*a))/d
```

mupad [B] time = 1.95, size = 50, normalized size = 0.93

$$\frac{2}{a\,d\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{a^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] 2/(a*d*(a + b/cos(c + d*x))^(1/2)) - (2*atanh((a + b/cos(c + d*x))^(1/2)/a^
(1/2)))/(a^(3/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

3.338 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-arctanh((a+b*sec(d*x+c)
)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2))/(a+b)^(3/2)/d+2*b^2/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3885, 898, 1287, 206}

$$\frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{b^2 \operatorname{Subst}\left(\int \frac{1}{x(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\operatorname{Subst}\left(\int \frac{1}{x^2(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\operatorname{Subst}\left(\int \left(\frac{1}{a(a^2-b^2)x^2} - \frac{1}{ab^2(a-x^2)} + \frac{1}{2(a-b)b^2(a-b-x^2)} + \frac{1}{2b^2(a+b)(a+b-x^2)}\right) dx, x,\right.}{d}$$

$$= \frac{2b^2}{a(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad} -$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} +$$

Mathematica [B] time = 6.94, size = 1020, normalized size = 7.18

$$\frac{(b+a\cos(c+dx))^2\left(-\frac{2b^3}{a^2(a^2-b^2)(b+a\cos(c+dx))} - \frac{2b^2}{a^2(b^2-a^2)}\right)\sec^2(c+dx)}{d(a+b\sec(c+dx))^{3/2}} - \frac{(b+a\cos(c+dx))^{3/2}\left(\frac{b\left(-\sqrt{-a^2}\sqrt{a+b}\log(\sqrt{\right.}{}\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((b + a*Cos[c + d*x])^(3/2)*((a^2*b*(-(Sqrt[-a^2]*Sqrt[a + b]*Log[-Sqr
t[-a + b] + Sqrt[b + a*Cos[c + d*x]]]) + Sqrt[-a^2]*Sqrt[a + b]*Log[Sqrt[-a
 + b] + Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[-Sqrt[a + b] + Sqrt[
b + a*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[Sqrt[a + b] + Sqrt[b + a*Cos[c +
d*x]]] + Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt[a]*Sqrt[-(a*Cos[c + d*x])] - S
qrt[-a + b]*Sqrt[b + a*Cos[c + d*x]]] - Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt
[a]*Sqrt[-(a*Cos[c + d*x])] + Sqrt[-a + b]*Sqrt[b + a*Cos[c + d*x]]] + a*Sq
rt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])] - Sqrt[a + b]*Sqrt[b +
a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])]
+ Sqrt[a + b]*Sqrt[b + a*Cos[c + d*x]]]))/((-a)^(3/2)*Sqrt[-a + b]*Sqrt[a +
 b]*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]]) - ((a^2 + b^2)*(Sqrt[a - b]
*(a + b)*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Co
s[c + d*x])])]] + (a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x
]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])])*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Se
c[c + d*x]])/(Sqrt[a]*(a - b)*(a + b)) - (a*(a^2 - b^2)*(4*Sqrt[a - b]*Sqrt
[a + b]*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]] - Sqrt[a]*
(Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]*Sqrt[-(
a*Cos[c + d*x])])] + Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/
(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])])]))*Sqrt[-(a*Cos[c + d*x])]*Cos[2*(c +
 d*x)]*Sqrt[Sec[c + d*x]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - 2*b^2 + 4*b*(b +
 a*Cos[c + d*x]) - 2*(b + a*Cos[c + d*x])^2)))*Sec[c + d*x]^(3/2))/(a*(-a +
 b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*((-2*b^
2)/(a^2*(-a^2 + b^2)) - (2*b^3)/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])))*Sec
[c + d*x]^2)/(d*(a + b*Sec[c + d*x])^(3/2))
```

fricas [B] time = 38.67, size = 3924, normalized size = 27.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
 c) + 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*
sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)
*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) + (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x
 + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2
*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a
^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 +
 a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sq
rt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a^4*b + 2*a^3*b^2 + a
^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 -
 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*
x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*
b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - (a^4*b - 2*a^3*
b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((
8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b
*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a
*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2
*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4
*(2*(a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*
sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sqrt((a*c
os(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - 2*(a^4*b - 2*a^2*b^3 + b^5 +
(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 -
 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (a^4*b - 2*a^3*b^2 + a^2*b^3 +
(a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b +
b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(
d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b
^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a
^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + b)) + 2*(a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d
*x + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - (a^4*b - 2*a^3*b^2 +
 a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2
 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(
d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b +
3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*
b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), 1/4*(2*(a
^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-
a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*
x + c)/((2*a + b)*cos(d*x + c) + b)) + 8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c))*cos(d*x + c) + 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 -
2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 +
```

```
2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a -
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*c
os(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a^2*b^3
+ b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)
) - 2*(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c)
)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
)*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a^4*b + 2*a^3*b^2 + a^2*b^3
 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b
 + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c)
)*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*c
os(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^
3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/2*((a^4*b + 2*
a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-a + b)*ar
ctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(
(2*a - b)*cos(d*x + c) + b)) - (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*
b + a^3*b^2)*cos(d*x + c))*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 4*(
a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - (a^
4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log
(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 +
 b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/((a^7 -
2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/
2*(2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c
)/(2*a*cos(d*x + c) + b)) + (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b +
 a^3*b^2)*cos(d*x + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^4*
b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-a -
 b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
 c)/((2*a + b)*cos(d*x + c) + b)) - 4*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))*cos(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x +
 c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration
 of abs or sign assumes constant sign by intervals (correct if the argument
 is real):Check [abs(cos(d*t_nostep+c))]Unable to check sign: (2*pi/t_noste
```

```
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Discontinuities at ze
roes of cos(d*t_nostep+c) were not checkedWarning, integration of abs or si
gn assumes constant sign by intervals (correct if the argument is real):Che
ck [abs(t_nostep^2-1)]Evaluation time: 0.88Error: Bad Argument Type
```

maple [B] time = 1.32, size = 2766, normalized size = 19.48

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))*(-2*a^(9/2)*cos(d*x+c)*(a-b)^(3/2)*ln(4*a^(1/2)*cos(d
*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)-2*a^(7/2
)*cos(d*x+c)*(a-b)^(3/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+
c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*
x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b-2*a^(7/2)*(a-b)^(3/2)*ln(4*a^(1/2)*cos
(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+
a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b+2*a^
(5/2)*cos(d*x+c)*(a-b)^(3/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(
d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+co
s(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^2-2*a^(5/2)*ln(4*a^(1/2)*cos(d*x+c
)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(
d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*(a-b)^(3/2)*
b^2+2*a^(3/2)*cos(d*x+c)*(a-b)^(3/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+
c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+
c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^3+(a+b)^(1/2)*cos(d*x+c)*(
a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+
cos(d*x+c))^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*a^4-(a+b)^(1/2
)*cos(d*x+c)*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*
cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2
)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*
a^3*b+2*a^(3/2)*(a-b)^(3/2)*ln(4*a^(1/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos
(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^3+(a+b)^(1/2)*ln(-2*(2*cos(d*x+c)*(
a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*a*cos(d*x
+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(1/2)+b)/(-1+cos(d*x+c)))*(a-b)^(3/2)*a^3*b-(a+b)^(1/2)*ln(-2*(2*cos(d*x
+c)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*a*co
s(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*(a-b)^(3/2)*a^2*b^2+2*a^(1/2)*ln(4*a^(1/2
)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*(
a-b)^(3/2)*b^4-4*(a-b)^(3/2)*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos
(d*x+c))^2)^(1/2)*a^2*b^2-4*cos(d*x+c)*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*
x+c)/(1+cos(d*x+c))^2)^(1/2)*a*b^3-cos(d*x+c)*ln(-(-1+cos(d*x+c))*(2*cos(d*
x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*c
os(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/
2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^6-cos(d*x+c)*ln(-(-1+cos(d*x+
c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^5*b+cos(d*x+c)*l
n(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(
```

```
1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(
d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4
*b^2+cos(d*x+c)*ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+c)^2
/(a-b)^(1/2))*a^3*b^3-4*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*a^2*b^2-4*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*a*b^3-ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)+2*(
(b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(d*x+
c)^2/(a-b)^(1/2))*a^5*b-ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)
+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b)/sin(
d*x+c)^2/(a-b)^(1/2))*a^4*b^2+ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+b*cos(
d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-b
)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^3+ln(-(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)
^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-2*a*cos(d*x+c)+
b*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(
1/2)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^4)*cos(d*x+c)*((b+a*cos(d*x+c))/cos
(d*x+c))^(1/2)*4^(1/2)/((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
/(b+a*cos(d*x+c))/sin(d*x+c)^2/(a-b)^(5/2)/(a+b)^2/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

3.339 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2b^4}{ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)^2(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+d}}{4d(a-b)^2(\sec(c+dx}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/4*(4*a-7*b)*arctanh(
(a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+1/4*(4*a+7*b)*arctanh((a+
b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d+2*b^4/a/(a^2-b^2)^2/d/(a+b*s
ec(d*x+c))^(1/2)+1/4*(a+b*sec(d*x+c))^(1/2)/(a+b)^2/d/(1-sec(d*x+c))+1/4*(a
+b*sec(d*x+c))^(1/2)/(a-b)^2/d/(1+sec(d*x+c))
```

Rubi [A] time = 0.38, antiderivative size = 316, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3885, 898, 1335, 206, 199}

$$\frac{2b^4}{ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)^2(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)^2(\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + ((2*a - 3*b)*A
rcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(2*(a - b)^(5/2)*d) - (b*ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*d) + (b*ArcTanh
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) + ((2*a + 3*b)*
ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(2*(a + b)^(5/2)*d) + (2*b^4
)/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + Sqrt[a + b*Sec[c + d*x]]/(
4*(a + b)^2*d*(1 - Sec[c + d*x])) + Sqrt[a + b*Sec[c + d*x]]/(4*(a - b)^2*d
*(1 + Sec[c + d*x]))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{\left(2b^4\right)\text{Subst}\left(\int \frac{1}{x^2\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{\left(2b^4\right)\text{Subst}\left(\int \left(-\frac{1}{a(a-b)^2(a+b)^2x^2} - \frac{1}{ab^4\left(a-x^2\right)} - \frac{1}{4(a-b)b^3\left(a-b-x^2\right)^2} + \frac{2a-3b}{4(a-b)^2b^4\left(a-b-x^2\right)} + \right.}{d}$$

$$= \frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad} +$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} + \frac{(2a+3b)\tanh}{2(a}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b}}{}\right.}{4(a-b)^5}$$

Mathematica [B] time = 7.53, size = 1114, normalized size = 4.72

$$\frac{(b+a\cos(c+dx))^2\left(-\frac{2b^5}{a^2\left(a^2-b^2\right)^2(b+a\cos(c+dx))} + \frac{\left(-a^2+2b\cos(c+dx)a-b^2\right)\csc^2(c+dx)}{2\left(b^2-a^2\right)^2} + \frac{a^4+b^2a^2+4b^4}{2a^2\left(b^2-a^2\right)^2}\right)\sec^2(c+dx)}{d(a+b\sec(c+dx))^{3/2}} - \frac{(b+a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/4*((b + a*Cos[c + d*x])^(3/2)*(-1/2*(a*(-(a^3*b) + 7*a*b^3)*(-(Sqrt[-a^2
]*Sqrt[a + b]*Log[-Sqrt[-a + b] + Sqrt[b + a*Cos[c + d*x]]]) + Sqrt[-a^2]*S
qrt[a + b]*Log[Sqrt[-a + b] + Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Lo
g[-Sqrt[a + b] + Sqrt[b + a*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[Sqrt[a + b]
 + Sqrt[b + a*Cos[c + d*x]]] + Sqrt[-a^2]*Sqrt[a + b]*Log[b + Sqrt[a]*Sqrt[
-(a*Cos[c + d*x])] - Sqrt[-a + b]*Sqrt[b + a*Cos[c + d*x]]] - Sqrt[-a^2]*Sq
rt[a + b]*Log[b + Sqrt[a]*Sqrt[-(a*Cos[c + d*x])] + Sqrt[-a + b]*Sqrt[b + a
*Cos[c + d*x]]] + a*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sqrt[-(a*Cos[c + d*x])] -
 Sqrt[a + b]*Sqrt[b + a*Cos[c + d*x]]] - a*Sqrt[-a + b]*Log[b + Sqrt[-a]*Sq
```

```
rt[-(a*Cos[c + d*x])] + Sqrt[a + b]*Sqrt[b + a*Cos[c + d*x]]]))/((-a)^(3/2)
*Sqrt[-a + b]*Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]]) - ((2
*a^4 - 6*a^2*b^2 - 2*b^4)*(Sqrt[a - b]*(a + b)*ArcTan[(Sqrt[a]*Sqrt[b + a*C
os[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cos[c + d*x])])] + (a - b)*Sqrt[a + b]*
ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cos[c + d*x
])])])*Sqrt[-(a*Cos[c + d*x])]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*(a - b)*(a + b)
) - (a*(2*a^4 - 4*a^2*b^2 + 2*b^4)*(4*Sqrt[a - b]*Sqrt[a + b]*ArcTan[Sqrt[b
 + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]] - Sqrt[a]*(Sqrt[a + b]*ArcTan[(
Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cos[c + d*x])])] +
Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a
*Cos[c + d*x])])]))*Sqrt[-(a*Cos[c + d*x])]*Cos[2*(c + d*x)]*Sqrt[Sec[c + d
*x]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - 2*b^2 + 4*b*(b + a*Cos[c + d*x]) - 2*
(b + a*Cos[c + d*x])^2)))*Sec[c + d*x]^(3/2))/(a*(a - b)^2*(a + b)^2*d*(a +
 b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*((a^4 + a^2*b^2 + 4*b^4)/
(2*a^2*(-a^2 + b^2)^2) - (2*b^5)/(a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) +
 ((-a^2 - b^2 + 2*a*b*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(-a^2 + b^2)^2))*Sec
[c + d*x]^2)/(d*(a + b*Sec[c + d*x])^(3/2))
```

fricas [B] time = 76.76, size = 8098, normalized size = 34.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (a^7 - 3*a^5*b^2 + 3*a^3*b
^4 - a*b^6)*cos(d*x + c)^3 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x
+ c)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*sqrt(a)*log(-8
*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*
cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (4*a^6*b +
 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 + 5*a^6*b - 9*a^5*
b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b + 5*a^5*b^2 - 9*a^4
*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 + 5*a^6*b - 9*a^5*b^
2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b
+ b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))
*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*co
s(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (4*a^6*b - 5*a^5*b^2 -
 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4
*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^
3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b
^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d
*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/
(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^7 + 3*a^3*b^4 - 4*a*b^6)*cos
(d*x + c)^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 2*(a^5*b^2 + a
^3*b^4 - 2*a*b^6)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((
a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3 + (a^8*b - 3*a^6*b^
3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x + c)^2 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 -
a^3*b^6)*d*cos(d*x + c) - (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d), -1/
16*(16*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6)*cos(d*x + c)^3 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c
)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*sqrt(-a)*arctan(2
*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x
 + c) + b)) - (4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 - (
4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a
^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*
a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(a -
b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x
+ c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
```

+ (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 - 5*a^
6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b - 5*a^5
*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 - 5*a^6*
b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(a + b)*log(-((8*
a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*c
os(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b
 + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^7 +
3*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x
 + c)^2 - 2*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c))*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c)))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x +
c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x + c)^2 - (a^9 -
3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c) - (a^8*b - 3*a^6*b^3 + 3*a^
4*b^5 - a^2*b^7)*d), -1/16*(2*(4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4
 - 7*a^2*b^5 - (4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d
*x + c)^3 - (4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cos(
d*x + c)^2 + (4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x
 + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*
x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) + 8*(a^6*b - 3*a^4*b^3 +
 3*a^2*b^5 - b^7 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (
a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^2 + (a^7 - 3*a^5*b^2 + 3*
a^3*b^4 - a*b^6)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) + (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^
3*b^4 - 7*a^2*b^5 - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*
cos(d*x + c)^3 - (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)
*cos(d*x + c)^2 + (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*co
s(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 +
4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 -
2*cos(d*x + c) + 1)) - 8*((a^7 + 3*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3 - (a^6
*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 2*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*
cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^9 - 3*a^7*b^2 +
3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^
2*b^7)*d*cos(d*x + c)^2 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x
 + c) - (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d), -1/16*(16*(a^6*b - 3*
a^4*b^3 + 3*a^2*b^5 - b^7 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x +
 c)^3 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^2 + (a^7 - 3*a^5
*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + 2*(4
*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 + 5*a^6*b
- 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b + 5*a^5*b^2
 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 + 5*a^6*b -
9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(-a + b)*arctan(-2*sq
rt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*
cos(d*x + c) + b)) + (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*
b^5 - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3
 - (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^
2 + (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sq
rt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*c
os(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d
x + c)) + 2(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c)
 + 1)) - 8*((a^7 + 3*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3 - (a^6*b - 2*a^4*b^3
 + a^2*b^5)*cos(d*x + c)^2 - 2*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c))*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^
3*b^6)*d*cos(d*x + c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d
*x + c)^2 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c) - (a^8*b
 - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d), 1/16*(2*(4*a^6*b - 5*a^5*b^2 - 9*a^
4*b^3 + 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3
- 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4

```
 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 -
7*a^3*b^4)*cos(d*x + c))*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 8*(a^
6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c
os(d*x + c)^3 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^2 + (a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x
+ c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))
*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) + (4*a^6*b + 5*a^5*b^2 -
9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*
b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3
*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^
3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*
x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 8*((a^7 + 3*a^3*b^4 - 4*a*b^6)*cos(
d*x + c)^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 2*(a^5*b^2 + a^
3*b^4 - 2*a*b^6)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a
^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3 + (a^8*b - 3*a^6*b^3
 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x + c)^2 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a
^3*b^6)*d*cos(d*x + c) - (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d), -1/1
6*(16*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
 a*b^6)*cos(d*x + c)^3 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)
^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x
+ c) + b)) - 2*(4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5 -
(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*
a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4
*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(-a
- b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
+ c)/((2*a + b)*cos(d*x + c) + b)) - (4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*
a^3*b^4 - 7*a^2*b^5 - (4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4
)*cos(d*x + c)^3 - (4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^
5)*cos(d*x + c)^2 + (4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*
cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2
- 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2
+ 2*cos(d*x + c) + 1)) - 8*((a^7 + 3*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3 - (a
^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 2*(a^5*b^2 + a^3*b^4 - 2*a*b^6
)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^9 - 3*a^7*b^2
+ 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 -
a^2*b^7)*d*cos(d*x + c)^2 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d
*x + c) - (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d), -1/8*((4*a^6*b + 5*
a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 + 5*a^6*b - 9*a^5*b^2
 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b + 5*a^5*b^2 - 9*a^4*b^
3 - 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 + 5*a^6*b - 9*a^5*b^2 -
 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c
) + b)) - (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5 - (4*a^
7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b
 - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7
- 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(-a - b)*
arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/
((2*a + b)*cos(d*x + c) + b)) + 4*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*cos(d*x + c)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(
d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*
(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c))) - 4*((a^7 + 3*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3 - (a^6*b - 2*a^
4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 2*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x +
```

```
c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4
 - a^3*b^6)*d*cos(d*x + c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*
cos(d*x + c)^2 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c) - (
a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d), -1/8*(8*(a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (a^
6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^2 + (a^7 - 3*a^5*b^2 + 3*a^
3*b^4 - a*b^6)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (4*a^6*b + 5*a^
5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 + 5*a^6*b - 9*a^5*b^2 -
 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3
- 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 + 5*a^6*b - 9*a^5*b^2 - 1
7*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sq
rt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c)
+ b)) - (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7
- 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b -
 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 -
5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(-a - b)*ar
ctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((
2*a + b)*cos(d*x + c) + b)) - 4*((a^7 + 3*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3
 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 2*(a^5*b^2 + a^3*b^4 - 2*
a*b^6)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^9 - 3*a^7
*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b
^5 - a^2*b^7)*d*cos(d*x + c)^2 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*
cos(d*x + c) - (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)]
```

giac **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(cos(d*t_nostep+c))]Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
```

```
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Discontinuities at
 zeroes of cos(d*t_nostep+c) were not checkedWarning, integration of abs or
 sign assumes constant sign by intervals (correct if the argument is real):
Check [abs(t_nostep^2-1)]Evaluation time: 1.2Unable to divide, perhaps due
to rounding error%%%{%%%{32,[2,6]%%%}+%%%{-32,[1,7]%%%},[6,1]%%%}+%%%{%%{[%
%%{64,[2,6]%%%}+%%%{64,[1,7]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%
%},[5,1]%%%}+%%%{%%%{-32,[3,6]%%%}+%%%{64,[2,7]%%%}+%%%{96,[1,8]%%%},[4,1]%
%%}+%%%{%%{[%%%{-128,[3,6]%%%}+%%%{-256,[2,7]%%%}+%%%{-128,[1,8]%%%},0]:[1,
0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[3,1]%%%}+%%%{%%%{-32,[4,6]%%%}+%%%{
-160,[3,7]%%%}+%%%{-224,[2,8]%%%}+%%%{-96,[1,9]%%%},[2,1]%%%}+%%%{%%{[%%%{6
4,[4,6]%%%}+%%%{192,[3,7]%%%}+%%%{192,[2,8]%%%}+%%%{64,[1,9]%%%},0]:[1,0,%%
%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[1,1]%%%}+%%%{%%%{32,[5,6]%%%}+%%%{128,[
4,7]%%%}+%%%{192,[3,8]%%%}+%%%{128,[2,9]%%%}+%%%{32,[1,10]%%%},[0,1]%%%} /
%%%{%%%{1,[2,0]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,2]%%%},[6,0]%%%}+%%%{%%{[%%%{
2,[2,0]%%%}+%%%{-2,[0,2]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[
5,0]%%%}+%%%{%%%{-1,[3,0]%%%}+%%%{3,[2,1]%%%}+%%%{1,[1,2]%%%}+%%%{-3,[0,3]%
%%},[4,0]%%%}+%%%{%%{[%%%{-4,[3,0]%%%}+%%%{-4,[2,1]%%%}+%%%{4,[1,2]%%%}+%%%
{4,[0,3]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[3,0]%%%}+%%%{%%%
{-1,[4,0]%%%}+%%%{-4,[3,1]%%%}+%%%{-2,[2,2]%%%}+%%%{4,[1,3]%%%}+%%%{3,[0,4]
%%%},[2,0]%%%}+%%%{%%{[%%%{2,[4,0]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[1,3]%%%}+%%%
{-2,[0,4]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[1,0]%%%}+%%%{%%
%{1,[5,0]%%%}+%%%{3,[4,1]%%%}+%%%{2,[3,2]%%%}+%%%{-2,[2,3]%%%}+%%%{-3,[1,4]
%%%}+%%%{-1,[0,5]%%%},[0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.96, size = 10977, normalized size = 46.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot\left(c+d\,x\right)^3}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{3}\left(c + dx\right)}{\left(a + b \sec\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

3.340 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=530

$$-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{4a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(4a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)}+$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a^2/d+2/3*(8*a^4-11*a^2*b^2+3*b^4)*cot(d*x+c)*EllipticE((a+b*
sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b
))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^4/d/(a+b)^(1/2)+2/3*(2*a+b)*(4
*a^2+a*b-3*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a
+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/b^3/d/(a+b)^(1/2)-4*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/
2)+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2*a^2*sec(d*x+c)*t
an(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*sec(d*x
+c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

Rubi [A] time = 1.30, antiderivative size = 907, normalized size of antiderivative = 1.71, number of steps used = 17, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3895, 3785, 4058, 3921, 3784, 3832, 4004, 3836, 4005, 3845, 4082}

$$-\frac{2\sec(c+dx)\tan(c+dx)a^2}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{4\tan(c+dx)a}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(8a^2-5b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{3b^4\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (4*a*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d)
 + (2*a*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) - (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(a*Sqrt[a + b]*d) - (4*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d) + (2*(2*a + b)*(
4*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) - (2*Sqrt[a + b]*Cot[c + d*x]*
EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(a^2*d) - (4*a*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) -
 (2*a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]
]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*(a^2 -
b^2)*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3895

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
```

```
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \left(\frac{1}{(a+b\sec(c+dx))^{3/2}} - \frac{2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}} + \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\right) dx$$

$$= -\left(2\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx\right) + \int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx + \int \frac{\sec^4(c+}{(a+b\sec(c+}$$

$$= -\frac{4a\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec(c+}{b\left(a^2-b^2\right)d}$$

$$= -\frac{4a\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec(c+}{b\left(a^2-b^2\right)d}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} - \frac{4a\cot}{}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} - \frac{4a\cot}{}$$

Mathematica [A] time = 17.19, size = 859, normalized size = 1.62

$$\frac{(b+a\cos(c+dx))^2\left(\frac{2(3b^2-8a^2)\sin(c+dx)}{3ab^3}-\frac{2(b^2\sin(c+dx)-a^2\sin(c+dx))}{ab^2(b+a\cos(c+dx))}+\frac{2\tan(c+dx)}{3b^2}\right)\sec^2(c+dx)}{d(a+b\sec(c+dx))^{3/2}}+\frac{2(b+a\cos(c+dx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]
^2)^(-1)]*(8*a^3*Tan[(c + d*x)/2] + 8*a^2*b*Tan[(c + d*x)/2] - 3*a*b^2*Tan[
(c + d*x)/2] - 3*b^3*Tan[(c + d*x)/2] - 16*a^3*Tan[(c + d*x)/2]^3 + 6*a*b^2
*Tan[(c + d*x)/2]^3 + 8*a^3*Tan[(c + d*x)/2]^5 - 8*a^2*b*Tan[(c + d*x)/2]^5
 - 3*a*b^2*Tan[(c + d*x)/2]^5 + 3*b^3*Tan[(c + d*x)/2]^5 + 6*b^3*EllipticPi
[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*b
^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/
2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Ta
n[(c + d*x)/2]^2)/(a + b)] + (8*a^3 + 8*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
+ Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - 2*a*b*(4*a + b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*a*b^3*d*(a +
 b*Sec[c + d*x])^(3/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + ((b + a*
Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-8*a^2 + 3*b^2)*Sin[c + d*x])/(3*a*b^3)
 - (2*(-(a^2*Sin[c + d*x]) + b^2*Sin[c + d*x]))/(a*b^2*(b + a*Cos[c + d*x])
) + (2*Tan[c + d*x])/(3*b^2)))/(d*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F] time = 23.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\tan(dx+c)^4}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.58, size = 1544, normalized size = 2.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x)

[Out] 1/3/d*4^(1/2)*(8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+3*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-6*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+3*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^3+8*cos(d*x+c)^3*a^3-4*a^2*cos(d*x+c)^3*b-3*a*b^2*cos(d*x+c)^3+3*cos(d*x+c)^3*b^3-8*cos(d*x+c)^2*a^3+8*cos(d*x+c)^2*a^2*b+2*cos(d*x+c)^2*a*b^2-3*cos(d*x+c)^2*b^3-4*cos(d*x+c)*a^2*b+b^2*a)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)/a/b^3

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan\left(c+d\,x\right)^4}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(3/2),x)

[Out] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x))**(3/2), x)
```

3.341 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=344

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-b)\sqrt{a+b}\cot(c+d}{}$$

```
[Out] 2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/a/b^2/d+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2)
,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/a/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b
))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*tan(d*x+c)/a/d/(a+b*sec(d*
x+c))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3894, 4061, 4059, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-b)\sqrt{a+b}\cot(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b^2*d) + (2*Sqrt[a + b]*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(a*b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*Tan[c +
d*x])/(a*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
```

```
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= \int \frac{-1+\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx \\
&= \frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(a^2-b^2\right)+\frac{1}{2}\left(a^2-b^2\right)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)} \\
&= \frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} - \frac{2\int \frac{\frac{1}{2}\left(a^2-b^2\right)-\frac{1}{2}\left(a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)} \\
&= \frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{ab^2d} \\
&= \frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{ab^2d}
\end{aligned}
$$

Mathematica [C] time = 23.59, size = 5162, normalized size = 15.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\tan\left(dx+c\right)^{2}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.19, size = 633, normalized size = 1.84

$$-\frac{\sqrt{4}\,\left(\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sin\left(dx+c\right)\mathrm{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)a+\cos\left(dx+c\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*4^(1/2)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a+cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-a*cos(d*x+
c)^2+cos(d*x+c)^2*b+a*cos(d*x+c)-b*cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c)
)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/a/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan\left(c+d\,x\right)^2}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}{\left(c + dx \right)}}{\left(a + b \sec{\left(c + dx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

3.342 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{a^2d}$$

```
[Out] 2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b
)^(1/2)-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/
a/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(
d*x+c))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3785, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (
2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
```

```
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int \frac{\sec(c+}{\sqrt{a}}}{a(}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} + \frac{}{a(}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} - \frac{2\,\mathrm{c}}{}$$

Mathematica [C] time = 6.16, size = 1249, normalized size = 3.60

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b*Sin[c + d*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Se
c[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[
```

```
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*
b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^5 - (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*Ell
ipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a -
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a^2 + a*b - 2*
b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/
(a + b)]*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)
*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c +
d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 23.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c)
+ a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

maple [B] time = 1.25, size = 1209, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
```

```
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*
b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*sin(d*x+c)*a*b-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*c
os(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)*cos(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)*cos(d*x+c)+(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b
-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*
a*b-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(
d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*a^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2-a*b*cos(d*x+c)+c
os(d*x+c)*b^2)/(b+a*cos(d*x+c))/sin(d*x+c)/a/(a+b)/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(1/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**(-3/2), x)
```

3.343 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=449

$$\frac{2b^2\left(a^2+b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{b^2\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\left(a^2-ab+2b^2\right)\cot(c+dx)\sqrt{-\frac{b(\sec(c+dx)-1)}{a+b}}}{ad(a-b)}$$

```
[Out] -cot(d*x+c)/d/(a+b*sec(d*x+c))^(3/2)+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x
+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*(a^2+b^2)*cot(d*x
+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(-b*(
-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(
3/2)/d-(a^2-a*b+2*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d+b^2*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*
x+c))^(3/2)+2*b^2*(a^2+b^2)*tan(d*x+c)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/
2)
```

Rubi [A] time = 0.95, antiderivative size = 664, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3896, 3785, 4058, 3921, 3784, 3832, 4004, 3875, 3833, 4003, 4005}

$$-\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{4ab^2\tan(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{b^2\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\sqrt{a+b}\ \mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a +
 b]*d) - ((3*a - b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) + (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) -
Cot[c + d*x]/(d*(a + b*Sec[c + d*x])^(3/2)) + (b^2*Tan[c + d*x])/((a^2 - b^
2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a*b^2*Tan[c + d*x])/((a^2 - b^2)^2*d*
Sqrt[a + b*Sec[c + d*x]]) - (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
```

```
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \left(-\frac{1}{(a+b\sec(c+dx))^{3/2}}+\frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\right)dx$$

$$= -\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx + \int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}} - \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{1}{2}(3b)\int \frac{\sec(c+ \cdots}{(a+b\sec(c \cdots}$$

$$= -\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}} + \frac{b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b^2\tan(c+ \cdots}{a\left(a^2-b^2\right)d\sqrt{a+b\,\mathrm{s}\cdots}}$$

$$= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} - \frac{\cdots}{d(a \cdots}$$

$$= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} + \frac{2\,\mathrm{co}\cdots}{\cdots}$$

$$= \frac{4a\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d} - \frac{2\cot \cdots}{\cdots}$$

Mathematica [C] time = 13.64, size = 663, normalized size = 1.48

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(-\frac{2b\left(a^2+b^2\right)\sin(c+dx)}{a\left(a^2-b^2\right)^2}+\frac{\csc(c+dx)\left(a^2(-\cos(c+dx))+2ab-b^2\cos(c+dx)\right)}{\left(b^2-a^2\right)^2}+\frac{2b^4\sin(c+dx)}{a\left(a^2-b^2\right)^2(a\cos(c+dx)+b)} \cdots\right)}{d(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(((2*a*b - a^2*Cos[c + d*x] - b^2*Co
s[c + d*x])*Csc[c + d*x])/(-a^2 + b^2)^2 - (2*b*(a^2 + b^2)*Sin[c + d*x])/(
a*(a^2 - b^2)^2) + (2*b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*(b + a*Cos[c + d*x
]))))/(d*(a + b*Sec[c + d*x])^(3/2)) - (2*Cos[(c + d*x)/2]^2*(b + a*Cos[c +
 d*x])*Sec[c + d*x]^2*((-2*I)*b*(-a^3 + a^2*b - a*b^2 + b^3)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)] + I*(2*a^4 - a^3*b - 2*a^2*b^2 - 3*a*b^3 + 4*b^4)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
 - b)] - (4*I)*(a^2 - b^2)^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b
)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] -
b*Sqrt[(-a + b)/(a + b)]*(a^2 + b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.32, size = 2238, normalized size = 4.98

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*(2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*a^3*b+cos(d*x+c)*a^3*b+3*cos(d*x+c)*a*b^3+2*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3-EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-6*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2-3*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*
b^3+8*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*a^2*b^2+2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*a^2*b^2+2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
```

```
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^4+2*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4-4*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a
^4-4*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*b^4-4*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*a^4-4*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^4+2*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^4+2*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4+2*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b+2*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*a^2*b^2+2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*a*b^3-EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-6*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2-3*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*a*b^3+8*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2-2*cos(d*x+c)^2*a^3*b-2*cos(d*x+c)^2*a*b^
3-2*cos(d*x+c)*a^2*b^2+cos(d*x+c)^2*a^2*b^2-2*cos(d*x+c)*b^4+cos(d*x+c)^2*a
^4+2*cos(d*x+c)^2*b^4)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos
(d*x+c))/sin(d*x+c)/a/(a-b)^2/(a+b)^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

3.344 $\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx$

Optimal. Leaf size=245

$$\frac{a^3(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{3a^2b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n}{}\right.}{df(n+1)}$$

```
[Out] 3*a*b^2*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+a^3*hypergeom([1, 1/2+1/2*n],[3/2+1/
2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+3*a^2*b*(cos(f*x+e)^2)^(
1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e
)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+b^3*(cos(f*x+e)^2)^(2+1/2*n)*hypergeom([2+
1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e)^3*(d*tan(f*x+e))^(1+
n)/d/f/(1+n)
```

Rubi [A] time = 0.27, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3886, 3476, 364, 2617, 2607, 32}

$$\frac{3a^2b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{df(n+1)}+\frac{a^3(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1\right.}{df(n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^3*(d*Tan[e + f*x])^n,x]
```

```
[Out] (3*a*b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^3*Hypergeometric2F1[1
, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1
+ n)) + (3*a^2*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2,
(2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n)
)/(d*f*(1 + n)) + (b^3*(Cos[e + f*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 +
n)/2, (4 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(d*Tan[e + f*x])
^(1 + n))/(d*f*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
```

```
!IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int \left(a^3(d\tan(e+fx))^n + 3a^2b\sec(e+fx)(d\tan(e+fx))^n + 3ab^2 \right.$$

$$= a^3\int (d\tan(e+fx))^n\,dx + \left(3a^2b\right)\int \sec(e+fx)(d\tan(e+fx))^n\,d$$

$$= \frac{3a^2b\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)\sec(e+fx)}{df(1+n)}$$

$$= \frac{3ab^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{a^3\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\,\text{ta}}{df(1+n)}$$

Mathematica [A] time = 3.52, size = 238, normalized size = 0.97

$$\frac{d\left(-\tan^2(e+fx)\right)^{-n/2}(d\tan(e+fx))^{n-1}\left(-3a^3\left(-\tan^2(e+fx)\right)^{\frac{n+2}{2}}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)+9a^2b(n\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^3*(d*Tan[e + f*x])^n,x]
```

```
[Out] (d*(d*Tan[e + f*x])^(-1 + n)*(9*a^2*b*(1 + n)*Hypergeometric2F1[1/2, (1 - n
)/2, 3/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2] + b^3*(1 + n)*
Hypergeometric2F1[3/2, (1 - n)/2, 5/2, Sec[e + f*x]^2]*Sec[e + f*x]^3*Sqrt[
-Tan[e + f*x]^2] - 3*a^3*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e
+ f*x]^2]*(-Tan[e + f*x]^2)^((2 + n)/2) + 9*a*b^2*(Sqrt[-Tan[e + f*x]^2] -
(-Tan[e + f*x]^2)^((2 + n)/2))))/(3*f*(1 + n)*(-Tan[e + f*x]^2)^(n/2))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\sec\left(fx+e\right)^3+3\,ab^2\sec\left(fx+e\right)^2+3\,a^2b\sec\left(fx+e\right)+a^3\right)\left(d\tan\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*(d*tan(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^3\left(d\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

maple [F] time = 1.80, size = 0, normalized size = 0.00

$$\int \left(a + b\sec\left(fx + e\right)\right)^{3} \left(d\tan\left(fx + e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x)
[Out] int((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx + e\right) + a\right)^{3} \left(d\tan\left(fx + e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d\tan\left(e + f\,x\right)\right)}^n\,{\left(a + \frac{b}{\cos\left(e + f\,x\right)}\right)}^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^3,x)
[Out] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(e + fx\right)\right)^{n} \left(a + b\sec\left(e + fx\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**3*(d*tan(f*x+e))**n,x)
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

3.345 $\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx$

Optimal. Leaf size=160

$$\frac{a^2(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{2ab\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\right.}{df(n+1)}$$

```
[Out] b^2*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+a^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n]
,-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+2*a*b*(cos(f*x+e)^2)^(1+1/2*
n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e)*(d*t
an(f*x+e))^(1+n)/d/f/(1+n)
```

Rubi [A] time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3886, 3476, 364, 2617, 2607, 32}

$$\frac{a^2(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{2ab\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\right.}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^2*(d*Tan[e + f*x])^n,x]
```

```
[Out] (b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^2*Hypergeometric2F1[1, (1
 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)
) + (2*a*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n
)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n))/(d*f
*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
```

```
IntegerQ[n]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx &= \int \left(a^2(d\tan(e+fx))^n + 2ab\sec(e+fx)(d\tan(e+fx))^n + b^2\sec^2(e\right.\\ &= a^2\int (d\tan(e+fx))^n\,dx + (2ab)\int \sec(e+fx)(d\tan(e+fx))^n\,dx +\\ &= \frac{2ab\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)\sec(e+fx)(d\,\mathrm{ta}}{df(1+n)}\\ &= \frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{a^2\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+}{df(1+n)}\end{aligned}$$

Mathematica [A] time = 1.30, size = 178, normalized size = 1.11

$$\frac{d\left(-\tan^2(e+fx)\right)^{-n/2}(d\tan(e+fx))^{n-1}\left(-a^2\left(-\tan^2(e+fx)\right)^{\frac{n+2}{2}}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)+2ab(n+1)\right.}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^2*(d*Tan[e + f*x])^n,x]
```

```
[Out] (d*(d*Tan[e + f*x])^(-1 + n)*(2*a*b*(1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, 3/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2] - a^2*Hypergeome
tric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(-Tan[e + f*x]^2)^((2 + n
)/2) + b^2*(Sqrt[-Tan[e + f*x]^2] - (-Tan[e + f*x]^2)^((2 + n)/2))))/(f*(1
+ n)*(-Tan[e + f*x]^2)^(n/2))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2\right)\left(d\tan\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*tan(f*x + e))^n
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^2\left(d\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

maple [F] time = 2.77, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(fx+e\right)\right)^{2}\left(d\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x)
[Out] int((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^{2}\left(d\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\tan\left(e+f\,x\right)\right)}^n\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^2,x)
[Out] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(e+fx\right)\right)^{n}\left(a+b\sec\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**2*(d*tan(f*x+e))**n,x)
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

3.346 $\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx$

Optimal. Leaf size=129

$$\frac{a(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n}{}\right.}{df(n+1)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/
d/f/(1+n)+b*(cos(f*x+e)^2)^(1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/
2*n],sin(f*x+e)^2)*sec(f*x+e)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3884, 3476, 364, 2617}

$$\frac{a(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n}{}\right.}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])*(d*Tan[e + f*x])^n,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f
*x])^(1 + n))/(d*f*(1 + n)) + (b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometri
c2F1[(1 + n)/2, (2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e
 + f*x])^(1 + n))/(d*f*(1 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3884

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = a\int (d\tan(e+fx))^n\,dx + b\int \sec(e+fx)(d\tan(e+fx))^n\,dx$$
$$= \frac{b\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)\sec(e+fx)(d\tan$$
$$df(1+n)$$
$$= \frac{a\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{b\cos^2(e+$$

Mathematica [A] time = 0.68, size = 106, normalized size = 0.82

$$\frac{(d\tan(e+fx))^n\left(\frac{a\tan(e+fx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{n+1} + b\csc(e+fx)\left(-\tan^2(e+fx)\right)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3}{2};\sec^2(e+\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])*(d*Tan[e + f*x])^n,x]
```

```
[Out] ((d*Tan[e + f*x])^n*((a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x])/(1 + n) + b*Csc[e + f*x]*Hypergeometric2F1[1/2, (1 -
 n)/2, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^((1 - n)/2)))/f
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sec\left(fx+e\right)+a\right)\left(d\tan\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)\left(d\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

maple [F] time = 2.54, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(fx+e\right)\right)\left(d\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x)
```

```
[Out] int((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)\left(d\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\tan\left(e+f\,x\right)\right)^n\,\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x)),x)
[Out] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(e+fx\right)\right)^n\left(a+b\sec\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))**n,x)
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

3.347 $\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=266

$$\frac{d\left(-\tan^2(e+fx)\right)^{\frac{1-n}{2}+\frac{n-1}{2}}(d\tan(e+fx))^{n-1}\left(-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}\right)^{\frac{1-n}{2}}\left(\frac{b(\sec(e+fx)+1)}{a+b\sec(e+fx)}\right)^{\frac{1-n}{2}}F_1\left(1-n;\frac{1-n}{2},\frac{1-n}{2};2-n;\frac{}{a+b}\right.}{af(1-n)}$$

```
[Out] d*AppellF1(1-n,1/2-1/2*n,1/2-1/2*n,2-n,(a-b)/(a+b*sec(f*x+e)),(a+b)/(a+b*se
c(f*x+e)))*(-b*(1-sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2-1/2*n)*(b*(1+sec(f*x+e
))/(a+b*sec(f*x+e)))^(1/2-1/2*n)*(d*tan(f*x+e))^(-1+n)/a/f/(1-n)+d*hypergeo
m([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(-1+n)*tan(f*x+e
)^2/a/f/(1+n)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]),x]
[Out] Defer[Int][(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]), x]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx=\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

Mathematica [B] time = 4.74, size = 786, normalized size = 2.95

$$\frac{}{f(a+b\sec(e+fx))\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\left((a+b)F_1\left(\frac{n+1}{2};n,1;\frac{n+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-bF_1\left(\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]),x]
[Out] (2*((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2,
 ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]*(d*Tan[e + f*x])^n
)/(f*(a + b*Sec[e + f*x])*(((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n
)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Sec[(e + f*
x)/2]^2 - 16*n*((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e
+ f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Cos[(e + f*x)/2]*Csc[e
+ f*x]^3*Sec[e + f*x]*Sin[(e + f*x)/2]^5 + 2*n*((a + b)*AppellF1[(1 + n)/2,
 n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1
+ n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(
```

```
a + b)])*Csc[e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2] - (2*(1 + n)*((a - b)*b
*AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e +
 f*x)/2]^2)/(a + b)] + (a + b)^2*(AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n, 1, (5 +
 n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]) + b*(a + b)*n*AppellF1[(3
+ n)/2, 1 + n, 1, (5 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^
2)/(a + b)])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^2)/((a + b)*(3 + n))))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\tan\left(fx+e\right)\right)^{n}}{b\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\tan\left(fx+e\right)\right)^{n}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

maple [F] time = 2.48, size = 0, normalized size = 0.00

$$\int\frac{\left(d\tan\left(fx+e\right)\right)^{n}}{a+b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x)
[Out] int((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\tan\left(fx+e\right)\right)^{n}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\cos\left(e+f\,x\right)\,{\left(d\,\tan\left(e+f\,x\right)\right)}^{n}}{b+a\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a + b/cos(e + f*x)),x)
```

```
[Out] int((cos(e + f*x)*(d*tan(e + f*x))^n)/(b + a*cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\tan\left(e+fx\right)\right)^n}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a+b*sec(f*x+e)),x)
[Out] Integral((d*tan(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

3.348 $\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=28

$$\text{Int}\left((a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Mathematica [A] time = 8.06, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

maple [A] time = 1.53, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e\tan(c+d\,x))^m\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\tan(c+dx))^m(a+b\sec(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(e*tan(d*x+c))**m,x)
[Out] Integral((e*tan(c + d*x))**m*(a + b*sec(c + d*x))**(3/2), x)
```

3.349 $\int \sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m\,dx=\int \sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$$

Mathematica [A] time = 0.71, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)+a}\,(e\tan(dx+c))^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

maple [A] time = 1.65, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(dx+c)}\,(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(dx+c)+a}\,(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e\tan(c+d\,x))^m\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\tan(c+dx))^m\,\sqrt{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*(e*tan(d*x+c))**m,x)
[Out] Integral((e*tan(c + d*x))**m*sqrt(a + b*sec(c + d*x)), x)
```

3.350 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 3.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\tan(dx+c))^m}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

maple [A] time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
[Out] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\tan(c+d\,x))^m}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral((e*tan(c + d*x))**m/sqrt(a + b*sec(c + d*x)), x)
```

3.351 $\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 3.89, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,(e\tan(dx+c))^m}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [A] time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{\left(e\tan\left(dx+c\right)\right)^{m}}{\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
[Out] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\tan\left(dx+c\right)\right)^{m}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(e\tan\left(c+d\,x\right)\right)^{m}}{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\tan\left(c+dx\right)\right)^{m}}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((e*tan(c + d*x))**m/(a + b*sec(c + d*x))**(3/2), x)
```

3.352 $\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((e\tan(c+dx))^m(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx=\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Mathematica [A] time = 3.49, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

fricas [A] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n(e\tan(dx+c))^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

maple [A] time = 2.12, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx+c)+a)^n (e \tan(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e \tan(c+d\,x))^m \left(a+\frac{b}{\cos(c+d\,x)}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^n,x)
[Out] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (e \tan(c+dx))^m (a+b \sec(c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*(e*tan(d*x+c))**m,x)
[Out] Integral((e*tan(c + d*x))**m*(a + b*sec(c + d*x))**n, x)
```

3.353 $\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx$

Optimal. Leaf size=177

$$-\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{n+1}}{b^4d(n+1)}+\frac{\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{n+2}}{b^4d(n+2)}-\frac{3a(a+b\sec(c+dx))^{n+3}}{b^4d(n+3)}+\frac{(a+b\sec(c-}{b^4d(n+}$$

```
[Out] -a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(1+n)/b^4/d/(1+n)-hypergeom([1, 1+n],[2+n],
1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n)+(3*a^2-2*b^2)*(a+b*sec(d
*x+c))^(2+n)/b^4/d/(2+n)-3*a*(a+b*sec(d*x+c))^(3+n)/b^4/d/(3+n)+(a+b*sec(d*
x+c))^(4+n)/b^4/d/(4+n)
```

Rubi [A] time = 0.20, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3885, 952, 1620, 65}

$$-\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{n+1}}{b^4d(n+1)}+\frac{\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{n+2}}{b^4d(n+2)}-\frac{3a(a+b\sec(c+dx))^{n+3}}{b^4d(n+3)}+\frac{(a+b\sec(c-}{b^4d(n+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] -((a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(1 + n))/(b^4*d*(1 + n))) - (Hyperg
eometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(
1 + n))/(a*d*(1 + n)) + ((3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(2 + n))/(b^4
*d*(2 + n)) - (3*a*(a + b*Sec[c + d*x])^(3 + n))/(b^4*d*(3 + n)) + (a + b*S
ec[c + d*x])^(4 + n)/(b^4*d*(4 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^n\left(b^2-x^2\right)^2}{x}\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$

$$= \frac{(a+b\sec(c+dx))^{4+n}}{b^4 d(4+n)} + \frac{\text{Subst}\left(\int \frac{(a+x)^n\left(b^4(4+n)-a^3(4+n)x-\left(3a^2+2b^2\right)(4+n)\right.}{x}\right.}{b^4 d(4+n)}$$

$$= \frac{(a+b\sec(c+dx))^{4+n}}{b^4 d(4+n)} + \frac{\text{Subst}\left(\int \left(-a\left(a^2-2b^2\right)(4+n)(a+x)^n + \right.\right.}{}$$

$$= -\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{1+n}}{b^4 d(1+n)} + \frac{\left(3a^2-2b^2\right)(a+b\sec(c+dx)}{b^4 d(2+n)}$$

$$= -\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{1+n}}{b^4 d(1+n)} - \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+}{a}\right.}{ad(1+}$$

Mathematica [A] time = 3.31, size = 298, normalized size = 1.68

$$-\frac{\sec^8\left(\frac{1}{2}(c+dx)\right)(a+b\sec(c+dx))^n\left(n(a\cos(c+dx)+b)\left(3a^3\cos(3(c+dx))+3a\left(3a^2+b^2\left(n^2-n-8\right)\right)c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] -1/2*((n*(b + a*Cos[c + d*x])*(-6*a^2*b + 12*b^3 - 6*a^2*b*n + 16*b^3*n + 4
*b^3*n^2 + 3*a*(3*a^2 + b^2*(-8 - n + n^2))*Cos[c + d*x] + 2*b*(1 + n)*(-3*
a^2 + b^2*(12 + 7*n + n^2))*Cos[2*(c + d*x)] + 3*a^3*Cos[3*(c + d*x)] - 12*
a*b^2*Cos[3*(c + d*x)] - 7*a*b^2*n*Cos[3*(c + d*x)] - a*b^2*n^2*Cos[3*(c +
d*x)]) - 2*b^4*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*Cos[c + d*x]^4*Hypergeom
etric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])*Sec[(c + d*x
)/2]^8*(a + b*Sec[c + d*x])^n)/(b^4*d*n*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(-1
 + Tan[(c + d*x)/2]^2)^4)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\tan(dx+c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

maple [F] time = 1.24, size = 0, normalized size = 0.00

$$\int (a + b\sec (dx + c))^n \left(\tan^5 (dx + c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (dx + c) + a)^n \tan (dx + c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan (c + d\,x)^5 \left(a + \frac{b}{\cos (c + d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))^n \tan^5 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**5, x)
```

3.354 $\int (a+b\sec(c+dx))^n\tan^3(c+dx)\,dx$

Optimal. Leaf size=102

$$-\frac{a(a+b\sec(c+dx))^{n+1}}{b^2d(n+1)}+\frac{(a+b\sec(c+dx))^{n+2}}{b^2d(n+2)}+\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

```
[Out] -a*(a+b*sec(d*x+c))^(1+n)/b^2/d/(1+n)+hypergeom([1, 1+n],[2+n],1+b*sec(d*x+
c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n)+(a+b*sec(d*x+c))^(2+n)/b^2/d/(2+n)
```

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3885, 952, 80, 65}

$$-\frac{a(a+b\sec(c+dx))^{n+1}}{b^2d(n+1)}+\frac{(a+b\sec(c+dx))^{n+2}}{b^2d(n+2)}+\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] -((a*(a + b*Sec[c + d*x])^(1 + n))/(b^2*d*(1 + n))) + (Hypergeometric2F1[1,
 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a*d*(
1 + n)) + (a + b*Sec[c + d*x])^(2 + n)/(b^2*d*(2 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a+x)^n(b^2-x^2)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2d}$$
$$= \frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)} - \frac{\text{Subst}\left(\int \frac{(a+x)^n(b^2(2+n)+a(2+n)x)}{x}\,dx, x, b\sec(c+a\right.}{b^2d(2+n)}$$
$$= -\frac{a(a+b\sec(c+dx))^{1+n}}{b^2d(1+n)} + \frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)} - \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx,\right.}{d}$$
$$= -\frac{a(a+b\sec(c+dx))^{1+n}}{b^2d(1+n)} + \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec}{ad(1+n)}$$

Mathematica [A] time = 1.35, size = 118, normalized size = 1.16

$$\frac{\sec^2(c+dx)(a+b\sec(c+dx))^n\left(n(a\cos(c+dx)+b)(-a\cos(c+dx)+bn+b)-b^2\left(n^2+3n+2\right)\cos^2(c+dx)\,{}_2\right.}{b^2dn(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] ((n*(b + b*n - a*Cos[c + d*x])*(b + a*Cos[c + d*x]) - b^2*(2 + 3*n + n^2)*C
os[c + d*x]^2*Hypergeometric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c
 + d*x])])*Sec[c + d*x]^2*(a + b*Sec[c + d*x])^n)/(b^2*d*n*(1 + n)*(2 + n))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\tan(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n\left(\tan^3(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx+c)+a)^n \tan(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^3 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**3,x)
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**3, x)
```

3.355 $\int (a+b\sec(c+dx))^n \tan(c+dx)\,dx$

Optimal. Leaf size=48

$$-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

```
[Out] -hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n
)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3885, 65}

$$-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a*d*(1 + n)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^n \tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= -\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

Mathematica [A] time = 0.45, size = 49, normalized size = 1.02

$$\frac{(a+b\sec(c+dx))^n\,{}_2F_1\left(1,-n;1-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])]*(a
+ b*Sec[c + d*x])^n)/(d*n)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\tan(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n\tan(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c),x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(c+d\,x)\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)*(a + b/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c),x)
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x), x)
```

3.356 $\int \cot(c + dx)(a + b\sec(c + dx))^n \, dx$

Optimal. Leaf size=162

$$-\frac{(a + b\sec(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)} - \frac{(a + b\sec(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)} \cdot$$

```
[Out] -1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n
)/(a-b)/d/(1+n)-1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*s
ec(d*x+c))^(1+n)/(a+b)/d/(1+n)+hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)*(
a+b*sec(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A] time = 0.18, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3885, 961, 65, 831, 68}

$$-\frac{(a + b\sec(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)} - \frac{(a + b\sec(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)} \cdot$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*S
ec[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n,
2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a +
b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]
*(a + b*Sec[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && !RationalQ[m
]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{b^2\,\mathrm{Subst}\left(\int \frac{(a+x)^n}{x\left(b^2-x^2\right)}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= -\frac{b^2\,\mathrm{Subst}\left(\int \left(\frac{(a+x)^n}{b^2x}-\frac{x(a+x)^n}{b^2\left(-b^2+x^2\right)}\right)dx, x, b\sec(c+dx)\right)}{d}$$
$$= -\frac{\mathrm{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d}+\frac{\mathrm{Subst}\left(\int \frac{x(a+x)^n}{-b^2+x^2}\,dx, x, b\sec(c\right.}{d}$$
$$= \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}+\frac{\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}-\frac{\mathrm{Subst}\left(\int\right.}{}$$
$$= -\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)}-\frac{{}_2F_1\left(1,1+\right.}{}$$

Mathematica [A] time = 1.69, size = 163, normalized size = 1.01

$$\frac{(a+b\sec(c+dx))^n\left(-2\,{}_2F_1\left(1,-n;1-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)+{}_2F_1\left(1,-n;1-n;\frac{(a+b)\cos(c+dx)}{b+a\cos(c+dx)}\right)+2^n\left(\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos}{b}\right.\right.}{2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((-2*Hypergeometric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])]
 + Hypergeometric2F1[1, -n, 1 - n, ((a + b)*Cos[c + d*x])/(b + a*Cos[c + d*
x])] + (2^n*Hypergeometric2F1[-n, -n, 1 - n, ((-a + b)*Cos[c + d*x]*Sec[(c
+ d*x)/2]^2)/(2*b)])/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/b)^n)*(a +
b*Sec[c + d*x])^n)/(2*d*n)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\sec(dx+c)+a)^n\cot(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

maple [F] time = 1.36, size = 0, normalized size = 0.00

$$\int \cot(dx+c)\,(a+b\sec(dx+c))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{n}\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)\left(a+\frac{b}{\cos(c+d\,x)}\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)*(a + b/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{n}\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**n,x)
[Out] Integral((a + b*sec(c + d*x))**n*cot(c + d*x), x)
```

3.357 $\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=279

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}+\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}$$

```
[Out] 1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)
/(a-b)/d/(1+n)+1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*se
c(d*x+c))^(1+n)/(a+b)/d/(1+n)-hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a
+b*sec(d*x+c))^(1+n)/a/d/(1+n)-1/4*b*hypergeom([2, 1+n],[2+n],(a+b*sec(d*x+
c))/(a-b))*(a+b*sec(d*x+c))^(1+n)/(a-b)^2/d/(1+n)+1/4*b*hypergeom([2, 1+n],
[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*sec(d*x+c))^(1+n)/(a+b)^2/d/(1+n)
```

Rubi [A] time = 0.23, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3885, 961, 68, 65, 831}

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}+\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a + b
)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*
(a + b*Sec[c + d*x])^(1 + n))/(a*d*(1 + n)) - (b*Hypergeometric2F1[2, 1 + n
, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a
- b)^2*d*(1 + n)) + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*
x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a + b)^2*d*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && !RationalQ[m
]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx = \frac{b^4\,\text{Subst}\left(\int \frac{(a+x)^n}{x\left(b^2-x^2\right)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{b^4\,\text{Subst}\left(\int \left(\frac{(a+x)^n}{4b^3(b-x)^2}+\frac{(a+x)^n}{b^4x}-\frac{(a+x)^n}{4b^3(b+x)^2}-\frac{x(a+x)^n}{b^4\left(-b^2+x^2\right)}\right)dx, x, b\sec(c+dx\ldots}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{x(a+x)^n}{-b^2+x^2}\,dx, x, b\sec(c+d\ldots}{d}$$

$$= -\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} - \frac{b\,{}_2F_1\left(2,1\ldots\right.}{\ldots}$$

$$= -\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} - \frac{b\,{}_2F_1\left(2,1\ldots\right.}{\ldots}$$

$$= \frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} + \frac{{}_2F_1\left(1,1+n;\ldots\right.}{\ldots}$$

Mathematica [A] time = 7.01, size = 256, normalized size = 0.92

$$\frac{\cot^2(c+dx)\left(a+b\sqrt{\sec^2(c+dx)}\right)(a+b\sec(c+dx))^n\left((a-b)\left(a(a-b)(2a-b(n-2))\tan^2(c+dx)\,{}_2F_1\left(1,n+1\ldots\right.\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n*(a + b*Sqrt[Sec[c + d*x]^2])*(a*(a +
 b)^2*(2*a + b*(-2 + n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sqrt[Sec
[c + d*x]^2])/(a - b)]*Tan[c + d*x]^2 + (a - b)*(a*(a - b)*(2*a - b*(-2 + n
))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sqrt[Sec[c + d*x]^2])/(a + b)]
*Tan[c + d*x]^2 - 2*(a + b)*(a*(1 + n)*(a - b*Sqrt[Sec[c + d*x]^2]) + 2*(a^
2 - b^2)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sqrt[Sec[c + d*x]^2])/a]
*Tan[c + d*x]^2))))/(4*a*(a - b)^2*(a + b)^2*d*(1 + n))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\cot(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \left(\cot^3(dx+c)\right)(a+b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^3 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \cot^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*cot(c + d*x)**3, x)
```

3.358 $\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\tan^4(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx = \int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$$

Mathematica [A] time = 6.65, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n \tan(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \tan(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

maple [A] time = 1.10, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n \left(\tan^4(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \tan(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^4 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \tan^4(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**4,x)
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**4, x)
```

3.359 $\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$

Optimal. Leaf size=237

$$-\text{Int}\left((a+b\sec(c+dx))^n,x\right)+\frac{\sqrt{2}\,(a+b)\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\frac{3}{2};\frac{1}{2}(1-\right.}{bd\sqrt{\sec(c+dx)+1}}$$

```
[Out] (a+b)*AppellF1(1/2,-1-n,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/b/d/(((a+b*sec(d*x+c))/(a+b))^n)/(1+s
ec(d*x+c))^(1/2)-a*AppellF1(1/2,-n,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*s
ec(d*x+c))*(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/b/d/(((a+b*sec(d*x+c))/(a+
b))^n)/(1+sec(d*x+c))^(1/2)-Unintegrable((a+b*sec(d*x+c))^n,x)
```

Rubi [A] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^n*Tan[c + d*x])/(b*d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^n) - (Sqrt[2]*a*AppellF1[1
/2, 1/2, -n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a
+ b*Sec[c + d*x])^n*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
 + d*x])/(a + b))^n) - Defer[Int][(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx &= \int (a+b\sec(c+dx))^n\left(-1+\sec^2(c+dx)\right)dx\\
&= \frac{\int (-b-a\sec(c+dx))(a+b\sec(c+dx))^n\,dx}{b}+\frac{\int \sec(c+dx)(a+b\sec(}{b}\\
&= -\frac{a\int \sec(c+dx)(a+b\sec(c+dx))^n\,dx}{b}-\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{(a+bx)^{1+}}{\sqrt{1-x}\sqrt{1-}}\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1}}\\
&= \frac{(a\tan(c+dx))\,\text{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}+\frac{\left((-a-b)(a+b\right.}{}\\
&= \frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec}{bd\sqrt{1+\sec(c+dx)}}\\
&= \frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec}{bd\sqrt{1+\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 3.55, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\tan(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\sec(dx+c)+a)^n\tan(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

maple [A] time = 0.84, size = 0, normalized size = 0.00

$$\int(a+b\sec(dx+c))^n\left(\tan^2(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\sec(dx+c)+a)^n\tan(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int\tan(c+d\,x)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))^n\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**2,x)
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**2, x)
```

3.360 $\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\cot^2(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
[Out] Defer[Int][Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx=\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [A] time = 3.72, size = 0, normalized size = 0.00

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
[Out] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\cot(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

maple [A] time = 1.05, size = 0, normalized size = 0.00

$$\int \left(\cot^2(dx+c)\right)(a+b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^n \cot (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cot (c + d\,x)^2 \left(a + \frac{b}{\cos (c + d\,x)}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^n \cot^2 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*cot(c + d*x)**2, x)
```

3.361 $\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\cot^4(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
[Out] Defer[Int][Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx=\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [A] time = 6.24, size = 0, normalized size = 0.00

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
[Out] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

fricas [A] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\cot(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\cot(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

maple [A] time = 1.20, size = 0, normalized size = 0.00

$$\int \left(\cot^4(dx+c)\right)(a+b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \cot(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cot(c+d\,x)^4 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c))**n,x)
[Out] Timed out
```

3.362 $\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\tan^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$$

Mathematica [A] time = 4.41, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2), x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

maple [A] time = 1.89, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^{n}\left(\tan^{\frac{3}{2}}(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{n}\tan(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**(3/2),x)
[Out] Timed out
```

3.363 $\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\sqrt{\tan(c+dx)}\,(a+b\sec(c+dx))^n,x\right)$$

[Out] Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]], x]

Rubi steps

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx=\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$$

Mathematica [A] time = 5.60, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]

[Out] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]], x]

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\sec(dx+c)+a)^n\sqrt{\tan(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\sqrt{\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)

maple [A] time = 1.65, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n\left(\sqrt{\tan}\,(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \sqrt{\tan(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\tan(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**n*sqrt(tan(c + d*x)), x)
```

3.364 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Mathematica [A] time = 5.09, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

maple [A] time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
[Out] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\tan(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(1/2),x)
[Out] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/tan(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**n/sqrt(tan(c + d*x)), x)
```

3.365 $\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Mathematica [A] time = 6.18, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

fricas [A] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

maple [A] time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(dx + c))^n}{\tan(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
[Out] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx + c) + a)^n}{\tan(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cos(c+d\,x)}\right)^n}{\tan(c + d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(3/2),x)
[Out] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/tan(d*x+c)**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**n/tan(c + d*x)**(3/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```